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Ejercicio 1.

1) Si $X \sim Ga(x|a,b) \Rightarrow Y \sim InvGa(y|a,b)$ + $Y = 1/X$

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \qquad x = 1/y$$

$$f_1(y) = f_x(1/y) \left| \frac{d}{dy} y^{-1} \right|$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{1}{y}\right)^{\alpha-1} e^{-\beta/y} \; y^{-2}$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{1}{y}\right)^{\alpha+1} e^{-\beta/y} = \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{1}{y}\right)^{\alpha+1} e^{-\beta/y}$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} y^{-\alpha-1} e^{-\beta/y} = f(y|\alpha,\beta) \sim IGa(y|\alpha,\beta).$$

2) $$\mathcal{E}(y) = \int_0^\infty y f(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty y\, y^{-\alpha+1} e^{-\beta/y} dy$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty y^{-\alpha} e^{-\beta/y} dy$$

no tar que
$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha-1)}{\beta^{\alpha-1}} = \frac{\beta \cancel{\Gamma(\alpha-1)}}{\alpha-1 \cancel{\Gamma(\alpha-1)}} = \frac{\beta}{\alpha-1} = \text{media}$$

$$\mathcal{E}(y^2) = \int_0^\infty y^2 f(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty y^{-\alpha+1} e^{-\frac{\beta}{y}} dy$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha-2)}{\beta^{\alpha-2}} = \frac{\beta^2 \cancel{\Gamma(\alpha-2)}}{(\alpha-1)(\alpha-2)\cancel{\Gamma(\alpha-2)}} = \frac{\beta^2}{(\alpha-1)(\alpha-2)}$$

$$Var(y) = \mathcal{E}(x^2) - \mathcal{E}(x) = \frac{\beta^2}{(\alpha-1)(\alpha-2)} - \frac{\beta^2}{(\alpha-1)^2}$$

$$\frac{\beta^2}{(\alpha-1)(\alpha-2)} - \frac{\beta^2}{(\alpha-1)^2} = \frac{\beta^2}{(\alpha-1)}\left\{\frac{1}{(\alpha-2)} - \frac{1}{(\alpha-1)}\right\} = \frac{\alpha-1-\alpha+2}{(\alpha-2)(\alpha-1)}$$

$$= \frac{\beta^2}{(\alpha-1)}\left\{\frac{1}{(\alpha-2)(\alpha-1)}\right\} = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)} = \text{Var}(y)$$

3) Sea $\sigma = 14$

y $\mu = 52.8$

$$\text{entonces } \frac{b}{a-1} = 52.8 \Rightarrow b = 52.8(a-1)$$

y $$\frac{b^2}{(a-1)^2(a-2)} = 14^2 \Rightarrow \frac{52.8^2(a-1)^2}{(a-1)^2(a-2)} = 196$$

$$\Rightarrow \frac{52.8^2}{196} = a-2 \Rightarrow 14.22 = a-2$$

$$a = 16.22$$

$$\Rightarrow b = 52.8(15.22)$$

$$\Rightarrow b = 803.616$$

$$a_\beta = 16.22$$

$$b_\beta = 803.616$$