



Tecnológico  
de Monterrey

Control engineering

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Second  
Simulation Project

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Birthday September 17<sup>th</sup>

- $A = 17$
- $B = 9$
- $C = 26$

I used the following formulas to obtain the values of  $K_c$ ,  $T_i$  and  $T_d$  of PID

## Sustained oscillations

Ziegler-Nichols (1942)

2. Obtain the values for  $K_c$ ,  $T_i$  and  $T_d$

	$K_c$	$T_i$	$T_d$
P	$\frac{K_u}{2}$		
PI	$\frac{K_u}{2.2}$	$\frac{t_u}{1.2}$	
PID	$\frac{K_u}{1.7}$	$\frac{t_u}{2}$	$\frac{t_u}{8}$

## Damped oscillations

Harriot (1957)

2. Obtain the values for  $K_c$ ,  $T_i$  and  $T_d$

	$K_c$	$T_i$	$T_d$
P	$K_o$		
PI	$K_o$	$t_o$	
PID	$K_o$	$\frac{t_o}{1.5}$	$\frac{t_o}{6}$

## System 1

### Proportional controller

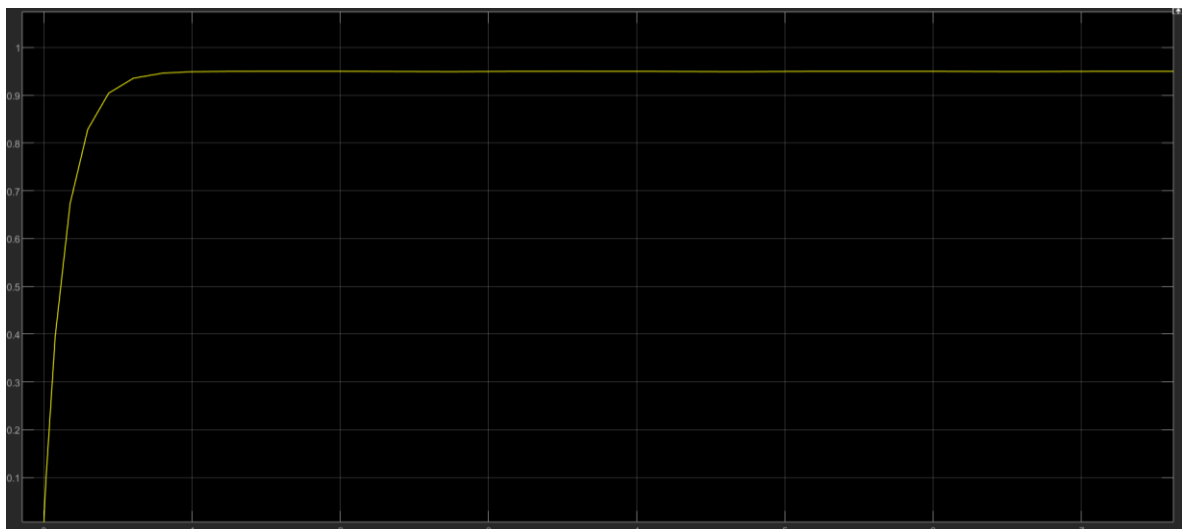
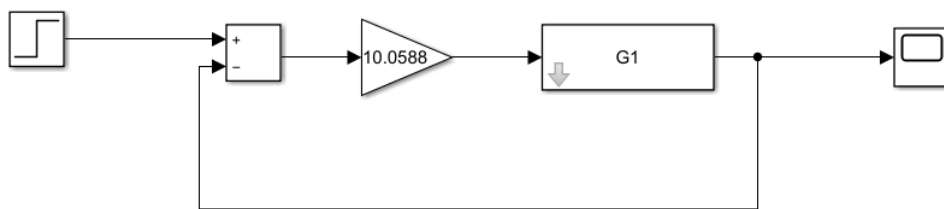
System 1

$$G(s) = \frac{17K}{26s + 9 + 17K} = \frac{17K}{9 + 17K} = .95 \quad K = 10.0588$$

$K > 0$   
 $K = 10.0588$

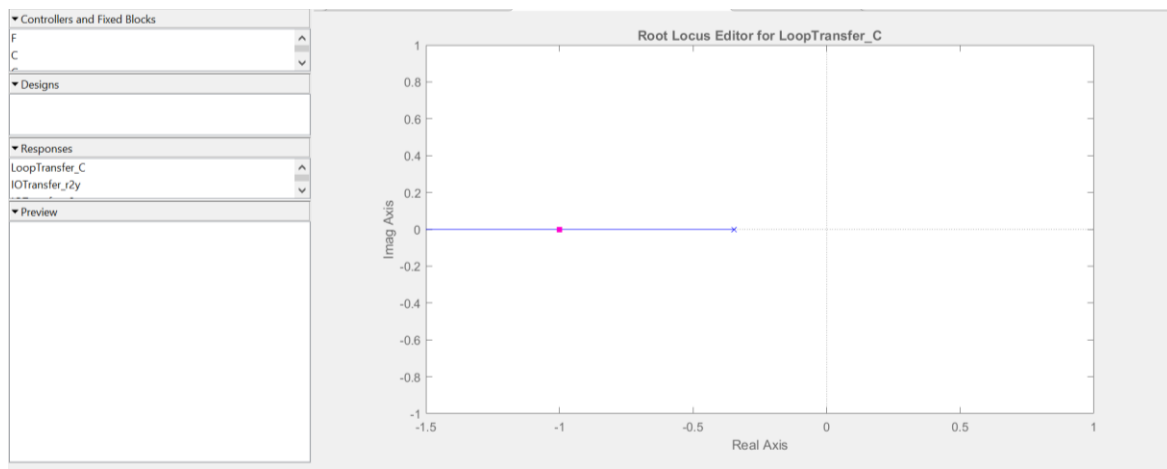
$$\frac{17K}{9 + 17K} = 1.05 \quad K = -11.1176$$

K = 10.0588



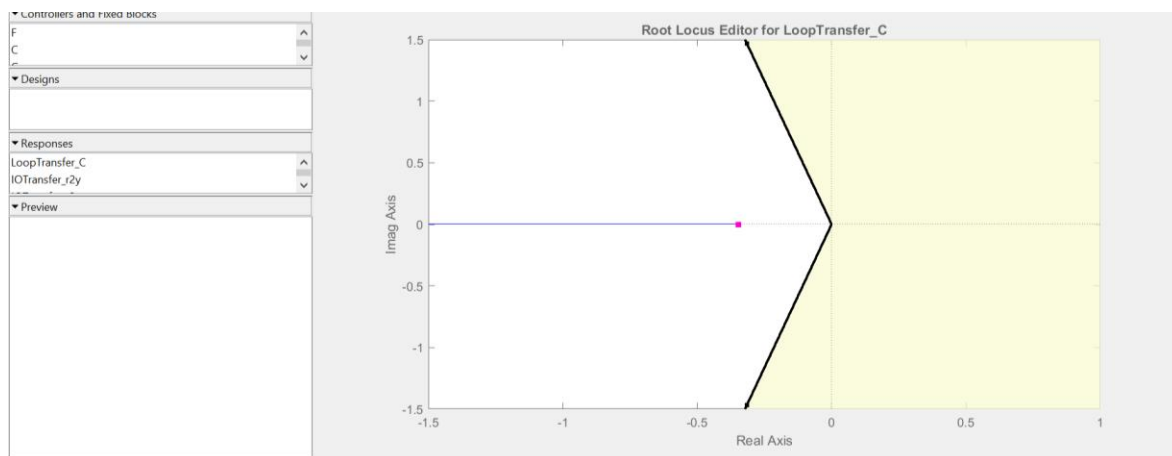
## *PID - Sustained Oscillations*

Point C can not be in the imaginary axis



## *PID - Damped Oscillations*

Point C can not be in the intersection of damping ratio = 0.21



## *PID - Algebraic method*

$K = .00011803$

$a = .6792$

$b = .194$

$kd = .00011803$

$kc = 1.0306e-04$

$ki = 1.5552e-05$

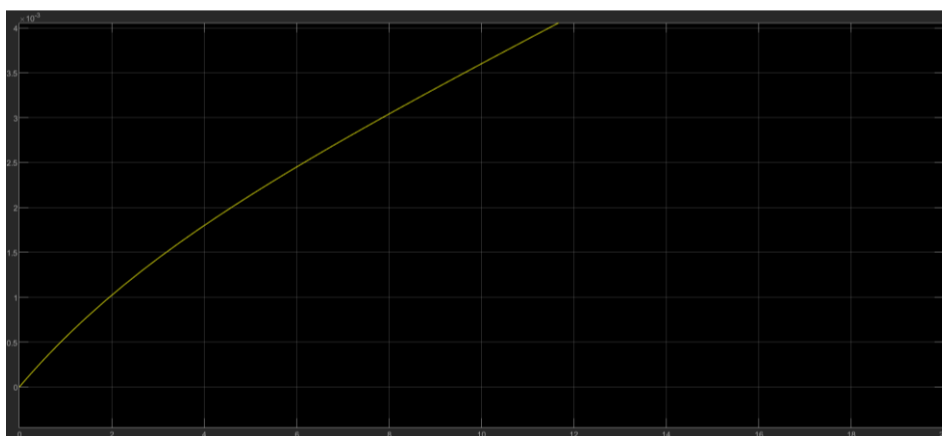
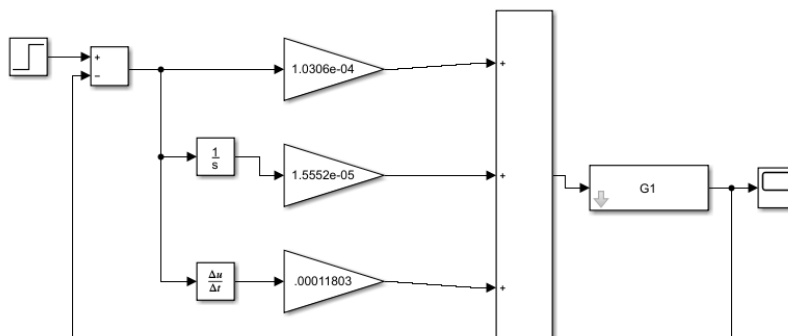
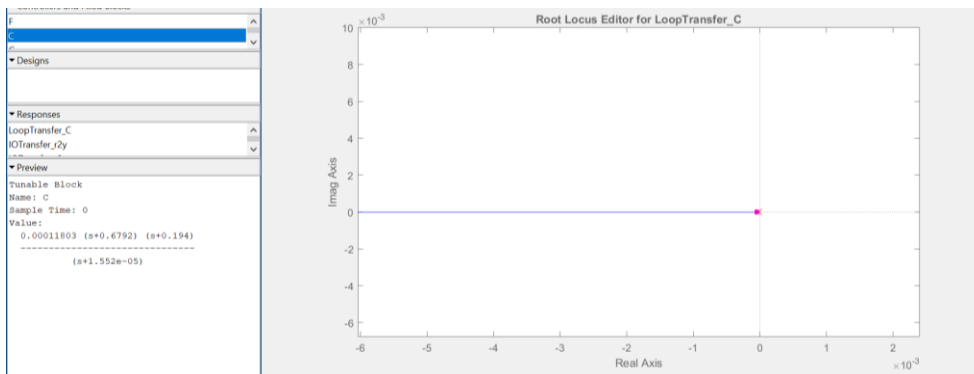
Algebraic method

