18.01 FINAL EXAM
DECEMBER 16, 2003

Name: ____

Problem 1:	__________	/20
Problem 2:	__________	/20
Problem 3:	__________	/30
Problem 4:	__________	/25
Problem 5:	__________	/25
Problem 6:	__________	/20
Problem 7:	__________	/30
Problem 8:	__________	/25
Problem 9:	__________	/25
Problem 10:	__________	/30
Total:	__________	/250

Please write the hour of your recitation.

Hour: ____________

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 3 hours for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand. Good luck!

Date: Fall 2003.

Name: ______________________________ **Problem 1**: ____________ /20

Problem 1(20 points) Use integration by parts to compute the following integral,

$$\int x \sec^2(x) dx.$$

Show all your work.

Name: ____ **Problem 2**: ________________ /20

Problem 2(20 points) Use integration by parts to compute the following integal,

$$\int (\ln(x))^2 dx.$$

Show all your work.

Name: ____ **Problem 3**: __________________ /30

Problem 3(30 points) A tennis ball can is designed so that sides form a cylinder of base radius r and height h, the bottom forms an inverted hemisphere of radius r (pointing into the can), and no top. The area of the can is a constant A.

Find an expression for A in terms of r and h, find an expression for the volume of the can in terms of r and h, and determine the ratio of h to r that maximizes the volume of the can.

The area of a hemisphere of radius r is $2\pi r^2$, and the volume is $\frac{2}{3}\pi r^3$. Show all your work.

Name: ______________________________ **Problem 4**: ____________ /25

Problem 4(25 points) A bowl of radius r and height h is made by rotating about the y-axis the region in the 1$^{\text{st}}$ quadrant of the xy-plane that is bounded on the left by the y-axis, on the bottom by the parabola $y = h(\frac{x}{r})^2$, and on the top by the parabola $y = \frac{h}{2}(1 - (\frac{x}{r})^2)$. Determine the volume of material needed to make the bowl. Show all your work.

Name: ____ **Problem 5**: _________________ /25

Problem 5(25 points) Using a trigonometric substitution to find the following integral,

$$\int x^2 \sqrt{a^2 - x^2} dx, \quad a > 0.$$

Show all your work.

Hint: Use the half-angle formulas, $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$.

Name: ____ **Problem 6**: __________________ /20

Problem 6(20 points) Use L'Hospital's rule to compute the following limits. Don't forget to check whether L'Hospital's rule applies to the limit. Show all your work.

(a)(10 points)

$$\lim_{x \to 0} \frac{\sin(x)}{e^x - 1}.$$

(b)(10 points)

$$\lim_{x \to -1^+} \frac{a^{x+1} - 1}{x+1}, \quad a > 0.$$

Name: ____ **Problem 7**: _________________ /30

Problem 7(30 points) Find the area of the region in the 1st quadrant bounded by the cardioid, i.e., the curve with polar equation $r = a(1 + \sin(\theta))$, $0 \leq \theta \leq \frac{\pi}{2}$. Show all your work.

Name: ____ **Problem 8**: _________________ /25

Problem 8(25 points) The curve C has equation $y = e^{2x}$. Find the equation of every tangent line to C that contains the point $(\frac{5}{2}, 0)$.

Name: ______________________________________ **Problem 9**: ________________ /25

Problem 9(25 points) Determine whether the following improper integral converges or diverges. Do not attempt to evaluate the integral. Show all your work and justify your answer.

$$\int_0^{1^-} \frac{dx}{\sqrt{1-x^4}}.$$

Name: ______________________ **Problem 10**: __________ /30

Problem 10(30 points) Evaluate each of the following integrals. Show all your work.

(a)(10 points)

$$\int_1^2 \frac{1}{x^2 + x^3} dx.$$

(b)(5 points)

$$\int_0^1 (x+1)e^{-(x^2+2x+1)} dx.$$

(c)(10 points)

$$\int_4^{12} \frac{dx}{\sqrt{x}(x+4)}.$$

(d)(5 points)

$$\int_0^{\frac{\pi}{4}} \tan(x)(1+\sec^2(x))dx.$$