

MATH 18.01, FALL 2014 - PROBLEM SET # 1

Professor: Jared Speck

Due: by 1:45pm on 9-12-14

(in the boxes outside of Room E17-131 during the day; write your name and recitation instructor on your homework)

18.01 Supplementary Notes (including Exercises and Solutions) are available on the course web page: http://math.mit.edu/~jspeck/18.01_Fall%202014/1801_CourseWebsite.html. This is where to find the exercises labeled 1A, 1B, etc. You will need these to do the homework.

Part I consists of exercises given and solved in the Supplementary Notes. It will be graded quickly, checking that all is there and the solutions not copied.

Part II consists of problems for which solutions are not given; it is worth more points. Some of these problems are longer multi-part exercises given here because they do not fit conveniently into an exam or short-answer format. See the guidelines below for what collaboration is acceptable, and follow them.

To encourage you to keep up with the lectures, both Part I and Part II tell you for each problem on which day you will have the needed background for it.

You are encouraged to use graphing calculators, software, etc. to check your answers and to explore calculus. However, (unless otherwise indicated) we strongly discourage you from using these tools to solve problems, perform computations, graph functions, etc. An extremely important aspect of learning calculus is developing these skills. You will not be allowed to use any such tools on the exams.

Part I (30 points)

Notation: The problems come from three sources: the Supplementary Notes, the Simmons book, and problems that are described in full detail inside of this pset. I refer to the former two sources using abbreviations such as the following ones: 2.1 = Section 2.1 of the Simmons textbook; Notes G = Section G of the Supplementary Notes; Notes 1A: 1a, 2 = Exercises 1a and 2 in the Exercise Section 1A of the Supplementary Notes; Section 2.4: 13 = Problem 13 in Section 2.4 of Simmons, etc.

Recitation 0. (Wed., Sept. 3) Graphing functions.

Read: Notes G: Sections 1-4.

Homework: Notes 1A:1a, 2a, 3abe, 6a, 7a.

Lecture 1. (Thurs., Sept. 4) Derivative; slope, velocity, rate of change.

Read: 2.1-2.4.

Homework: Notes 1C: 3abe, 4ab (use 3), 5, 6 (trace axes onto your answer sheet).

Homework: Notes 1B: 2; Notes 1C: 1a (start from the definition of derivative).

Lecture 2. (Fri., Sept. 5) Limits and continuity; some trigonometric limits.

Read: 2.5 (bottom pp. 70-73; concentrate on examples, skip the $\epsilon - \delta$ definition).

Read: 2.6 to p. 75; learn definition (1) and the proof that “diff. \implies cont.” at the end.

Read: Notes C.

Homework: Notes 1D: 1bcefg, 4a; Notes 1C: 2; Notes 1D: 3ade, 6a, 8a (hint: “diff. \implies cont.”).

Lecture 3. (Tues., Sept. 9) Differentiation formulas: products and quotients; Derivatives of trigonometric functions.

In the exercises, an antiderivative of $f(x)$ is any function $F(x)$ for which $F'(x) = f(x)$.

Read: 3.1, 3.2, 3.4.

Homework: Notes 1E: 1ac, 2b, 3, 4a, 5a; Notes 1J: 1e, 2.

Part II (50 points)

Directions and Rules: Collaboration on problem sets is encouraged, but:

i) Attempt each part of each problem yourself. Read each portion of the problem before asking for help. If you don’t understand what is being asked, then ask for help interpreting the problem and then make an honest attempt to solve it.

ii) Write up each problem independently. On both Part I and II exercises, you are expected to write the answer in your own words. *You must show your work; “bare” solutions will receive very little credit.*

iii) Write on your problem set whom you consulted and the sources you used. *This includes graphing calculators, software, etc.* (again, we strongly discourage you from relying on these tools to solve problems). If you fail to do so, you may be charged with plagiarism and subject to serious penalties.

iv) It is illegal to consult materials from previous semesters.

0. (not until due date; 3 points) Write the names of all the people whom you consulted or with whom you collaborated and the resources you used, or say “none” or “no consultation.” This includes visits outside recitation to your recitation instructor. If you don’t know a name, you must nevertheless identify the person, as in, “tutor in Room 2-106,” or “the student next to me in recitation.” Optional: note which of these people or resources, if any, were particularly helpful to you.

This “Problem 0” will be assigned with every problem set. Its purpose is to make sure that you acknowledge (to yourself as well as others) what kind of help you require and to encourage you to pay attention to how you learn best (with a tutor, in a group, alone). It will help us by letting us know what resources you use.

