STAT5002 Lab10 Solution Sheet

Introduction to Statistics

STAT5002

Two-sample T-tests

The Marrickville Golf Club has been collecting weather data (on and off) since 1904. Daily rainfall data may be downloaded by selecting the link “All years of data” from this page within the Bureau of Meteorology website (note: the actual URL of the “All years of data” link seems to change from day to day).

1 Data processing

In the first part of this lab, we demonstrate some basic data processing steps using the file `IDCJAC0009_066036_1800_Data.csv`. This section is intended solely to illustrate how to prepare the original dataset, so that you can use the processed data to generate additional examples for your own practice. **The questions begin on Page 6.**

```
mgc = read.csv("IDCJAC0009_066036_1800_Data.csv")
str(mgc)
```

```
'data.frame':   43980 obs. of  8 variables:
 $ Product.code                                   : chr  "IDCJAC0009" "IDCJAC0009" "IDCJAC000
 $ Bureau.of.Meteorology.station.number           : int  66036 66036 66036 66036 66036 66036 6
 $ Year                                           : int  1904 1904 1904 1904 1904 1904 1904 19
 $ Month                                          : int  1 1 1 1 1 1 1 1 1 1 ...
 $ Day                                            : int  1 2 3 4 5 6 7 8 9 10 ...
 $ Rainfall.amount..millimetres.                  : num  NA NA NA NA NA NA NA NA NA NA ...
 $ Period.over.which.rainfall.was.measured..days.: int  NA NA NA NA NA NA NA NA NA NA ...
 $ Quality                                        : chr  "" "" "" "" ...
```

The code below determines monthly totals of rainfall (note that `na.rm=T` ignores NAs, treating them as zero); rows are years, columns are months:

```
rf = mgc$Rainfall.amount..millimetres.
monthly.rf = tapply(rf, list(mgc$Year, mgc$Month), sum, na.rm=T)
monthly.rf
```

	1	2	3	4	5	6	7	8	9	10	11	12
1904	0.0	0.0	0.0	0.0	87.3	0.5	303.1	37.3	26.7	38.6	0.0	25.4
1905	42.0	68.1	227.2	151.1	162.3	52.6	9.1	7.6	35.5	46.3	6.4	77.9
1906	46.1	7.0	134.5	8.9	115.3	28.0	3.8	121.7	35.2	41.2	93.4	59.7
1907	67.9	84.5	192.6	29.7	31.8	195.5	4.5	6.3	5.1	7.2	26.6	50.7
1908	25.8	191.5	31.0	55.0	46.4	10.6	212.7	219.3	53.0	17.0	2.0	15.2
1909	17.8	148.0	19.1	30.5	20.6	110.0	15.1	29.6	104.0	35.8	72.8	91.5
1910	135.2	16.6	156.4	73.2	107.5	61.9	224.8	5.6	48.2	53.3	19.1	149.9
1911	348.0	121.3	55.9	61.0	38.1	2.5	163.8	164.4	46.2	19.5	39.3	53.4
1912	27.9	158.6	112.1	139.7	80.1	43.2	218.3	54.6	14.0	21.1	74.6	44.1
1913	13.8	34.3	257.1	173.1	411.2	0.0	199.7	0.0	42.6	34.3	10.9	8.9
1914	14.4	25.0	203.7	40.1	96.8	131.1	220.8	54.4	97.1	148.9	76.1	157.6
1915	23.4	26.8	92.2	222.6	96.8	26.2	122.0	26.7	28.6	16.7	0.0	63.8
1916	21.8	54.3	52.6	120.1	38.4	42.4	70.6	97.0	123.5	312.4	69.7	78.9
1917	56.5	169.9	8.7	319.0	82.8	131.8	9.0	39.6	101.4	97.7	186.4	40.6
1918	229.5	90.8	29.0	161.9	7.4	3.9	211.0	46.9	69.9	19.2	20.7	11.4
1919	28.9	96.1	84.8	57.4	416.7	32.4	31.7	4.6	79.8	46.6	80.1	63.6
1920	137.5	32.3	27.7	59.3	5.1	57.4	125.9	24.7	29.4	24.3	42.2	341.0
1921	67.4	23.9	75.6	154.3	145.6	17.1	152.0	24.4	84.1	55.4	73.3	145.0
1922	133.5	75.1	41.4	29.2	89.0	25.0	243.1	41.0	99.5	49.3	14.6	40.7
1923	51.6	13.8	20.0	150.7	26.7	101.2	174.2	139.4	42.5	33.2	26.7	41.4
1924	111.9	58.5	95.8	134.3	49.3	48.2	38.4	57.1	74.9	25.9	79.6	67.4
1925	70.7	41.3	44.1	29.4	430.8	154.0	4.1	86.4	18.0	16.3	99.7	16.0
1926	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1927	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1928	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1929	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1930	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1931	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1932	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1933	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1934	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1935	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1936	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1937	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

