

Exploring Data

Numerical Summaries

STAT5002

The University of Sydney

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Exploring Data

Topic 1: Data & Graphical Summaries

What type of data do we have & how can we visualise it?

Topic 2: Numerical Summaries

What are the main features of the data?

Outline

Centre

- Sample mean
- Sample median
- Robustness and comparisons

Spread

- Standard deviation
- Interquartile range

Write functions in R

Data story

How much does a property in Newtown cost?

cobden & hayson

Save [View Details](#)

New Open Sat 1 Jul

Buyers Guide \$600-\$650k

Auction Sat 22 Jul

205w/138 Carillon Avenue, Newtown, NSW 2042

1 1 1

Jim Nikolopoulos

Save Details >

Data on Newtown property sales

- Data is taken from [domain.com.au](#):
 - ➡ All properties sold in Newtown (NSW 2042) between April-June 2017.
 - ➡ The variable `Sold` has price in \$1000s.

```
1 data <- read.csv("data/NewtownJune2017.csv", header = T)
2 head(data, n = 2)

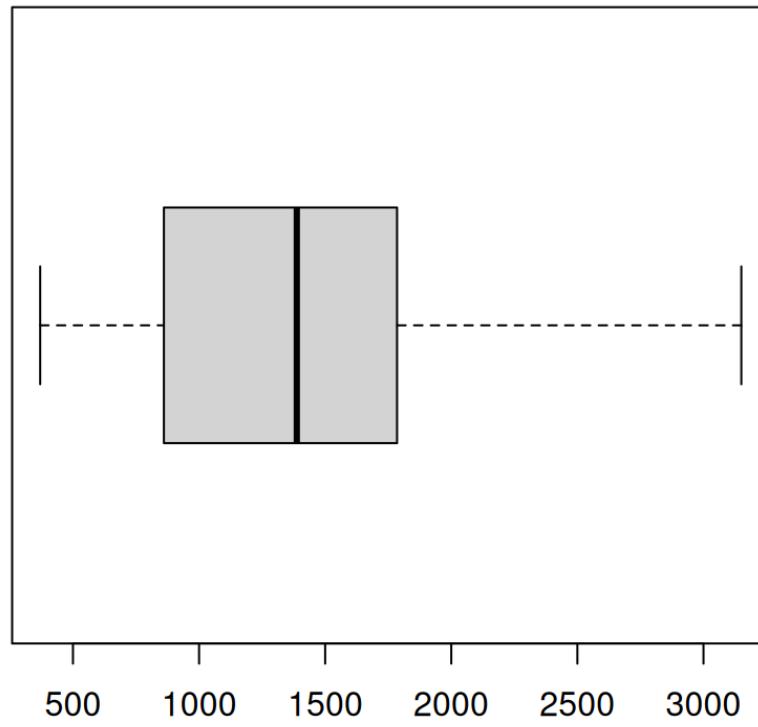
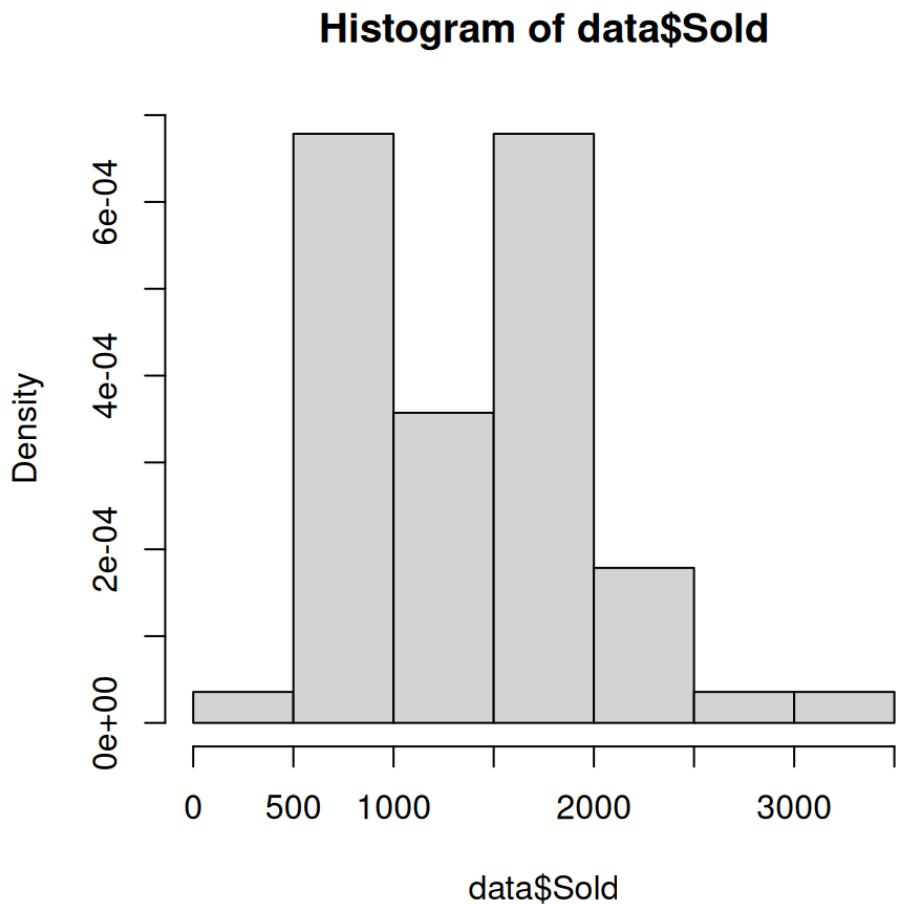
      Property Type     Agent Bedrooms Bathrooms Carspots Sold
1 19 Watkin Street Newtown House RayWhite        4          1       1 1975
2 30 Pearl Street Newtown House RayWhite        2          1       0 1250
  Date
1 23/6/17
2 23/6/17
```

Structure of Newtown data

```
1 dim(data)
[1] 56 8
1 str(data)
'data.frame': 56 obs. of 8 variables:
 $ Property : chr "19 Watkin Street Newtown" "30 Pearl Street Newtown" "26 John Street Newtowmn" "23/617-623
King Street Newtown" ...
 $ Type      : chr "House" "House" "House" "Apartment" ...
 $ Agent     : chr "RayWhite" "RayWhite" "Belle" "RayWhite" ...
 $ Bedrooms : int 4 2 2 1 1 5 1 1 1 3 ...
 $ Bathrooms: int 1 1 1 1 1 1 1 1 2 ...
 $ Carspots : int 1 0 0 1 1 1 0 1 1 0 ...
 $ Sold      : int 1975 1250 1280 780 650 2100 675 740 625 1950 ...
 $ Date      : chr "23/6/17" "23/6/17" "17/6/17" "17/6/17" ...
```

Graphical summaries

```
1 par(mfrow = c(1, 2))
2 hist(data$Sold, freq = F)
3 boxplot(data$Sold, horizontal = T)
```



Numerical summaries

Advantages of numerical summaries

- A numerical summary reduces all the data to one simple number (“statistic”).
 - ➡ This loses a lot of information.
 - ➡ However it allows easy communication and comparisons.
- Major features that we can summarise numerically are:
 - ➡ Maximum
 - ➡ Minimum
 - ➡ Centre [sample mean, median]
 - ➡ Spread [standard deviation, range, interquartile range]

Note

Which summaries might be useful for talking about Newtown house prices?

