

Linear Algebra: Foundations to Frontiers (LAFF)

The University of Texas at Austin (UTAustinX)

- **WEEK 0: GETTING STARTED**

- ***Opening Remarks***

- Welcome to LAFF
 - Outline
 - What You Will Learn

- ***How to LAFF***

- When to LAFF
 - How to Navigate LAFF
 - Homework and LAFF
 - Grading and LAFF
 - Programming and LAFF
 - Proving and LAFF
 - Setting Up to LAFF

- ***Software to LAFF***

- Why MATLAB
 - Activating MATLAB Online
 - MATLAB Basics
 - Setting Up MATLAB Online to LAFF

- ***Enrichments***

- The Origins of MATLAB
 - Intrigued by MATLAB and Want to Learn More?

- ***Wrap Up***

- Additional Homework
 - Summary

- **WEEK 1: VECTORS IN LINEAR ALGEBRA**

- ***1.1 Opening Remarks***

- 1.1.1 Takeoff
 - 1.1.2 Outline
 - 1.1.3 What You Will Learn

- **1.2 What is a Vector**
 - 1.2.1 Notation
 - 1.2.2 Unit Basis Vectors
- **1.3 Simple Vector Operations**
 - 1.3.1 Equality (=), Assignment (:=), and Copy
 - 1.3.2 Vector Addition
 - 1.3.3 Scaling
 - 1.3.4 Vector Subtraction
- **1.4 Advanced Vector Operations**
 - 1.4.1 Scaled Vector Addition (AXPY)
 - 1.4.2 Linear Combinations of Vectors
 - 1.4.3 Dot or Inner Product (DOT)
 - 1.4.4 Vector Length (norm2)
 - 1.4.5 Vector Functions
 - 1.4.6 Vector Functions That Map a Vector to a Vector
- **1.5 LAFF Software Package Development: Vectors**
 - 1.5.1 Starting the Package
 - 1.5.2 A Copy Routine (copy)
 - 1.5.3 A Routine That Scales a Vector (scal)
 - 1.5.4 A Scaled Vector Addition Routine (axpy)
 - 1.5.5 An Inner Product Routine (dot)
 - 1.5.6 A Vector Length Routine (norm2)
- **1.6 Slicing and Dicing Vectors**
 - 1.6.1 Slicing and Dicing: Dot Product
 - 1.6.2 Algorithms with Slicing and Dicing: Dot Product
 - 1.6.3 Coding with Slicing and Redicing: Dot Product
 - 1.6.4 Slicing and Dicing: axpy
 - 1.6.5 Algorithms with Slicing and Redicing: axpy
 - 1.6.6 Coding with Slicing and Redicing: axpy
- **1.7 Enrichment**
 - 1.7.1 Learn the Greek Alphabet
 - 1.7.2 Other Norms
 - 1.7.3 Overflow and Underflow
 - 1.7.4 A Bit of History
- **1.8 Wrap Up**
 - 1.8.1 Homework
 - Summary of Vector Operations
 - Summary of the Properties for Vector Operations
 - Summary of the Routines for Vector Operations
- **1.9 Week 1 Proofs**
 - Subsection 1.3

- Subsection 1.4

- **WEEK 2: LINEAR TRANSFORMATIONS AND MATRICES**

- **2.1 Opening Remarks**
 - 2.1.1 Rotations in 2D
 - 2.1.2 Outline Week 2
 - 2.1.3 What you will learn
- **2.2 Linear Transformations**
 - 2.2.1 What makes a linear transformation so special
 - 2.2.2 What is a linear transformation?
 - 2.2.3 Of linear transformations and linear combinations
- **2.3 Mathematical Induction**
 - 2.3.1 What is the Principle of Mathematical Induction?
 - 2.3.2 Examples
- **2.4 Representing Linear Transformations as Matrices**
 - 2.4.1 From Linear Transformation to Matrix-Vector Multiplication
 - 2.4.2 Practice with Matrix-Vector Multiplication
 - 2.4.3 It Goes Both Ways
 - 2.4.4 Rotations and Reflections, Revisited
- **2.5 Enrichment**
 - 2.5.1 The Importance of the Principle of Mathematical Induction for Programming
 - 2.5.2 Puzzles and Paradoxes in Mathematical Induction
- **2.6 Wrap Up**
 - 2.6.1 Additional Homework
 - 2.6.2 Summary
- **2.7 Week 2 Proofs**
 - 2.1 Homekorks
 - 2.2 Homeworks
 - 2.3 Homeworks
 - 2.4 Homewoks

