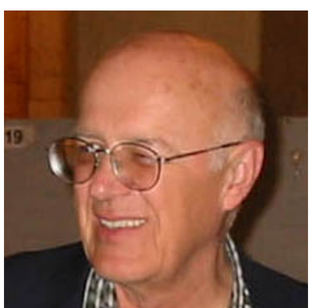
21st International Congress on Acoustics
165th Meeting of the Acoustical Society of America
52nd Meeting of the Canadian Acoustical Association

2-7 June 2013
Montréal, Canada

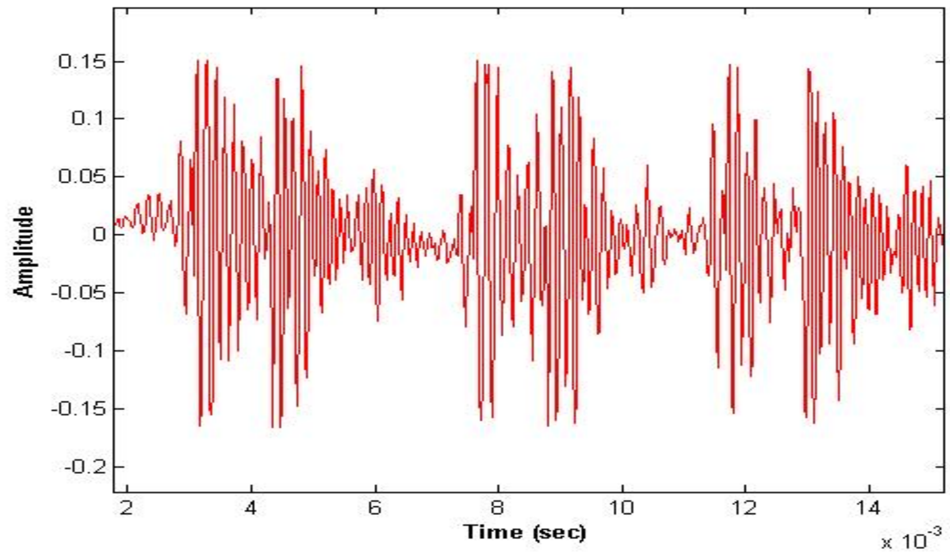
# *Buckling as a source of sound, with application to the modeling of cicada sound generation*

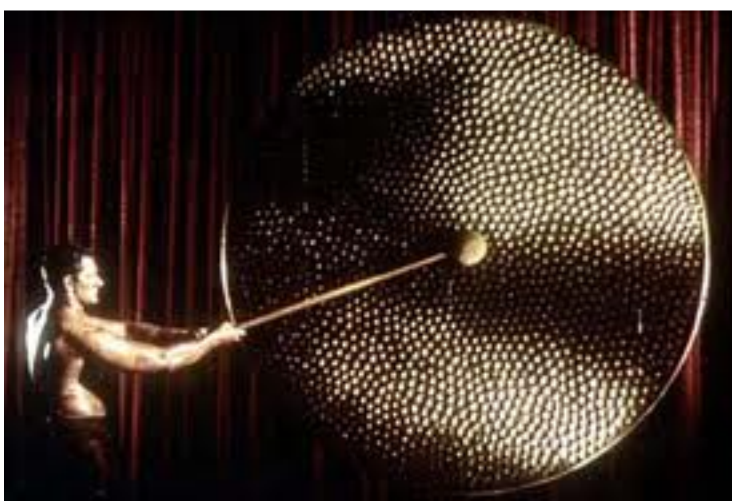
**Allan D. Pierce, Derke R. Hughes, Kossi Edoh, Richard A. Katz, Robert M. Koch**



