Linear Algebra:
Foundations to Frontiers (LAFF)

The University of Texas at Austin (UTAustinX)

- <u>**WEEK 0: GETTING STARTED**</u>
  - ***Opening Remarks***
    - Welcome to LAFF
    - Outline
    - What You Will Learn
  - ***How to LAFF***
    - When to LAFF
    - How to Navigate LAFF
    - Homework and LAFF
    - Grading and LAFF
    - Programming and LAFF
    - Proving and LAFF
    - Setting Up to LAFF
  - ***Software to LAFF***
    - Why MATLAB
    - Activating MATLAB Online
    - MATLAB Basics
    - Setting Up MATLAB Online to LAFF
  - ***Enrichments***
    - The Origins of MATLAB
    - Intrigued by MATLAB and Want to Learn More?
  - ***Wrap Up***
    - Additional Homework
    - Summary

- <u>**WEEK 1: VECTORS IN LINEAR ALGEBRA**</u>
  - ***1.1 Opening Remarks***
    - 1.1.1 Takeoff
    - 1.1.2 Outline
    - 1.1.3 What You Will Learn

- Subsection 1.4

- **<u>WEEK 2: LINEAR TRANSFORMATIONS AND MATRICES</u>**

  - ***2.1 Openning Remarks***
    - 2.1.1 Rotations in 2D
    - 2.1.2 Outline Week 2
    - 2.1.3 What you will learn

  - ***2.2 Linear Transformations***
    - 2.2.1 What makes a linear transformation so special
    - 2.2.2 What is a linear transformation?
    - 2.2.3 Of linear transformations and linear combinations

  - ***2.3 Mathematical Induction***
    - 2.3.1 What is the Principle of Mathematical Induction?
    - 2.3.2 Examples

  - ***2.4 Representing Linear Transformations as Matrices***
    - 2.4.1 From Linear Transformation to Matrix-Vector Multiplication
    - 2.4.2 Practice with Matrix-Vector Multiplication
    - 2.4.3 It Goes Both Ways
    - 2.4.4 Rotations and Reflections, Revisited

  - ***2.5 Enrichment***
    - 2.5.1 The Importance of the Principle of Mathematical Induction for Programming
    - 2.5.2 Puzzles and Paradoxes in Mathematical Induction

  - ***2.6 Wrap Up***
    - 2.6.1 Additional Homework
    - 2.6.2 Summary

  - ***2.7 Week 2 Proofs***
    - 2.1 Homekorks
    - 2.2 Homeworks
    - 2.3 Homeworks
    - 2.4 Homewoks

- **<u>WEEK 3: MATRIX-VECTOR OPERATIONS</u>**

  - ***3.1 Opening Remarks***
    - 3.1.1 Timmy Two Space
    - 3.1.2 Outline Week 3
    - 3.1.3 What You will Learn

- ***3.2 Special Matrices***
  - 3.2.1 The Zero Matrix
  - 3.2.2 The Identity Matrix
  - 3.2.3 Diagonal matrices
  - 3.2.4 Triangular Matrices
  - 3.2.5 Transpose Matrix
  - 3.2.6 Symmetric Matrices

- ***3.3 Operations with Matrices***
  - 3.3.1 Scaling a Matrix
  - 3.3.2 Adding matrices

- ***3.4 Matrix-Vector Multiplication Algorithms***
  - 3.4.1 Via dot products
  - 3.4.2 Via AXPY Operations
  - 3.4.3 Compare and Contrast
  - 3.4.4 Cost of Matrix-vector Multiplication

- ***3.5 Wrap Up***
  - 3.5.1 Homework
  - 3.5.2 Summary

- ***3.6 Week 3 Proofs***
  - 3.3.1 Videos with Transcripts
  - 3.3.2 Videos with Transcripts

- **WEEK 4: FROM MATRIX-VECTOR TO MATRIX-MATRIX MULTIPLICATION**

  - ***4.1 Opening Remarks***
    - 4.1.1 Predicting the Weather
    - 4.1.2 Outline
    - 4.1.3 What you will learn

  - ***4.2 Preparation***
    - 4.2.1 Partitioned Matrix-vector Multiplication
    - 4.2.2 Transposing a Partitioned Matrix
    - 4.2.3 Matrix-Vector Multiplication, again…

