Tecnológico de Monterrey

Control engineering

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Fourth Simulation Project

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System 1 – Inverted Pendulum

M mass of cart

m mass of pendulum

b coefficient of friction for cart

l length to pendulum

I mass moment of inertia of pendulum = .006 kg.m²

$u = F =$ force applied

$x =$ cart position coordinate

θ - pendulum angle

$M\ddot{x} + b\dot{x} + N = F$ → Sum forces and substitute

- $(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F$

Second equation: sum perpendicular forces

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\dot{\theta} + m\ddot{x}\cos\theta$$

$$-Pl\sin\theta - Nl\cos\theta = I\theta$$

Combine last two expressions

$$(I+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta$$

$$\theta = \pi + \phi$$

Let ϕ represent the deviation of the pendulum

$$\cos\theta = \cos(\pi+\phi) \approx -1$$

$$\sin\theta = \sin(\pi+\phi) = -\phi$$

$$\ddot{\theta}^2 = \phi^2 \approx 0$$

Substitude

$$(I+ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x}$$

$$(M+m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u$$

To obtain the tf, first take the Laplace transform

$$(I+ml^2)\phi(s)s^2 - mgl\phi(s) = mlX(s)s^2$$

$$(M+m)X(s)s^2 + bX(s)s - ml\phi(s)s^2 = U(s)$$

Solve X(S)

$$X(s) = \left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)$$

Substitute in the second equation

$$(M+m)\left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)s^2 + b\left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)s - ml\,\phi(s)s^2 = U(s)$$

$$\phi(s) = \frac{ml}{q}s^2$$

$$U(s) = s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s$$

where $q = [(M+m)(I+ml^2) - (ml)^2]$

There is a pole and zero, these can be cancelled

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}}$$

Now we can derived in a similar manner

$$X(s) = \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s}$$

```
M = 0.228;
m = 0.091;
b = 0.1;
I = 0.006;
g = 9.81;
l = 0.24;
q = (M+m)*(I+m*l^2)-(m*l)^2;
s = tf('s');
us = (((I+m*l^2)/q)*s^2 - (m*g*l/q));
ys = (s^4 + ((b*(I + m*l^2))*s^3/q) - (((M + m)*m*g*l)*s^2/q)
- (b*m*g*l*s/q));
ys = s^4 + (.3617*s^3) - (21.98*s^2) - (6.89184*s);
system1 = us/ys;
A = [0 1 0 0;0 0 1 0; 0 0 0 1; -.0317 21.98 6.892 0];
B = [0 0 0 1]';
C = [-68.91 0 3.616 0];
Areq = [0 1 0 0;0 0 1 0; 0 0 0 1; -120 -154 -71 -14];
poly([-5 -3 -2 -4])
g=system1;
```

```
system1 =

              3.616 s^2 - 68.91
  --------------------------------------
  s^4 + 0.3617 s^3 - 21.98 s^2 - 6.892 s

Continuous-time transfer function.
```

State Feedback – Step unit = c = 26

```
>> A

A =

        0    1.0000         0         0
        0         0    1.0000         0
        0         0         0    1.0000
  -0.0317   21.9800    6.8920         0

>> B

B =

     0
     0
     0
     1

>> C

C =

  -68.9100         0    3.6160         0
```

$$\frac{3.616\, s^2 - 68.01}{s^4 + .0317\, s^3 - 21.98\, s^2 - 6.892\, s}$$

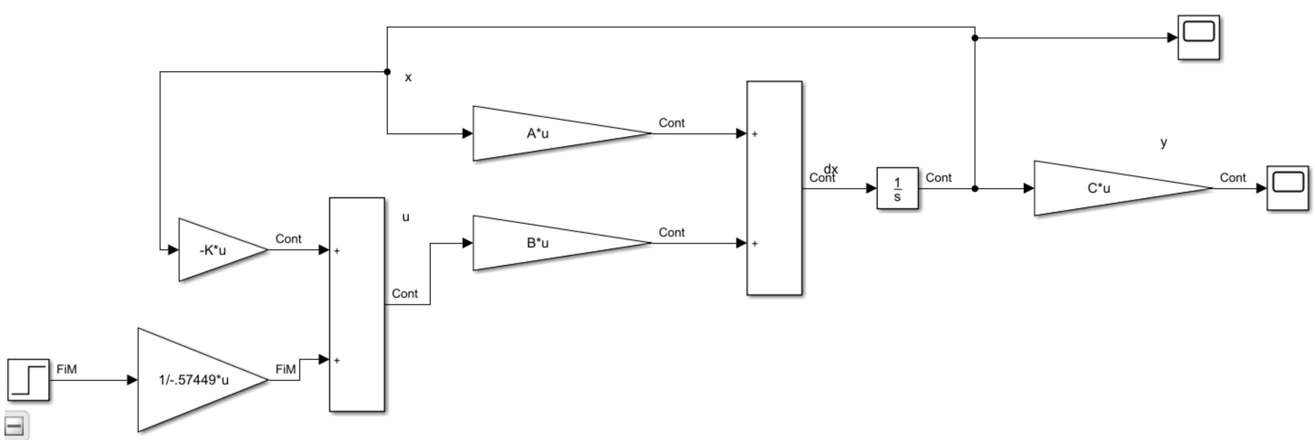
$-.0317$	$-K_1 = -120$		119.9683
$+21.98$	$-K_2 = -154$		175.98
6.892	$-K_3 = -71$		77.892
0	$-K_4 = -14$		14