***Noisy little (not necessarily) bugs***

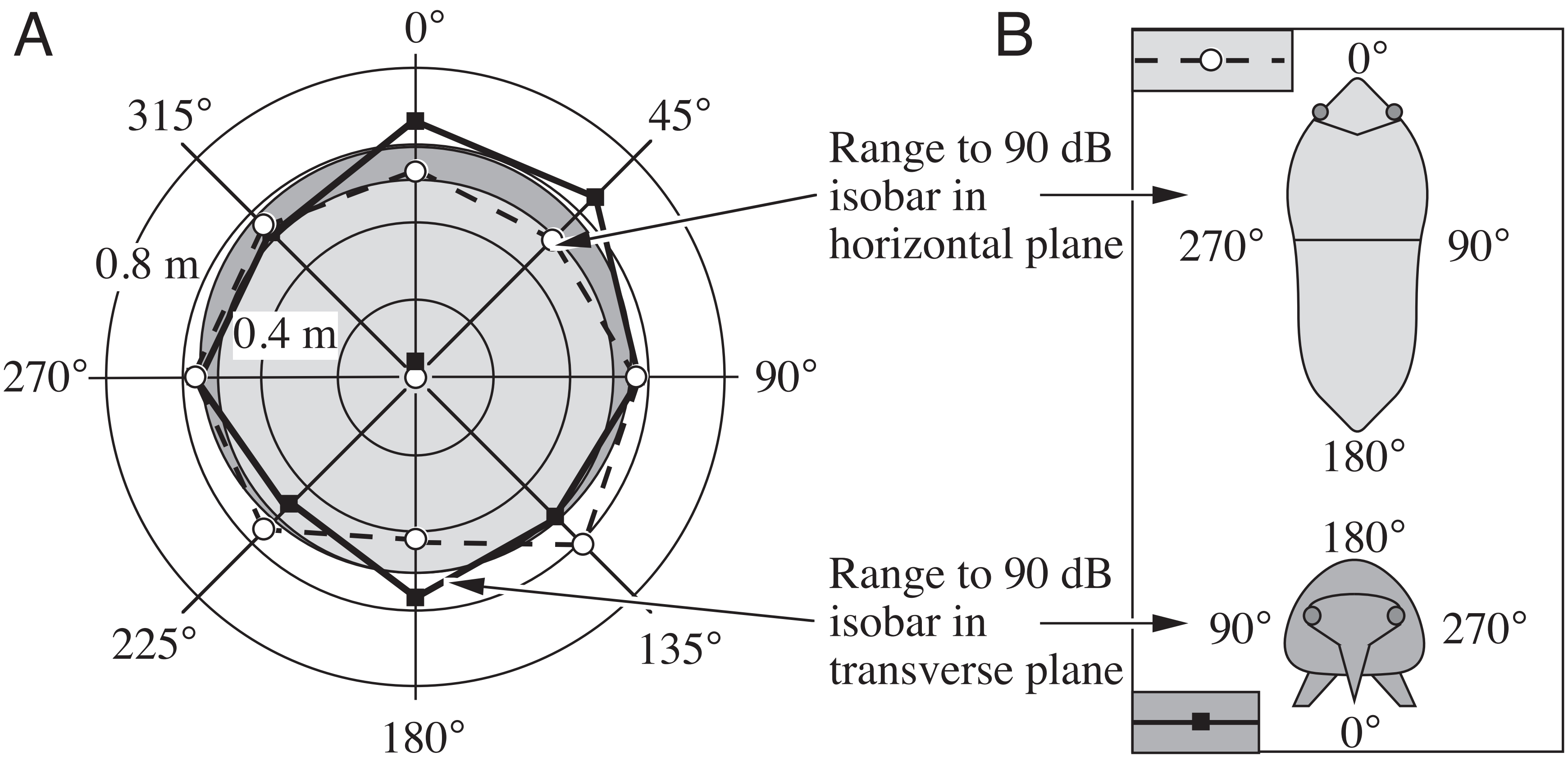
*Source: Massachusetts Cicada*



cymbals

# 90 dB in air at a distance of 0.8 m




Bennet-Clark and Daws, J. Exper. Biol. ( 1999)

A PHYSIOLOGICAL ANALYSIS OF CICADA SONG

By J. W. S. PRINGLE
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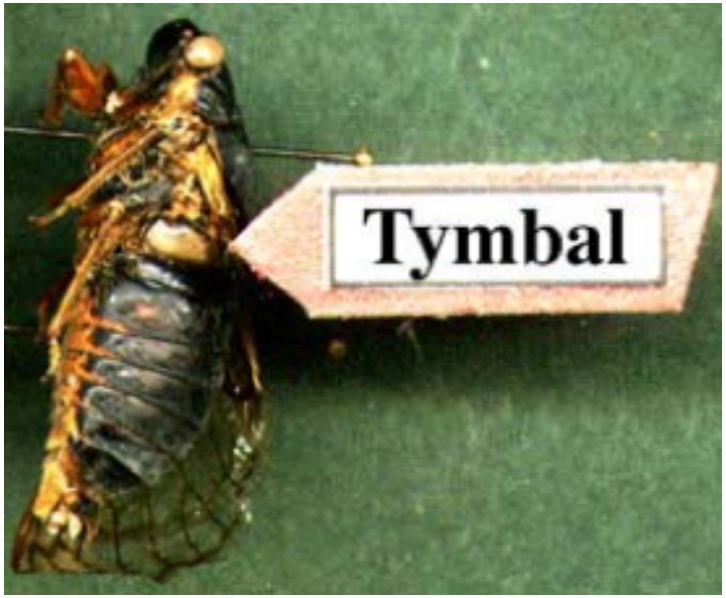
(*Received* 10 *February* 1954)

John William Sutton Pringle, FRS (1912-1982)

