Kirchhoff's Laws

Kirchhoff's laws are particularly useful

i. In determining the equivalent resistance of a complicated network of conductors
ii. For calculating the current flowing in various conductors.

- **Kirchhoff's First Law**

In any electrical network, the algebraic sum of the current meeting at point (or junction) is zero. It simply means that the total current leaving a junction is equal to the total current entering the junction.

Diagram A

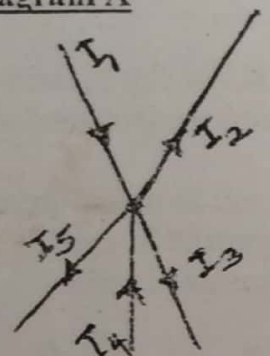
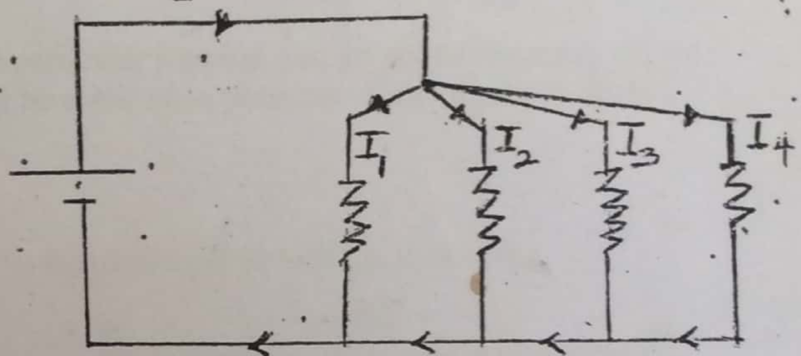


Diagram B



$I_1 + I_4 = I_2 + I_3 + I_5,$

$I_1 - I_2 - I_3 + I_4 - I_5 = 0$

$I = I_1 + I_2 + I_3 + I_4$

$I - I_1 - I_2 - I_3 - I_4 = 0$

- **Kirchhoff's Second Law (Mesh or Voltage Law)**

The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (mesh) in a network plus the algebraic sum of the emfs in that part is zero.

$$\sum IR + \sum emf = 0$$

Note: algebraic sum takes into account the polarities of the voltage drops.

Diagram A:

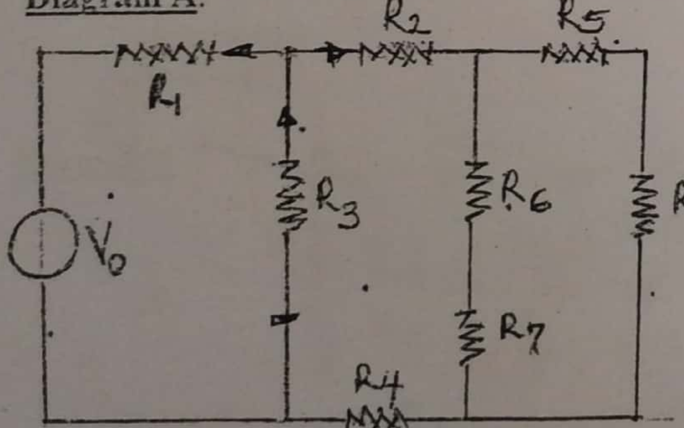
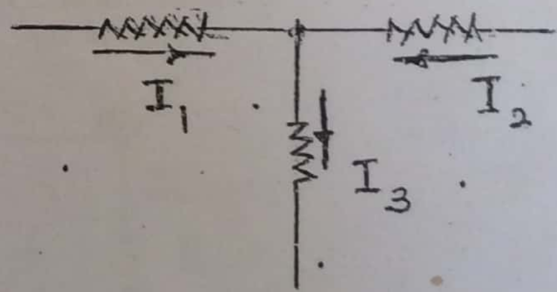


Diagram B

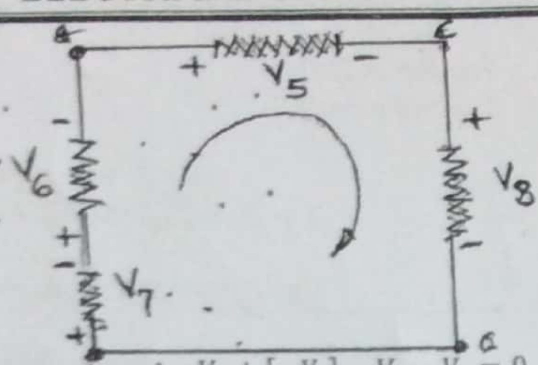


Sum current IN $= I_1 + I_2,$ sum current OUT $= I_3$

Therefore, $I_1 + I_2 = I_3$

$= +V_5 + V_6 + V_7 + V_8 = 0.$

Diagram C



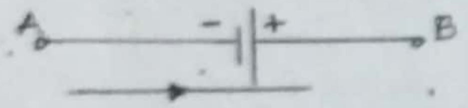
$-V_5 + [-V_8] - V_7 - V_6 = 0$

The basis of this law is that if we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started.

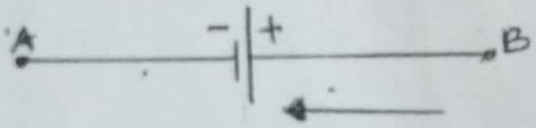
Determination of the voltage sign

i. **Sign of battery emf**

A rise in voltage should be given a +ve sign and a fall in voltage a −ve sign.

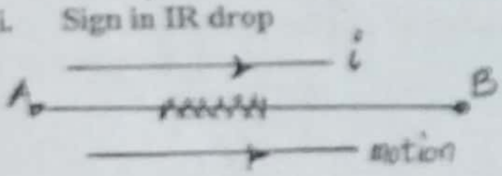


Rise in voltage
+E

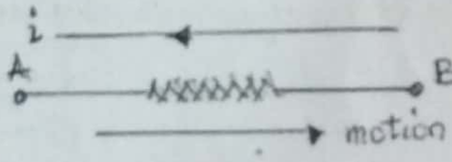


Fall in voltage
-E

ii. **Sign in IR drop**

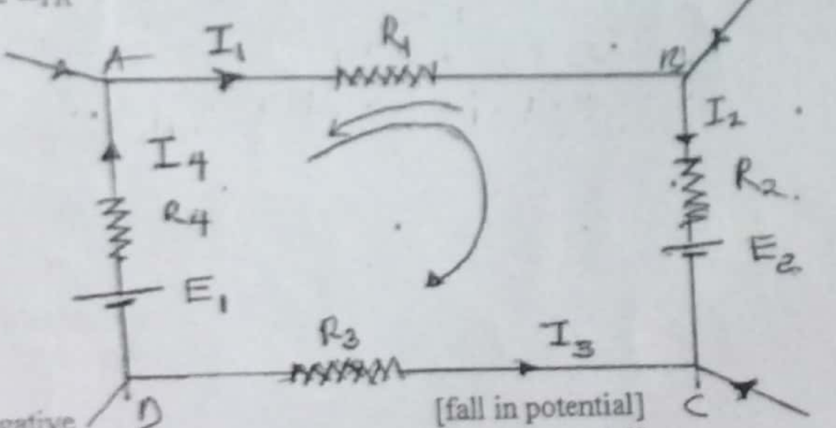


Fall in voltage
$-V = -IR$



Rise in voltage
$+V = +IR$

Example 1



I_1R_1 is negative [fall in potential]
I_2R_2 is negative [fall in potential]
I_3R_3 is positive [rise in potential]
I_4R_4 is negative [fall in potential]

$BADCB = I_1R_1 + I_4R_4 - E_1 - I_3R_3 + E_2 + E$
$= 0.$

$BADCB = I_1R_1 + I_4R_4 + I_2R_2 - I_3R_3 - E_1 + E_2 = 0$

$= I_1R_1 + I_4R_4 + I_2R_2 - I_3R_3 - E_2 + E_1 = 0.$

E_2 is negative [fall in voltage]
E_1 is positive [rise in voltage]

Using Kirchhoff's voltage law

$$-I_1R_1 - I_2R_2 + I_3R_3 - I_4R_4 - E_2 + E_1$$

$$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$

Assumed Direction of Current
If the assumed direction of current is not the actual direction, then on solving the question, this current will be found to have a minus sign.

