

LISTA 01 VOLUME – MÉTODO DO DISCO

1 – Esboce a região delimitada pelos gráficos das equações de cada item abaixo e calcule o volume do sólido gerado pela revolução desta área em torno do eixo dado, utilizando o método do disco:

- a) Área entre $y=x+1$, $x=0$, $x=2$, $y=0$, girando em torno no eixo x
- b) Área entre $y=x$, $y=2$ e o eixo y , girando em torno no eixo y
- c) Área entre $y=x^2+1$, $y=0$, $x=1$, $x=2$, girando em torno no eixo x
- d) Área entre $x=\sqrt{2y}$, $x=0$ e $y=2$, girando em torno no eixo y
- e) Área entre $y=\sqrt{x+4}$, $x=0$ e $y=0$, girando em torno no eixo x
- f) Área entre $x=y^2-4$ e $x=0$, girando em torno no eixo y
- g) Área entre $y=1/x$, $x=1$, $x=4$ e $y=0$, girando em torno no eixo x
- h) Área entre $y=x^2-4x$ e $y=0$, girando em torno no eixo x

2 – Esboce a região delimitada pelos gráficos das equações de cada item abaixo e calcule o volume do sólido gerado pela revolução desta área em torno do eixo dado, utilizando o método da arruela:

- a) Área entre $y=x+1$, $y=4$ e $x=0$, girando em torno no eixo x
- b) Área entre $y=x$, $y=0$, $x=2$, girando em torno no eixo y
- c) Área entre $y=x^2+1$ e $y=5$, girando em torno no eixo x
- d) Área entre $x=y^2$ e $x=4$, girando em torno no eixo y
- e) Área entre $x=1-y^2$, $x=y+2$, $y=-1$ e $y=1$, girando em torno no eixo y
- f) Área entre $y=x^2$ e $y=\sqrt{x}$, girando em torno no eixo x
- g) Área entre $x=y^2$ e $x=2y$, girando em torno no eixo y
- h) Área entre $y=4x-x^2$ e $y=x$, girando em torno no eixo x
- i) Área entre $x=y^2$ e $x=y$, girando em torno no eixo y

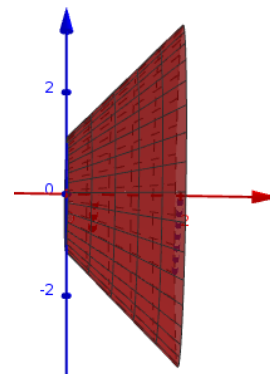
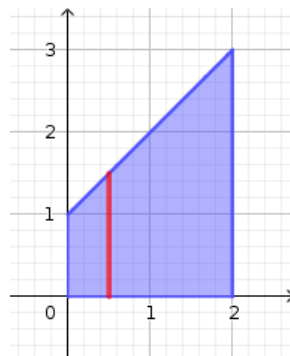
GABARITO

1 – a) Em torno do eixo x, devo usar dx, então:

$$R = x + 1 \Rightarrow R^2 = (x + 1)^2 = x^2 + 2x + 1$$

$$0 \leq x \leq 2$$

$$V = \pi \int_0^2 (x^2 + 2x + 1) dx = \pi \left[\frac{x^3}{3} + x^2 + x \right]_0^2 = \frac{26\pi}{3} \text{ u.v.}$$

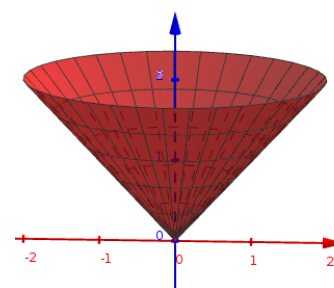
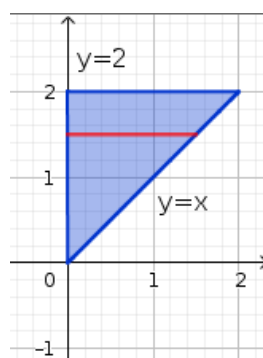


b) Em torno do eixo y, devo usar dy, então:

$$R = y \Rightarrow R^2 = y^2$$

$$0 \leq y \leq 2$$

$$V = \pi \int_0^2 y^2 dy = \pi \left[\frac{y^3}{3} \right]_0^2 = \frac{8\pi}{3} \text{ u.v.}$$



