18.01, September 26, 2003 Max-Min Problems

2 C-5, 2 C-10, 2 C-12, 2 D-3, 2 D-4

1. General strategy: (1) Label all params, (2) Draw a diagram, (3) Write equation of qtt'y to be max'ed/min'ed, (4) Write equations of constraints, (5) Use (4) to reduce to 1 free variable, x, (6) Rewrite qtt'y to be max'ed as a function of free car, f=f(x), (7) Write interval of x where f(x) defined, (8) If possible, sketch f(x), (9) Compute critical pts of f, (10) Find critical values of f(x) and f on endpts, pts of discontinuity, pts where f' not defined, (11) Find the absolute max, (12) Back substitute to find values of all parameters at maximum.

2. Fixed length L of fence, find dim'ns x,y so that area enclosed is max: $\left.\begin{array}{l} x+y=L \\ x \bullet y=A \end{array}\right\}$

$x=y=\dfrac{L}{2}$

3. Runner on land, water

Find (x,0) so path has shortest time

$c_1 \dfrac{(x-x_1)}{d_1}=c_2 \dfrac{(x_2-x)}{d_2}$ or $c_1 \cos\theta_1 = c_2 \cos\theta_2$

Snell's law: Refraction, reflection, special case $c_1 = c_2$

4. For h fixed, find length of radius so that total perimeter (=2xradius+length of circular arc) is minimum. Answer: $\tan\theta = \theta + 1, 0 < \theta < \dfrac{\pi}{2}$