MATH 18.01, FALL 2017 - PROBLEM SET # 5

Professor: Jared Speck

Due: by Friday 1:45pm on 10-27-17
(in the boxes outside of Room 4-174; write your name, recitation instructor, and recitation meeting days/time on your homework)

18.01 Supplementary Notes (including Exercises and Solutions) are available on the course web page: `http://math.mit.edu/~jspeck/18.01_Fall%202017/1801_CourseWebsite.html`. This is where to find the exercises labeled 1A, 1B, etc. You will need these to do the homework.

Part I consists of exercises given and solved in the Supplementary Notes. It will be graded quickly, checking that all is there and the solutions not copied.

Part II consists of problems for which solutions are not given; it is worth more points. Some of these problems are longer multi-part exercises given here because they do not fit conveniently into an exam or short-answer format. See the guidelines below for what collaboration is acceptable, and follow them.

To encourage you to keep up with the lectures, both Part I and Part II tell you for each problem on which day you will have the needed background for it.

You are encouraged to use graphing calculators, software, etc. to <u>check</u> your answers and to explore calculus. However, (unless otherwise indicated) we strongly discourage you from using these tools to <u>solve</u> problems, perform computations, graph functions, etc. An extremely important aspect of learning calculus is developing these skills. You will not be allowed to use any such tools on the exams.

Part I (30 points)

Notation: The problems come from three sources: the Supplementary Notes, the Simmons book, and problems that are described in full detail inside of this pset. I refer to the former two sources using abbreviations such as the following ones: 2.1 = Section 2.1 of the Simmons textbook; Notes G = Section G of the Supplementary Notes; Notes 1A: 1a, 2 = Exercises 1a and 2 in the Exercise Section 1A of the Supplementary Notes; Section 2.4: 13 = Problem 13 in Section 2.4 of Simmons, etc.

Lecture 13. (Thurs., Oct. 12) Differentials and antiderivatives.
  Read: 5.2, 5.3.
  Homework: Notes 3A: 1de, 2acegik, 3aceg.

Material beyond Lecture 13 will not appear on Midterm # 2

Lecture 14. (Fri., Oct. 13) Differential equations; separating variables. Inequalities.
  Read: 5.4, 8.5.
  Homework: Notes 3F: 1cd, 2ae, 4bcd, 8b.

Lecture 15. (Tuesday, Oct. 17) Definite integrals; summation notation.
  Read: 6.3 through formula (4); skip proofs; 6.4, 6.5.
  Homework: Notes 3B: 2ab, 3b, 4a, 5.

Lecture 16. (Tues., Oct. 24) First fundamental theorem; properties of integrals.
  Read: 6.6, 6.7.
  Homework: Notes 3C: 1, 2a, 3a, 5a; Notes 3E: 6bc; Notes 4J: 2.

Part II (50 points)

Directions and Rules: Collaboration on problem sets is encouraged, but:

i) Attempt each part of each problem yourself. Read each portion of the problem before asking for help. If you don't understand what is being asked, ask for help interpreting the problem and then make an honest attempt to solve it.

ii) Write up each problem independently. On both Part I and II exercises you are expected to write the answer in your own words. *You must show your work; "bare" solutions will receive very little credit.*

iii) Write on your problem set whom you consulted and the sources you used. If you fail to do so, you may be charged with plagiarism and subject to serious penalties.

iv) It is illegal to consult materials from previous semesters.

0. (not until due date; 3 points) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation." This includes visits outside recitation to your recitation instructor. If you don't know a name, you must nevertheless identify the person, as in, "tutor in Room 2-106," or "the student next to me in recitation." Optional: note which of these people or resources, if any, were particularly helpful to you.

This "Problem **0**" will be assigned with every problem set. Its purpose is to make sure that you acknowledge (to yourself as well as others) what kind of help you require and to encourage you to pay attention to how you learn best (with a tutor, in a group, alone). It will help us by letting us know what resources you use.

1. (Oct. 12; Antiderivatives; $2 + 2 + 3 = 7$ points) Recall that in pset 3A, you showed that $(d/dx) \tanh^{-1} x = \dfrac{1}{1 - x^2}$. Here, $\tanh^{-1}(x)$ denotes the inverse to the hyperbolic tangent function.

a) Find the two constants A and B such that
$$\frac{1}{1 - x^2} = \frac{A}{1 + x} + \frac{B}{1 - x}.$$
This is called the *method of partial fractions.* We will discuss this method in much more detail later in the course.

b) Use part a) to compute
$$\int \frac{1}{1 - x^2} \, dx.$$
Simplify your answer as much as possible.

c) Explain the connection between the function $\tanh^{-1}(x)$ and your answer to part b).

2. (Oct. 12; Antiderivatives; 6 points) Suppose that $E(x) = \int e^{-x^2} \, dx$. That is, suppose that $E(x)$ is a function such that $E'(x) = e^{-x^2}$. *Remark: $E(x)$ cannot be expressed in terms of simpler functions that you already know. Hence, you should think of $E(x)$ as a "new function."* Express the antiderivative
$$\int x e^{-x^4 - 2x^2 - 1} \, dx$$
in terms of the function E and polynomials.

3. (Oct. 13; Differential equations and separation of variables; $2 + 1 + 4 = 7$ points) Section 5.4: 20, Section 8.5: 8, pg. 291 problem 54ab; find the "blow-up" time t_0 in part b.

4. (Oct. 13; Differential equations; $6 + 4 + 2 = 12$ points) In this problem, you will learn about terminal velocity. Consider an object of mass m dropped (straight down) off of a very tall cliff. Its downward velocity is initially 0. Let $v(t)$ denote its *downward* velocity at time t (i.e. $v > 0$ means the object is falling towards the earth). In a very simple model of air resistance, the air provides a drag force that is proportional to v^2. That is, the air exerts an *upward* force of magnitude κv^2 on the object, where $\kappa > 0$ is a constant (which has dimensions of $\frac{\text{mass}}{\text{length}}$). Let $g > 0$ denote the downward acceleration of gravity. For the purposes of this problem, we can assume that g is a constant (approximately equal to 9.8 meter/sec^2). Under these assumptions, Newton's laws of motion imply that

$$(1) \qquad\qquad \frac{d}{dt}v = -\frac{\kappa}{m}v^2 + g.$$

a) Use separation of variables to solve the differential equation with the initial condition $v(0) = 0$. *Hint: A modified version of the method of partial fractions from problem 1 might be useful...*

b) Does $\lim_{t\to\infty} v(t)$ exist? If so, explain what happens when you plug the limiting velocity into the right-hand side of the differential equation (1) and why the result makes sense.

c) Explain how your answer to part b) changes if the object is thrown off the cliff (straight down) with initial velocity $v_0 > 0$.

5. (Oct. 17; Definite integrals; $1 + 1 + 1 + 5 = 8$ points) First do Section 6.3: 9. Then use this problem to help you calculate $\int_0^{\pi/2} \cos x \, dx$ by using right Riemann sums. By a "right Riemann sum," we mean that the right endpoint of the approximating rectangles lies on the graph of $y = \cos x$.

6. (Oct. 24; First fundamental theorem; properties of integrals; 4 (for finding the equation using geometry) + 3 (for checking the equation using integrals) $= 7$ points) Section 6.7: 12