1. (First recitation; Nonexistence of sin and cos cancellations; $2 + 1 = 3$ points)

a) Suppose that $E(x)$ is an even function and that $O(x)$ is an odd function. Suppose furthermore that $E(x) + O(x) = 0$. Show that for all x , $E(x) = 0$ and $O(x) = 0$.

b) Use part a) to show that if $A \sin(ax) + B \cos(bx) = 0$ for all x , where A, B, a, b are real numbers, then $B = 0$ and one of either A or a is also equal to 0.

2. (Thursday, Sept. 4; error analysis; $2 + 3 + 3 + 2 = 10$ points) Suppose that $f(x)$ is a differentiable function. Let x_0 be a fixed number, let x be a number near x_0 , and let $\Delta x = x - x_0$, $\Delta y = f(x) - f(x_0)$. If Δx is small, then as we saw in class, $\frac{\Delta y}{\Delta x}$ is approximately equal to $f'(x_0)$. This leads to the approximate formula: $\Delta y \approx f'(x_0)\Delta x$. This is an elementary form of *error analysis*. Roughly, this means that if

you know that x is near x_0 within an error Δx , then you know that $f(x)$ is near $f(x_0)$ within an approximate error $\Delta y \approx f'(x_0)\Delta x$. Use these ideas to answer the following questions:

Your very picky niece is visiting you and you are serving her ice cream in a conical shaped dish (whose tip is at the bottom). The cone has circular cross sections and is such that it looks like a perfect equilateral triangle if you stare at it from the side. You plan to partially fill the dish with ice cream to a height h . Assume that you are able to smooth the ice cream so that it lies perfectly flat in the dish. The volume V of ice cream in the dish depends on h according to the following classical volume formula: $V = \frac{\pi}{9}h^3$. Your niece has demanded that you fill her dish to a height of $h_0 = 1$, but she generously grants you a height margin of error $|\Delta h|$ as long as it is no larger than .02. In principle, it is possible to satisfy her wishes because the dish has a total height of 2. Unfortunately, the dish has no height markings. However, she plans to measure the height using a calibrated laser after you serve her the ice cream (so you better get the height right within the margin of error). Your only tools are a collection of scoops that are capable of measuring volumes within an error $|\Delta V|$ that is no larger than .1.

a) Solve for h as a function of V and calculate $h'(V)$ using the analytic definition of a derivative. *Hint: You may find the identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ to be useful.*

b) Find the exact volume V_0 of ice cream that you need to serve your niece. Then derive an approximate formula for the height error Δh in terms of the volume error ΔV .

c) Decide whether or not your collection of scoops is good enough to satisfy your niece within the margin of error.

d) Repeat the second half of part b), but this time, write V as a function of h and use the corresponding approximate error formula $\Delta V \approx V'(h_0)\Delta h$. In particular, you will have to calculate $V'(h_0)$ (you do not have to use the analytic definition of a derivative this time). Does this alternative approximate error formula change your answer to part c)?

3. (Thursday, Sept. 4; $4 + 4 + 3 + 3 + 4 = 18$ points) Section 3.1: 14, 18, 21a-c

4. (Friday, Sept. 5; $1 + 1 + 1 + 1 + 1 + 1 + 2 = 8$ points) Section 2.5: 19f), 19g), 20a), 20b), 20g), 22a), 22b) (use the proof on p. 73 for inspiration)

5. (Tuesday, Sept. 9; $1 + 5 + 2 = 8$ points)

a) Use the quotient rule to show that $\frac{d}{dx}x^{-1} = -x^{-2}$.

b) Let $n > 0$ be an integer. Use the product rule and the method of mathematical induction to show that $\frac{d}{dx}x^{-n} = -nx^{-(n+1)}$. More precisely, use the following strategy. Assume that you have shown that $\frac{d}{dx}x^{-n} = -nx^{-(n+1)}$ holds for a certain n value (by part a), you have shown that this formula is true when $n = 1$). To show that the formula is true for the next largest integer $n + 1$, first write $x^{-(n+1)} = x^{-1}x^{-n}$. Then apply the product rule to the product $x^{-1}x^{-n}$ and use the formula for the case n in order to show that $\frac{d}{dx}x^{-(n+1)} = -(n+1)x^{-(n+2)}$. You will have shown that case $n \implies$ case $n + 1$. Thus, this formula must be true for all positive integers. This technique is called *the method of induction*.

c) Use induction to show that $\frac{d}{dx}x^n = nx^{n-1}$ whenever $n \geq 1$ is an integer.