- It depends!
- Reporting the centre without the spread can be misleading!

Useful notation for data

- Observations of a single variable of size n can be represented by

$$x_1, x_2, \dots, x_n$$

- The ranked observations (ordered from smallest to largest) are

$$x_{(1)}, x_{(2)}, \dots, x_{(n)}$$

such that $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$

- The sum of the observations are

$$\sum_{i=1}^n x_i$$

Sample mean

Sample mean

The sample mean is the average of the data.

$$\text{sample mean} = \frac{\text{sum of data}}{\text{size of data}}$$

or

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Note that the sample mean involves **all** of the data.

- The sample mean of all the properties sold in Newtown is:

```
1 mean(data$Sold)
```

```
[1] 1407.143
```

- Focusing specifically on houses with 4 bedrooms (large), the sample mean is:

```
1 mean(data$Sold[data$type == "House" & data$Bedrooms == "4"])
```

```
[1] 2198.857
```

Deviation from the mean

Given a data point x_i , its deviation from the sample mean \bar{x} is

$$D_i = x_i - \bar{x}$$

For example,

- 19 Watkin St sold for \$1950 (thousands).
 - ➡ This gives a gap of $(\$1950 - \$1407.143) = \$542.857$ (thousands)
 - ➡ $\$542.857$ (thousands) **above** the sample mean
- 30 Pearl St sold for \$1250 (thousands).
 - ➡ This gives a gap of $(\$1250 - \$1407.143) = -\$157.143$ (thousands)
 - ➡ $\$157.143$ (thousands) **below** the sample mean

Sample mean as a balancing point

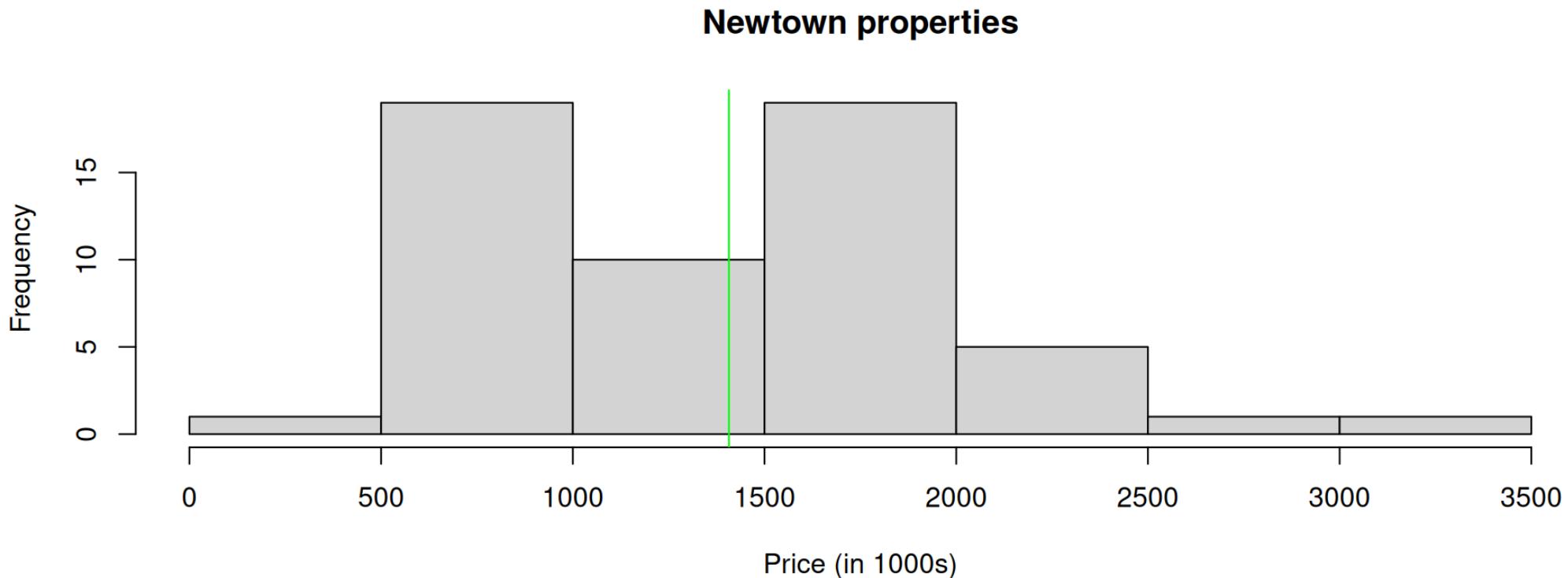
The sample mean is the point at which the data is **balanced** in the sense the sum of the **absolute deviations** for values to the left of the mean is the same as the sum of absolute deviations to the right of the mean.

$$\sum_{x_i < \bar{x}} |x_i - \bar{x}| = \sum_{x_i > \bar{x}} |x_i - \bar{x}|$$

Sample mean on the histogram

However, sample mean may **not** be balancing point of a histogram, the area to the left of the mean may not be the same as the area to the right of the mean.

