CONTROL SYSTEM

- S Hasan Saeed -

→ UNIT :

I CONTROL SYSTEM AND THEIR REPRESENTATION:

- Terminology and basic structure of control system
- open loop and closed loop system,
- servomechanism,
- regulatory system,
- analogous systems.
- physical systems and their models,
- electromechanical systems,
- electrical analogy of physical systems.

- Transfer function,
- Block diagram representation of physical systems,
- Block diagram algebra,
- signal flow graph and
- mason's formula.

II TIME RESPONSES:

- Types of test inputs,
- Response of first and second order system,
- Time domain specifications.
- error coefficients,
- generalized error series.

- STABILITY: · concepts of stability,
· locations of root in s - plane for stability,

- asymplotic stability and
- relative stability,
- Routh - Hurwitz stability criterion.

III ROOT LOCUS:
- Root locus plot,
- Properties of Root loci and applications,
- stability range from the loci.
- Determination of roots of the closed loop system,
- Effect of pole zero addition.

NYQUIST PLOTS:
- Polar plots,
- Nyquist plots and
- Nyquist stability criterion.

IV BODE PLOTS:
- concepts of gain margin and
- phase margin,
- Bode plots.

- frequency - domain specifications
- M and N loci,
- Nichols chart

V CONTROLLERS:
- Introduction of PID and
- lag - lead type controllers.

- STATE VARIABLE ANALYSIS:- page 12.1 Dinesh sethi
  - concepts of state,
  - state variable and
  - state model.
  - State variable models for LTI system
- canonical representations,
- Transfer function to state space and vice versa.
- solution to state equations.
- Concepts of controllability & observability

- COMPENSATION DESIGN:-
  - Compensation design using frequency domain techniques.

CONTROL SYSTEM

- Control play an important role in our day to day life.

"Automatic control systems play and important role of in a advancement and im-provement of engineering skills.

→ BASIC DEFINATIONS:

1. SYSTEM: A system is a combination or an arrangement of different physical components which act together as an entire unit to achieve certain objective.

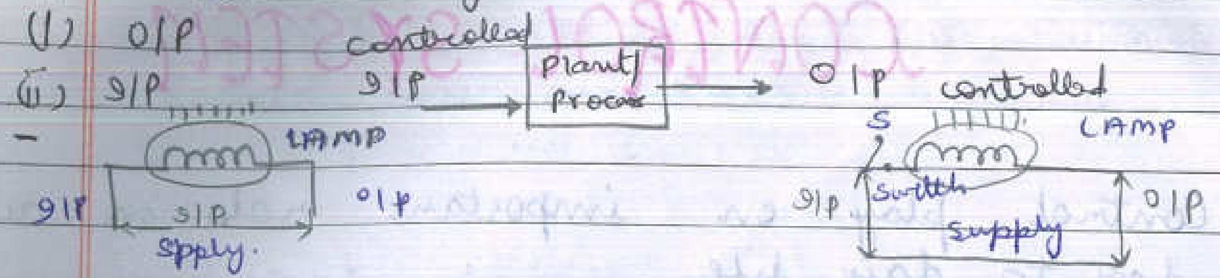
   ex: classroom: combinat" of Banches, Blackboard, fans, lighting arrangement etc.

   ex: lamp: made up of glass, filament is a physical system.

2. CONTROL: The meaning of control is to regulate or command a system so that a desired objective is obtain.

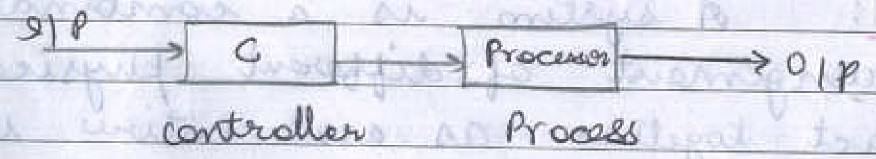
3. CONTROL SYSTEM: It is an arrangement of different physical elements connected in a manner so as to regulate, direct or command itself to obtain a certain objective

- A control system must have
(i) o/p controlled
(ii) i/p



"physical system" "control system"

4. → PLANT / PROCESS: The portion of the system which is to be control or regulated is called the 'plant or process.'



5 Controller C: The element of the system itself or external to the system which control the plant or the process is called controller.

6 Input:
It is an applied signal or an excitation signal applied to control system from an external energy source inorder to produce a specific o/p.

7 Output: It is the particular signals of interest or actual response obtain from a control system when i/p is applied it.

8 Disturbance:
Disturbance it is signal which tends to adversely affect the value of the o/p of a system.

- Disturbance is generated outside the system act an extra i/p to the system in addition to its normal i/p affecting the o/p advaresly is called external disturbance.
- Disturbance is generated within itself called internal disturbance.

→ Classificat^n of Control system:
1. Natural control system:
The system inside a human being or a biological system are known as Natural control system.

2. Man Made Natural system.

3. Time variant and invariant system

4. Combinational control system

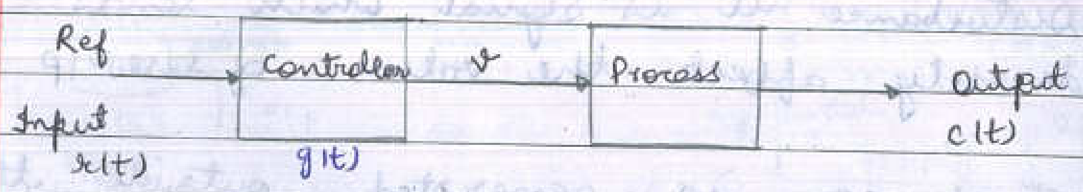
5. Linear and Non linear system

6. Continue and Discrete time control system

7. Distributed Parameter control system

8. Signal i/p single o/p and multiple i/p without (MIMO) system open loop and closed loop system.

→ OPEN LOOP SYSTEM:



ϑ = actuating signal

- Defination:

A system in which o/p is dependent on i/p but controlling action or i/p independent of o/p or change of the system is called an 'open loop system'.

• Advantage of open loop system:

1. simple in construction
2. easy from maintaine point of view.
3. economical

• Disadvantage of open loop system:

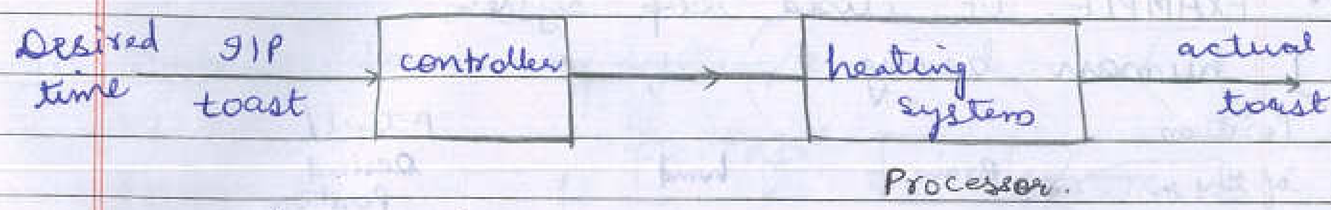
1. In accurate and unreliable because accuracy of the system depend on accurate pre-calibration of the controller.
2. Such system cannot sense enviorment change.
3. Cannot sense internal disturbances in the system.
4. To maintain the quality and accuracy recalibration at the controller is necessary time to time.

• EXAMPLE OF OPEN LOOP SYSTEM:

1. sprinker used to water. The system is adjusted to water a given area by opening the water

value and observing the resulting pattern.

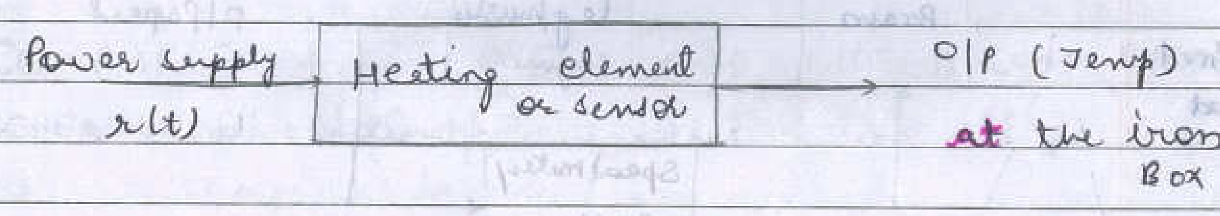
2. Automatic toaster system:



3. Traffic light system:

Open loop system

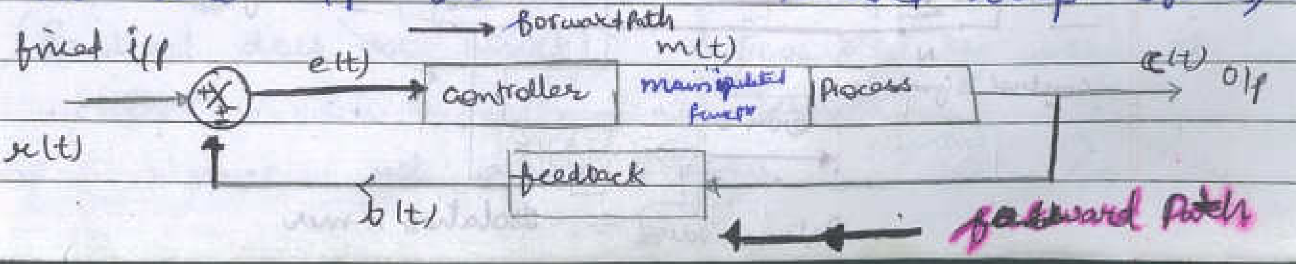
4. Electric iron Box (without automatic)



→ CLOSED LOOP SYSTEM:

- Definations:

A system in which the controlling system or i/p is dependent of the o/p or change in the o/p is called 'closed loop system'.

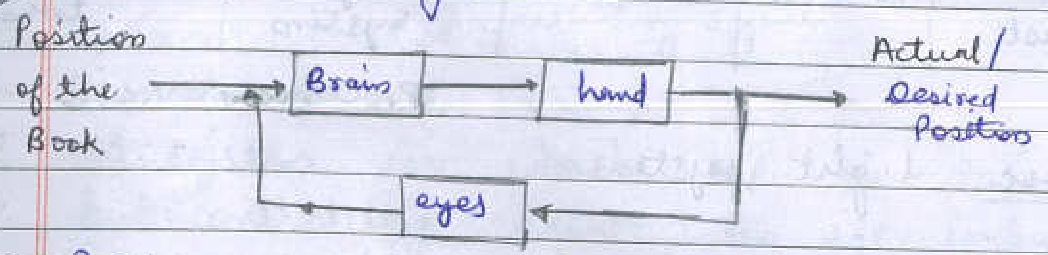


$$\begin{cases} e(t) = r(t) + b(t) & \text{+ve feedback} \\ e(t) = r(t) - b(t) & \text{-ve feedback} \end{cases}$$

Imp. 3 Marks 4

• EXAMPLE OF closed loop system:

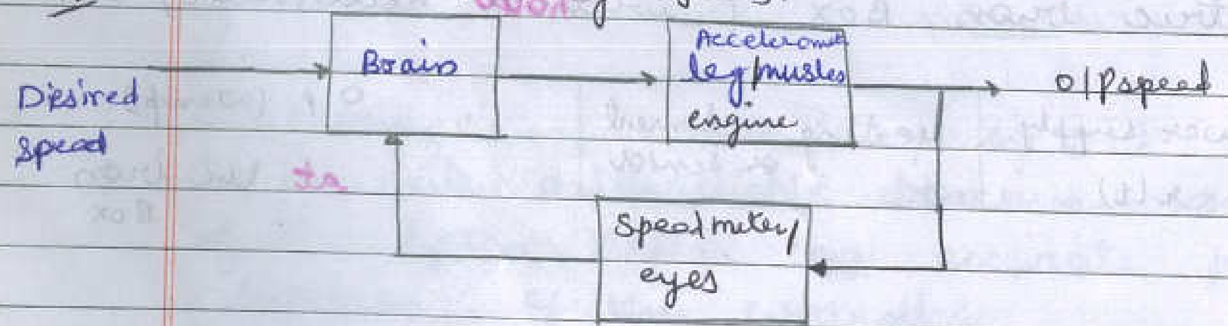
1. human being.



COMPONENT: Position of Book, Brain, Hand, eyes, Desired position (BRAIN)

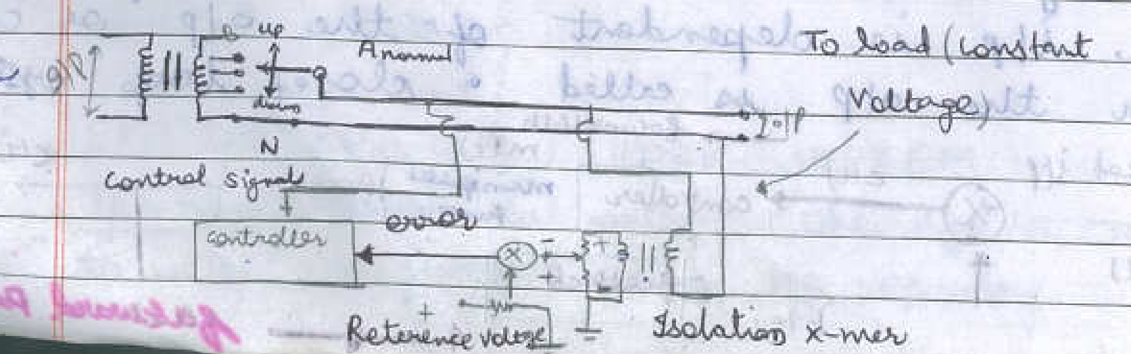
2. Automatic driving system:
3. Voltage stabilizer:

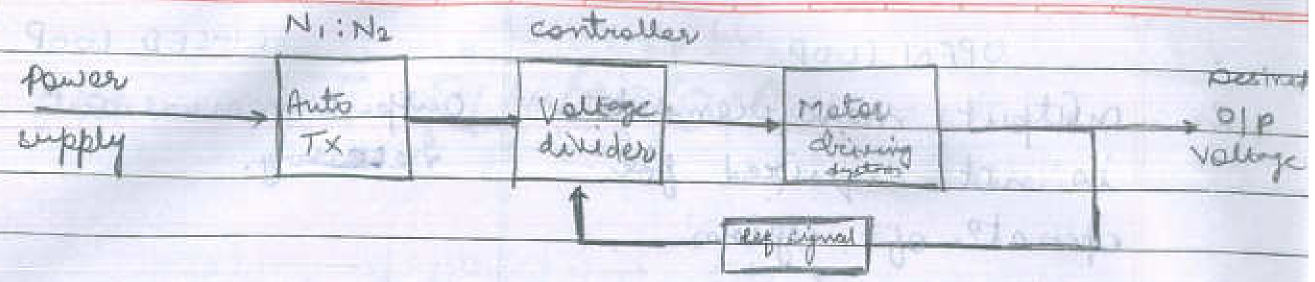
2. Automatic driving system:



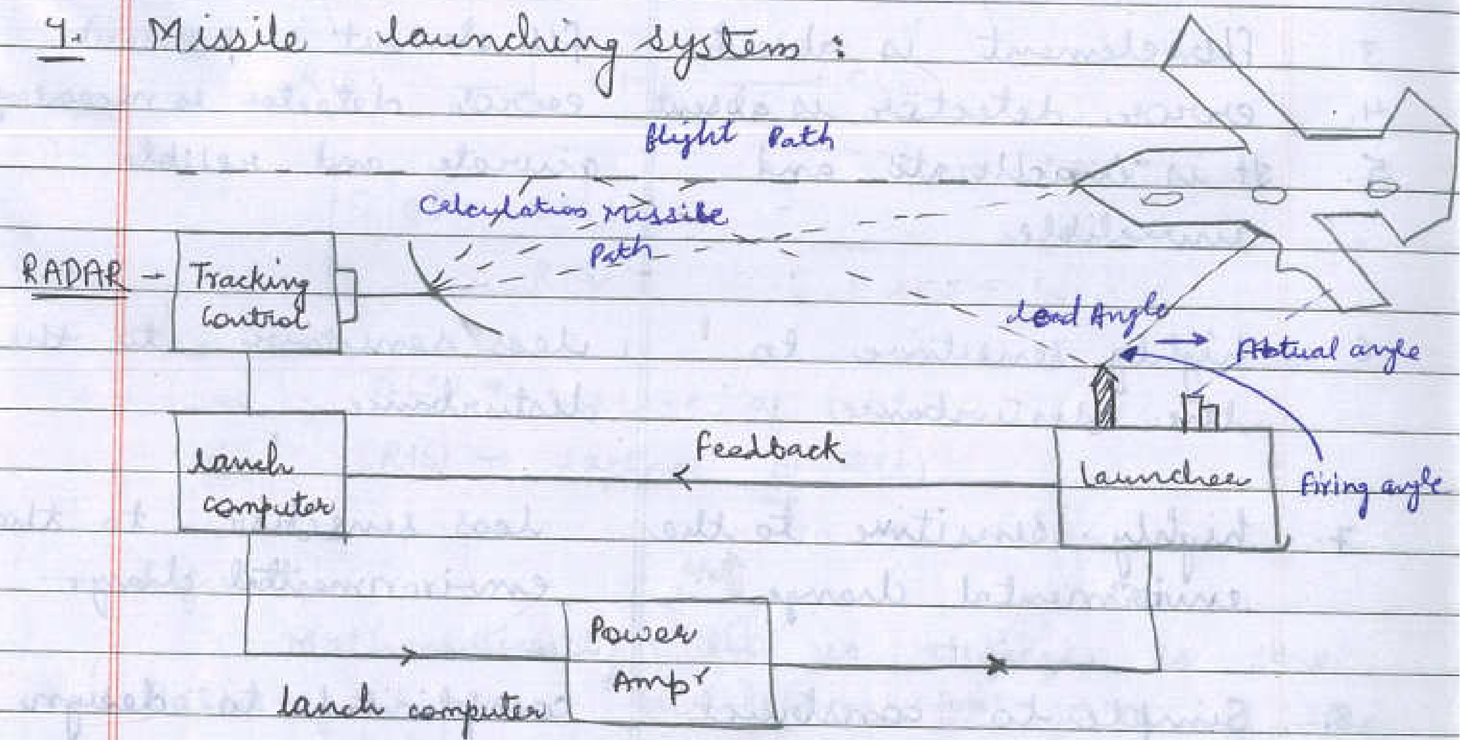
Desired speed, o/p speed, speedometer, eyes, brain, leg muscles engine, Accelerometer.

3. voltage stabilizer:





4. Missile launching systems:



COMPARITION OF OPEN AND CLOSED LOOP SYSTEM:

• OPEN LOOP: definat^n
 ckt diagram

• closed loop: definat^n
 ckt diagram

OPEN LOOP	CLOSED LOOP
1. Any change in o/p has no effect on the i/p (feedback does not exist)	change in o/p affect the i/p which is possible by used of feedback.

	OPEN LOOP	CLOSED LOOP
2.	output measurement is not required for operatⁿ of system	Output measurement is necessary.
3.	f/b element is absent	f/b element is present.
4.	error detector is absent	error detector is necessary.
5.	It is inaccurate and unrealible.	accurate and relible
6.	highly sensitive to the disturbance	less sensitive to the disturbance.
7.	highly sensitive to the environmental change	less sensitive to the environmental change
8.	Simple to construct and cheap	complicated to design and costly.
9.	stable in Nature	stability in the measure consideration (design Point of view).
10.	system is affected due to non linearity present in the element.	System adjust to the affect of non linearity Present in the system.

→ **TRANSFER FUNCTION :**

$r(t) \longrightarrow \boxed{\text{system}} \longrightarrow c(t)$

laplace

$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$

$$\boxed{G(s) = \frac{C(s)}{R(s)}}$$

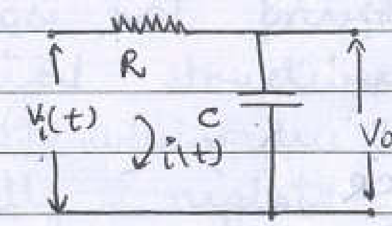
$C(s) \rightarrow$ laplace of $c(t)$

$R(s) \rightarrow$ laplace of $r(t)$

→ Defination :

Mathematically, it is define is the Ratio of laplace transform of O/P of the system. laplace transform of i/p under the assumption that all initial condⁿ are zero.

Q.



find out laplace of $\frac{V_o(s)}{V_i(s)} = ?$

$V_o(s) = X_c \, i(s)$

$V_i(s) = (R + X_c) \, i(s)$

$$\frac{V_o(s)}{V_i(s)} = \frac{X_c}{R + X_c} \Rightarrow \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \Rightarrow \frac{1}{sC \times \frac{R}{sC} + 1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sCR}$$

OR $V_i(t) = i(t)R + \frac{1}{C}\int i(t)\,dt$

$V_i(t) = i(t)R + \frac{1}{C}\int i(t)\,dt$

$V_i(s) = i(s)R + \int \frac{1}{C} \times I(s)\,s$

$V_i(s) = I(s)R + \frac{I(s)\,s}{Cs}$

$V_i(s) = R + \frac{s\,1}{Cs}$

$V_i(s) = R + \frac{1}{Cs}$ —(i)

$V_o(t) = \frac{1}{C}\int i(t)\,dt$

$V_o(s) = \frac{1}{sC} I(s)$ —(ii)

$\frac{V_o(s)}{V_i(s)} = \frac{1/sC}{\left(R + \frac{1}{Cs}\right) I(s)}$

$\frac{V_o(s)}{V_i(s)} = \frac{1}{RSC + 1}$

$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + SCR}$

Transfer funct^n $\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + SCR}$

→ ADVANTAGE OF TRANSFER FUNCTION:

1. It give Mathematical model of all system compo-nents of over all system.

2. It use laplace apporach integrat^n, differential eq^n with relate o/p to input.

3. Ones transfer funct^n is known output response for any type of reference i/p can be calculated.

4. It help in the stability analysis of the system.

5. The system differential eq^n can be obtain by replacing the variable 's' by 'd/dt'.

→ DISADVANTAGE:

1. Only aplicable to linear time invariant system.
2. It does not provide any informat^n for the physical structure of the system.
3. effect arising due to initial conditios are totally neglate.

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→ POLES AND ZEROES:

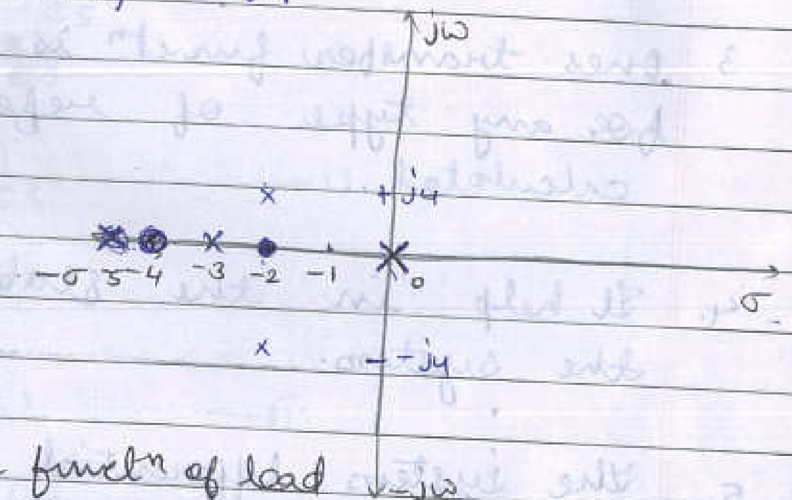
Transfer function
$$G(s) = \frac{(s+2)(s+4)}{s(s+3)(s+5)(s+2-j4)(s+2+j4)}$$

poles: $0, -3, -5, -2+j4, -2-j4$

zeroes: $-2, -4$

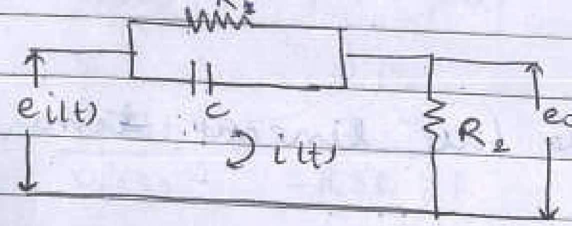
No of poles: 5

No of zeroes: 2



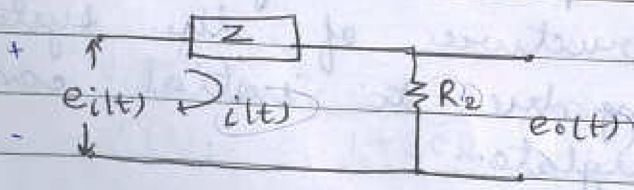
$$\boxed{\begin{aligned} V_R &= i(t)\,R \\ V_L &= L\frac{di}{dt} \\ V_C &= \frac{1}{C}\int i(t)\,dt \end{aligned}}$$

Q. Obtain the transfer funct^n of lead N/W as shown in the figure



$\frac{E_o(s)}{E_i(s)} = ?$

Sol^n:



$$Z = \frac{R_1 \times 1/sC}{R_1 + 1/sC}$$

$$\boxed{Z = \frac{R_1}{1+R_1sC}}$$

$$e_i(t) - z\,i(t) - R_2\,i(t) = 0$$
$$E_i(s) - Z\,I(s) - R_2\,I(s) = 0$$

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$$E_i(s) = (Z + R_2)\,I(s)$$
$$E_o(s) = R_2\,I(s)$$

$$1/\left(\frac{E_o(s)}{E_i(s)}\right) = \frac{Z+R_2}{R_2}$$

$$\frac{E_i(s)}{E_o(s)} = 1/\left(\frac{E_o(s)}{E_i(s)}\right) = \frac{\frac{R_1}{1+R_1sC} + R_2}{R_2}$$

$$1/\left(\frac{E_o(s)}{E_i(s)}\right) = \frac{R_1 + R_2 + R_1R_2sC}{(1+R_1sC)R_2}$$

$$\frac{E_i(s)}{E_o(s)} = \frac{R_1 + R_2 + R_1R_2sC}{R_2 + R_1R_2sC}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{R_2 + R_1R_2sC}{R_1 + R_2 + R_1R_2sC}} = \boxed{\frac{s+\alpha}{s+\beta}}$$

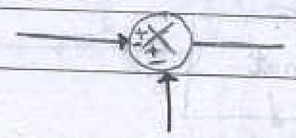
$$\boxed{\begin{aligned} \alpha &= \frac{R_2}{R_1R_2C} \\ \beta &= \frac{R_1+R_2}{R_1R_2C} \end{aligned}}$$

This ckt is called lead compensat^n.

→ BLOCK DIAGRAM:

Block Diagram is pictorial representat^n of given system. It is very simple way of representing the given complicated practical system.

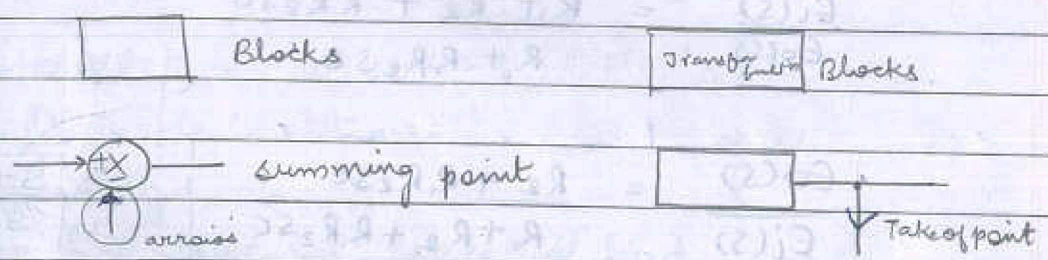
for a close loop system the funct^n comparing different signal indicated by summing point.



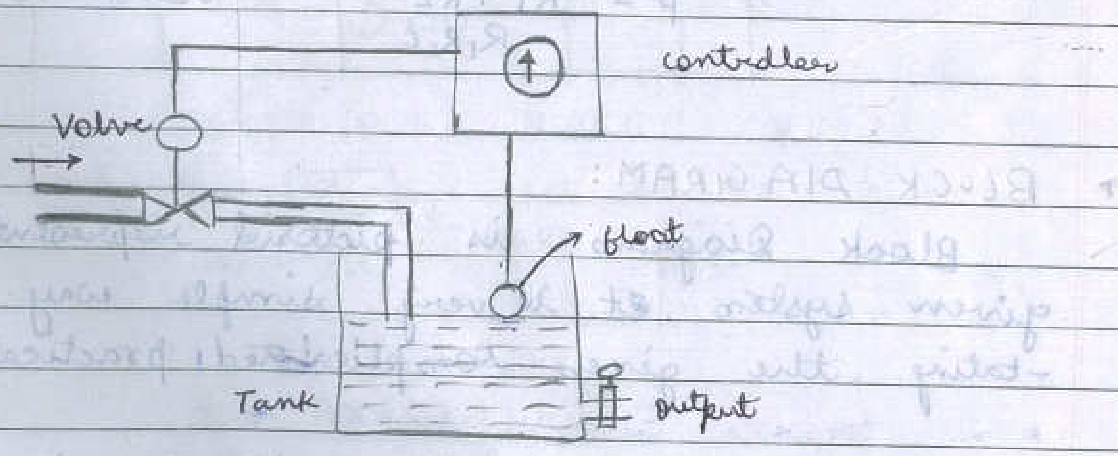
summing point

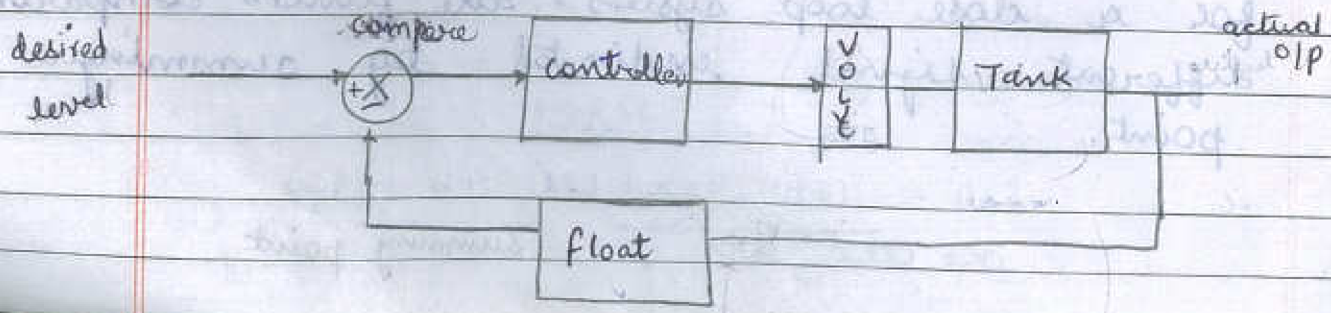
- A point from which signal is taken for the feedback purpose is indicated by take of point.
- Block diagram has following 5 basic elements

1. Blocks
2. Transfer funct^n of element shown inside blocks
3. Summing point (compare the signal)
4. Take of point
5. Arrows.



• Q. ex: Liquid level system:





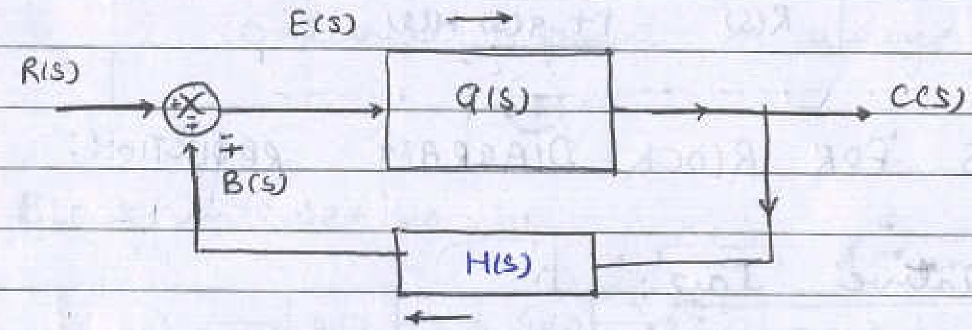
→ ADVATAGE OF BLOCK DIAGRAM:

1. Simple to construct the block diagram for complicated system.
2. The funct^n of individual element can be visualized from block diagram
3. over all closed loop transfer funct^n can be easily calculate by block diagram reduction Rule

→ DISADVANTAGE:

1. Block diagram does not include any informat^n above the physical construction of the system.
2. Source of energy is generally not show in the block diagram.

→ Derivation of transfer function:



R(S): laplace of Reference input r(t)
B(S): laplace of f/b signal b(t)
E(S): laplace of error signal e(t)
C(S): laplace of o/p c(t)
G(S) : laplace of equivalent forward path.
H(S) : laplace of equivalent f/b signal

$E(s) = R(s) \mp B(s)$ —(i)
$B(s) = C(s) \cdot H(s)$ —(ii)
$C(s) = E(s) \cdot G(s)$ —(iii)

$E(s) = R(s) \mp [C(s) \cdot H(s)]$
$E(s) = R(s) \mp [[E(s)\ G(s)] H(s)]$
or $C(s) = [R(s) \mp B(s)]\ G(s) = [R(s) \mp C(s)\ H(s)]\ G(s)$
$C(s) = G(s)\ R(s) \mp H(s)\ G(s) \Rightarrow R(s)G(s) \mp C(s)H(s)G(s) = C(s)$

$C(s)[1 \pm H(s)G(s)] = G(s)R(s)$

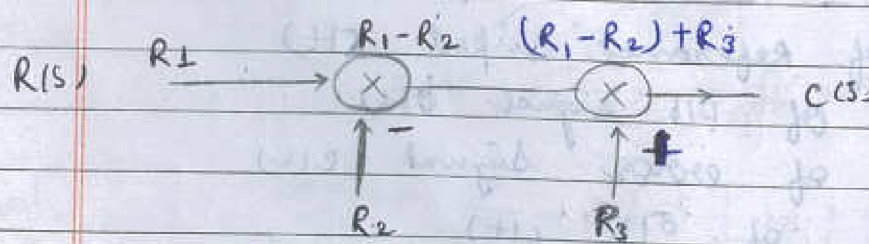
$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)\,H(s)}}$$ + ve f/b, − ve f/b

(i) +ve f/b : $\boxed{\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1 - G(s)H(s)}}$

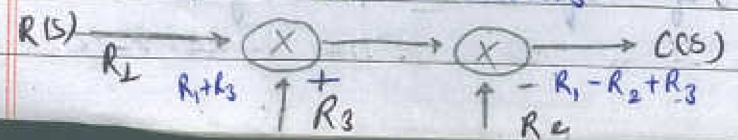
(ii) -ve f/b : $\boxed{\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1 + G(s)H(s)}}$

• RULES FOR BLOCK DIAGRAM REDUCTION:

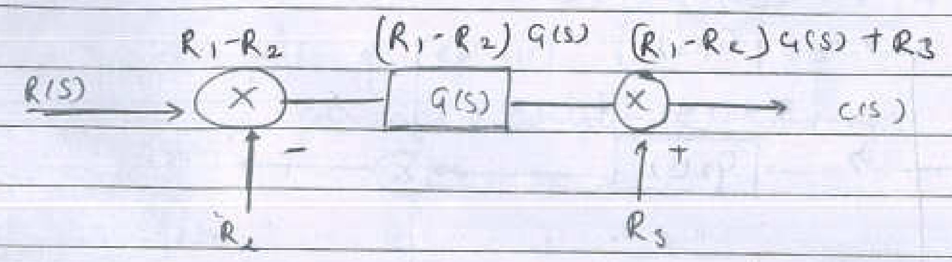
1. Associative law:

eg1:


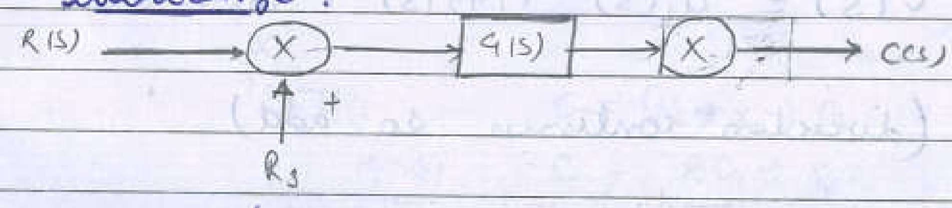
$C(s) = R_1 - R_2 + R_3$ —(i) interchange summing point



$C(s) = R_1 - R_2 + R_3$ —(ii)
eqn(i) = eqn(ii) so summing point change

eg2


$C(s) = (R_1 - R_2)\,G(s) + R_3$ —(i)

• interchange:


$C(s) = (R_1 + R_3)\ G(s) + (-R_2)$ —(ii)
eqn(i) ≠ eqn(ii), not interchange blocks and take of point

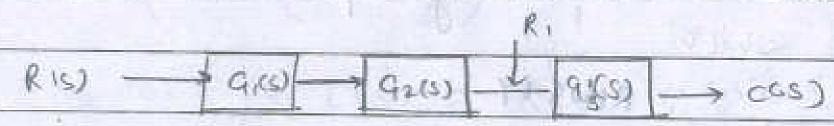
$\boxed{\text{2 continues point is always exchange}}$

2. Block in series:

eg1:


series → multiple

$C(s) = G_1(s) \cdot G_2(s) \cdot G_3(s)$

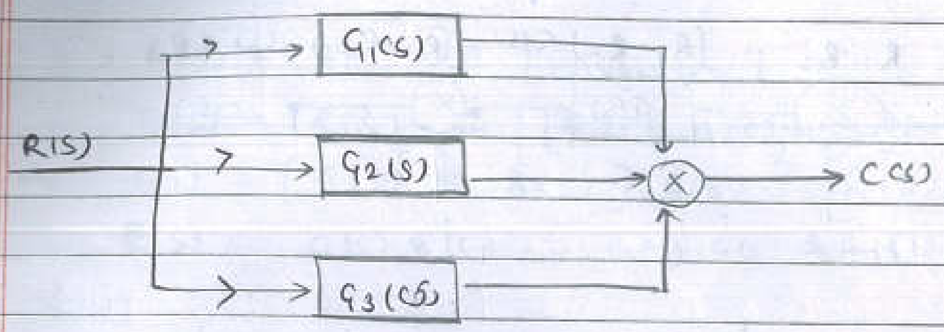
eg2:


not a series so not multiple

$\boxed{\text{continues blocks are always multiple.}}$

3 Block in Parallel:

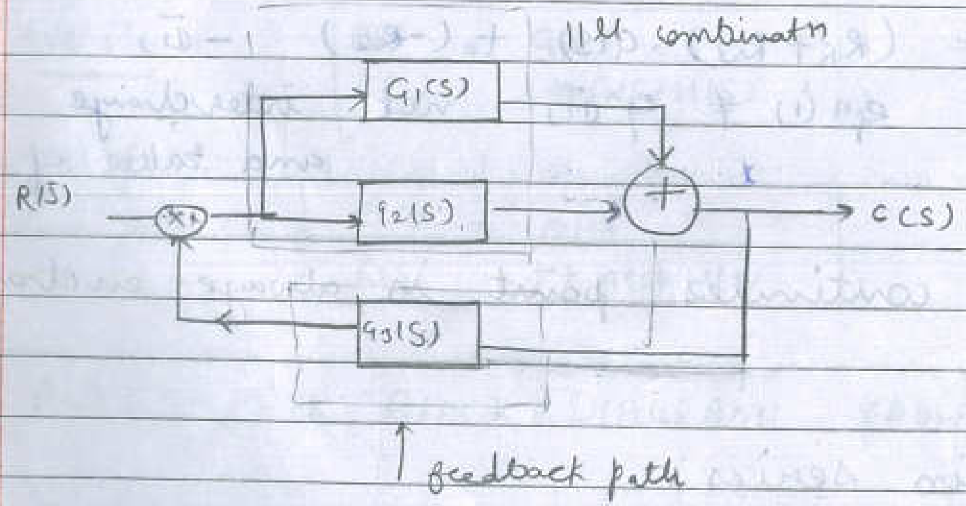
eg1.



$C(s) = G_1(s) + G_2(s) + G_3(s)$ —(i)

(direction continues so add)

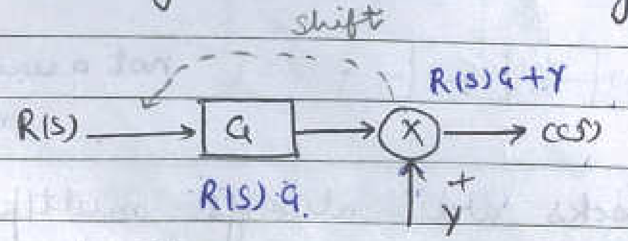
eg2.



$\boxed{\text{2 and 3 direction is same so add}}$

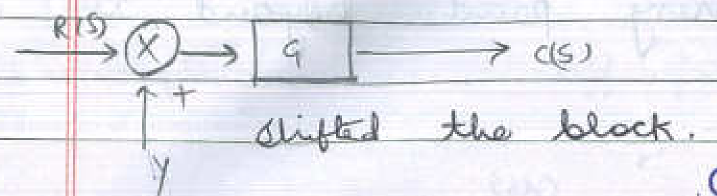
4. Shifting a summing point: behind the block

eg1.



not shift o/p: $C(s) = R(s)G + Y$ —(i)

eq^n(a)



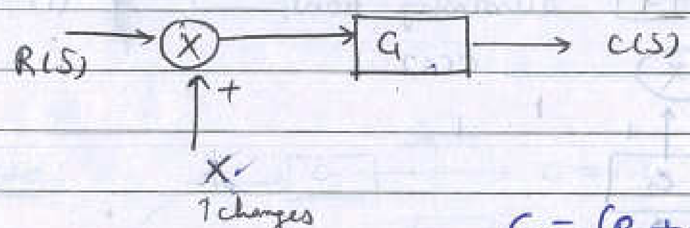
shifted the block.

$C(s) = (R(s) + Y)G$ —(ii)

$eq^n(i) \neq eq^n(ii)$

so not shift o/p change

- Modification:



$C = (R + X)G$ —(i)

eq^n(a) $C = RG + Y$ —(ii)

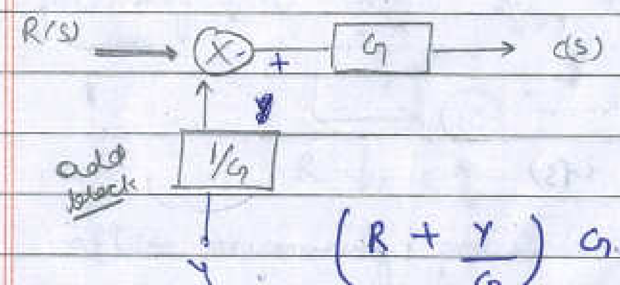
$(R + X)G = RG + X$

$RG + XG = RG + Y$

$Y = XG$

$\boxed{X = \frac{Y}{G}}$

- Shift the summing point add the block is $\boxed{1/G}$



$\left(R + \frac{Y}{G}\right)G = C(s)$

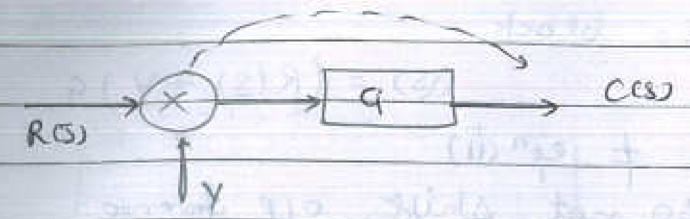
$C(s) = RG + Y \Rightarrow C(s)$

$C(s) = RG + Y$ —(b)

$eq^n(a) = eq^n(b)$ shift the summing point

5 → Shifting a summing point beyond the block:

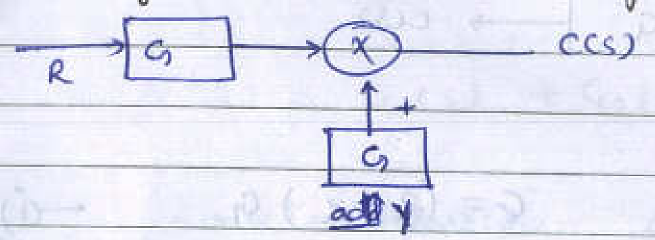
eg1



$C(S) = (R+Y)G$

$C(S) = RG + YG$ —(i)

• shifting: add $\boxed{G}$ summing point $eq^n (i) = eq^n (ii)$

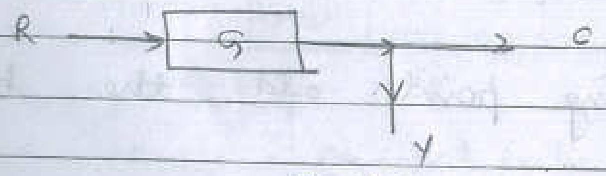


$C(S) = RG + YG$ —(ii)

eq^n (i) and (ii) equal

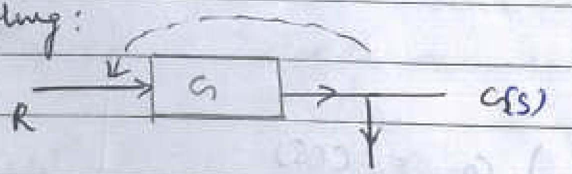
6 Shifting a take off point behind the block:

eg1



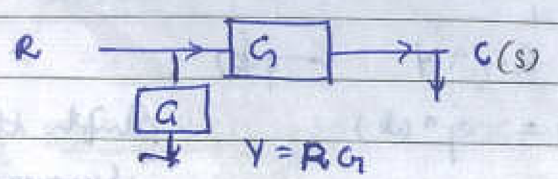
$C = RG$

• shifting:



R

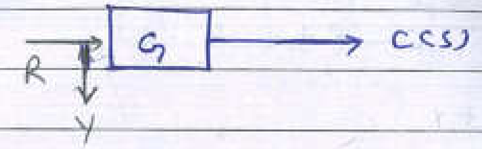
add:



$Y = RG$

7 Shifting the Takeoff point beyond the block. (ahead)

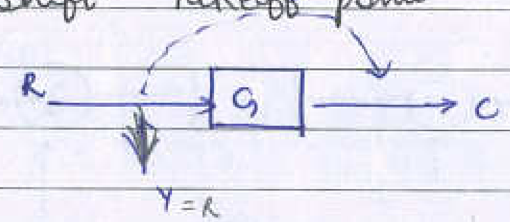
eg1



$C = RG$ —(i)

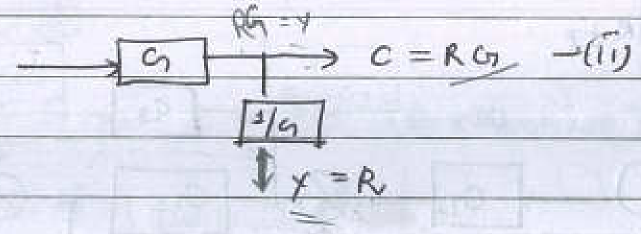
$Y = R$

• Shift: Takeoff point

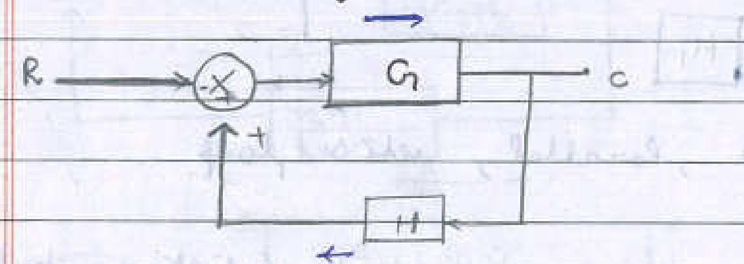


$C = RG$

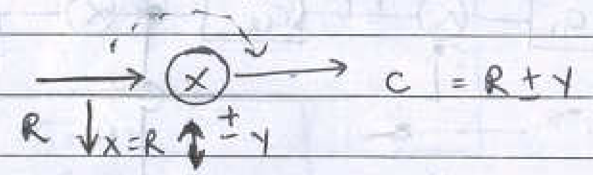
Add:



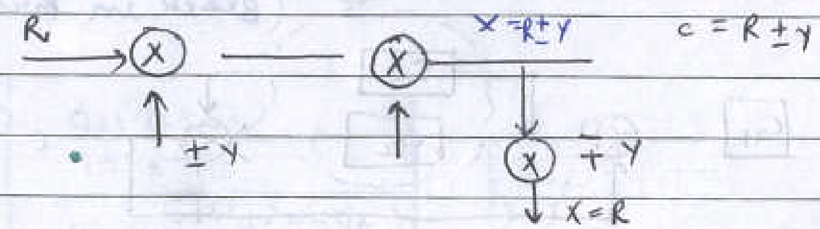
8 → closed loop system:



9. Shifting a take off point after a summing point:

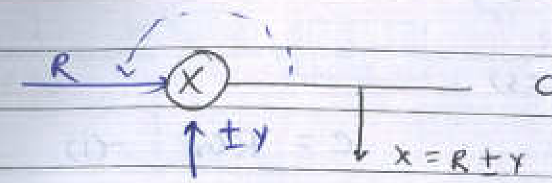


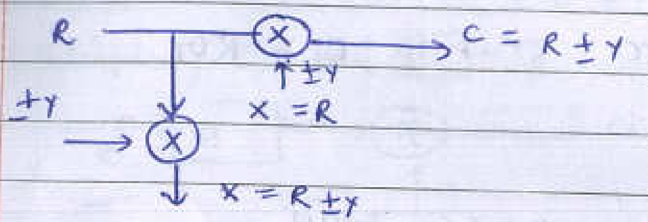
after summing point:



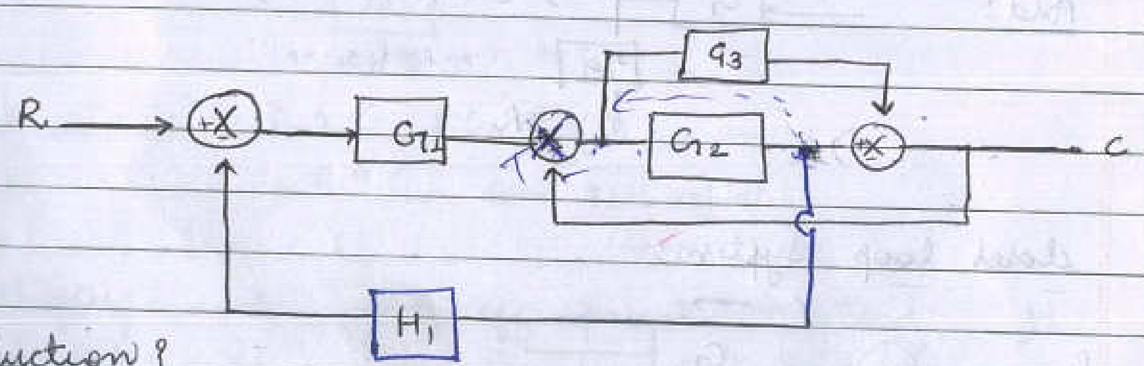
→ avoid takeoff point before and after summing point because one summing point is add.