Solving the Simultaneous Equation
Solving two unknown

$$ax + by = c$$
$$dx + ey = f$$

i. Write the two equations in the matrix form as

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$$

ii. Find the common determinant

$$D = \begin{bmatrix} a & b \\ d & e \end{bmatrix}$$

$$D = [ae - bd]$$

iii. Find the determinant of x by replacing the coefficient of x in the original matrix by the constants

$$D_x = \begin{bmatrix} c & b \\ f & e \end{bmatrix}$$

$$D_x = [ce - bf]$$

iv. Find the determinant of y by the same method

$$D_y = \begin{bmatrix} a & c \\ d & f \end{bmatrix}$$

$$D_y = [af - cd]$$

v. Apply Cramer's rule

$$x = \frac{D_x}{D}$$
$$y = \frac{D_y}{D}$$

Solving three unknown

$$ax + by + cz = d$$
$$ex + fy + gz = h$$
$$jx + ky + lz = m$$

i. Put in matrix form

$$\begin{bmatrix} a & b & c \\ e & f & g \\ j & k & l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \\ h \\ m \end{bmatrix}$$

ii. Find the common determinant

$$D = \begin{bmatrix} a & b & c \\ e & f & g \\ j & k & l \end{bmatrix}$$

$$D = a[fl - gk] - e[bl - ck] + j[bg - cf]$$

iii. Find determinant of x

$$D_x = \begin{bmatrix} d & b & c \\ h & f & g \\ m & k & l \end{bmatrix}$$

$$D_x = d[fl - gk] - h[bl - ck] + m[bg - cf]$$

iv. Similarly

$$D_y = \begin{bmatrix} a & d & c \\ e & h & g \\ j & m & l \end{bmatrix}$$

$$D_y = a[hl - mg] - e[dl - mc] + j[dg - hc]$$

$$D_z = \begin{bmatrix} a & b & d \\ e & f & h \\ j & k & m \end{bmatrix}$$

$$D_z = a[fm - hk] - e[bm - dk] + j[bh - df]$$

v. Using Cramer's rule

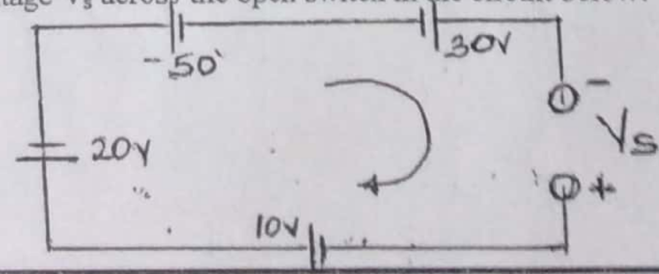
$$x = \frac{D_x}{D}$$

$$y = \frac{D_y}{D}$$

$$z = \frac{D_z}{D}$$

Example 2

What is the voltage V_s across the open switch in the circuit below.



$-30+50+20-10 = 30v$

$V_s = 30v.$

$V_c = +30V$

$V_1 = -50V$

$V_s = +10V$

$-50+30+20-20$

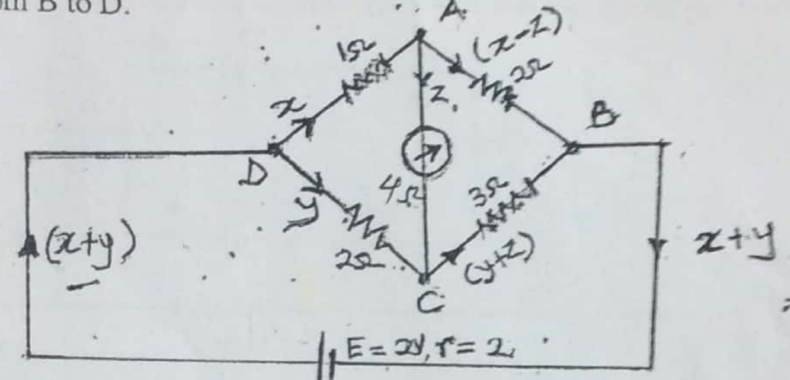
$=$

Soln.

$$+V_s + 10v - 20v - 50v + 30v = 0$$
$$V_s = 30v$$

Example 3

Determine the current in the unbalanced bridge circuit. Also determine the p.d. across BD and the resistance from B to D.



Soln.

Assuming current direction as shown above, and applying the second law to **DACD**

$$[-1 * x) + [-4z] + 2y = 0$$
$$x - 2y + 4z = 0 \quad \text{.................[1]}$$

ABCA

$$[-2 * (x - z)] + 3(y + z) + 4z = 0$$
$$2x - 3y - 9z = 0 \quad \text{.................[2]}$$

DABED DABED

$$[-x * 1] + [-2(x - z)] - 2(x + y) + 2 = 0$$
$$5x + 2y - 2z = 2 \quad \text{.................[3]}$$

Using Cramer's rule

$$x = \frac{30}{91}, \qquad y = \frac{17}{91}, \qquad z = \frac{1}{91}$$

And

$$AB\ current = \frac{29}{91}\text{A}, \quad CB\ current = \frac{18}{91}\text{A}, \quad external\ current = \frac{47}{91}\text{A}$$

Internal voltage drop = Ir = Voltage lost i.e x+y × V

$$\frac{17}{91} + \frac{30}{91} \times 2 = \frac{47}{91} * 2 =$$

$$= \frac{94}{91}\text{V} = 1.033V$$

P.d. across point D and B

$$= 2 - \frac{94}{91} = 2 - 1.033$$

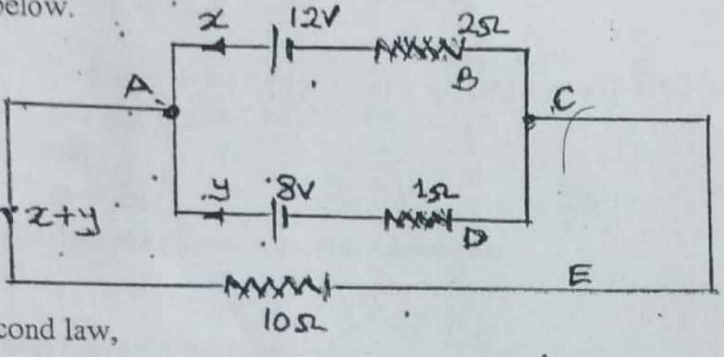
$$= \frac{88}{91}V = 0.967V$$

$2x-y=4$
$10x+11y=8$

$$Equivalent\ resistance = \frac{p.d.\ between\ B\ and\ D}{current\ between\ B\ and\ D}$$
$$= 1.87\Omega$$

Example 3

Determine the current flowing in the external resistance and P.d. across the external resistance in the network below.



Soln.

Applying second law,

ABCDA

$$-12v + 2x - (1 * y) + 8 = 0$$
$$2x - y = 4v \qquad \text{..............[1]}$$

ADCEA

$$-8v + (1 * y) + 10(x + y) = 0$$
$$10x + 11y = 8v. \qquad \text{..............[2]}$$

Using Cramer's rule

$$x = 1.625A, \qquad y = -0.75A$$

The –ve sign indicates that the current is flowing into 8V battery, charging it.

Current flowing in the external resistance $= x + y$

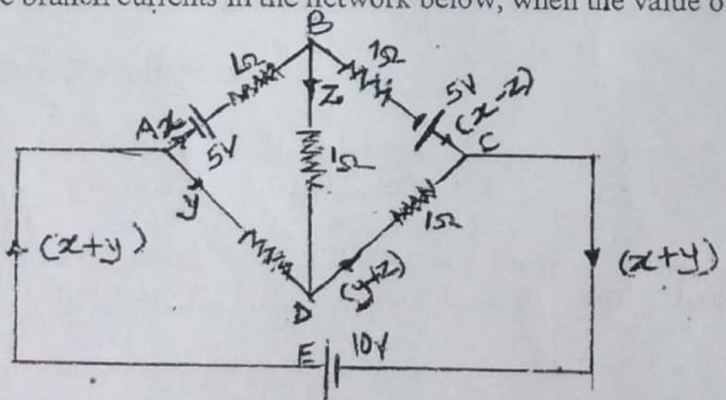
$$= 1.625 - 0.75$$
$$= 0.875A$$

The P.d. across the external resistance is

$$= 10 * 0.875$$
$$= 8.75V$$

Example 4

Determine the branch currents in the network below, when the value of each branch resistance is 1Ω.



ABDA= $-5V + (1\times x) + (1\times z) - y$

$x - z + y = 5$ --- (1)

BCDB

$5+(x-z)+(y+z)+z$

$5+x-z+y+z+z$

$5+x-3z+y$

Soln.

Applying the second law

ABDA

$$5v + (-x * 1) + (-z * 1) + (y * 1) = 0$$
$$x - y + z = 5$$

BCDB

$$[-1 * (x - z)] + 5v + [1 * (x + y)] + (1 * z) = 0$$
$$x - y - 3z = 5$$

ADCEA

$$[-1 * y] + [-1 * (x + z)] + [-1 * (x + y)] + 10 = 0$$
$$x + 3y + z = 10$$

Using Cramer's rule

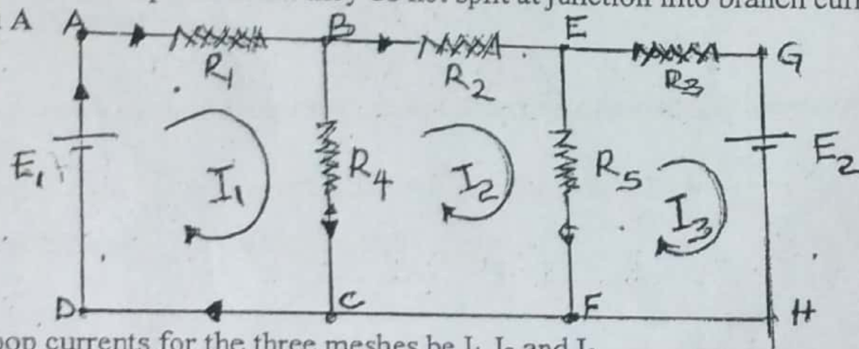
$$z = 0A, \qquad y = \frac{5}{4}A, \qquad x = \frac{25}{4}A$$

student should find the currents in other branches

Maxwell's Loop Current Method

This method which is particularly well suited to coupled circuit solutions employs a system of loop or mesh currents instead of branch currents. Here the currents in different meshes are assigned continuous path so that they do not split at junction into branch current.

Diagram A



Let the loop currents for the three meshes be I_1, I_2 and I_3.

When R_4 is considered as part of first loop, the current through it is given as $[I_1-I_2]$.

When R_5 is considered as part of second loop, the current through it is given as $[I_2-I_3]$.

