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Cálculo III

2ª lista de exercícios

Aluno: Abrantes Araújo Silva Filho

23) $f(x,y,z) = 1 + xy^2 - 2z^2$

$$\frac{\partial f}{\partial x}(1 + xy^2 - 2z^2) = \frac{\partial}{\partial x}1 + y^2\frac{\partial}{\partial x}x - 2\frac{\partial}{\partial x}z^2$$

$$= 0 + y^2(1) - 0$$

$$= y^2 //$$

$$\frac{\partial}{\partial y}(1 + xy^2 - 2z^2) = \frac{\partial}{\partial y}1 + x\frac{\partial}{\partial y}y^2 - 2\frac{\partial}{\partial y}z^2$$

$$= 0 + 2xy - 0$$

$$= 2xy //$$

$$\frac{\partial}{\partial z}(1 + xy^2 - 2z^2) = \frac{\partial}{\partial z}1 + \frac{\partial}{\partial z}(xy^2) - 2\frac{\partial}{\partial z}z^2$$

$$= 0 + 0 - 4z$$

$$= (-4z) //$$

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24) $f(x,y,z) = xy + yz + xz$

$$\frac{\partial}{\partial x}(xy+yz+xz) = y\frac{\partial}{\partial x}x + \frac{\partial}{\partial x}(yz) + z\frac{\partial}{\partial x}x$$

$$= y + 0 + z$$

$$= y + z \;//$$

$$\frac{\partial}{\partial y}(xy+yz+xz) = x\frac{\partial}{\partial y}y + z\frac{\partial}{\partial y}y + \frac{\partial}{\partial y}(xz)$$

$$= x + z + 0$$

$$= x + z \;//$$

$$\frac{\partial}{\partial z}(xy+yz+xz) = \frac{\partial}{\partial z}(xy) + y\frac{\partial}{\partial z}z + x\frac{\partial}{\partial z}z$$

$$= 0 + y + x$$

$$= y + x \;//$$

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25) $f(x,y,z) = x - \sqrt{y^2+z^2}$

$$\frac{\partial}{\partial x}\left(x-(y^2+z^2)^{1/2}\right) = \frac{\partial}{\partial x}x - \frac{\partial}{\partial x}(y^2+z^2)^{1/2}$$

$$= 1 - 0$$

$$= 1$$

$$\frac{\partial}{\partial y}\left(x-(y^2-z^2)^{1/2}\right) = \frac{\partial}{\partial y}x - \frac{\partial}{\partial y}(y^2-z^2)^{1/2}$$

$$= 0 - \frac{\partial}{\partial u}u^{1/2}\frac{\partial}{\partial y}(y^2-z^2) \quad | u = y^2-z^2$$

$$= -\frac{1}{2\sqrt{u}}\left(\frac{\partial}{\partial y}y^2 - \frac{\partial}{\partial y}z^2\right)$$

$$= -\frac{1}{2\sqrt{u}}(2y-0) = \frac{-2y}{2\sqrt{y^2-z^2}} = \frac{-y}{\sqrt{y^2-z^2}}$$

$$\frac{\partial}{\partial z}\left(x-(y^2-z^2)^{1/2}\right) = \frac{\partial}{\partial z}x - \frac{\partial}{\partial z}(y^2-z^2)^{1/2}$$

$$= 0 - \frac{\partial}{\partial u}u^{1/2}\frac{\partial}{\partial z}(y^2-z^2) \quad | u = y^2-z^2$$

$$= -\frac{1}{2\sqrt{u}}\left(\frac{\partial}{\partial z}y^2 - \frac{\partial}{\partial z}z^2\right) = -\frac{1}{2\sqrt{u}}(0-2z)$$

$$= \frac{2z}{2\sqrt{y^2-z^2}} = \frac{z}{\sqrt{y^2-z^2}}$$

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26) $f(x,y,z) = (x^2+y^2+z^2)^{-1/2}$

$$\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2} = \frac{\partial}{\partial u} u^{-1/2}\left(\frac{\partial}{\partial x}(x^2+y^2+z^2)\right) \quad | \; u = x^2+y^2+z^2$$

$$= -\frac{1}{2}u^{-3/2}\left(\frac{\partial}{\partial x}x^2 + \frac{\partial}{\partial x}y^2 + \frac{\partial}{\partial x}z^2\right)$$

$$= \frac{-1}{2\sqrt{u^3}}(2x) = \frac{-x}{\sqrt{(x^2+y^2+z^2)^3}}$$

$\frac{\partial}{\partial y}(x^2+y^2+z^2)^{-1/2}$ = por semelhança com a derivada parcial em x, sabe-se que $\frac{\partial}{\partial y} = \frac{-y}{\sqrt{(x^2+y^2+z^2)^3}}$

$\frac{\partial}{\partial z}(x^2+y^2+z^2)^{-1/2}$ = por semelhança com a derivada parcial em x, sabe-se que $\frac{\partial}{\partial z} = \frac{-z}{\sqrt{(x^2+y^2+z^2)^3}}$

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29) $f(x,y,z) = \ln(x+2y+3z)$

$$\frac{\partial}{\partial x}\ln(x+2y+3z) = \frac{\partial}{\partial u}\ln(u)\frac{\partial}{\partial x}(x+2y+3z) \quad | u = x+2y+3z$$

$$= \frac{1}{u}\left(\frac{\partial}{\partial x}x + 2\frac{\partial}{\partial x}y + 3\frac{\partial}{\partial x}z\right)$$

$$= \frac{1}{u}(1) = \frac{1}{x+2y+3z}$$

$$\frac{\partial}{\partial y}\ln(x+2y+3z) = \frac{\partial}{\partial u}\ln(u)\frac{\partial}{\partial y}(x+2y+3z) \quad | u = x+2y+3z$$

$$= \frac{1}{u}\left(\frac{\partial}{\partial y}x + 2\frac{\partial}{\partial y}y + 3\frac{\partial}{\partial y}z\right)$$

$$= \frac{1}{u}(2) = \frac{2}{x+2y+3z}$$

$$\frac{\partial}{\partial z}\ln(x+2y+3z) = \frac{\partial}{\partial u}\ln(u)\frac{\partial}{\partial z}(x+2y+3z) \quad | u = x+2y+3z$$

$$= \frac{1}{u}\left(\frac{\partial}{\partial z}x + 2\frac{\partial}{\partial z}y + 3\frac{\partial}{\partial z}z\right)$$

$$= \frac{1}{u}(3) = \frac{3}{x+2y+3z}$$

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Ferramentas utilizadas para estudo e verificação dos resultados:

- Wolfram Alpha
- James Stewart, Cálculo, 8ª edição.