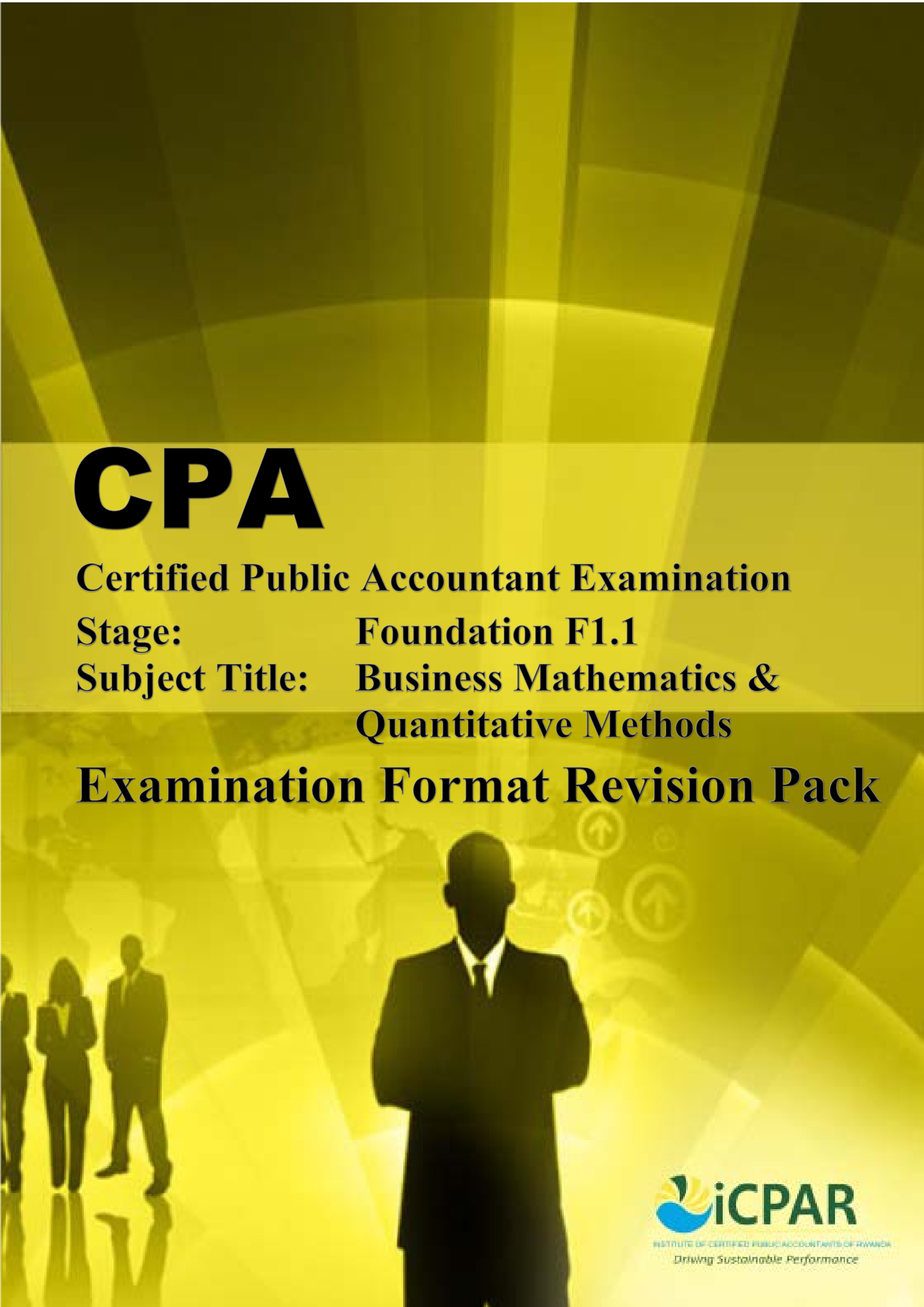
CPA

Certified Public Accountant Examination

Stage: Foundation F1.1

Subject Title: Business Mathematics & Quantitative Methods

Examination Format Revision Pack



ICPAR
INSTITUTE OF CERTIFIED PUBLIC ACCOUNTANTS OF RWANDA
Driving Sustainable Performance

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F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

EXAMINATION FORMAT REVISION QUESTIONS & SOLUTIONS

NOTES:
You are required to answer 5 questions.

(If you provide answers to all questions, you must draw a clearly distinguishable line through the answer not to be marked. Otherwise, only the first 5 answers to hand will be marked).

All questions carry equal marks.

STATISTICAL FORMULAE AND MATHEMATICS TABLES ARE PROVIDED AT BACK OF THIS PACK

TIME ALLOWED:
3 hours, plus 10 minutes to read the paper.

INSTRUCTIONS:
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Marks for each question are shown. The pass mark required is 50% in total over the whole paper.

Start your answer to each question on a new page.

You are reminded that candidates are expected to pay particular attention to their communication skills and care must be taken regarding the format and literacy of the solutions. The marking system will take into account the content of the candidates' answers and the extent to which answers are supported with relevant legislation, case law or examples where appropriate.

List on the cover of each answer booklet, in the space provided, the number of each question(s)attempted.

F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

QUESTION 1. A machine will be purchased by DYY Ltd. at a cost of RWF30,000. The suppliers also quote a trade-in value on the machine of RWF5,000 at the end of 5 years. The company has a number of different ways of obtaining the machine which are outlined below.

(i) An immediate purchase for cash. The bank will provide a loan at an interest rate of 12% p.a.
(ii) The machine can be purchased for cash with 20% paid as a deposit and the balance payable by 5 annual installments of RWF6,720 each.
(iii) The machine can be leased for 5 years at an annual rent of 25% of the cash price.

[Note: Use a discount rate of 12% for parts (ii) and (iii). Assume the cash flows take place at the end of the appropriate years].

You are asked by the Managing Director of DYY Ltd. to calculate the most cost effective means of obtaining the machine and comment on your result.

[Total: 20 Marks]

QUESTION 2. In a dispute with pharmacists, the Ministry of Health in Rwanda decided to collect sample data on the number of prescriptions filled and the value of these prescriptions over the past 4 weeks. It collected the following data from a local pharmacy:

Value of prescriptions RWF	Number
50 – 59	1
60 – 69	2
70 – 79	3
80 – 89	11
90 – 99	14
100 – 109	12
110 – 119	9
120 – 129	5
130 - 139	3

You are required to:

(i) Calculate the mean and standard deviation of the value of prescriptions.

(8 Marks)

(ii) Demonstrate the data on a diagram and hence derive the median value of prescriptions.

(8 Marks)

(iii) Discuss the difference between the 'averages' derived.

(4 Marks)

[Total: 20 Marks]

QUESTION 3. The management accountant wishes to derive a relationship between the units produced each month and the energy costs of a laser production machine.

The following data for the six month period from January to June shows the energy costs and the number of units produced:

Month	**Units Produced, x**	**Energy Costs (RWF), y**
January	1300	500
February	1600	700
March	1100	600
April	1500	700
May	1600	800
June	1300	900

Show how she would:

(i) Use regression analysis to derive an equation for the energy costs per month.

(8 Marks)

(ii) Calculate the correlation co-efficient for the data.

(6 Marks)

(iii) Calculate the fixed and variable costs for an estimated production level of 1,200 units in July and comment on the accuracy of your forecast.

(6 Marks)

[Total: 20 Marks]

QUESTION 4. Your firm, iCPAR Consultants & Associates, surveyed a random sample of 35 companies in order to determine the average number of staff leaving companies last year. The results showed an average staff turnover of 72. However, your test at DYY Ltd. shows a staff turnover of 75. Your colleagues don't believe that such a small difference is important. However, you decide to test the statement that the staff turnover in this client company is normal for the industry by using a 5% level of significance. Information in similar studies gives a standard deviation of 7.0.

(i) Explain the principles that you propose to use in your study.

(10 Marks)

(ii) Carry out a significance test at the 5% level and advise your colleagues on the outcome.

(10 Marks)

[Total: 20 Marks]

QUESTION 5.

(i) Superior Products has been granted a loan to purchase office equipment. The bank has promoted the loan at a nominal interest rate of 16% but the small print on the contract states that the APR is approximately 17%. Using this data, describe the difference between 'nominal interest rates' and 'annual percentage rate' (APR).

(6 Marks)

(ii) Your Managing Director has been advised that it will cost RWF25,000 to provide a number of networked printers for the office. The bank will provide finance for the job at an interest rate of 16%. The loan may be repaid at the end of 4 years. The normal cost of borrowing by the company is 12%. What is the increase in cost to the company by using this means of financing the project?

(6 Marks)

(iii) You are advised that the average deposit in the Office national des postes is RWF100 with a standard deviation of 15. You are asked to find the 95% confidence interval for the deposit of an 'average' account. Provide an interpretation of your result. Assume that the deposits follow a normal distribution.

(8 Marks)

[Total: 20 Marks]

QUESTION 6. "Network analysis incorporates a variety of techniques used to help plan, manage, allocate resources and review costs. Accountants play a major role in this process". In this context, you have planned the following activities for the refurbishment of your existing offices.

Task	Activity	Preceding Activity	Duration (weeks)
Advise staff	A	-	2
Consult with clients	B	-	3
Consult property agents	C	A	3
Plan temporary office	D	B	5
Install furniture	E	B	4
Equip new office	F	B	2
Move to new office	G	C,D	3
Train staff	H	F	5

You are required to:

(i) Draw a network of the above activities.

(10 Marks)

(ii) Estimate the maximum time available to complete the project and show the critical path.

(6 Marks)

(iii) Provide a description of the process that you have used.

(4 Marks)

[Total: 20 Marks]

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SUGGESTED SOLUTIONS

SOLUTION 1

(i) Purchase for cash

	Actual Price RWF	Discount Factor @ 12%	Present Value RWF
Purchase price	30,000	1	30,000
Less trade-in at end year 5	5,000	0.5674	2,837
Net Present Value	25,000		27,163
	2 Marks		**(4 Marks)**

(ii) Purchase by instalments

Deposit	6,000	1	6,000
Instalment			
Year 1	6,720	0.8929	6,000
Year 2	6,720	0.7972	5,357
Year 3	6,720	0.7118	4,783
Year 4	6,720	0.6355	4,271
Year 5	6,720	0.5674	3,813
	39,600		30,224
Less trade-in at end year 5	5,000	0.5674	2,837
Net Present Value	34,600		27,387
	2 Marks		**(4 Marks)**

(iii) Leasing (In this case there is no trade-in value, since the machine is not owned by the company as in (i) and (ii) above.)

Payment at end of

Year 1	7,500	0.8929	6,697
Year 2	7,500	0.7972	5,979
Year 3	7,500	0.7118	5,338
Year 4	7,500	0.6355	4,766
Year 5	7,500	0.5674	4,256
Net Present Value	**37,500**		**30,224**
	2 Marks		**(4 Marks)**

Summary: On the basis of the above calculations the machine should be leased rather than purchased for immediate cash or instalments since, with this option, the cost to the company is least. **(2 Marks)**

[Total : 20 Marks]

SOLUTION 2

(i) The mean and standard deviation are calculated from the data in the following table.

