STAT5002 Lab9 Question Sheet

Introduction to Statistics

STAT5002

One-sample T-test

In his original paper W.S. Gossett ("Student") demonstrated his new testing technique on various examples. One was where 10 patients tried two different drugs designed to increase sleep time. An image of the table is linked to below.

Additional hours' sleep gained by the use of hyoscyamine hydrobromide.

Patient	1 (Dextro-)	2 (Laevo-)	Difference (2–1)
1.	+ ·7	+ 1·9	+ 1·2
2.	− 1·6	+ ·8	+ 2·4
3.	− ·2	+ 1·1	+ 1·3
4.	− 1·2	+ ·1	+ 1·3
5.	− 1	− ·1	0
6.	+ 3·4	+ 4·4	+ 1·0
7.	+ 3·7	+ 5·5	+ 1·8
8.	+ ·8	+ 1·6	+ ·8
9.	0	+ 4·6	+ 4·6
10.	+ 2·0	+ 3·4	+ 1·4
	Mean + ·75	Mean + 2·33	Mean + 1·58
	S. D. 1·70	S. D. 1·90	S. D. 1·17

from "The Probable Error of a Mean", Student(1908), Biometrika

Note that there is a typographical error: the −1 for patient 5 under "Dextro-" should be −0.1. Also, the SDs presented under the table are actually computed using the population SD, not the sample SD!

Below is the corrected version of the first column of the data (labelled "Dextro-") and the the second column labelled "Laevo-" in R. We will use these two variables in this workshop.

```
dextro = c(0.7, -1.6, -0.2, -1.2, -0.1, 3.4, 3.7, 0.8, 0, 2)
laevo = c(1.9, 0.8, 1.1, 0.1, -0.1, 4.4, 5.5, 1.6, 4.6, 3.4)
```

For the following questions (next page), you may either carry out the computations directly in RStudio or use the accompanying R Markdown worksheet.

Beyond the calculations, **it is important to interpret your results (confidence intervals and P-values) for each question.**

For Question 5.1, it is highly recommended that you write down the testing procedure before performing any calculations.

1 The `dextro` data

1.1 Determine the sample mean and SD (note the original paper uses the population SD)

1.2 Is it reasonable to assume that this is a sample from an "approximately normal box"? Use some plots to check this.

1.3 Provide a 95% confidence interval for the average increase in hours' sleep using Dextro-.

1.4 Would we reject a two-sided test that Dextro makes no difference to the duration of sleep at the 5% level of significance? Explain.

2 The `laevo` data

2.1 Verify the calculations of the sample mean and SD

2.2 Is it reasonable to assume that this is a sample from an "approximately normal box"? Use some plots to check this.

2.3 Provide a 95% confidence interval for the average increase in hours' sleep using Laevo-.

2.4 Would we reject a two-side test that Laevo makes no difference to the duration of sleep at the 5% level of significance? Explain.

3 Are the two treatments being equally effective?

Are your results consistent with the two treatments being equally effective? Discuss with reference to **overlap between confidence intervals**.

4 Test the difference

Repeat the steps of question 1 again but this time using the third column of "Laevo – Dextro" differences for each patient.

- You can obtain a vector of differences by `d = laevo - dextro`

4.1 Verify the calculations of the sample mean and SD of the difference

4.2 Is it reasonable to assume that these differences are like a sample from an "approximately normal box"? Use some plots to check this.

4.3 Provide a 95% confidence interval for the average difference in increase in hours' sleep:

4.4 Is this consistent with the two treatments being equally effective? In your answer, refer to your answer to question 3 above.

5 Perform a formal t-test on the difference.

Perform a formal t-test that the "true" average difference over all (potential) patients is zero. Stating and checking any required assumptions. Will you reject based on a 5% level of significance?

5.1 Write down the testing procedure step-by-step

Let μ_d denote the true average difference over all potential patients.

- **Null hypothesis:**
- **Alternative hypothesis:**
- **Assumptions:**
- **Test statistic:**
- **P-value:**
- **Conclusion:**

5.2 Check your answers using `t.test()`.

5.3 Using simulation

Now use a bootstrap simulation to construct a 95% confidence interval and perform a test. How do your results here differ from those given by `t.test()`, and why?

- Check the R markdown worksheet for steps.