September 15, 2014

1

(a)
$E(x) = \frac{1}{2}(f(x) + f(-x)) = \frac{x^4}{1-x^2}$
$O(x) = \frac{1}{2}(f(x) - f(-x)) = -\frac{x^5}{1-x^2}$

(b)
See figures in Appendix.

2

$g''(x) = (f^2(x))'' = (2f(x)f'(x))' = 2(f'(x))^2 + 2f(x)f''(x)$
Thus $g''(x_0) = 2(f'(x_0))^2 + 2f(x_0)f''(x_0) > 0$

3

(a)
$\frac{d}{dx}(\arctan(x^2+x+1))^{-3}$
$= -3(\arctan(x^2+x+1))^{-4}\frac{d}{dx}\arctan(x^2+x+1)$
$= -3(\arctan(x^2+x+1))^{-4}\frac{1}{1+(x^2+x+1)^2}\frac{d}{dx}(x^2+x+1)$
$= -3(\arctan(x^2+x+1))^{-4}\frac{1}{1+(x^2+x+1)^2}(2x+1)$

(b)
$\frac{d}{dy}\sin y\cos y$
$= (\sin y)'\cos y + \sin y(\cos y)'$
$= \cos^2 y - \sin^2 y = \cos 2y$

$\frac{d}{dy}\sin y\cos y$
$= \frac{d}{dy}\frac{1}{2}\sin 2y$
$= \frac{1}{2}\cos 2y \cdot 2$
$= \cos 2y$

4

(a)
$\mathrm{d}(\frac{1}{3}y^3 + xy^2 + x^2y + x^3 + x - \frac{5}{3})$
$= y^2\mathrm{d}y + y^2\mathrm{d}x + 2xy\mathrm{d}y + 2xy\mathrm{d}x + x^2\mathrm{d}y + 3x^2\mathrm{d}x + \mathrm{d}x$
$= (y^2 + 2xy + 3x^2 + 1)\mathrm{d}x + (y^2 + 2xy + x^2)\mathrm{d}y$
Set $y^2 + 2xy + 3x^2 + 1 = 0$. Thus we see that there isn't any point at which $\frac{dy}{dx} = 0$, i.e. the tangent line is horizontal.

(b)
Set $x^2 + 2xy + y^2 = 0$. We see that y=-x. Plugging this into the equation, we have $\frac{2}{3}x^3 + x = \frac{5}{3}$. $x = 1$ is one solution to it. Thus at the point $(1, -1)$ the tangent line is vertical.

5

(a)
$y = \cos^{-1}(x)$
$\cos y = x$
$-\sin y \frac{dy}{dx} = 1$
$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-x^2}}$

(b)
$\sin(\pi/2 - \cos^{-1}(x)) = \cos(\cos^{-1}(x)) = x$
Thus $\pi/2 - \cos^{-1}(x) = \sin^{-1}(x)$
$\sin^{-1}(x) + \cos^{-1}(x) = \pi/2$

(c)
We know that $\sin^{-1}(x) + \cos^{-1}(x)$ is a constant. Taking derivatives of both sides, we see that $\frac{d}{dx}\sin^{-1}(x) = -\frac{d}{dx}\cos^{-1}(x)$

6

(a)
$\ln(uv) = \ln u + \ln v$
$(\ln(uv))' = (\ln u)' + (\ln v)'$
$\frac{1}{uv}(uv)' = \frac{1}{u}u' + \frac{1}{v}v'$
This implies $(uv)' = uv' + u'v$

(b)
$(\ln(\frac{u}{v}))' = (\ln u)' - (\ln v)'$
$\frac{v}{u}(\frac{u}{v})' = \frac{1}{u}u' - \frac{1}{v}v'$
$(\frac{u}{v})' = \frac{1}{v}u' - \frac{uv'}{v^2}$

$(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$

(c)
$\ln(u_1u_2...u_n) = \ln u_1 + \ln u(2) + ... + \ln u(n)$
$(\ln(u_1u_2...u_n))' = (\ln u_1)' + (\ln u_2)' + ... + (\ln u_n)'$
$\frac{1}{u_1u_2...u_n}(u_1u_2...u_n)' = \frac{1}{u_1}u_1' + \frac{1}{u_2}u_2' + ... + \frac{1}{u_n}u_n'$
$(u_1u_2...u_n)' = u_1'u_2...u_n + u_1u_2'...u_n + ... + u_1u_2...u_n'$

7

(8a)
$M_1 = \frac{2}{3}\log_{10}\frac{E_1}{E_0}$, $M_2 = \frac{2}{3}\log_{10}\frac{E_2}{E_0}$
$1 = M_2 - M_1 = \frac{2}{3}\log_{10}\frac{E_2}{E_1}$
$\frac{E_2}{E_1} = 10^{3/2} \approx 31.62$

(8c)
Energy released by an earthquake of magnitude 6 is $(10^{3/2})^6 E_0 = 10^9 E_0 = 7\times 10^6$
The number of days is $(7\times 10^6)/(3\times 10^5) \approx 23.3$

(10)
Assume $\log_3 2 = \frac{p}{q}$, p and q are positive integers. Then we have $3^{\frac{p}{q}} = 2$. Thus $3^p = 2^q$. This is impossible because 3^p is odd while 2^q is even.

(11)
The first inequality is true only when the base is between 0 and 1, while the last step is true only when the base is bigger than 1. Thus no matter what the base is, one of the two steps is false.