Value of prescriptions RWF	Number f	Cum frequency	x	fx	$(x - \bar{x})$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
50 – 59	1	1	54.5	54.5	- 45	2025	2025
60 – 69	2	3	64.5	129.0	- 35	1225	2450
70 – 79	3	6	74.5	223.5	- 25	625	1875
80 – 89	11	17	84.5	929.5	- 15	225	2475
90 – 99	14	31	94.5	1323.0	- 5	25	350
100 – 109	12	43	104.5	1254.0	5	25	300
110 – 119	9	52	114.5	1030.5	15	225	2025
120 – 129	5	57	124.5	622.5	25	625	3125
130 - 139	3	60	134.5	403.5	35	1125	3675
$\sum$	60			5970.0			18300

Mean $= \bar{x} = \frac{\sum fx}{\sum f} = \frac{5970}{60} =$ RWF99.5

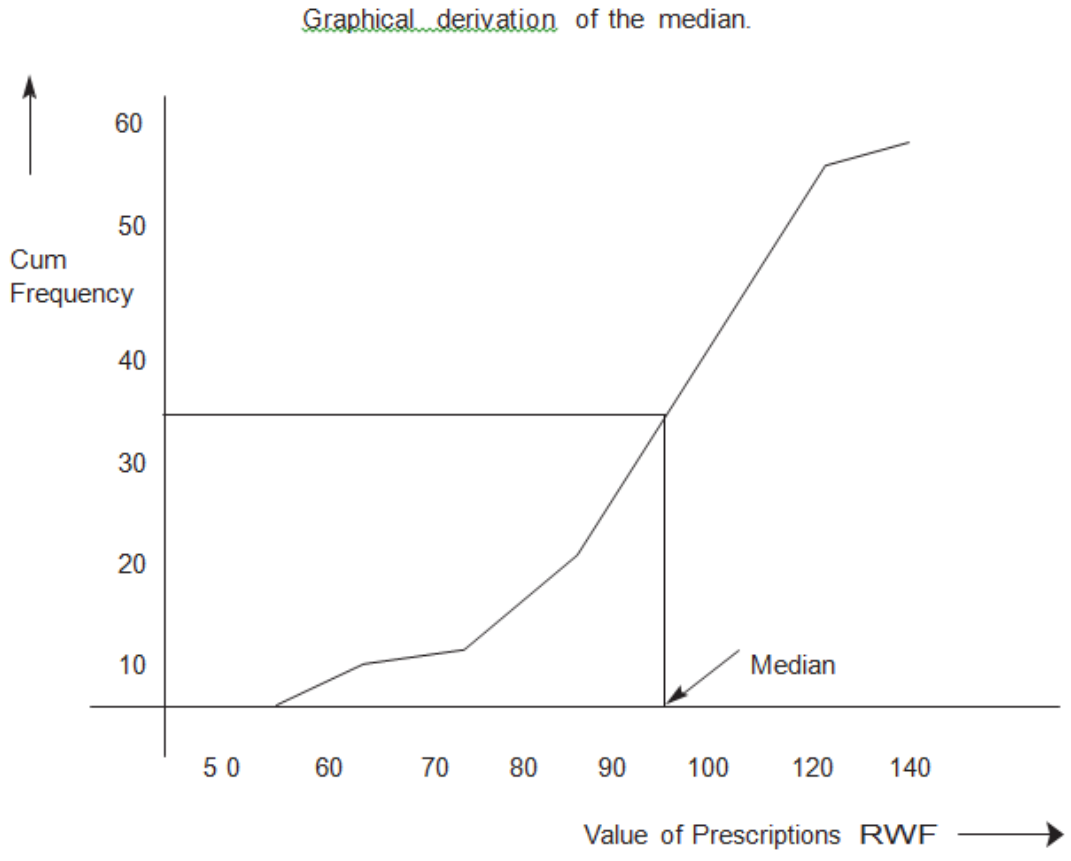
Standard deviation $= \sigma = \frac{\sqrt{\sum f (x - x)^2}}{\sum f} = \frac{\sqrt{18300}}{60}$

$= \sqrt{306}$

$=$ =RWF17.46

ii) Median; the median is described as the 'centre' of a set of data. Since you are asked to derive the median from the graph, the value is approximately RWF99. The median is the value of the data such that 50% lies above and below this value.

(4 Marks)



(4 Marks)

iii) In the particular data, both the mean and the median are approximately the same. The median is an 'average' in that it splits the data into two parts. It does not generally equal the mean since it defines the data in a different way. There are equally as many values above and below the median. The mean is average in that, if each value was replaced by a constant while the total remains unchanged, this number is the mean. In this way the mean is comparable to a centre of gravity. Like the mean the median always exists and is unique for any set of data. In certain situations the median may be a better measure of central tendency than the mean. It is less sensitive to extremely large or small measurements. Extreme values affect the mean since these values are used explicitly to calculate the mean. A comparison of the mean and median gives a general method for detecting skewness in data sets. However, the mean and median provide more descriptive information than the mode.

(4 Marks)

[Total : 20 Marks]

SOLUTION 3

(i) The relationship can be demonstrated by developing a linear regression equation where the relationship is

$y = a + bx$, where

x = no. units produced / month y = total energy costs / month

a = fixed costs / month b = variable costs / unit

n = no. of pairs of x and y variables

Month	Units Produced (00s) x	Energy Costs RWF00s y	x2	xy	y2
January	13	5	169	65	25
February	16	7	256	112	49
March	11	6	121	66	36
April	15	7	225	105	49
May	16	8	256	128	64
June	13	9	169	117	81
∑	84	42	1196	593	304

(2 Marks)

A linear regression equation may be written as: where

$\sum y = na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$; where

$$a = \frac{\sum y}{n} - \frac{b\sum x;}{n}$$

$$b = \frac{\sum xy - \sum x\sum y/n,}{\sum x^2 - (\sum x)^2/n}$$

Inserting values gives

$$b = \frac{593 - 84 \times 42/6}{1196 - 84^2/6}$$

$$= \frac{593 - 588}{1196 - 1176} = \frac{5}{20} = 0.25$$

That is, RWF0.25/unit (variable cost), RWF25 per 100 units.

$$a = \frac{42}{6} - \frac{0.25 \times 84}{6} = 7 - 3.5 = 3.5$$

That is, RWF350/month (fixed cost). **(2 Marks)**

Therefore, $y = 350 + 25x$. **(2 Marks)**

(ii) Correlation co-efficient is derived from the formula:

$$r = \frac{\sum xy/n - \sum x/n \sum y/n}{\sqrt{\{\sum x^2/n - (\sum x/n)^2\}\{\sum y^2/n - (\sum y/n)^2\}}}$$

Inserting values from the table above: $\sum xy = 593$; $\sum x = 84$; $\sum y = 42$;

$\sum x2 = 1196$; $\sum y2 = 304$.

Therefore, $r = \dfrac{593/6 - 84/6 \times 42/6}{\sqrt{(1196/6 - 196)(304/6 - 49)}} = \dfrac{98.8 - 98}{\sqrt{3.33 \times 1.66}}$

$= 0.8/2.35$

$= 0.34$ **(3 Marks)**

This value indicates that the relationship between the variables is not high. This is obvious from an inspection of the data where the costs, over such a short time period, obviously do not correlate closely to the production level. There are obviously other factors involved in the data which are not obvious at a superficial level. It would therefore be somewhat inaccurate to rely on forecasts based on this data.

(3 Marks)

(iii) In order to predict the costs for following months, the established regression equation could be used with the health warning set out above.

For a production level of 1200 units, the fixed costs are RWF350. Therefore the total costs for July are RWF 650

(4 marks)

In the regression equation the values of a and b are estimates based on sample data and can be expected to vary from sample to sample. If the estimate is based on a different sample, the method of least squares would probably yield different values for a and b and a different value for y. In linear regression analysis we assume that that the x's are constants, not values of random variables, and that for each value of x the variable to be predicted, y, has a certain distribution. We assume that these distributions are all normal distributions with the same standard deviation, σ

(4 Marks)

[Total : 20 Marks]

SOLUTION 4

(i) Hypothesis Testing.

This test considers the principles of hypothesis testing. If we postulate that there is no difference between the data, this is set up as the Null hypothesis, Ho. The hypothesis is set up to see if it can be rejected. Together with this hypothesis an alternative hypothesis, H1, must be formulated, that is, the hypothesis that is accepted if the Ho is rejected. In the present case we set up Ho = 72. H1 is set up as H1 $\neq$ 72 so that Ho will be rejected if the sample mean is much greater than or much less than 72. H1 usually specifies that the population mean is less than, greater than, or not equal to the value assumed under Ho. Since, in this case, the sample is large (n $\geq$30), the sampling distribution of the mean can be approximated with a normal distribution and the test statistic (z) is a value of a random variable having the standard normal distribution. Using two values, we can base tests on the Null hypothesis $\mu = \mu o$. Since the level of significance is 5%, that is, 0.05, the critical values (from the Normal tables) are -1.96 to +1.96 for a two sided alternative.

(10 Marks)

(ii) Setting down the process as described:

1. Ho: $\mu = 72$

   H1: $\mu \neq 72$ **(2 Marks)**

2. The level of significance, $\alpha = 0.05$

3. Reject the Null hypothesis if $z \leq -1.96$ or $z \geq +1.96$

   where $z = \dfrac{x - \mu o}{\sigma/\sqrt{n}}$, $n = 35$, $\sigma = 7$ **(2 Marks)**

4. $z = \dfrac{75 - 72}{7/\sqrt{35}} = \dfrac{3}{7/5.916} = \dfrac{3}{1.18} = 2.54$

5. Since $z = 2.54$ exceeds 1.96 the Null hypothesis must be rejected. **(2 Marks)**

You advise your colleagues that, based on the data provided, the difference between the numbers of exits in DYY Ltd. and the industry is statistically significant and the level of exits requires further examination by the company.

(4 Marks)

[Total : 20 Marks]

SOLUTION 5

(i) The majority of financial institutions express rates of interest as 'rate per annum'. The interest rate may be compounded over time periods less than a year. This annual rate is the 'nominal rate'. By compounding the interest rate over quarterly periods, the real rate of interest, that is, the effective rate or 'annual percentage rate' will always be greater than the nominal rate. The standard method of determining the APR is to make the effective time period equal to the compounding period and actually compounded over a period of 1 year.

(3 Marks)

The APR can be calculated using APR $= (1 + i/n)^n - 1$, where 1 = nominal interest rate, n = number of equal compounding periods in 1 year.

Therefore, in the problem the APR is $[1 + (0.16/4)]^4 - 1 = 1.04^4 - 1$

$= 1.1698 - 1 = 16.98\%$ **(3 Marks)**

(ii) The maturity value of the debt at the end of 4 years is RWF25,000 $(1.16)^4$ = RWF45,250. However, the present value is obtained by discounting this value at 12%, that is, $45,250/1.12^4 = 45,250/1.57 =$ RWF28,821.

(3 Marks)

This is the value that the original debt will cost in to-days terms. This is the present value of an interest bearing debt. Therefore, the real cost of the debt, to repay it in to-days terms is, RWF28,821 - RWF25,000 =

RWF3,821. This is based on the premise that the borrowing rate of 16% that the company pays will be greater than the investment rate that it could receive on its money; in other words, the present value of the debt will always be greater than the original amount. **(3 Marks)**

(iii) The 95% confidence interval for a population is

$\mu - 1.96\sigma < \bar{x} < \mu + 1.96\sigma$

where x represents the variable of interest, that is, deposits

Inserting values: $100 - 1.96\ (15) < \bar{x} < 100 + 1.96\ (15)$

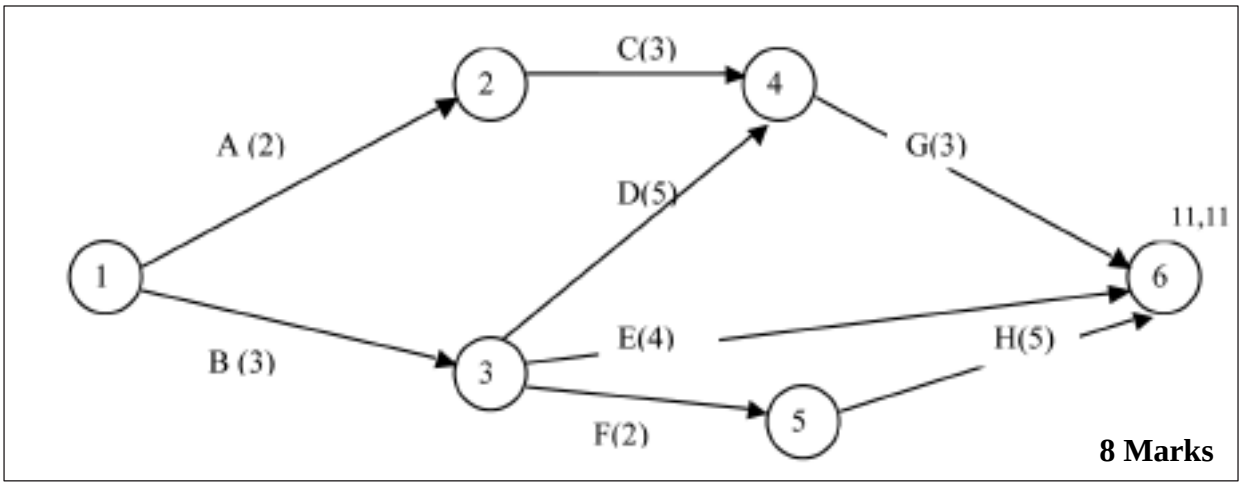
$70.6 < \bar{x} < 129.4$. **(4 Marks)**

Interpretation: There is a 95% confidence that a value from a normal distribution will lie between will lie within 1.96 standard deviations of the mean value. In the present case, 95% of the population of depositors has a deposit of between RWF70.6 and RWF129.2. **(4 Marks)**

[Total :20 Marks]

6. (i) The network for the activities is outlined below. It illustrates the sequence of activities and the steps involved.

Task	Activity	Preceding Activity	Duration (weeks)
Advise staff	A	-	2
Consult with clients	B	-	3
Consult property agents	C	A	3
Plan temporary office	D	B	5
Install furniture	E	B	4
Equip new office	F	B	2
Move to new office	G	C,D	3
Train staff	H	F	5



8 Marks

(ii) The total project duration is an important factor when managing projects. The overall duration can be calculated from the network providing the duration of each activity is known. The estimated time to complete the project is 11 weeks.

