Homework 2: Principal Component Analysis

S&DS 5360 | Multivariate Statistics

Brandon Tran (bat53)

February 01, 2026

```
library(tidyverse)
library(knitr)
library(kableExtra)
library(haven)
library(corrplot)
library(PerformanceAnalytics)
library(heplots)
source("https://raw.githubusercontent.com/jreuning/sds363_code/refs/heads/main/pcaThreshold.r
source("https://raw.githubusercontent.com/jreuning/sds363_code/refs/heads/main/ciscoreplot.r
```

Setup & Data Overview

Over the course of the semester, we will be using the Healthy Minds Network's latest 2024–2025 Healthy Minds Study (HMS) student survey. The HMS is a population-level, web-based survey administered by the HMN to assess mental health, service utilization, and related factors among post-secondary students. Since its inception over 15 years ago, the study has collected more than 935,000 responses from over 675 institution nationwide.

The latest 2024–2025 iteration was graciously made available to us following a successful application for access, which proposed the following research questions (framed in the context of S&DS 5360's curriculum):

> *The proposed research aims to examine the multivariate relationship between sexual and gender minority (GSM) identities, digital environment exposure, and mental health outcomes in the 2024-25 college student population. Utilizing the Demographics and Mental Health Status standard modules, this study will investigate how latent factors of psychological distress—comprising the PHQ-9 (Depression), GAD-7 (Anxiety), and UCLA Loneliness Scale—vary across intersectional identities. Additional exploratory analysis may include (1) factor analysis / PCA to*

identify if "Psychological Distress" and "Academic Impairment" (e.g., deprawsc, anx_score, and aca_impa) load onto distinct latent constructs for different GSM generations, (2) MANOVA to determine if the "vector of well-being" (comprising flourishing, loneliness, and distress scores) differs significantly based on the interaction between categorical identity markers and experiences of unfair treatment at the institutional or digital level, and (3) cluster analysis to identify "Student Behavioral Archetypes" based on a combination of lifestyle factors (sleep_wknight, exerc) and digital engagement.

The resulting dataset features $n = 84,735$ individual respondents across $1,608$ questions. These questions are organized into three module types:

- **Standard Modules:** Fielded at all institutions and provide core data on demographics, mental health status (e.g., depression and anxiety screenings), and help-seeking behaviors.
- **Elective Modules:** Specific topics requested by select institutions for in-depth assessment (e.g., substance use, eating and body image, AI attitudes).
- **Special Modules:** Other unique modules tailored for specific cohorts (e.g., Black College Student Mental Health Module for HBCUs).

To ensure that the results are representative of the broader student population at each school, the dataset includes a non-response weight variable, `nrweight`. It adjusts for the fact that female students typically respond at higher rates than male students, and it gives equal aggregate weight to each school in the national estimates, preventing results from being dominated by very large institutions.

For the purpose of Homework 2, which focuses on Principal Component Analysis, we will focus on questions that fall under the Standard Modules. Specifically, we will use composite variables that are created during cleaning and are the sum of a series of Likert scale questions on the same topic. The composite variables are defined as the sum of each nested question:

- **`flourish`:** Below are 8 statements with which you may agree or disagree. Using the 1-7 scale, indicate your agreement with each item by indicating that response for each statement. *($min = 8$, $max = 56$)*

    1. I lead a purposeful and meaningful life.
    2. My social relationships are supportive and rewarding.
    3. My social relationships are supportive and rewarding.
    4. I am engaged and interested in my daily activities.
    5. I actively contribute to the happiness and well-being of others.
    6. I am competent and capable in the activities that are important to me.
    7. I am a good person and live a good life.
    8. I am optimistic about my future.

- **dprawsc:** Over the last 2 weeks, how often have you been bothered by any of the following problems? *(min = 0, max = 27)*

    1. Little interest or pleasure in doing things
    2. Feeling down, depressed or hopeless
    3. Trouble falling or staying asleep, or sleeping too much
    4. Feeling tired or having little energy
    5. Poor appetite or overeating
    6. Feeling bad about yourself—or that you are a failure or have let yourself or your family down
    7. Trouble concentrating on things, such as reading the newspaper or watching television
    8. Moving or speaking so slowly that other people could have noticed; or the opposite—being so fidgety or restless that you have been moving around a lot more than usual
    9. Thoughts that you would be better off dead or of hurting yourself in some way

- **anx_score:** Over the last 2 weeks, how often have you been bothered by the following problems? *(min = 0, max = 21)*

    1. Feeling nervous, anxious or on edge
    2. Not being able to stop or control worrying
    3. Worrying too much about different things
    4. Trouble relaxing
    5. Becoming easily annoyed or irritable
    6. Being so restless that it's hard to sit still
    7. Feeling afraid as if something awful might happen

- **lonesc:** Please answer the following: *(min = 3, max = 9)*

    1. How often do you feel that you lack companionship?
    2. How often do you feel left out?
    3. How often do you feel isolated from others?

