For example, if $G(j\omega) = 1/j\omega$ then $G(j\omega)^{-1} = j\omega$

$$G(j\omega)^{-1} = \omega \angle 90^\circ$$

$$\lim_{\omega \to 0} \left|G(j\omega)^{-1}\right| = 0$$

$$\lim_{\omega \to \infty} \left|G(j\omega)^{-1}\right| = \infty$$

The inverse polar plot is shown in the Fig. 4.17(*b*).

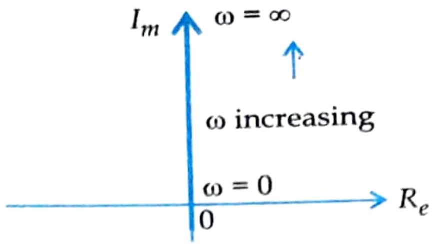


Fig. 4.17 (b). Inverse polar plot of 1/jω

EXAMPLE 4.7. Sketch the inverse polar plot of $G(j\omega) = j\omega T/1 + j\omega T$.

Solution :

$$G(j\omega)^{-1} = \frac{1}{G(j\omega)} = \frac{1 + j\omega T}{j\omega T} = \frac{1}{j\omega} + 1$$

$$\lim_{\omega \to 0} \left|G(j\omega)^{-1}\right| = \infty$$

$$\lim_{\omega \to \infty} \left|G(j\omega)^{-1}\right| = 1$$

$$\lim_{\omega \to 0} \angle G(j\omega)^{-1} = -90^\circ$$

$$\lim_{\omega \to \infty} \angle G(j\omega)^{-1} = 0^\circ$$

The polar plot is shown in Fig. 4.18.

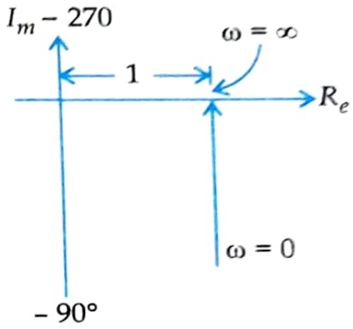


Fig. 4.18. Inverse plot of jωT/1+jωT

EXAMPLE 4.8. Sketch the inverse polar plot of $G(s) = \dfrac{1 + sT}{sT}$.

Solution :

$$G(s)^{-1} = \frac{1}{G(s)} = \frac{sT}{1 + sT}$$

Put $s = j\omega$

$$G(j\omega)^{-1} = \frac{1}{G(j\omega)} = \frac{j\omega T}{1 + j\omega T}$$

$$\lim_{\omega \to 0} \left|G(j\omega)^{-1}\right| = 0 \qquad \lim_{\omega \to 0} \angle G(j\omega)^{-1} = 90^\circ$$

$$\lim_{\omega \to \infty} \left|G(j\omega)^{-1}\right| = 1 \qquad \lim_{\omega \to \infty} \angle G(j\omega)^{-1} = 0^\circ$$

Inverse polar plot is shown in Fig. 4.19.

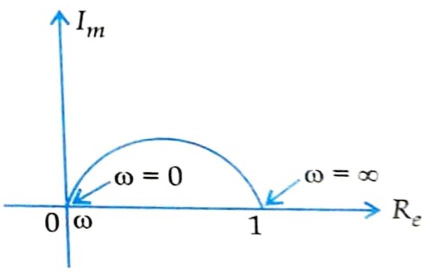


Fig. 4.19. Inverse polar plot of

4.10. BODE PLOT

Bode plot is a graphical representation of the transfer function for determining the stability of the control system. Bode plot consists of two separate plots. One is a plot of the logarithm of the magnitude of a sinusoidal transfer function, the other is a plot of the phase angle, both plots are plotted against the frequency. The curves are drawn on semilog graph paper, using the log scale for frequency and linear scale for magnitude (in decibles) or phase angle (in degrees). The magnitude is represented in decibles. Thus, Bode plot consists of :

(*i*) $20 \log_{10} |G(j\omega)|$ V_s $\log \omega$.

(*ii*) Phase shift V_s $\log \omega$

The main advantage of using Bode plot is that multiplication of magnitudes can be converted into addition.

Consider open loop transfer function of a closed loop control system

$$G(s)\ H(s) = \frac{K(1+sT_a)(1+sT_b)....}{s^N(1+sT_1)(1+sT_2)....}$$

Put $$s = j\omega$$

$$G(j\omega)\ H(j\omega) = \frac{K(1+j\omega T_a)(1+j\omega T_b).......}{(j\omega)^N\ (1+j\omega T_1)(1+j\omega T_2)......}$$

$$20\log_{10}|G(j\omega)\ H(j\omega)| = \left(20\log K + 20\log\sqrt{1+\omega^2 T^2{}_a} + 20\log\sqrt{1+\omega^2 T^2{}_b}\right)....)$$

$$-\left(20N\log\omega + 20\log\sqrt{1+\omega^2 T_1^2} + 20\log\sqrt{1+\omega^2 T_2^2}\right)....)$$

Hence, in order to get $|G(j\omega)\ H(j\omega)|$ we will have to obtain the individual plots and adding individual components, the resultant can be obtained. Suppose, $H(s) = 1$.

Case 1. The Gain K

$$G(s) = K$$

Put $$s = j\omega$$

$$G(j\omega) = K$$

$$20\log_{10}|G(j\omega)| = 20\log_{10}K \quad ...(4.1)$$

$$\text{Phase angle } \phi = \angle G(j\omega) = 0° \quad ...(4.2)$$

From equations (4.1) and (4.2) it is clear that the magnitude is independent of $\log_{10}\omega$ and phase angle always zero. The plots are shown in Fig. (4.20).

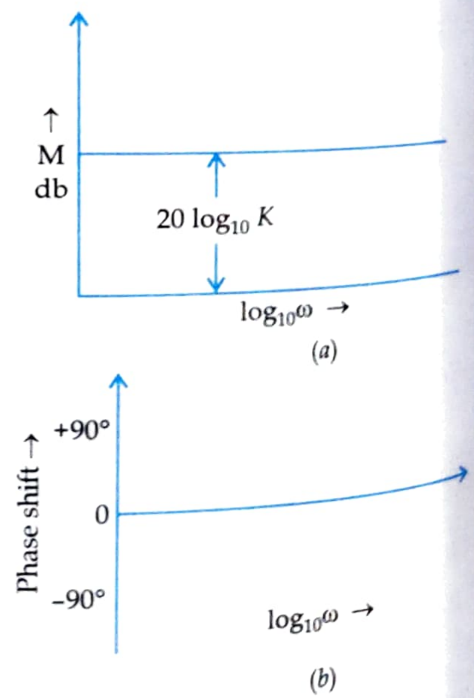


Fig. 4.20.