However,

When R_4 is considered as part of the second loop, the current is given as $[I_2-I_1]$.

When R_5 is considered as part of the third loop, the current is given as $[I_3-I_2]$.

Applying Kirchhoff's voltage law

Loop 1

$$E_1 - I_1R_1 - R_4[I_1 - I_2] = 0$$
$$I_1[R_1 + R_4] - I_2R_4 - E_1 = 0$$

Loop 2

$$-I_2R_2 - R_5[I_2 - I_3] - R_4[I_2 - I_1] = 0$$
$$I_2R_4 - I_2[R_2 + R_4 + R_5] + I_3R_5 = 0 \Rightarrow I_1R_4 - I_2[R_2 + R_4 + R_5] + I_3R_5 = 0$$

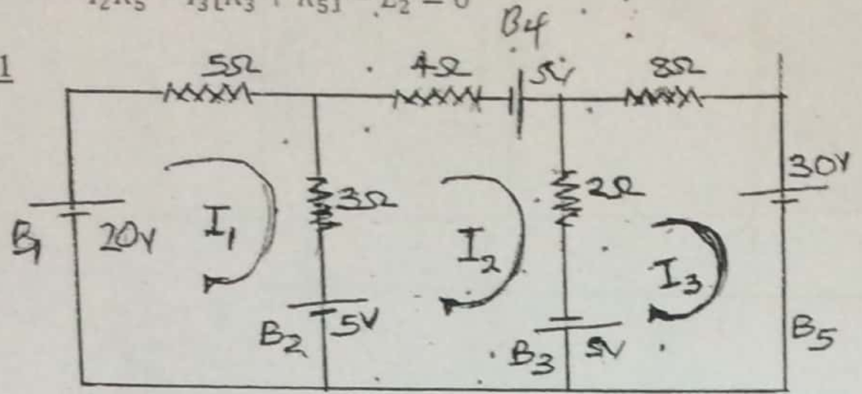
Loop 3

$$-I_3R_3 - E_2 - R_5[I_3 - I_2] = 0$$

$$I_2R_5 - I_3[R_5 + R_3] - E_2 = 0$$

$$I_2R_5 - I_3[R_3 + R_5] - E_2 = 0$$

Example 1



$B_2 = I_1 - I_2$
$B_3 = I_2 - I_3$ Sinc
B_3 is -ve, $B_2 =$
$I_2 + I_3$
$B_4 = I_2$
$B_5 = I_3$

Loop 1

$$20v - 5I_1 - 3[I_1 - I_2] - 5 = 0 \quad = I_1[5+3] - 3I_2 = 20V - 5V$$
$$8I_1 - 3I_2 = 15 \quad \text{.................[1]} \quad = 8I_1 - 3I_2 = 15V$$

Loop 2

$$-4I_2 + 5 - 2[I_2 - I_3] + 5 + 5 - 3[I_2 - I_1] = 0$$
$$3I_1 - 9I_2 + 2I_3 = -15 \quad \text{.................[2]}$$

Loop 3

$$-8I_3 - 30 - 5 - 2[I_3 - I_2] = 0$$
$$2I_2 - 10I_3 = 35 \quad \text{...................[3]}$$

Applying Cramer's rule,

$$I_2 = \frac{545}{299}A, \quad I_3 = \frac{-1875}{598}A, \quad I_1 = \frac{765}{299}A,$$

Since I_3 turns out to be negative, actual direction of current is opposite to the direction taken

Current in $B_1 = \frac{765}{299}A$, Current in $B_2 = \frac{220}{299}A$, Current in $B_3 = \frac{2965}{598}A$
Current in $B_4 = \frac{545}{299}A$, Current in $B_5 = \frac{1875}{598}A$

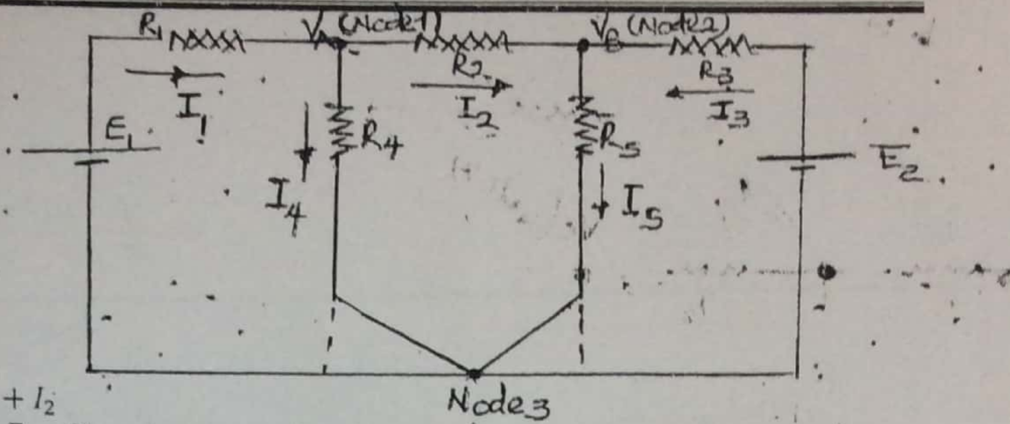
Nodal Analysis with Sources

The node equation is based directly on Kirchhoff's current law unlike loop-current method which is based on Kirchhoff's voltage law. However, like loop current method, nodal method also has the advantage that a minimum number of equation need be written to determine the unknown quantities. Moreover, it is particularly suited for networks having many parallel circuits with common ground connected such as electronic circuits.

For the application of this method, every junction in the network where three or more branches meet is regarded as node. One of these is regarded as the reference node or datum node or potential node. These node equations often become simplified if all voltages sources are converted into current sources.

First Case

V_a represents the potential of node1 with reference to node 3
V_b is the potential difference between node2 and node3
Note: current direction is arbitrarily choosen



For node 1

$$I_1 = I_4 + I_2$$
$$I_1R_1 = E_1 - V_A$$
$$I_1 = \frac{[E_1 - V_A]}{R_1}$$

Obviously,

$$I_4 = {}^{V_A}/_{R_4}$$

$$I_2R_2 = V_A - V_B$$
$$I_2 = {}^{[V_A - V_B]}/_{R_2}$$

Substituting

$$[E_1 - V_A]/R_1 = {}^{V_A}/_{R_4} + \frac{[V_A - V_B]}{R_2}$$

Simplifying,

$$V_A\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right] - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0 \qquad \text{..................[i]}$$

For node 2

$$I_5 = I_3 + I_2$$

$$I_5R_5 = V_B$$

$$I_5 = {}^{V_B}/_{R_5}$$

$$I_3R_3 = E_2 - V_B$$

$$I_3 = {}^{[E_2 - V_B]}/_{R_3}$$

$$I_2R_2 = V_A - V_B$$

Substituting

$$[V_B]/R_5 = {}^{[V_A - V_B]}/_{R_2} + \frac{[E_2 - V_B]}{R_3}$$

Simplifying,

$$V_B\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right] - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0 \qquad \text{................[ii]}$$

Similarly

$$V_A\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_8}\right]-\frac{V_C}{R_2}-\frac{V_B}{R_8}-\frac{E_1}{R_1}=0 \quad \text{..................... [i]}$$

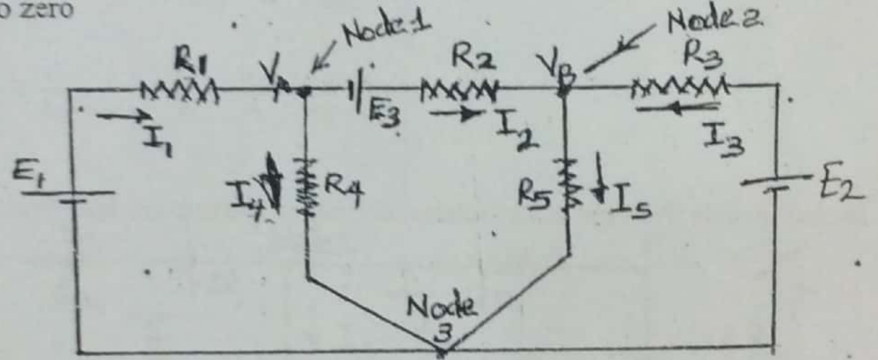
$$V_C\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right]-\frac{V_B}{R_3}-\frac{V_A}{R_2}=0 \quad \text{..................... [ii]}$$

$$V_B\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_7}+\frac{1}{R_8}\right]-\frac{V_C}{R_3}-\frac{V_A}{R_8}-\frac{E_2}{R_4}=0 \quad \text{..................... [iii]}$$

Equation at the node is represented by

i. the product of node potential and the sum of branch resistances connected to the node
ii. minus the ratio of adjacent potential and interconnecting resistances
iii. minus the ratio of adjacent battery(or generator) voltage and interconnecting resistance
iv. all above set to zero

Second Case



Note:

As we travel from node 1 to node 2, we go from the –ve terminal of E_3 to its positive terminal, E_3 must be taken as positive. However, if we travel from node 2 to node1, E_3 is taken as negative.