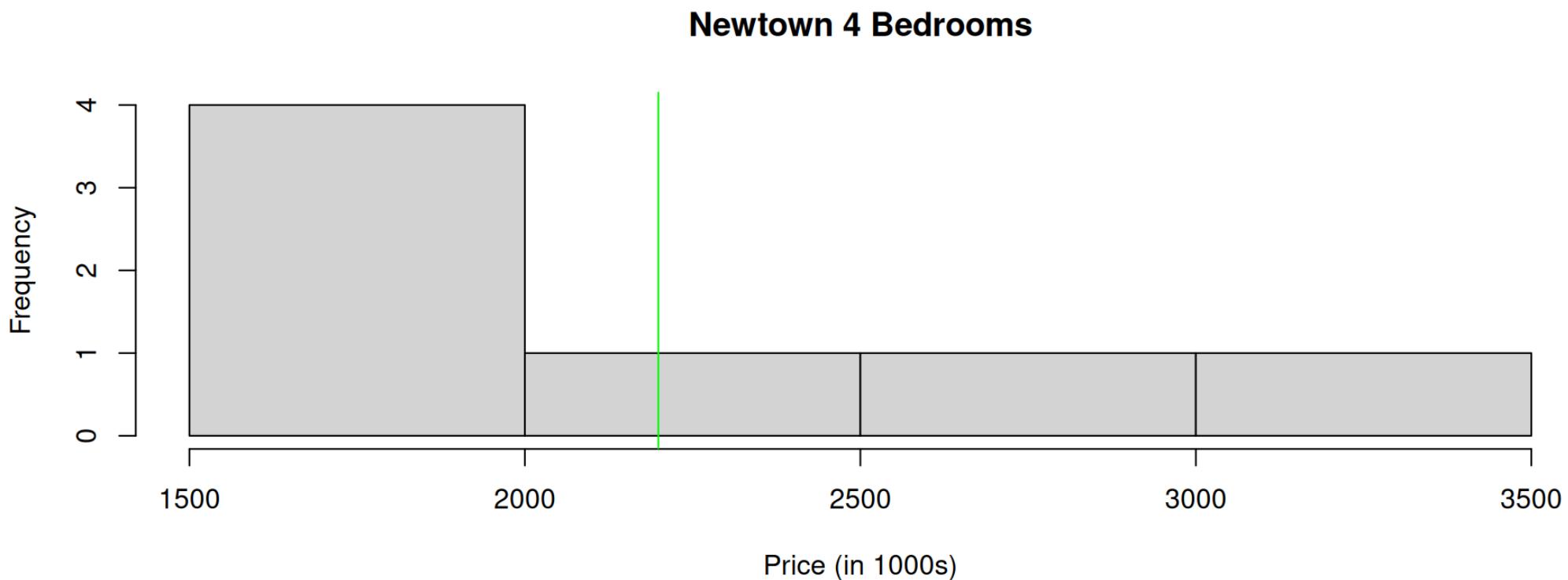
```
1 hist(data$Sold, main = "Newtown properties", xlab = "Price (in 1000s)")  
2 abline(v = mean(data$Sold), col = "green")
```



Skewed data

When the data is skewed, this effect is more significant.

```
1 hist(data$Sold[data>Type == "House" & data$Bedrooms == "4"], main = "Newtown 4 Bedrooms",
2       xlab = "Price (in 1000s)")
3 abline(v = mean(data$Sold[data>Type == "House" & data$Bedrooms == "4"]), col = "green")
```



Sample median

Sample median

The sample median \tilde{x} is the **middle data point**, when the observations are ordered from smallest to largest.

- For an odd sized number of observations:

$$\text{sample median} = \text{the unique middle point} = x_{\left(\frac{n+1}{2}\right)}$$

- For an even sized number of observations:

$$\text{sample median} = \text{average of the 2 middle points} = \frac{x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n}{2}+1\right)}}{2}$$

Ordering observations

The ordered observations are:

```
1 sort(data$Sold)
```

```
[1] 370 625 645 650 675 692 720 740 740 755 770 780 812 860 861  
[16] 920 935 955 955 999 1100 1240 1250 1280 1309 1315 1370 1375 1400 1460  
[31] 1553 1575 1590 1600 1600 1605 1662 1701 1710 1750 1780 1790 1806 1850  
[46] 1940 1950 1975 2000 2100 2200 2235 2300 2410 2810 3150
```

```
1 length(data$Sold)
```

```
[1] 56
```

As we have $n = 56$ observations (even), the sample median is found between the $(\frac{n}{2}) = 28$ th and $(\frac{n}{2} + 1) = 29$ th prices, or $\frac{1375+1400}{2} = 1387.5$.

- The sample median of all the properties sold in Newtown is:

```
1 median(data$Sold)
```

```
[1] 1387.5
```

- Focusing specifically on houses with 4 bedrooms (large), the sample median is:

```
1 median(data$Sold[data$type == "House" & data$Bedrooms == "4"])
```

```
[1] 1975
```

Sample median on the histogram

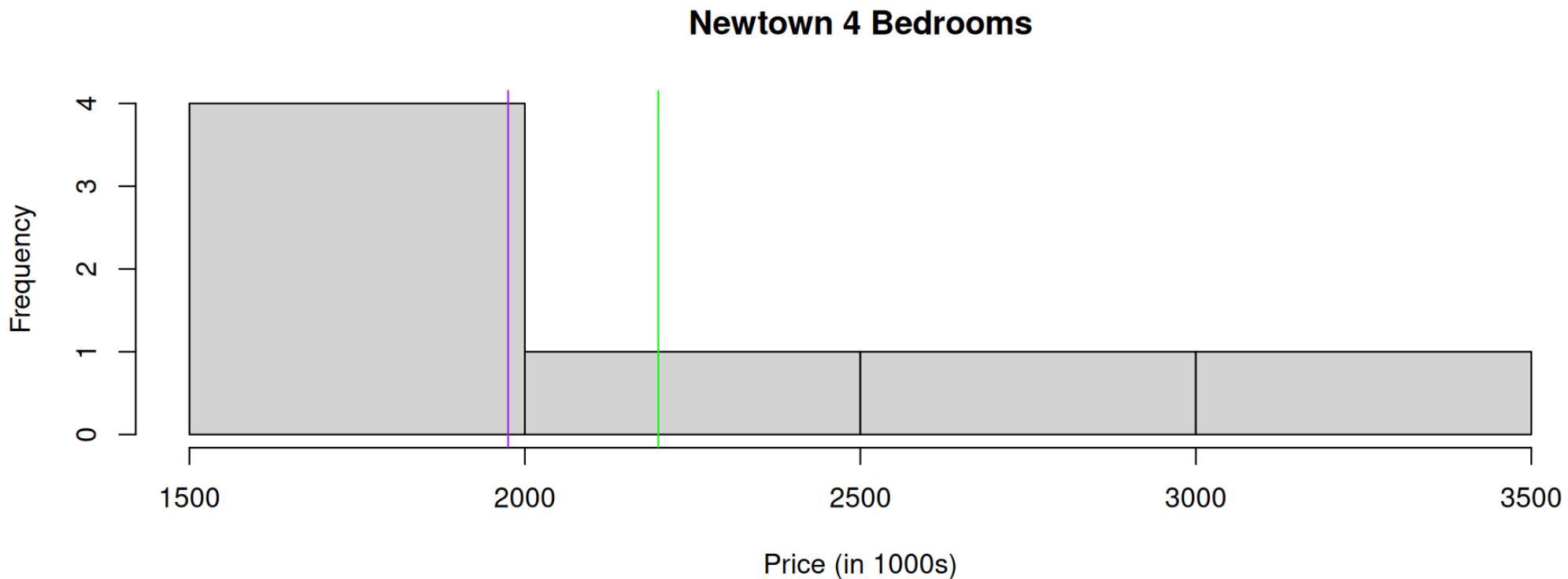
- The sample median is the **half way point** on the histogram - i.e., 50% of the houses sold are below and above \$1.3875 million.

```
1 hist(data$Sold, xlab = "Price (in 1000s)")  
2 abline(v = mean(data$Sold), col = "green") # create a green line for the mean  
3 abline(v = median(data$Sold), col = "purple") # create a purple line for the median
```



