number of orbiting electrons. Thus, an atom is electrically neutral. If an atom loses an electron, it loses the negative charge and becomes a *positive ion*. Similarly if an atom gains an additional electron, it becomes a *negative ion*. However, nearly all atoms have some electrons which are loosely bound to their nuclei. These electrons are known as *free electrons* and may be dislodged by one means or another and transferred from one atom to another. The body, that contains unequal number of electrons and protons is said to be *electrically charged.* If a body contains electrons more than its normal number then the body is said to be – *vely charged.* Similarly a body containing electrons less than its normal number is said to be + *vely charged.*

The atoms of some materials such as silver, copper, aluminium and zinc, known as *conductors*, have many free electrons. These free electrons of a metal move about haphazardly in all directions from atom to atom but when a certain electrical pressure or potential is applied to such metals at the two ends, the electrons move only in one direction. The drift of electrons in a conductor in one direction is known as the *electric current i.e.* the flow of electric current takes place by the movement of the electrons in a conductor. Since electrons are negatively charged, the direction of their motion is opposite to the direction of conventional current which is from higher potential to the lower potential. In non-metallic materials, such as glass, mica, slate and porcelain, the electrons are very closely bound to the nucleus and it is difficult to remove the electrons from the atoms. Such materials are known as *non-conductors* or *insulators*.

1.2 ELECTRIC CURRENT

Electric current may be defined as the time rate of net motion of an electric charge across a cross-sectional boundary (Fig. 1.2). A random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge with time.

i.e. electric current,

$$i = \text{Rate of transfer of electric charge}$$

$$= \frac{\text{Quantity of electric charge transferred during a given time duration}}{\text{Time duration}} = \frac{dQ}{dt} \qquad \text{...(1.1)}$$

Fig. 1.2

Coulomb is the practical as well as SI unit for measurement of electric charge. One coulomb is approximately equal to 624×10^{16} electrons.

Since current is the rate of flow of electric charge through a conductor and coulomb is the unit of electric charge, the current may be specified in coulombs per second. In practice the term coulomb per second is seldom used, a shorter term, *ampere* is used instead.

1.3 ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

Electromotive force (emf) is the force that causes an electric current to flow in an electric circuit while the *potential difference* (pd) between two points in an electric circuit is that difference in their electrical state which tends to cause flow of electric current between them.

Volt is a unit of electromotive force as well as potential difference in practical as well as in SI system of units.

The volt is defined as that potential difference between two points of a conductor carrying a current of one ampere when the power dissipated between these points is equal to one watt.

1.4 RESISTANCE

Resistance may be defined as that property of a substance which opposes (or restricts) the flow of an electric current (or electrons) through it.

The practical as well as MKS (or SI) unit of resistance is ohm (Ω), which is defined as that resistance between two points of a conductor when a potential difference of one volt, applied between these points, produces in this conductor a current of one ampere, the conductor not being a source of any emf.

For insulators having high resistance, much bigger units kilo ohm or kΩ (10^3 ohm) and megohm or MΩ (10^6 ohm) are used. In case of very small resistances smaller units like milli ohm (10^{-3} ohm) or micro ohm (10^{-6} ohm) are employed.

1.5 OHM'S LAW

As the rate of flow of water through a pipe is directly proportional to the effective pressure (*i.e.* difference of pressure at two ends) and inversely proportional to the frictional resistance, similarly the current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor and inversely proportional to the conductor resistance. This relation was discovered by Georg Simon Ohm and so it is known as *Ohm's law*.

If I is the current flowing through a conductor of resistance R across which a potential difference V is applied then according to Ohm's law

$$I \propto V \text{ and } I \propto \frac{1}{R} \quad \text{or} \quad I \propto \frac{V}{R} \quad \text{or} \quad I = \frac{V}{R} \qquad ...(1.2)$$

where V is in volts, R is in ohms and I is in amperes.

Ohm's law may be defined as follows:

Physical state i.e. temperature etc. remaining the same, the current flowing through a conductor is directly proportional to the potential difference applied across its ends.

Or

The ratio of potential difference applied across a conductor and current flowing through it remains constant provided physical state i.e. temperature etc. of the conductor remains unchanged.

$$i.e. \quad \frac{V}{I} = \text{Constant} = R \qquad ...(1.3)$$

where R is known as the resistance of the conductor.

Ohm's law may be alternatively expressed as

$$V = IR \qquad ...(1.4)$$

Equations (1.2), (1.3) and (1.4) give Ohm's law in three forms with which the student should be familiar.

Ohm's law cannot be applied to circuits consisting of electronic tubes or transistors because such elements are not bilateral *i.e.* they behave in different way when the direction of flow of current is reversed as in case of a diode. Ohm's law also cannot be applied to circuits consisting of nonlinear elements such as powdered carbon, thyrite, electric arc etc. For example, for silicon carbide, the relationship between applied voltage (or potential difference) V and current flowing I is given as $V = K I^m$ where K and m are constants and m is less than unity.

1.6 LAWS OF RESISTANCE

The resistance of a wire depends upon its length, area of x-section, type of material, purity and hardness of material of which it is made of and the operating temperature.

Resistance of a wire is

(*a*) directly proportional to its length, l *i.e.* $R \propto l$

(*b*) inversely proportional to its area of x-section, a *i.e.* $R \propto 1/a$

Combining above two facts we have $R \propto l/a$

$$\text{or } R = \rho \frac{l}{a} \qquad ...(1.5)$$

where ρ (rho) is a constant depending upon the nature of the material and is known as the *specific resistance or resistivity of* the material of the wire.

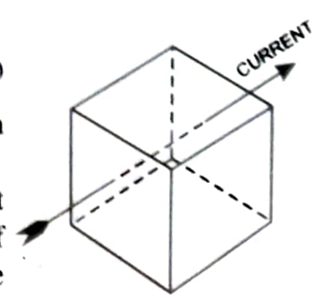


Fig. 1.3

To determine the nature of the constant ρ let us imagine a conductor of unit length and unit cross-sectional area, for example a cube whose edges are each of length one unit, and let the current flow into the cube at right angles to one face and out at the other face. Then putting $l = 1$ and $a = 1$ in Eq. (1.5) we have $R = \rho$.

Hence resistance of a material of unit length having unit cross-sectional area is defined as the resistivity or specific resistance of the material.

Specific resistance or resistivity of a material is also defined as the resistance between opposite faces of a unit cube of that material.

Resistivity is measured in ohm-metres (Ω-m) or ohms per metre cube in mks (or SI) system and ohm-cm (Ω-cm) or ohm per cm cube in cgs system.

$$1\ \Omega\text{-m} = 100\ \Omega\text{-cm}$$

1.7 CONDUCTANCE AND CONDUCTIVITY

The reciprocal of resistance *i.e.* 1/R is called the *conductance* and is denoted by English letter G. It is defined as the inducement offered by the conductor to the flow of current and is measured in siemens (S). Earlier, the unit of conductance was mho (℧).

$$1 \text{ siemen} = 1 \text{ mho}$$

From Eq. (1.5)

$$G = \frac{1}{R} = \frac{1}{\rho \frac{l}{a}} = \frac{1}{\rho} \cdot \frac{a}{l} = \sigma \frac{a}{l} \qquad ...(1.6)$$

where $\sigma = 1/\rho$ and is known as *specific conductance* or *conductivity* of the material. Hence *conductivity* is the reciprocal of the resistivity and is defined as the conductance between the two opposite faces of a unit cube. The unit of conductivity is siemens/metre (S/m).

Example 1.1. A coil consists of 2,000 turns of copper wire having a cross-sectional area of 0.8 mm². The mean length per turn is 80 cm, and the resistivity of copper is 0.02 μ Ω-m. Find the resistance of the coil.

Solution: Length of the coil, l = Number of turns × mean length per turn = 2,000 × 0.8 = 1,600 m

Cross-sectional area of wire, $a = 0.8 \text{ mm}^2 = 0.8 \times 10^{-6} \text{ m}^2$

Resistivity of copper, $\rho = 0.02\ \mu\Omega\text{-m} = 2 \times 10^{-2} \times 10^{-6}\ \Omega\text{-m} = 2 \times 10^{-8}\ \Omega\text{-m}$

Resistance of the coil, $R = \rho \dfrac{l}{a} = 2 \times 10^{-8} \times \dfrac{1{,}600}{0.8 \times 10^{-6}} = 40\ \Omega$ **Ans.**

Example 1.2. A heater element is made of nichrome wire having resistivity equal to 100 × 10⁻⁸ Ω-m. The diameter of the wire is 0.4 mm. Calculate the length of the wire required to get a resistance of 40 Ω.

Solution: Resistance of nichrome wire, $R = 40\ \Omega$

Resistivity of nichrome wire, $\rho = 100 \times 10^{-8}\ \Omega\text{-m}$

Diameter of nichrome wire, $d = 0.4 \text{ mm} = 4 \times 10^{-4} \text{ m}$

Cross-sectional area of nichrome wire, $a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}\left(4 \times 10^{-4}\right)^2 = 4\pi \times 10^{-8} \text{ m}^2$

Length of nichrome wire required, $l = \dfrac{R \times a}{\rho} = \dfrac{40 \times 4\pi \times 10^{-8}}{100 \times 10^{-8}} = 5.03 \text{ m}$ **Ans.**

Example 1.3. The resistance of a conductor 1 mm² in cross-section and 20 m long is 0.346 Ω. Determine the specific resistance of the conductor material.

Solution: Resistance of conductor, $R = 0.346\ \Omega$

Length of conductor, $l = 20$ m

Cross-sectional area of conductor, $a = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$

Specific resistance of conductor material, $\rho = \dfrac{R \times a}{l} = \dfrac{0.346 \times 1 \times 10^{-6}}{20} = 1.73 \times 10^{-8}\ \Omega\text{-m}$ **Ans.**

Example 1.4. A wire of length 1 m has a resistance of 2 Ω. Obtain the resistance if specific resistance is doubled, diameter is doubled and the length is made three times of the first.

[Pb. Technical Univ. Basic Electrical and Electronics Engineering Second Semester 2004-05; 2009-10]

Solution:

New specific resistance, $\rho_2 = 2\rho_1$

New length of wire, $l_2 = 3l_1$

New cross-sectional area, $a_2 = \frac{\pi}{4}\left(d_2^2\right) = \frac{\pi}{4}(2d_1)^2 = 4a_1$

New resistance, $R_2 = \frac{\rho_2 l_2}{a_2} = \frac{2\rho_1 \times 3l_1}{4a_1} = 1.5\frac{\rho_1 l_1}{a_1} = 1.5R_1 = 1.5 \times 2 = 3\Omega$ **Ans.**

Example 1.5. Determine the resistance of a copper tube having external diameter 8 cm; thickness 5 mm and length 5 metres. It is given that specific resistance of copper is 1.70 μΩ-cm. *[M.D. Univ. Electrical Technology, May-2009]*

Solution:

Resistivity of copper, $\rho = 1.7 \times 10^{-8}\ \Omega\text{-m}$

Length of copper tube, $l = 5$ metres

External diameter of tube, D = 8 cm = 0.08 metre

Internal diameter of tube, $d = D - 2 \times$ thickness of tube $= 0.08 - 2 \times 0.005 = 0.07$ m

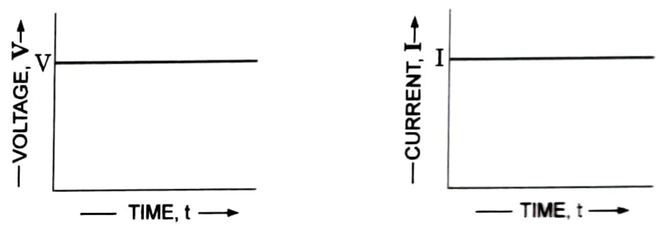
Cross-sectional area of tube, $a = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(0.08^2 - 0.07^2) = 0.001178\ m^2$

Resistance of copper tube, $R = \rho\frac{l}{a} = \frac{1.7 \times 10^{-8} \times 5}{0.001178} = 72.15\ \mu\Omega$ **Ans.**

1.8 TYPES OF SUPPLY

There are two types of supply viz. dc and ac.

The voltage or current available from batteries or solar cells is *direct* in the sense that the polarity remains the same. Such sources are called *direct current sources*. A plot of output of such sources with respect to time is a straight line parallel to time axis as illustrated in Fig. 1.4. This shows that the voltage / current output of a dc source is constant with respect to time, unless the chemicals in the battery are exhausted or the light incident on solar cell varies.



(a) Output Characteristic of Battery *(b) Output Characteristic of Solar Cell*

Fig. 1.4

Power obtainable from batteries or solar cells is very small as compared to the total power we need. The dc power in bulk can be generated by using dc generators, and will be discussed in detail later on. Standard dc voltages are 230 and 460 V.

A current (or voltage) is called *alternating* if it periodically changes its direction and magnitude whereas direct currents are steady and in one direction. In more restricted sense, alternating current is a periodically varying current, the average value of which, over a period, is zero.

At present a large percentage of electrical energy (nearly all) being used for domestic, commercial and industrial purposes is generated as alternating current because of technical and economical reasons (refer to Art. 4.1).

For industrial applications of direct current like electrolytic processes, welding processes and variable speed motor drives, the present trend is to generate ac and then convert it into dc by converters. For conversion of ac into dc the dc generator as a part of motor (ac)- generator (dc) set has to compete the SCR rectifiers and various other power controlled devices which usually are cheaper in cost, compact in size, relatively noise-free in operation and require minimum maintenance, though they suffer from the disadvantages of having poor power factor, harmonic generation, poor braking etc.

In our country the standard voltage for low tension (*lt*) distribution is 240 / 415 V (240 V between phase and neutral wires and 415 V from line-to-line). This is a reasonably safe voltage. Higher the voltage used, lower is the installation cost *i.e.* economy is achieved but increase of voltage means more hazard to human life and property. Hence, a compromise is made. All over the world the standard voltage used for *lt* distribution is in the range of 200 / 350 V to 250 / 440 V except in American continent where 110 /190 V is the distribution voltage. Generation, transmission and *ht* distribution voltages are governed by economy and operation.

1.9 RESISTANCE VARIATION WITH TEMPERATURE

The resistance of all pure metallic conductors increases with the increase in temperature but the resistance of the insulators and non-metallic materials generally decreases with the increase in temperature.

If the resistance of any pure metal is plotted on a temperature base, it is found that over the range of temperature from 0 to 100 °C the graph is practically a straight line, as illustrated in Fig. 1.5. If this straight line is extended, it cuts the temperature axis at some temperature, $-t_0$ °C, known as *inferred zero resistance temperature*. This does not mean that the resistance of the metal is actually zero at that temperature, but $-t_0$ °C is the temperature at which the resistance would be zero if the rate of decrease between 100 and 0°C were maintained constant at all temperatures. From the similarity of the triangles in Fig. 1.5.

$$\frac{R_2}{R_1} = \frac{t_0 + t_2}{t_0 + t_1} \qquad ...(1.7)$$

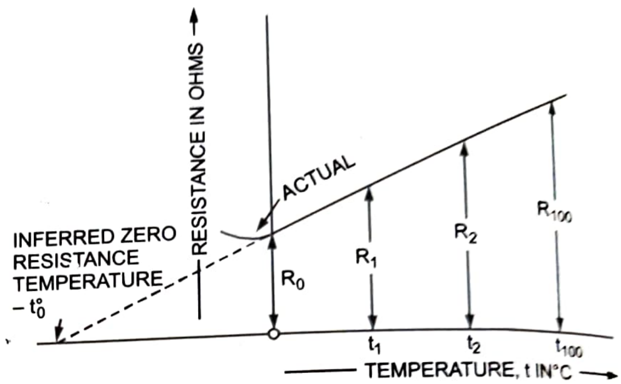


Fig. 1.5

where R_1 and R_2 are the resistances at temperatures t_1 °C and t_2 °C respectively. Thus if the resistance R_1 for any temperature t_1 °C is known, then resistance for any other temperature t_2 can be computed from above equation provided that t_0 for that particular material is known.

The variation of resistance with temperature is often utilized in determining temperature variations. For example, in testing of an electric machine, the resistance of the coil is measured both before and after the test run, and the increase in resistance is a measure of the rise in temperature. For computation of temperature rise Eq. (1.7) may be transposed to the following form

$$t_2 - t_1 = \frac{R_2 - R_1}{R_1}(t_0 + t_1) \qquad ...(1.8)$$

A temperature of 20 °C has been adopted as the *standard reference temperature* for measurement of resistance, and the handbooks give the resistance of the various materials at that temperature. Consequently, when a designer is computing the resistance of any conductor from its dimensions, the initial temperature t, at which the resistance is known, is generally 20 °C.

1.10 TEMPERATURE COEFFICIENT OF RESISTANCE

Let a metallic conductor having a resistance of R_0 at 0°C be heated to t°C and let its resistance at this temperature be R_t. From Eq. (1.7)

$$\frac{R_t}{R_0} = \frac{t_0 + t}{t_0 + 0} \quad \text{or} \quad R_t = R_0 + \frac{1}{t_0}R_0 t$$

$$\text{or change in resistance, } \Delta R = R_t - R_0 = \frac{1}{t_0}R_0\, t = \alpha_0\, R_0\, t \qquad ...(1.9)$$

where $\alpha_0 = \dfrac{1}{t_0}$ and is called the *temperature coefficient of resistance of the material at 0°C*

From Eq. (1.9) it may be concluded that change in resistance due to change in temperature

(*a*) varies directly as its initial resistance,

(*b*) varies directly as rise in temperature and

(*c*) depends on the nature of the material of the conductor.

The Eq. (1.9) may be rewritten as

$$\alpha_0 = \frac{\Delta R}{R_0\, t} \qquad \text{...(1.10)}$$

So the temperature coefficient of resistance may be defined as the ratio of increase in resistance per degree rise of temperature to the original resistance.

If R_0 is the resistance of any conductor at 0°C and α_0 is the temperature coefficient of resistance at 0°, then resistance at t°C is given as

$$R_t = \text{Original resistance} + \text{increase in resistance}$$
$$= R_0 + R_0\, \alpha_0\, t = R_0\,(1 + \alpha_0\, t) \qquad \text{...(1.11)}$$

The above expression holds good for both increase as well as decrease in temperature.

It is to be noted that

(*i*) Temperature coefficient of resistance for all pure metallic conductors is positive *i.e.* the resistance of all pure metallic conductors increase with the increase in temperature, that of non-metallic materials such as of carbon is negative *i.e.* the resistance of non-metallic materials such as of carbon decreases with the increase in temperature. The temperature coefficient of resistance of alloys like constantan and manganin is negligible.

(*ii*) Temperature coefficient of resistance is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then the temperature coefficient of resistance has the value of α_0. At any other temperature t°C, value of temperature coefficient of resistance is α_t and so on. For any material the temperature coefficient of resistance at 0 °C *i.e.* α_0 has the maximum value.

The temperature coefficient of resistance at any temperature t_1 is given as

$$\alpha_1 = \frac{1}{\dfrac{1}{\alpha_0} + t_1} \qquad \text{...(1.12)}$$

The temperature coefficient of resistance at temperature t_2 in terms of temperature coefficient of resistance at temperature t_1 is given as

$$\alpha_2 = \frac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)} \qquad \text{...(1.13)}$$

If R_1 is the resistance of any conductor at t_1°C and α_1 is the temperature coefficient of resistance at t_1°C, then resistance of the conductor at t_2°C is given

$$R_2 = R_1\,[1 + \alpha_1\,(t_2 - t_1)] \qquad \text{...(1.14)}$$

(*iii*) As the resistance of the material changes with the change in temperature so it is obvious that resistivity of the material depends on temperature.

Knowledge of temperature coefficient of resistance is utilized in determining the temperature rise of electrical machines.

Example 1.6. A coil of relay is made of copper wire. At a temperature of 20°C, the resistance of the coil is 400 Ω. Calculate the resistance of the coil at temperature of 80°C. The temperature coefficient of copper is 0.0038 Ω/Ω/°C at 0°C.

Solution: Temperature coefficient of resistance at 0°C,

$$\alpha_0 = 0.0038\ \Omega/\Omega/°C$$

Temperature coefficient of resistance at 20°C,

$$\alpha_{20} = \frac{1}{\dfrac{1}{\alpha_0} + t} = \frac{1}{\dfrac{1}{0.0038} + 20} = 0.003532$$

Resistance of wire at 20°C, $R_1 = 400\ \Omega$

Resistance of wire at 80°C, $R_2 = R_1\,[1 + \alpha_{20}\,(t_2 - t_1)] = 400\,[1 + 0.003532\,(80 - 20)] = 484.8\ \Omega$ **Ans.**

Example 1.7. Find the temperature at which resistance of the conductor becomes double to that at 0°C.

[Pb. Technical Univ. Basic Electrical and Electronics Engineering, May-2008]

Solution : Resistance of a conductor at t°C is given by equation

$R_t = R_0(1 + \alpha_0 t)$ where R_0 is temperature at 0°C and α_0 is temperature coefficient of resistance at 0°C

Substituting $R_t = 2R_0$ in above equation, we have

$$2R_0 = R_0(1 + \alpha_0 t)$$

$$\text{or } t = \frac{1}{\alpha_0} \quad \textbf{Ans.}$$

Example 1.8. A coil has resistance of 18 Ω when its mean temperature is 20°C and of 20 Ω when its mean temperature is 50°C. Find its mean temperature rise when its resistance is 21 Ω and the surrounding temperature is 15°C.

[Pb. Technical Univ. Basic Electrical and Electronics Engineering, June-2013]

Solution: Let R_0 be the resistance of the coil at 0°C and α_0 be its temperature coefficient of resistance at 0°C

$$\text{Resistance at 20°C, } 18 = R_0\,(1 + 20\,\alpha_0) \quad ...(i)$$

$$\text{Resistance at 50°C, } 20 = R_0\,(1 + 50\,\alpha_0) \quad ...(ii)$$

Dividing Eq. (*ii*) by Eq. (*i*) we have

$$\frac{20}{18} = \frac{1+50\,\alpha_0}{1+20\,\alpha_0}$$

$$\text{or} \quad \alpha_0 = \frac{1}{250} = 0.004$$

If t °C is the temperature of coil when its resistance is 21 Ω, then

$$21 = R_0\,(1 + 0.004\,t) \quad ...(iii)$$

Dividing Eq. (*iii*) by Eq. (*ii*) we have

$$\frac{21}{20} = \frac{1+0.004t}{1+50 \times 0.004}$$

$$\text{or} \quad t = 65\text{ °C}$$

$$\text{Temperature rise} = t - \text{surrounding temperature} = 65 - 15 = 50\text{ °C} \quad \textbf{Ans.}$$

Example 1.9. A motor winding has a resistance of 80 Ω at room temperature of 20°C before switching on to a 230 V. After 4 hour run the winding resistance is 100 Ω. Find the temperature rise if the material temperature coefficient is 1/234.5/°C. *[Pb. Technical Univ. Basic Electrical and Electronics Engineering, May-2006]*

Solution: If the temperature coefficient of resistance of winding material at room temperature of 20°C is α_1 and resistance of winding material at 20°C is R_1, then resistance of winding material at t°C is given by the equation

$$R_2 = R_1[1 + \alpha_1(t_2 - t_1)]$$

$$\text{or} \quad 100 = 80\left[1 + \frac{1}{234.5}(t_2 - 20)\right]$$

$$\because \quad R_2 = 100\ \Omega,\ R_1 = 80\ \Omega,\ t_1 = 20°C \text{ and } \alpha_1 = \frac{1}{234.5}\ /°C$$

$$\text{or Temperature rise, } (t_2 - 20) = \left(\frac{100}{80} - 1\right) \times 234.5 = 58.625°C \quad \textbf{Ans.}$$

1.11 SI SYSTEM OF UNITS

SI is the latest form of metric system and absorbs in it the rationalized MKSA system. SI stands for "Systeme International d' Unites" in French. This abbreviation is now adopted by the International Standardising Organisation as the abbreviated name of this new system of units in all languages.

The SI system is, in fact, simply the RMKSA* system expanded by adding the degree Kelvin, Candela and mole as basic units of temperature, luminous intensity and amount of substance respectively. The SI system is a

* RMKSA stands for rationalized metre, kilogram, second, ampere.

comprehensive, logical and coherent system, designed for use in all branches of science, engineering and technology.

This system derives all the units from the following seven base units.

Quantity	*Unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Intensity of electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

The SI system, besides seven base units, has following supplementary units.

Quantity	*Unit*	*Symbol*
Plane angle	radian	rad
Solid angle	steradian	sr

Recommended prefixes for formation of multiples and submultiples of units are given below:

Multiple	Prefix	Symbol	Fraction	Prefix	Symbol
10	deca	*da**	0.1	deci	*d**
100	hecto	*h**	0.01	centi	*c**
1000	kilo	k	0.001	milli	*m*
1000 000	mega	M	0.000 001	micro	μ
1000 000 000	giga	G	0.000 000 001	nano	*n*
1000 000 000 000	tera	T	0.000 000 000 001	pico	*p*
			0.000 000 000 000 001	fento	*f*
			0.000 000 000 000 000 001	atto	*a*

1.12 WORK, POWER AND ENERGY

Work is said to be done by or against a force, when its point of application moves in or opposite to the direction of the force and is measured by the product of the force and the displacement of the point of application in the direction of force.

i.e. Work done, W = Force [F] × distance [d]

The SI or MKS unit of work is the joule, which is defined as the work done when a force of one newton acts through a distance of one metre in the direction of the force. Hence, if a force F acts through distance d in its own direction,

$$W = F\,[\text{newtons}] \times d\,[\text{metres}] = Fd \text{ joules}$$

Power is defined as the rate of doing work or the amount of work done in unit time.

The MKS or SI unit of power is the joule/second or watt. In practice, the watt is often found to be inconveniently small and so a bigger unit, the *kilowatt* is frequently used.

1 kilowatt = 1,000 watts

The bigger unit of power, most commonly used in engineering practice (not at all in SI system) is horse power defined as below :

Metric Horse Power : It is the practical unit of power in MKS system (not in SI system) which according to ISI specifications is equal to 75 kgf-m of work done per second.

* Should be restricted as much as possible.

Energy is defined as the capacity of doing work. Its units are same as those of work, mentioned above. If a body having mass m, in kg, is moving with velocity v, in metres/second,

$$\text{Kinetic energy} = \frac{1}{2} mv^2 \text{ joules}$$

If a body having mass m, in kg, is lifted vertically through height h, in metres, and if g is the gravitational acceleration, in metres/second^2 in that region, potential energy acquired by the body

$$= \text{Work done in lifting the body} = mgh \text{ joules} = 9.81\ mh \text{ joules}$$

As already stated, in SI system the unit of energy of all forms is joule. Bigger unit of energy is mega joules (MJ) where 1 MJ = 10^6 J.

The thermal units of energy, calorie (gm-calorie) and kilo-calorie (kilogram-calorie), are defined below:

Calorie : It is the amount of heat required to raise the temperature of one gram of water through 1 °C.

$$1 \text{ calorie} = 4.18 \text{ J} \simeq 4.2 \text{ J}$$

Kilocalorie : It is the amount of heat required to raise the temperature of 1 kg of water through 1 °C.

$$1 \text{ k. calorie} = 1{,}000 \text{ calories} = 4{,}180 \text{ joules} \simeq 4{,}200 \text{ J}$$

1.13 ELECTRICAL UNITS OF WORK, POWER AND ENERGY

The unit of work done and of energy expended is joule. It is equal to the energy expended in passing 1 coulomb of charge through a resistance of 1 ohm *i.e.* the energy expended in passing one ampere current for 1 second through a resistance of one ohm is taken as one joule. It may also be expressed as 1 watt-second *i.e.* one watt of power consumed for one second.

i.e. 1 joule = 1 watt-second

The unit of energy, joule or watt-second is too small for practical purposes, so a bigger unit Mega joule (MJ) or kilowatt-hour (kWh) is used in electrical engineering.

$$1 \text{ kWh} = 1{,}000 \text{ watt-hours} = 1{,}000 \times 3{,}600 \text{ watt-seconds or joules} = 3.6 \text{ MJ}$$

The kWh, also called the *Board of Trade* (BOT) *unit,* is the energy absorbed by supplying a load of 1 kW or 1,000 watts for the period of one hour. This is legal unit on which charges for electrical energy are made, and, therefore, it is called the Board of Trade (BOT) unit.

***Watt* :** It is defined as the power expended when there is an unvarying current of one ampere between two points having a potential difference of one volt. As already stated, the bigger unit of power is kW or Mega watt.

$$1 \text{ kW} = 1{,}000 \text{ watts}$$
$$1 \text{ MW} = 1{,}000 \text{ kW} = 1 \times 10^6 \text{ watts}$$

1.14 CONVERSION OF ELECTRICAL UNITS INTO MECHANICAL AND THERMAL UNITS OR VICE VERSA

$$1 \text{ watt} = 1 \text{ joule/second} = 1 \text{ N-m/s}$$

$$1 \text{ kW} = 1{,}000 \text{ watts or J/s or N-m/s} = \frac{1{,}000}{735.5} \textit{ i.e. } 1.36 \text{ hp (metric)}$$

$$1 \text{ kWh} = 1{,}000 \text{ watt-hours} = 3{,}600{,}000 \text{ watt-seconds or joules} = 1.36 \text{ hp-hour (metric)}$$

$$1 \text{ calorie} = 4.18 \text{ J or watt-seconds}$$

$$1 \text{ k cal} = 4{,}180 \text{ J or watt-seconds} = \frac{4{,}180}{3{,}600{,}000} \textit{ i.e. } \frac{1}{860} \text{ kWh}$$

$$1 \text{ kWh} = 36 \times 10^5 \text{ watt-seconds} = \frac{36 \times 10^5}{4{,}180} \text{ i.e. } 860 \text{ kcals}$$

Example 1.18. A house is supplied with electricity at 250 V. It has the following electrical loads:

1. **20 in number, 100 W lamps each.**
2. **5 in number, 2 kW radiators each.**
3. **2 motors, each taking 50 A.**

If the electricity is kept in use for 5 hours a day, find the cost of energy consumed for the month of February 2007, at the rate of ₹ 4.50 per unit.

Solution : Lamp load, W_L = Number of lamps × rating of each lamp in watts = 20 × 100 = 2,000 W or 2 kW

Radiator load, $W_H = 5 \times 2 = 10$ kW

$$\text{Motor load for each motor} = \frac{V \times I}{1{,}000} = \frac{250 \times 50}{1{,}000} = 12.5 \text{ kW}$$

Motor load, $W_M = 2 \times 12.5 = 25$ kW

Total load, $W_T = W_L + W_H + W_M = 2.0 + 10.0 + 25 = 37$ kW

Hours of use per day = 5

Days in month of February, 2007 = 28

Total hours of use in Feb. 2007 = 5 × 28 = 140

Total energy consumption in Feb. 2007 = 37.0 × 140 = 5,180 kWh

Cost of energy consumed = ₹ 4.50 × 5,180 = ₹ 23,310.00 **Ans.**

1.18 SERIES CIRCUITS

When the resistors are connected end to end, so that they form only one path for the flow of current, then resistors are said to be connected in series and such circuits are known as *series circuits*.

Let resistors R_1, R_2 and R_3 be connected in series, as shown in Fig. 1.6, and the potential difference of V volts be applied between extreme ends A and D to cause flow of current of I amperes through all the resistors R_1, R_2 and R_3.

Now according to Ohm's law

Voltage drop across resistor R_1, $V_1 = I R_1$

Voltage drop across resistor R_2, $V_2 = I R_2$

Voltage drop across resistor R_3, $V_3 = I R_3$

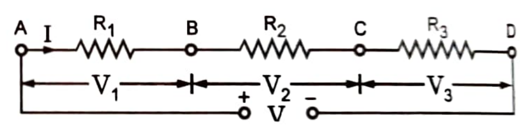


Fig. 1.6 *Series Circuit*

Voltage drop across whole circuit,

V = Voltage drop across resistor R_1 + Voltage drop across resistor R_2 + Voltage drop across resistor R_3

i.e. $V = I R_1 + I R_2 + I R_3 = I (R_1 + R_2 + R_3)$

$$\text{or} \quad \frac{V}{I} = R_1 + R_2 + R_3 \quad \text{...(1.17)}$$

And according to Ohm's law $\frac{V}{I}$ gives the whole circuit resistance, say R

∴ Effective resistance of the series circuit,

$$R = R_1 + R_2 + R_3 \quad \text{...(1.18)}$$

Thus, when a number of resistors are connected in series, the equivalent resistance is given by the arithmetic sum of their individual resistances.

$$i.e.\ R = R_1 + R_2 + R_3 + \ldots R_n \quad \text{...(1.19)}$$

From the above discussions for a series circuit we conclude that

1. same current flows through all parts of the circuit,
2. applied voltage is equal to the sum of voltage drops across the different parts of the circuit,
3. different resistors have their individual voltage drops,
4. voltage drop across individual resistor is directly proportional to its resistance, current being the same in each resistor,
5. voltage drops are additive,
6. resistances are additive,
7. powers are additive.

Series circuits are common in electrical equipment. The tube filaments in small radios are usually in series. Current controlling devices are wired in series with the controlled equipment. Fuses are in series with the equipment they protect. A thermostat switch is in series with the heating element in an electric iron. Automatic house-heating equipment has a thermostat, electromagnet coils, and safety cut-outs in series with a voltage source. Rheostats are placed in series with the coils in large motors for motor current control.

Example 1.19. Four coils having resistances of 3, 5, 10 and 12 ohms are connected in series across 120 V. Determine (*i*) equivalent resistance of the circuit (*ii*) current flowing through the circuit and (*iii*) voltage drop across individual coils.

Solution: The circuit diagram is shown in Fig. 1.7.