10 Shifting a take off point before summing point:





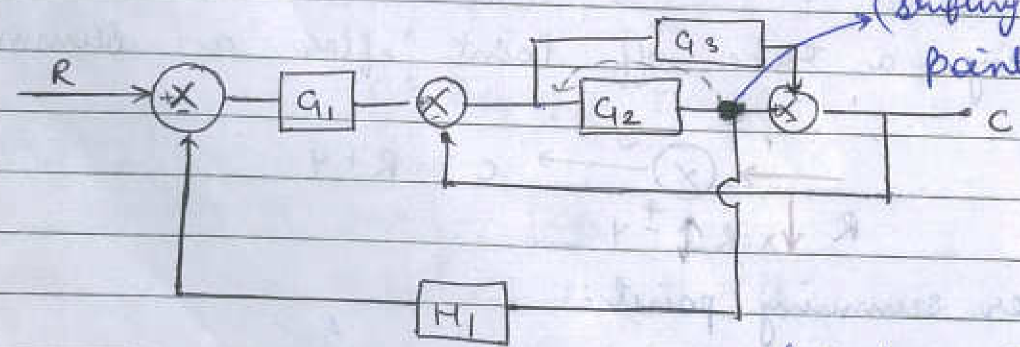
Nr: 1
Imp



Deduction?

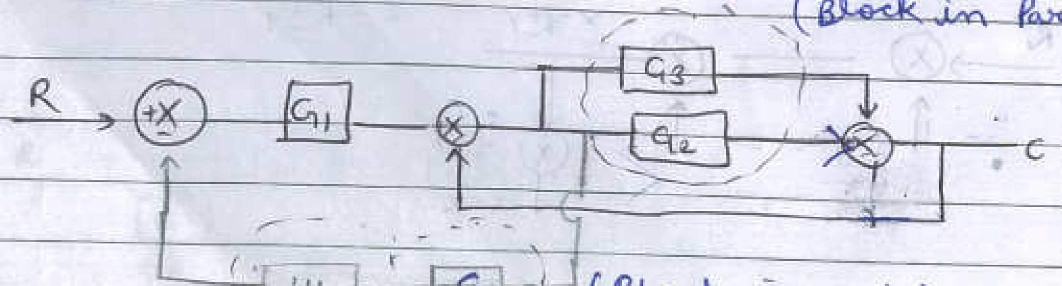
Sol^n: Rules: series, parallel, closed loop convert

Case 1:

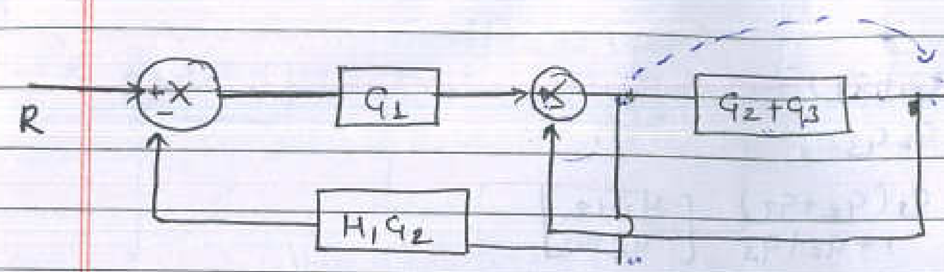


(shifting a take off point before a block)

(Block in Parallel)

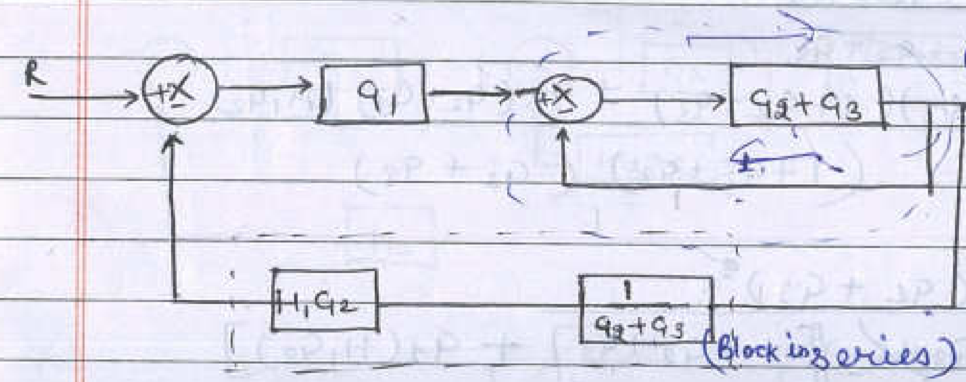


(Block in series)



Shifting the take off point after the block

(closed loop)

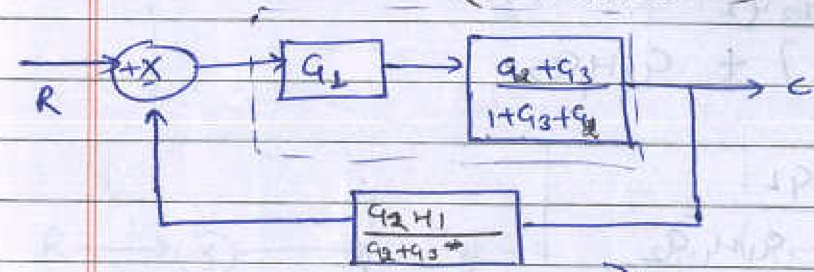


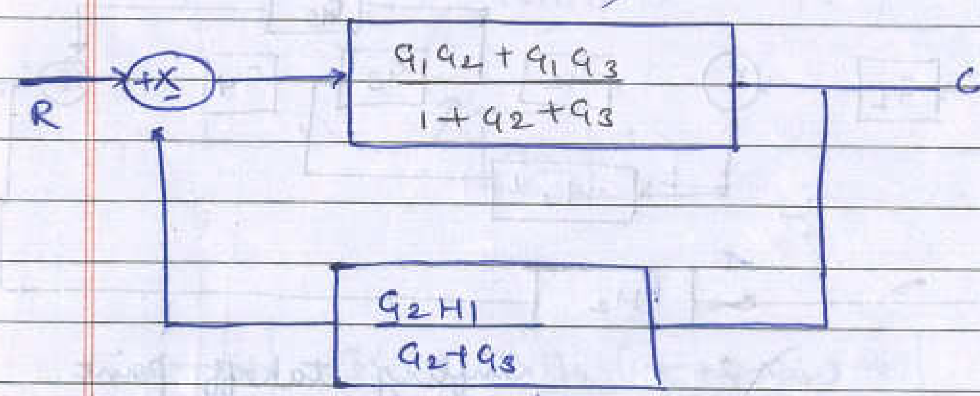
(Block in series)

$\boxed{\frac{C}{R}} = \frac{G_1G_2+G_1G_3}{1+G_2+G_3+G_1G_2}$

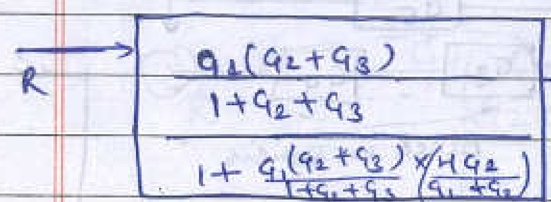
Ans

(Block in series)





(closed loop)



$$\frac{\frac{G_1(G_2+G_3)}{1+G_2+G_3}}{1+\frac{G_1(G_2+G_3)}{1+G_2+G_3}\times\frac{H_1G_2}{(G_2+G_3)}}$$

$$\frac{C}{R} = \frac{\dfrac{G_1(G_2+G_3)}{1+G_2G_3}}{1+\dfrac{G_1(G_2+G_3)}{1+G_2+G_3}\left[\dfrac{H_1G_2}{G_2+G_3}\right]}$$

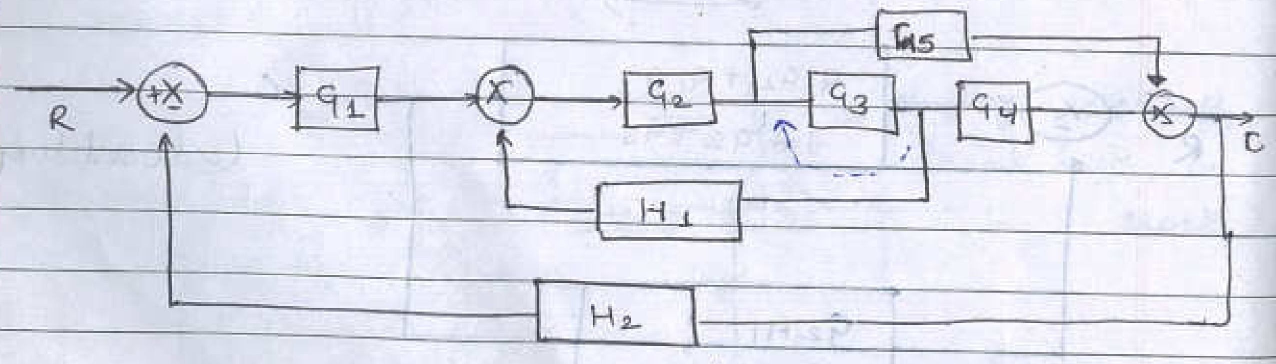
$$= \frac{\dfrac{G_1(G_2+G_3)}{\cancel{1+G_2+G_3}}}{\dfrac{(G_2+G_3)(1+G_2+G_3)+G_1(G_2+G_3)(H_1G_2)}{(\cancel{1+G_2+G_3})(G_2+G_3)}}$$

$$= \frac{G_1(G_2+G_3)}{(G_2+G_3)\left[[1+G_2+G_3]+G_1(H_1G_2)\right]}$$

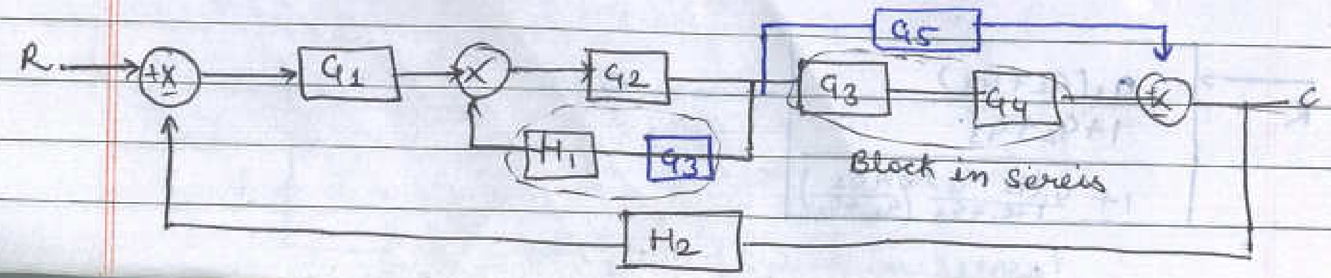
$$= \frac{G_1G_2+G_3G_1}{[1+G_2+G_3]+G_1H_1G_2}$$

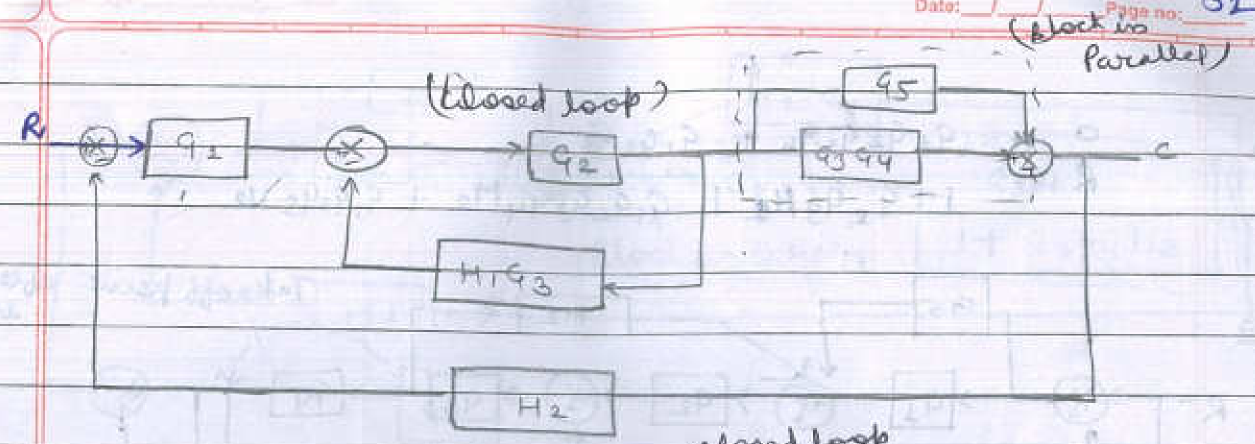
$$\boxed{\frac{C}{R} = \frac{G_1G_2+G_3G_1}{1+G_2+G_3+G_1H_1G_2}}$$

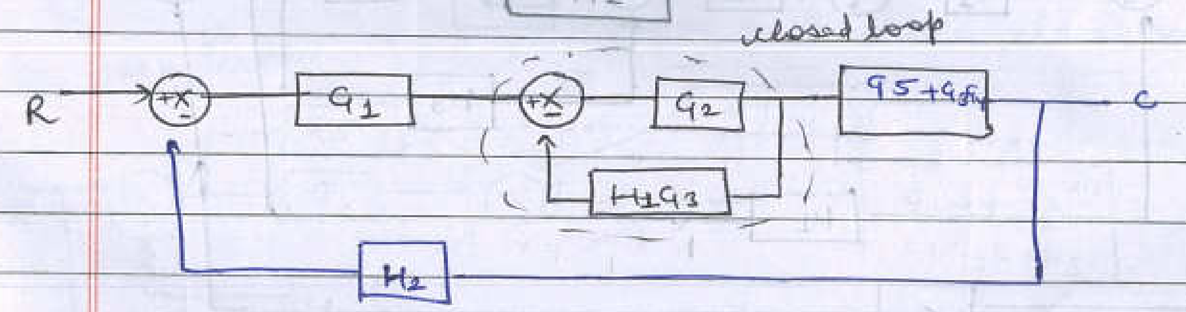
Nr.2 imp

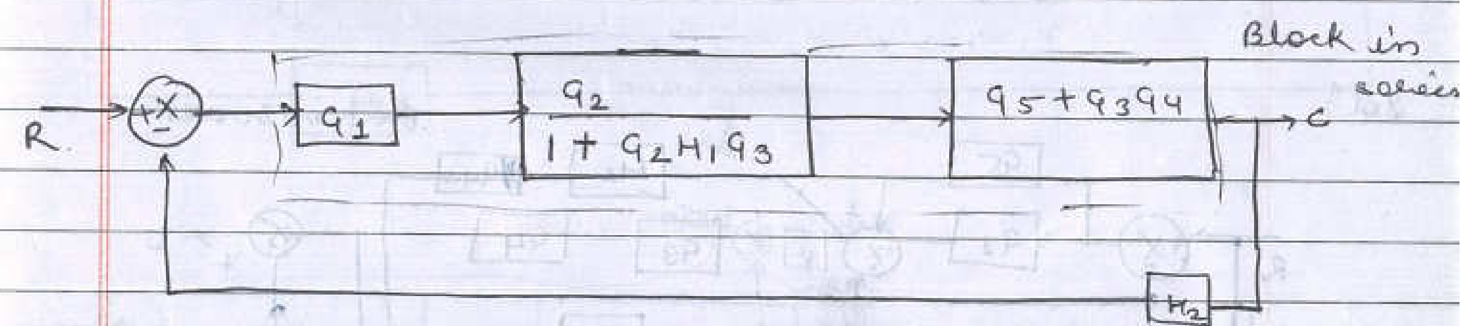


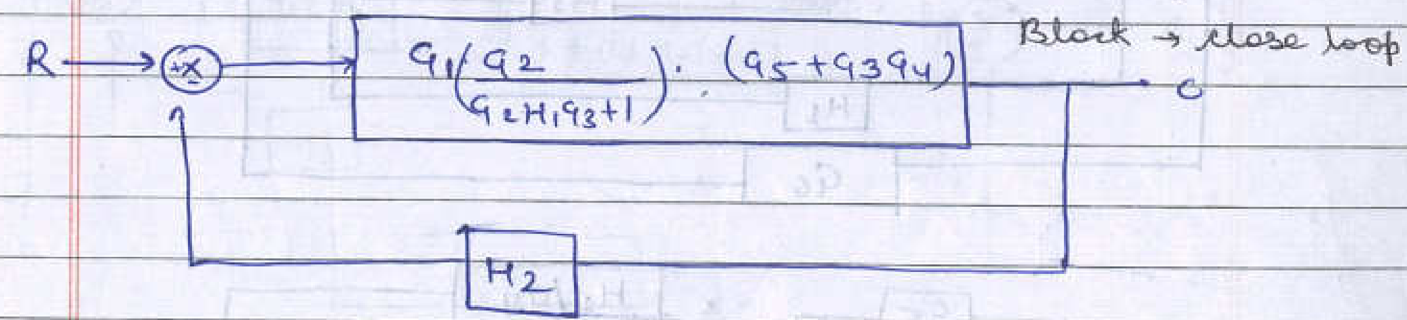
Shifting takeoff point before an block

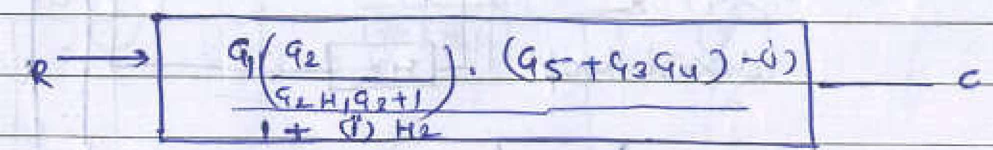












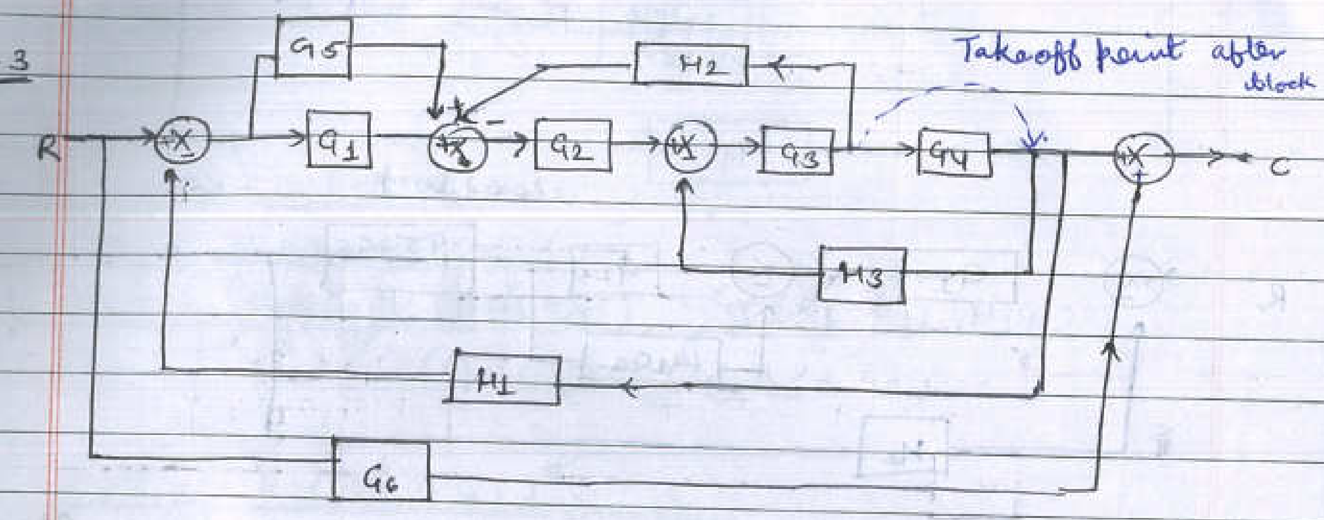
$$\frac{C}{R} = \frac{G_1\left(\dfrac{G_2}{G_2H_1G_3+1}\right)(G_5+G_3G_4)}{1+\left[G_1\dfrac{G_2}{G_2H_1G_3+1}(G_5+G_3G_4).H_2\right]}$$

Ans

$$\frac{C}{R} = \frac{G_1G_2G_3G_4 + G_1G_2G_3}{1+G_2G_3H_3 + G_1G_2G_3G_4H_2 + G_1G_2G_5H_2}$$

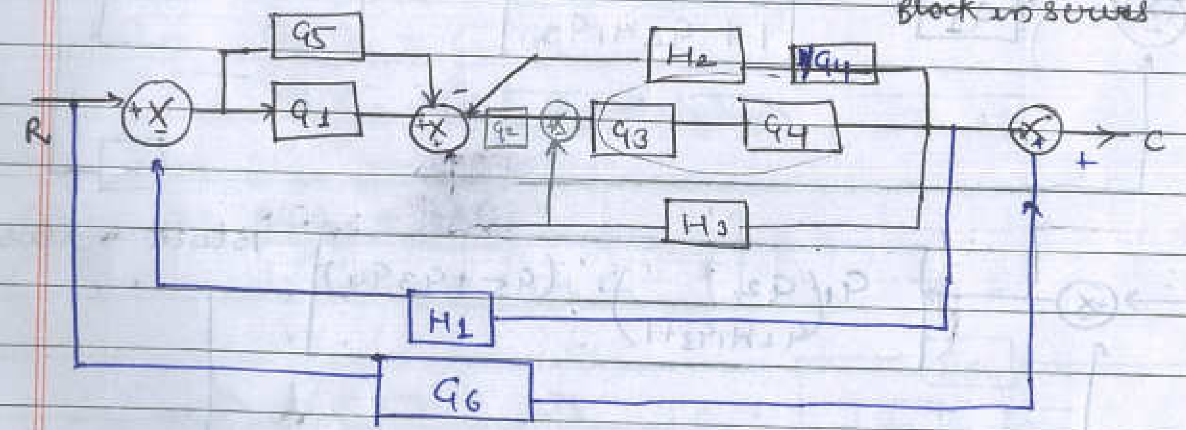
Nr: 3

Take off point after block

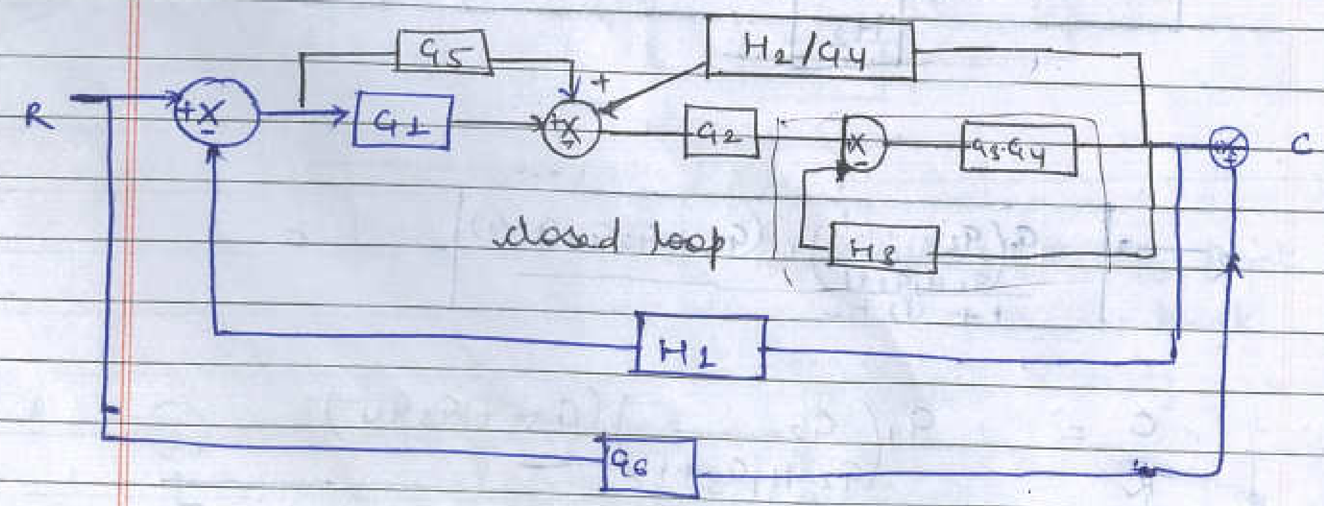


Sol^n:

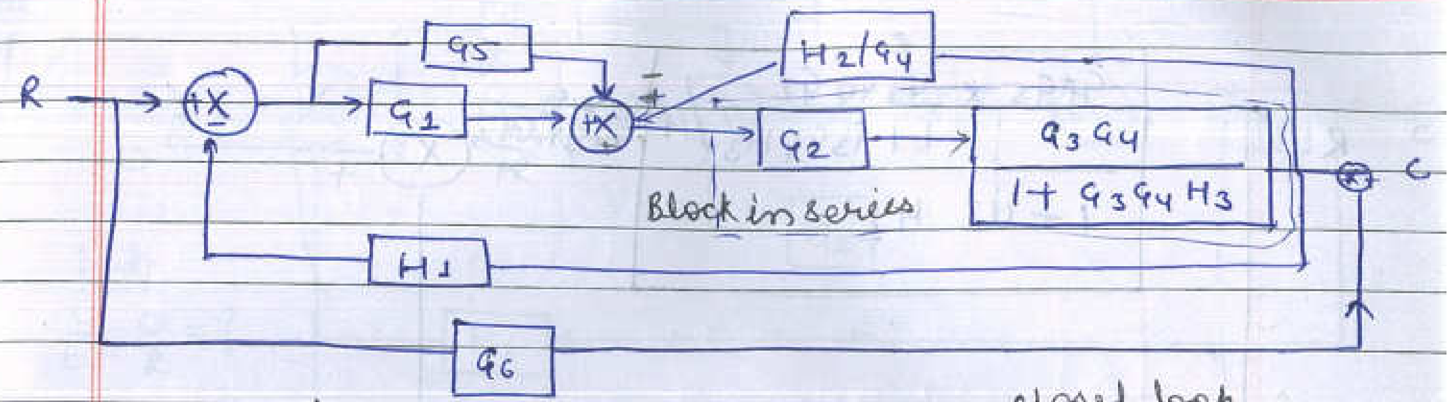
Block in series



closed loop

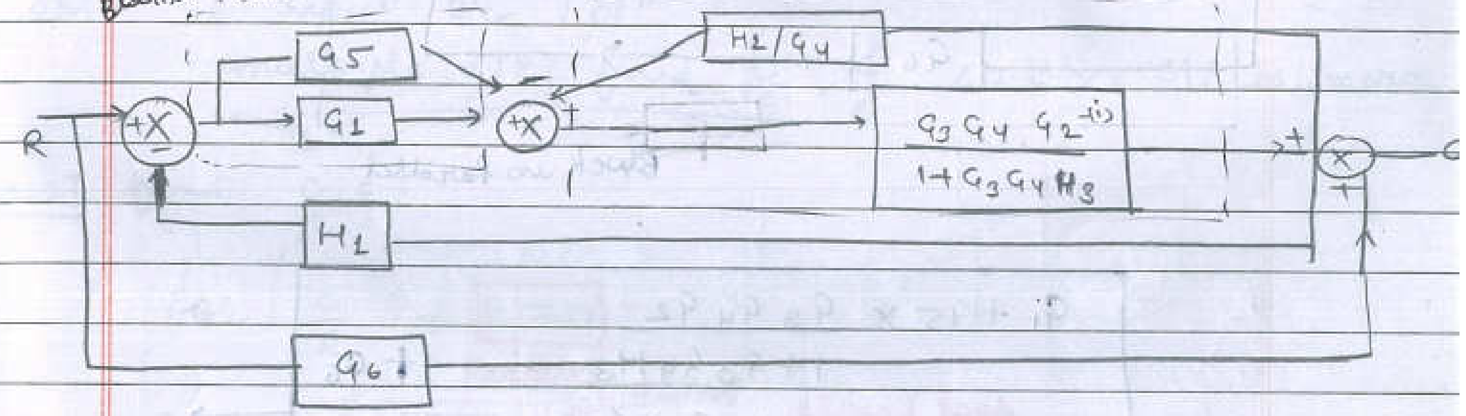


Block in series

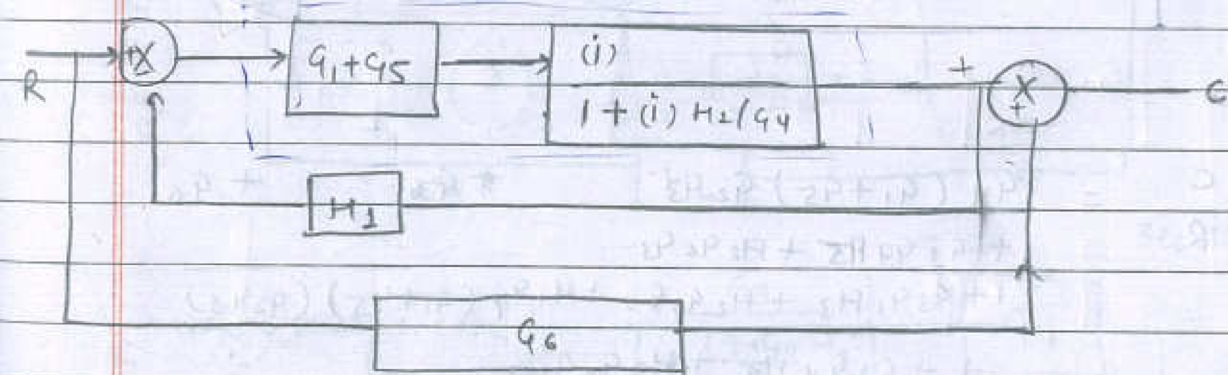


closed loop

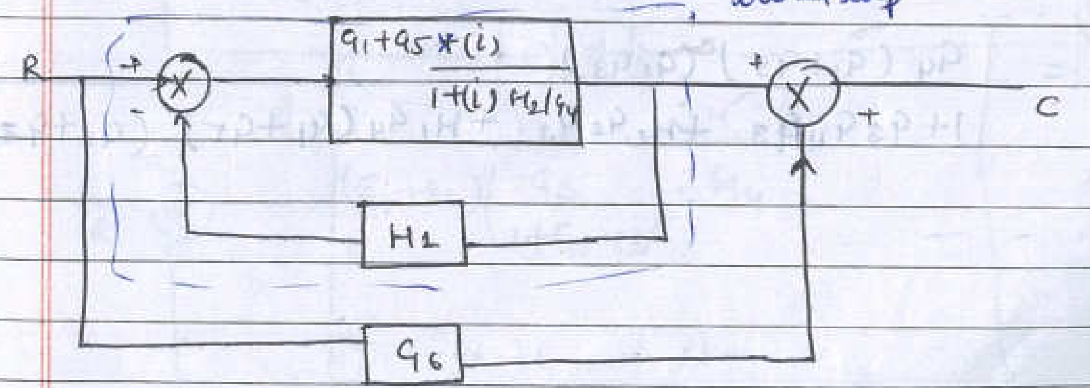
Block in Parallel

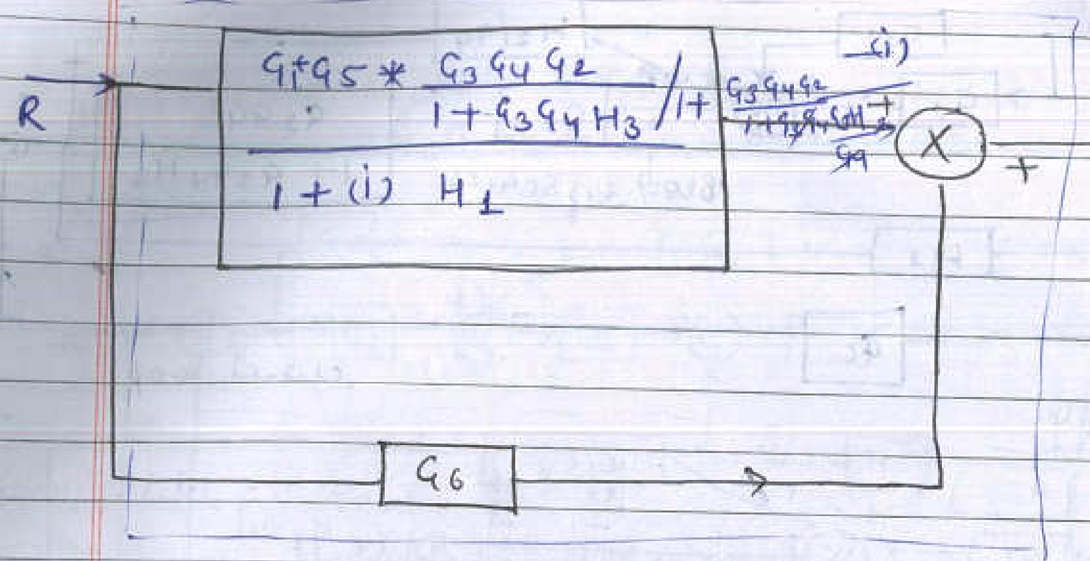


Block in series

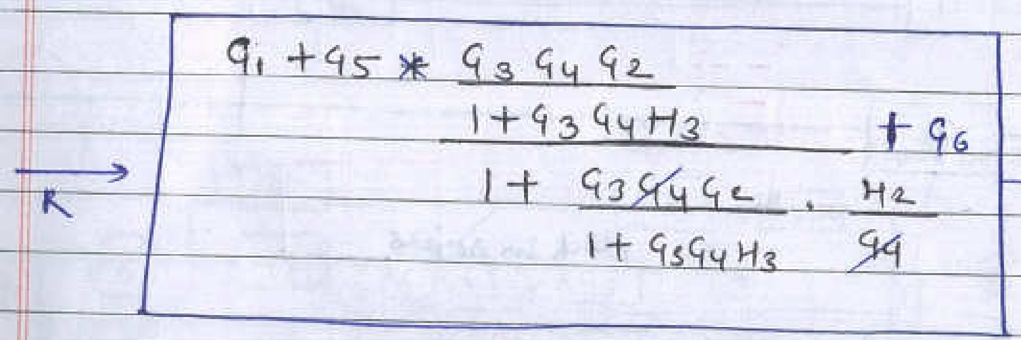


closed loop





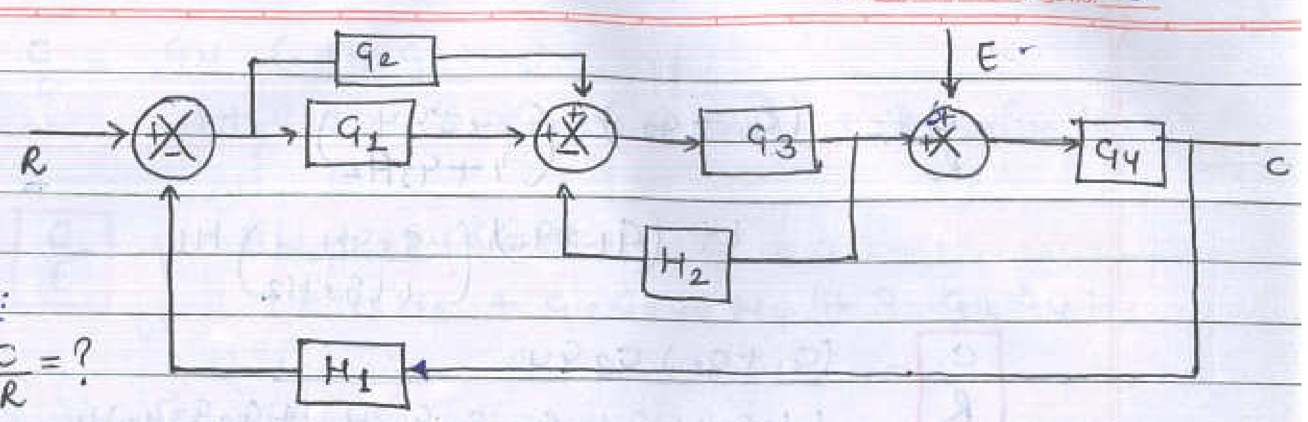
Block in Parallel



$$\frac{C}{R} = \frac{\dfrac{G_4(G_1+G_5)G_2G_3}{1+G_3G_4\cancel{H_3}+H_2G_2G_3}}{\dfrac{1+G_3G_1H_3+H_2G_2G_3+H_1G_4(G_1+G_5)(G_2G_3)}{1+G_3G_4\cancel{H_3}+H_2G_2G_3}} \cancel{* G_6} + G_6$$

$$\frac{C}{R} = \frac{G_4(G_1+G_5)(G_2G_3)}{1+G_3G_4H_3+H_2G_2G_3+H_1G_4(G_1+G_5)(G_2+G_3)} + G_6$$

Q.



find: $\frac{C}{E}, \frac{C}{R} = ?$

Sol^n 2 input or 1 o/p

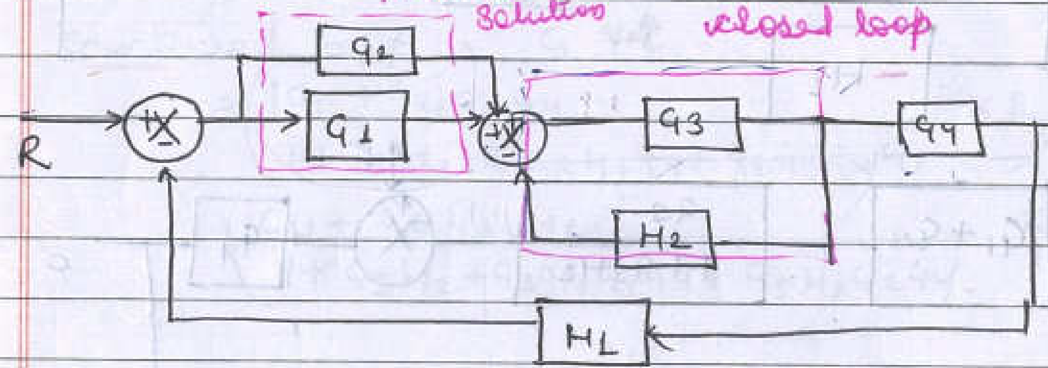
multiple i/p find $\frac{C}{E}, \frac{C}{R}$ so other i/p is zero.

Case I: find $\frac{C}{R} = ?$

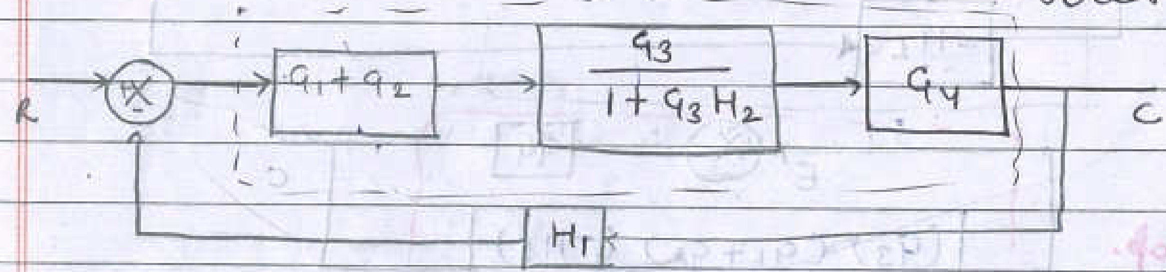
for $\frac{C}{R}$, $\boxed{E=0}$

Parallel solution

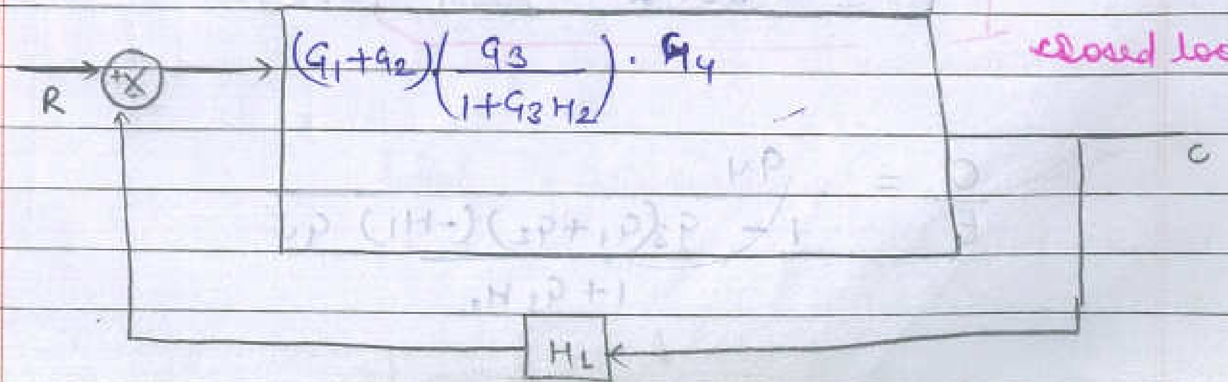
closed loop



series sol^n



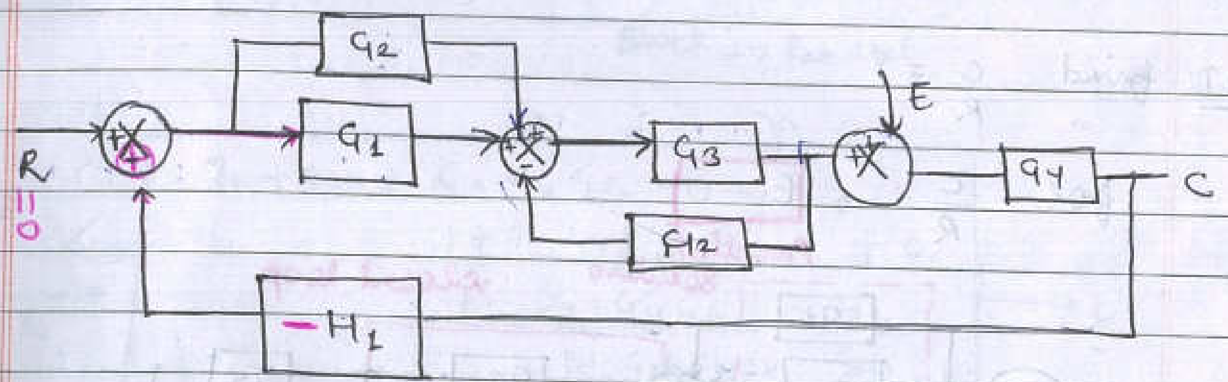
closed loop



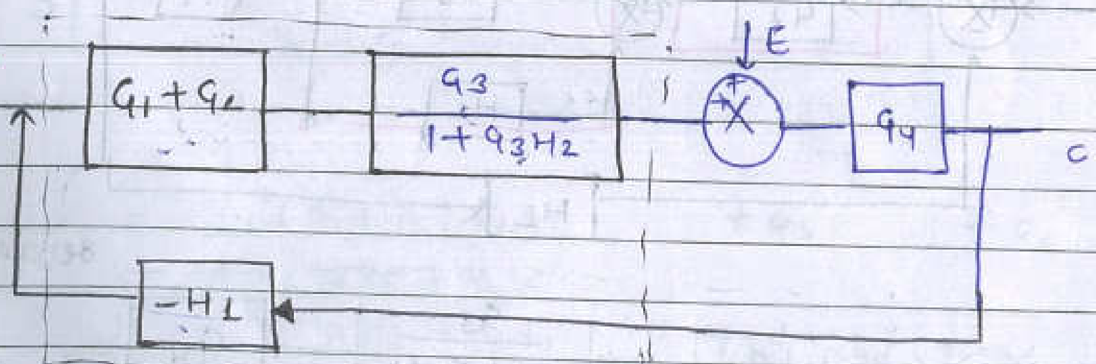
$$\frac{C}{R} = \frac{(G_1+G_2)\cdot\left(\frac{G_3G_4}{1+G_3H_2}\right)}{1+(G_1+G_2)\left(\frac{G_3G_4}{1+G_3H_2}\right)H_1}$$

$$\boxed{\frac{C}{R}} = \frac{(G_1+G_2)G_3G_4}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1}$$

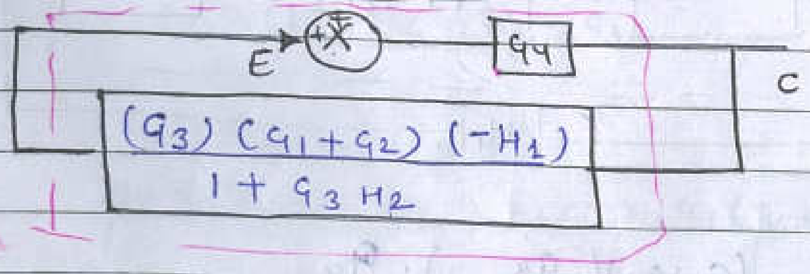
case II : for $\frac{C}{E}$; $\boxed{R=0}$



Block in series



closed loop.



$$\frac{C}{E} = \frac{G_4}{1-\frac{G_3(G_1+G_2)(-H_1)G_4}{1+G_3H_2}}$$

$$\frac{C}{E} = \frac{G_4(1+G_3H_2)}{1+G_3H_2+G_3G_1G_4H_1+G_3G_2G_4H_1}$$

$$\boxed{\frac{C}{E}} = \frac{G_4+G_4G_3H_2}{1+G_3H_2+G_3G_1G_4H_1+G_3G_2G_4H_1}$$

→ combined o/p = $(C_1 + C_2)$

$$\frac{C_1}{R} + \frac{C_2}{E}$$

$$\frac{C_1}{R} = \frac{(G_1+G_2)G_3G_4}{1+G_3H_2+G_1G_3H_1G_4+G_2G_3G_4H_1}$$

$$\frac{C_1}{E} = \frac{G_4+G_4G_3H_2}{1+G_3H_2+G_3G_1G_4H_1+G_3G_2G_4H_1}$$

combined o/p = $C_1 + C_2$

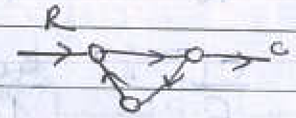
$$= \left(\frac{(G_1+G_2)G_3G_4}{1+G_3H_2+G_1G_3H_1G_4+G_2G_3G_4H_1} \times R\right) + \left(\frac{G_4+G_4G_3H_2}{1+G_3H_2+G_3H_1G_1G_4+G_3H_1G_2G_4} \times E\right)$$

Signal flow graphs ; (SFG):

Signal flow graph developed by 'S.J. Mason's' which does not required any reduction process because of available flow graph gain formulla which relate the i/p and o/p system variables.