- **WEEK 3: MATRIX-VECTOR OPERATIONS**

- **3.1 Opening Remarks**
 - 3.1.1 Timmy Two Space
 - 3.1.2 Outline Week 3
 - 3.1.3 What You will Learn

- **3.2 Special Matrices**
 - 3.2.1 The Zero Matrix
 - 3.2.2 The Identity Matrix
 - 3.2.3 Diagonal matrices
 - 3.2.4 Triangular Matrices
 - 3.2.5 Transpose Matrix
 - 3.2.6 Symmetric Matrices
- **3.3 Operations with Matrices**
 - 3.3.1 Scaling a Matrix
 - 3.3.2 Adding matrices
- **3.4 Matrix-Vector Multiplication Algorithms**
 - 3.4.1 Via dot products
 - 3.4.2 Via AXPY Operations
 - 3.4.3 Compare and Contrast
 - 3.4.4 Cost of Matrix-vector Multiplication
- **3.5 Wrap Up**
 - 3.5.1 Homework
 - 3.5.2 Summary
- **3.6 Week 3 Proofs**
 - 3.3.1 Videos with Transcripts
 - 3.3.2 Videos with Transcripts
- **WEEK 4: FROM MATRIX-VECTOR TO MATRIX-MATRIX MULTIPLICATION**
 - **4.1 Opening Remarks**
 - 4.1.1 Predicting the Weather
 - 4.1.2 Outline
 - 4.1.3 What you will learn
 - **4.2 Preparation**
 - 4.2.1 Partitioned Matrix-vector Multiplication
 - 4.2.2 Transposing a Partitioned Matrix
 - 4.2.3 Matrix-Vector Multiplication, again...
 - **4.3 Matrix-Vector Multiplication with Special Matrices**
 - 4.3.1 Transpose Matrix-Vector Multiplication
 - 4.3.2 Triangular Matrix-Vector Multiplication
 - 4.3.3 Symmetric Matrix-Vector Multiplication
 - **4.4 Matrix-Matrix Multiplication (Product)**
 - 4.4.1 Motivation
 - 4.4.2 Linear Transformations to Matrix-Matrix Multiplication

- 4.4.3 Computing the Matrix-Matrix Product
 - 4.4.4 Special Shapes
 - 4.4.5 Cost
 - **4.5 *Enrichment***
 - 4.5.1 Markov Chains: Their Application
 - **4.6 *Wrap Up***
 - 4.6.1 Homework
 - 4.6.2 Summary
 - 4.6.3 LAFF Routines (so far)
- **EXAM 1**
 - ***E1.1 How to Review for Exam 1***
 - E1.1.1
 - ***E1.2 Sample Exam 1***
 - E1.2.0 Sample Exam 1 (PDF to print out)
 - E1.2.1 Sample Question 1
 - E1.2.2 Sample Question 2
 - E1.2.3 Sample Question 3
 - E1.2.4 Sample Question 4
 - E1.2.5 Sample Question 5
 - E1.2.6 Sample Question 6
 - E1.2.7 Sample Question 7
 - E1.2.8 Sample Question 8
 - ***E1.3 Real Exam 1***
 - E1.3.0 Exam 1 (PDF to print out)
 - E1.3.1 Exam Question 1
 - E1.3.2 Exam Question 2
 - E1.3.3 Exam Question 3
 - E1.3.4 Exam Question 4
 - E1.3.5 Exam Question 5
 - E1.3.6 Exam Question 6
 - E1.3.7 Exam Question 7
- **WEEK 5: MATRIX-MATRIX MULTIPLICATION**
 - ***5.1 Opening Remarks***
 - 5.1.1 Composing Rotations
 - 5.1.2 Outline
 - 5.1.3 What you will Learn

