1ª Lista de Exercícios - Integrais Indefinidas

1 – Calcule as integrais indefinidas abaixo:

1) $\int 2x^3 dx =$	2) $\int (x^2+3x)dx =$	3) $\int (x^2-3x)dx =$
4) $\int (5-x)dx =$	5) $\int (3x^3-2x^2+8x-6)dx =$	6) $\int \frac{5}{x} dx =$
7) $\int (\operatorname{sen} x+\cos x)dx =$	8) $\int \sqrt{x}\, dx =$	9) $\int (\sqrt{x}+\sqrt[3]{x})dx =$
10) $\int \left(\frac{x^2-3x+5}{x^2}\right)dx =$	11) $\int 2e^x dx =$	12) $\int (3e^x+x^3)dx =$
13) $\int (3x^2+5+\sqrt{x})dx =$	14) $\int \frac{\sec^2 x}{\operatorname{cossec} x} dx =$	15) $\int (\sqrt[3]{x^2}+\frac{1}{3x})dx =$
16) $\int \frac{x^4+3x^{-\frac{1}{2}}+4}{\sqrt[3]{x}} dx =$	17) $\int \left(2\cos x+\frac{1}{\sqrt{x}}\right)dx =$	18) $\int \left(2e^x-\frac{1}{4\sec x}+\frac{2}{x^7}\right)dx =$
19) $\int \frac{dx}{x^3} =$	20) $\int (ax^4+bx^3+3c)dx =$	21) $\int (2x^2-3)^2 dx =$
22) $\int (\sqrt{2y}-\frac{1}{\sqrt{2y}})dy =$	23) $\int x^3 . \sqrt{x}\, dx =$	24) $\int \left(9t^2+\frac{1}{\sqrt{t^3}}\right)dt =$
25) $\int \left(\frac{1}{\sqrt{x}}+\frac{x\sqrt{x}}{3}\right)dx =$	26) $\int \frac{x^5+2x^2-1}{x^4} dx =$	27) $\int \left(\frac{e^t}{2}+\sqrt{t}+\frac{1}{t}\right)dt =$
28) $\int \frac{1}{sen^2 x} dx =$	29) $\int (t+\sqrt{t}+\sqrt[3]{t}+\sqrt[4]{t}+\sqrt[5]{t})dt =$	30) $\int \frac{sec\, w\; sen\, w}{\cos w} dw =$

2 – Encontre as primitivas que satisfazem às condições dadas:

1) $f'(x)=12x^2-6x+1$; $f(1)=5$	2) $f'(x)=9x^2+x-8$; $f(-1)=1$
3) $f'(x)=4x^{1/2}$; $f(4)=21$	4) $f'(x)=5x^{-1/3}$; $f(27)=70$
5) $f''(x)=4x-1$; $f'(2)=4$; $f(1)=0$	6) $f''(x)=6x-4$; $f'(2)=5$; $f(2)=4$
7) $f''(x)=3sen\ x-4\cos\ x$; $f'(0)=2$; $f(0)=7$	8) $f''(x)=2\cos\ x-5sen\ x$; $f'(\pi)=3$; $f(\pi)=2+6\pi$

RESOLUÇÃO

1 – 1) $\int 2x^3 dx = \frac{2x^4}{4} + C = \frac{x^4}{2} + C$

2) $\int (x^2 + 3x) dx = \frac{x^3}{3} + \frac{3x^2}{2} + C$

3) $\int (x^2 - 3x) dx = \frac{x^3}{3} - \frac{3x^2}{2} + C$

4) $\int (5 - x)\, dx = 5x - \frac{x^2}{2} + C$

5) $\int (3x^3 - 2x^2 + 8x - 6) dx = \frac{3x^4}{4} - \frac{2x^3}{3} + \frac{8x^2}{2} - 6x + C = \frac{3x^4}{4} - \frac{2x^3}{3} + 4x^2 - 6x + C$

6) $\int \frac{5}{x} dx = 5 \ln|x| + C$

7) $\int (sen x + \cos x) dx = -\cos\ x + sen\ x + C$

8) $\int \sqrt{x} dx = \int x^{1/2} dx = \frac{2x^{3/2}}{3} + C$

9) $\int (\sqrt{x} + \sqrt[3]{x}) dx = \int (x^{1/2} + x^{1/3}) dx = \frac{2x^{3/2}}{3} + \frac{3x^{4/3}}{4} + C$