- definations :

1. Nodes: sum of i/p and o/p



2. Branch:
3. node as a summing point:

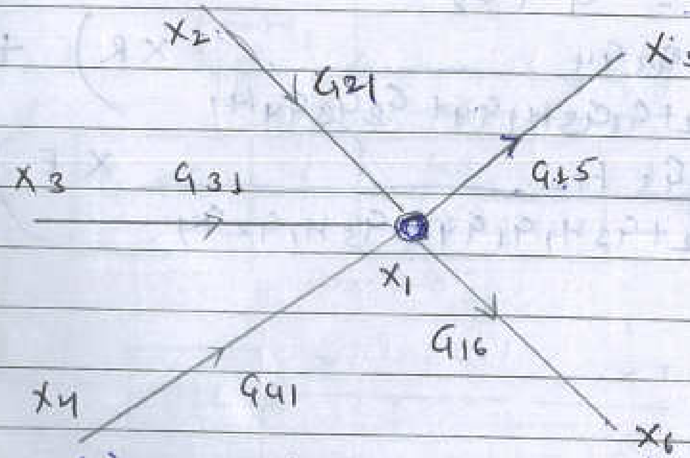
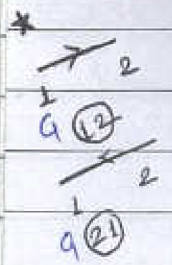


figure: node as a summing point as a Transmitting point



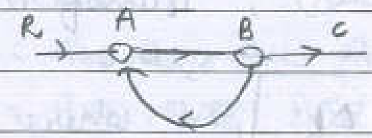
$$X_1 = X_2 G_{21} + X_2 G_{31} + G_{41} X_4$$

node as a summing point

$$X_5 = X_1 G_{15} \; ; \; X_6 = G_{16} X_1$$

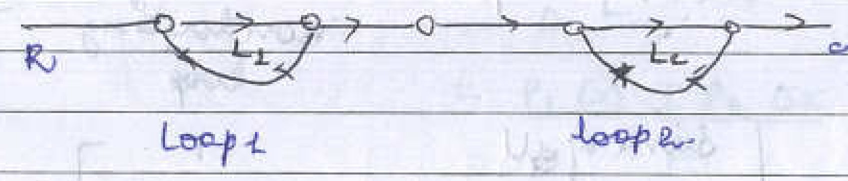
node as a transmitting point

4. Notations: a_{ij} $i \rightarrow j$
5. I/P node or source:
   incoming one node other outgoing node
6. O/P node or load:
7. Path: one way follow but not repeated
8. forward path: I/P → O/P
9. feedback path: O/P → I/P
10. loop:



- It is the path which starting and ending at the same node.
  - 2 path → forward path and feedback path.

11. Non touching loop:



combinats of 2 non touching loop

loops are not touching at a point so its called non touching loop.

→ Numerical:

Q 1. SET of the equation:

$$X_2 = a_{12} X_1 + a_{32} X_3 + a_{42} X_4 + a_{52} X_5$$
$$X_3 = a_{23} X_2$$
$$X_4 = a_{34} X_3 + a_{44} X_4$$
$$X_5 = a_{35} X_3 + a_{45} X_4$$

where X_1 is I/P variable and X_5 O/P variable forward path I/P → O/P not Repetition

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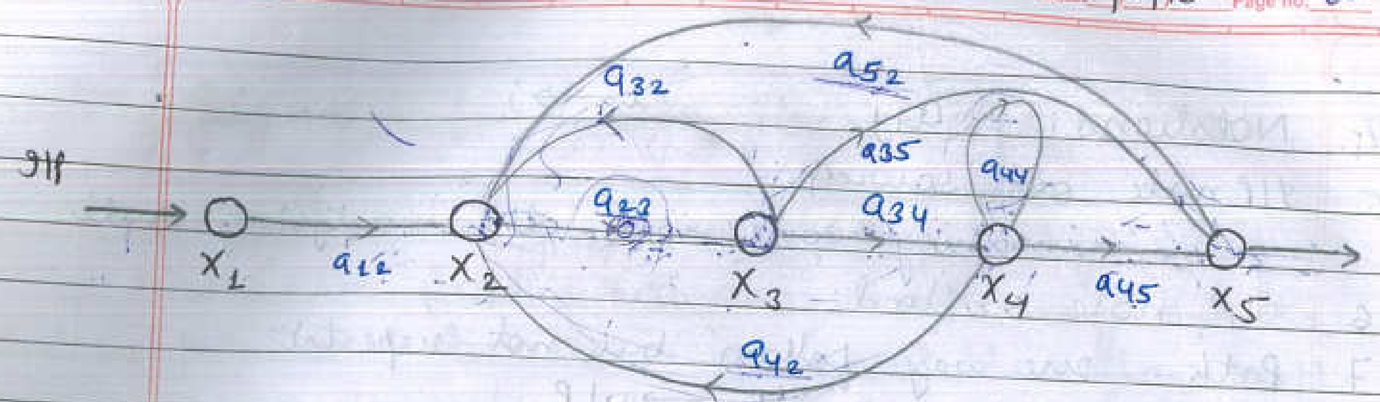

The overall gain from I/P to O/P obtain by Mason's gain formula', Transfer function

$$\boxed{T = \frac{1}{\Delta} \sum_K P_K \Delta_K}$$

where P_K: K^{th} forward path

Δ : 1 − [sum of all individual loop] + [Sum of all combination of all two non touching loop] − [combination of 3 non touching loop] ...

$\Delta = 1 -$ [sum of all individual loop] + [sum of all combination of all two non touching loop] − [Sum of all combination of 3 non touching loop]

$\Delta_K = 1 -$ [sum of all individual loop] + [sum of all combination of 2 non touching loop] − [Sum of all combination of 3 non touching loop]

↓ forward Path

Δ-1-

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- Forward path:

$P_1 = a_{12}\, a_{23}\, a_{34}\, a_{45}$ $\boxed{K=2}$

$P_2 = a_{12}\, a_{23}\, a_{35}$ no of forward path = 2

- loop:

$L_1 = a_{23}\, a_{32}$

$L_2 = a_{23}\, a_{34}\, a_{42}$

$L_3 = a_{44}$ no of loop = 5

$L_4 = a_{23}\, a_{34}\, a_{45}\, a_{32}$

$L_5 = \cancel{a_{23}}\, \cancel{a_{35}}\, \cancel{a_{52}} = a_{23}\, a_{35}\, a_{52}$

Mason's gain formula:

$$\boxed{T = \frac{1}{\Delta} \sum_K [P_K \Delta_K]}$$

$$T_F = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] \quad \text{-(ii)}$$

$$= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5] + [L_1L_3 + L_3L_5] - 0 \quad \text{-(i)}$$

- L_1L_3, L_3L_5 → (combination of 2 non touching loop)

$L_1L_3 = a_{23}\, a_{32}\, a_{44}$

$L_3L_5 = a_{44}\, a_{23}\, a_{35}\, a_{52}$

- Combination of 3 non touching loop = 0

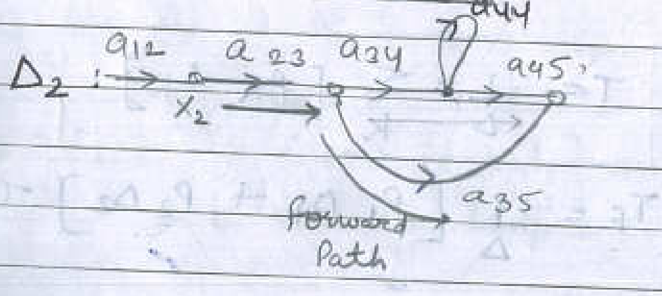
(i) ⇒ $\Delta = 1 - [L_1+L_2+L_3+L_4+L_5] + [L_1L_3 + L_3L_5]$

$$\boxed{\Delta = 1 - [a_{23}a_{32} + a_{23}a_{34}a_{42} + a_{44} + a_{23}a_{34}a_{45}a_{32} + a_{23}a_{35}a_{52}] + [a_{23}a_{32}a_{44} + a_{44}a_{23}a_{35}a_{52}]}$$

Δ_1: combinatn of indvival loop = 0
(all loops are touch in forward path)

$\boxed{\Delta_1 = 0}$ $\boxed{\Delta_1 = 0}$

Δ_2:



non touching loop → forward path $= a_{44}$

$$\boxed{\Delta_2 = 1 - [a_{44}]}$$

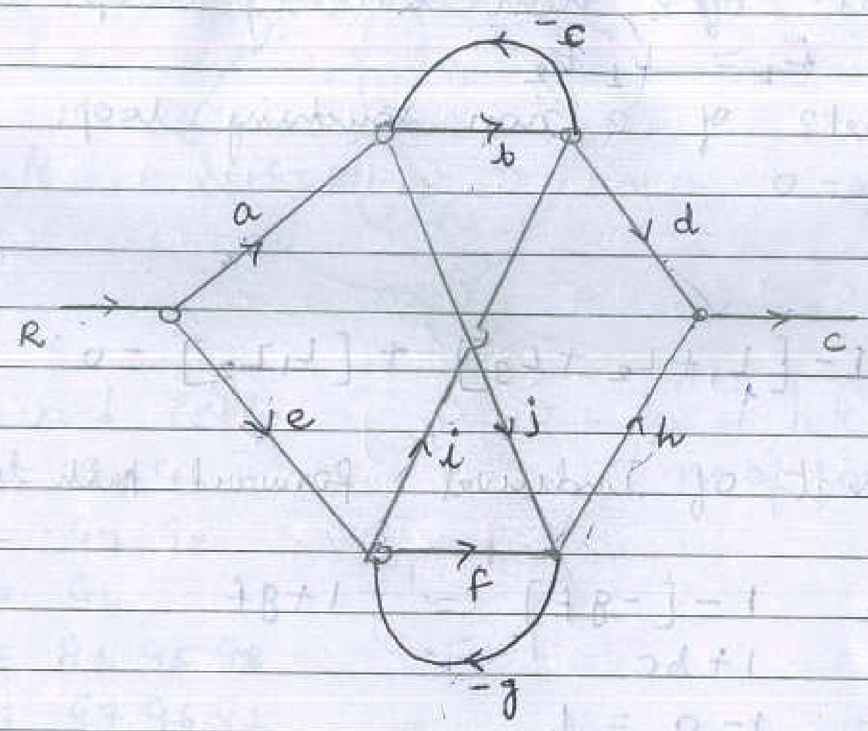
(ii): $T = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$

$$T = \frac{P_1 \times 0 + P_2[1 - a_{44}]}{\Delta}$$

($P_2 = a_{12}a_{23}a_{35}$ put)

$$T = \frac{P_2 - P_2 a_{44}}{1 - [a_{23}a_{32} + a_{23}a_{34}a_{42} + a_{44} + a_{23}a_{34}a_{45}a_{32} + a_{23}a_{35}a_{52}] + [a_{23}a_{32}a_{44} + a_{44}a_{35}a_{23}a_{52}]}$$

Q.2
Imp



Mason's gain formula:

$$\boxed{T = \frac{1}{\Delta}\sum_K P_K \Delta_K}$$

→ Forward Path:

$P_1 = R\,a\,b\,d\,C = abd$
$P_2 = R\,e\,f\,h\,C = efh$
$P_3 = R\,a\,i\,h\,C = aih$
$P_4 = R\,a\,j\,h\,C = \cancel{aih} - eibd$
$P_5 = e\,i\,d = eid$
$P_6 = -a\,j\,g\,i\,d = -ajgid$ $\boxed{K=6}$

no of loop = 6

→ loop: $L_1 = -bc$. no of loop = 3
$L_2 = -fg$.
$L_3 = cjgi = icjg$

→ combinat^n of 2 non touching loop:

$= L_1 L_2$

→ combinat^n of 3 non touching loop:

$= 0$

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_2] - 0 \quad -(1)$$

combinat^n of indivil forward path loop

$\Delta_1 = 1 - [-gf] = 1 + gf$

$\Delta_2 = 1 + bc$

$\Delta_3 = 1 - 0 = 1$

$\Delta_4 = 1$

$\Delta_5 = 1$

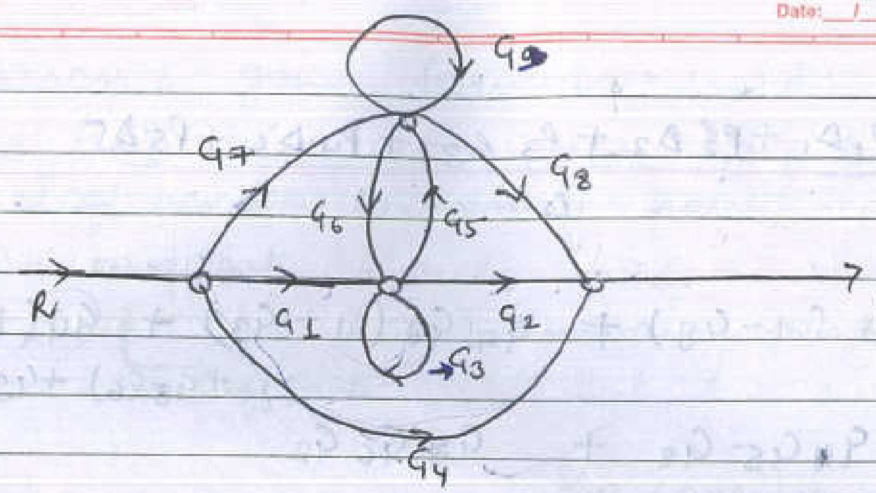
$\Delta_6 = 1$

$$T = \frac{1}{\Delta} \sum_K [P_K \Delta_K]$$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

$$T = \frac{abd(1+gf) + efh(1+bc) + aih(1) + 1(ajh) + ajd(1) + (-ajgid)\times 1}{1 - [-bc - fg + icjg] + [bcfg]}$$

Q3:



find the Transfer function:

→ Forward Path $\boxed{k = 5}$

no of forward path = 5

$P_1 = G_1 G_2$

$P_2 = G_7 G_8$

$P_3 = G_4$

$P_4 = G_1 G_5 G_8$

$P_5 = G_7 G_6 G_2$

$$\Delta = 1 - (G_9 + G_3 + G_5 G_6) + G_9 G_3$$

→ loop: no of loop: 5

$L_1 = G_5 G_6$

$L_2 = G_9$

$L_3 = G_3$

$L_4 = G_7 G_1 G_5$ (not i/p → o/p) not loop

$L_5 = G_6 G_2 G_8$ not loop

→ Combination of 2 non touching loop: '1'

$= L_2 L_3$

→ Combinat^n of 3 non touching loop = 0

→ combinat^n of indivial forward path loop:

$\Delta_1 = 1 - G_9$

$\Delta_2 = 1 - G_3$

$\Delta_3 = 1 - (G_9 + G_3 + G_5 G_6) + G_9 G_3$

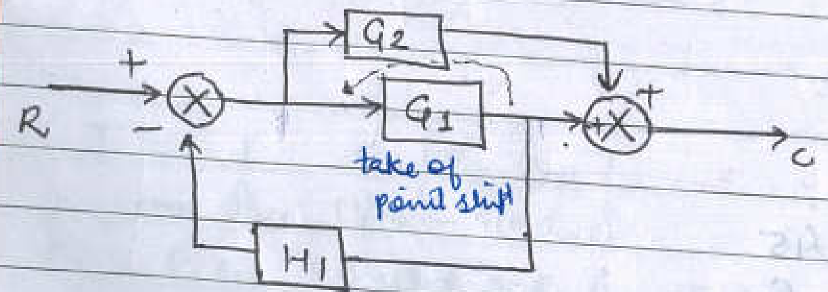
$\Delta_4 = 1 - 0 = 1$

$\Delta_5 = 1 - 0 = 1$

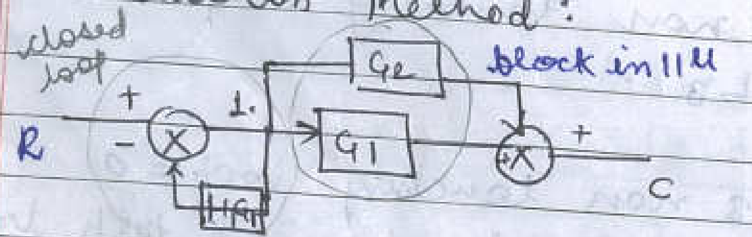
$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5}{\Delta}$$

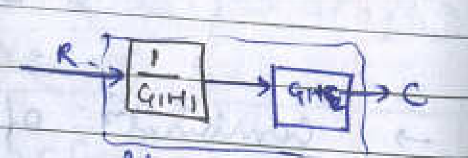
$$T = \frac{G_1G_2(1-G_9) + G_7G_8(1-G_3) + G_4(1-(G_9+G_3+G_5G_6)+G_9G_3)) + G_4G_5G_2 + G_7G_6G_2}{1-[G_9+G_3+G_5G_6]+G_9G_3}$$

eg: 4 → Draw the signal flow graph of given system and find out transfer function:



sol: I: Reduction Method:





block in series

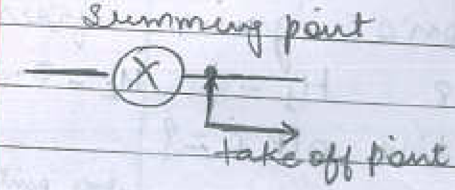
$$\boxed{\frac{C}{R} = \frac{G_1+G_2}{1+G_1H_1}}$$ Transfer function

II → Mason's gain formula used:

RULE:-

* Take off point, summing point consider: node

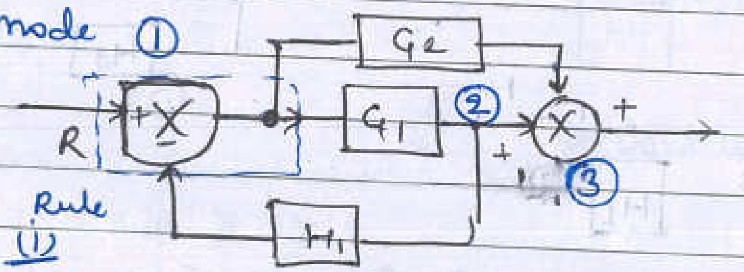
*
(i)



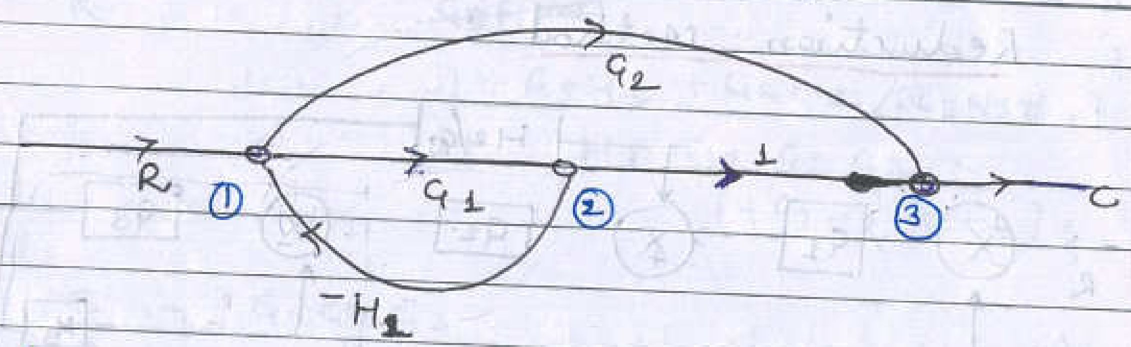
summing point → after take off point

So consider one node

II. Signal flow:



summing → before take off point not consider one node



forward path: $P_1 = G_1$
$P_2 = G_2$

$\boxed{K=2}$

loop $L_1 = -G_1H_1$

$\Delta = 1 - (-G_1H_1) = 1 + G_1H_1$

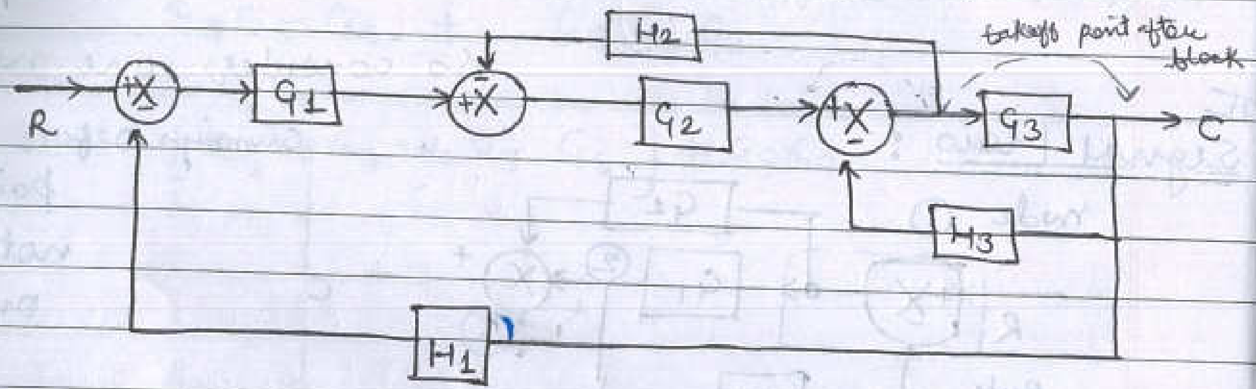
$\Delta_1 = 1$

$\Delta_2 = 1$

$$T.F = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 \times 1 + G_2 \times 1}{1+G_1H_1}$$

$$\boxed{T_F = \frac{G_1+G_2}{1+G_1H_1}}$$

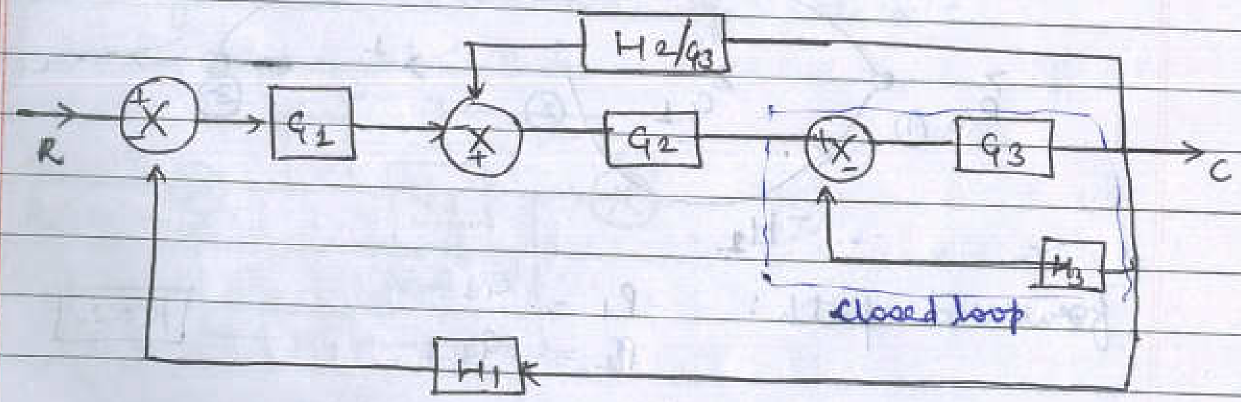
eg:5 For the system find the C/R using block diagram reduction method and find the C/R using mason's gain formula: If $G_1 = 10$, $G_2 = 5$, $G_3 = 8$, $H_1 = 1$, $H_2 = 0.25$, $H_3 = 0.2$, $R = 10.1$ find the o/p c=?

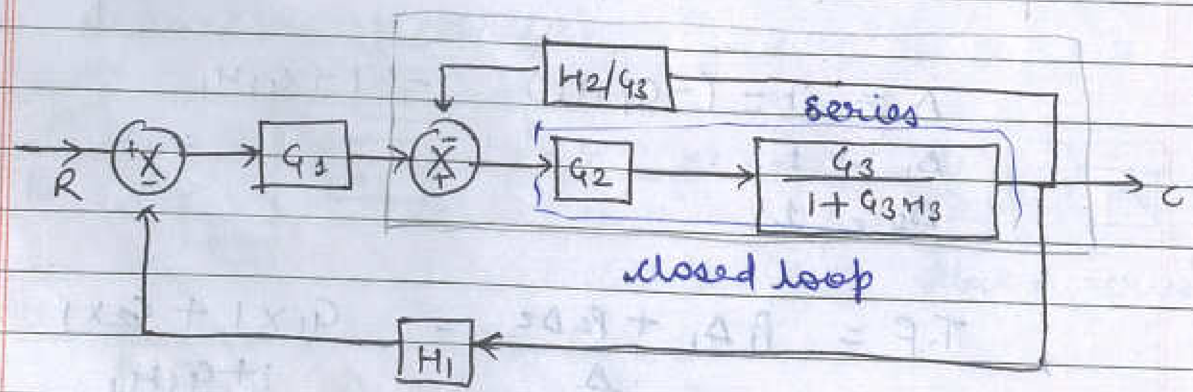


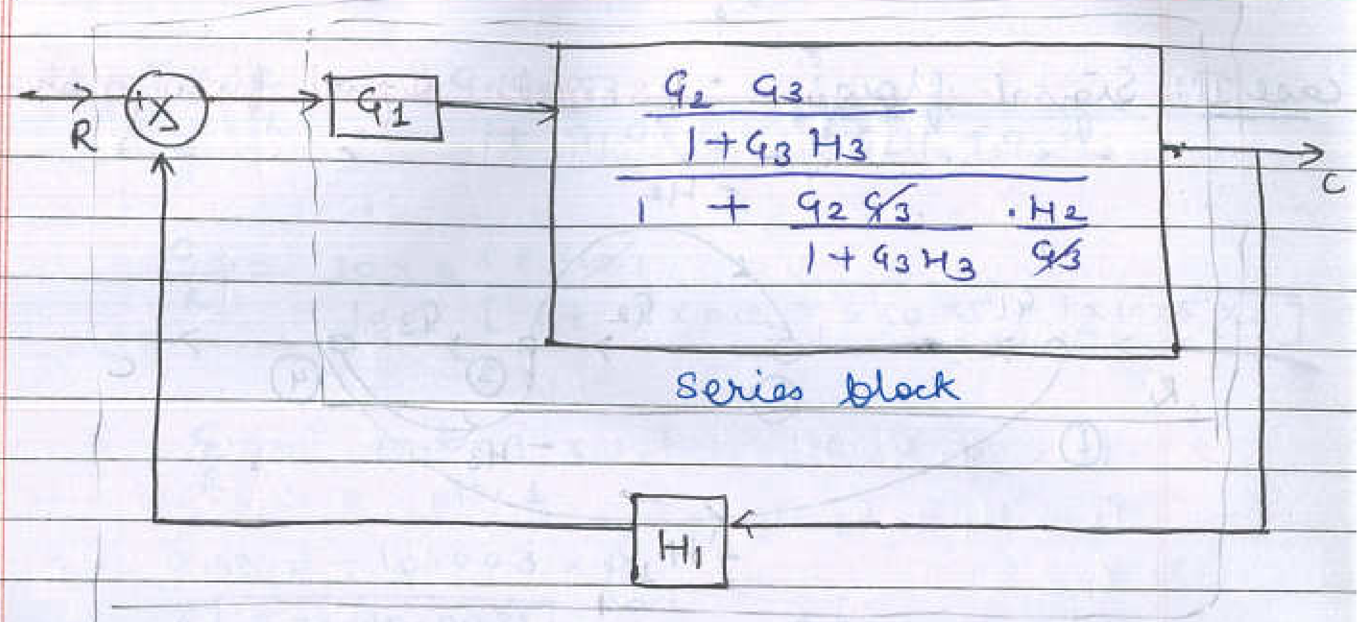
Sol^n:

case I:

Reduction Method :







$$\frac{C}{R} = \frac{G_1 \frac{G_2 G_3}{G_3H_3+1} \Big/ \frac{1+G_3H_3+G_2H_2}{G_3H_3+1}}{1 + H_1 \frac{G_1 G_2 G_3}{1+G_3H_3-G_2H_2}}$$

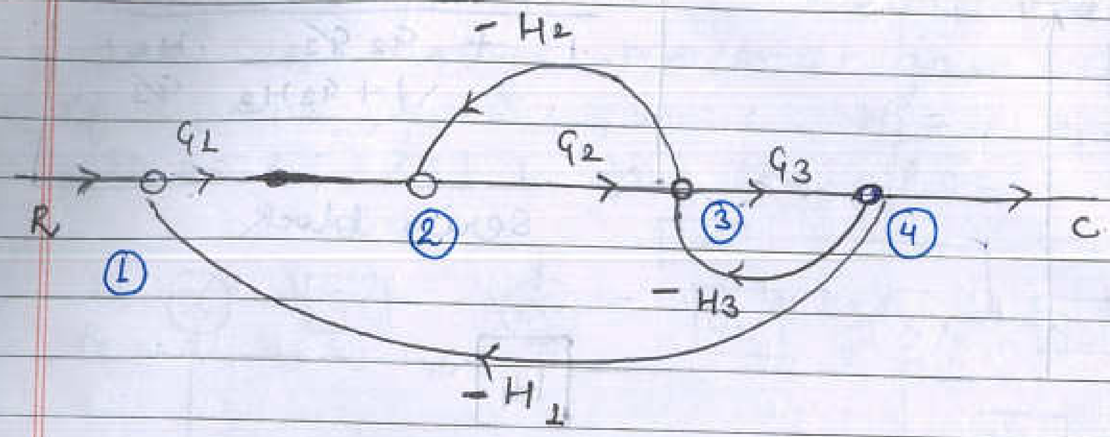
$$\frac{C}{R} = \frac{\frac{G_1 G_2 G_3}{1+G_3H_3+G_2H_2}}{1 + \frac{H_1 G_1 G_2 G_3}{1+G_3H_3-G_2H_2}}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3}{1+G_3H_3+G_2H_2+H_1G_1G_2G_3}$$

$$C = \frac{10 \times 5 \times 8 \times 10.1}{1 + 8 \times 0.2 + 5 \times 0.25 + 1 \times 10 \times 5 \times 8}$$

$$\boxed{C = 10.003}$$

case II: Signal flow: (SFG): Signal flow graph:



→1 forward Path:

no of forward Path = $\boxed{K=1}$

→ loop:

no of loop: 3

$P_1 = G_1 G_2 G_3$

$P_2 = 0$

$L_1 = -G_1 G_3 H_1 G_2$

$L_2 = -G_2 H_2$

$L_3 = -G_3 H_3$

$\Delta = 1 - [L_1 + L_2 + L_3] + [0] + 0$

$\Delta = 1 - [L_1 + L_2 + L_3]$

combinato of induvival forward path loop:

$\Delta_1 = 1 - 0 = 1$ (one forward path only)

$\Delta_2 = 1 - 0 = 1 = 1$

$\Delta_3 = 1 - 0 = 1$

$$T = \frac{1}{\Delta} \sum_K [P_K \Delta_K] = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3 \times (1)}{1 + G_1 G_2 G_3 H_1 + G_2 H_2 + G_3 H_3}$$

$$T = \frac{C}{R} = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 H_1 + G_2 H_2 + G_3 H_3}$$

$$\frac{C}{R} = \frac{10 \times 5 \times 8 \times 1/0.1}{10/1\,[1 + 8 \times 0.2 + 5 \times 0.25 + 1 \times 10 \times 5 \times 8]}$$

$$\frac{C}{R} = \frac{10.003 \times 1}{10.1}$$

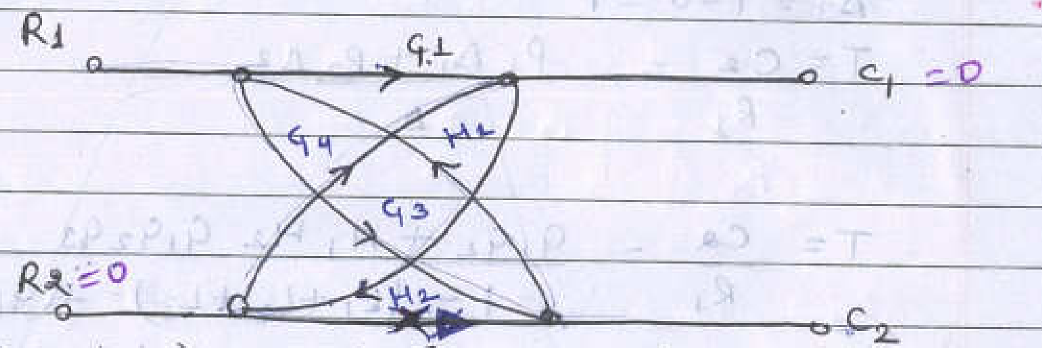
$$C = \frac{10.003 \times R}{10.1}$$

$\boxed{C = 10.003}$ Ans

$\boxed{C = 10.003}$

→

ex: 5.

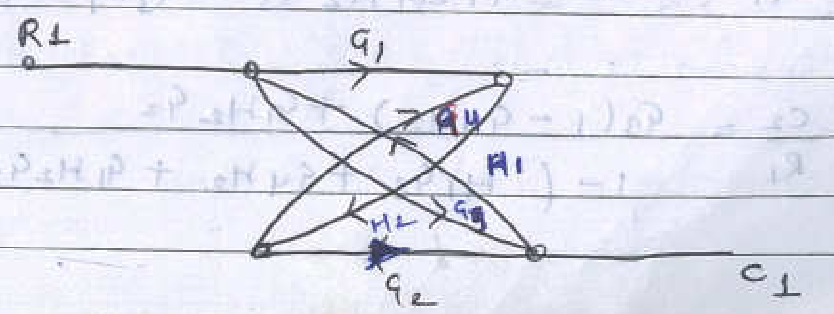


using mason's gain formula find out the $\boxed{\frac{C_2}{R_1}}$

Sol?

Signal flow graph: (SFG):

find $\frac{C_2}{R_1}$ so for $\frac{C_2}{R_1}$ $R_2, C_1 = 0$



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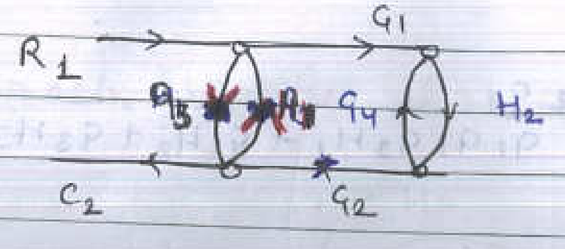
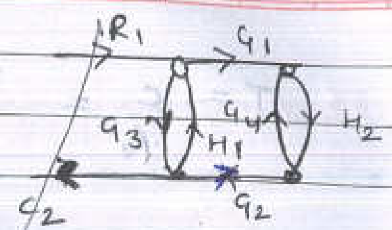


$\rightarrow$



→ forward path:

$P_1 = G_1 G_2 H_2$

~~$P_2 = H_1 G_3 G_1 H_2 G_2$~~

$P_3 = G_3$

loop:

$L_1 = G_3 H_1$

$L_2 = G_4 H_2$

$L_3 = G_1 H_2 G_3 H_1$

$\Delta = 1 - (L_1 + L_2 + L_3) + (G_3 H_1 + G_4 H_2) + (G_1 H_2 G_2 H_1 + H_1 G_3 G_4 H_2)$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

~~$\Delta_3 = 1 - 0 = 1$~~

$\Delta_1 = 1 - H_2 G_4$

$\Delta_2 = 1 - 0$

$T = \frac{C_2}{R_1} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

$T = \frac{C_2}{R_1} = \frac{G_1 G_2 + H_1 H_2 G_1 G_2 G_3}{1 - (L_1 + L_2 + L_3) + (G_1 H_1) + (G_4 H_2) + G_1 H_2 G_2 H_1 + H_1 G_3 G_4 H_2}$

$\frac{C_2}{R_1} = \frac{G_1 G_2 \cancel{H_2} + H_1 H_2 G_1 G_2 G_3}{1 - \cancel{G_3} H_1 - G_4 \cancel{H_2} - G_1 H_2 H_1 G_3 + \cancel{G_1 H_1} + G_4 \cancel{H_2} + G_1 H_2 G_2 H_1 + H_1 G_3 G_4 H_2}$

$\frac{C_2}{R_1} = \frac{G_1 G_2 \cancel{H_2} + H_1 H_2 G_1 G_2 G_3 + G_3}{1 - G_1 H_1 H_2 G_3 - G_3 H_1 - G_4 H_2 + H_1 G_3 G_4 H_2}$

$\frac{C_2}{R_1} = \frac{G_3 (1 - G_4 H_2) + G_1 H_2 G_2}{1 - (H_1 G_3 + G_4 H_2 + G_1 H_2 G_2 H_1 + H_1 G_3 G_4 H_2)}$

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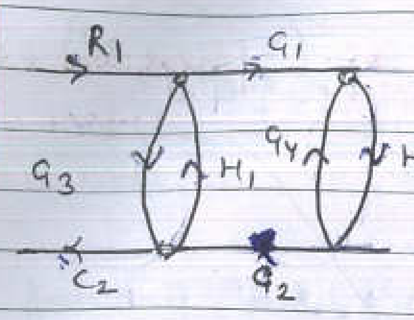
• → Mathematical Modelling of physical system:



forward path:

$$P_1 = G_1 G_2 H_2 = G_1 H_2 G_2$$
$$P_2 = G_3$$

loop:
$$L_1 = G_3 H_1$$
$$L_2 = G_4 \cdot H_2$$
$$L_3 = G_1 H_2 G_2 H_1$$

$$\Delta = 1 - (G_3 H_1 + G_4 H_2 + G_1 H_2 G_2 H_1)$$

$$T = \frac{C_2}{R_1} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\Delta_2 = 1 - H_2 G_4$$
$$\Delta_1 = 1 - 0 = 1$$

combinat^n of ~~non~~ two non-touching loop: $H_2 G_4 \; G_3 H_1$

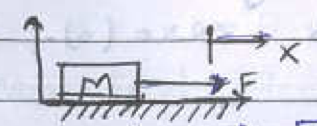
$$\frac{C_2}{R_1} = \frac{G_3(1 - H_2 G_4) + G_1 G_2 H_2 \times 1}{1 - \{[G_3 H_1 + G_4 H_2 + G_1 H_2 G_3 H_1] + [H_1 G_3 G_4 H_2]\}}$$

$$\frac{C_2}{R_1} = \frac{G_3(1 - G_4 H_2) + G_1 H_2 G_2}{1 - [H_1 G_3 + G_4 H_2 + G_1 H_2 G_2 H_1 + H_1 G_3 G_4 H_2]}$$

→ Mathematical Modelling of physical system:

1. classified as translational } (I) Mechanical system
2. Rotational

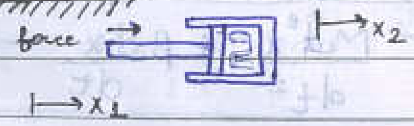
1. Trans-lation: (a) Mass →



$$\boxed{F = ma}$$

(b) Spring

(c) Damper →



$$\boxed{F = \beta v}$$

$$\boxed{F = kx}$$

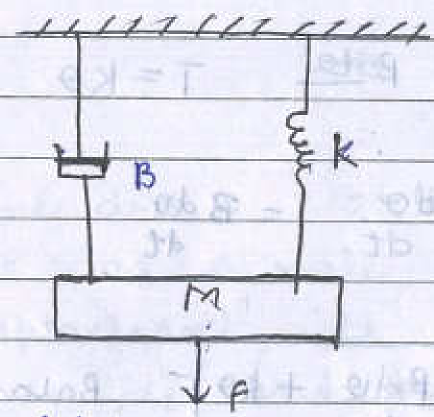
Mass ⇒ $F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$

Damper ⇒ $F = \beta v = \beta \frac{dx}{dt} = \beta \frac{d}{dt}(x_1 - x_2)$; β = damping constant

Spring ⇒ $F = kx = k(x_1 - x_2)$

k: spring constant

→



Balanced condition (stationary):

$$F = ma + Bv + kx \quad \text{—— (i)}$$

(D-ALEMBRTS) Rule

$$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$

→ D-ALEMBRTS RULE:

It state that for any body the aligbric

sum of externally applied force and the force resisting the motion in given direction is zero (0).

$$\boxed{F - M\frac{d^2x}{dt^2} - \beta\frac{dx}{dt} - Kx = 0}$$

2. Rotational :

(a) inertia torque
(b) spring
(c) Damper

inertia torque :

$$T = \frac{J d^2\theta}{dt}$$

spring :

~~Bdθ~~ $T = K\theta$

Damper : $T = \cancel{B}\frac{d\theta}{dt} = B\frac{d\theta}{dt}$

$$\boxed{T = \frac{Jd^2\theta}{dt^2} + \beta\frac{d\theta}{dt} + K\theta}$$ Balanced condition

where : T : torque

J : moment of inertia

θ = angular displacement

angular velocity

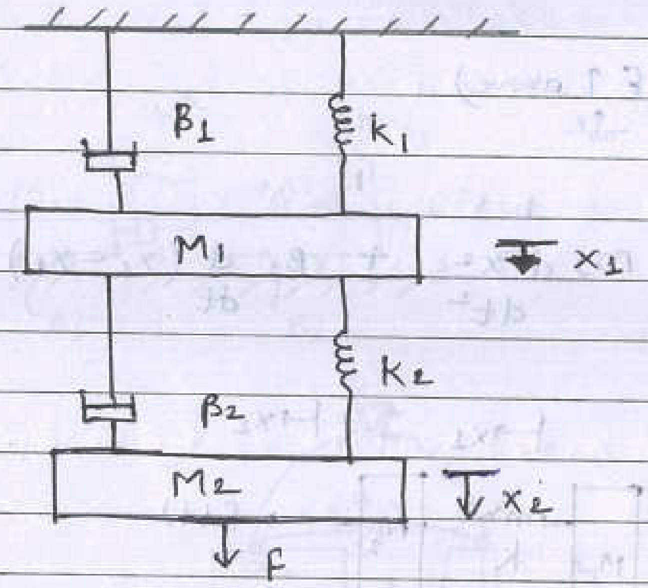
$\frac{d\theta}{dt} = \omega$

$\frac{d^2\theta}{dt} = \alpha$ angular acceleration

* Numerical:

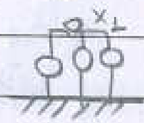
→ Physical system: Mechanical system

Q.1 Obtain the Mathematical Model of given Mechanical system

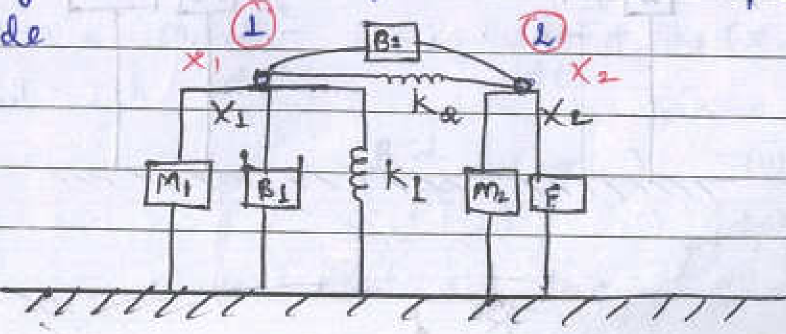


• STEPS:

1. Reference line draw
2. displacement as a node consider x_1 x_2
3. check displacement with connected system between x_1, x_2 and Reference.



4. Mass always connect reference and displacement.
5. applied node analysis.



→ Nodal Analysis:

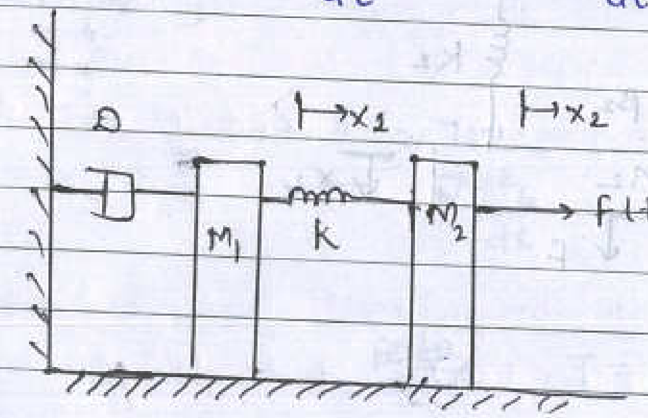
(i) Node 1:

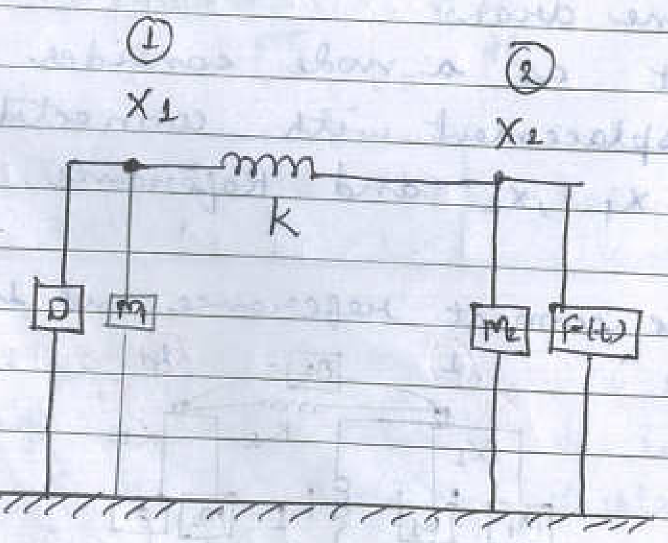
$$0 = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 x_1 + B_2 \frac{d}{dt}(x_1 - x_2) + k_2 (x_1 - x_2)$$

(ii) Node 2: (∵ F ↑ above)

$$F = M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{d}{dt}(x_2 - x_1) + K_2(x_2 - x_1)$$

Q.2.





