Chapter 4

AC Fundamentals

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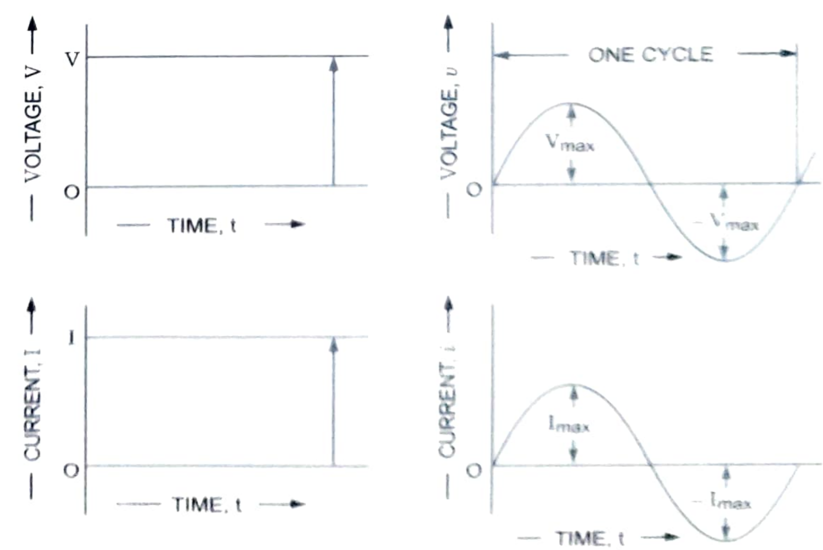
4.1 INTRODUCTION

A current (or voltage) is called *alternating* if it reverses periodically in direction, and its magnitude undergoes a definite cycle of changes in definite intervals of time. Each cycle of alternating current (or voltage) consists of two half cycles, during one of which the current (or voltage) acts in one direction; while during the other in opposite direction. In more restricted sense, alternating current is a periodically varying current, the average value of which, over a period, is zero. The direct current always flows in one direction, and its magnitude remains unaltered. In order to produce an alternating current through an electric circuit, a source capable of reversing the emf periodically (ac generator) is required while for generating dc in an electric circuit, a source capable of developing a constant emf is required such as a battery or dc generator. The graphical representations of direct current and alternating current are given in Figs. 4.1(*a*) and (*b*) respectively.

At present a large percentage of the electrical energy (nearly all) being used for domestic and commercial purposes is generated as alternating current. In fact, almost the whole of the vast amount of electrical energy used throughout the world for every imaginable purpose is generated by alternating current generators. This is not due to any superiority of alternating current over direct current in the sphere of applicability to industrial and domestic use. In fact, there are certain types of works for which alternating current is unsuitable and, therefore, direct current is absolutely necessary such as for electroplating, charging



(*a*) *DC Voltage and Current* (*b*) *AC Voltage and Current*

Fig. 4.1

of storage batteries, refining of copper, refining of aluminium, electrotyping, production of industrial gases by electrolysis, municipal traction etc. In some power applications, the ac motor is unsatisfactory such as for metal rolling mills, paper making machines, high- speed gearless elevators, automatic machine tools and high-speed printing presses. Direct current required for these applications is nowadays derived from an ac supply by the use of suitable convertors or rectifiers. For lighting and heating dc and ac are equally useful. The reasons for generation of electrical energy in the form of alternating current are given below:

1. AC generators have no commutator and can, therefore, be built in very large units to run at high speeds producing high voltages (as high as 33,000 volts), so that the construction and operating cost per kW is low, whereas dc generator capacities and voltages are limited to comparatively low values.
2. Alternating current can be generated at comparatively high voltages and can be raised and lowered readily by a static machine called the *transformer*, which makes the transmission and distribution of electrical energy economical. In direct current use of transformers is not possible.
3. AC induction motor is cheaper in initial cost and in maintenance since it has got no commutator and is more efficient than dc motor for constant speed work, so it is desirable to generate power as alternating current.
4. The high transmission efficiency in ac makes the generation of electrical energy economical by generating it in large quantities in a single station and distributing over a large territory.
5. The switchgear (*e.g.* switches, circuit breakers etc.) for ac system is simpler than that required in a dc system.
6. The maintenance cost of ac equipment is less.

4.2 GENERATION OF ALTERNATING EMF

We know that an alternating emf can be generated either by rotating a coil within a stationary magnetic field, as illustrated in Fig. 4.2 (*a*) or by rotating a magnetic field within a stationary coil, as illustrated in Fig. 4.2 (*b*). The emf generated, in either case, will be of sinusoidal waveform. The magnitude of emf generated in the coil

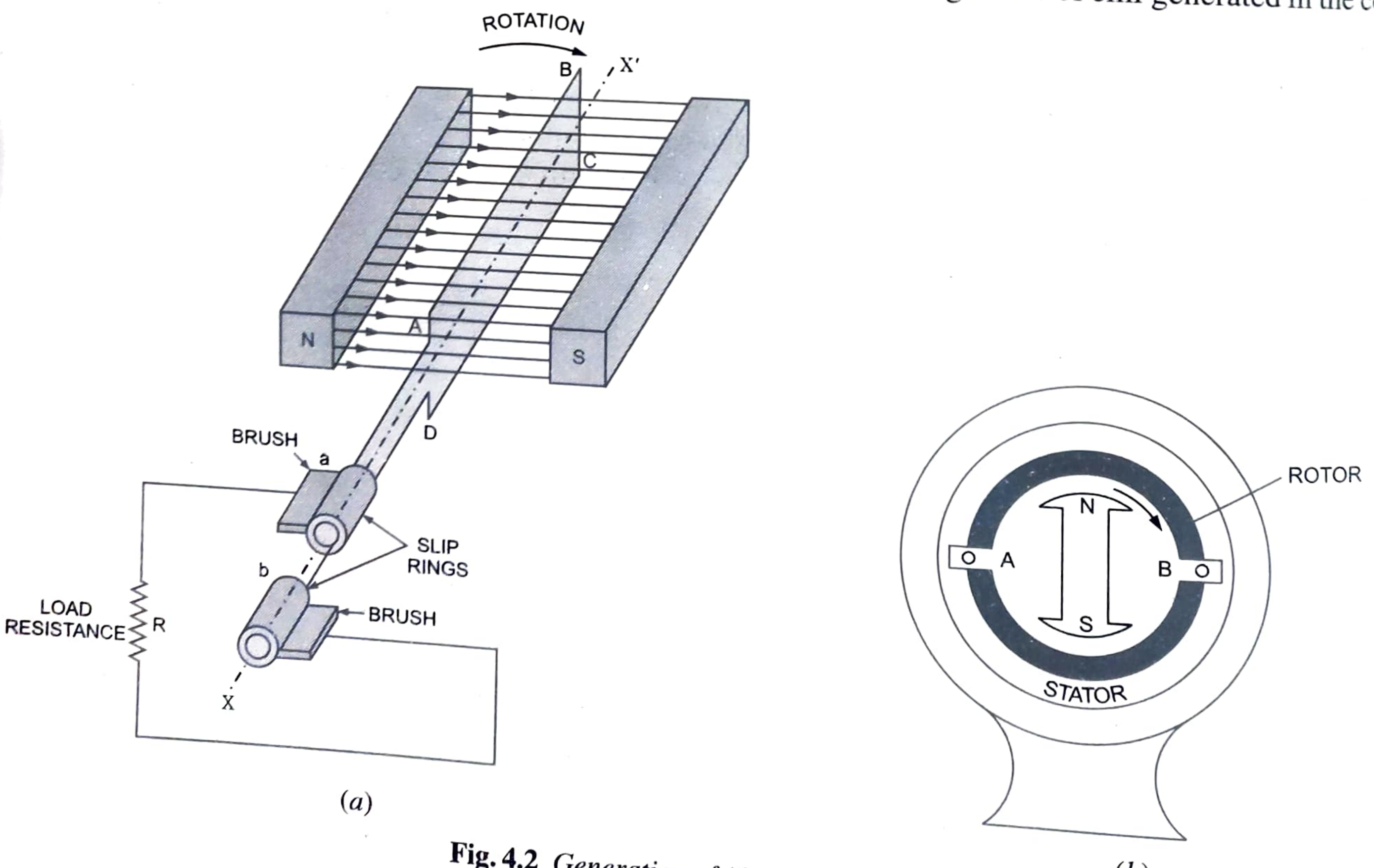


Fig. 4.2 *Generation of Alternating EMF*

depends upon the number of turns on the coil, the strength of magnetic field and the speed at which the coil or magnetic field rotates. The former method is employed in case of small ac generators while the later one is employed for large sized ac generators.

Now consider a rectangular coil of N turns rotating in counter-clockwise direction with angular velocity of ω radians per second in a uniform magnetic field, as illustrated in Fig. 4.3.

Let the time be measured from the instant of coincidence of the plane of the coil with the X-axis. At this instant maximum flux, Φ_{max} links with the coil. Let the coil assume the position, as shown in Fig. 4.3. after moving in counter-clockwise direction for t seconds.

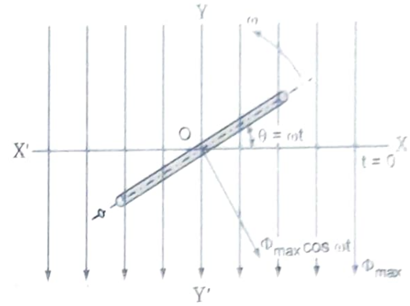


Fig. 4.3

The angle θ through which the coil has rotated in t seconds

$$= \omega t$$

In this position, the component of flux along perpendicular to the plane of coil $= \Phi_{max} \cos \omega t$.

Hence flux linkages of the coil at this instant

= Number of turns on coil × linking flux

i.e. instantaneous flux linkages $= N\, \Phi_{max} \cos \omega t$

Since emf induced in a coil is equal to the rate of change of flux linkages with minus sign,

EMF induced at any instant,

$$e = -\frac{d}{dt}\left[N\, \Phi_{max} \cos \omega t\right]$$

$$= \Phi_{max} N \frac{d}{dt}\left[-\cos \omega t\right] = \Phi_{max} N\, \omega \sin \omega t \quad ...(4.1)$$

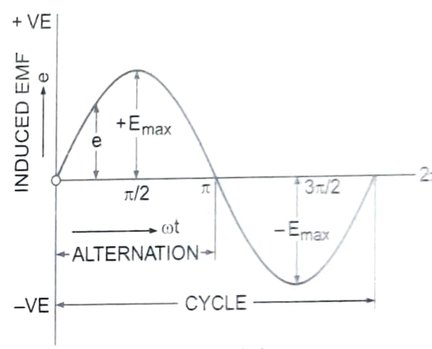


Fig. 4.4

when $\omega t = 0$, $\sin \omega t = 0$, therefore, induced emf is zero, when $\omega t = \frac{\pi}{2}$, $\sin \frac{\pi}{2} = 1$, therefore, induced emf is maximum, which is denoted by E_{max} and is equal to $\Phi_{max} N \omega$

Substituting $\Phi_{max} N\omega = E_{max}$ in Eq. (4.1) we have

$$\text{Instantaneous emf, } e = E_{max} \sin \omega t \quad ...(4.2)$$

So the emf induced varies as the sine function of the time angle ωt, and if emf induced is plotted against time, a curve of sine waveshape is obtained as illustrated in Fig. 4.4. Such an emf is called the *sinusoidal* emf. The sine curve is completed when the coil rotates through an angle of 2π radians. The induced emf e will have maximum value, represented by E_{max}, when the coil has turned through $\frac{\pi}{2}$ radians (or 90°) in counter-clockwise direction from the reference axis (*i.e.* OX axis).