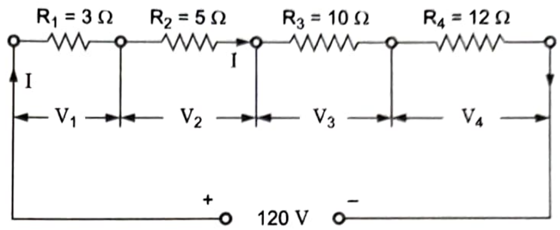


Fig. 1.7

(*i*) Equivalent resistance of the circuit,

$R = R_1 + R_2 + R_3 + R_4 = 3 + 5 + 10 + 12 = 30\ \Omega$ **Ans.**

PD applied across the circuit, V = 120 volts

(*ii*) Current flowing through the circuit,

$$I = \frac{V}{R} = \frac{120}{30} = 4\ \text{A} \quad \textbf{Ans.}$$

(*iii*) Voltage drop in 3 Ω resistance coil, $V_1 = I R_1 = 4 \times 3 = 12$ V **Ans.**

Voltage drop in 5 Ω resistance coil, $V_2 = I R_2 = 4 \times 5 = 20$ V **Ans.**

Voltage drop in 10 Ω resistance coil, $V_3 = I R_3 = 4 \times 10 = 40$ V **Ans.**

Voltage drop in 12 Ω resistance coil, $V_4 = I R_4 = 4 \times 12 = 48$ V **Ans.**

Example 1.20. A 100 V, 60 watt bulb is to be operated from a 220 V supply. What is the resistance to be connected in series with the bulb to glow normally ?

Solution: Rated power of lamp, P = 60 W

Rated voltage of lamp, V = 100 V

Current drawn by the lamp, when operated on rated voltage, *i.e.*

$$\text{Rated current, } I = \frac{P}{V} = \frac{60}{100} = 0.6\ \text{A}$$

Lamp will operate normally on 220 V also if the current flowing through the lamp remains the rated current *i.e.* 0.6 A.

Let the resistance connected in series with the lamp to make it glow normally on 220 V be of R ohms, as shown in Fig. 1.8.

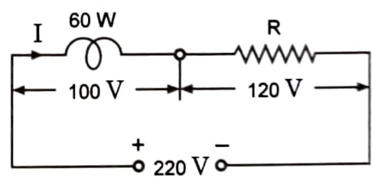


Fig. 1.8

Now since the resistance R is in series with the lamp, the same current will flow through the resistance R, as in the lamp *i.e.* 0.6 A and voltage drop across series resistance R will be equal to supply voltage less voltage drop across the lamp (*i.e.* rated voltage of the lamp)

or Voltage drop across the series resistance,

$$I R = \text{Supply voltage} - \text{rated voltage of the lamp} = 220 - 100 = 120\ \text{V}$$

$$\text{or } R = \frac{120}{I} = \frac{120}{0.6} = 200\ \Omega \quad \textbf{Ans.}$$

1.19 PARALLEL CIRCUITS

When a number of resistors are connected in such a way that one end of each of them is joined to a common point and the other ends being joined to another common point, as shown in Fig. 1.9, then resistors are said to be connected in parallel and such circuits are known as *parallel circuits*. In these circuits current is divided into as many paths as the number of resistances.

Let the resistors R_1, R_2 and R_3 be connected in parallel, as shown in Fig. 1.9, and the potential difference of V volts be applied across the circuit.

Since potential difference across each resistor is same and equal to potential difference applied to the circuit *i.e.* V

∴ According to Ohm's law

$$\text{Current in resistor } R_1,\ I_1 = \frac{V}{R_1} \qquad \text{...(1.20)}$$

$$\text{Current in resistor } R_2,\ I_2 = \frac{V}{R_2} \qquad \text{...(1.21)}$$

$$\text{Current is resistor } R_3,\ I_3 = \frac{V}{R_3} \qquad \text{...(1.22)}$$

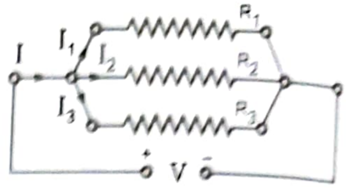


Fig. 1.9 *Parallel Circuit*

Adding Eqs. (1.20), (1.21) and (1.22) we have

$$I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad \text{...(1.23)}$$

and since $I_1 + I_2 + I_3 = I$, the total current flowing through the circuit

$$\text{so } I = I_1 + I_2 + I_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\text{or } \frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

and since $\frac{I}{V} = \frac{1}{R}$ where R is the equivalent resistance of the whole circuit.

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{...(1.24)}$$

Thus, when a number of resistors are connected in parallel, the reciprocal of the equivalent resistance is given by the arithmetic sum of the reciprocals of their individual resistances.

In general if *n* resistors of resistances $R_1, R_2, R_3, \ldots\ldots R_n$ are connected in parallel, then equivalent resistance R of the circuit is given by the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \qquad \text{...(1.25)}$$

$$\text{Also } G = G_1 + G_2 + G_3 +, \ldots\ldots G_n \qquad \text{...(1.26)}$$

where $G = \frac{1}{R}$, $G_1 = \frac{1}{R_1}$, $G_2 = \frac{1}{R_2}$, $G_3 = \frac{1}{R_3}$ and so on.

From the above discussions for a parallel circuit we conclude that

1. same voltage acts across all branches of the circuit,
2. different resistors (or branches) have their individual currents,
3. total circuit current is equal to the sum of individual currents through the various resistors (or branches),
4. branch currents are additive,
5. conductances are additive,
6. powers are additive,
7. the reciprocal of the equivalent or combined resistance is equal to the sum of the reciprocals of the resistances of the individual branches.

Parallel circuits are very common in use. Various lamps and appliances in a house are connected in parallel, so that each one can be operated independently. A series circuit is an "all or none" circuit, in which either every thing operates or nothing operates. For individual control, devices are wired in parallel.

Example 1.21. The equivalent resistance of four resistors joined in parallel is 20 Ω. The currents flowing through them are 0.6, 0.3, 0.2 and 0.1 ampere. Find the value of each resistor.

Solution : Equivalent resistance of the whole circuit, R = 20 Ω

Total current flowing through the circuit,

$$I = I_1 + I_2 + I_3 + I_4$$

$$= 0.6 + 0.3 + 0.2 + 0.1 = 1.2 \text{ A}$$

PD across the circuit, $V = IR = 1.2 \times 20 = 24$ V

$$\text{Resistance } R_1 = \frac{V}{I_1} = \frac{24}{0.6} = 40\ \Omega \textbf{ Ans.}$$

$$\text{Resistance } R_2 = \frac{V}{I_2} = \frac{24}{0.3} = 80\ \Omega \textbf{ Ans.}$$

$$\text{Resistance } R_3 = \frac{V}{I_3} = \frac{24}{0.2} = 120\ \Omega \textbf{ Ans.}$$

$$\text{Resistance } R_4 = \frac{V}{I_4} = \frac{24}{0.1} = 240\ \Omega \textbf{ Ans.}$$

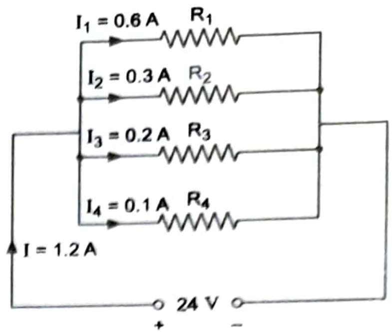


Fig. 1.10

Example 1.22. Three loads A, B, C are connected in parallel to a 240 V source. Load A takes 9.6 kW, load B takes 60 A and load C has a resistance of 4.8 Ω. Calculate (*i*) R_A and R_B (*ii*) total current (*iii*) total power, and (*iv*) equivalent resistance. *[Anna Univ. Circuit Theory, May/June-2013]*

Solution: Current drawn by load A, $I_A = \frac{P}{V} = \frac{9.6 \times 1{,}000}{240} = 40$ A

(*i*)
$$R_A = \frac{V}{I_A} = \frac{240}{40} = 6\ \Omega \textbf{ Ans.}$$

$$R_B = \frac{V}{I_B} = \frac{240}{60} = 4\ \Omega \textbf{ Ans.} \qquad \because I_B = 60 \text{ A ...(given)}$$

Power drawn by load B, $P_B = V \times I_B = \frac{240 \times 60}{1{,}000} = 14.4$ kW

Load resistance, $R_C = 4.8\ \Omega$

Current drawn by load C, $I_C = \frac{V}{R_C} = \frac{240}{4.8} = 50$ A

Power drawn by load C, $P_C = V \times I_C = \frac{240 \times 50}{1{,}000} = 12$ kW

(*ii*) Total current, $I = I_A + I_B + I_C = 40 + 60 + 50 = 150$ A **Ans.**

(*iii*) Total power, $P = P_A + P_B + P_C = 9.6 + 14.4 + 12 = 36$ kW **Ans.**

Also,
$$P = V \times I = \frac{240 \times 150}{1{,}000} = 36 \text{ kW, same as above}$$

(*iv*) Equivalent resistance,
$$R = \frac{1}{\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}} = \frac{1}{\frac{1}{6} + \frac{1}{4} + \frac{1}{4.8}} = \frac{24}{4+6+5} = 1.6\ \Omega \textbf{ Ans.}$$

Also,
$$R = \frac{V}{I} = \frac{240}{150} = 1.6\ \Omega \text{ same as above}$$

and
$$R = \frac{P}{I^2} = \frac{36 \times 1{,}000}{150^2} = 1.6\ \Omega \text{ same as above.}$$

Example 1.23. When a resistor is placed across 230 V supply (dc) the current is 12 A. What is the value of resistor that must be connected in parallel to increase the load current to 16 A ?

[Pb. Technical Univ. Basic Electrical and Electronics Engineering Dec. 2005]

Solution: Resistance of the given resistor,

$$R_1 = \frac{V}{I} = \frac{230}{12} = 19.167\ \Omega$$

Equivalent resistance of parallel combination,

$$R_P = \frac{V}{I_P} = \frac{230}{16} = 14.375\ \Omega$$

Let the value of the resistor connected in parallel be R_2 Ω then

$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

$$\text{or } R_P(R_1 + R_2) = R_1 R_2$$

$$\text{or } R_2 = \frac{R_P R_1}{R_1 - R_P} = \frac{14.375 \times 19.167}{19.167 - 14.375} = 57.5\,\Omega \textbf{ Ans.}$$

Example 1.24. A wire of 100 Ω resistance is cut into how many pieces so that when they are connected in parallel resultant is 1 ohm. *[Pb. Technical Univ. Basic Electrical and Electronics Engineering May 2008]*

Solution: Let the wire be cut into n pieces of equal length.

∵ R α l

Now the resistance of each piece of wire, $R = \frac{100}{n}\,\Omega$

Equivalent resistance when the pieces are connected in parallel,

$$R_{eq} = \frac{\text{Resistance of each path}}{\text{Number of parallel paths}} = \frac{100/n}{n} = \frac{100}{n^2}.$$

Substituting $R_{eq} = 1\ \Omega$, we have

$$1 = \frac{100}{n^2} \quad \text{or} \quad n^2 = 100 \quad \text{or} \quad n = \sqrt{100} = 10 \textbf{ Ans.}$$

Example 1.25. Find the current I and voltage across 30 Ω of the circuit shown in Fig. 1.11(*a*). *[Anna Univ. Circuit Theory May-June-2014]*

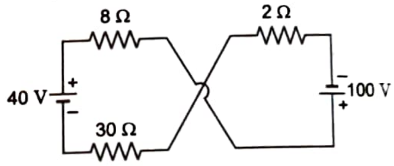


Fig. 1.11(*a*)

Solution : The circuit may be redrawn as shown in Fig. 1.11(*a*).

Total resistance in the circuit,

$$R = 30 + 2 + 8 = 40\ \Omega$$

Net voltage acting in the circuit,

$$V = 100 - 40 = 60\ V$$

Current through 30 Ω resistor

= Current flowing through the series circuit shown in Fig. 1.11(*b*)

$$= I = \frac{V}{R} = \frac{60}{40} = 1.5\ A \textbf{ Ans.}$$

The direction of flow of current is shown in Fig. 1.11(*b*)

Voltage drop across 30 Ω resistor

$$= I \times 30 = 1.5 \times 30 = 45\ V \textbf{ Ans.}$$

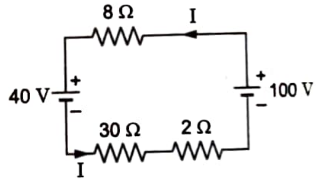


Fig. 1.11(*b*)

1.20 CURRENT DISTRIBUTION IN PARALLEL CIRCUITS

Let two resistors of resistances R_1 and R_2 be connected in parallel across a pd of V volts, According to Ohm's law

Current flowing through resistor R_1, $I_1 = \frac{V}{R_1}$ ...(1.27)

Current flowing through resistor R_2, $I_2 = \frac{V}{R_2}$ ...(1.28)

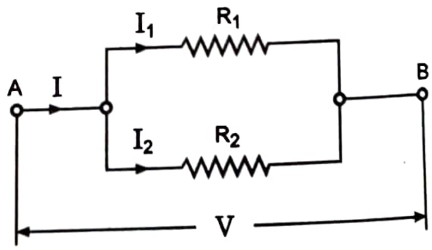


Fig. 1.12

Dividing Eq. (1.27) by Eq. (1.28) we have

$$\frac{I_1}{I_2} = \frac{V/R_1}{V/R_2} = \frac{R_2}{R_1} \qquad ...(1.29)$$

Hence current flowing through each resistor, when connected in parallel, is inversely proportional to their respective resistances.

Since conductance is reciprocal of resistance and if G_1 and G_2 are the respective conductances of resistors R_1 and R_2 then

$$\frac{1}{G_1} = R_1$$

and $$\frac{1}{G_2} = R_2$$

Substituting the above values of R_1 and R_2 in Eq. (1.29) we have

$$\frac{I_1}{I_2} = \frac{1/G_2}{1/G_1} = \frac{G_1}{G_2} \quad ...(1.30)$$

Adding 1 on both sides of the above equation we have

$$\frac{I_1}{I_2} + 1 = \frac{G_1}{G_2} + 1$$

or $$\frac{I_1 + I_2}{I_2} = \frac{G_1 + G_2}{G_2}$$

or $$I_2 = \frac{I_1 + I_2}{G_1 + G_2} \times G_2$$

Since $I_1 + I_2 = I$ and $G_1 + G_2 = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R} = G$

$$\therefore \quad I_2 = I \times \frac{G_2}{G} \quad ...(1.31)$$

and $$I_1 = I \times \frac{G_1}{G} \quad ...(1.32)$$

Hence, current in any branch of a parallel circuit is directly proportional to its respective conductance and is equal to the total current flowing through the circuit multiplied by the ratio of the conductance of the branch to that of the circuit.

The same relation holds good for parallel circuit consisting of more than two resistors and is very useful for its solution.

Example 1.26. Determine the current in all the resistors of the circuit shown in Fig. 1.13.

[Anna Univ. Circuit Theory May-June-2014]

Solution: Total supply current, $i = 50$ A

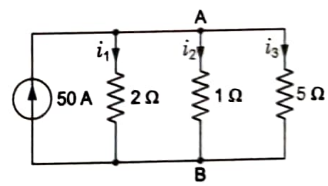


Fig. 1.13

Conductance of 2 Ω resistor, $G_1 = \frac{1}{2} = 0.5$ S

Conductance of 1 Ω resistor, $G_2 = \frac{1}{1} = 1.0$ S

Conductance of 5 Ω resistor, $G_3 = \frac{1}{5} = 0.2$ S

Total conductance, $G = G_1 + G_2 + G_3 = 0.5 + 1 + 0.2 = 1.7$ S

Current, $i_1 = i \times \frac{G_1}{G} = 50 \times \frac{0.5}{1.7} = 14.706$ A **Ans.**

Current, $i_2 = i \times \frac{G_2}{G} = \frac{50 \times 1}{1.7} = 29.412$ A **Ans.**

Current, $i_3 = i \times \frac{G_3}{G} = \frac{50 \times 0.2}{1.7} = 5.882$ A **Ans.**

Example 1.27. Four wires of same material, the same cross-sectional area and the same length when placed in parallel give a total resistance of 0.25 Ω. If the same four wires are connected in series what will be the effective resistance?

Solution: Since the length, area of cross-section and material of each wire are the same, therefore, resistance of each wire will be same, let it be R ohms

When four wires, each of resistance R ohms are connected in parallel, the equivalent resistance will be

$$\frac{1}{1/R + 1/R + 1/R + 1/R} = \frac{R}{4}\ \Omega$$

Since the total resistance of the circuit consisting of four similar wires is given as 0.25

$$\text{so}\quad \frac{R}{4} = 0.25$$

$$\text{or}\quad R = 4 \times 0.25 = 1\ \Omega$$

When these wires are connected in series, their equivalent resistance will become equal to R + R + R + R *i.e.* 4 R or $4 \times 1 = 4\ \Omega$ **Ans.**

Example 1.28. The resistance of two coils is 25 Ω, when these are connected in series and 6 Ω, when connected in parallel. Determine the individual resistance of the two coils. [*M.D. Univ. Electrical Technology, December-2006*]

Solution: Let the resistances of the coils be R_1 and R_2 ohms

Equivalent resistance of coils when connected in series,

$$R_1 + R_2 = 25 \qquad ...(i)$$

Equivalent resistance of coils when connected in parallel,

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} = 6 \qquad ...(ii)$$

Multiplying Eqs. (*i*) and (*ii*) we have

$$R_1 R_2 = 150 \qquad ...(iii)$$

$$\text{and}\quad R_1 - R_2 = \sqrt{(R_1 + R_2)^2 - 4 R_1 R_2} = \sqrt{25^2 - 4 \times 150} = \pm 5\ \Omega \qquad ...(iv)$$

Solving Eqs. (*i*) and (*iv*) we have

$$R_1 = 15 \text{ or } 10\ \Omega \quad \text{and} \quad R_2 = 10 \text{ or } 15\ \Omega \quad \textbf{Ans.}$$

Example 1.29. A 100 V, 60 W bulb connected in series with a 100 V, 100 W bulb and the combination is connected across the 200 V mains. Find the value of resistance that should be connected across the first bulb, so that each bulb may get proper current at the proper voltage.

[*Gujarat Technology Univ. Elements of Electrical Engineering January-2011*]

Solution : Current rating of 60 W bulb, $I_1 = \frac{\text{Wattage of lamp, } W_1}{\text{Rated voltage, } V_1} = \frac{60}{100} = 0.6$ A

Resistance of 60 W bulb, $R = \frac{V_1}{I_1} = \frac{100}{0.6} = 166.67\ \Omega$

Current rating of 100 W bulb, $I_2 = \frac{W_2}{V_2} = \frac{100}{100} = 1.0$ A

Resistance of 100 W bulb, $R_2 = \frac{V^2}{W} = \frac{100^2}{100} = 100\ \Omega$

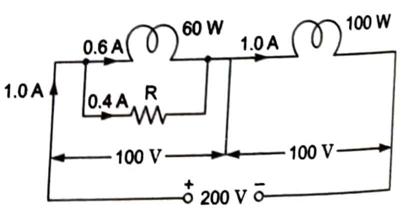


Fig. 1.14

Let the resistance of R Ω be connected across the first bulb so that each bulb may get proper current at the proper voltage when connected in series across 200 V main, as shown in Fig. 1.14.

Since 60 W bulb shunted by resistor R is connected in series with 100 W bulb, the current flowing through combination of 60 W bulb and resistor will be the same, as the current flowing through 100 W bulb, *i.e.*, 1.0 A and voltage across this combination would be 200 – 100 = 100 V, *i.e.*, the voltage rating of 60 W bulb

Current flowing through resistor, R = I – rated current of 60 W bulb = 1.0 – 0.6 = 0.4 A

So resistance of resistor, $R = \frac{100}{0.4} = 250\ \Omega$ **Ans.**

Equivalent resistance of 60 W bulb (*i.e.*, 166.67 Ω) connected across 250 Ω resistor

$$= \frac{1}{\frac{1}{166.7} + \frac{1}{250}} = \frac{250}{1.5 + 1} = 100\,\Omega \quad \textit{i.e.}, \text{ the same as of 100 W, 100 V bulb.}$$

For sharing of 200 V equally, the resistance of the parallel combination of 60 W bulb and resistor R should be same as that of 100 V, 100 W bulb which is there.

1.21 SERIES-PARALLEL CIRCUITS

So far, only simple series and simple parallel circuits have been considered. Practical electric circuits very often consist of combinations of series and parallel resistances. Such circuits may be solved by the proper application of Ohm's law and the rules for series and parallel circuits to the various parts of the complex circuit. There is no definite procedure to be followed in solving complex circuits, the solution depends on the known facts concerning the circuit and the quantities which one desires to find. One simple rule may usually be followed, however—reduce the parallel branches to an equivalent series branch and then solve the circuit as a simple series circuit.

For example, consider a series-parallel circuit shown in Fig. 1.15 for solution.

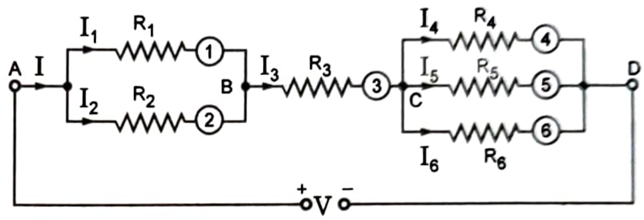


Fig. 1.15

First of all equivalent resistances of all parallel branches are determined separately *e.g.* of branches AB and CD by the law of parallel circuits discussed in Art. 1.19.

Equivalent resistance of parallel branch AB,

$$R_{AB} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

and equivalent resistance of parallel branch CD,

$$R_{CD} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}} = \frac{R_4 R_5 R_6}{R_5 R_6 + R_4 R_6 + R_4 R_5}$$

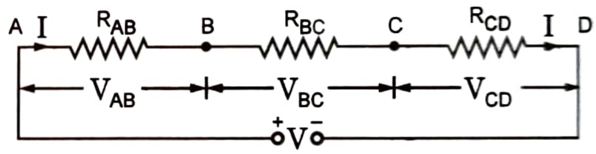


Fig. 1.16

Now the circuit shown in Fig. 1.15 gets reduced to a simple series circuit shown in Fig. 1.16 consisting of three resistances,

$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2}, \quad R_{BC} = R_3$$

$$\text{and } R_{CD} = \frac{R_4 R_5 R_6}{R_5 R_6 + R_4 R_6 + R_4 R_5}$$

Total resistance of circuit, $R_T = R_{AB} + R_{BC} + R_{CD}$

Now circuit current may be determined from the relation

$$I = \frac{V}{R_T}$$

After knowing I, potential differences across branches AB, BC and CD are determined from the relations

$$\text{PD across branch AB, } V_{AB} = I.R_{AB} = \frac{V}{R_T} R_{AB}$$

$$\text{PD across branch BC, } V_{BC} = I.R_{BC} = \frac{V}{R_T} R_{BC}$$

$$\text{and PD across branch CD, } V_{CD} = IR_{CD} = \frac{V}{R_T} R_{CD}.$$

After determination of potential difference across each parallel branch, the currents in the various resistances are determined from the relations

$$\text{Current in resistance } R_1 = I_1 = \frac{V_{AB}}{R_1}$$

$$\text{Current in resistance } R_2 = I_2 = \frac{V_{AB}}{R_2}$$

$$\text{Current in resistance } R_3 = I_3 = I$$

$$\text{Current in resistance } R_4 = I_4 = \frac{V_{CD}}{R_4}$$

$$\text{Current in resistance } R_5 = I_5 = \frac{V_{CD}}{R_5}$$

$$\text{Current in resistance } R_6 = I_6 = \frac{V_{CD}}{R_6}$$

Thus equivalent resistance of the whole circuit, voltage drop across each branch and currents in the various resistors may be determined.

Example 1.30. A circuit consists of two parallel resistors having resistances of 20 Ω and 30 Ω respectively connected in series with a 15 Ω resistor. If the current through 30 Ω resistor is 1.2 A, find (i) currents in 20 Ω and 15 Ω resistors (ii) the voltage across the whole circuit (iii) voltage across 15 Ω resistor and 20 Ω resistor (iv) total power consumed in the circuit.

[V.T.U. Karnataka, Basic Electrical Engineering, First Semester 2014-15]

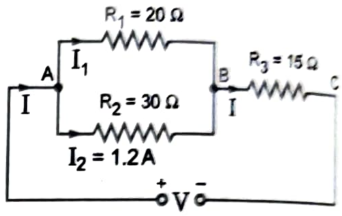


Fig. 1.17

Solution: Voltage drop across branch AB, $V_{AB} = I_2 R_2 = 1.2 \times 30 = 36$ V

$$(i) \text{ Current in } 20\ \Omega \text{ resistor, } I_1 = \frac{V_{AB}}{R_1} = \frac{36}{20} = 1.8 \text{ A} \quad \textbf{Ans.}$$

$$\text{Current in } 15\ \Omega \text{ resistor} = \text{Line current, } I = I_1 + I_2 = 1.8 + 1.2 = 3.0 \text{ A} \quad \textbf{Ans.}$$

$$(ii) \text{ Voltage across the whole circuit, } V = V_{AB} + V_{BC} = V_{AB} + IR_3 = 36 + 3 \times 15 = 81 \text{ V} \quad \textbf{Ans.}$$

$$(iii) \text{ Voltage across } 15\ \Omega \text{ resistor} = V_{BC} = 3 \times 15 = 45 \text{ V} \quad \textbf{Ans.}$$

$$\text{Voltage across } 20\ \Omega \text{ resistor} = V_{AB} = 36 \text{ V} \quad \textbf{Ans.}$$

$$\text{(iv) Total power consumed, } P = V \times I = 81 \times 3 = 243 \text{ W} \quad \textbf{Ans.}$$

Example 1.31. In the circuit shown 100 V dc voltage is applied across terminal A-B. Calculate power dissipated in each resistor and the reading of a voltmeter connected across the 5 Ω resistor.

[Uttarakhand Technical Univ. Basic Electrical Engineering, 2012]

Solution: Let the currents flowing through resistor of 5 Ω, 10 Ω and 20 Ω be I_1, I_2 and I_3 respectively as shown in Fig. 1.18 and reading of voltmeter connected across 5 Ω resistor be V_1 volts

Equivalent resistance of parallel combination of resistors of 10 Ω and 20 Ω

$$R_{A'B'} = \frac{1}{\frac{1}{10} + \frac{1}{20}} = \frac{20}{2+1} = \frac{20}{3}\ \Omega$$

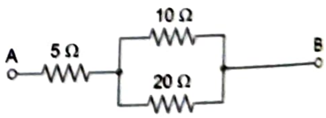


Fig. 1.18(a)

Total resistance of the circuit,

$$R = 5 + R_{A'B'} = 5 + \frac{20}{3} = \frac{35}{3}\ \Omega$$

Current drawn from supply main,

$$I = \frac{V}{R} = \frac{100}{35/3} = \frac{60}{7} \text{ A}$$

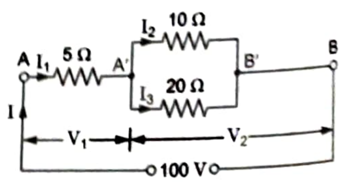


Fig. 1.18(b)

Current flowing through 5 Ω resistor,

$$I_1 = I = \frac{60}{7} \text{ A}$$

Reading of voltmeter connected across 5 Ω resistor = Voltage drop across 5 Ω resistor

$$= 5\,I_1 = 5 \times \frac{60}{7} = \frac{300}{7} \quad i.e., \text{ 42.857 V } \textbf{Ans.}$$

$$\text{Power dissipated in 5 }\Omega\text{ resistor} = I_1^2 \times 5 = \left(\frac{60}{7}\right)^2 \times 5 = 367.347 \text{ W } \textbf{Ans.}$$

$$\text{Voltage drop across parallel combination, } V_2 = V - V_1 = 100 - 42.857 = 57.143 \text{ V}$$

$$\text{Current flowing through 10 }\Omega\text{ resistor, } I_2 = \frac{V_2}{R_2} = \frac{57.143}{10} = 5.7143 \text{ A}$$

$$\text{Power dissipated through 10 }\Omega\text{ resistor} = I_2^2 R_2 = (5.7143)^2 \times 10 = 326.53 \text{ W } \textbf{Ans.}$$

$$\text{Current flowing through 20 }\Omega\text{ resistor} = I_3 = \frac{V_2}{R_3} = \frac{57.143}{20} = 2.857 \text{ A}$$

$$\text{Power dissipated through 20 }\Omega\text{ resistor} = I_3^2 R_3 = (2.857)^2 \times 20 = 163.266 \text{ W } \textbf{Ans.}$$

Example 1.32. In the given circuit shown in Fig. 1.19 find the resistance between the points B and C.

[G.B. Technical Univ. Electrical Engineering Even Semester 2009-10]

Solution: Resistance in branch AB, $R_{AB} = 4 \,||\, 2 = \dfrac{4}{1+2} = 1.333\,\Omega$

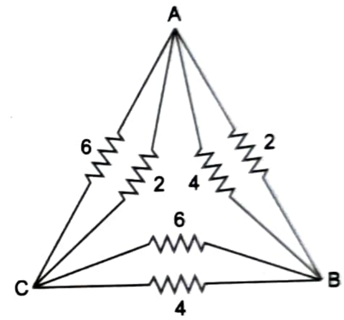


Fig. 1.19

$$\text{Resistance in branch, BC, } R_{BC} = 6\,||\,4 = \frac{6 \times 4}{4+6} = 2.4\,\Omega$$

$$\text{Resistance is branch CA, } R_{CA} = 6\,||\,2 = \frac{6 \times 2}{6+2} = 1.5\,\Omega$$

$$\text{Resistance between points B and C} = R_{BC} \,||\, (R_{AB} + R_{CA})$$

$$= 2.4 \,||\, (1.333 + 1.5)$$

$$= 2.4 \,||\, 2.833$$

$$= \frac{2.4 \times 2.833}{2.4 + 2.833} = 1.3\,\Omega \ \textbf{Ans.}$$

Example 1.33. Twelve identical wires of resistance 6 Ω each are arranged to form the edges of cube. A current of 40 milliamperes is led into the cube at one corner and out at opposite corner. Calculate the pd developed between these corners and the effective resistance of the circuit. *[M.D. Univ. Electrical Technology, December-2008]*

Solution: Let the current of 40 mA be led into the cube at corner A and out at the opposite corner E.

When the current comes at junction A, it divides itself into three paths, namely AB, AC and AH. Since the resistances are equal, current in wire AB =

Current in wire AC = Current in wire AH = $\dfrac{40}{3}$ mA.

When the current reaches next junction points B, C and H currents are divided again along two paths at each junction points *i.e.* BD and BG at B, CD and CF at C and HG and HF at H. So, each section will carry half of the current of $\dfrac{40}{3}$ mA *i.e.* $\dfrac{20}{3}$ mA.

At next junction points F, D and G currents coming from two sides add up and begin to flow in FE, DE and GE respectively. Each section carries currents of 40/3 mA. All these three conductors meet at junction point E and current from all paths come and total current of 40 mA gets out at junction point E.

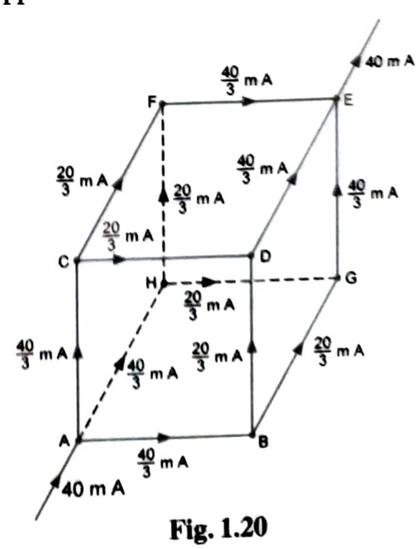


Fig. 1.20

One of the parallel paths is along AB, BD and DE. Taking this path and adding voltage drops in each conductor of the path, we shall get potential drop across A and E.

$$\text{PD across A and E} = \text{Voltage drop in AB} + \text{voltage drop in BD} + \text{voltage drop in DE}$$

$$= \frac{0.04}{3} \times 6 + \frac{0.02}{3} \times 6 + \frac{0.04}{3} \times 6 = 0.2 \text{ V } \textbf{Ans.}$$

$$\text{Current between A and E} = 40 \text{ mA} = 0.04 \text{ A}$$

$$\text{Equivalent resistance between A and E} = \frac{\text{PD across A and E}}{\text{Current between A and E}} = \frac{0.2}{0.04} = 5\ \Omega \textbf{ Ans.}$$

Example 1.34. Find the voltage drop between terminals AB, CB and AD in Fig. 1.21.

[Rajasthan Technical Univ. June-July 2011]

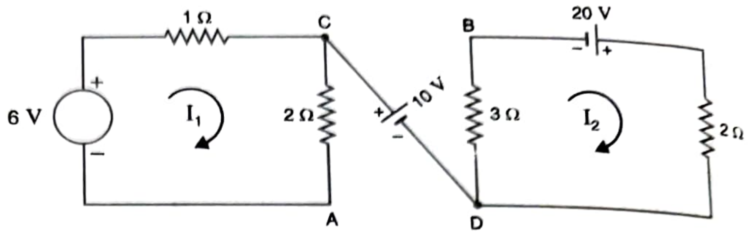


Fig. 1.21

Solution: $\text{Current } I_1 = \frac{6}{1+2} = 2 \text{ A}$

$$V_{CA} = 2 \times 2 = 4 \text{ V}$$

$$\text{Current } I_2 = \frac{20}{2+3} = 4 \text{ A}$$

$$V_{DB} = 3 \times 4 = 12 \text{ V}$$

The circuit between terminals A and B becomes as shown in Fig. 1.22 (*b*).

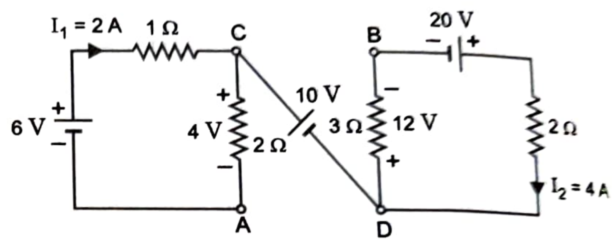


Fig. 1.22 (*a*)

$$V_{AB} = V_{AC} + V_{CD} + V_{DB} = -4 + 10 + 12 = 18 \text{ V}$$

i.e., terminal A is 18 V above terminal B. **Ans.**

$$V_{CB} = V_{CD} + V_{DB} = 10 + 12 = 22 \text{ V}$$

i.e., terminal C is 22 V above terminal B. **Ans.**

$$V_{AD} = V_{AC} + V_{CD} = -4 + 10 = 6 \text{ V}$$

i.e., terminal A is 6 V above terminal D **Ans.**

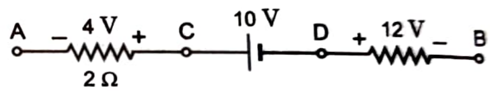


Fig. 1.22 (*b*)

1.22 NETWORK SIMPLIFICATION (OR REDUCTION)

Sometimes we come across so complicated circuits that they cannot be solved simply by applying Ohm's law. Hence for solution of such a circuit, first of all the circuit is reduced to simple series or simple parallel or series-parallel circuit and then solved by applying Ohm's law.