(4 Marks)

This is the critical path – the critical times that cannot be delayed that cannot be delayed, otherwise the overall duration of the project will be extended. It is designated on the diagram by 1-3-4-6.

(4 Marks)

(iii) Each activity is represented by an arrow on the diagram. Circles are numbered and drawn to indicate the start and of finish activity. The diagram shows the relationships between the activities as set down in the table. In order to produce a network diagram a list of activities is required and the interdependence between activities, that is, the activities that precede other activities. In order to calculate the overall duration of the project it is necessary to estimate the earliest and latest event times. The earliest time is determined by the longest route through the network and the largest value is used. This gives the earliest time in which the project can be completed. The latest event times are then calculated. Preceding latest event times are calculated by subtracting an activity's duration from the subsequent latest event time. If two or more activities start from an event, the latest time for each route is calculated and the lowest value is used. The times show that the project can be completed in 11 weeks.

(4 Marks)

[Total : 20 Marks]

F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

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F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

QUESTIONS

Q1. Your company has received three quotations from financial institutions for the purchase of computer equipment.

The quotations are as follows:

(a) The equipment can be purchased for RWF350,000 today with a maintenance contract of RWF30,000 per annum payable in advance. The equipment has an expected life of 8 years and will be sold for RWF25,000 at the end of year 8.

(b) The equipment can be rented for RWF90,000 per annum, payable in advance.

(c) The equipment can be obtained on hire purchase with a deposit of RWF70,000 today and 7 equal annual payments of RWF95,000 starting at the end of year 1. At the end of 8 years, the equipment will be owned by the company and has a value of RWF20,000.

You are required to:

(i) Set out the cash flows for the three options. **(10 marks)**

(ii) Using Net Present Value recommend the best option for the company with a cost of capital of 16%.

(10 marks)

(Total: 20 Marks)

Q2. The local union representative has approached the manager of the Do-It-Better Manufacturing company and claims that the wages paid to its employees are inferior to the wages paid to another local company, Compact Ltd. He provides the company with the data on employee wages from both companies. The manager asks you to analyse the data and to present it for comparison at the next management meeting.

You decide to:

(i) Present the data on histograms. **(8 marks)**

(ii) Calculate the mean and standard deviations for both companies. **(8 marks)**

(iii) Comment on the differences between the wages of both companies using the information derived in (i) and (ii). **(4 marks)**

Employee wages (RWF per week)	Do-It-Better company	Compact Ltd
300 < 310	3	4
310 < 320	7	12
320 < 330	32	16
330 < 340	26	23
340 < 350	23	39
350 < 360	20	26
360 < 370	18	16
370 < 380	16	11
380 < 400	5	3

(Total: 20 Marks)

Q3. A tenant claims to the tenancy board that the average rent for an apartment in the area is RWF7,200 per annum. However, the developer carries out a survey of 49 apartments and claims that the average annual rents in the area are RWF7,500 per annum with a standard deviation of RWF840.

(i) Using a 95% confidence interval does the developer's data support the claim? **(10 marks)**

(ii) Describe by means of diagrams the principles underpinning your calculations **(10 marks)**

(Total 20 Marks)

Q4. The advertising expenditure and sales data for the Superior Products company over the past 9 months is set out in the table below. The company wants to determine the effect that advertising has on the levels of actual and projected sales over the period.

Month	**J**	**F**	**M**	**A**	**M**	**J**	**J**	**A**	**S**
Advertising RWF000	25	50	20	45	50	35	20	25	20
Sales RWF000	100	140	130	120	150	170	120	100	80

In this context, using linear regression analysis:

(i) Derive the equation of a linear relationship between sales and advertising expenditure.

(8 marks)

(ii) Plot the data and regression equation on a graph **(8 marks)**

(iii) Forecast the level of sales based on advertising expenditure of RWF4,000

(4 marks)

(Total 20 Marks)

Q5. As the Financial Adviser with iCPAR Consultants, you have been consulted by a client who requests you to provide details of the principles, and formulae if used, underpinning your calculations with respect to the following:

(i) Your client wishes to invest RWF20,000 starting now and continuing with the payments at the end of each year until he retires in 20 years. He wishes to know the total value of his investment at the end of this period at the current rate of interest of 5%.

(6 marks)

(ii) He states that he wishes to make an investment now so that his wife could withdraw a guaranteed pension of RWF20,000 for 20 years. How much should be invested now to cater for this payment, if the rate of interest is 5%?

(8 marks)

(iii) At retirement he wishes to purchase an annuity, that is, an annual pension for the rest of his life for the sum of RWF1,061,319. The insurance company uses an interest rate of 3% for pension purposes. What annual pension can he expect?

(6 marks)

(Total: 20 Marks)

Q6. Your company is considering building an extension to its financial services centre. The developer proposes using network analysis to give the management team a perspective on the range of activities involved and the progress of the development. You are asked by the Managing Director to provide a management briefing on Network Analysis and how this process operates. Your briefing should include the following elements:

(i) Key information required to draw a network

(ii) Use of dummy activities

(iii) Time calculations for earliest and latest event times

(iv) Derivation of the critical path

(v) Limitations of the method

(Total 20 Marks)

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SUGGESTED SOLUTIONS

SOLUTION 1

(i) Cash flows for the three options

Year	Purchase RWF		Rent RWF		Hire Purchase RWF	
	Cash Outflow	Cash Inflow	Cash Outflow	Cash Inflow	Cash Outflow	Cash Inflow
0	(350,000) (30,000)		(90,000)		(70,000)	
1-7	(30,000)		(90,000)		(95,000)	
8		25,000		----------	(95,000)	20,000
	2 marks	**2 marks**	**2 marks**		**2 marks**	**2 marks**

In the HP option, since the company retains the machine it is assumed that it can sell it for a value of RWF20,000 in year 8.

(ii) Calculation of NPV using a discount factor of 16%

Since the company is making equal annual payments for years 1 – 7, the annuity factor can be used rather than the discount factor for each year.

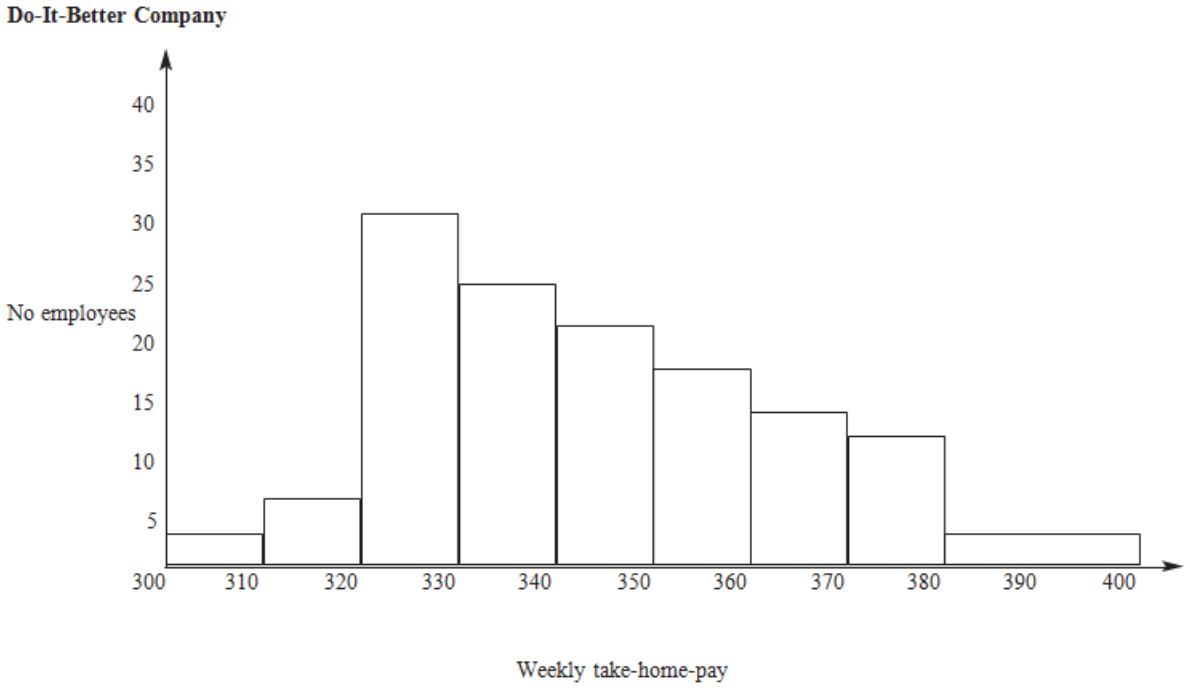
Year	Discount Factor 16%	Purchase, RWF		Rent RWF		Hire Purchase RWF	
		Net Cash Flow	PV	Net Cash Flow	PV	Net Cash Flow	PV
0	1	(380,000)	(380,000)	(90,000)	(90,000)	(70,000)	(70,000)
1-7	4.039	(30,000)	(121,170)	(90,000)	(363,510)	(95,000)	(383,705)
8	0.305	7,625	7,625			(75,000)	22,875
NPV			**(493,545)**		**(453,510)**		**(430,830)**
			2 **marks**		**2 marks**		**2 marks**

Since the HP provides the minimum cash outflow for the company it is recommended as the best option.

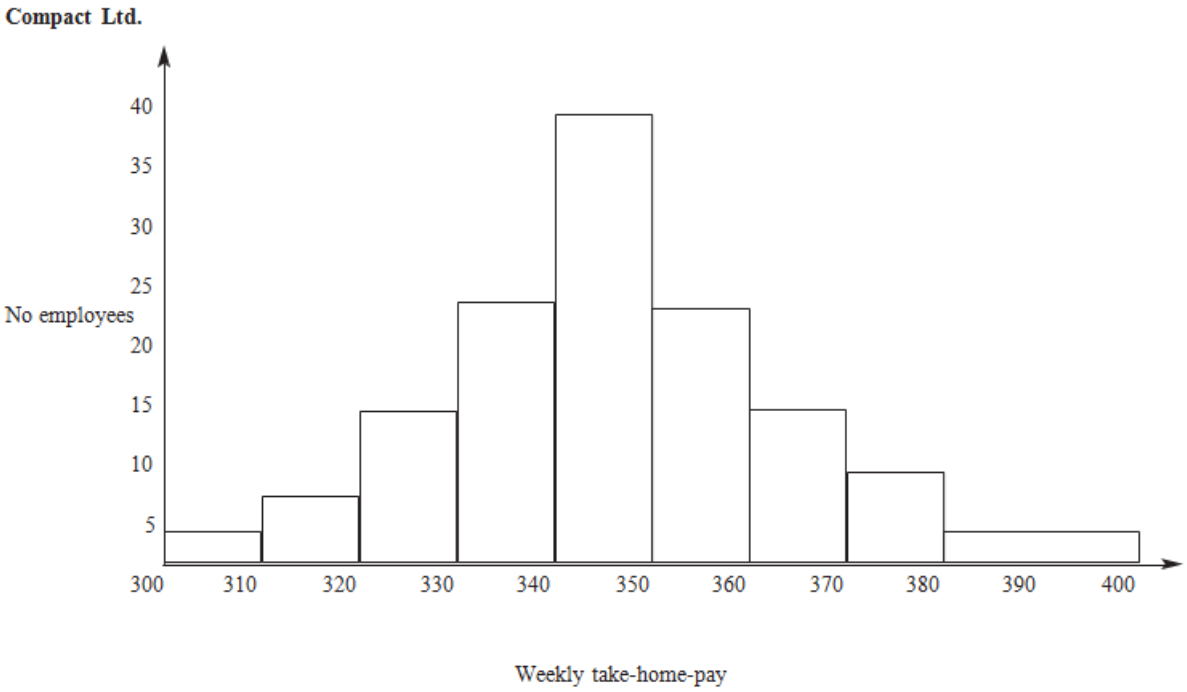
(4 marks)

SOLUTION 2

(i) The histograms are plotted below.



(4 marks)



(4 marks)

(ii) The mean and standard deviation are calculated from the data in the following tables.

Do-It-Better Company

Take home pay RWF	Mid point x	Employees f	f(x)	(x – x)	$(x - x)^2$	$(x - x)^2$
300 < 310	305	3	915	-40	1600	4800
310 < 320	315	7	2205	-30	900	6300
320 < 330	325	32	10400	-20	400	12800
330 < 340	335	26	8710	-10	100	2600
340 < 350	345	23	7935	0	0	0
350 < 360	355	20	7100	10	100	2000
360 < 370	365	18	6570	20	400	7200
370 < 380	375	16	6000	30	900	14400
380 < 400	390	5	1950	45	2025	10125
		150	51785			60225

Mean = $\bar{x} = \frac{\sum fx}{\sum f} = \frac{51785}{150} = \text{RWF}345.2 \approx \text{RWF}345$

(2 marks)

Standard deviation = $\sigma = \frac{\sqrt{\sum f(x - \bar{x})^2}}{\sum f} = \frac{\sqrt{60225}}{150}$

$= \sqrt{401.5} = \text{RWF}20.03$ **(2 marks)**

Compact Co. Ltd.