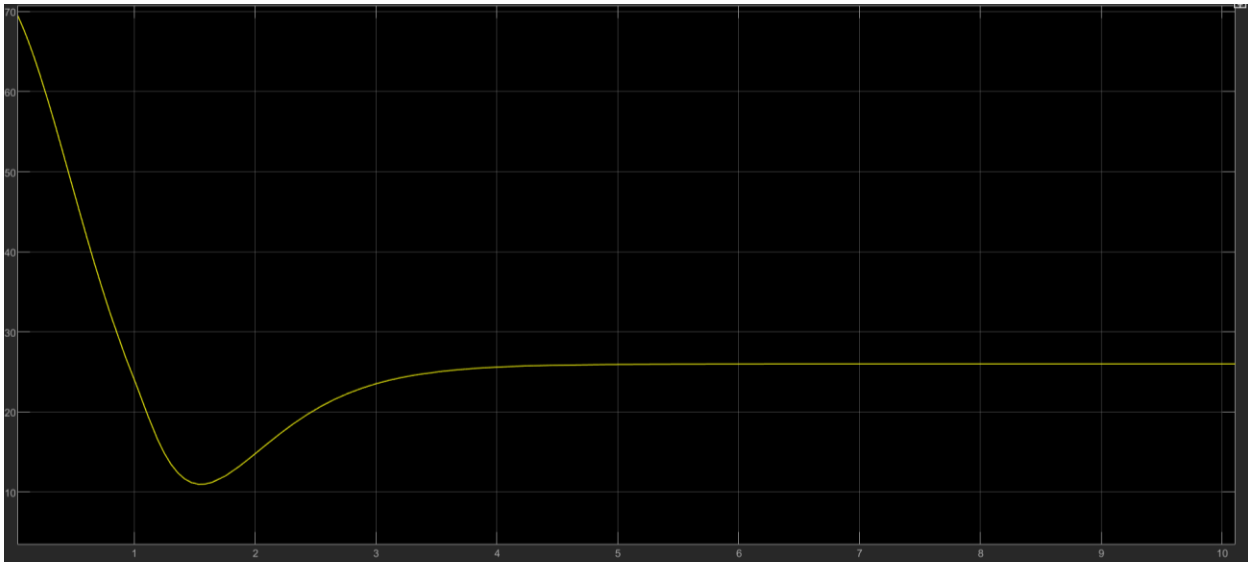
$(A - B * K) = A_{req}$

```
K =

   110   153    69    11
```

Areq = (A – B *K)

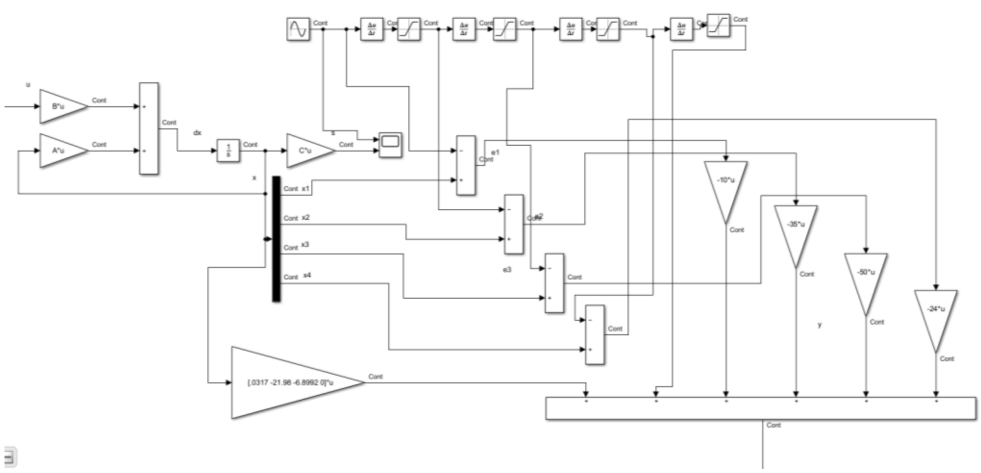
```
>> Areq

Areq =

     0     1     0     0
     0     0     1     0
     0     0     0     1
  -120  -154   -71   -14

>> A-B*K

ans =

     0     1     0     0
     0     0     1     0
     0     0     0     1
  -120  -154   -71   -14
```