Network reduction or simplification is a process by which currents and voltages acting in a circuit composed of resistors having a series, parallel or series-parallel combinations can be determined.

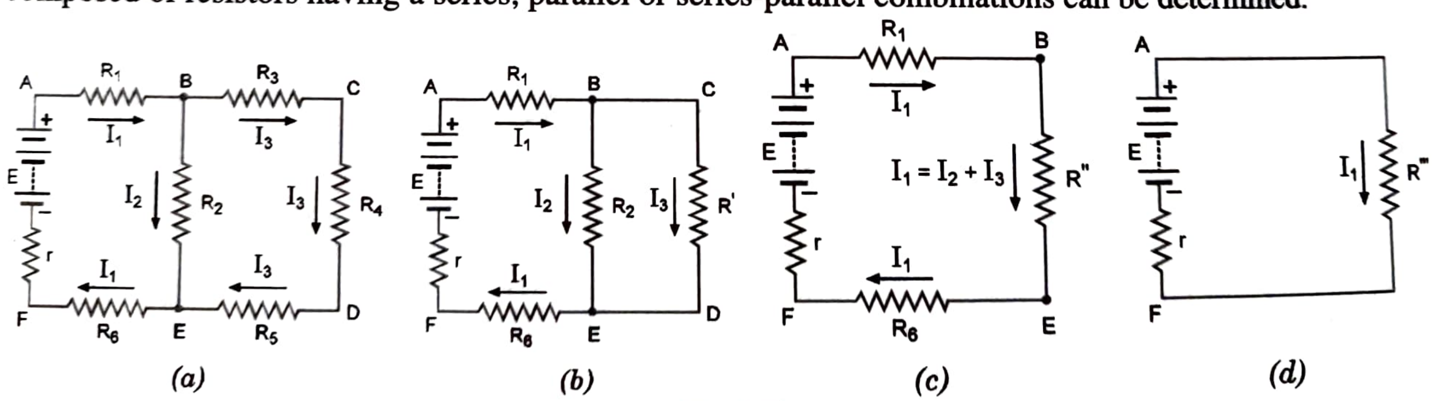


Fig. 1.23

It may be seen from Fig. 1.23 (*a*) that resistances R_3, R_4 and R_5 are connected in series and the same current I_3 flows through them because the circuit does not branch off at points C and D. By virtue of this fact, the three resistances R_3, R_4 and R_5 can be combined into a single equivalent resistance given by the equation

$$R' = R_3 + R_4 + R_5$$

Thus, a simpler equivalent circuit shown in Fig. 1.23 (*b*) is obtained. From circuit diagram shown in Fig. 1.23 (*b*) it can be seen that resistance R_2 is connected in parallel with the series combination of resistances R_3, R_4 and R_5 *i.e.* equivalent resistance R'. Resistances R_2 and R' can be combined into a single equivalent resistance given by the equation

MODULE 1

DC CIRCUITS

Chapter 2

DC Circuits

Inside this Chapter

2.1 CONCEPT OF NETWORK

An electric circuit (or network) is an interconnection of physical electrical devices. The purpose of electric networks is to distribute and convert electrical energy into some other forms. Accordingly, the basic circuit components are an energy source (or sources), an energy converter (or converters), and conductors connecting them.

An *energy source* (or source), such as a primary or secondary cell, a generator, and the like, is a device that converts chemical, mechanical, thermal or some other form of energy into electrical energy.

An *energy convertor,* also called the *load*, (such as lamp, heating appliance, or an electric motor) converts electrical energy into light, heat, mechanical work and so on.

Events in an electrical circuit may be defined in terms of emf (or voltage) and current.

When electrical energy is generated, transmitted and converted under conditions such that the currents and voltages involved remain constant with time, the electric circuit is identified as *direct current* (dc) *circuit.* If the currents and voltages do change with time, the circuit is defined as *alternating current* (ac) *circuit.*

A graphic representation of an electric circuit is called a *circuit diagram* (Fig. 2.1). Such a diagram consists of interconnected symbols called *circuit elements or circuit parameters.* Two elements are necessary to represent processes in a dc circuit. These are source of emf E_s and of internal (or source) resistance R_s and the load resistance (which includes the resistance of the conductors) R.

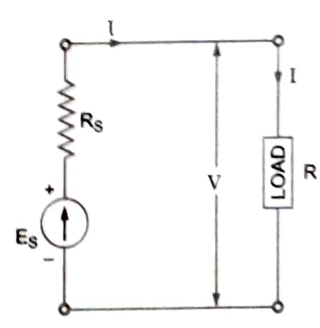


Fig. 2.1

In any electric circuit the energy convertor (or load) and the conductors connecting it to the source make up the *external circuit* in which current flows from the +ve side to the –ve side of the source whereas inside the source, current flows in the opposite direction, *i.e.* from the –ve side to the +ve side. The source emf is directed from the terminal at a lower potential to that at a higher one. In diagrams this is shown by arrows.

The *source emf* (or open-circuit voltage) is the voltage that appears across the source when no load is connected across it.

When a load is connected to the source terminals and the circuit is closed, an electric current starts flowing through the circuit. Now voltage across source terminals (called the *terminal voltage*) is not equal to source emf. It is due to voltage drop inside the source, *i.e.* across the source resistance.

$$\text{Voltage drop inside the source} = IR_s$$

The relationship between the current through a resistance and the voltage across the same resistance is called its *volt-ampere (or voltage-current) characteristic*. When represented graphically, voltages are laid off as abscissae and currents as ordinates.

There are two types of volt-ampere characteristics– straight line and nonlinear (curve), as shown in Figs. 2.2 (a) and 2.2 (b) respectively.

Resistive elements for which the volt-ampere characteristic is a straight line [Fig. 2.2 (a)] are called *linear*, and the electric circuits containing only linear resistances are called *linear circuits*.

Resistive elements for which the volt-ampere characteristic is other than a straight line are termed *non-linear*, and so the electric circuits containing them are called *nonlinear circuits*. Examples of nonlinear elements are tungsten lamps, vacuum tubes and transistors, etc.

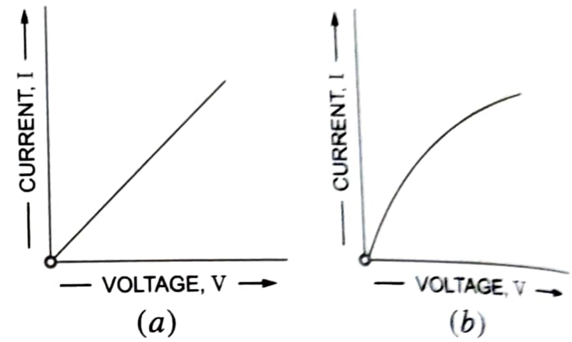


Fig. 2.2

An electric circuit, whose characteristics or properties are same in either direction (*e.g.* a distribution or transmission line), is called the *bilateral circuit*. The distribution or transmission line can be made to perform its function equally well in either direction.

An electric circuit, whose characteristics or properties change with the direction of its operation (*e.g.* a diode rectifier), is called the *unilateral circuit*. A diode rectifier cannot perform rectification in both directions.

A network is said to be *passive* if it contains no source of emf in it. The *equivalent resistance* between any two terminals of a passive network is the ratio of potential difference across the two terminals to the current flowing into (or out of) the network. When a network contains one or more sources of emf and/or current, it is said to be *active*.

A circuit consisting of a limited (finite) number of circuit elements is referred to as '*lumped parameter circuit*' while a circuit containing unlimited (infinite) number of circuit elements is called a *distributed parameter circuit*. Transmission line is an example of distributed parameter network.

In case, a branch is removed from an electric network, the remainder of the network is left with a pair of terminals. The part of the network, which is considered with respect to the removed branch or terminal pair or port is termed as *one-port network*. When two branches are removed so that the network is left with four terminals or two pairs of terminals, the remainder network is called the *two-port network*. Usually one port accepts a source and the other port is coupled to a load, so that there is an input port and an output port in any two-port system.

2.2 SINGLE- AND MULTI-MESH NETWORK

Electric network may provide a single closed path (known as a *mesh or loop)* or several closed paths for the flow of current. An elementary *single-mesh* network, in which all the elements carry the same current, is shown in Fig. 2.1. An elementary multi-mesh network is shown in Fig. 2.3. It has 6 nodes, seven branches, three loops and two meshes. A *junction* (*or node**) is a point in a network where two or more branches meet. A *branch* is any section of a network which joins two nodes directly,

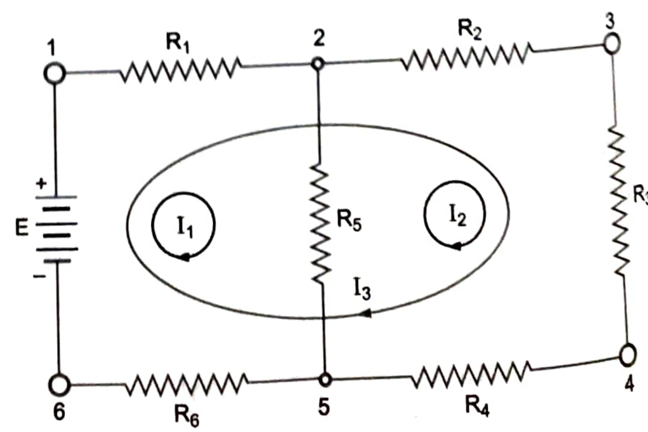


Fig. 2.3

* Nodes are considered to be resistanceless and infinitesimal

that is, without passing through a third node. A *loop* is a closed path in a network formed by a number of connected branches. Mesh is a loop that contains no other loop within it.

If in the circuit diagram, as in Fig. 2.4 (a), there is a bold dot at the intersection of two branches, these branches are electrically connected and have a common node. Otherwise they simply cross, as in Fig. 2.4. (b), and are not connected electrically.

Fig. 2.4

2.3 ACTIVE AND PASSIVE ELEMENTS

Network elements may be classified into two categories viz. active elements and passive elements.

The elements which supply energy to the network are known as *active elements*. The voltage sources like batteries, dc generators, ac generators and current sources like photoelectric cells, metadyne generators fall under the category of active elements. Most of the semiconductor devices like transistors are treated as current sources.

The components which dissipate or store energy are known as *passive components*. Resistors, inductors and capacitors fall under the category of passive elements. The resistor is the only component which dissipates electrical energy. The inductors and capacitors are the components which store energy, the inductor stores energy by virtue of a current passing through it whereas the capacitor stores energy by virtue of potential difference across it.

2.4 RESISTANCE

Resistance is a dissipative element, which converts electrical energy into heat, when the current flows through it in any direction. The process of energy conversion is irreversible.

The circuit element used to represent energy dissipation is most commonly described by requiring the voltage across the element be directly proportional to the current through it. Mathematically, the voltage is

$$v = \mathrm{R}i \text{ volts} \quad \text{...(2.1)}$$

where i is the current in amperes. The constant of proportionality R is the resistance of the element and is measured in ohms (abbreviated Ω). The voltage-current relation expressed by Eq. (2.1) is known as Ohm's law. A physical device whose principal electrical characteristic is resistance is known as a *resistor*.

Since an electric charge gives up energy when passing through a resistor, the voltage v in Eq. (2.1) is a voltage drop in the direction of current. Alternatively, v is a voltage rise in the direction opposite to the current. The conventional diagrammatic representation of a resistance, together with designations of the current direction and voltage polarity, is shown in Fig. 2.5. The plus and minus signs denote decrease of potential, and hence a voltage drop, from left to right (or plus to minus). The element has two terminals (also called nodes).

The power dissipated by resistance may be given by expression

$$p = vi = i^2\mathrm{R} = \frac{v^2}{\mathrm{R}} \text{ watts} \quad \text{...(2.2)}$$

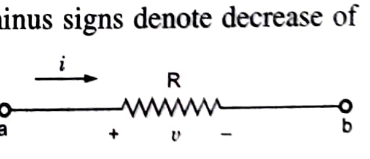


Fig. 2.5 *Schematic Representation of Resistance*

Eq. (2.1) gives the voltage across a resistor in terms of its current. A reciprocal relationship providing the current in terms of voltage is often of equal or greater value in a particular case. As a result, Ohm's law is often expressed as

$$i = \mathrm{G}v \text{ amperes} \quad \text{...(2.3)}$$

$$\text{where } \mathrm{G} = \frac{1}{\mathrm{R}} \quad \text{...(2.4)}$$

Reciprocal of resistance R *i.e.* G is called *conductance* and is measured in mhos or siemens (SI unit of conductance is siemens, but mho is more frequently used).

Power dissipated can then be expressed in the alternative form

$$p = vi = v\,(\mathrm{G}v) = v^2\mathrm{G} = i \times \frac{i}{\mathrm{G}} = \frac{i^2}{\mathrm{G}} \text{ watts} \quad \text{...(2.5)}$$

2.5 CAPACITANCE

Capacitance is a two-terminal element that has the capability of charge storage and, consequently, energy storage. The stored energy can be fully retrieved.

The current through the capacitor is proportional to the derivative of voltage across it and is given by expression

$$i = C\frac{dv}{dt} \quad ...(2.6)$$

where C has the unit of farads, the practical unit being a microfarad (μF) because a farad is physically a large unit. Integrating above Eq. (2.6) we have

$$v = \frac{1}{C}\int_0^t i\,dt + v_c(0) \quad ...(2.7)$$

where $v_c(0)$ = Capacitance voltage at $t = 0$

For an initially uncharged capacitor $v_c(0) = 0$, so that

$$v = \frac{1}{C}\int_0^t i\,dt = \frac{q}{C} \text{ or } C = \frac{q}{v} \text{ or } q = Cv \quad ...(2.8)$$

The proportionality constant C expresses the charge-storing property of the element and is called the *capacitance*. With q in coulombs and v in volts, the capacitance C is in farads (abbreviated F). A capacitor is a physical element which exhibits the property of capacitance.

The schematic representation of capacitance, in which current and voltage reference directions are indicated, is depicted in Fig. 2.6. In this figure and in Eqs. (2.6) and (2.8), a voltage drop exists in the direction of flow of current. Charge flow from a higher potential to a lower potential *i.e.* from plus to minus, signifies that energy can be removed from the circuit and stored. The capacitive effect may be thought of as opposing a change of voltage.

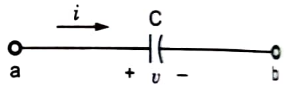


Fig. 2.6 *Schematic Representation of Capacitance*

The power associated with a capacitance is

$$p = vi = Cv\frac{dv}{dt} \text{ watts} \quad ...(2.9)$$

Energy stored in the capacitance may be had by integrating above Eq. (2.9) as

$$W = \int p\,dt = \int Cv\frac{dv}{dt}\,dt = \frac{1}{2}Cv^2 \text{ joules} \quad ...(2.10)$$

A capacitor offers low impedance to ac but very high impedance to dc. So capacitors are used to couple alternating voltage from one circuit to another circuit and at the same time to block dc voltage from reaching the next circuit. It is also employed as a *bypass capacitor* where it passes the ac through it without letting the dc to go through the circuit across which it is connected. A capacitor forms a *tuned circuit* in series or in parallel with an inductor.

2.6 INDUCTANCE

The circuit element used to represent the energy stored in a magnetic field is defined by the relation

$$v = L\frac{di}{dt} \quad ...(2.11)$$

The above expression describes a situation in which the voltage across the element is proportional to the time rate of change of current through it. The constant of proportionality L is the *self-inductance* or simply the *inductance* of the element, and is measured in *henrys* (abbreviated H).

The voltage v in Eq. (2.11) is a voltage drop in the direction of current and can be considered to oppose an increase in current. Fig. 2.7 depicts the schematic representation of an inductance and its associated reference direction for current and voltage polarity.

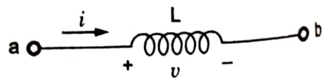


Fig. 2.7 *Schematic Representation of Inductance*

Integrating Eq. (2.11) we have

$$i = \frac{1}{L}\int_0^t v\,dt + i(0) \qquad \text{...(2.12)}$$

where $i(0)$ = Inductance current at $t = 0$.

According to Eq. (2.12) *current through an inductance* cannot change instantly (compared with capacitance voltage) as it would require infinite voltage.

Because the effect of inductance is to oppose the change in the magnitude of current, inductance is analogous to mass or inertia in a mechanical system and to the mass of liquid in hydraulics. Inductance prevents the current from changing instantly as it requires infinite voltage to cause an instantaneous change in current, just as the mass of an automobile prevents it from stopping or starting instantaneously.

The power associated with the inductive effect in a circuit is

$$p = vi = Li\frac{di}{dt} \text{ watts} \qquad \text{...(2.13)}$$

and the energy stored is

$$W = \int p\,dt = \int Li\frac{di}{dt}dt = \int Li\,di = \frac{1}{2}Li^2 \text{ joules} \qquad \text{...(2.14)}$$

unlike the resistive energy, which is transformed into heat, the inductive energy is stored in the same sense that kinetic energy is stored in a moving mass. Eq. (2.14) reveals that the magnitude of stored energy depends on the magnitude of current and not in the manner of attaining that magnitude. The stored inductive energy reappears in the circuit as the current is reduced to zero. For example, if a switch is opened in a current carrying inductive circuit, the current decays rapidly, but not instantaneously. In accordance with Eq. (2.11), a relatively high voltage appears across the separating contacts of the switch, and an arc may form. The arc makes it possible for the stored energy to be dissipated as heat in the arc and the circuit resistances.

In case of an inductor, current does not change instantaneously. It offers high impedance to ac but very low impedance to dc *i.e.* it blocks ac signal but passes dc signal.

A piece of wire, or a conductor of any type, has inductance *i.e.* a property of opposing the change of current through it. By coiling the wire the inductance is increased as the square of the number of turns. The inductance is represented by English capital letter L and measured in henrys.

2.7 SUMMARY OF RELATIONSHIPS FOR THE PARAMETERS

Some of the relationships discussed so far in this chapter are summarised in tabular form (Table 2.1). These equations are encountered so frequently in the study of electrical engineering that they should be memorized.

TABLE 2.1 Summary of Relationships For The Parameters

Parameter	*Basic Relationship*	*Voltage-Current Relationships*		*Energy*
Resistance, R	$v = Ri$	$v_R = Ri_R$	$i_R = Gv_R$	$W_R = \int_{-\infty}^{t} v_R i_R\,dt$
$G = \frac{1}{R}$				
L (or M)	$\psi = Li$	$v_L = L\frac{di_L}{dt}$	$i_L = \frac{1}{L}\int_{-\infty}^{t} v_L dt$	$W_L = \frac{1}{2}LI^2$
C	$q = Cv$	$v_C = \frac{1}{C}\int_{-\infty}^{t} i_C\,dt$	$i_C = C\frac{dv_C}{dt}$	$W_C = \frac{1}{2}CV^2$

2.8 VOLTAGE AND CURRENT SOURCES

Most of the sources encountered in everyday life (such as batteries, dynamos, alternators etc.) are voltage sources but some current sources do exist. Some examples are; photoelectric cells as used in light meters, metadyne generators as used in military gun controls. Other devices may be regarded as current sources, such as the collector circuits of transistors and the anode circuits of pentode thermionic tubes.

2.8.1 Independent and Dependent Sources. The source (voltage or current) may be independent or dependent. A source is said to be independent when it does not depend on any other quantity in the circuit. Fig. 2.8(*a*) shows an independent dc voltage source whereas Fig. 2.8(*b*) depicts a time varying voltage source. The positive sign indicates that terminal A is positive with respect to terminal B *i.e.* the potential of terminal A is V volts higher than that of terminal B.

Similarly an ideal constant current source is shown in Fig. 2.8(*c*) whereas time varying current source is shown in Fig. 2.8(*d*). The arrow indicates the direction of current flow at any moment under consideration.

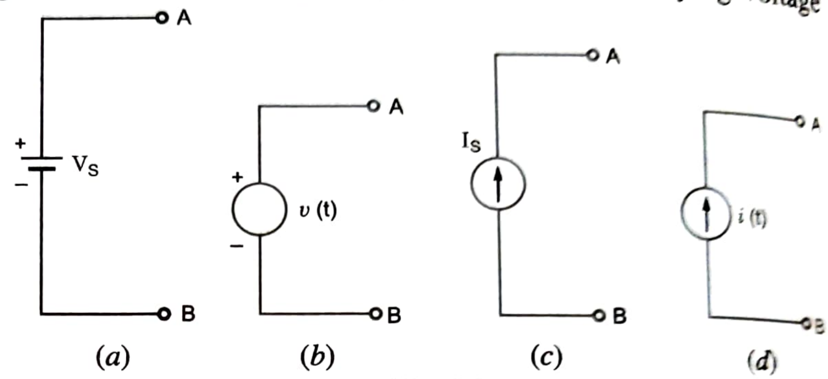
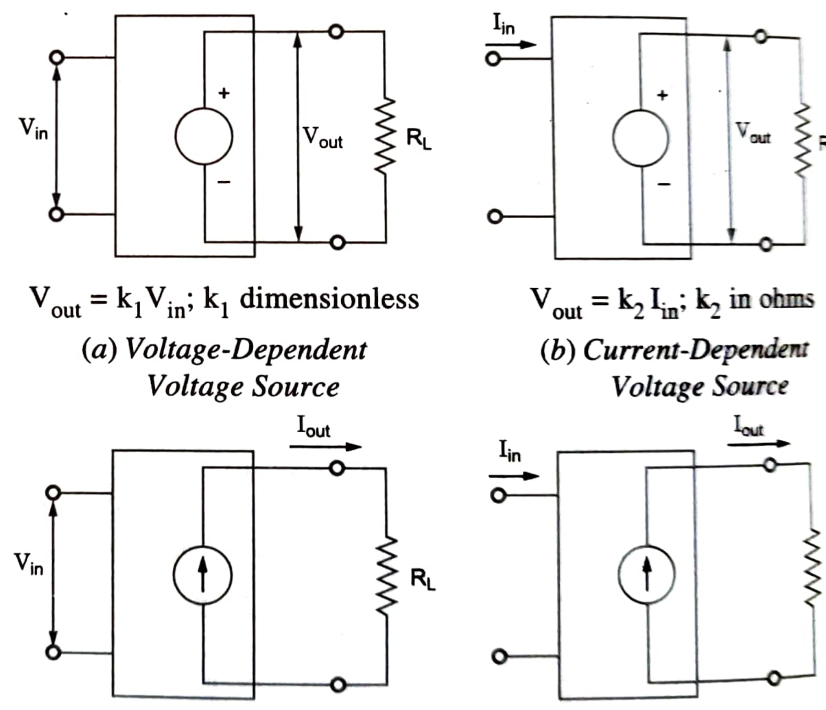


Fig. 2.8

A *dependent source* is one which depends on some other quantity in the circuit which may be either a voltage or a current. In electronic circuits, we very often find that the current through an element, (say collector current through a bipolar junction transistor) is dependent on a current through some other element or in a MOSFET it is dependent on the voltage across some other element. Such a source is called a *dependent source*. In a dependent source the output voltage (or current) depends on another voltage (or current). The relationship may be linear or nonlinear. There are four possible dependent sources as are represented in Fig. 2.9.

Such sources can also be either constant sources or time varying sources.

Independent sources actually exist as physical entities such as an accumulator, a dc generator and an alternator. But dependent sources are parts of *models* that are used to represent electrical properties of electronic devices such as operational amplifiers and transistors etc.



$V_{out} = k_1 V_{in}$; k_1 dimensionless

(*a*) *Voltage-Dependent Voltage Source*

$V_{out} = k_2 I_{in}$; k_2 in ohms

(*b*) *Current-Dependent Voltage Source*

$I_{out} = k_3 V_{in}$; k_3 in mhos

(*c*) *Voltage-Dependent Current Source*

$I_{out} = k_4 I_{in}$: k_4 dimensionless

(*d*) *Current Dependent Current Source*

Fig. 2.9 *Dependent Sources*

Another major difference is that four terminals are required to define a controlled source; whereas only two are required for an independent source. Of the four dependent source terminals, one pair provides the control and the second pair exhibits the properties of the source.

2.8.2. Ideal Voltage Source. A constant voltage source is an ideal source element capable of supplying any current at a given voltage. If the internal resistance of a voltage source is zero, the terminal voltage

(voltage across the load) is equal to the voltage across the source (the source emf), and is independent of the amount of load current, or in other words the voltage of an ideal voltage source is independent of load current supplied by it. For example, if the terminals are connected together the source will supply an infinite current. The symbolic representations of dc and ac ideal voltage sources are given in Figs. 2.10 and 2.11 respectively.

There are two noteworthy points regarding ideal voltage sources. First, ideal voltage source cannot be short circuited (because this will be contrary to the definition of the ideal voltage source itself). Secondly (and for the same reason) two ideal voltage sources of unequal output voltages cannot be placed in parallel.

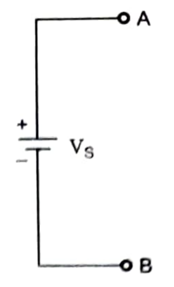


Fig. 2.10

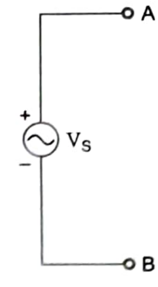
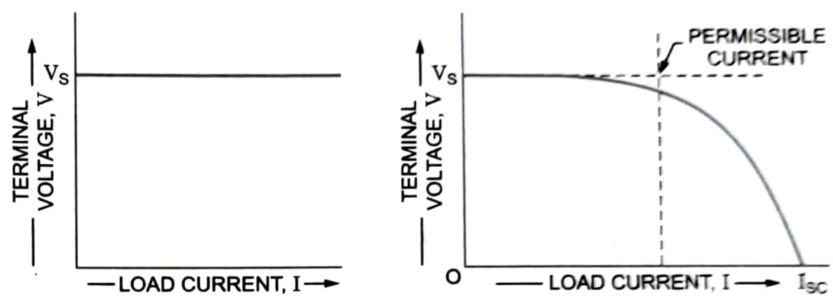


Fig. 2.11

An ideal voltage source is not practically possible. There is no voltage source which can maintain its terminal voltage constant even when its terminals are short circuited.

A lead-acid battery or a dry cell are examples of a constant voltage source when the current drawn is below a certain limit. However, a practical voltage source always shows a drop in its terminal voltage which increases with load current. A dc generator or a rectifier operating on mains supply is a voltage source (the output voltage is contaminated with ripples) and exhibit a voltage drop which is load dependent.



(*a*) *For Ideal Voltage Source* (*b*) *For a Practical Voltage Source*

Fig. 2.12 *V-I Characteristics of Voltage Sources*

The volt-ampere (V-I) characteristics of an ideal voltage source are depicted in Fig. 2.12(*a*) in comparison to that of a practical voltage source shown in Fig. 2.12(*b*). Dotted line is that of an ideal voltage source for differentiation. Within a permissible range of current a practical dc voltage source maintains the terminal (output) voltage within a narrow range of its nominal voltage. Beyond this value of current, called the *rated current*; the voltage drops rapidly till the short-circuit current I_{SC}, where the terminal voltage drops to zero. In contrast, an ideal dc voltage source would have maintained the voltage even up to infinitely large current.

2.8.3. Ideal Current Source. Like a constant voltage source, there may be a constant current source—a source that supplies a constant current to a load even if its impedance varies. Ideally, the current supplied by such a source should remain constant irrespective of the load impedance. A symbolic representation of such an ideal constant current source is shown in Fig. 2.8(*c*). The arrow inside the circle indicates the direction of flow of current in the circuit, when a load is connected to the source.

There are two noteworthy points regarding ideal current sources. First, an ideal current source cannot be open circuited (because this will be contrary to the definition of the constant current source itself). Again, for the same reason, two ideal current sources of different output currents cannot be placed in series.

An ideal current source, like an ideal voltage source, is not practically possible. There is no current source which can maintain current supplied by it constant even when its terminals are open circuited. An ideal current source does not exist in practice. A practical current source can be represented as shown in Fig. 2.13 (*a*).

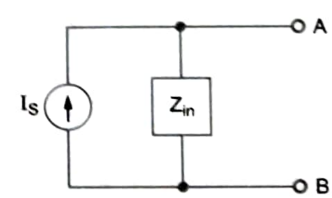


Fig. 2.13 (*a*) *Practical Current Source*

A solar cell is an example of *current source*. It provides constant current to a resistance within a specified range of output voltage. The value of the current delivered by a current source depends on the flux incident on the cell. An ideal current source provides a constant current to a resistor of resistance R ohms ($0 < R < \infty$). V-I characteristics of a practical current source is compared with that of an ideal current source (dotted vertical lines) in Fig. 2.13(*b*). The limit V_R up to which a current source maintains the current is drawn by horizontal dotted line. Beyond this value of output voltage (hence load resistance) the current falls. A current source can also be had by connecting a high resistance R_{se} in series with a high voltage dc source. It will behave as a current source in a range R_L is less than 5 to 10% of series resistance R_{se}. In that case, the current is primarily decided by R_{se}. The current through load resistance R_L is nearly constant unless R_L is significant in comparison to R_{se}.

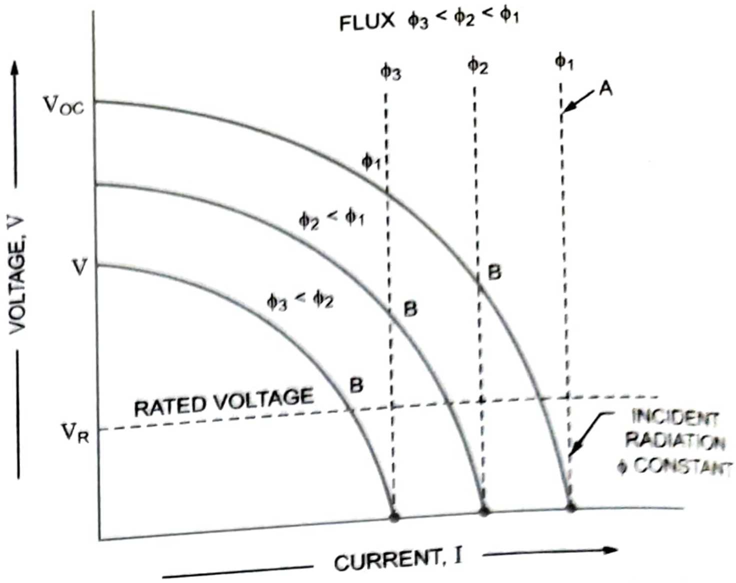
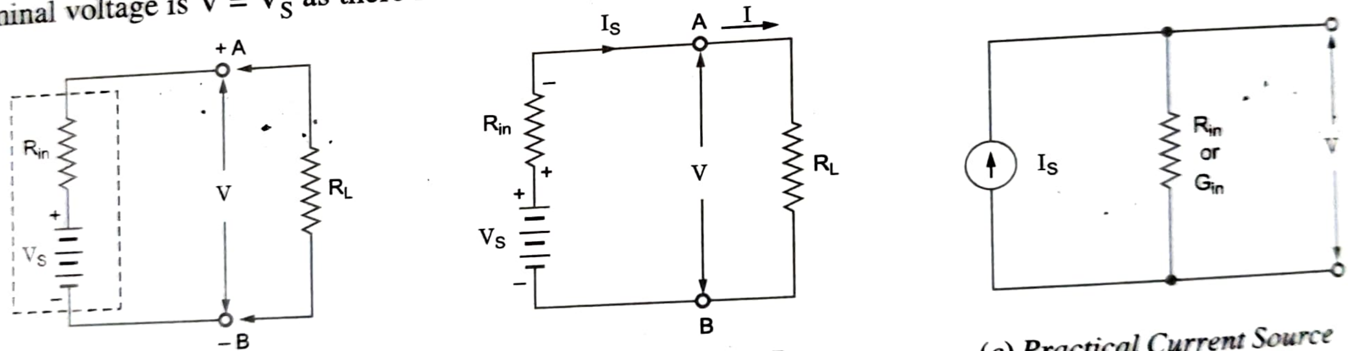


Fig. 2.13 *(b) Comparison of V-I Characteristics of an Ideal Current Source (A) With That of a Practical Current Source (B) For Different Values of Radiation Intensity*

2.8.4. Practical Sources. In most applications, ideal sources are approximation. The internal resistance R_{in} of a voltage source, which is responsible for a drop in terminal voltage of a source on load is quite small in comparison to the load resistance of a network connected across the voltage source. Hence voltage drop is considered negligible. Internal resistance needs to be taken into account when it is significant in comparison to load resistance R_L.

A practical dc voltage source is represented in Fig. 2.14(*a*). V_S is the internal voltage of the voltage source and R_{in} is the internal resistance of the voltage source. When we measure the voltage across the source terminals without any load (no resistance connected to the output terminals of the source) the terminal voltage is $V = V_S$ as there is no current through R_{in} to cause a voltage drop.



(a) A Practical Voltage Source *(b) Voltage Drop in R_{in}* *(c) Practical Current Source*

Fig. 2.14 *Practical Sources*

Assuming V_S to be constant, the terminal voltage falls on loading by an external or load resistance R_L. The resulting current,

$$I = \frac{V_S}{R_{in} + R_L} \quad ...(2.15)$$

and the terminal voltage is then $$V = IR_L = \frac{V_S}{R_{in} + R_L} \times R_L \quad ...(2.16)$$

As the load resistance R_L reduces (load on the source increases), terminal voltage V falls linearly, if R_{in} and V_S are assumed constant.

