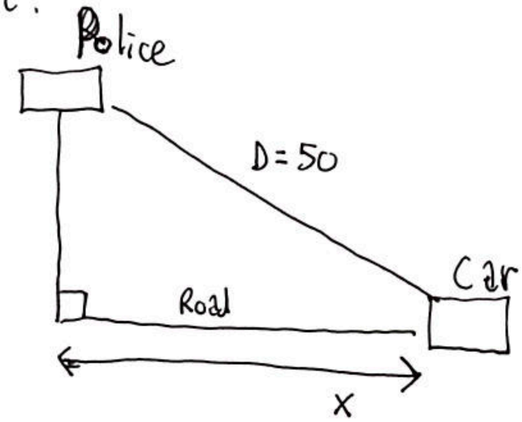
• Related rates

• <u>Ex.</u> Police are 30 ft. from the side of the road. Their radar sees your car approaching at 80 ft/sec when your car is 50 ft. from the radar gun. The speed limit is 65 miles/hr. $\approx$ 95 ft/sec

Picture:



• Note that we have given names to the variables.

(*) It is very important to figure out which variables are changing, and which are constant. In this problem, x and D are changing as t varies, where t = time.

• We are given: $D' = -80$ (D is decreasing in time)

• Relationships between variables:

By the Pythagorean thm, $x^2 + 30^2 = D^2$ (*)

• When $D = 50$, we solve for x: $\boxed{x = 40}$

• We then implicitly differentiate the eqn. (*) with respect to t:

$$\frac{d}{dt}(x^2 + 30^2) = \frac{d}{dt}(D^2)$$

• Using the chain rule, we have: $2x\frac{dx}{dt} = 2D\frac{dD}{dt}$

• Thus, $x' = \frac{D}{x}D'$, where $' = \frac{d}{dt}$.

- Now we simply plug in $X=40$, $D=50$, $D'=-80$ to deduce:

$$X' = \frac{50}{40}(-80) = -100\ \frac{ft.}{sec} \Rightarrow \text{You are speeding.}$$

- Note that $X'<0$, which makes sense since X is decreasing.

- You could also have solved this problem by first solving for D in terms of X:

$$D = (30^2 + X^2)^{1/2}.$$

You could then differentiate this equation with respect to t and use the chain rule to deduce that

$$\frac{dD}{dt} = \cancel{\tfrac{1}{2}}(30^2 + x^2)^{-1/2} \cdot \cancel{2}x \cdot \frac{dx}{dt}.$$

- You could then plug in as above to find $\frac{dx}{dt}$.

- However, the algebra is often more complicated when you try to explicitly solve for the one variable in terms of the other.

- Overall Strategy

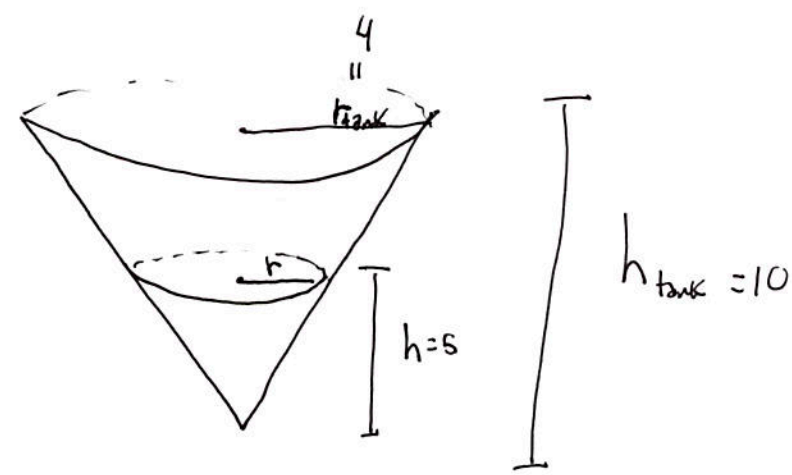
① Draw picture

② Set up variables + equations

③ Take derivatives

④ Plug in the values <u>after taking the derivatives</u>.

Ex:



- Consider a conical tank with $r_{tank} = 4$, $h_{tank} = 10$. Suppose it is being filled with water at a rate of 2 ft^3/min. How fast is the water rising when it is 5 ft. high?

- The volume of water in the tank is $V = \frac{1}{3}\pi r^2 h$.
- Using similar triangles (side view), we have: $\frac{r}{h} = \frac{4}{10}$

$$\Rightarrow r = \frac{2}{5}h$$

- Plugging into V, we have

  - $V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 \cdot h = \frac{4}{75}\pi h^3$

  - Differentiating with respect to t and using the chain rule, we have :

$$\frac{dV}{dt} = \frac{4}{25}\pi h^2 \frac{dh}{dt}$$

- Plugging in $\frac{dV}{dt} = 2$ and $h = 5$, we have

$$2 = \left(\frac{4}{\cancel{25}}\pi\right) \cdot \cancel{5^2} \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{2\pi}$$