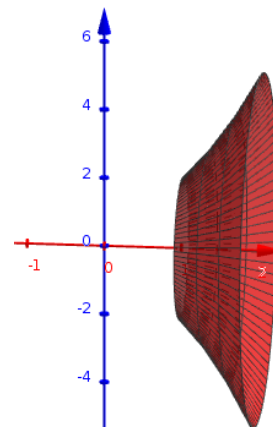
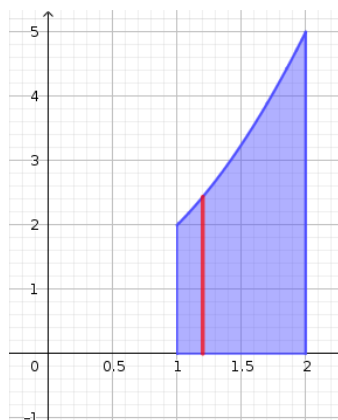
c) Em torno do eixo x, devo usar dx, então:

$$R = x^2 + 1 \Rightarrow R^2 = (x^2 + 1)^2 = x^4 + 2x^2 + 1$$

$$1 \leq x \leq 2$$

$$V = \pi \int_1^2 (x^4 + 2x^2 + 1) dx = \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_1^2$$

$$V = \frac{178\pi}{15} \text{ u.v.}$$

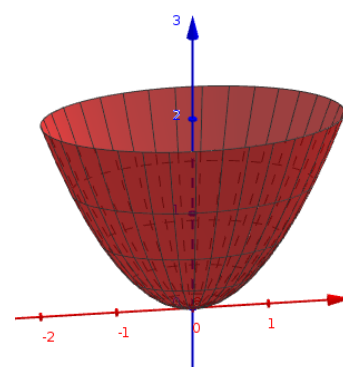
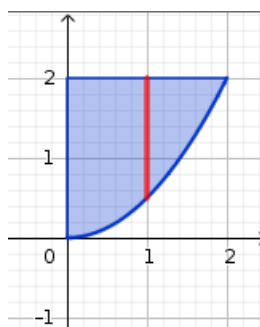


d) Em torno do eixo y, devo usar dy, então:

$$R = \sqrt{2y} \Rightarrow R^2 = 2y$$

$$0 \leq y \leq 2$$

$$V = \pi \int_0^2 (2y) dy = \pi y^2 \Big|_0^2 = 4\pi \text{ u.v.}$$



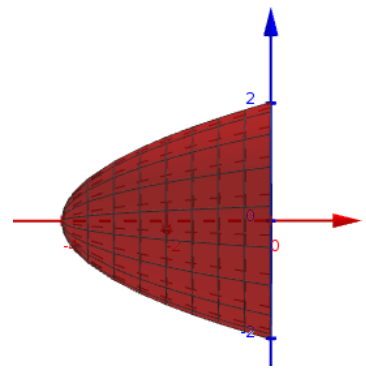
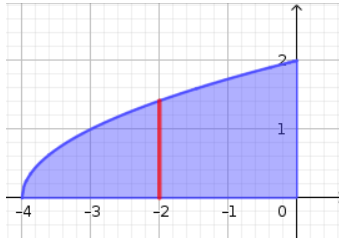
e) Em torno do eixo x, devo usar dx, então:

$$R = \sqrt{x+4} \Rightarrow R^2 = x+4$$

$$-4 \leq x \leq 0$$

$$V = \pi \int_{-4}^0 (x+4) dx = \left[\frac{x^2}{2} + 4x \right]_{-4}^0$$

$$V = \pi [0 - (8 - 16)] = 8\pi \text{ u.V}$$



f) Em torno do eixo y, devo usar dy, então:

$$R = x_d - x_e = 0 - (y^2 - 4) = 4 - y^2$$

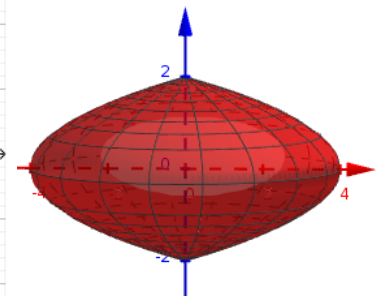
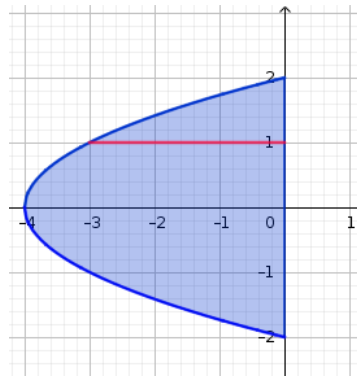
$$R^2 = (4 - y^2)^2 = 16 - 8y^2 + y^4$$

$$-2 \leq y \leq 2$$

$$V = \pi \int_{-2}^2 (16 - 8y^2 + y^4) dy$$

$$V = \pi \left[16y - \frac{8y^3}{3} + \frac{y^5}{5} \right]_{-2}^2 = \frac{512\pi}{15} \text{ u.V}$$

$$V = \pi \left[\left(32 - \frac{64}{3} + \frac{32}{5} \right) - \left(-32 + \frac{64}{3} - \frac{32}{5} \right) \right] = \frac{512\pi}{15} \text{ u.V}$$



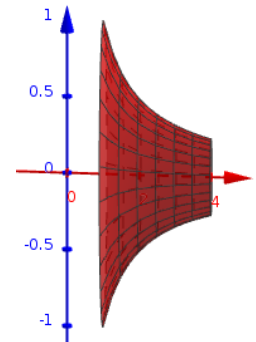
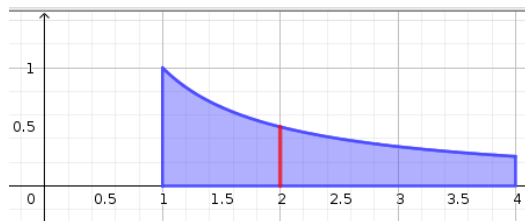
g) Em torno do eixo x, devo usar dx, então:

$$R = \frac{1}{x} \Rightarrow R^2 = \frac{1}{x^2}$$

$$1 \leq x \leq 4$$

$$V = \pi \int_1^4 \frac{1}{x^2} dx = \pi \int_1^4 x^{-2} dx$$

$$V = \pi \left[-x^{-1} \right]_1^4 = \pi \left[-\frac{1}{x} \right]_1^4 = \pi \left(-\frac{1}{4} - (-1) \right) = \frac{3\pi}{4} \text{ u.V}$$



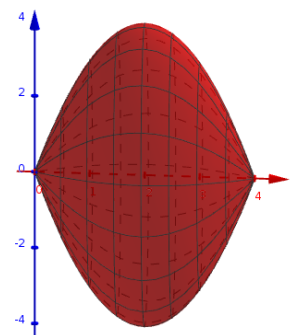
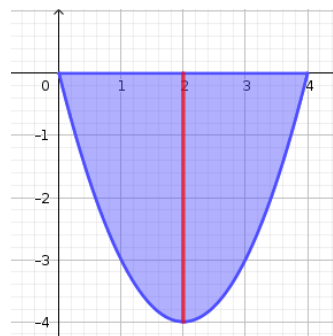
h) Em torno do eixo x, devo usar dx, então:

$$R = 0 - (x^2 - 4x) = 4x - x^2$$

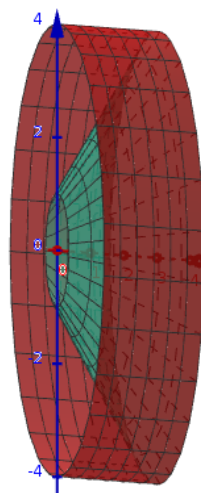
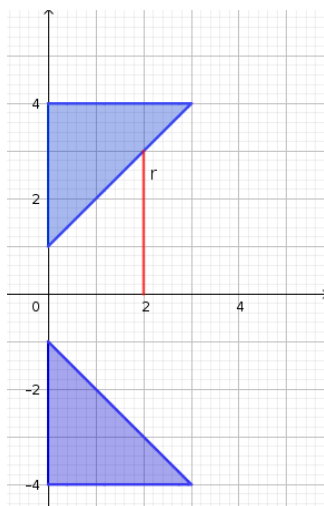
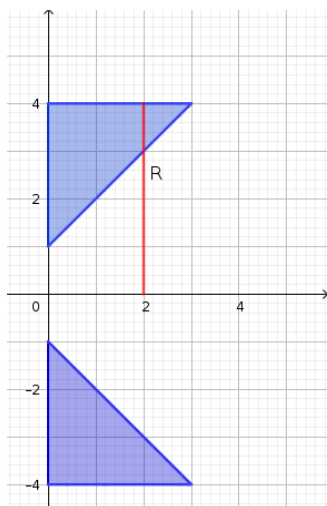
$$R^2 = (4x - x^2)^2 = 16x^2 - 8x^3 + x^4 \quad 0 \leq x \leq 4$$

$$V = \pi \int_0^4 (16x^2 - 8x^3 + x^4) dx = \pi \left[\frac{16x^3}{3} - 2x^4 + \frac{x^5}{5} \right]_0^4$$

$$V = \pi \left(\frac{1024}{3} - 512 + \frac{1024}{5} \right) = \frac{512\pi}{15} \text{ u.V}$$



2 – a)



$$R=4 \Rightarrow R^2=16$$

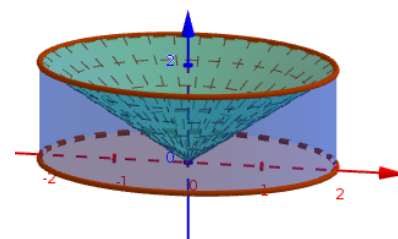
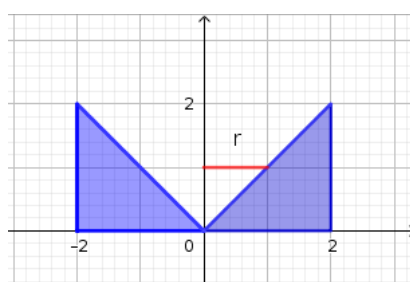
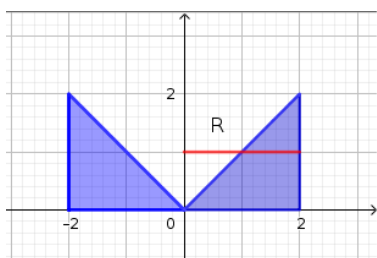
$$r=(x+1) \Rightarrow r^2=(x+1)^2=x^2+2x+1$$

$$R^2-r^2=16-x^2-2x-1=15-x^2-2x$$

$$0 \leq x \leq 3$$

$$V = \pi \int_0^3 (15-x^2-2x) dx = \pi \left[15x - \frac{x^3}{3} - x^2 \right]_0^3 = \pi(45-9-9) = 27\pi \text{ u.V}$$

b)



$$R=2 \Rightarrow R^2=4$$

$$r=y \Rightarrow r^2=y^2$$

$$R^2-r^2=4-y^2$$

$$0 \leq y \leq 2$$

$$V = \pi \int_0^2 (4-y^2) dy = \pi \left[4y - \frac{y^3}{3} \right]_0^2 = \pi \left(8 - \frac{8}{3} \right) = \frac{16\pi}{3} \text{ u.V}$$