Take home pay RWF	Mid point x	Employees f	f(x)	(x – x)	$(x - x)^2$	$(x - x)^2$
300 < 310	305	4	1220	-40	1600	6400
310 < 320	315	12	3780	-30	900	10800
320 < 330	325	16	5200	-20	400	6400
330 < 340	335	23	7705	-10	100	2300
340 < 350	345	39	13455	0	0	0
350 < 360	355	26	9230	10	100	2600
360 < 370	365	16	5840	20	400	6400
370 < 380	375	11	4125	30	900	9900
380 < 400	390	3	1170	45	2025	6075
		150	51725			50875

(2 marks)

Mean = $\bar{x} = \frac{\sum fx}{\sum f} = \frac{51725}{150} = RWF344.8 \approx RWF345$

Standard deviation = $\sigma = \frac{\sqrt{\sum f(x - x)^2}}{\sum f} = \frac{\sqrt{50875}}{150}$

$= \sqrt{339.17} = RWF18.4$ **(2 marks)**

(iii) Comment on the data presented on the histograms and the calculations.

The histograms give a good pictorial representation of the data. The first graph represents a skewed distribution to the left which would indicated that a larger proportion of employees are receiving lower pay. The second graph is a more symmetrical distribution which indicates that a larger number in this company appear to receive higher wages. The modal class for company 1is RWF320 - RWF330 while for company 2 is RWF340 - RWF350. This would tend to support the unions claim that the employees of this company are 'worse off'. However the management of the company quantify the data by calculating the mean wages received and the distribution of both. They claim that the employees of both companies earn the same mean wage and the distribution is small indicating that the unions analysis is incorrect and that the histograms distort the data.

To support their claim the union should subject the data to further analysis by calculating the median and the quartiles. These would support the union's claim that the employees of the Do-It-Better company are 'worse' off.

(4 marks)

SOLUTION 3

(i) 95% Confidence interval.

To construct a confidence interval, X and Sx are required where the confidence interval is $X \pm ZSx$. **(2 marks)**

In this case, X = RWF7,500 and $Sx = \frac{S}{\sqrt{n}} = \frac{840}{\sqrt{49}} = \frac{840}{7} = 120.0$

(2 marks)

The value of Z for the Normal tables is 1.96.

Therefore, confidence interval $7500 \pm 1.96 \times 120 = 7500 \pm 235.2$, **(2 marks)**

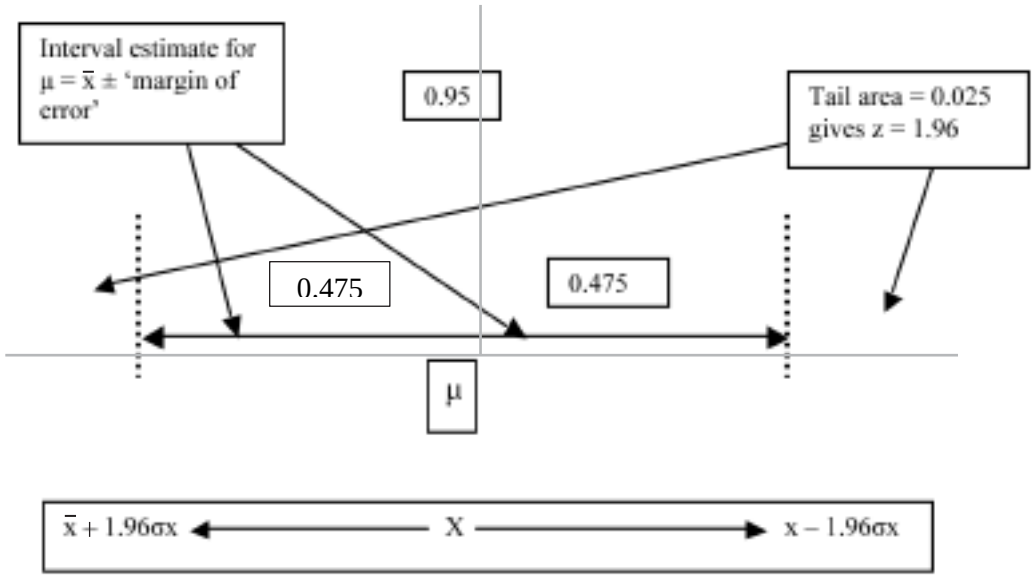
That is, 7264.8 to 7735.2.

Since RWF7,500 lies within the interval the data supports the developers claim. **(4 marks)**

(ii) The principles underpinning the calculations

Parameter such as the population mean is estimated by a sample mean, calculated from a random sample. In this case it is assumed that the sample means are randomly distributed and that the mean of the sample means is the same as the population mean. It is therefore possible to calculate probabilities about the sample means and calculate intervals that contain a given percentage of all sample means. Sample statistics are used to make inferences about population parameters. The mean and variance calculated from sample data are used to estimate the population mean and variance. The standard deviation of all sample means is the 'standard error or the mean' given by $\sigma x = \sigma/\sqrt{n}$. The difference between μ and its point estimate X is the 'sampling error. Therefore the interval that contains 95% of sample means ($n \geq 30$) is $\mu \pm 1.96\sigma x$. Since μ is unknown it is replaced by the point estimator X. The interval estimate $X \pm 1.96\sigma x$ will contain μ if the sample mean, X, is one of the 95% of X's within the interval. An interval estimate will not contain μ if it falls in either tail. The 5% of sample means that fall outside these limits are equally divided between the two tails with tail areas of 0.025; hence Z = 1.96 and the error is $1.96\sigma x$.

(4 marks)



(4 marks)

The interval μ ± 1.96σx contains 95% of all sample means, that is, we are confident that μ is somewhere within the interval.

Confidence intervals can be constructed for any level of confidence – the most common are 90%, 95%, and 99%. The upper confidence limit means that we are 95% confident that the mean is at most RWF7,735.

The lower confidence limit means that we are 95% confident that the mean is at least RWF7,264. **(2 marks)**

SOLUTION 4

(i) Derivation of linear regression relationship.

Month	Advertising X	Actual Sales, Y	XY	X2
J	25	100	2500	625
F	50	140	7000	2500
M	20	130	2600	400
A	45	120	5400	2025
M	50	50	7500	2500
J	35	170	5950	1225
J	20	120	2400	400
A	25	100	2500	625
S	20	80	1600	400
Total	290	1110	37,450	10,700

Linear regression equation: $y = a + bx$ where

$$b = \frac{\sum xy - \sum x \sum y/n}{\sum x^2 - (\sum x)^2/n}, \qquad a = \sum y/n - b\sum x/n$$

Inserting the above values: $b = \frac{37,500 - 290 \times 1110/9}{10,700 - (290)^2/9}$ **(2 marks)**

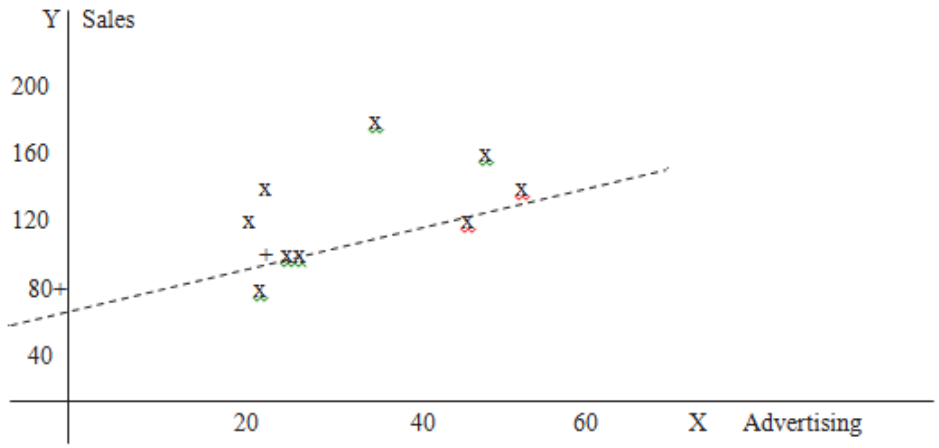
$$= \frac{37,450 - 35,767}{10,700 - 9,344} = \frac{1683}{1356} = 1.24$$

a = 1110/9 - 1.24 x 290/9

= 123.3 - 39.95 = 83.35 **(2 marks)**

y = 83.35 + 1.24x **(4 marks)**

(ii) Scatter graph and graphing the relationship.



The regression line is indicated by the 'dashed' line on the graph. There is appositive relationship between the variables – as advertising expenditure increases sales increase.

(8 marks)

(iii) Forecast sales based on advertising expenditure of RWF4,000.

The expenditure is derived from the regression equation y = 83.35 + 1.24x where

x = 40, that is, y = 83.35 + 1.24 x 40 = 132.95, that is, RWF132,950. **(4 marks)**

SOLUTION 5

(i) This is an annuity where regular payments are made into the pension fund until retirement. The means to derive the value of the is investment over a period of t years is

A0 + $A_0(1 + i) + A_0(1 + i)^2$ -------- $A_0(1 + i)^{t-1}$ where A_0 is the annual payment, I is the interest rate and t is the time period over the investment life. This is the compound interest for fixed deposits at regular intervals of time. This is a geometric series where a = A_0, r = (1 + i) to give the total value of the investment $S = a(r^{t-1})/(r - 1)$.

(3 marks)

That is, Investment Value = $20,000(1.05^{20} - 1)/(1.05 - 1)$

= $20,000(2.653 - 1)/0.05$

= RWF1,061,319 **(3 marks)**

(ii) If a series of equal withdrawals will be made in the future it is necessary to get the present value of the annuity.

This is based on the premise that the current investment V0(1 + i)t is adequate to provide for series of payments Vt at the end of t years, A0[(1 + i)t – 1]/i, where V0 is the amount invested now and A0 is the amount invested at the end of each year.

(2 marks)

Then $V_0(1 + i)t = A_0[(1 + i)t - 1]/i$;

Simplifying this expression gives $V_0 = A_0[1 - (1 + i)-t]/I$

(2 marks)

Therefore, V_0 = $20,000[1 – (1 + 0.05)-20/0.05$

= $20,000[1 - 1/1.05^{20}]/0.05$

= $20,000[1 - 1/2.653]/0.05$

= $20,000[1 - 0.3769]/0.05$

= 400,000 x 0.6231 = RWF249,240 **(4 marks)**

(iii) The present value of the annuity is RWF1,061,319. This can be described as

$V_0 = A_0[1 - (1 + i)\text{-}t]/i.$ **(2 marks)**

However, the client wants a perpetual annuity, that is, an annual payment that continues until his death without any

predetermined end time – an indefinite time limit. In this case t becomes ∞ and the expression (1 + i) becomes 0.

(2 marks)

Therefore $V_0 = A_0/i$; that is, $A_0 = V_0 i = 1{,}063{,}319 \times 0.03 =$ RWF31,839pa

(2 marks)

SOLUTION 6

Briefing note on Network Analysis.

Network analysis includes a variety of techniques which managers, specialists, accountants use to plan and manage the scheduling of a range of interrelated activities. These techniques are particularly useful in project management and multi- disciplinary projects where networks are used to monitor the progress of projects, allocate resources – money, material and personnel – where appropriate and review costs. In large projects the application of computer analysis is essential due to the size and complexity of such projects. Constructing a network by hand is not an option for the majority of projects.

A network consists of a series of activities and events. Each activity can be illustrated by an arrow. Events denoting the start and finish of each activity are illustrated by nodes (designated by circles) and are normally numbered. A network has one start and one end event, that is, all activities must be connected in one complete sequence.

A node indicates the completion of all activities leading into it. More than one activity can commence and finish at one node. Using the network diagram allows complex relationships between a range of activities to be illustrated. Therefore, a network is a diagram illustrating the relationship between a range of activities.

(4 marks)

Key information required. In order to produce a network diagram the following information must be supplied or available:

(i) A list of activities.

(ii) The interdependence between activities – the activities which precede other activities so that the logical sequence of the process becomes clear. Two columns are drawn up designated as 'Activity' and 'Preceding activity'. Each of the preceding activities leads directly into a following activity. These activities must be completed before the following activity can commence and are normally listed in a 'precedence table'.

(iii) The nodes are numbered so that, for every activity, the start event number is less than the finish event number.

(4 marks)

Occasionally the relationship between activities cannot be easily illustrated because of 'unusual' dependencies – in other words the correct precedence is not possible to produce. To implement the correct precedence and maintain the logic of the process, it is necessary to include a 'dummy' activity – this is an activity which does not require resources or time. It is used to continue the logic of the sequence without adding additional time – it has no duration.