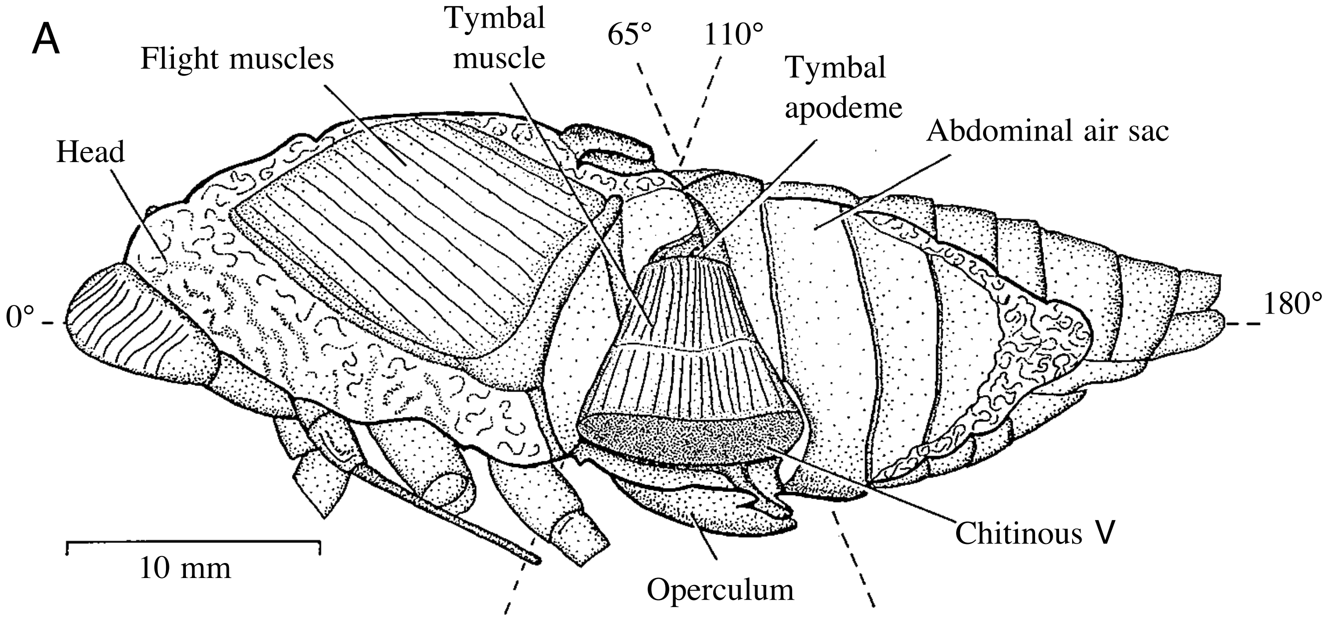
***Pringle was the first to explicitly mention the word "buckling"***

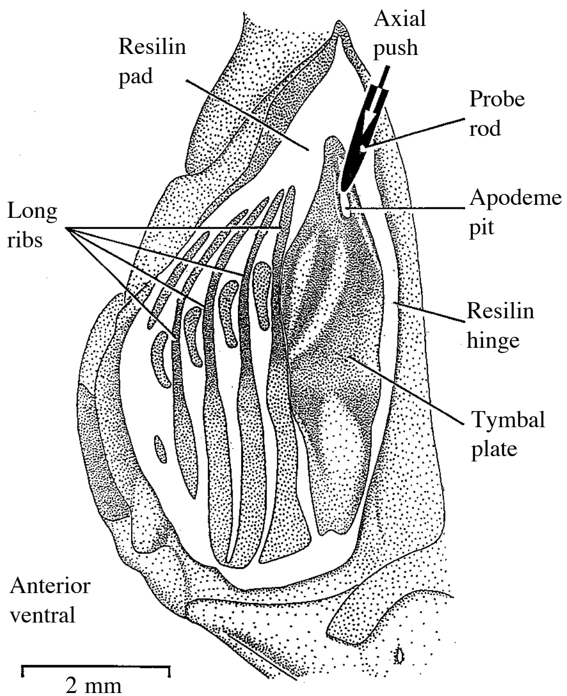
A pulse of sound is emitted when the tymbal suddenly buckles or is restored to its resting position by its natural elasticity; in the song of some species both movements are effective. The tymbal muscles, which are responsible for the buckling, have a myogenic rhythm of activity, initiated, but only slightly controlled in frequency, by impulses in the single nerve fibre supplying each muscle. The two tymbals normally act together.




The word ***tymbal*** is defined in Merriam-Webster Unabridged Dictionary as the `"***vibrating membrane in the shrilling organ of a cicada***"'--- so it is a term invented by biologists to describe part of the anatomy of cicadas.  OED attributes its origin to someone named Bushman in 1854