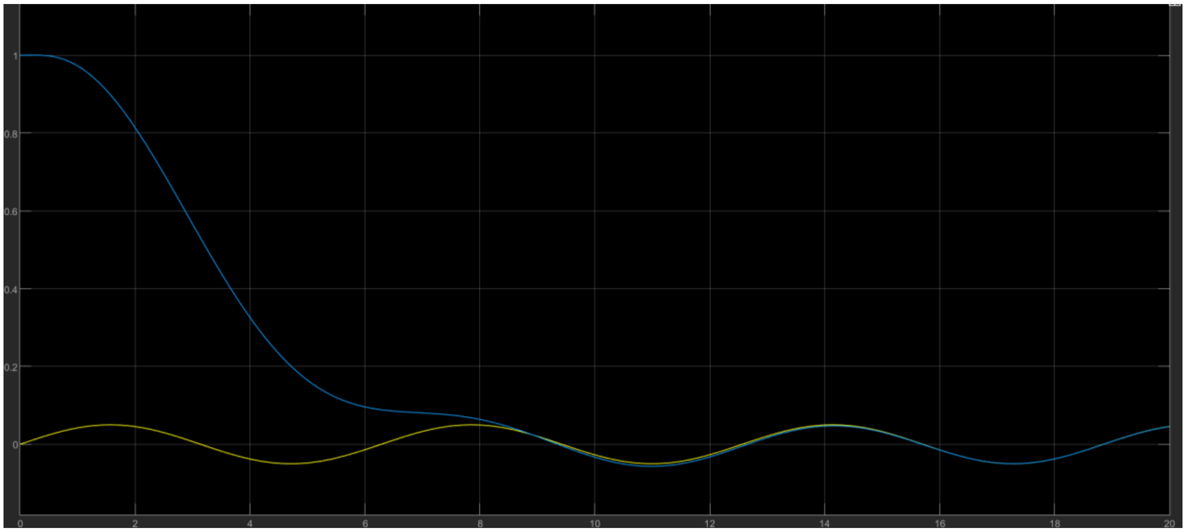




State Feedback r(t) = 0.05sin (t)