Histogram for 4 Bedroom Houses

```
1 hist(data$Sold[data>Type == "House" & data$Bedrooms == "4"], main = "Newtown 4 Bedrooms",
2      xlab = "Price (in 1000s)")
3 abline(v = mean(data$Sold[data>Type == "House" & data$Bedrooms == "4"]), col = "green")
4 abline(v = median(data$Sold[data>Type == "House" & data$Bedrooms == "4"]), col = "purple")
```



Comparison between sample mean and median

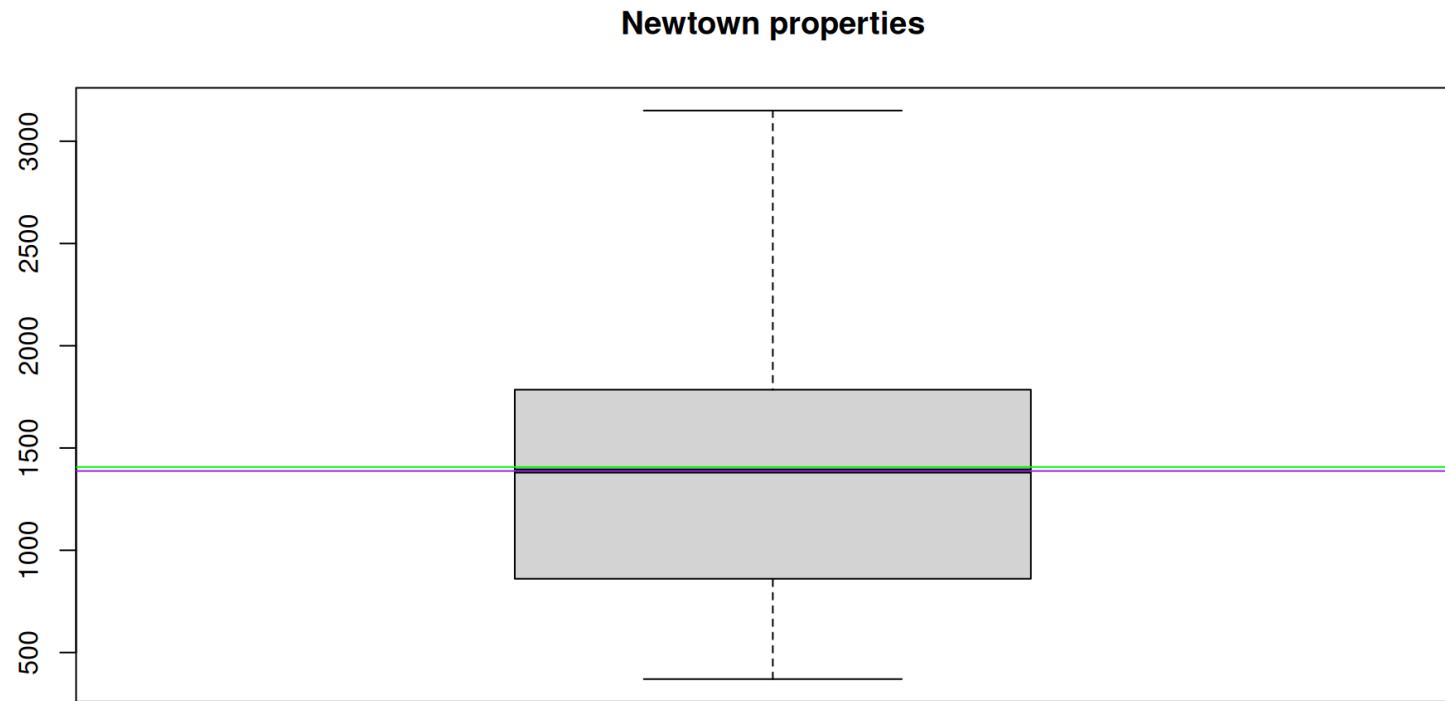
If you had to choose between reporting the sample mean or sample median for Newtown properties, which would you choose and why?

- For the full property portfolio, the sample mean and the sample median are fairly similar.
- For the 4 bedroom houses, the sample mean is higher than the sample median because it is being “pulled up” by some very expensive houses.
- For the average buyer, the sample median would be more useful as an indication of the sort of price needed to get into the market.
- For any agent selling houses in the area, the sample mean might be more useful in order to predict their average commissions!
- In practice, we can report both!