When the terminals of a practical voltage source are short circuited by a thick wire of zero resistance resulting in a short circuit, source current I_S is given as

$$I_S = I_{SC} = \frac{V_S}{R_{in}} \quad ...(2.17)$$ In Fig. 2.14 (*b*), R_L being zero.

$$\text{Hence, } R_{in} = \frac{V_S}{I_{SC}} = \frac{\text{Open-circuit voltage of the source, } V_{OC}}{\text{Short-circuit current, } I_{SC}} \quad ...(2.18)$$

Similarly an ideal current source must produce infinite voltage on open circuit. A practical current source will have a finite output voltage. A practical current source is represented as shown in Fig. 2.14(*c*). In this case short-circuit current,

$$I_{SC} = I_S \quad \text{...(2.19)}$$

and the open-circuit voltage, $V_{OC} = I_S R_{in}$ ...(2.20)

$$\text{Hence, } R_{in} = \frac{V_{OC}}{I_{SC}} = \frac{\text{Open-circuit voltage}}{\text{Short-circuit current}} \quad \text{...(2.21)}$$

$$\text{or } G_{in} = \frac{1}{R_{in}} = \frac{I_{SC}}{V_{OC}} = \frac{\text{Short-circuit current}}{\text{Open-circuit voltage}} \quad \text{...(2.22)}$$

2.9 SOURCE EQUIVALENCE OR TRANSFORMATION

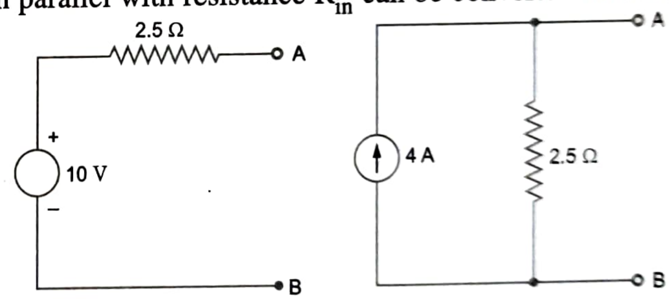
Practically, a voltage source is not different from a current source. In fact, a source can either operate as a current source or as a voltage source. It merely depends upon its operating conditions. If load impedance is very large in comparison to internal impedance of the source, it will be advantageous to treat the source as a voltage source. On the other hand, if the load impedance is very small in comparison to the internal impedance of the source, it is better to represent the source as a current source. From the circuit point of view it does not matter at all whether the source is treated as a voltage source or a current source. In fact, it is possible to convert a voltage source into a current source and vice versa.

Consider a voltage source of voltage V_S and internal resistance R_{in} shown in Fig. 2.14(*a*) for conversion into an equivalent current source. The current supplied by this voltage source, when a short circuit is put across terminals A and B, will be equal to V_S/R_{in}. A current source supplying this current $I_S = V_S/R_{in}$ and having the same resistance across it will represent the equivalent current source [Fig. 2.14(*c*)].

Similarly a current source of output current I_S in parallel with resistance R_{in} can be converted into an equivalent voltage source of voltage $V_S = I_S R_{in}$ and a resistance R_{in} in series with it [Fig. 2.14(*a*)].

It should be noted that a voltage source-series resistance combination is equivalent to a current source–parallel resistance combination if, and only if their respective open-circuit voltages are equal, and their respective short-circuit currents are equal.

For example, a voltage source branch consisting of a 10 V source in series with a resistance of 2.5 Ω may be replaced by a current source branch consisting of a 4 A source in parallel with a 2.5 Ω resistance and vice versa, as shown in Figs. 2.15(*a*) and 2.15(*b*) respectively.

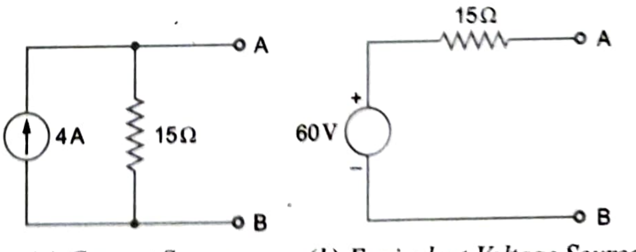


(*a*) (*b*)

Fig. 2.15

Example 2.1. Convert 4A source with its parallel resistance of 15 Ω into its equivalent voltage source.

Solution: 4 A current source with its parallel resistance of 15 Ω will be equivalent of a voltage source of 4 × 15 *i.e.*, 60 V in series with a resistance of 15 Ω as shown in Fig. 2.16(*b*).



(*a*) *Current Source* (*b*) *Equivalent Voltage Source*

Fig. 2.16

Example 2.2. State voltage to current transformation theorem.

It is required to replace network N in figure (Fig. 2.17) by a suitable equivalent network. Which of the networks of the Fig. 2.18 could be valid equivalent network(s)?

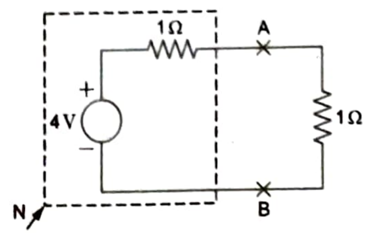


Fig. 2.17

Solution: For any voltage source, if the ideal voltage is V volts and internal resistance is R ohms, the voltage source can be replaced by current source I with the internal resistance in parallel to the current source and the magnitude of I is given by

$$I = \frac{V}{R}$$

A voltage source of 4 V in series with a resistance of 1 Ω will be equivalent to a current source of $\frac{4}{1}$ *i.e.* 4 A in parallel with a resistance of 1 Ω, as shown in Fig. 2.18(*ii*).

Hence network shown in Fig. 2.18(*ii*) is the equivalent valid network.

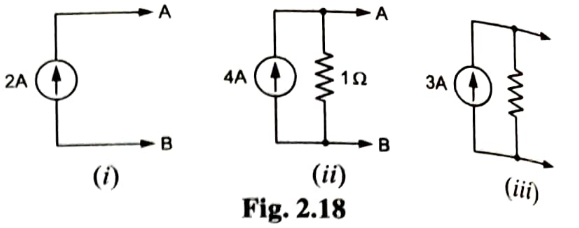


Fig. 2.18

Example 2.3. Use source conversion technique to find the voltage V_0 in the circuit as shown in Fig. 2.19.
[*Rajasthan Univ. 2004*]

Solution: Converting current source of 5 A in parallel with 7 Ω resistor and current source 5 A in parallel with 10 Ω resistor into equivalent voltage sources, the given circuit becomes as depicted in Fig. 2.20.

When current and voltage sources are connected in parallel, current source does not mean anything, so

$$\text{Output voltage, } V_0 = 15 + 50 + 35$$
$$= 100 \text{ V} \quad \textbf{Ans.}$$

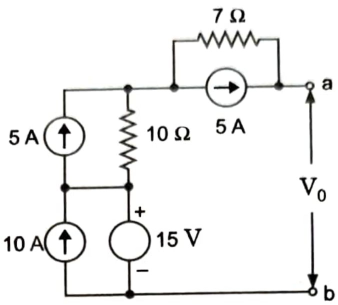


Fig. 2.19

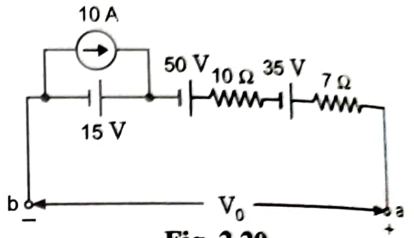


Fig. 2.20

Example 2.4. Use the source transformation to find V_1 of Fig. 2.21(*a*).
[*G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009*]

Solution: Converting 4 A current source in parallel with a resistance of 10 Ω into equivalent voltage source of 40 V in series with a resistance of 10 Ω and 15 A current source in parallel with a parallel combination of resistances of 4 Ω and (3 + 1) Ω into equivalent voltage source of 15 × [4 || (3 + 1)] *i.e.* 15 × 2 or 30 V in series with a resistance of 2 Ω we have the circuit, that is shown in Fig. 2.21(*b*).

From equivalent circuit shown in Fig. 2.21(*b*) we have current flowing through the circuit,

$$I = \frac{30 + 40}{2 + 10 + 1} = \frac{70}{13} \text{ A}$$

$$V_1 = \text{Voltage drop across resistance of } 1\ \Omega$$
$$= \frac{70}{13} \times 1 = \frac{70}{13} \text{ or } 5.38 \text{ V } \textbf{Ans.}$$

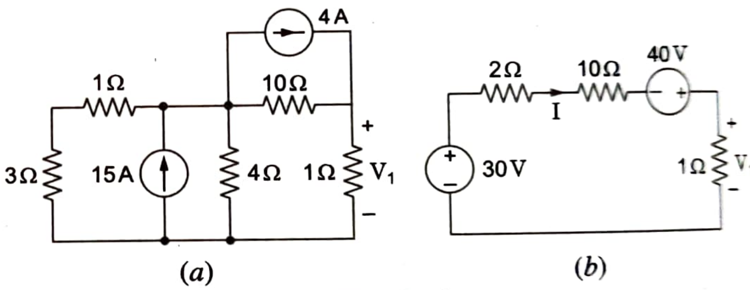


Fig. 2.21

2.10 KIRCHHOFF'S LAWS

The basic laws, that electric circuits follow rationally from the nature of the electrical quantities, hav already been defined. They lead directly to methods for the systematic analysis of electric circuits. Thes laws are known as *Kirchhoff's laws*, and they describe the relationships among circuit voltages and circu currents that must be satisfied. These laws are very helpful in determining the equivalent resistance of complex network and the current flowing in the various branches of the network.

Gustov Robert Kirchhoff derived two basic laws governing networks, one commonly known as *first la* or *current law* (KCL) or *point law*, whereas the second is called *second law* or *voltage law* (KVL) or *mesh la*

1. Kirchhoff's First Law or Current Law (KCL) or Point Law. According to this law in any network of wires carrying currents, the algebraic sum of all currents meeting at a point (or junction) is zero or the sum of incoming currents towards any point is equal to the sum of outgoing currents away from that point. If I_1, I_2, I_3, I_4, I_5, and I_6 are the currents meeting at junction O, flowing in the directions of arrowheads marked on them (Fig. 2.22), taking incoming currents as positive and outgoing currents as negative, according to Kirchhoff's first law (KCL)

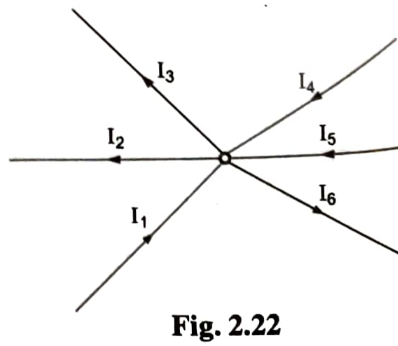


Fig. 2.22

$$I_1 - I_2 - I_3 + I_4 + I_5 - I_6 = 0$$
$$\text{or } I_1 + I_4 + I_5 = I_2 + I_3 + I_6$$

2. Kirchhoff's Second Law or Voltage Law (KVL) or Mesh Law. According to this law in any closed circuit or mesh the algebraic sum of emfs acting in that circuit or mesh is equal to the algebraic sum of the products of the currents and resistances of each part of the circuit.

If the circuit shown in Fig. 2.23 is considered, then according to Kirchhoff's second law (or KVL)

In mesh AFCBA,

$$E_1 = R_1 (I_1 + I_2) + R_2 (I_1 + I_2) + R_5 I_1$$
$$\text{or } E_1 = (R_1 + R_2)(I_1 + I_2) + R_5 I_1$$

In mesh FEDCF,

$$-E_2 = -R_3 I_2 - R_4 I_2 - R_2 (I_1 + I_2) - R_1 (I_1 + I_2)$$
$$\text{or } E_2 = R_3 I_2 + R_4 I_2 + (R_1 + R_2)(I_1 + I_2)$$

and in mesh AFEDCBA,

$$E_1 - E_2 = -R_3 I_2 - R_4 I_2 + R_5 I_1$$
$$\text{or } E_1 - E_2 = R_5 I_1 - (R_3 + R_4) I_2$$

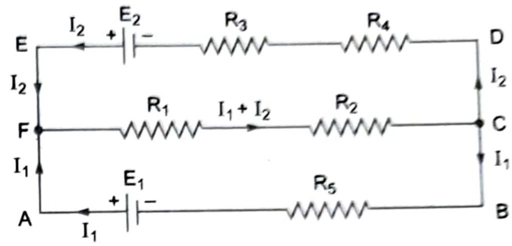


Fig. 2.23

2.10.1. Application of Kirchhoff's Laws To Circuits. First of all, the current distribution in various branches of the circuit is made with directions of their flow complying with first law of Kirchhoff. Then Kirchhoff's second law is applied to each mesh (one by one) separately and algebraic equations are obtained by equating the algebraic sum of emfs acting in a mesh equal to the algebraic sum of respective drops in the same mesh. By solving the equations so obtained unknown quantities can be determined. While applying Kirchhoff's second law, the question of algebraic signs may be troublesome and is a frequent source of error. If, however, the following rules are kept in mind, no difficulty should occur.

The resistive drops in a mesh due to current flowing in clockwise direction must be taken positive drops.

The resistive drops in a mesh due to current flowing in counter-clockwise direction must be taken as negative drops.

Similarly the battery emf causing current to flow in clockwise direction in a mesh must be taken as positive emf and the battery emf causing current to flow in counter-clockwise direction in a mesh must be taken as negative emf.

For example, for the circuit shown in Fig. 2.23 let the current distribution be made as shown, which satisfy Kirchhoff's first law fully.

Taking first, mesh AFCBA for the application of Kirchhoff's second law, we see that there is only one emf acting in the mesh (E_1) and since it tries to send current in clockwise direction so E_1 be taken as positive, similarly all the resistive drops *i.e.* $R_1 (I_1 + I_2)$, $R_2 (I_1 + I_2)$ and $R_5 I_1$ are clockwise, so these must be taken as positive.

∴ According to Kirchhoff's second law in mesh AFCBA

$$E_1 = R_1 (I_1 + I_2) + R_2 (I_1 + I_2) + R_5 I_1$$

In mesh FEDCF, there is only one emf acting in the mesh (E_2) and since it tries to send current in counter-clockwise direction through the mesh under consideration, it may be taken as negative. Since all of the resistive drops $R_3 I_2$, $R_4 I_2$, $R_2 (I_1 + I_2)$ and $R_1 (I_1 + I_2)$ are counter-clockwise, these may be taken as negative.

Hence according to Kirchhoff's second law in mesh FEDCF we get,

$$-E_2 = -R_3 I_2 - R_4 I_2 - R_2 (I_1 + I_2) - R_1 (I_1 + I_2)$$
$$\text{or } E_2 = (R_3 + R_4) I_2 + (R_1 + R_2)(I_1 + I_2)$$

In mesh AFEDCBA, emf E_1 tries to cause current in clockwise direction so be taken as positive and emf E_2 tries to cause current in counter-clockwise direction so be taken as negative. Similarly resistive drop $R_5 I_1$ being clockwise be taken as positive and resistive drops $R_3 I_2$ and $R_4 I_2$ being counter-clockwise be taken as negative.

$$\text{Hence } E_1 - E_2 = -R_3 I_2 - R_4 I_2 + R_5 I_1 = R_5 I_1 - (R_3 + R_4) I_2$$

Example 2.5. With reference to Fig. 2.24, given $i_1 = -\frac{1}{2}e^{-2t}$, $V_3 = 2e^{-2t}$, $V_4 = 2e^{-2t}$. Find V_2.

[*G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007*]

Solution: Applying Kirchhoff's current law at junction O, we have

$$i_1 - i_2 + i_3 - 6i_1 + i_4 = 0$$

$$\text{or } i_2 = -5i_1 + i_3 + i_4 \quad ...(i)$$

$$\text{Now current } i_1 = -\frac{1}{2}e^{-2t}$$

$$\text{Current } i_3 = C\frac{dV_3}{dt} = \frac{1}{2}\frac{d}{dt}(2e^{-2t}) = -2e^{-2t}$$

$$i_4 = \frac{V_4}{R} = \frac{2e^{-2t}}{10} = 0.2e^{-2t}$$

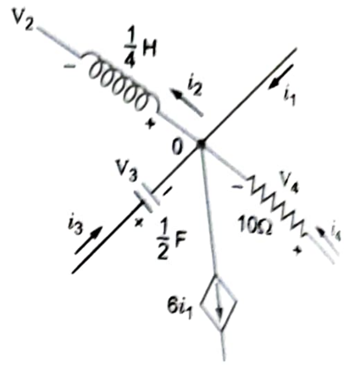


Fig. 2.24

Substituting the values of i_1, i_3 and i_4 in Eq. (*i*), we have

$$i_2 = -5\times\left(-\frac{1}{2}e^{-2t}\right) + (-2e^{-2t}) + 0.2e^{-2t} = 0.7e^{-2t}$$

The voltage V_2 developed across the coil is

$$V_2 = L\frac{di_2}{dt} = \frac{1}{4}\frac{d}{dt}(0.7e^{-2t}) = \frac{1}{4}\times 0.7\times(-2)e^{-2t} = -0.35e^{-2t} \quad \textbf{Ans.}$$

Example 2.6. In the network given in Fig. 2.25, find i_4 if $V_1 = 6$ V, $V_2 = 2\sin 4t$ and $i_3 = \frac{1}{2}e^{-2t}$.

Solution: Applying Kirchhoff's voltage law (KVL) to the closed mesh ADCBA, we have

$$-V_4 + V_3 + V_2 - V_1 = 0 \quad ...(i)$$

$$\text{Now } V_3 = L\frac{di_3}{dt} = 4\frac{d}{dt}\left(\frac{1}{2}e^{-2t}\right)$$

$$= 4\times\frac{1}{2}\times(-2)\,e^{-2t} = -4e^{-2t}$$

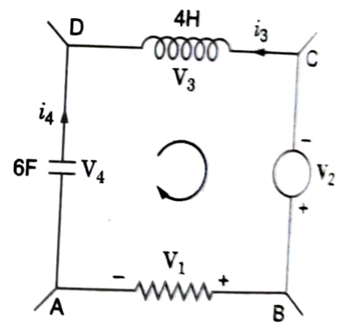


Fig. 2.25

Substituting values of V_1, V_2 and V_3 in Eq. (*i*), we have

$$-V_4 + (-4e^{-2t}) + 2\sin 4t - 6 = 0$$

$$\text{or } V_4 = 2\sin 4t - 4e^{-2t} - 6$$

$$\text{Now current } i_4 = C\frac{dV_4}{dt} = 6\frac{d}{dt}[2\sin 4t - 4e^{-2t} - 6] = 6[2\times 4\cos 4t - 4\times(-2)e^{-2t}]$$

$$= 48\cos 4t + 48e^{-2t} \quad \textbf{Ans.}$$

Example 2.7. Obtain I_R in terms of I_s in Fig. 2.26. [*G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007*]

Solution: Applying Kirchhoff's current law (KCL) to node A, we have

$$I_s = i_r + I_R \quad ...(i)$$

Applying KVL to closed mesh ABCD, we have

$$I_R R - i_r r + \alpha v = 0$$

$$\text{or } I_R R - i_r r + \alpha i_r r = 0 \qquad \because v = i_r r$$

$$\text{or } I_R R = r i_r(1-\alpha) = r(I_s - I_R)(1-\alpha) \qquad \because \text{From Eq. } (i)\ I_r = I_s - I_R$$

$$\text{or } I_R = \frac{r(1-\alpha)}{R + r(1-\alpha)}\cdot I_s \quad \textbf{Ans.}$$

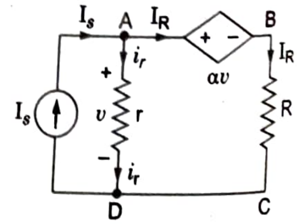


Fig. 2.26

Example 2.8. Determine the current through the 5Ω resistor in Fig. 2.27.

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007]

Solution: Applying KCL at junction A respectively, we have

$$I = 0.5i + i = 1.5i \quad ...(i)$$

Applying KVL to outer closed loop, we have

$$2I + 1 \times i + 5i = 6$$

$$\text{or } 2 \times 1.5i + 6i = 6 \quad \because \text{ From Eq. } (i)\ I = 1.5i$$

$$\text{or } i = \frac{6}{9} = \frac{2}{3} \text{ A} \quad \textbf{Ans.}$$

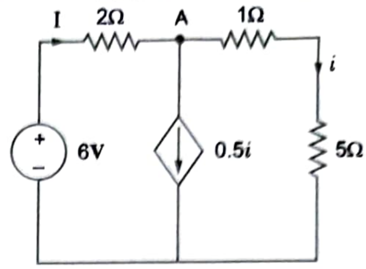


Fig. 2.27

Example 2.9. Find current in 2 Ω resistor. *[U.P. Technical Univ. Basic Electrical Engineering Odd Semester 2013-14]*

Solution: Let the current distribution in the network be as shown in Fig. 2.29.

Applying KVL to meshes ABEFA and BCDEB we have respectively

$$4 I_1 + 2 (I_1 + I_2) = 24 \quad ...(i)$$

$$\text{and } I_2 \times 1 + 2 (I_1 + I_2) = 6 \quad ...(ii)$$

$$\text{or } 3 I_1 + I_2 = 12 \quad ...(iii)$$

$$\text{and } 2 I_1 + 3 I_2 = 6 \quad ...(iv)$$

Solving Eqs (*iii*) and (*iv*) we have

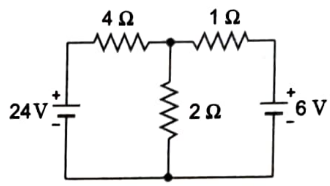


Fig. 2.28

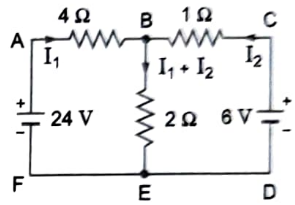


Fig. 2.29

$$I_1 = \frac{30}{7} \text{ A} \quad \text{and} \quad I_2 = \frac{-6}{7} \text{ A}$$

$$\text{Current in 2 Ω resistor} = I_1 + I_2 = \frac{30}{7} + \left(\frac{-6}{7}\right) = \frac{24}{7} \text{ A} \quad \textbf{Ans.}$$

Example 2.10. Determine the current in the 5 Ω resistor in the network shown in Fig. 2.30.

[Anna Univ. Circuit Theory May/June-2014]

Solution: Let the current distribution, complying KCL, in the network be as shown in Fig. 2.31.

Applying KVL to the left hand loop ABGFA we have

$$10 I_1 + 5 (I_1 + 2) = 50$$

$$\text{or } 3 I_1 = 8$$

$$\text{or } I_1 = \frac{8}{3} \text{ A}$$

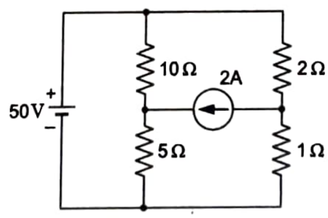


Fig. 2.30

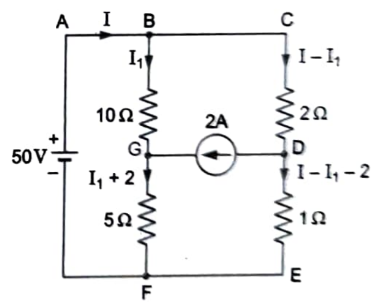


Fig. 2.31

$$\text{Current through 5 Ω resistor} = I_1 + 2 = \frac{8}{3} + 2$$

$$= \frac{14}{3} \text{ or } 4.667 \text{ A} \quad \textbf{Ans.}$$

Example 2.11. Find the value of R and current through each branch if current in branch AO is zero (Fig. 2.32).

[Pb. Technical Univ. Basic Electrical and Electronics Engineering Dec. 2007]

Solution: Let the current through branches BA and BO of the circuit be I_1 and I_2 amperes respectively. Applying KCL to junctions B, A and O, we have

Current supplied by battery or current in branch CB = $I_1 + I_2$

Current through branch AC = I_1 $\quad \because$ Current through branch AO is zero

Current through branch OC = I_2

Now applying Kirchhoff 's voltage law to meshes ABOA, CAOC and BCOB respectively we have

$$-I_1 + 4I_2 = 0$$
$$\text{or } I_1 = 4I_2 \qquad ...(i)$$
$$1.5I_1 - RI_2 = 0$$
$$\text{or } R = \frac{1.5I_1}{I_2}$$
$$= 1.5 \times 4 = 6\ \Omega \textbf{ Ans.}$$
$$\text{and } 4I_2 + RI_2 + 2(I_1 + I_2) = 10$$
$$\text{or } 4I_2 + 6I_2 + 2(4I_2 + I_2) = 10$$
$$20I_2 = 10$$
$$\text{or } I_2 = 0.5 \text{ A}$$

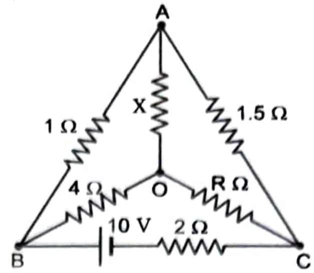


Fig. 2.32

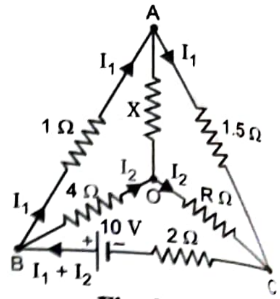


Fig. 2.33

Now Current in branch BA $= I_1 = 4I_2 = 4 \times 0.5 = 2$ A **Ans.**

Current in branch AC $= I_1 = 2$ A **Ans.**

Current in branch BO $= I_2 = 0.5$ A **Ans.**

Current in branch OC $= I_2 = 0.5$ A **Ans.**

Current in branch CB $= I_1 + I_2 = 2 + 0.5 = 2.5$ A **Ans.**

Example 2.12. Determine the current drawn from the 5 volt battery in the network shown Fig. 2.34.

[R.G. Technical Univ. Basic Electrical and Electronics Engineering, June-2012]

Solution: Let the current distribution be as shown in Fig. 2.35.

Applying KVL to meshes ABCDA, BCEB and CDEC, we have

$$3I + 2(I - I_1) + 2(I - I_1 - I_2) = 5$$
$$\text{or } 7I - 4I_1 - 2I_2 = 5 \quad ...(i)$$
$$2(I - I_1) + 3I_2 - I_1 = 0$$
$$\text{or } 2I - 3I_1 + 3I_2 = 0 \quad ...(ii)$$
$$\text{and } 2(I - I_1 - I_2) - 4(I_1 + I_2) - 3I_2 = 0$$
$$\text{or } 2I - 6I_1 - 9I_2 = 0 \quad ...(iii)$$

Solving Eqs. (*i*), (*ii*) and (*iii*), we have

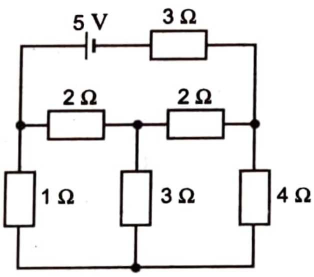


Fig. 2.34

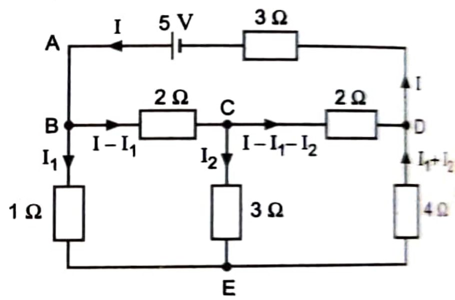


Fig. 2.35

$$I = \frac{7.5}{7.7} \text{ A}, \; I_1 = \frac{4}{7.7} \text{ A and } I_2 = \frac{-1}{7.7} \text{ A}$$

$$\text{Current drawn from 5 V source} = I = \frac{7.5}{7.7} = 0.974 \text{ A } \textbf{Ans.}$$

Example 2.13. Find the current through the 5 Ω resistor in the circuit shown in Fig. 2.36.

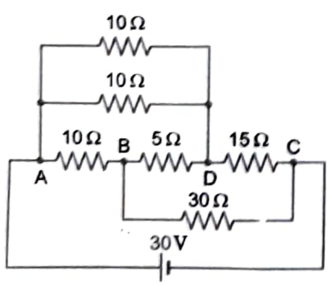


Fig. 2.36

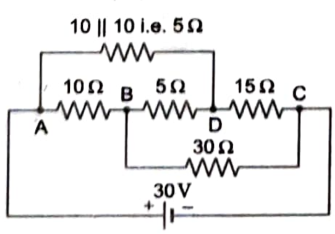


Fig. 2.37

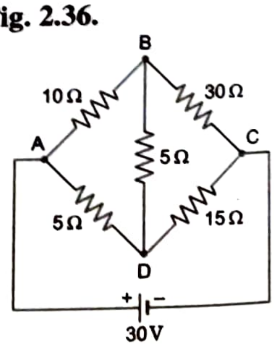


Fig. 2.38

Solution: The given circuit may be redrawn as shown in Fig. 2.36 and be rearranged again as shown in Fig. 2.37.

$$\text{Since } \frac{R_{AB}}{R_{AD}} = \frac{R_{BC}}{R_{CD}} \quad i.e. \quad \frac{10}{5} = \frac{30}{15}$$

It means that the circuit is balanced Wheatstone bridge. Hence the current through 5 Ω resistance (connected between B and D) is 0 ampere. **Ans.**

Example 2.14. Use source transformation method to compute the current through 6 Ω resistor of Fig. 2.39(*a*).

[*U.P. Technical Univ. Basic Electrical Engineering Second Semester, 2014-15*]

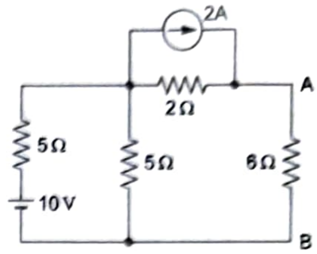


Fig. 2.39 (*a*)

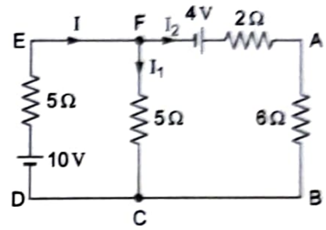


Fig. 2.39 (*b*)

Solution: Converting current source into equivalent voltage source, the given circuit becomes as shown in Fig. 2.39(*b*). Let the current distribution be as shown in Fig. 2.39(*b*).

Applying KCL to node F we have

$$I = I_1 + I_2 \qquad ...(i)$$

Applying KVL to meshes CDEFC and FABCF respectively, we have

$$5I + 5I_1 = 10$$

$$\text{or} \quad 5(I_1 + I_2) + 5I_1 = 10 \qquad \because \text{ From Eq. (1) } I = I_1 + I_2$$

$$\text{or} \quad 10I_1 + 5I_2 = 10$$

$$\text{or} \quad 2I_1 + I_2 = 2 \qquad ...(ii)$$

$$\text{and} \quad 2I_2 + 6I_2 - 5I_1 = 4$$

$$\text{or} \quad 8I_2 - 5I_1 = 4 \qquad ...(iii)$$

Solving Eqs. (ii) and (iii) we have

$$I_1 = \frac{4}{7}\text{A and } I_2 = \frac{6}{7}\text{A}$$

So current flowing through 6 Ω resistor = $I_2 = \frac{6}{7}$A **Ans.** 0.857A

Example 2.15. In Fig. 2.40 compute the voltage required between terminal *a-b* so that a voltage drop of 45 V occurs across 15 Ω resistor. [*G.B. Technical Univ. Electrical Engineering Second Semester, 2010-11*]

Solution: Given circuit is redrawn as shown in Fig. 2.41. Let the currents flowing through the various branches be as shown in the circuit.

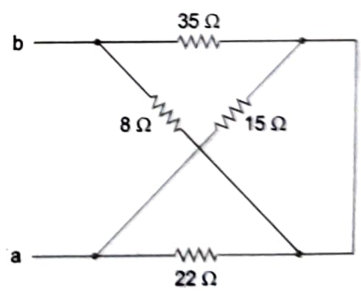


Fig. 2.40

Voltage drop across 15 Ω resistor = $(I_1 - I_3) \times 15 = 45$ V

$$\text{or} \quad I_1 - I_3 = 3 \qquad ...(i)$$

Applying KVL to loop ABCA, we have

$$35\, I_1 - 8\, I_2 = 0$$

$$\text{or} \quad I_2 = \frac{35}{8} I_1 = 4.375\, I_1 \qquad ...(ii)$$

Applying KVL to loop BCDB, we have

$$22\,(I_2 + I_3) - 15\,(I_1 - I_3) = 0$$

$$\text{or} \quad 22\,(I_2 + I_3) = 15\,(I_1 - I_3) = 45$$

$$\because \text{ From Eq. } (i),\ 15\,(I_1 - I_3) = 45$$

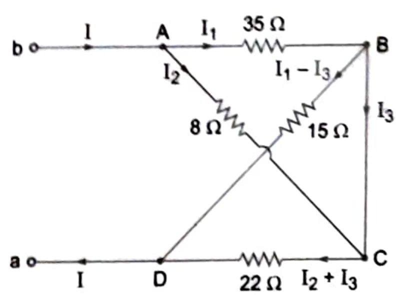


Fig. 2.41

$$\text{or} \quad I_2 + I_3 = \frac{45}{22} = 2.0454545$$

Substituting $I_2 = 4.375\, I_1$ from Eq. (*ii*) in above equation, we have

$$4.375\, I_1 + I_3 = 2.0454545 \qquad ...(iii)$$

Adding Eqs. (*i*) and (*iii*), we have

$$5.375\, I_1 = 5.0454545$$

$$\text{or} \quad I_1 = \frac{5.0454545}{5.375} = 0.9387 \text{ A}$$

Voltage required between terminal $ba = 35\,I_1 + 15\,(I_1 - I_3)$

$= 35 \times 0.9387 + 45$

$= 77.854$ V **Ans.**

So $V_{ab} = -V_{ba} = -77.854$ V **Ans.**

Example 2.16. Calculate the current through the galvanometer in the following bridge [Fig. 2.42 (*a*)].

[*U.P. Technical Univ. Electrical Engineering, January-2003*]

Solution: Assume current distribution in the bridge network as shown in Fig. 2.42 (*b*).

