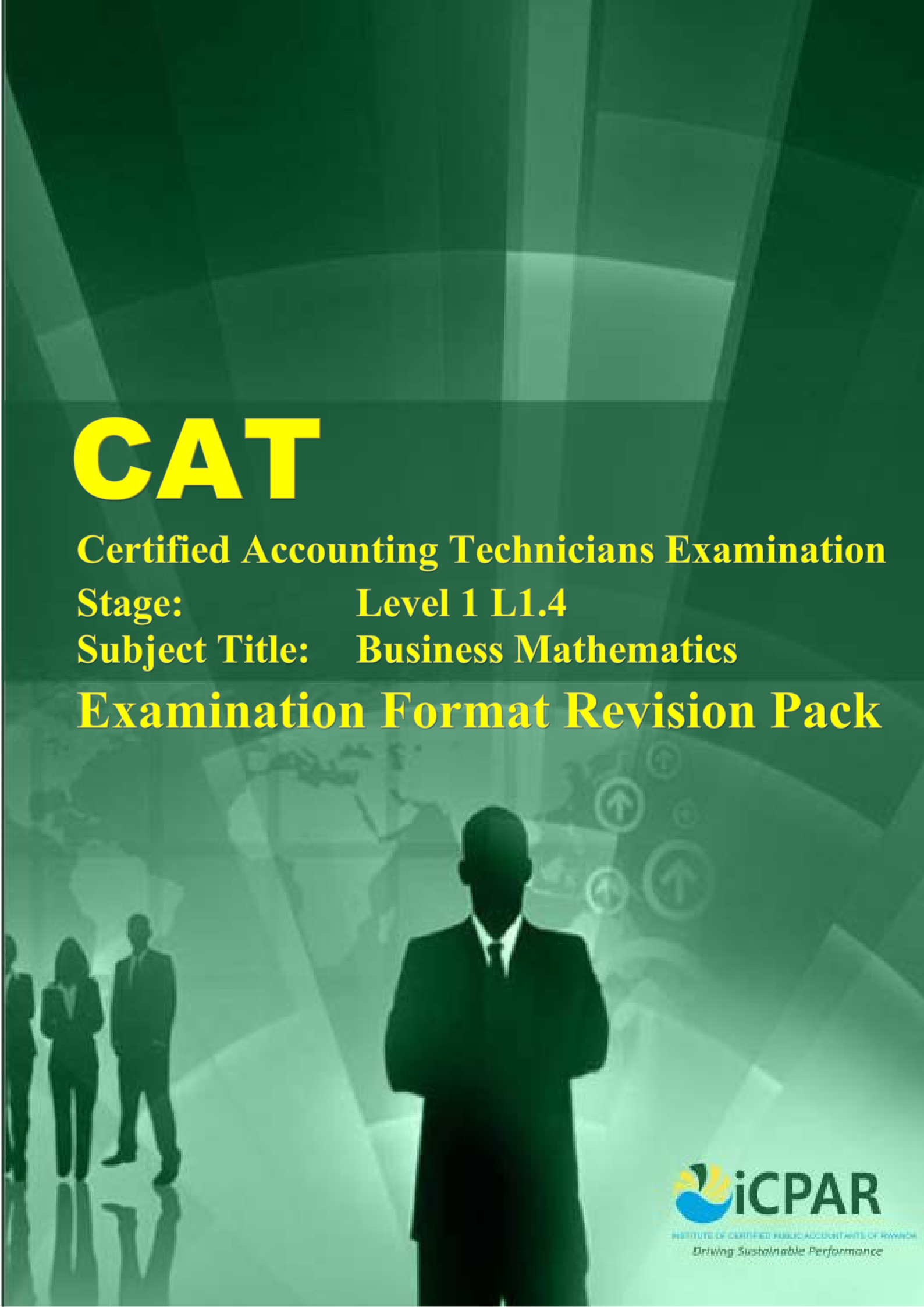
CAT

Certified Accounting Technicians Examination

Stage: Level 1 L1.4

Subject Title: Business Mathematics

Examination Format Revision Pack



ICPAR
INSTITUTE OF CERTIFIED PUBLIC ACCOUNTANTS OF RWANDA
Driving Sustainable Performance

INSIDE COVER - BLANK

L1.4 BUSINESS MATHEMATICS

LEVEL 1

EXAMINATION FORMAT REVISION QUESTIONS & SOLUTIONS

NOTES:
You are required to answer 5 questions.

(If you provide answers to all questions, you must draw a clearly distinguishable line through the answer not to be marked. Otherwise, only the first 5 answers to hand will be marked).

All questions carry equal marks.

STATISTICAL FORMULAE AND MATHEMATICS TABLES ARE PROVIDED AT BACK OF THIS PACK

TIME ALLOWED:
3 hours, plus 10 minutes to read the paper.

INSTRUCTIONS:
During the reading time you may write notes on the examination paper but you may not commence writing in your answer book.

Marks for each question are shown. The pass mark required is 50% in total over the whole paper.

Start your answer to each question on a new page.

You are reminded that candidates are expected to pay particular attention to their communication skills and care must be taken regarding the format and literacy of the solutions. The marking system will take into account the content of the candidates' answers and the extent to which answers are supported with relevant legislation, case law or examples where appropriate.

List on the cover of each answer booklet, in the space provided, the number of each question(s)attempted.

L1.4 BUSINESS MATHEMATICS

LEVEL 1

QUESTION 1.

A machine will be purchased by DYY Ltd. at a cost of RWF30,000. The suppliers also quote a trade-in value on the machine of RWF5,000 at the end of 5 years. The company has a number of different ways of obtaining the machine which are outlined below.

(i) An immediate purchase for cash. The bank will provide a loan at an interest rate of 12% p.a.

(ii) The machine can be purchased for cash with 20% paid as a deposit and the balance payable by 5 annual installments of RWF6,720 each.

(iii) The machine can be leased for 5 years at an annual rent of 25% of the cash price.

[Note: Use a discount rate of 12% for parts (ii) and (iii). Assume the cash flows take place at the end of the appropriate years].

You are asked by the Managing Director of DYY Ltd. to calculate the most cost effective means of obtaining the machine and comment on your result.

(Total: 20 Marks)

QUESTION 2.

In a dispute with pharmacists, the Ministry of Health in Rwanda decided to collect sample data on the number of prescriptions filled and the value of these prescriptions over the past 4 weeks. It collected the following data from a local pharmacy:

Value of prescriptions RWF	Number
50 – 59	1
60 – 69	2
70 – 79	3
80 – 89	11
90 – 99	14
100 – 109	12
110 – 119	9
120 – 129	5
130 - 139	3

You are required to:

(i) Calculate the mean and standard deviation of the value of prescriptions.

(8 Marks)

(ii) Demonstrate the data on a diagram and hence derive the median value of prescriptions.

(8 Marks)

(iii) Discuss the difference between the 'averages' derived.

(4 Marks)

(Total: 20 Marks)

QUESTION 3.