c) $R=5 \Rightarrow R^2=25$

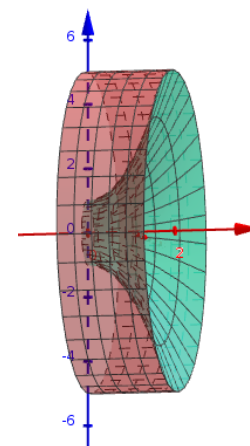
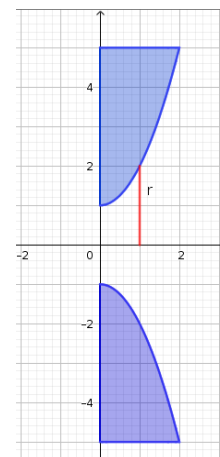
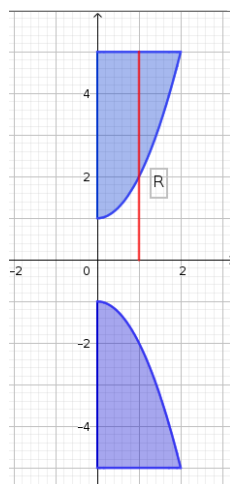
$$r=(x^2+1) \Rightarrow r^2=(x^2+1)^2=x^4+2x^2+1$$

$$R^2-r^2=25-x^4-2x^2-1$$

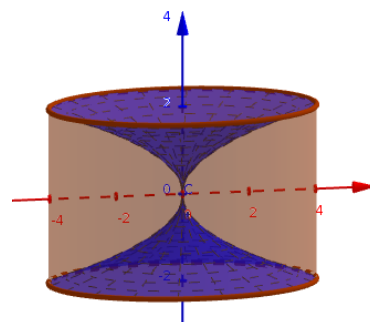
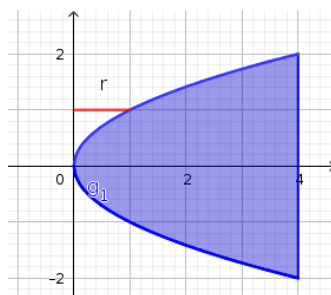
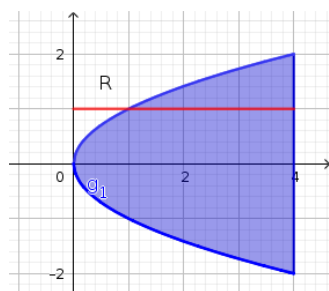
$$R^2-r^2=24-x^4-2x^2 \quad -2 \leq x \leq 2$$

$$V = \pi \int_{-2}^2 (24-x^4-2x^2) dx$$

$$V = \pi \left[24x - \frac{x^5}{5} - \frac{2x^3}{3} \right]_{-2}^2 = \frac{1088\pi}{15} \text{ u.V}$$



d)



$$R = 4 \Rightarrow R^2 = 16$$

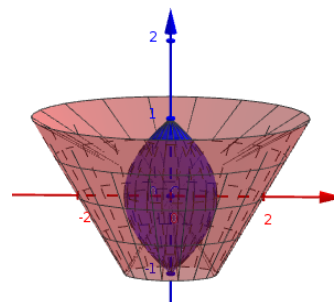
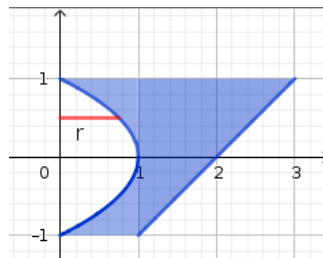
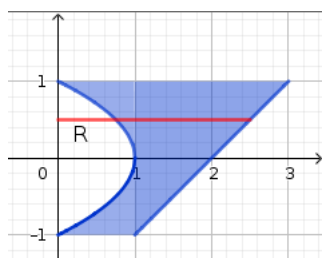
$$r = y^2 \Rightarrow r^2 = (y^2)^2 = y^4$$

