18.01, September 12, 2003 Lecture Notes
<u>Practice Problems</u>
1G-3, 1G-4, 1H-1, 1H-3, 1H-5

1. Higher derivatives, notations:

$\frac{d^k}{dx^k}\upsilon = \frac{d}{dx}(...(\frac{d\upsilon}{dx})...)$, k copies of $\frac{d}{dx}$, $\upsilon' = \frac{d\upsilon}{dx}$, $\upsilon'' = \frac{d^2\upsilon}{dx^2}$, $\upsilon''' = \frac{d^3\upsilon}{dx^3}$, but generally

$\upsilon^{(k)} = \frac{d^k}{dx^k}\upsilon$ (simpler than $\upsilon^{\prime ... \prime}$ $,^{\prime ... \prime}$ k times).

2. General formulas: $\frac{d^k}{dx^k}(x'') = n$ (n-1)…(n-k+1)x^{n-t}

In fact if n is a general fraction! (Proof by induction on b.)

If n is a positive integer = $\frac{n!}{(n-k)!}$ x^{n-t}

If n=-m, = $(-1)^m \frac{(m+k-1)!}{(m-1)!}\frac{1}{x^{m+k}}$ DIDN'T WRITE THIS FORMULA!!

DID WRITE: $\frac{d^k}{dx^k}\frac{1}{x+q} = \frac{(-1)^k k!}{(x+q)^{k+1}}$

<u>LEIBNITZ FORMULA</u>: $(UV)^{(n)} = U^{(n)}\bullet V + nU^{(n-1)}V^{(1)} + ... + \binom{n}{k}U^{(n-k)}\bullet V^{(k)} + ... + UV^{(n)}$

<u>Exponentials</u>: $\alpha^x := \lim_{e \to x} \alpha^e$ where $\alpha^{\frac{\rho}{2}} = (\sqrt[2]{\alpha})^\rho$, $\alpha > 1, \lim_{x \to -\infty} \alpha^x = 0, \lim_{x \to \infty} \alpha^x = +\infty$.

Should α^x increasing (α>1!!) and <u>claimed</u> α^x continuous $\Rightarrow \forall y > 0, \exists x$ s.t. y=α^x .
Define $x := \log_\alpha(y)$

$$\begin{cases} \alpha^0 = 1 \\ \alpha^1 = \alpha \\ \alpha^x \alpha^y = \alpha^{x+y} \\ (\alpha^x)^y = \alpha^{(xy)} \end{cases}$$

$$\Updownarrow$$

$\log_\alpha(1) = 0$

$\log_\alpha(a) = 1$

$\log_\alpha(xy) = \log_\alpha(x) + \log_\alpha(y)$

$\log_\alpha(x^y) = y\log_\alpha(x)$

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

(1) $\log(x) = \log_{10}(x)$ *common* log

(2) $\ln(x) = log_e(x)$ *natural* log

Mention $\frac{d}{dx}(\alpha^x) = ??\bullet\alpha^x$ and ??=1 for <u>e</u>.