Example 4.1. A coil having 200 turns and area of x-section 250 cm² is rotated about its axis at right angle to a uniform magnetic field of strength 0.5 T at a speed of 1,200 rpm. Determine (*i*) maximum value of emf induced (*ii*) equation for instantaneous induced emf and (*iii*) instantaneous values of induced emf when (*a*) the plane of the coil is at right angle to the field. (*b*) the plane of the coil is parallel to the field and (*c*) the plane of the coil is at an angle of 60° to the field.

Solution: Angular velocity, $\omega = \frac{2\pi \times RPM}{60} = \frac{2\pi \times 1{,}200}{60} = 40\,\pi$ radians/s

Maximum flux linking with the coil, $\Phi_{max} = B_{max} \times$ area of coil $= 0.5 \times 250 \times 10^{-4} = 0.0125$ Wb

(*i*) Maximum value of induced emf, $E_{max} = \Phi_{max} N\omega = 0.0125 \times 200 \times 40\,\pi = 314$ V **Ans.**

(*ii*) Equation for instantaneous induced emf $= E_{max} \sin \omega t = 314 \sin 40\,\pi\, t$ **Ans.**

(iii) Instantaneous values of induced emf when

(a) the plane of the coil is at right angle to the field *i.e.* when

$$\theta = \omega t = 0$$

$$\therefore \quad e_0 = 314 \sin 0^\circ = 0 \textbf{ Ans.}$$

(b) the plane of the coil is parallel to the field, *i.e.* when

$$\theta = \omega t = \frac{\pi}{2} \text{ radians}$$

$$\therefore \quad e' = 314 \sin \frac{\pi}{2} = 314 \text{ V } \textbf{Ans.}$$

(c) the plane of the coil makes an angle of 60° to the field *i.e.* when

$$\theta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \text{ radians}$$

$$\therefore \quad e'' = 314 \sin \frac{\pi}{6} = 314 \times \frac{1}{2} = 157 \text{ V } \textbf{Ans.}$$

4.3 SINUSOIDAL QUANTITIES (EMF, VOLTAGE OR CURRENT)

It is not an accident that the bulk of electric power generated in electric power stations throughout the world and distributed to the consumers appears in the form of sinusoidal variations of voltage and current. There are many technical and economical advantages associated with the use of sinusoidal voltages and currents. For example, it will be learned that the use of sinusoidal voltages applied to appropriately designed coils results in a revolving magnetic field which has the capacity to do work. As a matter of fact it is this principle which underlies the operation of almost all the electric motors found in home appliances and about 90% of all electric motors found in commercial and industrial applications. Although other waveforms can be used in such devices, none leads to an operation which is as efficient and economical as that achieved through the use of sinusoidal quantities.

The other advantages of using sinusoidal voltages and currents are:

1. The waveform from generation to utilization remains the same if a sinusoidal waveform is generated.
2. Electromagnetic torque developed in three-phase machines (generators and motors) with balanced three-phase currents is uniform (constant), and therefore, there are no oscillations in developed torque and absence of noise in operation.
3. Nonsinusoidal voltages which contain harmonic frequencies, according to Fourier analysis, are harmful to the system on account of
   (i) increased losses in generators, motors, transformers, and transmission and distribution systems.
   (ii) more interference (noise) to nearby communication circuits,
   (iii) resonance may result in overvoltages or overcurrents at many pockets on the way from generating station to consumer's premises which may damage the equipment and increase losses.
   (iv) increased current through power factor improvement capacitors.

In practical electrical engineering it is assumed that the alternating voltages and currents are sinusoidal though they may slightly deviate from it. The advantage of this assumption is that calculations become simple. *It may be noted that alternating voltage and current mean sinusoidal voltage and current unless stated otherwise.*

Alternating emf following sine law (*i.e.* sinusoidal emf) is illustrated in Fig. 4.4 and is expressed in the form

$$e = E_{max} \sin \omega t \qquad ...(4.3)$$

where e is the instantaneous value of alternating emf (or voltage), E_{max} is the maximum value of the alternating emf (or voltage) and ω is angular velocity of the coil.

The rotating coil moves through an angle of 2π radians in one cycle, so angular velocity $\omega = 2\pi f$ where f is the number of cycles completed per second.

Substituting $\omega = 2\pi f$ in Eq. (4.3) we have

$$e = E_{max} \sin 2\pi ft \quad ...(4.4)$$

If the alternating emf (or voltage) given by Eq. (4.3) is applied across a load, alternating current flows through the circuit which would also vary sinusoidally *i.e.* following a sine law. The expression for alternating current is given as

$$i = I_{max} \sin \omega t = I_{max} \sin 2\pi ft \quad ...(4.5)$$

provided the load is pure resistive.*

4.4 TERMINOLOGY

An *alternating quantity* (voltage or current) is one which changes continuously in magnitude and alternates in direction at regular intervals of time. It rises from zero to maximum positive value, falls to zero, increases to a maximum in the reverse direction and falls back to zero again, as illustrated in Fig. 4.4. The emf (or voltage) and current repeat the procedure.

The important ac terms are defined below:

4.4.1. Waveform. The shape of the curve of the voltage or current when plotted against time as abcissa (base) is called the *waveform*. The waveform of an alternating voltage varying sinusoidally is shown in Fig. 4.4. The waveform of the induced emf in an alternator differs slightly from that of sine wave but for calculation purposes it is treated as such.

4.4.2. Instantaneous Value. The value of alternating quantity (emf, voltage or current) at any particular instant is called the *instantaneous value* and is designated by a small italic letter (e for emf, v for voltage and i for current). The instantaneous values of an alternating quantity can be determined either from the curve or from an equation of the alternating quantity. For example, the instantaneous values of emf represented by the curve shown in Fig. 4.4 at 0, $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$ are zero, $+E_{max}$, zero and $-E_{max}$ respectively.

4.4.3. Alternation and Cycle. When a periodic wave, such as sinusoidal wave, goes through one complete set of positive or negative values, it completes one *alternation* and when it goes through one complete set of positive and negative values it is said to have completed one *cycle*.

Alternation and cycle can also be defined in terms of angular measure. One alternation corresponds to 180°(or π radians) while one cycle corresponds to 360° (or 2π radians).

4.4.4. Time Period and Frequency. The time taken in seconds by an alternating quantity to complete one cycle is known as *time period* or *periodic time* and is denoted by T.

The number of cycles completed per second by an alternating quantity is known as *frequency* and is denoted by f. In SI system the frequency is expressed in hertz (pronounced as hurts). One hertz (or Hz) is equal to one cycle per second.

The number of cycles completed per second $= f$

Time period, T = Time taken in completing one cycle $= 1/f$

$$\text{or } f = 1/T \quad ...(4.6)$$

The commercial ac power is generated at frequency of 50 Hz or 60 Hz.** The reasons of suitability of frequency of this range are:

1. The output of the equipment increases with the increase in frequency. For a given output, smaller size machines are required as compared to those for lower frequency output. Because of high power-weight ratio, relative cost of the equipment is also reduced.

* The load may be resistive, inductive or capacitive. It will be shown in chapter 5 that if the load is inductive or capacitive the current equation differs in time angle.

** In India the standard frequency for power supply is 50 Hz.

2. Lower regulation, lower skin effect resulting in lower ohmic losses, lower magnetic and dielectric losses resulting in higher efficiency, lower corona loss and higher power transmission line capacity as compared to those at higher frequency.

4.4.5. Angular Velocity and Frequency. A glance at Fig. 4.4 indicates that each cycle spans 2π radians. Hence, if this quantity is divided by the time period, *angular velocity* of the sine function is obtained. It is denoted by ω and is expressed in radians per second.

$$\therefore \quad \text{Angular velocity, } \omega = \frac{\text{Angle turned}}{\text{Time taken}} = \frac{2\pi}{T} = \frac{2\pi}{1/f} = 2\pi f \text{ radians per second} \qquad ...(4.7)$$

4.4.6. Electrical Time Degrees and Mechanical Degrees. It is seen that the coil must revolve past a pair of poles in order to carry the generated emf through one complete cycle. In circuit work, one complete cycle of voltage or current is designated as 360 electrical degrees or 2 π electrical radians. To correlate with this designation the arc through which a coil of dynamo must rotate in order to generate one cycle of emf is called 360 electrical degrees. In a 2-pole machine one complete revolution of coil produces one cycle of emf. But in a multipolar machine, such as four, six or eight pole machine, the emf completes one cycle or 360 electrical degrees or 2 π electrical radians as soon as the coil passes a pair of poles and a mechanical degree will be equal to as many electrical degrees as there are pairs of poles in the dynamo structure. In a multipolar machine, the number of cycles completed per second by generated emf,

$$f = \text{Pair of poles} \times \text{number of revolutions made per second}$$

$$= \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \qquad ...(4.8)$$

where N is the speed of rotation of the coil in rpm.

4.4.7. Amplitude. The maximum value, positive or negative, which an alternating quantity attains during one cycle is called the *amplitude* of the alternating quantity. The amplitude of an alternating emf (or voltage) and current is designated by E_{max} (or V_{max}) and I_{max} respectively.

Example 4.2. What is the time period of the wave produced by a 6-pole alternator which is driven at 1,000 rpm ?

[Pb Technical Univ. Basic Electrical Engineering, June 2003]

Solution: Number of poles on alternator, P = 6

Speed of alternator, N = 1,000 rpm

Number of cycles completed per second by generated emf,

$$f = \frac{PN}{120} = \frac{6 \times 1,000}{120} = 50$$

$$\text{Time period, } T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ second} \quad \textbf{Ans.}$$

4.5 DETERMINATION OF MAXIMUM VALUE AND FREQUENCY FROM EMF OR CURRENT EQUATIONS

From Eq. (4.4)

$$e = E_{max} \sin 2\pi f t$$

From above expression we observe that

(i) *the maximum value of an alternating emf is given by the coefficient of the sine of the time angle.*

(ii) *the frequency is given by coefficient of time t divided by 2π*

$$\text{i.e. } f = \frac{\text{Coefficient of time, } t}{2\pi} \qquad ...(4.9)$$

Similarly we can also find the maximum value and frequency of the current from the equation of instantaneous values of current.

Example 4.3. An ac voltage of 50 Hz frequency has a peak value of 220 V. Write down the expression for the instantaneous value of this voltage. *[Pb Technical Univ. Electrical Engineering January 2000]*

Solution: Supply frequency, $f = 50$ Hz

Peak value of ac voltage, $V_{max} = 220$ V

Expression for instantaneous value of ac voltage (assumed sinusoidal) with θ as zero when time is zero, is given as

$$v = V_{max} \sin \omega t = V_{max} \sin 2\pi f t = 220 \sin 2\pi \times 50\, t = 220 \sin 314t \quad \textbf{Ans.}$$

Example 4.4. An alternating current of frequency 50 Hz has a maximum value of 100 A. Calculate (*a*) its value 1/600 second after the instant the current is zero and its value decreasing there afterwards (*b*) how many seconds after the instant the current is zero (increasing there afterwards) will the current attain the value of 86.6 A ?

Solution: The current waveform is shown in Fig. 4.5.

