THE EVOLUTION OF THE UNIVERSE

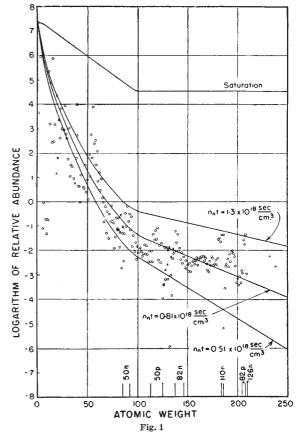
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THE discovery of the red shift in the spectra of I distant stellar galaxies revealed the important fact that our universe is in the state of uniform expansion, and raised an interesting question as to whether the present features of the universe could be understood as the result of its evolutionary development, which must have started a few thousand million years ago from a homogeneous state of extremely high density and temperature. clude first of all that the relative abundances of various atomic species (which were found to be essentially the same all over the observed region of the universe) must represent the most ancient archæological document pertaining to the history of the universe. These abundances must have been established during the earliest stages of expansion when the temperature of the primordial matter was still sufficiently high to permit nuclear transformations to run through the entire range of chemical elements. It is also interesting to notice that the observed relative amounts of natural radioactive elements suggest that their nuclei must have been formed (presumably along with all other stable nuclei) rather soon after the beginning of the universal expansion. In fact, we notice that natural radio-active isotopes with the decay periods of many thousand million years (such as uranium-238, thorium-232 and samarium-148) are comparatively abundant, whereas those with decay periods measuring only several hundred million years are extremely rare (as uranium-235 and potassium-40). If, using the known decay periods and natural abundances of these isotopes, we try to calculate the date when they have been about as abundant as the corresponding isotopes of longer life, we find that it must have been a few thousand million years ago, in general agreement with the astronomically determined age of the universe.

The early attempts to explain the observed relative abundances of the elements1,2 were based on the assumption that the present distribution represents a 'frozen equilibrium state' corresponding to some very high temperature and density in an early stage of universal expansion. Such equilibrium theories lead, however, to the result that the logarithm of the relative abundance must be a linear function of the nuclear binding energy, which in its turn is known to be a linear function of atomic weight. Thus, according to that picture, we would expect a rapid exponential decrease of relative abundances all the way from hydrogen to uranium, in direct contradiction to the observed distribution (circles in Fig. 1), which shows a rapid decrease for the first half of the natural sequence of elements, but levels up almost to a constant value in the second half.

As the result of this difficulty, I suggested that the observed abundances do not correspond to any equilibrium state at all, but, quite on the contrary, represent a dynamical building-up process which was arrested in a certain stage of its development by a rapid expansion of the universe. According to this point of view, one should imagine the original state of matter as a very dense over-heated neutron gas which could have originated (if one lets one's imagina-



tion fly beyond any limit) as the result of hypothetical universal collapse preceding the present expansion. In fact, the extremely high pressures obtaining near the point of complete collapse (singular point at t = 0) would have squeezed the free electrons into the protons, turning the matter into the state of over-heated neutron fluid. When the expansion began, and the density of neutron gas dropped, the neutrons would be expected to begin decaying again into protons, and more and more complex nuclear aggregates could be built up as the result of the union between the newly formed protons and the neutrons still remaining. Such a building-up process must have started when the temperature of the neutron-proton mixture dropped below a few times 10¹⁰ ° K., which corresponds to the mutual binding energies of these nuclear particles. The equations governing such a gradual building-up process can evidently be written in the form:

$$\frac{dn_i}{dt} = \lambda_{i-1} n_{i-1} - \lambda_i n_i \ (i = 1, 2, 3, \ldots), \ (1)$$

where n_i is the number of atomic nuclei of atomic weight i, and λ 's are the coefficients depending on the collision frequency, and the capture cross-sections for fast neutrons in the nuclei of various atomic weight.

The numerical study of these equations was carried out by R. Alpher^{4,5}, who used the recent experimental data on the capture cross-sections of fast neutrons. These cross-sections are known to increase very rapidly by a factor of several hundred for the first half of the atomic weights, and to remain more or less constant for the second half; a fact which is

of paramount importance for understanding the general shape of the abundance curve. Suppose that the building-up process goes at a constant density, p, for a limited period of time, Δt ; the resulting abundances will depend apparently only on the product $\rho \Delta t$, which determines the total number of collisions between the developing nucleus and the free neutrons.