→ Nodal Analysis!

node 1:

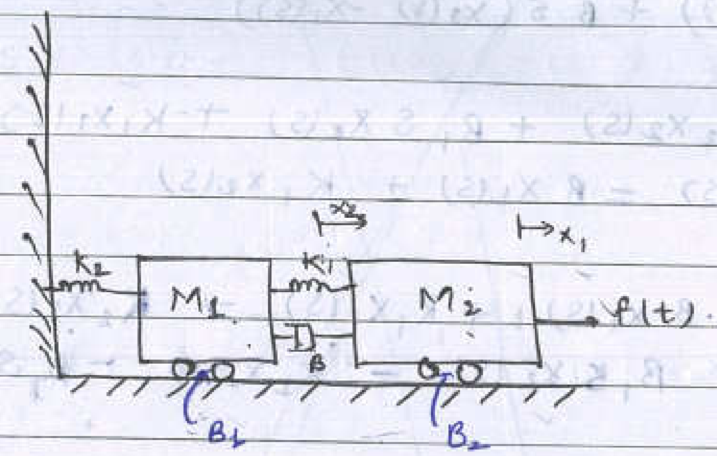
$$0 = B\frac{dx_1}{dt} + M_1 \frac{d^2x_1}{dt^2} + K(x_1 - x_2)$$

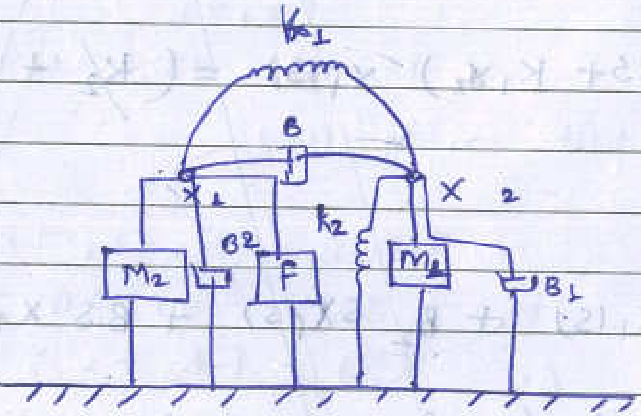
node 2:

$$F = M_2 \frac{dx_2^2}{dt^2} + K(x_2 - x_1)$$

Q3

Determine TF $\frac{X_1(s)}{F(s)}$ for given ~~trans~~ mechanical system





→ node1:

$$F = M_2 \frac{dx_1^2}{dt^2} + B_2 \frac{dx_1}{dt} + K_1(x_1 - x_2) + B\frac{d}{dt}(x_1 - x_2) \quad \text{—(i)}$$

$$0 = M_1 \frac{dx_2^2}{dt^2} + B_1 \frac{dx_2}{dt} + K_1(x_2 - x_1) + k_2 x_2 + B\frac{d}{dt}(x_2 - x_1) \quad \text{—(ii)}$$

taking laplace Transform of eq^n (i) and (ii)

$$F(S) = S^2 M_2 X_1(S) + B_2 S X_1(S) + K_1(X_1(S) - X_2(S)) + BS(X_1(S) - X_2(S)) \quad \text{—(iii)}$$

$$0 = s^2 M_1 X_2(S) + K_2 X_2(S) + B_1 S X_2(S) + K_1(X_2(S) - X_1(S)) + Bs(X_2(S) - X_1(S)) \quad \text{—(iv)}$$

$$0 = s^2 M_1 X_2(S) + k_2 X_2(S) + B_1 S X_2(S) + K_1 X_2(S) + K_1 X_1(S) + BS X_2(S) + (-BS X_1(S))$$

$$K_1 X_1(S) + BS X_1(S) = X_2(S)(M_1 S^2 + K_2 + B_1 S + K_1 + BS)$$

$$X_2(S) = \frac{X_1(S)(K_1 + BS)}{M_1 S^2 + K_2 + B_1 S + K_1 + BS} \quad \text{—(v)}$$

eq^n(v) put in eq^n (iii)

$$\frac{X_1(S)}{F(S)} = \frac{M_1 S^2 + B_1 S + K_2 + K_1 + BS}{(M_1 S^2 + B_1 S + K_1 + K_2 + BS)(M_2 S^2 + B_2 S + BS + K_1) - (Bs + K_1)^2}$$

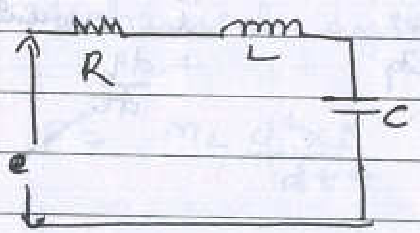
→ Analogous system:

mechanical system → electrical system change

$$F = M\frac{d^2x}{dt} + B\frac{dx}{dt} + Kx \quad \text{—(i)}$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta \quad \text{—(ii)}$$

- There are 3 basic element: Resistor (R), capacitor (L), inductor (C)

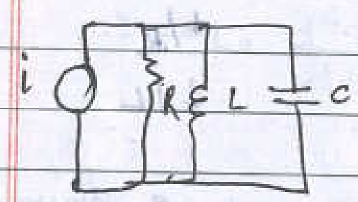


$$e = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$e = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$i = \frac{dq}{dt}$$

$$e = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C} \quad (\because \text{voltage } eq^n) \quad \text{—(iii)}$$



$$i = i_R + i_L + i_C$$

$$i = \frac{e}{R} + \frac{1}{L}\int e\,dt + C\frac{de}{dt}$$

$$e = \frac{d\phi}{dt}$$

$$i = \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} + C\frac{d^2\phi}{dt^2}$$

$$i = \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} + C\frac{d^2\phi}{dt^2} \quad (c/n\ eq^n) \quad \text{—(iv)}$$

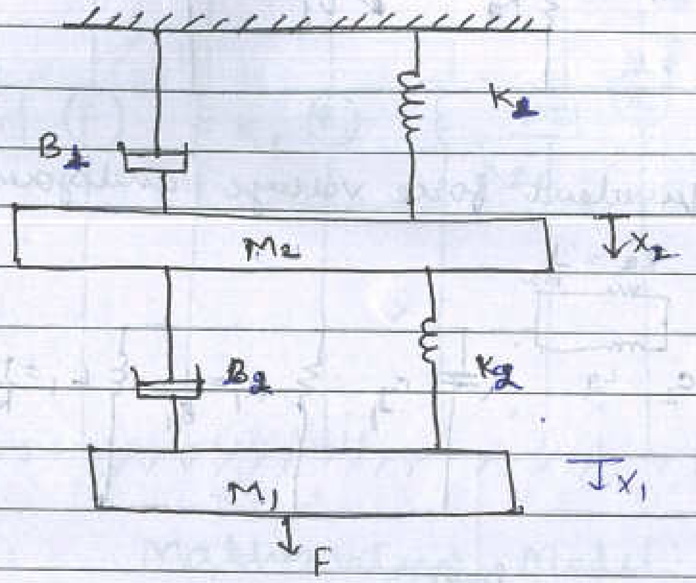
→ force voltage Analogous :

Translation (T)	Rotational (R)	electrical. (E)
F force	T torque	e voltage
M Mass	J inertia	L inductor
x displacement	θ angular displacement	q charge
B damping coefficient	B damping coefficient	R Resistor
K spring constant	K spring constant	$1/C$ capacitance
$v = \frac{dx}{dt}$ velocity	$\frac{d\theta}{dt} = \omega$ angular velocity	i current $= \frac{dq}{dt}$

→ force current Analogous:

Translation (T)	Rotational	electrical
F	T	i
M	J	C
x	θ	ϕ flux
B	B	$1/R$
K	K	$1/L$
v	$\frac{d\theta}{dt} = \omega$	e voltage $= \frac{d\phi}{dt}$

Q.3. Draw the analogous system of given mechanical system:



→ Mechanical system: eqn: (∵ reference page 62)

$$F = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d}{dt}(x_2 - x_1) + K_2 (x_2 - x_1)$$

$$0 = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B_2 \frac{d}{dt}(x_1 - x_2) + K_2 (x_1 - x_2)$$

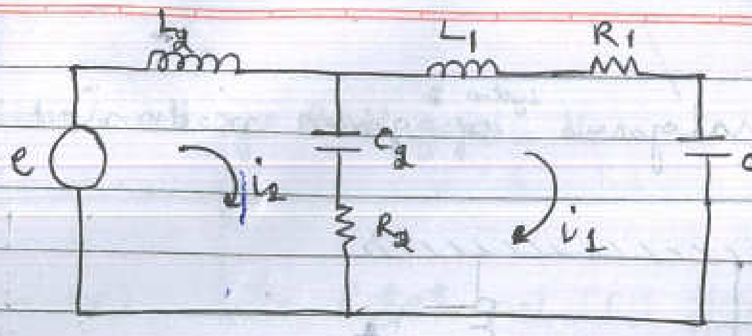
→ force voltage analogous:

$$e = L_2 \frac{d^2 q_2}{dt^2} + \frac{1}{C_2}(q_2 - q_1) + R_2 \frac{d}{dt}(q_2 - q_1)$$

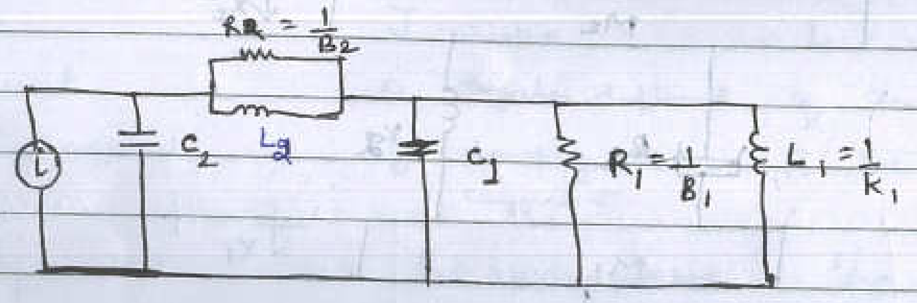
$$e = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1)\, dt + R_2 (i_2 - i_1) \quad \text{— (iii)}$$

$$0 = L_1 \frac{d^2 q_1}{dt^2} + \frac{1}{C_1}\frac{dq_1}{dt} + R_1 q_1 + R_2 \frac{d}{dt}(q_1 - q_2)$$

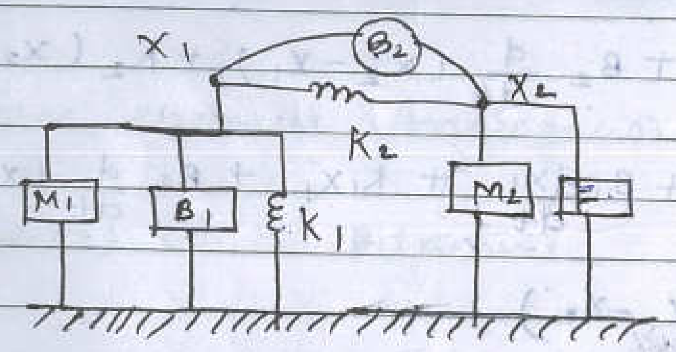
$$0 = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1\, dt + R_2 (i_1 - i_2) + \frac{1}{C_2}\int (i_1 - i_2)$$



equivalent force voltage analogous



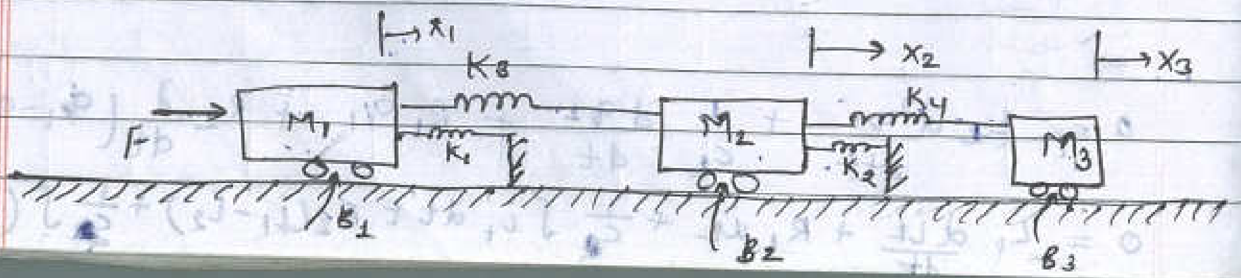
equivalent force c/n analogous

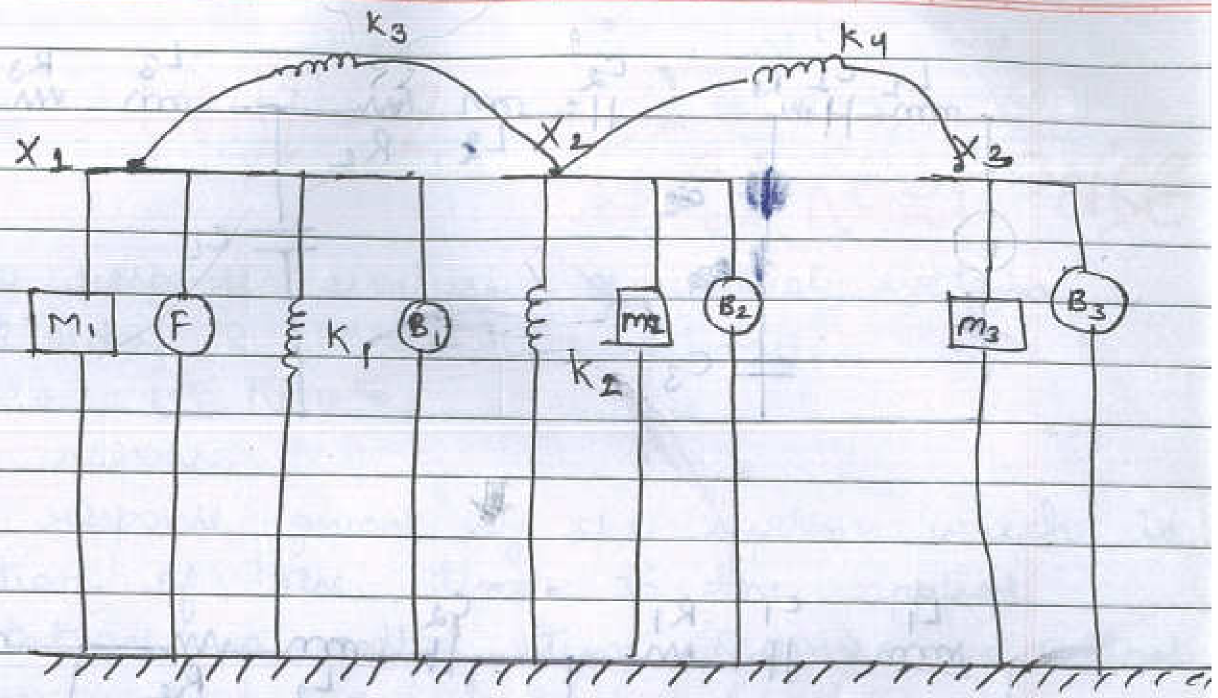


node diagram

* force c/n analogous direct change node diagram according.

Q.4. Obtain the mathematical model and electrical analogous of given mechanical system:





1. node 1 : Mathematical Model:

$$f = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d}{dt} x_1 + K_1 x_1 + K_3 (x_1 - x_2) \quad \text{—(i)}$$

$$0 = K_2 x_2 + M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_4 (x_2 - x_3) + K_3 (x_2 - x_1) \quad \text{—(ii)}$$

$$0 = M_3 \frac{d^2x_3}{dt^2} + B_3 \frac{dx_3}{dt} + K_4 (x_3 - x_2) \quad \text{—(iii)}$$

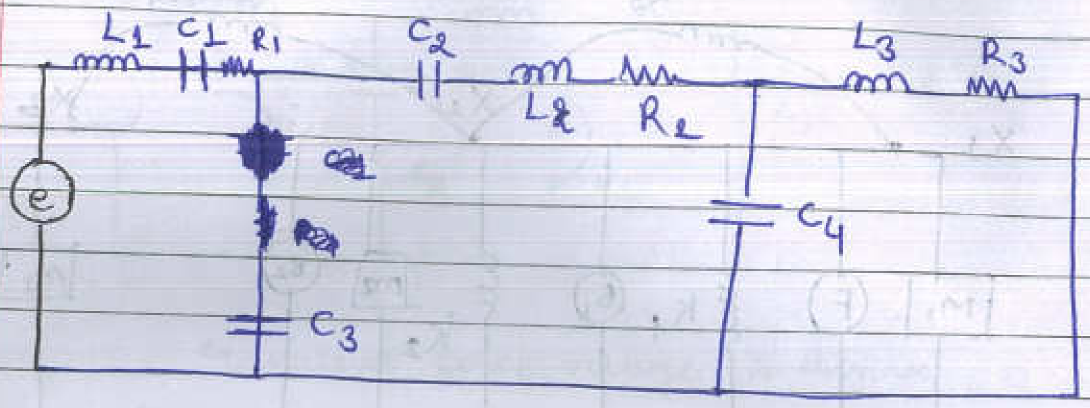
• force voltage analogous

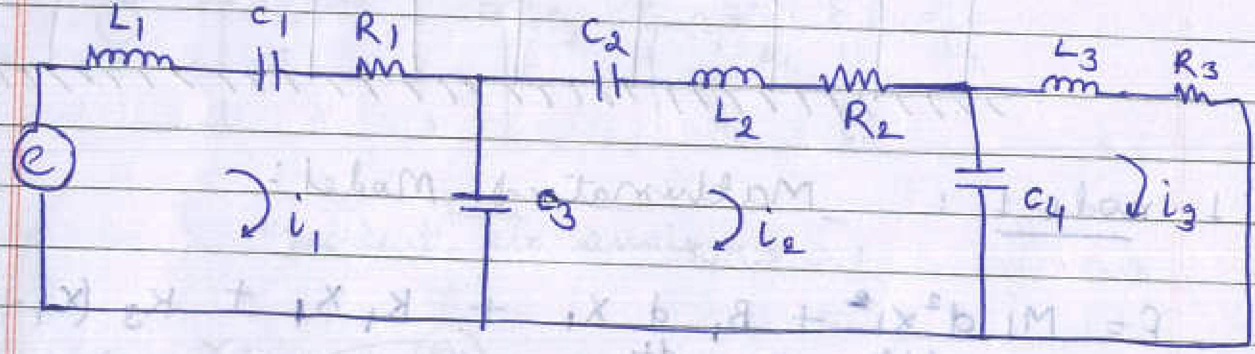
$$e = L_1 \frac{d^2q_1}{dt^2} + R_1 \frac{d}{dt} q_1 + \frac{1}{C_1} q_1 + \frac{1}{C_3} (q_1 - q_2)$$

$$e = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + \frac{1}{C_3} \left(\int (i_1 - i_2)\right)$$

$$0 = \frac{1}{C_2} q_2 + L_2 \frac{d^2q_2}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{1}{C_4} (q_2 - q_3) + \frac{1}{C_3} (q_2 - q_1)$$

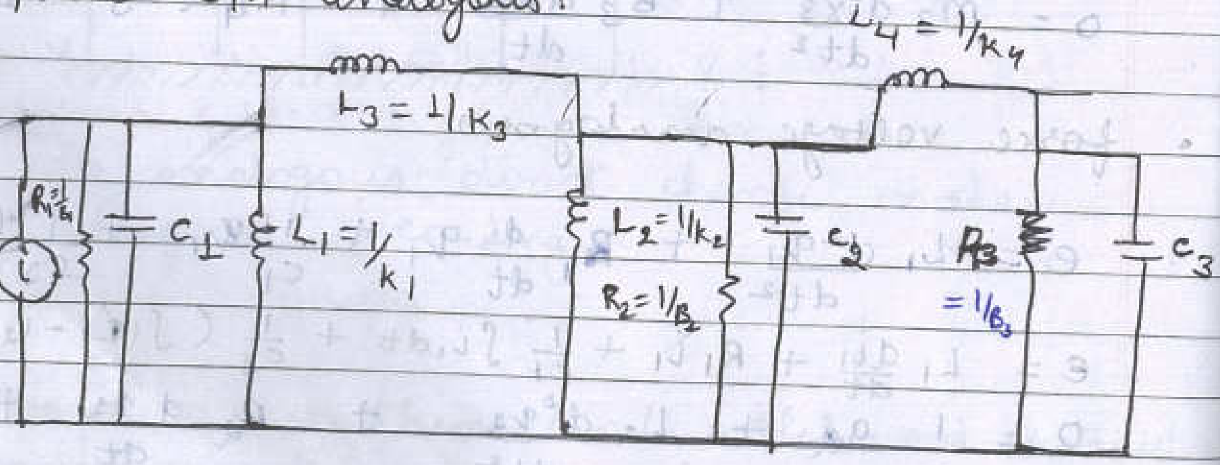
$$0 = L_3 \frac{d^2q_3}{dt^2} + R_3 \frac{dq_3}{dt} + \frac{1}{C_4} (q_3 - q_2)$$

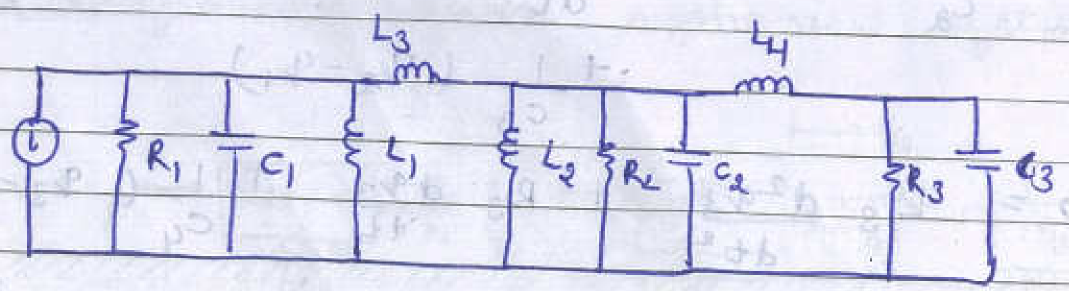




equivalent force voltage analogous

• force c/n analogous:





UNIT - 2

TIME RESPONSE

• Time response analysis of control system (1st order & 2nd order system).

Time response:

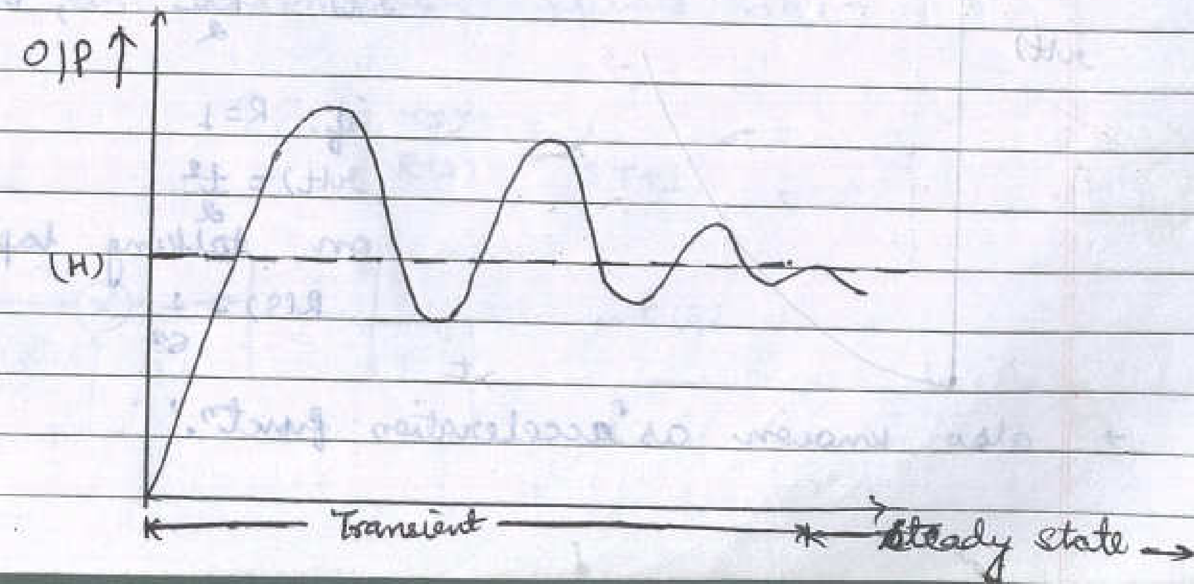
The response given by the system which is function of the time, to the applied excitation is called time Response of control system.

Transient response:

The O/P variation during the time it take to achieve its final value called transient response.

Steady time Response:

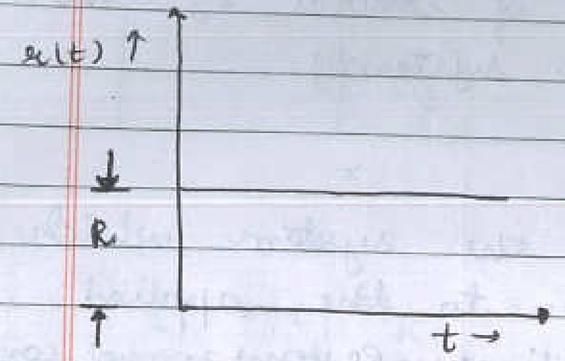
It is that part of time Response which remains after complete transient response vanish from the system O/P.



Input test signal:

1) Step function :-

It is desribed as sudden applied of i/p signal



$r(t) = R \quad , t \geq 0$

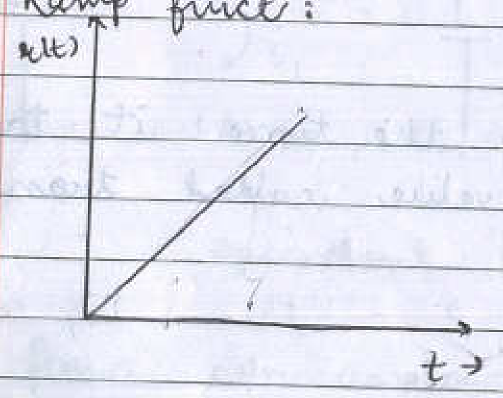
if $\Rightarrow R = 1$

→ unit step functn on taking laplace

$$R(s) = \frac{1}{s}$$

also known as 'displacement functn.

2) Ramp funct^n:



$r(t) = Rt \quad t \geq 0$

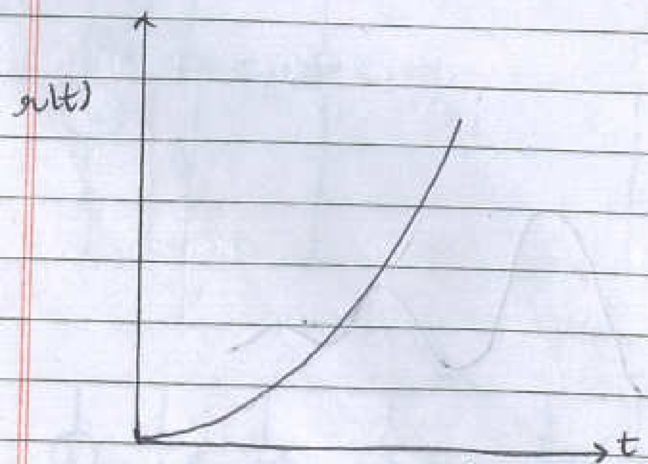
if $R = 1$

→ unit ramp functn on taking laplace

$$R(s) = \frac{1}{s^2}$$

This functn is called 'velocity functn.'

3) Parabolic function:



$r(t) = \frac{Rt^2}{2} \quad , t \geq 0$

if $R = 1$

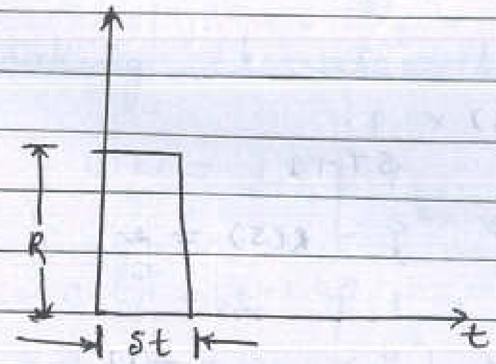
$r(t) = \frac{t^2}{2}$

on talking laplace

$$R(s) = \frac{1}{s^3}$$

→ also known as 'acceleration functn.'

4) Impulse function:



Input is suddenly applied for a very short durat- of time.

If the magnitude of impulse functn is unity, the functn is called unit impulse functn.

Impulse functn is the first time derivative of unit step functn.

unit impulse functn = $\frac{d}{dt}$ [unit step functn]

L[unit impulse functn] = L[unit step functn]

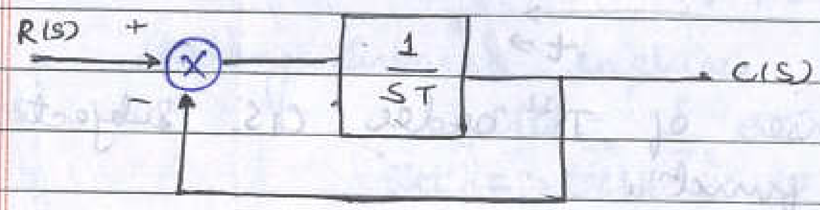
$R(s) = s\,L$ (unit step functn) $\quad \frac{d}{dt} \xrightarrow{laplace} s$

$R(s) = s \times \frac{1}{s} = 1$

Time Response of 1st order C.S:

1st order control system has highest power of s. in the denominator equals to 1.

$$T.F. = \frac{C(s)}{R(s)} = \frac{1}{sT+1}$$



(a) Time Response of 1^{st} order c.s subjected to unit step funct^n:

$$C(s) = R(s) \times \frac{1}{sT+1}$$

Here $r(t) = 1$; $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s} \times \frac{1}{sT+1} \quad \text{—(i)}$$

taking inverse laplace both side

$$c(t) = L^{-1}\left[\frac{1}{s^2T+s}\right]$$

$$= L^{-1}\left[\frac{1}{s} - \frac{T}{sT+1}\right]$$

$$c(t) = 1 - e^{-t/T} \quad \text{—(ii)}$$

$$e(t) = r(t) - c(t)$$

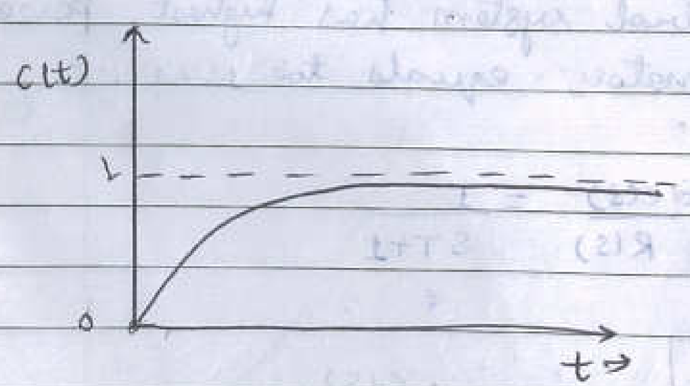
$$= 1 - [1 - e^{-t/T}]$$

$$e(t) = e^{-t/T} \quad \text{—(iii)}$$

In eq^n (ii)

if $t = 0$, $c(t) = 1 - e^0 = 1 - 1 = 0$

If $t = \infty$, $c(t) = 1 - 0 = 1$



(b) Time response of Ist order c.s. subject to unit ramp funct^n.

Here $r(t) = t$; $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{1}{s^2} \cdot \frac{1}{sT+1} \quad \text{—(i)}$$

taking inverse laplace both side

$$c(t) = L^{-1}\left[\frac{1}{s^2} \cdot \frac{1}{sT+1}\right]$$

$$c(t) = L^{-1}\left[\frac{1}{s^2} - \frac{T}{s} + \frac{T}{sT+1}\right]$$

$$c(t) = t - T + T e^{-t/T} \quad \text{—(ii)}$$

$$e(t) = r(t) - c(t)$$

$$= t - t + T - T e^{-t/T}$$

$$e(t) = r(t) - c(t)$$

$$= t - t + T - T e^{-t/T}$$

$$e(t) = T(1 - e^{-t/T}) \quad \text{—(iii)}$$

In eq^n (ii)

If $t = 0$, $c(t) = -T$

$t = \infty$, $c(t) = -T$

During the steady state the o/p velocity matched with the i/p velocity but lag behind the i/p by time T & positional error of T unit exist in the system.

(c) Time Response of Ist order c.s subject to unit impulse funct^n:

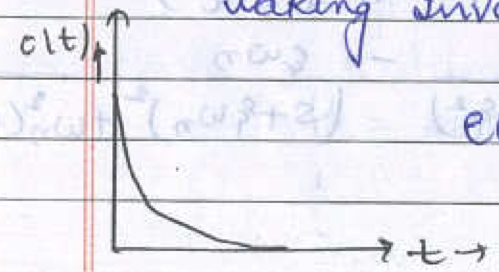
Here, $R(s) = 1$

$$C(s) = 1 \cdot \frac{1}{sT+1} \quad \text{—(i)}$$

taking Inverse laplace both side.

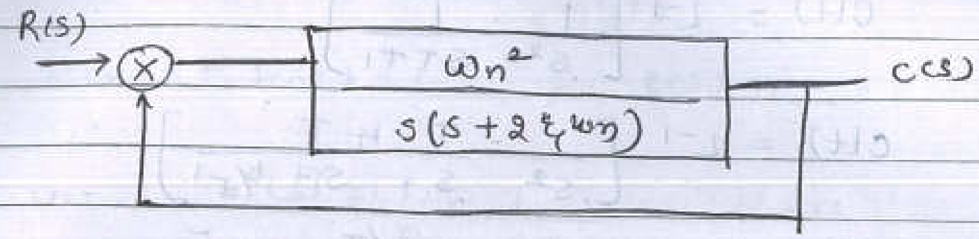
$$c(t) = \frac{1}{T} e^{-t/T} \quad \text{—(ii)}$$

$$e(t) = r(t) - c(t)$$



Time Response of 2nd order c.s:-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



(a) Time Response of 2nd order c.s subject to unit step funct^n:

$$r(t) = 1 \quad t \geq 0$$

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 \Rightarrow$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 + \omega_n^2\xi^2 - \omega_n^2\xi^2$$

(i) — $$s^2 + 2\xi\omega_n s + \omega_n^2\xi^2 + \omega_n^2 - \omega_n^2\xi^2$$

$$(s + \omega_n\xi)^2 + \omega_n^2(1-\xi^2)$$

$$C(s) = \frac{1}{s} * \frac{\omega_n^2}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

by partial fraction

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$= \frac{1}{s} - \frac{s + \xi\omega_n + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

Taking inverse laplace Transform both side:

$$c(t) = 1 - L^{-1}\left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}\right] - L^{-1}\left[\frac{\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}\right]$$

let $\omega_n^2(1-\xi^2) = \omega_d^2$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$c(t) = 1 - L^{-1}\left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}\right] - L^{-1}\left[\frac{\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}\right] \times \frac{\omega_d}{\omega_d}$$

$$= 1 - L^{-1}\left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}\right] - L^{-1}\left[\frac{\xi\omega_n}{\omega_d} \times \frac{\omega_d}{(s+\xi\omega_n)^2 + \omega_d^2}\right]$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t}\sin\omega_d t.$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t,$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right] \quad \text{—(ii)}$$

put

$$\sin\phi = \sqrt{1-\xi^2}$$
$$\cos\phi = \xi$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi) \quad \text{—(iii)}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} \; ; \quad \phi = ?$$

$$\tan\phi = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\phi = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right] \quad \text{(iv)}$$

error, $e(t) = r(t) - c(t)$

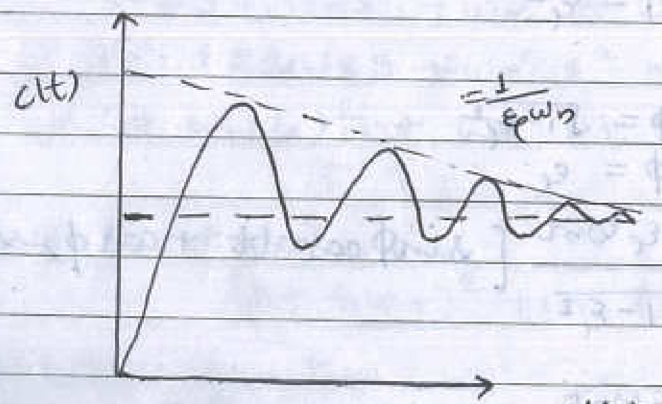
$$= 1 - \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left\{\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right\}\right]$$

$$e(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

steady state error,

$$e_{ss} / e(s) = \lim_{t\to\infty} e(t)$$

Case I: $\xi < 1$: underdamped case the response exponentially decay oscillation having a frequency $\omega_n\sqrt{1-\xi^2}$ & time constant exponentially decay is $\frac{1}{\xi\omega_n}$.



$c(t) = 1 - 1(\sin(\omega_n + \pi/2))$

$c(t) = 1 - \cos\omega_n t$

Case II: $\xi = 0$ (undamped, sustain Oscillation)

Case III: $\xi = 1$ (critical damping):

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t)$$

$$c(t) = \lim_{\xi\to 1}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left\{\sin\omega_n\sqrt{1-\xi^2}\,t\,\xi + \sqrt{1-\xi^2}\cos\omega_n\sqrt{1-\xi^2}\right.\right.$$

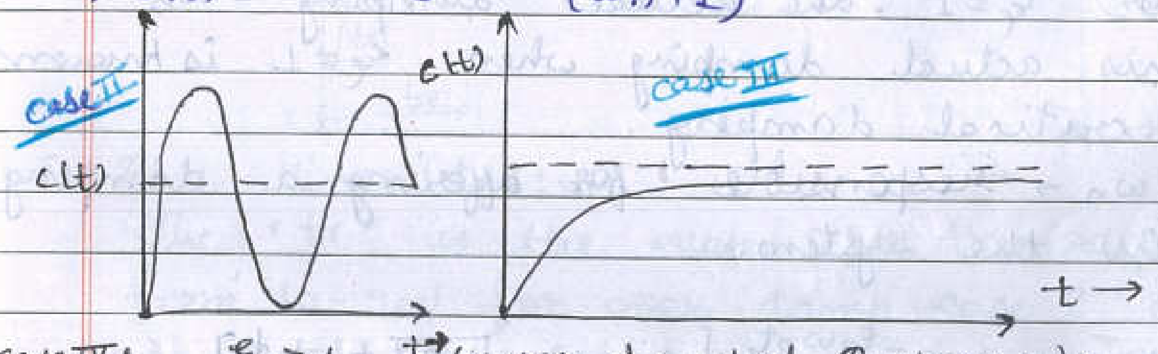
$$= \lim_{\xi\to 1}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left\{\sin\omega_n\sqrt{1-\xi^2}\,t + \sqrt{1-\xi^2}\right\}\right]$$

$$= \lim_{\xi\to 1}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left\{\omega_n\sqrt{1-\xi^2}\,t + \sqrt{1-\xi^2}\right\}\right]$$

$$= \lim_{\xi\to 1}\left[\left\{1 - \frac{e^{-\xi\omega_n t}}{\cancel{\sqrt{1-\xi^2}}} \times \cancel{\sqrt{1-\xi^2}}(\omega_n + 1)\right\}\right]$$

$$= \lim_{\xi\to 1}\left[1 - e^{-\xi\omega_n t}(\omega_n + 1)\right]$$

$$\Rightarrow c(t) = 1 - e^{-\omega t}(\omega_n + 1)$$



Case IV: $\xi > 1$ (over damped Response):

$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2\xi^2 - \omega_n^2\xi^2 + \omega_n^2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2\xi^2 + \omega_n^2 - \omega_n^2\xi^2 \Rightarrow$$

$$s^2 + 2\xi\omega_n + \omega_n^2\xi^2 - \omega_n^2(\xi^2 - 1)$$

$$(s + \xi\omega_n)^2 - \omega_n^2(\xi^2 - 1)$$

$$(s + \xi\omega_n)^2 - (\omega_n\sqrt{\xi^2-1})^2$$

$$(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})$$

$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})}$$

$$= \frac{1}{s} - \frac{1}{2\sqrt{\xi^2-1}(\xi - \sqrt{\xi^2-1})(s + (\xi - \sqrt{\xi^2-1})\omega_n)} + \frac{1}{2\sqrt{\xi^2-1}(\xi + \sqrt{\xi^2-1})(s + (\xi + \sqrt{\xi^2-1})\omega_n)}$$

taking ILT both side:

$$c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}(\xi - \sqrt{\xi^2-1})} + \frac{e^{-(\xi + \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}(\xi + \sqrt{\xi^2-1})}$$

$$c(t) = \frac{2 - e^{-(\xi - \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})}$$

$$c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})}$$

critical damping:

For $\xi = 1$, the actual damping $= \omega_n$.
This actual damping when $\xi = 1$ is known as critical damping.
$\xi\omega_n \rightarrow$ responsible for offering a damping in the system.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin \omega_n\sqrt{1-\xi^2}\,t + \phi\right]$$

ξ :-

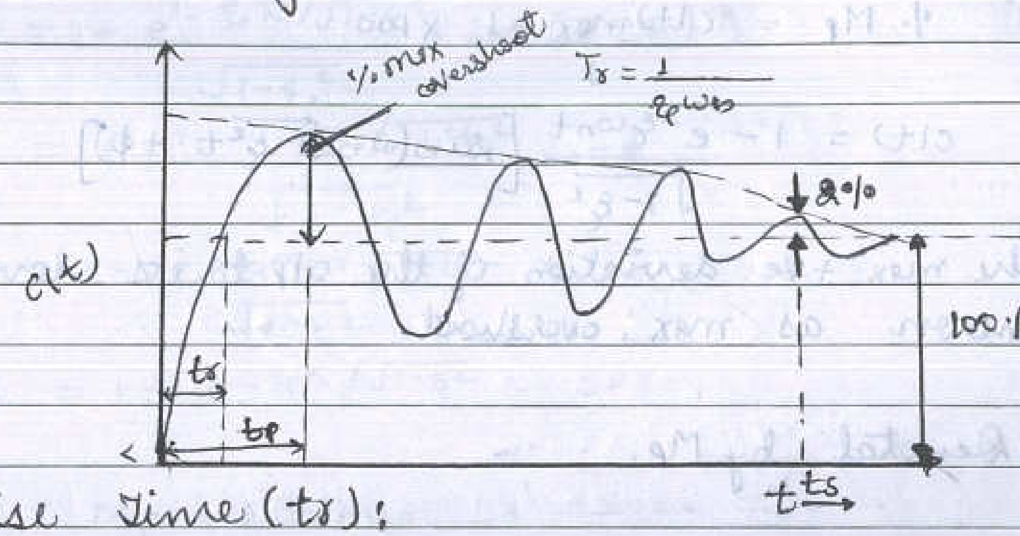
$$\xi = \frac{\text{Actual damping}}{\text{critical damping}} = \frac{\xi\omega_n}{\omega_n}$$

characterstics equation:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{G(s)}{1 + G(s)\cdot H(s)}$$

$$1 + G(s).H(s) = s^2 + 2\xi\omega_n s + \omega_n^2$$
characterstics eq^n.

Transient Response specification of 2nd order control system:



• Rise Time (tr):
The 'tr' is the needed for the response reach from 10-90% for over damp case & 0-100% is used for under damp system.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin \omega_n\sqrt{1-\xi^2}\,t + \phi\right]$$

at 1st instant, when the time response reach 100% of desired value $c(t) = 1$, time = 'tr'.

$$\cancel{1} = \cancel{1} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin \omega_n\sqrt{1-\xi^2}\,t + \phi\right] + \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$$
$$\left[\sin \omega_n\sqrt{1-\xi^2}\,t + \phi\right] = 0$$

$$\sin \omega_n\sqrt{1-\xi^2}\,t + \phi = 0$$

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\omega_n\sqrt{1-\xi^2}\,t_r + \phi = \sin^{-1} 0$$

$$\boxed{t_r = \frac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}}}$$

• Max overshoot (% Mp):

$$M_p = c(t)_{max} - 1$$

$$\% M_p = \frac{c(t)_{max} - 1}{1} \times 100$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_n\sqrt{1-\xi^2}\,t + \phi)\right]$$

The max +ve deviation of the o/p to its device value known as max. overshoot.

Denoted by Mp.

$$c(t)_{max} = \frac{d\,c(t)}{dt}$$

$$= \left[0 - \left\{\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\cos(\omega_n\sqrt{1-\xi^2}\,t+\phi)\right]\omega_n\sqrt{1-\xi^2} + \sin(\omega_n\sqrt{1-\xi^2}\,t+\phi)\left(\frac{\xi\omega_n}{\sqrt{1-\xi^2}}\,e^{-\omega_n\xi t}\right)\right]\right.$$

$$0 = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[-\cos(\omega_n\sqrt{1-\xi^2}\,t+\phi)\,\omega_n\sqrt{1-\xi^2} + \sin(\omega_n\sqrt{1-\xi^2}\,t+\phi)\,\xi\omega_n\right]$$

$$\sin(\omega_n\sqrt{1-\xi^2}\,t+\phi)\,\xi\omega_n = \cos(\omega_n\sqrt{1-\xi^2}\,t+\phi)\,\omega_n\sqrt{1-\xi^2}$$

$$\tan(\omega_n\sqrt{1-\xi^2}\,t_p+\phi) = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\omega_n\sqrt{1-\xi^2}\,t_p+\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\omega_n\sqrt{1-\xi^2}\,t_p = n\pi \qquad n = 1,2,3\ldots$$

$$\boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$c(t)_{max} = 1 - e^{-\xi\omega_n\pi/\omega_n\sqrt{1-\xi^2}}\left(\sin\frac{\omega_n\sqrt{1-\xi^2}\,\pi}{\omega_n\sqrt{1-\xi^2}} + \phi\right)$$

$$= 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin(\pi+\phi)$$

$$= 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin\phi$$

$$= 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \times \sqrt{1-\xi^2}$$

$$c(t)_{max} = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$M_p = c(t)_{max} - 1$$

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

• Settling time (Settling time) ts:
The time read to settle down the oscillation with-in 2% of desire value of the o/p known by settling time and denoted by

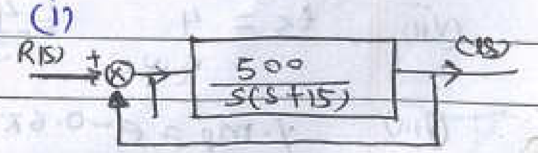
$$\boxed{t_s = \frac{4}{\xi\omega_n}}$$

Ques: A 2nd order system unity feedback and open loop transfer function $g(s) = \frac{500}{s(s+15)}$ (i) Draw the block diagram of the system (ii) write down the characterstics eqn. (iii) Find out the natural freqn (ω_n), & damping ratio(ξ) (iv) Find the value of % Mp (max. overshoot) & peak time tp. (v) Find out the value of settling time (ts).

Soln, given $H(s) = 1$

$$G(s) = \frac{500}{s(s+\cancel{15})} = \frac{500}{s(s+15)}$$

(i)



(ii) $$\frac{C(s)}{R(s)} = \frac{500}{s^2+15s+500}$$

$1+G(s)\cdot H(s)=0$

$\Rightarrow s^2+15s+500=0$ —(i)

as we know that $\Rightarrow s^2+2\xi\omega_n s+\omega_n^2=0$ —(ii)

(iii) $\omega_n^2=500$

$\omega_n=\sqrt{500}=22.36$ rad/s.

$2\xi\omega_n=15$

$\xi=\frac{15}{2\times 22.36}=0.335$

(iv) $\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}\times 100$

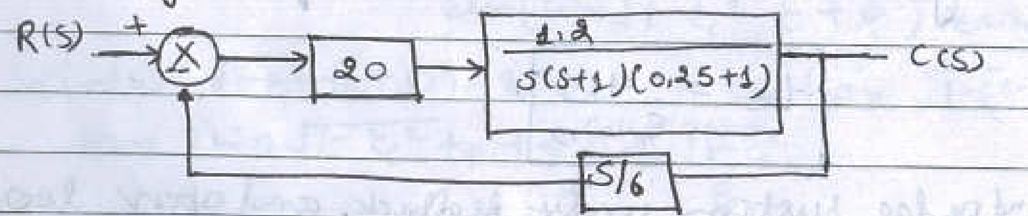
$=e^{-0.335\pi/\sqrt{1-(0.335)^2}}\times 100$

$=0.3269\times 100 = 32.69\%$

(iv) $t_p=\frac{\pi}{\omega_n\sqrt{1-\xi^2}}=\frac{\pi}{22.36\sqrt{1-(0.335)^2}}=0.149$ sec.

(v) $t_s=\frac{4}{\xi\omega_n}=0.5340$ sec.

Ques 2: A 2nd order system ~~unity~~ f: find the (i) characterstics eqn (ii) undamped freqn ω_n (iii) Damp freqn ω_d (iv) Damping factor (v) Damping Ratio & (vi) max. overshoot (vii) Settling time & peak time



Soln:

Given $H(s)=s/6$

$G(s)=\frac{1.2}{s(s+1)(0.25s+1)}*20$

(i) $1+G(s)\cdot H(s)=0$

$1+\frac{24}{s(s+1)(0.25s+1)}\cdot\frac{s}{6}=0$

$s^2+6s+25=0$

(i) $s^2+6s+25=0$

$s^2+2\xi\omega_n s+\omega_n^2=0$ $\boxed{\omega_n=5}$

(iii) $2\xi\omega_n=6$ $\xi=0.6$

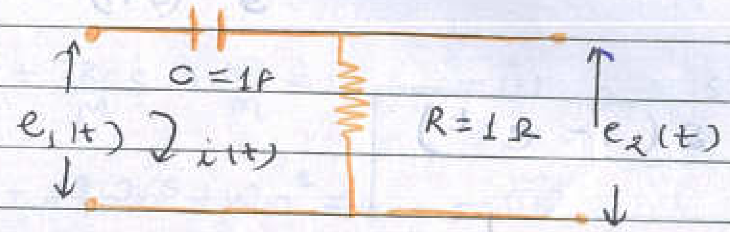
(iv) $\xi\omega_n=3$ (v) $\omega_d=\omega_n\sqrt{1-\xi^2}=4$ (vi) $t_p=\frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

(vii) $t_s=\frac{4}{\xi\omega_n}=\frac{4}{3}=1.33$ sec

$t_p=\frac{\pi}{4}=0.785$ sec

(viii) $\% M_p=e^{-0.6\pi/\sqrt{1-0.6^2}}\times 100=9.48\%$

Q.2. for the electrical n/w the voltage $e_2 t$ consider as o/p and $e_1(t)$ consider as a input. determine the transfer funcn of the system and calculate the voltage $e_2(t)$ when $e_1(t)=2t$.



$\frac{E_2(s)}{E_1(s)}=?$

$E_2(s)=R\,i(s)$

$E_1(s)=(X_c+R)\,i(s)$

$\frac{E_2(s)}{E_1(s)}=\frac{R}{(R+X_c)}$

$=\frac{R}{R+\frac{1}{sC}}$

$=\frac{RCs}{RCs+1}$

$=\frac{1Cs}{s+1}=\frac{s}{s+1}$

$$\boxed{\frac{E_2(s)}{E_1(s)}=\frac{s}{s+1}}$$

$e_1(t)=2t$

$E_1(s)=\frac{2}{s^2}$

$E_2(s)=\frac{2}{s^2}\times\frac{s}{s+1}$

$E_2(s)=\frac{2}{s(s+1)}$

$1 + G(s) \cdot H(s) = 0$

$\Rightarrow s^2 + 15s + 500 = 0$ —(i)

as we know that $\Rightarrow s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ —(ii)

$\omega_n^2 = 500$

(iii) $\omega_n = \sqrt{500} = 22.36$ rad/s.

$2\xi\omega_n = 15$

$\xi = \frac{15}{2 \times 22.36} = 0.335$

(iv) $\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

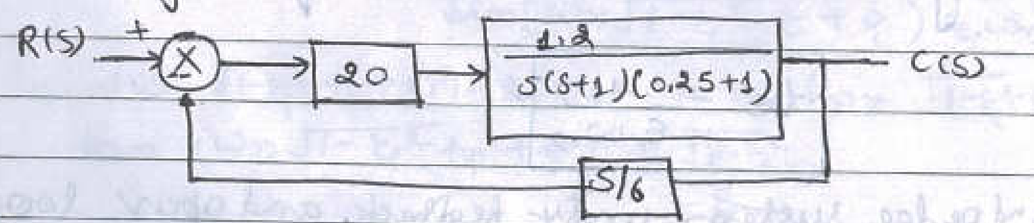
$= e^{-0.335\pi/\sqrt{1-(0.335)^2}} \times 100$

$= 0.3269 \times 100 = 32.69\%$

(iv) $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{22.36\sqrt{1-(0.335)^2}} = 0.149$ sec.

(v) $t_s = \frac{4}{\xi\omega_n} = 0.5340$ sec.

