18.01 PROBLEM SET 3

Due date: Tuesday, October 14 **before 1pm**. Write your name, the date, the name of your recitation instructor, and the hour of your recitation on the top of your paper. Late work will be accepted only with a medical note or for another Institute-approved reason. You are encouraged to work with others, but the final write-up should be entirely your own and based on your understanding.

Problem 1(15 points) Evaluate the following integrals.

(a) $\int_0^{100} \frac{x}{e^{(x^2)}} dx$,

(b) $\int \cos(ax)\cos(bx)dx$ where a, b are two distinct positive numbers,
(Hint: Compute $\cos((a+b)x), \cos((a-b)x)$ by the angle addition formulas.)

(c) $\int \cot(z)\frac{1}{\sin(z)} dz$,

(d) $\int_1^4 \frac{1}{\sqrt{x+\sqrt{x^3}}} dx$,
(Hint: Let $u = 1 + \sqrt{x}$.)

(e) $\int \left(\frac{\sin(t)}{(\sqrt{\cos(t)})-1} - \frac{\sin(t)}{(\sqrt{\cos(t)})+1} \right) dt$,
(Hint: First clear denominators, then make a substitution.)

Problem 2(15 points) This problem is a special case of a more general method for computing antiderivatives that will be developed systematically later in the semester.

(a)(10 points) Compute the antiderivative $\int \frac{1}{1-u^2} du$ as follows. Consider the sum $\frac{a}{u+1} + \frac{b}{u-1}$ and clear denominators. Find a choice of a and b such that you get $\frac{1}{1-u^2}$. Now make the substitution and use $\ln(u+1)$, $\ln(u-1)$ to compute the antiderivative.

(b)(5 points) Reduce the computation of the antiderivative $\int \frac{1}{\cos(x)} dx$ to (a) by multiplying numerator and denominator of the integrand by $\cos(x)$, using the trig identity $(\sin(x))^2 + (\cos(x))^2 = 1$ and substituting $u = \sin(x)$. Don't forget to back-substitute. Also say for what range of x your antiderivative is valid ($\ln(y)$ isn't defined if y is negative!).

Problem 3(10 points) Using the definition of the Riemann integral, upper sums, and the formulas for special sums in the textbook, prove that

$$\int_0^a x(a-x)dx = \frac{a^3}{6}.$$

Problem 4(10 points) Define the function $f(x)$ for $x > 0$ by the formula

$$f(x) = \int_x^{x^2} \frac{\ln(u)}{\sqrt{u}} du.$$

Compute the derivative of $f(x)$ (do not attempt to evaluate the integral directly, instead apply the Fundamental Theorem of Calculus).