cymbals



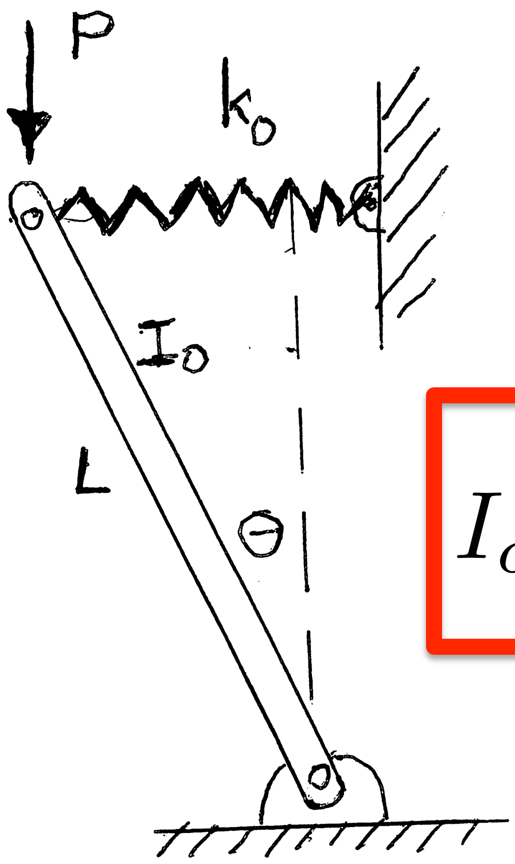




# *Long ribs do the actual buckling*

# What is buckling?

***Buckling caused the recent Bangladesh building collapse***

# Exposition to explain how buckling generates sound




$$I_o \frac{d^2\theta}{dt^2} = PL\sin\theta - k_0 L^2 \sin\theta.$$

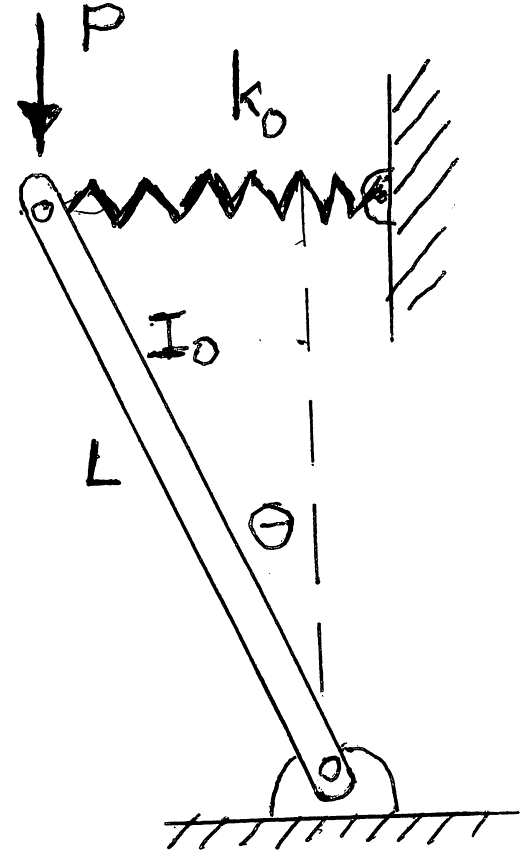
approximates to

$$I_o \frac{d^2\theta}{dt^2} + \left(k_0 L^2 - PL\right)\theta = 0$$

# *Buckling (critical) load*

$$P_{\mathrm{cr}} = k_o L$$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{L}{I_o}\left(P_{\mathrm{cr}} - P\right)\theta = 0}$$



If (muscle) applied compressive force is ***below the buckling load***:

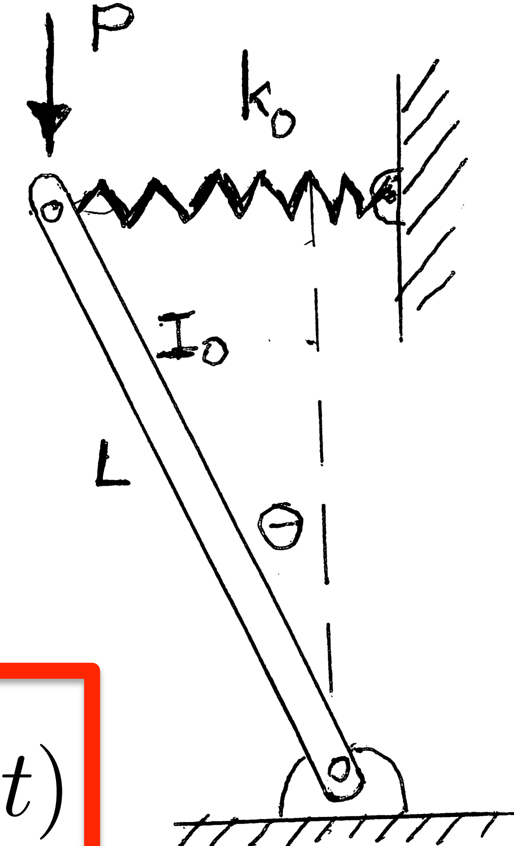
$$P < P_{\mathrm{cr}}$$

The rod (rib) ***oscillates*** back and forth

$$\boxed{\theta = A\cos(\omega_o t) + B\sin(\omega_o t)}$$

with the angular frequency

$$\omega_o = \left(\frac{L}{I_o}\right)^{1/2} (P_{\mathrm{cr}} - P)^{1/2}$$



If (muscle) applied compressive force is ***above the buckling load***:

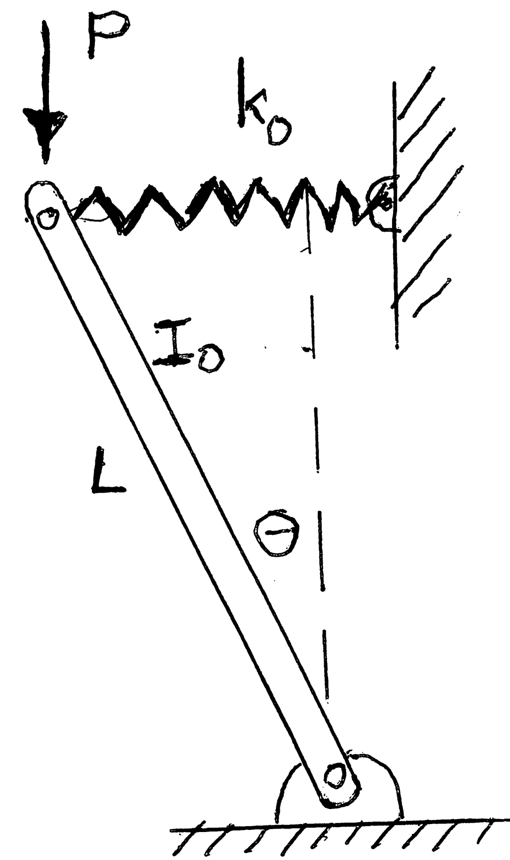
$$P > P_{\rm cr}$$

The displacement of the rod (rib) grows exponentially

$$\boxed{\theta = Ae^{\alpha_o t} + Be^{-\alpha_o t}}$$

with the exponential growth rate

$$\alpha_o = \left(\frac{L}{I_o}\right)^{1/2} (P - P_{\rm cr})^{1/2}$$

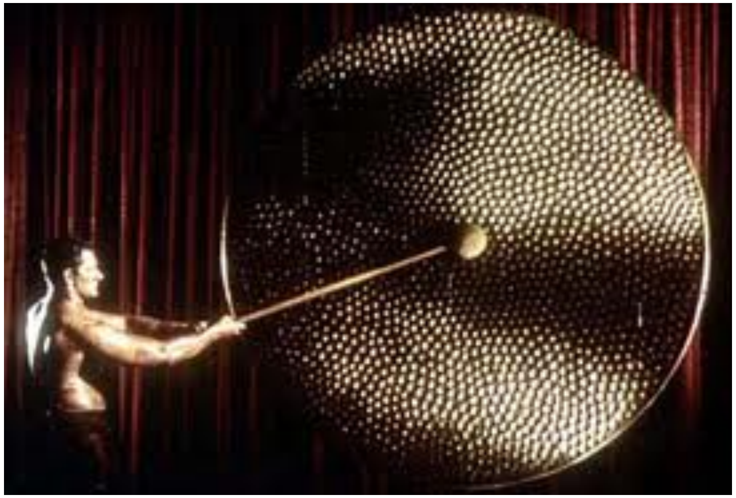


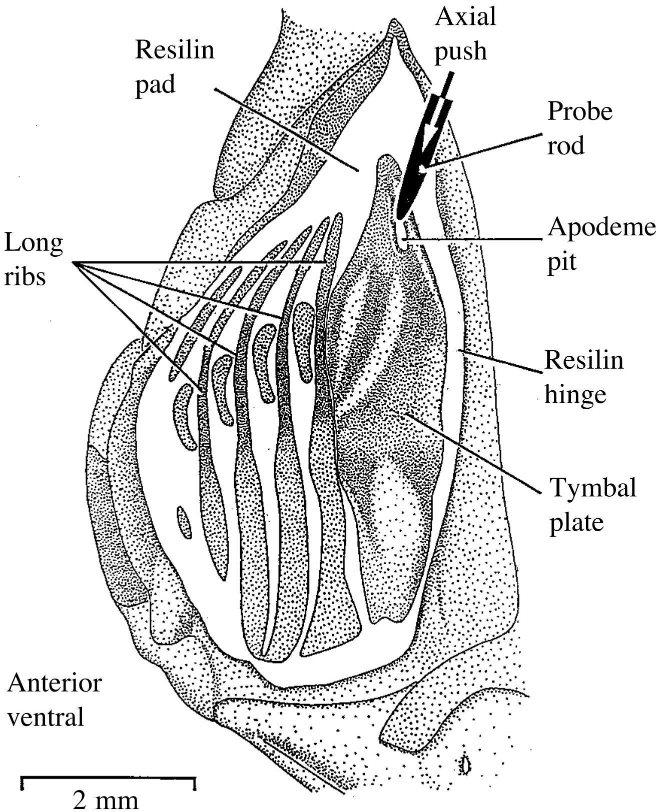
If this exponential growth isn't arrested ***then the rib will break***.

Something like what happened in Bangladesh.

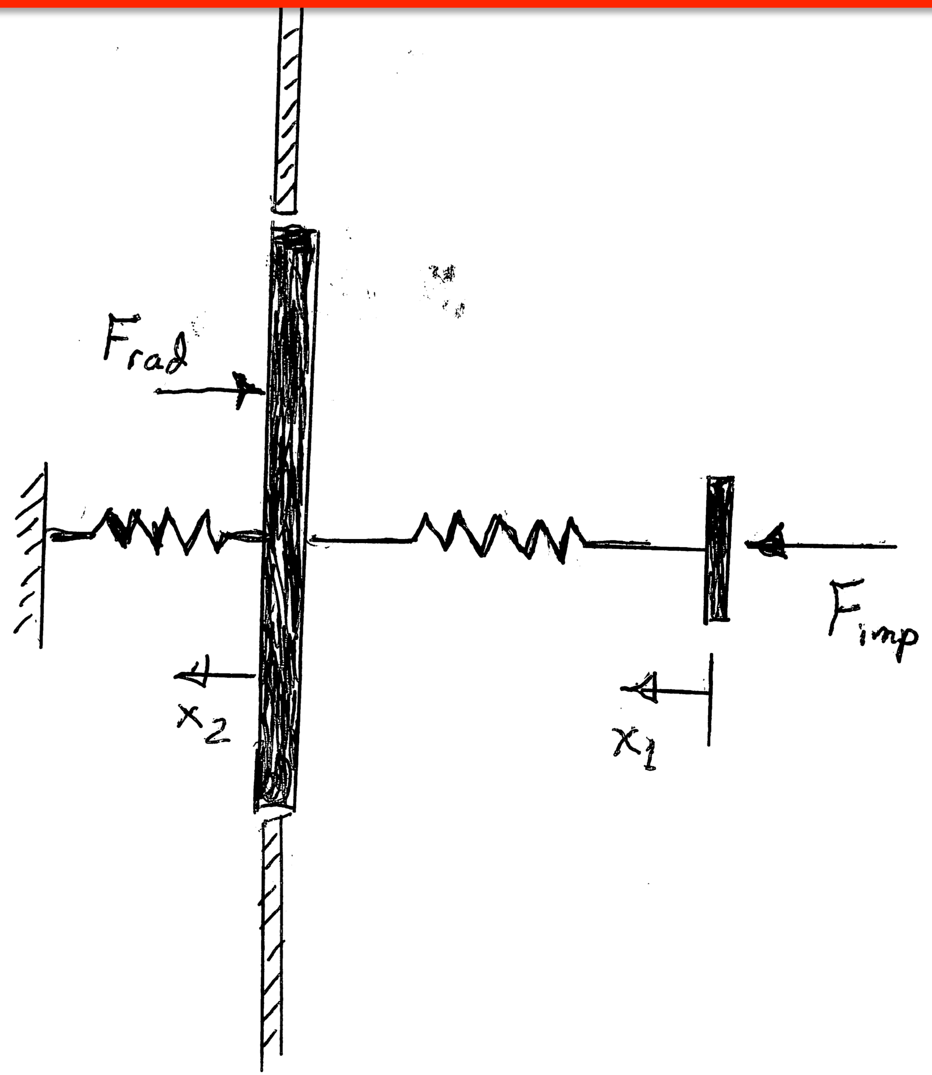
In the cicada the exponential growth of the buckling rib is ***arrested***.

