

18.01, September 12, 2003 Lecture Notes

Practice Problems

1G-3, 1G-4, 1H-1, 1H-3, 1H-5

1. Higher derivatives, notations:

$\frac{d^k}{dx^k} v = \frac{d}{dx} (\dots (\frac{dv}{dx}) \dots)$, k copies of $\frac{d}{dx}$, $v' = \frac{dv}{dx}$, $v'' = \frac{d^2 v}{dx^2}$, $v''' = \frac{d^3 v}{dx^3}$, but generally

$v^{(k)} = \frac{d^k}{dx^k} v$ (simpler than $v^{'\dots'}$, $'\dots'$ k times).

2. General formulas: $\frac{d^k}{dx^k} (x^n) = n(n-1)\dots(n-k+1)x^{n-k}$

In fact if n is a general fraction! (Proof by induction on b.)

If n is a positive integer = $\frac{n!}{(n-k)!} x^{n-k}$

If $n = -m$, = $(-1)^m \frac{(m+k-1)!}{(m-1)!} \frac{1}{x^{m+k}}$ DIDN'T WRITE THIS FORMULA!!

DID WRITE: $\frac{d^k}{dx^k} \frac{1}{x+q} = \frac{(-1)^k k!}{(x+q)^{k+1}}$

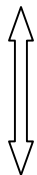
LEIBNITZ FORMULA: $(UV)^{(n)} = U^{(n)} \bullet V + n U^{(n-1)} V^{(1)} + \dots + \binom{n}{k} U^{(n-k)} \bullet V^{(k)} + \dots + UV^{(n)}$

Exponentials: $\alpha^x := \lim_{e \rightarrow x} \alpha^e$ where $\alpha^{\frac{p}{2}} = (\sqrt[2]{\alpha})^p$, $\alpha > 1$, $\lim_{x \rightarrow -\infty} \alpha^x = 0$, $\lim_{x \rightarrow \infty} \alpha^x = +\infty$.

Should α^x increasing ($\alpha > 1$!!) and claimed α^x continuous $\Rightarrow \forall y > 0, \exists x$ s.t. $y = \alpha^x$.

Define $x := \log_{\alpha}(y)$

$$\begin{cases} \alpha^0 = 1 \\ \alpha^1 = \alpha \\ \alpha^x \alpha^y = \alpha^{x+y} \\ (\alpha^x)^y = \alpha^{(xy)} \end{cases}$$



$$\log_{\alpha}(1) = 0$$

$$\log_{\alpha}(\alpha) = 1$$

$$\log_{\alpha}(xy) = \log_{\alpha}(x) + \log_{\alpha}(y)$$

$$\log_{\alpha}(x^y) = y \log_{\alpha}(x)$$

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

$$(1) \log(x) = \log_{10}(x) \text{ } common \log$$

$$(2) \ln(x) = \log_e(x) \text{ } natural \log$$

Mention $\frac{d}{dx}(\alpha^x) = ?? \cdot \alpha^x$ and $??=1$ for e.