Case 2 :

$$G(s) = \frac{1}{S^N}$$

Put $$s = (j\omega)^N$$

$$\therefore \quad G(j\omega) = \frac{1}{(j\omega)^N}$$

$$20\log_{10}|G(j\omega)| = 20\log_{10}\frac{1}{(j\omega)^N}$$

$$= 20\log_{10}(j\omega)^{-N}$$

$$= -20\ N\log_{10}(\omega)$$

$$\angle G(j\omega)^N = -90\ N°$$

Where $N = 1,2,3,$ ....

The plot M Vs $\log_{10}\omega$ is a straight line. For $N = 1$ the line has a slope of -20 db/decade and angle $-90°$. For $N = 2$, the slope of the line will be -40 db/decade and angle will be $-180°$ and so on.

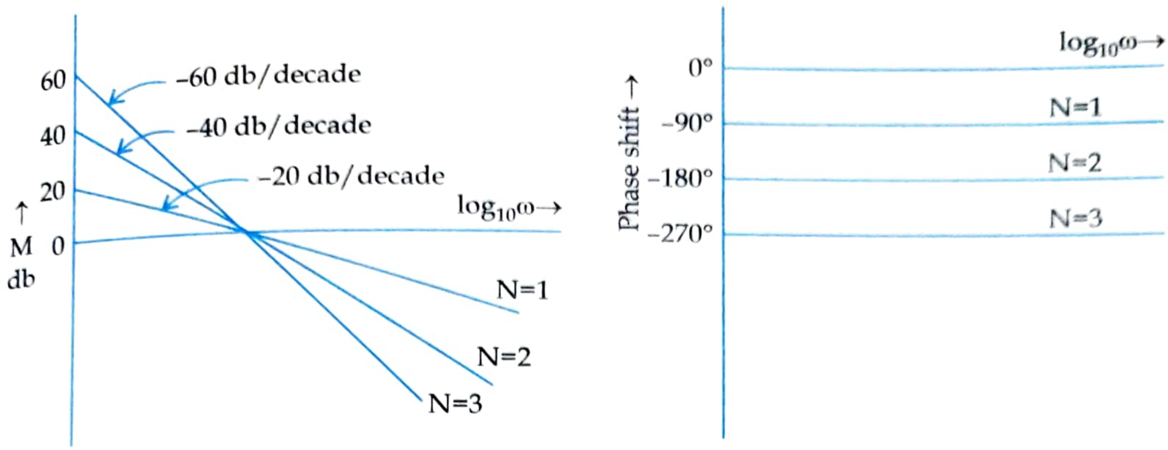


Fig. 4.21.

Case 3 :

$$G(s) = s$$

Put

$$s = j\omega$$

$$G(j\omega) = j\omega$$

$$M = 20\, log_{10} |G(j\omega)| = 20 \log_{10} \omega$$

$$\angle G(j\omega) = +\ 90°$$

The plot $M\ V_s \log_{10} \omega$ is a straight line having a slope of $+20\ db/$dec and angular phase shift of + 90°.

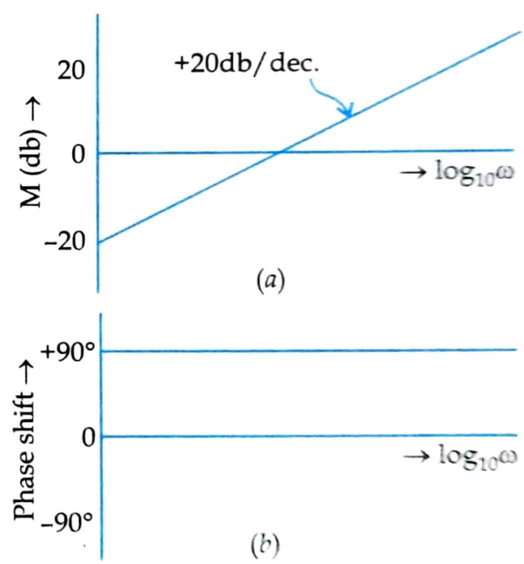


Fig. 4.22.

Case 4 :

$$G(s) = \frac{1}{1+sT}$$

Put

$$s = (j\omega)$$

$\therefore$

$$G(j\omega) = \frac{1}{1+j\omega T}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2T^2}}$$

$$20 \log_{10} |G(j\omega)| = 20\log_{10}\left[\frac{1}{\sqrt{1+\omega^2T^2}}\right]$$

$$= 20\log_{10} 1 - 20\log_{10}\sqrt{1+\omega^2T^2}$$

$$= -20\log_{10}\sqrt{1+\omega^2T^2} \qquad \because \log_{10} 1 = 0$$

Put the different values of ω, we will get $|G(j\omega)|$ consider following two cases.

(a) For $\omega T << 1$ (very low frequencies)

$$-20\log_{10}\sqrt{1+\omega^2T^2} = -20\log_{10}\sqrt{1} = 0$$

$\therefore$

$$M = 0 \text{ for } \omega T << 1 \text{ or } \omega \leq \frac{1}{T}$$

(b) For $\omega T >> 1$ (very high frequencies)

$$-20\log_{10}\sqrt{1+\omega^2T^2} = -20\log_{10}\sqrt{\omega^2T^2}$$

$$= -\ 20 \log_{10} \omega T \text{ for } \omega >> 1/T$$

Hence, M Vs $\log_{10} \omega$ has two parts

(i) One part having $M = 0$ for $\omega << 1/T$

(ii) In other part M varies as a straight line with slope of – 20 db/decade for $\omega >> \frac{1}{T}$

$\omega = \frac{1}{T}$ is called break frequency or corner frequency

$$M = -20 \log_{10} \omega T = -20 (\log_{10} \omega + \log_{10} T)$$
$$M = -20 \log_{10} \omega - 20 \log_{10} T$$
$$= -20 \log_{10} \omega + 20 \log_{10} 1/T \quad ...(4.3)$$

The above two parts of the graph intersect 0 db axis is determined by equating the eqⁿ (4.3) to zero

$$0 = -20 \log_{10} \omega + 20 \log_{10} 1/T$$

$\therefore$ $\omega = 1/T$ is called break frequency.

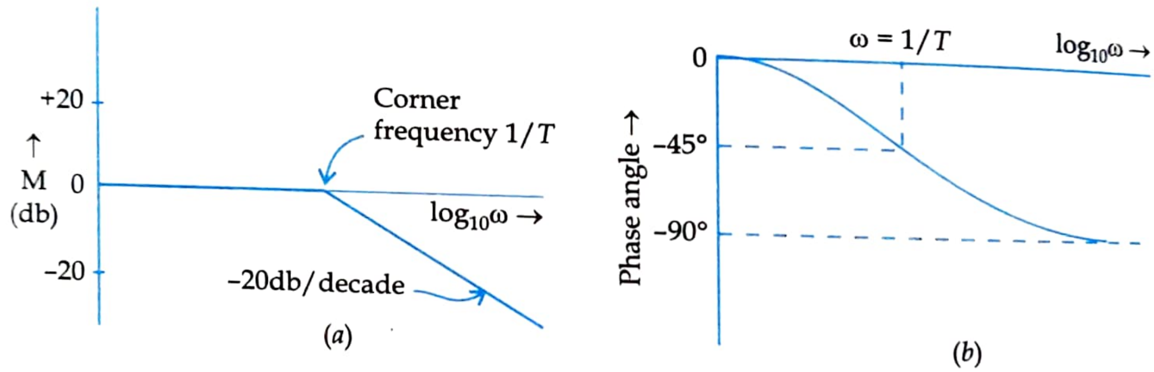


Fig. 4.23.