  - ***4.3 Matrix-Vector Multiplication with Special Matrices***
    - 4.3.1 Transpose Matrix-Vector Multiplication
    - 4.3.2 Triangular Matrix-Vector Multiplication
    - 4.3.3 Symmetric Matrix-Vector Multiplication

  - ***4.4 Matrix-Matrix Multiplication (Product)***
    - 4.4.1 Motivation
    - 4.4.2 Linear Transformations to Matrix-Matrix Multiplication

- 4.4.3 Computing the Matrix-Matrix Product
- 4.4.4 Special Shapes
- 4.4.5 Cost

- ***4.5 Enrichment***
  - 4.5.1 Markov Chains: Their Application

- ***4.6 Wrap Up***
  - 4.6.1 Homework
  - 4.6.2 Summary
  - 4.6.3 LAFF Routines (so far)

EXAM 1

- ***E1.1 How to Review for Exam 1***
  - E1.1.1

- ***E1.2 Sample Exam 1***
  - E1.2.0 Sample Exam 1 (PDF to print out)
  - E1.2.1 Sample Question 1
  - E1.2.2 Sample Question 2
  - E1.2.3 Sample Question 3
  - E1.2.4 Sample Question 4
  - E1.2.5 Sample Question 5
  - E1.2.6 Sample Question 6
  - E1.2.7 Sample Question 7
  - E1.2.8 Sample Question 8

- ***E1.3 Real Exam 1***
  - E1.3.0 Exam 1 (PDF to print out)
  - E1.3.1 Exam Question 1
  - E1.3.2 Exam Question 2
  - E1.3.3 Exam Question 3
  - E1.3.4 Exam Question 4
  - E1.3.5 Exam Question 5
  - E1.3.6 Exam Question 6
  - E1.3.7 Exam Question 7

WEEK 5: MATRIX-MATRIX MULTIPLICATION

- ***5.1 Opening Remarks***
  - 5.1.1 Composing Rotations
  - 5.1.2 Outline
  - 5.1.3 What you will Learn

- ***5.2 Observations***
  - 5.2.1 Partitioned Matrix-Matrix Multiplication
  - 5.2.2 Properties
  - 5.2.3 Transposing a Product of Matrices
  - 5.2.4 Matrix-Matrix Multiplication with Special Matrices

- ***5.3 Algorithms for Computing Matrix-Matrix Multiplications***
  - 5.3.1 Lots of Loops
  - 5.3.2 Matrix-matrix multiplication by columns
  - 5.3.3 Matrix-matrix multiplication by rows
  - 5.3.4 Matrix-Matrix Multiplication with Rank-1 Updates

- ***5.4 Updates***
  - 5.4.1 Slicing and Dicing for Performance
  - 5.4.2 How It is Really Done

- ***5.5 Wrap Up***
  - 5.5.1 Homework
  - 5.5.2 Summary

- ***5.6 Week 5 Proofs***
  - Proofs with Transcripts Unit 5.2.4
  - Proofs with Transcripts Unit 5.3.2

- **WEEK 6: GAUSSIAN ELIMINATION**

  - ***6.1 Opening Remarks***
    - 6.1.1 Opening Remarks
    - 6.1.2 Outline
    - 6.1.3 What You will Learn

  - ***6.2 Gaussian Elimination***
    - 6.2.1 Reducing a System of Linear Equations to an Upper Triangular System
    - 6.2.2 Appended Matrices
    - 6.2.3 Gauss Transforms
    - 6.2.4 Computing Separately with the Matrix and Right-Hand Side (Forward Substition)
    - 6.2.5 Towards an Algorithm

  - ***6.3 Solving Ax = b via LU Factorization***
    - 6.3.1 LU factorization (Gaussian elimination)
    - 6.3.2 Solving Lz = b (Forward substitution)
    - 6.3.3 Solving Ux = b (Back substitution)
    - 6.3.4 Putting it all together to solve Ax = b
    - 6.3.5 Cost

- *6.4 Enrichment*
  - 6.4.1 Blocked LU Factorization
  - 6.4.2 How ordinary elimination became Gaussian elimination
  - 6.4.3 Formal derivation of LU factorization
  - 6.4.4 High-performance matrix-matrix multiplication, again