Sample mean and median on the boxplot

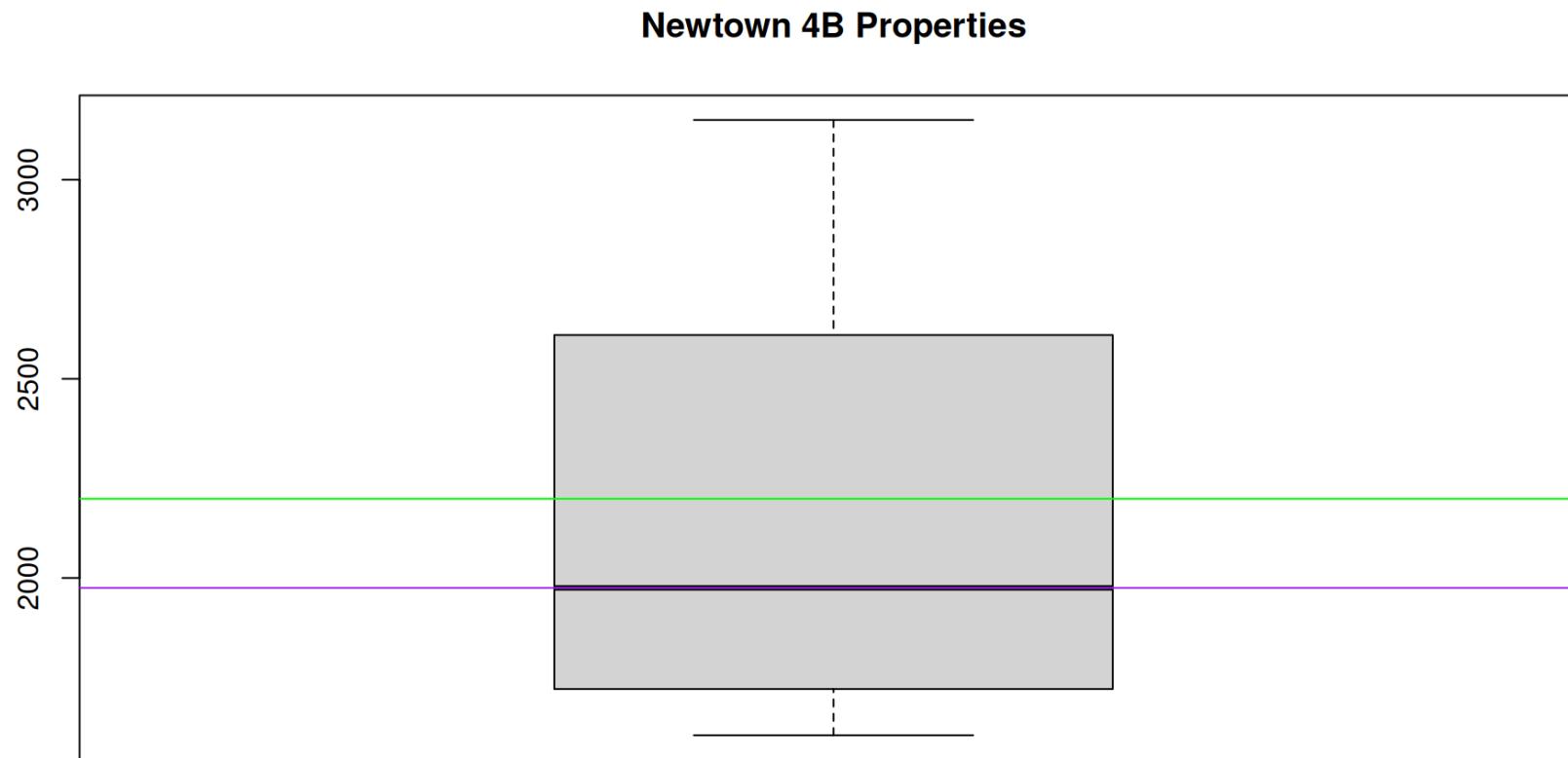
The sample median is the centre line on the boxplot.

```
1 boxplot(data$Sold, main = "Newtown properties")
2 abline(h = mean(data$Sold), col = "green")
3 abline(h = median(data$Sold), col = "purple")
```



Boxplot for 4 Bedroom Houses

```
1 boxplot(data$Sold[data>Type == "House" & data$Bedrooms == "4"], main = "Newtown 4B Properties")
2 abline(h = mean(data$Sold[data>Type == "House" & data$Bedrooms == "4"]), col = "green")
3 abline(h = median(data$Sold[data>Type == "House" & data$Bedrooms == "4"]), col = "purple")
```



Robustness and comparisons

The sample median is said to be **robust** and is a good summary for skewed data as it is not affected by **outliers** (extreme data values).

Example

Recently a heritage building was sold for 13 million in Newtown.



i Note

How would the sample mean and sample median change if it was added to the data?

- The sample mean would be a lot higher.
- The sample median would be a bit higher: it moves from the average of the 28th and 29th points to the 29th point.

Adding in a large outlier

```
1 data2 = c(data$Sold, 13000)
2 sort(data2)
```

```
[1] 370 625 645 650 675 692 720 740 740 755 770 780
[13] 812 860 861 920 935 955 955 999 1100 1240 1250 1280
[25] 1309 1315 1370 1375 1400 1460 1553 1575 1590 1600 1600 1600
[37] 1605 1662 1701 1710 1750 1780 1790 1806 1850 1940 1950 1975
[49] 2000 2100 2200 2235 2300 2410 2810 3150 13000
```

```
1 mean(data2)
```

```
[1] 1610.526
```

```
1 median(data2)
```

```
[1] 1400
```

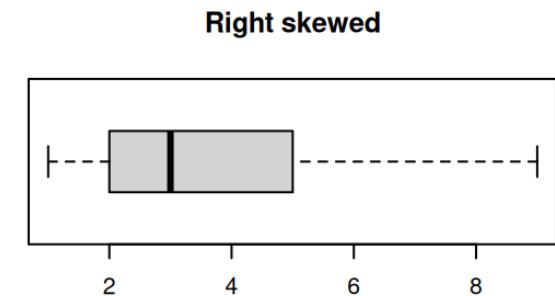
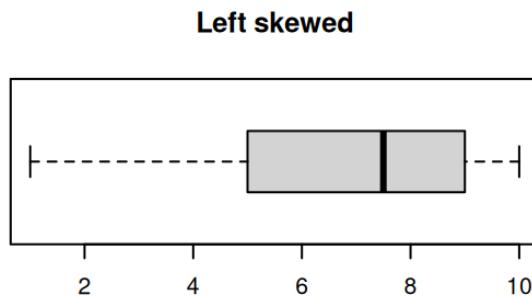
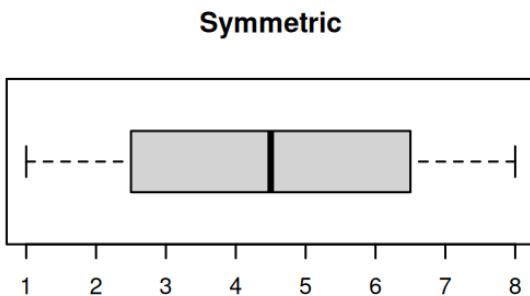
Summary of changes

Change in data	Sample mean	Sample median
Original data	1407.143	1387.5
Extra property of 13000	1610.526	1400

Skewness

The difference between the sample mean and the sample median can be an indication of the **shape** of the data.

- For symmetric data, the sample mean and sample median are the same: $\bar{x} = \tilde{x}$.
- For left skewed data (the most frequent data are concentrated on the right, with a left tail), the sample mean is smaller than the sample median: $\bar{x} < \tilde{x}$.
- For right skewed data (the most frequent data are concentrated on the left, with a right tail), the sample mean is larger than the sample median: $\bar{x} > \tilde{x}$.