- **5.2 Observations**
 - 5.2.1 Partitioned Matrix-Matrix Multiplication
 - 5.2.2 Properties
 - 5.2.3 Transposing a Product of Matrices
 - 5.2.4 Matrix-Matrix Multiplication with Special Matrices
- **5.3 Algorithms for Computing Matrix-Matrix Multiplications**
 - 5.3.1 Lots of Loops
 - 5.3.2 Matrix-matrix multiplication by columns
 - 5.3.3 Matrix-matrix multiplication by rows
 - 5.3.4 Matrix-Matrix Multiplication with Rank-1 Updates
- **5.4 Updates**
 - 5.4.1 Slicing and Dicing for Performance
 - 5.4.2 How It is Really Done
- **5.5 Wrap Up**
 - 5.5.1 Homework
 - 5.5.2 Summary
- **5.6 Week 5 Proofs**
 - Proofs with Transcripts Unit 5.2.4
 - Proofs with Transcripts Unit 5.3.2

- **WEEK 6: GAUSSIAN ELIMINATION**

- **6.1 Opening Remarks**
 - 6.1.1 Opening Remarks
 - 6.1.2 Outline
 - 6.1.3 What You will Learn
- **6.2 Gaussian Elimination**
 - 6.2.1 Reducing a System of Linear Equations to an Upper Triangular System
 - 6.2.2 Appended Matrices
 - 6.2.3 Gauss Transforms
 - 6.2.4 Computing Separately with the Matrix and Right-Hand Side (Forward Substitution)
 - 6.2.5 Towards an Algorithm
- **6.3 Solving $Ax = b$ via LU Factorization**
 - 6.3.1 LU factorization (Gaussian elimination)
 - 6.3.2 Solving $Lz = b$ (Forward substitution)
 - 6.3.3 Solving $Ux = b$ (Back substitution)
 - 6.3.4 Putting it all together to solve $Ax = b$
 - 6.3.5 Cost

- **6.4 Enrichment**
 - 6.4.1 Blocked LU Factorization
 - 6.4.2 How ordinary elimination became Gaussian elimination
 - 6.4.3 Formal derivation of LU factorization
 - 6.4.4 High-performance matrix-matrix multiplication, again
- **6.5 Wrap Up**
 - 6.5.1 Homework
 - 6.5.2 Summary
- **WEEK 7: MORE GAUSSIAN ELIMINATION AND MATRIX INVERSION**
 - **7.1 Opening Remarks**
 - 7.1.1 Introduction
 - 7.1.2 Outline
 - 7.1.3 What you will learn
 - **7.2 When Gaussian Elimination Breaks Down**
 - 7.2.1 When Gaussian Elimination Works
 - 7.2.2 The Problem
 - 7.2.3 Permutations
 - 7.2.4 Gaussian Elimination with Row Swapping (LU Factorization with Partial Pivoting)
 - 7.2.5 When Gaussian Elimination Fails Altogether
 - **7.3 The Inverse Matrix**
 - 7.3.1 Inverse Functions in 1D
 - 7.3.2 Back to Linear Transformations
 - 7.3.3 Simple Examples
 - 7.3.4 More Advanced (But Still Simple) Examples
 - 7.3.5 Properties
 - **7.4 Enrichment**
 - 7.4.1 Library Routines for LU with Partial Pivoting
 - **7.5 Wrap Up**
 - 7.5.1 Homework
 - 7.5.2 Summary
- **WEEK 8: MORE ON MATRIX INVERSION**
 - **8.1 Opening Remarks**
 - 8.1.1 When LU Factorization with Row Pivoting Fails
 - 8.1.2 Outline