Ques 2: A 2nd order ~~system unity~~ : find the (i) characteristics eqn (ii) undamped freqn ω_n (iii) Damp freqn ω_d (iv) Damping factor (v) Damping ratio & (vi) max. overshoot (vii) settling time & peak time



Soln:

Given $H(s) = S/6$

$G(s) = \frac{1.2}{s(s+1)(0.25+1)} \times 20$

(i) $1 + G(s) \cdot H(s) = 0$

$1 + \frac{24}{s(s+1)(0.25+1)} \cdot \frac{S}{6} = 0$

(i) $S^2 + 6S + 25 = 0$

$S^2 + 2\xi\omega_n s + \omega_n^2 = 0$ $\boxed{\omega_n = 5}$

$S^2 + 6S + 25 = 0$

(ii) $2\xi\omega_n = 6$ $\xi = 0.6$

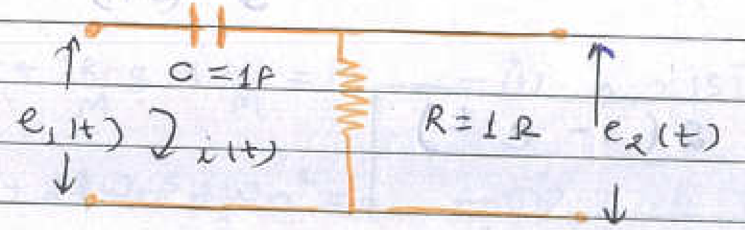
(iv) $\xi\omega_n = 3$ (v) $\omega_d = \omega_n\sqrt{1-\xi^2} = 4$ (vi) $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

(vii) $t_s = \frac{4}{\xi\omega_n} = \frac{4}{3} = 1.33$ sec

$t_p = \frac{\pi}{4} = 0.785$ sec.

(viii) $\% M_p = e^{-0.6\pi/\sqrt{1-0.6^2}} \times 100 = 9.48\%$

Q. 2. for the electrical n/w the voltage $e_2 t$ consider as a o/p and $e_1(t)$ consider as a input. determine the transfer function of the system and calculate the voltage $e_2(t)$ when $e_1(t) = 2t$.



$\frac{E_2(s)}{E_1(s)} = ?$

$E_2(s) = R\, i(s)$

$E_1(s) = (X_c + R)\, i(s)$

$\frac{E_2(s)}{E_1(s)} = \frac{R}{(R + X_c)}$

$= \frac{R}{R + \frac{1}{sC}}$

$= \frac{RCs}{RCs + 1}$

$= \frac{1Cs}{s+1} = \frac{s}{s+1}$

$$\boxed{\frac{E_2(s)}{E_1(s)} = \frac{s}{s+1}}$$

$e_1(t) = 2t$

$E_1(s) = \frac{2}{s^2}$

$E_2(s) = \frac{2}{s^{\cancel{2}}} \times \frac{\cancel{s}}{s+1}$

$E_2(s) = \frac{2}{s(s+1)}$

$$\frac{2}{s(s+1)} = \frac{A}{(s)} + \frac{B}{(s+1)}$$

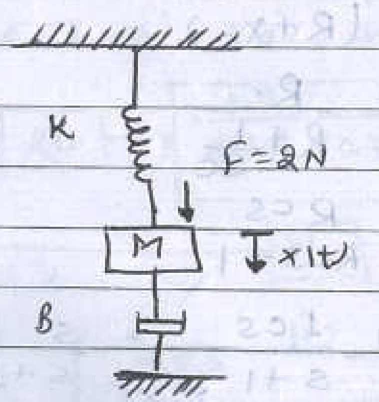
$$B = -\frac{2}{2}$$

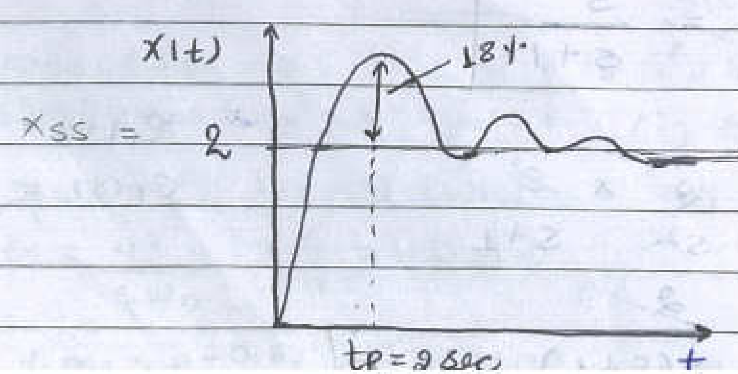
$$E_2(s) = \frac{2}{s} - \frac{2}{(s+1)}$$

$$A = 2$$

$$\boxed{e_2(t) = 2(1 - e^{-t})}$$

Q.3 for an spring Mass damper system shown the figure an experiment is connected by a appling force of 2 N to the mass the response x(t) recode by using XY ploter and experiment result show in the figure. find the value of M, K, B





$f(t) = 2$

$$f(s) = \frac{2}{s} \quad ; \quad f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

Taking Laplace Transform:

$$f(t) = Ms^2X(s) + BsX(s) + KX(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} = \frac{1/M}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

$$s^2 + \frac{B}{M}s + \frac{K}{M} = 0 \quad -(i) \quad ; \quad X(s) = F(s) \times \frac{1/M}{s^2 + \frac{B}{M}s + \frac{K}{M}} \quad -(iii)$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad -(ii)$$

$$\boxed{\omega_n = \sqrt{\frac{K}{M}}} \quad -(iv)$$

$$2\xi\omega_n = \frac{B}{M}$$

$$2\xi = \frac{B \times 1}{M \times \omega_n}$$

$$\xi = \frac{B}{M} \times \sqrt{\frac{M}{K}} \frac{1}{2}$$

$$\boxed{\xi = \frac{B}{2\sqrt{MK}}} \quad -(v)$$

→ Steady state value $X_{ss} = 2$

$$\boxed{X_{ss} = \lim_{s \to 0} sX(s)}$$

$$2 = \lim_{s \to 0} \cancel{s} \times \frac{2}{\cancel{s}} \left[\frac{1/M}{s^2 + B/M\, s + K/M}\right]$$

$$\Rightarrow 2 = \frac{2}{K}$$

$$\boxed{K = 1}$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

$$\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

$$\frac{18}{100} = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

taking ln

$$-1.71 = -\frac{\xi\pi}{\sqrt{1-\xi^2}}$$

$$= -0.5461 = -\frac{\xi}{\sqrt{1-\xi^2}}$$

Taking square both side

$$= 0.5461(1-\xi^2) = \xi^2$$

$$0.5461 = 1.5461\xi^2$$

$$\boxed{\xi^2 = 0.4707}$$

$$\boxed{\omega_n = 1.789}$$

$$\omega_n = \sqrt{\frac{K}{M}}$$

$$\xi = \frac{B}{2\sqrt{MK}}$$

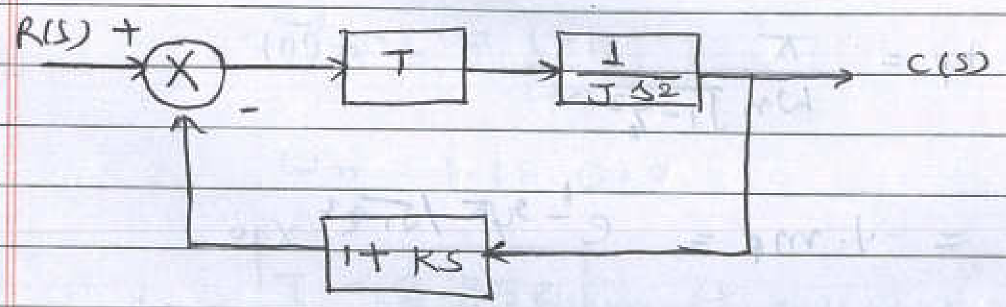
$$(1.789)^2 = \frac{1}{M}$$

$$\boxed{m = 0.3125} \qquad \boxed{K = 1}$$

$$\boxed{B = 0.5354}$$

Ans

Q. Calculate the value of T and k for the block diagram of a closeloop system show below for which maxm overshoot is unit step response is 25% and Peak time is 2 sec. Assume $J = 1\ kg\ m^2$



Solⁿ:

$$2\,sec = t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$25\% = \% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

$$G(S) = T \times \frac{1}{Js^2}$$

$$H(S) = 1 + KS$$

second order eqⁿ $= 1 + G(S)H(S) = 0$

$$\frac{T}{Js^2} \times (1+KS) + 1 = 0$$

$$T(1+KS) + Js^2 = 0$$

$$Js^2 + T + KTS = 0$$

$$S^2 + T + KTS = 0 \quad \text{—(i)}$$

$$S^2 + 2\xi\omega_n S + \omega_n^2 = 0$$

$$\omega_n^2 = T$$

$$\omega_n = \sqrt{T} \qquad 2\xi\omega_n = KT$$

$2\xi\omega_n = KT$

$2\xi\sqrt{T} = K\cancel{T}$

$\boxed{\xi = \frac{K\sqrt{T}}{2}}$ —(i) $\boxed{\omega_n = \sqrt{T}}$

$2 = t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$ —(iii)

$25\% = \%m_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 10$

$0.25 = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$-1.3862 = -\frac{\xi\pi}{\sqrt{1-\xi^2}}$

$-0.44127 = -\frac{\xi}{\sqrt{1-\xi^2}}$

$0.44127(1-\xi^2) = \xi^2$

$0.44127 = \xi^2 + 0.44127\xi^2$

$0.44127 = 1.44127\xi^2$

$0.3061 = \xi^2$

$\boxed{\xi = 0.40432}$ —(ii) $\boxed{\xi = 0.40432}$

eqⁿ (ii) compare eqⁿ (i)

$2 \times 0.40432 = K\sqrt{T}$

$0.80664 = K\sqrt{T}$ —(iv)

(iii) $\Rightarrow$ $2 = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\frac{0.8329 \times 2}{3.14} = \frac{1}{\omega_n}$

$\frac{1.6659}{\pi} = \frac{1}{\omega_n}$

$0.5302 = \frac{1}{\omega_n}$

$\omega_n = 1.8860808$

$\boxed{T = 3.958}$

(iv) $\Rightarrow$ $1.10664 = K\sqrt{T}$

$\boxed{K = 0.47}$

→ Dynamic error and dynamic error coefficient:

Dynamic error and dynamic error coefficient
One of the drawback of error constant is do not provide informatⁿ on the steady state error when i/p are other than the basic signal. consider A unity f/b control system where the error is laplace domain is given by

$E(s) = T_e(s)\,R(s)$

where $T_e(s) = \frac{1}{1+G(s)(1)}$ $T_e(s)$: Transfer functⁿ of error

$\because H(s) = 1$

Talayer series expension:

$$T_e(s) = T_e(0) + \frac{d\,T_e(0)\,s}{ds} + \frac{1}{\lfloor 2}\frac{d^2 T_e(0)}{ds^2} s^2 + \frac{1}{\lfloor 3}\frac{d^3 T_e(0)\,s^3}{ds^3} + \cdots$$

$$E(s) = T_e(0)R(s) + \frac{d\,T_e(0)}{ds} s R(s) + \frac{1}{\lfloor 2}\frac{d^2 T_e(0)\,s^2}{ds^2} R(s) + \frac{1}{\lfloor 3}\frac{d^3 T_e(0)\,s^3}{ds^3} + \cdots$$

Taking inverse Laplace Transform:

$$e(t) = T_e(0)\,r(t) + \frac{d\,T_e(0)}{ds}[r(t)] + \frac{1}{\lfloor 2}\frac{d^2 T_e(0)}{ds^2} + [r(t)] + \frac{1}{\lfloor 3}\frac{t^2 d^3 T_e(0)}{ds^3}[r(t)] + \cdots$$

$$e(t) = c_0 r(t) + c_1 r(t) + \frac{c_2 r(t)}{\lfloor 2} + \frac{c_3}{\lfloor 3} r(t) + \cdots$$

c_0, c_1, c_2 --- are called dynamic error coefficients.

$$c_0 = \lim_{s\to 0} T_e(0)$$

$$c_1 = \lim_{s\to 0} \frac{d\,T_e(0)}{ds}$$

$$c_2 = \lim_{s\to 0} \frac{d^2 T_e(0)}{ds^2}$$

$$\vdots$$

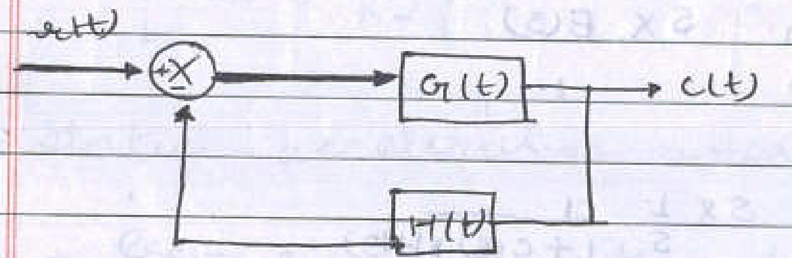
$$c_n = \lim_{s\to 0} \frac{d^n T_e(0)}{ds^n}$$

→ Static error coefficient:

Position error (K_p), unit step $= \left(R(s) = \frac{1}{s}\right)$

velocity (Ramp) error coefficient (K_v), unit step $= \left(R(s) = \frac{1}{s^2}\right)$

Acceleratn error coefficient (K_a), unit step $= \left(R(s) = \frac{1}{s^3}\right)$



$$e(t) = r(t) - H(t)\,c(t)$$

$$e(t) = r(t) - H(t)\cdot r(t)\frac{c(t)}{r(t)}$$

$$e(t) = r(t) - H\left[\frac{G(t)}{1+G(t)H(t)}\right] r(t)$$

$$e(t) = r(t)\left[1 - \frac{H(t)G(t)}{1+G(t)H(t)}\right]$$

$$e(t) = r(t)\left[\frac{1}{GH+1}\right]$$

$$E(s) = \left[\frac{1}{1+G(s)H(s)}\right] \times R(s)$$

$$E(s) = R(s) \times \frac{1}{1+G(s)H(s)}$$

→ Static Position error coefficients: (K_p)

$$E(S) = R(S) \times \frac{1}{1+G(S)H(S)}$$

$$E(S) = \frac{1}{S} \times \frac{1}{1+G(S)H(S)}$$

$$e_{ss} = \lim_{S\to 0} S \times E(S)$$

$$e_{ss} = \lim_{S\to 0} S \times \frac{1}{S} \frac{1}{1+G(S)H(S)}$$

$$= \lim_{S\to 0} \frac{1}{1+G(S)H(S)}$$

$$e_{ss} = \frac{1}{1+\lim_{S\to 0} G(S)H(S)} = \frac{1}{1+K_p}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}} \qquad K_p = \lim_{S\to 0} G(S)\cdot H(S)$$

error

static position coefficient

→ Static velocity error coefficient:

$$e_{ss} = \lim_{S\to 0} S \times E(S)$$

$$e_{ss} = \lim_{S\to 0} S \times \frac{R(S)}{1+G(S)H(S)}$$

$$e_{ss} = \lim_{S\to 0} S \times \frac{R(S)}{1+G(S)H(S)}$$

$$e_{ss} = \lim_{S\to 0} S \times \frac{1}{S^2} \times \frac{1}{1+G(S)H(S)}$$

$$e_{ss} = \lim_{S\to 0} \frac{1}{S+SG(S)H(S)}$$

$$e_{ss} = \frac{1}{\lim_{S\to 0} SG(S)H(S)}$$

$$\boxed{e_{ss} = \frac{1}{K_V}} \qquad \boxed{K_V = \lim_{S\to 0} SG(S)H(S)}$$

→ Static acceleration error coefficient:

$$e_{ss} = \frac{1}{K_a} \qquad K_a = \lim_{S\to 0} S^2 G(S)H(S)$$

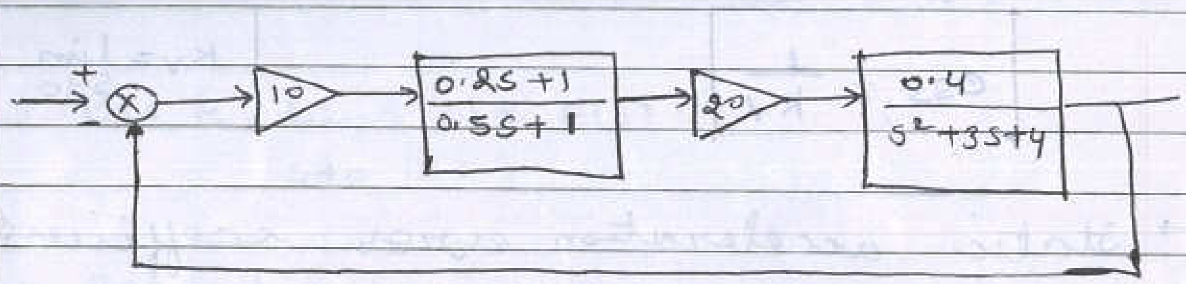
$$E(S) = \frac{1}{S^3} \times \frac{1}{1+G(S)H(S)}$$

$$e_{ss} = \lim_{S\to 0} S \times \frac{1}{S^3} \frac{1}{1+G(S)H(S)}$$

Q. for the system find out the static steady state error for unit step i/p and unit ramp i/p.

unit step input $R(s) = 1/s \rightarrow K_p$

unit Ramp Input $R(s) = 1/s^2 \rightarrow K_v$



Sol: $e_{ss} = \frac{1}{1+K_p}$

$$K_p = \lim_{s\to 0} G(s)\cdot H(s)$$

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} s\cdot G(s)\cdot H(s)$$

$$G(s) = 10 \times \left(\frac{0.2s+1}{0.5s+1}\right) \times 20 \times \frac{0.4}{s^2+3s+4}$$

$$G(s) = 200\left(\frac{0.2s+1}{0.5s+1}\right) \times \frac{0.4}{s^2+3s+4}$$

$$G(s) = \frac{16s+80}{(0.5s+1)(s^2+3s+4)}$$

$$G(s) = \frac{16s+80}{0.5s^3+1.5s^2+2s+s^2+3s+4}$$

$$G(s) = \frac{16s+80}{0.5s^3+2.5s^2+5s+4}$$

$$K_p = \lim_{s\to 0} \frac{16s+80}{0.5s^3+2.5s^2+5s+4} \times 1$$

$$\boxed{\begin{aligned} K_p &= 20 \\ e_{ss} &= \frac{1}{1+20} = 0.0476 \end{aligned}}$$

$$K_v = \lim s \times K_p$$

$$\boxed{\begin{aligned} K_v &= 0 \\ e_{ss} &= \infty \end{aligned}}$$

Q. A Thermometer required 1 mint to indicate 95% of the response to a step input assume the thermometer to be a first order system find the time constant.

$T = ?$

$$c(t) = 1 - e^{-t/T}$$

where $T \rightarrow$ time constant

$t = 1$ mint

$t = 60$ sec

$1 \rightarrow 95\% \rightarrow 0.95$

$$0.95 = 1 - e^{-t/T}$$

$$e^{-60/T} = 0.05$$

$$\cancel{-}\frac{60}{T} = \cancel{-}2.9957$$

$$\boxed{T = 20.03}$$

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→ Measurement conducted on servo mechanism the system response
Q.
$$c(t) = 1 + 0.2e^{-30t} - 1.2e^{-5t}$$
when subjected to unit step input $R(S) = 1/s$
(i) obtain the expression for closed loop system
(ii) Determine the ω_n and ξ of given system.

Soln: $c(t) = 1 + 0.2e^{-30t} - 1.2e^{-5t}$

$$C(S) = \frac{1}{S} + \frac{0.2 \times 1}{(S+30)} - \frac{1.2 \times 1}{(S+5)}$$

$$C(S) = \frac{(S+30)(S+5) + 0.2(S)(S+5) - 1.2(S)(S+30)}{S \times (S+30)(S+5)}$$

$$C(S) = \frac{S^2 + 5S + 30S + 150 + 0.2S^2 + 1S - 1.2S^2 - 36S}{S \times (S+30)(S+5)}$$

$$C(S) = \frac{0 \times 2.4S^2 + 0 \times 72S + 150}{S(S+30)(S+5)}$$

$$C(S) = \frac{150}{S(S^2 + 5S + 30S + 150)}$$

$$C(S) = \frac{150}{S(S+30)(S+5)}$$

$$\frac{C(S)}{R(S)} = \frac{150}{S^2(S+30)(S+5)}$$

Pressure ↑ temperature ↓
electricity → workdone (Pressure) → heat (out) → temp ↓

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closed loop expression: $1 + G(S)H(S) = 0$
$$S^2(S+30)(S+5) = 0$$
$$(S+30)(S+5) = 0$$
$$S^2 + 30S + 5S + 150 = 0$$
$$S^2 + 35S + 150 = 0$$

$$\omega_n = \sqrt{150}$$

$$\omega_n = 12.24$$

$$2\omega_n\xi = 35$$

$$\xi = 1.4297$$

$$\boxed{\begin{aligned}\omega_n &= 12.24 \\ \xi &= 1.4297\end{aligned}}$$ Radian, Radian/sec

→ Stability of the system:
Stability is the one of the most property of the system.

- A linear time invariant system is stable if the following conditions are satisfied

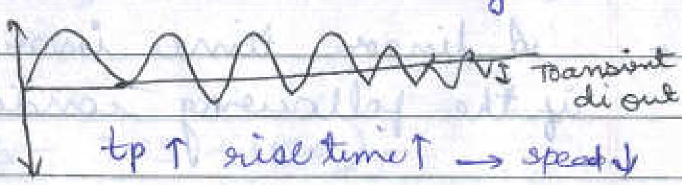
1. bounded i/p bounded o/p stability concept (BIBO)
- when the system exited by BIBO
- In the absence of i/p the o/p → 0
- A system is stable if is all routes of characteristics eqn in the left end side.

stable LHS ⟶ unstable RHS

- A system is called marginally stable or limit stable
- If roots of the characteristics eqn have zero Real part and pole is not repeated.
- A system is stable only if the system experience (a) positive damping Ratio $\xi \geq 0$ and for stable
  (b) for $\xi < 0$ unstable

2 marks

1. **Absolute stability:**
Its referes to the system stability with respect to variations in system parameters if the system is stable for all the values of the parameter it is said to be absolute stability it can determine from the locatn of the roots of characteristics eqn in s plane.

2. **Relative stability:**
It is quantity measure of how fast the transient diout in the system.
The maxm overshoot, 

damping Ratio, gain Margin, phase margin are the mesure of Relative stability.

2015 EE 16 marks

→ **Routh Hurwitz Criterion (Routh):**

- Polynomic eqn:

$$\Rightarrow a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

$a_0, a_2, a_4 \ldots$

$a_1, a_3, a_5 \ldots$

High Power → s^n	a_0	a_2	a_4
s^{n-1}	a_1	a_3	a_5
s^{n-2}	b_1	b_2	b_3
s^{n-3}	c_1	c_2	c_3
s^{n-4}	d_1	d_2	d_3
⋮			

$$b_1 = \frac{-a_0 a_3 + a_1 a_2}{a_1} = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = 0$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \qquad c_2 = \frac{b_1 a_5 - b_3 a_1}{b_1}$$

$$c_3 = 0$$

→ **To check the stability:**

1. determine the total number of sign in the first column of the array

1. The number of sign change indicate the number of roots of characterstics eqn is the Right half of s plane.

2 If there is no sign change in first column of array is indicate that system is stable.

- Cases: Numerial:

→ Numericals:

Q.1. A closed loop c.s has the characteristics eqn given by

$$S^3 + 4.5S^2 + 3.5S + 1.5 = 0$$

Envastegate the stability using routh Harwitz creterias.

Solⁿ, $S^3 + 4.5S^2 + 3.5S + 1.5 = 0$

S^3	1	3.5
S^2	4.5	1.5
S^1	3.16 $\frac{4.5 \times 3.5 - 1.5 \times 1}{4.5} = \frac{14.25}{4.5} = 3.16$	0
S^0	1.5	

Stable: first column of array all the sign of +ve so its stable system.

ex:

S^3	1
S^2	0
S^1	0
S^0	1.5

solution → Numerial:

Method I

Q. A closed loop system has the characteristics eqn given by

$$S^5 + 2S^4 + 4S^3 + 8S^2 + 3S + 1 = 0$$

examine the stability:

S^5	1	4	3
S^4	2	8	1
S^3	0 → ϵ (put)	2.5	0
S^2	$-\infty$ problem $\frac{8\epsilon - 5}{\epsilon} = -\infty$		
S^1	$\frac{\frac{8\epsilon-5}{\epsilon} \times 2.5 - \epsilon}{\frac{8\epsilon-5}{\epsilon}} = 2.5$		
S^0	1		

NOTE: columns zero put zero → ϵ unresolve

sign change

system unstable

zero consider ; $\epsilon = 0$

$$\left(\lim_{\epsilon \to 0}\right)$$

$$S^2 \Rightarrow \left(\lim_{\epsilon \to 0}\right) \to \frac{8\epsilon - 5}{\epsilon} \Rightarrow -\infty$$

$S1 \rightarrow \frac{\frac{8\xi-5}{\xi} \times 2.5 - \xi}{\frac{8\xi-5}{\xi}} + \left(\lim_{\xi \to 0}\right)$

$\lim_{\xi \to 0} \frac{2.5 \times 8 - \frac{5}{\xi} \times 2.5 - \xi}{8 - \frac{5}{\xi}}$

$= 2.5$

"(system is unstable)"
(−ve sign change)
first column.

Method II:

→ NOTE: first Row and first column (first column) zero solution put eqn $s = 1/z$

→Q. $s^5 + 2s^4 + 4s^3 + 8s^2 + 3s + 1 = 0$

put $s = 1/z$

$\frac{1}{z^5} + \frac{2}{z^4} + \frac{4}{z^3} + \frac{8}{z^2} + \frac{3}{z'} + 1 = 0$

$\Rightarrow 1 + 2z + 4z^2 + 8z^3 + 3z^4 + z^5 = 0$

$\Rightarrow z^5 + 3z^4 + 8z^3 + 4z^2 + 2z + 1 = 0$

z^5	1	8	2
z^4	3	4	1
z^3	6.6	1.6	
z^2	3.27	1	
z^1	−0.91	0	
z^0	1		

⇒ (system unstable)

→ NOTE: all Rows (R) are zero then, zero just before power + eqn put value. in zero. replace by eqn (differentiate)

Q. $s^6 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0$

s^6	1	5	2	−8
s^5	1	3	−4	0
s^4	2	6	−8	
s^3	0→(8)	0→(12)	0→(0)	
s^2	3	−8		
s^1	100/3	0		
s^0	−8			

→ eqn: $K = 2s^4 + 6s^2 - 8 = 0$

$\frac{dK}{ds} = 8s^3 + 12s^1 + 0$

$\frac{dK}{ds} = 8s^3 + 12s$

↓

replace zero by 8, 12, 0

• unstable system (−ve sign)

Q. $s(s^2+2s+3)(s+2)+K=0$

find the range of k the system having following characteristics e^n will be stable.

Sol^n: $(s^3+2s^2+3s)(s+2)+K=0$

$s^4+2s^3+3s^2+2s^3+4s^2+6s+K=0$

$K+s^4+4s^3+7s^2+6s=0$

$s^4+4s^3+7s^2+6s+K=0$ —(i)

s^4	1	7	K
s^3	4	6	0
s^2	5.5	K	
s^1	$\frac{33-4K}{5.5}$	0	
s^0	K		

Case I: $K>0$

case II: $\frac{33-4K}{5.5}>0$

$33-4K>0$

$+4K<+33$

$K<8.25$

$\boxed{8.25>K>0}$ Range of k

→ ROOT LOCUS:

The root locus technique for the s plane has been stablish as in important tool for and analysis and design or linear control system

• general steps to plot root locus

1. denominator $=0$ poles

Numerator $=0$ zeroes

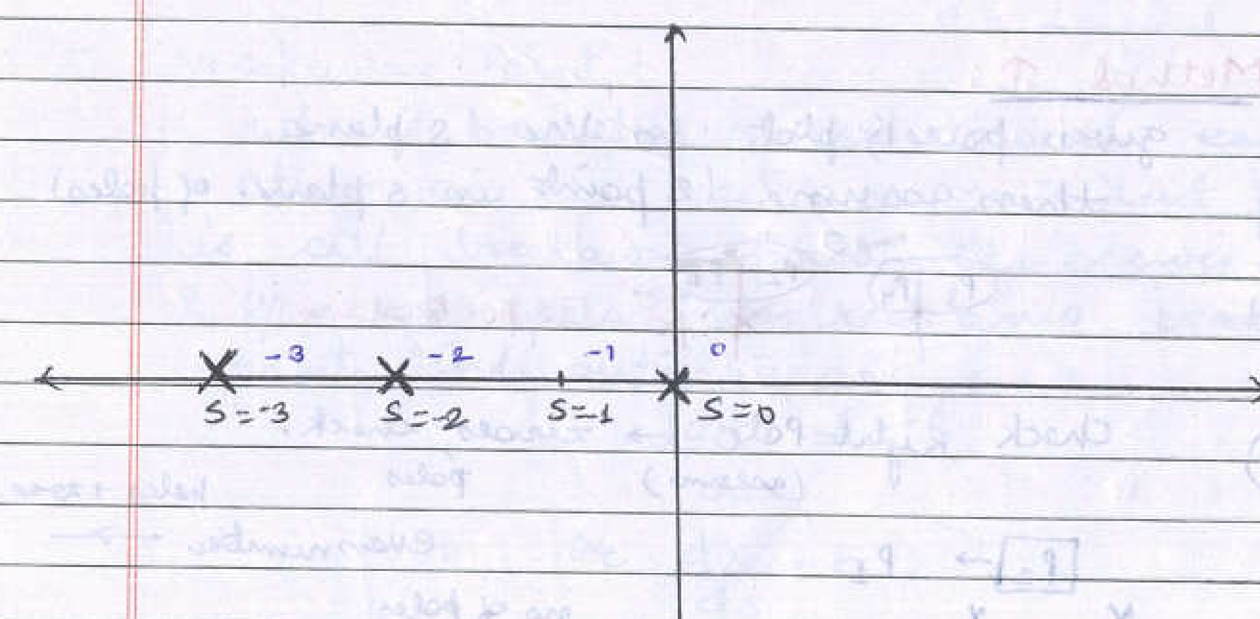
determine all open loop poles and zeroes of open loop transfer functio and plot in s plane.

$G(s)=\frac{k}{s(s+2)(s+3)}$ gain not defined so used k

zeroes → $k=0$, NON ⊙

poles → $s=0, s=-2, s=-3$

$P=0,-2,-3$ ⊗



Number of zeroes $Z = 0$
Number of poles $P = 3$

2. <u>Starting and ending point</u>:

The root locus start from the open loop poles $K=0$ and terminate on either finite open loop zero(0) and $K=\infty$

pole $= K=0$ terminate $K=\infty$

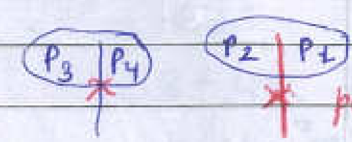
3. <u>Number of branches</u>:

no. of poles $= P$
no of zeroes $= Z$

if $P > Z$ no of branches = no of poles
if $Z > P$ no of branches = no of zeroes

4. Determine the directn of root loci on real axis:

(i) **Method I**:
given poles, plot in the s plane then assume (2 point in s plane of poles)

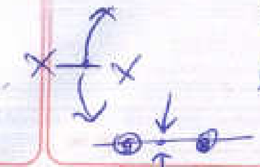


(Real axis) Check Right Pole (assume) → zeroes, poles check.

poles → zero
even number →

$\boxed{P_2} \rightarrow P_1$

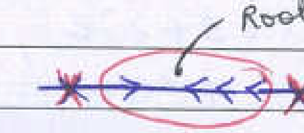
X X no. of poles



Poles → break system → out
zeroes → break system → IN

no. of zeroes and poles
{ odd : ∞ ← poles
even : poles → zero

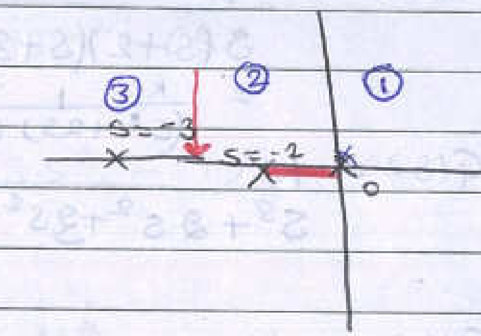
(Consider assume right poles)



system Break root locus adjust

→ **Method II**:

write poles and zeroes → counting



NRL RL
④ ← ③ ← ② ← ①

odd → even RL
even → odd NRL

Intersect point root locus adjust ② ← ①
Not adjust → not intersect. ③ ← ②

· Breakin point adjust & poles and & zeroes b/w adjust

5. Breakaway Point:

when 2 branches move towards each other on the real axis coincident point is call breakaway point. Its occurs b/w 2 open loop poles on real axis breakover point find out by using:

$$\frac{dG(s)}{ds} = 0 \quad \text{or} \quad \frac{dK}{ds} = 0$$

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$$G(s) = \frac{K}{s(s+1)(s+3)}$$

Method I:

$$\frac{dG(s)}{ds} = 0$$

$$G(s) = \frac{K}{s(s+2)(s+3)} = \frac{K}{s^3+2s^2+6s+3s^2}$$

$$= \frac{K}{(s^2+2s)(s+3)} = \frac{K}{s^3+3s^2+2s^2+6s}$$

$$G(s) = \frac{K}{s^3+2s^2+3s^2+6s}$$

$$G(s) = \frac{K}{s^3+4s^2+6s+1s^2}$$

$$G(s) = K(s^3+4s^2+6s)^{-1}$$ (+1s²)

$$\frac{dG(s)}{ds} = -K(s^3+4s^2+6s)^{-2}(3s^2+9s+6)$$ (+1s², +1s)

$$\frac{dG(s)}{ds} = 0$$

$$\boxed{3s^2+10s+6+1s=0}$$ NRL Not adjust Root locus

$$\boxed{3s^2+10s+6=0}$$ ③ ← ②

$$\boxed{s = -0.784, \; -2.549}$$

RL ② ← ①

Breakover point is $\boxed{-0.784}$

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• Method II: characteristics eqⁿ find out

Solve method because eqⁿ (i) is used further

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+3)} \cdot 1 = 0$$

$$s(s+2)(s+3) + K = 0$$

$$(s^2+2s)(s+3) + K = 0$$

$$\boxed{s^3+5s^2+6s+K=0} \quad \text{—(i)}$$

$$K = -s^3-5s^2-6s$$

$$\frac{dK}{ds} = -3s^2-10s-6$$

$$\frac{dK}{ds} = 0$$

$$-3s^2-10s-6 = 0$$

$$3s^2+10s+6 = 0$$

$$\boxed{s = -0.784}$$ breakover point

6. Asymptotes of real axis:

(i) Intersection point of asymptotes

$$\boxed{x = \frac{\Sigma P - \Sigma Z}{P - Z}}$$

ΣZ = all zeroes add with sign

ΣP = (all) add poles with sign

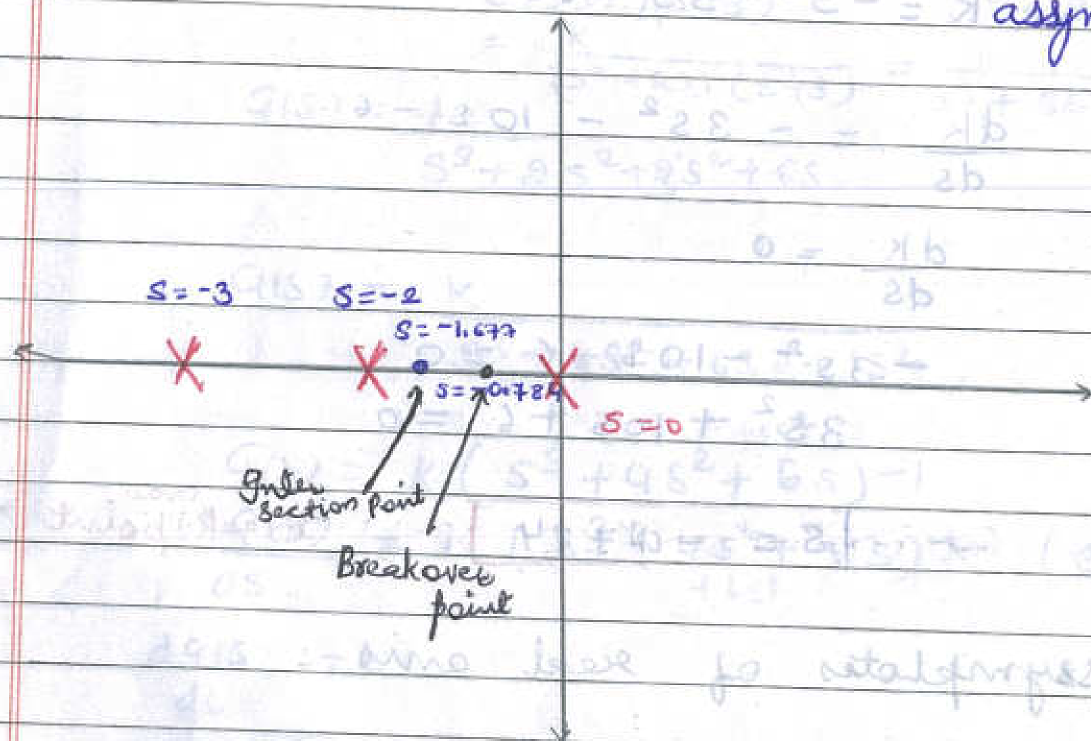
P = no of poles

Z = no of zeroes

$$X = \frac{(0-2-3)-0}{3-0}$$

$$X = \frac{-5}{3} = -1.667$$

$\boxed{X = -1.677}$ This is the intersection point of asymptotes.



6. Angle of asymptotes:

$$\boxed{\theta = \frac{(2m+1)\times 180}{P-Z}}$$

P=3
Z=0

m = 0, 1, 2 ... so on ... (P-Z-1)

m = 3-0-1
m = 2

P=3
Z=0

m = 0, 1, 2

put
m = 0 $\theta_1 = \frac{1\times 180}{(3-0)} = \frac{180}{3} = 60$

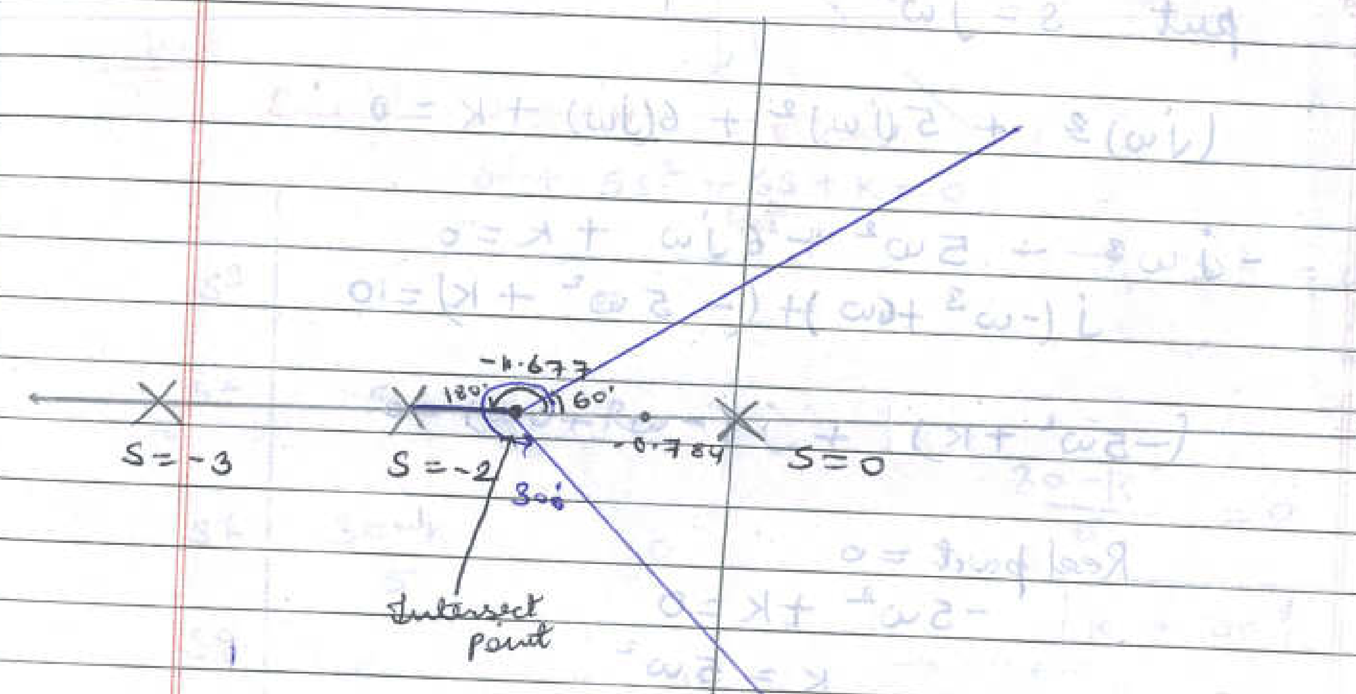
m = 1 $\theta_2 = \frac{(2\times 1+1)}{3}\times 180 = \frac{4\times 180}{3} = 60\times 4$

$\theta_2 = 180° = 180°$

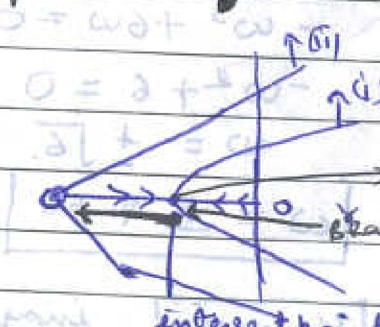
m $\theta_2 = \frac{(2\times 2+1)\times 180}{3} = 300°$

$\theta_1 = 60°$
$\theta_2 = 180°$
$\theta_3 = 300°$

intersection point all asymptotes draws: (angle)



7. Intersect point of imaginary axis:



system Break then Not intersect

break'd angle (ii) & (i)

But Intersect ...

Point where the branch intersect the imaginary axis. determine by the 2 method

(i) $S = j\omega$ → complex eqⁿ solⁿ thus eqⁿ is used

(ii) Routh Hurwitz criteria (solve II method)

Characterstics eqⁿ:

$1 + G(s)H(s) = 0$

$S^3 + 5S^2 + 6S + K = 0$

Method I: put $S = j\omega$

$(j\omega)^3 + 5(j\omega)^2 + 6(j\omega) + K = 0$

$-j\omega^3 - 5\omega^2 + 6j\omega + K = 0$

$j(-\omega^3 + 6\omega) + (-5\omega^2 + K) = 0$

$(-5\omega^2 + K) + j(-\omega^3 + 6\omega) = 0$

Real part $= 0$

$-5\omega^2 + K = 0$

$K = 5\omega^2$

$\boxed{K = 5\omega^2}$

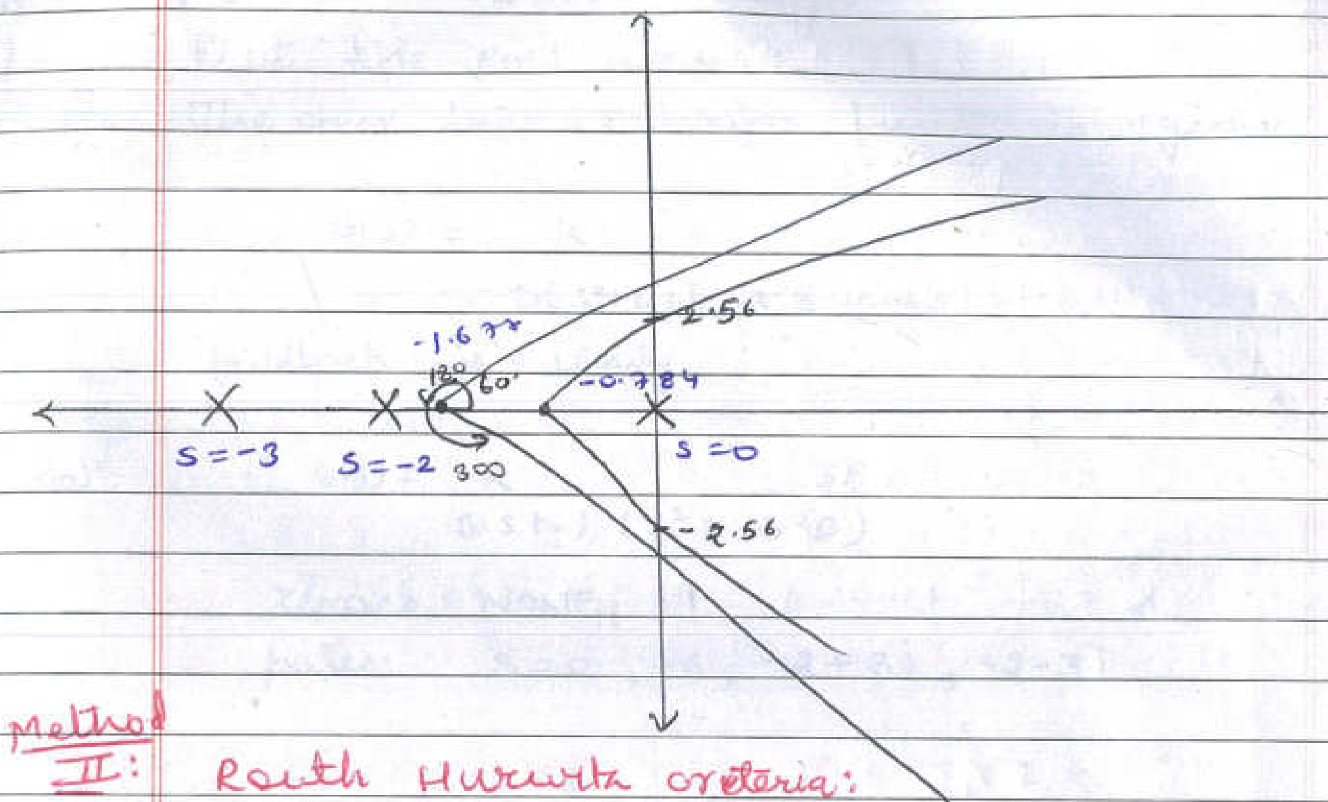
Imaginary $= 0$

$-\omega^3 + 6\omega = 0$

$-\omega^2 + 6 = 0$

$\omega = \pm\sqrt{6}$

$\boxed{\omega = \pm 2.56}$



Method II: Routh Hurwitz criteria:

$S^3 + 5S^2 + 6S + K = 0$

S^3	1	6
S^2	5	K
S^1	$\frac{30-K}{5}$	0
S^0	K	

→ $S^1 = 0$ put solve

$\frac{30-K}{5} = 0$

$\boxed{K = 30}$

→ S^2 eqⁿ:

$\boxed{5S^2 + K = 0}$

$5S^2 + 30 = 0$

$S = \pm j\sqrt{6}$

put $S = j\omega = S$

$\omega = \pm\sqrt{6}$

$\boxed{\omega = \pm 2.56}$

Imp exam

Plot the Root locus :

Q. The open loop transfer funct^n is given by

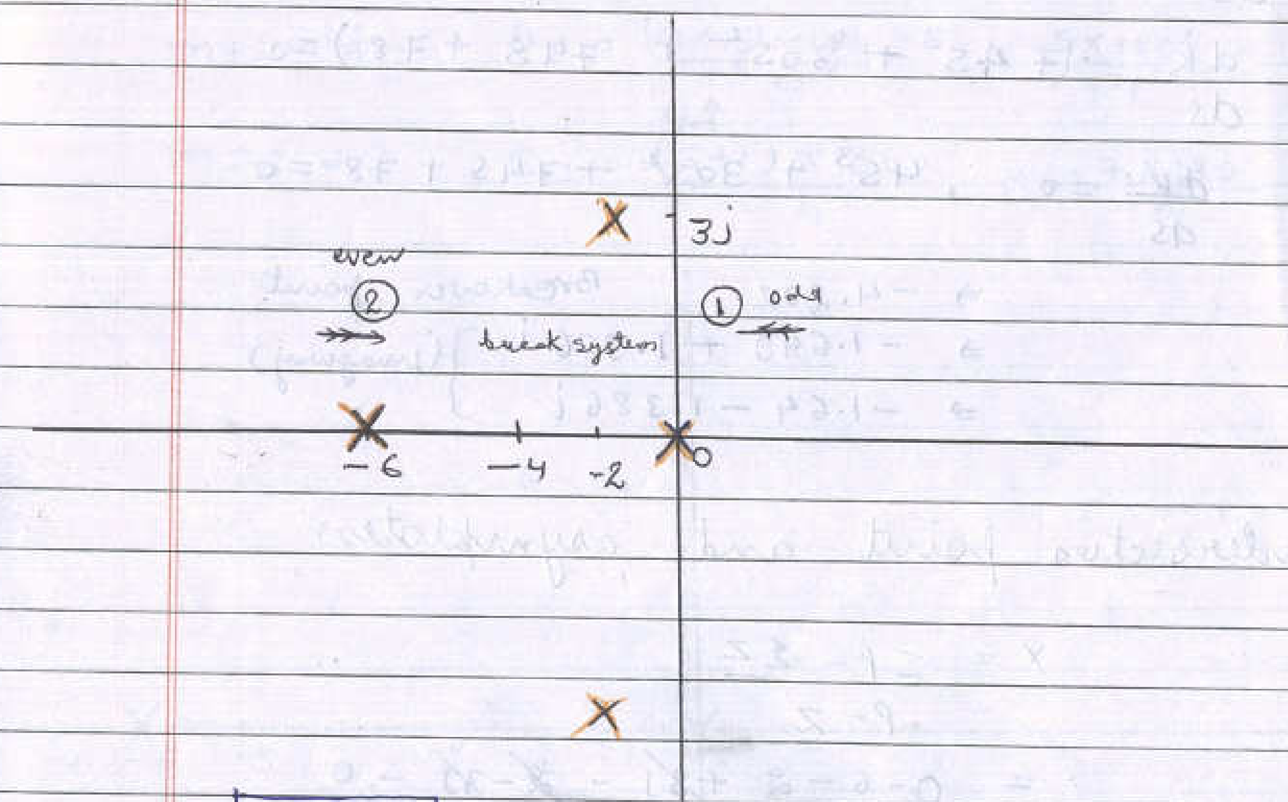
$$G(s) = \frac{K}{S(S+6)(S^2+4S+3)}$$

feedback is unity.

