

18.01, Oct. 8, 2003 Recitation Suggestions

1. Do a little calculation of diff. Maybe do an “implicit diff.” example, e.g. if $x^2 + y^2 = 1$, then $dy = -\frac{x}{y} dx$. If $x = \frac{2t}{t^2 + 1}$, $y = \frac{t^2 - 1}{t^2 + 1}$, for instance, get $d(\frac{t^2 - 1}{t^2 + 1}) = -\frac{2t}{t^2 - 1} d(\frac{2t}{t^2 + 1})$ (which is true)

2. Do lots of examples of antideriv's and make students do them

i) $\int \sqrt{x^3 + 3x^2 + 1} \cdot (x^2 + 2x) dx$

ii) $\int (1 + x + x^2 + x^3 + \dots) dx$ and use to explain lin+quadratic expansion of

$$\ln\left(\frac{1}{1-x}\right) = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \text{ (probably takes too much time)}$$

iii) $\int (\sqrt{x} + \frac{1}{\sqrt{x}}) dx$, could be directly or sub $U = x^{\frac{1}{2}}$, $dU = \frac{1}{2\sqrt{x}} dx$ so

$$= \int (U^2 + 1) 2dU = 2\left(\frac{U^3}{3} + U\right) + C = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

iv) $\int \frac{\sin(x)}{\cos(x)} dx$, $U = \cos(x)$, $\rightarrow \ln|U| = \ln|\cos(x)| + C$.

v) Maybe do a ln problem where C becomes a constant inside, i.e. $\frac{d}{dx} \ln(2x) = \frac{1}{x}$.

vi) $\int x e^{-x^2/2} dx$, if really ambitious $\int x e^{-tx^2/2} dx = \dots = -\frac{1}{t} e^{-tx^2/2}$ and diff. W.R.T. “var” t to

get formulas for $\int x^{2k+1} e^{-tx^2/2} dx$ (instead of integ. by parts).