The management accountant wishes to derive a relationship between the units produced each month and the energy costs of a laser production machine.

The following data for the six month period from January to June shows the energy costs and the number of units produced:

Month	**Units Produced, x**	**Energy Costs (RWF), y**
January	1300	500
February	1600	700
March	1100	600
April	1500	700
May	1600	800
June	1300	900

Show how she would:

(i) Use regression analysis to derive an equation for the energy costs per month.

(8 Marks)

(ii) Calculate the correlation co-efficient for the data.

(6 Marks)

(iii) Calculate the fixed and variable costs for an estimated production level of 1,200 units in July and comment on the accuracy of your forecast.

(6 Marks)

(Total: 20 Marks)

QUESTION 4.

Your firm, iCPAR Consultants & Associates, surveyed a random sample of 35 companies in order to determine the average number of staff leaving companies last year. The results showed an average staff turnover of 72. However, your test at DYY Ltd. shows a staff turnover of 75. Your colleagues don't believe that such a small difference is important. However, you decide to test the statement that the staff turnover in this client company is normal for the industry by using a 5% level of significance. Information in similar studies gives a standard deviation of 7.0.

(i) Explain the principles that you propose to use in your study.

(10 Marks)

(ii) Carry out a significance test at the 5% level and advise your colleagues on the outcome.

(10 Marks)

(Total: 20 Marks)

QUESTION 5.

(i) Superior Products has been granted a loan to purchase office equipment. The bank has promoted the loan at a nominal interest rate of 16% but the small print on the contract states that the APR is approximately 17%. Using this data, describe the difference between 'nominal interest rates' and 'annual percentage rate' (APR).

(6 Marks)

(ii) Your Managing Director has been advised that it will cost RWF25,000 to provide a number of networked printers for the office. The bank will provide finance for the job at an interest rate of 16%. The loan may be repaid at the end of 4 years. The normal cost of borrowing by the company is 12%. What is the increase in cost to the company by using this means of financing the project?

(6 Marks)

(iii) You are advised that the average deposit in the Office national des postes is RWF100 with a standard deviation of 15. You are asked to find the 95% confidence interval for the deposit of an 'average' account. Provide an interpretation of your result. Assume that the deposits follow a normal distribution.

(8 Marks)
(Total: 20 Marks)

QUESTION 6.

"Network analysis incorporates a variety of techniques used to help plan, manage, allocate resources and review costs. Accountants play a major role in this process". In this context, you have planned the following activities for the refurbishment of your existing offices.

Task	Activity	Preceding Activity	Duration (weeks)
Advise staff	A	-	2
Consult with clients	B	-	3
Consult property agents	C	A	3
Plan temporary office	D	B	5
Install furniture	E	B	4
Equip new office	F	B	2
Move to new office	G	C,D	3
Train staff	H	F	5

You are required to:

(i) Draw a network of the above activities.

(10 Marks)

(ii) Estimate the maximum time available to complete the project and show the critical path.

(6 Marks)

(iii) Provide a description of the process that you have used.

(4 Marks)

(Total: 20 Marks)

END OF PAPER

SUGGESTED SOLUTIONS

SOLUTION 1

(i) Purchase for cash

	Actual Price RWF	Discount Factor @ 12%	Present Value RWF
Purchase price	30,000	1	30,000
Less trade-in at end year 5	5,000	0.5674	2,837
Net Present Value	25,000		27,163
	2 Marks		**(4 Marks)**

(ii) Purchase by instalments

Deposit	6,000	1	6,000
Instalment			
Year 1	6,720	0.8929	6,000
Year 2	6,720	0.7972	5,357
Year 3	6,720	0.7118	4,783
Year 4	6,720	0.6355	4,271
Year 5	6,720	0.5674	3,813
	39,600		30,224
Less trade-in at end year 5	5,000	0.5674	2,837
Net Present Value	34,600		27,387
	2 Marks		**(4 Marks)**

(iii) Leasing (In this case there is no trade-in value, since the machine is not owned by the company as in (i) and (ii) above.)

Payment at end of

Year 1	7,500	0.8929	6,697
Year 2	7,500	0.7972	5,979
Year 3	7,500	0.7118	5,338
Year 4	7,500	0.6355	4,766
Year 5	7,500	0.5674	4,256
Net Present Value	**37,500**		**30,224**
	2 Marks		**(4 Marks)**

Summary: On the basis of the above calculations the machine should be leased rather than purchased for immediate cash or instalments since, with this option, the cost to the company is least. **(2 Marks)**

[Total : 20 Marks]

SOLUTION 2

(i) The mean and standard deviation are calculated from the data in the following table.

Value of prescriptions RWF	Number f	Cum frequency	x	fx	$(x - \bar{x})$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
50 – 59	1	1	54.5	54.5	- 45	2025	2025
60 – 69	2	3	64.5	129.0	- 35	1225	2450
70 – 79	3	6	74.5	223.5	- 25	625	1875
80 – 89	11	17	84.5	929.5	- 15	225	2475
90 – 99	14	31	94.5	1323.0	- 5	25	350
100 – 109	12	43	104.5	1254.0	5	25	300
110 – 119	9	52	114.5	1030.5	15	225	2025
120 – 129	5	57	124.5	622.5	25	625	3125
130 - 139	3	60	134.5	403.5	35	1125	3675
∑	60			5970.0			18300

$$\text{Mean} = \bar{x} = \frac{\sum fx}{\sum f} : \frac{5970}{60} = \text{RWF}99.5$$