Sol^n: $$G(s) = \frac{K}{s(s+6)(s^2+4s+13)}$$

Zeroes : NONE

poles: $s = 0, -6, -2+3j, -2-3j$



$\boxed{\begin{array}{l} Z = 0 \\ P = 4 \end{array}}$ no of Branches

- Breakover point :

$1 + G(S)H(S) = 0$

$$1 + \frac{K}{s(s+6)\,(s^2+4s+13)} = 0$$

$$1 + \frac{K}{s(s+6)(s^2+4s+13)} = 0$$

$$s(s+6)\,(s^2+4s+13) + K = 0$$

$$(s^2+6s)\,(s^2+4s+13) + K = 0$$

$$s^4 + 4s^3 + 13s^2 + 6s^3 + 24s^2 + 13\times 6s + k = 0$$

$$s^4 + 10s^3 + 37s^2 + 78s + K = 0$$

$$\rightarrow s^4 + 10s^3 + 37s^2 + 78s + K = 0$$

eqn(1) → $K = -(s^4 + 10s^3 + 37s^2 + 78s) = 0$

$$\frac{dk}{ds} = 0$$

$$\frac{dk}{ds} = -(4s^3 + 30s^2 + 74s + 78) = 0$$

$$\frac{dk}{ds} = 0, \quad 4s^3 + 30s^2 + 74s + 78 = 0$$

$\Rightarrow -4.202$ Breakover point

$\Rightarrow -1.649 + 1.386\,i$
$\Rightarrow -1.64 - 1.386\,i$ } (imaginary)

→ Intersection point and asymptotes:

$$X = \frac{\Sigma P - \Sigma Z}{P - Z}$$

$$= \frac{0 - 6 - 2 + \cancel{3j} - 2 - \cancel{3j} - 0}{4 - 0}$$

$$X = \frac{-6-4}{4} = \frac{-10}{4} = \frac{-5}{2}$$

$$\boxed{X = \frac{-5}{2}}$$

→ Angle of Asymptotes:

$$\theta = \frac{(2m+1) \times 180°}{P - Z}$$

$m = 0, 1, 2, 3$ $\quad P = 4, \; Z = 0$

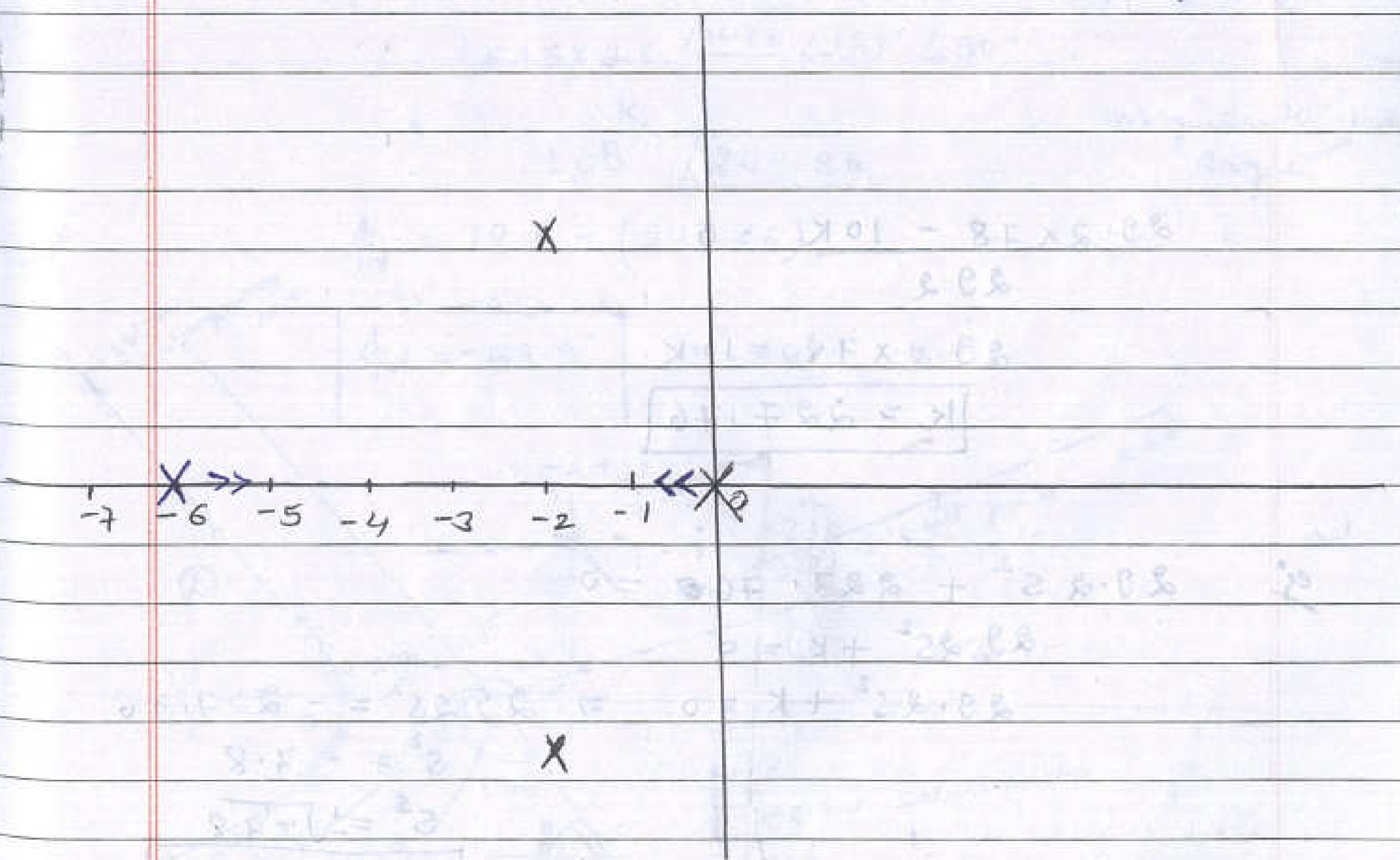
$P - Z - 1$
$P = 3$

put $m = 0$ $\quad \theta_1 = \frac{(0+1)\,180°}{4-0} = \frac{180°}{4} = 45°$

put $m = 1 \Rightarrow \theta_2 = \frac{(2+1)\times 180}{4} = \frac{3\times 180'}{4} = 135°$

$m = 2 \Rightarrow \theta_3 = \frac{(4+1)\times 180}{4} = \frac{5\times 180'}{4} = 225'$

$m = 3 \Rightarrow \theta_3 = \frac{(6+1)\times 180}{4} = \frac{7\times 180'}{4} = 315'$



→ Intersect point:

Routh Hurwitz criteria:

s^4	1	37	K
s^3	10	78	0
s^2	29.2	$\frac{10\times K-0}{10}=K$	
s^1	$\frac{29.2\times 78-10\times K}{29.2}=227.76$ $=K$ $=0$		
s^0	K		

$$\frac{29.2\times 78-10K}{29.2}=0$$

$$29.2\times 78=10K$$

$$\boxed{K=227.76}$$

eq: $29.2s^2+227.76=0$

$$29.2s^2+K=0$$

$$29.2s^2+K=0 \Rightarrow 29.2s^2=-227.76$$

$$s^2=-7.8$$

$$s=\pm\sqrt{-7.8}$$

put $s=j\omega$

$$j\omega=\pm 2.79j$$

$$\boxed{\omega=\pm 2.79}$$ Intersect point

→ angle of Departure:

$$\phi_d=180°-(\phi_p-\phi_z)$$

$$\phi_p''=\frac{K}{s(s+6)(s^2+4s+13)}=\frac{K}{s(s+6)(s+2-3j)(s+2+3j)}$$

$s=-2+3j$

$$=\frac{K}{(-2+3j)(-2+3j+6)(-2+3j+2+3j)}$$

$$\theta p''=\frac{K}{(-2+3j)(4+3j)(6j)}$$

$\Downarrow 90°$ $\tan^{-1}(b/a)$

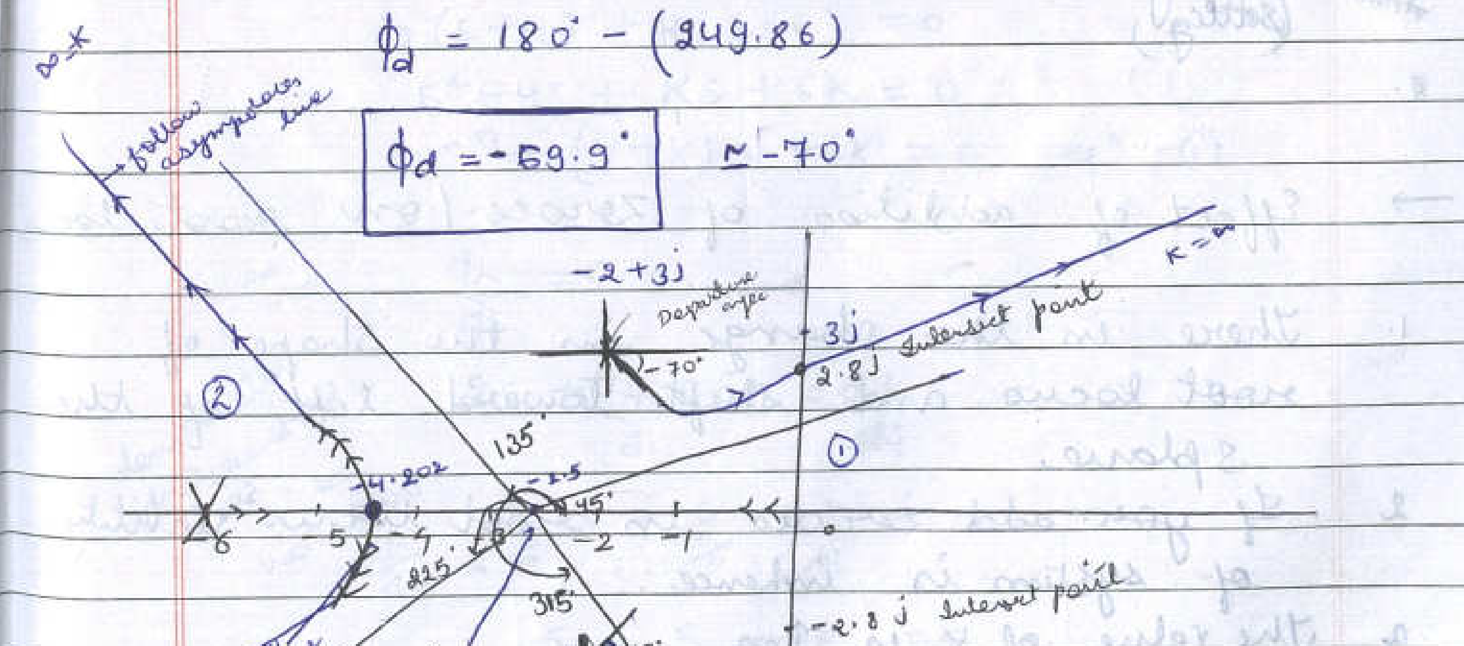
$$\theta p''=\frac{K}{6\times 5\times 3.6\ \angle 36.86\ \angle 123°\ \angle 90°}$$

$$\frac{K}{108\ \angle 249.86}$$

magnitude multiple, angle add

$$\phi_d=180°-(249.86)$$

$$\boxed{\phi_d=-69.9°}\ \simeq -70°$$



→ Effect of addition of poles: on root locus:

$$G(s) \cdot H(s) = \frac{K}{s(s+a)(s+b)}$$

• effect of addition of poles:

1. change in shape of root locus and it shift toward the imaginary axis.
2. The intersect on the jω axis occurs for a lower value of K.

poles ↑ K value ↓

3. System becomes oscillatory.
4. gain margin and relative stability decrease.
5. Reduction in the Range of K.
6. Settling time increase. (settling)
7.

→ Effect of addition of zeroes / on root locus

1. There is the change in the shape of root locus and shift toward left of the s plane.
2. If you add zeroes in root locus so stability of system is inhence.
3. The value of K is ↑es.
4. The settling time speedup

Nr. Q. The Loop Transfer function of feedback control system is

loop transfer function

$$G(s)H(s) = \frac{K(s+6)}{s(s+4)}$$ sketch the root locus

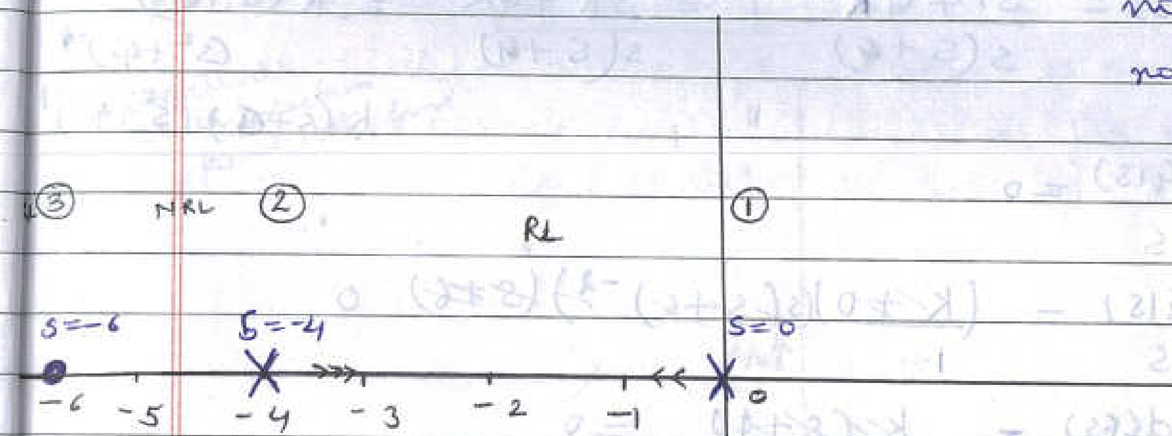
Sol^n:

zeroes $s = -6$

poles $s = 0, -4$

no of poles $P = 2$

no of zeroes $Z = 1$



$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+6)}{s(s+4)} = 0$$

$$s(s+4) + K(s+6) = 0$$

$$s^2 + 4s + Ks + 6K = 0$$

$$s^2 + (4+K)s + 6K = 0 \quad eq^n - (i)$$

Not solve.

$$\frac{dK}{ds} = 0$$

$$2s + \frac{(4+K)}{6} = \frac{dK}{ds} = 0$$

denominator s second order so eq^n not used $\frac{dK}{ds}$

$$\frac{2}{6}s + \frac{4}{6} + \frac{K}{6} = 0$$

$$s + \frac{2}{\ } + 2K = 0$$

$$\frac{dG(s)}{ds} = 0$$

$$G(s)\cdot H(s) = \frac{K(s+6)}{s(s+4)} = \frac{K(s+6)}{s(s+4)}$$

$$G(s) = \frac{K(s+6)\cdot 1}{s(s+4)} = \frac{K(s+6)}{(s^2+4s)}$$

$$G(s) = \frac{sK+6K}{s(s+4)} = \frac{sK+6K}{s(s+4)} = K(s+6)(s^2+4s)^{-1}$$

$$K(s+6)(s^2+4s)^{-1}$$

$$\frac{dG(s)}{ds} = 0$$

$$\frac{dG(s)}{ds} = (K+0)(s(s+6)^{-2})(s+6) \quad 0$$

$$\frac{dG(s)}{ds} = \frac{K(s+4)}{(s+6)^2} = 0$$

$$\frac{dG(s)}{ds} = \frac{s(s+4)\cdot K - K(s+6)(2s+4)}{(s(s+4))^2} = 0$$

$$\frac{dG(s)}{ds} = (s^2+4s)K - K(s+6)(2s+4) = 0$$

$$\Rightarrow s^2+4s-(2s^2+4s+12s+24) = 0$$

$$-s^2 + \cancel{4s} - \cancel{4s} - 12s - 24 = 0$$

$$s^2+12s+24 = 0$$

$$\boxed{s = -2.53,\ -9.464}$$ Break over point

→ angle of asymptotes:

$$\theta = \frac{(2m+1)\times 180^\circ}{P-Z}$$

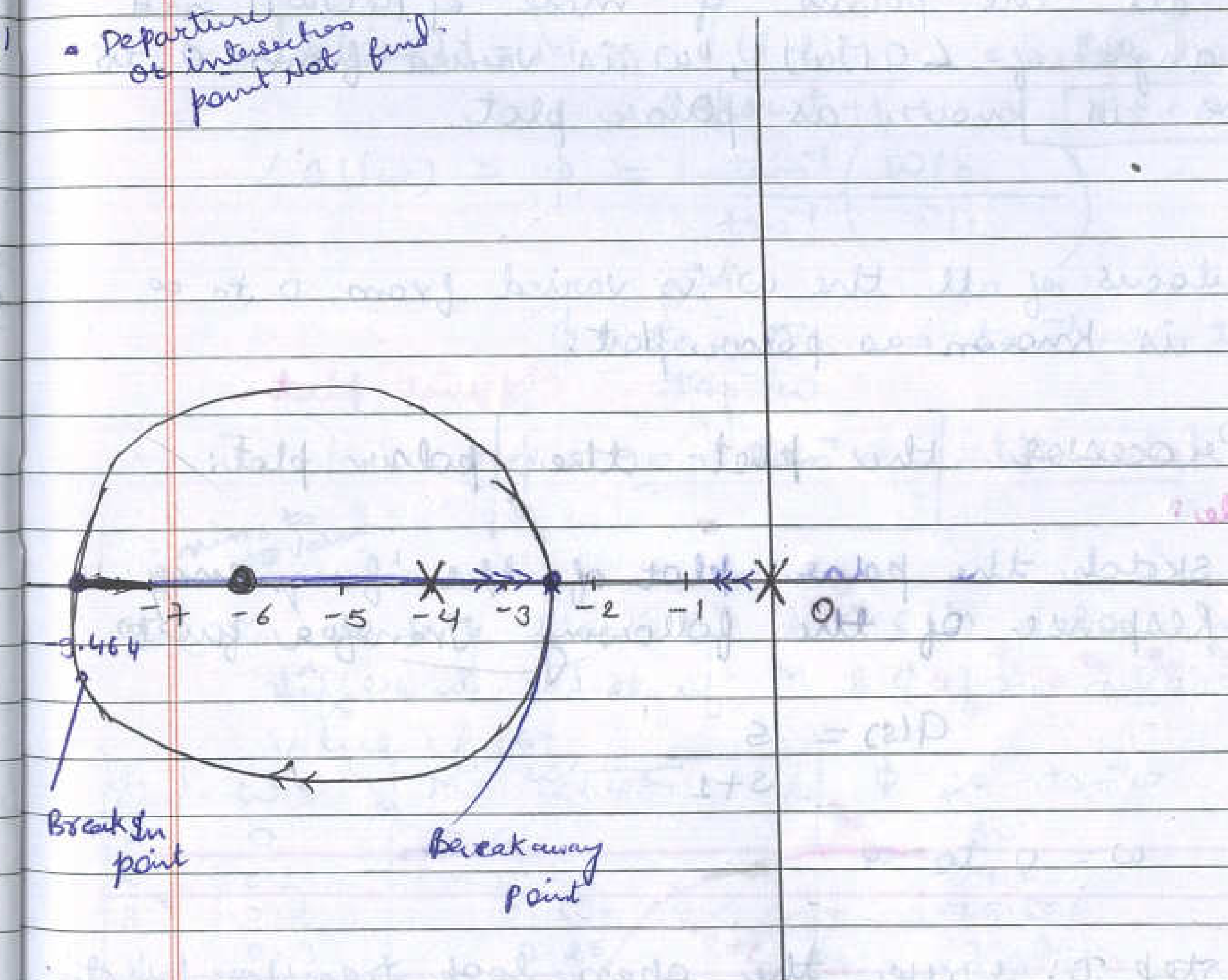
$$\theta_1 = \frac{1\times 180}{2-1} = \frac{90^\circ\times 2}{2-1} = 180^\circ \qquad m = 0, 1$$

$$P-Z-1 = 2-1-1 \Rightarrow 0$$

$$\theta = \frac{(2+1)\times 180^\circ}{1} = \frac{3\times 180}{1} =$$

put $m=0$ $\boxed{\theta_1 = 180^\circ}$

• Departure or intersection point not find.



$\omega = 0$ to ∞ → Polar plot] Mirror image
$\omega = -\infty$ to ∞ → Nyquist plot

→ Polar Plots:

Any sinusoidal transfer function $G(j\omega)$ having magnitude mode of $|G(j\omega)|$ and phase angle $\angle G(j\omega)$ can be ploted on polar cordinate varying ω from 0 to ∞. The plot of both mode of $G(j\omega)$ and angle of $G(j\omega)$ a funct^n of ω give a polar plot, the locus of all the points of mode of $|G(j\omega)|$ and angle of $\angle G(j\omega)$, ω is varied from 0 to ∞ is known as polar plot.

locus of all the ω is varied from 0 to ∞ is known as polar plot.

→ Processor the plot the polar plot:

example:

Sketch the polar plot of the frequency Response of the following Transfer funct^n

$$G(s) = \frac{s}{s+1}$$

$\omega = 0$ to ∞

Sol^n. step I: write the open loop transfer function $G(s)$ of the system

$$G(s) = \frac{s}{s+1}$$

• step II: for sinusiodal form put $s = j\omega$

$$G(s) = \frac{j\omega}{j\omega+1} \qquad j\omega \to a+ib \quad a=0 \Rightarrow \sqrt{a^2+b^2}$$

• step III: write the eq^n for magnitude $|G(j\omega)|$ and phase angle of $\angle G(j\omega)$

$$|G(j\omega)| = M = \frac{\sqrt{(j\omega)^2}}{\sqrt{(j\omega)^2+1}} = \boxed{\frac{j\omega}{\sqrt{1+\omega^2}}} \quad \text{—(i)}$$

$$\angle G(j\omega) = \phi = \frac{\tan^{-1}\left(\omega/0\right)}{\tan^{-1}\left(\omega/1\right)}$$

$$\phi = \frac{\tan^{-1}\infty}{\tan^{-1}\omega} \Rightarrow \tan^{-1}\infty - \tan^{-1}\omega$$

$$\boxed{\phi = 90° - \tan^{-1}\omega} \quad \text{—(ii)}$$

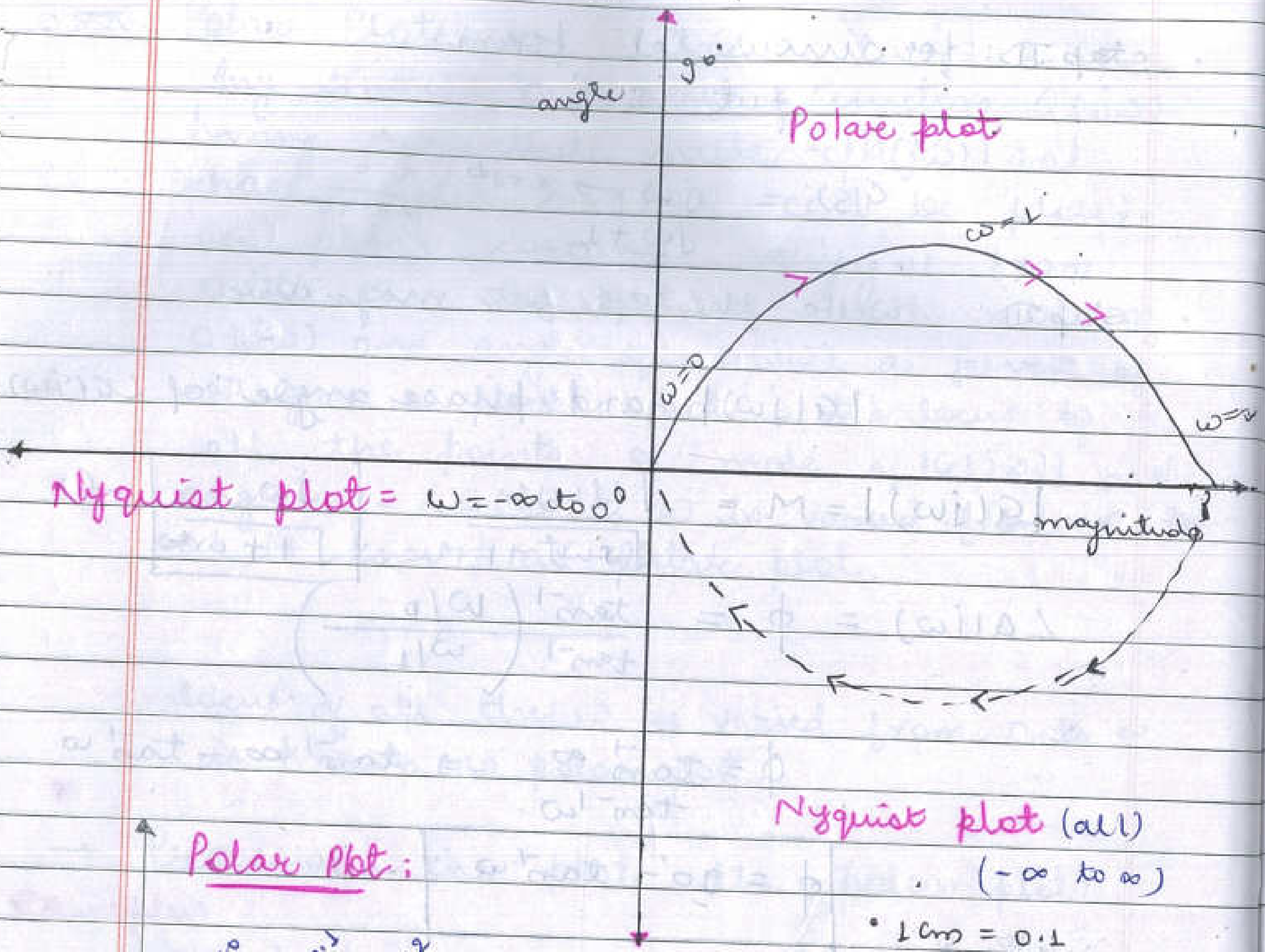
miss to 10 value

Step IV: using the eq^n no.(i) & (ii) determine the different value of M & ϕ for different value of ω

S.No	ω	$M = \omega/\sqrt{\omega^2+1}$	$\phi = 90° - \tan^{-1}\omega$
1.	0	0	90°
2	0.1	0.1	84.28°
3	0.2	0.196 ≃ 0.2	78.69°
4	0.3	0.25	73.30°
5	0.5	0.44	63.43°
6	1	0.707	45°
7	2	0.89	26.56°
8	5	0.980	11.30°
9	10	0.995	5.71°
10	∞	1	0°

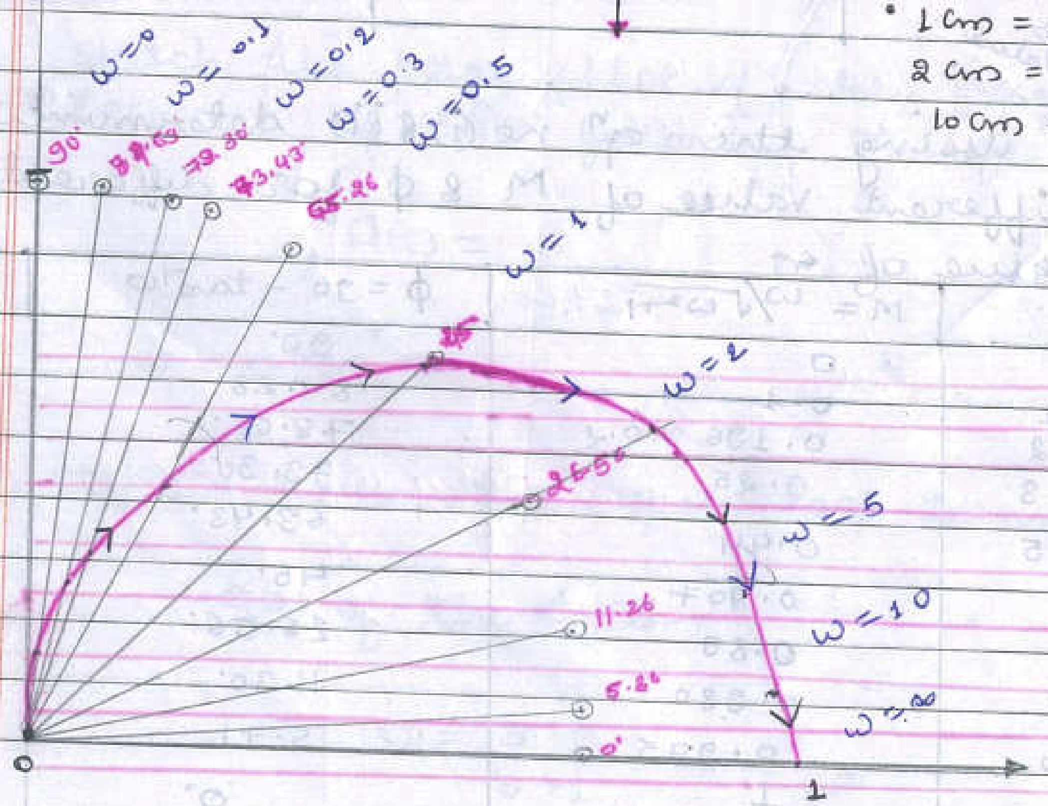
$\omega = -\infty$ to ∞

→ **Polar Plot:** $\omega = 0$ to ∞



Nyquist plot (all) $(-\infty$ to $\infty)$

Polar Plot:



$1\,cm = 0.1$

$2\,cm = 0.2$

$10\,cm = 1$

$\boxed{\tan^{-1}(-\infty) = 180°}$

Q.2 Sketch the polar plot of following transfer functn,

$$G(s) = \frac{1}{(1+s)(1+2s)}$$

$$G(s) = \frac{1}{(\sqrt{1+s})^2(1+2s)} = \frac{1}{(1+s)(1+2s)}$$

$s = j\omega$

$$G(j\omega) = \frac{1}{(1+j\omega)\cdot(1+2j\omega)}$$

$$|G(j\omega)| = \frac{1 \;\angle \tan^{-1}(0/1)}{\sqrt{1+\omega^2}\,\sqrt{1+4\omega^2} \;\angle\tan^{-1}(\omega/1) * \;\angle\tan^{-1}(2\omega/1)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \quad \tan^{-1}(0) - \{\tan^{-1}(\omega) + \tan^{-1}(2\omega)\}$$

$$\boxed{|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}}$$

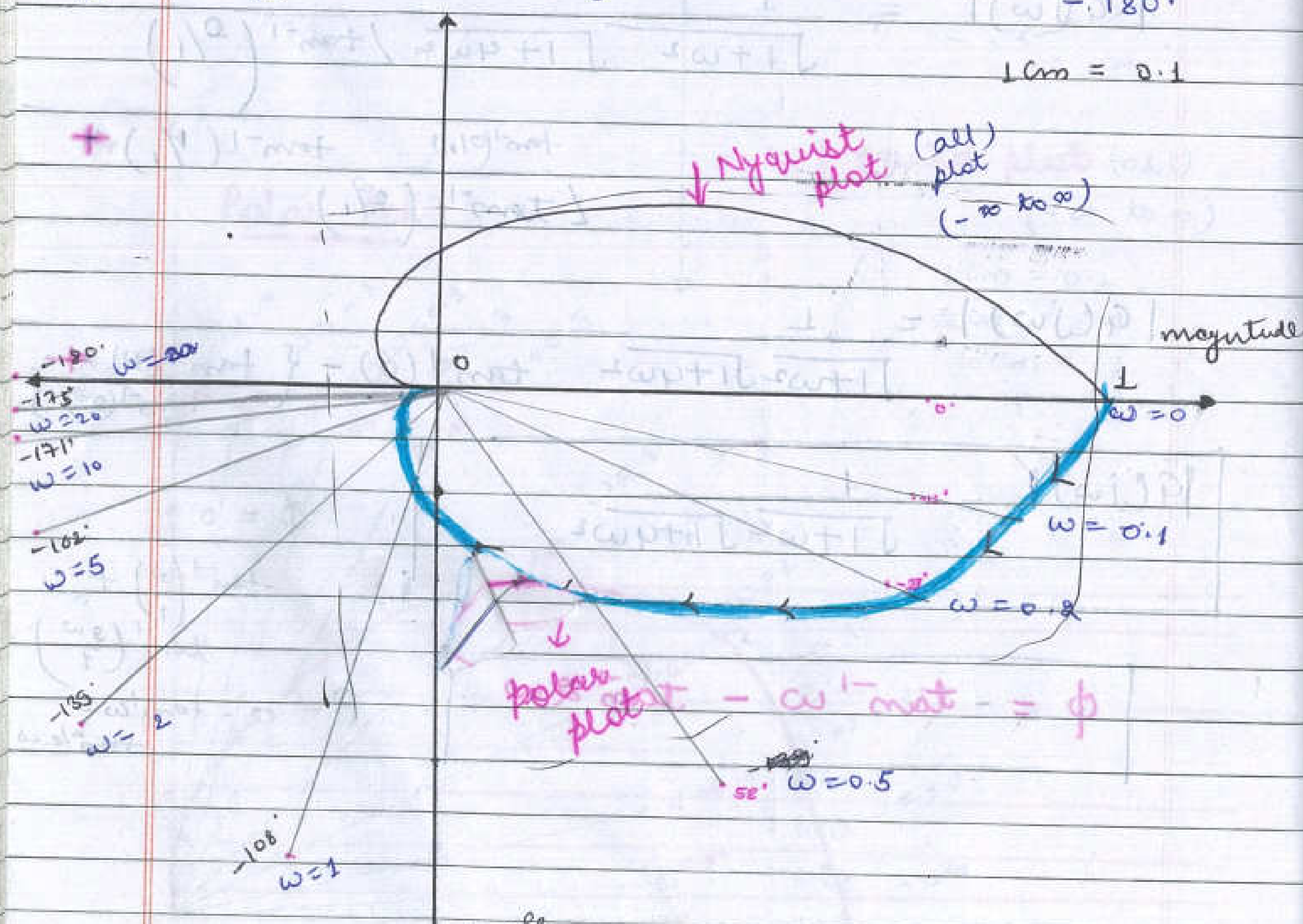
$\phi = 0 - \tan^{-1}\left(\frac{\omega}{1}\right) + \tan^{-1}\left(\frac{2\omega}{1}\right)$

$$\boxed{\phi = -\tan^{-1}\omega - \tan^{-1}2\omega}$$

$\phi = 0 - \tan^{-1}\omega - \tan^{-1}2\omega$

S.NO.	ω	$M = \frac{1}{\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$	$\phi = -\tan^{-1}\omega - \tan^{-1}2\omega$
1.	0	1	0°
2	0.1	0.9766	-12.05°
3	0.2	0.8453	-23.99°
4	0.5	0.4472	-58.28°
5	1	0.14142	-108.43°
6	2	0.0548	-139.39°
7	5		-162.27°
8	10		-171.42
9	20	6.17×10^{-4}	-175.70
10	∞	0	-180°

1cm = 0.1



Q. Sketch the polar plot: of given transfer functn

$$G(S) = \frac{1}{S(1+S)}$$

put $S = j\omega$

$$G(S) = \frac{1}{j\omega(1+j\omega)}$$

$$|G(S)|_{j\omega} = \frac{1}{\omega\sqrt{1+\omega^2}} = M$$

$$\angle G(s)_{j\omega} = \frac{1}{j\omega(1+j\omega)} = \frac{\tan^{-1}(0/1)}{\tan^{-1}\left(\frac{\omega}{0}\right) + \tan^{-1}\left(\omega/1\right)}$$

$$= \tan^{-1}(0) - \tan^{-1}(\infty) - \tan^{-1}(\omega)$$

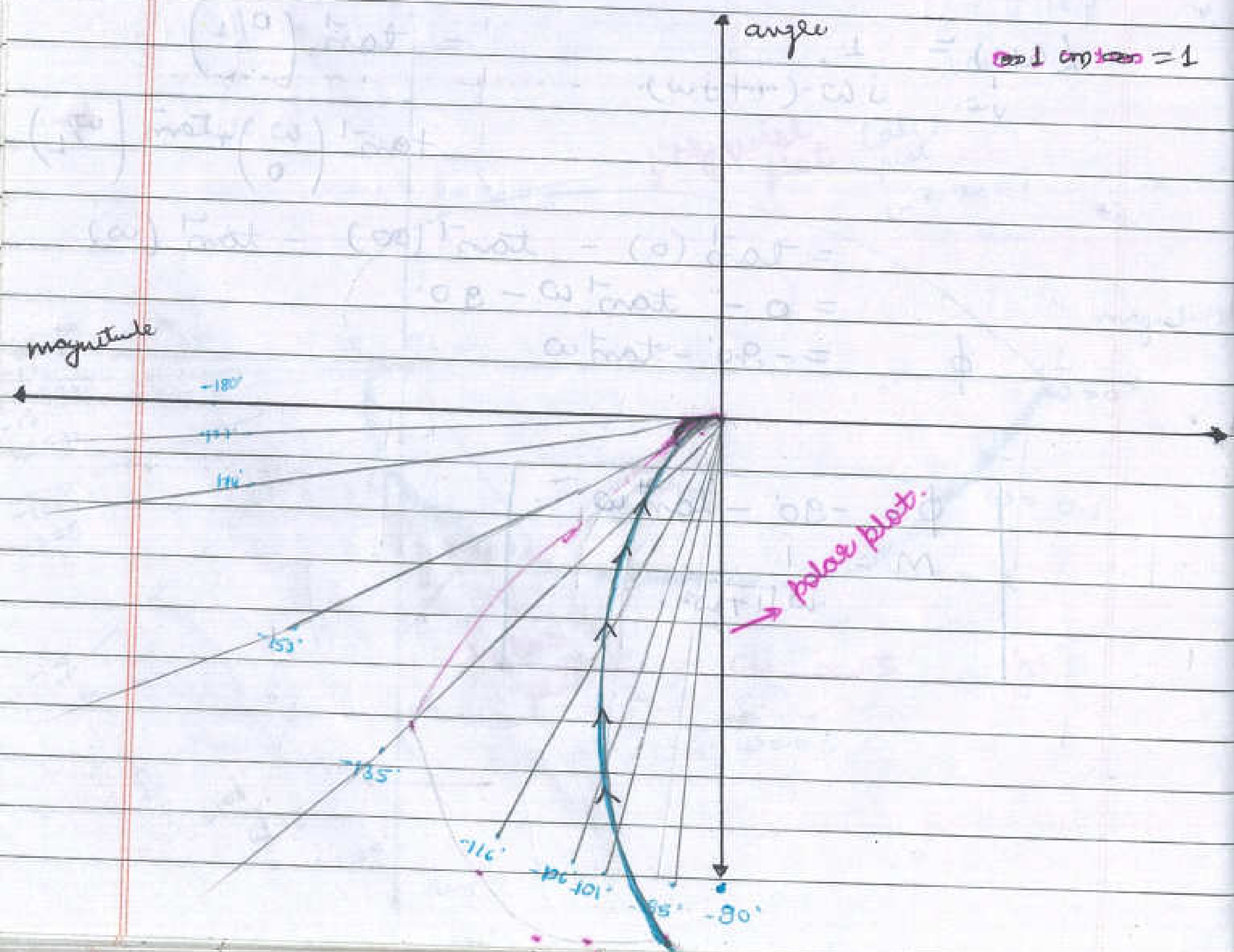
$$= 0 - \tan^{-1}\omega - 90°$$

$$\phi = -90° - \tan^{-1}\omega$$

$$\boxed{\begin{aligned} \phi &= -90° - \tan^{-1}\omega \\ M &= \frac{1}{\omega\sqrt{1+\omega^2}} \end{aligned}}$$

$\{ \tan^{-1}\infty = 90^\circ \}$

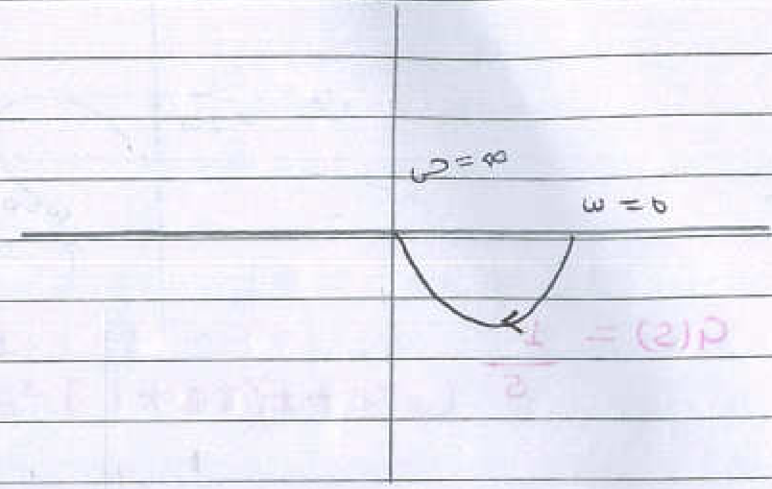
S.No.	ω	$M = \frac{1}{\omega\sqrt{1+\omega^2}}$	$\phi = -90^\circ - \tan^{-1}\omega$
1	0	∞	-90°
2	0.1	9.9568	-95.71°
3	0.2	0.9824	-101.30°
4	0.3	0.953	-106.69
5	0.5	0.849	-116.56
6	1	0.707	-135°
7	2	0.150	-153.43°
8	10	9.900×10^{-3}	-174.28
9	20	5.587×10^{-4}	-177.13
10	∞	0	-180°



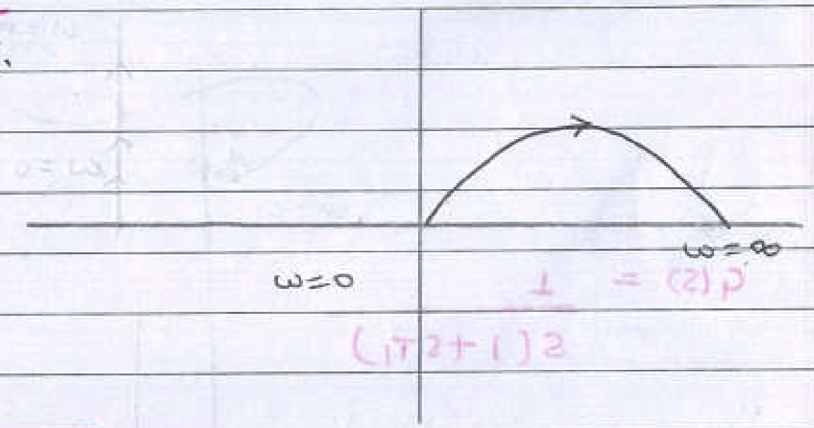
→ Function → polar plot:

- plot (polar) for typical transfer function :

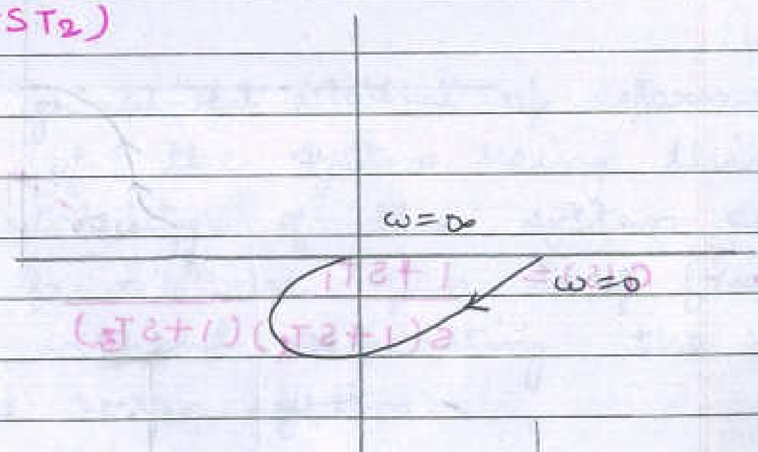
1. $G(s) = \frac{1}{sT+1}$



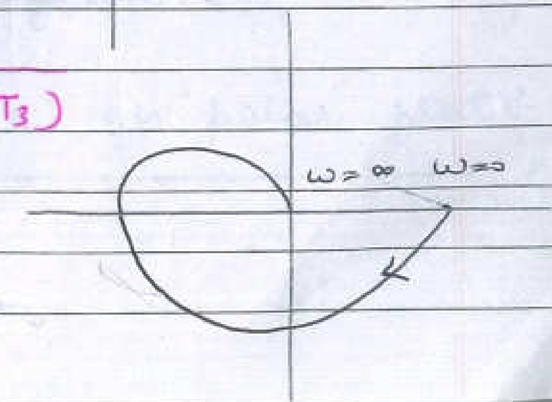
2. $G(s) = \frac{sT}{1+sT}$



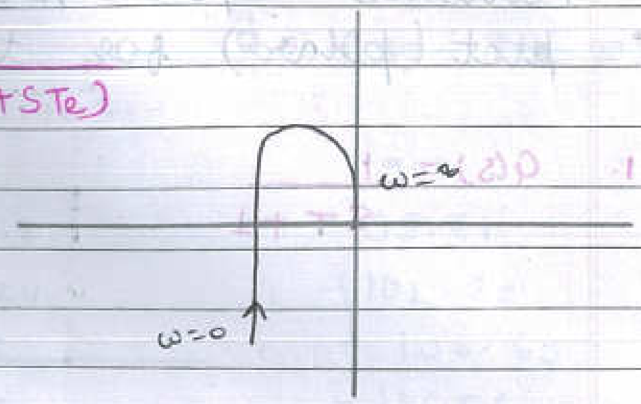
3. $G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$



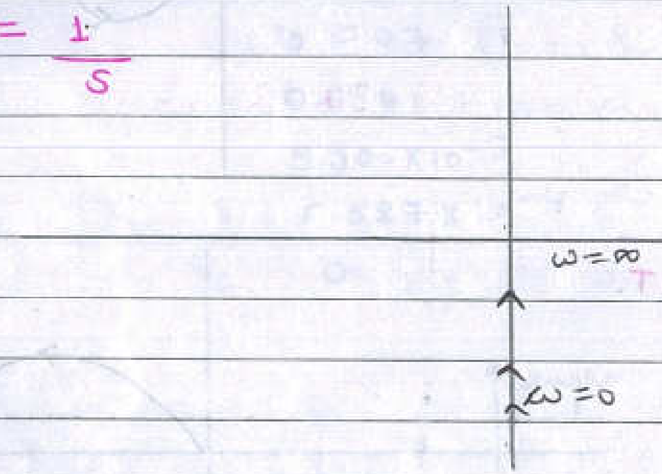
4. $G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$



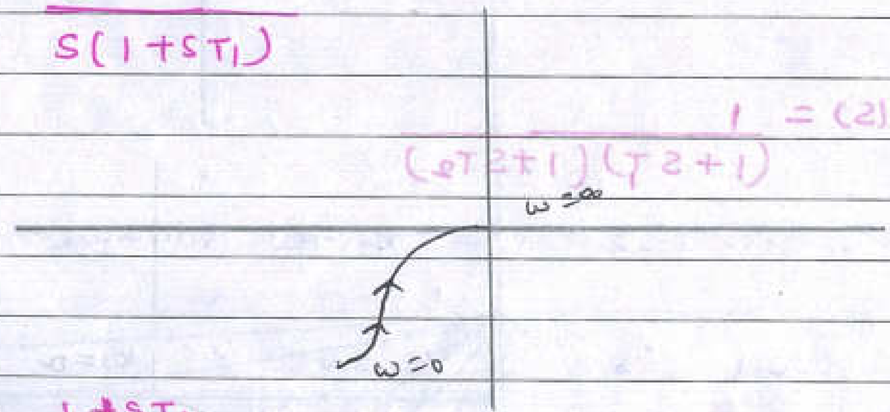
5. $G(s) = \dfrac{1}{s(1+sT_1)(1+sT_2)}$



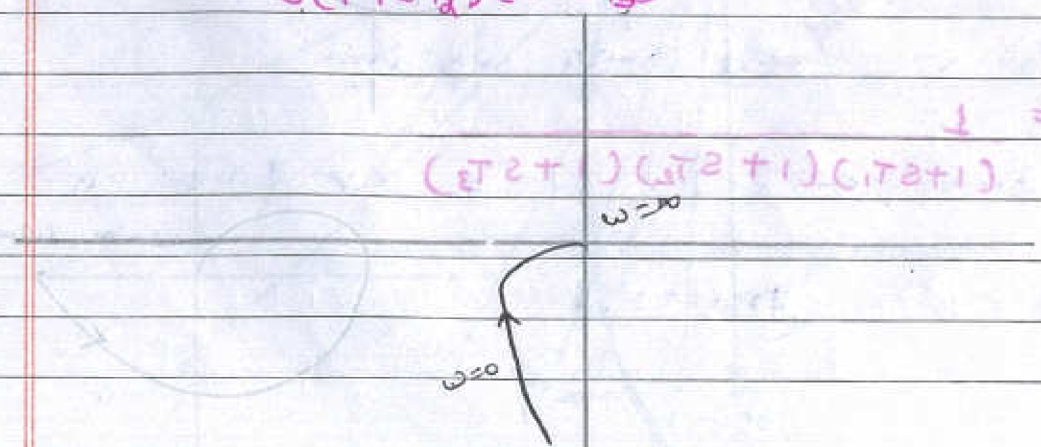
6. $G(s) = \dfrac{1}{s}$



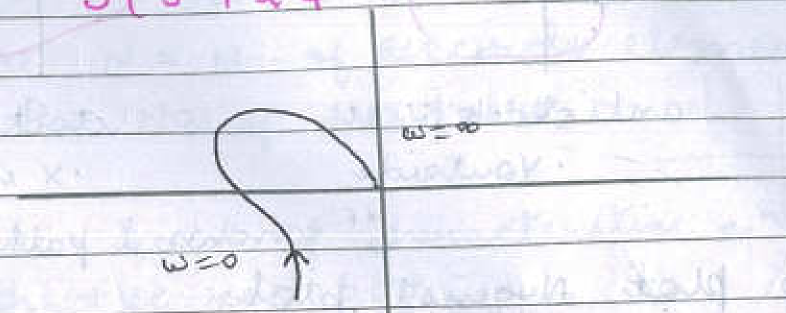
7. $G(s) = \dfrac{1}{s(1+sT_1)}$



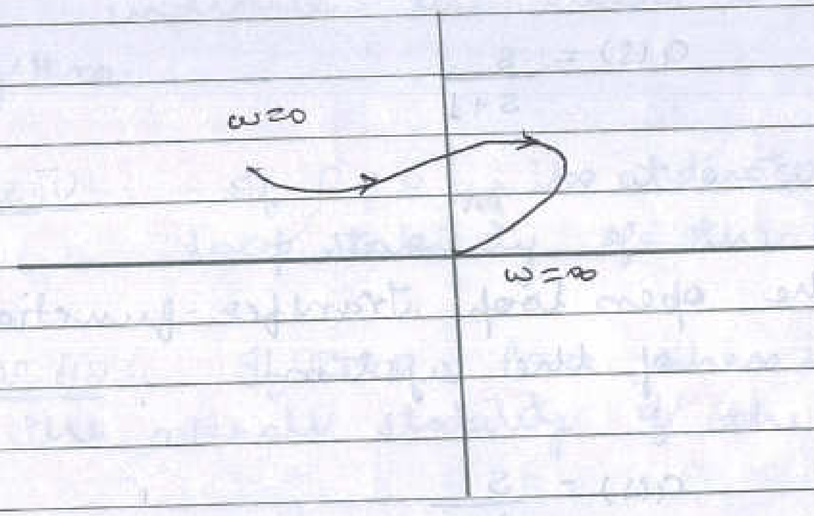
8. $G(s) = \dfrac{1+sT_1}{s(1+sT_2)(1+sT_3)}$



9. $G(s) = \dfrac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$



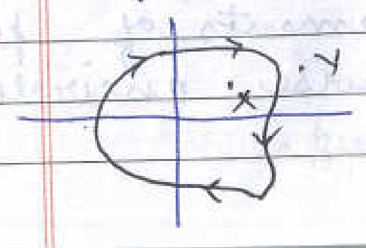
10. $G(s) = \dfrac{1}{s^2(1+sT_1)(1+sT_2)}$



→ NYQUIST PLOT:

Nyquist plot is graphical Method of determine the stability of the system using this Method the stability of the system can be determine from the open loop frequency Response without calculating the Roots of closed loop system.