$$R^2 - r^2 = 16 - y^4$$

$$-2 \leq y \leq 2$$

$$V = \pi \int_{-2}^2 (16 - y^4) dy = \pi \left[16y - \frac{y^5}{5} \right]_{-2}^2 = \frac{256\pi}{5} \text{ u.v.}$$

e)



$$R = y + 2 \Rightarrow R^2 = (y + 2)^2 = y^2 + 4y + 4$$

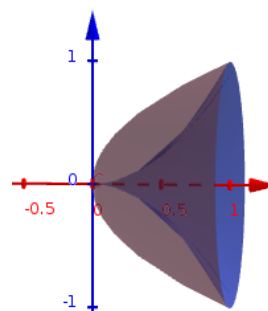
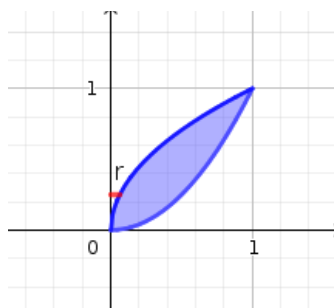
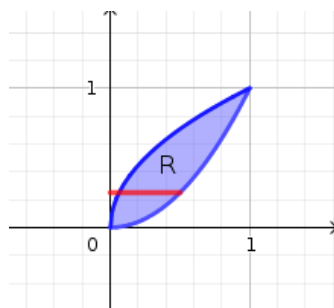
$$r = (1 - y^2) \Rightarrow r^2 = (1 - y^2)^2 = 1 - 2y^2 + y^4$$

$$R^2 - r^2 = y^2 + 4y + 4 - 1 + 2y^2 - y^4 = -y^4 + 3y^2 + 4y + 3$$

$$-1 \leq y \leq 1$$

$$V = \pi \int_{-1}^1 (-y^4 + 3y^2 + 4y + 3) dy = \pi \left[-\frac{y^5}{5} + y^3 + 2y^2 + 3y \right]_{-1}^1 = \frac{38\pi}{5} \text{ u.v.}$$

f)



$$R = \sqrt{x} \Rightarrow R^2 = x$$

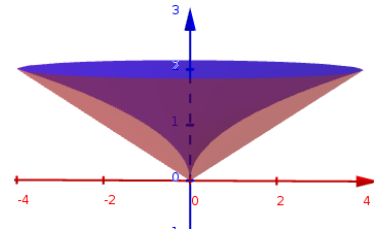
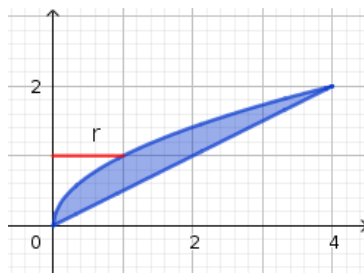
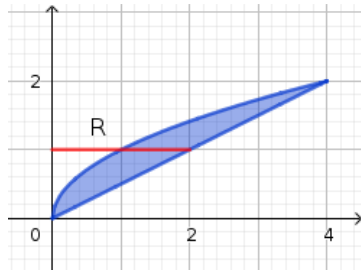
$$r = x^2 \Rightarrow r^2 = x^4$$

$$R^2 - r^2 = x - x^4$$

Ponto de interseção: $\sqrt{x} = x^2 \Rightarrow x = x^4 \Rightarrow x - x^4 = 0 \Rightarrow x(1 - x^3) = 0 \Rightarrow \begin{cases} x' = 0 \\ x'' = 1 \end{cases}$

$$V = \pi \int_0^1 (x - x^4) dx = \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1 = \frac{3\pi}{10} \text{ u.v.}$$

g)



$$R=2y \Rightarrow R^2=(2y)^2 \Rightarrow R^2=4y^2 \quad r=y^2 \Rightarrow r^2=(y^2)^2 \Rightarrow r^2=y^4 \quad R^2-r^2=4y^2-y^4$$