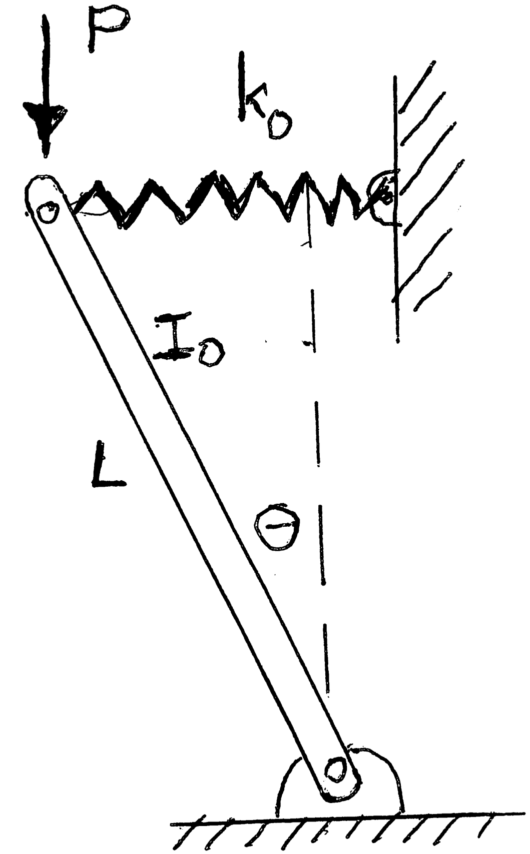
***Result is analogous to a hammer striking a gong.***