Case 5 :

$$G(s) = (1 + sT)$$

Put

$$s = j\omega$$
$$G(j\omega) = (1 + j\omega T)$$
$$|G(j\omega)| = \sqrt{1+\omega^2 T^2}$$
$$20 \log_{10} |G(j\omega)| = 20 \log_{10} \sqrt{1+\omega^2 T^2}$$

(i) When $\omega T << 1$

$$M = 20 \log_{10} \sqrt{1} = 0\ db$$

(ii) When $\omega T >> 1$

$$M = 20 \log_{10} \omega T$$
$$M = 20 \log_{10} \omega T = 20 \log_{10} \frac{\omega}{1/T}$$
$$= 20 \log_{10} \omega - 20 \log_{10} \frac{1}{T}$$

Equate the above equation to zero

$$0 = 20 \log_{10} \omega - 20 \log_{10} \frac{1}{T}$$

$\therefore$ $\omega = \frac{1}{T}$ corner frequency.

Thus, the two parts of the graph intersects the '0' db axis at $\omega = \frac{1}{T}$. The second part is a straight line having the slope of + 20 db/decade.

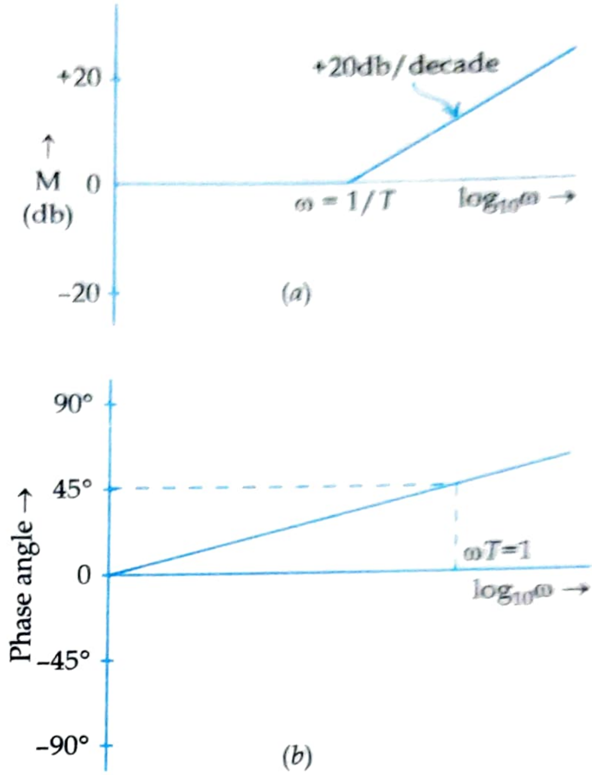


Fig. 4.24.

Phase Angle Plot

$$\phi = \angle G(j\omega) = \tan^{-1}\omega T$$

(i) At very low frequencies ωT is very very small

$$\phi = \tan^{-1}(0) = 0°$$

(ii) At $\omega T = 1$

$$\phi = \tan^{-1}1 = 45°$$

(iii) At very high frequencies

$$\phi = \tan^{-1}(\infty) = 90°$$

Thus, the value of ϕ gradually changes from 0° to 90° as ω increases from 0 to very high values.

Case 6 : General second order system

$$G(s) = \frac{\omega^2_n}{s^2 + 2\xi\omega_n s + \omega^2_n}$$

Put $s = j\omega$

$$G(j\omega) = \frac{\omega^2_n}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega^2_n} = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}}$$

$$20\log_{10}|G(j\omega)| = 20\log_{10}\left|\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}}\right| = -20\log_{10}\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}$$

Suppose $\frac{\omega}{\omega_n} = u$

$$\therefore \quad 20\log_{10}|G(j\omega)| = M = -20\log_{10}\sqrt{(1-u^2)^2 + 4\zeta^2u^2}$$

Consider the two cases

1. $u << 1$ i.e., $\frac{\omega}{\omega_n} << 1$

$$M = -20\log_{10}\sqrt{1} = 0 \text{ db.}$$

2. $u >> 1$ i.e., $\frac{\omega}{\omega_n} >> 1$

$$M = -20\log_{10}\sqrt{(u^2)^2} = -20\log_{10}u^2 = -40\log_{10}u$$

So, it is a straight line having slope of – 40 db/dec. and passing through the point u.

Therefore, the asymptotic plot consists of

(i) $M = 0$ $\quad u << 1$

(ii) $M = -40 \log_{10} u$ $\quad u >> 1$

Phase Angle Plot

$$\phi = \angle G(j\omega) = -\tan^{-1} \frac{2\xi u}{1-u^2}$$

(i) For small value of u, u^2 is small

$$\therefore \quad \phi = -\tan^{-1} 2\xi u$$

(ii) For large value of u, $u^2 >> 1$

$$\therefore \quad \phi = +\tan^{-1} \frac{2\zeta}{u}$$

(iii) When $u = 1$

$\phi = -\tan^{-1} \infty = -90°$

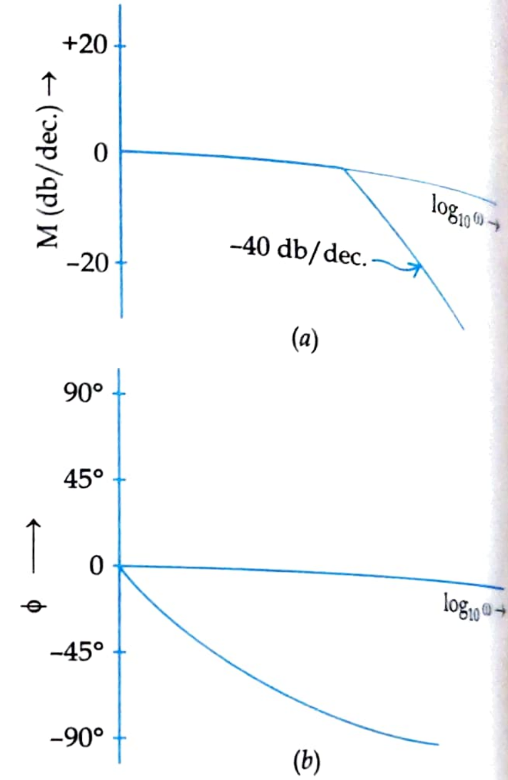


Fig. 4.25.

