MATH 18.01 - MIDTERM 3 REVIEW: SUMMARY OF SOME KEY CONCEPTS

18.01 Calculus, Fall 2017	Professor: Jared Speck

a. Differential equations: $\frac{dy}{dx} = F(x, y)$
  (a) Separation of variables: allows one to solve equations of the form $\frac{dy}{dx} = g(x)h(y)$
    (i) Solution: $\int \frac{dy}{h(y)} = \int g(x)\, dx + C$
  (b) In general there are infinitely many solutions
  (c) Initial value problem
    (i) Finding the particular solution $y(x)$ with $y(x_0) = y_0$

b. Riemann sums for a function $f(x)$ on an interval $[a, b]$: are sums of the form $\sum_{i=1}^{n} f(c_i)\Delta x$
  (a) $f(c_i)\Delta x$ is the area of a thin rectangle that approximates a small portion of the area under the graph of $y = f(x)$
  (b) $\Delta x = (b - a)/n$ when the rectangles have equal width
  (c) When the rectangles have equal width, we partition $[a, b]$ into the sub-intervals $[x_0, x_1]$, $[x_1, x_2]$, $\cdots$, $[x_{n-1}, x_n]$, where $x_0 = a$, $x_n = b$, and $x_i - x_{i-1} = \Delta x$ for $1 \leq i \leq n$.
  (d) Generally, the c_i can be *any* points in the sub-interval $[x_{i-1}, x_i]$. $c_i = x_i$ is a "right Riemann sum" while $c_i = x_{i-1}$ is a "left Riemann sum."
  (e) As $n \to \infty$, we have that $\Delta x \to 0$ and that $\sum_{i=1}^{n} f(c_i)\Delta x$ converges to $\displaystyle\int_a^b f(x)\, dx$, which represents the *signed* area under the graph of $y = f(x)$ over the interval $[a, b]$

c. First fundamental theorem
  (a) If $F' = f$, then $\int_a^b f(x)\, dx = F(b) - F(a)$

d. Second fundamental theorem
  (a) $\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$

e. $L(x) = \int_1^x \frac{dt}{t}$ is an alternate definition of $\ln x$

f. Areas between curves
  (a) Area $= \int_a^b f(x) - g(x)\, dx$ (using vertical rectangles)
    (i) f is the upper function, g is the lower function
    (ii) a and b are the x coordinates of the intersection points
  (b) Area $= \int_c^d \widetilde{f}(y) - \widetilde{g}(y)\, dy$ (using horizontal rectangles)
    (i) $\widetilde{f}$ is the right function, $\widetilde{g}$ is the left function
    (ii) c and d are the y coordinates of the intersection points

g. Volume of a solid
  (a) Total solid volume $= \int$(Cross sectional area) $d$width

h. Volume of a solid of revolution
  (a) Disk method
    (i) Volume of a thin disk: $dV = \pi r^2\, dx$, where r is the disk radius and dx is the disk thickness
    (ii) Total solid volume $= \int dV$
  (b) Shell method
    (i) Volume of a thin shell of radius x, height h, and thickness dx is $dV = 2\pi x h\, dx$
    (ii) Total solid volume $= \int dV$

i. Average value of $f(x)$ on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\, dx$