- 8.1.3 What you will learn
 - **8.2 Gauss-Jordan Elimination**
 - 8.2.1 Solving $Ax = b$ via Gauss-Jordan elimination
 - 8.2.2 Solving $Ax = b$ via Gauss-Jordan Elimination, Gauss Transforms
 - 8.2.3 Solving $Ax = b$ via Gauss-Jordan Elimination: Multiple Right-Hand Sides
 - 8.2.4 Computing the Inverse of A via Gauss-Jordan Elimination
 - 8.2.5 Computing the Inverse of A via Gauss-Jordan Elimination, Alternative
 - 8.2.6 Pivoting
 - 8.2.7 Cost of Matrix Inversion
 - **8.3 (Almost) Never, Ever, Invert a Matrix!**
 - 8.3.1 Solving $Ax = b$
 - 8.3.2 But...
 - **8.4 (Very Important) Enrichment**
 - 8.4.1 Symmetric Positive Definite Matrices
 - 8.4.2 Solving $Ax=b$ when A is Symmetric Positive Definite
 - 8.4.3 Other Factorizations
 - 8.4.4 Welcome to the Frontier
 - **8.5 Wrap Up**
 - 8.5.1 Homework
 - 8.5.2 Summary
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- **EXAM 2**
 - **E2.1 How to Review for Exam 2**
 - E2.1.1 Suggestions
 - **E2.2 Sample Exam 2**
 - E2.2.1 Sample Midterm (PDF to Print Out)
 - E2.2.2 Sample Exam Answers and Videos Questions 1-2
 - E2.2.3 Sample Exam Answers and Videos Questions 3-4
 - E2.2.4 Sample Exam Answers and Videos Questions 5-6
 - E2.2.5 Sample Exam Answers and Videos Questions 7-8
 - E2.2.6 Sample Exam Answers and Videos Questions 9-10
 - E2.2.7 Sample Exam Answers and Videos Questions 11-12
 - **E2.3 Real Exam 2**
 - E2.3.1 Questions 1-2
 - E2.3.2 Questions 3-4
 - E2.3.3 Questions 5-6
 - E2.3.4 Questions 7-8
 - E2.3.5 Question 9

- **WEEK 9: VECTOR SPACES**

- **9.1 Opening Remarks**

- 9.1.1 Solvable or Not Solvable, That's the Question
 - 9.1.2 Outline
 - 9.1.3 What You Will Learn

- **9.2 When Systems Don't Have a Unique Solution**

- 9.2.1 When Solutions Are Not Unique
 - 9.2.2 When Linear Systems Have No Solutions
 - 9.2.3 When Linear Systems Have Many Solutions
 - 9.2.4 What is Going On?
 - 9.2.5 Toward a Systematic Approach to Finding All Solutions

- **9.3 Review of Sets**

- 9.3.1 Definition and Notation
 - 9.3.2 Examples
 - 9.3.3 Operations with Sets

- **9.4 Vector Spaces**

- 9.4.1 What is a Vector Space?
 - 9.4.2 Subspaces
 - 9.4.3 The Column Space
 - 9.4.4 The Null Space

- **9.5 Span, Linear Independence, and Bases**

- 9.5.1 Span
 - 9.5.2 Linear Independence
 - 9.5.3 Bases for Subspaces
 - 9.5.4 The Dimension of a Subspace

- **9.6 Enrichment**

- 9.6.1 Typesetting Algorithms with the FLAME Notation

- **9.7 Wrap Up**

- 9.7.1 Homework
 - 9.7.2 Summary

- **WEEK 10: VECTOR SPACES, ORTHOGONALLITY, AND LINEAR LEAST-SQUARES**

- **10.1 Opening Remarks**

- 10.1.1 Visualizing Planes, Lines, and Solutions
 - 10.1.2 Outline
 - 10.1.3 What You Will Learn