u = .3617x1 + -2198x2 - 68992x3 - 0x4 - cos(t)
- 10e1 - 35e2 - 50e3 - 24e4

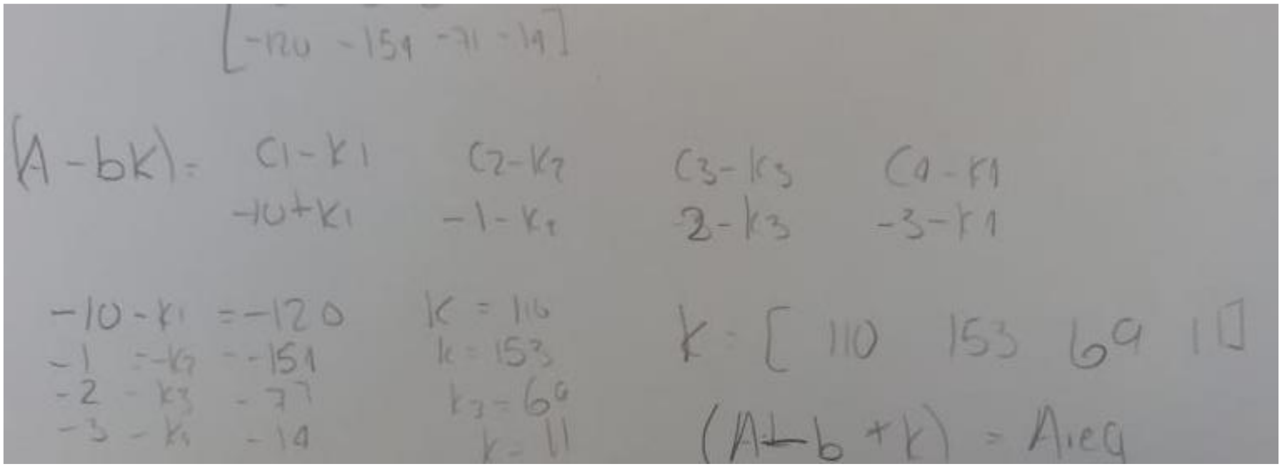




System 2 – Transfer fuction

State Feedback - r(t) = c = 26

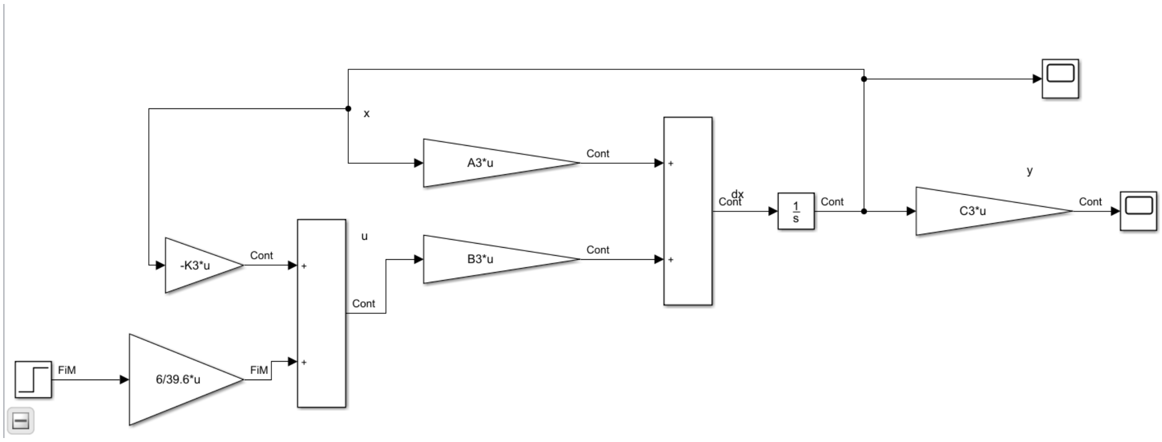
```
G3 =

          4 s + 39.6
  --------------------------
  s^3 + 36 s^2 + 81 s - 4374

Continuous-time transfer function.

>> A3

A3 =

           0            1            0
           0            0            1
        4374          -81          -36

>> B3

B3 =

     0
     0
     1
```

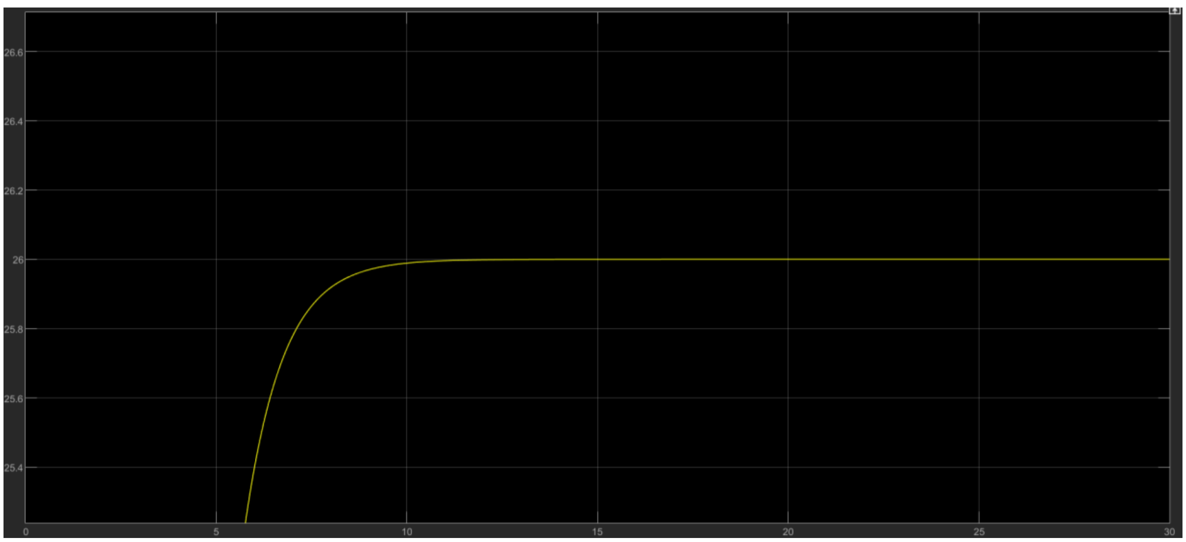



```
K =

   110   153    69    11
```