1938	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1939	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1940	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1941	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1942	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1943	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1944	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1945	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1946	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1947	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1948	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	58.8
1949	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1950	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1951	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1952	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1953	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1954	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1955	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1956	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1957	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1958	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1959	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1960	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1961	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1962	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1963	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1964	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1965	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1966	18.8	113.9	203.3	183.7	50.1	113.3	11.4	67.6	50.6	38.0	140.2	68.2
1967	153.1	164.4	101.8	46.8	31.0	214.0	20.8	211.9	68.3	55.4	78.6	18.8
1968	109.5	15.0	86.1	11.2	85.8	20.3	47.8	22.2	3.1	4.2	17.5	76.1
1969	51.7	195.6	104.0	169.7	44.2	149.3	29.8	155.4	45.0	49.0	255.5	37.4
1970	94.1	60.4	140.1	58.7	15.2	27.2	0.0	31.9	132.9	18.1	0.0	0.0
1971	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1972	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1973	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1974	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1975	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1976	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1977	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1978	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1979	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1980	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

1981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1982	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1983	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1984	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1985	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1986	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1988	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1989	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1990	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1991	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1993	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1994	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1996	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1998	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1999	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	39.0	57.0	29.0	74.0	18.0
2002	67.0	313.0	33.0	26.0	75.0	18.0	9.0	13.0	20.0	19.0	19.0	88.0
2003	5.0	53.0	70.0	242.0	335.0	57.0	42.0	34.0	8.0	61.0	81.0	52.0
2004	51.0	43.0	4.0	2.0	8.0	27.0	28.0	86.0	54.0	200.0	42.0	71.0
2005	60.0	95.0	45.0	17.0	29.0	64.0	61.0	0.0	40.0	56.0	92.0	37.0
2006	61.0	33.0	29.0	2.0	20.0	157.0	98.0	68.0	147.0	8.0	22.0	13.0
2007	10.0	105.0	49.0	113.0	12.0	333.0	31.0	99.0	48.0	33.0	122.0	77.0
2008	53.0	325.0	64.0	155.0	4.0	109.0	55.0	34.0	73.0	44.0	44.0	73.0
2009	18.0	125.0	17.0	50.0	115.0	75.0	48.0	4.0	17.0	158.0	24.0	54.0
2010	24.0	164.0	50.0	32.0	149.0	93.0	30.0	18.0	46.0	79.0	165.0	75.0
2011	29.0	16.0	153.0	216.0	112.0	63.0	264.0	38.0	71.0	32.0	169.0	154.0
2012	112.0	140.0	221.0	163.0	23.0	211.0	52.0	8.0	20.0	24.0	48.0	38.0
2013	133.0	169.0	71.0	109.0	2.0	308.0	30.0	14.0	47.0	18.0	183.0	31.0
2014	7.0	41.0	117.0	61.0	9.0	84.0	12.0	243.0	48.0	131.0	18.0	161.0
2015	159.0	101.0	49.0	413.0	102.0	90.0	54.0	60.0	56.0	31.0	81.0	69.0
2016	251.0	29.0	117.0	86.0	13.0	300.0	108.0	82.0	0.0	27.0	34.0	66.0
2017	48.0	177.0	267.0	71.0	19.0	116.0	11.0	21.0	0.0	59.0	40.0	52.0
2018	25.0	116.0	77.0	15.0	14.0	146.0	8.0	6.0	80.0	181.0	110.0	93.0
2019	69.0	88.0	170.0	16.0	9.0	146.0	43.0	53.0	103.0	27.0	25.0	1.0
2020	66.0	435.0	138.0	25.0	99.0	82.0	140.0	60.0	6.0	4.0	45.0	89.0
2021	71.0	108.0	407.0	27.0	99.0	70.0	32.0	80.0	52.0	61.0	160.0	91.0
2022	91.0	394.0	626.0	221.0	178.0	4.0	348.0	42.0	79.0	173.0	43.0	42.0
2023	157.0	191.0	61.0	131.0	36.0	15.0	38.0	49.0	0.0	23.0	143.0	81.0

```
2024  76.0 180.0  51.0 249.0 163.0    NA    NA    NA    NA    NA    NA    NA
```

