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Disciplina: Cálculo III
Professor: Kennedy
Exercício: Atividade Avaliativa

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1a) $f(x,y) = \tan(xy)$

a) Cálculo de f_x:

$$\frac{\partial}{\partial x}\tan(xy) =$$

Pela regra da cadeia, $u = xy$, temos:

$$= \frac{\partial}{\partial u}\tan(u)\,\frac{\partial}{\partial x}(xy) = \sec^2(u)\,y\,\frac{\partial}{\partial x}x = \boxed{y\sec^2(xy)}$$

b) Cálculo de f_y:

$$\frac{\partial}{\partial y}\tan(xy) = \frac{\partial}{\partial u}\tan(u)\,\frac{\partial}{\partial y}(xy) = \sec^2(u)\,x\,\frac{\partial}{\partial y}y = \boxed{x\sec^2(xy)}$$

1b) $f(x,\theta) = \sqrt{x}\ln(\theta)$

a) Cálculo de f_x:

$$\frac{\partial}{\partial x}\sqrt{x}\ln(\theta) = \ln(\theta)\frac{\partial}{\partial x}x^{1/2} + \cancel{\sqrt{x}\frac{\partial}{\partial x}\ln(\theta)}^{0} = \boxed{\frac{\ln(\theta)}{2\sqrt{x}}}$$

b) Cálculo de f_θ:

$$\frac{\partial}{\partial \theta}\sqrt{x}\ln(\theta) = \cancel{\ln(\theta)\frac{\partial}{\partial \theta}\sqrt{x}}^{0} + \sqrt{x}\frac{\partial}{\partial \theta}\ln(\theta) = \boxed{\frac{\sqrt{x}}{\theta}}$$

②

2a) $f(\alpha,\beta,\gamma) = \dfrac{\beta}{\alpha+\beta+\gamma}$, $f_\beta(2,1,-1)$

$$\frac{\partial}{\partial\beta}\left(\frac{\beta}{\alpha+\beta+\gamma}\right) = \frac{(\alpha+\beta+\gamma)\frac{\partial}{\partial\beta}\beta - \beta\frac{\partial}{\partial\beta}(\alpha+\beta+\gamma)}{(\alpha+\beta+\gamma)^2}$$

$$= \frac{(\alpha+\beta+\gamma)-\beta}{(\alpha+\beta+\gamma)^2} = \frac{\alpha+\gamma}{(\alpha+\beta+\gamma)^2}$$

$$\therefore f_\beta(2,1,-1) = \frac{2-1}{(2+1-1)^2} = \boxed{\frac{1}{4}}$$

2b) $f(x,y) = x^3y^3 + x^2y + xy^2 + 10$, $f_x(2,1)$

$$\frac{\partial}{\partial x}(x^3y^3 + x^2y + xy^2 + 10) = y^3\frac{\partial}{\partial x}x^3 + y\frac{\partial}{\partial x}x^2 + y^2\frac{\partial}{\partial x}x + \frac{\partial}{\partial x}10$$

$$= 3y^3x^2 + 2yx + y^2$$

$$\therefore f_x(2,1) = 3(1^3)(2^2) + 2(1)(2) + 1^2$$
$$= 3(4) + 4 + 1$$
$$= 12 + 5$$
$$= \boxed{17}$$

3a) $f(x,y) = \dfrac{xy}{x-y}$

Cálculo de f_x:

$$\frac{\partial}{\partial x}\left(\frac{xy}{x-y}\right) = \frac{(x-y)\frac{\partial}{\partial x}xy - xy\frac{\partial}{\partial x}(x-y)}{(x-y)^2}$$

$$= \frac{(x-y)\,y\frac{\partial}{\partial x}x - xy\left(\frac{\partial}{\partial x}x - \frac{\partial}{\partial x}y\right)}{(x-y)^2}$$

$$= \frac{y(x-y) - xy}{(x-y)^2} = \frac{\cancel{xy} - y^2 - \cancel{xy}}{(x-y)^2} = \boxed{\frac{-y^2}{(x-y)^2} = f_x}$$

Cálculo de f_y:

$$\frac{\partial}{\partial y}\left(\frac{xy}{x-y}\right) = \frac{(x-y)\frac{\partial}{\partial y}xy - xy\frac{\partial}{\partial y}(x-y)}{(x-y)^2}$$

$$= \frac{(x-y)\,x\frac{\partial}{\partial y}y - xy\left(\frac{\partial}{\partial y}x - \frac{\partial}{\partial y}y\right)}{(x-y)^2}$$

$$= \frac{x(x-y) - xy(-1)}{(x-y)^2} = \frac{x^2 - \cancel{xy} + \cancel{xy}}{(x-y)^2} = \boxed{\frac{x^2}{(x-y)^2} = f_y}$$