Initial Slope of Bode Plot

Let $$G(s)\,H(s) = \frac{K}{S^N}$$

Put $$s = j\omega$$

$$G(j\omega)\,H(j\omega) = \frac{K}{(j\omega)^N}$$

$$20 \log_{10} |G(j\omega)\,H(j\omega)| = 20 \log_{10} \left|\frac{K}{(jw)^N}\right| = 20 \log_{10} K - 20\,N \log_{10}\omega \quad ...(4.4)$$

1. For $N = 0$ (Type zero system)

$20 \log_0 |G(j\omega)\,H(j\omega)| = 20 \log_{10} K.$

This is a straight line. The graph is shown in Fig. 4.26.

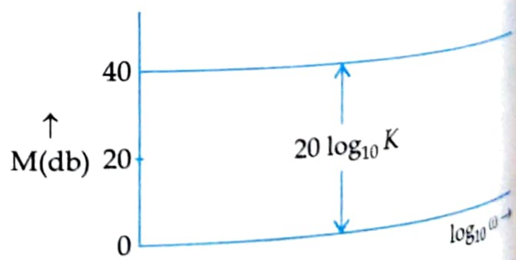


Fig. 4.26.

2. For $N = 1$ (type one system)

Put $N = 1$ in equation (4.4)

$$20\log_{10} |G(j\omega)\,H(j\omega)| = 20 \log_{10} K - 20 \log_{10}\omega$$

Intersection with 0 db axis

$$0 = 20 \log_{10} K - 20 \log_{10}\omega$$

$$\therefore \quad K = \omega$$

locate $\omega = K$ on 0 db axis and at this point draw a line of – 20 db/decade produce it till it intersect the y-axis that will be the starting point on Bode plot.

3. For $N = 2$ (type two system)

Put $N = 2$ in equation (4.4)

$$20 \log_{10} |G(j\omega)\,H(j\omega)| = 20 \log_{10} K - 20.\,2 \log_{10} \omega$$
$$= 20 \log_{10} K - 40 \log_{10} \omega$$

Intersection with 0 db axis

$$0 = 20 \log_{10} K - 40 \log_{10} \omega$$

$$20 \log_{10} K = 40 \log_{10} \omega$$
$$20 \log_{10} K = 20 \log_{10} \omega^2$$
$$\omega^2 = K$$
$$\omega = \sqrt{K}$$

Hence, graph intersect the 0 db axis at $\omega = \sqrt{K}$. Locate $\omega = \sqrt{K}$ on 0 db axis and draw a line – 40 db/dec. and produce it to the y-axis. Graph having the slope of – 40 db/decade is shown in Fig. 4.27.

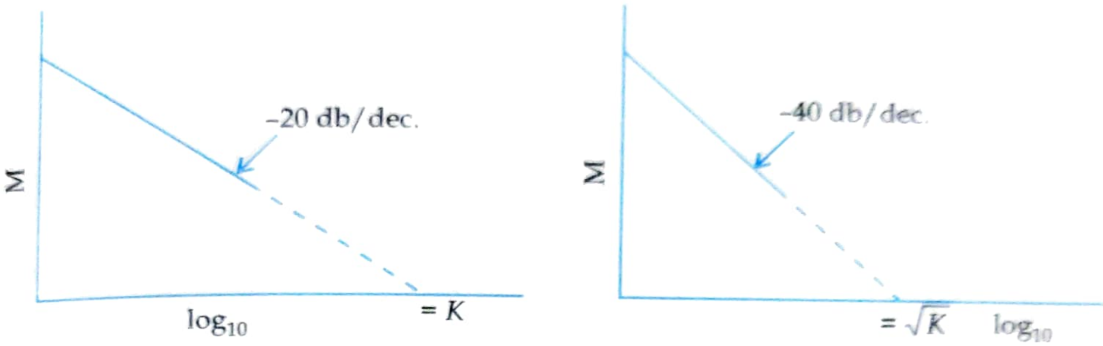


Fig. 4.27.

Table 4.3.

Type of the System N	Initial Slope 0 db Axis	Intersection with
0	0 db/decade	Parallel to 0 axis
1	– 20 db/dec.	$= K$
2	– 40 db/dec.	$= \sqrt{K}$
3	– 60 db/dec.	$= K^{1/3}$
⋮	⋮	⋮
N	–20N db/dec.	$K^{1/N}$

4.11. MINIMUM PHASE SYSTEMS AND NON-MINIMUM PHASE SYSTEMS

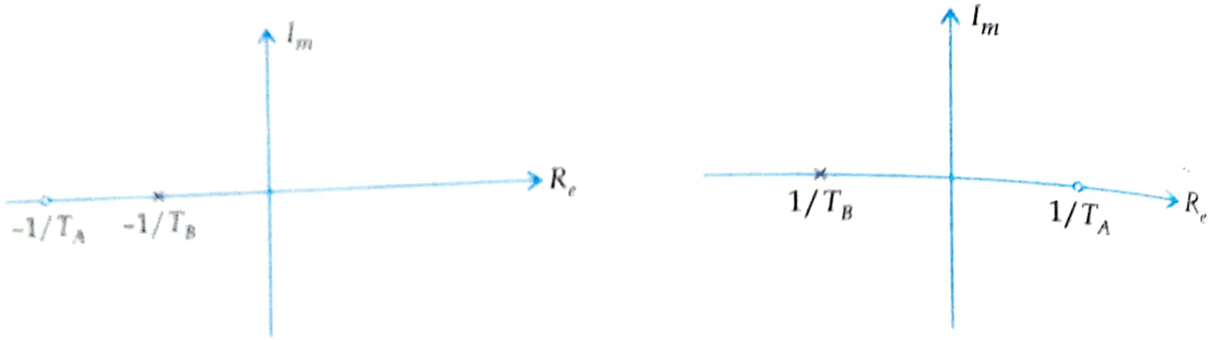
The transfer functions having no poles and zeros in the right half s-plane are called minimum phase transfer functions. Systems with minimum phase transfer functions are called *minimum phase systems*.

The transfer functions having poles and/or zeros in the right half s-plane are called non-minimum phase transfer functions. Systems with non-minimum phase transfer functions are called *non-minimum-phase systems*.

Let
$$G_1(j\omega) = \frac{1 + j\omega T_A}{1 + j\omega T_B} \quad \text{...(4.5)}$$

$$G_2(j\omega) = \frac{1 - j\omega T_A}{1 + j\omega T_B} \quad \text{...(4.6)}$$

The transfer function given by equation (4.5) is a minimum phase transfer function and transfer function given by equation (4.6) is non-minimum phase type transfer function.



Minimum phase   Non-minimum phase

Fig. 4.28.

For minimum phase system, the magnitude and phase angle plots are uniquely related. It means that if the magnitude curve is specified for the frequency from zero to infinity, then the phase angle curve is uniquely related. This is not applicable for non-minimum phase system.

For minimum phase system, the phase angle at $\omega = \infty$ is $-90° (q-p)$ where p and q are the degrees of the numerator and denominator polynomials of transfer function. For both minimum and non-minimum phase system the slope is $-20\ (q-p)$ db/decade. But the phase angle for non-minimum system is differ from $-90°\ (q-p)$. Thus it is possible to determine whether the system is minimum phase or non-minimum phase system. If the slope of the log-magnitude curve as ω approaches infinity is $-20\ (q-p)$ db/dec. and the phase angle is equal to $-90°\ (q-p)$ at $\omega = \infty$ then the system is minimum phase otherwise not.