The equation of the alternating current (assumed sinusoidal) with respect to the origin O is

$$i = 100 \sin 100\,\pi\, t \qquad ...(i)$$

(*a*) Since the current is measured from the instant the current is zero and is decreasing there afterwards (*i.e.* from point A in Fig. 3.5), the equation for the alternating current with respect to the point A becomes

$$i = 100 \sin (100\,\pi\, t + \pi) = -100 \sin 100\,\pi\, t \qquad ...(ii)$$

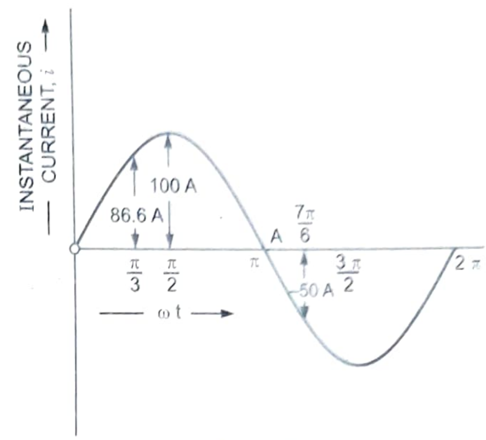


Fig. 4.5

Substituting $t = \frac{1}{600}$ second in above equation we get the instantaneous value of current $\frac{1}{600}$ second after the instant the current is zero and decreasing there afterward

$$\text{So } i = -100 \sin 100\,\pi \times \frac{1}{600} = -100 \sin \frac{\pi}{6} = -50 \text{ A} \quad \textbf{Ans.}$$

(*b*) Let the current attain the value of 86.6 A, *t* seconds after the zero value of the current. Now substituting $i = 86.6$ A in Eq. (*i*) we get

$$86.6 = 100 \sin 100\,\pi\, t$$

$$\text{or } \quad t = \frac{1}{100\,\pi} \sin^{-1} \frac{86.6}{100} = \frac{1}{100\,\pi} \times \frac{\pi}{3} = \frac{1}{300} \text{ second } \textbf{Ans.}$$

4.6 PLOTTING OF SINE WAVEFORM

Sine curve may be graphically drawn, as illustrated in Fig. 4.6. Draw a circle of radius equal to the maximum value of sinusoidal quantity. Divide the circumference of the circle drawn so into any number of equal parts, say 12, and draw a horizontal line AB (the base on which the sine wave is to be drawn) passing through the centre of the circle. Divide the line AB into the same number of equal parts *i.e.* 12 and number the points correspondingly. Draw perpendicular ordinates from each point. Project the points on the circle horizontally to meet the perpendicular ordinates having corresponding numbers. Draw smooth curve through these points. Curve so drawn will be of sine waveform.

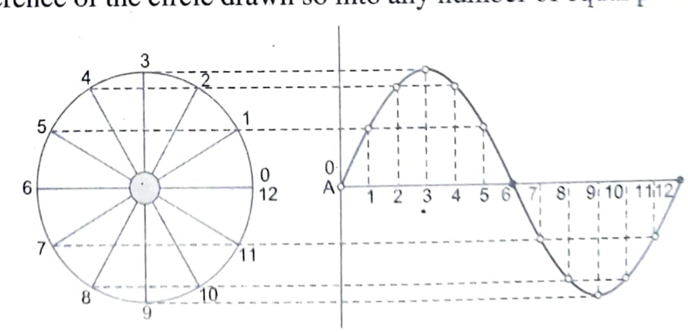


Fig. 4.6 *Simple Method of Plotting Sine Wave*

4.7 AVERAGE AND EFFECTIVE (RMS) VALUES OF ALTERNATING VOLTAGE AND CURRENT

In a dc system, the voltage and current are constant and, therefore, there is no problem in specifying their magnitude. But in case of ac system, an alternating voltage or current varies from instant to instant and so poses a problem how to specify the magnitude of an alternating voltage or current. An alternating voltage or current may possibly be expressed in terms of peak (maximum) value, average (mean) value or effective (rms) value.

In specifying an alternating voltage or current, its peak or maximum value is rarely used because it has that value only twice each cycle. Furthermore, the average or mean value cannot be used because it is positive as much as it is negative, so the average value is zero. Although the average value over half cycle might be used, it would not be as logical a choice as what we shall find *effective* (*virtual or rms*) *value* which is related to the power developed in a resistance by the alternating voltage or current.

4.7.1. Average Value of Alternating Current. The average (or mean) value of an alternating current is equal to the value of direct current which transfers across any circuit the same charge as is transferred by that alternating current during a given time.

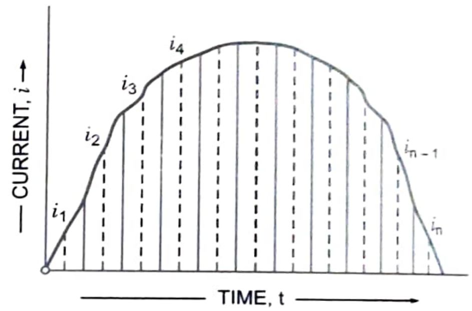


Fig. 4.7

Since in the case of a symmetrical alternating current (*i.e.* one whose two half cycles are exactly similar, whether sinusoidal or nonsinusoidal) the average or mean value over a complete cycle is zero hence for such alternating quantities average or mean value means the value determined by taking the average of instantaneous values during half cycle or one alternation only. However, for un-symmetrical alternating current, as half-wave rectified current, the average value means the value determined by taking the mean of instantaneous values over the whole cycle.

The average value is determined by measuring the lengths of a number of equidistant ordinates and then taking their mean *i.e.* of $i_1, i_2, i_3 \ldots\ldots i_n$ etc. which are mid-ordinates.

$$\because \quad \text{Average value of alternating current, } I_{av} = \frac{i_1 + i_2 + i_3 + i_4 \ldots + i_n}{n}$$

$$= \frac{\text{Area of one alternation (or half cycle)}}{\text{Length of base over one alternation (or half cycle)}}$$

Using the integral calculus the average (or mean) value of a function $f(t)$ over a specific interval of time between t_1 and t_2 is given by

$$F_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t)\, dt \qquad \ldots(4.10)$$

Any function whose cycle is repeated continuously, irrespective of its waveshape, is termed as *periodic function*, such as sinusoidal function, and its average value is given by

$$F_{av} = \frac{1}{T} \int_0^T f(t)\, dt \qquad \ldots(4.11)$$

where T is time period of periodic function.

In case of a symmetrical alternating current, whether sinusoidal or nonsinusoidal the average value is determined by taking average of one half cycle or one alternation only.

$$\textit{i.e. } \text{for symmetrical waveforms, } F_{av} = \frac{2}{T} \int_0^{T/2} f(t)\, dt \qquad \ldots(4.12)$$

4.7.2. RMS Value or Effective Value of Alternating Current. The rms or effective value of an alternating current or voltage is given by that steady current or voltage which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage is flowing or applied to the same resistance for the same time.

Consider an alternating current of waveform shown in Fig. 4.8 flowing through a resistor of R ohms. Divide the base of one alternation into n equal parts and let the mid-ordinates be $i_1, i_2, i_3 \ldots i_n$ etc.

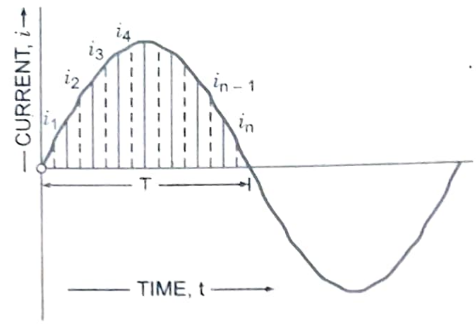


Fig. 4.8

$$\text{Heat produced during 1st interval} = i_1^2 \text{ R} \times \frac{\text{T}}{n} \text{ joules}$$

$$\text{Heat produced during 2nd interval} = i_2^2 \text{ R} \times \frac{\text{T}}{n} \text{ joules}$$

$$\text{Heat produced during 3rd interval} = i_3^2 \text{ R} \times \frac{\text{T}}{n} \text{ joules}$$

$$\text{Heat produced during } n\text{th interval} = i_n^2 \text{ R} \times \frac{\text{T}}{n} \text{ joules}$$

$$\text{Total heat produced in time T} = \text{RT}\left(\frac{i_1^2 + i_2^2 + i_3^2 + \ldots . i_n^2}{n}\right) \text{ joules}$$

Now if I_{eff} is the effective current, then heat produced by this current in time T = I_{eff}^2 RT joules. By definition these two expressions are equal

$$\therefore \quad \text{I}_{eff}^2 \text{ RT} = \text{RT}\left(\frac{i_1^2 + i_2^2 + i_3^2 + \ldots . i_n^2}{n}\right)$$

$$\text{or} \quad \text{I}_{eff}^2 = \frac{i_1^2 + i_2^2 + i_3^2 + \ldots . i_n^2}{n}$$

$$\text{or} \quad \text{I}_{eff} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots . i_n^2}{n}}$$

Hence the *effective* or *virtual value* of alternating current or voltage is equal to the square root of the mean of the squares of successive ordinates and that is why it is known as *root mean square (rms) value.*

Using the integral calculus the root mean square (rms) or effective value of an alternating quantity over a time period is given by

$$\text{F}_{\text{rms}} = \sqrt{\frac{1}{\text{T}}\int_0^{\text{T}} f^2(t)\, dt} \qquad (4.13)$$

4.8 AVERAGE AND EFFECTIVE (RMS) VALUES OF SINUSOIDAL CURRENT AND VOLTAGE

4.8.1. Average Value For Sinusoidal Current or Voltage. The average value of a sine wave over a complete cycle is zero. Therefore, the half cycle average value is intended.

Instantaneous value of sinusoidal current is given by

$$i = \text{I}_{\text{max}} \sin \omega t$$

Considering first half cycle i.e when ωt varies from 0 to π we get,

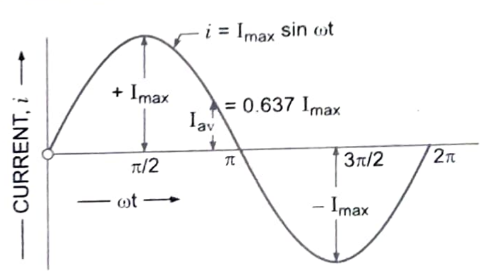


Fig. 4.9

$$\text{I}_{\text{av}} = \frac{\text{Area of first half cycle}}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i\, d(\omega t) = \frac{1}{\pi}\int_0^{\pi} \text{I}_{\text{max}} \sin \omega t \;\; d(\omega t)$$

$$\text{or } \text{I}_{av} = \frac{\text{I}_{\text{max}}}{\pi}\left[-\cos \omega t\right]_0^{\pi} = \frac{2}{\pi}\,\text{I}_{\text{max}} = 0.637\,\text{I}_{\text{max}} \qquad \ldots(4.14)$$

$$\text{Similarly } \text{E}_{av} = 0.637\,\text{E}_{\text{max}}$$

4.8.2. Effective (RMS) Value For Sinusoidal Current or Voltage. A sinusoidal alternating current i, represented by

$$i = I_{max} \sin \omega t$$

$$I_{rms}^2 = \frac{\text{Area of first half cycle of } i^2}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 \, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} I_{max}^2 \sin^2 \omega t \, d(\omega t)$$

$$= \frac{I_{max}^2}{2\pi}\int_0^{\pi} (1 - \cos 2\omega t)\, d(\omega t)$$

$$= \frac{I_{max}^2}{2\pi}\left[\omega t - \frac{1}{2}\sin 2\omega t\right]_0^{\pi}$$

$$= \frac{I_{max}^2}{2\pi} \times \pi = \frac{I_{max}^2}{2}$$

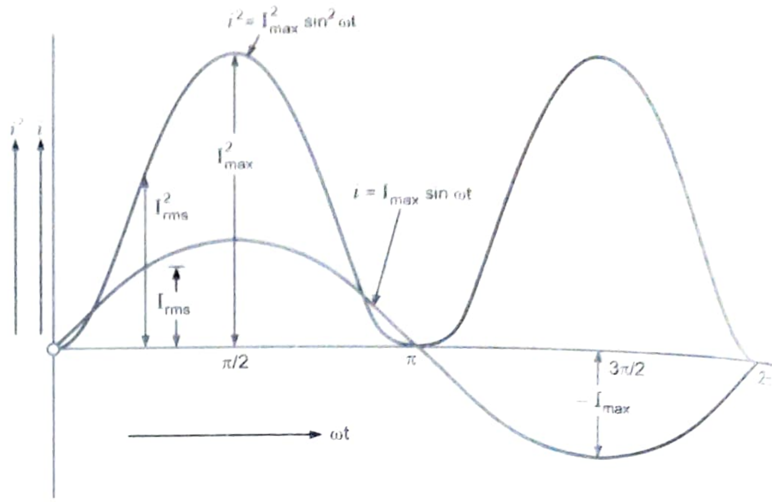


Fig. 4.10

$$\text{or } I_{rms} = \sqrt{\frac{I_{max}^2}{2}} = \frac{I_{max}}{\sqrt{2}} \qquad ...(4.15)$$

$$\text{Similarly, } E_{rms} = \frac{E_{max}}{\sqrt{2}}$$

Example 4.5. Write the mathematical expression for a 50 Hz sinusoidal voltage supplied for domestic purpose at 230 V.
[Pb. Technical Univ. Basic Electrical and Electronics Engineering Dec. 2005]

