Linear Algebra 2

Homework 1.5 – Roots of Unity

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1. (a) If z is a root of the equation $x^8 = 1$, then it is not necessarily true that $\mathrm{Re}(z) > 0$, since as shown in figure 1, this is only true for the three rightmost of the eight roots.

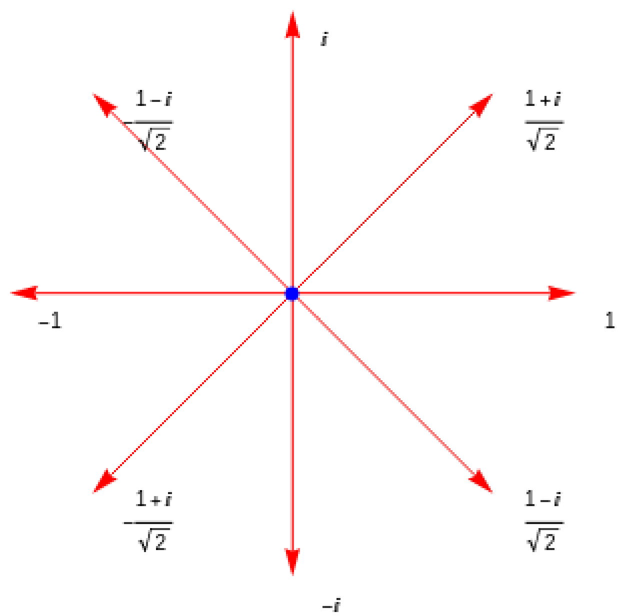

Figure 1: The roots of unity of order 8.

   (b) If z is a root of the equation $x^6 = 1$, then it is not guaranteed to be the case that $\mathrm{Im}(z) \neq 0$, since as is visible in figure 2, there are in fact two real solutions, namely 1 and -1.

   (c) For every even n, there must be two real solutions to the equation $x^n = 1$. This is because of the following which shows that both 1

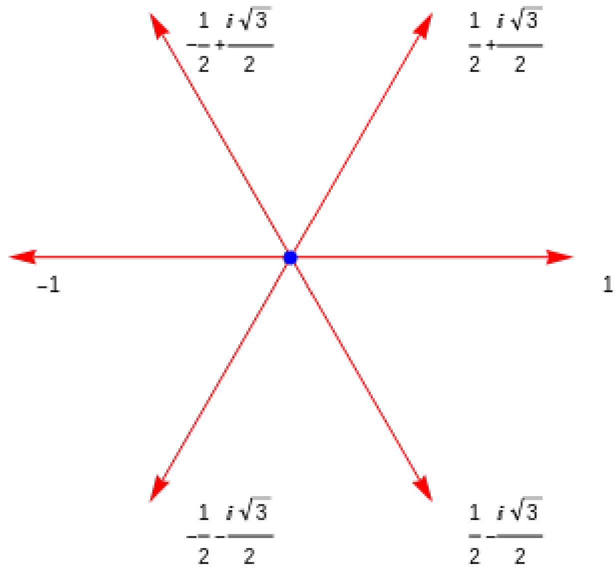

Figure 2: The roots of unity of order 6.

and -1 are real solutions the equation.

$$\forall n \in \mathbb{N},\ (-1)^{2n} = 1^{2n} = 1$$

(d) For every odd n there must be exactly one real solutions to the equation $x^n = 1$. For any n, $x = 1$ must be a solution, then adding $\frac{2\pi}{n}$ radians about the unit circle gives another, and another until we arrive again at 1. Since n here is odd, $\frac{2k\pi}{n}$ can never be equal to π, else we would have $n = 2k$, contradicting its oddity. Thus none of the solutions are precisely π radians around the unit circle from 1, denying us the solution $x = -1$ which is the only other real number on the unit circle.