- **ed_sde:** Please answer the following questions as honestly as possible. *(min = 0, max = 5)*

    1. Do you often feel the desire to eat when you are emotionally upset or stressed?
    2. Do you often feel that you can't control what or how much you eat?
    3. Do you sometimes make yourself throw up (vomit) to control your weight?
    4. Are you often preoccupied with a desire to be thinner?
    5. Do you believe yourself to be fat when others say you are thin?

Omitting NAs, we fall from $84,735$ to $68,062$ unique responses to each of the above modules. With our variables defined, we proceed to load in our data for subsequent analyses.

```r
hms_data <- read_sav("../data/hms_data.sav", encoding = 'latin1')
```

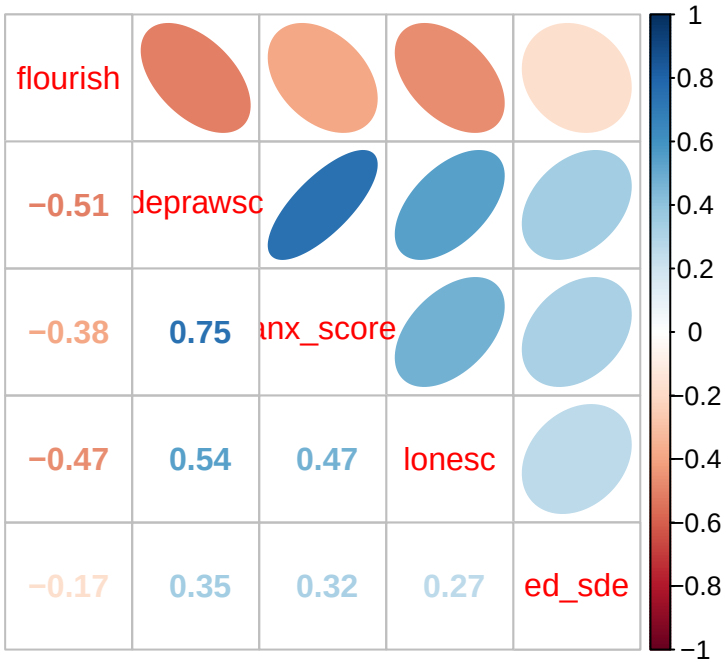
```r
pca_data <- hms_data %>%
    select(
        flourish,
        deprawsc,
        anx_score,
        lonesc,
        ed_sde
    ) %>%
    drop_na()

head(pca_data)   # Including for grader to easily orient to data :)
```

```
# A tibble: 6 x 5
  flourish deprawsc anx_score lonesc ed_sde
     <dbl>    <dbl>     <dbl>  <dbl>  <dbl>
1       26       24        13      7      2
2       32        8         7      6      1
3       47        8         9      6      3
4       48        6         6      5      0
5       28        7        14      9      2
6       41        7         8      7      3
```

Part 1 | Exploratory Data Analysis

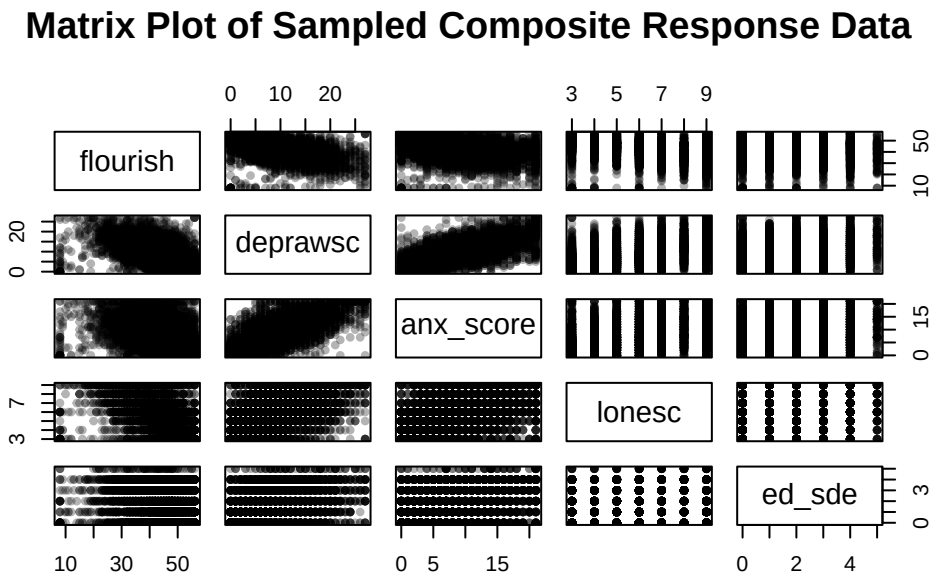
High-Dimensional Linearity

```r
corr_matrix <- cor(pca_data)
corrplot.mixed(corr_matrix, lower = "number", upper = "ellipse")
```