Areq3 = (A3 – B3 *K3)

```
C3 =

   39.6000    4.0000         0

>> Areq3

Areq3 =

     0     1     0
     0     0     1
    -6   -11    -6

>> A3-B3*K3

ans =

     0     1     0
     0     0     1
    -6   -11    -6
```

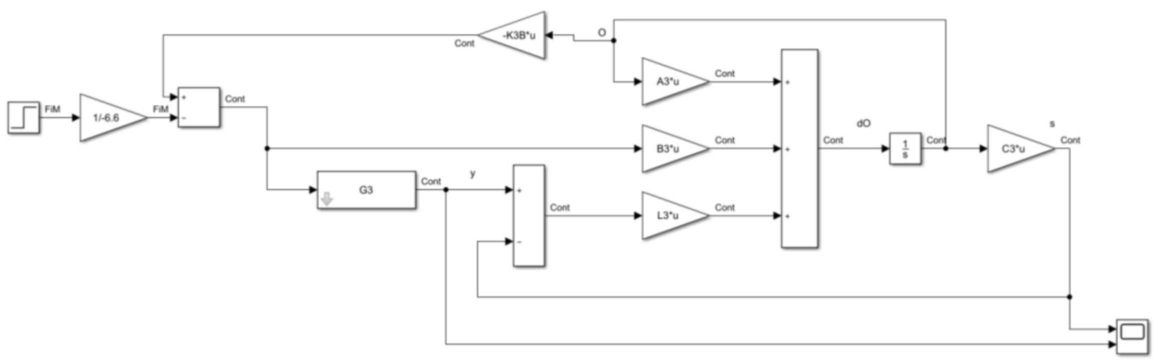


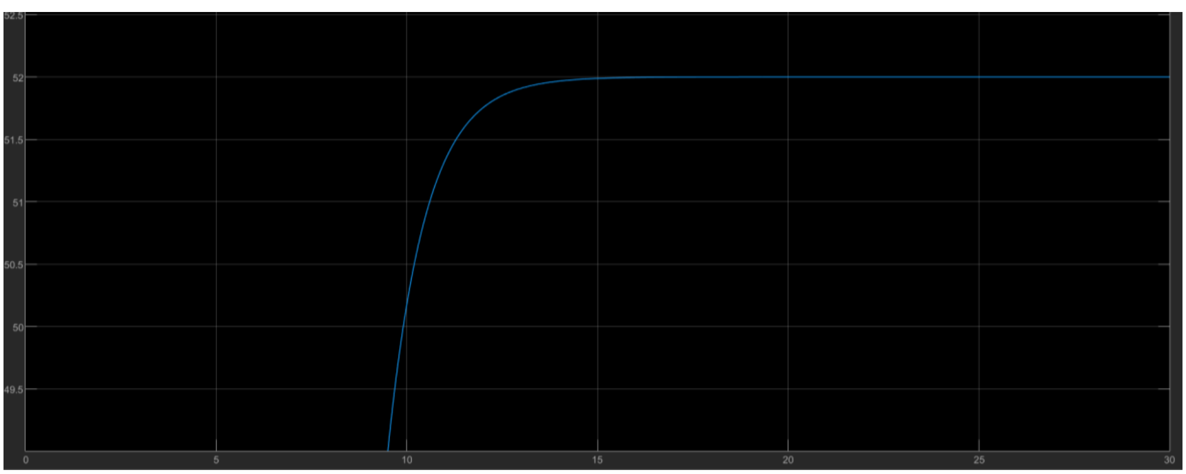


State Feedback with observer – r(t) = 2c = 52

```
G3 =

          4 s + 39.6
  --------------------------
  s^3 + 36 s^2 + 81 s - 4374

Continuous-time transfer function.

>> A3

A3 =

           0           1           0
           0           0           1
        4374         -81         -36

>> B3

B3 =

     0
     0
     1
```

```
C3 =

   39.6000    4.0000         0

>> L3

L3 =

    0.2500
   -3.2252
  129.1797

>> -K3B

ans =

       -4380          70          30
```





State Feedback r(t) = sin (t)