Applying Kirchhoff's second law to meshes ABDA, BCDB and ABCA respectively we have

$$I_1 + 4\,I_3 - 2\,I_2 = 0 \quad ...(i)$$

$$2(I_1 - I_3) - 3\,(I_2 + I_3) - 4\,I_3 = 0$$

or $$2\,I_1 - 3\,I_2 - 9\,I_3 = 0 \quad ...(ii)$$

and $$I_1 + 2\,(I_1 - I_3) = 2$$

or $$3\,I_1 - 2\,I_3 = 2 \quad ...(iii)$$

Solving Eqs. (*i*), (*ii*) and (*iii*) we get

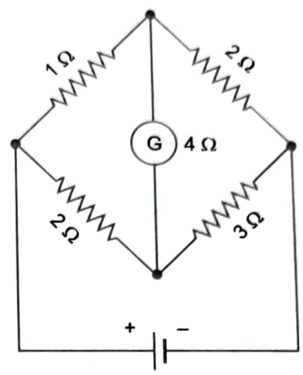


Fig. 2.42 (*a*)

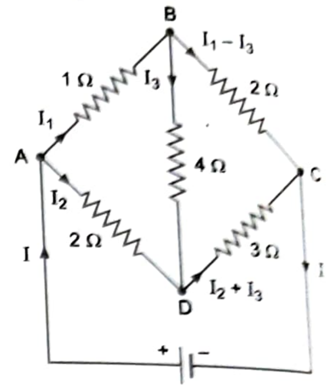


Fig. 2.42 (*b*)

$$I_3 = \frac{1}{44}\text{ A} ; I_2 = \frac{17}{44}\text{ A and } I_1 = \frac{30}{44}\text{ A}$$

So current through galvanometer $= I_3 = \dfrac{1}{44}$ A **Ans.**

2.11 NETWORK THEOREMS

The circuit variables, we are interested in determining, generally are (*i*) current through or (*ii*) voltage across a resistance of interest or a group of resistances. Sometimes it is required to work out the necessary source voltage or source current which will cause a specific current through or voltage across a given resistor or result into a specified power dissipation by it.

The current flowing through a circuit is governed by basic law called the *Ohm's law*, already discussed in chapter 1. Distribution of current or the voltage over a circuit is governed by Kirchhoff's laws, already discussed in the preceding Art (Art. 2.10).

The circuit to be analysed may be simple or quite complex. In case of complex networks the solution procedure may be too tedious and time consuming. Certain techniques for solution of such networks have been developed which reduces the networks to simpler form for quick solution. This may be accomplished through the use of what are called as *network theorems*. Few of these network theorems, which are relevant at this stage, will be discussed here.

2.12 SUPERPOSITION THEOREM

This theorem is applied when we are to determine the current in one particular branch of a network containing several voltage sources or current sources or both voltage sources and current sources. This scheme is to determine how much current each of the individual source contributes to the branch in question, and then add algebraically these component currents.

If there are several sources of emfs acting simultaneously in an electric circuit, then according to this theorem emf of each source acts independently of those of other sources, *i.e.* as if the other sources of emf did not exist and current in any branch or conductor of a network is equal to the algebraic sum of the currents due to each source of emf separately, all other emfs being taken equal to zero. This theorem is applicable only in linear circuits, *i.e.* circuits consisting of resistances in which Ohm's law is valid. In circuits having nonlinear resistances such as thermionic valves and metal rectifiers, this theorem is not applicable. However, superposition theorem can be applied to a circuit containing current sources and even to circuits containing both voltage sources and current sources. To remove a current source from the circuit, circuit of the source is opened leaving in place any conductance that may be in parallel with it, just as series resistance is kept in place when voltage source is removed.

Though the application of the above theorem requires a little more work than other methods such as the circulating current method but it avoids the solution of two or more simultaneous equations. After a little practice with this method, equations can be written directly from the original circuit diagram and labour in drawing extra diagrams is saved.

The superposition theorem can be stated as below:

In a linear resistive network containing two or more voltage sources, the current through any element (resistance or source) may be determined by adding together algebraically the currents produced by each source acting alone, when all other voltage sources are replaced by their internal resistances. If a voltage source has no internal resistance, the terminals to which it was connected are joined together. If there are current sources present they are removed and the network terminals to which they were connected are left open.

The procedure for applying superposition theorem is as follows:

1. Replace all but one of the sources of supply by their internal resistances. If the internal resistance of any source is very small as compared to other resistances existing in the network, the source is replaced by a short circuit. In case of a current source open the circuit leaving in place any conductance that may be in parallel with it.
2. Determine the currents in various branches using Ohm's law.
3. Repeat the process using each of the emfs turn by turn as the sole emf each time. Now the total current in any branch of the circuit is the algebraic sum of currents due to each source.

The details of the above procedure can best be understood by examining its application to the following solved examples.

Example 2.17. In the circuit shown in Fig. 2.43, find the current through the 6 Ω resistor using superposition theorem.
[*U.P. Technical Univ. Electrical Engineering Second Semester, 2007-08*]

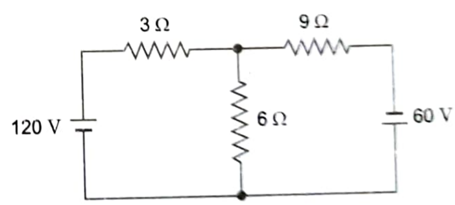


Fig. 2.43

Solution: Replacing 60 V source by a short circuit, the circuit is reduced to a simple circuit shown in Fig. 2.44(*a*).

Equivalent resistance of the circuit shown in Fig. 2.44(*a*).

$$R' = R_{AB} + R_{BC} \parallel R_{BD}$$
$$= 3\ \Omega + 6 \parallel 9 = 3 + 3.6 = 6.6\ \Omega$$

Current supplied by 120 V source

$$I_1 = \frac{120}{R'} = \frac{120}{6.6} = \frac{200}{11}\ \text{A}$$

Current in resistance, R_{BD}, $I' = I_1 \times \dfrac{R_{BC}}{R_{BC} + R_{CD}}$ By current division rule

$$= \frac{200}{11} \times \frac{9}{9+6} = \frac{120}{11}\ \text{A}$$

Similarly on replacement of 120 V source by a short circuit, the given circuit is reduced to a simple circuit shown in Fig. 2.44(*b*).

Equivalent resistance of this circuit is

$$R'' = R_{CB} + R_{BD} \parallel R_{BA} = 9 + 6 \parallel 3 = 9 + 2 = 11\ \Omega$$

Current supplied by 60 V source,

$$I_2 = \frac{60}{11} = \frac{60}{11}\ \text{A}$$

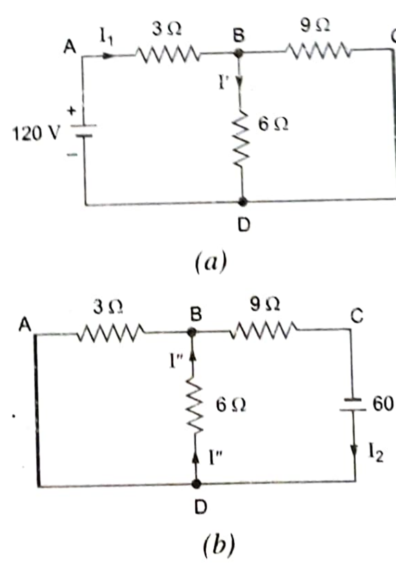


Fig. 2.44

Current in resistor BD, $I'' = I_2 \times \dfrac{R_{BA}}{R_{BA} + R_{CB}} = \dfrac{60}{11} \times \dfrac{3}{3+6} = \dfrac{20}{11}$ A

Total current, through 6 Ω resistor is equal to the algebraic sum of currents I′ and I″

$$I = I' + I'' = \frac{120}{11} + \frac{-20}{11} = \frac{100}{11} \text{ A} \quad \textbf{Ans.} \qquad \because I'' \text{ is in opposite direction to that of } I'.$$

Example 2.18. Find the current flowing through 10 Ω resistance in the following circuit (Fig. 2.45) use superposition theorem.

[Electrical Engineering G.B. Technical Univ. Second Semester 2011-12; U.P. Technical Univ. Even Semester 2013-14]

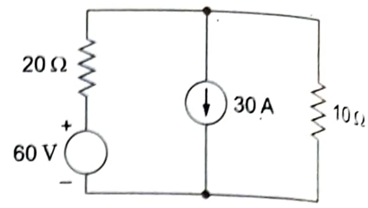


Fig. 2.45

Solution: Replacing 60 V source by a short circuit, the circuit is reduced to a simple circuit shown in Fig. 2.46.

Current flowing through 10 Ω resistance,

$$I_1 = 30 \times \frac{20}{20+10} = 20 \text{ A upward}$$

Replacing 30 A current source by an open circuit, the given circuit is reduced to a simple circuit shown in Fig. 2.47.

Current flowing through 10 Ω resistance,

$$I_2 = \frac{60}{20+10}$$

$$= 2 \text{ A downward}$$

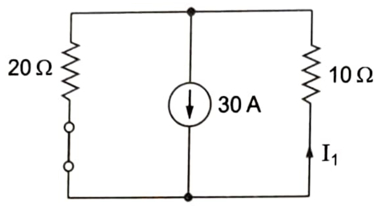


Fig. 2.46

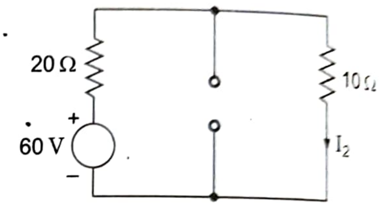


Fig. 2.47

Total current flowing through 10 Ω resistance,

$$I = I_1 + I_2$$

$$= 20 + (-2) = 18 \text{ A upward} \quad \textbf{Ans.}$$

Example 2.19. Using superposition theorem find the current flowing through resistor R in Fig. 2.48(*a*).

[Mahamaya Technical Univ. Electrical Engineering First Semester 2011-12]

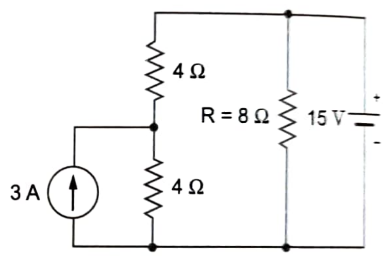
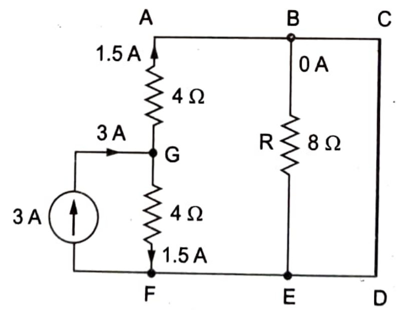


Fig. 2.48(*a*)

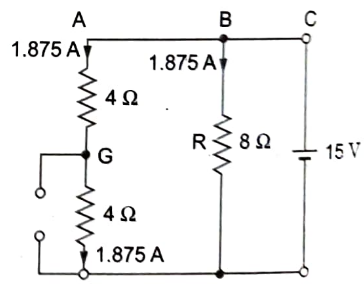
Solution: Replacing 15 V source by a short circuit, the circuit is reduced to a simple circuit as shown in Fig. 2.48(*b*). The current of 3 A is divided into resistances of 4 Ω and 4 Ω equally, as shown in Fig. 2.48(*b*). The current in resistor R will be zero, being short circuited.

Replacing 3 A current source by an open circuit, the circuit is reduced to a simple circuit as shown in Fig. 2.48(*c*). Current in resistor R will be equal to 15/8 *i.e.* 1.875 A.

Thus current through resistor R in Fig. 2.48(*a*) = 0 + 1.875 = 1.875 A **Ans.**



(*b*)



(*c*)

Fig. 2.48

Example 2.20. State superposition theorem. Apply the same for finding the current in 50 Ω resistor with the reference direction shown in circuit (Fig. 2.49).

[R.G. Technical Univ. Basic Electrical and Electronics Engineering, June-2012]

Solution: Refer to Art. 2.12.

Shorting the voltage sources, we have the circuit as shown in Fig. 2.50.

Current in 50 Ω resistor,

$$I' = 0.1 \times \frac{1/50}{\frac{1}{50} + \frac{1}{100} + \frac{1}{150}} = \frac{0.002}{0.02 + 0.01 + 0.006667} = 0.05455 \text{ A}$$

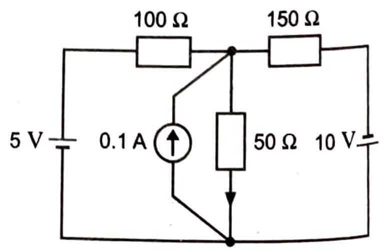


Fig. 2.49

On shorting 10 V source and opening the current source, the circuit becomes as shown in Fig. 2.51.

Equivalent resistance of the circuit, $R'' = 100 + 50 \parallel 150 = 100 + \dfrac{50 \times 150}{50 + 150} = 137.5\ \Omega$

Current supplied by 5 V source, $I = \dfrac{5}{137.5} = \dfrac{1}{27.5}$ A

Current through 50 Ω resistor, $I'' = \dfrac{1}{27.5} \times \dfrac{150}{50 + 150}$

$= \dfrac{1}{27.5} \times \dfrac{3}{4} = \dfrac{3}{110}$ A ...By current division rule

On shorting 5 V source and opening the current source, the circuit becomes as shown in Fig. 2.52.

Equivalent resistance of the circuit,

$R''' = 150 + 100 \parallel 50$

$= 150 + \dfrac{100 \times 50}{100 + 50} = 183.33\ \Omega$

Current supplied by 10 V source

$= \dfrac{10}{183.33} = \dfrac{3}{55}$ A

Current through 50 Ω resistor,

$I''' = \dfrac{3}{55} \times \dfrac{100}{100 + 50} = \dfrac{2}{55}$ A

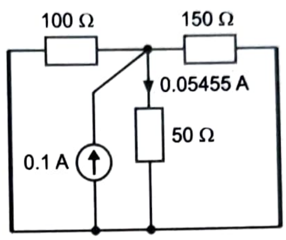


Fig. 2.50

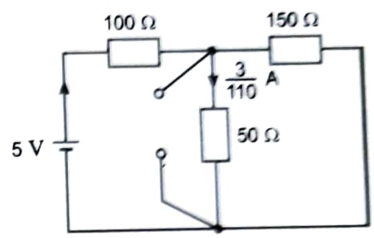


Fig. 2.51

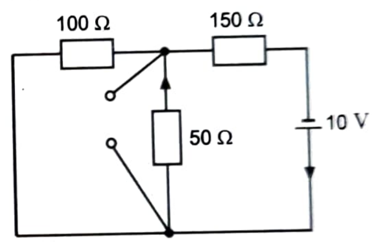


Fig. 2.52

Resultant current with the reference direction shown in the circuit,

$I = I' + I'' + I''' = 0.05455 + \dfrac{3}{110} - \dfrac{2}{55}$

$= 0.04546$ A **Ans.**

Example 2.21. Using superposition theorem, calculate the current in the AB in the circuit shown in Fig. 2.53.

[G.B. Technical Univ. Electrical Engineering Odd Semester, 2012-13]

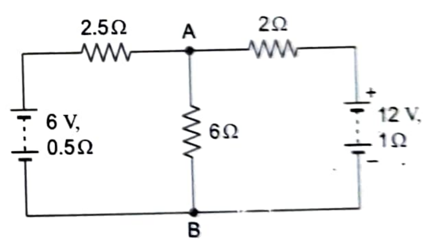


Fig. 2.53

Solution: Taking 12 V battery out of the circuit, the circuit is reduced to a simple circuit shown in Fig. 2.54 in which resistance of 6 Ω and (2 + 1) Ω are in parallel and the combination is in series with a resistor of 2.5 Ω and battery of 6 V and 0.5 Ω internal resistance.

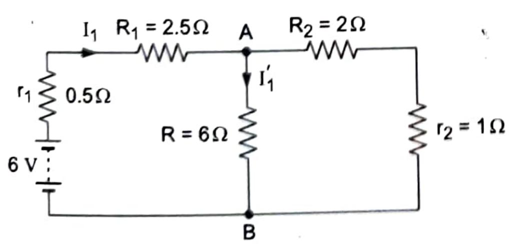


Fig. 2.54

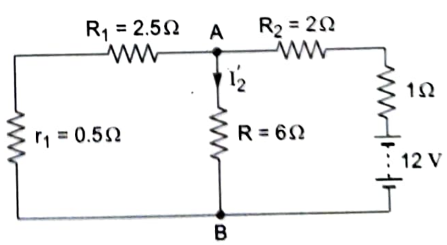


Fig. 2.55

Equivalent resistance of the circuit shown in Fig. 2.54.

$$R' = (R_1 + r_1) + R \parallel (R_2 + r_2) = (2.5 + 0.5) + 6 \parallel (2 + 1) = 3 + 6 \parallel 3 = 3 + 2 = 5\ \Omega$$

Current supplied by 6 V battery, $I_1 = \dfrac{E_1}{R'} = \dfrac{6}{5} = 1.2$ A

Current in branch AB, $I_1' = I_1 \times \dfrac{R_2 + r_2}{R_2 + r_2 + R} = 1.2 \times \dfrac{2+1}{2+1+6} = 0.4$ A from node A to node B

Taking 6 V battery out of circuit, the circuit is reduced to a simple circuit shown in Fig. 2.55 in which resistances of 6 Ω and (0.5 + 2.5) Ω are in parallel and the combination is in series with resistor of 2.5 Ω and battery of 12 V and 1 Ω internal resistance.

Equivalent resistance of the circuit shown in Fig. 2.55.

$$R'' = (R_2 + r_2) + R \parallel (R_1 + r_1) = (1 + 2) + 6 \parallel (2.5 + 0.5) = 3 + 6 \parallel 3 = 5\ \Omega$$

Current supplied by 12 V battery, $I_2 = \dfrac{E_2}{R''} = \dfrac{12}{5} = 2.4$ A

Current in branch AB, $I_2' = I_2 \times \dfrac{R_1 + r_1}{R_1 + r_1 + R} = 2.4 \times \dfrac{2.5 + 0.5}{2.5 + 0.5 + 6} = 0.8$ A from node A to node B

Total current through branch AB $= I_2' + I_2' = 0.4 + 0.8 = 1.2$ A **Ans.**

Example 2.22. Find the current in 5 Ω resistance using superposition principle.

[Rajasthan Technical Univ. 2007]

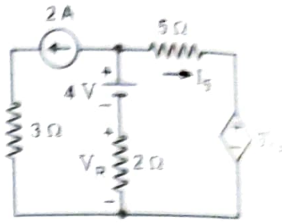


Fig. 2.56

Solution: In the given circuit, there are three sources, out of which one source is dependent source. The magnitude of dependent source becomes zero when all other sources are reduced to zero. Therefore, dependent source alone needs not to be considered in applying superposition theorem.

Short circuiting the voltage source the circuit becomes as shown in Fig. 2.57.

Applying KVL to closed loop II, we have

$$2(2 + I_5') + 5I_5' + 3V_R = 0$$

or $\quad 4 + 7I_5' + 3V_R = 0 \quad ...(i)$

and $\quad 2(2 + I_5') = -V_R$

Substituting $V_R = -(2I_5' + 4)$ in Eq. (*i*), we have

$$4 + 7I_5' + 3[-(2I_5' + 4)] = 0$$

or $\quad I_5' = 8$ A

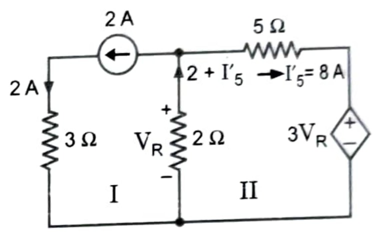


Fig. 2.57

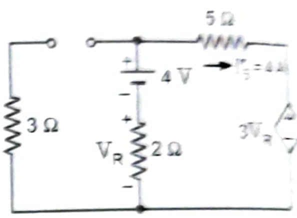


Fig. 2.58

Open circuiting the 2 A current source the circuit becomes as shown in Fig. 2.58.

Applying KVL in the closed circuit, we have

$$2I_5'' + 5I_5'' = 4 - 3V_R$$

or $\quad 7I_5'' = 4 - 3V_R = 4 + 6I_5'' \qquad \because V_R = -2I_5''$

or $\quad 7I_5'' = 4$ A

Total current through 5 Ω resistor, $I_5 = I_5' + I_5'' = 8 + 4 = 12$ A **Ans.**

2.13 MAXWELL CIRCULATING CURRENT THEOREM

If a network with several sources has more than two nodes the current in it may be determined by this theorem. This is one of the most universal methods for solving networks.

In a number of cases, a network may be considered as consisting of a set of adjoining loops (two in Fig. 2.3), each of which forms a polygon made up of several branches of the network (without any diagonals). Some branches of the network are common to two adjacent loops, while others form an external circuit where each branch occurs in one loop only.

This theorem involves representing a current that is assumed to circulate around a closed loop by a curved arrow and labelling the arrow with its identifying current symbol I with a subscript. By this theorem the current flowing through the branch common to two meshes will be equal to the algebraic sum of the two loop currents flowing through it. The direction of any loop current may be taken either as clockwise or counter-clockwise but for systematic solution the directions of all loop currents are assumed to be the same (say clockwise). Then Kirchhoff's second law is applied to each mesh and algebraic equations are obtained. The total number of independent equations is equal to the number of meshes (*i.e.* there are fewer equations than in a purely Kirchhoffian solution). Therefore, they can be solved as simultaneous equations to give the circulating currents and then the branch currents. Thus, this method eliminates a great deal of tedious calculation work involved in the branch current method, discussed in Art 2.10.

Application of Maxwell circulating current theorem will be more clear from the following illustrations.

Example 2.23. Find the currents in all the resistive branches of the circuit shown in Fig. 2.59 by KVL.

[U.P. Technical Univ. Electrical Engineering Second Semester, 2007-08]

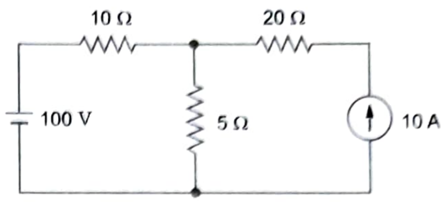


Fig. 2.59

Solution: The network is redrawn as shown in Fig. 2.60. The individual branch currents along with their directions of flow are shown in the circuit diagram.

Applying KVL to loop 1 we have

$$10\,I_1 + 5\,(I_1 + 10) = 100$$

$$\text{or} \quad 15\,I_1 + 50 = 100$$

$$\text{or} \quad I_1 = \frac{50}{15} = 3.33\text{ A}$$

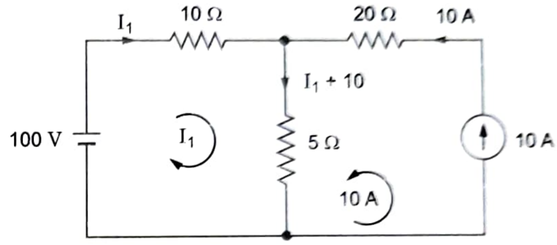


Fig. 2.60

Thus current in 10 Ω resistance,

$$I_1 = 3.33\text{ A} \quad \textbf{Ans.}$$

$$\text{Current in 5 Ω resistance} = I_1 + 10$$

$$= 3.33 + 10 = 13.33\text{ A} \quad \textbf{Ans.}$$

$$\text{Current in 20 Ω resistance} = 10\text{ A} \quad \textbf{Ans.}$$

Example 2.24. Find the current in 2 Ω resistance in the following figure (Fig. 2.61) using loop analysis method.

[A.K. Technical Univ. Basic Electrical Engineering Second Semester, 2015-16]

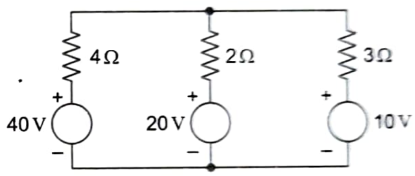


Fig. 2.61

Solution: The network is redrawn, as illustrated in Fig. 2.62. There are two independent loops. Loop currents have been taken clockwise as marked in the figure. The individual branch currents along with their directions of flow are also shown in the circuit diagram.

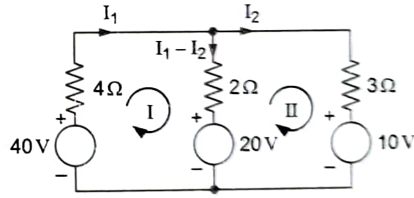


Fig. 2.62

Applying KVL to meshes I and II respectively, we have

$$4I_1 + 2(I_1 - I_2) = 40 - 20$$

$$\text{or} \quad 6I_1 - 2I_2 = 20$$

$$\text{or} \quad 3I_1 - I_2 = 10 \qquad \text{...(i)}$$

$$\text{and} \quad 3I_2 - 2(I_1 - I_2) = 20 - 10$$

$$\text{or} \quad -2I_1 + 5I_2 = 10 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii) we have

$$I_1 = \frac{60}{13}\text{A and } I_2 = \frac{50}{13}\text{A}$$

$$\text{Current in 2 Ω resistor} = I_1 - I_2 = \frac{60}{13} - \frac{50}{13} = \frac{10}{13}\text{A} \quad \textbf{Ans.}$$

Example 2.25. Using mesh analysis, calculate the currents I_1 and I_2 in Fig. 2.63.

[G.B. Technical Univ. Electrical Engineering First Semester, 2011-12]

Solution: The network shown in Fig. 2.63 is redrawn, as shown in Fig. 2.64. These are three independents loops. The loop current have been taken as shown in Fig. 2.64 satisfying the currents marked in Fig. 2.63. The individual branch currents along with their directions of flow are also shown in the circuit diagram. The resistor carrying current I_1, whose value is not given, may be assumed of 8 Ω.

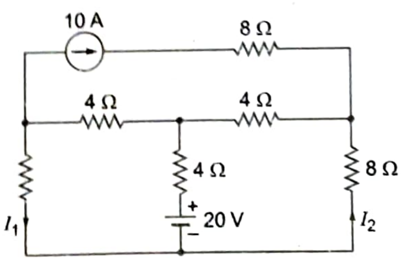


Fig. 2.63

Applying KVL to meshes I and II respectively, we have

$$8I_1 + 4(I_1 - 10) + 4(I_1 + I_2) = -20$$

$$\text{or} \quad 16I_1 + 4I_2 = 20 \qquad \text{...}(i)$$

and $8I_2 + 4(I_2 + 10) + 4(I_1 + I_2) = -20$

$$\text{or } 4I_1 + 16I_2 = -60$$

$$\text{or } I_1 + 4I_2 = -15 \qquad ...(ii)$$

Subtracting Eq. (*ii*) from Eq. (*i*), we have

$$15I_1 = 35$$

$$\text{or } I_1 = \frac{7}{3} \text{ A } \textbf{Ans.}$$

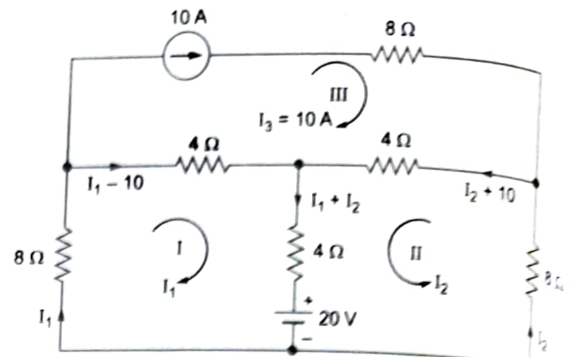


Fig. 2.64

Substituting $I_1 = \frac{7}{3}$ A in Eq. (*ii*), we have

$$I_2 = \frac{-15 - I_1}{4} = \frac{-15 - \frac{7}{3}}{4} = \frac{-13}{3} \text{ A } \textbf{Ans.}$$

Example 2.26. In Fig. 2.65, if all the voltages are equal each being 10 volts, and if all the resistances are also equal each being 5 Ω, find the currents I_1, I_2 and I_3 using Maxwell current loop method.

[Rajasthan Technical Univ. March 2009]

Solution: Applying KVL to meshes 1, 2 and 3, we have

$$I_1 \times 5 + (I_1 - I_3) \times 5 + (I_1 - I_2) \times 5 = 10$$

$$I_2 \times 5 + (I_2 - I_3) \times 5 + (I_2 - I_1) \times 5 = 10$$

and $$I_3 \times 5 + (I_3 - I_1) \times 5 + (I_3 - I_2) \times 5 = 10$$

$$\text{or } 3I_1 - I_2 - I_3 = 2 \qquad ...(i)$$

$$3I_2 - I_1 - I_3 = 2 \qquad ...(ii)$$

$$3I_3 - I_1 - I_2 = 2 \qquad ...(iii)$$

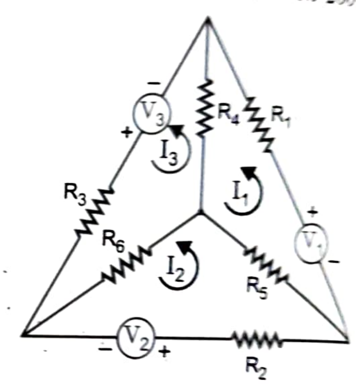


Fig. 2.65

Solving Eqs. (*i*), (*ii*) and (*iii*), we have,

$$I_1 = I_2 = I_3 = 2 \text{ A. } \textbf{Ans.}$$

Example 2.27. Applying mesh analysis, obtain the current through 5 Ω resistance in the following circuit (Fig. 2.66).

[Electrical Engineering G.B. Technical Univ. Second Semester, 2011-12
U.P. Technical Univ. Even Semester, 2013-14]

Solution: The circuit is redrawn with branch currents marked with the flow of direction, as illustrated in Fig. 2.67.

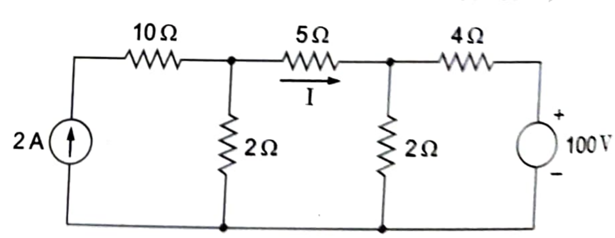


Fig. 2.66

From circuit diagram shown in Fig. 2.67, we have

$$I_1 = 2 \text{ A}$$

$$I_2 = \text{I A}$$

Applying KVL to meshes BCFGB and CDEFC respectively we have

$$5\,I_2 + 2\,(I - I_3) - 2\,(I_1 - I_2) = 0$$

$$\text{or } 9\,I - 2\,I_3 = 2 \times 2 = 4 \quad ...(i)$$

$$\because I_1 = 2 \text{ A and } I_2 = I$$

$$\text{and } 4\,I_3 - 2\,(I_2 - I_3) = -100$$

$$\text{or } 2\,I - 6\,I_3 = 100 \qquad ...(ii)$$

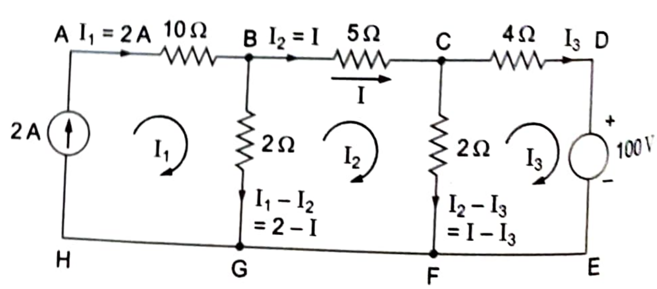


Fig. 2.67

Solving Eqs (*i*) and (*ii*) we have

$$I = \frac{-88}{25} = -3.52 \text{ A} \quad \textbf{Ans.}$$

Example 2.28. Find voltage V_1 across 6 Ω resistance in the following circuit (Fig. 2.68) using loop analysis method.

[*Electrical Engineering G.B. Technical Univ. Odd Semester, 2009-10; U.P. Technical Univ. Odd Semester, 2013-14*]

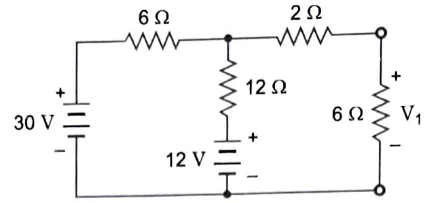


Fig. 2.68

Solution: Current distribution in different branches of the circuit satisfying KCL is shown in Fig. 2.69.

Applying KVL to meshes I and II respectively we have

$$6 I_1 + 12 (I_1 - I_2) = 30 - 12$$

$$\text{or } 3 I_1 - 2 I_2 = 3 \quad ...(i)$$

$$\text{and } 2 I_2 + 6 I_2 - 12 (I_1 - I_2) = 12$$

$$\text{or } 5 I_2 - 3 I_1 = 3 \quad ...(ii)$$

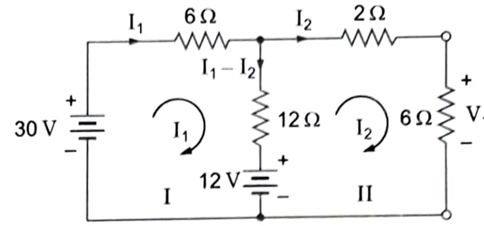


Fig. 2.69

Solving Eqs (i) and (ii) we have

$$I_2 = 2 \text{ A}$$

$$\text{and } V_1 = 6 I_2$$

$$= 6 \times 2 = 12 \text{ V} \quad \textbf{Ans.}$$

2.14 NODE–VOLTAGE THEOREM OR NODAL ANALYSIS

The direct use of Kirchhoff's laws for determining the currents in complicated networks necessitates the simultaneous solution of a considerable number of equations, making the computations extremely time-consuming. However, there are a number of methods (such as loop-current method, node-voltage method etc.), based on the same Kirchhoff's laws, that obviate the solving of a set of equations or reduce the number of equations and, therefore, significantly cut the computation work.

For application of node voltage theorem one of the node is taken as *reference* or *zero potential* or *datum node* and the potential difference between each of the other nodes and the reference node is expressed in terms of an unknown voltage (symbolized as V_1, V_2 or V_A, V_B or V_x, V_y etc.) and at every node Kirchhoff's first (or current) law is applied assuming the possible directions of branch currents. This assumption does not change the statement of problem, since the branch currents are determined by the potential difference between respective nodes and not by absolute values of node potentials. Like Maxwell's circulating current theorem, node-voltage theorem reduces the number of equations to be solved to determine the unknown quantities. If there are *n* number of nodes, there shall be $(n - 1)$ number of nodal equations in terms of $(n - 1)$ number of unknown variables of nodal voltages. By solving these equations, nodal voltages are known to compute the branch currents.