There is substantial scope in deciding what activities precede others. This can be a major difficulty in major project planning. Some activities (where one cannot commence before another finishes) are obviously consecutive but others are concurrent and the difficulty is to determine which activities can be implemented simultaneously or concurrently. This requires experience and knowledge of the project and the ability to construct a network from the table of activities.

(2 marks)

Time calculations – earliest and latest times. The total project duration is an important factor when managing projects involving many activities. The overall duration can be calculated from the network providing the duration of each activity in the project is known. However, in order to derive the overall duration it is necessary to consider the earliest and latest times at which activities can start and end. The ‘earliest event time’ is derived by working through the project from the start position and adding an activity’s duration onto the preceding earliest event time. If two or more activities lead into an event, the earliest time on each event is calculated and the largest value is used.

The ‘latest event time’ for which an activity can start is determined by starting at the end position and working backwards through the project, that is, in the reverse direction as previously. These times are calculated by deducting the activity’s duration from the subsequent latest event time. If two or more activities start from an event the latest event time from each activity is calculated and the lowest value is used.

(4 marks)

In many projects some activities may be delayed for a variety of reasons. It may be necessary to estimate the free time or flexibility available to allow these delays to be incorporated in the network without increasing the overall time of the project. This is a float, that is, the amount of time an activity can be delayed without delaying a subsequent activity or the amount of time by which its duration can be increased. This must be done without affecting either the total project time or the time available for subsequent activities. Floats may be designated as total, free and independent – these give the amount of flexibility for a specific activity based on their effect of previous and subsequent activities.

Critical path analysis. The critical path is the sequence of activities that defines the overall duration of the project. Activities on the critical path have no flexibility if the project is to finish on time – an increase in the duration of a critical activity will increase overall project time. An increase in the duration of a non critical activity will, up to a certain limit, have no impact on total project time. Critical activities can be identified by comparing a number of the time periods in the nodes demonstrated in the example below - the earliest and latest start times are equal, the earliest and latest finish times are equal and the difference between the start and finish times equal the activity’s duration. For large projects this is easily done by computers to determine the critical path.

(4 marks)

Limitations of the method.

(i) Differences may occur about the activities that precede each other.

(ii) Duration of activities are estimates – it is unlikely that the durations of all activities are known. They are generally based on experience on similar projects, the availability and quality of staff and the level of resources available.

(iii) It is often necessary to make decisions on employing extra staff to accelerate progress of the project or specific elements of it. This increase in cost has to be off-set against potential savings which may accrue. This may arise due to unforeseen circumstances.

(2marks)

F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

EXAMINATION FORMAT REVISION QUESTIONS & SOLUTIONS

NOTES:
You are required to answer 5 questions.

(If you provide answers to all questions, you must draw a clearly distinguishable line through the answer not to be marked. Otherwise, only the first 5 answers to hand will be marked).

All questions carry equal marks.

STATISTICAL FORMULAE AND MATHEMATICS TABLES ARE PROVIDED AT BACK OF THIS PACK

TIME ALLOWED:
3 hours, plus 10 minutes to read the paper.

INSTRUCTIONS:
During the reading time you may write notes on the examination paper but you may not commence writing in your answer book.

Marks for each question are shown. The pass mark required is 50% in total over the whole paper.

Start your answer to each question on a new page.

You are reminded that candidates are expected to pay particular attention to their communication skills and care must be taken regarding the format and literacy of the solutions. The marking system will take into account the content of the candidates' answers and the extent to which answers are supported with relevant legislation, case law or examples where appropriate.

List on the cover of each answer booklet, in the space provided, the number of each question(s)attempted.

F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

QUESTIONS

Q1. Your company, CPL Products plc is considering two capital projects. These involve the purchase, use and disposal of two production machines. Machine 1 costs RWF50,000 and Machine 2 costs RWF45,000. The net cash flows of both machines are set out in the following table. Provide a report to the company on the most economical purchase. Your report should include the Net Present Value and the Internal Rate of Return (IRR) for each machine. The bank will provide an overdraft at a rate of 12% to the company.

Year	Net Cash Flows RWF	
	Machine 1	Machine 2
1	25,000	13,000
2	24,000	15,000
3	16,000	22,000
4	15,000	35,000

[Total: 20 Marks]

Q2. The Equality Officer carried out a survey in 2010 of the income of supervisory and middle management staff in a number of SMEs by their highest educational qualification. The data is set out below.

Weekly Income RWF	Educational Qualification	
	3rd Level	2nd Level
400 – 500	5	20
500 – 600	6	30
600 – 700	10	25
700 – 800	18	15
800 – 900	21	5
900 – 1000	14	3
1000 – 1100	10	1
1100 - 1200	16	1

You are required to

(i) Calculate the mean and standard deviation for those with 3rd and 2nd level qualifications. **(12 Marks)**

(ii) Compare the two distributions and comment on your results. **(8 Marks)**

[Total: 20 Marks]

Q3. DIB Ltd produces specialised glass panels for stores and shops. It is necessary that the panels have sufficient thickness to avoid accidental breakage. The panels are produced with a mean thickness of 4 cms and a standard deviation of 1 cm. The quality control department takes a large number of samples of 100 from the production line. To confirm the quality of the product you are asked to provide the following:

(i) The 95% and 99% confidence intervals and confidence limits for the panels. **(10 Marks)**

(ii) An explanation, using diagrams, of what these confidence limits mean. **(10 Marks)**

(Total 20 Marks)

Q4. The following data was produced by the management accountant to demonstrate the effectiveness of the internal audit function. He claimed that the number of errors discovered in company processes reduced as the frequency of audits increased.

Number of errors (y)	Number of audits (x)
19	1
18	2
16	3
16	4
20	5
13	6
6	7
6	8
11	9
9	10

By means of this data:

(i) Confirm the management accountants claim by calculating the correlation coefficient between the number of errors and the number of audits.

(8 Marks)

(ii) Calculate a linear equation based on the data.

(6 Marks)

(iii) Explain your result and the basis of his claim.

(6 Marks)

(TOTAL 20 MARKS)

Q5. As a consultant to CPAR Ltd. you are continuing to address a diverse range of business problems. Provide advice to our clients on the following problems.

(i) A small business wishes to set up a pension plan for its employees. You are aware that personal pensions are calculated on the basis of annual annuities. Calculate how much should be paid as a lump sum now to get an annuity of RWF20,000 for 6 years at a discount rate of 5%.

(8 Marks)

(ii) The company wishes to purchase a van for the business and is quoted an annual percentage rate (APR) of interest of 20% on a loan of RWF15,000 over 3 years. Calculate the total interest paid over three years assuming interest is paid at the end of each year.

(6 Marks)

(iii) The leasing company states that the tyres on the van should last for more than 45,000 miles. The tyre production company suggests that the mean life of the tyre is 42,000 miles with a standard deviation of 4,000 miles. Tyre life is normally distributed. Calculate the % of tyres that will last more than 45,000 miles.

(6 Marks)

[Total: 20 Marks]

Q6. You are a member of the management team who has attended a seminar on modern business techniques including the use of business mathematics and quantitative methods. The following terms have been used during presentations at the seminar – 'Statistical Hypotheses', 'The Normal Distribution', 'Internal Rate of Return', 'The appropriate selection of a Base Period in Index Numbers'.

Provide a brief outline on the meaning of these terms for the management team.

[Total: 20 Marks]

END OF PAPER

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SUGGESTED SOLUTIONS

SOLUTION 1

(i) Net Present Value for each proposal.

The scrap values for the machines are included in the 'Net Cash Flows' and should not be further considered.

A discount value of 20% is included in the table to assist in the calculation of the internal rate of return

Cash flows for Machine 1.

Year	Initial cost	Net Cash Flows	PV Factor @ 12%	PV RWF	PV Factor @ 20%	PV RWF
0	50,000	(50,000)	1.000	(50,000)	1.000	(50,000)
1		25,000	0.893	22,325	0.833	20,825
2		24,000	0.797	19,128	0.694	16,656
3		16,000	0.712	11,392	0.579	9,264
4		15,000	0.636	9,540	0.482	7,230
NPV				12,385		3,975

4 Marks

Year	Initial cost	Net Cash Flow	PV Factor @ 12%	PV RWF	PV Factor @ 20%	PV RWF
0	45,000	(45,000)	1.000	(45,000)	1.000	(45,000)
1		13,000	0.893	11.609	0.833	10,829
2		15,000	0.797	11,955	0.694	10.410
3		22,000	0.712	15,664	0.579	12,738
4		35,000	0.636	22,260	0.482	16,870
NPV				12,385		5,847

4 Marks

Due to the higher net present value, with this analysis, Machine 2 represents the best proposal for the company.

(ii) Internal rates of return.

The IRR can be derived by calculation or graphically. Using the formula

$\frac{N_1I_2 - N_2I_1}{N_1 - N_2}$, where discount rate I_1 gives NPV N_1 and discount rate I_2 gives NPV N_2.

Machine 1. N1 = RWF12,385, I1 = 12%; N2 = RWF3,975, I_2 = 20%

$$IRR = \frac{12,385 \times 0.2 - 3,975 \times 0.12}{12,385 - 3,975} = \frac{2000}{8410} = 23.8\%$$

4 Marks

Machine 2. N1 = RWF16,488, I1 = 12%; N2 = RWF5,847, I_2 = 20%

$$IRR = \frac{16,488 \times 0.2 - 5,847 \times 0.12}{16,488 - 5,857} = \frac{2,596}{10,641} = 24.4\%$$

4 Marks

Summary.

	NPV RWF	IRR %
Machine 1	12,385	23.8
Machine 2	16,488	24.4

Machine 2 is the most profitable investment for the company. This is confirmed by both the values of NPV and IRR. Both projects give a return substantially greater than the cost of the overdraft facility provided by the bank **4 Marks**

[Total: 20 Marks]

SOLUTION 2.

(i) The mean and standard deviation.

To compare the two groups in part (ii) it is necessary to derive the mean and standard deviation for both groups.

3rd level qualifications.

Class Interval	f	Mid point x	fx	$x - \bar{x}$	$(x - x)^2$	$f(x - x)^2$
400 – 500	5	450	2,250	- 406	164,836	824,180
500 – 600	6	550	3,300	- 306	93,636	561,816
600 – 700	10	650	6,500	- 206	42,436	424,360
700 – 800	18	750	13,500	- 106	11,236	202,248
800 – 900	21	850	17,850	- 6	36	756
900 – 1000	14	950	13,300	94	8,836	123,704
1000 – 1100	10	1,050	10,500	194	37,636	376,360
1100 - 1200	16	1,150	18,400	294	86,436	1,382,976
∑	100		85,600			3,896,400

Mean = $\bar{x} = \frac{\sum fx}{\sum f} = \frac{85,600}{100} = 856$ **2 Marks**

Standard Deviation.

$\sigma = \frac{\sqrt{\sum f(x - \bar{x})^2}}{\sum f} = \frac{\sqrt{3,896,400}}{100}$

$= 38964$

$= 197.4$ **3 Marks**

2nd level qualifications

Class Interval	f	Mid point x	fx	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
400 – 500	20	450	9,000	- 173	29,929	598,580
500 – 600	30	550	16,500	- 73	5,329	159,870
600 – 700	25	650	16,250	27	729	18,225
700 – 800	15	750	11,250	127	16,129	241,935
800 – 900	5	850	4,250	227	51,529	257,645
900 – 1000	3	950	2,850	327	106,929	320,787
1000 – 1100	1	1,050	1,050	427	183,329	182,329
1100 - 1200	1	1,150	1,150	527	277,729	277,729
∑	100		62,300			2,057,100

Mean = $\bar{x} = \frac{\sum fx}{\sum f} = \frac{62,300}{100} = 623$ **2 Marks**

Standard Deviation.

$\sigma = \frac{\sqrt{\sum f (x - \bar{x})^2}}{\sum f} = \frac{\sqrt{2,057,100}}{100}$

$= \sqrt{20,571} = 143.4$ 3 Marks

(ii) To compare the two groups the most appropriate method is to use the coefficient of variation. This measures the relative dispersion of the two groups.

C of $V_{3rd\ level} = \sigma / \bar{x} = 197.4/856 = 23.06\%$

C of $V_{2nd\ level} = \sigma / \bar{x} = 143.4/623 = 23.01\%$

5 Marks

The co-efficient of variation of both distributions is similar. The data with the highest coefficient of variation has the greatest relative dispersion. In this case the relative dispersion between both groups is minimal. Both distributions are skewed showing that a higher number of third level graduates have a greater income. The median would be a more representative average in this particular case.

5 Marks

(Total: 20 Marks)

SOLUTION 3.

i) The general form of a confidence interval is

$x = \mu \pm z(\sigma/\sqrt{n})$, where

$z1 = 1.96$ for the 95% confidence interval

$z2 = 2.58$ for the 99% confidence interval

and $\mu = 4$, $\sigma = 1$, $n = 100$.