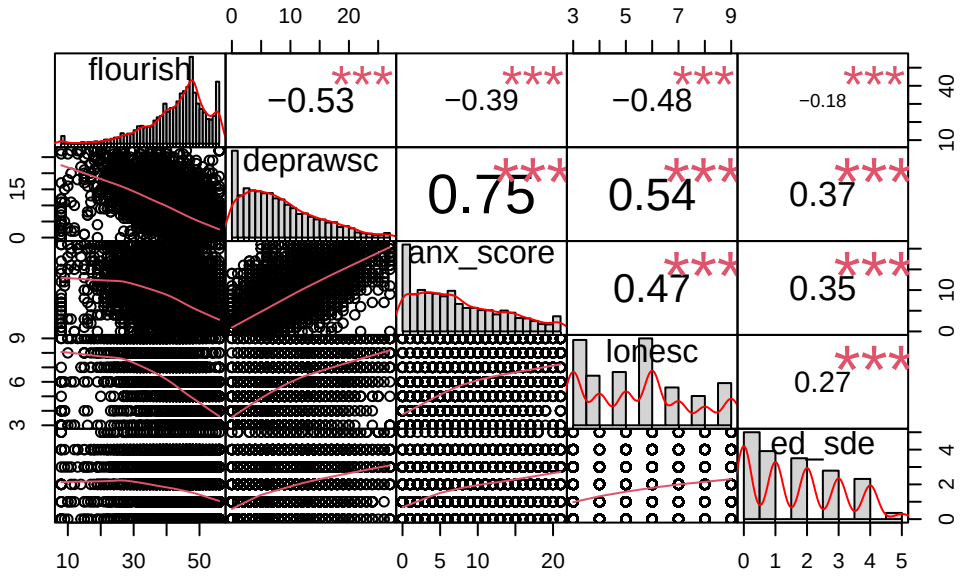
Our initial correlation plot suggests strong linear relationships between our composite variables. Observe the dark reds and blues: there appears to be a distinct cluster of positive correlations between depression (`deprawsc`), anxiety (`anx_score`), and loneliness (`lonesc`), which are all strongly negatively correlated with flourishing (`flourish`). The strength of these coefficients (generally, $r >| 0.5 |$) suggests that the data exhibits linearity, making it a strong candidate for PCA. However, we will create a matrix plot to specifically confirm high-dimensional linearity.

```
pca_plot_sample <- pca_data %>%
    slice_sample(n = 5000)

plot(
    pca_plot_sample,
    pch = 19,
    cex = 0.7,
    col = alpha("black", 0.3),
     main = "Matrix Plot of Sampled Composite Response Data")
```