$$\frac{.00011803}{s} (s + .6792)(s + .194) = \frac{K(s+a)(s+b)}{s}$$

$K = .00011803$   
 $a = .6792$   
 $b = .194$

$$\frac{Ks^2 + K(a+b)s + Kab}{Kd \quad Kc \quad Ki}$$

$Kd = .00011803$   
 $Kc = 1.0306e-04$   
 $Ki = 1.5552e-05$



*PID controller  $r(t)=at$*

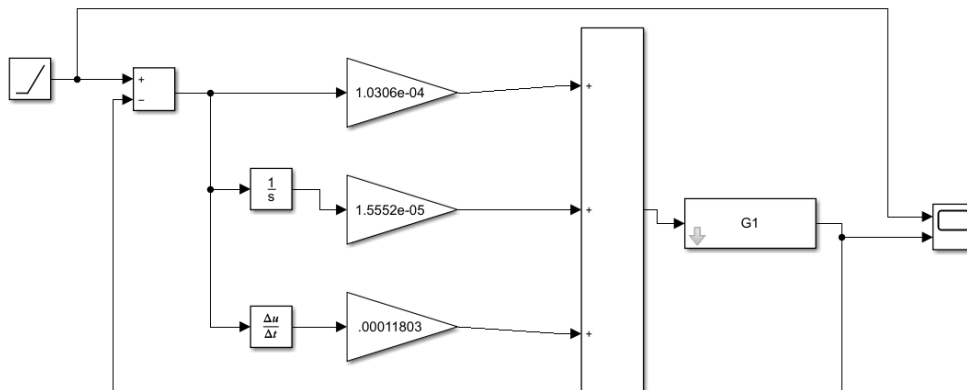
$k_v = 2.93e-05$

$ess = 34.041e03$

PID for ramp

$$\lim_{s \rightarrow 0} \frac{s(k_v s + k_i + k_d s^2)}{s} \frac{17}{9} = \frac{k_i 17}{9} = 2.9376 \times 10^{-5}$$

$$ess = \frac{1}{K_v} = 34041.3943 \rightarrow$$



## System 2

### Proportional controller

$K=29.0588$  and it is unstable, so we have to design PID controller

System 2

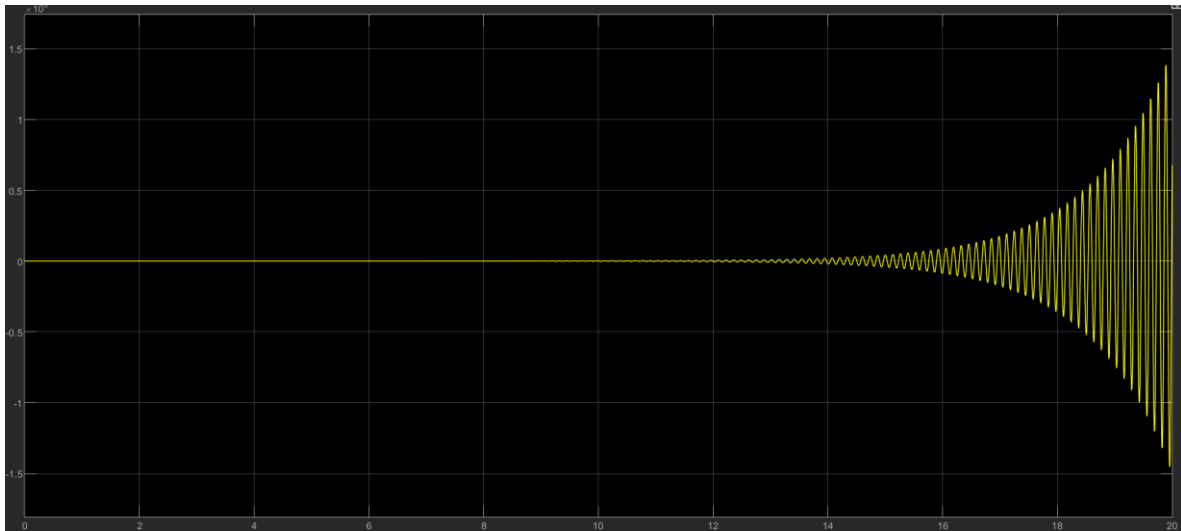
$$G(s) = \frac{-0.884s + 442}{6s^2 + 17.68s + 676}$$