As can be seen, the rainfall was only recorded May 1904 – Dec 1925, then Jan 1966 – Oct 1970, then Aug 2001 to the present. Let us extract the rainfall for the month of May in the two periods 1904-1925 and 2002-2024:

```
yrs = as.numeric(rownames(monthly.rf))
yrs
```

```
  [1] 1904 1905 1906 1907 1908 1909 1910 1911 1912 1913 1914 1915 1916 1917 1918
 [16] 1919 1920 1921 1922 1923 1924 1925 1926 1927 1928 1929 1930 1931 1932 1933
 [31] 1934 1935 1936 1937 1938 1939 1940 1941 1942 1943 1944 1945 1946 1947 1948
 [46] 1949 1950 1951 1952 1953 1954 1955 1956 1957 1958 1959 1960 1961 1962 1963
 [61] 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978
 [76] 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993
 [91] 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008
[106] 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023
[121] 2024
```

```
yr.19 = (yrs < 1926)
yr.20 = (yrs > 2001)
may.19 = monthly.rf[yr.19,5]
may.19
```

```
 1904  1905  1906  1907  1908  1909  1910  1911  1912  1913  1914  1915  1916
 87.3 162.3 115.3  31.8  46.4  20.6 107.5  38.1  80.1 411.2  96.8  96.8  38.4
 1917  1918  1919  1920  1921  1922  1923  1924  1925
 82.8   7.4 416.7   5.1 145.6  89.0  26.7  49.3 430.8
```

```
may.20 = monthly.rf[yr.20,5]
may.20
```

```
2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017
  75  335    8   29   20   12    4  115  149  112   23    2    9  102   13   19
2018 2019 2020 2021 2022 2023 2024
  14    9   99   99  178   36  163
```

We will use `may.19` and `may.20` in the rest of sections to investigate if the differences in mean rainfall significant. To address this question we assume that

- the 1904-1925 May rainfall totals are like a random sample $X_1, \ldots, X_{22}$ taken with replacement from a box with mean μ_X;
- the 2002-2024 May rainfall totals are like a random sample $Y_1, \ldots, Y_{23}$ taken with replacement from a box with mean μ_X;
- both samples are taken independently of each other.

2 Classical two-sample T-test

2.1 Hypotheses

Formally state the hypotheses being considered here. (Hint: is this a one-sided or two-sided test?)

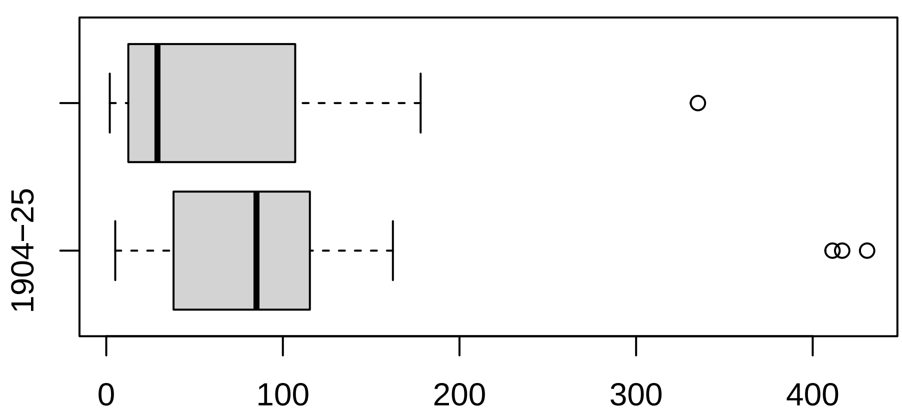
Answer:

- null hypothesis $H_0\colon \mu_X = \mu_Y$;
- alternative hypothesis $H_1\colon \mu_X \neq \mu_Y$.

