Linear Algebra 2

Homework 1 – Complex Numbers

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2. We are tasked with solving the following equations for z.

(b)
$$\begin{aligned}
z^2 &= -10 + 20i \\
(a+bi)^2 &= -10 + 20i \\
a^2 + 2abi - b^2 &= -10 + 20i \\
a^2 - b^2 &= -10 \\
2ab &= 20 \\
b &= \frac{10}{a} \\
a^2 - \left(\frac{10}{a}\right)^2 &= -10 \\
a^2 - \frac{100}{a^2} &= -10 \\
a^4 + 10a^2 - 100 &= 0 \\
\text{Let } t &= a^2 \\
t^2 + 10t - 100 &= 0 \\
t &= -5 \pm 5\sqrt{5} \\
a = \pm\sqrt{-5+5\sqrt{5}} &= \pm\sqrt{5}\sqrt{\sqrt{5}-1} \\
b = \pm\frac{10}{\sqrt{5}\sqrt{\sqrt{5}-1}} &= \pm\frac{2\sqrt{5}}{\sqrt{\sqrt{5}-1}} \\
z_1 &= \sqrt{5}\sqrt{\sqrt{5}-1} + \frac{2\sqrt{5}}{\sqrt{\sqrt{5}-1}}i \\
z_2 &= -\sqrt{5}\sqrt{\sqrt{5}-1} - \frac{2\sqrt{5}}{\sqrt{\sqrt{5}-1}}i
\end{aligned}$$