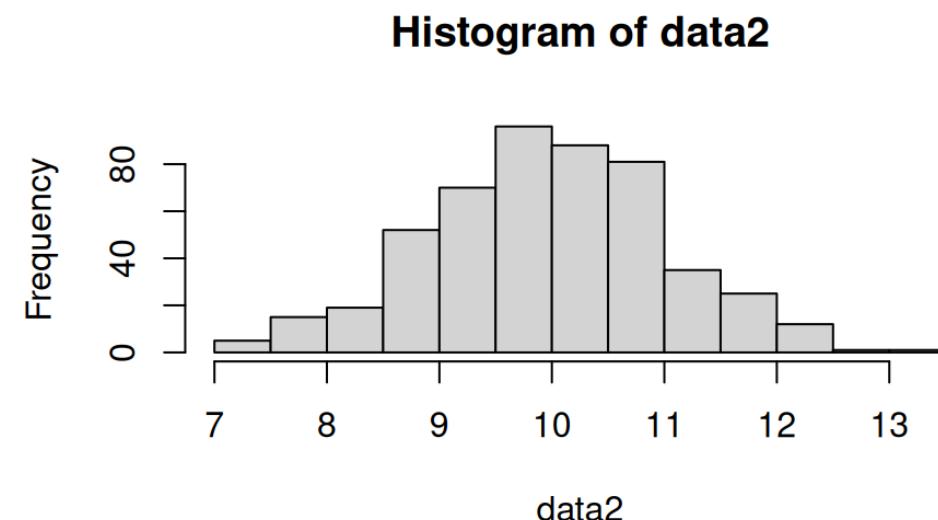
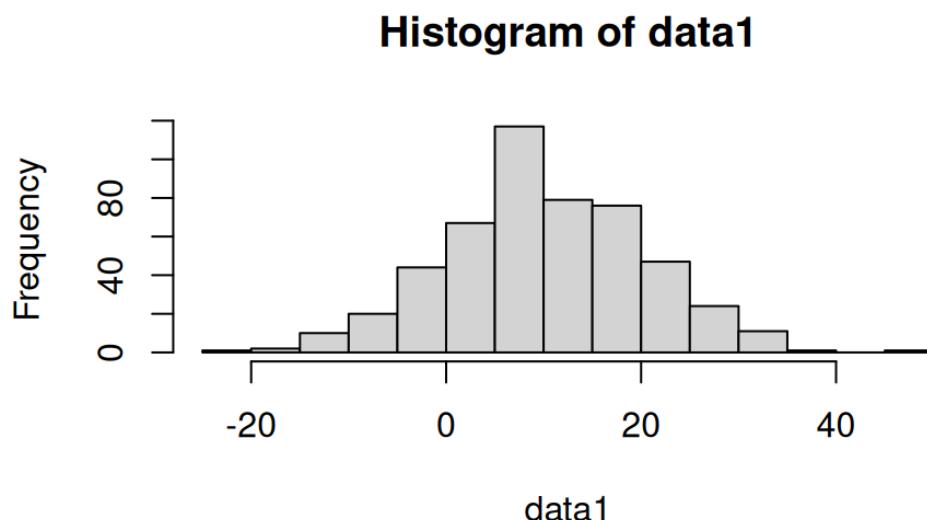
Which is optimal for describing centre?

- Both have strengths and weaknesses depending on the nature of the data.
- Sometimes neither gives a sensible sense of location, for example if the data is **bimodal**.
- As the **sample median is robust**, it is preferable for data which is skewed or has many outliers, like Sydney house prices.
- The **sample mean** is helpful for data which is **basically symmetric**, with not too many outliers, and for theoretical analysis.

Limitations of both?

- Both the sample mean and sample median allow easy comparisons.
- However, they need to be paired with a measure of **spread**.
- In the following example, the sample means are the same, but the data are very different.

Or, consider two data sets $\{-1, 0, 1\}$ and $\{-100, 0, 100\}$.



```
[1] 10.22644 9.95406
```

Standard deviation

How to measure spread?

For each property sold, we could calculate the **deviation** (or the gap) from the sample mean, $D_i = x_i - \bar{x}$, between the house and the sample mean \$1407 (thousands).

Property	Sold	Gap	Conclusion
19 Watkin Street	\$1950 (thousands)	1950-1407=543	More than half a million dollars more expensive than the average house price
30 Pearl St	\$1250 (thousands)	1250-1407=-157	Cheaper than the average house price

Deviations in the dataset

```
1 gaps = data$Sold - mean(data$Sold)
2 gaps
```

```
[1] 567.857143 -157.142857 -127.142857 -627.142857 -757.142857
[6] 692.857143 -732.142857 -667.142857 -782.142857 542.857143
[11] -32.142857 167.857143 -408.142857 -452.142857 -547.142857
[16] 197.857143 182.857143 -167.142857 -1037.142857 532.857143
[21] -687.142857 -452.142857 -487.142857 442.857143 192.857143
[26] -652.142857 -7.142857 145.857143 1402.857143 192.857143
[31] 792.857143 372.857143 398.857143 293.857143 -98.142857
[36] -307.142857 -472.142857 -762.142857 52.857143 -37.142857
[41] -715.142857 1742.857143 1002.857143 -637.142857 254.857143
[46] 827.857143 592.857143 382.857143 342.857143 302.857143
[51] 192.857143 -546.142857 -667.142857 -92.142857 892.857143
[56] -595.142857
```

```
1 max(gaps)
```

```
[1] 1742.857
```

(i) Note

What are the biggest and smallest deviations?

How do we **summarise** all the deviations into **1 number** ("spread")?

1st attempt: The mean gap

We could calculate the **average** of the deviations.

$$\text{mean deviation} = \text{sample mean}(\text{data} - \text{sample mean}(\text{data}))$$

```
1 round(mean(gaps))  
[1] 0
```

Note

What's the problem?

Note: It will always be 0.

- From the definition, the mean deviation must be 0, as the mean is the **balancing point** of the deviations.
- The mean deviation is

$$\frac{\sum_{i=1}^n D_i}{n} = \frac{\sum_{i=1}^n (x_i - \bar{x})}{n} = \frac{\sum_{i=1}^n x_i}{n} - \frac{n\bar{x}}{n} = 0.$$

Better option: Standard deviation

Standard deviation is a measure of spread that is based on the average.

First define the **root mean square** (RMS).

- The RMS measures the **average** of a set of numbers, regardless of the signs.
- The steps are: *Square* the numbers, then *Mean* the result, then *Root* the result.

$$\text{RMS}(\text{numbers}) = \sqrt{\text{sample mean} (\text{numbers}^2)}$$

- So effectively, the *Square* and *Root* operations “reverse” each other.
- RMS retain the same unit as the unit of the sample mean.

- Applying RMS to the deviations, we get

$$\text{RMS of deviations} = \sqrt{\text{sample mean } (\text{deviations}^2)} = \sqrt{\frac{\sum_{i=1}^n D_i^2}{n}}$$

- To avoid the cancellation of the deviations, another possible method is to consider the average of the absolute values of the deviations:

$$\text{mean absolute deviation (MAD)} = \frac{\sum_{i=1}^n |D_i|}{n}.$$

However, MAD is much harder to analyse.