For node 1

$$I_1 = I_4 + I_2$$
$$I_1 - I_2 - I_4 = 0$$

$$I_1 = \frac{[E_1 - V_A]}{R_1}$$

$I_4R_4 = V_A$

Obviously,

$$I_4 = {V_A}/{R_4}$$

$$I_2R_2 = V_A + E_3 - V_B$$

$$I_2 = {[V_A + E_3 - V_B]}/{R_2}$$

~~$I_1R_1 = E_1 - V_A$~~

~~$I_1 = \frac{E_1 - V_A}{R_1}$~~

~~$I_4R_4 = V_A$~~

~~$I_4 = \frac{V_A}{R_4}$~~

~~$I_2R_2 = V_A - V_B + E_3$~~

~~$I_2 = \frac{V_A + E_3 - V_B}{R_2}$~~

Substituting

$$[E_1 - V_A]/R_1 = {V_A}/{R_4} + \frac{[V_A + E_3 - V_B]}{R_2}$$

Simplifying,

$$V_A\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right]-\frac{V_B}{R_2}-\frac{E_1}{R_1}+\frac{E_3}{R_2}=0$$

using gramers rule

$$\begin{bmatrix}7 & -2\\ 2 & -4\end{bmatrix}=\begin{bmatrix}6\\ 4\end{bmatrix}$$

For node 2

$$I_5 = I_3 + I_2$$

$$I_2 + I_3 - I_5 = 0$$

$$I_2 = \frac{[V_A+E_3-V_B]}{R_2}$$

$$I_5 = V_B/R_5$$

Substituting

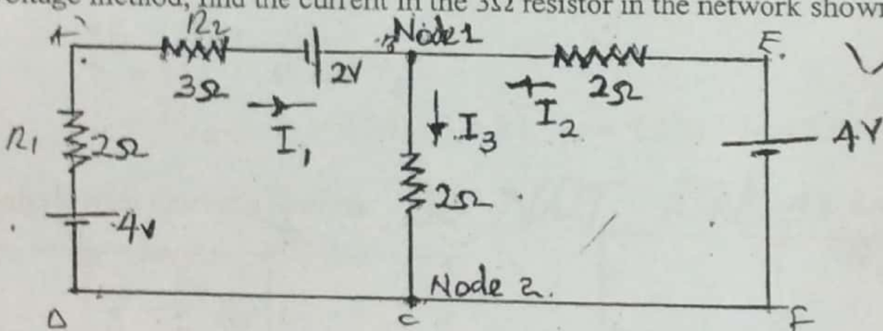
$$[V_B]/R_5 = [E_2-V_B]/R_3 + \frac{[V_A+E_3-V_B]}{R_2}$$

Simplifying,

$$V_B\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right]-\frac{E_1}{R_1}-\frac{V_A}{R_2}-\frac{E_2}{R_2}=0$$

Example 1

Using node voltage method, find the current in the 3Ω resistor in the network shown below.



$V = V_1 + V_2$
$= 2+4 = 6$

$R = R_1 + R_2$
$= 2+3 = 5Ω$

$$V_1\left[\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right]-\frac{4}{2}-\left[\frac{4+2}{5}\right]=0$$

~~ABCDA = 4 - 2x5~~ - LCM = 10

Note:
The 3Ω and 2Ω are connected in series
The 2V and 4V are also additive

$$V_1 = 8/3\,V$$

$V_1\left(\frac{12}{10}\right) = \frac{6}{5} + \frac{4}{2} = \frac{12+20}{10}$

$V_1\,\frac{12}{10} = \frac{32}{10}$

The current through 3Ω resistance towards node1 is

$$= \frac{\left[6-\left(\frac{8}{3}\right)\right]}{3+2}$$

$= \frac{32}{10} \times \frac{10}{12} = 8/3$

$$= 2/3\,A$$

$V_1 = 8/3$

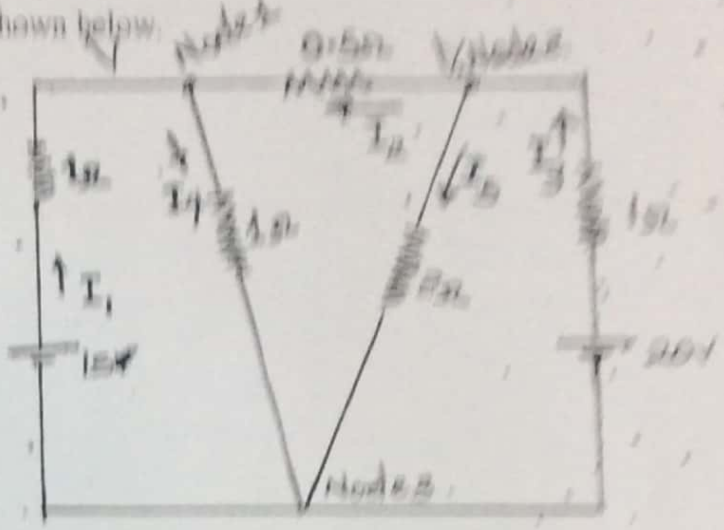
$IR = 6V - 8/3\,V$

$\frac{2+5+5}{10} = \frac{12}{10}$

$I = \frac{6V - 8/3\,V}{5} = 2/3\,A$.

Example 2

Frame and solve the node equations of the network shown below.



Node 1

$$V_1\left[\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_2}\right]-\frac{V_2}{R_2}-\frac{15V}{R_1}=0$$

$$V_1\left[1+1+\frac{1}{0.5}\right]-\frac{V_2}{0.5}-\frac{15V}{1}=0$$

$$4V_1-2V_2=15$$

$$V_1\left[1+1+\frac{1}{0.5}\right]-\frac{V_2}{0.5}-\frac{15V}{1}=0$$

Node2

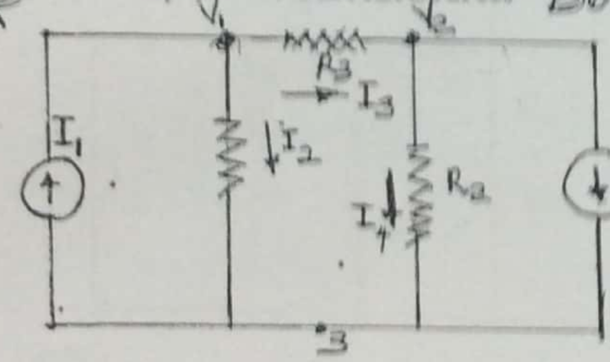
$$V_2\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right]-\frac{V_1}{R_2}-\frac{20V}{1}=0$$

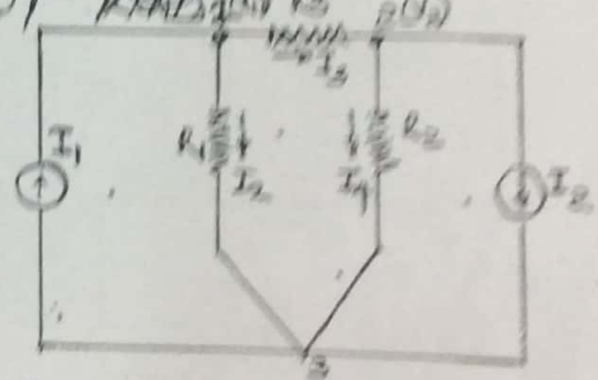
$$4V_1-7V_2=-40$$

$$V_2=11,\qquad V_1={}^{37}/_{4}$$

$$I_1={}^{23}/_{4}A,\ I_2=3.5A,\ I_3=9A,\ I_4=9.25A,\ I_5=5.5A$$

Nodal Analysis with Current Sources DO NOT READ





Consider the network which has two current sources and three nodes out of which 1 and 2 are independent ones.

i. Both V_1 and V_2 are positive with respect to reference node. That is why their current flow into node 3

ii. V_1 is positive with respect to V_2 because current has been shown flowing from node 1 to node 2.

Positive result will confirm the assumption whereas negative one will indicate that actual direction is opposite.

Node1

$$I_1 - I_2 - I_3 = 0$$
$$I_1 = I_2 + I_3$$
$$I_2 = {V_1}/{R_1},\ I_3 = {[V_1 - V_2]}/{R_3}$$
$$I_1 = \frac{V_1}{R_1} + {[V_1 - V_2]}/{R_3}$$
$$I_1 = V_1\left[\frac{1}{R_1} + \frac{1}{R_3}\right] - \frac{V_2}{R_3} \qquad \text{.............. [1]}$$

Node 2

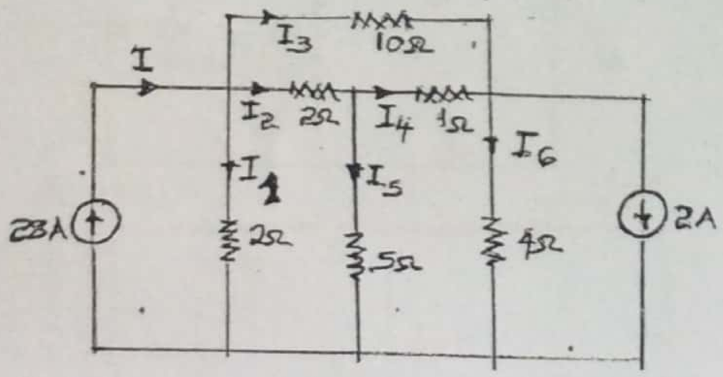
$$I_3 - I_2 - I_4 = 0$$
$$I_3 = I_2 + I_4$$
$$I_4 = \frac{V_2}{R_2},\quad I_3 = \frac{[V_1 - V_2]}{R_3},$$
$$\frac{[V_1 - V_2]}{R_3} = I_2 + \frac{V_2}{R_2}$$
$$V_2\left[\frac{1}{R_3} + \frac{1}{R_2}\right] - \frac{V_1}{R_3} = -I_2 \qquad \text{........... [2]}$$