95% confidence interval: $x1 = 4 \pm 1.96\,(1/\sqrt{100}) = 4 \pm 0.196$

95% confidence limits are: 4.196 and 3.804. **5 Marks**

99% confidence interval: $x2 = 4 \pm 2.58\,(1/\sqrt{10}) = 4 \pm 0.258$

99% confidence limits are: 4.258 and 3.742. **5 Marks**

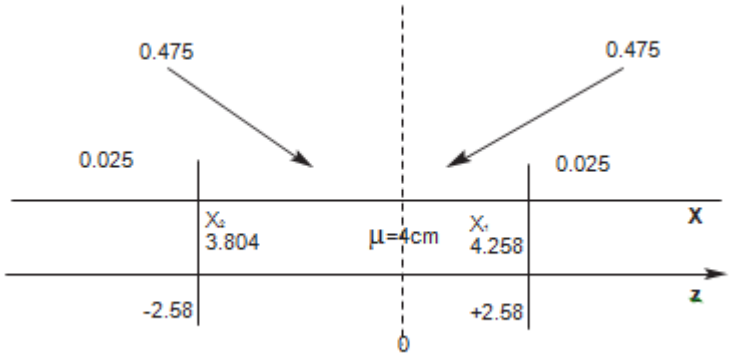
(ii) A confidence interval is a range of values within which a certain level of confidence (95% or 99%) can be stated, that is, a range within which a particular value of a variable will lie. The confidence interval for a sample mean is the range of values

around the population mean (μ) within which it can be stated, normally with 95% or 99% confidence, that a particular sample mean ($\bar{x}$) will lie.

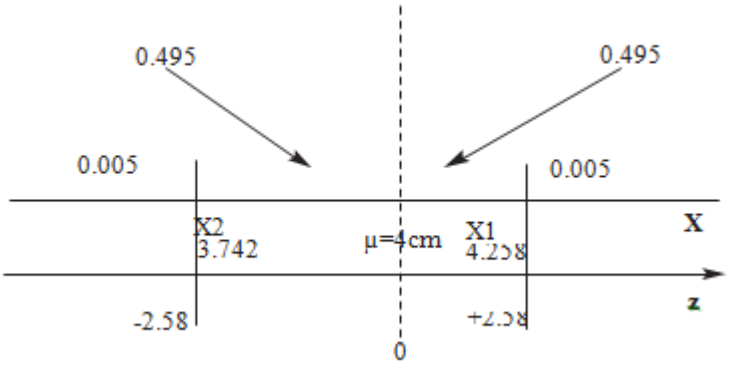
5 Marks

The above problem is represented by the following diagrams.

95% confidence interval.



95% confidence interval.



5 Marks

(Total: 20 Marks)

SOLUTION 4.

(i) Correlation co-efficient is derived form the formula:

$$r = \frac{\sum xy/n - \sum x/n \sum y/n}{\sqrt{\{\sum x^2/n - (\sum x/n)^2\}\{\sum y^2/n - (\sum y/n)^2\}}}$$

Therefore, $$r = \frac{624/10 - 55/10 \times 134/10}{\sqrt{385/10 - 30.25)(2040/10 - 179.56)}}$$

$$= -11.3/14.07 = -0.8$$

(ii) The relationship can be demonstrated by developing a linear regression equation and analysing it.

Number of errors y	Number of audits x	x^2	xy	y^2
19	1	1	19	361
18	2	4	36	324
16	3	9	48	256
16	4	16	64	256
20	5	25	100	400
13	6	36	78	169
6	7	49	42	36
6	8	64	48	36
11	9	81	99	121
9	10	100	90	81
134	55	385	624	2040

A linear regression equation may be written as: $y = a + bx$ where

$\sum y = na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$; where

$$a = \frac{\sum y}{n} - \frac{b\sum x}{n}$$

$$b = \frac{n\sum xy - \sum x\sum y,}{n\sum x^2 - (\sum x)^2}$$

Inserting values gives

$$b = \frac{10 \times 624 - 55 \times 134}{10 \times 385 - 552}$$

$$= \frac{6240 - 7370}{3850 - 3025} = \frac{-1130}{825} = -1.37$$

$$a = \frac{134}{10} - \frac{(-1.37) \times 55}{10} = 13.4 + 7.54 = 20.94$$

Therefore, $y = 20.94 - 1.37x$. **6 Marks**

(iii) The regression equation shows that the number of errors reduce as the number of audits increases – this is demonstrated by the negative slope of the regression equation and confirmed by the correlation co- efficient. This show that every time the independent variable x rises by one unit, the dependent variable y falls by 1.37 units, that is, as the number of audits increase the number of errors reduce. A graph of the variables will show this obvious relationship.

6 Marks

(Total: 20 Marks)

SOLUTION 5.

(i) The beneficiary will receive RWF20,000 per year for 6 years. It is necessary to find the present value (PV) of these amounts.

$$PV = 20{,}000 \times 1/(1 + 0.05)^1 + 20{,}000 \times 1/(1 + 0.05)^2 +$$
$$20{,}000 \times 1/(1 + 0.05)^3 + 20{,}000 \times 1/(1 + 0.05)^4 +$$
$$20{,}000 \times 1/(1 + 0.05)^5 + 20{,}000 \times 1/(1 + 0.05)^6$$

This can be calculated for each year or use geometric progression formula where

PV of annuity $= S_n = a \times (1 - r^n)/(1 - r)$ and $a = 20{,}000/ 1.05$, $r = 1/1.05$

Therefore, $PV = 20{,}000/1.05 \times 1 - (1/1.05)^6 / 1 - (1/1.05)$

$= 19{,}047.6 \times (1 - 0.677)/(1 - 0.952)$

$= 19{,}047.6 \times 6.729$

$=$ RWF128,171

8 Marks

(ii) The interest will be compounded over the three years. The compound interest is derived as follows

$CI = P(1 + i)^t - P$ where I = 20%, P = RWF15,000, t = 3.

$CI = 15{,}000(1 + 0.2)^3 - 15{,}000$

$= 15{,}000 \times 1.728 - 15{,}000$

$=$ 25,920 - 15,000 = RWF10,920 **6 Marks**

(iii) To find the probability

$$\text{Prob}\,(x > 45{,}000),\ z = \frac{x1 - \mu}{\sigma} = \frac{45{,}000 - 42{,}000}{4{,}000} = 0.75$$

From the normal tables, z = 0.75 gives 0.7734, that is, the area in the right hand tail is

1 – 0.7734 = 0.2266.

Nearly 23% of tyres will last for more than 45,000 miles. **6 Marks**

(Total: 20 Marks)

SOLUTION 6.

An explanation of the terms is provided below.

Statistical Hypotheses. A statistical hypothesis is an assertion about an attribute of a population which may also concern the type or nature of a population. To develop procedures for testing statistical hypotheses it is essential to know exactly what to expect when an hypothesis is true and we often hypothesize the opposite of what we hope to prove. For example if we wish to show that one method of teaching is more effective than another, we would hypotesize that the two methods are equally effective. Since we hypothesize that there is no difference in the two teaching methods, these hypotheses are called 'null hypotheses' and are denoted by Ho. The term 'null hypothesis' is used for any hypothesis that is set up primarily to see whether it can be rejected. The hypothesis that is used as an alternative to the null hypothesis, that is, the hypothesis that is accepted when the null hypothesis is rejected is the 'alternative hypothesis' and is denoted by H1. It must always be formulated with the null hypothesis otherwise we would not know when to reject Ho. Alternative hypotheses usually specify that the population mean (or whatever other attribute may be of interest) is less than, greater than, or not equal to the value assumed under the null hypothesis. For any given problem, the choice of one of these alternatives depends on what we hope to be able to show, or where we want to put the burden of proof.

5 Marks

The Normal Distribution. Among many different continuous distributions used in statistics the most important is the normal distribution. It plays a very important role in the science of statistical inference since many business phenomena generate random variables with probability distributions that are well approximated by a normal distribution. The graph of a normal distribution is a bell-shaped curve that extends indefinitely in both directions. An important feature of normal distributions is that they depend only on two quantities μ and σ, that is, the mean and standard deviation. There are different curves depending on the value of μ and σ In the majority of work undertaken in this area the concern is with the 'standard normal distribution'. The standard normal distribution is a normal distribution with $\mu = 0$ and $\sigma = 1$. Areas under any normal curve are obtained by performing a change of scale that converts the units of measurement from the original scale, or x-scale, into standard units or 'z-scores' by means of the formula $z = x - \mu / \sigma$ In this new scale a value of z states how many standard deviations the corresponding value of x lies above or below the mean of its distribution. These values are obtained from standard normal tables to enable calculations to be easily performed.

5 Marks

Internal Rate of Return. The internal rate of return (IRR) or the Yield is an alternative method of investment appraisal to Net Present Value. It can be described as the rate that a project earns. The decision rule when using IRR is that a firm should undertake a project if the annualised return in the form of the IRR is greater that the annual cost of capital (the rate of interest). If the IRR is less than the interest rate used to calculate NPV then the project would not be undertaken. In other words, if the IRR is less than the annual cost of capital then the company should avoid such capital expenditure. There is no precise formula for calculating the IRR of a given project. However, it can be estimated either graphically or by formula. Both of these techniques need the NPV calculated using two different discount rates. An advantage of using the IRR is that it does not depend on any external rates of interest. A disadvantage however is that it returns a relative (percentage) value and does not differentiate between the scale of projects, that is, one project could involve substantially larger cash flows than another. This could be of significance for some project comparisons.

5 Marks

The Appropriate Selection of a Base Period in Index Numbers. An index number measures the percentage change in value of an item relative to its value at a predetermined historical point known as the

'base period'. At this base period the index number is equal to 100 and all subsequent index numbers are calculated as percentages of that base period. In a business application the choice of base period is very important. The base year should be a 'normal' representative period when no abnormal changes have occurred. If the period is abnormal, then future

periods of relatively minor changes in price or quantity will be very difficult to reflex in the index number; then then cover up any significant change in the value of the business variable being considered. It would be in appropriate to measure a variable such as the change in house prices if the base year selected was in the late 1980s as the boom in house prices over the past decade represented a time of boom for this sector of the economy. When making comparisons over long periods of time index numbers can become too large, too small or too similar to be meaningful. It is appropriate therefore, at times to reset the base period. If a comparison of index numbers is required over a period of time where there has been a change of base, appropriate calculations are made to reflect the change.

5 Marks

F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

EXAMINATION FORMAT REVISION QUESTIONS & SOLUTIONS

NOTES:
You are required to answer 5 questions.

(If you provide answers to all questions, you must draw a clearly distinguishable line through the answer not to be marked. Otherwise, only the first 5 answers to hand will be marked).

All questions carry equal marks.

STATISTICAL FORMULAE AND MATHEMATICS TABLES ARE PROVIDED AT BACK OF THIS PACK

TIME ALLOWED:
3 hours, plus 10 minutes to read the paper.

INSTRUCTIONS:
During the reading time you may write notes on the examination paper but you may not commence writing in your answer book.

Marks for each question are shown. The pass mark required is 50% in total over the whole paper.

Start your answer to each question on a new page.

You are reminded that candidates are expected to pay particular attention to their communication skills and care must be taken regarding the format and literacy of the solutions. The marking system will take into account the content of the candidates' answers and the extent to which answers are supported with relevant legislation, case law or examples where appropriate.

List on the cover of each answer booklet, in the space provided, the number of each question(s)attempted.

F1.1 BUSINESS MATHEMATICS & QUANTITATIVE METHODS

FOUNDATION 1

QUESTIONS

Q1. As the Management Accountant to a company, you are assessing a purchasing proposal for the Production Department. The Manager has proposed the purchase of a machine for RWF50,000. He projects that the cash inflows and costs shown in the following table will arise:

Year	Sale Revenues RWF	Maintenance Costs RWF
1	10,000	500
2	15,000	500
3	20,000	1,000
4	15,000	1,000
5	10,000	1,500
6	5,000	2,000

The machine can be sold after 6 years for RWF10,000. All cash flows are at the end of the year. The cost of capital to the company is 10%. To assess the proposal you are required to:

(i) Calculate the net present value of the project **(8 Marks)**

(ii) Calculate the discounted payback period. **(6 Marks)**

(iii) Provide an explanation of these terms and the most appropriate method of appraisal of a project of this type.

(6 Marks)

(Total: 20 Marks)

Q2. The percentage annual growth of 50 small pension funds over the year 2011 was collected by the Pensions Board.