Non-minimum phase systems are slow in response. In control system excessive phase lag should be avoided.

4.12. PROCEDURE FOR DRAWING THE BODE PLOTS

Consider the transfer function

$$G(s) = \frac{K(1+sT_a)(1+sT_b)\ldots\ldots}{s^N(1+sT_1)(1+sT_2)\ldots\ldots\left[1+2\zeta\left(\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2\right]} \qquad \ldots(4.7)$$

where N is the number of poles at the origin *i.e.*, N defines the type of system.

For type zero system $K = K_p$

For type one system $K = K_v$

For type two system $K = K_a$

In above transfer function put $s = j\omega$

$$G(j\omega) = \frac{K(1+j\omega T_a)(1+j\omega T_b)\ldots\ldots}{(j\omega)^N(1+j\omega T_1)(1+j\omega T_2)\ldots\ldots\left[1+2\zeta(\omega/\omega_n)+(j\omega/\omega_n)^2\right]} \qquad \ldots(4.8)$$

$$20\log_{10}|G(j\omega)| = 20\log K + 20\log\sqrt{1+\omega^2T_a^2} + 20\log\sqrt{1+\omega^2T_b^2} + \ldots\ldots -20N\log\omega$$

$$-20\log\sqrt{1+\omega^2T_1^2} - 20\log\sqrt{1+\omega^2T_2^2} - 20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+4\xi^2\left(\frac{\omega}{\omega_n}\right)^2} \qquad \ldots(4.9)$$

Phase angle

$$\angle G(j\omega) = \tan^{-1}\omega T_a + \tan^{-1}\omega T_b + \ldots\ldots N(90°) - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 \ldots\ldots \tan^{-1}\left[\frac{2\zeta\omega\omega_n}{\omega^2_n-\omega^2}\right] \qquad \ldots(4.10)$$

Step 1 : I dentify the corner frequency.

Step 2 : Draw the asymptotic magnitude plot. The slope will change at each corner frequency by + 20 db / dec. for zero and – 20 db / dec for pole. For complex conjugate pole and zero the slope will change by $\mp$ 40 db/ decade.

Step 3 : (*i*) For type zero system draw a line upto first (lowest) corner frequency having 0 db dec. slope.

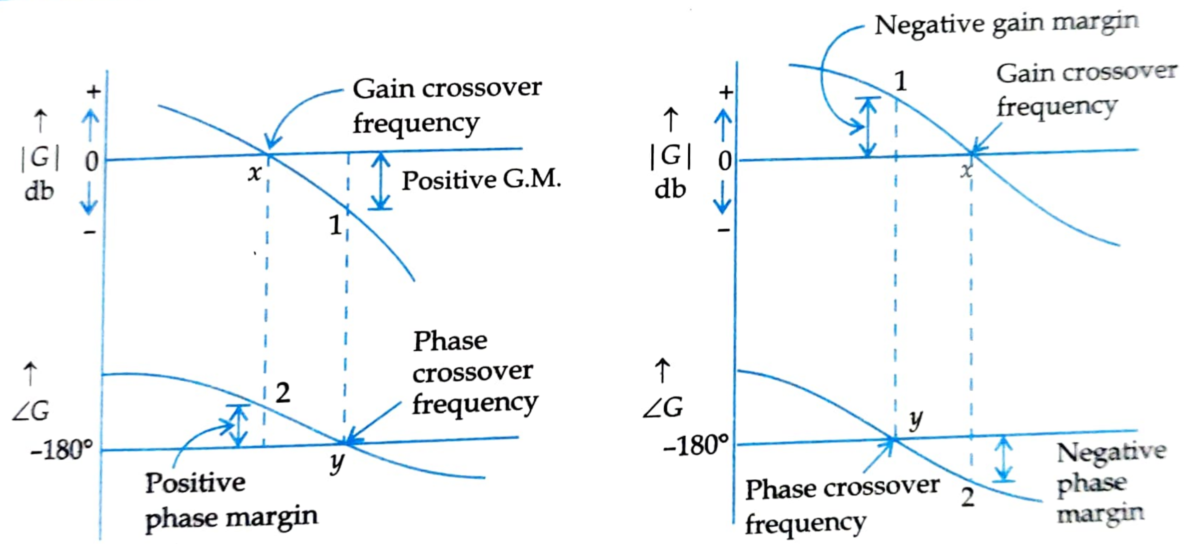
(*ii*) For type one system draw a line having slope – 20 db/ dec. upto $\omega = K$. Mark first (lowest) corner frequency.

(*iii*) For type two system draw the line having slope – 40 db/ dec. upto $\omega = \sqrt{K}$ and so on. Mark first corner frequency.

Step 4 : Draw a line upto second corner frequency by adding the slope of next pole or zero to the previous slope and so on.

Step 5 : Calculate phase angle for different values of ω from the equation (4.10) and join all points.

4.13. PHASE MARGIN AND GAIN MARGIN



(*a*) *Stable system* (*b*) *Unstable system*

Fig. 4.29.

Positive gain margin means the system is stable and negative gain margin means the system is unstable. For minimum phase system both phase margin and gain margin must be positive for the system to be stable.

The point at which the magnitude curve crosses the 0 db line is the gain crossover frequency. The phase crossover frequency is the point where the phase curve crosses the 180° line.

Gain Margin : Gain margin is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability. Mathematically gain margin is defined as the reciprocal of the magnitude of the $G(j\omega)\, H(j\omega)$ at phase cross over frequency.

$$\therefore \qquad \text{G.M.} = \frac{1}{|G(j\omega)\, H(j\omega)|_{\omega = \omega_{c_2}}}$$

where ω_{c_2} = phase crossover frequency.

Generally, G.M. is expressed in decibels

$\therefore$ In decibels $\quad$ G.M. $= 20 \log \dfrac{1}{|G(j\omega)\,H(j\omega)|_{\omega=\omega_{c2}}}$

or $\quad$ G.M. $= -20 \log_{10} |G(j\omega)\,H(j\omega)|_{\omega=\omega_{c2}}$

Phase Margin : For gain the additional phase lag can be introduced without affecting the magnitude plot. Therefore, phase margin can be defined as the amount of additional phase lag which can be introduced in the system till system reaches on the verge of instability is called as phase margin (P.M.). Mathematically phase margin can be defined as

$$\text{P.M.} = \left[\angle G(j\omega)\,H(j\omega)\big|_{\omega=\omega_{c1}}\right] - (-180°)$$

$$\text{P.M.} = 180° + \angle G(j\omega)\,H(j\omega)\big|_{\omega=\omega_{c1}}$$

where ω_{c_1} = Gain crossover frequency.

The above expressions for gain margin and phase margin can be directly used for mathematical determination of G.M. and P.M. For example, for the given transfer function $G(s)\,H(s) = \dfrac{2}{s(1+0.5s)(1+0.05s)}$ determine phase crossover frequency, gain margin, gain cross-over frequency and phase margin.