$K = 29.0588$

Proportional Controller

$$\frac{442K}{676 + 442K} = 0.95 \quad K = 29.0588$$

$$\frac{442K}{676 + 442K} = 1.05 \quad K = -32.11$$



### PID - Sustained Oscillations

Sustained Oscillations

$K_u = 19.868$  For P controller

First  $K_u/2$  for calculate  $t_u = 9.934$

$t_u = 0.21973$

PID

$K_c = \frac{K_u}{1.7}$

$K_c = 11.6871$

$K_i = \frac{K_c}{t_i}$

$K_i = 106.3430$

$T_i = \frac{t_u}{2}$

$T_i = 0.1099$

$T_d = t_u/8$

$T_d = 0.0275$

$K_d = (K_c)(T_d)$

$K_d = 0.3214$

*P*

$K_u = 19.868$

$k_c = 9.934$

$t_u = .21973$

PID

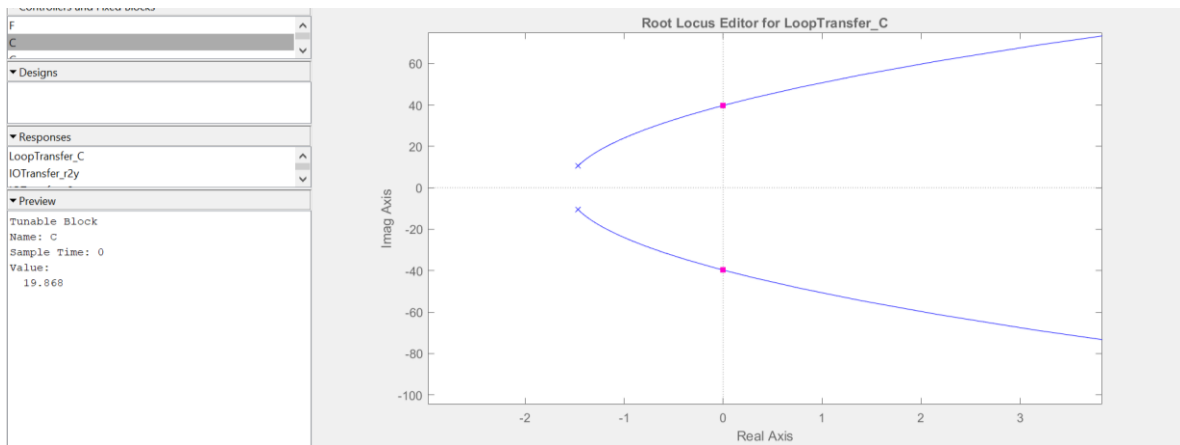
$k_c = 11.6871$

$t_i = .1099$

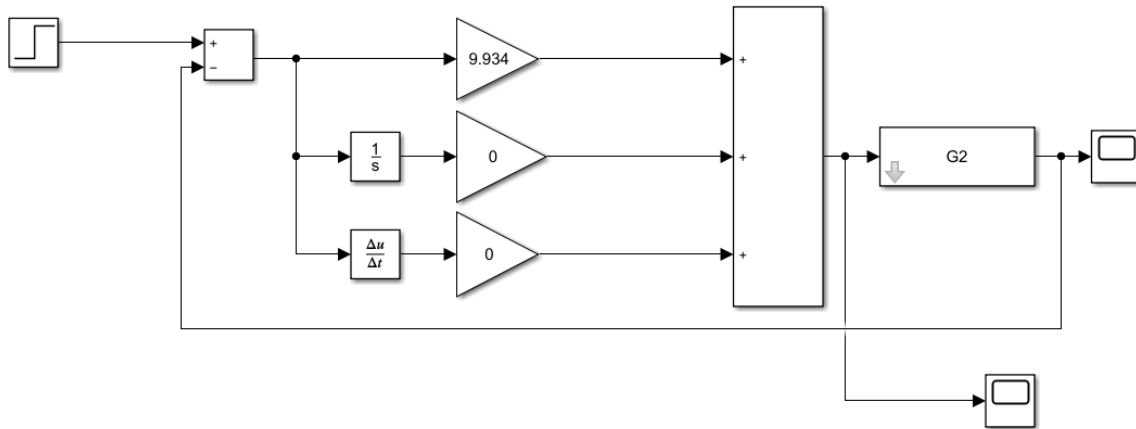
$t_d = .0275$

$k_i = 106.343$

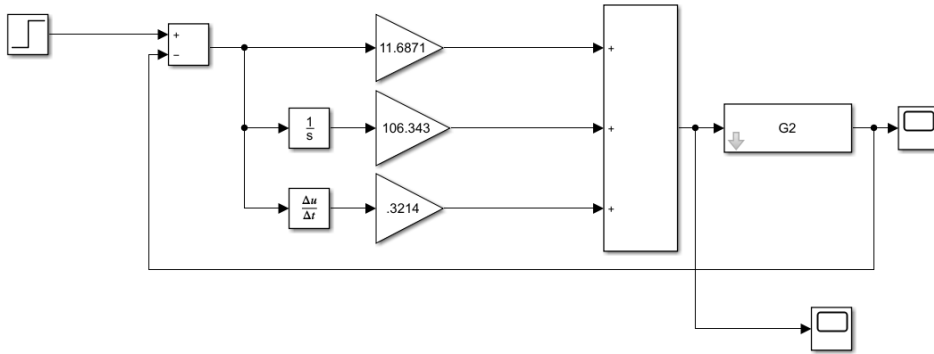
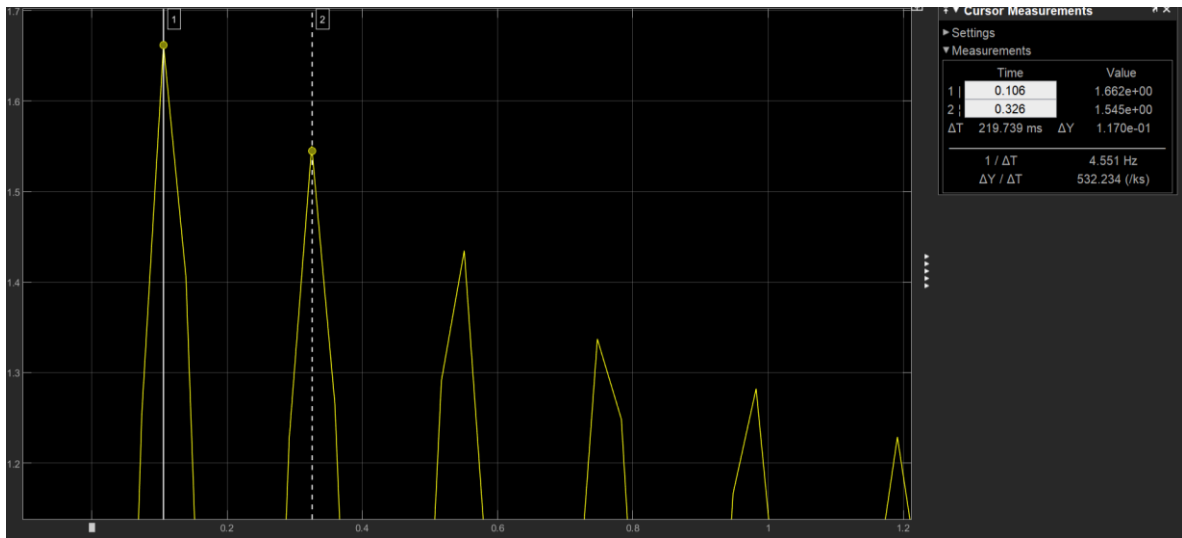
$k_d = .3214$



First design a P controller

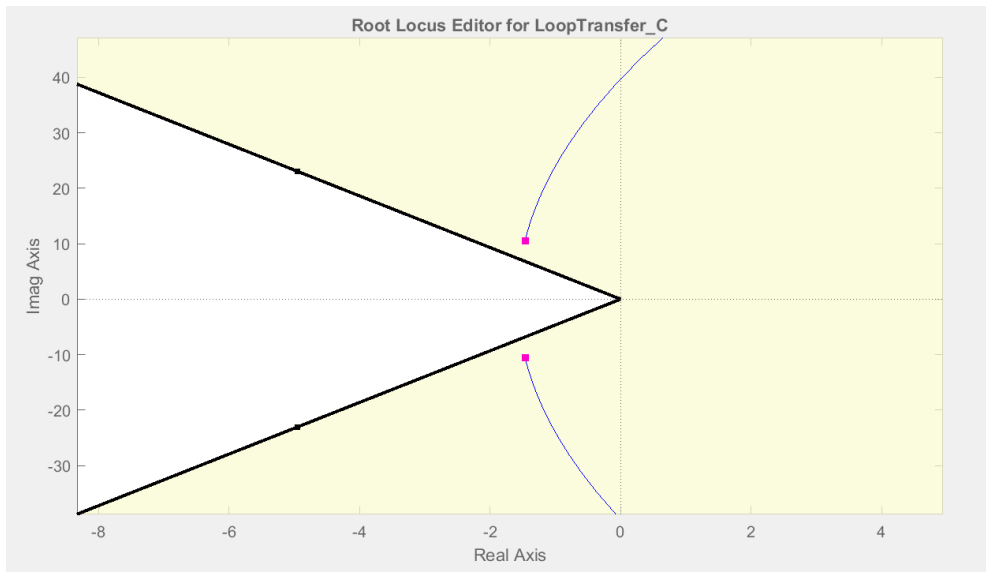






## PID - Damped Oscillations

It is not possible to do a damped oscillations PID controller



## PID - Algebraic method

Algebraic method

$$\frac{2.5814e-08 (s+125.4)(s+382.8)}{s}$$

$K = 2.5814e-08$   
 $a = 125.4$   
 $b = 382.8$

$$\frac{Ks^2}{K_d} + \frac{K(a+b)s}{K_c} + \frac{Kab}{K_i}$$

$K_d = 2.5814e-08$   
 $K_c = 1.3118 \times 10^{-5}$   
 $K_i = 1.2391 \times 10^{-3}$

$$K = 2.5814e-08$$

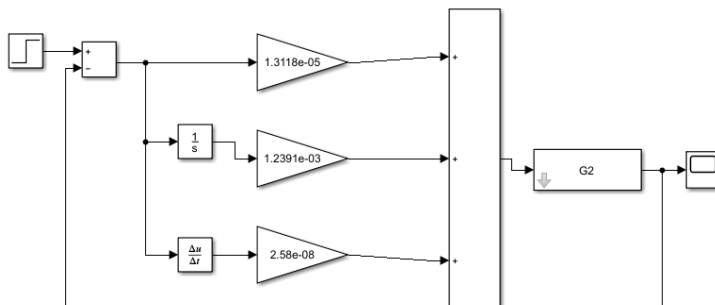
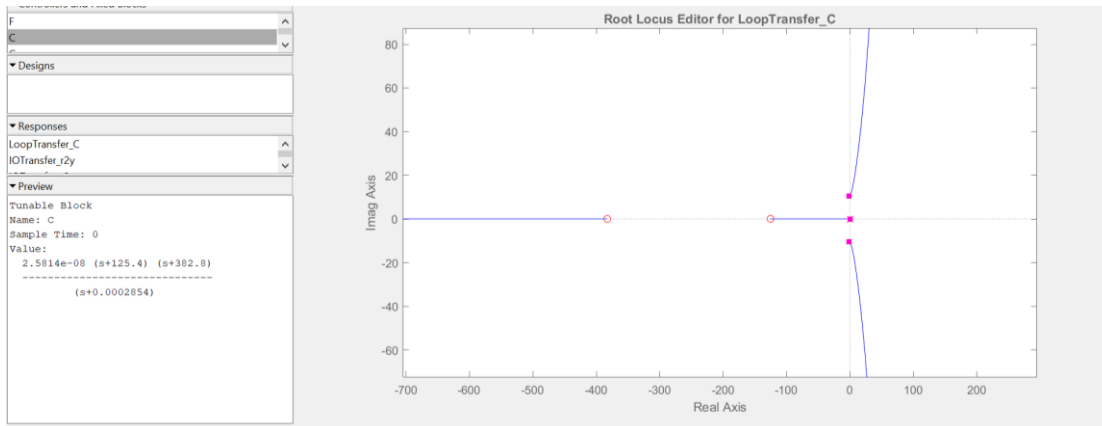
$$a = 125.4$$

$$b = 382.8$$

$k_d = 2.5814 \times 10^{-8}$

$k_c = 1.3118 \times 10^{-5}$

$k_i = 1.239 \times 10^{-3}$



*PID controller  $r(t) = at$*

$K_v = 69.5319$

$ess = .01438$

PID ramp

$$\lim_{s \rightarrow 0} s \frac{(K_p + K_i + K_d s)}{s} = \frac{-0.8815 + 442}{63 + 173 + 676}$$