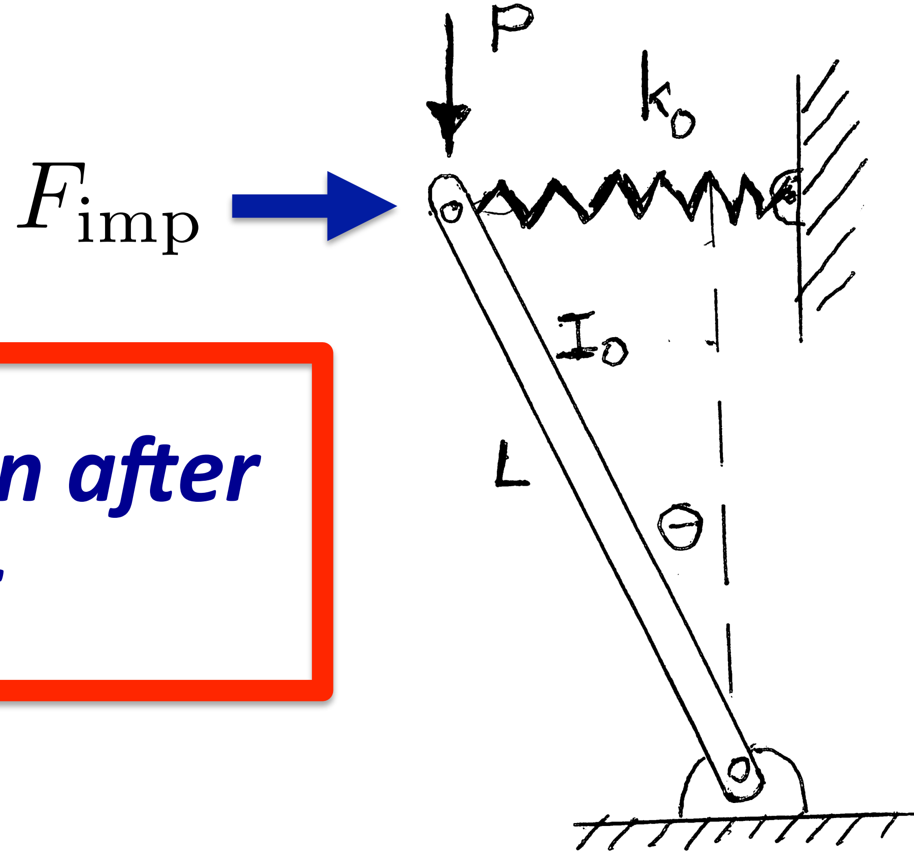




# Sketch of a model for an arresting mechanism



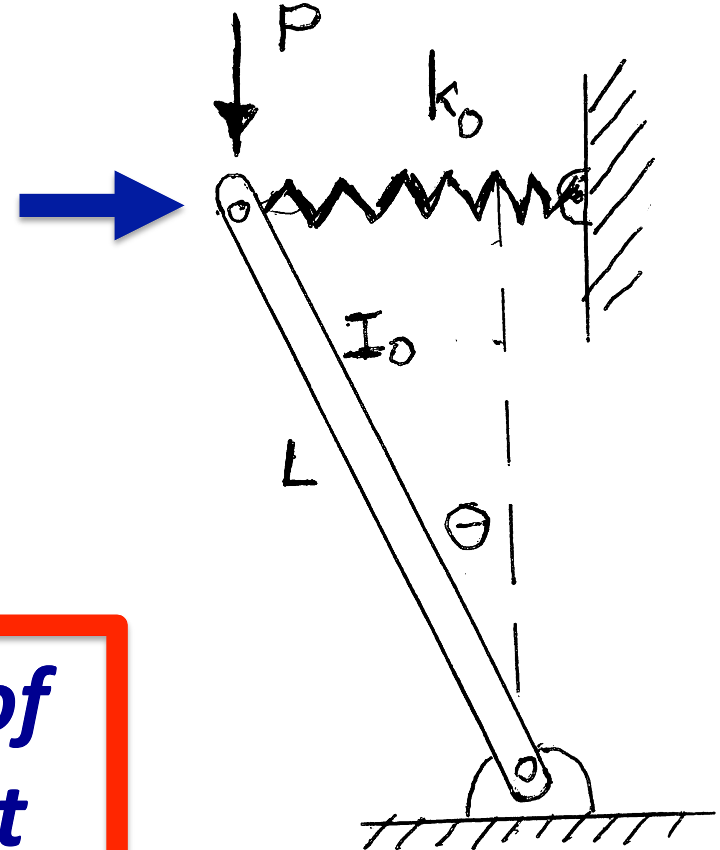






***Motion of rod motion after impact with arrester***

$$I_o \frac{d^2\theta}{dt^2} + \left(k_0 L^2 - PL\right)\theta = -F_{\text{imp}} L$$

$F_{\mathrm{imp}}$




$$x_1 = L\left(\theta - \theta_{\mathrm{ar}}\right)$$

**Displacement of the end of the rod after initial impact with the arrester**

# Coupled differential equations

$$\frac{I_o}{L^2}\frac{d^2x_1}{dt^2} + \left(k_o - \frac{P}{L}\right)(x_1 + L\theta_{\mathrm{ar}}) + k_1(x_1 - x_2) = 0$$

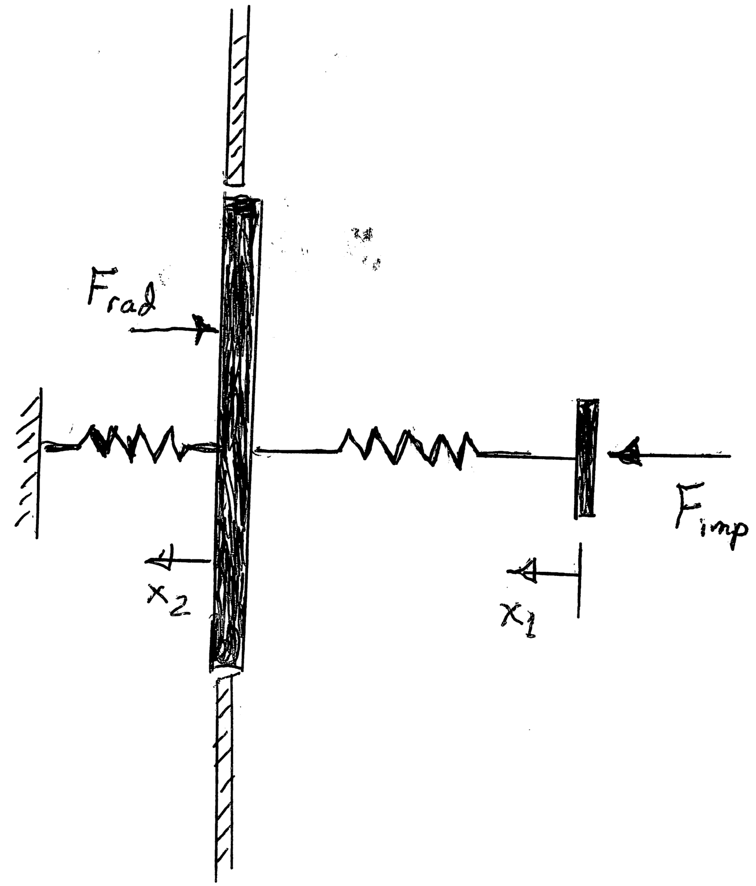
$$M\frac{d^2x_2}{dt^2} + k_2x_2 + k_1(x_2 - x_1) = -F_{\mathrm{rad}}$$

**The second displacement variable can be regarded as a representative tymbal displacement**

# *Initial conditions at time of impact*

$$x_1 = 0$$

$$\frac{dx_1}{dt} = V_0 \qquad \text{at} \quad t = 0$$



***After the impact, the second mass (M) is quickly accelerated to the velocity***

$$V_{\mathrm{begin}} = \frac{(I_o/L^2)}{M + (I_o/L^2)} V_o$$

***so the second mass (tymbal) starts to oscillate with an initial energy***

$$E_{\mathrm{begin}} = \frac{1}{2} \frac{M(I_o/L^2)^2}{[M + (I_o/L^2)]^2} V_o^2$$