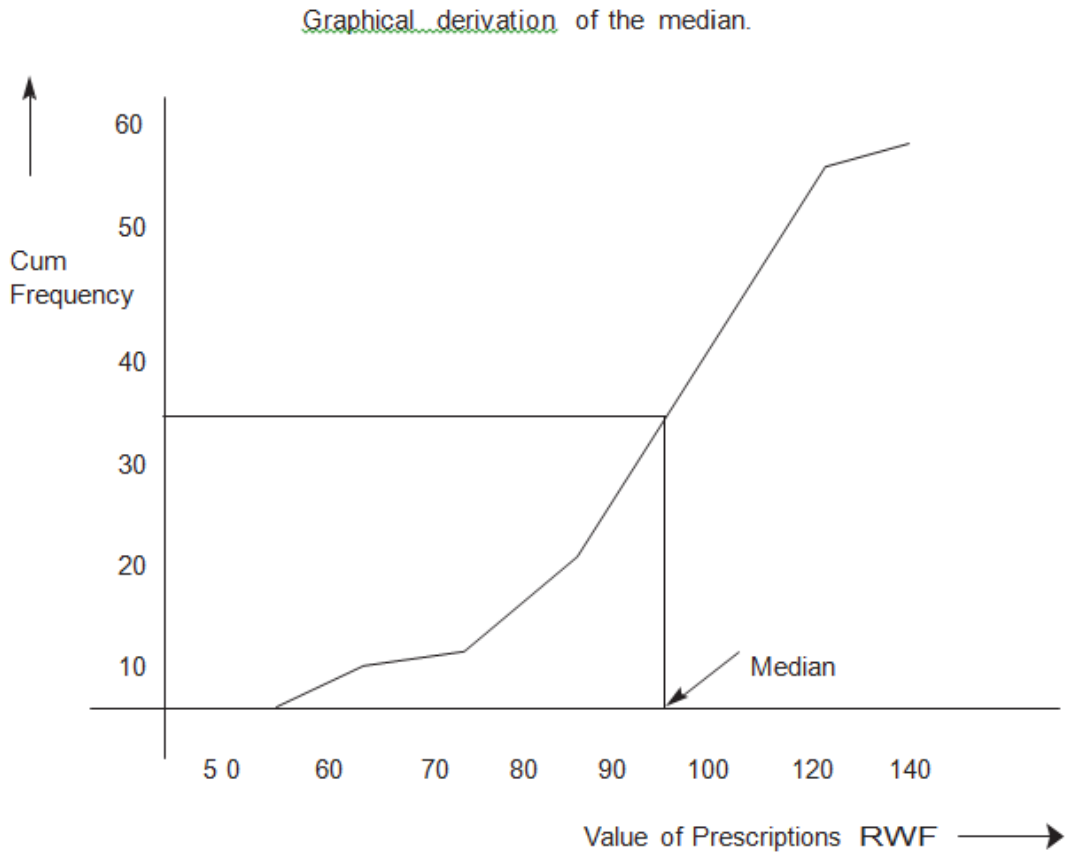
$$\text{Standard deviation} = \sigma = \frac{\sqrt{\sum f (x - x)^2}}{\sum f} = \frac{\sqrt{18300}}{60}$$

$$= \sqrt{306}$$

$$= \text{RWF}17.46$$

ii) Median; the median is described as the ‘centre’ of a set of data. Since you are asked to derive the median from the graph, the value is approximately RWF99. The median is the value of the data such that 50% lies above and below this value.

(4 Marks)



(4 Marks)

iii) In the particular data, both the mean and the median are approximately the same. The median is an 'average' in that it splits the data into two parts. It does not generally equal the mean since it defines the data in a different way. There are equally as many values above and below the median. The mean is average in that, if each value was replaced by a constant while the total remains unchanged, this number is the mean. In this way the mean is comparable to a centre of gravity. Like the mean the median always exists and is unique for any set of data. In certain situations the median may be a better measure of central tendency than the mean. It is less sensitive to extremely large or small measurements. Extreme values affect the mean since these values are used explicitly to calculate the mean. A comparison of the mean and median gives a general method for detecting skewness in data sets. However, the mean and median provide more descriptive information than the mode.

(4 Marks)

(Total : 20 Marks)

SOLUTION 3

(i) The relationship can be demonstrated by developing a linear regression equation where the relationship is

$y = a + bx$, where

x = no. units produced / month y = total energy costs / month

a = fixed costs / month b = variable costs / unit

n = no. of pairs of x and y variables

Month	Units Produced (00s) x	Energy Costs RWF00s y	x2	xy	y2
January	13	5	169	65	25
February	16	7	256	112	49
March	11	6	121	66	36
April	15	7	225	105	49
May	16	8	256	128	64
June	13	9	169	117	81
∑	84	42	1196	593	304

(2 Marks)

A linear regression equation may be written as: where

$\sum y = na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$; where

$$a = \frac{\sum y}{n} - \frac{b\sum x;}{n}$$

$$b = \frac{\sum xy - \sum x\sum y/n,}{\sum x^2 - (\sum x)^2 /n}$$

Inserting values gives

$$b = \frac{593 - 84 \times 42/6}{}$$

$$1196 - 84^2/6$$

$$= \frac{593 - 588}{1196 - 1176} = \frac{5}{20} = 0.25$$

That is, RWF0.25/unit (variable cost), RWF25 per 100 units.

$$a = \frac{42}{6} - \frac{0.25 \times 84}{6} = 7 - 3.5 = 3.5$$

That is, RWF350/month (fixed cost). **(2 Marks)**

Therefore, $y = 350 + 25x$. **(2 Marks)**

(ii) Correlation co-efficient is derived from the formula:

$$r = \frac{\sum xy/n - \sum x/n \sum y/n}{\sqrt{\{\sum x^2/n - (\sum x/n)^2\}\{\sum y^2/n - (\sum y/n)^2\}}}$$

Inserting values from the table above: $\sum xy = 593$; $\sum x = 84$; $\sum y = 42$;

$\sum x2 = 1196$; $\sum y2 = 304$.

$$\text{Therefore, } r = \frac{593/6 - 84/6 \times 42/6}{\sqrt{(1196/6 - 196)(304/6 - 49)}} = \frac{98.8 - 98}{\sqrt{3.33 \times 1.66}}$$

$$= 0.8/2.35$$

$$= 0.34$$ **(3 Marks)**

This value indicates that the relationship between the variables is not high. This is obvious from an inspection of the data where the costs, over such a short time period, obviously do not correlate closely to the production level. There are obviously other factors involved in the data which are not obvious at a superficial level. It would therefore be somewhat inaccurate to rely on forecasts based on this data.

(3 Marks)

(iii) In order to predict the costs for following months, the established regression equation could be used with the health warning set out above.

For a production level of 1200 units, the fixed costs are RWF350. Therefore the total costs for July are RWF 650

(4 marks)

In the regression equation the values of a and b are estimates based on sample data and can be expected to vary from sample to sample. If the estimate is based on a different sample, the method of least squares would probably yield different values for a and b and a different value for y. In linear regression analysis we assume that that the x's are constants, not values of random variables, and that for each value of x the variable to be predicted, y, has a certain distribution. We assume that these distributions are all normal distributions with the same standard deviation, σ

(4 Marks)
(Total : 20 Marks)

SOLUTION 4

(i) Hypothesis Testing.

This test considers the principles of hypothesis testing. If we postulate that there is no difference between the data, this is set up as the Null hypothesis, Ho. The hypothesis is set up to see if it can be rejected. Together with this hypothesis an alternative hypothesis, H1, must be formulated, that is, the hypothesis that is accepted if the Ho is rejected. In the present case we set up Ho = 72. H1 is set up as H1 ≠ 72 so that Ho will be rejected if the sample mean is much greater than or much less than 72. H1 usually specifies that the population mean is less than, greater than, or not equal to the value assumed under Ho. Since, in this case, the sample is large ($n \geq 30$), the sampling distribution of the mean can be approximated with a normal distribution and the test statistic (z) is a value of a random variable having the standard normal distribution. Using two values, we can base tests on the Null hypothesis $\mu = \mu o$. Since the level of significance is 5%, that is, 0.05, the critical values (from the Normal tables) are -1.96 to +1.96 for a two sided alternative.