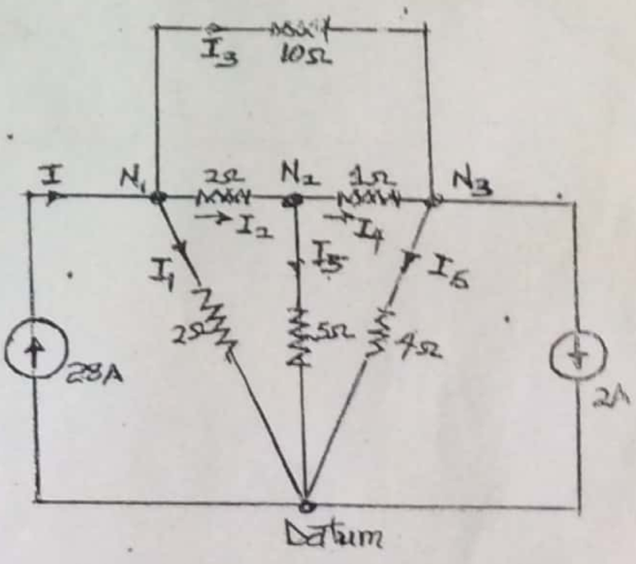
Note:

i. Product of potential and sum of the reciprocal of the branch resistances connected to the node
ii. Minus the ratio of adjoining potential and the interconnected resistances
iii. All equated to the current supplied by the current source connected to this node. It is positive if flowing into the node and negative if flowing out of it.

Example 1

Use nodal analysis method to find currents in the various resistors of the circuit shown below.





Node 1

$$V_1\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{10}\right] - \frac{V_2}{2} - \frac{V_3}{10} = 8$$
$$11V_1 - 5V_2 - V_3 = 280$$

Node2

$$V_2\left[\frac{1}{2} + \frac{1}{5} + \frac{1}{1}\right] - \frac{V_1}{2} - \frac{V_3}{1} = 0$$
$$5V_1 - 17V_2 + 10V_3 = 0$$

Node3

$$V_3\left[\frac{1}{10}+\frac{1}{4}+\frac{1}{1}\right]-\frac{V_2}{1}-\frac{V_1}{10}=-2$$

$$V_1+10V_2-13.5V_3-20=0$$

$V_1 = 36V$, $\quad V_2 = 20V$, $\quad V_3 = 16V$

$I_1 = 18A,\ I_2 = 8A,\ I_3 = 2A,\ I_4 = 4A,\ I_5 = 4A,\ I_6 = 4A$

Superposition Theorem

According to this theorem, if there are a number of emfs acting simultaneously in any linear bilateral network, then each emfs acts independently of others. The value of current in any conductor is the algebraic sum of the currents due to each emfs. Similarly, voltage across any conductor is the algebraic sum of the voltages which emfs would have produced while acting singly. In other words, the current in or voltage across, any conductor of the network is obtained by superimposing the currents and voltages due to each emf in the network.

It is important to keep in mind that this theorem is applicable only to linear networks where current is linearly related to voltage as per ohm's law.

Hence the theorem may be stated as follow: "*in a network of linear resistances containing more than one generator (or sources of emf), the current which flow at any point is the sum of all the currents which would flow at that point if each generators were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistances*".

Diagram A

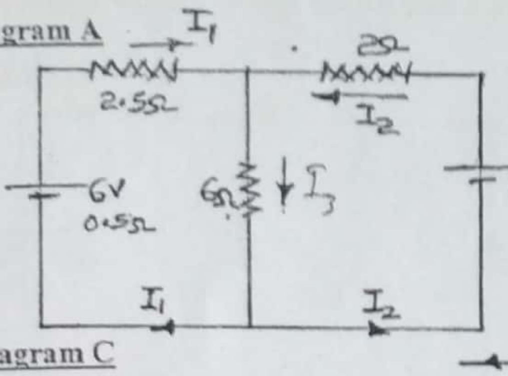


Diagram B

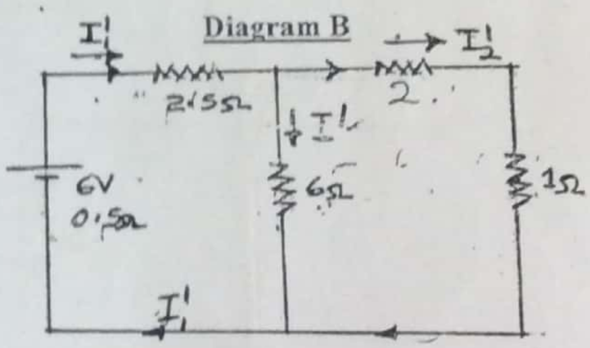
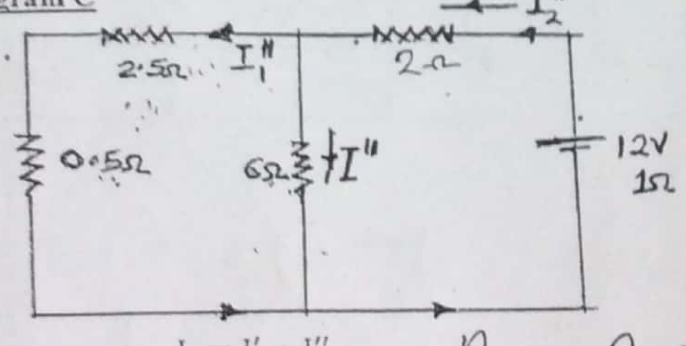


Diagram C



$$I_1 = I_1' - I_1''$$
$$I_2 = I_2' - I_2''$$
$$I_3 = I_3' + I_3''$$

$R_T = 2.5 + \cancel{\frac{6}{6}} + \frac{1}{3} + 0.5$

$0.5 + 2.5 + \frac{3}{6} = 3 + 2 = 5.0\Omega$

For b

Total resistance = 0.5 + 2.5 + 2

$= 5\Omega$

$I_1' = 6/5 = 1.2A$

$I' = 1.2 * 3/9 = 0.4A$

$I_2' = 1.2 * 6/9 = 0.8A$

$V = IR$

$I_1' = \frac{V'}{R} = \frac{6}{5} = 1.2$Am

For c

Total resistance $= 1 + 2 + 2$

$= 5\Omega$

$I_1'' = 2.4 * 6/9 = 1.6A$

$I'' = 2.4 * 3/9 = 0.8A$

$I_2'' = 12/5 = 2.4A$

$R_T = 1+2+6//3$

$1+2+\frac{1}{6}+\frac{1}{3}$

$1+2+\frac{3}{6} = 1+2+2 = 5$

So,

$I_1 = 1.2 - 1.6 = -0.4A$ (Charging current)

$I_2 = 2.4 - 0.8 = 1.6A$ current $= I_2 - I_2'' - I_2$

$I = 0.4 + 0.8 = 1.2A$

$= 2.4 - 0.8 = 1.6A$

Voltage drop across 6Ω resistor

$= 6 * 1.2 = 7.2V$

Thevenin Theorem

It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by a single voltage source with a series resistance.

Diagram A

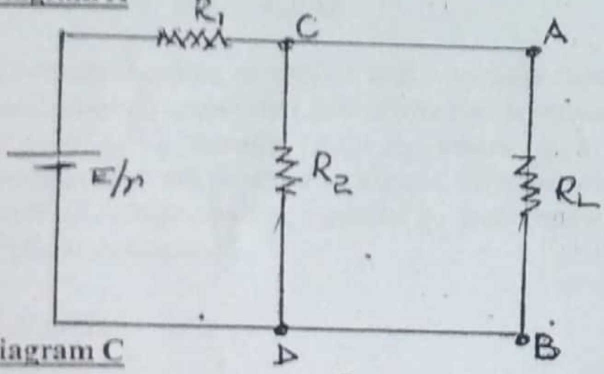


Diagram B

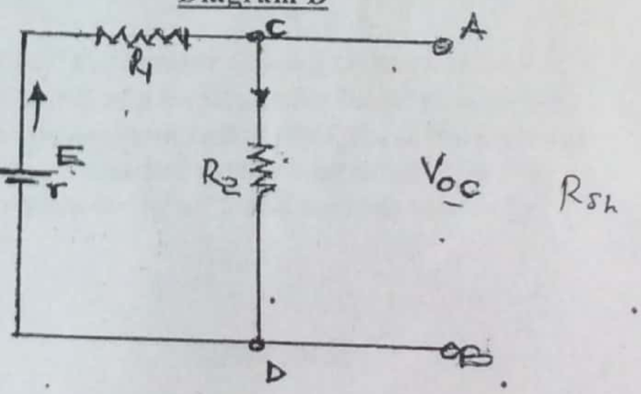
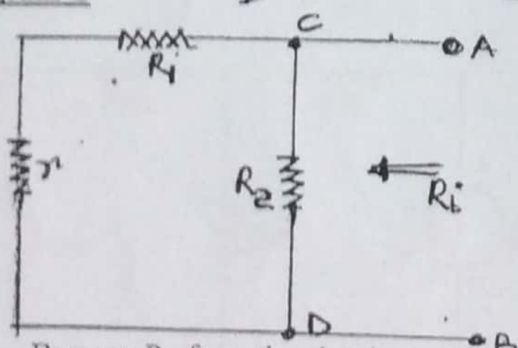


