

# Differential Equations

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## 1 Chapters 1 and 2

A. Given a differential equation and a function  $y(x)$ , we are to check that  $y$  is a solution to the differential equation.

1. For the following differential equation and function  $y(x)$ ,

$$\frac{dy}{dx} = 3y, \quad y = 4e^{3x}$$

Since we have  $y$ , it can be differentiated to find  $\frac{dy}{dx}$ , then verify that it in fact is equal to  $3y$ .

$$\frac{dy}{dx} = 12e^{3x} = 3 \cdot 4e^{3x} = 3y$$

3. For the following differential equation and function  $y(x)$ ,

$$\frac{d^2y}{dx^2} + 16y = 0, \quad y = \sin(4x)$$

This time we must differentiate  $y$  twice to find  $\frac{d^2y}{dx^2}$ .

$$\frac{dy}{dx} = 4\cos(4x) \Rightarrow \frac{d^2y}{dx^2} = -16\sin(4x)$$

Now we can substitute this back into the original equation, and it quite obviously satisfies it.

$$-16\sin(4x) + 16\sin(4x) = 0$$

5. For the following differential equation and function  $y(x)$ ,

$$\frac{dy}{dx} + 2xy = 1, \quad y = e^{-x^2} \int_0^x e^{t^2} dt + ce^{-x^2}$$

we must find  $\frac{dy}{dx}$ .

$$\begin{aligned} \frac{dy}{dx} &= e^{-x^2} e^{x^2} - 2xe^{-x^2} \int_0^x e^{t^2} dt - 2cxe^{-x^2} \\ &= e^0 - 2x \left( e^{-x^2} \int_0^x e^{t^2} dt + ce^{-x^2} \right) \\ &= 1 - 2xy \end{aligned}$$

Now we can substitute this into the original differential equation, and we see that is in fact satisfies it.

$$\frac{dy}{dx} + 2xy = 1 - 2xy + 2xy = 1$$

B. 2. We are given the following differential equation.

$$\frac{dy}{dx} = 4xe^{2x}$$

To find a solution, both sides can be integrated.

$$y = 4 \int x e^{2x} dx$$

Now integration by parts can be applied, with  $f(x) = x$  and  $g'(x) = e^{2x}$ . This implies that  $f'(x) = 1$  and  $g(x) = \frac{1}{2}e^{2x}$ . Therefore, by the rule of integration by parts,

$$\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx$$

The next step is to calculate the integral of  $e^{2x}$ . To do so, we will substitute  $u = 2x$ .

$$\int e^{2x} dx = \frac{1}{2} \int e^u du = \frac{1}{2} \cdot \frac{e^u}{\ln(e)} = \frac{1}{2} e^{2x} + c$$

Therefore we have

$$\begin{aligned} y &= 4 \left( \frac{1}{2} x e^{2x} - \frac{1}{2} \left( \frac{1}{2} e^{2x} \right) \right) + c = 2x e^{2x} - e^{2x} + c \\ &= e^{2x} (2x - 1) + c \end{aligned}$$

The graphs for  $y$  when  $c = 0, 1$ , and  $-6$  can be seen in

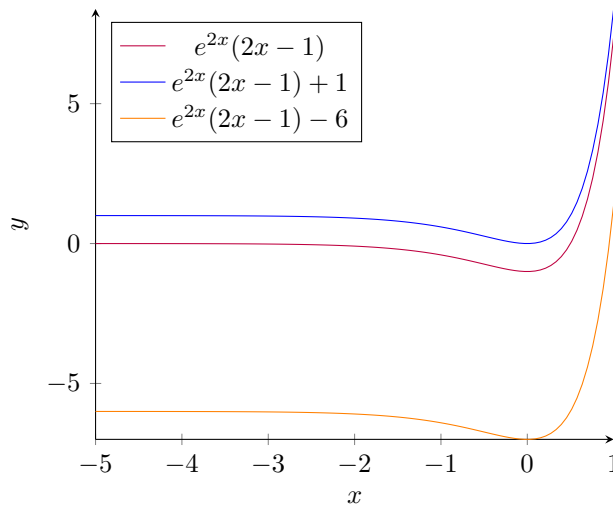


Figure 1:  $y = e^{2x}(2x - 1) + c$  when  $c = 0, 1$ , or  $-6$