MATH 18.01 - MIDTERM 4 REVIEW: SUMMARY OF SOME KEY CONCEPTS

18.01 Calculus, Fall 2014	Professor: Jared Speck

a. Numerical integration
- (a) Riemann sums $\int_a^b f(x)\,dx \approx \sum_{i=1}^n y_i\Delta x$
  - (i) $y_i = f(x_i)$
  - (ii) The x_i belong to the i^{th} subinterval (the precise definition of x_i depends on whether you are using right sums, left sums, upper sums, lower sums, etc.)
  - (iii) $\Delta x = \frac{b-a}{n}$
- (b) Trapezoid rule $\int_a^b f(x)\,dx \approx \Delta x \left(\frac{y_0}{2} + y_1 + y_2 + \cdots + y_{n-1} + \frac{y_n}{2}\right)$
  - (i) $\Delta x = \frac{b-a}{n}$
  - (ii) $x_0 = a$, $x_1 = a + \Delta x$, $x_2 = a + 2\Delta x$, $\cdots$, $x_n = a + n\Delta x = b$
- (c) Simpson's method (n must be even)
  - (i) $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-3} + 2y_{n-2} + 4y_{n-1} + y_n)$
  - (ii) $\Delta x = \frac{b-a}{n}$
  - (iii) $x_0 = a$, $x_1 = a + \Delta x$, $x_2 = a + 2\Delta x$, $\cdots$, $x_n = a + n\Delta x = b$

b. Computing $\int (\sin x)^n(\cos x)^m\,dx$
- (a) If m is odd, let $u = \sin x$, $du = \cos x\,dx$, and substitute $(\cos x)^2 = 1 - (\sin x)^2$ to transform the integral into a u integral.
- (b) If n is odd, interchange the roles of $\sin x$ and $\cos x$ and proceed as above.
- (c) If m, n both even, make repeated use of the trig identities $(\cos x)^2 = \frac{1}{2}[1 + \cos(2x)]$ and $(\sin x)^2 = \frac{1}{2}[1 - \cos(2x)]$.

c. Computing $\int (\sec x)^n(\tan x)^m\,dx$
- (a) $\int \tan x\,dx = -\ln|\cos x| + C$.
- (b) $\int \sec x\,dx = \ln|\sec x + \tan x| + C$.
- (c) $\int (\sec x)^2\,dx = \tan x + C$.
- (d) $\int \sec x \tan x\,dx = \sec x + C$.
- (e) If m is odd, let $u = \sec x, du = \sec x \tan x$ and substitute $(\tan x)^2 = (\sec x)^2 - 1$ to transform the integral into a u integral.
- (f) If n is even, let $u = \tan x, du = (\sec x)^2$ and substitute $(\sec x)^2 = 1 + (\tan x)^2$ to transform the integral into a u integral.
- (g) If m is even and n is odd, then we haven't studied how to evaluate the integral.

d. Inverse trig substitution

(a) Is useful for evaluating $\int \sqrt{ax^2+bx+c}\,dx$ because it gets rid of the square root (a, b, c are constants).
(b) To evaluate $\int \frac{dx}{\sqrt{x^2+1}}$, let $x = \tan u$, $dx = (\sec u)^2\,du$, and substitute $(\tan u)^2 + 1 = (\sec u)^2$.
(c) To evaluate $\int \frac{dx}{\sqrt{x^2-1}}$, let $x = \sec u$, $dx = \sec u \tan u\,du$, and substitute $secu)^2 - 1 = (\tan u)^2$.
(d) To evaluate $\int \frac{dx}{\sqrt{1-x^2}}$, let $x = \sin u$, $dx = \cos u\,dx$, and substitute $1 - (\sin u)^2 = (\cos u)^2$.
(e) To evaluate e.g. $\int \frac{dx}{\sqrt{x^2+2x+2}}$, first complete the square: $x^2+2x+2 = (x+1)^2+1$. Then let $v = x+1$, $dv = dx$, and proceed as above.
(f) You can draw a suitable right triangle to help you express the final answer in terms of x.

e. Partial fractions
(a) Is a strategy for evaluation $\int \frac{P(x)}{Q(x)}$, where P, Q are polynomials and the degree of P is $<$ the degree of Q.
(b) You have to factor $Q(x)$ to its fullest extent.
(c) If $Q(x) = (x+a)(x+b)$, guess $\frac{P(x)}{Q(x)} = \frac{A}{x+a} + \frac{B}{x+b}$ and solve for the constants A, B using e.g. the cover-up method. Then integrate the right-hand side using prior techniques.
(d) If $Q(x) = (x+a)(x+b)^2$, guess $\frac{P(x)}{Q(x)} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{(x+b)^2}$ and solve for the constants A, B, C (the cover-up method does not work for B.) Then integrate the right-hand side using prior techniques.
(e) If $Q(x) = (x+a)^2(x+b)^3$, guess $\frac{P(x)}{Q(x)} = \frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{x+b} + \frac{D}{(x+b)^2} + \frac{E}{(x+b)^3}$, etc.
(f) If $Q(x) = (x+a)(x^2+b)$, guess $\frac{P(x)}{Q(x)} = \frac{A}{x+a} + \frac{B_0+B_1x}{x^2+b}$ and solve for the constants A, B_0, B_1 (the cover-up method works only on A.) Then integrate the right-hand side using prior techniques. A similar idea would allow you to treat other quadratic factors (with no real roots) in place of x^2+b (you might have to complete the square first).
(g) If $Q(x) = (x+a)^2(x^2+b)^2$, guess $\frac{P(x)}{Q(x)} = \frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C_0+C_1x}{x^2+b} + \frac{D_0+D_1x}{(x^2+b)^2}$, etc

f. Integration by parts
(a) Is simply the product rule in reverse
(b) $\int u\,dv = uv - \int v\,du$