18.01, September 30, 2003 Related Rates

2 E-2, 2 E-4, 2 E-8, 2 E-9

Example 1: Spring displaced x units from equilibrium has potential energy
$V = \frac{1}{2}kx^2$ (k=const.). At time t_0, $x(t_0)$=5cm and $\frac{dx}{dt}(t_0) = 5\frac{cm}{s}$

What is $\frac{dv}{dt}$? Setup and solve, thereby showing general method: $\frac{dv}{dt} = \frac{1}{2}k \bullet 2x\frac{dx}{dt} = \underline{25k}$.

General method: Almost same as min/max problems rate-of-change required, rate-of-change known.
Implicitly differentiate constraint eq'n to get relation between 2 rates of change.

Ex 2: State trooper problem

State trooper aims radar gun at car at 45° angle to road and clocks $\frac{dr}{dt} = 50mph$.

Q: Is $\frac{dx}{dt} > 60mph$ (speed limit)

A. $r^2 = y^2 + x^2, 2r\frac{dr}{dt} = 2y\frac{dx}{dt} + 0, \frac{dy}{dt} = \frac{r}{y}\frac{dr}{dt}$ $t_0 : r = \sqrt{2}y$ so

$\frac{dy}{dt} = \sqrt{2} \bullet 50 \approx 70mph > 60mph$

Ex.3: rate of change of distance from horizon to launch pt as rocket moves away from Earth

$$\left.\begin{aligned} l &= r\theta \\ \cos\theta &= \frac{r}{y} \end{aligned}\right\} \longrightarrow \begin{aligned} \frac{dl}{dt} &= r\frac{d\theta}{dt} \\ -\sin\theta\frac{d\theta}{dt} &= -\frac{r}{y^2}\frac{dy}{dt} \end{aligned}$$