18.01 EXAM 3
OCTOBER 21, 2003

Name: ____

Problem 1:	__________	/25
Problem 2:	__________	/25
Problem 3:	__________	/25
Problem 4:	__________	/25
Total:	__________	/100

Please write the hour of your recitation.

Hour: ____________

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Date: Fall 2003.

Name: ____ **Problem 1**: __________________ /25

Problem 1(30 points) Evaluate each definite and indefinite integral. Use whatever method you like, but show all work. If you use a basic integral formula, write the formula on your paper.

(a)(6 points):

$$\int_0^{\frac{\pi}{4}} \frac{2\sin(\theta)\cos(\theta)}{1+\cos^2(\theta)} d\theta.$$

(b)(6 points):

$$\int \frac{x(x+2)}{\sqrt{x^3+3x^2+1}} dx.$$

(c)(3 points):

$$\int_0^{\ln(1)} e^{-t^2}\,dt.$$

(d)(10 points):

$$\int_0^1 \frac{(x-1)(x+1)x}{(x^2+1)^3}\,dx.$$

Name: ____ **Problem 2**: ________________ /25

Problem 2(25 points) Define the function $F(x)$, $0 \leq x < \frac{\pi}{2}$, by the formula,

$$F(x) = \int_0^{\tan(x)} \frac{1}{1+t^2} dt.$$

(a)(15 points): Using the Fundamental Theorem of Calculus (not a table of antiderivatives), compute $F'(x)$. Simplify your answer as much as possible and show all work.

(b)(5 points): Using (a), give a formula for $F(x)$.

(c)(5 points): Use the formula for $F(x)$ to find an antiderivative of $\frac{1}{1+x^2}$ (do not simply copy the antiderivative from your index card – explain why the antiderivative follows from (b)).

Name: ______________________________________ **Problem 3**: ________________ /25

Problem 3(25 points) Find the unique function $y(x)$ satisfying the differential equation with initial condition,

$$\frac{dy}{dx} = x^2 y, \quad y(1) = 1.$$

Name: ____ **Problem 4**: __________________ /25

Problem 4(25 points) Interpret the following limit as a limit of Riemann sums and compute a Riemann integral to find the value of the limit.

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \sin\left(\frac{kb}{n}\right), \quad b > 0.$$