- *6.5 Wrap Up*
  - 6.5.1 Homework
  - 6.5.2 Summary

WEEK 7: MORE GAUSSIAN ELIMINATION AND MATRIX INVERSION

- *7.1 Opening Remarks*
  - 7.1.1 Introduction
  - 7.1.2 Outline
  - 7.1.3 What you will learn

- *7.2 When Gaussian Elimination Breaks Down*
  - 7.2.1 When Gaussian Elimination Works
  - 7.2.2 The Problem
  - 7.2.3 Permutations
  - 7.2.4 Gaussian Elimination with Row Swapping (LU Factorization with Partial Pivoting)
  - 7.2.5 When Gaussian Elimination Fails Altogether

- *7.3 The Inverse Matrix*
  - 7.3.1 Inverse Functions in 1D
  - 7.3.2 Back to Linear Transformations
  - 7.3.3 Simple Examples
  - 7.3.4 More Advanced (But Still Simple) Examples
  - 7.3.5 Properties

- *7.4 Enrichment*
  - 7.4.1 Library Routines for LU with Partial Pivoting

- *7.5 Wrap Up*
  - 7.5.1 Homework
  - 7.5.2 Summary

WEEK 8: MORE ON MATRIX INVERSION

- *8.1 Opening Remarks*
  - 8.1.1 When LU Factorization with Row Pivoting Fails
  - 8.1.2 Outline

- 8.1.3 What you will learn

- ***8.2 Gauss-Jordan Elimination***
  - 8.2.1 Solving Ax = b via Gauss-Jordan elimination
  - 8.2.2 Solving Ax = b via Gauss-Jordan Elimination, Gauss Transforms
  - 8.2.3 Solving A x = b via Gauss-Jordan Elimination: Multiple Right-Hand Sides
  - 8.2.4 Computing the Inverse of A via Gauss-Jordan Elimination
  - 8.2.5 Computing the Inverse of A via Gauss-Jordan Elimination, Alternative
  - 8.2.6 Pivoting
  - 8.2.7 Cost of Matrix Inversion

- ***8.3 (Almost) Never, Ever, Invert a Matrix!***
  - 8.3.1 Solving Ax = b
  - 8.3.2 But…

- ***8.4 (Very Important) Enrichment***
  - 8.4.1 Symmetric Positive Definite Matrices
  - 8.4.2 Solving Ax=b when A is Symmetric Positive Definite
  - 8.4.3 Other Factorizations
  - 8.4.4 Welcome to the Frontier

- ***8.5 Wrap Up***
  - 8.5.1 Homework
  - 8.5.2 Summary

- **EXAM 2**

  - ***E2.1 How to Review for Exam 2***
    - E2.1.1 Suggestions

  - ***E2.2 Sample Exam 2***
    - E2.2.1 Sample Midterm (PDF to Print Out)
    - E2.2.2 Sample Exam Answers and Videos Questions 1-2
    - E2.2.3 Sample Exam Answers and Videos Questions 3-4
    - E2.2.4 Sample Exam Answers and Videos Questions 5-6
    - E2.2.5 Sample Exam Answers and Videos Questions 7-8
    - E2.2.6 Sample Exam Answers and Videos Questions 9-10
    - E2.2.7 Sample Exam Answers and Videos Questions 11-12

  - ***E2.3 Real Exam 2***
    - E2.3.1 Questions 1-2
    - E2.3.2 Questions 3-4
    - E2.3.3 Questions 5-6
    - E2.3.4 Questions 7-8
    - E2.3.5 Question 9

- **<u>WEEK 9: VECTOR SPACES</u>**

  - ***9.1 Opening Remarks***
    - 9.1.1 Solvable or Not Solvable, That's the Question
    - 9.1.2 Outline
    - 9.1.3 What You Will Learn

  - ***9.2 When Systems Don’t Have a Unique Solution***
    - 9.2.1 When Solutions Are Not Unique
    - 9.2.2 When Linear Systems Have No Solutions
    - 9.2.3 When Linear Systems Have Many Solutions
    - 9.2.4 What is Going On?
    - 9.2.5 Toward a Systematic Approach to Finding All Solutions