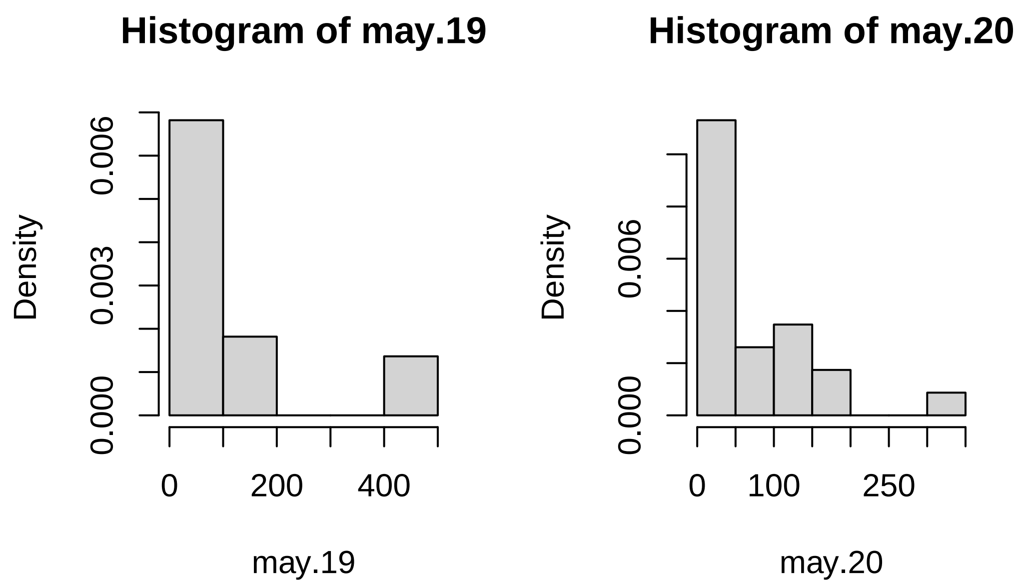
2.2 Checking assumptions

State the additional assumptions underlying the use of the Classical two-sample T-test, and check the assumptions using graphical summaries.

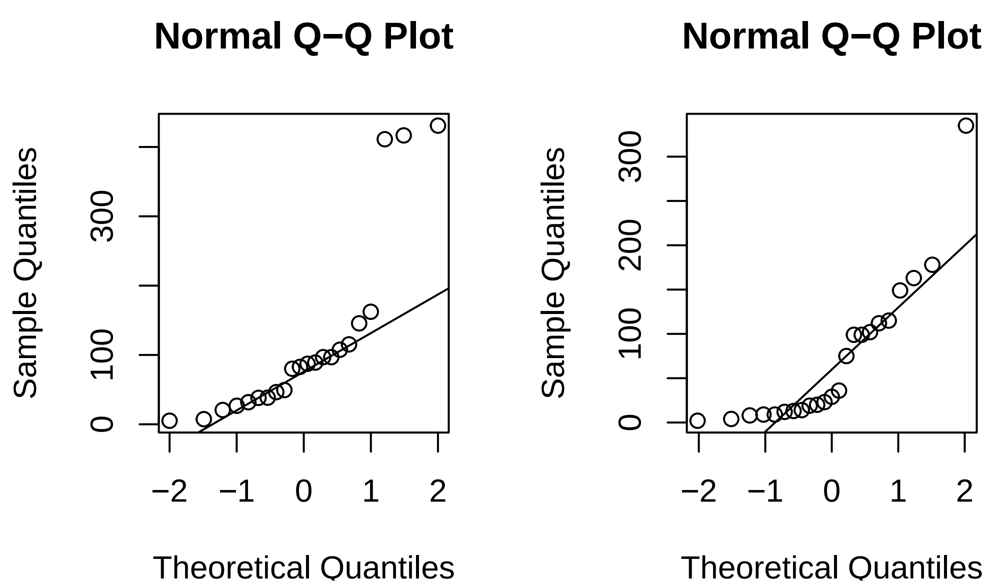
```r
# Write your code here
boxplot(may.19, may.20, names=c("1904-25", "2002-24"), horizontal=T)
```




```r
par(mfrow=c(1,2))
hist(may.19, freq=F)
hist(may.20, freq=F)
```




```
par(mfrow=c(1,2))
qqnorm(may.19)
qqline(may.19)
qqnorm(may.20)
qqline(may.20)
```



Answer: In addition to the box model assumptions listed above, we have that

- both boxes have the same SD;
- both boxes are (at least approximately) normal-shaped.

While the spreads (IQRs) of the two samples seem to be similar as shown by the boxplots, the shapes don't necessarily look normal, because

- there are outliers
- histograms suggest possible multi-modality.
- QQ plots don't follow the QQ lines.