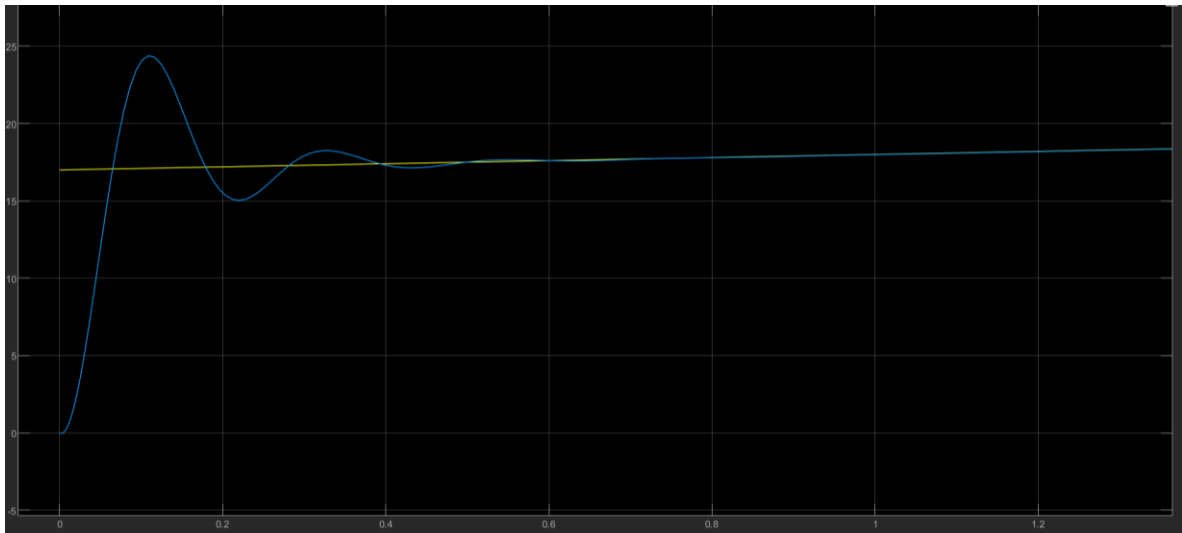
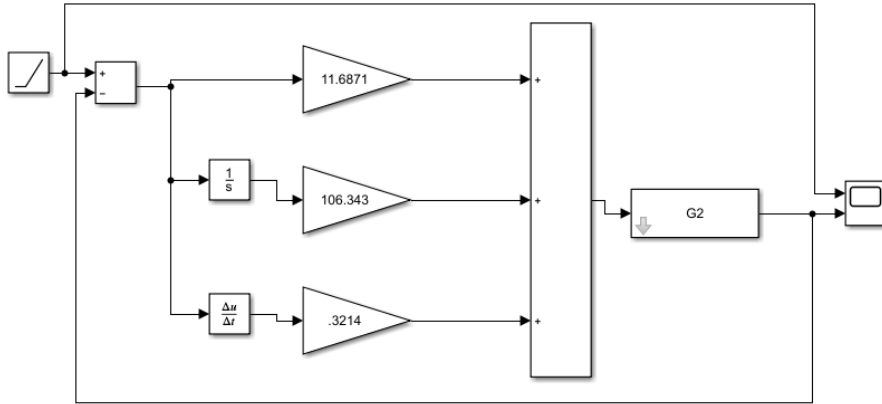
$$= K_i \left( \frac{442}{676} \right)$$

$$K_i = 106.3430$$

$$K_v = 69.5319$$

$$ess = \frac{1}{K_v}$$

$$ess = .01438$$



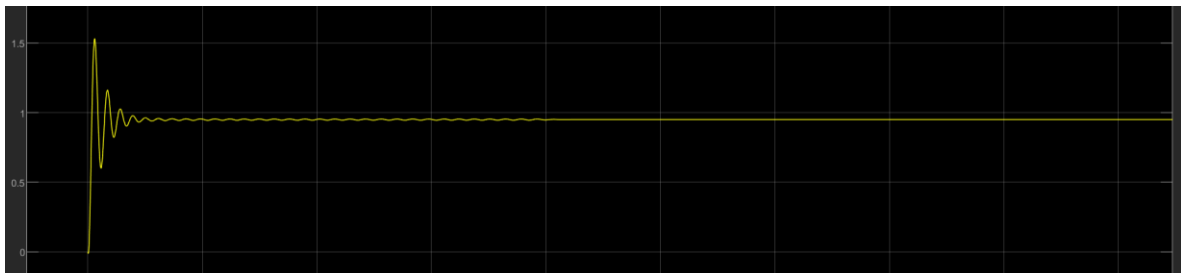
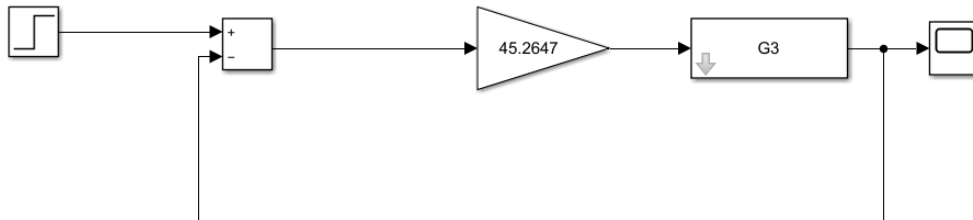
### System 3

*Proportional controller*

System 3

$$G(s) = \frac{-.204s + 68K}{s^2 + 27s + 162 + 68K}$$
$$= \frac{68K}{162 + 68K} = .95 \quad K = 45.2647$$
$$\frac{68K}{162 + 68K} = 1.05 \quad K = -51.0294$$

K = 45.2647



*PID - Sustained Oscillations*

*P*

Ku = 135.09

kc = 67.545

tu = .096875

PID

kc = 79.4647

ti = .04843

td = .012109

ki = 1640-8156

kd = 1.6759

# PID oscillations method

$K_u = C$  in imaginary axis  $C = 135.09$

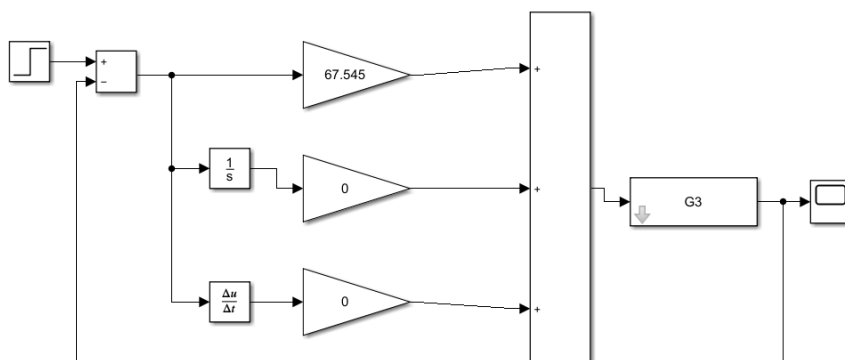
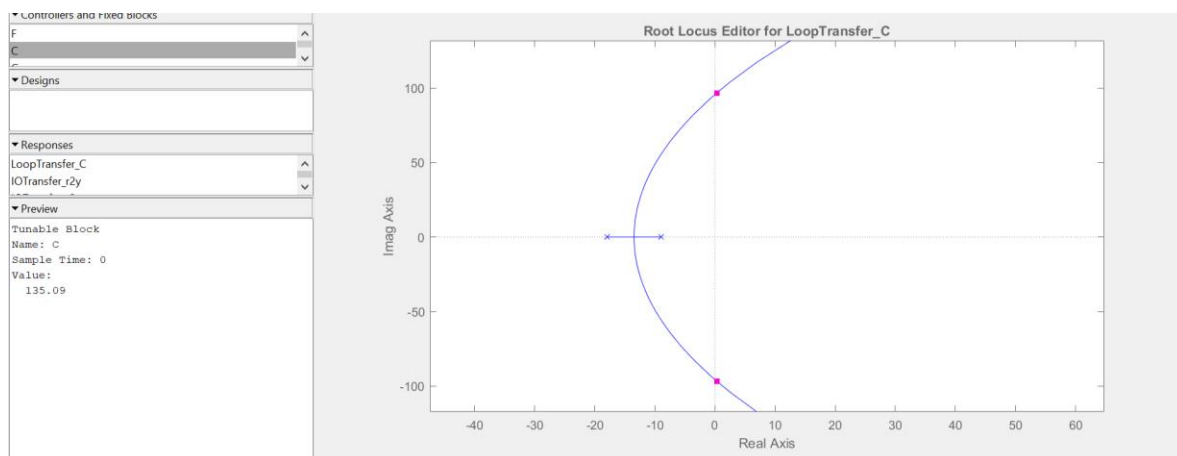
For P  $k_c = K_u / 2 = 67.545$

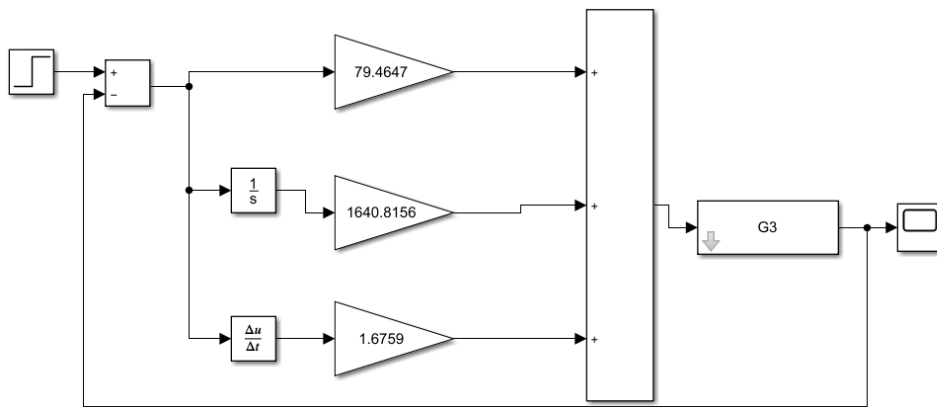
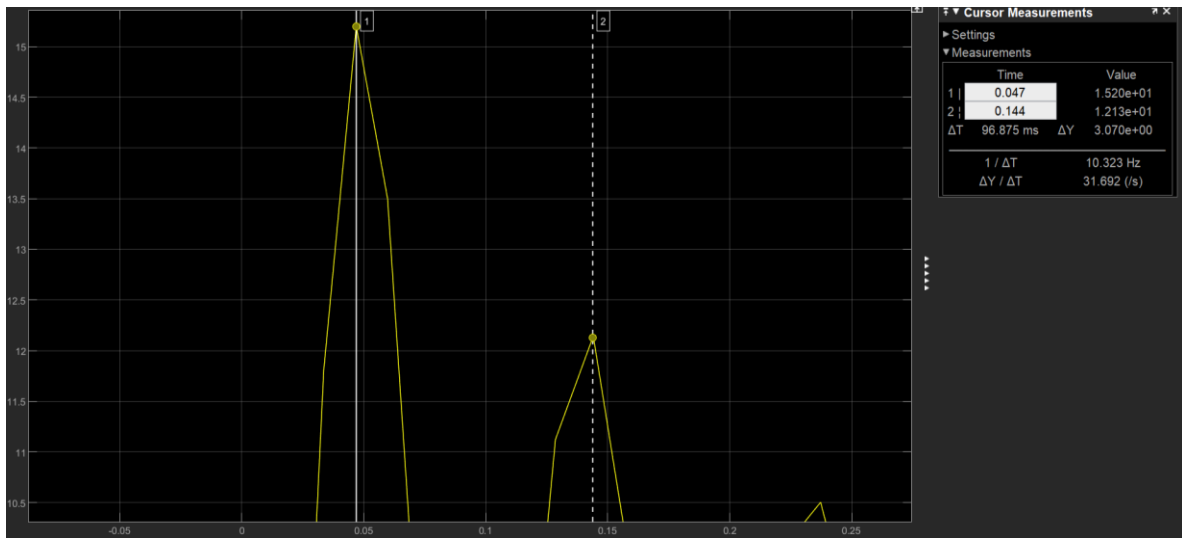
$t_u = .096875$