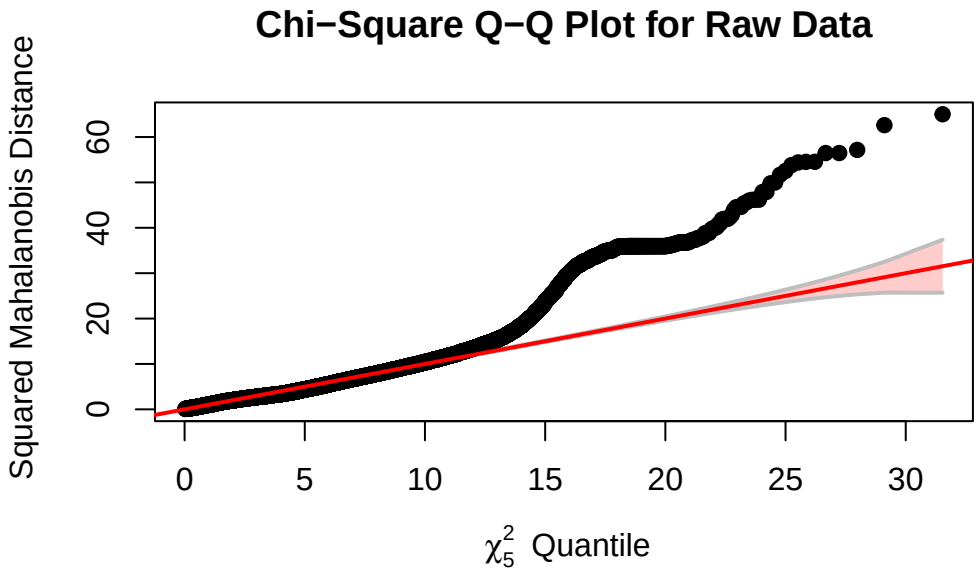
We sample $5,000$ observations for our matrix plot, as plotting all $68,062$ observations would result in overplotting and be computationally intensive. Immediately, we see the same strong linear correlations that we saw in the correlation plots: a positive correlation between depression and anxiety and a negative correlation between flourishing versus depression and anxiety. This is unsurprising. However, it is harder to see a clear relationship amongst `lonesc` and `ed_sde`. The grid-like striations do not necessarily reflect non-linearity (just discreteness), but we can further analyze with the correlation chart capabilities of the `PerformanceAnalytics` package.

```
chart.Correlation(pca_plot_sample)
```



The trends we see across flourishing, depression, anxiety, and loneliness remain consistent with earlier analyses, with added specificity on statistically significant relationships ($p < 0.001$). The relationships between `flourish` and both `deprawsc` and `anx_score` appear to be slightly nonlinear when viewed via LOESS curves, whereas the relationship between `deprawsc` and `anx_score` remains highly linear. The LOESS curves for relationships with `ed_sde` and `lonesc` appear roughly linear, though with a potential quadratic component. Before considering transformations, we test for multivariate normality.

```
cqplot(pca_data, main = "Chi-Square Q-Q Plot for Raw Data")
```



We see strong structure and skewness in the Q-Q plot, suggesting that the data are not multivariate normal. At the cost of interpretability, we transform the variables, not only to hopefully attain multivariate normality (which is not required for PCA), but to improve linearity and reduce outlier influence.

Data Transformations

Given the output in our `PerformanceAnalytics` plot, we propose the following transformations:

- **Square Root:** `deprawsc` and `anx_score` to correct for right-skew
- **Power Transformation** (x^2)**:** `flourish` to correct for left-skew

Since `lonesc` and `ed_sde` have fewer levels, we do not propose transformations for these variables. Note that for the variables we take the square root of, we first increment by 0.5 to stabilize the variance given the plethora of zeroes in our data.

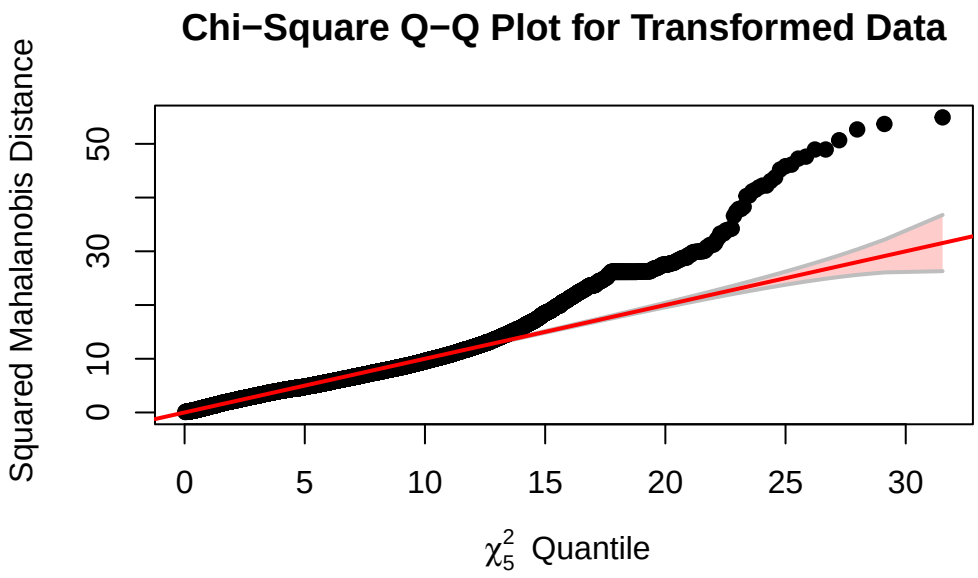
```
pca_data_transform <- pca_data %>%
    mutate(
        deprawsc_sqrt = sqrt(deprawsc + 0.5),
        anx_sqrt = sqrt(anx_score + 0.5),
        flourish_sq = flourish^2
    ) %>%
```

```
    select(flourish_sq, deprawsc_sqrt, anx_sqrt, lonesc, ed_sde)
```