④

Cálculo de f_{xx}:

$$\frac{\partial}{\partial x}\left\{\frac{-y^2}{(x-y)^2}\right\} = \frac{\partial}{\partial x} -y^2(x-y)^{-2}$$

$$= \cancel{(x-y)^{-2}\frac{\partial}{\partial x}(-y^2)}^{0} + (-y^2)\frac{\partial}{\partial x}(x-y)^{-2}$$

$$= -y^2\left[\frac{\partial}{\partial u}u^{-2}\,\frac{\partial}{\cancel{\partial x}}(\cancel{x}-y)^{1}\right] = -y^2\left[\frac{-2}{u^3}\right] = \boxed{\frac{2y^2}{(x-y)^3} = f_{xx}}$$

Cálculo de f_{yy}:

$$\frac{\partial}{\partial y}\left\{\frac{x^2}{(x-y)^2}\right\} = \frac{\partial}{\partial y} x^2(x-y)^{-2}$$

$$= \cancel{(x-y)^{-2}\frac{\partial}{\partial y}x^2}^{0} + x^2\frac{\partial}{\partial y}(x-y)^{-2}$$

$$= x^2\left[\frac{\partial}{\partial u}u^{-2}\,\frac{\partial}{\cancel{\partial y}}(x\cancel{-y})^{-1}\right] = x^2\left[\frac{-2\,(-1)}{u^3}\right] = \boxed{\frac{2x^2}{(x-y)^3} = f_{yy}}$$

Cálculo de f_{yx}:

$$\frac{\partial}{\partial x}\left\{\frac{x^2}{(x-y)^2}\right\} = \frac{\partial}{\partial x} x^2(x-y)^{-2}$$

$$= (x-y)^{-2}\frac{\partial}{\partial x}x^2 + x^2\frac{\partial}{\partial x}(x-y)^{-2}$$

$$= 2x(x-y)^{-2} + x^2\left[\frac{\partial}{\partial u}u^{-2}\,\frac{\partial}{\cancel{\partial x}}(\cancel{x}-y)^{1}\right]$$

$$= 2x(x-y)^{-2} + x^2\left[\frac{-2}{u^3}\right]$$

$$= 2x(x-y)^{-2} - \frac{2x^2}{(x-y)^3}$$

$$= \frac{2x}{(x-y)^2} - \frac{2x^2}{(x-y)^3} = \frac{2x(x-y) - 2x^2}{(x-y)^3}$$

$$= \frac{\cancel{2x^2} - 2xy - \cancel{2x^2}}{(x-y)^3} = \boxed{-\frac{2xy}{(x-y)^3} = f_{yx} = f_{xy}}$$

3b) $f(x,y) = \sin(x^2y)$

Cálculo de f_x:

$$\frac{\partial}{\partial x}\sin(x^2y) = \frac{\partial}{\partial u}\sin(u)\frac{\partial}{\partial x}(x^2y) = \cos(u)\cdot y2x = \boxed{\overset{f_x =}{2xy\cos(x^2y)}}$$

Cálculo de f_y:

$$\frac{\partial}{\partial y}\sin(x^2y) = \frac{\partial}{\partial u}\sin(u)\frac{\partial}{\partial y}(x^2y) = \cos(u)x^2 = \boxed{x^2\cos(x^2y) = f_y}$$

Cálculo de f_{xx}:

$$\frac{\partial}{\partial x}2xy\cos(x^2y) = 2y\frac{\partial}{\partial x}x\cos(x^2y)$$

$$= 2y\left\{\cos(x^2y)\frac{\partial}{\partial x}x + x\frac{\partial}{\partial x}\cos(x^2y)\right\}$$

$$= 2y\left[\cos(x^2y) + x\left(\frac{\partial}{\partial u}\cos(u)\frac{\partial}{\partial x}x^2y\right)\right\}$$

$$= 2y\left[\cos(x^2y) + x(-\sin(x^2y)\,2yx)\right]$$

$$= 2y(\cos(x^2y) + x(-2xy\sin(x^2y)))$$

$$= 2y(\cos(x^2y) - 2x^2y\sin(x^2y))$$

$$= \boxed{2y\cos(x^2y) - 4x^2y^2\sin(x^2y) = f_{xx}}$$

Cálculo de f_{yy}:

$$\frac{\partial}{\partial y}(x^2\cos(x^2y)) = \cancelto{0}{\cos(x^2y)\frac{\partial}{\partial y}x^2} + x^2\frac{\partial}{\partial y}\cos(x^2y)$$