The frequency table of the data is presented below. For a company management meeting you are required to perform the following:

(i) Draw a histogram and cumulative frequency graph of the data. **(10 Marks)**

(ii) Estimate the median growth in the funds from either of the graphs and compare it to the calculated median. **(6 Marks)**

(iii) Comment on the value of the graphs for presentation purposes. **(4 Marks)**

% Growth per annum	Frequency
$0 < X < 4$	6
$4 < X < 6$	12
$6 < X < 8$	11
$8 < X < 10$	9
$10 < X < 12$	5
$12 < X < 14$	4
$14 < X < 16$	3

(Total: 20 Marks)

Q3. As the Manager of DIY Manufacturing Ltd. you use sample statistics to make inferences about population parameters. You have been assigned the task of confirming that the production of its miniature batteries is statistically acceptable. The company claims that the average weight of the batteries is 30 grams. The batteries are filled with powder to an average weight of 29.5 grams with a standard deviation of 2.1 grams. From a random sample of 36 batteries:

(i) Calculate the probability that the average weight is 30 grams or more. **(8 Marks)**

(ii) Calculate the limits within which 95% of all batteries weigh. **(8 Marks)**

(iii) If the sample size was smaller than 36 how would this affect the results you derived in (i) and (ii). **(4 Marks)**

(You are required to use appropriate diagrams to support your calculations)

(Total: 20 Marks)

Q4. Over the past 5 years the % level of unemployment has been tracked by the Statistics Office in Rwanda and is set out in the table below. Political opponents claim that the underlying trend is increasing and is disguised by the seasonal nature of the data. In order to present an accurate assessment of the data you are required to:

(i) Smooth the data by means of a four quarterly moving average and briefly describe the method used.

(10 Marks)

(ii) Plot the trend on a graph. **(6 Marks)**

(iii) Comment on the claim that the "underlying trend is increasing". **(4 Marks)**

Year	March	June	September	December
2005	2.0	2.1	2.4	2.4
2006	2.6	3.0	3.4	3.6
2007	3.6	3.2	3.3	2.9
2008	2.5	2.7	2.5	2.1
2009	2.1	2.3		

(Total: 20 Marks)

Q5. As a Financial Consultant for CPA International Investments you are required to provide advice to the company's clients on the following:

(i) Client A wishes to invest in a supplementary pensions product to support his retirement income when he retires in 10 years time. He wishes to invest a lump sum each year of RWF2,000. He expects to receive an interest rate of 6% per annum. Advise him on the value of the fund at the end of the 10 year period.

(8 Marks)

(ii) Client B has been offered the opportunity to invest in a property development. He believes that the average rental income for an apartment is at most RWF10,000 per year. A random sample of 36 apartments gives a mean rent of RWF10,200. The sample standard deviation is RWF1,450. You are asked to calculate a 90%

confidence interval for the mean annual rental income. Do the sample results support Client B's belief?

(6 Marks)

(iii) The Superior Products company will only supply jeans when a price greater than RWF25 per unit is available.

It will increase output by 2 units for every unit increase in price. Write the equation of the supply function for the company so that it can estimate the units to produce when the price is RWF80.

(6 Marks)

(Total: 20 Marks)

Q6. "Without doubt an index is an extremely fashionable tool". In the context of this statement explain the following terms with regard to index numbers.

- Indices – their use and construction.
- An expenditure index.
- A price index.
- A volume index.
- The base period.

(Total: 20 Marks)

END OF PAPER

SUGGESTED SOLUTIONS

SOLUTION 1

(i) Net Present Value of the project.

The cash inflows, cash outflows and net cash flow is provided in the following table. Applying the discount factor of 10% provides a Net Present Value of RWF7,054.5. On this basis the project is viable and should proceed.

Year	Cash Outflow RWF	Cash Inflow RWF	Net Cash Flow	Discount Factor @ 10%	Discounted Cash Flow	Discounted Cash Outflow	Discounted Cash Inflow
0	50,000	----	(50,000)	1.000	(50,000.0)	(50,000)	-----
1	500	10,000	9,500	0.909	8,635.5	(454.5)	9,090.0
2	500	15,000	14,500	0.826	11,977.0	(413.0)	12,390.0
3	1,000	20,000	19,000	0.751	14,269.0	(751.0)	15,020.0
4	1,000	15,000	14,000	0.683	9,562.0	(683.0)	10,245.0
5	1,500	10,000	8,500	0.621	5,278.5	(931.5)	6,210.0
6	2,000	5,000	13,000	0.564	7,332.0	(1128.0)	8,460.0
		10,000					
NPV					7,054.5	54,360.0	61415.0
	(2 Marks)	(2 Marks)			(4 Marks)	(2 Marks)	(2 Marks)

(ii) Discounted payback is the time it takes for the discounted incoming cash flow to recover (or break even with) the discounted outgoing cash flows. It is regarded as the time it takes for the project to become profitable. In the present case the breakeven occurs in the first quarter of year 6. This is in agreement with the NPV.

(6 Marks)

(iii) Explanation and comparison of the terms.

The NPV measures the added value to the company of undertaking the project. It is calculated by using a rate of interest equal to the rate of return that the company would expect to pay to finance the project. NPV measures the overall profit of the project. The risk of the project is taken into account by means of the cost of capital employed. Cash flows should, therefore, not include interest payments as these are emplicitely taken into account in the discounting process. If cash inflows occur later than cash outflows, the cash outflows, the cash inflows will be discounted more in the NPV calculations.

Discounted payback is often useful if capital is scarce. A project which becomes profitable earlier may be considered better than one which becomes profitable later. It allows capital / loans to be repaid and other investments to be processed. A project which takes longer but which repays capital more slowly might have a higher NPV> however, if the cost of capital used in the NPV calculations is correct, it should not be of concern when the project becomes profitable. Later cash flows are discounted more. However, it may be better to use a higher interest rate for discounting cash flows from projects that take a greater length of time. This is a better way to take time preferences into account than using discounted payback. It is often considered useful to know the discounted payback period for the process of financial planning.

'Payback period' is the same as discounted payback except that cash flows are not discounted. The discounted payback period for investment appraisal is not often used. It is a method, however, of ensuring the preference for early cash flows over later cash flows is taken into account but is better to do so using the NPV approach.

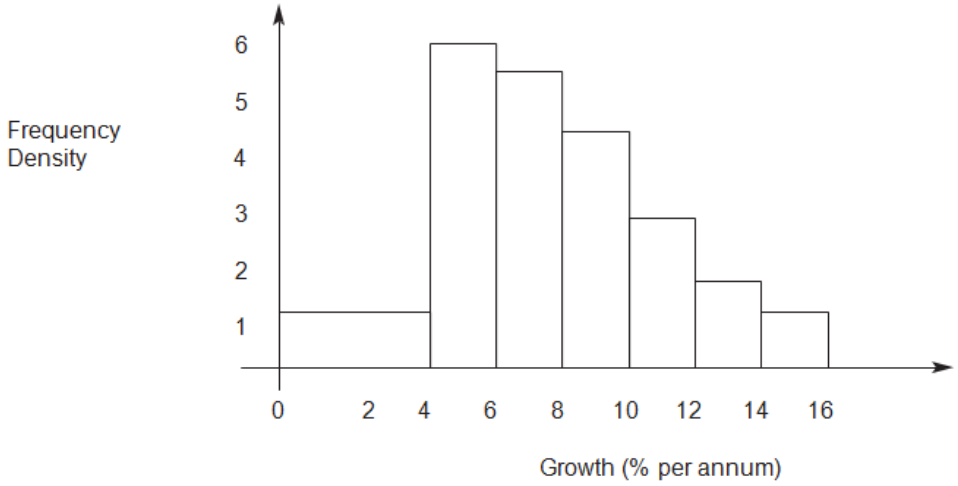
(6 Marks)

(Total: 20 Marks)

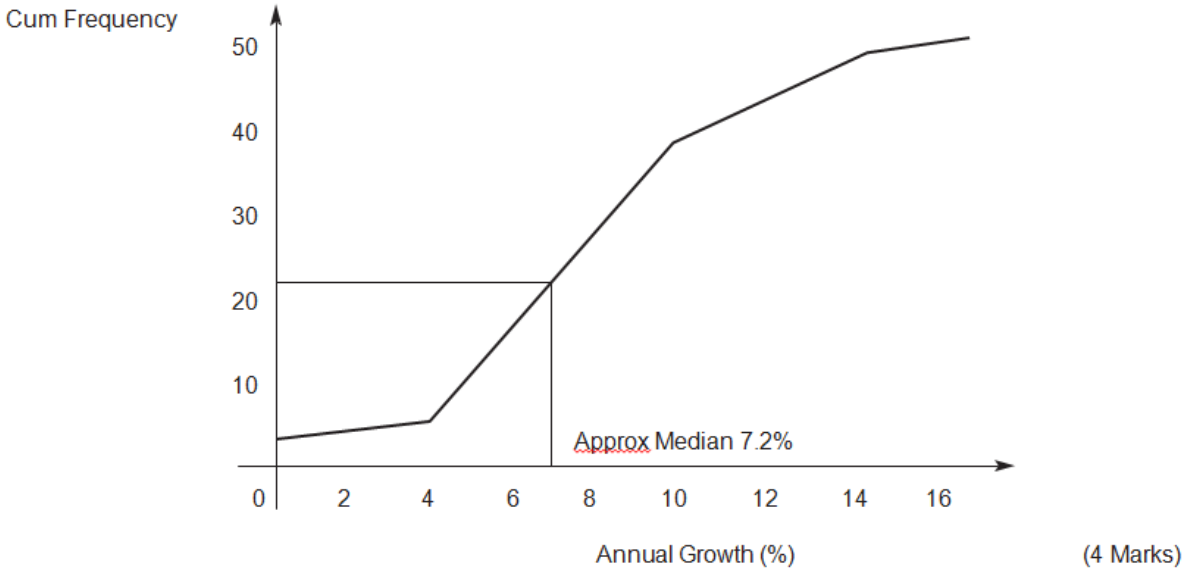
SOLUTION 2

(i) A histogram and cumulative frequency graph.

% Growth per annum	Frequency	Cumulative frequency	Frequency density
$0 \leq X \leq 4$	6	6	1.5
$4 \leq X \leq 6$	12	18	6.0
$6 \leq X \leq 8$	11	29	5.5
$8 \leq X \leq 10$	9	38	4.5
$10 \leq X \leq 12$	5	43	2.5
$12 \leq X \leq 14$	4	47	2.0
$14 \leq X \leq 16$	3	50	1.5



(4 Marks)



(4 Marks)

(ii) The median can be calculated or derived from the graph and is approx. 7.2%.

(4 Marks)

The calculated value of median is: $(N + ½)^{th}$ observation. Since there are 18 observations up to this interval, the

$25½^{th}$ value is the $7½^{th}$ value in the interval.

Therefore, median is 6 + (7.5/11) x 2 = 7.36%. **(4 Marks)**

(iii) Value of graphs for presentation purposes.

Cumulative frequency tables are suitable for discrete and continuous data and show the number of observations below the end of the current interval. This allows us to see that 29% of funds (the majority) reported levels of growth below 8%. A bar chart or histogram is an effective way of displaying discrete or categorical data. In a histogram frequency is represented by area rather than height. For this reason it is not necessary to have intervals of equal width. Histograms depict continuous data so the rectangles are drawn touching each

other. The vertical scale represents 'frequency density' (frequency / class width) rather than frequency otherwise the first bar would give a misleading representation of growth if actual frequencies were used rather than relative frequencies.

The histogram gives a better visual representation of the data for presentation.

The 'ogive' gives a greater facility for calculating a range of values, eg. How many funds had growth in excess of 12% etc. or what is the interquartile range (measure of dispersion not affected by extreme values) 9.9% - 4.3% = 4.6%.

(4 Marks)

[Total: 20 Marks]

SOLUTION 3

(i) The question asks for the probability that the mean is greater than or equal to 30; $P(x \geq 30)$.

Since the sample size is greater than 30, the sample is normally distributed. Therefore, the mean and standard error of the distribution of mean is

$\mu x = \mu = 29.5$

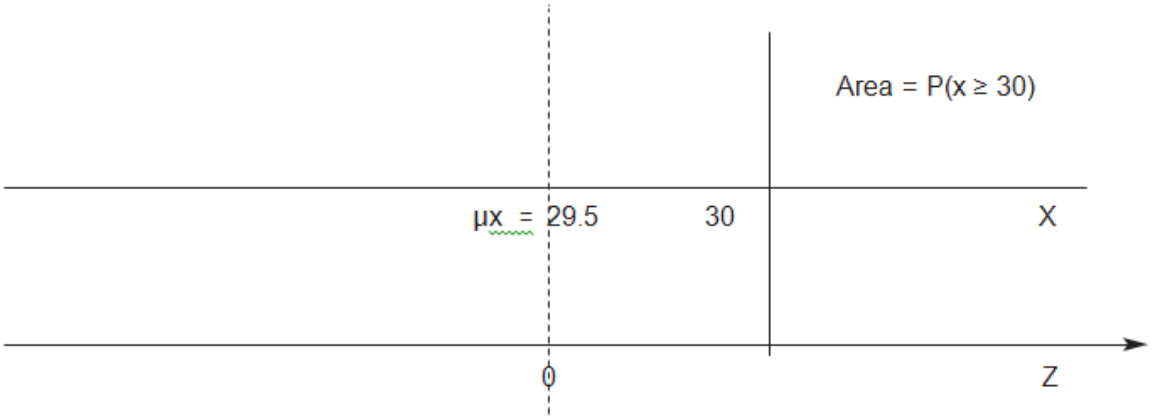
$\sigma x = \sigma/\sqrt{n} = 2.1/\sqrt{36} = 2.1/6 = 0.35$ **(2 Marks)**

Therefore, for x = 30, $z = \frac{30 - 29.5}{0.35} = 1.43$ **(2 Marks)**

From the Normal tables, the area in the tail of the distribution = 0.0764.

