Analysis of Algorithms

Homework 7 – *NP*

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1 Subgraph Isomorphism

We are given two graphs, $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. The subgraph isomorphism problem (SGI) is to determine if there exists a subgraph of G_2 which is isomorphic to G_1.

Part A

To prove that SGI $\in NP$ we must show that there is a way to verify a solution to the problem in polynomial time.

If we are given a solution to an SGI problem in the form of $G_3 = (V_3, E_3)$, which is a subgraph of G_2; and a bijection $f : V_1 \to V_3$,which is a map from the vertices of G_1 to the vertices of G_3, then we can verify that they are in fact isomorphic as follows.

```
function Isomorphic(G1, G3, f)
    for (u, v) ∈ E1 do
        if (f(u), f(v)) ∉ E3 then return false
    return true
```

This algorithm is clearly $O(E)$, so is polynomial. Therefore $SCI \in NP$.

Part B

To prove that SGI is *NP*-complete, we will show that the Clique problem (which is known to be *NP*-complete) is reducible to SGI.

The Clique problem is to determine, given a graph $G = (V, E)$ and a constant $c \leq |V|$, whether or not K_c (a complete graph with c vertices) is a subgraph of G.

To reduce the Clique problem to SGI, we must convert every instance of the Clique problem into one of SGI, such that the solution will be the same for both.

Consider some inputs to the Clique problem; a graph G, and a constant c. We can feed K_c and G as the inputs G_1 and G_2 of the SGI problem respectively. Then any SGI algorithm will tell us whether or not $G = G_2$ has a subgraph isomorphic to $K_c = G_1$, which is precisely the definition of the Clique problem.