Diagram C



$R_T = R_1+R_2+r$

Total current will be given by

$$I = \frac{E}{R_1+R_2+r} = \frac{E}{R_T}$$

i. Remove R_L from the circuit terminal A and B

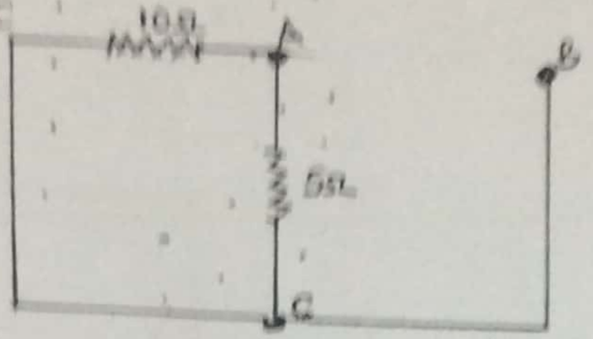
ii. The terminals have been open circuited, calculate the open-circuit voltage V_{oc}

To get $V_{oc} = IR_2$

V_{oc} = thevenin voltage

STOP

Diagram C



From B

From voltage divider rule, potential drop across 5Ω resistor

$= 15 \times {}^{5}/_{5+10}$

$= 5V$

For finding V_{AB}, we go from point B to point A

$V_{AB} = -6 + 5$

$= -1V$

It means point B is at higher potential than A

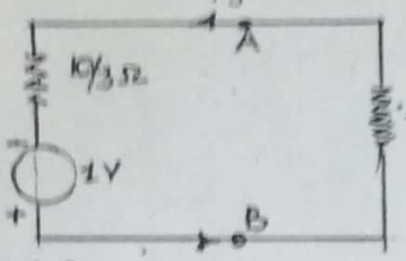
From C

$R_{AB} = R_{AC} = 5 \parallel 10$

$= {}^{10}/_{3}\,\Omega$

$R_T = \frac{10}{3} + 4 = \frac{22}{3} = 7\frac{1}{3}$

$V = 1v. \quad I = \frac{1}{22/3} = \frac{3}{22}$ Amp



Example 2

Apply Thevenin theorem on the circuit below and find current in 15Ω.

Diagram A

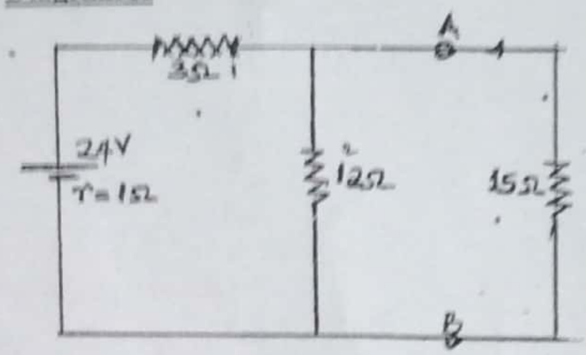
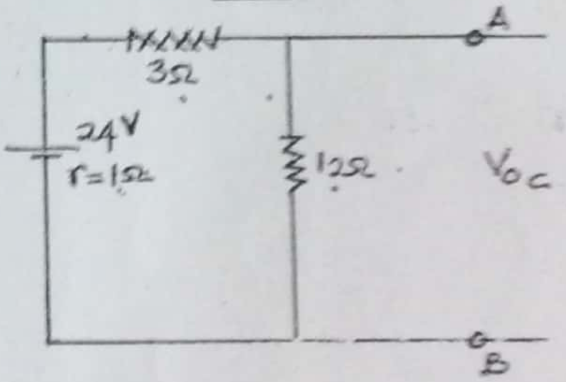


Diagram B



The voltage drop across 12Ω resistor

$R = [1 + 3 + 12]$

$= 16\ Ohms$

$I = {}^{24}/_{16} \quad I = \frac{V}{R}$

$= {}^{3}/_{2} A = 1.5A$

$V_{oc} = 1.5 \times 12 = 18v.$

$R_{TH} = \frac{1}{4} + \frac{1}{12} = \frac{3+1}{12} = \frac{4}{12}$

$R_{TH} = 3\ ohm = 18\ ohm$

$3 + 15 = 18\ ohm$

$V_{OC} = drop\ across\ R_2 = IR_2$

$I = \frac{E}{R_1 + R_2 + r}$

$V_{OC} = \frac{ER_2}{R_1 + R_2 + r}$

V_{OC} is Thevenin voltage (V_{th})

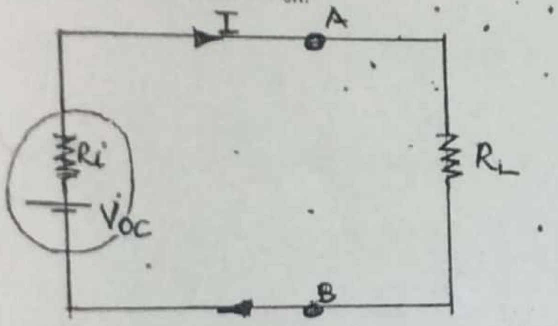
iii. Imagine the battery to be removed from the circuit leaving its internal resistance r

$R = R_2\ parallel\ (R_1 + r)$

$R = R_2(R_1 + r) \Big/ R_2 + (R_1 + r)$

This is called Thevenin resistance R_{sh}

The whole network can be reduced to a single source called Thevenin source whose emf equal V_{th} and internal resistance R_{sh}.



iv. R_L is now connected back across terminal A and B. current flowing through R_L is given by

$I = \frac{V_{th}}{R_{sh} + R_L}$

Thevenin theorem, as applied to d.c. circuits states that " *the current flowing through a load resistance R_L connected across any two terminal A and B of a linear, active bilateral network is given by V_{OC} parallel ($R_{sh} + R_L$) where V_{OC} is the open-circuit voltage and R_{sh} is the internal resistance of the network as viewed back into the open- circuited network terminal A and B with all voltage sources replaced by their internal resistance (if any) and current source by infinite resistance*".

Example 1

Diagram A

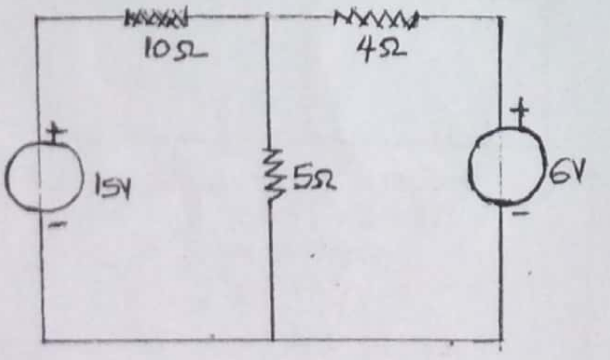
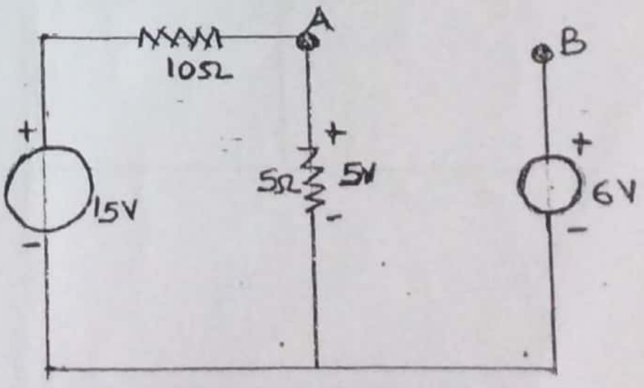


Diagram B



$\frac{1}{10} + \frac{1}{5} = \frac{10}{3}$

The P.d. across 12 ohms resistor

$$V_{12} = \frac{3}{2} * 12$$
$$= 18\ Volts$$

Diagram C

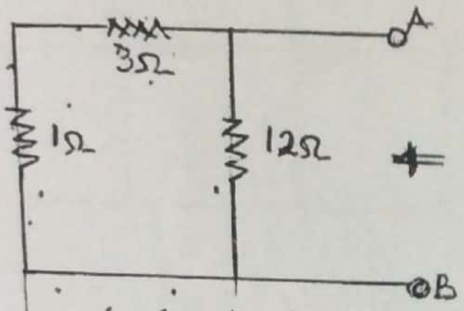
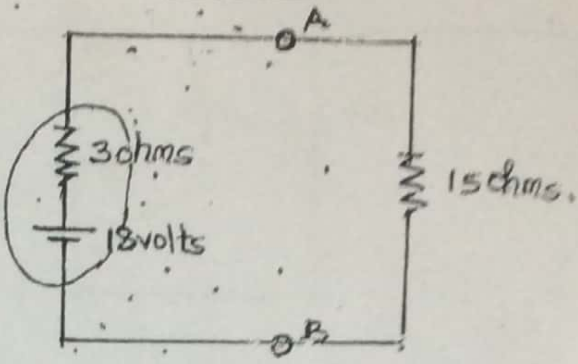


Diagram D



$$\frac{1}{R_{th}} = \frac{1}{4} + \frac{1}{12}$$
$$I = {}^{18}/_{15+3}$$
$$I = 1Amp$$

Example 3

Using Thevenin theorem, calculate the current flowing through the 4Ω.

