MATH 18.01, FALL 2017 - PROBLEM SET # 4

Professor: Jared Speck

Due: by Friday 1:45pm on 10-13-17
(in the boxes outside of Room 4-174; write your name, recitation instructor, and recitation meeting days/time on your homework)

18.01 Supplementary Notes (including Exercises and Solutions) are available on the course web page: `http://math.mit.edu/~jspeck/18.01_Fall%202017/1801_CourseWebsite.html`. This is where to find the exercises labeled 1A, 1B, etc. You will need these to do the homework.

Part I consists of exercises given and solved in the Supplementary Notes. It will be graded quickly, checking that all is there and the solutions not copied.

Part II consists of problems for which solutions are not given; it is worth more points. Some of these problems are longer multi-part exercises given here because they do not fit conveniently into an exam or short-answer format. See the guidelines below for what collaboration is acceptable, and follow them.

To encourage you to keep up with the lectures, both Part I and Part II tell you for each problem on which day you will have the needed background for it.

You are encouraged to use graphing calculators, software, etc. to check your answers and to explore calculus. However, (unless otherwise indicated) we strongly discourage you from using these tools to solve problems, perform computations, graph functions, etc. An extremely important aspect of learning calculus is developing these skills. You will not be allowed to use any such tools on the exams.

Part I (5 points)

Notation: The problems come from three sources: the Supplementary Notes, the Simmons book, and problems that are described in full detail inside of this pset. I refer to the former two sources using abbreviations such as the following ones: 2.1 = Section 2.1 of the Simmons textbook; Notes G = Section G of the Supplementary Notes; Notes 1A: 1a, 2 = Exercises 1a and 2 in the Exercise Section 1A of the Supplementary Notes; Section 2.4: 13 = Problem 13 in Section 2.4 of Simmons, etc.

Lecture 11. (Thurs., Oct. 5) Newton's method.
Read: 4.6, (4.7 is optional).

Lecture 12. (Fr., Oct. 6) Mean-value theorem. Inequalities.
Read: 2.6 to middle of p. 77; Notes MVT.
Homework: Notes 2G: 1b, 2b, 5, 6.

Part II (30 points)

Directions and Rules: Collaboration on problem sets is encouraged, but:

i) Attempt each part of each problem yourself. Read each portion of the problem before asking for help. If you don't understand what is being asked, ask for help interpreting the problem and then make an honest attempt to solve it.

ii) Write up each problem independently. On both Part I and II exercises you are expected to write the answer in your own words. *You must show your work; "bare" solutions will receive very little credit.*

iii) Write on your problem set whom you consulted and the sources you used. If you fail to do so, you may be charged with plagiarism and subject to serious penalties.

iv) It is illegal to consult materials from previous semesters.

0. (not until due date; 3 points) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation." This includes visits outside recitation to your recitation instructor. If you don't know a name, you must nevertheless identify the person, as in, "tutor in Room 2-106," or "the student next to me in recitation." Optional: note which of these people or resources, if any, were particularly helpful to you.

This "Problem **0**" will be assigned with every problem set. Its purpose is to make sure that you acknowledge (to yourself as well as others) what kind of help you require and to encourage you to pay attention to how you learn best (with a tutor, in a group, alone). It will help us by letting us know what resources you use.

1. (Oct. 5; Newton's method; $1 + 2 + 2 = 5$ points) This problem will show you why it is important to choose a good starting point x_0 when you are applying Newton's method. Consider the function $f(x) = x^3 - 2x + 2$.

a) Plot the graph of $f(x)$. The graph will help you make sense of the rest of the problem.

b) Use Newton's method with the base point $x_0 = 0$ in an effort to locate a zero of $f(x)$. Compute the first 4 iterates an explain what is happening.

c) Using a calculator, compute the first 4 Newton iterates starting with a base point x_0 near 0, but not equal to 0. The repeat your calculations for a slightly different choice of x_0 (but still near 0). Explain how the behavior of the iterates compares to the behavior you observed in part b). In particular, do the iterates converge to the solution x to $f(x) = 0$?

2. (Oct. 5; Newton's method; $1 + 1 + 2 + 2 = 6$ points)

a) Sketch the graph of the function $f(x) = x^5 - 33$. The graph will help you make sense of the rest of the problem.

b) With the help of a computer, compute $33^{1/5}$ to 15 significant figures by using Newton's method starting with $x_0 = 2$. How many steps does it take to achieve this degree of accuracy?

c) For each of x_0, x_1, x_2, x_3, indicate whether x_k is larger or smaller than $33^{1/5}$. Then indicate whether x_k is larger or smaller than x_{k-1}. For the points x_k that are bigger than $33^{1/5}$, explain how your answers are connected to graph you sketched in part a).

d) Find a quadratic approximation to $33^{1/5}$ and estimate the error between your quadratic approximation and the precise value of $33^{1/5}$. How many digits of accuracy does your quadratic approximation achieve? Remark: when constructing the quadratic approximation, it is helpful to note that $33^{1/5} = \{32(1 + 1/32)\}^{1/5}$.

3. (Oct. 6; Mean value theorem; 3 + 3 = 6 points)

a) Decide whether or not there is a differentiable function $f(x)$ with the following properties: $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x. Explain your answer.

b) Let $f(x)$ be a differentiable function. A point a is said to be a *fixed point* if $f(a) = a$. Suppose that there is no point x such that $f'(x) = 1$. Show that there exists at most one fixed point.

4. (Oct. 6; Mean value theorem; 2 + 2 + 2 + 2 + 2 = 10 points)

a) Let $f(x)$ be a differentiable function. Use the mean value theorem to show that if $f(0) = 0$ and $f'(x) \geq 0$, then $f(x) \geq 0$ for all $x \geq 0$.

b) Using part a), show that $\ln(1 + x) \leq x$ for $x \geq 0$. *Hint: Use* $f(x) = x - \ln(1 + x)$.

c) Use the same method as in b) to show that $\ln(1+x) \geq x - x^2/2$ and $\ln(1+x) \leq x - x^2/2 + x^3/3$ for $x \geq 0$.

d) Find the pattern in b) and c) and make a general conjecture; you do not have to prove your conjecture.

e) Show that $\ln(1 + x) \leq x$ for $-1 < x \leq 0$. (Use the change of variable $u = -x$.)