For PID

$k_c = \frac{K_u}{1.7} = 79.4647$   $T_i = \frac{t_u}{2} = .04843$   $T_d = \frac{t_u}{8} = .012109$

$K_i = \frac{k_c}{T_i} = 1640.8156$   $k_d = k_c T_d = 1.6759$





## PID - Damped Oscillations

P

$K_0 = 32.24$

$k_c = 32.24$

$t_o = 0.125597$

PID

$k_c = 32.24$

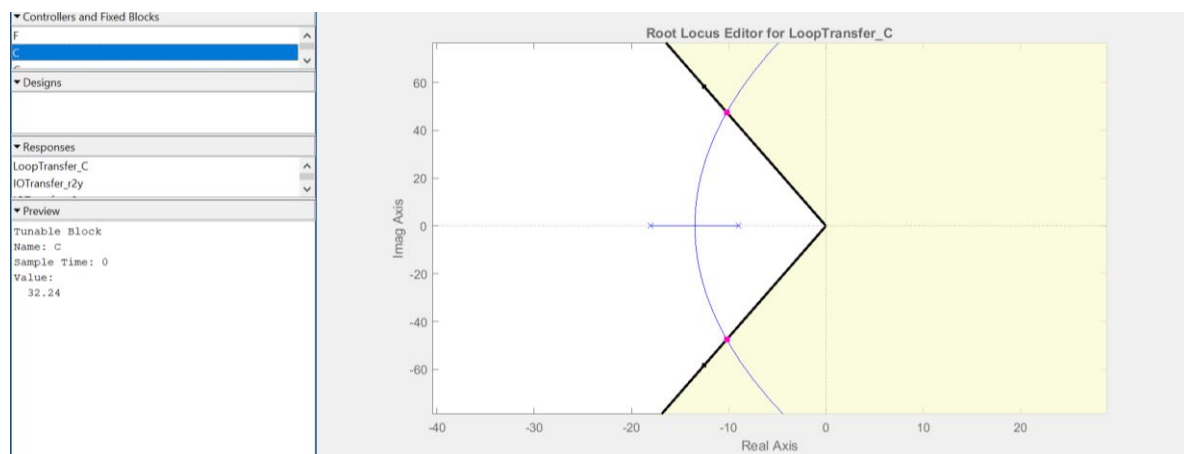
$t_i = 0.0837$

$t_d = 0.0209$

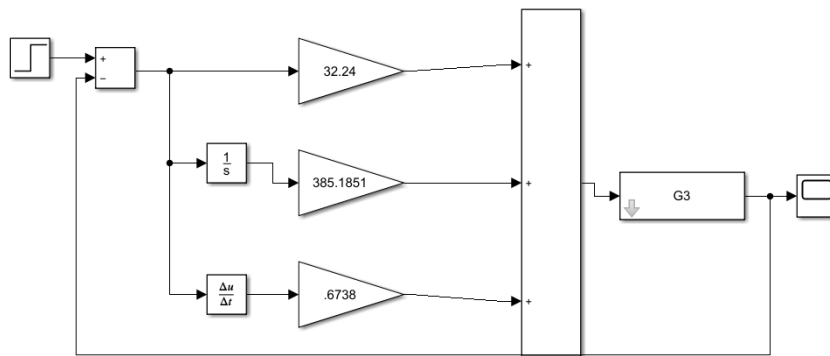
$k_i = 385.1851$

$k_d = 0.6738$

PID Damped oscillators  
 $K_0 = 32.24$   
 $t_o = 0.125597$   
For PID  
 $K_c = 32.24$   $T_i = \frac{t_o}{1.5} = 0.0837$   $T_d = \frac{t_o}{6} = 0.0209$   
 $k_i = \frac{K_c}{T_i} = 385.1851$   $k_d = K_c T_d = 0.6738$





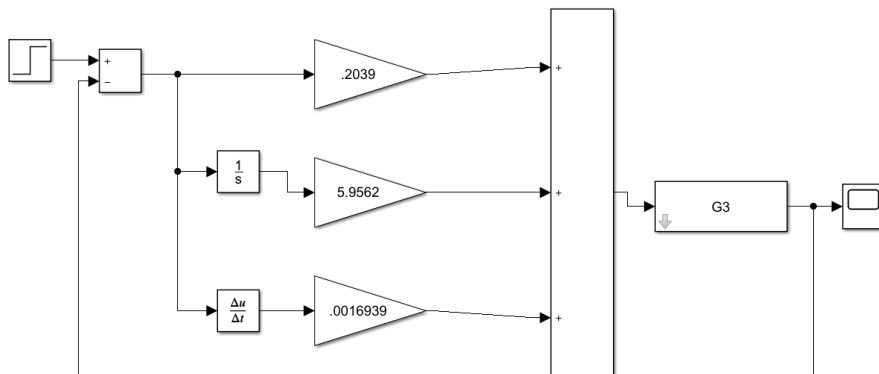
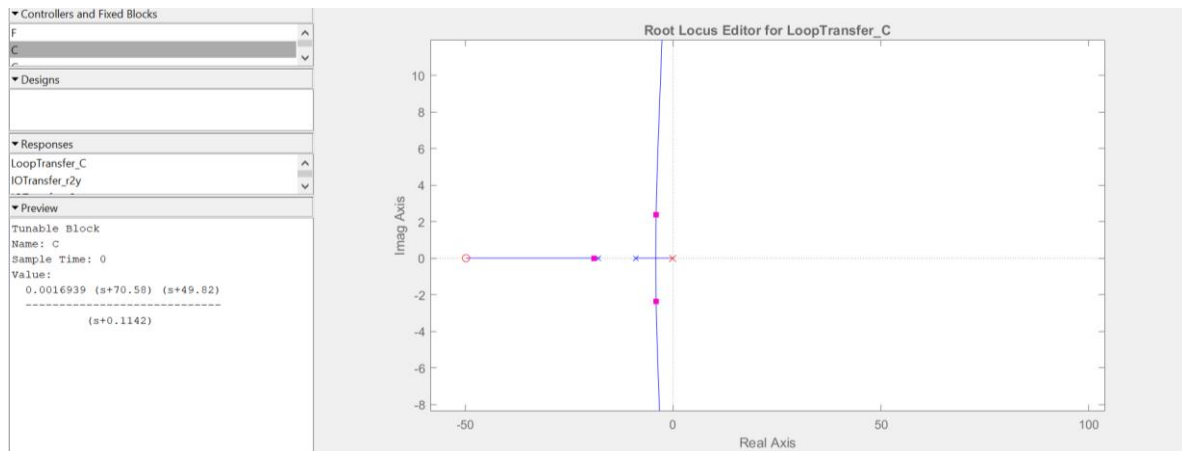


### *PID - Algebraic method*

$$\frac{.0016939 (s+70.58)(s+49.82)}{s} = \frac{K (s+a)(s+b)}{s}$$

$K = .0016939$	$K_d = .0016939$
$a = 70.58$	$K_c = .2039$
$b = 49.82$	$K_i = 5.9562$

$K = .0016939$   
 $a = 70.58$   
 $b = 49.82$   
 $kd = .0016939$   
 $kc = .2039$   
 $ki = 5.9562$

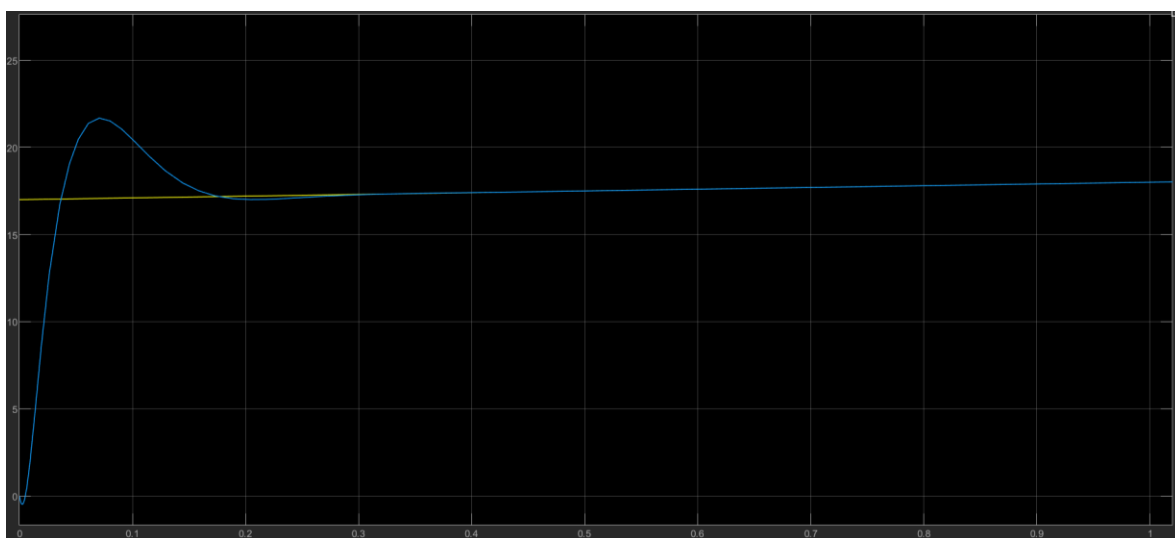
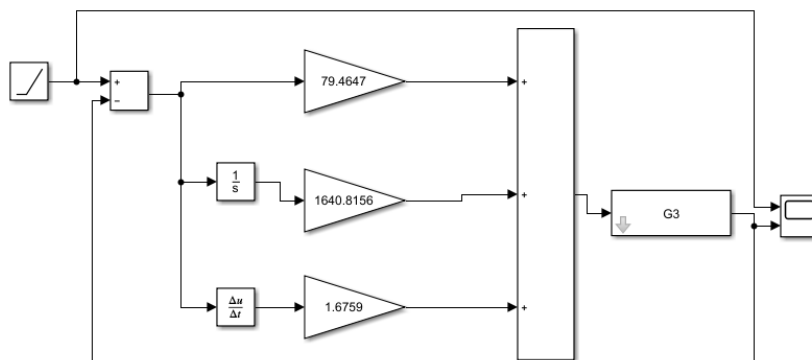


