MATH 18.01 - MIDTERM 2 REVIEW: SUMMARY OF SOME KEY CONCEPTS

18.01 Calculus, Fall 2017 Professor: Jared Speck

a. Linear approximations
  (a) $f(x) = f(x_0) + f'(x_0)(x - x_0) + O((x - x_0)^2)$ for x near x_0

b. Quadratic approximations
  (a) $f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f'(x_0)(x - x_0)^2 + O((x - x_0)^3)$ for x near x_0

c. Graphing
  (a) Zeros
    (i) Are points x where $f(x) = 0$
  (b) Critical points
    (i) Are points x where $f'(x) = 0$
    (ii) Help identify local mins and maxes
    (iii) Not all critical points correspond to a min or a max
  (c) Concavity
    (i) $f''(x) > 0 \implies$ concave up
    (ii) $f''(x) < 0 \implies$ concave down
    (iii) Inflection points
      (A) Are points where $f''(x) = 0$
      (B) Help identify points where the concavity changes from up to down or vice versa
      (C) Not all inflection points correspond to changes in concavity
  (d) Second derivative test
    (i) If x is a critical point and $f''(x) > 0$, then x is a local min
    (ii) If x is a critical point and $f''(x) < 0$, then x is a local max
    (iii) If x is a critical point and $f''(x) = 0$, then more information is needed to determine whether x is a local min or a local max (or neither)
  (e) Watch out for points of discontinuity or non-differentiability
  (f) Make sure to indicate limiting behavior as $x \to \pm\infty$ or $x \to$ a point of discontinuity

d. Max-min problems
  (a) If $f(x)$ is continuous on $[a, b]$, then the min and max occur at an endpoint, a critical point, or a "bad point" (i.e., a point of non-differentiability)

e. Related rates of change
  (a) The key to these problems is to differentiate the variable relationships and to apply the chain rule.

f. Newton's method
  (a) $x_{k+1} = x_k - f(x_k)/f'(x_k)$
g. Mean value theorem
  (a) $f(b) - f(a) = f'(c)(b-a)$ for some c in between a and b
  (b) $\frac{f(b)-f(a)}{b-a} = f'(c)$; average rate of change of f is equal to the instantaneous rate of change of f at some point in between
  (c) $f' \geq 0$ implies that f is increasing
  (d) $f' \leq 0$ implies that f is decreasing
  (e) If $f'(x) = 0$ for all x, then f is constant-valued
h. Differentials
  (a) Are essentially equivalent to linear approximation
  (b) $\Delta y \approx dy = f'(x)dx$
i. Antiderivatives
  (a) $F' = f$ means that F is an antiderivative of f
  (b) If $F' = f$ and $G' = f$, then $F(x) = G(x) + c$ for some constant c
  (c) Substitution method
    (i) $\int f(u(x))u'(x)dx = \int f(u)du$
  (d) Basic examples (this is only a partial list)
    (i) $\int \sin x\, dx = -\cos x + c$
    (ii) $\int \cos x\, dx = \sin x + c$
    (iii) $\int x^n\, dx = \frac{1}{n+1}x^{n+1} + c$, $n \neq -1$
    (iv) $\int x^{-1}\, dx = \ln|x| + c$
    (v) $\int (1+x^2)^{-1}\, dx = \arctan x + c$
    (vi) $\int (1-x^2)^{-1/2}\, dx = \arcsin x + c_1 = -\arccos x + c_2$
    (vii) $\int e^x\, dx = e^x + c$