Solution: RMS value for a 50 Hz sinusoidal voltage supplied for domestic purposes,

$$V_{rms} = 230 \text{ V}$$

Maximum value, $V_{max} = \sqrt{2} \times V_{rms} = \sqrt{2} \times 230 = 325.27$ V

Expression for instantaneous value with θ as zero when time is zero,

$$v = V_{max} \sin \omega t = V_{max} \sin 2\pi ft = 325.27 \sin 2\pi \times 50t = 325.27 \sin 314t \quad \textbf{Ans.}$$

Example 4.6. The equation of an alternating current is i = 141.4 sin 314t. What is rms value of current and frequency?
[A.K. Technical Univ. Basic Electrical Engineering Second Semester, 2015-16]

Solution: Maximum value of current, I_{max} = Coefficient of the sine of the time angle = 141.4 A

(*i*) RMS value of current, $I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100$ A **Ans.**

(*ii*) Frequency, $f = \frac{\text{Coefficient of time, } t}{2\pi} = \frac{314}{2\pi} = 50$ Hz **Ans.**

Example 4.7. An alternating current when passed through a resistance immersed in water for 5 minutes, just raised the temperature of water to boiling point. When a direct current of 4 amperes was passed through the same resistance under identical conditions it took 8 minutes to boil the water. Find the rms value of the alternating current.

Solution: Let the rms value of alternating current passed through the resistance be I_{rms} amperes.

Heat produced when an alternating current of I_{rms} amperes is passed through a resistance R immersed in water for 5 minutes

$$= (I_{rms})^2 \times R \times 5 \times 60 = 300\, I_{rms}^2 \text{ R joules} \qquad ...(i)$$

Heat produced when a direct current of 4 A is passed through the same resistance R immersed in the water for 8 minutes

$$= (4)^2 \times R \times 8 \times 60 = 7{,}680 \text{ R joules} \qquad ...(ii)$$

Since heat produced in both of the cases is same, therefore, equating Eq. (i) and (ii) we get

$$300\, I_{rms}^2\, R = 7{,}680\, R$$

$$\text{or } I_{rms} = \sqrt{\frac{7{,}680}{300}} = \sqrt{25.6} = 5.06 \text{ A} \quad \textbf{Ans.}$$

Example 4.8. An ac has frequency 50 Hz and rms current 25 amp. Write equation of instantaneous current and find (i) current at time 0.0025 second, (ii) Time at which current is 14.14 amp.

[Pb. Technical Univ. Basic Electrical and Electronics Engineering, May-2008; December-2012]

Solution: Peak value of current, $I_{max} = \sqrt{2}\, I_{rms} = \sqrt{2} \times 25 = 35.36$ A

Supply frequency, $f = 50$ Hz

The equation of alternating current is given by expression

$$i = I_{max} \sin 2\pi f t = 35.36 \sin 2\pi \times 50t = 35.36 \sin 314t \quad \textbf{Ans.}$$

(i) Current at time $t = 0.0025$ second

$$= 35.36 \sin 2\pi \times 50 \times 0.0025$$

$$= 35.36 \sin \frac{\pi}{4} = 35.36 \times \frac{1}{\sqrt{2}} = 25 \text{ A} \quad \textbf{Ans.}$$

Substituting $i = 14.14$ A in above equation, we have

$$14.14 = 35.36 \times \sin 100\pi t$$

or Time at which current is 14.14 A is given by the expression

$$(ii)\ t = \frac{1}{100\pi} \sin^{-1} \frac{14.14}{35.36} = \frac{1}{100\pi} \sin^{-1} 0.4 = \frac{1}{100\pi} \times 0.131\pi$$

$$= 0.00131 \text{ second or } 1.31 \text{ ms} \quad \textbf{Ans.}$$

Example 4.9. An alternating voltage is given by $v = 141.4 \sin 314\, t$. Find (i) frequency (ii) rms value (iii) average value (iv) the instantaneous value of voltage when 't' is 3 ms (v) the time taken for the voltage to reach 100 V for the first time after passing through zero value.

[U.P. Technical Univ. Electrical Engineering First Semester, 2006-07]

Solution: Instantaneous value of alternating voltage is given by expression ...(i)

$$v = 141.4 \sin 314\, t$$

(i) The frequency is given by the coefficient of time divided by 2π, therefore,

$$\text{frequency, } f = \frac{314}{2\pi} = 50 \text{ Hz} \quad \textbf{Ans.}$$

Peak value of an alternating voltage is given by the coefficient of sine of time angle, therefore,

$$V_{max} = 141.4 \text{ volts}$$

(ii) RMS value of given alternating voltage, $V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100$ V **Ans.**

(iii) Average value of given alternating voltage, $V_{av} = 0.637\, V_{max} = 0.637 \times 141.4 = 90$ V **Ans.**

(iv) Instantaneous value of voltage at 3 ms (0.003 s) after the instant the voltage is zero and increasing in positive direction may be obtained by substituting $t = 0.003$ second in Eq. (i). So we have

$$v = 141.4 \sin 314 \times 0.003 = 141.4 \sin 0.942 = 114.4 \text{ V} \quad \textbf{Ans.}$$

(v) Time taken from $t = 0$ for the voltage to reach 100 V

$$t = \frac{1}{314} \sin^{-1} \frac{100}{141.4} = \frac{\pi/4}{314} \text{ second} = \frac{1}{400}\, s = 2.5 \text{ ms} \quad \textbf{Ans.}$$

Example 4.10. An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of 20 A. Write down the equation for the instantaneous value and find this value (a) 0.0025 second (b) 0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A ?

Solution: The current waveform is shown in Fig. 4.11

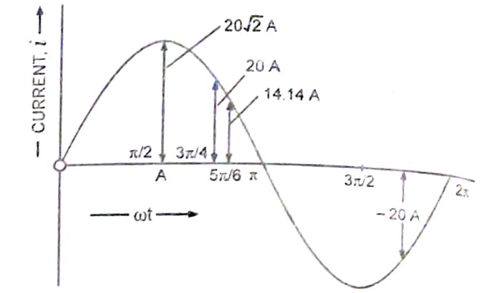


Fig. 4.11

Peak value of current,

$$I_{max} = \sqrt{2} I_{rms} = \sqrt{2} \times 20 = 28.28 \text{ A}$$

The equation of alternating current with respect to origin O is

$$i = I_{max} \sin 2\pi f t = 28.28 \sin 2\pi \times 50t = 28.28 \sin 100\pi t$$

The above equation is valid when time is measured from the instant the current is zero and increasing in positive direction. Since the time is measured from the positive maximum value (point A in Fig. 4.11), the above equation is modified to

$$i = 28.28 \sin\left(100\pi t + \frac{\pi}{2}\right) = 28.28 \cos 100\pi t \quad \textbf{Ans.}$$

(*a*) When $t = 0.0025$ s, the instantaneous value of current,

$$i_1 = 28.28 \cos 100\pi \times 0.0025 = 28.28 \cos \frac{\pi}{4} = 28.28 \times \frac{1}{\sqrt{2}} = 20 \text{ A} \quad \textbf{Ans.}$$

(*b*) When $t = 0.0125$ s, the instantaneous value of current,

$$i_2 = 28.28 \cos 100\pi \times 0.0125 = 28.28 \cos \frac{5\pi}{4} = 28.28\left(\frac{-1}{\sqrt{2}}\right) = -20 \text{ A} \quad \textbf{Ans.}$$

(*c*) Substituting $i = 14.14$ A in expression

$$i = 28.28 \cos 100\pi t \text{ we have}$$

$$14.14 = 28.28 \cos 100\pi t$$

$$\text{or } t = \frac{1}{100\pi} \cos^{-1}\frac{14.14}{28.28} = \frac{1}{100\pi}\cos^{-1} 0.5 = \frac{1}{100\pi} \times \frac{\pi}{3} = \frac{1}{300} \text{ second} \quad \textbf{Ans.}$$

4.9 FORM FACTOR AND PEAK FACTOR

4.9.1. Form Factor. In certain cases it is convenient to have calculations at first upon the mean value of the emf over half a period, therefore, it becomes essential to have some means of connecting this mean value with the effective or rms value. The knowledge of form factor, which is defined as the ratio of effective value to the average or mean value of periodic wave is, therefore, necessary.

Mathematically form factor is given by the relation

$$\text{Form factor} = \frac{\text{Effective value}}{\text{Average value}}$$

$$\text{Form factor for sinusoidal wave, } K_f = \frac{E_{rms}}{E_{av}} = \frac{\dfrac{E_{max}}{\sqrt{2}}}{\dfrac{E_{max}}{\pi/2}} = 1.11$$

4.9.2. Peak Factor. Knowledge of peak factor of an alternating voltage is very essential in connection with determining the dielectric strength since the dielectric stress developed in any insulating material is proportional to the maximum value of the voltage applied to it.

Peak or crest or amplitude factor of a periodic wave is defined as the ratio of maximum or peak to the effective or rms value of the wave

$$\textit{i.e.} \text{ Peak factor, } K_p = \frac{\text{Maximum value}}{\text{Effective value}}$$

$$\text{Peak factor for sinusoidal wave, } K_p = \frac{E_{max}}{E_{rms}} = \frac{E_{max}}{E_{max}/\sqrt{2}} = 1.414$$

Example 4.11. An alternating voltage is $v = 100 \sin 100t$. Find (*i*) Amplitude (*ii*) Time period and frequency (*iii*) Angular velocity (*iv*) Form factor (*v*) Peak factor.

[U.P. Technical Univ. Electrical Engineering Second Semester, 2007-08]

Solution: Instantaneous value of alternating voltage is given by expression

$$v = 100 \sin 100\, t$$

(*i*) Amplitude of alternating voltage is given by the coefficient of the sine of the time angle, so

$$\text{Amplitude of given wave} = V_{max} = 100 \text{ V} \quad \textbf{Ans.}$$

(*ii*) Frequency is given by coefficient of time, t divided by 2π

$$\therefore \text{ Frequency}, f = \frac{100}{2\pi} = 15.9 \text{ Hz} \quad \textbf{Ans.}$$

$$\text{Time period, T} = \frac{1}{f} = \frac{1}{15.9} = 0.063 \text{ second or } 63 \text{ ms} \quad \textbf{Ans.}$$

(*iii*) Angular velocity, $\omega = 2\,\pi f = 2\,\pi \times 15.9 = 100$ radians/second **Ans.**

$$\text{RMS value, } V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71 \text{ V}$$

$$\text{Average value } V_{av} = 0.637\, V_{max} = 0.637 \times 100 = 63.7 \text{ V}$$

(*iv*) Form factor, $K_f = \dfrac{V_{rms}}{V_{av}} = \dfrac{70.71}{63.7} = 1.11$ **Ans.**

(*v*) Peak factor, $K_p = \dfrac{V_{max}}{V_{rms}} = \dfrac{100}{70.71} = 1.4142$ **Ans.**

Example 4.12. Find the rms value, average value and form factor of the voltage waveform shown in Fig. 4.12.