So the assumptions underlying the Classical Two-sample T-test may not be reasonable.

2.3 Pooled estimates

First determine the sample sizes, sample means, and sample SDs for `may.19` and `may.20`. Then determine the pooled estimated standard deviation and the standard error of the difference.

```
# Write your code here
m.19 = mean(may.19)
m.20 = mean(may.20)
s.19 = sd(may.19)
s.20 = sd(may.20)
n.19 = length(may.19)
n.19
```

```
[1] 22
```

```
n.20 = length(may.20)
n.20
```

```
[1] 23
```

```
#
sp = sqrt(( (n.19-1)*(s.19^2) + (n.20-1)*(s.20^2))/(n.19+n.20-2))
sp
```

```
[1] 107.5603
```

```
#
SE.p = sp*sqrt(1/n.19 + 1/n.20)
SE.p
```

```
[1] 32.0762
```

Answer: Under the assumption that both boxes have the same SD σ, the theoretical standard error of the mean difference is given by

$$SE(\bar{X}-\bar{Y}) = \sqrt{SE(\bar{X})^2 + SE(\bar{Y})^2} = \sqrt{\frac{\sigma^2}{22} + \frac{\sigma^2}{23}} = \sigma\sqrt{\frac{1}{22} + \frac{1}{23}}\,.$$

The Classical Two-sample T-test estimates the common σ using the "pooled'' estimate given by

$$\hat{\sigma}_p = \sqrt{\frac{21\hat{\sigma}_X^2 + 22\hat{\sigma}_Y^2}{43}}\,,$$

where $\hat{\sigma}_X$ and $\hat{\sigma}_Y$ are the two sample SDs. This pooled estimate of σ is given by 107.56, so the estimated standard error is

$$\hat{\sigma}_p = 107.56\sqrt{\frac{1}{22} + \frac{1}{23}} \approx 32.076\,.$$

2.4 Test statistic

Determine the test distribution and the observed value of T-statistic.

```r
# Write your code here
tobs = (m.19-m.20)/SE.p
tobs
```

```
[1] 1.461934
```

Answer: The test distribution is Student's T with 43 degrees of freedom. Both small and large values of the test statistic argue against the null hypothesis, so it does not matter the direction we take the difference in the sample mean. Here we use

$$T = \frac{\bar{X}-\bar{Y}}{SE(\bar{X}-\bar{Y})} = \frac{\bar{X}-\bar{Y}}{\hat{\sigma}_p\sqrt{\frac{1}{22}+\frac{1}{23}}}.$$

2.5 P-value and conclusion

Is the apparent mean difference significantly different from zero at the 5% significance level?

```r
# Write your code here
P.value = 2*pt(abs(tobs), df=43, lower.tail=F)
P.value
```

```
[1] 0.1510297
```

Answer: It's a two-sided test. So we take $P(|T| > |t|) * 2$. We don't reject reject at the 5% significance level.

2.6 Critical region of rejection

What is the critical region of rejection for a 5% significance level?

```
# Write your code here
qt(0.975, df=43)
```

```
[1] 2.016692
```

Answer: $|T| > 2.016692$

3 Welch test

3.1 Assumption

This test relaxes one of the assumptions of the Classical two-sample T-test. Which one exactly?

Answer: That the two boxes have the same SD.

3.2 Standard error

This test uses a different estimate of the standard error of the mean difference, compared to the Classical two-sample T-test. Determine the value of this estimated standard error.

```
# Write your code here
SE.welch = sqrt(s.19^2/n.19 + s.20^2/n.20)
SE.welch
```

```
[1] 32.40043
```

Answer: When the two box SDs, say σ_X and σ_Y are possibly different, the theoretical standard error is

$$SE(\bar{X} - \bar{Y}) = \sqrt{SE(\bar{X})^2 + SE(\bar{Y})^2} = \sqrt{\frac{\sigma_X^2}{22} + \frac{\sigma_Y^2}{23}}$$

which may be estimated by plugging in each of the sample SDs, giving

$$\sqrt{\frac{\hat{\sigma}_X^2}{22} + \frac{\hat{\sigma}_Y^2}{23}} = \sqrt{\frac{129.84^2}{22} + \frac{80.75^2}{23}} \approx 32.4\,.$$