Diagram A

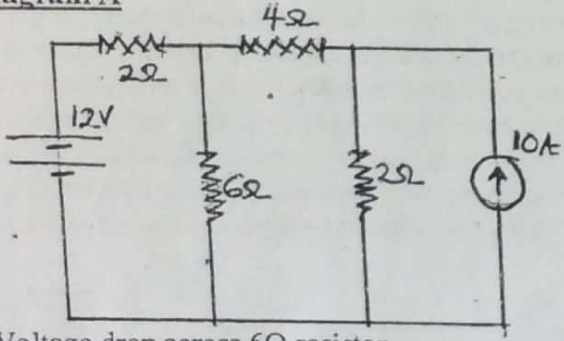
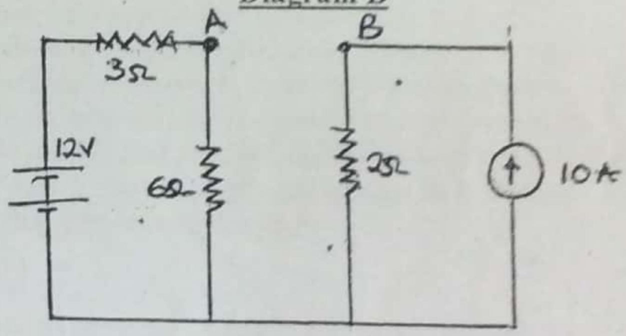


Diagram B



Voltage drop across 6Ω resistor,

$$R = 3\Omega + 6\Omega$$
$$= 9\Omega$$
$$I = {}^{12V}/_{9\Omega}$$
$$= {}^{4}/_{3}\ A$$
$$V_{6\Omega} = \frac{4}{3} * 6$$
$$= 8\ Volts$$

Voltage across 2Ω

$$V_{2\Omega} = 10 * 2$$
$$= 20V$$

$V_{th} = V_B - V_A$ [B is at high potential than A]

$$= 20 - 8$$
$$= 12V$$

Diagram C

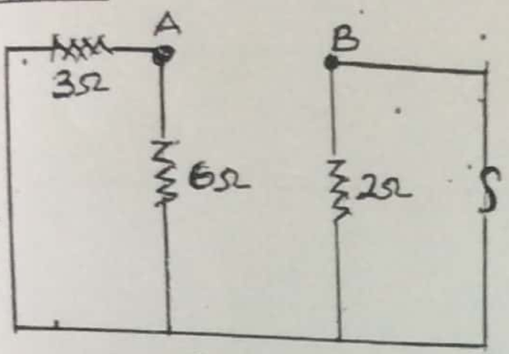
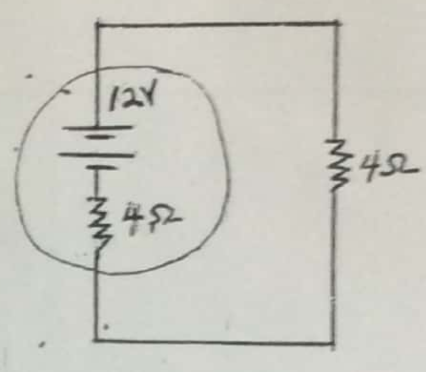


Diagram D



$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6}$$

$$R_{sh} = 2 + 2 = 4\Omega$$

$$I = {}^{12}/_{4+4}$$

$$= 1.5A$$

Norton's Theorem

This theorem is alternative to Thevenin's theorem. In fact, it is dual of Thevenin's theorem. Whereas Thevenin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant-current source and a parallel resistance.

This theorem may be stated as follow: "***any two- terminal active network containing voltage sources and resistance when viewed from its output terminal is equivalent to a constant-current source and a parallel resistance. The constant current is equal to the current which will flow in a short circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from these open circuit terminal after all voltage and current sources have been removed and replaced by their internal resistances***".

Example 1

Determine the Norton's equivalent circuit between terminals A and B for the voltage divider circuit below.

Diagram A

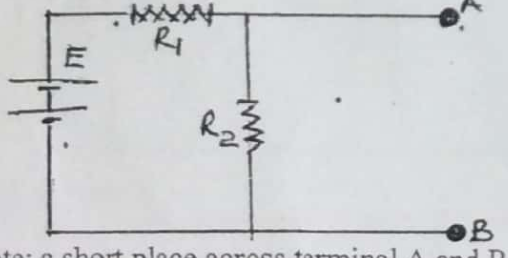
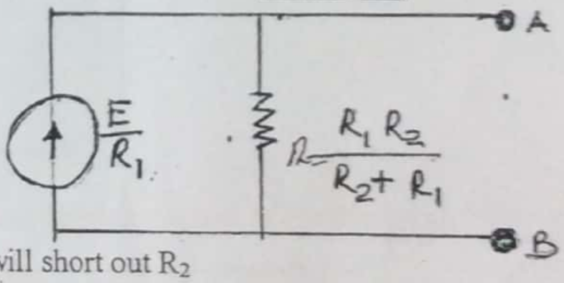


Diagram B



Note: a short place across terminal A and B will short out R_2

$$I_{SC} = {}^{E}/_{R_1}$$

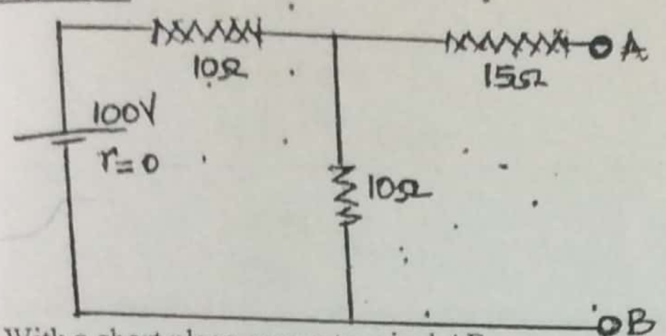
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{[R_2 + R_1]}{R_1 R_2} \quad \Rightarrow R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Example 2

Using Norton's theorem; find the constant-current equivalent of the circuit shown.

Diagram A



With a short place across terminal AB

$\frac{1}{R} = \frac{1}{15} + \frac{1}{10}$

$R = {}^{150}/_{25}$

$R = 6\,Ohms$

$R_T = 10\Omega + 6\Omega$

$\doteq 16\Omega$

Battery current $I = {}^{100}/_{6}$

$= 6.25A$

Isc is the current through AB (15 Ohms resistor)

$= 6.25 * {}^{10}/_{15 + 10}$ $\quad I \times \frac{R}{R_1 + R_2}$

$\doteq 6.25A$ $\quad = 2.5A.$

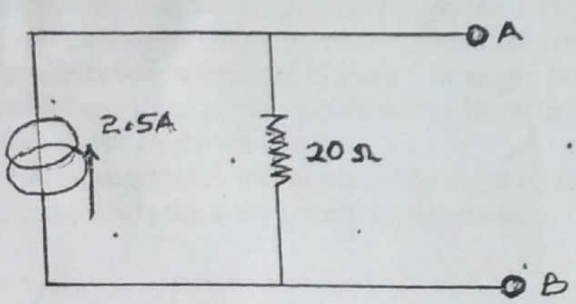
Input resistance of the network as viewed from A and B

$\frac{1}{R} = \frac{1}{10} + \frac{1}{10}$

$R = 5\,Ohms$

$R_T = 15 + 5$

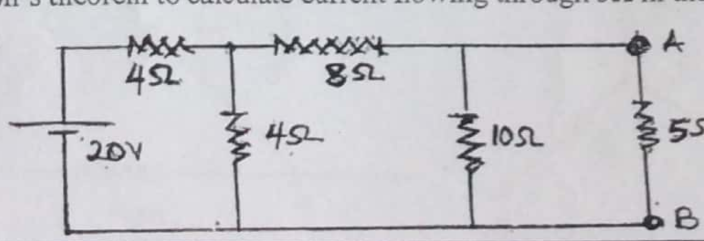
$= 20\,Ohms$



Example 3

Apply Norton's theorem to calculate current flowing through 5Ω in the circuit below.

Diagram A



A short place across terminal AB will short the 10Ω resistor out.

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{4}$$

$$R = 8/3$$

$$R_T = \frac{8}{3} + 4$$

$$R_T = 20/3 \ Ohms$$

$$I = 20 * \frac{20}{3}$$

$$= 3A$$

Isc is the current flowing through AB (through the 8Ω resistor)

$$= 3A * 4/8 + 4$$

$$= 1A$$

Resistance as viewed from AB

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{4}$$

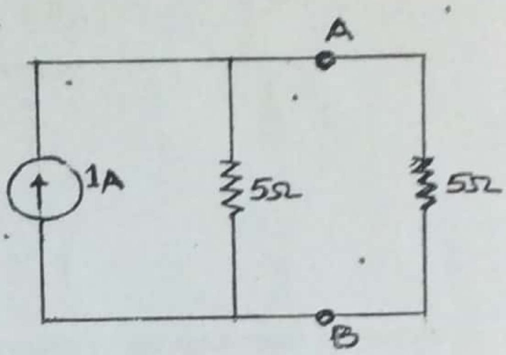
$$R = 2 \ Ohms$$

$$R_2 = 2 + 8$$

$$= 10 \ Ohms$$

$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{10}$$

$$R_T = 5 \ Ohms$$



The current flowing through the 5Ω resistor

$$= 1A * 5/5 + .5$$

$$= 0.5A$$

Delta/Star Transformation

In solving networks by the application of Kirchhoff's law, one sometimes experience great difficulty due to a large number of simultaneous equations that have to be solved. However, such complicated network can be simplified by successively replacing delta meshes by equivalent star system and vice versa. Suppose we are given three resistances R_{12}, R_{23}, and R_{31} connected in delta fashion between terminal 1, 2 and 3 as shown below. So far as the respective terminals are concerned, these three given resistances can be replaced by the three resistances R_1, R2 and R_3 connected in star as shown below.

These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both arrangements.