When the number of nodes minus one is less than the number of independent meshes in the network, it is, in fact more advantageous. Moreover, it is particularly suited for networks having many parallel circuits with common ground connected node such as in electronic circuits.

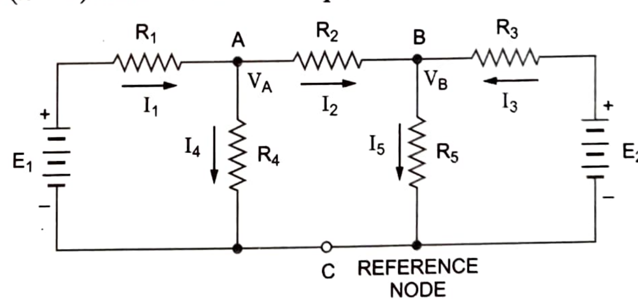


Fig. 2.70

Illustration. Consider, for example, a two node network, as illustrated in Fig. 2.70.

Node C has been taken as reference node. Let V_A and V_B be the voltages of nodes A and B respectively with respect to node C. Let the current distribution be as shown on the circuit diagram (Fig. 2.70) arbitrarily. Now let us get independent equations for these two nodes.

Node A is the junction of resistors R_1, R_2 and R_4. So current equation for node A is

$$I_1 = I_2 + I_4$$

$$\text{or} \quad \frac{E_1 - V_A}{R_1} = \frac{V_A - V_B}{R_2} + \frac{V_A}{R_4}$$

$$\text{or} \quad V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)-\frac{V_B}{R_2}=\frac{E_1}{R_1} \quad \text{...(2.23)}$$

Node B is the junction of resistors R_2, R_3 and R_5. So current equation for node B is

$$I_5 = I_2 + I_3$$

$$\text{or} \quad \frac{V_B}{R_5}=\frac{V_A-V_B}{R_2}+\frac{E_2-V_B}{R_3}$$

$$\text{or} \quad \frac{-V_A}{R_2}+V_B\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right]=\frac{E_2}{R_3} \quad \text{...(2.24)}$$

The Eqs. (2.23) and (2.24) can now be solved to get the values of V_A and V_B and then the values of currents I_1, I_2, I_3, I_4 and I_5 can be computed easily.

Example 2.29. Find the current through the 1 ohm resistor using node voltage method for the circuit shown in Fig. 2.71. [*Rajasthan Technical Univ. June-2010; Jan./Feb. 2013*]

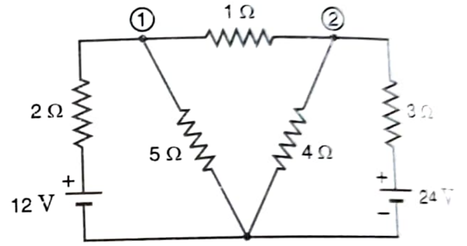


Fig. 2.71

Solution: Let the circuit be redrawn with its different nodes marked A, B and C, the last one being taken as reference or datum node (Fig. 2.72). Applying KCL to nodes A and B respectively, we have

$$I = I_1 + I_2$$

$$\text{or} \quad \frac{12-V_A}{2}=\frac{V_A}{5}+\frac{V_A-V_B}{1}$$

$$\text{or} \quad 17V_A - 10V_B = 60 \quad \text{...(i)}$$

$$\text{and} \quad I_2 = I_3 - I_4$$

$$\frac{V_A-V_B}{1}=\frac{V_B}{4}-\frac{24-V_B}{3}$$

$$\text{or} \quad 12V_A - 19V_B = -96 \quad \text{...(ii)}$$

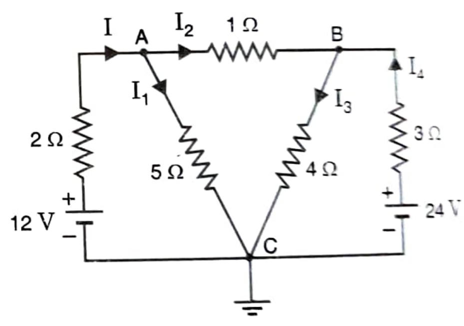


Fig. 2.72

Solving Eqs. (*i*) and (*ii*), we have $=10.34$

$$V_A = \frac{2{,}100}{203}\text{ V} \quad \text{and} \quad V_B = \frac{2{,}352}{203} = 11.58$$

Current through 1 Ω resistor

$$= I_2 = \frac{V_A - V_B}{1} = \frac{2{,}100-2{,}352}{203} = -1.24 \text{ A}$$

i.e., 1.24 A from node (2) to node (1) **Ans.**

Example 2.30. Use nodal analysis method to find currents in the various resistors of the circuit shown in Fig. 2.73. [*Chhattisgarh Vivekanand Technical Univ. 2006-07*]

Solution : Let the circuit be redrawn with its different nodes marked A, B, C and D, the last one being taken as reference or datum node. Applying KCL to nodes A, B and C respectively, we have

$$I_1 + I_2 + I_3 = I$$

$$\text{or} \quad \frac{V_A}{2}+\frac{V_A-V_B}{2}+\frac{V_A-V_C}{10}=28$$

$$\text{or} \quad 11\,V_A - 5\,V_B - V_C = 280 \quad \text{...(i)}$$

$$I_2 = I_4 + I_5$$

$$\text{or} \quad \frac{V_A-V_B}{2}=\frac{V_B-V_C}{1}+\frac{V_B}{5}$$

$$\text{or} \quad 5\,V_A - 17\,V_B + 10\,V_C = 0 \quad \text{...(ii)}$$

$$I_3 + I_4 - I_6 = 2$$

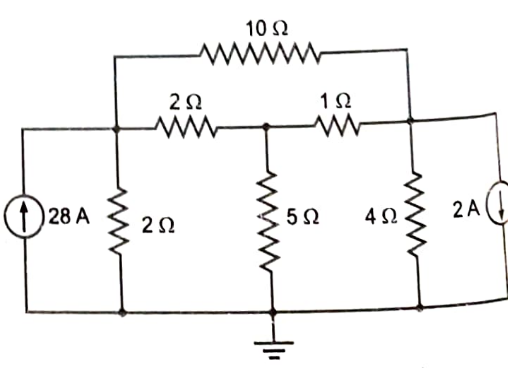


Fig. 2.73

$$\text{or} \quad \frac{V_A - V_C}{10} + \frac{V_B - V_C}{1} - \frac{V_C}{4} = 2$$

$$\text{or } 2\,V_A + 20\,V_B - 27\,V_C = 40 \qquad ...(iii)$$

Solving Eqs. (*i*), (*ii*) and (*iii*) we have

$$V_A = 36\text{ V},\ V_B = 20 \text{ and } V_C = 16\text{ V}$$

The various currents in the circuit shown in Fig. 2.74 are determined as below.

$$I_1 = \frac{V_A}{2} = \frac{36}{2} = 18\text{ A } \textbf{Ans.}$$

$$I_2 = \frac{V_A - V_B}{2} = \frac{36-20}{2} = 8\text{ A } \textbf{Ans.}$$

$$I_3 = \frac{V_A - V_C}{10} = \frac{36-16}{10} = 2\text{ A } \textbf{Ans.}$$

$$I_4 = \frac{V_B - V_C}{1} = \frac{20-16}{1} = 4\text{ A } \textbf{Ans.}$$

$$I_5 = \frac{V_B}{5} = \frac{20}{5} = 4\text{ A } \textbf{Ans.}$$

$$\text{and } I_6 = \frac{V_C}{4} = \frac{16}{4} = 4\text{ A } \textbf{Ans.}$$

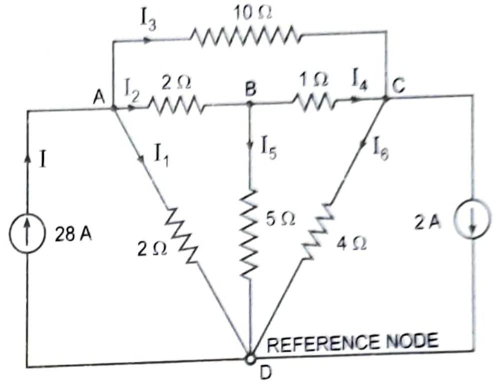


Fig. 2.74

Example 2.31. Determine i_1, i_2, and i_3 using nodal analysis for a given circuit shown in Fig. 2.75.
[U.P. Technical Univ. Electrical Engineering Odd Semester 2013-14]

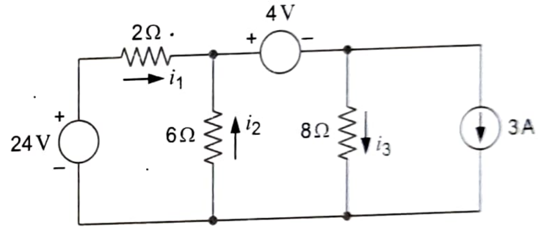


Fig. 2.75

Solution: Let the circuit given in Fig. 2.75 be redrawn with terminal G as the reference node as shown in Fig. 2.76. Let the current flowing through branch AB be i_x amperes flowing from node A to node B.

Applying Kirchhoff's current law to nodes A and B respectively we have

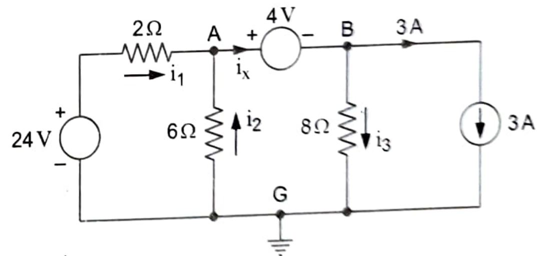


Fig. 2.76

$$i_1 + i_2 = i_x$$

$$\text{or} \quad \frac{24 - V_A}{2} + \frac{0 - V_A}{6} = i_x \qquad ...(i)$$

$$\text{and } i_x - 3 = i_3$$

$$\text{or } i_3 + 3 = i_x$$

$$\text{or} \quad \frac{V_B}{8} + 3 = i_x \qquad ..(ii)$$

$$\text{Node voltage } V_B = \text{Node voltage } V_A - 4 \qquad ...(iii)$$

Substituting $V_B = V_A - 4$ in Eq. (*ii*) we have

$$\frac{V_A - 4}{8} + 3 = i_x \qquad ...(iv)$$

Comparing Eqs. (*i*) and (*iv*) we have

$$\frac{24 - V_A}{2} - \frac{V_A}{6} = \frac{V_A - 4}{8} + 3$$

$$\text{or } 288 - 12V_A - 4V_A = 3V_A - 12 + 72$$

$$\text{or } 19V_A = 288 + 12 - 72 = 228$$

$$\text{or } V_A = \frac{228}{19} = 12\text{ V}$$

$$V_B = V_A - 4 = 12 - 4 = 8\text{ V}$$

$$\text{Current } i_1 = \frac{24 - V_A}{2} = \frac{24-12}{2} = 6\text{ A} \quad \textbf{Ans.}$$

$$\text{Current } i_2 = \frac{0 - V_A}{6} = \frac{0 - 12}{6} = -2 \text{ A} \quad \textit{i.e.} \text{ 2 A flowing away from node A} \quad \textbf{Ans.}$$

$$\text{Current } i_3 = \frac{V_B}{8} = \frac{8}{8} = 1 \text{ A} \quad \textbf{Ans.}$$

Example 2.32. Determine the voltage V_1 and V_2 in Fig. 2.77.

[*Mahamaya Technical Univ. Electrical Engineering Second Semester, 2011-12*]

Solution: Let the circuit be redrawn with terminal O as the reference node and arbitrarily assumed distribution of currents, as shown in Fig. 2.78.

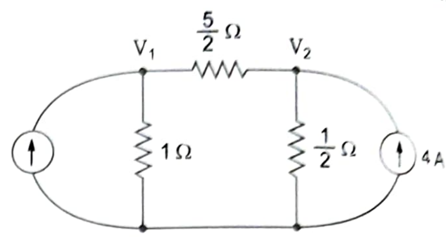


Fig. 2.77

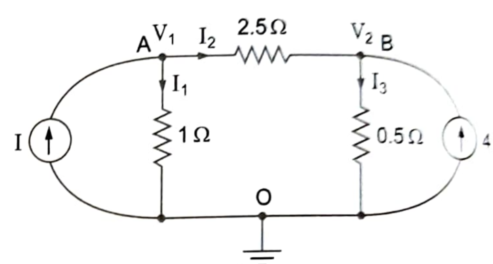


Fig. 2.78

Assume I = 5 A (missing data)

Applying KCL to nodes A and B respectively we have

$$I_1 + I_2 = I$$

$$\text{or} \quad \frac{V_1}{1} + \frac{V_1 - V_2}{2.5} = 5 \quad \text{...}(i)$$

$$\text{and } I_3 - I_2 = 4$$

$$\text{or} \quad \frac{V_2}{0.5} - \frac{V_1 - V_2}{2.5} = 4 \quad \text{...}(ii)$$

$$\text{or } 7\,V_1 - 2\,V_2 = 25 \quad \text{...}(iii)$$

$$\text{and } -V_1 + 6\,V_2 = 10 \quad \text{...}(iv)$$

Solving Eqs. (iii) and (iv) we have

$$V_1 = 4.25 \text{ V} \quad \textbf{Ans.}$$

$$\text{and } V_2 = 2.375 \text{ V} \quad \textbf{Ans.}$$

Example 2.33. Calculate currents in all the resistances of the circuit shown in Fig. 2.79 (*a*) using node analysis method.

[*U.P. Technical Univ. Electrical Engineering First Semester, 2006-07*]

Solution: Let the given circuit be redrawn with terminal G as the reference node and arbitrarily assumed distribution of currents, as shown in Fig. 2.79 (*b*).

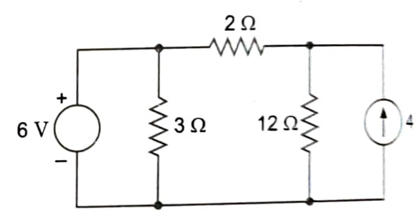


Fig. 2.79(*a*)

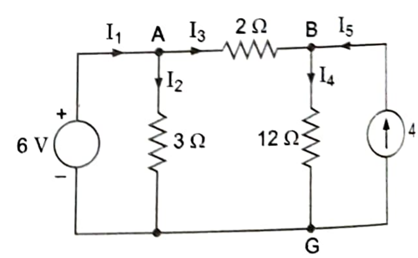


Fig. 2.79 (*b*)

Applying Kirchhoff's current law to node B we have

$$I_4 = I_3 + I_5$$

$$\text{or} \quad \frac{V_B}{12} = \frac{V_A - V_B}{2} + 4$$

$$\text{or} \quad \frac{V_B}{12} = \frac{6 - V_B}{2} + 4 \qquad \because V_A = 6 \text{ V}$$

$$\text{or } V_B = 36 - 6\,V_B + 48$$

$$\text{or } 7\,V_B = 84$$

$$\text{or } V_B = 12 \text{ V}$$

So we have

$$\text{Current in 3 } \Omega \text{ resistance, } I_2 = \frac{V_A}{3} = \frac{6}{3} = 2 \text{ A} \quad \textbf{Ans.}$$

$$\text{Current in 2 } \Omega \text{ resistance, } I_3 = \frac{V_A - V_B}{2} = \frac{6 - 12}{2} = -3 \text{ A} \quad \textbf{Ans.} \quad \textit{i.e.} \text{ 3 A from node B to node A} \quad \textbf{Ans.}$$

$$\text{Current in 12 } \Omega \text{ resistance, } I_4 = \frac{V_B}{12} = \frac{12}{12} = 1 \text{ A} \quad \textbf{Ans.}$$

Example 2.34. Find current in each branch by using nodal analysis. Also calculate total power loss.

[U.P. Technical Univ. Basic Electrical Engineering Odd Semester, 2013-14]

Solution: Taking node G as reference node and the potentials of nodes A, B and C be V_A, V_B and V_C respectively and current distribution as shown in Fig. 2.80 (*b*) arbitrarily.

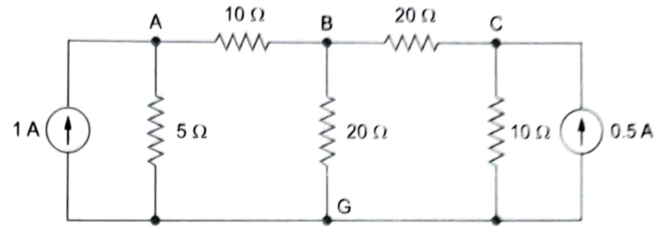


Fig. 2.80 (*a*)

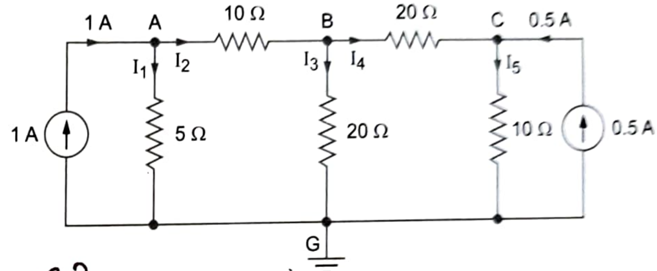


Fig. 2.80 (*b*)

Applying Kirchhoff's current law to nodes A, B and C respectively we have

$$1 = I_1 + I_2$$

$$\text{or } 1 = \frac{V_A}{5} + \frac{V_A - V_B}{10}$$

$$\text{or } 3\,V_A - V_B = 10 \qquad ...(i)$$

$$I_2 = I_3 + I_4$$

$$\text{or } \frac{V_A - V_B}{10} = \frac{V_B}{20} + \frac{V_B - V_C}{20}$$

$$\text{or } 2\,V_A - 4\,V_B + V_C = 0 \qquad ...(ii)$$

$$\text{and } I_4 + 0.5 = I_5$$

$$\text{or } \frac{V_B - V_C}{20} + 0.5 = \frac{V_C}{10}$$

$$\text{or } V_B - 3\,V_C = -10 \qquad ...(iii)$$

Solving Eqs. (*i*), (*ii*) and (*iii*) we have

$$V_A = \frac{40}{9}\text{ V} \rightarrow 4.44, \quad V_B = \frac{10}{3}\text{ V} \quad 3.33$$

$$\text{and } V_C = \frac{40}{9}\text{ V} \quad 4.44$$

So current through 5 Ω resistor, $I_1 = \frac{V_A}{5} = \frac{40}{9} \times \frac{1}{5} = \frac{8}{9}$ A **Ans.**

Current through 10 Ω resistor, $I_2 = \frac{V_A - V_B}{10} = \left(\frac{40}{9} - \frac{10}{3}\right) \times \frac{1}{10} = \frac{1}{9}$ A **Ans.**

Current through 20 Ω resistor, $I_3 = \frac{V_B}{20} = \frac{10}{3} \times \frac{1}{20} = \frac{1}{6}$ A

Current through 20 Ω resistor, $I_4 = \frac{V_B - V_C}{20} = \left(\frac{10}{3} - \frac{40}{9}\right) \times \frac{1}{20} = \frac{-1}{18}$ A *i.e.* $\frac{1}{18}$ A from node C to node B

Current through 10 Ω resistor, $I_5 = \frac{V_C}{10} = \frac{4}{9}$ A **Ans.**

$$\text{Total power loss} = I_1^2 \times 5 + I_2^2 \times 10 + I_3^2 \times 20 + I_4^2 \times 20 + I_5^2 \times 10$$

$$= \left(\frac{8}{9}\right)^2 \times 5 + \left(\frac{1}{9}\right)^2 \times 10 + \left(\frac{1}{6}\right)^2 \times 20 + \left(\frac{-1}{18}\right)^2 \times 20 + \left(\frac{4}{9}\right)^2 \times 10 = \frac{20}{3} \text{ watts } \textbf{Ans.}$$

Example 2.35. Carryout the nodal analysis for the circuit given below (Fig. 2.81) and then compute the voltage across the 2 ℧ conductance.

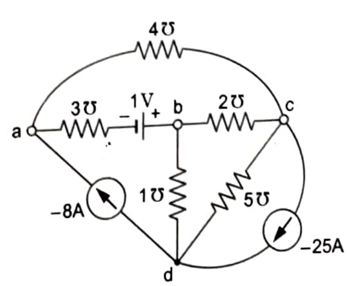


Fig. 2.81

Solution: Let node *d* be taken as reference node and the potentials of terminals *a*, *b* and *c* be V_a, V_b and V_c respectively and current distribution as shown in Fig. 2.82 arbitrarily.

Applying KCL at node *a* we have

$$I_1 + 8 = I_2$$

$$\text{or } (V_a + 1 - V_b) \times 3 + 8 = (V_c - V_a) \times 4 \qquad ...(i)$$

$$\text{or } 7\,V_a - 3\,V_b - 4\,V_c = -11$$

Applying KCL at node b we have

$$I_1 + I_3 = I_4$$

or $(V_a + 1 - V_b) \times 3 + (V_c - V_b) \times 2 = V_b \times 1$

or $3\,V_a - 6\,V_b + 2\,V_c = -3$ ...(ii)

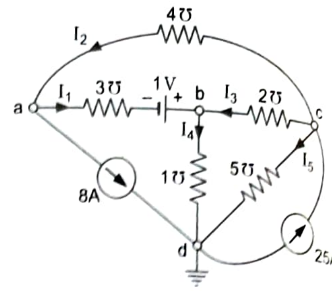


Fig. 2.82

Applying KCL at node c we have

$$I_2 + I_3 + I_5 = 25$$

$$4\,(V_c - V_a) + 2\,(V_c - V_b) + 5\,V_c = 25$$

or $-4\,V_a - 2\,V_b + 11\,V_c = 25$ ...(iii)

Solving Eqs. (*i*), (*ii*) and (*iii*) we have

$$V_a = 1\text{ V}$$
$$V_b = 2\text{ V}$$
$$V_c = 3\text{ V}$$

Voltage across 2 ℧ conductance $= V_{cb} = V_c - V_b = 3 - 2 = 1$ V **ans.**

2.15 THEVENIN'S THEOREM

This theorem provides a mathematical technique for replacing a two-terminal network by a voltage sourc V_T, and resistance R_T connected in series. The voltage source V_T (called the *Thevenin's equivalent voltage*) is the open-circuit voltage that appears across the load terminals when the load is removed or disconnected and resistance R_T, called the *Thevenin's equivalent resistance,* is equal to the resistance of the network looking back into the load terminals. A Thevenin's equivalent circuit is shown in Fig. 2.83. The steady-state current will be given as

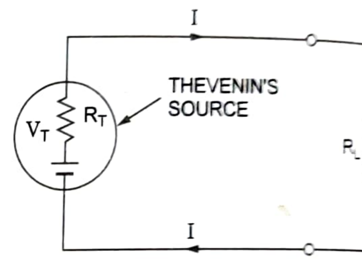


Fig. 2.83

$$I = \frac{V_T}{R_T + R_L}$$

For visualizing the application of Thevenin's theorem, let us consider a circuit shown in Fig. 2.8 which consists of a source of emf E volts and internal resistance r ohms connected to an external circui consisting of resistances R_1 and R_2 ohms in series.

So far as terminals AB across which a resistance of R_2 ohms is connected the network acts as source of open-circuit voltage V_{OC} (also called the Thevenin's equivalent voltage V_T) and internal resistance R_{in} (also called the Thevenin's resistance R_T).

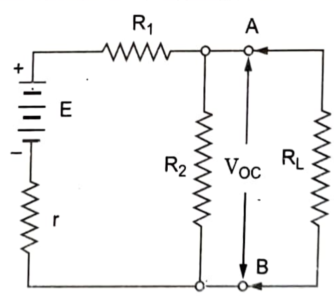


Fig. 2.84

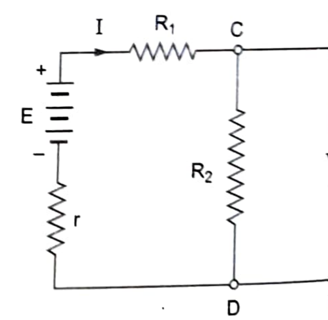


Fig. 2.85

For determination of open-circuit voltage V_{OC} (or V_T), disconnect the load resistance R_L from the terminals A and B to provide open circui [Fig. 2.85].

Now current through resistance R_2, $I = \dfrac{E}{R_1 + R_2 + r}$

and open-circuit voltage V_{OC} or V_T = Voltage across terminals AB = Voltage drop across resistance R

$$= IR_2 = \frac{ER_2}{R_1 + R_2 + r}$$

For determination of internal resistance R_{in} (or R_T) of the network under consideration re voltage source from the circuit, leaving behind only its internal resistance r, as illustrated in Fig view the circuit inwards from the open terminals A and B. It is found that the circuit [Fig. 2.86] now consists of two parallel paths—one consisting of resistance R_2 only, and the other consisting of resistance R_1 and r in series. Thus, the equivalent resistance (R_T), as viewed from the open terminals A and B, is given as

$$R_T = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)} \qquad ...(2.26)$$

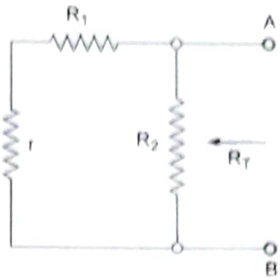


Fig. 2.86

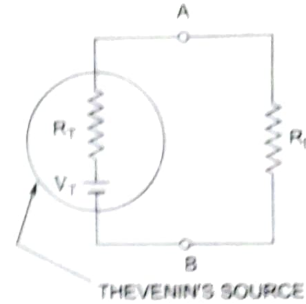


Fig. 2.87

Now when load resistance R_L is connected across terminals A and B, the network behaves as a source of voltage V_T and internal resistance R_T [Fig. 2.87] and current flowing through the load resistance R_L is given as

$$I_L = \frac{V_T}{R_T + R_L} = \frac{ER_2/[R_1 + R_2 + r]}{\dfrac{R_2(R_1 + r)}{R_1 + r + R_2} + R_L} = \frac{ER_2}{R_2(R_1 + r) + R_L(R_1 + R_2 + r)} \qquad ...(2.27)$$

The Thevenin's theorem can be stated as follows:

The current in any *passive circuit element* (which may be called R_L) in a network is the same as would be obtained if R_L were supplied with a voltage source V_{OC} (or V_T) in series with an equivalent resistance R_{in} (or R_T). V_{OC} being the open-circuit voltage at the terminals from which R_L has been removed and R_{in} (or R_T) being the resistance that would be measured at these terminals after all sources have been removed and each has been replaced by its internal resistance.

This theorem is advantageous when we are to determine the current in a particular element of a linear bilateral network particularly when it is desired to find the current which flows through a resistor for its different values. It makes the solution of the complicated networks (particularly electronic networks) quite simple.

Example 2.36. Find the Thevenin's equivalent of the network (Fig. 2.88).

[U.P. Technical Univ. Electrical Engineering Second Semester, 2004-05]

Solution: Replacing 5 A current source by an open circuit and replacing 10 V voltage source by a short circuit we have $R_{Th} = 10\ \Omega$.

Let us convert 10 V voltage source with its series resistance of 10 Ω into an equivalent current source of 1 A with a parallel resistance of 10 Ω as shown in Fig. 2.89.

From the circuit it is obvious that 1 + 5 = 6 A current flows through 10 Ω resistor, thereby producing a voltage drop of 6 × 10 = 60 V. Hence V_{Th} = 60 V.

So Thevenin equivalent of the given network is equivalent to a voltage source of 60 V with 10 Ω resistance in series with it.

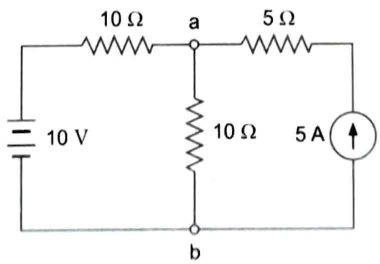


Fig. 2.88

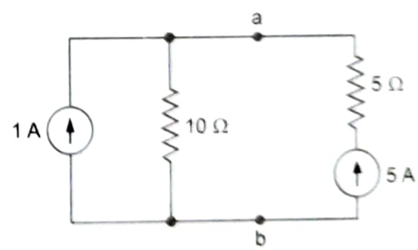


Fig. 2.89

Example 2.37. Find the current through 2 Ω using Thevenin's theorem.

[J.N. Technological Univ. Basic Electrical Engineering, December-2012]

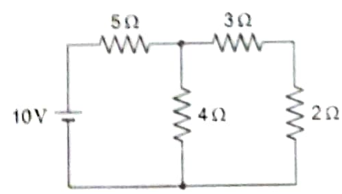


Fig. 2.90

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals A and B, the voltage source is short circuited, as shown in Fig. 2.91.

$$\text{Thevenin's equivalent resistance, } R_T = 3 + 4 \parallel 5 = 3 + \frac{4 \times 5}{4 + 5} = 5.222\ \Omega$$

For determination of open-circuit voltage, the resistance of 2 Ω is removed, as shown in Fig. 2.92.

Current flowing through closed mesh,

$$I = \frac{10}{5 + 4} = \frac{10}{9}\ \text{A}$$

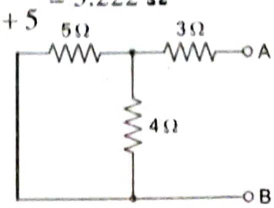


Fig. 2.91

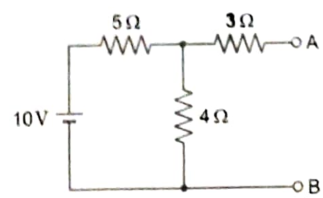


Fig. 2.92

Thevenin's voltage, V_T = Voltage drop across 4 Ω resistance $= 4\,I = 4 \times \frac{10}{9} = 4.444$ V

Current through 2 Ω resistance, $I_L = \frac{V_T}{R_L + R_T} = \frac{4.444}{2 + 5.222} = 0.6154$ A **Ans.**

Example 2.38. Draw the Thevenin's equivalent of the circuit shown in the following figure (Fig. 2.93).

[R.G. Technical Univ. Network Analysis, June-2014]

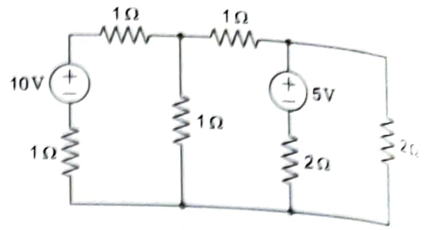


Fig. 2.93

Solution: For determining the Thevenin's equivalent resistance of the circuit w.r.t. terminals AB, the voltage sources are replaced by short circuits, and 2 Ω resistance is removed as shown in Fig. 2.94(*a*).

Thevenin's equivalent resistance,

$R_T = [\{(1 + 1) \,||\, 1\} + 1] \,||\, 2$

$$= \left(\frac{1}{\frac{1}{2}+1} + 1\right) || 2 = \left(\frac{2}{3}+1\right) || 2 = \frac{5}{3} || 2 = \frac{\frac{5}{3}\times 2}{\frac{5}{3}+2} = \frac{10}{11}\ \Omega$$

For determination of open-circuit voltage, the resistance of 2 Ω is removed from the circuit, the circuit becomes as shown in Fig. 2.94(*b*).

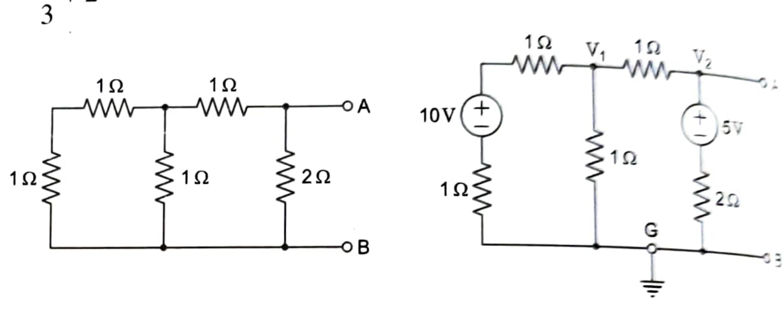


Fig. 2.94

Let node G be the reference node and potential of nodes 1 and 2 be V_1 and V_2 respectively. Applying KCL at node 1 we have

$$\frac{10 - V_1}{1+1} = \frac{V_1}{1} + \frac{V_1 - V_2}{1}$$

or $5\,V_1 - 2\,V_2 = 10$ ...(*i*)

Applying KCL at node 2 we have

$$\frac{V_1 - V_2}{1} = \frac{V_2 - 5}{2}$$

or $2\,V_1 - 3\,V_2 = -5$ ...(*ii*)

Solving Eqs. (*i*) and (*ii*) we have

$$V_1 = \frac{40}{11}\text{ V and } V_2 = \frac{45}{11}\text{ V}$$

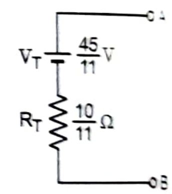


Fig. 2.94(*c*)

Open-circuit or Thevenin's voltage, $V_T = V_2 = \frac{45}{11}$ V

Thevenin's equivalent circuit is shown in Fig. 2.94(*c*).

Example 2.39. Find V_{Th} and R_{Th} for the circuit shown in Fig. 2.95.

[U.P. Technical Univ. Electrical Engineering Odd Semester 2013-14]

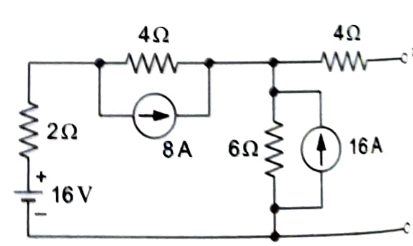


Fig. 2.95

Solution: The equivalent resistance of the network (with voltage source replaced by a short circuit and current sources replaced by open-circuits with reference to terminals P-Q [Fig. 2.96]

$R_T = 4 + 6 \,||\, (4 + 2)$

$= 4 + 6 \,||\, 6 = 4 + 3 = 7\ \Omega$ **Ans.**

The given circuit may be redrawn, as illustrated in Fig. 2.97. The current sources are converted into their equivalent voltage sources, the circuit with terminals PQ kept open.