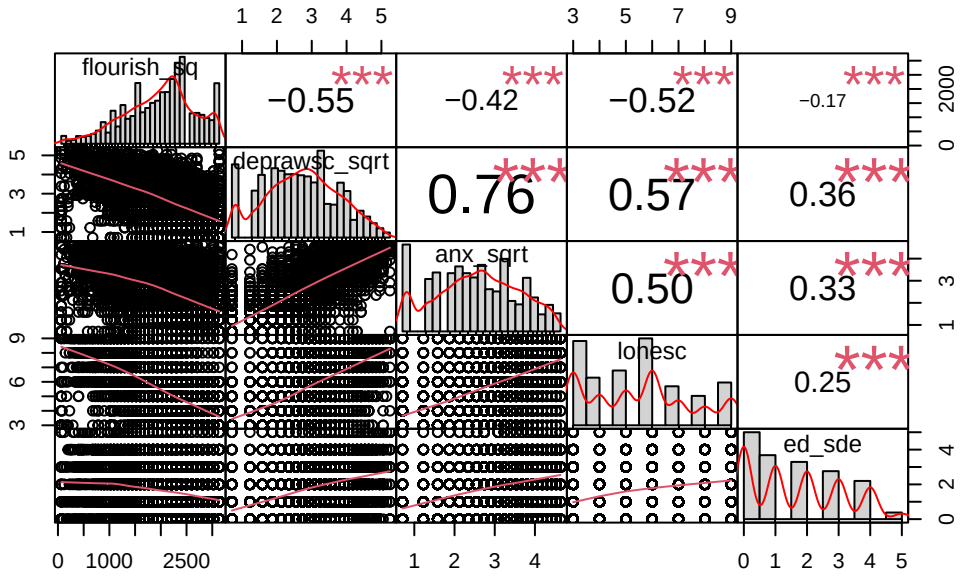
We review the same Chi-Square Q-Q and correlation plots for our transformed data.

Multivariate Normality

```
cqplot(pca_data_transform, main = "Chi-Square Q-Q Plot for Transformed Data")
```




```
chart.Correlation(slice_sample(pca_data_transform, n = 5000))
```



While our data are still not multivariate normal, we notice much clearer linear trends in the scatterplots. We feel comfortable proceeding with PCA given our large sample size, high correlations, and high-dimensional linearity.

As an aside, we also considered a log transformation of `deprawsc` and `anx_score` but found that this introduced new artifacts due to the discrete nature of the data and generally over-corrected.

Part 2 | Correlations

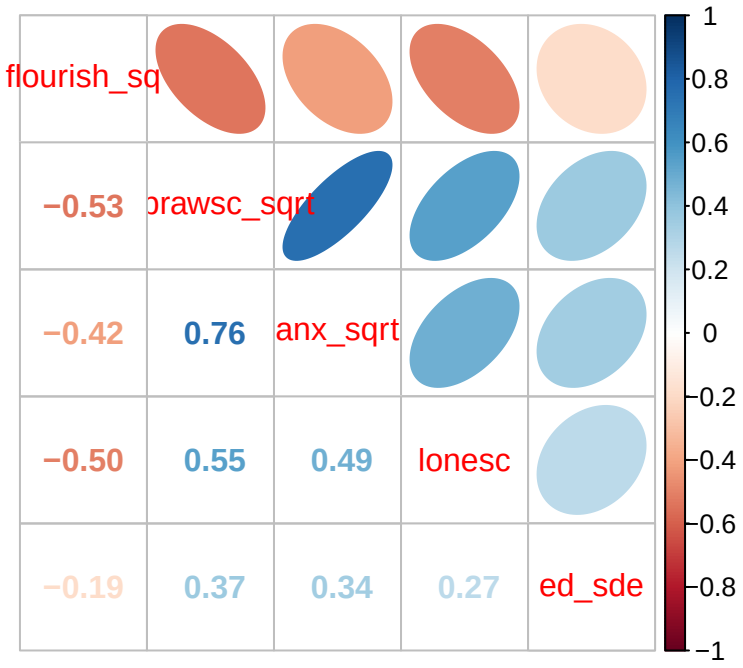
We presented a variety of correlation plots in Part 1 as part of our EDA but reproduce key visualizations here for post-transformation analysis. To begin, we present a side-by-side comparison of correlations by variable pair before and after transformations. For the grader's ease of review, we omit the code for this table and render results via `echo = FALSE` (see Table 1).