Diagram A

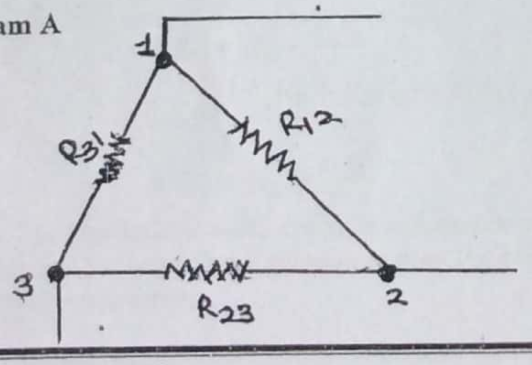
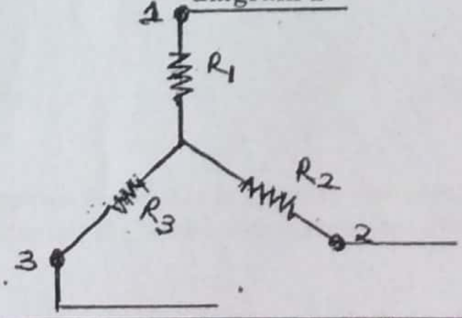


diagram B



First, take delta connection: Between terminals 1 and 2, there are two parallel paths; one having a resistance of R_{12} and the other having a resistance of $(R_{23} + R_{31})$

Resistance between terminals 1 and 2

$$= {(R_{12} * (R_{23} + R_{31})} / {R_{12} + (R_{23} + R_{31})}$$

Now, take star connection: The resistance between the same terminal 1 and 2 is

$$= R_1 + R_2$$

The terminals resistances have to be the same

$$R_1 + R_2 = {(R_{12} * (R_{23} + R_{31})} / {R_{12} + R_{23} + R_{31}} \quad \text{.............. [i]}$$

Similarly for terminal 2 and 3

$$R_2 + R_3 = {(R_{23} * (R_{12} + R_{31})} / {R_{12} + R_{23} + R_{31}} \quad \text{.............. [ii]}$$

$$R_3 + R_1 = {(R_{31} * (R_{23} + R_{12})} / {R_{12} + R_{23} + R_{31}} \quad \text{.............. [iii]}$$

Now subtracting [ii] from [i] and adding the result to [iii], we get

$$R_1 = {(R_{12} * R_{31}} / {R_{12} + R_{23} + R_{31}}$$

$$R_2 = {(R_{12} * R_{23}} / {R_{12} + R_{23} + R_{31}}$$

$$R_3 = {R_{23} * R_{31}} / {R_{12} + R_{23} + R_{31}}$$

Note: resistance of each arm of the star is given by the product of the resistances of the two deltas sides that meet at its end divided by the sum of three delta resistances.

Star /Delta Transformation

This transformation can be easily done by using equation [i], [ii], [iii] given above. Multiplying [i] and [ii], [ii] and [iii], [iii] and [i] adding them together and then simplifying, we get

$$R_{12} = {(R_1R_2 + R_2R_3 + R_3R_1)} / {R_3}$$

$$= R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_{23} = {(R_1R_2 + R_2R_3 + R_3R_1)} / {R_1}$$

$$= R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = {(R_1R_2 + R_2R_3 + R_3R_1)} / {R_2}$$

$$= R_1 + R_3 + \frac{R_1R_3}{R_2}$$

Note: The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistances.

Example 1

Find the input resistance of the circuit between points A and B in the diagram below.

Diagram A

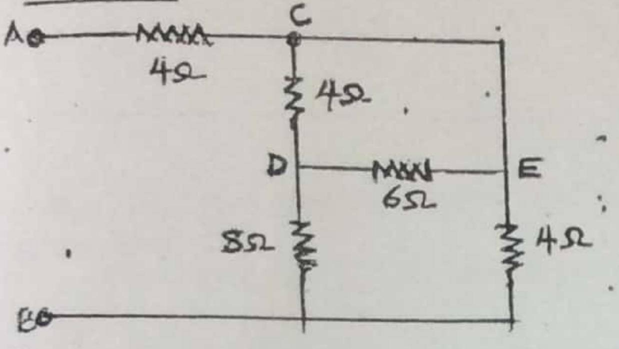
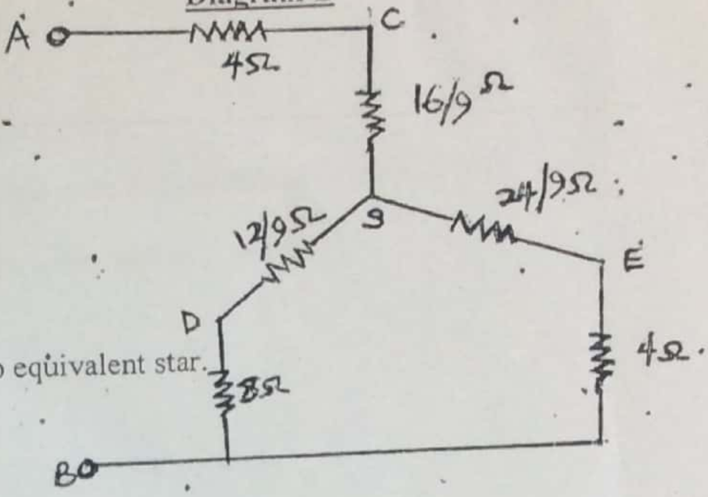


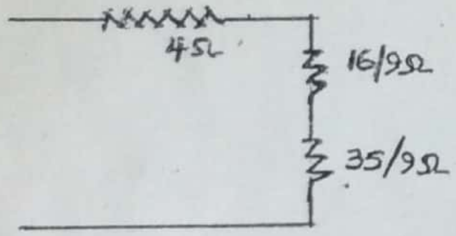
Diagram B



For finding R_{AB}, we will convert the delta CDE into equivalent star.

$R_{CS} = 8 * {}^{4}/_{8}$
$= 16/9\,\Omega$
$R_{ES} = 8 * {}^{6}/_{18}$
$= 24/9\Omega$
$R_{DS} = 8 * {}^{4}/_{18}$
$= 12/9\Omega$

The two parallels between S and B can be reduced to a single resistance of ${}^{35}/_{9}\,\Omega$



$R_{AB} = 4 + {}^{16}/_{9} + {}^{35}/_{9}$
$= {}^{87}/_{9}\,\Omega$

Example 2

A network of resistances is formed as shown below. Compute the network resistance measured between (i) A and B, (ii) B and C and (iii) C and A.

Diagram A

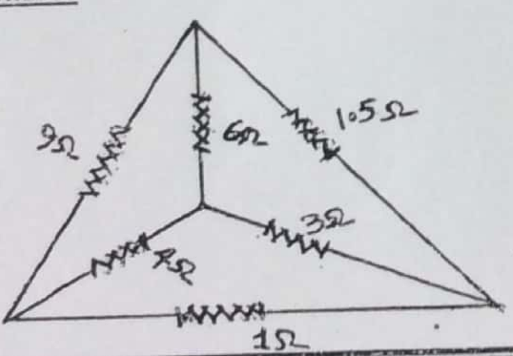


Diagram B

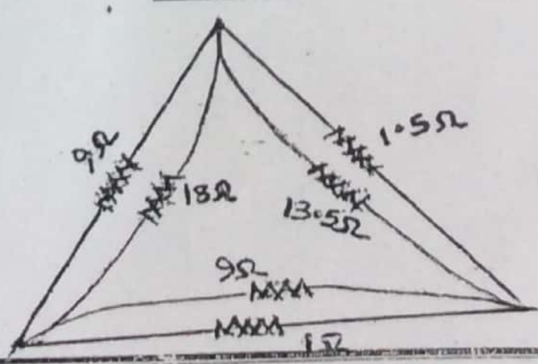
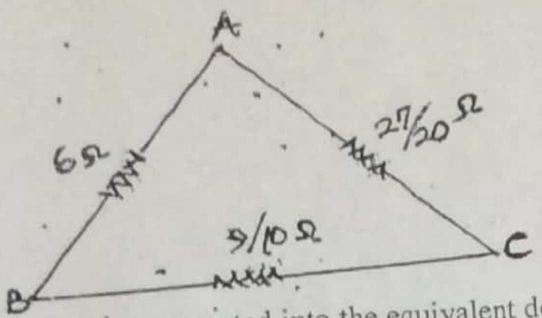


Diagram C



The star of diagram A may be converted into the equivalent delta and combined in parallel with the given delta ABC.

i. As seen in diagram C, there are two parallel paths across points A and B

a) One directly from A to B
$= 6\Omega$

b) The other via C
$= {}^{27}/_{20} + {}^{9}/_{10}$

$$R_{AB} = \left[6 * \left[\frac{9}{10} + \frac{27}{20}\right]\right] \Big/ \left[6 + {}^{9}/_{10} + {}^{27}/_{20}\right]$$

$$= {}^{18}/_{11}\,\Omega$$

Similarly,

$$R_{BC} = \left[\frac{9}{10} * \left[6 + \frac{27}{20}\right]\right] \Big/ \left[6 + {}^{9}/_{10} + {}^{27}/_{20}\right]$$

$$= {}^{441}/_{550}\,\Omega$$

$$R_{CA} = \left[\frac{27}{20} * \left[6 + \frac{9}{10}\right]\right] \Big/ \left[6 + {}^{9}/_{10} + {}^{27}/_{20}\right]$$

$$= {}^{621}/_{550}\,\Omega$$