The results of the computations carried out by R. Alpher are shown by continuous curves in Fig. 1,

with the values of $n_n \Delta t = \frac{\rho}{m} \Delta t$ marked on each

curve. It is seen that for very large values of $n_n \Delta t$, the process reaches saturation, and the number of the various nuclei becomes inversely proportional to the corresponding capture cross-sections. For smaller values of $n_n \Delta t$, heavier nuclei do not have time to build up to the saturation point, and we see from the figure that for $\rho \Delta t \cong 1.3 \times 10^{-6}$ gm. cm.⁻³ sec. $(n_n \Delta t \simeq 0.8 \times 10^{18} \text{ sec. cm.}^{-3})$ the calculated curve stands in very good agreement with the observational

The agreement obtained is amplified by the fact that the observed abundances show the abnormally high values for the isotopes containing the completed shells of neutrons or protons (as indicated along the atomic weight axis in Fig. 1); in fact, it is known that such nuclei possess abnormally small capture cross-sections, which would cause the accumulation of the material at these particular atomic weights. Since the building-up process must have been accomplished within a time period comparable with the decay period of neutrons, we have $\Delta t \cong 30$ min. $\cong 2 \times 10^3$ sec., from which it follows that during that period the density of matter must have been of the order of magnitude 10-9 gm. cm.-3. On the other hand, since the temperature must have been of the order of 109°K., the mass-density of radiation, aT^4/c^2 , was comparable with the density of water. Thus we come to the important conclusion that, at that time, the expansion of the universe was governed entirely by radiation and not by matter.

In this case the relativistic formula for the expansion can be written in the form⁶,

$$dl/dt = \sqrt{\frac{8\pi G}{3} \frac{aT^4}{c^2} l^2}, \qquad (2)$$

where l is an arbitrary distance in the expanding space, and the constant term containing the radius of curvature is neglected because of the high value of the density. Remembering that for the adiabatic expansion of the radiation $T \sim 1/l$, we can integrate (2) into the form: $T = \sqrt[4]{\frac{3c^2}{32\pi Ga}} \cdot \frac{1}{t^{1/2}} = \frac{2 \cdot 14 \times 10^{10}}{t^{1/2}} \text{ °K.} \tag{3}$

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 (3)

For the mass-density of radiation we have:

$$\rho_{\text{rad.}} = \frac{3}{32\pi G} \cdot \frac{1}{t^2} = \frac{4 \cdot 5 \times 10^5}{t^2} \text{ gm.cm.}^{-3}.$$
 (4)

For the density of matter we must evidently write:

$$\rho_{\text{mat.}} = \frac{\rho_3}{t^{3/2}} \text{ gm.cm.}^{-3}, \tag{5}$$

where ρ_0 is to be determined from the conditions of the nuclear building-up process. It can be done in the simplest way by considering the building up of deuterons by proton-neutron collisions. Writing X for the concentration of neutrons (with X(0) = 1), and Y for the concentration of protons (with Y(0)=0), we obtain the equations:

$$\frac{dX}{dt} = -\lambda X - \frac{XY}{m} \rho v\sigma,
\frac{dY}{dt} = +\lambda X - \frac{XY}{m} \rho v\sigma,$$
(6)

where v is the thermal velocity, and σ the capture cross-section which (for the energies in question) can be sufficiently accurately represented by the formula?:

$$\sigma = \frac{2^{3/2}\pi e^2 \hbar \varepsilon^{5/4}}{m^3 c^5} \cdot \frac{1}{E^{3/2}}, \tag{7}$$

 $\varepsilon = 2.19$ MeV. being the binding energy of the deuteron. Expressing v and E through the temperature, and using (3), we can rewrite (6) as:

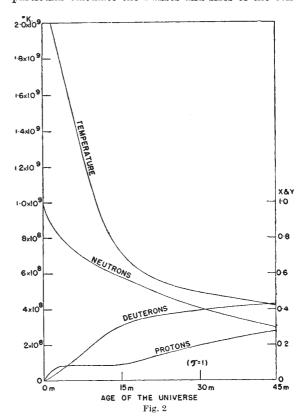
$$\frac{dX}{d\tau} = -X - \frac{\alpha XY}{\tau},
\frac{dY}{dz} = +X - \frac{\alpha XY}{\tau},$$
(8)

where $\tau = \lambda t$ and:

$$\alpha = \frac{2^{13/2\pi^{5/4}e^{2}\hbar\varepsilon^{5/2}G^{1/4}a^{1/4}}}{3^{5/2}m^{9/2}c^{11/2}k} \cdot \rho_0.$$
 (9)

In order that the equations (8) should yield $Y \cong 0.5$ for $\tau \to \infty$ (since hydrogen is known to form about 50 per cent of all matter), the coefficient α must be set equal to 0.5. The change of X and Y with time in this case is shown in Fig. 2, which also indicates the corresponding variation of temperature. Assuming $\alpha=0.5$, we find from equation (9) that $\rho_0=$ 0.72×10^{-2} , which fixes the dependence of material density on the age of the universe.

Once we have $\rho_{rad.}$ and $\rho_{mat.}$ as functions of time, we can follow the physical processes taking place during the further expansion of the universe, and in particular calculate the masses and sizes of the con-



densations of that primordial gas which must have originated sooner or later according to Jeans' principle of gravitational instability8. Jeans' classical formula gives the diameter D of the condensations which will be formed in a gas of temperature T and density ρ in the form:

$$D^2 = \frac{10\pi}{9mG_{\rm p}} \cdot \frac{3}{2} kT. \tag{10}$$

Using the expressions (3) and (5), we get:

$$M = \rho D^3 = \frac{2^{31/8} 5^{7/4} \pi^{5/4} e \hbar^{5/4} e^{5/4}}{3^{17/8} m^{15/4} e^{5/4} G^{7/4}}$$
(11)

(where a has been expressed through other fundamental constants).