(18)
$\ln y = \frac{1}{3}(\ln(x+1) + \ln(x-2) + \ln(2x+7))$ Differentiating this equation with respect to x, we get $\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}(\frac{1}{x+1} + \frac{1}{x-2} + \frac{2}{2x+7})$. Thus we have $\frac{dy}{dx} = \frac{y}{3}(\frac{1}{x+1} + \frac{1}{x-2} + \frac{2}{2x+7})$

(20)
$y = e^{x\ln x}$
$y' = e^{x\ln x}(\ln x + 1)$

$\ln y = x\ln x$
$\frac{1}{y}y' = \ln x + 1$
$y' = y(\ln x + 1)$

8 Appendix

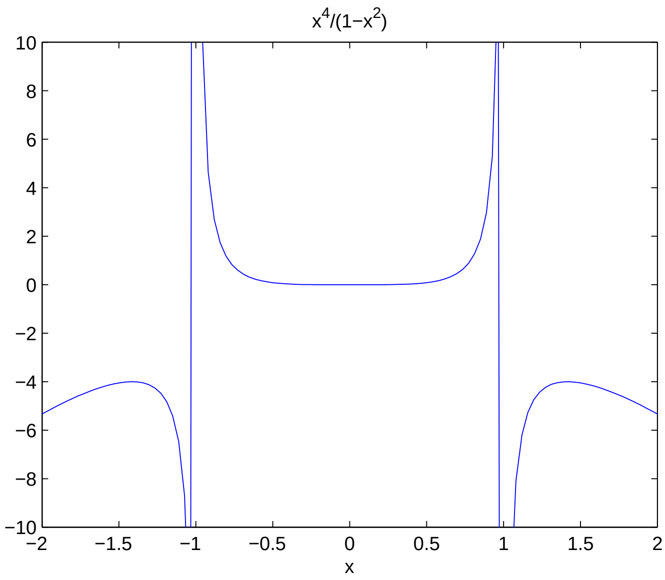

Figure 1: $E(x)$

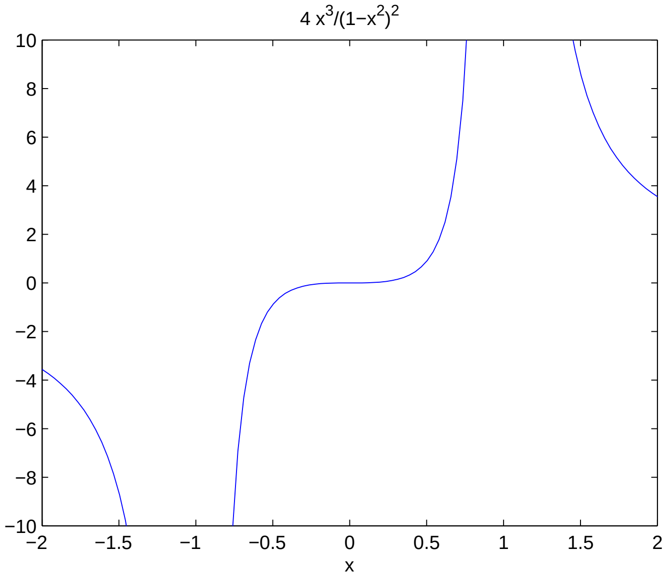

Figure 2: $E'(x)$

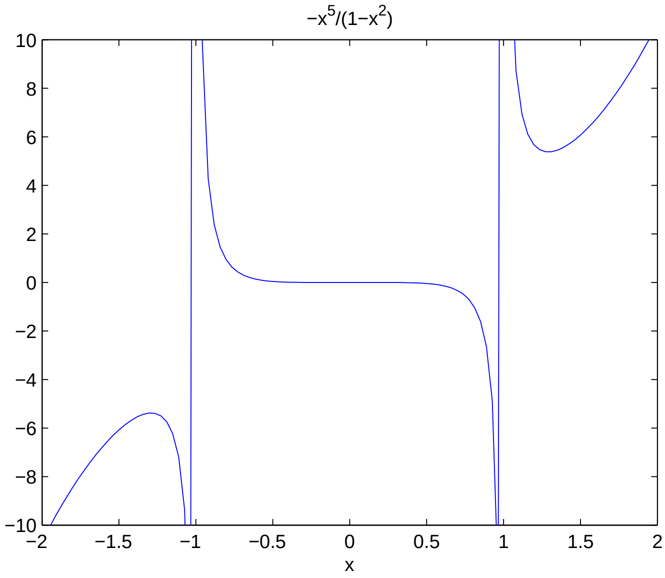

Figure 3: $O(x)$

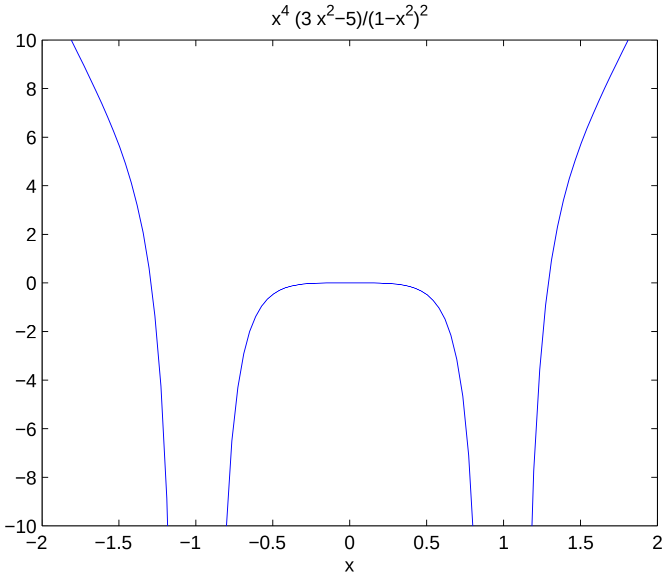

Figure 4: $O'(x)$