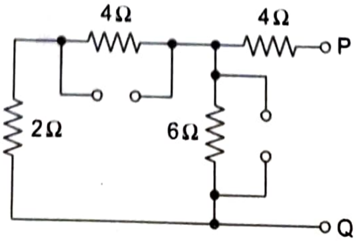


Fig. 2.96

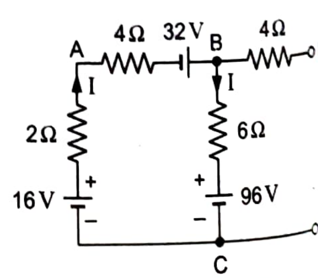


Fig. 2.97

Current flowing through closed loop, $I = \dfrac{16+32-96}{2+4+6} = \dfrac{-48}{12} = -4A$

Thevenin's voltage, V_{Th} = Voltage across branch BC $= 96 + 6I = 96 + 6 \times (-4) = 72$ V **Ans.**

Example 2.40. Using Thevenin's theorem find the current flowing through 6 Ω resistor of the network shown in Fig. 2.98.
[R.G.T.U. Basic Electrical and Electronics Engg., December-2012]

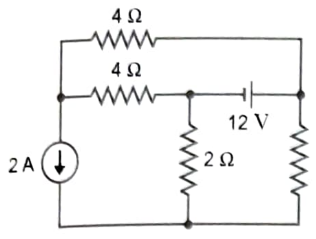


Fig. 2.98

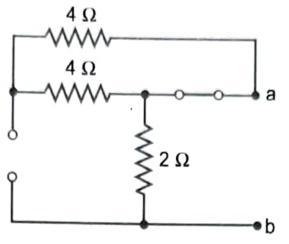


Fig. 2.99

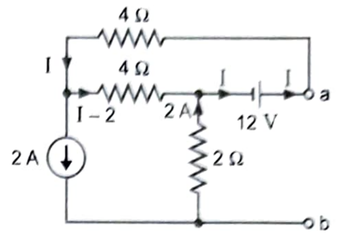


Fig. 2.100

Solution: For determination of equivalent resistance of the circuit w.r.t. terminals *a* and *b*, the voltage source is short circuited and current source is open circuited and 6 Ω resistance is removed as shown in Fig. 2.99.

So, $R_T = 2\ \Omega$

For determination of open-circuit voltage across terminals *a* and *b*, 6 Ω resistor is removed and the circuit becomes as shown in Fig. 2.100.

V_{oc} = Open-circuit voltage across terminals *a* and *b*, $V_{ab} = 12 - 2 \times 2 = 8$ V

Current flowing through 6 Ω resistor

$$= \frac{V_{oc}}{R_T + R_L} = \frac{8}{2+6} = 1 \text{ A}$$ from terminal *a* to terminal *b*. **Ans.**

Example 2.41. Find the Thevenin's equivalent circuit of the circuit shown in Fig. 2.101 to the left of terminals *a-b*. Then find the current through R_L = 6 Ω, 16 Ω and 36 Ω.
[DCRUSTM Electrical Technology, December-2011/January-2012]

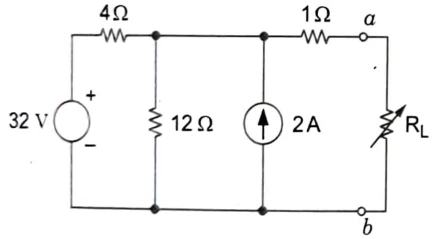
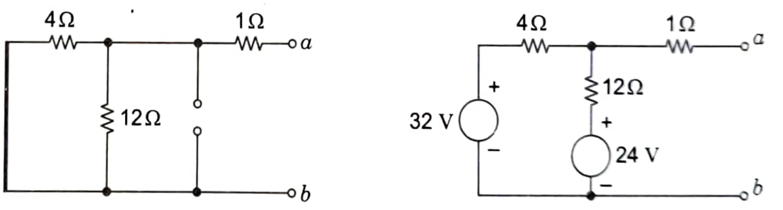


Fig. 2.101



(*a*) (*b*)

Fig. 2.102

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals *a* and *b*, the voltage source is short circuited and current source is open circuited, as shown in Fig. 2.102(*a*).

Thevenin's equivalent resistance, $R_T = 1 + 12 \,||\, 4 = 1 + \dfrac{1}{\dfrac{1}{12} + \dfrac{1}{4}} = 4\ \Omega$

Converting current source into equivalent voltage, the circuit becomes as shown in Fig. 2.102(*b*). When terminals *a* and *b* are open, the current flowing through the closed mesh.

$$I = \frac{32-24}{4+12} = 0.5 \text{ A}$$

So open-circuit voltage, $V_T = 32 - 4 \times 0.5 = 30$ V

Thevenin's equivalent circuit is shown in Fig. 2.102(*c*).

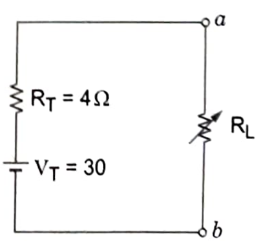


Fig. 2.102(*c*)

Current through R_L, $I = \dfrac{V_T}{R_T + R_L}$

$$I = \frac{30}{4+6} = 3 \text{ A } \textbf{Ans.} \qquad \text{for } R_L = 6\ \Omega$$

$$I = \frac{30}{4+16} = 1.5 \text{ A } \textbf{Ans.} \qquad \text{for } R_L = 16\ \Omega$$

$$I = \frac{30}{4+36} = 0.75 \text{ A } \textbf{Ans.} \qquad \text{for } R_L = 36\ \Omega.$$

Example 2.42. Applying Thevenin's theorem to the circuit given below (Fig. 2.103), calculate the current through 2 Ω resistor.

[B.P. Univ. of Technology Basic Electrical Engineering, 2010]

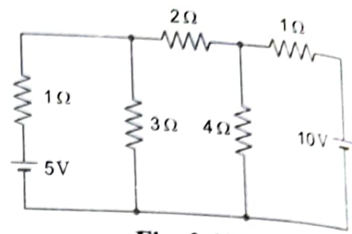


Fig. 2.103

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals A and B, the voltage sources are replaced by short circuits and 2 Ω resistor is removed as shown in Fig. 2.104(*a*).

Thevenin's equivalent resistance,

$$R_T = (3 \parallel 1) + (4 \parallel 1)$$

$$= \frac{3}{4} + \frac{4}{5} = \frac{31}{20}\ \Omega$$

For determination of Thevenin's voltage across terminals A and B, let us determine the potentials of terminals A and B (Fig. 2.104(*b*)).

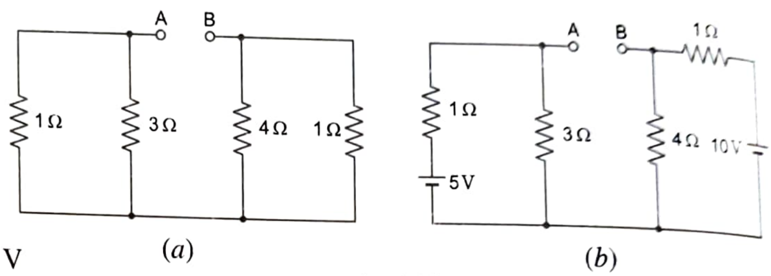


Fig. 2.104

$$\text{Potential of terminal A, } V_A = \frac{5}{1+3} \times 3 = \frac{15}{4} \text{ V}$$

$$\text{Potential of terminal B, } V_B = \frac{10}{1+4} \times 4 = 8 \text{ V}$$

$$\text{Thevenin's voltage, } V_{BA} = 8 - \frac{15}{4} = \frac{17}{4} \text{ V}$$

Current through 2 Ω resistor connected between terminals A and B,

$$I_L = \frac{V_T}{R_T + R_L} = \frac{V_{BA}}{R_T + R} = \frac{17/4}{\frac{31}{20} + 2} = \frac{85}{71} \text{ A from terminal B to terminal A } \textbf{Ans.}$$

Example 2.43. Calculate current in branch AB in the unbalanced bridge in Fig. 2.105 by Thevenin's theorem.

[Rajasthan Technical Univ. January-February 2011]

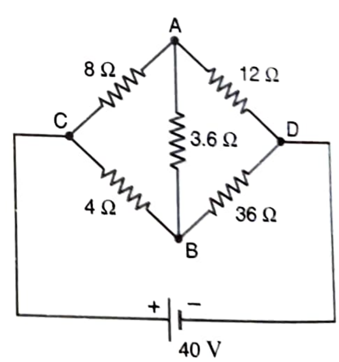


Fig. 2.105

Solution: After removing the 3.6 Ω resistor between terminals A and B, the circuit takes the form shown in Fig. 2.106(*a*).

Now in the circuit shown in Fig. 2.106(*a*), potential of terminal A w.r.t. terminal D,

$$V_{AD} = 40 \times \frac{12}{8+12} = 24 \text{ V}$$

$$\text{Similarly } V_{BD} = 40 \times \frac{36}{4+36} = 36 \text{ V}$$

PD between terminals A and B,

$$V_T = V_{AD} - V_{BD} = 24 - 36 = -12 \text{ V}$$

Short circuiting the 40 V battery, equivalent resistance of the network with reference to terminals A and B [Fig. 2.106(*b*)],

$$R_T = 8\ \Omega \,||\, 12\ \Omega + 4\ \Omega \,||\, 36\ \Omega$$

$$= \frac{8 \times 12}{8+12} + \frac{4 \times 36}{4+36} = 4.8 + 3.6 = 8.4\ \Omega$$

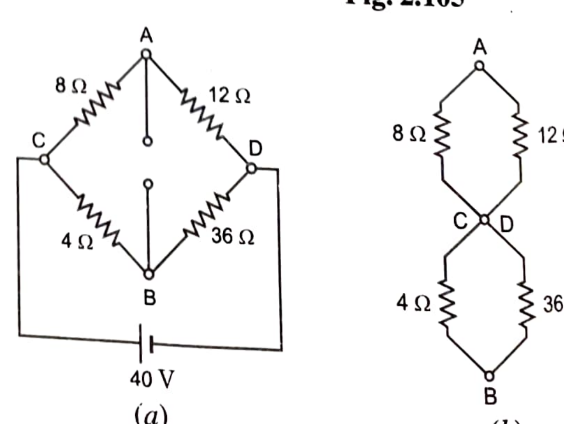


Fig. 2.106

Current in branch AB, $I = \dfrac{V_T}{R_T + R_{AB}}$

$= \dfrac{-12}{8.4 + 3.6} = -1$ A *i.e.* 1 A from terminal B to terminal A **Ans.**

Example 2.44. State Thevenin's theorem and calculate current in a 1,000 Ω resistor connected between terminals A and B, as shown in Fig. 2.107.

[*Electrical Engineering U.P. Technical Univ., June-2001; G.B. Technical Univ. Odd Semester, 2012-13*]

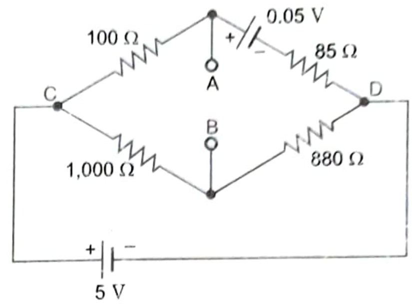


Fig. 2.107

Solution: Now in the circuit shown in Fig. 2.107,

Potential of point B w.r.t. terminal D

$$V_{BD} = \frac{5 \times 880}{1{,}000 + 880} = 2.340426 \text{ V}$$

Potential of point A w.r.t. to terminal D

$$V_{AD} = \frac{5 - 0.05}{100 + 85} \times 85 + 0.05 = 2.324324 \text{ V}$$

PD between terminals B and A,

$$V_T = 2.340426 - 2.324324 = 0.0161 \text{ V}$$

After replacing batteries by short circuits in the circuit, equivalent resistance of the network with reference to terminals A and B (Fig. 2.108),

$$R_T = 100 \,||\, 85 + 1{,}000 \,||\, 880$$

$$= \frac{1700}{17 + 20} + \frac{22{,}000}{22 + 25} = 514 \; \Omega$$

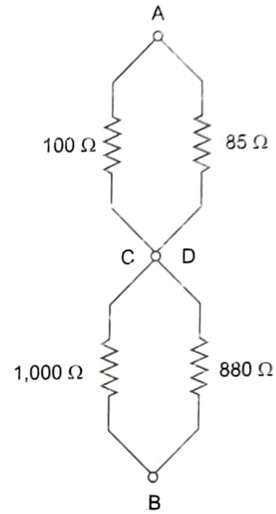


Fig. 2.108

Current in 1,000 Ω resistor connected between terminals A and B,

$$I = \frac{V_T}{R_T + R_L} = \frac{0.0161}{1{,}000 + 514}$$

$= 10.625$ μ A from terminals B to A **Ans.**

Example 2.45. How Norton's theorem is equivalent to Thevenin's theorem? Also write the limitations of Thevenin's theorem and find the voltage across load resistance R_L using Thevenin's theorem, when load resistance is 2 kΩ.

[*A.K. Technical Univ. Basic Electrical Engineering First Semester, 2015-16*]

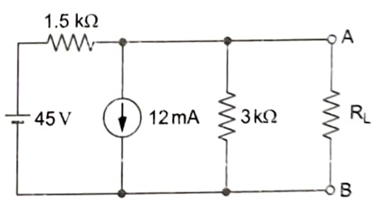


Fig. 2.109

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals A and B in Fig. 2.109, the voltage source is replaced by a short and current source is open circuited, as shown in Fig. 2.110(*a*).

Thevenin's equivalent resistance, $R_T = 1.5 \,||\, 3 = \dfrac{1}{\dfrac{1}{1.5} + \dfrac{1}{3}} = 1 \text{ k}\Omega$

Removing load resistance R_L and applying KVL to the loop formed by 45 V source, 1.5 kΩ resistor and 3 kΩ resistor we have, if the current supplied by the 45 V source is I mA, as shown in Fig. 2.110(*b*).

$$1.5 \times 10^3 \times I \times 10^{-3} + 3 \times 10^3 \times (I - 12) \times 10^{-3} = 45$$

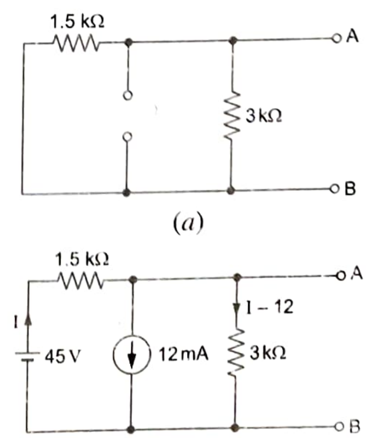
$$\text{or } 1.5 I \times 3 I - 36 = 45$$

$$\text{or } \quad I = \frac{45 + 36}{4.5} = 18 \text{ mA}$$

Current through 3 kΩ resistor $= I - 12 = 18 - 12 = 6$ mA .

Thevenin's voltage, V_T = Voltage drop across 3 kΩ resistor (keeping terminals A and B open)

$= 6 \times 10^{-3} \times 3 \times 10^3 = 18$ V



(*a*)

(*b*)

Fig. 2.110

Current through load resistor of 2 kΩ, $I_L = \frac{V_T}{R_L + R_T} = \frac{18}{(2+1)\times 10^3} = 6$ mA

Voltage across load resistance of 2 kΩ, $V_L = I_L \times R_L = 6 \times 10^{-3} \times 2 \times 10^3 = 12$ V **Ans.**

Example 2.46. Consider the circuit shown in Fig. 2.111. Determine open-circuit voltage across AB terminals shown in Fig. 2.111 by Thevenin's theorem.

[*U.P. Technical Univ. Electrical Engineering Odd Semester, 2013-14*]

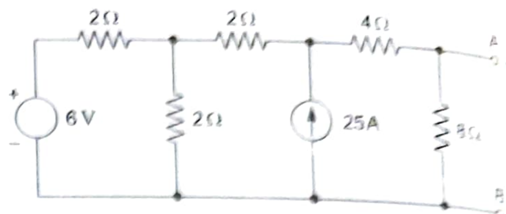


Fig. 2.111

Solution: For determining the Thevenin's equivalent resistance of the circuit with reference to terminals AB, the 6 V source is replaced by a short circuit and 25 A source is replaced by an open circuit, as shown in Fig. 2.112.

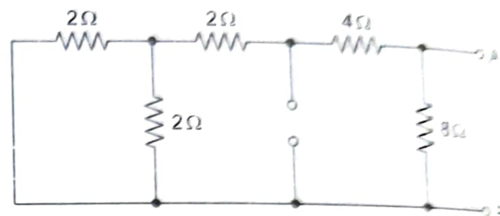


Fig. 2.112

Thevenin's equivalent resistance, $R_T = 8 \parallel (4 + 2 + 2 \parallel 2)$

$= 8 \parallel (4 + 2 + 1)$

$= 8 \parallel 7 = \frac{8 \times 7}{8 + 7} = \frac{56}{15}$ Ω

For determination of short-circuit current I_{SC} *i.e.* current in zero resistance connected across terminals AB [Fig. 2.113], Kirchhoff's voltage law is applied to loops I and II.

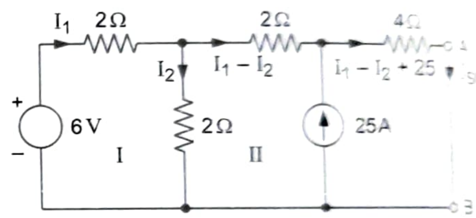


Fig. 2.113

$$2 I_1 + 2 I_2 = 6$$

$$\text{or } I_1 + I_2 = 3 \quad ...(i)$$

$$\text{and } 2 (I_1 - I_2) + 4 (I_1 - I_2 + 25) - 2 I_2 = 0$$

$$\text{or } 6 I_1 - 8 I_2 = -100 \quad ...(ii)$$

Solving Eqs. (*i*) and (*ii*) we have

$$I_2 = \frac{59}{7} \text{ A}$$

$$\text{and } I_1 = \frac{-38}{7} \text{ A}$$

Short-circuit current, I_{SC} = Current flowing through 4 Ω resistor

$$= I_1 - I_2 + 25 = \frac{-38}{7} - \frac{59}{7} + 25 = \frac{78}{7} \text{ A}$$

Open-circuit voltage, $V_{OC} = I_{SC} \times R_T = \frac{78}{7} \times \frac{56}{15} = 41.6$ V **Ans.**

Example 2.47. Determine the current in the 1 Ω resistor connected across AB of the network of Fig. 2.114 using Thevenin's theorem.

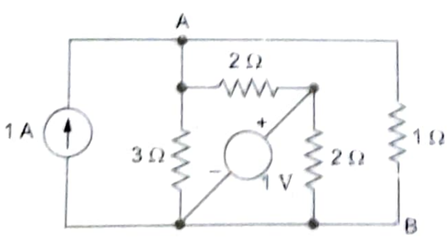
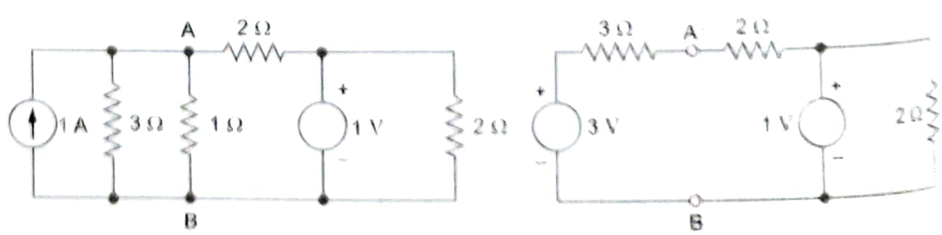


Fig. 2.114



(*a*) (*b*)

Fig. 2.115

Solution: The given circuit may be redrawn, as illustrated in Fig. 2.115 (*a*). Now the current source is converted into its equivalent voltage source, the circuit, with terminals AB kept open, becomes as shown in Fig. 2.115 (*b*).

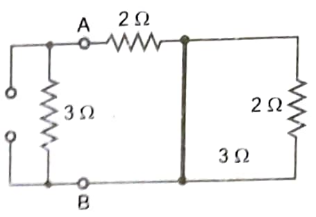
Open-circuited voltage across terminals AB,

$$V_T = 3 - \frac{3-1}{3+2} \times 3 = 1.8 \text{ V}.$$

The equivalent resistance of the network (with voltage source replaced by a short circuit and current source replaced by an open circuit) with reference to terminals A-B, [Fig. 2.115 (*c*)]

$$R_T = 3 \parallel 2 = \frac{3 \times 2}{3+2} = 1.2\ \Omega$$

$$\text{Current in } 1\ \Omega \text{ resistor} = \frac{V_T}{R_T + 1} = \frac{1.8}{1.2+1} = 0.82\ \text{A}\ \textbf{Ans.}$$



(*c*)

Fig. 2.115

2.16 NORTON'S THEOREM

This theorem is in fact, an alternative to the Thevenin's theorem. Whereas by Thevenin's theorem a complex two-terminal network may be simplified for solution by reducing it into a simple circuit in which the so called open-circuit voltage and looking back resistance are connected in series with the load resistance, by Norton's theorem network is reduced into a simple circuit in which a parallel combination of constant current source and looking back resistance feeds the load resistance.

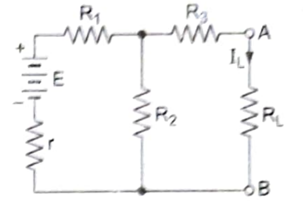
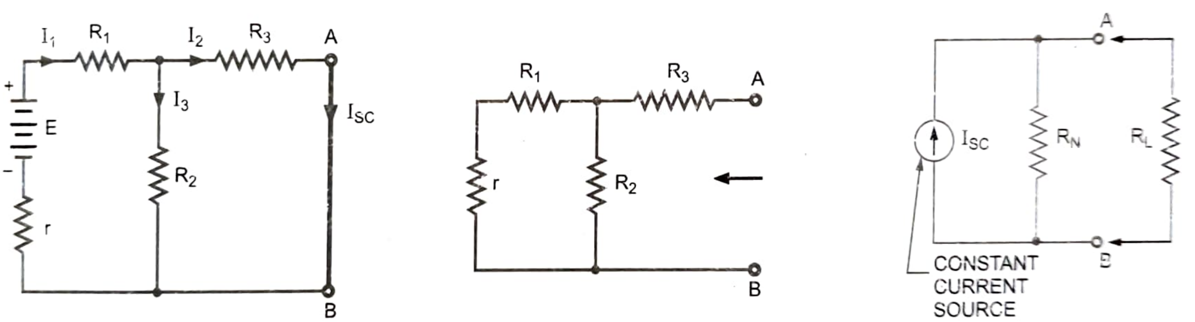


Fig. 2.116

In both theorems use of resistance looking back into the network from the load terminals, with all sources removed leaving their internal resistances in the circuit, is made. However, while solving circuit by Thevenin's theorem, the open-circuit voltage is determined at the load terminals with the load removed whereas in Norton's method use of a fictitious constant current source is made, the constant current delivered being equal to the current that would pass into a short circuit connected across the output terminals of the given network.

Now for understanding this theorem let us consider a circuit shown in Fig. 2.116 in which load current I_L is to be determined.



(*a*) (*b*) (c)

Fig. 2.117

For *determination of short-circuit current* I_{SC}, terminals A and B are short circuited by zero resistance thick wire, as illustrated in Fig. 2.117(*a*).

$$\text{Equivalent resistance of the network, } R = (R_1 + r) + R_2 \parallel R_3 = (R_1 + r) + \frac{R_2 R_3}{R_2 + R_3} \quad \text{...(2.28)}$$

$$\text{Current supplied by battery, } I_1 = \frac{E}{R} = \frac{E}{R_1 + r + \dfrac{R_2 R_3}{R_2 + R_3}} \quad \text{...(2.29)}$$

Short-circuit current, I_{SC} = Current flowing through resistance R_3,

$$I_2 = I_1 \times \frac{R_2}{R_2 + R_3} \qquad \text{By current division rule}$$

$$= \frac{E}{R_1 + r + \dfrac{R_2 R_3}{R_2 + R_3}} \times \frac{R_2}{R_2 + R_3} = \frac{ER_2}{(r + R_1)(R_2 + R_3) + R_2 R_3} \quad \text{...(2.30)}$$

For determination of internal resistance R_{in} (or R_N) of the network under consideration, remove the load resistance R_L from terminals A and B and also remove voltage source from the circuit leaving behind only its internal resistance, as illustrated in Fig. 2.117(*b*)

Equivalent resistance (R_N), as viewed from the open-terminals A and B is given as

$$R_N = R_3 + R_2 \,||\, (R_1 + r)$$

$$= R_3 + \frac{R_2\,(R_1 + r)}{R_2 + R_1 + r} = \frac{R_2 R_3 + (R_1 + r)(R_2 + R_3)}{R_1 + R_2 + r} \quad ...(2.31)$$

Now when load resistance R_L is connected across terminals A and B, the network behaves as constant current source of current I_{SC} in parallel with a resistance R_N, as shown in Fig. 2.117(*c*) and current flowing through the load resistance is given as

$$I_L = \frac{I_{SC}\,R_N}{R_N + R_L} \qquad \text{By current division rule}$$

$$= \frac{\dfrac{E\,R_2}{(r+R_1)(R_2+R_3)+R_2R_3} \times \dfrac{R_2R_3+(R_1+r)(R_2+R_3)}{R_1+R_2+r}}{\dfrac{R_2R_3+(R_1+r)(R_2+R_3)}{R_1+R_2+r}+R_L}$$

$$= \frac{E\,R_2}{R_2R_3+(R_1+r)(R_2+R_3)+R_L(R_1+R_2+r)} \quad ...(2.32)$$

Norton's equivalent circuit is illustrated in Fig. 2.117(*c*).

Norton's theorem can be stated as follows:

The current in any *passive circuit element* (which may be called R_L) in a network is the same as would flow in it if it were connected in parallel with R_N and the parallel pair were supplied with a constant current I_{SC}. R_N is the resistance measured "looking back" into the original circuit after R_L has been disconnected and all the sources have been replaced by their internal resistances: I_{SC} is the current which will flow in a short placed at the terminals of R_L in the original circuit.

2.17 CONVERSION OF THEVENIN'S EQUIVALENT INTO NORTON'S EQUIVALENT AND VICE VERSA

A Thevenin's equivalent can be converted into its Norton's equivalent and vice versa. A Thevenin's equivalent is depicted in Fig. 2.118. According to statement made in Art 2.16. Norton's current source equals the current I_{SC} which flows through a short across terminals A and B.

$$\text{Hence } I_{SC} = \frac{V_{OC}}{R_{in}} \quad ...(2.33)$$

Likewise a Norton's circuit can be converted into its Thevenin's equivalent. The Thevenin's equivalent source V_{OC} or V_T is the voltage on open circuit and is given as

$$V_{OC} \text{ or } V_T = I_{SC}\,R_{in} \quad ...(2.34)$$

Each theorem is dual of the other.

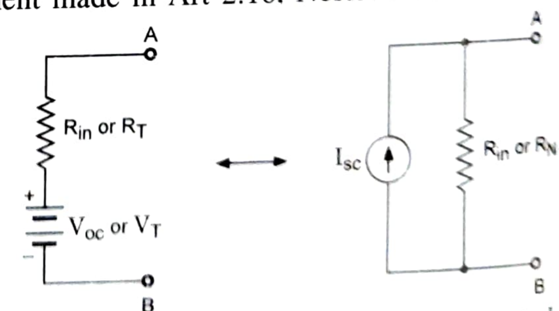


Fig. 2.118 *Thevenin's Equivalent Compared With Norton's Equivalent*

Example 2.48. Find the value of current flowing through 4 Ω resistance in the given circuit (Fig. 2.119) by using Norton's theorem.

[M.D. Univ. Electrical Technology, May-2012]

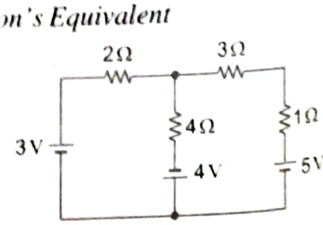


Fig. 2.119

Solution: For determination of Norton's equivalent resistance of the given circuit, 4 Ω resistor is removed and all the batteries are short circuited, as shown in Fig. 2.120(*a*).

Norton's equivalent resistance,

$$R_N = 2 \parallel (3+1)\ \Omega = \frac{1}{\frac{1}{2}+\frac{1}{4}} = \frac{4}{3}\ \Omega$$

For determination of short-circuit current I_{sc}, let us determine V_T first. After removal of 4 Ω resistance from the circuit shown in Fig. 2.119 we have circuit as shown in Fig. 2.120(*b*).

Current flowing through the closed mesh of circuit shown in Fig. 2.120(*b*)

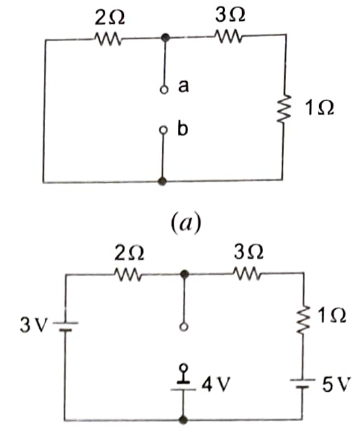
$$I = \frac{5-3}{1+3+2} = \frac{1}{3}\ \text{A}$$

$$\text{Open-circuit voltage, } V_T = \left(5 - \frac{1}{3}\times 4\right) - (-4) = \frac{11}{3} + 4 = \frac{23}{4}\ \text{V}$$

$$\text{Short-circuit current, } I_{SC} = \frac{V_T}{R_N} = \frac{23/4}{4/3} = \frac{69}{16}\ \text{A}$$

Current through 4 Ω resistance,

$$I = \frac{I_{sc}}{R_N + R_L}\times R_N = \frac{69/16}{\frac{4}{3}+4}\times\frac{4}{3} = \frac{69\times 4}{16\times 16} = \frac{69}{64} = 1.078\ \text{A}\ \textbf{Ans.}$$



(*a*)

(*b*)

Fig. 2.120

Example 2.49. Find Norton's equivalent circuit to the left of terminals X-Y in the figure shown in Fig. 2.121.

[*M.D. Univ. Electrical Technology, May-2010*]

Solution: As the circuit to the left of X-Y in the given figure does not have any independent source, therefore I_{SC} or $I_N = 0$.

For determination of R_N of the given circuit w.r.t. terminals X-Y, let X-Y be kept open circuited and a dc voltage V_{dc} be applied, input current being I_{dc}. Now, we have

$$I_{dc} = \frac{V_{dc}}{4} + 0.5i + \frac{V_{dc}}{6}$$

$$= \frac{V_{dc}}{4} + 0.5\times\frac{V_{dc}}{6} + \frac{V_{dc}}{6} = \frac{V_{dc}}{2} \quad \because i = \frac{V_{dc}}{6}$$

$$\text{or} \quad R_N = \frac{V_{dc}}{I_{dc}} = 2\ \Omega \quad \textbf{Ans.}$$

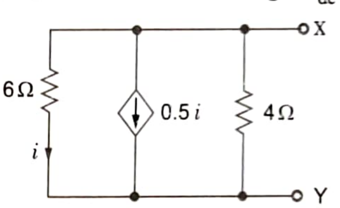


Fig. 2.121

Example 2.50. Using Norton's theorem, find the current which would flow in a 15 Ω resistor connected between points A and B in the following figure:

[*Chhattisgarh Vivekanand Technical Univ. 2006-07*]

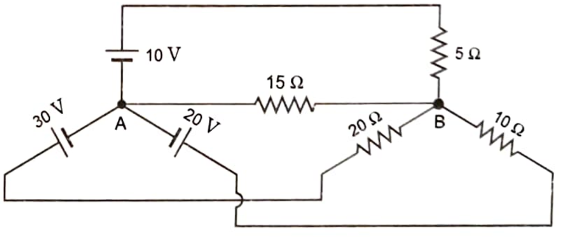
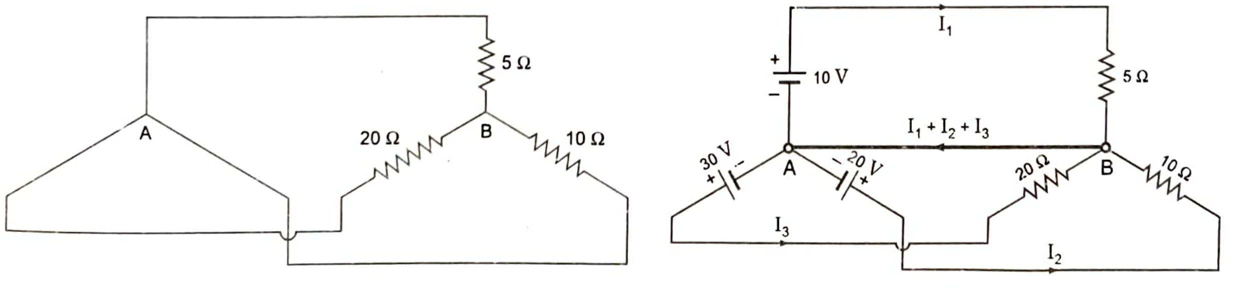


Fig. 2.122

Solution: Equivalent resistance of network when viewed from terminals A and B, keeping all the voltage short-circuited, [Fig. 2.123(*a*)]

$$R_A = 5 \parallel 10 \parallel 20$$

$$= \frac{1}{\frac{1}{5}+\frac{1}{10}+\frac{1}{20}} = \frac{20}{7}\ \Omega$$



(*a*)

(*b*)

Fig. 2.123

Short-circuit current *i.e.* the current in zero resistance conductor connected across terminals AB [Fig. 2.123(*b*)]

$$I_{SC} = I_1 + I_2 + I_3 = \frac{10}{5} + \frac{20}{10} + \frac{30}{20} = 5.5 \text{ A}$$

Current through a resistance of 15 Ω connected between points A and B.

$$I = \frac{I_{SC}}{R_N + R_L} \times R_N = \frac{5.5 \times \frac{20}{7}}{\frac{20}{7} + 15} \text{ A} = 0.88 \text{ A} \quad \textbf{Ans.}$$

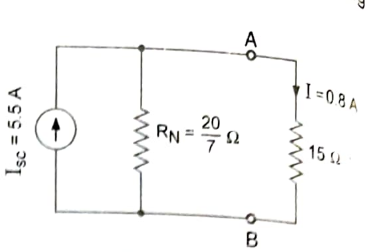


Fig. 2.123(*c*)

Example 2.51. Refer to Example 2.42 and verify your answer with the help of Norton's theorem.