It is interesting to notice that the time-factor cancels out in the calculation of M, so that the mass of the condensations comes out the same, independent of the epoch when they were formed. It seems, however, reasonable to assume that the effect of gravitational instability became important only when the mass-density of radiation became comparable with the density of matter, since it is hard to imagine a 'gravitational condensation of pure radiation'. Using (4) and (5), we find that $\rho_{\text{rad.}} = \rho_{\text{mat.}} =$ 3×10^{-26} gm.cm.⁻³ at $t=3.9\times10^{15}$ sec. = 1.3×10^8 years, at which point $T=340^\circ$ K. For this value of t we obtain:

$$D = \frac{2^{45/8} 5^{1/4} \pi^{7/4} e^{3 \hbar^{3/4} \epsilon^{15/4}}}{3^{27/8} m^{29/4} e^{35/4} G^{5/4}}.$$
 (12)

Substituting numerical values, we have:

$$M = 5.5 \times 10^{40} \text{ gm.} = 2.7 \times 10^7 \text{ sun-masses}$$

 $D = 1.3 \times 10^{22} \text{ cm.} = 13,000 \text{ light-years,}$ (13)

which must represent the masses and the diameters of the original galaxies.

The above estimate of galactic masses falls short by a factor of about one hundred from the massvalues of galaxies obtained from astronomical data. But it must be remembered that the simple Jeans' formula used in these calculations does not take into account the effect of radiation pressure, and also is applicable only to the gravitational condensations in non-expanding space. The effect of additional radiation pressure (which is quite important according to the previous considerations) and the tearing force of expansion will lead to considerably larger condensation masses. The detailed study of this question will require, however, the extension of Jeans' classical arguments for the case of a mixture of gas and radiation in the expanding space. At the present stage one should be satisfied with the fact that, by such comparatively simple and rather natural considerations, masses and sizes comparable to those of stellar galaxies can be expressed in terms of funda-mental constants, and the basic quantities of nuclear

We may add that, according to the above picture, the galaxies have been originally formed in the purely gaseous form (including a certain amount of solid dust particles), which must account for the regular shapes of rotating bodies. The formation of individual stars within the galactic bodies must have taken place at a somewhat later stage, probably along the lines of the Spitzer - Whipple theories 9,10. stars were formed by the condensation process within the rotating gaseous mass, their tangential velocities, being equal to the original velocities of the gasmasses, were clearly not high enough to maintain them on circular Kepler orbits, so that the newly formed stars must have been moving along elongated elliptical orbits with the points of maximum elongation in the places of their origin. This situation must have remained essentially unchanged even when all the material of originally gaseous galaxies was used up in the formation of stars.

These considerations give a simple explanation of the otherwise mysterious fact that the elliptical galaxies and the central bodies of spirals rotate 'as solid bodies' with the linear velocities proportional to the distance from the axis. In fact, according to our picture, the maximum Doppler displacements observed at various distances from the axis correspond to the velocities of stars passing through 'aphelion' at these particular distances, and, according to the previous argument, are equal to the velocities which the gas-masses must have had in these regions prior to their condensation into the stars.

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COASTAL WAVES*

R ECENT investigations by the Admiralty have shown that storm waves and the swell that leaves the storm area are composed of a mixture of wave-trains, the wave-lengths of which range from a few feet up to a maximum which depends on the greatest wind strength, and may be as much as 3,000 feet. It seems remarkable that such component wave-trains should travel independently across the ocean, and that all except the very shortest should be recognizable after travelling thousands of miles; but such close agreement with theory has been demonstrated, and the component wave-trains have been found to advance across the ocean with the theoretical velocities appropriate to their lengths.

The short waves formed at the beginning of a storm are overtaken and outdistanced by long waves formed when the wind is strongest. With the help of new wave-recording and analysing apparatus, each wave-length can be detected and its amplitude measured when it arrives at a distant coast, and the waves recorded on the coast of Cornwall are found at times to be a mixture of waves generated near the coast with swell-components from more than one North Atlantic storm, and with other swell, probably a few inches high, which was generated in a storm so far away as Cape Horn.

The visible crests and troughs are the result of the combination of trains of waves of different lengths, and since such waves travel with different velocities, the wave-pattern is continually changing. some of the component wave-trains get into step and reinforce each other, they produce a group of typical waves with relatively high crests and deep troughs; but when they get out of step and tend to neutralize

* An account of the symposium held at the meeting of Section A (Mathematics and Physics) of the British Association on September 9, in which Brigadier R. A. Bagnold, Mr. N. F. Barber, Dr. G. E. R. Deacon and Major W. W. Williams took part.