[*U.P. Technical Univ. Electrical Engineering Second Semester 2002-03*]

Solution: Average value of voltage over one cycle

$$= \frac{2\int_0^{\pi} V_{max} \sin\theta\, d\theta}{2\pi} = \frac{V_{max}}{\pi}\int_0^{\pi} \sin\theta\, d\theta = \frac{2\,V_{max}}{\pi} = \frac{2}{\pi} \times 100 = 63.66 \text{ V } \textbf{Ans.}$$

RMS value of voltage over one cycle

$$= \sqrt{\frac{2\int_0^{\pi} V_{max}^2 \sin^2 d\theta}{2\pi}}$$

$$= \sqrt{\frac{V_{max}^2}{2\pi}\int_0^{\pi} 2\sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_{max}^2}{2\pi}\int_0^{\pi} (1-\cos 2\theta)\, d\theta}$$

$$= \sqrt{\frac{V_{max}^2}{2\pi} \times \pi} = \frac{V_{max}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71 \text{ V } \textbf{Ans.}$$

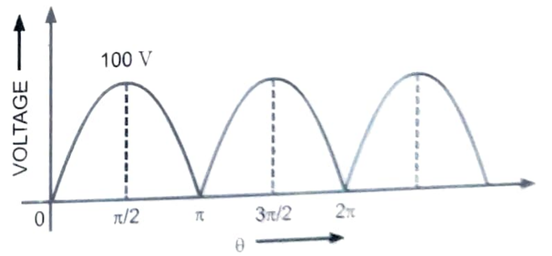


Fig. 4.12

Form factor of the waveform shown

$$= \frac{\text{RMS value}}{\text{Average value}} = \frac{70.7}{63.66} = 1.11 \textbf{ Ans.}$$

Example 4.13. Calculate the average and root mean square values, the form factor and peak factor of a periodic current wave having the following values for equal time intervals over half cycle, changing suddenly from one value to the next: 0, 40, 60, 80, 100, 80, 60, 40, 0.

Solution: Since the wave is symmetrical, so considering one half cycle only. Average value

$$I_{av} = \frac{i_1 + i_2 + i_3 + i_4 + i_5 + i_6 + i_7 + i_8}{8} = \frac{0 + 40 + 60 + 80 + 100 + 80 + 60 + 40}{8} = 57.5 \text{ A } \textbf{Ans.}$$

RMS value, I_{rms}

$$= \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + i_4^2 + i_5^2 + i_6^2 + i_7^2 + i_8^2}{8}}$$

$$= \sqrt{\frac{0 + 1,600 + 3,600 + 6,400 + 10,000 + 6,400 + 3,600 + 1,600}{8}}$$

= 64.42 A **Ans.**

Peak value, I_{max} = 100 A

$$\text{Form factor} = \frac{I_{rms}}{I_{av}} = \frac{64.42}{57.5} = 1.12 \text{ Ans.}$$

$$\text{Peak factor} = \frac{I_{max}}{I_{rms}} = \frac{100}{64.42} = 1.552 \text{ Ans.}$$

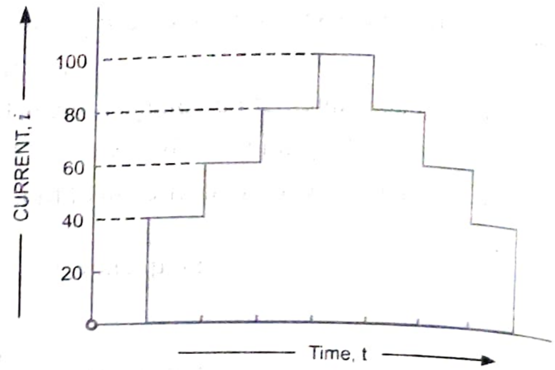


Fig. 4.13

4.10 RMS VALUE, AVERAGE VALUE, PEAK FACTOR AND FORM FACTOR OF HALF-WAVE RECTIFIED ALTERNATING CURRENT

Half-wave rectified alternating current is one whose one-half cycle has been suppressed *i.e.* one which flows for half the time during one cycle. It is illustrated in Fig. 4.14 where suppressed half cycle is shown dotted.

As mentioned earlier, for determining rms and average values of such an alternating current summation would be carried over the period for which current actually flows *i.e.* 0 to π but would be averaged for the whole cycle *i.e.* from 0 to 2π.

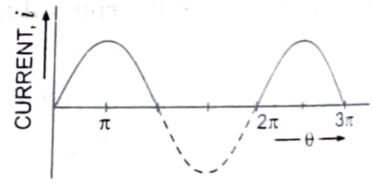


Fig. 4.14

$$\therefore \text{ RMS value of current, } I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i^2\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_{max}^2 \sin^2\theta\, d\theta}$$

$$= \frac{I_{max}}{\sqrt{2\pi}}\sqrt{\int_0^{\pi}\sin^2\theta\, d\theta}$$

$$= \frac{I_{max}}{\sqrt{4\pi}}\sqrt{\int_0^{\pi}(1-\cos 2\theta)\, d\theta}$$

$$= \frac{I_{max}}{\sqrt{4\pi}}\sqrt{\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi}} = \frac{I_{max}}{\sqrt{4\pi}} \times \sqrt{\pi} = \frac{I_{max}}{2}$$

$$\text{Average value of current, } I_{av} = \frac{1}{2\pi}\int_0^{\pi} i\, d\theta = \frac{1}{2\pi}\int_0^{\pi} I_{max}\sin\theta\, d\theta = \frac{I_{max}}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_{max}}{\pi}$$

$$\text{Peak Factor} = \frac{I_{max}}{I_{rms}} = \frac{I_{max}}{I_{max}/2} = 2$$

$$\text{Form factor} = \frac{I_{rms}}{I_{av}} = \frac{I_{max}/2}{I_{max}/\pi} = \frac{\pi}{2} = 1.57$$

Example 4.14. A voltage $v(t) = 220\sqrt{2}\sin 100t$ is applied to the circuit shown. What is the rms value of current through the resistor R of 100 Ω? Derive the formula used.

[Pb. Technical Univ. Basic Electrical Engineering May-2002]

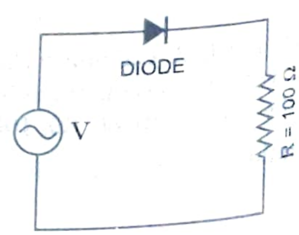


Fig. 4.15

Solution: The maximum value of voltage applied to the circuit

$$V_{max} = 220\sqrt{2} \text{ V}$$

$$\text{Maximum value of circuit current, } I_{max} = \frac{V_{max}}{R} = \frac{220\sqrt{2}}{100} = 3.11 \text{ A}$$

$$\text{RMS value of circuit current, } I_{rms} = \frac{I_{max}}{2} = \frac{3.11}{2} = 1.55 \text{ A } \textbf{Ans.}$$

Example 4.15. Define average value of alternating current. Find this value for the waveform shown in Fig. 4.16.

[Pb Technical Univ. Basic Electrical Engineering December-2002]

Solution: Maximum value of alternating current, $I_{max} = 10$ A

$$\text{Average value, } I_{av} = \frac{I_{max}}{\pi} = \frac{10}{\pi} = 3.183 \text{ A } \textbf{Ans.}$$

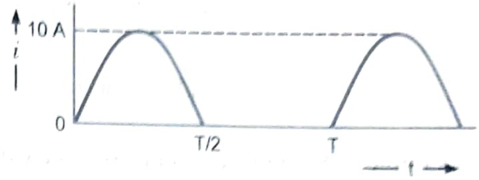


Fig. 4.16

Example 4.16. Find the average and effective values of voltage for sinusoidal waveform shown in Fig. 4.17.

Solution: The equation of the given sinusoidal waveform is $v = 100 \sin \theta$

$$V_{av} = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100 \sin\theta \, d\theta = \frac{100}{2\pi}\left[-\cos\theta\right]_{\pi/4}^{\pi} = 27.17 \text{ V } \textbf{Ans.}$$

$$V_{rms}^2 = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100^2 \sin^2\theta \, d\theta$$

$$= \frac{10{,}000}{4\pi}\int_{\pi/4}^{\pi} (1 - \cos 2\theta)\, d\theta$$

$$= \frac{2{,}500}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi} = \frac{2{,}500}{\pi}\left[\pi - \frac{\pi}{4} + \frac{1}{2}\right] = 2{,}273$$

$$\text{or } V_{rms} = \sqrt{2{,}273} = 47.67 \text{ V } \textbf{Ans.}$$

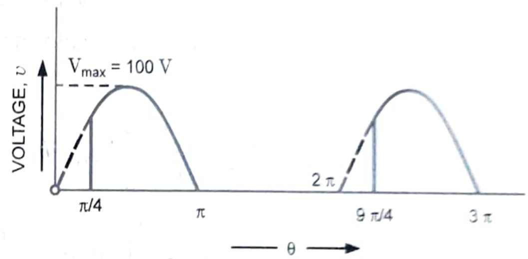


Fig. 4.17

Example 4.17. A circuit carries a current which is the resultant of direct current of 20 A, and a sinusoidal alternating current having a peak value of 20 A. Find the rms value of current in the circuit.

[Pb. Technical Univ. Basic Electrical Engineering June-2003]

Solution: The resultant current waveform is shown in Fig. 4.18.

The instantaneous value of the resultant current wave is given by the expression

$$i = 20 + 20 \sin \omega t = 20(1 + \sin \omega t) = 20(1 + \sin\theta)$$

$$\text{and } i^2 = 400(1 + \sin\theta)^2 = 400 + 800 \sin\theta + 400 \sin^2\theta$$

$$\text{or } i^2 = 400 + 800 \sin\theta + 200(1 - \cos 2\theta) = 600 + 800 \sin\theta - 200 \cos 2\theta$$

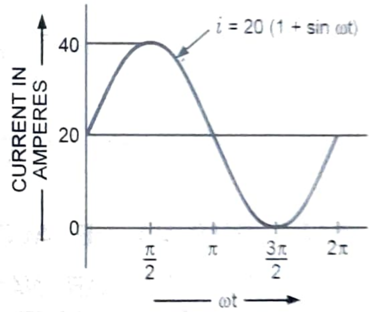


Fig. 4.18

$$I_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta \text{ considering complete cycle } = \frac{1}{2\pi}\int_0^{2\pi}(600 + 800\sin\theta - 200\cos 2\theta)\, d\theta$$

$$= \frac{1}{2\pi} \times 600 \times 2\pi = 600$$

$$\text{or } I_{rms} = \sqrt{600} = 24.5 \text{ A } \textbf{Ans.}$$

4.11 RMS VALUE, AVERAGE VALUE, PEAK FACTOR AND FORM FACTOR OF SQUARE WAVEFORM

Square wave current is shown in Fig. 4.19. Instantaneous value of current can be expressed as

$$i = I_{max} \quad \text{for} \quad 0 < \theta < \pi$$

Average value of the square current wave is given by expression

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i \, d\theta = \frac{1}{\pi}\int_0^{\pi} I_{max}\, d\theta = I_{max}\frac{(\pi - 0)}{\pi} = I_{max}$$

RMS value of the square current wave is given by the expression

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2 d\theta} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_{max}^2 \, d\theta} = \sqrt{\frac{1}{\pi} I_{max}^2 (\pi - 0)} = I_{max}$$

$$\text{Form factor, } K_f = \frac{\text{RMS value}}{\text{Average value}} = \frac{I_{max}}{I_{max}} = 1$$

$$\text{Peak factor, } K_p = \frac{\text{Peak value}}{\text{RMS value}} = \frac{I_{max}}{I_{max}} = 1$$

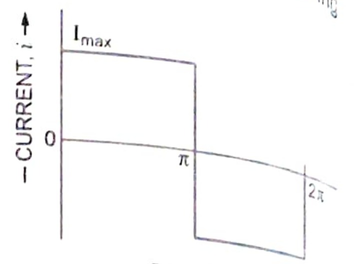


Fig. 4.19

Example 4.18. Find rms value, average value and form factor of the wave shown in Fig. 4.20.