Standard deviation in terms of RMS

Population Standard deviation

- The standard deviation measures the **spread** of the data.

$$\text{SD}_{pop} = \text{RMS of (deviations from the mean)}$$

- Formally, $\text{SD}_{pop} = \sqrt{\text{Mean of (deviations from the mean)}^2} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$

```
1 sqrt(mean(gaps^2))
```

```
[1] 593.7166
```

Standard deviation in R?

It is easy to calculate in R.

```
1 sd(data$Sold)
```

```
[1] 599.0897
```



Note

But why is this slightly different?

Adjusting the standard deviation

- There are **two** different formulas for the standard deviation, depending on whether the data is the **population** or a **sample**.
- The `sd` command in R always gives the **sample** version, as we most commonly have samples.
- Formally, $SD_{pop} = \sqrt{\frac{1}{n} \sum_{i=1}^n D_i^2}$ and $SD_{sample} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n D_i^2}$, where $D_i = x_i - \bar{x}$ is the deviation.

```
1 sd(data$Sold) * sqrt(55/56) # adjust by sqrt((n-1)/n), it calculates the population SD.
```

```
[1] 593.7166
```

```
1 gaps = data$Sold - mean(data$Sold) # calculate the gaps  
2 sqrt(mean(gaps^2)) # calculates the population SD.
```

```
[1] 593.7166
```

Why does the sample SD use the adjustment $\sqrt{(n - 1)/n}$?

- It is an **unbiased estimator** of the standard deviation (beyond the scope of this unit, will be covered in Year 2)
- Estimating the sample mean uses all of the n data points. The sum (or the mean) of n deviations is zero

$$\sum_{i=1}^n D_i = \sum_{i=1}^n (x_i - \bar{x}) = 0.$$

This means, given the first $n - 1$ deviations, we know the n -th deviation, because

$$\left(\sum_{i=1}^{n-1} D_i \right) + D_n = 0 \quad \Rightarrow \quad D_n = - \sum_{i=1}^{n-1} D_i.$$

Hence, there are only $n - 1$ effective pieces of information in the deviations.

Summary: population and sample

Summary	Formula	In R
Population or Sample mean	Sample Mean (Average)	<code>mean(data)</code>
Population standard deviation SD_{pop}	RMS of gaps from the sample mean	<code>sd(data)*sqrt((n-1)/n)</code>
Sample standard deviation SD_{sample}	Adjusted RMS of gaps from the sample mean	<code>sd(data)</code>

- The population standard deviation is always smaller than a sample standard deviation, ($\{pop\}$ $\{sample\}$), why? Extra variability due to sampling.
- Note for large sample sizes, the difference becomes negligible.

How to tell the difference?

- It can be tricky to work out whether your data is a population or sample!
- Look at the information about the data story and the research questions.
 - ➡ If we are just interested in the Newtown property prices during April-June 2017, then the **data** is the whole **population**.
 - ➡ If we are studying the property prices during April-June 2017 as a window into more general property prices (for the rest of the year or for the Inner West area) , then the **data** could be considered a **sample**.
- Population SD and sample SD get closer with increasing sample size **n** .

Variance

The squared standard deviation is called the **variance**. Similar to the sample SD and the population SD, there are two versions of the variance

$$\text{Var}_{\text{sample}} = \text{SD}_{\text{sample}}^2 \quad \text{and} \quad \text{Var}_{\text{pop}} = \text{SD}_{\text{pop}}^2.$$

- For summarising spread, we often prefer SD, as it has the same unit as the data points and the mean.
- In some situations, e.g., dealing with random variables and understanding the property of sample mean, using the variance can be much simpler.

Standard units (“Z score”)

Standard units of a data point = how many standard deviations is it below or above the mean

$$\text{standard units} = \frac{\text{data point} - \text{mean}}{\text{SD}}$$

This means that

$$\text{data point} = \text{mean} + \text{SD} \times \text{standard units}$$

It gives the relative location of a data point in the data set. It also have other benefits in data modelling (see later lectures).

Comparing 2 data points

To compare 2 data points, we can compare the standard units.

Property	Sold	Standard units	Conclusion
19 Watkin Street	\$1950 (thousands)	$\frac{1950 - 1407}{599} = 0.91$	Almost 1 SD higher than the average house price
30 Pearl St	\$1250 (thousands)	$\frac{1250 - 1407}{599} = -0.26$	0.26 SDs cheaper than the average house price

So 19 Watkin is a more unusual purchase than 30 Pearl St, relative to the mean.

Interquartile range

Interquartile range (IQR)

The IQR is another measure of spread by **ordering** the data.

IQR = range of the middle 50% of the data

More formally, $\text{IQR} = Q_3 - Q_1$, where

- Q_1 is the 25-th percentile (1st quartile) and Q_3 is the 75-th percentile (3rd quartile).
- The median is the 50-th percentile, or 2nd quartile $\tilde{x} = Q_2$.
- p-th percentile: there are p% of **ordered** data below the value of p-th percentile.

Quantile, quartile, percentile

The set of **q -quantiles** divides the **ordered** data into q equal size sets (in terms of percentage of data).

Percentile is 100-quantile, so the set of percentiles divides the data into 100 equal parts.

The set of **quartiles** divides the data into four quarters.

```
1 summary(data$Sold)
Min. 1st Qu. Median Mean 3rd Qu. Max.
370.0 860.8 1387.5 1407.1 1782.5 3150.0
1 summary(data$Sold)[5] - summary(data$Sold)[2] # one way to calculate IQR
3rd Qu.
921.75
1 IQR(data$Sold) # use the built-in function
[1] 921.75
```

So the range of the middle 50% of properties sold is almost a million dollars!

Reporting

- Like the median, the IQR is **robust**, so it's suitable as a summary of spread for skewed data.
- We report in pairs: (mean,SD) or (median,IQR).

IQR on the boxplot and outliers

- The IQR is the length of the box in the boxplot. It represents the span of the middle 50% of the houses sold.
- The **lower** and **upper thresholds** (expected minimum and maximum) are a distance of 1.5IQR from the 1st and 3rd quartiles (by **Tukey**'s convention).

$$LT = Q_1 - 1.5IQR$$

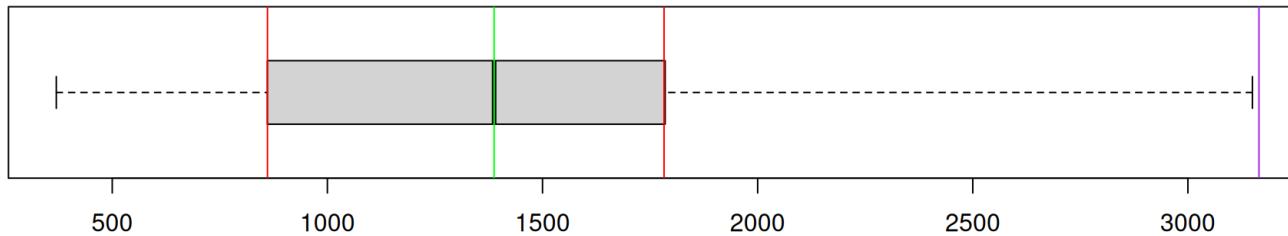
and

$$UT = Q_3 + 1.5IQR$$

- Data outside these thresholds is considered an **outlier** ("extreme reading").

