MATH 18.01 - MIDTERM 3 REVIEW: SUMMARY OF SOME KEY CONCEPTS

18.01 Calculus, Fall 2014	Professor: Jared Speck

a. Differential equations: $\frac{dy}{dx} = F(x, y)$
  (a) Separation of variables: allows one to solve equations of the form $\frac{dy}{dx} = g(x)h(y)$
    (i) Solution: $\int \frac{dy}{h(y)} = \int g(x)\,dx + C$
  (b) In general there are infinitely many solutions
  (c) Initial value problem
    (i) Finding the particular solution $y(x)$ with $y(x_0) = y_0$

b. First fundamental theorem
  (a) If $F' = f$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

c. Second fundamental theorem
  (a) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

d. $L(x) = \int_1^x \frac{dt}{t}$ is an alternate definition of $\ln x$

e. Areas between curves
  (a) Area $= \int_a^b f(x) - g(x)\,dx$ (using vertical rectangles)
    (i) f is the upper function, g is the lower function
    (ii) a and b are the x coordinates of the intersection points
  (b) Area $= \int_c^d \widetilde{f}(y) - \widetilde{g}(y)\,dy$ (using horizontal rectangles)
    (i) $\widetilde{f}$ is the right function, $\widetilde{g}$ is the left function
    (ii) c and d are the y coordinates of the intersection points

f. Volume of a solid
  (a) Total solid volume $= \int$ Cross sectional area $d$width

g. Volume of a solid of revolution
  (a) Disk method
    (i) Volume of a thin disk: $dV = \pi r^2\,dx$, where r is the disk radius and dx is the disk thickness
    (ii) Total solid volume $= \int dV$
  (b) Shell method
    (i) Volume of a thin shell of radius x, height h, and thickness dx is $dV = 2\pi x h\,dx$
    (ii) Total solid volume $= \int dV$

h. Average value of $f(x)$ on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$