[U.P. Technical Univ. Electrical Engineering Odd Semester, 2013-14]

Solution: The given voltage waveform may be expressed as

$$v = V_{max} \text{ for } 0 < t < 0.1 = 30 \text{ V for } 0 < t < 0.1$$

Average value of the given voltage waveform is given as

$$V_{av} = \frac{1}{0.3}\int_0^{0.1} V_{max}\, dt$$

$$= \frac{1}{0.3}\int_0^{0.1} 30\, dt = \frac{1}{0.3}[30t]_0^{0.1} = \frac{3}{0.3} = 10 \text{ V} \quad \textbf{Ans.}$$

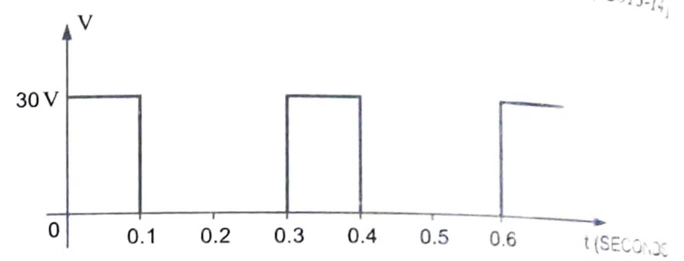


Fig. 4.20

RMS value of the given voltage waveform is given as

$$V_{rms} = \sqrt{\frac{1}{0.3}\int_0^{0.1} V_{max}^2\, dt} = \sqrt{\frac{1}{0.3}\int_0^{0.1} 30^2\, dt} = \sqrt{\frac{1}{0.3}[900t]_0^{0.1}} = \sqrt{300} = 17.32 \text{ V} \quad \textbf{Ans.}$$

$$\text{Form factor, } k_f = \frac{V_{rms}}{V_{av}} = \frac{17.32}{10} = 1.732 \quad \textbf{Ans.}$$

4.12 RMS VALUE, AVERAGE VALUE, FORM FACTOR AND PEAK FACTOR OF A TRIANGULAR WAVEFORM

Let the maximum value of the current be I_{max} amperes.

Since $i = I_{max}$ when $\theta = \pi$. Hence, expression for the instantaneous current can be written as

$$i = \frac{I_{max}}{\pi}\theta \quad \text{for } 0 < \theta < \pi$$

RMS value of the current wave,

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d\theta} = \sqrt{\frac{1}{\pi}\int_0^{\pi} \frac{I_{max}^2}{\pi^2}\theta^2\, d\theta} = \frac{I_{max}}{\sqrt{3}}$$

Average value of the current wave,

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta = \frac{1}{\pi}\int_0^{\pi} \frac{I_{max}}{\pi}\theta\, d\theta = \frac{I_{max}}{2}.$$

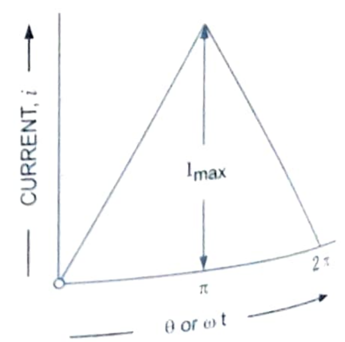


Fig. 4.21

$$\text{Form factor, } K_f = \frac{I_{rms}}{I_{av}} = \frac{I_{max}/\sqrt{3}}{I_{max}/2} = \frac{2}{\sqrt{3}} = 1.155 \quad \textbf{Ans.}$$

$$\text{Peak factor, } K_p = \frac{I_{max}}{I_{rms}} = \frac{I_{max}}{I_{max}/\sqrt{3}} = \sqrt{3} \quad \textbf{Ans.}$$

Example 4.19. A periodic voltage waveform has been shown in Fig. 4.22. Determine (*i*) frequency of the waveform; (*ii*) wave equation for $0 < t < 100$ m second; (*iii*) rms value; (*iv*) average value and (*v*) form factor.

[Rajasthan Univ. 2004]

Solution: Time period, T = 100 ms

(*i*) Frequency, $f = \frac{1}{T} = \frac{1}{100 \times 10^{-3}} = 10$ Hz **Ans.**

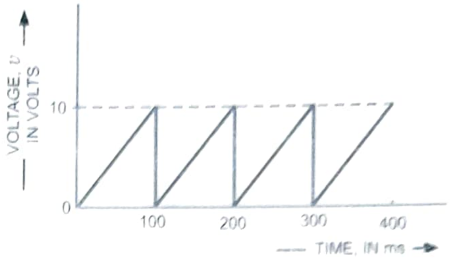


Fig. 4.22

As seen from Fig. 4.22, the instantaneous value of voltage can be expressed as

$v = Kt$ where K is the slope of the curve

Maximum value of voltage, $V_{max} = 10$ V

Since $v = V_{max}$ when $t = 100$ ms = 0.1 second so

$$10 = 0.1 \text{ K or K} = \frac{10}{0.1} = 100$$

(*ii*) Thus wave equation is $v = 100t$ **Ans.**

(*iii*) RMS value of the voltage wave, $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\, dt} = \sqrt{\frac{1}{0.1}\int_0^{0.1} (100t)^2\, dt}$

$$= \sqrt{\frac{10{,}000}{0.1} \times \left[\frac{t^3}{3}\right]_0^{0.1}} = \sqrt{\frac{100}{3}} = 5.774 \text{ V } \textbf{Ans.}$$

(*iv*) Average value of the voltage wave

$$V_{av} = \frac{1}{T}\int_0^T v\, dt = \frac{1}{0.1}\int_0^{0.1} 100t\, dt = \frac{100}{0.1} \times \left[\frac{t^2}{2}\right]_0^{0.1} = \frac{10}{2} = 5 \text{ V } \textbf{Ans.}$$

(*v*) Form factor, $K_f = \frac{V_{rms}}{V_{av}} = \frac{5.774}{5} = 1.155$ **Ans.**

4.13 GRAPHICAL OR PHASOR REPRESENTATION OF ALTERNATING (SINUSOIDAL) QUANTITIES (VOLTAGES AND CURRENTS)

It has already been pointed out that it is assumed that alternating voltages and currents follow sine law and generators are designed to give emfs having sine waveform. The above said assumption makes the calculations simple. The method of representing alternating quantities by waveform or by the equations giving instantaneous values is quite cumbersome. For solution of ac problems it is advantageous to represent a sinusoidal quantity (voltage or current) by a line of definite length rotating in counter-clockwise* direction with the same angular velocity as that of the sinusoidal quantity. Such a rotating line is called the *phasor*.**

* It is a standard convention that the phasor is rotated in counter-clockwise direction — a convention that is in harmony with the general use of polar coordinates.

** Strictly speaking, sinusoidal quantities are scalar quantities varying periodically with time and, according to the definition of vector quantities, they are not true vectors. Voltage is simply energy or work per coulomb and cannot be classified as a vector. Current is also not a vector quantity because it is merely the flow of electrons through a wire. But keeping in mind that any sinusoidal quantity at a given frequency is completely specified by its amplitude and phase angle, its similarity to a vector quantity is evident, since the amplitude may be considered as the magnitude and the phase angle as the direction of a vector. To account for the difference the term phasor has been adopted, instead of the term vector, for representing graphically the magnitude and phase of a sinusoidal current or voltage.

Consider a line OA (or phasor as it is called) representing to scale the maximum value of an alternating quantity, say emf *i.e.* OA = E_{max} and rotating in counter-clockwise direction at an angular velocity ω radians/second about the point O, as shown in Fig. 4.23. An arrow head is put at the outer end of the phasor, partly to indicate which end is assumed to move and partly to indicate the precise length of the phasor when two or more phasors happen to coincide.

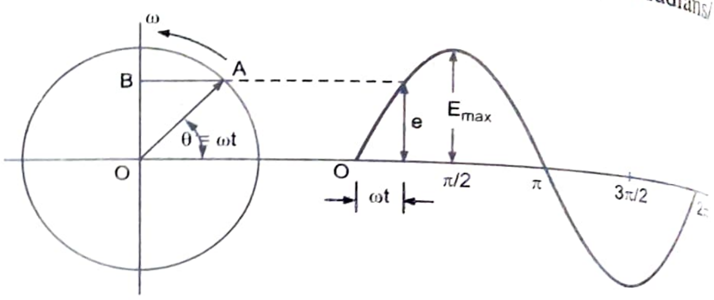


Fig. 4.23 *Phasor Representation of An Alternating Quantity*

Figure 4.23 shows OA when it has rotated through an angle θ, being equal to ω*t*, from the position occupied when the emf was passing through its zero value. The projection of OA on Y-axis,

$OB = OA \sin\theta = E_{max} \sin \omega t = e$, the value of the emf at that instant

Thus, the projection of OA on the vertical axis represents to scale the instantaneous value of emf.

It will be seen that the phasor OA rotating in counter-clockwise direction will represent a sinusoidal quantity (voltage or current) if

(*i*) its length is equal to the peak or maximum value of the sinusoidal voltage or current to a suitable scale.

(*ii*) it is in horizontal position at the instant the alternating quantity (voltage or current) is zero and increasing and

(*iii*) its angular velocity is such that it completes one revolution in the same time as taken by the alternating quantity (voltage or current) to complete one cycle.

4.14 PHASOR DIAGRAM USING RMS VALUES

Since there is definite relation between maximum value and rms value ($E_{max} = \sqrt{2}\ E_{rms}$), the length of phasor OA can be taken equal to rms value if desired. But it should be noted that in such cases, the projection of the rotating phasor on the vertical axis will not give the instantaneous value of that alternating quantity.

The phasor diagram drawn in rms values of the alternating quantities helps in understanding the behaviour of the ac machines under different loading conditions.

4.15 PHASE AND PHASE ANGLE

By phase of an alternating current is meant the fraction of the time period of that alternating current that has elapsed since the current last passed through the zero position of reference. The phase angle of any quantity means the angle the phasor representing the quantity makes with the reference line (which is taken to be at zero degrees or radians). For example the phase angle of current I_2 in Fig. 4.24 is (– ϕ).

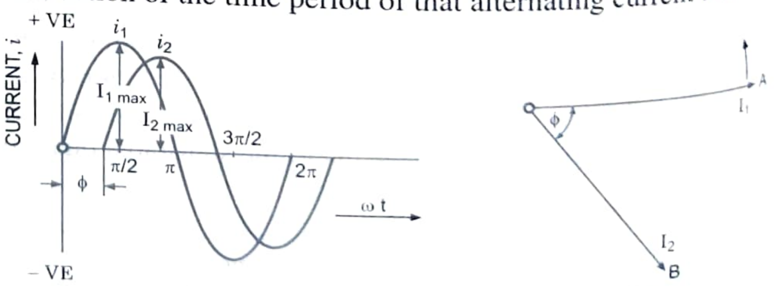


Fig. 4.24

4.16 PHASE DIFFERENCE

When two alternating quantities, say, two emfs, or two currents or one voltage and one current are considered simultaneously, the frequency being the same, they may not pass through a particular point at the same instant. One may pass through its maximum value at the instant when the other passes through the value other than its maximum one. These two quantities are said to have a *phase difference*. Phase difference is always given either in degrees or in radians.