(10 Marks)

(ii) Setting down the process as described:

1. Ho: $\mu = 72$

   H1: $\mu \neq 72$ **(2 Marks)**

2. The level of significance, $\alpha = 0.05$

3. Reject the Null hypothesis if $z \leq -1.96$ or $z \geq +1.96$

   where $z = \frac{x - \mu o}{\sigma/\sqrt{n}}$, $n = 35$, $\sigma = 7$ **(2 Marks)**

4. $z = \frac{75 - 72}{7/\sqrt{35}} = \frac{3}{7/5.916} = \frac{3}{1.18} = 2.54$

5. Since $z = 2.54$ exceeds 1.96 the Null hypothesis must be rejected. **(2 Marks)**

You advise your colleagues that, based on the data provided, the difference between the numbers of exits in DYY Ltd. and the industry is statistically significant and the level of exits requires further examination by the company.

(4 Marks)
(Total : 20 Marks)

SOLUTION 5

(i) The majority of financial institutions express rates of interest as 'rate per annum'. The interest rate may be compounded over time periods less than a year. This annual rate is the 'nominal rate'. By compounding the interest rate over quarterly periods, the real rate of interest, that is, the effective rate or 'annual percentage rate' will always be greater than the nominal rate. The standard method of determining the APR is to make the effective time period equal to the compounding period and actually compounded over a period of 1 year.

(3 Marks)

The APR can be calculated using APR $= (1 + i/n)^n - 1$, where 1 = nominal interest rate, n = number of equal compounding periods in 1 year.

Therefore, in the problem the APR is $[1 + (0.16/4)]^4 - 1 = 1.04^4 - 1$

$= 1.1698 - 1 = 16.98\%$ **(3 Marks)**

(ii) The maturity value of the debt at the end of 4 years is RWF25,000 $(1.16)^4$ = RWF45,250. However, the present value is obtained by discounting this value at 12%, that is, $45,250/1.12^4 = 45,250/1.57 =$ RWF28,821.

(3 Marks)

This is the value that the original debt will cost in to-days terms. This is the present value of an interest bearing debt. Therefore, the real cost of the debt, to repay it in to-days terms is, RWF28,821 - RWF25,000 =

RWF3,821. This is based on the premise that the borrowing rate of 16% that the company pays will be greater than the investment rate that it could receive on its money; in other words, the present value of the debt will always be greater than the original amount. **(3 Marks)**

(iii) The 95% confidence interval for a population is

$\mu - 1.96\sigma < \bar{x} < \mu + 1.96\sigma$

where x represents the variable of interest, that is, deposits

Inserting values: $100 - 1.96\ (15) < \bar{x} < 100 + 1.96\ (15)$

$70.6 < \bar{x} < 129.4.$ **(4 Marks)**

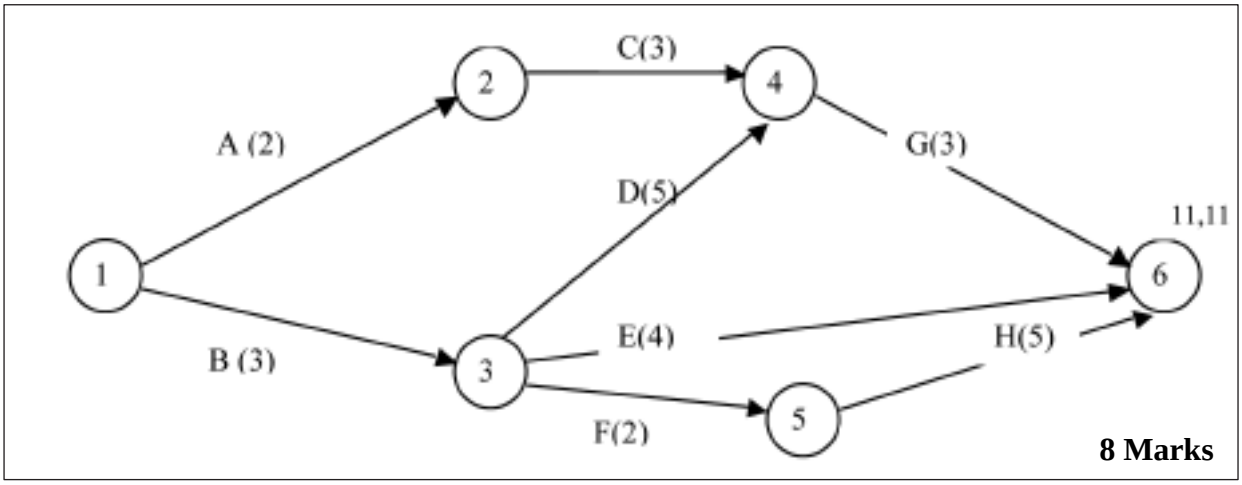
Interpretation: There is a 95% confidence that a value from a normal distribution will lie between will lie within 1.96 standard deviations of the mean value. In the present case, 95% of the population of depositors has a deposit of between RWF70.6 and RWF129.2. **(4 Marks)**

(Total :20 Marks)

SOLUTION 6

(i) The network for the activities is outlined below. It illustrates the sequence of activities and the steps involved.

Task	Activity	Preceding Activity	Duration (weeks)
Advise staff	A	-	2
Consult with clients	B	-	3
Consult property agents	C	A	3
Plan temporary office	D	B	5
Install furniture	E	B	4
Equip new office	F	B	2
Move to new office	G	C,D	3
Train staff	H	F	5



8 Marks

(ii) The total project duration is an important factor when managing projects. The overall duration can be calculated from the network providing the duration of each activity is known. The estimated time to complete the project is 11 weeks.

(4 Marks)

This is the critical path – the critical times that cannot be delayed that cannot be delayed, otherwise the overall duration of the project will be extended. It is designated on the diagram by 1-3-4-6.

(4 Marks)

(iii) Each activity is represented by an arrow on the diagram. Circles are numbered and drawn to indicate the start and of finish activity. The diagram shows the relationships between the activities as set down in the table. In order to produce a network diagram a list of activities is required and the interdependence between activities, that is, the activities that precede other activities. In order to calculate the overall duration of the project it is necessary to estimate the earliest and latest event times. The earliest time is determined by the longest route through the network and the largest value is used. This gives the earliest time in which the project can be completed. The latest event times are then calculated. Preceding latest event times are calculated by subtracting an activity's duration from the subsequent latest event time. If two or more activities start from an event, the latest time for each route is calculated and the lowest value is used. The times show that the project can be completed in 11 weeks.

(4 Marks)
(Total : 20 Marks)

END OF SOLUTIONS

L1.4 BUSINESS MATHEMATICS

LEVEL 1

EXAMINATION FORMAT REVISION QUESTIONS & SOLUTIONS

NOTES:
You are required to answer 5 questions.

(If you provide answers to all questions, you must draw a clearly distinguishable line through the answer not to be marked. Otherwise, only the first 5 answers to hand will be marked).

All questions carry equal marks.

STATISTICAL FORMULAE AND MATHEMATICS TABLES ARE PROVIDED AT BACK OF THIS PACK

TIME ALLOWED:
3 hours, plus 10 minutes to read the paper.

INSTRUCTIONS:
During the reading time you may write notes on the examination paper but you may not commence writing in your answer book.