- **10.2 How the Row Echelon Form Answers (Almost) Everything**
 - 10.2.1 Example
 - 10.2.2 The Important Attributes of a Linear System
- **10.3 Orthogonal Vectors and Spaces**
 - 10.3.1 Orthogonal Vectors
 - 10.3.2 Orthogonal Spaces
 - 10.3.3 Fundamental Spaces
- **10.4 Approximating a Solution**
 - 10.4.1 A Motivating Example
 - 10.4.2 Finding the Best Solution
 - 10.4.3 Why it is Called Linear Least-squares
- **10.5 Enrichment**
 - 10.5.1 Solving the Normal Equations
- **10.6 Wrap Up**
 - 10.6.1 Homework
 - 10.6.2 Summary
- **WEEK 11: ORTHOGONAL PROJECTION, LOW RANK APPROXIMATION, AND ORTHOGONAL BASES**
 - **11.1 Opening Remarks**
 - 11.1.1 Low Rank Approximation
 - 11.1.2 Outline
 - 11.1.3 What You Will Learn
 - **11.2 Projecting a Vector onto a Subspace**
 - 11.2.1 Component in the Direction of ...
 - 11.2.2 An Application: Rank-1 Approximation
 - 11.2.3 Projection onto a Subspace
 - 11.2.4 An Application: Rank-2 Approximation
 - 11.2.5 An Application: Rank-k Approximation
 - **11.3 Orthonormal Bases**
 - 11.3.1 The Unit Basis Vectors, Again
 - 11.3.2 Orthonormal Vectors
 - 11.3.2 Orthonormal Vectors (Continued)
 - 11.3.3 Orthogonal Bases
 - 11.3.4 Orthogonal Bases (Alternative Explanation)
 - 11.3.5 The QR Factorization
 - 11.3.6 Solving the Linear Least-Squares Problem via QR Factorization
 - 11.3.7 The QR Factorization (Again)

- **11.4 Change of Bases**
 - 11.4.1 The Unit Basis Vectors, One More Time
 - 11.4.2 Change of Basis
- **11.5 Singular Value Decomposition**
 - 11.5.1 The Best Low Rank Approximation
- **11.6 Enrichment**
 - 11.6.1 The Problem with Computing the QR Factorization
 - 11.6.2 QR Factorization Via Householder Transformations (Reflections)
 - 11.6.3 More on SVD
- **11.7 Wrap Up**
 - 11.7.1 Homework
 - 11.7.2 Summary

- **WEEK 12: EIGENVALUES AND EIGENVECTORS**

- **12.1 Opening Remarks**
 - 12.1.1 Predicting the Weather, Again
 - 12.1.2 Outline
 - 12.1.3 What You Will Learn
- **12.2 Getting Started**
 - 12.2.1 The Algebraic Eigenvalue Problem
 - 12.2.2 Simple Examples
 - 12.2.2 Simple Examples (continued)
 - 12.2.3 Diagonalization
 - 12.2.4 Eigenvalues and Vectors of 3 x 3 Matrices
- **12.3 The General Case**
 - 12.3.1 Eigenvalues and Eigenvectors of $n \times n$ Matrices: Special Cases
 - 12.3.2 Eigenvalues of $n \times n$ Matrices
 - 12.3.3 Diagonalization, Again
 - 12.3.4 Properties
- **12.4 Practical Methods**
 - 12.4.1 Predicting the Weather, One Last Time
 - 12.4.2 The Power Method
 - 12.4.3 In Preparation for this Week's Enrichment
- **12.5 Enrichment**
 - 12.5.1 The Inverse Power Method
 - 12.5.2 The Rayleigh Quotient Iteration
 - 12.5.3 More Advanced Techniques

- **12.6 Wrap Up**
 - 12.6.1 Homework
 - 12.6.2 Summary

- **FINAL EXAM**
 - ***F.1 How to Review for the Final***
 - F.1.1 Suggestions

 - ***F.2 Sample Final Exam***
 - F.2.1 Sample Final (PDF to Print Out)
 - F.2.2 Sample Exam Answers and Videos Questions 1-2
 - F.2.3 Sample Exam Answers and Videos Questions 3-4
 - F.2.4 Sample Exam Answers and Videos Questions 5-6
 - F.2.5 Sample Exam Answers and Videos Questions 7-8
 - F.2.6 Sample Exam Answers and Videos Questions 9

 - ***F.3 Real Final Exam***
 - F.3.1 Final Questions 1-2
 - F.3.2 Final Questions 3-4
 - F.3.3 Final Questions 5-6
 - F.3.4 Final Questions 7-8

 - ***F.4 Congratulations! (and Further Reading!!!)***
 - F.4.1 Congratulations!
 - F.4.2 Some Recommended Materials for Further Learning
 - F.4.3 You are Ready for the Cutting Edge Research
 - F.4.4 Write a Review!