PID controller  $r(t)=at$

$k_v = 688.7374$

$ess=1.43e-03$

PID ramp  
lin  $\lim_{s \rightarrow 0} \frac{s(Ks^2 + Kcs + Ki)}{s} = \frac{-0.2045 + 68}{s^2 + 27s + 162}$   
 $K_v = K_i \left( \frac{68}{162} \right) \quad K_i = 1640.8156 = 688.7374$   
 $ess = \frac{1}{K_v} = 1.45 \times 10^{-3}$



## System 4

With the following script I obtain the transfer function

```
close all
t=zeros(); %arreglo de tiempo
k=zeros(); % arreglo de la ganancia
[fil,col]=size(Out); % declaro el arreglo para leer la posicion de los valores de la funcion
t=Out(:,1);%Datos del tiempo obtenidos de la tf G mediante simulink
k=Out(:,2);%Datos de la ganancia obtenidos de la tf G mediante simulink
figure;
plot(t,k); %grafica de la tf

%identificar el #orden del sistema
pico=max(k);
final=k(end);
if pico == final
    z = 1;

else
    z = 2;

end
%implementacion una vez conocido el orden del sistema
if z==1
    Orden=1
    for y=1:fil
        if Out(y,2) > final*0.9933 %Encontrar el valor de 5Tao
            pos=y %Posicion del arreglo donde esta 5tao
            break

        else
            end

        end

        Tss=Out(pos,1);
        Tao=Tss/5;
        ktf=final;
        G1=tf(ktf,[Tao 1])
        figure;
        plot(t,k);
        hold on
        step(G1)
    else
        Orden=2
        for y=1:fil
            if Out(y,2) == pico
                tpa=y;
                break

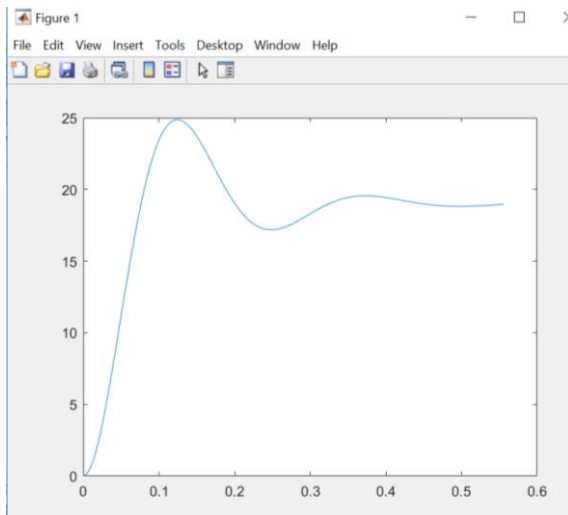
            else
                end

            end

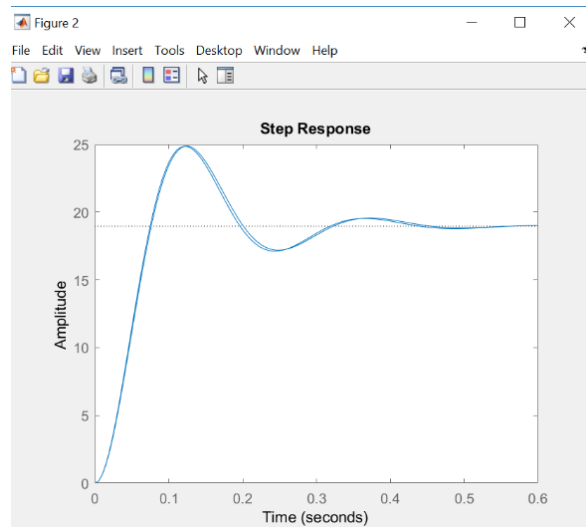
            tp=Out(tpa,1);
            wd=pi/tp;
            mp=((pico-final)/final);
            delta=-(log(mp))/(sqrt(log(mp)^2 + pi^2));
            wn=wd/sqrt(1-delta^2);
            ks2=final;
            G2=tf((ks2*wn^2),[1 (2*delta*wn) (wn^2)])
            figure;
            plot(t,k);
            hold on
            step(G2)
        end
    end
```

```
>> G4
G4 =
      14340
-----
s^2 + 19.14 s + 756.5
Continuous-time transfer function.
```

## Original function



## Approximate function



## Proportional controller

System 4  
Proportional controller

$$g(s) = \frac{1.434e04 K}{s^2 + 19.14s + 756.5 + 1.434e04 K}$$

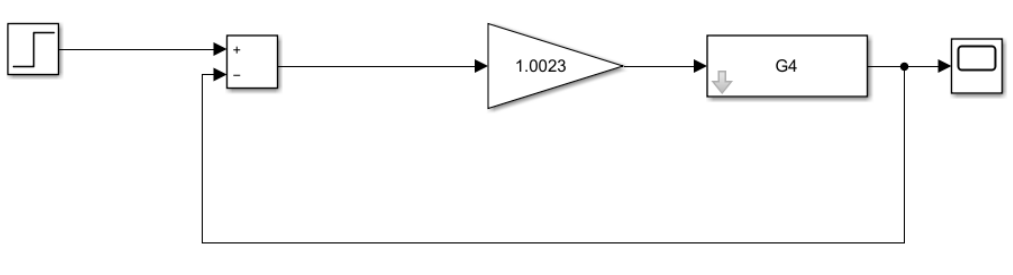
$$K = 1.0023$$

PID controller

$$\frac{1.434e04 K}{1.434e04 K + 756.5} = 1.05 \quad K = -1.107$$

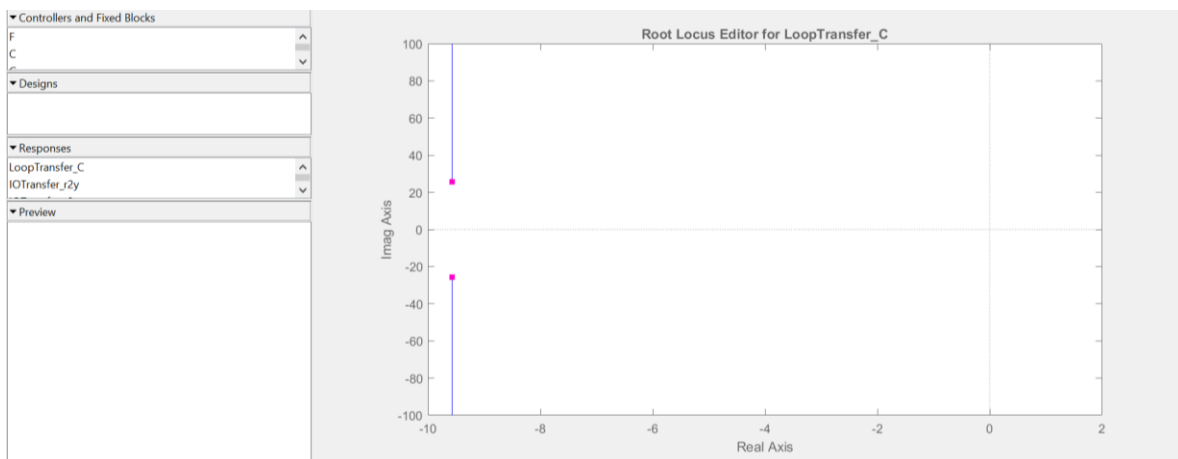
$$\frac{1.434e04 K}{1.434e04 K + 756.5} = .95 \quad K = 1.0023$$

