18.01, October 2, 2003 Newton's method and the Mean Value theorem

2F-3, 2F-6, 2G-3, 2G-7

1. Exlained Newton's method for finding approximate values of the zeros of a function
$x_{n+1} = x_n - f(x_n) / f'(x_n)$

Example 1: $f(x) = x^2 - 4x + 1$. Approx x_1=4. After four iterations get that one zero is x=3.7320 ± 0.0001. Compare to actual answer $2+\sqrt{3}$, and point out other zero and dependence of algorithm on starting point.

Example 2: Numerical computation related to cookie-cutter prob. From max/min lecture: Find θ with $0 \le \theta \le \frac{\pi}{2}$ such that $\tan(\theta) = \theta + 1$. Explained why bad to use $f(\theta) = \tan(\theta) - (\theta + 1)$ (blowup at $\theta = \frac{\pi}{2}$), so used $f(\theta) = \sin(\theta) - (\theta + 1)\cos(\theta)$

Approx. $\theta_1 = \frac{\pi}{3}$. After 3 iterations, $\theta = 1.13227 \pm 0.00001.$ Point out, no exact method of solution –Newton's method is best we round.

2. M.V.T.: If f(x) cts. +diff on [a,b], then these exists $c \in (a,b)$ such that $f'(c) = \frac{f(b) - f(a)}{b-a}$, i.e. tgt line to y=f(x) at c is parallel to serant line thru (a,f(a)), (b,f(b))

Reduced to Extreme Value Theorem: f(x) achieves max/min on [a,b]. Also mentioned the Intermediate Value Theorem: Give applic. to error analysis: If $|f'(c) - f'(a)| < M$ on [a,b], then for all $x \in [a,b]$, $|f(x) - (f(a) + f'(a)(x-a))| < M \bullet |x-a|$. "Error of lines approx."