  - ***9.3 Review of Sets***
    - 9.3.1 Definition and Notation
    - 9.3.2 Examples
    - 9.3.3 Operations with Sets

  - ***9.4 Vector Spaces***
    - 9.4.1 What is a Vector Space?
    - 9.4.2 Subspaces
    - 9.4.3 The Column Space
    - 9.4.4 The Null Space

  - ***9.5 Span, Linear Independance, and Bases***
    - 9.5.1 Span
    - 9.5.2 Linear Independence
    - 9.5.3 Bases for Subspaces
    - 9.5.4 The Dimension of a Subspace

  - ***9.6 Enrichment***
    - 9.6.1 Typesetting Algorithms with the FLAME Notation

  - ***9.7 Wrap Up***
    - 9.7.1 Homework
    - 9.7.2 Summary

- **<u>WEEK 10: VECTOR SPACES, ORTHOGONALLITY, AND LINEAR LEAST-SQUARES</u>**

  - ***10.1 Opening Remarks***
    - 10.1.1 Visualizing Planes, Lines, and Solutions
    - 10.1.2 Outline
    - 10.1.3 What You Will Learn

- ***11.4 Change of Bases***
  - 11.4.1 The Unit Basis Vectors, One More Time
  - 11.4.2 Change of Basis

- ***11.5 Singular Value Decomposition***
  - 11.5.1 The Best Low Rank Approximation

- ***11.6 Enrichment***
  - 11.6.1 The Problem with Computing the QR Factorization
  - 11.6.2 QR Factorization Via Householder Transformations (Reflections)
  - 11.6.3 More on SVD

- ***11.7 Wrap Up***
  - 11.7.1 Homework
  - 11.7.2 Summary

- **WEEK 12: EIGENVALUES AND EIGENVECTORS**

  - ***12.1 Opening Remarks***
    - 12.1.1 Predicting the Weather, Again
    - 12.1.2 Outline
    - 12.1.3 What You Will Learn

  - ***12.2 Getting Started***
    - 12.2.1 The Algebraic Eigenvalue Problem
    - 12.2.2 Simple Examples
    - 12.2.2 Simple Examples (continued)
    - 12.2.3 Diagonalization
    - 12.2.4 Eigenvalues and Vectors of 3 x 3 Matrices

  - ***12.3 The General Case***
    - 12.3.1 Eigenvalues and Eigenvectors of n x n Matrices: Special Cases
    - 12.3.2 Eigenvalues of n x n Matrices
    - 12.3.3 Diagonalization, Again
    - 12.3.4 Properties

  - ***12.4 Practical Methods***
    - 12.4.1 Predicting the Weather, One Last Time
    - 12.4.2 The Power Method
    - 12.4.3 In Preparation for this Week's Enrichment

  - ***12.5 Enrichment***
    - 12.5.1 The Inverse Power Method
    - 12.5.2 The Rayleigh Quotient Iteration
    - 12.5.3 More Advanced Techniques

- ***12.6 Wrap Up***
  - 12.6.1 Homework
  - 12.6.2 Summary

- **<u>FINAL EXAM</u>**

  - ***F.1 How to Review for the Final***
    - F.1.1 Suggestions

  - ***F.2 Sample Final Exam***
    - F.2.1 Sample Final (PDF to Print Out)
    - F.2.2 Sample Exam Answers and Videos Questions 1-2
    - F.2.3 Sample Exam Answers and Videos Questions 3-4
    - F.2.4 Sample Exam Answers and Videos Questions 5-6
    - F.2.5 Sample Exam Answers and Videos Questions 7-8
    - F.2.6 Sample Exam Answers and Videos Questions 9

  - ***F.3 Real Final Exam***
    - F.3.1 Final Questions 1-2
    - F.3.2 Final Questions 3-4
    - F.3.3 Final Questions 5-6
    - F.3.4 Final Questions 7-8

  - ***F.4 Congratulations! (and Further Reading!!!)***
    - F.4.1 Congratulations!
    - F.4.2 Some Recommended Materials for Further Learning
    - F.4.3 You are Ready for the Cutting Edge Research
    - F.4.4 Write a Review!