Table 1: Correlation Comparison (Pre vs. Post Transformation)

Pair	Original	Transformed	Change
deprawsc_sqrt vs flourish	-0.508	-0.534	-0.025
anx_sqrt vs flourish	-0.382	-0.416	-0.035
lonesc vs flourish	-0.467	-0.504	-0.036
ed_sde vs flourish	-0.172	-0.189	-0.016
anx_sqrt vs deprawsc	0.748	0.759	0.012
lonesc vs deprawsc	0.540	0.550	0.010
ed_sde vs deprawsc	0.348	0.365	0.018
lonesc vs anx_score	0.475	0.487	0.013
ed_sde vs anx_score	0.323	0.340	0.018
ed_sde vs lonesc	0.265	0.265	0.000

Evidently, there is a general increase in correlation with every pair of variables, aside from `ed_sde` versus `lonesc`, where the correlation remains unchanged because the variables, themselves, were not transformed. We present a visualization of the transformed correlations for ease of analysis.

```
corrplot.mixed(corr_matrix_2, lower = "number", upper = "ellipse")
```



Again, the correlation structure indicates high multicollinearity, particularly with a strong, positive $r = 0.76$ between anxiety and depression. Flourishing is inversely correlated to all other metrics, as expected, and eating disorder remains loosely correlated. Based on these correlations, combined with the aforementioned ratio of $n = 68,062$ observations against $p = 5$ variables (a response-to-variable ratio of over $13,600 : 1$), PCA is expected to perform well. The strong correlations indicate that a large proportion of the total variance is shared, and as observed in Part 1, the relationships mostly appear to be linear. Thus, we feel comfortable proceeding with PCA.

Part 3 | Principal Component Analysis

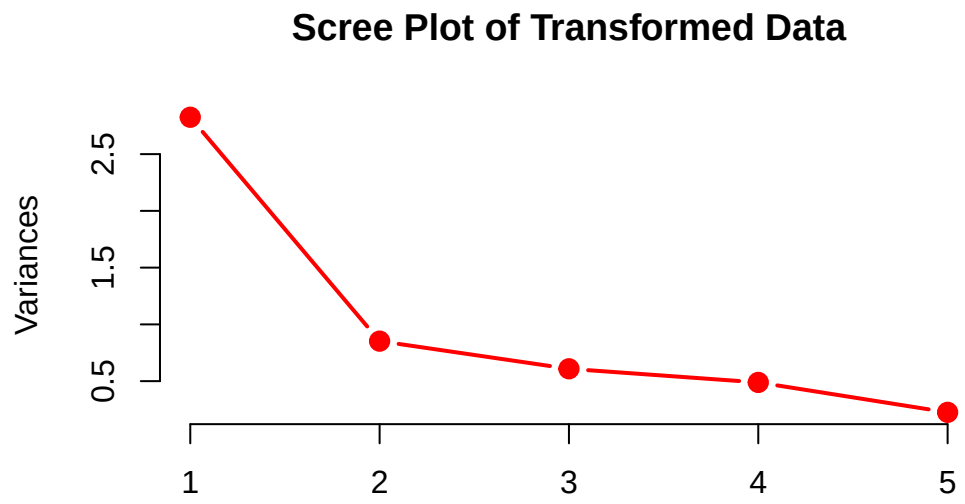
We begin by analyzing total variance by a given number of principal components.

```
pca <- prcomp(pca_data_transform, scale. = T)

summary.PCA.JDRS <- function(x){
  sum_JDRS <- summary(x)$importance
  sum_JDRS[1, ] <- sum_JDRS[1, ]^2
  attr(sum_JDRS, "dimnames")[[1]][1] <- "Eigenvals (Variance)"
  sum_JDRS
}

round(summary.PCA.JDRS(pca), 2)
```

```
                        PC1  PC2  PC3  PC4  PC5
Eigenvals (Variance)   2.82 0.85 0.61 0.49 0.23
Proportion of Variance 0.56 0.17 0.12 0.10 0.05
Cumulative Proportion  0.56 0.74 0.86 0.95 1.00
```