According to definition, the phase cross-over frequency is the frequency at which phase angle of $G(j\omega)\,H(j\omega)$ is $-180°$.

For given transfer function

$$\angle G(j\omega)\,H(j\omega) = -90° - \tan^{-1} 0.5\,\omega - \tan^{-1} 0.05\,\omega$$

According to definition

$$-180° = -90° - \tan^{-1} 0.5\,\omega - \tan^{-1} 0.05\,\omega$$

or $\tan^{-1} 0.5\omega + \tan^{-1} 0.05\,\omega = 90°$

taking tangent on both sides

$\tan[\tan^{-1} 0.5\,\omega + \tan^{-1} 0.05\,\omega] = \tan 90°$

$$\frac{0.5\omega + 0.05\omega}{1 - 0.5\omega . 0.05\omega} = \infty$$

or $\quad 1 - 0.025\,\omega^2 = 0$

$$\omega = 6.32 \text{ rad/sec.}$$

This is the phase crossover frequency.

$$\omega_{c_2} = 6.32 \text{ rad/sec.}$$

Now calculate magnitude at this frequency

$$G(j\omega)\,H(j\omega) = \frac{2}{j6.32\,(1 + j0.5 \times 6.32)\,(1 + j0.05 \times 6.32)}$$

$$|G(j\omega)\,H(j\omega)|_{\omega=\omega_{c_2}} = 0.0918$$

$\therefore$

$$\text{G.M.} = -20 \log_{10} |G(j\omega)\,H(j\omega)|_{\omega=\omega_{c_2}}$$

$$\text{G.M.} = -20 \log_{10} 0.0918 = 20.74 \text{ db.}$$

At gain crossover frequency the magnitude of $G(j\omega)\,H(j\omega)$ is unity.

$$|G(j\omega)\,H(j\omega)| = \frac{2}{\omega\sqrt{1+0.25\omega^2}\,\sqrt{1+0.0025\omega^2}}$$

or
$$1 = \frac{2}{\omega\sqrt{1+0.25\omega^2}\,\sqrt{1+0.0025\omega^2}}$$

$$\omega = 2 \text{ rad/sec.}$$

$\therefore$ this is the gain crossover frequency

$$\omega_{c_1} = 2 \text{ rad/sec.}$$

$\therefore$

$$G(j\omega)\,H(j\omega)\Big|_{\omega=\omega_{c_1}} = \frac{2}{j2\,(1+j\,1)\,(1+j\,0.1)} = 0.707\ \underline{\angle -140.71^\circ}$$

$$\underline{\angle G(j\omega)\,H(j\omega)}\Big|_{\omega=\omega_{c_1}} = -\,140.71^\circ$$

According to definition

$$\text{P.M.} = 180^\circ + \underline{\angle G(j\omega)\,H(j\omega)}\Big|_{\omega=\omega_{c_1}} = 180^\circ - 140.71^\circ$$

$$\text{P.M.} = 39.28^\circ$$

From Bode's plot, phase margin and gain margin can be determined as follows.

Draw a line downwards from gain crossover point 'x' till it intersects phase angle plot. This point of intersection is represented by '2' the distance between point '2' and – 180° line is the phase margin. If point '2' on phase angle plot is above – 180° line, phase margin is positive and if point '2' is below – 180° line, phase margin is negative (Fig. 4.29).

From phase crossover point 'y' draw a line upwards till it intersects magnitude plot at point '1'. The difference between magnitude corresponding to point '1' and '0' db is gain margin. If point '1' is below 0 db G.M. is positive and if point '1' is above 0 db line G.M. is negative as shown in Fig. 4.29.

For positive G.M. and P.M. the system is stable and the gain crossover frequency<phase cross-over frequency. If gain crossover frequency > phase crossover frequency both P.M. and G.M. will be negative and the system is unstable. If gain crossover frequency = phase crossover frequency, the system is marginally stable.

The transfer function changes with the change of temperature and pressure. The gain of the system is also affected by supply voltage, supply frequency, air pressures. If the gain is high, the gain margin is low and for step response the settling time is large. If the gain is low G.M. and P.M. will be high and settling time will be more *i.e.*, the system response will be sluggish and also the rise time and steady state error will be high.

EXAMPLE 4.9. Sketch the Bodes plot for the transfer function

$$G(s) = \frac{1000}{(1+0.1s)(1+0.001s)}$$

Determine the (*a*) **P.M**

(*b*) **Gain margin**

(*c*) **Stability of the system.**

Solution : Step 1 : Put $s = j\omega$

$$G(j\omega) = \frac{1000}{(1+j0.1\omega)(1+j0.001\omega)}$$

The given transfer function is of type 'o' system. Therefore the initial slope of the Bode plot is 0 db/decade. The starting point is given by.

$$20 \log_{10} K = 20 \log_{10} 1000 = 60 \text{ db}$$

Corner frequencies $\omega_1 = \frac{1}{0.1} = 10$ rad/sec.

$$\omega_2 = \frac{1}{0.001} = 1000 \text{ rad/sec.}$$

Step 2 : Mark the starting point 60 db on y-axis and draw a line of slope 0 db/decade upto first corner frequency.

Step 3 : From first corner frequency to second corner frequency draw a line with slope $(0-20=-20$ db/decade).

Note : Draw the line parallel to slope marker of – 20 db/dec.

Step 4 : From second corner frequency to next corner frequency (if given) draw a line having the slope $-20+(-20)=-40$ db/decade.

Note : Draw the line parallel to slope marker of – 40 db/dec.

Step 5 : The magnitude plot is complete and now draw the phase plot by calculating the phase at different frequencies (as given in table).

Step 6 : From the bode plot

From the point of intersection of magnitude curve with 0 db axis draw a line on phase curve. This line cuts the phase curve at – 154°

$$\therefore \quad P.M = -154-(-180)$$
$$= +26°$$

Step 7 : Gain margin $G.M = \infty$

Since, $P.M = +26°$ and gain margin $= \infty$, the system is inherently stable.