Ponto de interseção: $2y=y^2 \Rightarrow 2y-y^2=0 \Rightarrow y(2-y)=0 \Rightarrow \begin{cases} y'=0 \\ y''=2 \end{cases}$

$$V=\pi \int_0^2 (4y^2-y^4) dy = \pi \left[\frac{4y^3}{3} - \frac{y^5}{5} \right]_0^2 = \frac{64\pi}{15} u.V$$

h) $R=4x-x^2$

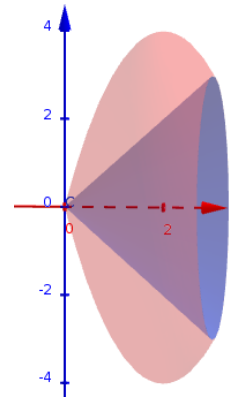
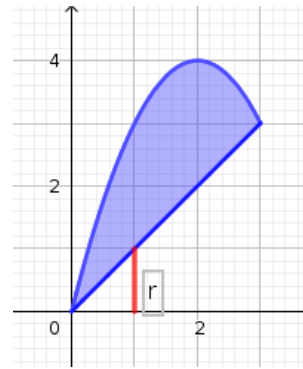
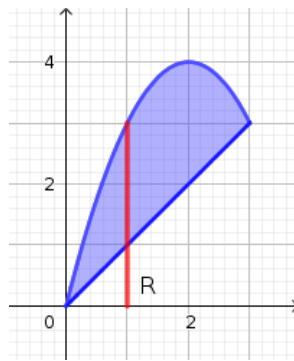
$$R^2=(4x-x^2)^2$$

$$R^2=16x^2-8x^3+x^4$$

$$r=x \Rightarrow r^2=x^2$$

$$R^2-r^2=16x^2-8x^3+x^4-x^2$$

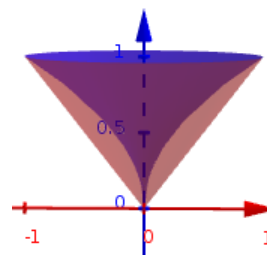
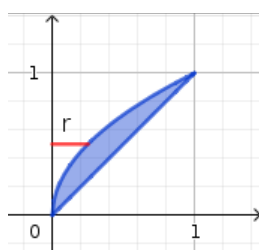
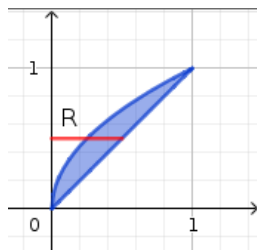
$$R^2-r^2=15x^2-8x^3+x^4$$



Ponto de interseção: $4x-x^2=x \Rightarrow 3x-x^2 \Rightarrow x(3-x)=0 \Rightarrow \begin{cases} x'=0 \\ x''=3 \end{cases}$

$$V=\pi \int_0^3 (15x^2-8x^3+x^4) dx = \pi \left[5x^3-2x^4+\frac{x^5}{5} \right]_0^3 = \frac{108\pi}{5} u.V$$

i)



$$R=y \Rightarrow R^2=y^2 \quad r=y^2 \Rightarrow r^2=(y^2)^2 \Rightarrow r^2=y^4 \quad R^2-r^2=y^2-y^4$$

Ponto de interseção: $y=y^2 \Rightarrow y-y^2=0 \Rightarrow y(1-y)=0 \Rightarrow \begin{cases} y'=0 \\ y''=1 \end{cases}$

$$V=\pi \int_0^1 (y^2-y^4) dy = \pi \left[\frac{y^3}{3} - \frac{y^5}{5} \right]_0^1 = \frac{2\pi}{15} u.V$$