Note that the first principal component (PC1) captures 56% of the variance. Adding PC2 increments this to 74% of the variance, and PC3 brings us to 86%. However, note that only one eigenvalue is greater than one: $\lambda_{\text{PC1}} = 2.82$. We proceed to viewing the scree plot to continue with our determination of the number of principal components to keep.

```
screeplot(pca, type = "lines", col = "red", lwd = 2, pch = 19, cex = 1.2,
          main = "Scree Plot of Transformed Data")
```



There is a clear elbow after the first principal component, which suggests keeping only PC1, just as suggested by the "eigenvalue > 1" " rule. We do not perform parallel analysis, as our data violate the multivariate normality assumption (see Part 1). While we would need three principal components to explain $> 80\%$ of the variance, we proceed with only PC1 based on the two other evaluation criteria. Though 56% of the variance is lower than the soft 80% threshold, it represents a dominant latent structure in this psychological dataset. Given that PC1 captures more than triple the variance of the next component (17%), we prioritize a parsimonious, one-dimensional interpretation over a more complex, three-dimensional model that would include components with eigenvalues below 1.

Part 4 | Interpretation of Loadings

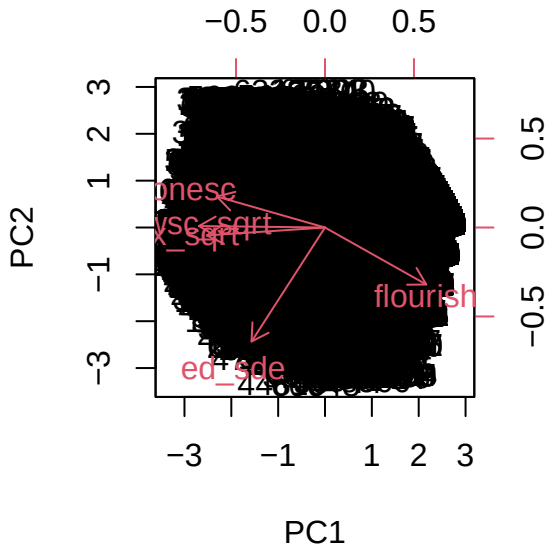
Now, we present and interpret the loadings for our principal components. While we will only proceed with PC1, we include the loadings for PC2–5 for reference.

```
round(pca$rotation, 2)
```

```
                PC1   PC2   PC3   PC4   PC5
flourish_sq    0.42 -0.43 -0.51  0.59 -0.16
deprawsc_sqrt -0.53  0.01 -0.36 -0.12 -0.76
anx_sqrt      -0.49 -0.06 -0.60 -0.03  0.63
```

```
lonesc        -0.45  0.24  0.34  0.79  0.03
ed_sde        -0.31 -0.87  0.38 -0.08  0.03
```

```r
biplot(pca, choices = c(1, 2), pc.biplot = T)
```

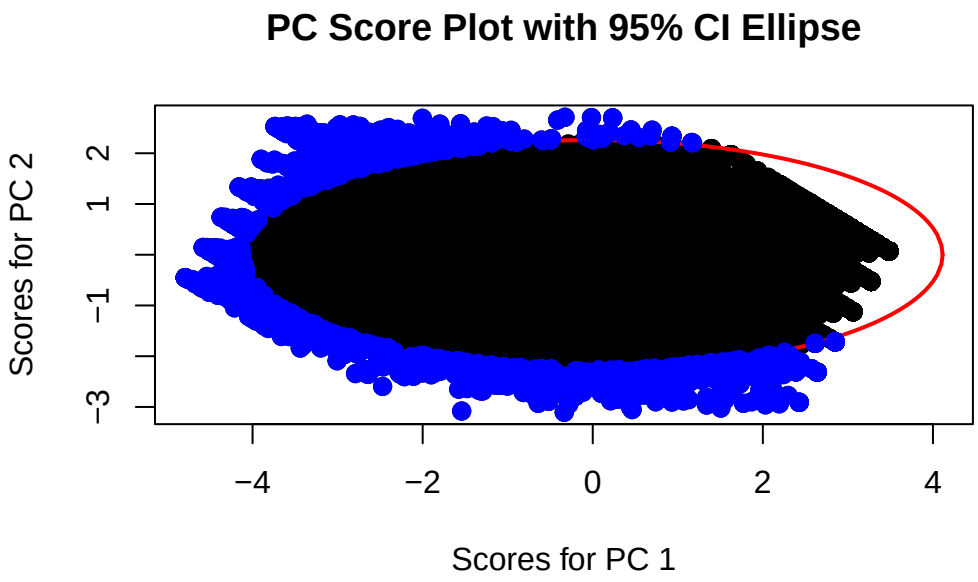


Based on the loadings for PC1, we see two predominant axes emerge: a measure of general well-being and a measure of psychological distress. Observe that flourishing is the only variable with a positive loading (0.42), indicating that a higher score on the flourishing scale contributes directly to a higher PC1 score. Conversely, the remaining variables have negative loadings, with depression having the largest effect (-0.53) on a reduced PC1 score. The biplot confirms our findings visually (we interpret this along the x-axis, we are only assessing PC1). Flourishing points right, whereas the remaining variables point left.