Table 4.4

ω	$-$ Arg $(1+j0.1\omega)$ $-\tan^{-1} 0.1\omega$	$-$Arg $(1+j0.001\omega)$ $-\tan^{-1} 0.001\omega$	Resultant
50	– 78.6°	– 2.86°	– 81.46°
100	– 84.2°	– 5.7°	– 90°
150	– 86.2°	– 8.5°	– 94°
200	– 87.13°	– 11.3°	– 98°
500	– 88.85°	– 26.56°	– 115.4°
800	– 89.28°	– 38.65°	– 127.93°
1000	– 89.48°	– 45°	– 134.42°
2000	– 89.72°	– 63.43°	– 153.15°
3000	– 89.8°	– 71.56°	– 161.36°
5000	– 89.88°	– 78.69°	– 168.57°
8000	– 89.92°	– 82.87°	– 172.79°

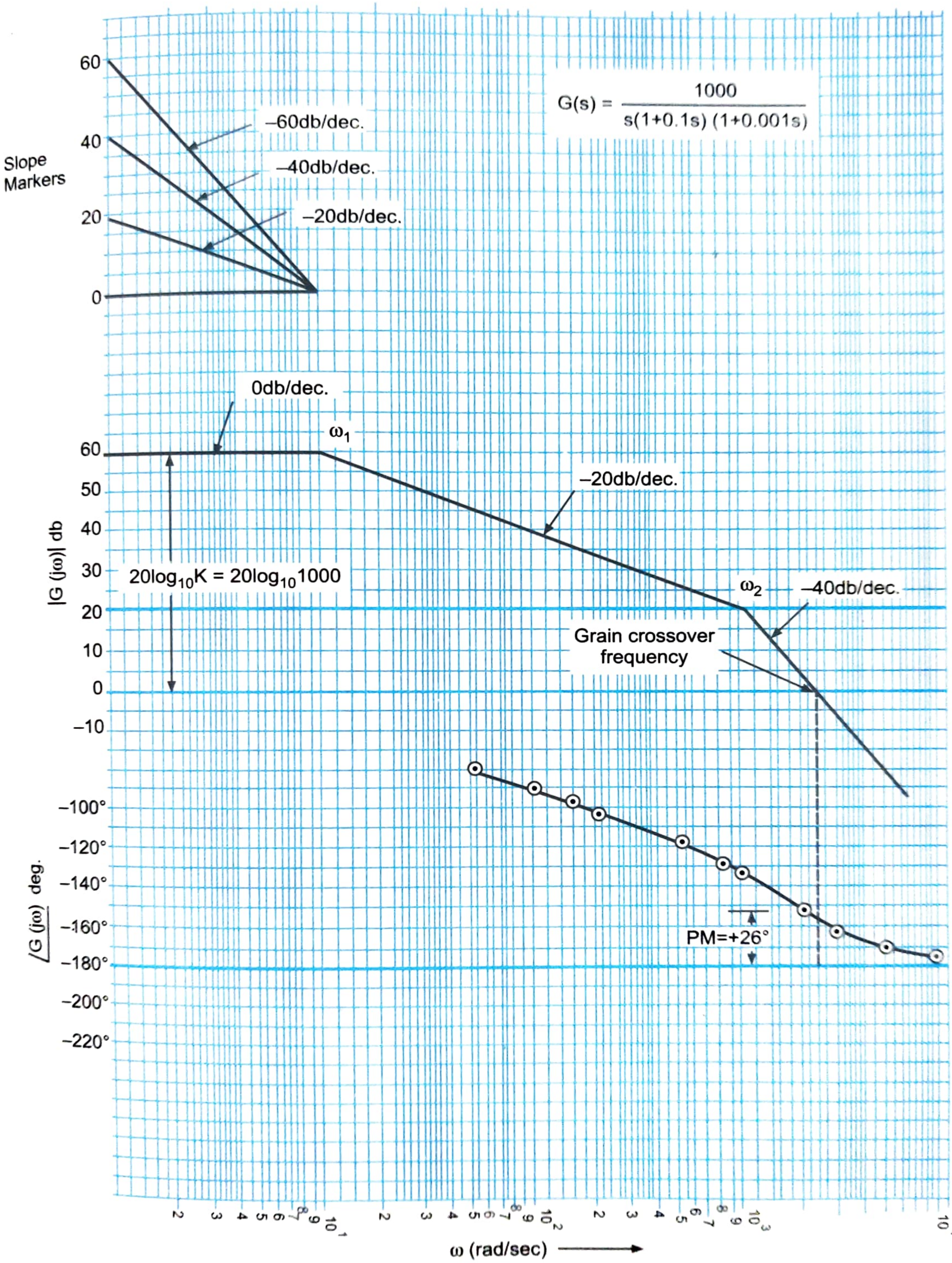

G(s) = 1000 / s(1+0.1s) (1+0.001s)
Slope Markers
–60db/dec.
–40db/dec.
–20db/dec.
0db/dec.
ω1
–20db/dec.
ω2
–40db/dec.
20log10K = 20log101000
Grain crossover frequency
PM=+26°
|G (jω)| db
/G (jω) deg.
ω (rad/sec)

Fig. 4.30.

EXAMPLE 4.10. Sketch the Bode plot for the transfer function.

$$G(s) = \frac{1000}{s(1+0.1s)(1+0.001s)}$$

Determine:

(i) Gain crossover frequency

(ii) Phase crossover frequency

(iii) G.M and P.M

(iv) Stability of the given system

Solution : Step 1 :

Put $s = j\omega$

$$G(j\omega) = 1000/\ (j\omega)\ (1 + j\ 01\omega)\ (1 + j\ 0.001\omega)$$

Step 2 : Draw the magnitude curve

Corner frequencies $\omega_1 = \frac{1}{0.1} = 10$ rad/sec.

$$\omega_2 = \frac{1}{0.001} = 1000 \text{ rad/sec.}$$

Initial slope of the curve will be – 20 db/decade due to $^1/_s$ term. On ω-axis mark ω = K = 1000. From this point draw a line having the slope – 20 db/decade to meet y-axis. This will be the starting point. From the starting point to the first corner frequency the slope will be – 20 db/decade, then from corner frequency 10 the slope will be – 20 + (–20) = – 40 db/dec: upto 1000.

After 1000, the slope will be – 40 + (–20) = – 60 db/decade.

Step 3 : Draw the phase curve

$$\phi = -90° - \tan^{-1} 0.1\omega - \tan^{-1} 0.001\omega$$

Table 4.5.

ω	Arg (jω)	Arg (1 + j 0. 1ω) $-\tan^{-1} 0.1\omega$	– Arg (1 + j 0.001ω) $-\tan^{-1} 0.001\omega$	Resultant φ
1	–90°	–5.7°	–0.06°	–95.7°
5	–90°	–26.5°	–0.28°	–116.5°
10	–90°	–45°	–0.57°	–135.63°
50	–90°	–78.6°	–2.86°	–171.46°
100	–90°	–84.2°	–5.7°	–179.9°
150	–90°	–86.2°	–8.5°	–184°
200	–90°	–87.13°	–11.3°	–188°
500	–90°	–88.85°	–26.56°	–205.41°

