18.01, October 7, 2003 Differentials and indefinite integration

3A-1, 3A-2, 3A-3

1. Talked about makeup exams, grading policy, etc.

2. Differentials. Just notation: if $F'(x) = f(x)$ then $d(F(x)) = f(x)dx$. Useful for transforming deriv. identities into integral indentities (e.g. chain rule in diff. notation is $d(F(U(x))) = F'(U)\bullet U'(x)dx$ suggests "integ. by substit.": $\int f(U(x))\bullet U'(x)dx = \int f(U)dU$)

3. Antiderivative=indef. integral $\int f(x)dx$

- Doesn't always have simple expression, e.g. $\frac{\sin x}{x}$.
- Guess-and-check, e.g. $\ln(x) \to guess \to x\ln(x)$ $(no) \to guess \to x\ln(x) - x$
- Linearity of $\int fdx$, $\int (f+g)dx = \int fdx + \int gdx, \int af(x)dx = a\int f(x)dx$ antideriv of $x^n = \frac{1}{n+1}x^{n+1} + C$ if n≠-1 $\int \frac{1}{x}dx = \ln(x) + C$ (or ln(-x)+C if x negative).

$\int e^x dx = e^x + C, \int \cos(x)dx = \sin(x) + C, \int \sin(x)dx = -\cos(x) + C$

4. Integr. By substitution. If a term U(x) appears in integrand and also $U'(x)$ appears substit and use $\int f(U(x))\bullet U'(x)dx = \int f(U)dU$.

e.g. $\int \frac{x}{\sqrt{x^2+a}}dx = \sqrt{x^2+a} + C$, $\int [\cos(x)]^n \sin(x)dx = -\frac{1}{n+1}[\cos(x)]^{n+1} + C$

More complicated: $\int \cos^3(x)dx = \int (1-\sin^2(x))\bullet\cos(x)dx = \sin(x) - \frac{1}{3}\sin^3(x) + C$.