[B.P. Univ. of Technology Basic Electrical Engineering, 2010]

Solution: Norton's equivalent circuit, $R_N = R_T = \frac{31}{20}$ Ω

$$\text{Thevenin's voltage, } V_{oc} = \frac{17}{4} \text{ V, as determined in Example 2.42}$$

$$\text{Short-circuit current, } I_s = \frac{V_{oc}}{R_N} = \frac{17/4}{31/20} = \frac{85}{31} \text{ A}$$

$$\text{and load current, } I_L = \frac{I_{sc}R_N}{R_N + R_L} = \frac{\frac{85}{31} \times \frac{31}{20}}{\frac{31}{20} + 2} = \frac{85/20}{71/20} = \frac{85}{71} \text{ A, same as worked out in Example 2.42}$$

Example 2.52. State Norton's theorem. Find current through 15 Ω by using Norton's theorem. *[U.P. Technical Univ. Basic Electrical Engineering Odd Semester, 2013-14]*

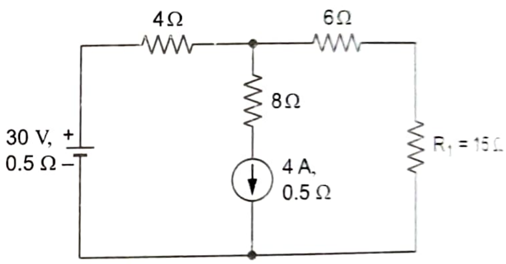


Fig. 2.124

Solution: Norton's equivalent resistance of the given network (*i.e.* equivalent resistance of the network after terminals A and B are open circuited, 30 V battery removed from the circuit leaving its internal resistance of 0.5 Ω in the circuit and removing 4 A current source leaving 0.5 Ω resistance in the circuit) as viewed from terminals A and B shown in Fig. 2.125 (*a*)

$$R_N = 6 + 4 + 0.5 = 10.5 \text{ Ω}$$

For determining short-circuit current I_{SC} *i.e.* current in zero resistance connected across terminals A and B, resistance R_1 of 15 Ω is replaced by a zero resistance as shown in Fig. 2.125 (*b*).

Applying Kirchhoff's voltage law to outer loop we have

$$(0.5 + 4)\,I + 6\,(I - 4) = 30$$

$$\text{or } 10.5\,I = 54$$

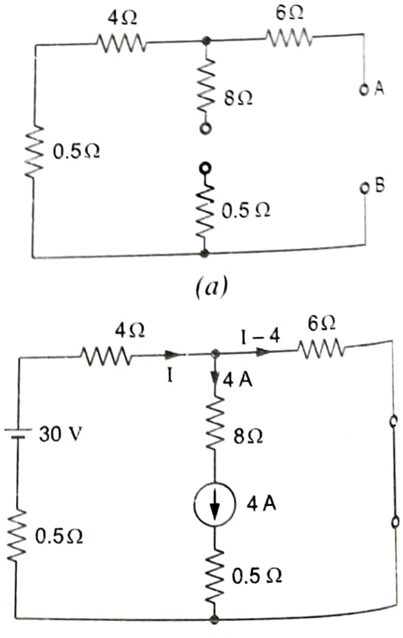
$$\text{or } I = \frac{36}{7} \text{ A}$$

$$\text{Short-circuit current, } I_{SC} = I - 4 = \frac{36}{7} - 4 = \frac{8}{7} \text{ A}$$

$$\text{Current through 15 Ω resistor} = \frac{I_{SC}}{R_N + R_1} \times R_N$$

$$= \frac{8/7}{10.5 + 15} \times 10.5$$

$$= \frac{8}{17} \text{ A} \quad \textbf{Ans.}$$



(*a*) (*b*)

Fig. 2.125

Example 2.53. Draw the Norton's equivalent circuit across AB, and determine current flowing through 12 Ω resistor for the network shown in Fig. 2.126.

[*U.P. Technical Univ. Electrical Engineering Second Semester, 2003-04*]

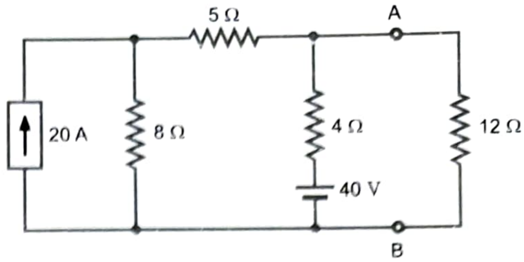
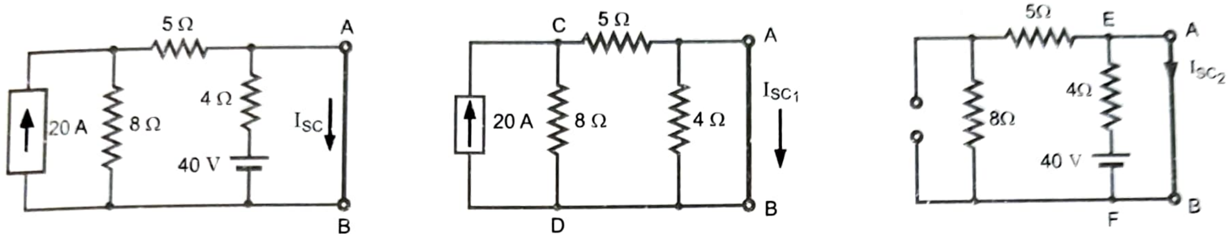


Fig. 2.126

Solution: As illustrated in Fig. 2.127 (*a*), terminals A and B have been shorted after removing 12 Ω resistor.

Now short-circuit current is determined by making use of superposition theorem.



(*a*) (*b*) (*c*)

Fig. 2.127

(*i*) *When only current source is present* [Fig. 2.127 (*b*)]

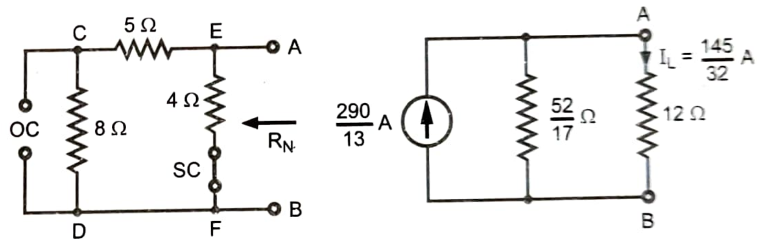
In this case 40 V battery is replaced by a short circuit. The 20 A current divides at point C between parallel combination of 8 Ω and 5 Ω (no current will flow through 4 Ω resistor being short circuited at terminals A and B).

I_{SC_1} = Current through 5 Ω resistor due to current source alone

$$= 20 \times \frac{8}{8+5} = \frac{160}{13} \text{ A} = 12.30\text{A}$$

(*ii*) *When only voltage source is present* [Fig. 2.127 (*c*)]

In this case, current source is replaced by an open circuit. Voltage across terminals EF is equal to voltage across terminals A and B *i.e.* zero. So short-circuit current,



(*d*) (*e*)

Fig. 2.127

$$I_{SC_2} = \text{Current through 4 Ω resistor} = \frac{40}{4} = 10 \text{ A}$$

$$\text{Short-circuit current, } I_{SC} = I_{SC_1} + I_{SC_2} = \frac{160}{13} + 10 = \frac{290}{13} \text{ A}$$

As seen from Fig. 2.127 (*d*), Norton's equivalent resistance of the network,

$$R_N = 4 \parallel (5+8) = 4 \parallel 13 = \frac{1}{\frac{1}{4} + \frac{1}{13}} = \frac{52}{17} \, \Omega$$

Fig. 2.127 (*e*) shows the Norton's equivalent circuit with the load resistor of 12 Ω.

$$\text{Load current, } I_L = I_{SC} \times \frac{R_N}{R_N + R_L} = \frac{290}{13} \times \frac{52/17}{\frac{52}{17} + 12} = \frac{1,160}{256} = 4.53125 \text{ A} \textbf{ Ans.}$$

2.18 NETWORK REDUCTION BY DELTA-STAR TRANSFORMATION OR VICE VERSA

Three resistances connected nose-to-tail as shown in Fig. 2.128(*a*), are said to be *delta* (or Δ) or *mesh-connected* (they form a mesh). Three resistances connected together at a common point O, as shown in Fig. 2.128(*b*), are said to *be star* (or Y)-*connected.* If the nodes (A, B, and C) to which the two sets of resistances are connected are part of a larger network, it is possible to assign values to the two sets of resistances so that they have exactly the same effect on the network. If, therefore, delta-connected resistances are part of a network it is possible to substitute them by the star-connected ones and vice versa. The obvious advantages are that a delta-star transformation eliminates a mesh which reduces by one the variables and equations necessary to solve a network by mesh analysis whereas star-delta transformation eliminates a node (node O) which reduces by one the variables and equations necessary to solve a network by node analysis.

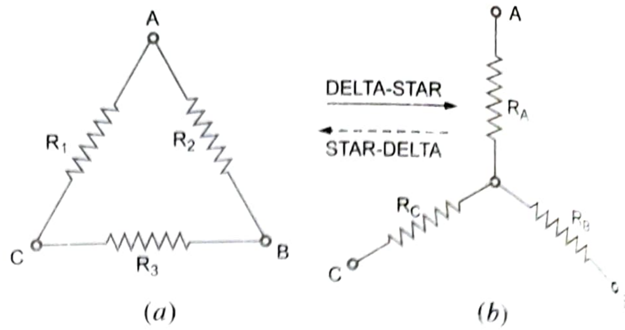


Fig. 2.128 *Delta-Star Transformation*

2.18.1. Delta-Star Transformation. *The replacement of delta or mesh by equivalent star system is known as delta-star transformation.*

The two systems will be equivalent or identical if the resistances measured between any pair of lines are same in both of the systems, when the third line is open.

Hence resistances between terminals B and C,

$$R_{BC} = R_3 \parallel (R_1 + R_2) = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \quad \text{in delta system}$$

$$\text{and } R_{BC} = R_B + R_C \quad \text{in star system}$$

Since the two systems are identical, resistances measured between terminals B and C in both of the systems must be equal.

$$\text{So } R_B + R_C = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \quad \text{...(2.35)}$$

Similarly resistances between terminals C and A being equal in the two systems

$$R_C + R_A = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \quad \text{...(2.36)}$$

And resistance between terminals A and B

$$R_A + R_B = \frac{R_2(R_1+R_3)}{R_1+R_2+R_3} \quad \text{...(2.37)}$$

Adding Eqs. (2.35), (2.36) and (2.37) we have

$$2(R_A + R_B + R_C) = \frac{2(R_1 R_3 + R_2 R_3 + R_1 R_2)}{R_1+R_2+R_3}$$

$$\text{or } R_A + R_B + R_C = \frac{R_1 R_3 + R_2 R_3 + R_1 R_2}{R_1+R_2+R_3} \quad \text{...(2.38)}$$

Subtracting Eqs. (2.35), (2.36) and (2.37) from Eq. (2.38) we have respectively

$$R_A = \frac{R_1 R_2}{R_1+R_2+R_3}$$

$$R_B = \frac{R_2 R_3}{R_1+R_2+R_3}$$

$$R_C = \frac{R_3 R_1}{R_1+R_2+R_3}$$

These relationships may be expressed as follows:

The equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta-connected resistances.

If the three delta-connected resistances have the same value R_D, the three resistances in the equivalent star for identical systems will be

$$R_S = \frac{R_D\,R_D}{R_D + R_D + R_D} = \frac{R_D}{3} \qquad \text{...(2.42)}$$

2.18.2. Star-Delta Transformation. Multiplying Eqs. (2.39) and (2.40), (2.40) and (2.41) and (2.41) and (2.39) and then adding them we get

$$R_A\,R_B + R_B\,R_C + R_C\,R_A = \frac{R_1\,R_2^2\,R_3 + R_1\,R_3^2\,R_2 + R_1^2\,R_2\,R_3}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1\,R_2\,R_3(R_2 + R_3 + R_1)}{(R_1 + R_2 + R_3)^2} = \frac{R_1\,R_2\,R_3}{R_1 + R_2 + R_3} \qquad \text{...(2.43)}$$

Dividing Eq. (2.43) by Eqs. (2.39), (2.40) and (2.41) separately we have

$$R_3 = \frac{R_A\,R_B + R_B\,R_C + R_C\,R_A}{R_A} = R_B + R_C + \frac{R_B\,R_C}{R_A} \qquad \text{...(2.44)}$$

$$R_1 = \frac{R_A\,R_B + R_B\,R_C + R_C\,R_A}{R_B} = R_A + R_C + \frac{R_A\,R_C}{R_B} \qquad \text{...(2.45)}$$

$$\text{and } R_2 = \frac{R_A\,R_B + R_B\,R_C + R_C\,R_A}{R_C} = R_A + R_B + \frac{R_A\,R_B}{R_C} \qquad \text{...(2.46)}$$

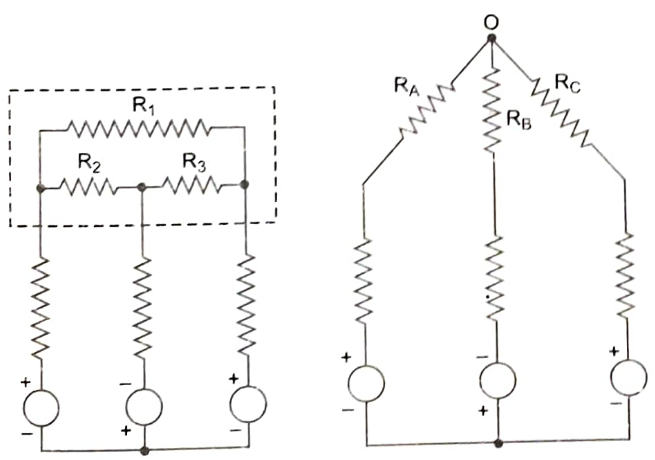
The above relationship may be expressed as below:

The equivalent delta resistance between two terminals is the sum of the two star resistances connected to those terminals plus the product of the same divided by the third star resistance.

If the three star-connected resistances have the same value, say R_S, the three resistances of the equivalent delta for identical systems will be

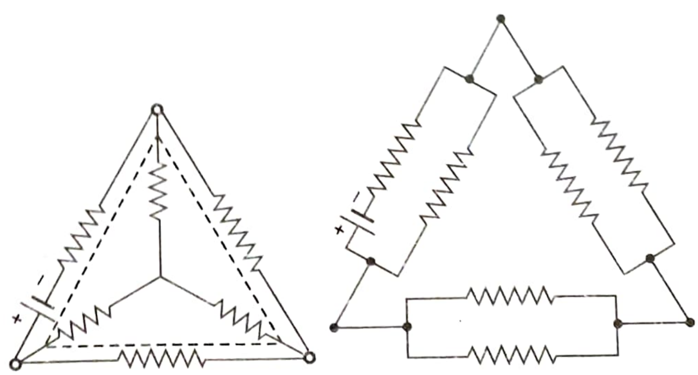
$$R_D = R_S + R_S + \frac{R_S\,R_S}{R_S} = 3\,R_S \qquad \text{...(2.47)}$$

The advantage of delta-star transformation may be shown by reference to network of Fig. 2.129. Fig. 2.129(*a*) illustrates the network before conversion, where the dotted lines are drawn around the delta connection to be transformed into a star. Fig. 2.129(*b*) illustrates the same network after transformation. The currents in the transformed form [Fig. 2.129(*b*)] are much simpler to determine.



(*a*) (*b*)

Fig. 2.129



(*a*) (*b*)

Fig. 2.130

The advantage of star-delta transformation can be illustrated by reference to network of Fig. 2.130. Figure 2.130(*a*) illustrates the network prior to transformation, with the dotted lines around the star to be transformed. Fig. 2.130(*b*) illustrates the same network after transformation. The original network is now reduced to a simple series parallel connection of resistances.

Example 2.54. Three resistors each of R Ω are connected in delta. If they are transferred to Y-connection, what will be the resistance of each resistor?

Solution: Three resistances in the equivalent star will be given as

$$R_s = \frac{R_D}{3} = \frac{R}{3} \ \Omega \ \textbf{Ans.} \qquad \text{Refer to Eq. (2.42)}$$

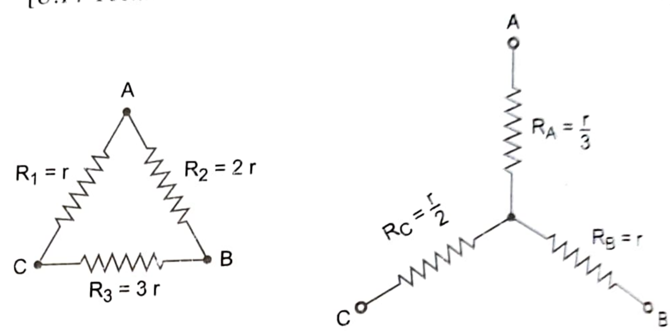
Example 2.55. Three resistances *r*, 2 *r*, and 3 *r* are connected in delta. Determine the resistances for an equivalent star connection. [*U.P. Technical Univ. Electrical Engineering January 2003*]

Solution: Resistances for the equivalent star-connection (Fig. 2.131) are worked out as below:

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{r \times 2r}{r + 2r + 3r} = \frac{r}{3} \quad \textbf{Ans.}$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2r \times 3r}{r + 2r + 3r} = r \quad \textbf{Ans.}$$

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3} = \frac{3r \times r}{r + 2r + 3r} = \frac{r}{2} \quad \textbf{Ans.}$$



(*a*) *Delta-Connected Resistors*   (*b*) *Equivalent Star-Connected Resistors*

Fig. 2.131

Example 2.56. Find the current in 10 Ω resistor in the network shown by star/delta transformation in Fig. 2.132.

[*G.G.S.I.P. Univ. Delhi Electrical Science May 2011*]

Solution: Converting delta ABC and delta DEF into equivalent star, we have

$$R_A = R_B = R_C = \frac{12}{3} = 4\ \Omega$$

$$R_D = R_E = R_F = \frac{30}{3} = 10\ \Omega$$

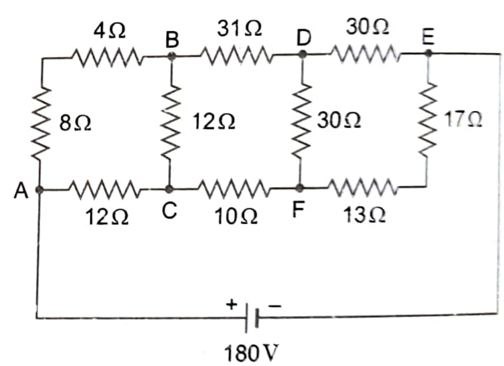


Fig. 2.132

The equivalent circuit becomes as shown in Fig. 2.133.

The equivalent resistance of the circuit shown in Fig. 2.133 is given as

$$R = 4 + [(4 + 31 + 10) \,||\, (4 + 10 + 10)] + 10$$

$$= 14 + \frac{45 \times 24}{45 + 24} = 29.65\ \Omega$$

$$\text{Current drawn, } I = \frac{180}{29.65} = 6.07\ \text{A}$$

Current through 10 Ω resistor

$$= I \times \frac{(4 + 31 + 10)}{(4 + 31 + 10) + (4 + 10 + 10)}$$

$$= 6.07 \times \frac{45}{69}$$

$$= 3.96\ \text{A}\ \textbf{Ans.}$$

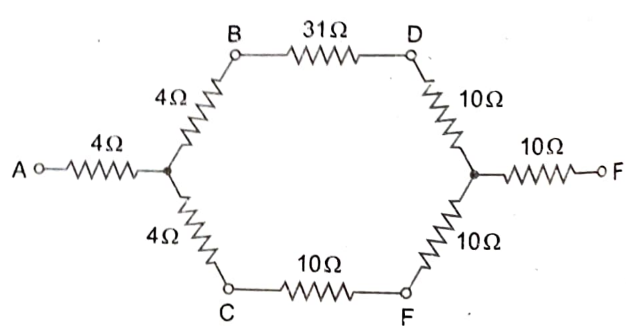


Fig. 2.133

Example 2.57. In the network in Fig. 2.134, find the resistances between the points A and B. *[M.D. Univ. Electrical Technology, May-2011]*

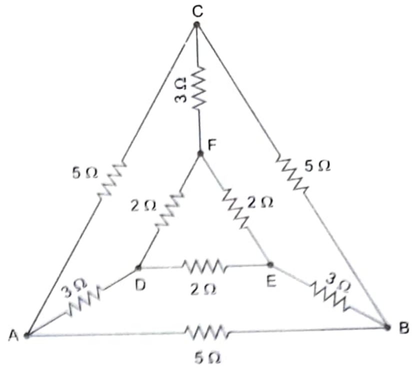


Fig. 2.134

Solution: Converting internal ΔDEF into equivalent star having resistance of

$$R_S = \frac{R_D}{3} = \frac{2}{3}\ \Omega.$$

Now internal star become as shown in Fig. 2.135(*a*). Now this star has an equivalent resistance of $\left(3 + \frac{2}{3}\right)\ \Omega$, *i.e.* $\frac{11}{3}\ \Omega$.

This star can be converted into equivalent delta having resistance

$$R'_D = 3R'_S = 3 \times \frac{11}{3} = 11\ \Omega.$$

When combined with external delta, the circuit becomes as shown in Fig. 2.135(*b*).

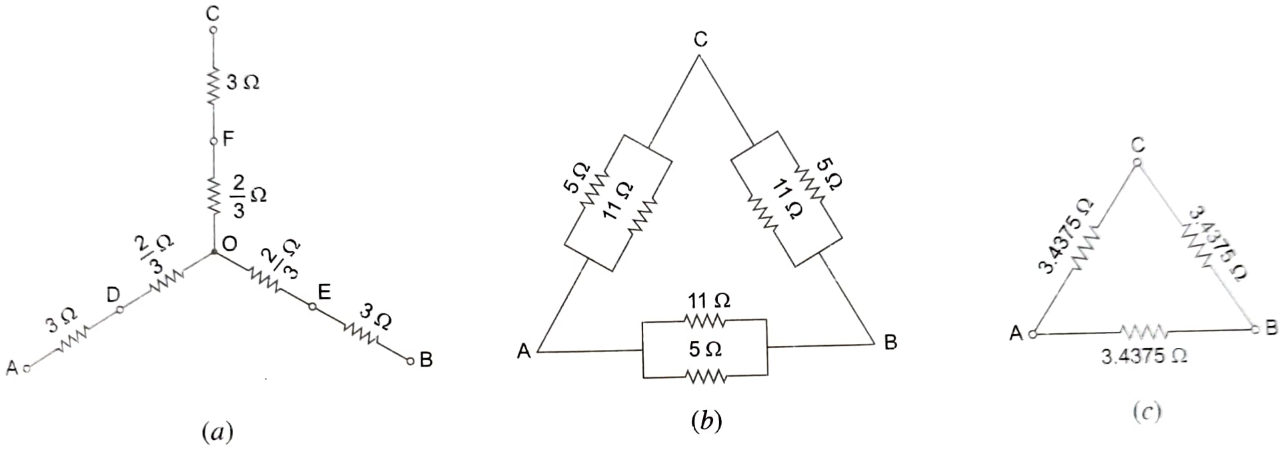


Fig. 2.135

The circuit shown in Fig. 2.135(*b*) is reduced to that shown in Fig. 2.135(c) having resistance of 5 Ω || 11 Ω, *i.e.*

$$\frac{5 \times 11}{5 + 11} = 3.4375\ \Omega \text{ in each arm.}$$

$$\text{Now resistance } R_{AB} = (3.4375)\ \Omega \parallel (3.4375 + 3.4375)\ \Omega$$

$$= 3.4375\ \Omega \parallel 6.875\ \Omega$$

$$= \frac{3.4375 \times 6.875}{3.4375 + 6.875} = 2.292\ \Omega \quad \textbf{Ans.}$$

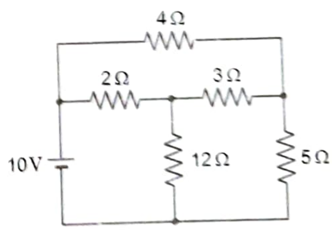


Fig. 2.136

Example 2.58. Find the current supplied by 10 V battery by using star-delta transformation. *[J.N. Technological Univ. Electrical Circuit Analysis, December-2012]*

Solution: By transforming delta formed by resistors of 4 Ω, 2 Ω and 3 Ω into star [Fig. 2.137(*a*) and (*b*)], we have

$$R_A = \frac{R_{AC} \times R_{AB}}{R_{AC} + R_{AB} + R_{BC}}$$

$$= \frac{2 \times 4}{2 + 4 + 3} = \frac{8}{9}\ \Omega$$

$$\text{Similarly } R_B = \frac{4 \times 3}{2 + 4 + 3} = \frac{4}{3}\ \Omega$$

$$\text{and } R_C = \frac{2 \times 3}{2 + 4 + 3} = \frac{2}{3}\ \Omega$$

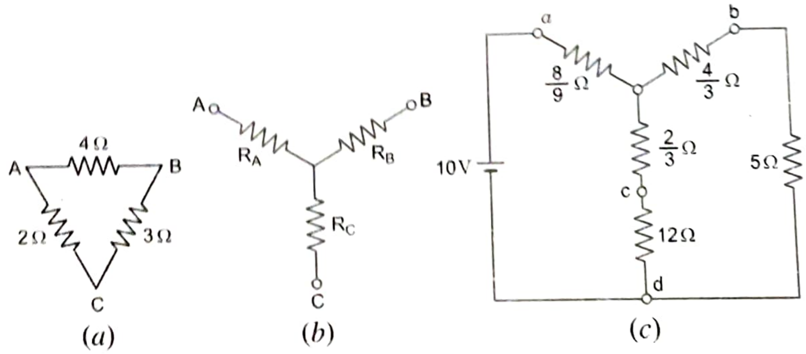


Fig. 2.137

Now the given circuit is reduced to that as shown in Fig. 2.137(*c*).

Resistance between terminals *a* and *d*

$$R = \frac{8}{9} + \left(\frac{2}{3} + 12\right) \| \left(\frac{4}{3} + 5\right) = \frac{8}{9} + \left(\frac{38}{3}\right) \| \frac{19}{3} = \frac{8}{9} + \frac{38}{3} \| \frac{19}{3} = \frac{8}{9} + \frac{\frac{38 \times 19}{9}}{\frac{38}{3} + \frac{19}{3}}$$

Current supplied by battery, $I = \frac{V}{R} = \frac{10}{46/9} = \frac{45}{23} = 1.957$ A **Ans.**

Example 2.59. Derive the relationship to convert delta-connected resistances to equivalent star. Also determine the equivalent resistance between the terminal A-B shown in Fig. 2.138.

[U.P. Technical Univ. Electrical Engineering First Semester 2014]

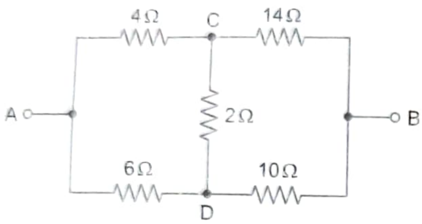


Fig. 2.138

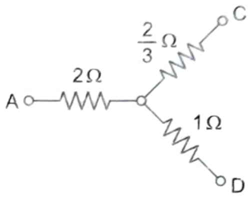


Fig. 2.139

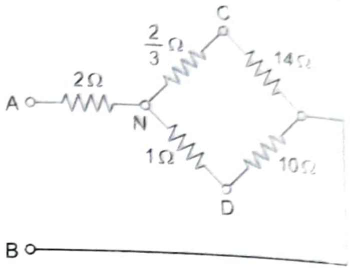


Fig. 2.140

Solution: As shown in Fig. 2.139, delta ACD has been reduced to its equivalent star.

$$R_A = \frac{R_{AC} R_{AD}}{R_{AC} + R_{CD} + R_{DA}} = \frac{4 \times 6}{4 + 2 + 6} = \frac{24}{12} = 2\ \Omega$$

$$\text{Similarly} \quad R_C = \frac{4 \times 2}{12} = \frac{2}{3} \Omega$$

$$\text{and} \quad R_D = \frac{6 \times 2}{12} = 1\ \Omega$$

Hence the given circuit is reduced to that shown in Fig. 2.140. As seen there are two parallel paths between points A and B. one is of resistance $\left(\frac{2}{3} + 14\right) \Omega$ *i.e.*, $\frac{44}{3} \Omega$ and the other is $(1 + 10)\ \Omega$ *i.e.* $11\ \Omega$

$$\text{Their combined resistance is } \frac{\frac{44}{3} \times 11}{\frac{44}{3} + 11} = \frac{44 \times 11}{77} = \frac{44}{7} \Omega$$

So total resistance of the network between terminals A-B is $2 + \frac{44}{7} = \frac{58}{7} \Omega$ **Ans.**

Example 2.60. Using delta to star transformation determine the resistance between terminals *a-b* and the total power drawn from the supply in the circuit shown in Fig. 2.141.

[U.P. Technical Univ. Electrical Engineering First Semester, 2006-07]

Solution: By transforming delta formed by resistors of 8 Ω, 7 Ω and 3 Ω into star we have

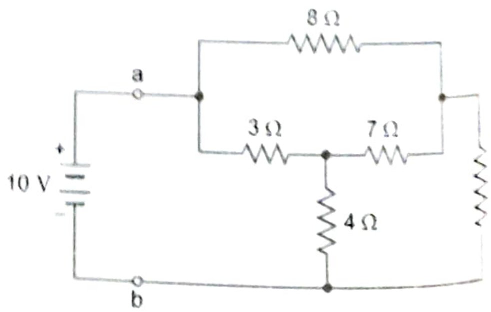


Fig. 2.141

$$R_A = \frac{R_{AC} \times R_{AB}}{R_{AC} + R_{AB} + R_{BC}} = \frac{3 \times 8}{3 + 8 + 7} = \frac{4}{3} \Omega$$

$$\text{Similarly } R_B = \frac{8 \times 7}{18} = \frac{28}{9} \Omega$$

$$\text{and } R_C = \frac{3 \times 7}{18} = \frac{7}{6} \Omega$$

Now the given circuit is reduced to that as shown in Fig. 2.142 (*c*).

Resistance between terminals *a* and *b*,

$$R = \frac{4}{3} + \left(\frac{7}{6} + 4\right) || \left(\frac{28}{9} + 10\right) = \frac{4}{3} + \frac{31}{7} || \frac{118}{9} = 1.3333 + 3.7062 = 5.04\ \Omega \ \textbf{Ans.}$$

$$\text{Total power drawn, } P = \frac{V^2}{R} = \frac{10^2}{5.04} = 19.84 \text{ watts } \textbf{Ans.}$$

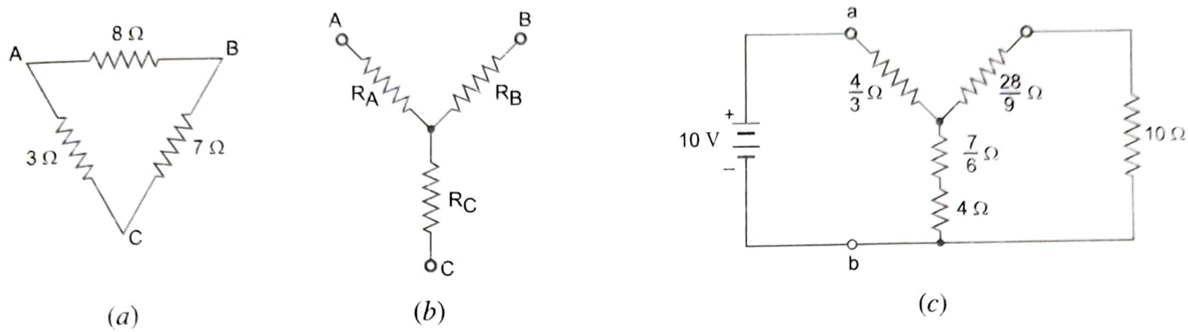


Fig. 2.142

Highlights

1. An *electric circuit* (or network) is an interconnection of physical electrical devices such as an *energy source* (or sources), an *energy convertor* or convertors (load or loads), and *conductors* that connect them.
2. A *junction (or node)* is a point in a network where two or more branches meet.
3. A *loop* is a closed path in a network formed by a number of connected branches. *Mesh* is a loop that contains no other loop within it.
4. Network elements may be classified into two categories viz. active elements and passive elements.

   The elements which supply energy to the network are known as *active elements*. The voltage sources like batteries, dc generators, ac generators and current sources like photoelectric cells, metadyne generators fall under the category of active elements. Most of the semiconductor devices like transistors are treated as current sources.

   The components which dissipate or store energy are known as *passive components*. Resistors, inductors and capacitors fall under the category of passive elements. The resistor is the only component which dissipates electrical energy. The inductors and capacitors are the components which store energy, the inductor stores energy by virtue of a current passing through it whereas the capacitor stores energy by virtue of potential difference across it.
5. A voltage source of voltage V_S and internal resistance R_{in} can be converted into an equivalent current source of current $I_S = V_S/R_{in}$ and a resistance R_{in} across it. Similarly a current source of output current I_S in parallel with resistance R_{in} can be converted into an equivalent voltage source of voltage $V_S = I_S R_{in}$ and a resistance R_{in} in series with it.

   A voltage source-series resistance combination is equivalent to a current source-parallel resistance combination if, and only if their respective open-circuit voltages are equal, and their respective short-circuit currents are equal.
6. According to *Kirchhoff's first law (or current law)*, the algebraic sum of currents in two or more conductors meeting at a point (junction) is always zero

   $$i.e.\ \Sigma I = 0$$

   While applying above law, incoming currents are taken as positive and outgoing currents as negative.

   According to *Kirchhoff's second law (or voltage law)*, the algebraic sum of emfs acting in any closed circuit or mesh is equal to the algebraic sum of the products of currents and resistances of each part of that closed circuit or mesh *i.e.*

   $$\Sigma IR = \Sigma \text{emf}$$
7. According to Superposition theorem if there are a number of voltage and current sources acting simultaneously in any linear bilateral network, then each source can be considered acting independently of the others.
8. In *loop method of analysis*, independent mesh currents are taken and the network is solved by framing equations according to Kirchhoff's second or voltage law (KVL).