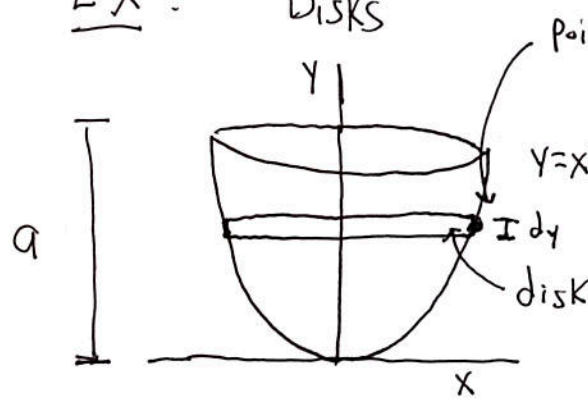
# • Volumes by Disks & Shells

**Ex:** Disks



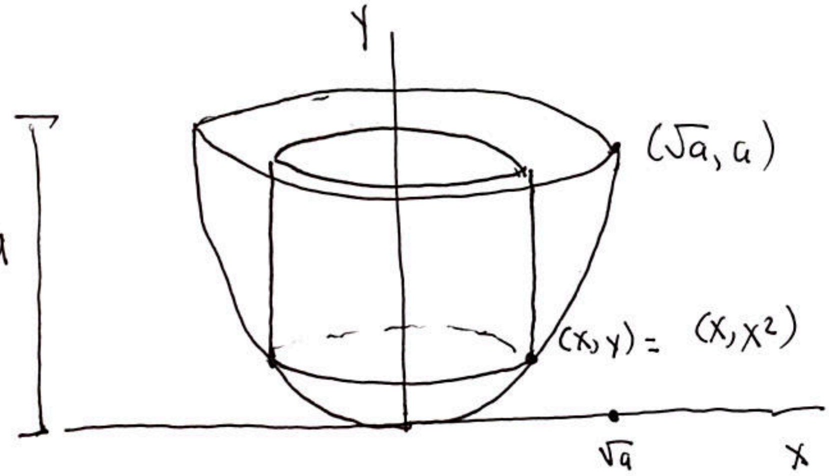

$y = x^2$ rotated about y-axis

· The volume of the disk is $dV = \overbrace{\pi x^2}^{\text{area of a disk of radius } x} dy$

The total volume is $V = \int_{y=0}^{y=a} \pi x^2 \, dy$

$$= \int_{y=0}^{y=a} \pi y \, dy = \frac{\pi y^2}{2}\Big]_0^a = \frac{\pi a^2}{2}$$

Ex: (Cylindrical Shells)




- Cylinder height $= a - y = a - x^2$
- Cylinder radius $= x$
- Cylinder thickness $= dx$

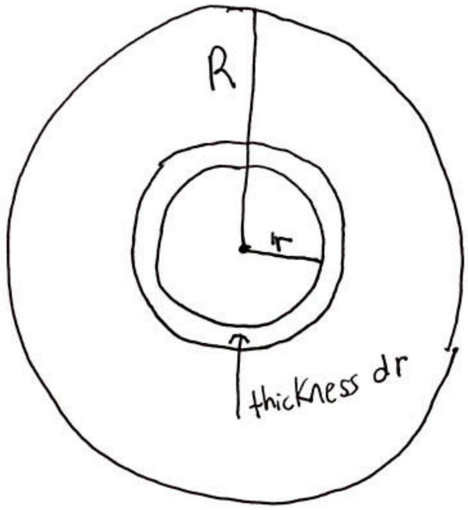
Cylinder volume $= dV = \underbrace{2\pi x \cdot (a-x^2)}_{\text{Surface area of the cylinder}} \cdot dx$

- $$V = \int_{x=0}^{x=\sqrt{a}} 2\pi x (a-x^2)\, dx = 2\pi \left[ \frac{ax^2}{2} - \frac{x^4}{4} \right]_0^{\sqrt{a}}$$

$$= 2\pi \left( \frac{a^2}{2} - \frac{a^2}{4} \right) = \frac{\pi}{2} a^2$$

**Ex:** Pipe flow

Consider a pipe of radius $R$




Poiseuille figured out that the velocity profile for a fluid flowing in the pipe is $V = C\,(R^2 - r^2)$, where $C$ is a constant (with dimensions)

$$V = \text{speed} = \frac{\text{distance}}{\text{time}}$$

- The flow through the ring is

  $\text{ring flow} = (\text{area of ring}) \times (\underbrace{\text{flow rate}}_{V})$

  - area of ring $= 2\pi r\,dr = (\text{circumference}) \times (\text{width})$

- Total flow through pipe $= \displaystyle\int_0^R (2\pi r\,dr) \times V = \int_0^R 2\pi C (R^2 - r^2)\, r\,dr$

$$= 2\pi C \left[R^2 \frac{r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{\pi}{2} C R^4.$$