The phase difference is measured by the angular distance between the points where the two curves cross the base or reference line in the same direction.

The quantity ahead in phase is said to lead the other quantity while the second quantity is said to lag behind the first one. In Fig. 4.24(*b*) current I_1 represented by phasor OA leads the current I_2 represented by phasor OB by ϕ or current I_2 lags behind the current I_1 by ϕ. The leading current I_1 goes through its zero and maximum values first and the current I_2 goes through its zero and maximum values after time angle ϕ. The two waves representing these two currents are shown in Fig. 4.24(*a*). If I_1 is taken as reference phasor, the two currents can be expressed as

$$i_1 = I_{1\,max} \sin \omega t$$

$$\text{and } i_2 = I_2 \sin (\omega t - \phi)$$

The two quantities are said to be in phase with each other if they pass through zero values at the same instant and rise in the same direction, as shown in Fig. 4.25. But the two quantities passing through zero values at the same instant but rising in opposite directions, as shown in Fig. 4.26 are said to be in phase opposition, *i.e.* phase difference is 180°. When the two alternating quantities have a phase difference of 90° or $\frac{\pi}{2}$ radians they are said to be in quadrature.

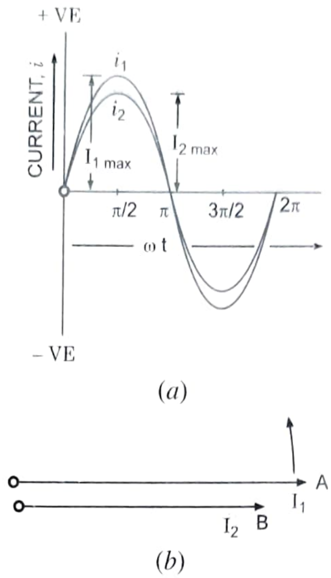


Fig. 4.25

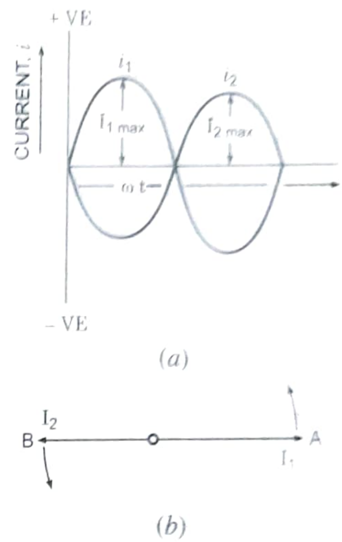


Fig. 4.26

4.17 CONVENTIONS FOR DRAWING PHASOR DIAGRAMS

As already mentioned, the alternating quantities (voltages and currents) in practice are represented by straight lines having definite direction and length. Such lines are called the *phasors* and the diagrams in which phasors represent currents, voltages and their phase difference are known as *phasor diagrams*.

Though phasor diagrams can be drawn to represent either maximum or effective values of voltages and currents but since effective values are of much more importance, phasor diagrams are mostly drawn to represent effective values.

In order to achieve consistent and accurate results it is essential to follow certain conventions. Some of the common conventions in this regard are enlisted below:

1. Counter-clockwise direction of rotation of phasors is usually taken as positive direction of rotation of phasors *i.e.* a phasor rotated in a counter-clockwise direction from a given phasor is said to lead the given phasor while a phasor rotated in clockwise direction is said to lag the given phasor.
2. For series circuits, in which the current is common to all parts of the circuit, the current phasor is usually taken as reference phasor for other phasors in the same diagram and drawn on horizontal line.
3. In parallel circuits in which the voltage is common to all branches of the circuit, the voltage phasor is usually taken as reference phasor and drawn on the horizontal line. Other phasors are referred to the common voltage phasor.
4. It is not necessary that current and voltage phasors are drawn to the same scale; in fact it is often desirable to draw the current phasor to a larger scale than the voltage phasor when the values of currents being represented are small. However, if several voltage phasors are to be used in the same phasor diagram, they should all be drawn to the same scale. Likewise all current phasors in the same diagram should be drawn to the same scale.

4.18 ADDITION AND SUBTRACTION OF ALTERNATING QUANTITIES

In most of the ac circuits, it is necessary to consider the combined action of several emfs or voltages acting in a series circuit and action of several currents flowing through the different branches of a parallel circuit.

Let it be required to add two currents given by the equations

$$i_1 = I_{1\ max} \sin \omega t \text{ and } i_2 = I_{2\ max} \sin (\omega t - \phi)$$

The resultant sum may be expressed as

$$i_r = i_1 + i_2 = I_{1\ max} \sin \omega t + I_{2\ max} \sin (\omega t - \phi)$$

but it is too awkward and gives no idea of the peak value and phase angle of the resultant current.

The currents may be added graphically by plotting their curves in the same system of coordinates and then adding the ordinates of i_1 and i_2 point by point, according to the equation $i_r = i_1 + i_2$. Evidently this method is also too cumbersome and unwieldy to be practical. This is particularly so when situation arise where more than two sinusoidal quantities are to be added.

A simpler and more direct method consists in adding the sinusoidal quantities as phasors.

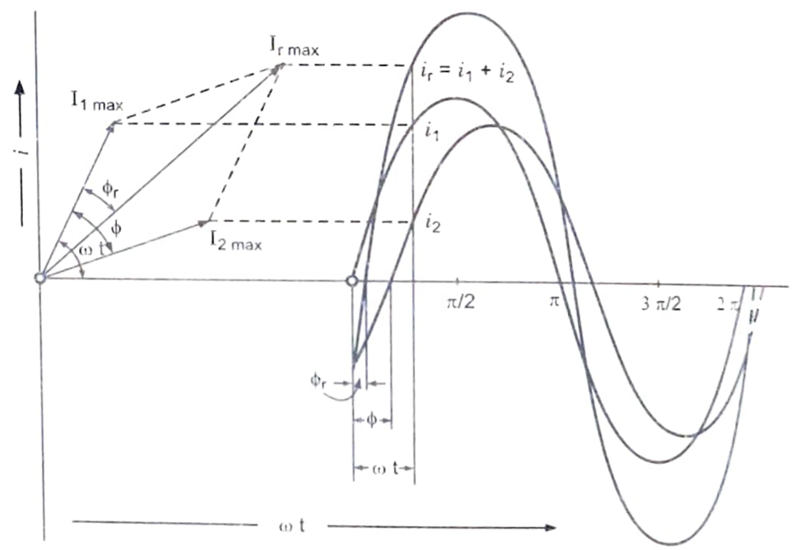


Fig. 4.27

Consider the phasors $I_{1\ max}$ and $I_{2\ max}$ that would generate the two curves i_1 and i_2 and let them be in a position, as shown in Fig. 4.27 at one particular instant of time. If we now add $I_{1\ max}$ and $I_{2\ max}$ by completing the parallelogram as shown, the diagonal $I_{r\ max}$ will, when rotated, generate a third sine curve. It remains to be shown that this third sine curve coincides with the waveform of i_r obtained by adding i_1 and i_2 point by point.

Now the vertical component of $I_{r\ max}$ is the sum of the vertical components of $I_{1\ max}$ and $I_{2\ max}$. Therefore, the waveform of i_r is the graph generated by rotating $I_{r\ max}$ in counter-clockwise direction.

It follows, therefore, that two or more alternating quantities may be added in the same way as forces are added, namely by constructing parallelograms or closed polygons and either measuring or calculating the lengths of the diagonals or closing sides and the magnitude of the phase angles.

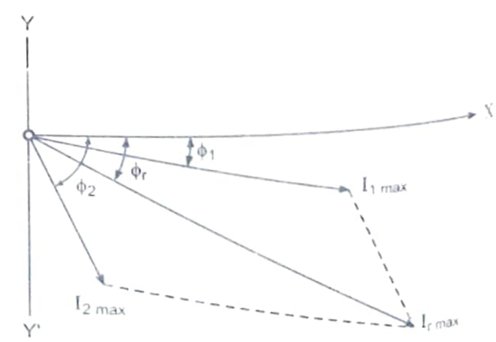


Fig. 4.28

Illustrations: The way in which the two given currents i_1 and i_2 can be added by the parallelogram rule of phasor addition is illustrated in Fig. 4.28 where the currents are shown as phasor drawn from the origin O of the system of coordinates. The resultant phasor is the diagonal of the parallelogram formed by the phasors $I_{1\ max}$ and $I_{2\ max}$.

This method is more convenient when more than two phasors are to be added, as shown in Fig. 4.29. From the end point of $I_{1\ max}$ a phasor is constructed parallel to $I_{2\ max}$ of the same magnitude and direction as the latter; then from the end point of $I_{2\ max}$ a phasor is constructed parallel to $I_{3\ max}$ and so on. Phasor $I_{r\ max}$ from the origin of the first phasor ($I_{1\ max}$) to the end point of the last phasor ($I_{5\ max}$) represents the sum of all the phasors.

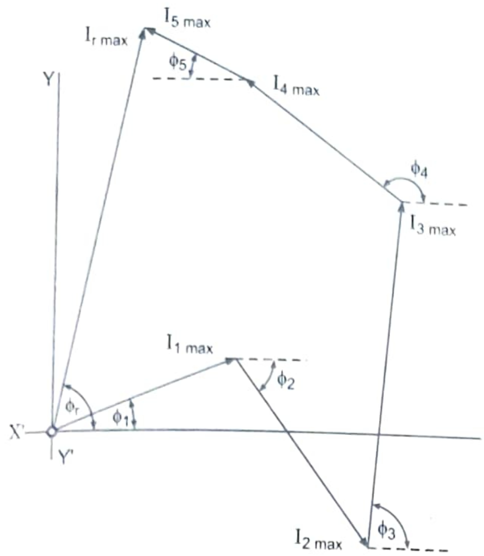


Fig. 4.29

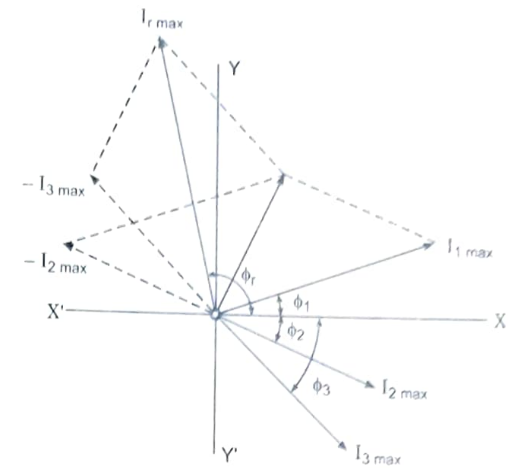


Fig. 4.30

Phasors may also be subtracted by the above method. For example if phasors $I_{2\,max}$ and $I_{3\,max}$ are to be subtracted from phasor $I_{1\,max}$ each of the two phasors be reversed in direction and then added as explained above [Fig. 4.30]. This time, the phasor drawn from the origin of the first phasor $I_{1\,max}$ to the terminal point of the last phasor $I_{3\,max}$ gives the difference of phasors $I_{2\,max}$ and $I_{3\,max}$ from phasor $I_{1\,max}$.

Example 4.20. The two branches of parallel circuit draws currents I_1 and I_2 such that $I_1 = 10\sqrt{2}\sin\omega t$ and $I_2 = 5\sqrt{2}\sin(\omega t - 60°)$. What is the total current drawn?