10) $\int \left(\frac{x^2 - 3x + 5}{x^2}\right) dx = \int \left(\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{5}{x^2}\right) dx = \int \left(1 - \frac{3}{x} + 5x^{-2}\right) dx = x - 3\ \ln|x| + \frac{5x^{-1}}{-1} + C$

$\int \left(\frac{x^2 - 3x + 5}{x^2}\right) dx = \ x - 3\ \ln|x| - \frac{5}{x} + C$

11) $\int 2e^x dx = 2e^x + C$

12) $\int (3e^x + x^3)\, dx = 3e^x + \frac{x^4}{4} + C$

13) $\int (3x^2 + 5 + \sqrt{x}) dx = \int (3x^2 + 5 + x^{1/2}) dx = \frac{3x^3}{3} + 5x + \frac{2x^{3/2}}{3} + C = x^3 + 5x + \frac{2x^{3/2}}{3} + C$

14) $\int \frac{\sec^2 x}{\mathrm{cossec}\, x} dx = \int \frac{\sec x . \sec x}{\mathrm{cossec}\, x} dx$, mas pelas identidades trigonométricas, temos que

$\frac{\sec\ x}{\mathrm{cossec}\ x} = \frac{1/\cos x}{1/sen\ x} = \frac{1}{\cos x} . sen\ x = tg\ x$. Então, a integral fica:

$\int \frac{\sec^2 x}{\mathrm{cossec}\, x} dx = \int tg\ x . \sec\ x\ dx = \sec\ x + C$

15) $\int (\sqrt[3]{x^2}+\frac{1}{3x})dx=\int (x^{2/3}+\frac{1}{3x})dx=\frac{3x^{5/3}}{5}+\frac{1}{3}\ln x+C$

16) $\int \frac{x^4+3x^{-\frac{1}{2}}+4}{\sqrt[3]{x}}dx=\int \left(\frac{x^4}{x^{1/3}}+\frac{3x^{-1/2}}{x^{1/3}}+\frac{4}{x^{1/3}}\right)dx=\int \left(x^{11/3}+3x^{-5/6}+4x^{-1/3}\right)dx=$

$=\frac{3x^{14/3}}{14}+\frac{3.6.x^{1/6}}{1}+\frac{4.3.x^{2/3}}{2}+C=\frac{3x^{14/3}}{14}+18x^{1/6}+6x^{2/3}+C$

17) $\int \left(2\cos x+\frac{1}{\sqrt{x}}\right)dx=\int \left(2\cos x+x^{-1/2}\right)dx=2\ sen\ x+\frac{2x^{1/2}}{1}+C=2\ sen\ x+2x^{1/2}+C$

18) $\int (2e^x-\frac{1}{4\sec x}+\frac{2}{x^7})dx=\int \left(2e^x-\frac{1}{4}\cos\ x+2x^{-7}\right)dx=2e^x-\frac{1}{4}sen\ x-\frac{2x^{-6}}{6}+C$

$2e^x-\frac{sen\ x}{4}-\frac{1}{3x^6}+C$

19) $\int \frac{dx}{x^3}=\int x^{-3}dx=\frac{x^{-2}}{-2}+C=-\frac{1}{2x^2}+C$

20) $\int (ax^4+bx^3+3c)dx=\frac{ax^5}{5}+\frac{bx^4}{4}+3cx+C$

21) $\int (2x^2-3)^2dx=\int (4x^4-12x^2+9)dx=\frac{4x^5}{5}-\frac{12x^3}{3}+9x+C=\frac{4x^5}{5}-4x^3+9x+C$

22) $\int \left(\sqrt{2y}-\frac{1}{\sqrt{2y}}\right)dy=\int \left(\sqrt{2}y^{1/2}-\frac{1}{\sqrt{2}}y^{-1/2}\right)dy=\sqrt{2}\frac{2y^{3/2}}{3}-\frac{1}{\sqrt{2}}\frac{2y^{1/2}}{1}+C$

$\frac{2\sqrt{2}y^{3/2}}{3}-\frac{2y^{1/2}}{\sqrt{2}}+C=\frac{2\sqrt{2}y^{3/2}}{3}-\frac{2y^{1/2}}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}+C=\frac{2\sqrt{2}y^{3/2}}{3}-\sqrt{2}\ y^{1/2}$

23) $\int x^3.\sqrt{x}\,dx=\int x^3.x^{1/2}dx=\int x^{7/2}dx=\frac{2x^{9/2}}{9}+C$

