

MATH 18.01, FALL 2017 - PROBLEM SET # 6

Professor: Jared Speck

Due: by Friday 1:45pm on 11-03-17

(in the boxes outside of Room 4-174; write your name, recitation instructor, and recitation meeting days/time on your homework)

18.01 Supplementary Notes (including Exercises and Solutions) are available on the course web page: http://math.mit.edu/~jspeck/18.01_Fall%202017/1801_CourseWebsite.html. This is where to find the exercises labeled 1A, 1B, etc. You will need these to do the homework.

Part I consists of exercises given and solved in the Supplementary Notes. It will be graded quickly, checking that all is there and the solutions not copied.

Part II consists of problems for which solutions are not given; it is worth more points. Some of these problems are longer multi-part exercises given here because they do not fit conveniently into an exam or short-answer format. See the guidelines below for what collaboration is acceptable, and follow them.

To encourage you to keep up with the lectures, both Part I and Part II tell you for each problem on which day you will have the needed background for it.

You are encouraged to use graphing calculators, software, etc. to check your answers and to explore calculus. However, (unless otherwise indicated) we strongly discourage you from using these tools to solve problems, perform computations, graph functions, etc. An extremely important aspect of learning calculus is developing these skills. You will not be allowed to use any such tools on the exams.

Part I (20 points)

Notation: The problems come from three sources: the Supplementary Notes, the Simmons book, and problems that are described in full detail inside of this pset. I refer to the former two sources using abbreviations such as the following ones: 2.1 = Section 2.1 of the Simmons textbook; Notes G = Section G of the Supplementary Notes; Notes 1A: 1a, 2 = Exercises 1a and 2 in the Exercise Section 1A of the Supplementary Notes; Section 2.4: 13 = Problem 13 in Section 2.4 of Simmons, etc.

Lecture 17. (Thurs., Oct. 26) Second Fundamental Theorem. New definition of $\ln(x)$.

Read: Notes PI, p.2 [eqn.(7) and example]; Notes FT.

Homework: 3E: 1, 3a; Notes 3D: 1, 4bc, 5, 8a; Notes 3E: 2ac.

Lecture 18. (Fri., Oct. 27) Areas between curves. Volumes by slicing.

Read: 7.1, 7.2, 7.3.

Homework: Notes 4A: 1b, 2, 4; Notes 4B: 1de, 6, 7.

Lecture 19. (Tues., Oct. 31) Volumes by disks and shells.

Read: 7.4.

Homework: Notes 4B: 2eg, 5; Notes 4C: 1, 2, 3; Notes 4J: 3.

Part II (50 points)

Directions and Rules: Collaboration on problem sets is encouraged, but:

i) Attempt each part of each problem yourself. Read each portion of the problem before asking for help. If you don't understand what is being asked, ask for help interpreting the problem and then make an honest attempt to solve it.

ii) Write up each problem independently. On both Part I and II exercises you are expected to write the answer in your own words. *You must show your work; "bare" solutions will receive very little credit.*

iii) Write on your problem set whom you consulted and the sources you used. If you fail to do so, you may be charged with plagiarism and subject to serious penalties.

iv) It is illegal to consult materials from previous semesters.

0. (not until due date; 3 points) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation." This includes visits outside recitation to your recitation instructor. If you don't know a name, you must nevertheless identify the person, as in, "tutor in Room 2-106," or "the student next to me in recitation." Optional: note which of these people or resources, if any, were particularly helpful to you.

This "Problem 0." will be assigned with every problem set. Its purpose is to make sure that you acknowledge (to yourself as well as others) what kind of help you require and to encourage you to pay attention to how you learn best (with a tutor, in a group, alone). It will help us by letting us know what resources you use.

1. (Thurs., Oct. 26; Second Fundamental Theorem; $3 + 2 + 2 + 3 + 3 + 3 = 16$ points) Let $\text{sinc}(x)$ denote the "sinc" function

$$\text{sinc}(x) = \begin{cases} 1 & \text{if } x = 0, \\ \frac{\sin x}{x} & \text{if } x \neq 0. \end{cases}$$

Now consider the "sine integral" function

$$\text{Si}(x) = \int_0^x \text{sinc}(t) dt.$$

Both of these functions frequently come up in Fourier analysis and signal processing and hence have been given their own names. *Remark: $\text{Si}(x)$ cannot be expressed in terms of standard elementary functions.*

a) Compute $\text{Si}'(x)$ and $\text{Si}''(x)$. You will have to compute $\text{Si}''(0)$ by using the definition of the derivative. *Hint: In computing $\text{Si}''(0)$, you can make use of the fact that $\sin(\Delta x) = \Delta x + O((\Delta x)^3)$.*

b) List the critical points of $\text{Si}(x)$ in the entire range $-\infty < x < \infty$. Which critical points are local maxima and which ones are local minima?

c) Draw a rough sketch of $\text{Si}'(x)$ and $\text{Si}''(x)$. The drawings only have to be qualitatively correct, but make sure that the zeros of $\text{Si}'(x)$ are accurately displayed.

d) Sketch the graph of $\text{Si}(x)$ on the interval $-4\pi \leq x \leq 4\pi$ with labels for the critical points and inflection points. The drawing should be qualitatively correct and should reflect the shape of the graphs you sketched in part c). *Note: The equation satisfied by the inflection points cannot be solved using pencil and paper. You should use a computer or calculator to approximate the inflection points.*

e) Let $r > 1$ be a real number, and define

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ \frac{\sin(x^r)}{x} & \text{if } x \neq 0. \end{cases}$$

Remark: It is not too hard to show that $f(x)$ is continuous, even at $x = 0$. Consider the function

$$h(x) = \int_0^x f(t) dt.$$

Show that $h(x)$ is a constant multiple of the composition of Si with another function.

f) Compute

$$\lim_{x \rightarrow 3} \frac{x^2}{x-3} \int_3^x \operatorname{sinc}(t) dt.$$

2. (Fri., Oct. 27; volumes by slicing; $4 + 1 = 5$ points) 7.3: 22

3. (Fri., Oct. 27; volumes by slicing; 10 points) Find the volume of the three-dimensional solid with $x > 0, y > 0, z > 0$ and

$$z^4 < x + y < z.$$

Hint: First find the area of the horizontal cross sections, which are perpendicular to the z axis.

4. (Tues., Oct. 31; shell and disk method; $2 + 1 + 2 + 1 = 6$ points) (Donut with triangular cross sections)

a) An equilateral triangle in the (x, y) plane of side length ℓ has a base that runs along the x axis. The center of the triangle is a distance R from the y axis, where $R > \frac{1}{2}\ell$ (and thus the triangle does not intersect the y axis). The triangle is revolved around the y axis to create a solid. Use the cylindrical shell method to express the volume of the solid in terms of an integral. Your answer should depend on ℓ and R .

b) Compute the integral from part a) to find a formula for the volume.

c) Repeat parts a) and b), but this time using the disk method.

5. (Tues., Oct. 31; shell method; $3 + 7 = 10$ points) 7.4: 12, 13.

Remark: Think of the “spherical ring” as a sphere that has been gored by a cylinder whose radius is smaller than the radius of the sphere, but whose length is infinite.