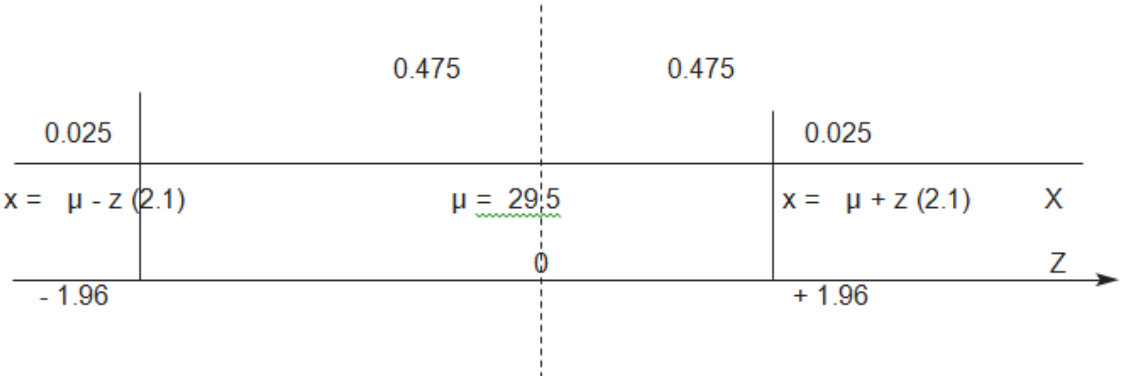
This is the required probability: $P(x \geq 30gm) = 0.0764 = 7.64\%$ **(2 Marks)**

The appropriate Normal Distribution is



(2 Marks)

(ii) Two limits that contain 95% of weights; σ = 2.1



(2 Marks)

Since we are dealing with individual values we use σ. **(2 Marks)**

From the Normal tables, the z value for the tail area 0.025 is + 1.96.

The number of units from the mean is z x σ = 1.96 (2.1) = 4.116 gms. **(2 Marks)**

Hence the two limits are $\mu \pm z\sigma$ = 29.5 ± 4.116

That is, 25.384 to 33.616. The weights of 95% of batteries are within these limits.

(2 Marks)

(iii) If the sample size is smaller than 36, this could involve the distribution for small samples – Student's distribution where it cannot be assumed that the population is normally distributed. However, if the assumption is made that the smaller population is normally distributed, smaller samples give a smaller standard error of the mean.

In (i) above the larger standard error will result in a smaller value of z giving a larger value of probability.

(2 Marks)

In (ii) since the question does not involve samples but individual batteries, there is no change.

(2 Marks)

[Total: 20 Marks]

SOLUTION 4

Year	March	June	September	December
2005	2.0	2.1	2.4	2.4
2006	2.6	3.0	3.4	3.6
2007	3.6	3.2	3.3	2.9
2008	2.5	2.7	2.5	2.1
2009	2.1	2.3		

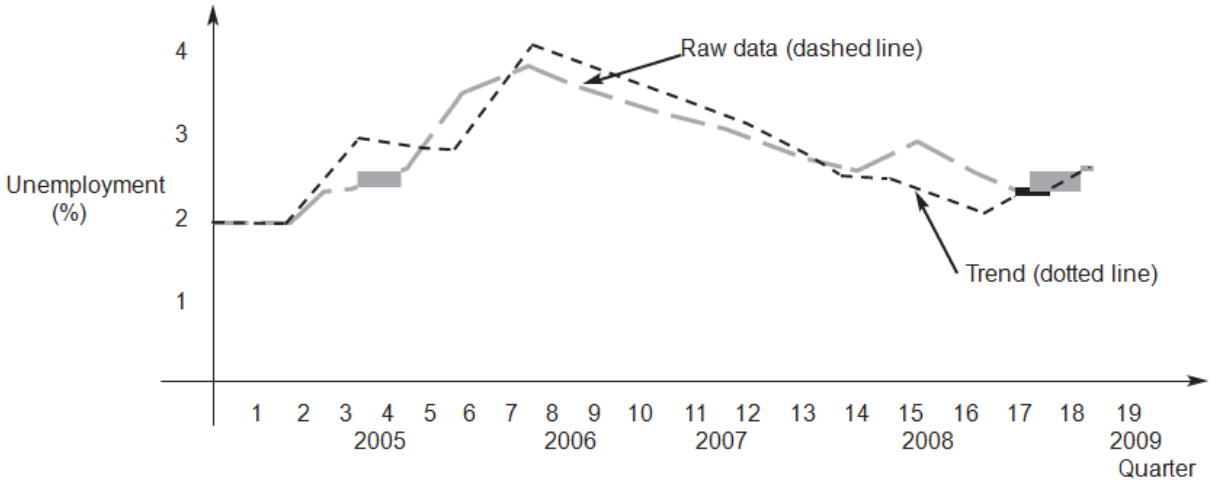
(i)

Year	Quarter	Unemployment	Annual Total	Pair Total	Trend
2005	M (1)	2.0			
	J (2)	2.1			
	S (3)	2.4	8.9	18.4	2.4250
	D (4)	2.4	9.5	19.9	2.4875
2006	M (1)	2.6	10.4	21.8	2.7250
	J (2)	3.0	11.4	24.0	3.0000
	S (3)	3.4	12.6	26.2	3.2750
	D (4)	3.6	13.6	27.4	3.4250
2007	M (1)	3.6	13.8	27.5	3.4375
	J (2)	3.2	13.7	26.7	3.3375
	S (3)	3.3	13.0	24.9	3.1125
	D (4)	2.9	11.9	23.3	2.9125
2008	M (1)	2.5	11.4	22.0	2.7500
	J (2)	2.7	10.6	20.4	2.5500
	S (3)	2.5	9.8	19.2	2.4000
	D (4)	2.1	9.4	18.4	2.3000
2009	M (1)	2.1	9.0		
	J (2)	2.3			
			(2 Marks)	(2 Marks)	(2 Marks)

The trend is calculated as the centred moving average and is plotted on the graph below. The four point moving average is taken as the natural time period of the series and data is quarterly. The centred moving average therefore gives the trend.

(2 Marks)

(ii)



(6 Marks)

The graph shows a seasonal affect for the first two years where it peaks at the end of 2006 and beginning of 2007. It tends to peak again at the beginning of 2008 but the trend is downwards for 2009. It is difficult to discriminate between the raw data and the trend since the variation in the percentage data is so small.

(2 Marks)

(iii) Although the raw data shows an increase in the trend over the period, the data for 2009 is reducing. The trend indicates that the data peaked over the period 2007/2008 and since that time shows a reducing or downward trend. The claim that "the underlying trend is increasing" cannot be supported by the data.

4 Marks

[Total: 20 Marks]

SOLUTION 5

(i) This payment is an annuity – a series of equal deposits made at equal intervals of time. The principle (P) of the investment is

Year 1 = RWF2,000

Year 2 = RWF2,000 (1 + 0.06) + RWF2,000

Year 3 = RWF2,000 (1 + 0.06)2 + RWF2,000 (1 + 0.06) + RWF2,000, etc.

(3 Marks)

This series is a geometric progression of the form $\frac{Sn = a(r^n - 1)}{r - 1}$

For 10 years, the value is $\frac{RWF2,000[(1 + 0.06)^{10} - 1]}{0.06}$ **(3 Marks)**

$= \frac{RWF2,000(1.7908 - 1)}{0.06}$

$=$ RWF26,360 **(2 Marks)**

(ii) To construct the confidence interval you require:

X, sx = s/√n, z (90%) **(2 Marks)**

The 90% confidence interval is $x \pm zs_x$, where x = RWF10,200, z = 1.6449,

$s_x = 1450/\sqrt{36} = 1450/6 = 241.6$

Hence, the 90% confidence interval is RWF10,200 ± 241 **(2 Marks)**

That is, RWF9,959 to RWF10,441. You are 90% confident that the mean lies within this range.

Therefore since the confidence interval contains RWF10,000 your client's belief is confirmed. **(2 Marks)**

(iii) The general form of a linear equation to represent this supply function is

$Y = aX + b$ where Y = Quantity and X = Price.

$Q = aP + b$.

From the data given, P = RWF25 when Q = 0 and the slope of the line is 2 (that is the change in Y per unit change in X). **(3 Marks)**

Therefore, $0 = 2 \times 25 + b$; $b = -50$

The supply equation is $Q = -50 + 2P$

When the price is RWF80, the quantity produced is 110 pairs. **(3 Marks)**

SOLUTION 6

Indices – Use and Construction

Economists, statisticians and business specialists have constructed a means to measure the magnitude of economic changes over time. This measure is an index number and is used for international comparisons of economic and business data. Because they work in a similar way to percentages they make such changes easier to compare. The Consumer Price Index is probably the most widely quoted index for everyday economic purposes. It attempts to measure the change in the price of a wide range of goods and services that we purchase regularly. The cost of living index is probably the most familiar to many people but a wide range of other indices are in use: Index of Retail Prices, Index of Industrial Production, Dow Jones, Nasdaq, etc. A particular time period – the base period - is chosen and the variable for that period is given a value of 100. An index is calculated for the remaining periods on the assumption that the base period has a value of 100. An index gives the change (percentage) that has taken place since the base period.

(4 Marks)

Simple Aggregate Index.

If we take account of unit prices over current and base periods, this gives the simple aggregate index. It is not very useful because the quantities may differ so much from each other over the periods and the unit prices may be for different quantities of products.

An Expenditure Index.

Expenditure is made up of two different elements, prices and quantities bought, any change in either will cause a change in expenditure. If base year prices and quantities (P_o, Q_o) and current year prices and quantities (Pn, Qn) are used an expenditure index can be calculated in the form $\sum P_nQ_n/\sum P_oQ_o$. In this case account has been taken of the different quantities by multiplying the unit prices by the corresponding quantities. This process is called weighting.

(4 Marks)

A Price Index.

To measure prices from a base year we can use the quantities purchased in the base year to weigh the unit prices in both years. If quantities are held constant any change in outlay or expenditure will be entirely due to the change in price. This is a base weighted price index or Laspeyres Price Index which is given by $\sum P_nQ_o/\sum P_oQ_o$. In practice, the Laspeyres price index is usually calculate using price relatives. For this method the expenditures in the base year are used as weights. This method is used because it is easier to obtain data on expenditure than on actual quantities bought when we are dealing with a large complicated index. The base weighted index has the advantage that the base year expenditures have to be worked out once. These can then be used in the calculation of the index in any subsequent period. However this index can be misleading. For example fluctuations in a particular element might have a considerable impact on an index. The popularity of a particular element could dramatically affect the quantities sold and an index which uses base year quantities from some time in the past could be misleading. This particular problem can be avoided by using Paasche's price index or the end year weighted price index. In this particular case we have $\sum P_nQ_n/\sum P_oQ_n$.

(4 Marks)

A Volume Index.

However, if the prices remain relatively stable and the quantities of items change, it is more useful to calculate an index based on quantities, using prices as weights. These are the base weighted or Laspeyres Volume index and the end weighted or Paasche's Volume index and are given by $\sum P_oQ_n/\sum P_oQ_o$. and $\sum P_nQ_n/\sum P_nQ_o$. **(4 Marks)**

The Fisher price index gives a geometric mean of Laspeyres and Paasche indexes. Fischer = √Laspeyres x Paasche.

The price indices answer the question: By how much have prices increased?

The Volume indices answer the question: By how much have quantities increased?

While a third question could be asked: How much more money is being spent? This can be answered by the Value index $\sum P_n Q_n / \sum P_o Q_o$

Base period.

One problem in the development of any index number is selecting a suitable base period. A base period should be chosen where prices or volumes are not unnaturally high or low. Also a base period should not be too far in the past. Since tastes and availability can change substantially over time such an index can be seriously misleading. One means to avoid such problems is to use a chain based system where, in calculating successive index numbers, the base used is the previous period. A chain-based index number is particularly suited for period by period comparisons while a fixed-base index number makes it easier to compare movement of prices over time.

(4 Marks)