3.3 T-test

Use the built-in R function `t.test()` to perform Welch's test with the 10% significance level. Note that you can set this by choosing the confidence level `conf.level = 0.90`. What is the P-value? how does it compare with the classical two-sample T-test?

```
# Write your code here
t.test(may.19, may.20, conf.level=0.90)
```

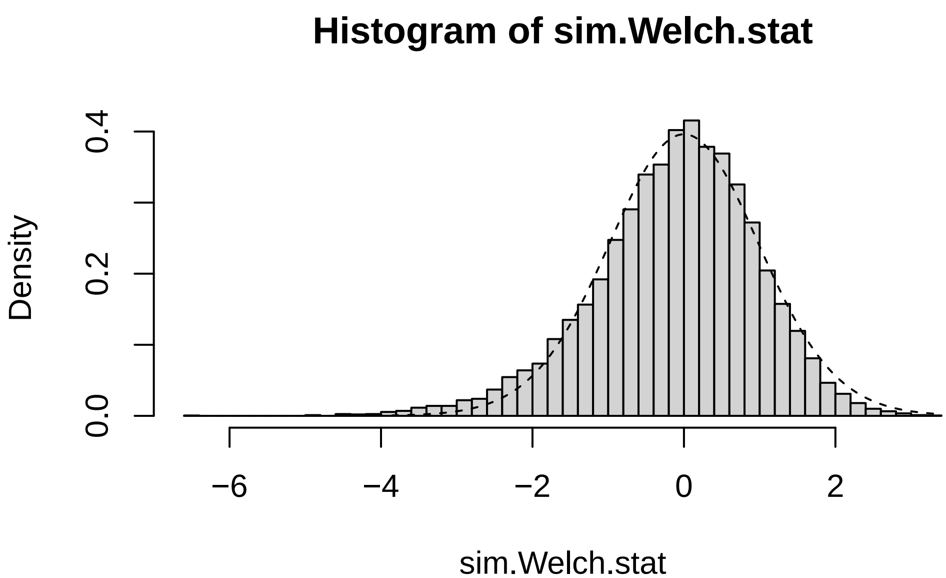
```

	Welch Two Sample t-test

data:  may.19 and may.20
t = 1.4473, df = 34.859, p-value = 0.1567
alternative hypothesis: true difference in means is not equal to 0
90 percent confidence interval:
  -7.855638 101.642200
sample estimates:
mean of x mean of y 
117.54545  70.65217 
```

4 Simulation-based two-sample T-test

4.1 Simulate test statistics

- Use a bootstrap simulation to simulate 10,000 samples of the Welch statistic, plot the histogram of the test statistics, and compare with the Student's T distribution with d.o.f. 34.859 (which is the one used in Welch's test).
- Note that we need to define two surrogate boxes `box.19 = may.19-mean(may.19)` and `box.20 = may.20-mean(may.20)`. Why are the means subtracted off?

```r
set.seed(23)
# Write your code here
n.19 = length(may.19)
n.20 = length(may.20)
box.19 = may.19-mean(may.19)
box.20 = may.20-mean(may.20)
sim.Welch.stat=0
for(i in 1:10000){
samp.19 = sample(box.19, size=n.19, replace=T)
samp.20 = sample(box.20, size=n.20, replace=T)
est.SE = sqrt( (sd(samp.19)^2)/n.19 + (sd(samp.20)^2)/n.20 )
sim.Welch.stat[i] = (mean(samp.19)-mean(samp.20))/est.SE
# Or sim.Welch.stat[i] = t.test(samp.19, samp.20)$stat
}
hist(sim.Welch.stat,pr=T, n=50)
curve(dt(x, df=34.859), lty=2, add=T, n=1001)
```



Answer: The idea of substracting the mean in building the surrogate boxes is to simulate the statistic under certain conditions:

- "shape" of boxes are more plausible given observed data
- the null hypothesis is true i.e. both boxes have the same mean. We subtract the means so the boxes we simulate from have the same mean (i.e. zero).

4.2 P-value

We use the simulated values of the Welch statistic to approximate the distribution of the test statistic when the null hypothesis is true. What is a simulation-based P-value? You can use an observed Welch statistic 1.4473 here (obtained by `t.test()`).

```
# Write your code here
( sum(sim.Welch.stat>=1.4473) + sum(sim.Welch.stat<=-1.4473) ) / 10000
```

```
[1] 0.1664
```

Answer: The simulation-based P-value is 0.1664.