Marks for each question are shown. The pass mark required is 50% in total over the whole paper.

Start your answer to each question on a new page.

You are reminded that candidates are expected to pay particular attention to their communication skills and care must be taken regarding the format and literacy of the solutions. The marking system will take into account the content of the candidates' answers and the extent to which answers are supported with relevant legislation, case law or examples where appropriate.

List on the cover of each answer booklet, in the space provided, the number of each question(s)attempted.

L1.4 BUSINESS MATHEMATICS

LEVEL 1

QUESTION 1

Your company has received three quotations from financial institutions for the purchase of computer equipment.

The quotations are as follows:

(a) The equipment can be purchased for RWF350,000 today with a maintenance contract of RWF30,000 per annum payable in advance. The equipment has an expected life of 8 years and will be sold for RWF25,000 at the end of year 8.

(b) The equipment can be rented for RWF90,000 per annum, payable in advance.

(c) The equipment can be obtained on hire purchase with a deposit of RWF70,000 today and 7 equal annual payments of RWF95,000 starting at the end of year 1. At the end of 8 years, the equipment will be owned by the company and has a value of RWF20,000.

You are required to:

(i) Set out the cash flows for the three options. **(10 marks)**

(ii) Using Net Present Value recommend the best option for the company with a cost of capital of 16%.

(10 marks)
(Total: 20 Marks)

QUESTION 2

The local union representative has approached the manager of the Do-It-Better Manufacturing company and claims that the wages paid to its employees are inferior to the wages paid to another local company, Compact Ltd. He provides the company with the data on employee wages from both companies. The manager asks you to analyse the data and to present it for comparison at the next management meeting.

You decide to:

(i) Present the data on histograms. **(8 marks)**

(ii) Calculate the mean and standard deviations for both companies. **(8 marks)**

(iii) Comment on the differences between the wages of both companies using the information derived in (i) and (ii). **(4 marks)**

Employee wages (RWF per week)	Do-It-Better company	Compact Ltd
300 < 310	3	4
310 < 320	7	12
320 < 330	32	16
330 < 340	26	23
340 < 350	23	39
350 < 360	20	26
360 < 370	18	16
370 < 380	16	11
380 < 400	5	3

(Total: 20 Marks)

QUESTION 3

A tenant claims to the tenancy board that the average rent for an apartment in the area is RWF7,200 per annum. However, the developer carries out a survey of 49 apartments and claims that the average annual rents in the area are RWF7,500 per annum with a standard deviation of RWF840.

(i) Using a 95% confidence interval does the developer's data support the claim?
(10 marks)

(ii) Describe by means of diagrams the principles underpinning your calculations
(10 marks)
(Total 20 Marks)

QUESTION 4

The advertising expenditure and sales data for the Superior Products company over the past 9 months is set out in the table below. The company wants to determine the effect that advertising has on the levels of actual and projected sales over the period.

Month	**J**	**F**	**M**	**A**	**M**	**J**	**J**	**A**	**S**
Advertising RWF000	25	50	20	45	50	35	20	25	20
Sales RWF000	100	140	130	120	150	170	120	100	80

In this context, using linear regression analysis:

(i) Derive the equation of a linear relationship between sales and advertising expenditure.

(8 marks)

(ii) Plot the data and regression equation on a graph **(8 marks)**

(iii) Forecast the level of sales based on advertising expenditure of RWF4,000

(4 marks)
(Total 20 Marks)

QUESTION 5

As the Financial Adviser with iCPAR Consultants, you have been consulted by a client who requests you to provide details of the principles, and formulae if used, underpinning your calculations with respect to the following:

(i) Your client wishes to invest RWF20,000 starting now and continuing with the payments at the end of each year until he retires in 20 years. He wishes to know the total value of his investment at the end of this period at the current rate of interest of 5%.

(6 marks)

(ii) He states that he wishes to make an investment now so that his wife could withdraw a guaranteed pension of RWF20,000 for 20 years. How much should be invested now to cater for this payment, if the rate of interest is 5%?

(8 marks)

(iii) At retirement he wishes to purchase an annuity, that is, an annual pension for the rest of his life for the sum of RWF1,061,319. The insurance company uses an interest rate of 3% for pension purposes. What annual pension can he expect?

(6 marks)

(Total: 20 Marks)

QUESTION 6

Your company is considering building an extension to its financial services centre. The developer proposes using network analysis to give the management team a perspective on the range of activities involved and the progress of the development. You are asked by the Managing Director to provide a management briefing on Network Analysis and how this process operates. Your briefing should include the following elements:

(i) Key information required to draw a network

(ii) Use of dummy activities

(iii) Time calculations for earliest and latest event times

(iv) Derivation of the critical path

(v) Limitations of the method

(Total 20 Marks)

END OF PAPER

SUGGESTED SOLUTIONS

SOLUTION 1

(i) Cash flows for the three options

Year	Purchase RWF		Rent RWF		Hire Purchase RWF	
	Cash Outflow	Cash Inflow	Cash Outflow	Cash Inflow	Cash Outflow	Cash Inflow
0	(350,000) (30,000)		(90,000)		(70,000)	
1-7	(30,000)		(90,000)		(95,000)	
8		25,000		----------	(95,000)	20,000
	2 marks	**2 marks**	**2 marks**		**2 marks**	**2 marks**

In the HP option, since the company retains the machine it is assumed that it can sell it for a value of RWF20,000 in year 8.

(ii) Calculation of NPV using a discount factor of 16%

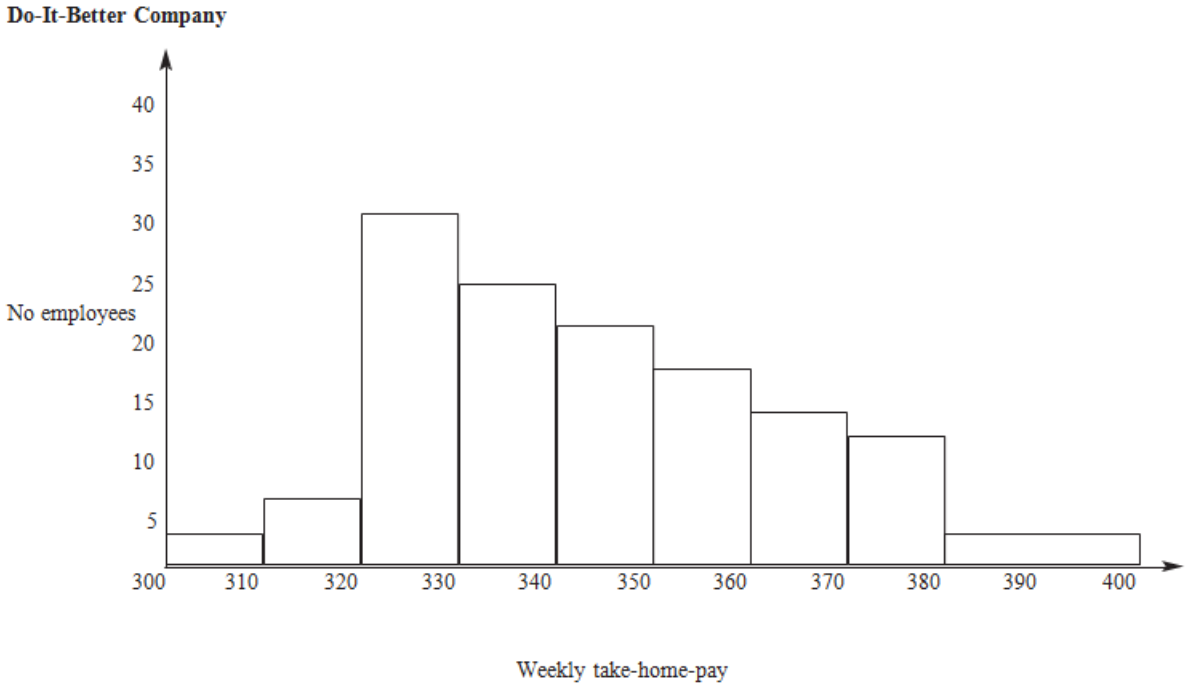
Since the company is making equal annual payments for years 1 – 7, the annuity factor can be used rather than the discount factor for each year.