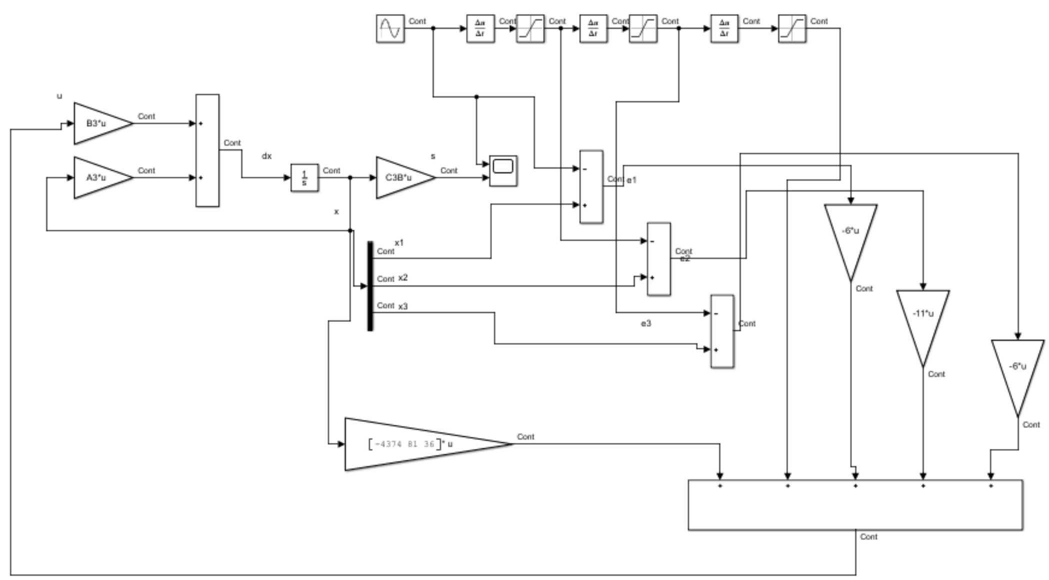
Poly = [-1 -2 -3]
ans = 1 6 11 6

$e_1 = y - r \qquad \dot{e}_1 = x$

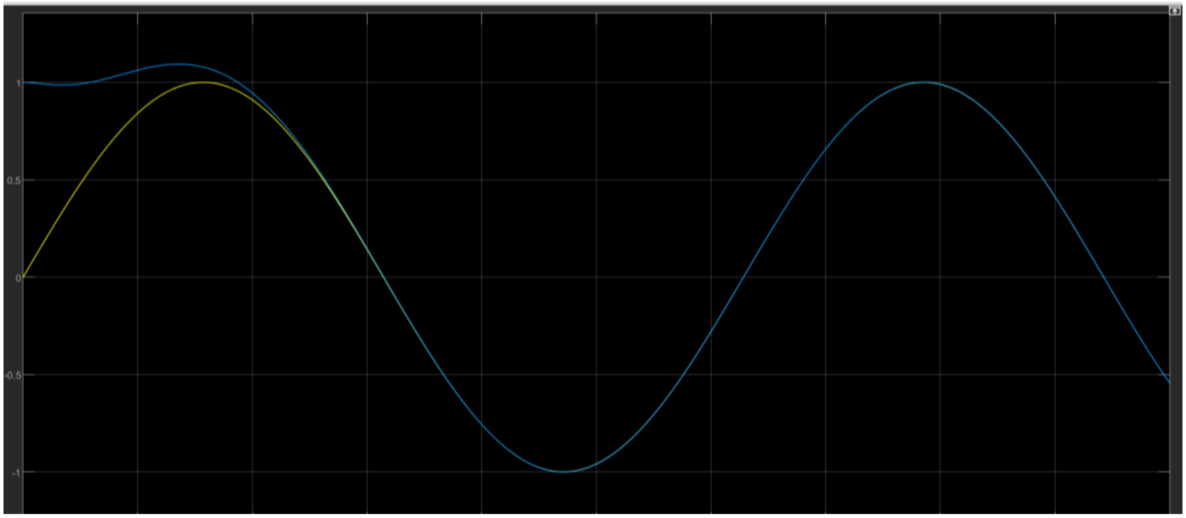
$\dot{e}_3 = +4374\, x_1 - 81 x_2 - 36 x_3 + u + \cos(t)$

$$\dot{e} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} e + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$u = -4374\, x_1 + 81 x_2 + 36 x_3 - \cos(t) - 6 e_1 - 11 e_2 - 6 e_3$



Amplitude = 1



Amplitude 0 .05