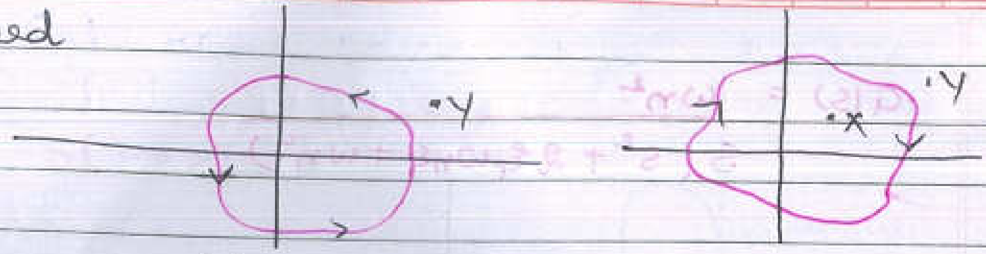
Nyquist plot is base on 'polar plot'.



(encircled)

x inside clockwise

• Enclosed



anticlockwise · Y outside

clockwise · X inside

"enclosed path"

→ STEP to plot Nyquist plot:

case 1: sketch the polar that of the frequency Response of following Transfer functn.

$$G(S) = \frac{S}{S+1}$$

$\omega = 0$ to ∞

case 2: write the open loop Transfer function $G(S)$ of the system

$$G(s) = \frac{S}{S+1}$$

case: 3. for sinusoidal form $s = j\omega$

$$G(S) = \frac{j\omega}{j\omega + 1}$$

case: 4. write the eq^n for magnitude and angle (phase) $|G(j\omega)|$, $\angle G(j\omega)|$

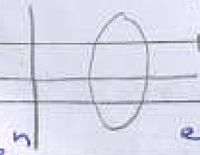
case: 5. → mirror image of polar plot → Nyquist plot $\omega = -\infty$ to ∞ total number of encircle elements of point $-1 + j0$ observed. clock-wise encircle elements are taken.

case 6. check the stability by using following eq^n

$$N = P - Z$$

where N = No. of encircle elements of $-1 + j0$

P = No. of +ve poles.



RHS +ve poles

eq^n $(s-1)$ $s = 1$

Z = no. of loops of characterstics eq^n with +ve real path.

• case(i): If $Z = 0$, $N = P$ then at indicate the closed stability of the system.

• case (ii): If $P = 0$, it indicate the open loop stability of the system.

• case (iii): If $Z = 0$, $P = 0$ Then it indicate the overall stability of the system.

Ques. A unity feedback system has a ~~fol~~ loop Transfer functn.

$$G(s) = \frac{50}{(S+1)(S+2)}$$

use Nyquist criteria to determine the system stability in the closed loop configuration is the open loop system stable.

Sol^n:

$$G(S) = \frac{50}{(S+1)(S+2)}$$

put $s = j\omega$

Nyquist plot = $-\infty$ to ∞

polar plot : 0 to ∞

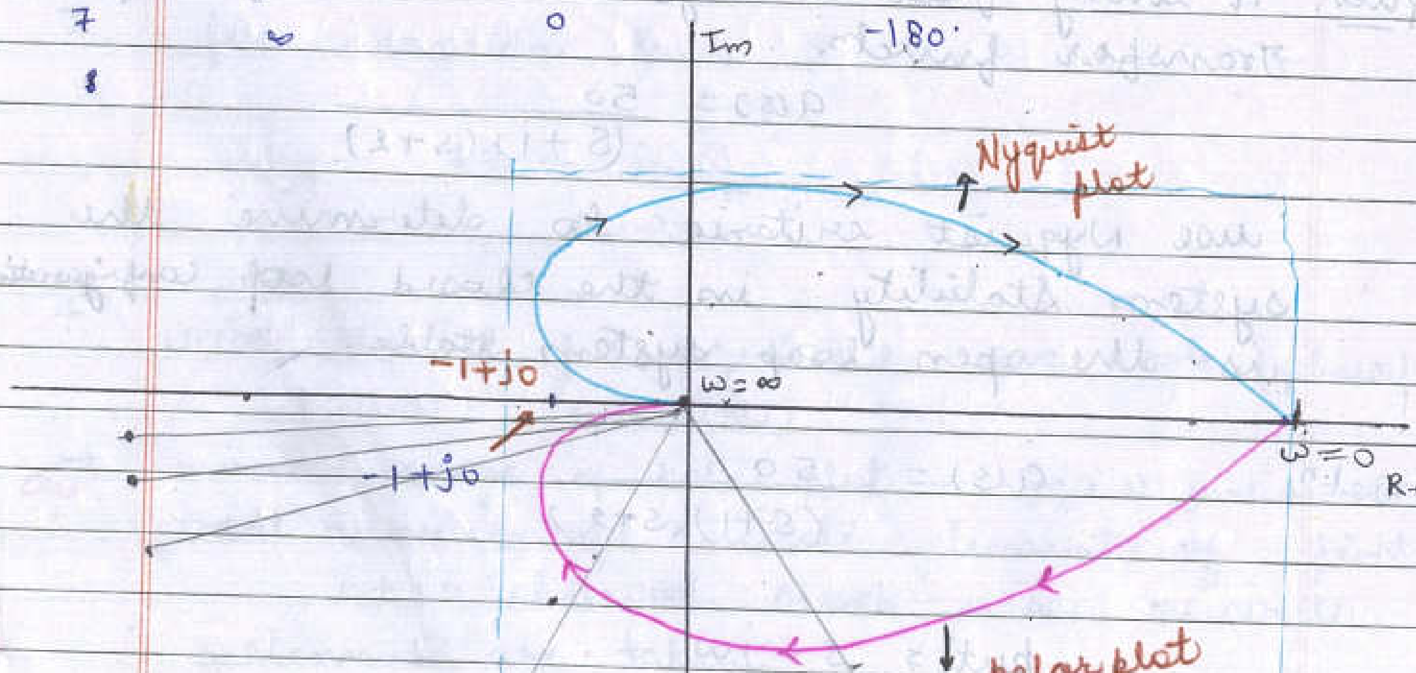
$$G(s)H(s) = \frac{50}{(j\omega+1)(j\omega+2)}$$

$$M = |G(j\omega)| = \frac{50}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$\angle G(j\omega) = \tan^{-1}(0/50) - \tan^{-1}(\omega/1) - \tan^{-1}(\omega/2)$$

$$\phi = \angle G(j\omega) = 0 - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

S.No.	ω	M	ϕ
1	0	25	0
2	1	16	$-72°$
3	2	8	$-108°$
4	10	0.5	$-163°$
5	20	0.1	$-171°$
6	100	0.005	$-178°$
7	∞	0	$-180°$
8			



Nyquist plot = polar plot + mirror image of polar plot

$-\infty$ to ∞ = ∞ to 0 + $-\infty$ to 0

Nyquist plot is obtain draw the mirror image of polar plot, $\omega = -\infty$ to ∞

Now check the stability

$N = P - Z$

$P = 0$ RHS Not zero open loop stable

$Z = 0$ RHS Not zero closed loop stable

$Z = P = 0$ overall stable

|| UNIT - 4 ||

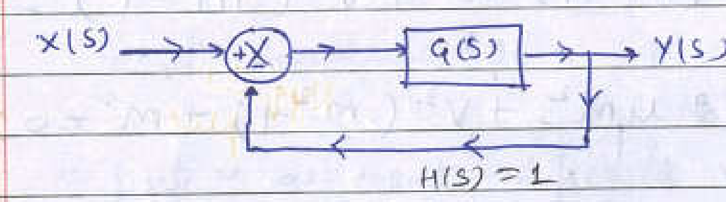
Short Note

→ (closed loop frequency Response): using M circle and N circle information about closed loop response can be easily be obtained.

1. M circle : (constant magnitude loci):

The closed loop transfer function of unity f/b system

$$T(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)}$$



Imp. **Derivation :** put $s = j\omega$

$$T(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$$

Now assume

put $G(j\omega) = u + jv$

find out the magnitude of $T(j\omega)$

$$M = |T(j\omega)| = \frac{\sqrt{u^2+v^2}}{\sqrt{(1+u)^2+v^2}}$$

Now square both the side

$$M^2 = \frac{u^2+v^2}{(1+u)^2+v^2} \quad -(i)$$

eqⁿ (i) modified the circle eqⁿ:

$$M^2((1+u)^2+v^2) = u^2+v^2$$

$$M^2(1+u^2+2u+v^2) = +u^2+v^2$$

$$M^2+M^2u^2+M^2 2u+v^2M^2 = u^2+v^2$$

$$M^2+M^2u^2-u^2+M^2 2u = v^2-v^2M^2$$

$$M^2+(M^2-1)u^2+M^2 2u = +v^2(1-M^2)$$

$$M^2(1+2u)+(M^2-1)u^2-v^2(1-M^2)=0$$

$$M^2(1+2u)+(M^2-1)u^2+v^2(M^2-1)=0$$

$$\frac{1}{(M^2-1)} \times u^2(M^2-1)+2uM^2+v^2(M^2-1)+M^2=0 \times \frac{1}{(M^2-1)}$$

$$u^2+\frac{2uM^2}{(M^2-1)}+v^2+\frac{M^2}{(M^2-1)}=0$$

$$\boxed{\left[\frac{M^2}{(1-M^2)}\right]^2+u^2-\frac{2uM^2}{(1-M^2)}+v^2} = \frac{M^2}{1-M^2}+\left[\frac{M^2}{(1-M^2)}\right]^2$$

$$\left(u-\frac{M^2}{1-M^2}\right)^2+v^2 = \frac{M^2}{1-M^2}+\frac{M^4}{(1-M^2)^2}$$

$$\left(u-\frac{M^2}{1-M^2}\right)^2+v^2 = \frac{M^4}{(1-M^2)^2}+\frac{M^2}{1-M^2}$$

$$\left(u-\frac{M^2}{1-M^2}\right)^2+v^2 = \left(\frac{M^2}{1-M^2}\right)\left[1+\frac{M^2}{1-M^2}\right]$$

$$\left(u-\frac{M^2}{1-M^2}\right)^2+v^2 = \left[\frac{M^2}{1-M^2}\times\frac{1}{1-M^2}\right]$$

$$\boxed{\left(u-\frac{M^2}{1-M^2}\right)^2+v^2 = \left(\frac{M}{1-M^2}\right)^2} \quad -(ii)$$

$$x^2+y^2=a^2 \quad -(iii)$$

compare eqⁿ (ii) and (iii), (ii) the represent the eqⁿ of circle and

center $u = \dfrac{M^2}{1-M^2}$ (u, v)

$v = 0$

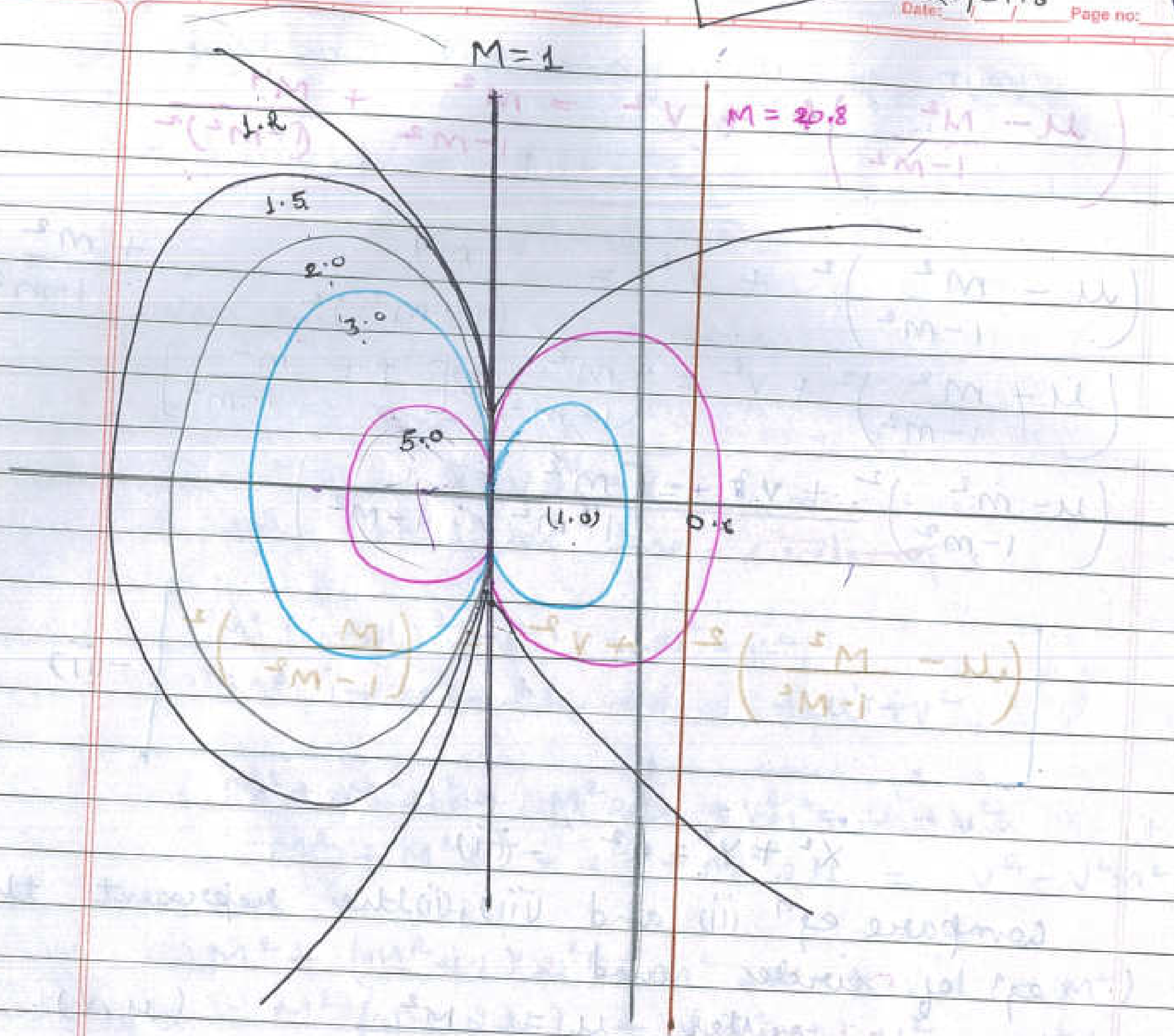
and radius $r = \dfrac{M}{1-M^2}$

put M different - different value center same but radius change u change.

~~Amot - A m~~ center (and) radius for various values of ~~A~~ M: ~~A mot I~~

M	center $\left(u=\frac{M^2}{1-M^2},\ v=0\right)$	Radius $r = M/1-M^2$
0	(1, 0)	0
0.5	(0.333, 0)	0.666
1	(∞, 0)	∞
5	(−1.041, 0)	+0.208

$M \propto \frac{1}{x}$

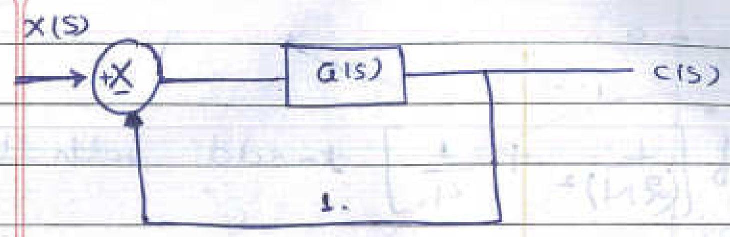


→ N- circles :

Short Notes

constant phase angle loci

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$



$$T(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)} = \frac{u+jv}{1+u+jv}$$

$G(s) = u+jv$ $(s=j\omega)$ put

$$\angle G(j\omega) = \theta = \tan^{-1} v/u - \tan^{-1}\frac{v}{u+1}$$

$$\theta = \tan^{-1} v/u - \tan^{-1} v/1+u$$

$$\tan\theta = \tan\left[\tan^{-1} v/u - \tan^{-1} v/u+1\right]$$

let $N = \tan\theta$

$$N = \tan\left[\tan^{-1} v/u - \tan^{-1} v/1+u\right]$$

$$N = \frac{\tan\tan^{-1} v/u - \tan\tan^{-1} v/1+u}{1+\tan\tan^{-1} v/u * \tan\tan^{-1} v/1+u}$$

$$N = \frac{v/u - v/1+u}{1+ v/u \times v/1+u}$$

$$\boxed{N = \frac{v}{u^2+v^2+u}}$$

$$\frac{u^2+v^2+u}{v} = \frac{1}{N}$$

$$\Rightarrow u^2+v^2+u = \frac{v}{N}$$

$$\Rightarrow u^2+v^2+u-\frac{v}{N} = 0$$

⇒ $v^2+u^2+u-\frac{v}{N}=0$

The addition of $\left[\frac{1}{(2N)^2}+\frac{1}{4}\right]$ to add both the side

⇒ $v^2+u^2+\frac{1}{(2N)^2}+\frac{1}{4}+u-\frac{v}{N}=0+\frac{1}{(2N)^2}+\frac{1}{4}$

⇒ $u^2+u+\frac{1}{4}+v^2-\frac{v}{N}+\frac{1}{(2N)^2}=\frac{1}{(2N)^2}+\frac{1}{4}$

$$\left(u+\frac{1}{2}\right)^2+\left(v-\frac{1}{2N}\right)^2=\left(\frac{1}{(2N)^2}+\frac{1}{4}\right)$$

$$\left(u+\frac{1}{2}\right)^2+\left(v-\frac{1}{2N}\right)^2=\left(\sqrt{\frac{1}{(2N)^2}+\frac{1}{4}}\right)^2 \quad \text{—(ii)}$$

$$x^2+y^2=a^2 \quad \text{—(i)}$$

eqn (i) and (ii) compare. $u=-\frac{1}{2}$, $v=\frac{1}{2N}$

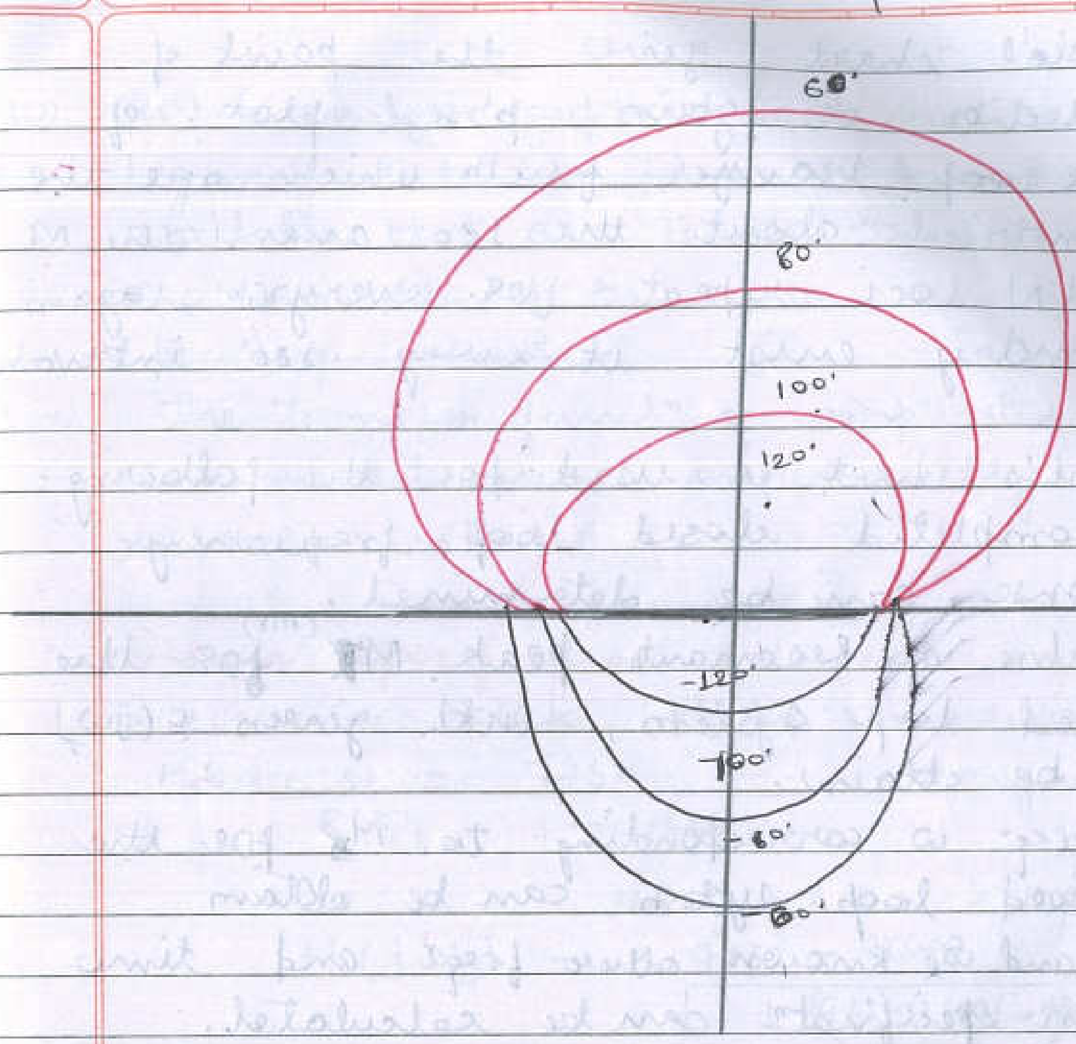
centre: c $\left(-\frac{1}{2}, \frac{1}{2N}\right)$

$r=\sqrt{\left(\frac{1}{2N}\right)^2+\frac{1}{4}}$

θ	N = tanθ	$u=-\frac{1}{2}$, $v=\frac{1}{2N}$	$r=\sqrt{\frac{1}{(2N)^2}+\frac{1}{4}}$
0°	0	$-1/2$, ∞	∞
30°	0.577	$-1/2$, 0.866	1
-30°	-0.577	$-1/2$, -0.860	1
60°	1.732	$-1/2$, 0.288	0.577
-60°	-1.732	$-1/2$, -0.288	0.577

$\left(M \propto \frac{1}{2u}\right)$

$\left(\theta \to N \propto \frac{1}{2v}\right)$



short Notes

→ (2+4 marks)

Nichol's chart:

It is possible to obtain the frequency Response by sketching the magnitude in dB against the phase angle for various frequency the plot is obtain is called gain phase plot or log magnitude v/s phase plot.

After Transfering the constant magnitude loci (M circle), constant phase angle loci (N circle) to the gain phase plot the resultant chart is known as Nichol's chart.

Nichol's chart give the point of intersection or gain phase plot of open loop transfer functn which are the symmetrical about the 180° axis. The M and N loci repeat for every 360°, again symmetry exist it every 180° interval.

- Nichol's chart is used for the following:

(i) The completed closed loop frequency response can be determined.

(ii) The value of Resonant peak M_r (M_r) for the closed loop system, with given $G(j\omega)$ can be obtain.

(iii) The freq ω corresponding to M_r for the closed loop system can be obtain

(iv) If M_R and ω_R known other freq^r and time domain specificatn can be calculated.

(v) for given M_R it is possible to design the values of K. ($\because K \rightarrow$ gain)

→ Frequency domain Analysis:

frequency domain Analysis is another technique that can be use for analysis or design. The frequency Response technique a control system analysis and design have the following advantages:

(i) In frequency Response analysis we need sinusoidal signal which is easily available.

(ii) In freq^r analysis graphical Tool such as bode plot, log magnitude V/s phase lock and Nyquist plot available that are limited to low order system

(iii) The bandwidth of the system can be varied as per requirement.

(iv) The Transfer functn of complicated component can be determine experimently by freq^r response test.

• Design specification in frequency domain:

1. Resonance peak (M_r): The resonance peak magnitude of $|M(j\omega)|$

$$M_r, \frac{C(S)}{R(S)} = \frac{G(S)}{1 \pm G(S)H(S)}$$

$$|M(j\omega)| = \frac{G(S) \rightarrow G(j\omega)}{\sqrt{1 \pm (G(j\omega)H(j\omega))^2}}$$

$$|M(j\omega)| = \left| \frac{G(j\omega)}{1 \pm G(j\omega)H(j\omega)} \right|$$

This is the max^m value of $M(j\omega)$ the relative stability of the closed loop system can be indicated by M_r, If M_r is large There are be large max^m overshoot in step response.

2. Resonant frequency (ω)

This is the freq^r at which resonance peak occurs.

3. Bandwidth: It is the range of freq^r for which the system gain is more than 3dB

BW gives and idea about Transient response.

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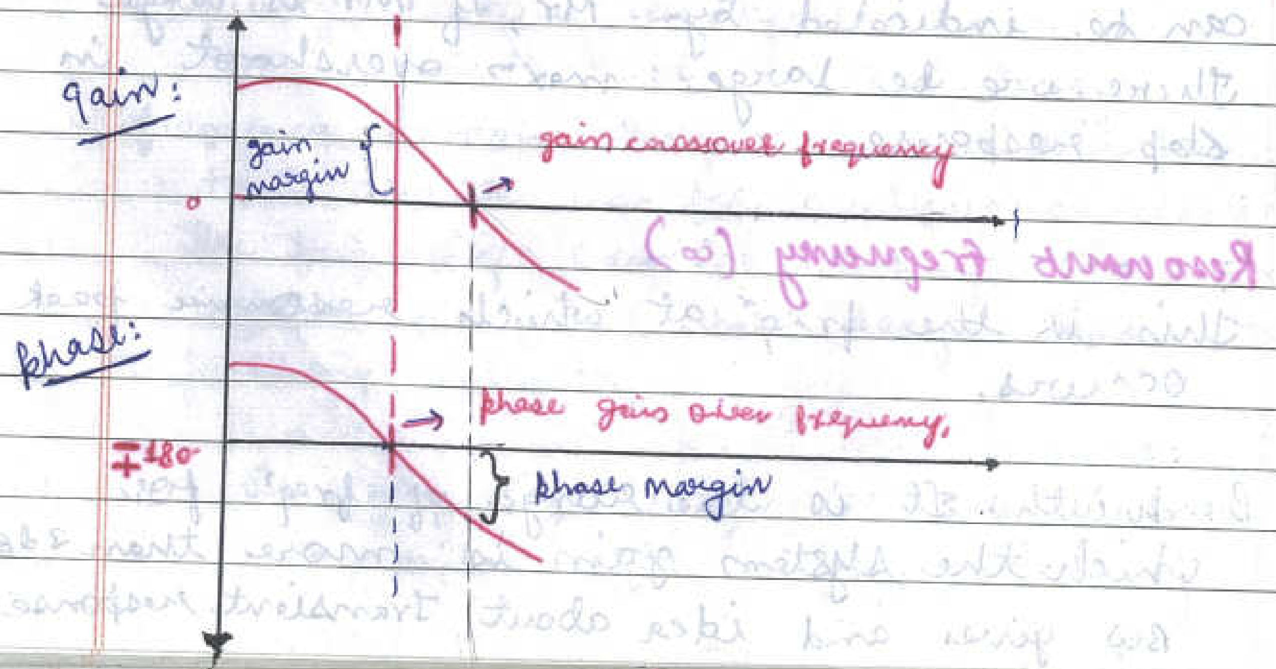
4. cut of Rate:
It is the slope of log magnitude curve near the cut off frequency.

5. phase Margin:
The phase margin is that the amount of addition phase log at the gain crossover freq^n required to bring the system to the words of in-stability
The gain crossover freq^n is the freq^n. at which the magnitude of open loop Transfer function is unity is given by $(180^\circ + \phi)$ where ϕ is the phase angle open loop Transfer funct^n at gain cross over freq^n.

6. Gain Margin:
gain Margin is the amount of gain in decibel that can be added to the open loop transfer function before the closed loop system become unstable.



• BODE PLOT:
The bode plot technique give a graphical approach to check the stability of a control system based on sinusoidal frequency Response

→ Bode plot Method:
1. plot of log arithematic of magnitute again frequency in log arithematic scale plot of $20\log_{10}|G(j\omega)H(j\omega)|$ V/s $\log_{10}\omega$

2. plot of phase angle against frequency in log airthematic scale plot of phase angle V/s $\log 10(\omega)$

$\angle G(j\omega)H(j\omega)$ V/s $\log_{10}\omega$

→ Procedure to plot bode diagrams:
Using log airthematic scale plotting of low and high frequency is possible in one diagram

In log airthematic scale the product term are converted into some terms

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\ldots}{s^N(1+sT_a)(1+sT_b)\ldots(s^2+2\xi\omega_n s+\omega_n^2)}$$

put $s=j\omega$

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega T_1)(1+j\omega T_2)\ldots}{(j\omega)^N(1+j\omega T_a)(1+j\omega T_b)\ldots((j\omega)^2+2\xi j\omega\omega_n+\omega_n^2)}$$

$j\omega \to 90^\circ$

magnitude Taking $20\log_{10}$ both the side.

$$20\log_{10}|G(j\omega)H(j\omega)| = \left[20\log_{10} k + 20\log_{10}\sqrt{1+(\omega T_1)^2} + 20\log_{10}\sqrt{1+(\omega T_2)^2} - - - - - -\right]$$

$$- 20N\log_{10}\omega - 20\log_{10}\sqrt{1+\omega T_a^2} - 20\log_{10}\sqrt{1+\omega T_b^2} - 20\log_{10}\sqrt{(2\xi_1\omega_n\omega)^2 + (\omega_n^2-\omega^2)^2} + ---$$

OR

$$20\log_{10}|G(j\omega)H(j\omega)| = \left[20\log_{10} k + 20\log\sqrt{1+(\omega T_1)^2} + -\right]$$

$$- 20N\log_{10}\omega - 20\log_{10}\sqrt{1+\omega T_a^2} - 20\log_{10}\sqrt{1+\omega T_b^2} + --- - 20\log_{10}\sqrt{(2\xi_1\omega_n\omega)^2 + (\omega_n^2-\omega^2)^2}$$

phase angle

$$\angle G(j\omega)H(j\omega) = \left[\tan^{-1}\omega T_1 + \tan^{-1}\omega T_2 + \tan^{-1}\omega T_3 + ---\right]$$
$$-N\angle -90^\circ - \tan^{-1}\omega T_a - \tan^{-1}\omega T_b + --- \tan^{-1}\frac{2\xi_1\omega\omega_n}{\omega_n^2-\omega^2}$$

extra →
$$\boxed{\begin{aligned} +N\tan^{-1}\left(\frac{-\omega}{0}\right) &= N\angle 90^\circ \\ -N\tan^{-1}(90^\circ) &= N\angle -90^\circ \end{aligned}}$$

$$\tan^{-1}(-\infty) = -90^\circ \Rightarrow -\tan^{-1}(\infty)$$

$$\tan^{-1}(\infty) = 90^\circ$$

$(s+10) \to +$ term is '1' $= (0.1s+1)$
$(s+5) \to +$ term is '1' $= \left(\frac{1}{5}s+1\right)$

- magnitude plot $\to 20\log_{10}$

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Q. Check the stability from bode plot in

(i) bode plot, the relative stability of the system checks on the gain margin and phase margin;

(ii) If gain crossover frequency is less than phase crossover frequency then gain margin and phase margin both are positive (stable) gain cross freq < phase crossover freq

(iii) If gain crossover frequency is greater than phase crossover frequency then both gain margin and phase are Negative. (unstable)

(iv) If gain crossover frequency and phase crossover are same then gain margin both are zero, then system is called marginally stable.

Q. Sketch the bode plot and check the stability system is unity feedback

$$G(s) = \frac{50}{s(1+s)(1+0.5s)}$$

$\to$ (i) ↓(ii) ↓(iii) ↓(iv) factor

sol^n: put $s = j\omega$

$$G(j\omega) = \frac{50}{j\omega(1+j\omega)(1+0.5j\omega)} \quad \text{—(i)}$$

- magnitude : {log scale}

$$|G(j\omega)| = \left|\frac{50}{j\omega(1+j\omega)(1+0.5j\omega)}\right|$$

• put ω = value then eqn change pole, zero also is '0' or '∞'
• factor order → corner freqr order (increasing order)

$$\boxed{20\log_{10}|G(j\omega)| = 20\log_{10}50 - 20\log_{10}\omega - 20\log_{10}\sqrt{1+\omega^2} - 20\log_{10}\sqrt{1+(0.5\omega)^2}} \quad \text{— (ii)}$$

3. phase:

corner frequency $1+j\omega = 0$, $\omega = 1$; $j\omega = 0$, $\omega = 0$; $1+0.5\omega = 0$, $\omega = 2$

$$\phi = \tan^{-1} 0/50 - \tan^{-1}\omega/0 - \tan^{-1}\omega/1 - \tan^{-1}0.5\omega$$

$$\boxed{\phi = -90 - \tan^{-1}\omega - \tan^{-1}0.5\omega} \quad \text{— (iii)}$$

4. Draw the magnitude plot:

→ ω value put $f = \infty$ (pole, zero) eqn zero

S.No.	Factor	Corner frequency	slope	Remarks
1.	$50 = 20\log_{10}50 = 34$	None	(not s factor) 0 dB/decade	
2.	$\frac{1}{j\omega} = -20\log_{10}$	None	(1/s) factor −20 dB/decade	
3.	$\frac{1}{1+j\omega} = -20\log\sqrt{1+\omega^2}$	1	ω_1 −20 dB/decade	
4.	$\frac{1}{1+0.5j\omega} = 20\log\sqrt{1+0.5\omega^2}$	2	−20 dB/decade	

• log(0) undefine so log graph not maintain (NONE)
• pole (−20 dB) } slope; zero (+20 dB) } slope — ($\frac{1}{s^2}$ = −40 dB/decade) ($\frac{1}{s}$ = −20 dB/decade)

13-3-18

• Draw the phase plot:

$$\phi = -90° - \tan^{-1}\omega - \tan^{-1}0.5\omega$$

S.No.	ω	φ
1	$\boxed{0}$	−90°
2	0.1	−98.6
3	0.5	−130.6
4	1	−161.6
5	1.4	−179.5
6	1.5	−183.5 → 180°
7	2.0	−198°
8	5.0	−236.88
9	10.0	−252.97
10	$\boxed{\infty}$	−270°

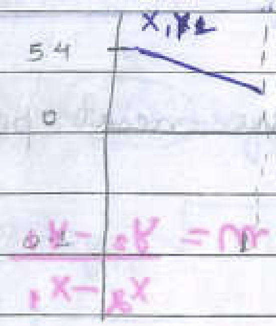
• Starting point find to this method:

s factor $\frac{1}{s^2} = \frac{K}{\omega^2}$; s factor $\frac{1}{s}$ → $j\omega$

$$20\log_{10}\frac{K}{\omega} = 20\log_{10}\frac{50}{0.1}$$ (K = constant value)

$$= 54\ dB$$

• find the next point: (20 dB scale down) not easily draw so formula used



slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

corner frequency

$X_1 = 0.1$

$X_2 = 1$

$Y_1 = 54$

$Y_2 = ?$

$\boxed{m = -20\ dB}$

$\frac{1}{j\omega} \times \frac{1}{j\omega} = -180$

$-90 \quad -90$

$$-20 = \frac{y_2 - 54}{\log 1 - \log 0.1}$$

$$20 = \frac{y_2 - 54}{\log 1 - \log 0.1}$$

$$\boxed{y_2 = 34 \text{ dB/decade}}$$

- find the next point $\omega = 1$ to 2

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y_1 = 34,\ y_2 = ?,\ x_1 = 1,\ x_2 = 2$$

$m =$ total slope

$-20\,dB - 20\,dB = (-40\,dB)$

$\uparrow \omega = 1 \quad \uparrow \omega = 2$

$$-40 = \frac{y_2 - 34}{\log 2 - \log 1}$$

$$\boxed{y_2 = 22 \text{ dB/decade}}$$

- find the next point $\omega = 2$ to 10 ↑ assume

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y_1 = 22,\ y_2 = ?,\ x_2 = 10,\ x_1 = 2$$

$$-60 = \frac{y_2 - 22}{\log 10 - \log 2}$$

$$\boxed{y_2 = -20 \text{ dB/decade}}$$

- corner freq^n slope change
- slope change ; corner freq^n.

$G(s) \cdot H(s) = -1$ ↓ $(\tan^{-1} -1) \Rightarrow -180$

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- find the next point $\omega = 10$ to 20

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_2 = 20,\ x_1 = 10,\ y_1 = -20,\ y_2 = ?$$

$$-80 = \frac{y_2 + 20}{\log 20 - \log 10}$$

$$-38 = y_2$$

$$\boxed{y_2 = -38 \text{ /decade}}$$

Q. Draw the bode plot

$$G(s) = \frac{40}{s^2 (s+2)(s+5)}$$

Sol^n, put $s = j\omega$

$$G(j\omega) = \frac{40}{(j\omega)^2 (j\omega + 2)(j\omega + 5)}$$

$$|G(j\omega)| = \left| \frac{40}{(j\omega)^2 (j\omega + 2)(j\omega + 5)} \right|$$

$$|G(j\omega)| = \left| \frac{40}{(j\omega)^2 \left(\frac{j\omega}{2} + 1\right)\left(\frac{j\omega}{5} + 1\right)} \right|$$

- magnitude:

$$20\log_{10} |G(j\omega)| = 20\log_{10} 40 - 2 \times 20 \log_{10} \omega - 20\log_{10} \sqrt{1 + \omega^2/4} - 20\log_{10} \sqrt{1 + \frac{\omega^2}{25}}$$

$\frac{1}{s} = -20 dB$

$(j\omega)^2 = (j\omega)(j\omega)$

$\frac{1}{j\omega} \times \frac{1}{j\omega} = -180$

$-\left(\tan^{-1}\left(\frac{\omega}{0}\right)\right)^2$

3 phase angle:

$-\tan^{-1}\omega/0 - \tan^{-1}\omega/0$

$\phi = \tan^{-1} 0/40 - \cancel{\tan^{-1}\omega/0}\ 180 -$

$\tan^{-1} \frac{0.5\omega}{10} - \tan^{-1} \frac{0.2\omega}{10}$

eq^n multiple by 10

$\phi = -180° - \tan^{-1} 0.5\omega - \tan^{-1} 0.2\omega$

- magnitude:

$20\log_{10} |G(j\omega)| = 20\log_{10} 4 - 20\log_{10}\omega - 20\log_{10}\omega -$

$20\log_{10}\sqrt{1+(0.5\omega)^2} - 20\log_{10}\sqrt{(0.2\omega)^2+1}$

→ Magnitude table:

S.No	Factor	corner freq^n	slope	Remarks
1	$4 = 20\log_{10} 4 = 12 dB$	None	0 dB/decade	
2	$\frac{1}{s^2} = -40\log_{10}\omega = -40\log_{10}\omega$	None	-40 dB/decade	
3	$\frac{1}{0.5\omega+1} = -20\log_{10}\sqrt{(0.5)^2+1}$	2	-20 dB/decade	
4	$\frac{1}{0.2\omega+1} = -20\log_{10}\sqrt{(0.2)^2+1}$	5	-20 dB/decade	

- phase angle eq^n table:

S.No	ω	ϕ
	0	-180
	0.1	-184
	0.2	188
	0.8	246
	2	293
	5	313
	8	332
	10	346
	20	353
	∞	360°

- Starting point:

$20\log_{10}\frac{K}{\omega.\omega} = 20\log_{10}\frac{4}{0.1 \times 0.1}$

$20\log_{10}\frac{K}{\omega^2} = 52.04$

- find the next point

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y_1 = 52$
$y_2 = ?$
$x_1 = 0.1$
$x_2 = 2$

$m = \frac{-52 + y_2}{\log_{10} 2 - \log_{10} 0.1}$

$-40 = \frac{y_2 - 52}{1.30}$

$-52.04 = y_2 - 52$

$y_2 = -0.4$ dB/decade

Statevariable Page 12.2

Dinesh Sethi

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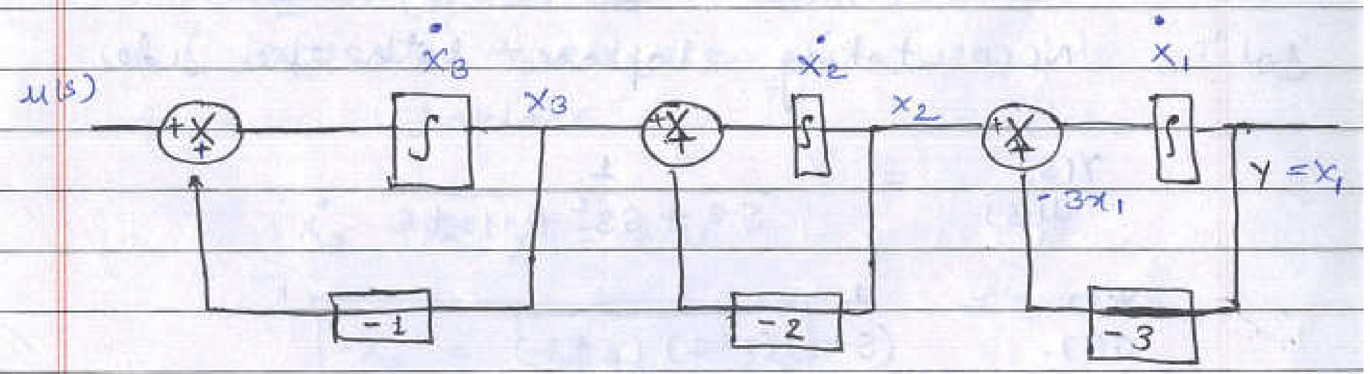
Q. $\frac{Y(S)}{G(S)} = \frac{1}{(S+1)} \times \frac{1}{(S+2)} \times \frac{1}{(S+3)}$

$\frac{Y(S)}{G(S)} = \frac{1}{(S+1)} \times \frac{1}{(S+2)} \times \frac{1}{(S+3)}$

cascade decomposition

Sol^n: $\frac{Y(S)}{G(S)} = \frac{1}{s^3+6s^2+11s+6}$

for the decomposition of transferfunctn the O/P of $\frac{1}{S+1}$ is given as i/p to $\frac{1}{S+2}$ and O/P of $\frac{1}{S+2}$ is given as i/p of $\frac{1}{S+3}$ is the O/P of $\frac{1}{S+3}$ is the final o/p



State eq^n:

$\dot{x}_1 = -3x_1 + x_2$

$\dot{x}_2 = -2x_2 + x_3$

$\dot{x}_3 = -x_3 + u$

State Matrix

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-3 & 1 & 0\\ 0 & -2 & 1\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-3 & 1 & 0\\ 0 & -2 & 1\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} u$$

$Y = x_1$ $\quad Y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$

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→ Parallel decomposition or diginal form or canonical form:

The $\parallel^{u}$ decomposition of Transfer function is carryout by spliting the transfer function into carry out by spereding partial fraction.

Q. construct the state model of given differential eqn.

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = u$$

by using $\parallel^{u}$ decomposition and given the block diagram representation.