***Radiated energy is maximized if tymbal mass is***

$$M_{\mathrm{opt}} = \frac{I_o}{L^2}$$

***Shortly after impact the tymbal is in damped oscillation***

$$M \frac{d^2 x_2}{dt^2} + k_2 x_2 = -F_{\mathrm{rad}}$$

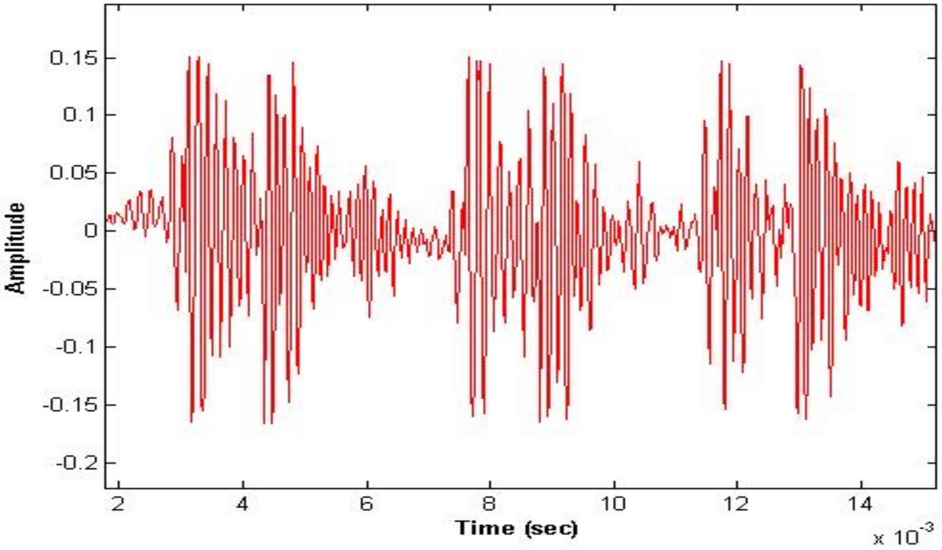
$$F_{\mathrm{rad}} \approx -\frac{\rho \pi a^4}{2c} \frac{d^3 x_2}{dt^3}$$

# Solution for tymbal motion

$$x_2 = \left(\frac{V_{\text{begin}}}{\omega_{\text{vib}}}\right) G(t) \sin(\omega_{\text{vib}} t)$$

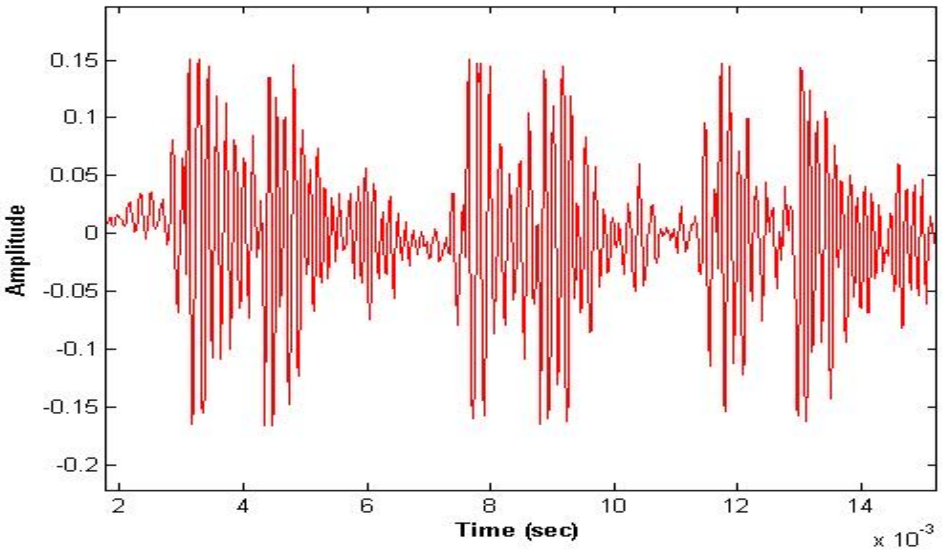
$$\omega_{\text{vib}} = \left(\frac{k_2}{M}\right)^{1/2}$$

$$G(t) = e^{-\alpha_{\text{rad}} t} H(t)$$



# Radiated pressure

$$p = -\omega_{\text{vib}} V_{\text{begin}} \frac{\rho A}{2\pi r} G\left(t - \frac{r}{c}\right) \sin\left[\omega_{\text{vib}}\left(t - \frac{r}{c}\right)\right]$$




# Radiated power

$$P_{\text{rad}} = \frac{\omega_{\text{vib}}^2 \rho A^2}{4\pi c} V_{\text{begin}}^2 G^2(t)$$

## *Total energy radiated*

$$E_{\mathrm{rad}} = \frac{1}{2\alpha_{\mathrm{rad}}} \frac{\omega_{\mathrm{vib}}^2 \rho A^2}{4\pi c} V_{\mathrm{begin}}^2$$

## *Conservation of energy*

$$E_{\mathrm{rad}} = E_{\mathrm{rad}}$$

## *Concluding remarks*

- *Energy initially stored in muscle transferred to rib motion*
- *Energy in rib motion transferred to tymbal motion*
- *Tymbal radiates nearly as a baffled piston*
- *Energy in vibrating piston transferred to acoustic energy*