Year	Discount Factor 16%	Purchase, RWF		Rent RWF		Hire Purchase RWF	
		Net Cash Flow	PV	Net Cash Flow	PV	Net Cash Flow	PV
0	1	(380,000)	(380,000)	(90,000)	(90,000)	(70,000)	(70,000)
1-7	4.039	(30,000)	(121,170)	(90,000)	(363,510)	(95,000)	(383,705)
8	0.305	7,625	7,625			(75,000)	22,875
NPV			**(493,545)**		**(453,510)**		**(430,830)**
			2 **marks**		**2 marks**		**2 marks**

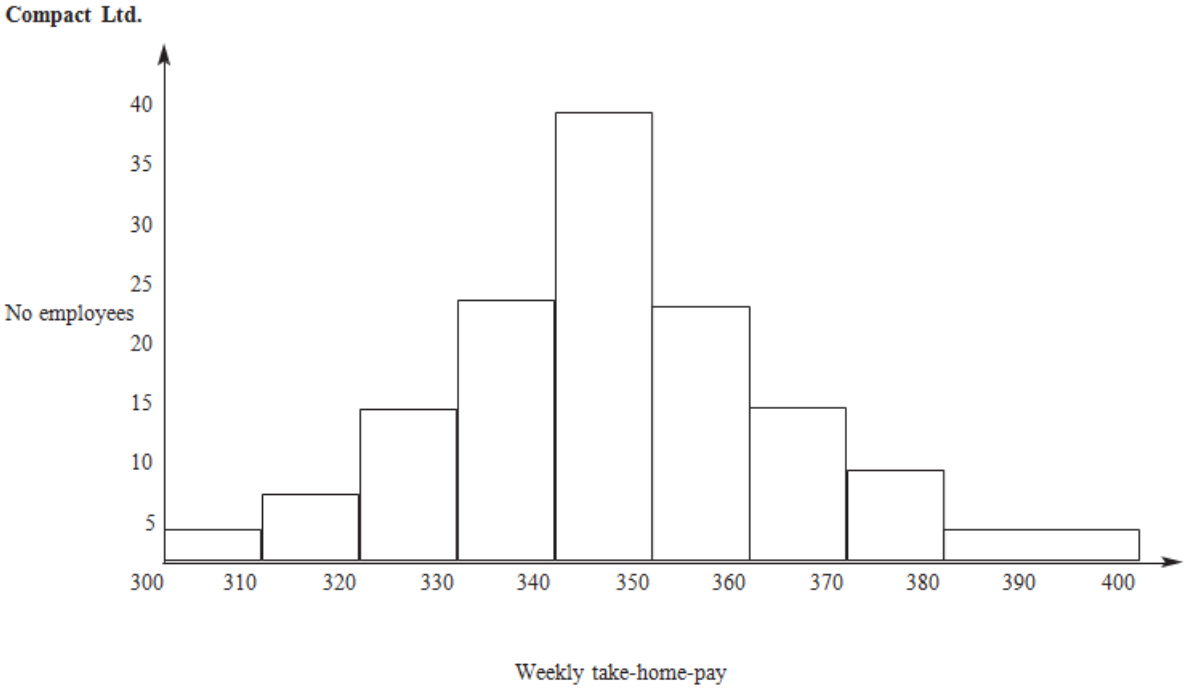
Since the HP provides the minimum cash outflow for the company it is recommended as the best option. **(4 Marks)**

SOLUTION 2

(i) The histograms are plotted below.



(4 marks)



(4 marks)

(ii) The mean and standard deviation are calculated from the data in the following tables.

Do-It-Better Company

Take home pay RWF	Mid point x	Employees f	f(x)	(x – x)	$(x - x)^2$	$(x - x)^2$
300 < 310	305	3	915	-40	1600	4800
310 < 320	315	7	2205	-30	900	6300
320 < 330	325	32	10400	-20	400	12800
330 < 340	335	26	8710	-10	100	2600
340 < 350	345	23	7935	0	0	0
350 < 360	355	20	7100	10	100	2000
360 < 370	365	18	6570	20	400	7200
370 < 380	375	16	6000	30	900	14400
380 < 400	390	5	1950	45	2025	10125
		150	51785			60225

Mean = $\bar{x} = \frac{\sum fx}{\sum f} = \frac{51785}{150} = RWF345.2 \approx RWF345$

(2 marks)

Standard deviation = $\sigma = \frac{\sqrt{\sum f(x - \bar{x})^2}}{\sum f} = \frac{\sqrt{60225}}{150}$

$= \sqrt{401.5} = RWF20.03$ **(2 marks)**

Compact Co. Ltd.

Take home pay RWF	Mid point x	Employees f	f(x)	(x – x)	$(x - x)^2$	$(x - x)^2$
300 < 310	305	4	1220	-40	1600	6400
310 < 320	315	12	3780	-30	900	10800
320 < 330	325	16	5200	-20	400	6400
330 < 340	335	23	7705	-10	100	2300
340 < 350	345	39	13455	0	0	0
350 < 360	355	26	9230	10	100	2600
360 < 370	365	16	5840	20	400	6400
370 < 380	375	11	4125	30	900	9900
380 < 400	390	3	1170	45	2025	6075
		150	51725			50875

(2 marks)

Mean = $\bar{x} = \frac{\sum fx}{\sum f} = \frac{51725}{150} = RWF344.8 \approx RWF345$

Standard deviation = $\sigma = \frac{\sqrt{\sum f(x - x)^2}}{\sum f} = \frac{\sqrt{50875}}{150}$

$= \sqrt{339.17} = RWF18.4$ **(2 marks)**

(iii) Comment on the data presented on the histograms and the calculations.