Lower and Upper Thresholds on the Boxplot

```
1 boxplot(data$Sold, horizontal = T)
2 iqr = quantile(data$Sold)[4] - quantile(data$Sold)[2]
3 abline(v = median(data$Sold), col = "green")
4 abline(v = quantile(data$Sold)[2], col = "red")
5 abline(v = quantile(data$Sold)[4], col = "red")
6 abline(v = quantile(data$Sold)[2] - 1.5 * iqr, col = "purple")
7 abline(v = quantile(data$Sold)[4] + 1.5 * iqr, col = "purple")
```

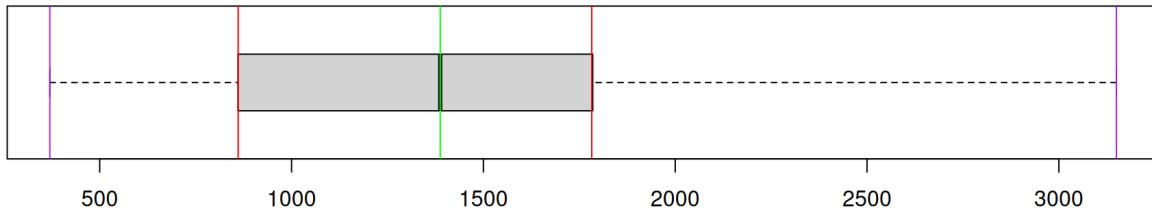


Note the lower threshold is not shown...why?

- The lower threshold should be the same difference from Q1 as the upper threshold is from Q3.
- So the lower threshold would be <0, which is not possible as we are dealing with house prices.

Thresholds can be outside of the data's range

```
1 boxplot(data$Sold, horizontal = T)
2 abline(v = median(data$Sold), col = "green")
3 abline(v = quantile(data$Sold)[2], col = "red")
4 abline(v = quantile(data$Sold)[4], col = "red")
5 abline(v = max(min(data$Sold), quantile(data$Sold)[2] - 1.5 * iqr), col = "purple")
6 abline(v = min(max(data$Sold), quantile(data$Sold)[4] + 1.5 * iqr), col = "purple")
```



To make the LT and UT staying within the range of data, R uses the convention

$$LT = \max(\min(x), Q_1 - 1.5IQR)$$

and

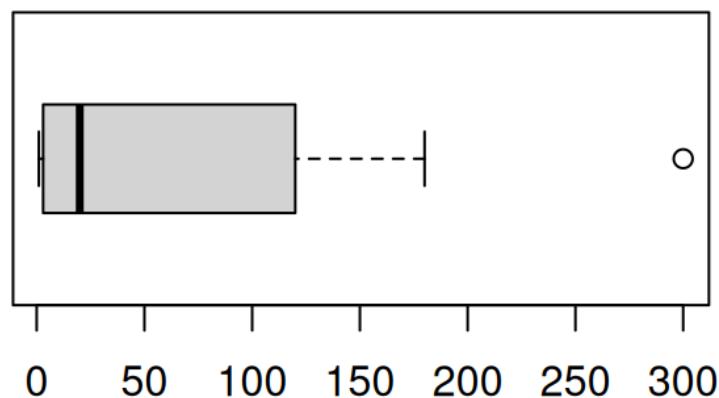
$$UT = \min(\max(x), Q_3 + 1.5IQR)$$

Dealing with outliers

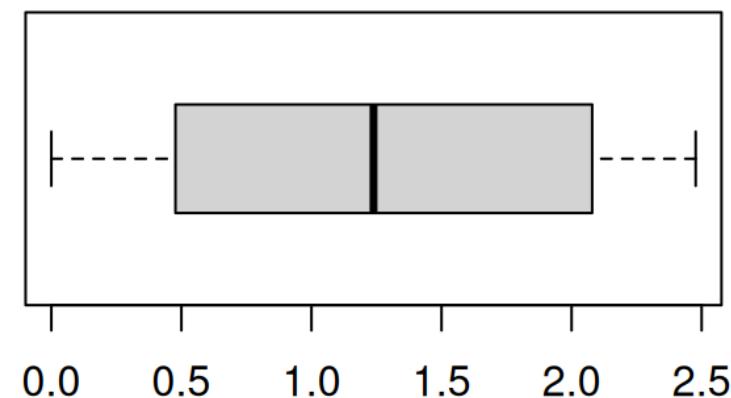
Sometimes outliers indicate that a better model is needed. We may remove outliers by transforming the data. For example, a right skewed data set with outliers can be transformed into the logarithmic scale.

```
1 w = c(1, 2, 3, 4, 10, 30, 60, 120, 180, 300)
2 w1 = log(w, 10)
3 par(mfrow = c(1, 2))
4 boxplot(w, main = "Data", horizontal = T)
5 boxplot(w1, main = "Log of Data", horizontal = T)
```

Data



Log of Data



Write a function in R

How to write a function in R

A function in R is one of the most used objects. For example, `mean`, `median`, `sd` are all R functions. It is very important to understand the purpose and syntax of R functions and knowing how to create or use them.

To declare a user-defined function in R, we use the keyword `function`.

```
1 function_name ← function(parameter1, parameter2) {  
2     # function body  
3     c = parameter1 + parameter2  
4     # return the outputs  
5     return(c)  
6 }
```

Here we declared a function with name `function_name`, the function takes inputs `parameter1`, `parameter2` and returns an output `c`. It can take any number of inputs but **only one** outputs.

Example

Here we want to write a function in R that calculates the sample mean and sample standard deviation

```
1 my_summary <- function(X) {  
2     # Write operations within the curly brackets  
3     m = sum(X)/length(X)  
4     s = sqrt(sum((X - m)^2)/(length(X) - 1))  
5     # put mean and sd in a vector, then return the vector as a single output  
6     return(c(m, s))  
7 }
```

Then we can reuse all the operations defined in the function.

```
1 w = c(1, 2, 3, 4, 10, 30, 60, 120, 180, 300) # a data vector  
2 my_summary(w) # our function  
[1] 71.0000 100.5651  
1 c(mean(w), sd(w)) # R built-in function  
[1] 71.0000 100.5651
```

Summary

Centre

- Sample mean
- Sample median
- Robustness and comparisons

Spread

- Standard deviation
- Interquartile range

Write functions in R