

Analysis of Algorithms

Homework 7 – *NP*

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1 Subgraph Isomorphism

We are given two graphs, $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. The subgraph isomorphism problem (*SGI*) is to determine if there exists a subgraph of G_2 which is isomorphic to G_1 .

Part A

To prove that $SGI \in NP$ we must show that there is a way to verify a solution to the problem in polynomial time.

If we are given a solution to an *SGI* problem in the form of a subgraph $G_3 = (V_3, E_3) \subseteq G_2$ and a bijection $f : V_1 \rightarrow V_3$, i.e. f maps from the vertices of G_1 to the vertices of G_3 , then we can verify that they are in fact isomorphic as follows.

```
function ISOMORPHIC( $G_1, G_3, f$ )  
  for  $(u, v) \in E_1$  do  
    if  $(f(u), f(v)) \notin E_3$  then return false  
  return true
```

This algorithm is clearly $O(E)$, so is polynomial. Therefore $SGI \in NP$.