[G.B. Technical Univ. Electrical Engineering Second Semester, 2010-11]

Solution:

$$i_1 = 10\sqrt{2}\sin\omega t$$

$$i_2 = 5\sqrt{2}\sin(\omega t - 60°)$$

Resultant current, $i = i_1 + i_2 = 10\sqrt{2}\sin\omega t + 5\sqrt{2}\sin(\omega t - 60°)$

$$= 10\sqrt{2}\sin\omega t + 5\sqrt{2}\sin\omega t\cos(-60°) + 5\sqrt{2}\cos\omega t\sin(-60°)$$

$$= 10\sqrt{2}\sin\omega t + 2.5\sqrt{2}\sin\omega t - 2.5\sqrt{6}\cos\omega t = 17.678\sin\omega t - 6.124\cos\omega t$$

Multiplying and dividing RHS of the above expression by $\sqrt{(17.678)^2 + (-6.124)^2}$ i.e. 18.708 we have

$$i_r = 18.708\left(\sin\omega t \times \frac{17.678}{18.708} + \cos\omega t \times \frac{-6.124}{18.708}\right)$$

$$= 18.708(\sin\omega t\cos\phi_r + \cos\omega t\sin\phi_r) \quad \text{where } \cos\phi_r = \frac{17.678}{18.708} \text{ and } \sin\phi_r = \frac{-6.124}{18.708}$$

$$= 18.708\sin(\omega t + \phi_r) \quad \text{where } \phi_r = \text{Tan}^{-1}\frac{-6.124}{17.678} = -19.1°$$

So $i_r = 18.708\sin(\omega t - 19.1°)$ **Ans.**

Analytical method. Resolving currents $I_{1\,max}$ and $I_{2\,max}$ along X-axis and Y-axis (Fig. 4.31) we have

Algebraic sum of X-components $= 10\sqrt{2}\cos 0° + 5\sqrt{2}\cos(-60°) = 14.1421 - 3.5355 = 17.678$

Algebraic sum of Y-components $= 10\sqrt{2}\sin 0° + 5\sqrt{2}\sin(-60°) = 0 + (-6.124) = -6.124$

Maximum value of resultant current, $I_{r\,max} = \sqrt{(\text{X-components})^2 + (\text{Y-components})^2}$

$$= \sqrt{(17.678)^2 + (-6.124)^2} = 18.708 \text{ A}$$

Phase angle, $\phi_r = \text{Tan}^{-1}\dfrac{\text{Y-components}}{\text{X-components}} = \text{Tan}^{-1}\dfrac{-6.124}{17.678} = -19.1°$

$\therefore$ Resultant current, $i_r = I_{r\,max}\sin(\omega t + \phi_r) = 18.708\sin(\omega t - 19.1°)$ **Ans.**

Graphical Method. Take OX as reference phasor. Draw phasor OA = 4 cm in length along OX as to represent current $I_{1\ max}$ of $10\sqrt{2}$ A (to the scale 1 cm = $2.5\sqrt{2}$ A *i.e.* 3.5355 A) in magnitude as well as in direction. Then draw AB = 2 cm in length and making an angle of $-60°$ with the reference phasor so as to represent $I_{2\ max}$ of $5\sqrt{2}$ A in magnitude as well as in direction. The phasor OB, obtained by joining points O and B, will represent the resultant current in magnitude as well as in direction. On measurement from Fig. 4.32, OB = 5.3 cm and angle AOB = 19.1°.

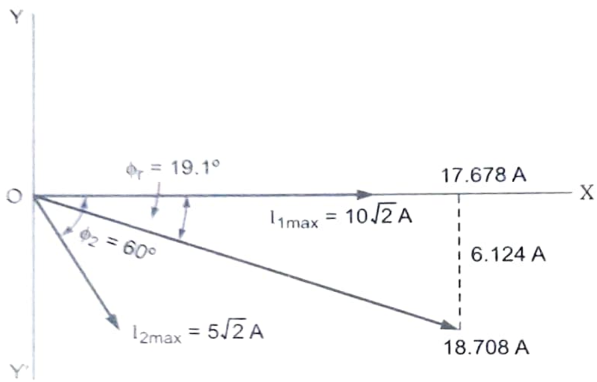


Fig. 4.31

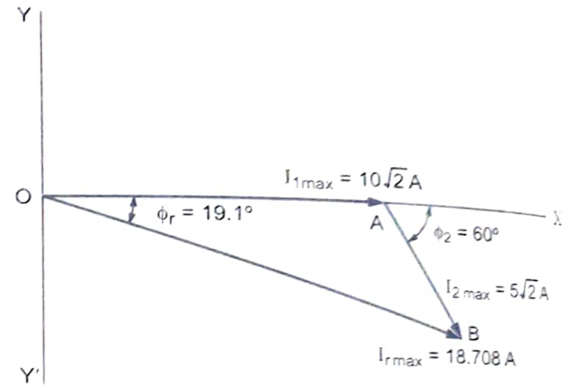


Fig. 4.32

Maximum value of resultant current, $I_{r\ max} = 5.3 \times 2.5\sqrt{2} = 18.7$ A

Phase angle $\phi_r = 19.1°$

Hence the resultant current may be expressed as

$$i_r = 18.7 \sin(\omega t - 19.1°) \quad \textbf{Ans.}$$

Example 4.21. Three sinusoidal currents flow into a junction $i_1 = 3\sqrt{2} \sin \omega t$, $i_2 = 5\sqrt{2} \sin(\omega t + 30°)$ and $i_3 = 6\sqrt{2} \sin(\omega t - 120°)$. Find the time expression for the resultant sinusoidal current which leaves the junction (use phasor diagram). [*G.G.S.I.P. Univ. Delhi Electrical Science, May/June-2009*]

Solution: Take OX as reference phasor. Draw phasor OA 3 cm in length along OX so as to represent $I_{1\ max}$ of $3\sqrt{2}$ A (to the scale of 1 cm = $\sqrt{2}$ A) in magnitude as well as in direction. Then draw line AB = 5 cm in length making an angle of 30° with the reference phasor so as to represent $I_{2\ max} = 5\sqrt{2}$ A in magnitude as well as in direction. Then draw line BC = 6 cm in length making an angle of $-120°$ with the reference phasor so as to represent $I_{3\ max} = 6\sqrt{2}$ A in magnitude as well as in direction. The phasor CO obtained by joining points C and O, will represent the resultant sinusoidal current which leaves the junction.

On measurement from Fig. 4.33.

Maximum value of resultant current,

$$I_{r\ max} = OC \times \sqrt{2} = 5.1\sqrt{2} \text{ A}$$

Phase angle $\phi_r = 148.1°$

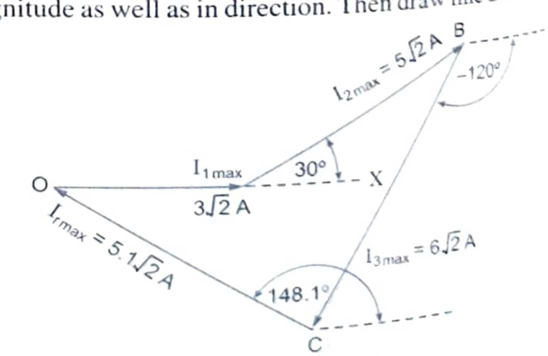


Fig. 4.33

Hence resultant current leaving the junction is given by expression

$$i_r = 5.1\sqrt{2} \sin(\omega t + 148.1°).$$

Example 4.22. Draw a phasor diagram showing the following voltages:

$$v_1 = 100 \sin 500\, t$$
$$v_2 = 200 \sin(500\, t + 45°)$$
$$v_3 = -50 \cos 500\, t$$

Find rms value of the resultant voltage.

[*U.P. Technical Univ. Basic Electrical Engineering Second Semester, 2013-14*]

Solution:

$$v_1 = 100 \sin 500\, t$$
$$v_2 = 200 \sin (500\, t + 45°)$$
$$v_3 = -50 \cos 500\, t = -50 \sin (90° - 500\, t)$$
$$= 50 \sin (500\, t - 90°)$$

Phasor diagram is shown in Fig. 3.34.

Resolving voltages V_1, V_2 and V_3 along X-axis and Y-axis we have

Algebraic sum of X-components $= 100 \cos 0° + 200 \cos 45° + 50 \cos 90°$

$$= 100 + 100\sqrt{2} = 241.42 \text{ V}$$

Algebraic sum of Y-components $= 100 \sin 0° + 200 \sin 45° + 50 \sin (-90°)$

$$= 0 + 141.42 - 50 = 91.42 \text{ V}$$

Maximum value of the resultant voltage,

$$V_{max} = \sqrt{(241.42)^2 + (91.42)^2} = 258.15 \text{ V}$$

RMS value of the resultant voltage,

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{258.15}{\sqrt{2}} = 182.54 \text{ V} \quad \textbf{Ans.}$$

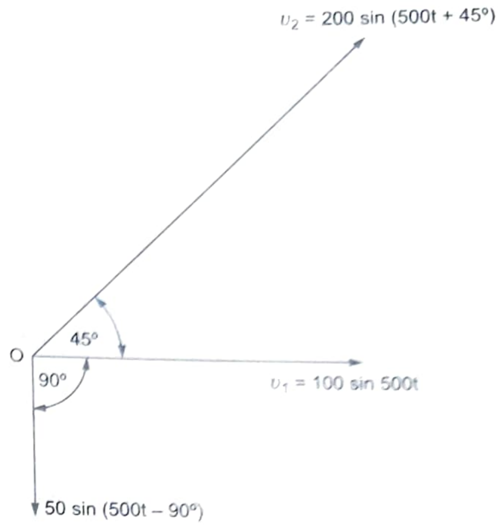


Fig. 4.34 *Phasor Diagram*

Highlights

1. An *alternating quantity* (voltage or current) is one which changes continuously in magnitude and alternates in direction at regular intervals of time.

   An alternating quantity that varies sinusoidally (*i.e.* according to the sin of angle θ) is called the *sinusoidal quantity*. Sinusoidal quantities are expressed as

   $$e = E_{max} \sin \theta = E_{max} \sin \omega t = E_{max} \sin 2\pi f t$$
   $$\text{and } i = I_{max} \sin \theta = I_{max} \sin \omega t = I_{max} \sin 2\pi f t$$

2. The shape of the curve of the voltage or current when plotted against time as abscissa (base) is called the *waveform*.
3. When a periodic wave, such as sinusoidal wave, goes through one complete set of positive and negative values it is said to have completed one *cycle*. One cycle corresponds to 360° or 2 π radians.

   Alternation is one half of cycle and corresponds to 180° or π radians.

4. The time taken in seconds by an alternating quantity to complete one cycle is known as *time period* or *periodic time* (T), while the number of cycles completed per second by an alternating quantity is known as *frequency* (f).

   $$f = \frac{1}{T}$$

   In India the standard frequency for power supply is 50 Hz (cycles per second).

5. Angular velocity, $\omega = 2\pi f$ radians per second.
6. In a multipolar machine, the number of cycles completed per second by generated emf,

   $$f = \text{Pairs of poles} \times \text{number of revolutions made per second} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120}$$

7. The maximum value, positive or negative, which an alternating quantity attains during one cycle is called the *amplitude* of the alternating quantity.
8. The general expression for an alternating voltage and current are given as

   $$v = V_{max} \sin (\omega t \pm \phi)\ ; \quad i = I_{max} \sin (\omega t \pm \phi)$$

   where φ is the phase angle in degrees or radians.

   The coefficient of the sine of the time angle gives maximum or peak value of the alternating quantity (emf, voltage or current)

   Coefficient of time t divided by 2 π gives the frequency of the periodic wave.