The histograms give a good pictorial representation of the data. The first graph represents a skewed distribution to the left which would indicated that a larger proportion of employees are receiving lower pay. The second graph is a more symmetrical distribution which indicates that a larger number in this company appear to receive higher wages. The modal class for company 1is RWF320 - RWF330 while for company 2 is RWF340 - RWF350. This would tend to support the unions claim that the employees of this company are 'worse off'. However the management of the company quantify the data by calculating the mean wages received and the distribution of both. They claim that the employees of both companies earn the same mean wage and the distribution is small indicating that the unions analysis is incorrect and that the histograms distort the data.

To support their claim the union should subject the data to further analysis by calculating the median and the quartiles. These would support the union's claim that the employees of the Do-It-Better company are 'worse' off.

(4 marks)

SOLUTION 3

(i) 95% Confidence interval.

To construct a confidence interval, X and Sx are required where the confidence interval is X ± ZSx. **(2 marks)**

In this case, X = RWF7,500 and $Sx = \frac{S}{\sqrt{n}} = \frac{840}{\sqrt{49}} = \frac{840}{7} = 120.0$

(2 marks)

The value of Z for the Normal tables is 1.96.

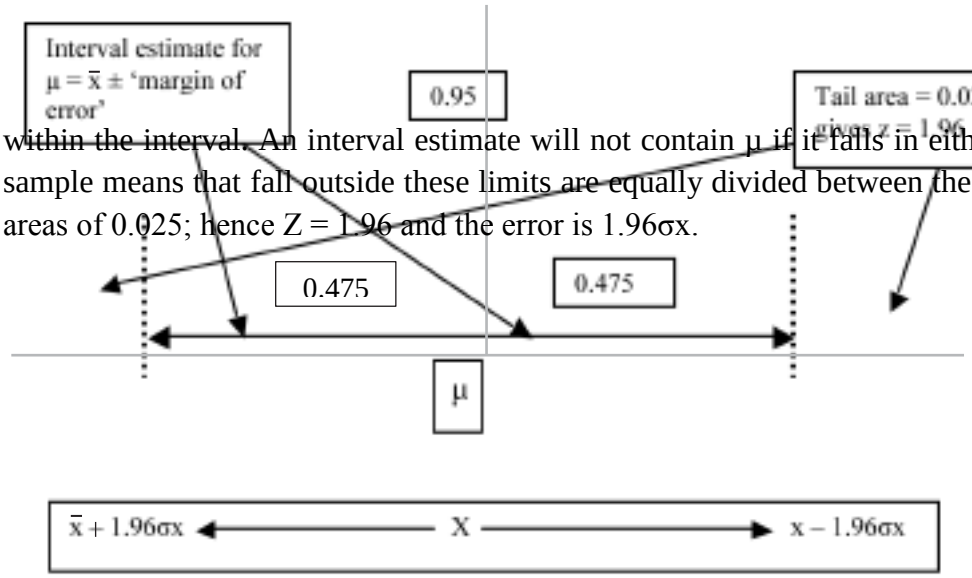
Therefore, confidence interval 7500 ± 1.96 x 120 = 7500 ± 235.2, **(2 marks)**

That is, 7264.8 to 7735.2.

Since RWF7,500 lies within the interval the data supports the developers claim. **(4 marks)**

(ii) The principles underpinning the calculations

Parameter such as the population mean is estimated by a sample mean, calculated from a random sample. In this case it is assumed that the sample means are randomly distributed and that the mean of the sample means is the same as the population mean. It is therefore possible to calculate probabilities about the sample means and calculate intervals that contain a given percentage of all sample means. Sample statistics are used to make inferences about population parameters. The mean and variance calculated from sample data are used to estimate the population mean and variance. The standard deviation of all sample means is the 'standard error or the mean' given by $\sigma x = \sigma/\sqrt{n}$. The difference between μ and its point estimate X is the 'sampling error. Therefore the interval that contains 95% of sample means ($n \geq 30$) is $\mu \pm 1.96\sigma x$. Since μ is unknown it is replaced by the point estimator X. The interval estimate $X \pm 1.96\sigma x$ will contain μ if the sample mean, X, is one of the 95% of X's



within the interval. An interval estimate will not contain μ if it falls in either tail. The 5% of sample means that fall outside these limits are equally divided between the two tails with tail areas of 0.025; hence Z = 1.96 and the error is 1.96σx.

(4 marks)

(4 marks)

The interval μ ± 1.96σx contains 95% of all sample means, that is, we are confident that μ is somewhere within the interval.

Confidence intervals can be constructed for any level of confidence – the most common are 90%, 95%, and 99%. The upper confidence limit means that we are 95% confident that the mean is at most RWF7,735.

The lower confidence limit means that we are 95% confident that the mean is at least RWF7,264. **(2 marks)**

SOLUTION 4

(i) Derivation of linear regression relationship.

Month	Advertising X	Actual Sales, Y	XY	X2
J	25	100	2500	625
F	50	140	7000	2500
M	20	130	2600	400
A	45	120	5400	2025
M	50	50	7500	2500
J	35	170	5950	1225
J	20	120	2400	400
A	25	100	2500	625
S	20	80	1600	400
Total	290	1110	37,450	10,700

Linear regression equation: $y = a + bx$ where

$$b = \frac{\sum xy - \sum x \sum y/n}{\sum x^2 - (\sum x)^2/n}, \qquad a = \sum y/n - b\sum x/n$$

Inserting the above values: $b = \frac{37,500 - 290 \times 1110/9}{10,700 - (290)^2/9}$ **(2 marks)**

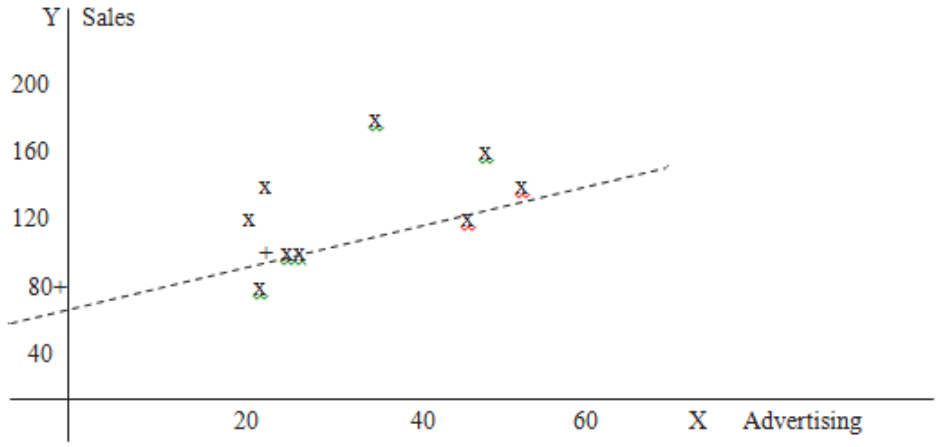
$$= \frac{37,450 - 35,767}{10,700 - 9,344} = \frac{1683}{1356} = 1.24$$

$$a = 1110/9 - 1.24 \times 290/9$$

= 123.3 - 39.95 = 83.35 **(2 marks)**

y = 83.35 + 1.24x **(4 marks)**

(ii) Scatter graph and graphing the relationship.



