

# Linear Algebra 2

## Homework 1 – Complex Numbers

Abraham Murciano

March 16, 2021

2. We are tasked with solving the following equations for  $z$ .

$$(b) \quad z^2 = -10 + 20i$$

$$(a + bi)^2 = -10 + 20i$$

$$a^2 + 2abi - b^2 = -10 + 20i$$

$$a^2 - b^2 = -10$$

$$2ab = 20$$

$$b = \frac{10}{a}$$

$$a^2 - \left(\frac{10}{a}\right)^2 = -10$$

$$a^2 - \frac{100}{a^2} = -10$$

$$a^4 + 10a^2 - 100 = 0$$

$$\text{Let } t = a^2$$

$$t^2 + 10t - 100 = 0$$

$$t = -5 + 5\sqrt{5}$$

$$a = \pm\sqrt{-5 + 5\sqrt{5}} = \pm\sqrt{5}\sqrt{\sqrt{5} - 1}$$

$$b = \pm\frac{10}{\sqrt{5}\sqrt{\sqrt{5} - 1}} = \pm\frac{2\sqrt{5}}{\sqrt{\sqrt{5} - 1}}$$

$$z_1 = \sqrt{5}\sqrt{\sqrt{5} - 1} + \frac{2\sqrt{5}}{\sqrt{\sqrt{5} - 1}}i$$

$$z_2 = -\sqrt{5}\sqrt{\sqrt{5} - 1} - \frac{2\sqrt{5}}{\sqrt{\sqrt{5} - 1}}i$$

$$\begin{aligned}
\text{(d)} \quad & z^2 + |z|^2 = 2 - 4i \\
& (a + bi)^2 + |a + bi|^2 = 2 - 4i \\
& (a + bi)^2 + \sqrt{a^2 + b^2}^2 = 2 - 4i \\
& a^2 - b^2 + 2abi + a^2 + b^2 = 2 - 4i \\
& 2a^2 + 2abi = 2 - 4i \\
& 2a^2 = 2 \\
& 2ab = -4 \\
& a^2 = 1 \\
& b = -\frac{2}{a} \\
& a = \pm 1 \\
& b = \mp 2 \\
& z_1 = 1 - 2i \\
& z_2 = -1 + 2i
\end{aligned}$$