$$K = 1.0023$$



## *PID - Sustained Oscillations*

It can not in imaginary axis



## PID - Damped Oscillations

P

$K_o = 0.092106$

$k_c = 0.092106$

$t_o = 0.13848$

PID

$k_c = .092106$

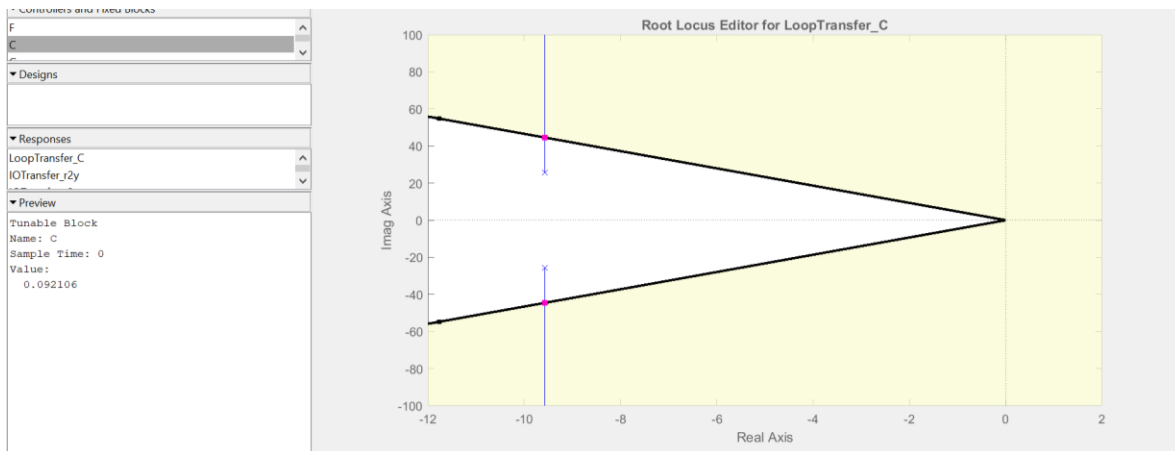
$t_i = .09232$

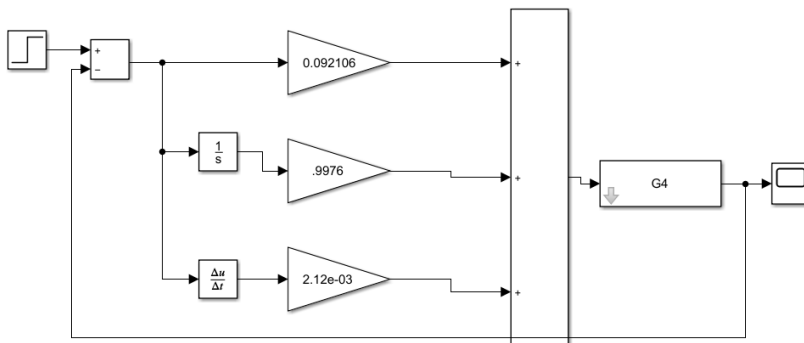
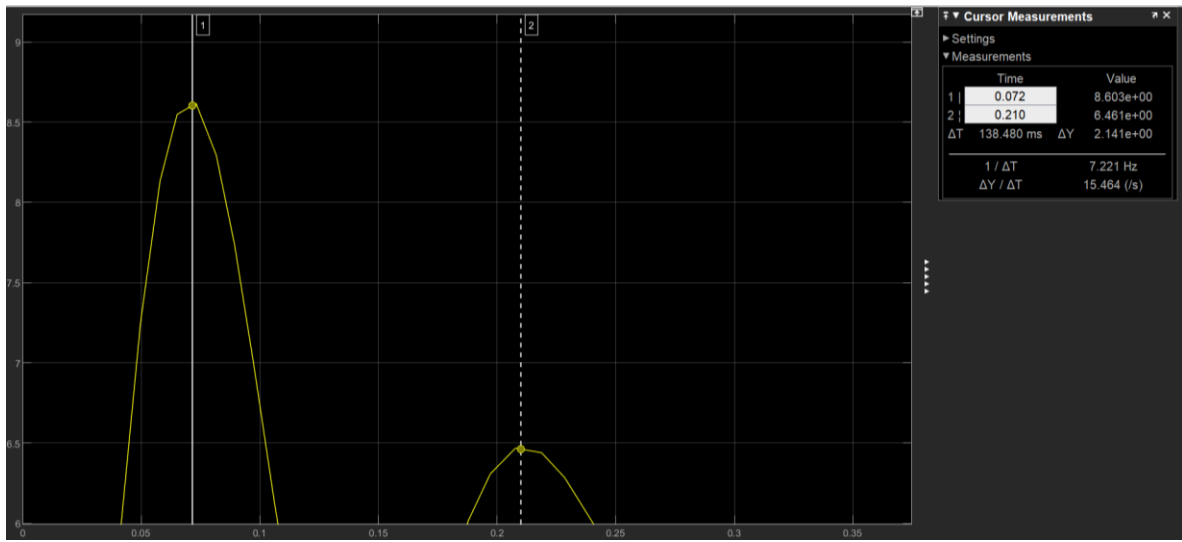
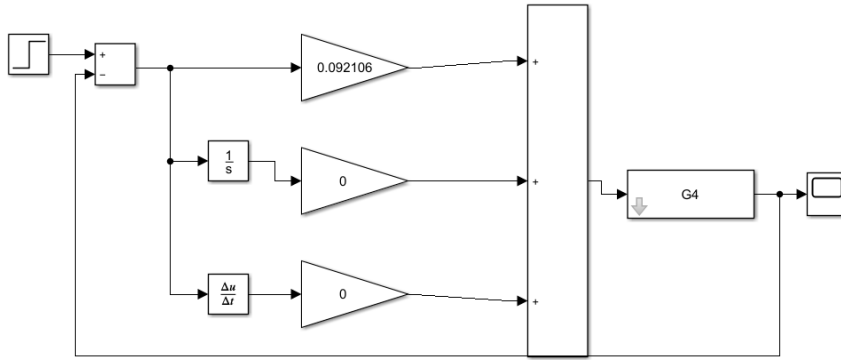
$t_d = .02308$

$k_i = .9976$

$k_d = 2.12e-03$

PID Damped oscillations  
 $k_o = .092106$  For P  
 $k_c = .092106$   
 $t_o = .13848$   
For PID  
 $K_c = k_o = .092106$   $T_i = t_o / 1.5 = .09232$   $T_d = t_o / 6 = .02308$   
 $k_i = \frac{k_c}{T_i} = .9976$   $k_d = k_c T_d = 2.12 \times 10^{-3}$









### *PID - Algebraic method*

$K = 6.0497e-09$

$a = 8.355$

$b = 1282$

$k_d = 6.0497e-09$

$k_c = 7.8062e-06$

$k_i = 6.4799e-05$

PID algebraic method

$$\frac{6.0497e-09 (s+8.355) (s+1282)}{s} \quad \frac{K(s+a)(s+b)}{s}$$

$K = 6.0497e-09$        $k_d = 6.0497e-09$        $k_i = 6.4799 \times 10^{-5}$   
 $a = 8.355$   
 $b = 1282$

$$= \frac{Ks^2}{k_d} + \frac{K(a+b)}{k_c} + \frac{Kab}{k_i}$$

$k_c = 7.8062 \times 10^{-6}$

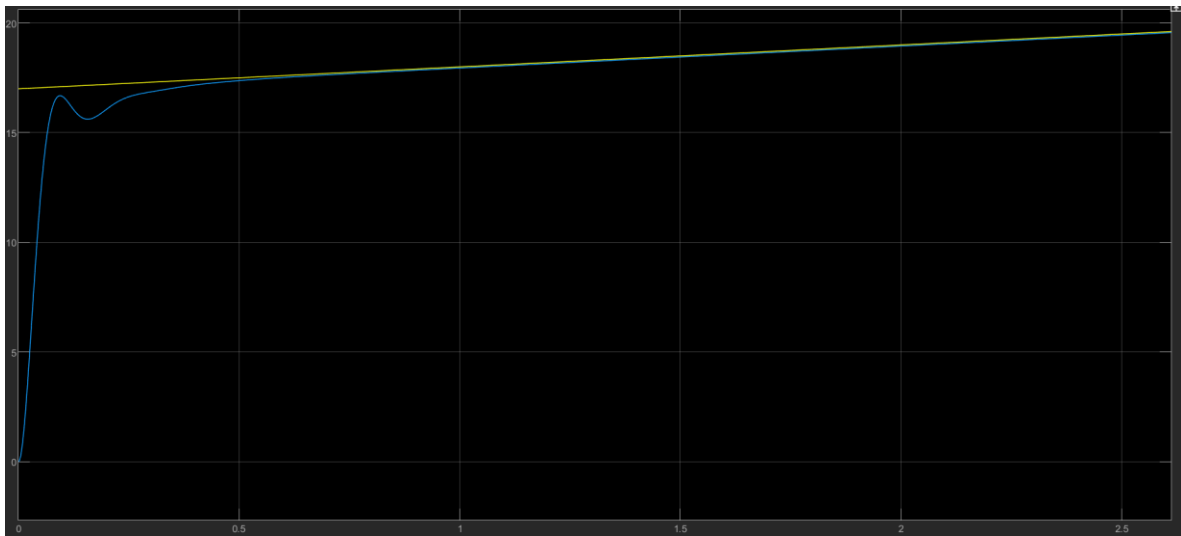
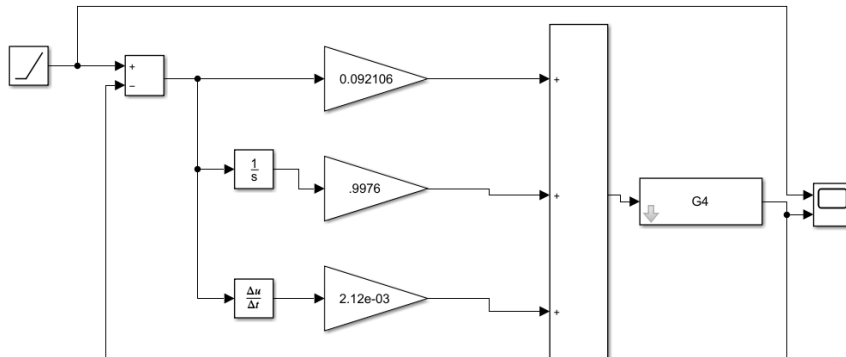


PID ramp

$$\lim_{s \rightarrow 0} \frac{s(Ks^2 + Ks + K_i)}{s^2 + 19.11s + 756.5} \cdot \frac{1.434e04}{s^2 + 19.11s + 756.5} = \frac{14340 K_i}{756.5}$$

$K_i = .9976 \quad K_v = 18.91$

$$ess = \frac{1}{K_v} = .0528$$



## System 5

{n = 21}

H1 Ref\_ 1 = 10.875

Symbolic transfer function

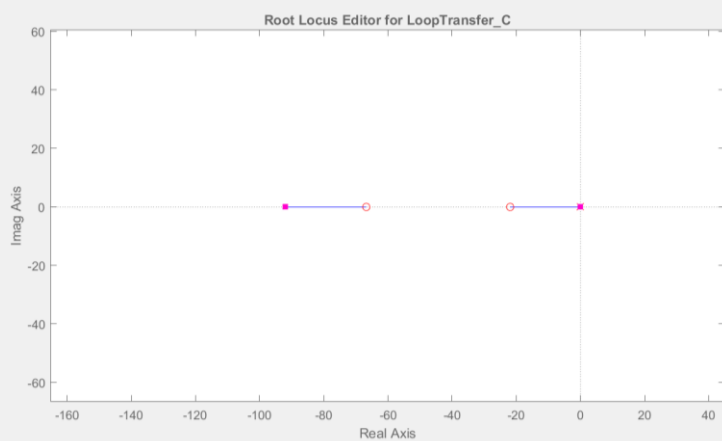
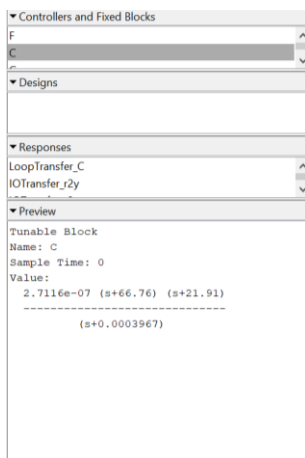
$$\frac{1}{Area1 + \frac{s}{R1}}$$