Effectively, we can interpret PC1 to be a measure of a student's psychological health. A higher score is commensurate with the flourishing Likert questions, wheras a lower score indicates trouble with one or more of: depression, anxiety, loneliness, and eating disorders.

Part 5 | Further Analysis

```r
# Pass in empty row names as they will be unintelligible with so many points.
ciscoreplot(pca, c(1, 2), rep("", nrow(pca_data_transform)))
```



The score plot reveals a single, continuous cloud of data points with no distinct sub-groupings, suggesting that psychological health amongst students varies along a continuum rather than categorical types. The superimposed 95% confidence ellipse highlights a significant deviation from multivariate normality, which we expect, driven by a heavy tail of outliers on the negative end of PC1. Based on our loading analysis, where negative PC1 scores correspond to high depression, anxiety, and loneliness, these outliers represent students with severe psychological distress.

While the violation of multivariate normality suggests that the 95% probability boundary is imperfect, we can more easily identify extreme points that are statistically far from the average student profile and potentially distribute critical care accordingly.

Note: the presentation and analysis of the biplot in the context of PC1 can be found in Part 4. If we interpret it in the context of both PC1 and PC2, we see the same near direct inverse relationship between flourishing versus loneliness, depression, and anxiety, but we see that the presentation of an eating disorder acts somewhat perpendicularly to this relationship. This is mirrored in the PC2 loadings, which very heavily penalize `ed_sde`, alongside `flourish_sq`. It appears that eating disorders have a unique variance structure that doesn't perfectly align with that of depression, anxiety, and loneliness. That is, a student with a high ED score may not necessarily suffer from low flourishing, high loneliness, and so on.

While this an interesting dimension to explore, we ultimately refrain from including PC2 in our analysis because its eigenvalue of 0.85 indicates PC2 explains less variance than a single original variable. Similarly, the scree plot is sharp and unambiguous after PC1, and under the

principle of parsimony, we avoid doubling the complexity of our model for a relatively small gain in variance explained (17%).

Part 6 | Conclusion and Discussion

Principal component analysis proved to be an effective tool for dimensionality reduction on the 2024–2025 HMS dataset (from which we selected five key composite variables that summarized series of questions pertaining to flourishing, depression, anxiety, loneliness, and eating disorders). We condensed the five variables into a single component that generally reflects psychological health (a higher score is tied to flourishing, whereas a lower score is tied to the remaining negative psychological conditions).

We determined the data to be eligible for PCA during exploratory data analysis because scatterplots, mixed correlation plots, and matrix plots revealed strong linear correlations. We enhanced linearity (and incidentally, correlation) by performing transformations on some of our variables, including square root and power transformations, with corrections for our Likert-scaled data that included a significant number of `0` responses. While the Chi-Square Q-Q plot revealed that transformations did not resolve our lack of multivariate normality, we proceeded as (1) MVN is not a requirement for PCA and (2) we were satisfied with the response-to-variable ratio of over $13,600 : 1$.

The loadings that resulted from our PCA helped confirm the story behind our variables. PC1 was abundantly clear: the loadings and biplot confirmed an inverse relationship between flourishing and the other negative psychological conditions, explaining 56% of the variability in our data. We elected to stop here under the principle of parsimony, as well as the technical thresholds we satisfied (eigenvalue > 1 and scree plot). However, further analysis of PC2 revealed a disparate relationship with eating disorders and the other variables. Namely, an increase in ED score did not necessarily mean a similar increase in psychological health (measured by PC1). Further analysis could be warranted on the impact of ED symptoms in future assignments.

Finally, while we do not fully trust the 95% confidence ellipse due to the lack of multivariate normality in our data, we note that outliers here do imply meaningful differences from the response mean. We could operationalize this to encourage schools to provide heightened emotional support for students who are identified as outliers, as their responses reflect that they are far behind their peers in terms of psychological health.