$$= x^2\left(\frac{\partial}{\partial u}\cos(u)\frac{\partial}{\partial y}(x^2y)\right) = x^2(-\sin(u)x^2)$$

$$= \boxed{-x^4\sin(x^2y) = f_{yy}}$$

Cálculo de f_{yx}:

$$\frac{\partial}{\partial x}(x^2\cos(x^2y)) = \cos(x^2y)\frac{\partial}{\partial x}x^2 + x^2\frac{\partial}{\partial x}\cos(x^2y)$$

$$= 2x\cos(x^2y) + x^2\left[\frac{\partial}{\partial u}\cos(u)\frac{\partial}{\partial x}(x^2y)\right]$$

$$= 2x\cos(x^2y) + x^2(-\sin(x^2y)\,2xy) = 2x\cos(x^2y) - 2x^3y\sin(x^2y)$$

$$= \boxed{2x(\cos(x^2y) - x^2y\sin(x^2y)) = f_{yx} = f_{xy}}$$

4a) $f(x,y) = x^{10}y + 5xy^2 + 1$

$$\frac{\partial}{\partial x}(x^{10}y + 5xy^2 + 1) = 10x^9y + 5y^2$$

$$\frac{\partial}{\partial y}(x^{10}y + 5xy^2 + 1) = x^{10} + 10xy$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{\partial}{\partial x}(x^{10} + 10xy) = 10x^9 + 10y$$

$$\boxed{f_{xy} = f_{yx}}$$

$$\frac{\partial^2}{\partial y \partial x} = \frac{\partial}{\partial y}(10x^9y + 5y^2) = 10x^9 + 10y$$

4b) $f(x,y) = \ln(x+2y)$

$$\frac{\partial}{\partial x}\ln(x+2y) = \frac{\partial}{\partial u}\ln(u)\,\frac{\partial}{\partial x}(x+2y) = \frac{1}{u} = \frac{1}{x+2y}$$

$$\frac{\partial}{\partial y}\ln(x+2y) = \frac{\partial}{\partial u}\ln(u)\,\frac{\partial}{\partial y}(x+2y) = \frac{1}{u}\cdot 2 = \frac{2}{x+2y}$$

$$\frac{\partial^2}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{1}{x+2y}\right) = \frac{\partial}{\partial y}(x+2y)^{-1} = \frac{\partial}{\partial u}u^{-1}\,\frac{\partial}{\partial y}(x+2y)$$

$$= \frac{-1}{u^2}\cdot 2 = \frac{-2}{(x+2y)^2}$$

$$\boxed{f_{xy} = f_{yx}}$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{2}{x+2y}\right) = 2\frac{\partial}{\partial x}(x+2y)^{-1} = 2\left(\frac{\partial}{\partial u}u^{-1}\,\frac{\partial}{\partial x}(x+2y)\right)$$

$$= 2\left(\frac{-1}{(x+2y)^2}\right) = \frac{-2}{(x+2y)^2}$$

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$f(x,y,z)=\sqrt{\sin^2(x)+\sin^2(y)+\sin^2(z)}$, $f_z(0,0,\pi/4)$ = ?

$\frac{\partial}{\partial z}\left(\sin^2(x)+\sin^2(y)+\sin^2(z)\right)^{1/2} =$

$\frac{\partial}{\partial u} u^{1/2} \frac{\partial}{\partial z}\left(\sin^2(x)+\sin^2(y)+\sin^2(z)\right)$

$= \frac{1}{2\sqrt{\sin^2(x)+\sin^2(y)+\sin^2(z)}}\left(\frac{\partial}{\partial z}(\sin(z))^2\right)$

$= \frac{1}{2\sqrt{\sin^2(x)+\sin^2(y)+\sin^2(z)}}\left(\frac{\partial u^2}{\partial u}\frac{\partial \sin(z)}{\partial z}\right)$

$= \frac{2\sin(z)\cos(z)}{2\sqrt{\sin^2(x)+\sin^2(y)+\sin^2(z)}} = \frac{\sin(z)\cos(z)}{\sqrt{\sin^2(x)+\sin^2(y)+\sin^2(z)}}$

$\therefore f_z(0,0,\pi/4) = \frac{\sin(\pi/4)\cos(\pi/4)}{\sqrt{\sin^2(\pi/4)}} = \frac{\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}}{\sin(\pi/4)} = \frac{\frac{1}{\sqrt{2}}\cdot\cancel{\frac{1}{\sqrt{2}}}}{\cancel{\frac{1}{\sqrt{2}}}} = \boxed{\frac{1}{\sqrt{2}}}$

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Ferramentas utilizadas para o estudo e resolução:

- Calculadora HP-50G
- WolframAlpha