MATH 18.01 - MIDTERM 1 REVIEW: SUMMARY OF SOME KEY CONCEPTS (WARNING: THERE MAY BE TYPOS, SO YOU SHOULD CHECK EVERYTHING ON YOUR OWN!!)

18.01 Calculus, Fall 2014 Professor: Jared Speck

a. Ways of thinking about derivatives
  (a) Analytic definition:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

  (b) Geometric interpretation: $f'(x_0) =$ slope of tangent line to the graph of f at $(x_0, f(x_0))$
  (c) $\frac{dy}{dx} =$ instantaneous rate of change of y with respect to x

b. Tangent lines:
  (a) $y - f(x_0) = f'(x_0)(x - x_0)$

c. Derivative rules (know how to prove them)
  (a) Sum: $(u + v)' = u' + v'$
  (b) Constant multiple: $(cu)' = cu'$ if c is a constant
  (c) Product: $(uv)' = uv' + u'v$
  (d) Quotient: $(u/v)' = (u'v - uv')/v^2$
  (e) Chain:

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x),$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

d. Limits including how to deduce them (here are some important examples)
  (a) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$
  (b) $\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 0$
  (c) $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta^2} = -\frac{1}{2}$
  (d) $\lim_{k \to \infty} (1 + \frac{1}{k})^k = e$

e. Continuity
  (a) Analytic definition: $\lim_{\Delta_x \to 0} f(x + \Delta x) = f(x)$
  (b) Jump discontinuities
  (c) Removable discontinuities
  (d) Discontinuities that are neither jumps nor removable
  (e) Differentiable $\implies$ continuous (know how to prove this)

f. Derivatives of elementary functions including how to deduce the formulas (here are some examples):
(a) $\frac{d}{dx} \sin x = \cos x$
(b) $\frac{d}{dx} \cos x = -\sin x$
(c) $\frac{d}{dx} \tan x = \sec^2 x$
(d) $\frac{d}{dx} x^r = rx^{r-1}$
(e) $\frac{d}{dx} e^x = e^x$
(f) $\frac{d}{dx} a^x = (\ln a)a^x$
(g) $\frac{d}{dx} \ln x = \frac{1}{x}$
(h) $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$
(i) $\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$
(j) $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$
(k) $\frac{d}{dx} \sinh x = \cosh x$
(l) $\frac{d}{dx} \cosh x = \sinh x$

g. Function inverses
(a) $f(f^{-1}(x)) = x$
(b) $f^{-1}(f(x)) = x$
(c) If $y = f(x)$ and $x = f^{-1}(y)$, then $\frac{d}{dy} f^{-1}(y) = \frac{1}{\frac{d}{dx} f(x)} = \frac{1}{f'(x)}$
(d) The graph of f^{-1} is the reflection of the graph of f through the line $y = x$
(e) Example: $\ln x$ and e^x are inverses of each other

h. Logarithmic differentiation
(a) Main point: if $y = f(x)$, then sometimes $\ln y$ is easier to differentiate than y
(b) If $y = f(x)$, then $\frac{d}{dx} \ln y = \frac{y'}{y}$