24) $\int (9t^2+\frac{1}{\sqrt{t^3}})dt=\int (9t^2+t^{-3/2})dt=\frac{9t^3}{3}-\frac{2t^{-1/2}}{1}+C=3t^3-\frac{2}{t^{1/2}}+C$

25) $\int \left(\frac{1}{\sqrt{x}}+\frac{x\sqrt{x}}{3}\right)dx=\int \left(x^{-1/2}+\frac{x^{3/2}}{3}\right)dx=\frac{2x^{1/2}}{1}+\frac{2x^{5/2}}{3.5}+C=2x^{1/2}+\frac{2x^{5/2}}{15}+C$

26) $\int \frac{x^5+2x^2-1}{x^4}\,dx=\int\left(\frac{x^5}{x^4}+\frac{2x^2}{x^4}-\frac{1}{x^4}\right)dx=\int\left(x+2x^{-2}-x^{-4}\right)dx=$

$=\frac{x^2}{2}+\frac{2x^{-1}}{-1}-\frac{x^{-3}}{-3}+C=\frac{x^2}{2}-\frac{2}{x}+\frac{1}{3x^3}+C$

27) $\int\left(\frac{e^t}{2}+\sqrt{t}+\frac{1}{t}\right)dt=\int\left(\frac{e^t}{2}+t^{1/2}+\frac{1}{t}\right)dt=\frac{e^t}{2}+\frac{2t^{3/2}}{3}+\ln|t|+C$

28) $\int \frac{1}{sen^2 x}\,dx=\int \cos ec^2\, x\; dx=-\cot g\; x+C$

29) $\int\left(t+\sqrt{t}+\sqrt[3]{t}+\sqrt[4]{t}+\sqrt[5]{t}\right)dt=\int\left(t+t^{1/2}+t^{1/3}+t^{1/4}+t^{1/5}\right)dt=\ \frac{t^2}{2}+\frac{2t^{3/2}}{3}+\frac{3t^{4/3}}{4}+\frac{4t^{5/4}}{5}+\frac{5t^{6/5}}{6}+C$

30) $\int \frac{\sec\, w\;\; sen\, w}{\cos\, w}\,dw=\int \sec\, w\;\; tg\; w\;\; dw=\sec\; w+c$

2 – 1) $f'(x)=12x^2-6x+1$; $\qquad f(1)=5$

$f(x)=\int\left(12x^2-6x+1\right)\,dx=\frac{12x^3}{3}-\frac{6x^2}{2}+x+C=4x^3-3x^2+x+C$

Como $f(1)=5$, podemos calcular o valor de C:

$f(1)=5\Rightarrow 4.1^3-3.1^2+1+C=5\Rightarrow 2+C=5\Rightarrow C=3$

Assim: $f(x)=4x^3-3x^2+x+3$

2) $f'(x)=9x^2+x-8$; $\qquad f(-1)=1$

$f(x)=\int\left(9x^2+x-8\right)\,dx=\frac{9x^3}{3}+\frac{x^2}{2}-8x+C=3x^3+\frac{x^2}{2}-8x+C$

Como $f(-1)=1$, podemos calcular o valor de C:

$f(-1)=1\Rightarrow 3.(-1)^3+\frac{(-1)^2}{2}-8.(-1)+C=1\Rightarrow -3+\frac{1}{2}+8+C=1\Rightarrow C=-\frac{9}{2}$

Assim: $f(x)=3x^3+\frac{x^2}{2}-8x-\frac{9}{2}$

3) $f'(x)=4x^{1/2}$; $\qquad f(4)=21$

$f(x)=\int 4x^{1/2}\;dx=4.\frac{2x^{3/2}}{3}+C=\frac{8x^{3/2}}{3}+C$

Como $f(4)=21$, podemos calcular o valor de C:

$f(4)=21 \Rightarrow \frac{8.4^{3/2}}{3}+C=21 \Rightarrow \frac{64}{3}+C=21 \Rightarrow C=-\frac{1}{3}$

Assim: $f(x)=\frac{8x^{3/2}}{3}-\frac{1}{3}=\frac{8x^{3/2}-1}{3}$

4) $f'(x)=5x^{-1/3}$; $f(27)=70$

$f(x)=\int 5x^{-1/3}\, dx=5.\frac{3x^{2/3}}{2}+C=\frac{15x^{2/3}}{2}+C$

Como $f(27)=70$, podemos calcular o valor de C:

$f(27)=70 \Rightarrow \frac{15.27^{2/3}}{2}+C=70 \Rightarrow \frac{135}{2}+C=70 \Rightarrow C=\frac{5}{2}$

Assim: $f(x)=\frac{15x^{2/3}}{2}+\frac{5}{2}=\frac{15x^{2/3}+5}{2}$

5) $f''(x)=4x-1$; $f'(2)=4$;

$f'(x)=\int (4x-1)\, dx=4.\frac{x^2}{2}-x+C=2x^2-x+C$

Como $f'(2)=4$, podemos calcular o valor de C:

$f'(2)=4 \Rightarrow 2.2^2-2+C=4 \Rightarrow 8-2+C=4 \Rightarrow C=-2$

Assim: $f'(x)=2x^2-x-2$, para calcular f(x), temos que $f(1)=0$

$f(x)=\int (2x^2-x-2)\, dx=\frac{2x^3}{3}-\frac{x^2}{2}-2x+C$

Como $f(1)=0$, podemos calcular o valor de C:

$f(1)=0 \Rightarrow \frac{2.1^3}{3}-\frac{1^2}{2}-2.1+C=0 \Rightarrow \frac{2}{3}-\frac{1}{2}-2+C=0 \Rightarrow C=\frac{11}{6}$

Assim: $f(x)=\frac{2x^3}{3}-\frac{x^2}{2}-2x+\frac{11}{6}$

6) $f''(x)=6x-4$; $f'(2)=5$;

$f'(x)=\int (6x-4)\, dx=6.\frac{x^2}{2}-4x+C=3x^2-4x+C$

Como $f'(2)=5$, podemos calcular o valor de C:

$f'(2)=5 \Rightarrow 3.2^2-4.2+C=5 \Rightarrow 12-8+C=5 \Rightarrow C=1$

Assim: $f'(x)=3x^2-4x+1$, para calcular f(x), temos que $f(2)=4$

$f(x)=\int (3x^2-4x+1)\ dx=\frac{3x^3}{3}-\frac{4x^2}{2}+x+C=x^3-2x^2+x+C$

Como $f(2)=4$, podemos calcular o valor de C:

$f(2)=4 \Rightarrow 2^3-2.2^2+2+C=4 \Rightarrow 8-8+2+C=4 \Rightarrow C=2$

Assim: $f(x)=x^3-2x^2+x+2$

7) $f''(x)=3 sen\ x-4\cos x$; $f'(0)=2$;

$f'(x)=\int (3 sen\ x-4\cos x)\ dx=-3\cos x-4\ sen\ x+C$

Como $f'(0)=2$, podemos calcular o valor de C:

$f'(0)=2 \Rightarrow -3\cos 0-4\ sen\ 0+C=2 \Rightarrow -3+C=2 \Rightarrow C=5$

Assim: $f'(x)=-3\cos x-4\ sen\ x+5$, para calcular f(x), temos que $f(0)=7$

$f(x)=\int (-3\cos x-4\ sen\ x+5)\ dx=-3\ sen\ x+4\cos x+5x+C$

Como $f(0)=7$, podemos calcular o valor de C:

$f(0)=7 \Rightarrow -3\ sen\ 0+4\cos 0+C=7 \Rightarrow 4+C=7 \Rightarrow C=3$

Assim: $f(x)=-3\ sen\ x+4\cos x+5x+3$

8) $f''(x)=2\cos x-5 sen\ x$; $f'(\pi)=3$;

$f'(x)=\int (2\cos x-5 sen\ x)\ dx=2\ sen\ x+5\cos x+C$

Como $f'(\pi)=3$, podemos calcular o valor de C:

$f'(\pi)=3 \Rightarrow 2\ sen\ \pi+5\cos \pi+C=3 \Rightarrow -5+C=3 \Rightarrow C=8$

Assim: $f'(x)=2\ sen\ x+5\cos x+8$, para calcular f(x), temos que $f(\pi)=2+6\pi$

$f(x)=\int (2\ sen\ x+5\cos x+8)\ dx=-2\cos x+5\ sen\ x+8x+C$

Como $f(\pi)=2+6\pi$, podemos calcular o valor de C:

$f(\pi)=2+6\pi \Rightarrow -2\cos \pi+5\ sen\ \pi+8\pi+C=2+6\pi \Rightarrow 2+8\pi+C=2+6\pi \Rightarrow C=-2\pi$

Assim: $f(x)=-2\cos x+5 sen\ x+8x-2\pi$