Soln: Now taking laplace both the side

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 6s^2 + 11s + 6}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+2)(s+3)}$$

$$\frac{Y(s)}{U(s)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = \frac{1}{(-2+1)(-3+1)} = 1/2$$

$B = -1$

$C = 1/2$

$$\frac{Y(s)}{U(s)} = \frac{1/2}{(s+1)} + \frac{(-1)}{(s+2)} + \frac{1/2}{(s+3)}$$

$$Y(s) = \frac{1}{2}U(s) + (-1)U(s) + \frac{1}{2}U(s)$$

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$$Y(s) = X_1(s) + X_2(s) + X_3(s)$$

$$X_1(s) = \frac{\frac{1}{2}U(s)}{s+1}$$

$$(s+1)X_1(s) = \frac{1}{2}U(s)$$

$$X_1(s)s + X_1(s) = \frac{1}{2}U(s)$$

laplace

$$\dot{X}_1 + X_1 = \frac{1}{2}u$$

$$\boxed{\dot{X}_1 = -X_1 + \frac{1}{2}u}$$

$$X_2(s) = \frac{-1 \times U(s)}{s+2}$$

$$X_2(s)(s+2) = -1\,U(s)$$

$$X_2(s)s + 2X_2(s) = -U(s)$$

laplace

$$\dot{X}_2 + 2X_2 = -u$$

$$\boxed{\dot{X}_2 = -2X_2 - u}$$

$$X_3(s) = \frac{+\frac{1}{2}U(s)}{s+3}$$

$$\boxed{\dot{X}_3 = -3X_3 + \frac{1}{2}u}$$

Inverse laplace

$$Y = X_1 + X_2 + X_3$$

$$Y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ & & \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1/2 \\ -1 \\ 1/2 \end{bmatrix} u$$

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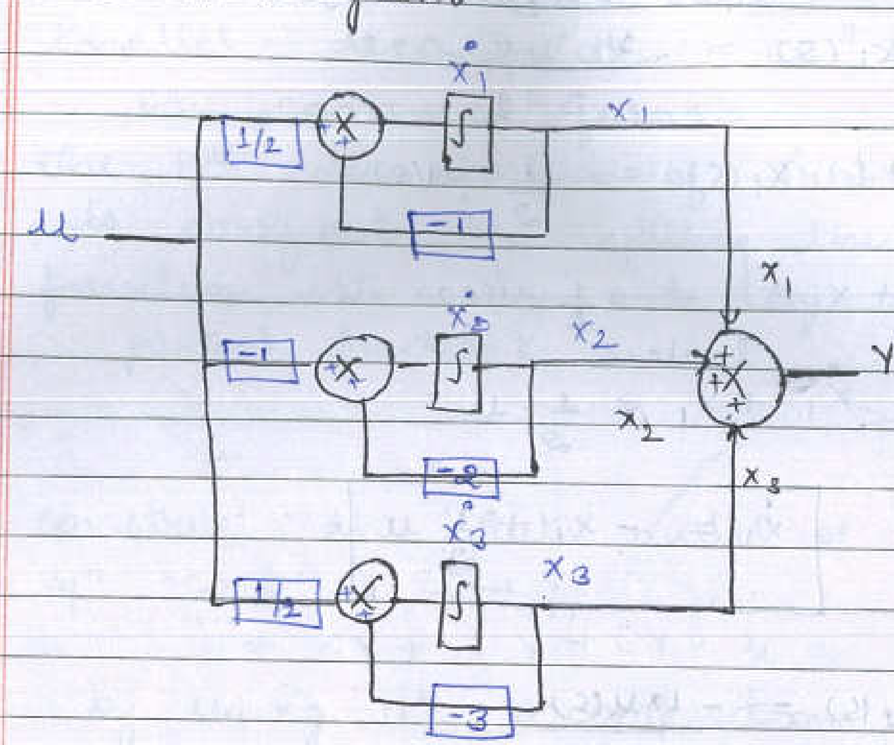
Block diagram :



→ Jordan canonical form: (state space representation by using 11^{n} decomposition :

Q. Construct the state model by a system (→ in Jordan canonical form) transfer funcn is given by

$$\frac{Y(S)}{U(S)} = \frac{10}{(S+1)^2(S+2)}$$

$$\frac{Y(S)}{U(S)} = \frac{A}{(S+2)} + \frac{B}{(S+1)} + \frac{C}{(S+1)^2}$$

$$A = \frac{10}{(-1+2)(-1)} = \frac{10}{1} = 10$$

$$B = \frac{10}{(-2+1)} = \frac{10}{(-1)} = -10$$

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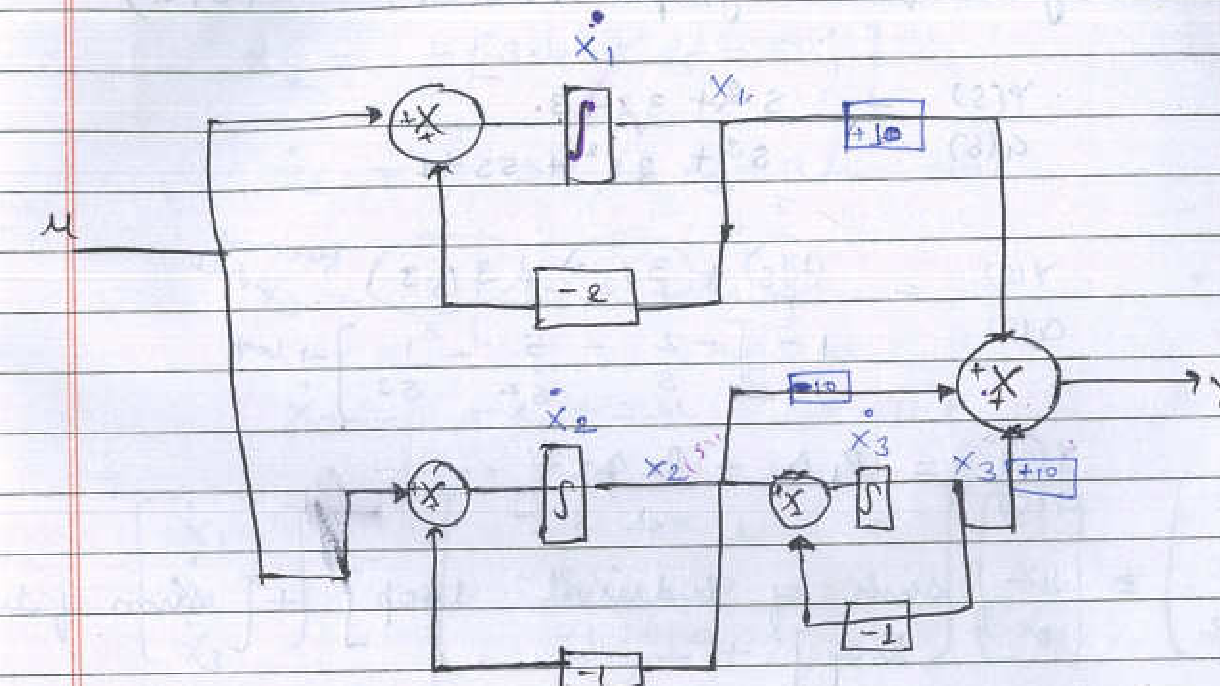
$$\frac{Y(S)}{U(S)} = \frac{10}{(S+2)} + \frac{-10}{(S+1)} + \frac{10}{(S+1)^2}$$



State model eqn:

$$\dot{x}_1 = -2x_1 + u$$
$$\dot{x}_2 = -1x_2 + u$$
$$\dot{x}_3 = -x_3 + x_2 + 0 = -x_3 + x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$$

$$Y(s) = 10x_1 + (-10x_2) + 10x_3$$

$$Y = 10x_1 - 10x_2 + 10x_3$$

$$Y = \begin{bmatrix} 10 & -10 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ find the state model eqn by using signal flow graph method: (SFG)

$$\frac{Y(S)}{G(S)} = \frac{S^2 + 3S + 3}{S^3 + 2S^2 + 5S + 1}$$

$$\frac{Y(s)}{G(s)} = \frac{1/s + 3/s^2 + 3/s^3}{1 - \left[-\frac{2}{s} - \frac{5}{s^2} - \frac{1}{s^3}\right]}$$ forward path → loop

$$\frac{Y(s)}{G(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + \cdots}{\Delta}$$

$= 1 - [\text{sum of indiviual loop}] + [\text{sum of two loop}] + \cdots$

$P_1 = 1/s$

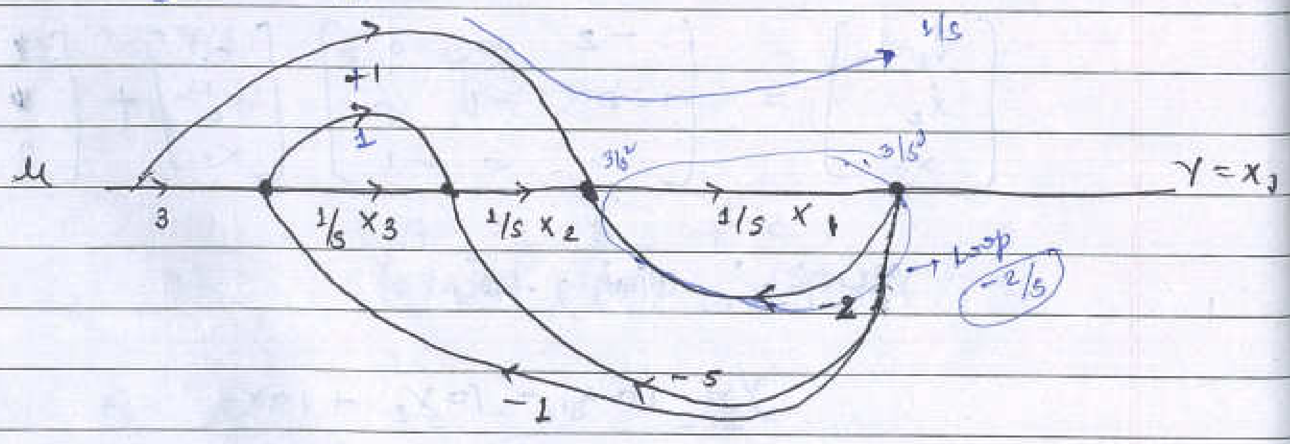
$P_2 = 3/s^2$

$P_3 = 3/s^3$

$L_1 = -2/s$

$L_2 = -5/s^2$

$L_3 = -1/s^3$



$$X_1 = \frac{1}{s}\left[u + X_2 - 2X_1\right]$$

$X_1 s = u + X_2 - 2X_1$

$\dot{x}_1 = x_2 - 2x_1 + u$ —(i)

$$X_2 = \frac{1}{s}\left[3u + X_3 - 5X_1\right]$$

$\dot{X}_2 = -5x_1 + x_3 + 3u$ —(ii)

$$X_3 = \frac{1}{s}\left[-x_1 + 3u\right]$$

$\dot{x}_3 = -x_1 + 3u$ —(iii)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -5 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} u$$

$Y = X_1$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ Solution of the state model eqn: OR Derivation of the transfer function by using state model eqn:

8 marks

state model eqn

$\dot{x}(t) = A\,x(t) + B\,u(t)$

$y(t) = C\,x(t) + D\,u(t)$

taking laplace both the side

$S\,X(S) = A\,X(S) + B\,U(S)$

matrix dimensions same.

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$$X(s)\left[S - A \right] = B\,U(s)$$

$$\frac{X(s)}{U(s)} = \frac{B}{(S-A)}$$

$$\frac{X(s)}{U(s)} = \frac{B}{[SI - A]}$$

identity matrix

$$X(s) = [SI - A]^{-1} B\,U(s)$$

Transfer functn : $\frac{Y(s)}{U(s)} = \frac{Y(s)}{U(s)} = ?$

$$Y(s) = C\,X(s) + D\,U(s)$$

$$Y(s) = C\left[[SI - A]^{-1} B\,U(s) \right] + D\,U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = C\,[SI - A]^{-1} B + D}$$ eqn of transfer functn

Q. Obtain the transfer functn if state model is given

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underset{(A)}{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underset{(B)}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} [u]$$

$$Y = \underset{(C)}{\begin{bmatrix} 1 & 0 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Sol^n: compare state model eq^n:

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$Y(t) = C\,x(t) + D\,u(t)$$

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$$\frac{Y(s)}{U(s)} = C[SI - A]^{-1} B + D$$

find $SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

$$= S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$$

$$[SI - A] = \begin{bmatrix} S+0 & -1 \\ 2 & S+3 \end{bmatrix}$$

$$[SI - A] = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$[SI - A]^{-1} = \begin{bmatrix} \frac{S+3}{S^2+3S+2} & \frac{1}{S^2+3S+2} \\ \frac{-2}{S^2+3S+2} & \frac{S}{S^2+3S+2} \end{bmatrix}$$

$$[SI - A]^{-1} = \begin{bmatrix} \frac{S+3}{S^2+3S+2} & \frac{1}{S^2+3S+2} \\ \frac{-2}{S^2+3S+2} & \frac{S}{S^2+3S+2} \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{S+3}{S^2+3S+2} & \frac{1}{S^2+3S+2} \\ \frac{-2}{S^2+3S+2} & \frac{S}{S^2+3S+2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + D = 0$$

$$adj = \frac{1}{|A|} \begin{bmatrix} +(S+3) & -1 \\ -2 & +S \end{bmatrix} \quad (-1)^{m+n}$$

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$$\frac{Y(s)}{u(s)} = \left[\frac{s+3}{s^3+3s+2} \quad -\frac{2}{s^2+3s+2} \quad 0 \right] \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{Y(s)}{u(s)} =$$

$$\frac{Y(s)}{u(s)} = \left[\frac{s+3}{s^3+3s+2} \quad \frac{1}{s^2+3s+2} \right] \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{Y(s)}{u(s)} = \frac{1}{s^2+3s+2} \quad \text{Ans}$$

Q consider the state model matrix

Imp.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}, \quad D = 0$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 40 & 10 & 0 \end{bmatrix}$$

find the transfer function.

Ans

$$\dot{X}(t) = A\,X(t) + B\,u(t)$$
$$Y(t) = C\,X(t) + D\,u(t)$$

$$Y(s) = C\,X(s) + D\,u(s)$$
$$X(s)\cdot s = A\,X(s) + B\,u(s)$$

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$$\frac{Y(s)}{u(s)} = C\,[sI - A]^{-1}B + D$$

$$[sI - A] = s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 3 & +4s \end{bmatrix} \qquad a_{11} =$$

$$|A| = s(\quad) - 1(\quad) + 0(\quad)$$

$$[sI - A]^{-1} = \frac{1}{|A|}\,\text{adj}\,[sI - A]$$

$$[sI - A]^{-1} =$$

$$|sI - A| = s[(4s+3)] - 1[0] + 0$$

$$(4s^2+3)s = 4s^3 + 3s^2$$

$a_{11} = 4s + s^2 + 3$

$a_{12} = 0$

$a_{13} = 0$

$a_{21} = -4 - 3$

$a_{22} = 4s + s^2$

$a_{23} = 3s$

$a_{31} = -1$

$a_{32} = -s$

$$= s[(+4s+3) + s^2] + 0$$

$$= 4s^2 + 3s + s^3 + 0$$

$$= s^3 + 4s^2 + 3s$$

$$= \begin{bmatrix} 4s+s^2+3 & 0 & 0 \\ 4+s & +(4s+s^2) & -3s \\ +1 & s & s^2 \end{bmatrix}$$

$$[SI-A]^{-1} = \begin{bmatrix} \dfrac{4s+s^2+3}{s^3+4s^2+3s} & \dfrac{4+s}{s^2+4s^2+3s} & \dfrac{+1}{s^3+4s^2+3s} \\ 0 & \dfrac{4s+s^2}{s^3+4s^2+3s} & \dfrac{s}{s^3+4s^2+3s} \\ 0 & \dfrac{-3s}{s^3+4s^2+3s} & \dfrac{s^2}{s^3+4s^2+3s} \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = C[SI-A]^{-1}B + D$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 40 & 10 & 0 \end{bmatrix} [SI-A]^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 0$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 40 & 10 & 0 \end{bmatrix} [SI-A]^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 0$$

$$= \cancel{\frac{40}{s}} + \cancel{\frac{10(s+4)}{(s^3+4s+3)}}$$

$$= \begin{bmatrix} 40 \times \dfrac{(4s+s^2+3)}{s^3+4s^2+3s} + 0\times 10 + 0 \times 0 & \dfrac{4s}{s^3+4s^2+3s} \times 40 + \dfrac{10 \times 4s+s^2}{s^3+4s^2+3s} & \dfrac{40}{s^2+4s^2+3s} + \dfrac{s\times 10}{s^3+4s^2+3s} + 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{40}{s^3+4s^2+3s} + \frac{10s}{s^3+4s^2+3s} = \frac{40+10s}{s^3+4s^2+3s}$$

Ans

→ Controlable test : (controbility)

for the system describe by the state eq^n to be complete state controvable the necessary and sufficient condition is that following Q_c matrix has rank of N

$$Q_c = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

$$Q_c = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

gilbert Method

$$\dot{x} = Ax + Bu$$
$$Y = cx + Du$$

(Q_c depend $\dot{x}$, Y)

? dimention A matrix

Q_c Rank → n full Rank

Rank find out

$[Q_c] = n$ Rank

OR

$|Q_c| \neq 0$ controvable.

Rank full controlable

→ OBSERVATIVE TEST → for the system describe by the state eq^n to be completly observable if a following matrix of Q_o is non singular of Rank N.

Kalman method

$$Q_o = \left[C^T \quad A^TC^T \quad (A^T)^2 C^T \ \dots \ (A^T)^{n-1} C^T \right]$$

$$Q_o = \left[C^T \quad A^TC^T \quad (A^T)^2 C^T \right] \quad \text{if } n = 3$$

$$Q_o = \left[C^T \quad A^TC^T \right] \quad \text{if } n = 2$$

$|Q_o| \neq 0$ observable.

Rank Q_o = N Rank (full)

* square matrix → find Rank determinant
* non square matrix → find Rank

Q. find the controlability of the system given by the state eqn.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 5 & 5 \\ 0 & -5 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 5 \\ 0 \end{bmatrix}} u$$

$$A = \begin{bmatrix} 5 & 5 \\ 0 & -5 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$n = 2$

$$Q_c = [B \quad AB]$$

$$|Q_c| = \left(\begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 5 & 5 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} \right)$$

Rank 1 $\quad |Q_c| = \begin{bmatrix} 5 & 25 \\ \dots \end{bmatrix} = \begin{vmatrix} 5 & 25 \\ \dots \end{vmatrix} = 0$

$|Q_c| = 0$ not controlable

Q. check observable?

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 9 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$Q_o = [C^T \quad A^TC^T]$$

$$Q_o = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} \qquad \text{Rank} = 1$$

$|Q_o| = 0$ not observable

Q. (Imp) The state eqn of system are given below determine if the system is completely controlable and observable. Matrix

$$\dot{X} = \overset{A}{\begin{bmatrix} -6 & 2 & -4 \\ -18 & 3 & -8 \\ 6 & 1 & -3 \end{bmatrix}} X + \overset{B}{\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}} u$$

$$y = \overset{C}{[1 \quad -1 \quad 2]} \; X$$

$$Q_c = [B \quad AB \quad A^2B]$$

$$Q_o = [C^T \quad A^TC^T \quad (A^T)^2 C^T]$$

$$Q_c = \begin{bmatrix} 1 & -6+6-4 & 14 \\ 3 & -18+9-8 & 69 \\ \dots \end{bmatrix} = \begin{bmatrix} 1 & -4 & 14 \\ 3 & -17 & 69 \\ \dots \end{bmatrix}$$

control system are accuracy, speed of response and stability. In order to meet these performance specifications, it is required to redesign a control system. An external device is added to the system called compensation (control action) external device know as the 'compensator'

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UNIT-V

• CONTROLLERS:

1. Analog PID controllers:

An automatic control scheme compares a controlled condition value with desired value and automatically correct any deviation.

• modes :

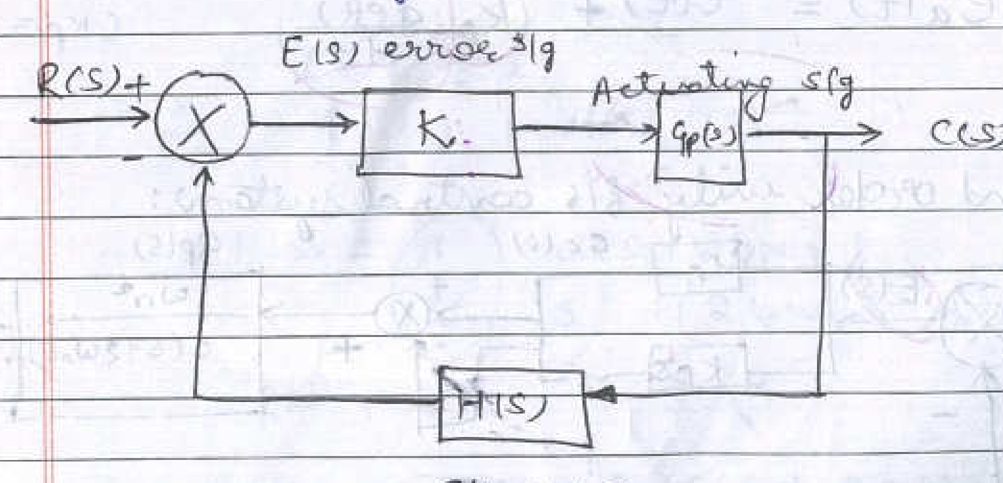
(i) Propostional control:

actuating signal $\propto$ error signal

error signal = feeback sig − reference sig (i/p)

used:

1. error signal is very weak and there need of amplificatn.
2. offset can be tolerated.
3. load changes are small.



• Advantages:

1. The sluggish overdamped response of a control system can be made faster by ↑sing forward path gain of the system.

This in turn reduces the steady state error.

2. without any change in steady state accuracy the max^m overshoot (mp) can be reduced to same extent by controlling the actuating signal.

• Disadvantages:

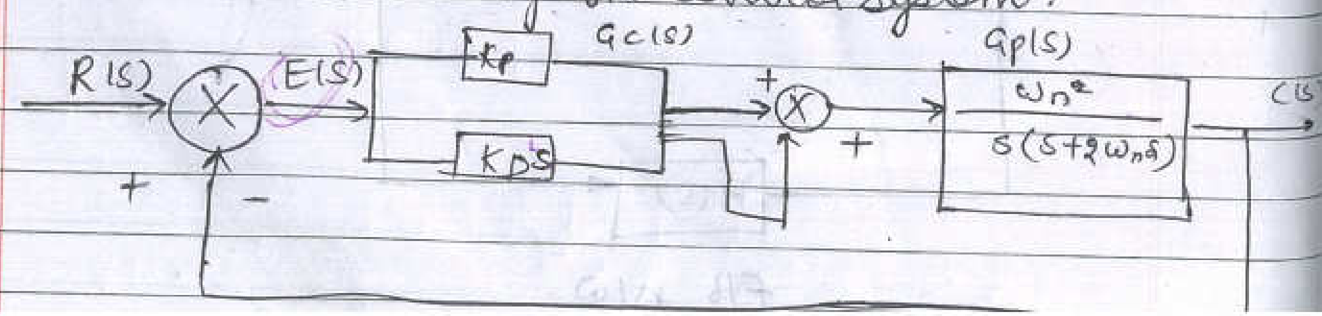
1. Mp ↑sed when forward path gain of the system is ↑sed.

(II) Derivative control (P.D control):

actuating signal ∝ error signal + derivative of error sig.

$$e_a(t) = e(t) + K_D \frac{de(t)}{dt} \qquad (K_P = 1)$$

Second order unity f/b control system:



$$\frac{C(S)}{R(S)} = \frac{(K_P + K_D S)\,\omega_n^2 / s^2 + 2s\xi\omega_n}{1 + \frac{\omega_n^2}{S^2 + 2\xi\omega_n S}(K_P + K_D s)}$$

$$\frac{C(S)}{R(S)} = \frac{(K_P + K_D S)\,\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2 (K_P + K_D s)}$$

$$\frac{C(S)}{R(S)} = \frac{(K_P + K_D S)\,\omega_n^2}{S^2 + (2\xi\omega_n + K_D\omega_n^2)S + K_P\omega_n^2}$$

characteristics eq^n.

$$S^2 + (2\xi\omega_n + K_D\omega_n^2)S + K_P\omega_n^2 = 0$$

$$2\xi'\omega_n = 2\xi\omega_n + \omega_n^2 K_D$$

$$\boxed{\xi' = \frac{\xi + \omega_n K_D}{2}}$$ damping Ratio

steady state error for unity Ramp input:

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$\frac{E(S)}{R(S)} = \frac{1}{1 + G(S)H(S)}$$

$$E(S) = \frac{1}{S^2} \cdot \frac{s(s + 2\xi\omega_n)}{s^2 + (2\xi\omega_n + \omega_n^2 K_D)s + K_P\omega_n^2}$$

R(S)

$$e_{ss} = \lim s \cdot \frac{1}{\;} \left[s(s + 2\xi\omega_n) \right.$$

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$$e_{ss} = \frac{2\xi\omega_n}{K_p\omega_n^2} = \boxed{\frac{2\xi}{K_p\omega_n}}$$

- used:
1. need of minimizats of amount of derivation caused by plant load changes
2. Max^m overshoot (M_p) has to be reduced.

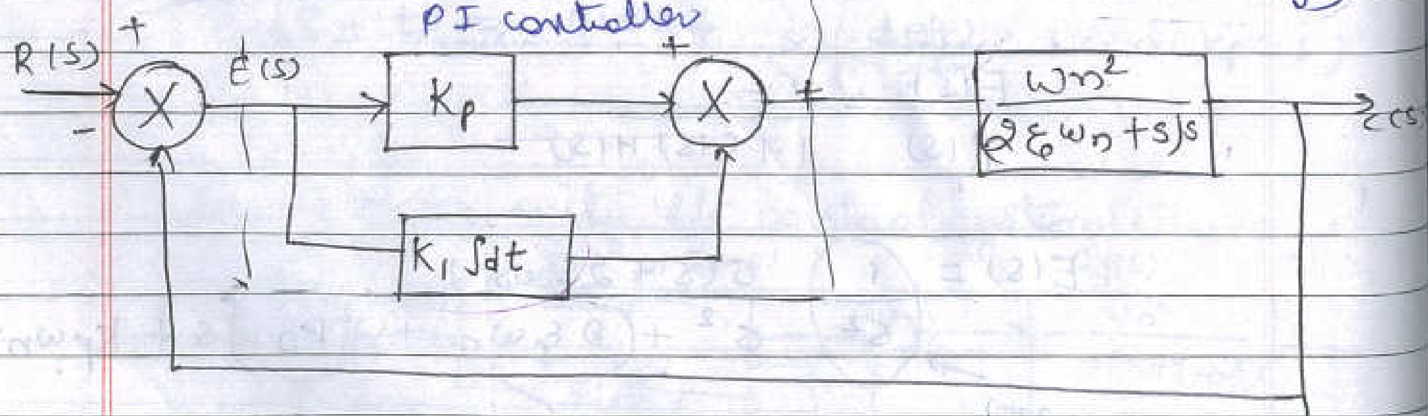
- Advantages:
1. M_p is reduced.
2. since zero is added, rise time (t_r) ↓es.

- Disadvantages:
Stedy state error is unchanged for ramp i/p.

(iii) Proportional Integral control: (P.I control)

actuating signal ∝ [error s/g + integral error s/g]

PI controller



$$\frac{C(s)}{R(s)} = \frac{\left(K_p + \frac{K_I}{s}\right)\left[\frac{\omega_n^2}{s(s+2\xi\omega_n)}\right]}{1 + \left(K_p + \frac{K_1}{s}\right)\left[\frac{\omega_n^2}{s(s+2\xi\omega_n)}\right]}$$

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$$\frac{C(s)}{R(s)} = \frac{(K_p s + K_1)\,\omega_n^2}{s^3 + s^2(2\xi\omega_n) + s(\omega_n^2 K_p) + K_1\omega_n^2}$$

characterstics eq^n:

$$s^3 + s^2(2\omega_n\xi) + s(\omega_n^2 K_p) + K_1\omega_n^2 = 0$$

$$\frac{E(s)}{R(s)} = \left(\frac{1}{1 + \frac{\omega_n^2(K_p s + K_1)}{s^2(s + 2\xi\omega_n)}}\right) \quad \text{-(i)}$$

$$E(s) = \frac{1}{s^3} * (i) \Rightarrow \frac{1}{s^3}\left[\frac{s^2(s + 2\xi\omega_n)}{s^3 + s^2(2\xi\omega_n) + s(\omega_n^2 K_p) + K_1\omega_n^2}\right]$$

R(s)

parabolic i/p

$$e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

$$e_{ss} = \frac{2\xi\omega_n}{K_1\omega_n^2} = \boxed{\frac{2\xi}{K_1\omega_n}}$$

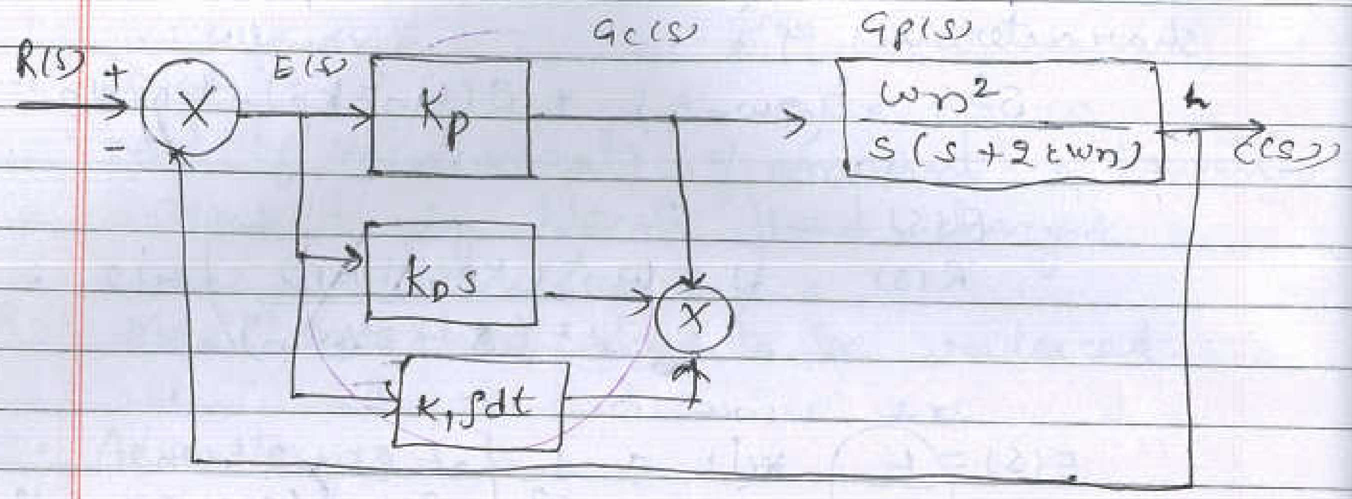
step i/p : ramp $e_{ss} = 0$

- used:
1. high accuracy is required.
2. Integral saturation due to sustained deviation should not be a problem.
3. offset must be eliminated.

- Application
1. System accuracy is ↑sed.
2. offset eliminates
3. Stedy state error is minimized.

- disadvantages:
1. Order of system ↑es, stability ↓es. no improment transient response, not suitable for rapid changes.

(v) Proportional Integral Derivative control (PID controller):



$$e_a(t) = e(t) + K_D \frac{de(t)}{dt} + K_I \int e(t)\,dt$$

$$\frac{C(S)}{R(S)} = \frac{\left(K_P + \frac{K_I}{S} + K_D S\right)\left[\frac{\omega_n^2}{S(2\xi\omega_n + S)}\right] \rightarrow G(S)}{1 + G(S)H(S)}$$

$$\frac{C(S)}{R(S)} = \frac{(S^2 K_D + S K_P + K_I)\,\omega_n^2}{S^2(S + 2\omega_n\xi)}$$

- Advantages

1. three types of indivial control actions
2. It has no offset problem.
3. high stability, accuracy and speed.

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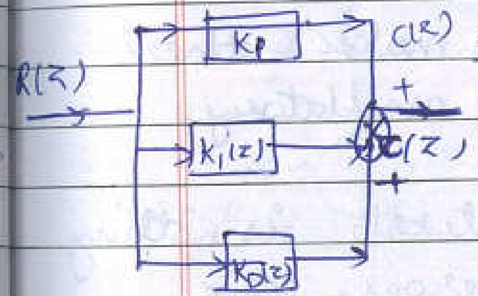
2. Digital PID controller:

can be designed by computer, digital N/ws, μP or digital signal processors.

gives better accuracy and control. Algorithms of digital controllers can be easily changed by changing the program of the controller where as changing the components of continous data controller is rather difficult once the controller has been implemented. Digital controllers also provide high resolutions and high speed airthmetic.



In digital control, the proportional control is implemented (same as in continous case) by a proportional constant K_p,

Integrated scheme

1. Backward rectangular scheme

$$K_I(z) = K_I(T)\frac{T}{z-1}$$

2. forward rectangular

$$K_I(z) = K_I(T)\frac{Tz}{z-1}$$

3. Bilinear Transform

$$K_I(z) = K_I(T)\frac{Tz+1}{2z-1}$$

Derivative scheme

$$\left[\frac{de(T)}{dt}\right]_{t=T} = \frac{[e(kT) - e(k-1)T]}{T}$$

$$K_D(z) = K_D(T)\frac{z-1}{Tz}$$

Compensation of control system

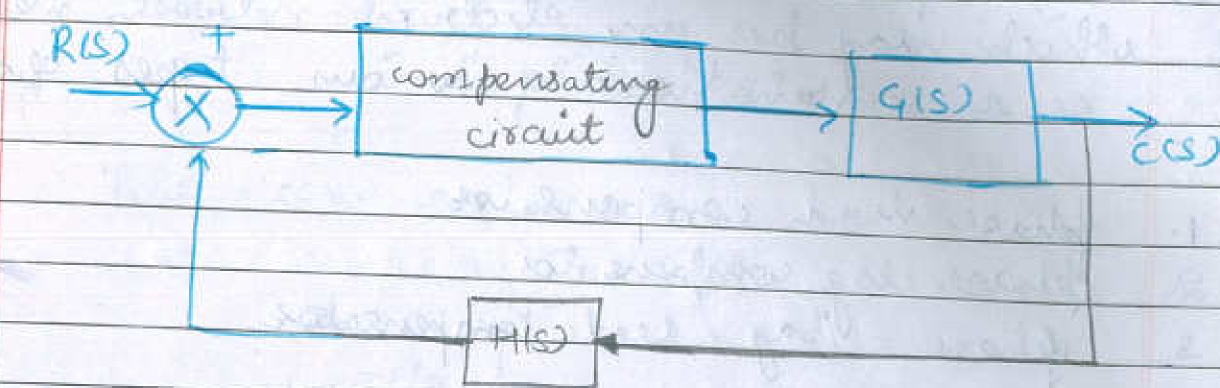
- compensate
  ↓
  unstable system $\xrightarrow{\text{make}}$ stable system

1. steady state error reduced
   ↓
   ↑ing forward pathgain
   ↓
   But on the other hand it makes the system the system unstable and oscillatory.
2. Introducing a compensation network is nothing but addition of poles and zeroes. The addition of zero to the open transfer function will lead the system towards stability. The speed of the response becomes fast. but the accuracy of the system is reduced and addition of a pole will lead the system towards instability. The speed of the system slows down, but the accuracy of the system ↑es.

design of system in s-domain, introduction of poles and zeroes at suitable places will give satisfactory performance.

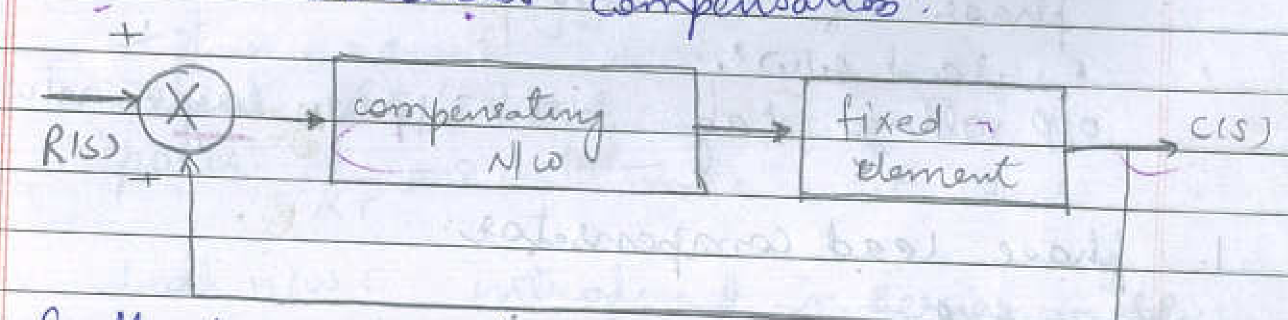
- mainly accuracy and stability.

- This can be achieved by connecting a n/w b/w error detector and plant is known as compensation network.

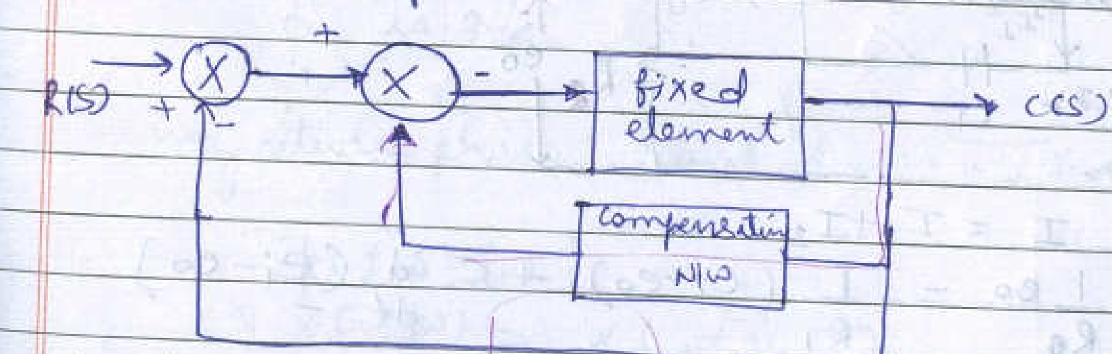


- Arrangement for introduction of compensat^n n/w.

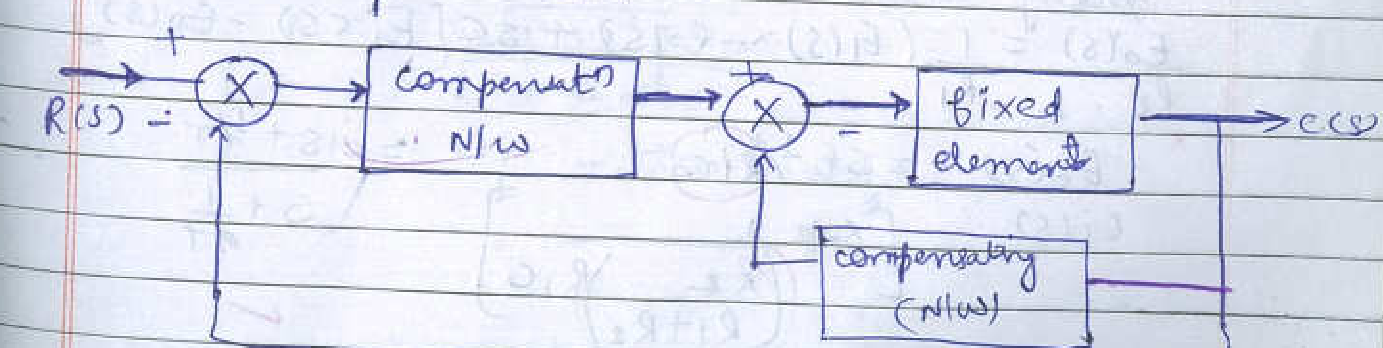
1. cascade or series compensation:



2. feedback compensation:



3. load compensation:



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• Compensating Network:

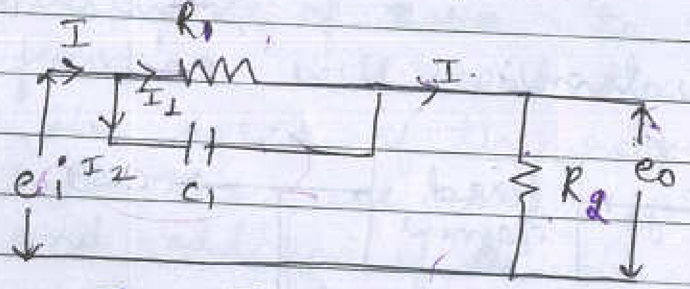
A compensator is a physical device which may be an electrical network, mechanical or a combination of various types of devices.

1. phase lead compensator
2. phase lag compensator
3. phase - lag - lead compensator

phase of the o/p voltage leads the phase of i/p the n/w is called 'lead n/w'.

o/p voltage lag the i/p lag n/w.

1. phase lead compensator:



$$I = I_1 + I_2$$

$$\frac{1}{R_2} e_o = \frac{1}{R_1}(e_i - e_o) + C\frac{d}{dt}(e_i - e_o)$$

taking laplace transform

$$\frac{E_o(S)}{R_2} = \frac{1}{R_1}(E_i(S) - E_o(S)) + SC[E_i(S) - E_o(S)]$$

$$\frac{E_o(S)}{E_i(S)} = \frac{S + 1/R_1C}{\left[S + \frac{1}{\left(\frac{R_2}{R_1+R_2}\right)R_1C}\right]} = \frac{S + 1/T}{S + \frac{1}{\alpha T}}$$

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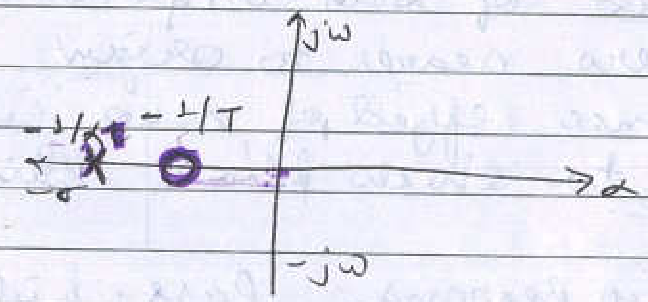
$$\frac{E_o(S)}{E_i(S)} = \frac{\alpha(1+ST)}{1+S\alpha T}$$

$$\alpha = \frac{R_2}{R_1+R_2} < 1 \qquad T = R_1C$$

Pole - zero configurato



zeroes : $S = -1/T$

poles: $\frac{-1}{\alpha T} = S$

lead n/w : introduced in series with forward path of transfer functn, the ϕ is +ve.

• magnitude phase lead ϕ_m) and α:

put $S = j\omega$

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{\alpha(1+j\omega T)}{(1+j\omega\alpha T)}$$

$$M = \frac{\alpha\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2\alpha^2T^2}}$$

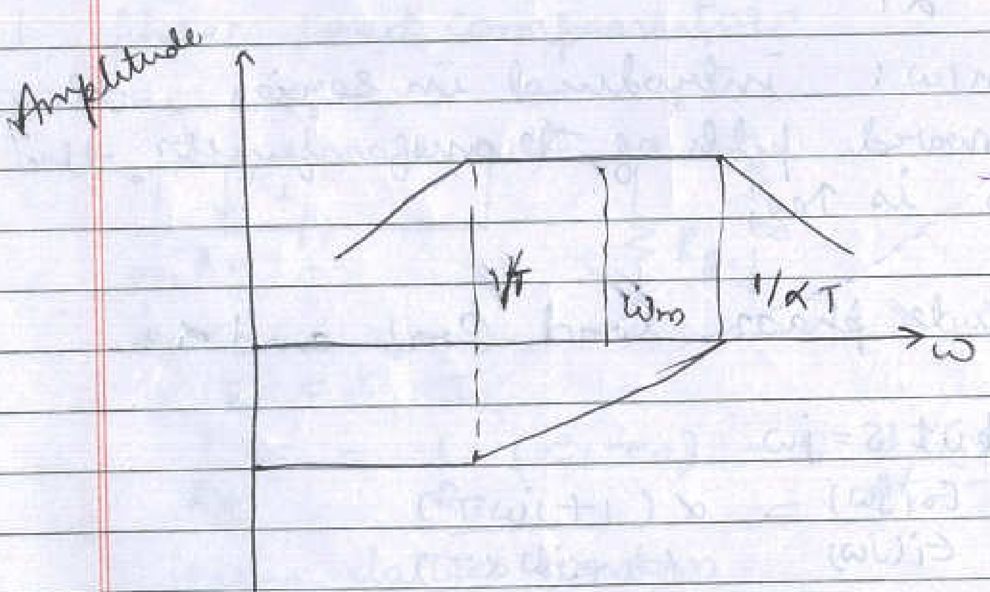
$$\phi = \tan^{-1}\frac{\omega T}{1} - \tan^{-1}\frac{\omega\alpha T}{1}$$

- Bode plot:
  high f pass
  low f attenuated.

- pre-emphasis ckt
- differentiator,
- $\boxed{\text{HPF}}$
- pole-zero of lead compensator
  zero nearer to origin
  hence effect of zero is domaint
  due to which phase shift ↑es.

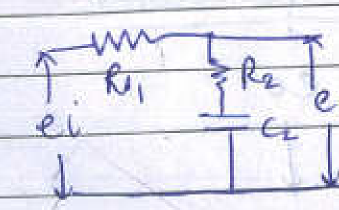
- Frequency Response: Pass: high frqⁿ
  attenuate: low frqⁿ

- rise time, BW ↑es



$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

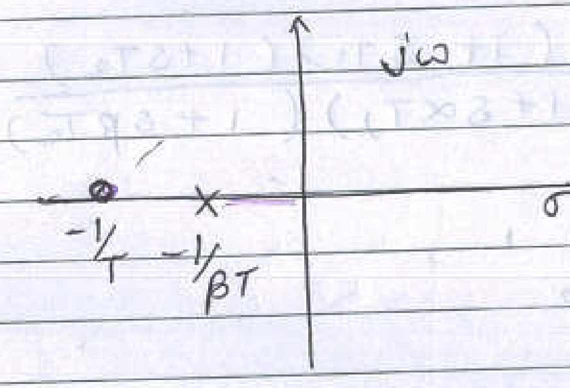
2. phase-lag compensation:



$$\frac{E_0(S)}{E_i(S)} = \frac{1+ST}{1+S\beta T}$$

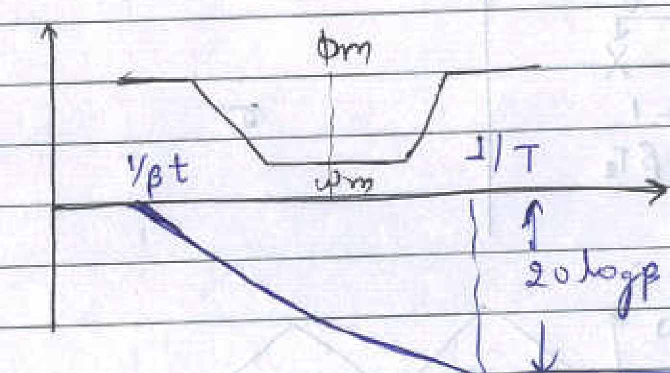
$$\beta = \frac{R_1+R_2}{R_2}$$

$$T = R_2 C$$



$$M = \frac{\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2\beta^2T^2}}$$

$$\phi = \tan^{-1}\frac{\omega T}{1} - \tan^{-1}\frac{\omega\beta T}{1}$$

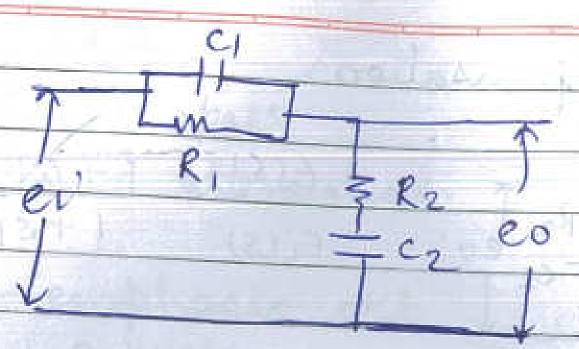


$$\omega_m = \frac{1}{T\sqrt{\beta}}$$

3. phase lag-lead compensation:

- good static accuracy
- ↑es low frqⁿ gain which improves the stedy state.

- LPF: low frqⁿ Pass
  attenuate high frqⁿ.

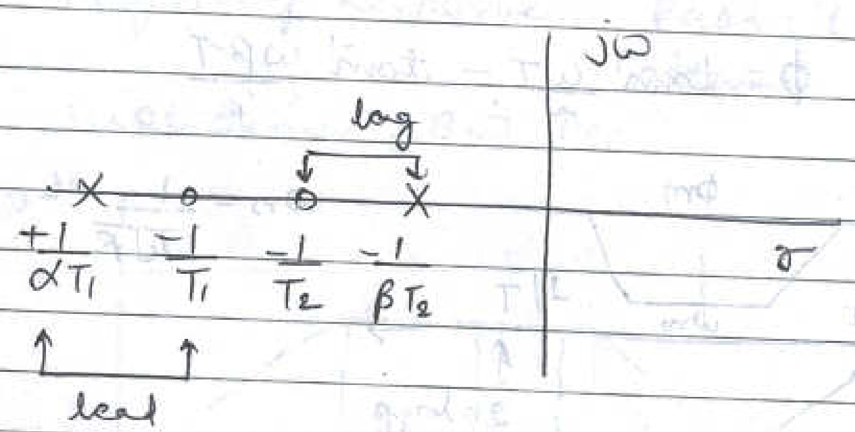


$$\frac{E_o(s)}{E_i(s)} = \frac{\alpha(1+sT_1)(1+sT_2)}{(1+s\alpha T_1)(1+s\beta T_2)}$$

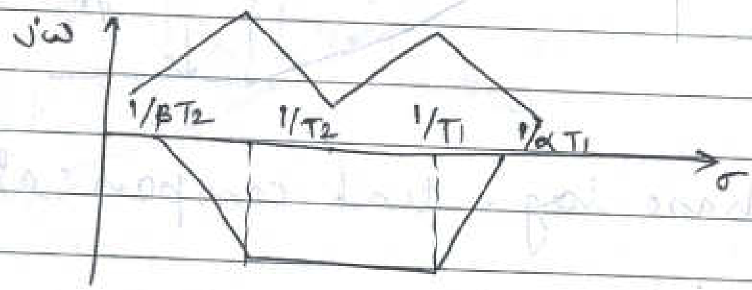
$T_1 = R_1 C_1$

$T_2 = R_2 C_2$

$\alpha\beta = 1$



Bode plot



- fast response as well as better steady state accuracy is desired.
- settling time reduces gain, phase margin can be adjusted.
- BW of the system ↑ed.

Canonical Representation