The regression line is indicated by the 'dashed' line on the graph. There is appositive relationship between the variables – as advertising expenditure increases sales increase.

(8 marks)

(iii) Forecast sales based on advertising expenditure of RWF4,000.

The expenditure is derived from the regression equation y = 83.35 + 1.24x where

x = 40, that is, y = 83.35 + 1.24 x 40 = 132.95, that is, RWF132,950. **(4 marks)**

SOLUTION 5

(i) This is an annuity where regular payments are made into the pension fund until retirement. The means to derive the value of the is investment over a period of t years is

A0 + $A_0(1 + i) + A_0(1 + i)^2$ -------- $A_0(1 + i)^{t-1}$ where A_0 is the annual payment, I is the interest rate and t is the time period over the investment life. This is the compound interest for fixed deposits at regular intervals of time. This is a geometric series where a = A_0, r = (1 + i) to give the total value of the investment $S = a(r^{t-1})/(r - 1)$.

(3 marks)

That is, Investment Value = $20,000(1.05^{20} - 1)/(1.05 - 1)$

= $20,000(2.653 - 1)/0.05$

= RWF1,061,319 **(3 marks)**

(ii) If a series of equal withdrawals will be made in the future it is necessary to get the present value of the annuity.

This is based on the premise that the current investment V0(1 + i)t is adequate to provide for series of payments Vt at the end of t years, A0[(1 + i)t – 1]/i, where V0 is the amount invested now and A0 is the amount invested at the end of each year.

(2 marks)

Then $V_0(1 + i)t = A_0[(1 + i)t - 1]/i$;

Simplifying this expression gives $V_0 = A_0[1 - (1 + i)-t]/I$

(2 marks)

Therefore, V_0 = 20,000[1 – (1 + 0.05)-20/0.05

= $20,000[1 - 1/1.05^{20}]/0.05$

= 20,000[1 - 1/2.653]/0.05

= 20,000[1 - 0.3769]/0.05

= 400,000 x 0.6231 = RWF249,240 **(4 marks)**

(iii) The present value of the annuity is RWF1,061,319. This can be described as

$V_0 = A_0[1 - (1 + i)\text{-}t]/i.$ **(2 marks)**

However, the client wants a perpetual annuity, that is, an annual payment that continues until his death without any

predetermined end time – an indefinite time limit. In this case t becomes ∞ and the expression (1 + i) becomes 0.

(2 marks)

Therefore $V_0 = A_0/i$; that is, $A_0 = V_0 i = 1{,}063{,}319 \times 0.03 =$ RWF31,839pa

(2 marks)

SOLUTION 6

Briefing note on Network Analysis.

Network analysis includes a variety of techniques which managers, specialists, accountants use to plan and manage the scheduling of a range of interrelated activities. These techniques are particularly useful in project management and multi- disciplinary projects where networks are used to monitor the progress of projects, allocate resources – money, material and personnel – where appropriate and review costs. In large projects the application of computer analysis is essential due to the size and complexity of such projects. Constructing a network by hand is not an option for the majority of projects.

A network consists of a series of activities and events. Each activity can be illustrated by an arrow. Events denoting the start and finish of each activity are illustrated by nodes (designated by circles) and are normally numbered. A network has one start and one end event, that is, all activities must be connected in one complete sequence.

A node indicates the completion of all activities leading into it. More than one activity can commence and finish at one node. Using the network diagram allows complex relationships between a range of activities to be illustrated. Therefore, a network is a diagram illustrating the relationship between a range of activities.

(4 marks)

Key information required. In order to produce a network diagram the following information must be supplied or available:

(i) A list of activities.

(ii) The interdependence between activities – the activities which precede other activities so that the logical sequence of the process becomes clear. Two columns are drawn up designated as ‘Activity’ and ‘Preceding activity’. Each of the preceding activities leads directly into a following activity. These activities must be completed before the following activity can commence and are normally listed in a ‘precedence table’.

(iii) The nodes are numbered so that, for every activity, the start event number is less than the finish event number.

(4 marks)

Occasionally the relationship between activities cannot be easily illustrated because of ‘unusual’ dependencies – in other words the correct precedence is not possible to produce. To implement the correct precedence and maintain the logic of the process, it is necessary to include a ‘dummy’ activity – this is an activity which does not require resources or time. It is used to continue the logic of the sequence without adding additional time – it has no duration.

There is substantial scope in deciding what activities precede others. This can be a major difficulty in major project planning. Some activities (where one cannot commence before another finishes) are obviously consecutive but others are concurrent and the difficulty is to determine which activities can be implemented simultaneously or concurrently. This requires experience and knowledge of the project and the ability to construct a network from the table of activities.

(2 marks)

Time calculations – earliest and latest times. The total project duration is an important factor when managing projects involving many activities. The overall duration can be calculated from the network providing the duration of each activity in the project is known. However, in order to derive the overall duration it is necessary to consider the earliest and latest times at which activities can start and end. The ‘earliest event time’ is derived by working through the project from the start position and adding an activity’s duration onto the preceding earliest event time. If two or more activities lead into an event, the earliest time on each event is calculated and the largest value is used.

The ‘latest event time’ for which an activity can start is determined by starting at the end position and working backwards through the project, that is, in the reverse direction as previously. These times are calculated by deducting the activity’s duration from the subsequent latest event time. If two or more activities start from an event the latest event time from each activity is calculated and the lowest value is used.

(4 marks)

In many projects some activities may be delayed for a variety of reasons. It may be necessary to estimate the free time or flexibility available to allow these delays to be incorporated in the network without increasing the overall time of the project. This is a float, that is, the amount of time an activity can be delayed without delaying a subsequent activity or the amount of time by which its duration can be increased. This must be done without affecting either the total project time or the time available for subsequent activities. Floats may be designated as

total, free and independent – these give the amount of flexibility for a specific activity based on their effect of previous and subsequent activities.

Critical path analysis. The critical path is the sequence of activities that defines the overall duration of the project. Activities on the critical path have no flexibility if the project is to finish on time – an increase in the duration of a critical activity will increase overall project time. An increase in the duration of a non critical activity will, up to a certain limit, have no impact on total project time. Critical activities can be identified by comparing a number of the time periods in the nodes demonstrated in the example below - the earliest and latest start times are equal, the earliest and latest finish times are equal and the difference between the start and finish times equal the activity's duration. For large projects this is easily done by computers to determine the critical path.

(4 marks)

Limitations of the method.

(i) Differences may occur about the activities that precede each other.

(ii) Duration of activities are estimates – it is unlikely that the durations of all activities are known. They are generally based on experience on similar projects, the availability and quality of staff and the level of resources available.

(iii) It is often necessary to make decisions on employing extra staff to accelerate progress of the project or specific elements of it. This increase in cost has to be off-set against potential savings which may accrue. This may arise due to unforeseen circumstances.

(2marks)