System 5 First tank

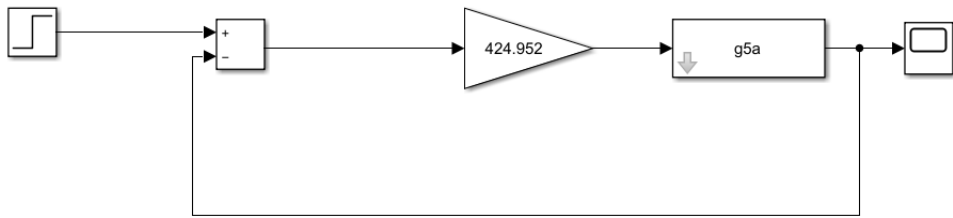
$$G(s) = \frac{Q_{in}}{Area + \frac{s}{R}} = \frac{3.5}{.5s + 46}$$

ess < 10%       $\frac{3.5K}{46 + 3.5k} = .97$       K = 424.952

ess = 3%       $\frac{3.5k}{46 + 3.5k} = 1.03$       K = -451.238



k = 424.952



### *PID - Algebraic method*

$K = 2.7116e-07$

$a = 66.76$

$b = 21.91$

$kd = 2.7116e-07$

$kc = 2.4043e-05$

$ki = 3.9662e-04$

# Algebraic PID

$$2.7116e-07 \frac{(s+66.76)(s+21.91)}{s} - \frac{K(s+a)(s+b)}{s}$$

$$K = 2.7116e-07$$

$$a = 66.76$$

$$b = 21.91$$

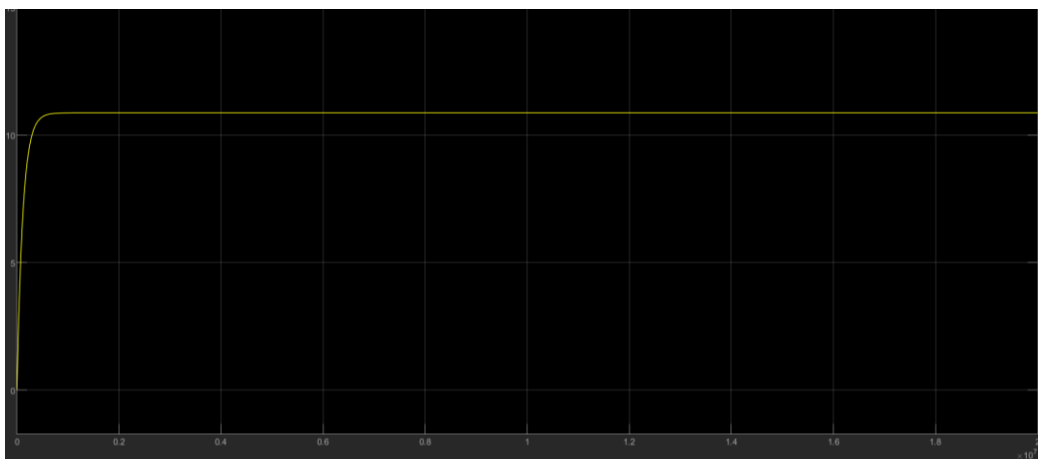
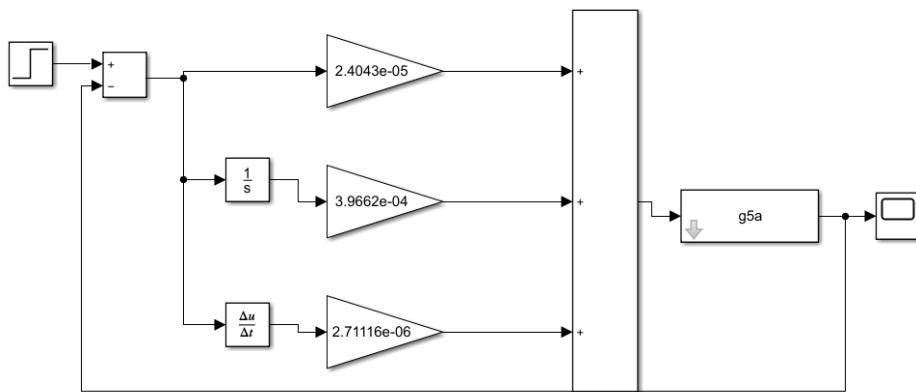
$$\frac{Ks^2}{K_d} + \frac{K(a+b)}{K_c} + \frac{Kab}{K_i}$$

$$K_d = 2.7116e-07$$

$$K_c = 2.4043e-05$$

$$K_i = 3.9662 \times 10^{-4}$$

$$Ref_{-1} = 10.875$$



$$H2 \text{ Ref\_2} = 13.775$$

Symbolic transfer function

$$\frac{Q_{in1}}{\frac{A2s}{R1} + A2A1 + \frac{s^2}{R1R2} + \frac{A1s}{R2}}$$

Second Tank

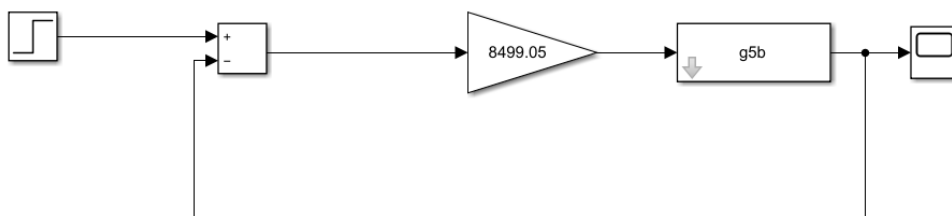
$$G(s) = \frac{Q_{in1}(s)}{A2 \frac{s}{R1} + A2A1 + \frac{s^2}{R1R2} + \frac{A1s}{R2}}$$

$$= \frac{3.5}{10s^2 + 920s + 13.775}$$

Proportional Controller  $K < 10\%$   $K = .03$

$$1 - \frac{3.5K}{3.5K + 920} = .97 \quad K = 8499.05$$

$$\frac{3.5K}{3.5K + 920} = 1.03 \quad K = -9024.76$$

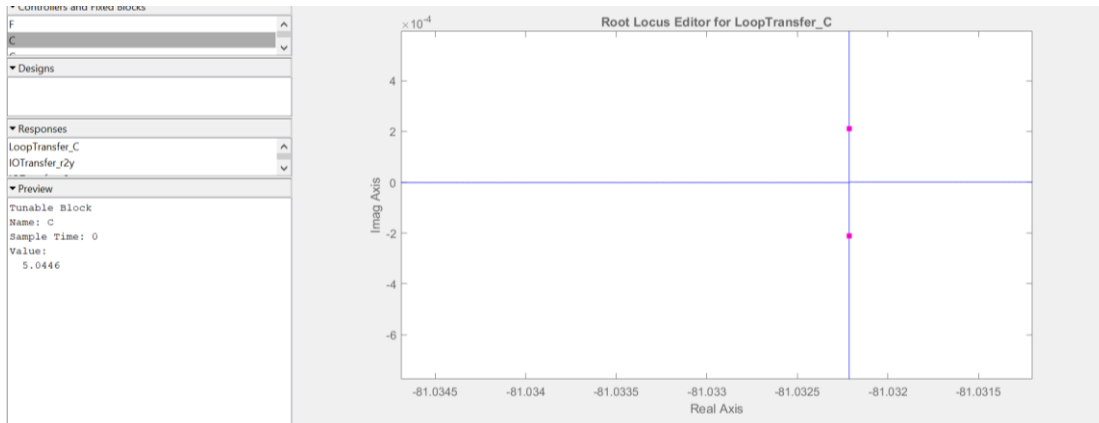


## PID – Oscillations Sustained

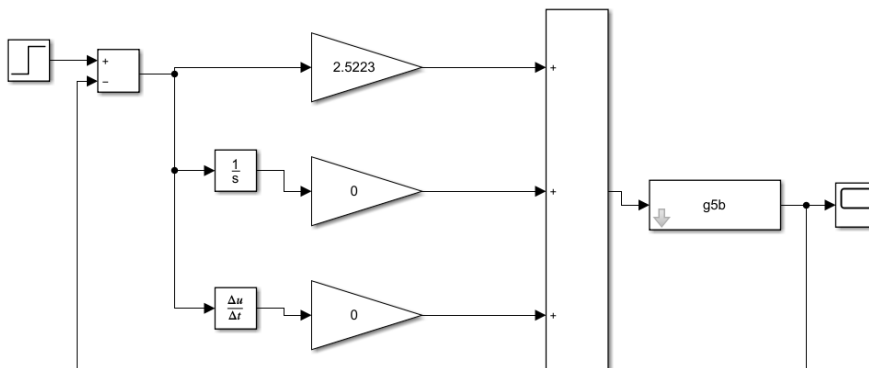
$$k_u = 5.0446$$

$$k_c = 2.5223$$

$t_u$  = Using a step as a reference I can not determinate  $t_u$



Oscillations Sustained PID Ref2 = 13.775  
 $k_u = 5.0446$   
 $k_c = k_u/2 = 2.5223$   
 $t_u$  = By the step I can not determinate, so I will do





## PID - Algebraic method

K = 4.3023e-06

a = 86.94

b = 61.29

kd = 4.3023e-06

kc = 6.3772e-04

ki = .0229

algebraic PID

$$\frac{4.3023e-06 (s+86.94)(s+61.29)}{s} = \frac{K (s+a)(s+b)}{s}$$

$K = 4.3023e-06$   
 $a = 86.94$   
 $b = 61.29$   
 $K_d = 4.3023e-06$   
 $K_c = 6.3772e-04$   
 $K_i = .0229$

$\frac{Ks^2}{K_d} + \frac{K(a+b)}{K_c} + \frac{Kab}{K_i}$