Step 4 : From Bode plot

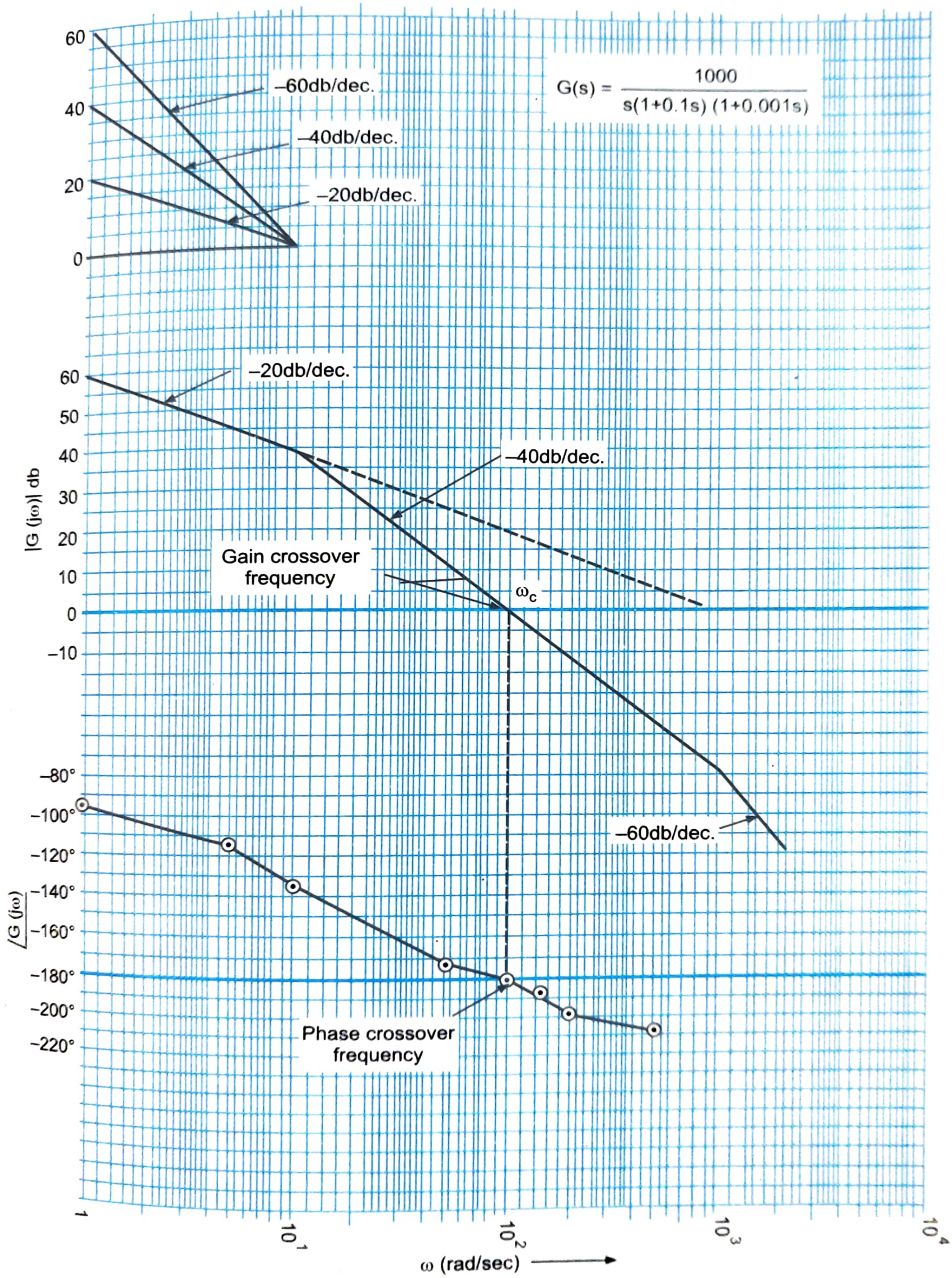
(i) Gain cross over frequency = 100 rad/sec.

(ii) Phase crossover frequency = 100 rad/sec.

(iii) G.M = 0

P.M = 0

(iv) Since G.M = P.M = 0, the system is marigrally stable



60
40
20
0
–60db/dec.
–40db/dec.
–20db/dec.
G(s) = 1000 / s(1+0.1s) (1+0.001s)
|G (jω)| db
60
50
40
30
20
10
0
–10
–20db/dec.
–40db/dec.
Gain crossover frequency
ω_c
–60db/dec.
–80°
–100°
–120°
–140°
–160°
–180°
–200°
–220°
∠G (jω)
Phase crossover frequency
1
10^1
10^2
10^3
10^4
ω (rad/sec)

Fig. 4.31.

EXAMPLE 4.11. **Draw the Bode plot for the transfer function**

$$G(s) = \frac{16(1+0.5s)}{s^2(1+0.125s)(1+0.1s)}$$

From the graph determine :

(*i*) **Phase crossover frequency** (*ii*) **Gain crossover frequency**

(*iii*) **P.M** (*iv*) **G.M**

(*v*) **Stability of the system**

Solution : Step 1 : Put $s = j\omega$

$$G(j\omega) = \frac{16(1+j0.5\omega)}{(j\omega)^2(1+j0.125\omega)(1+jo.1\omega)}$$

Step 2 : Draw the magnitude plot

Corner frequencies

$$\omega_1 = 1/0.5 = 2 \text{ rad/sec.}$$
$$\omega_2 = 1/0.125 = 8 \text{ rad/sec.}$$
$$\omega_3 = 1/0.1 = 10 \text{ rad/sec.}$$

At ω-axis mark $\omega = \sqrt{K} = \sqrt{16} = 4$ rad/sec. because this is a type two system. From 4 rad/sec. draw a line having the slope of – 40 db/decade to meet the y-axis. This will be the starting point. From the starting point to the first corner frequency the slope will be – 40 db/dec. from first corner frequency (2 rad/sec.) to the second corner frequency (8 rad/sec.) the slope of the line will be –40 + (+20) = –20 db/decade. From second corner frequency to the third corner frequency (10 rad/sec.) the slope of the line will be –20 + (–20) = – 40 db/decade. After 10 rad/sec. the slope will be – 40 + (–20) = –60 db/decade.

Step 3 : Draw the phase diagram

Step 4 : Phase cross over frequency = 6.5 rad/sec (ω_{c2})

Gain cross over frequency = ω_{c1} = 8 rad/sec.

$$P.M = -8°$$
$$G.M = -2 \text{ db}$$

(*v*) Since gain crossover frequency > phase crossover frequency hence the system is unstable. Also both *P.M* & *G.M* are negative.

Table 4.6.

ω	$-(j\omega)^2$	$-\tan^{-1} 0.125\omega$	$-\tan^{-1} 0.1\omega$	$+\tan^{-1} 0.5\omega$	Resultant
0.1	–180°	–0.716°	–0.57°	+2.86°	–178.42°
0.2	–180°	–1.43°	–1.15°	+5.71°	–176.86°
0.5	–180°	–3.57°	–2.86°	+14°	–172.43°
0.8	–180°	–5.71°	–4.57°	+21.8°	–168.48°
1.0	–180°	–7.13°	–5.71°	+26.56°	–166.28°
2.0	–180°	–14°	–11.3°	+ 45°	–160.3°
5.0	–180°	–32°	–26.56°	+ 68.19°	–170.37°
8.0	–180°	–45°	–38.66°	+76°	–187°
10.0	–180°	–51.34°	–45°	+78.69°	–197.65°
20.0	–180°	–68.19°	–63.43°	+ 84.28°	–227.34°
30.0	–180°	– 75°	–71.56°	+86.18°	–240.38°

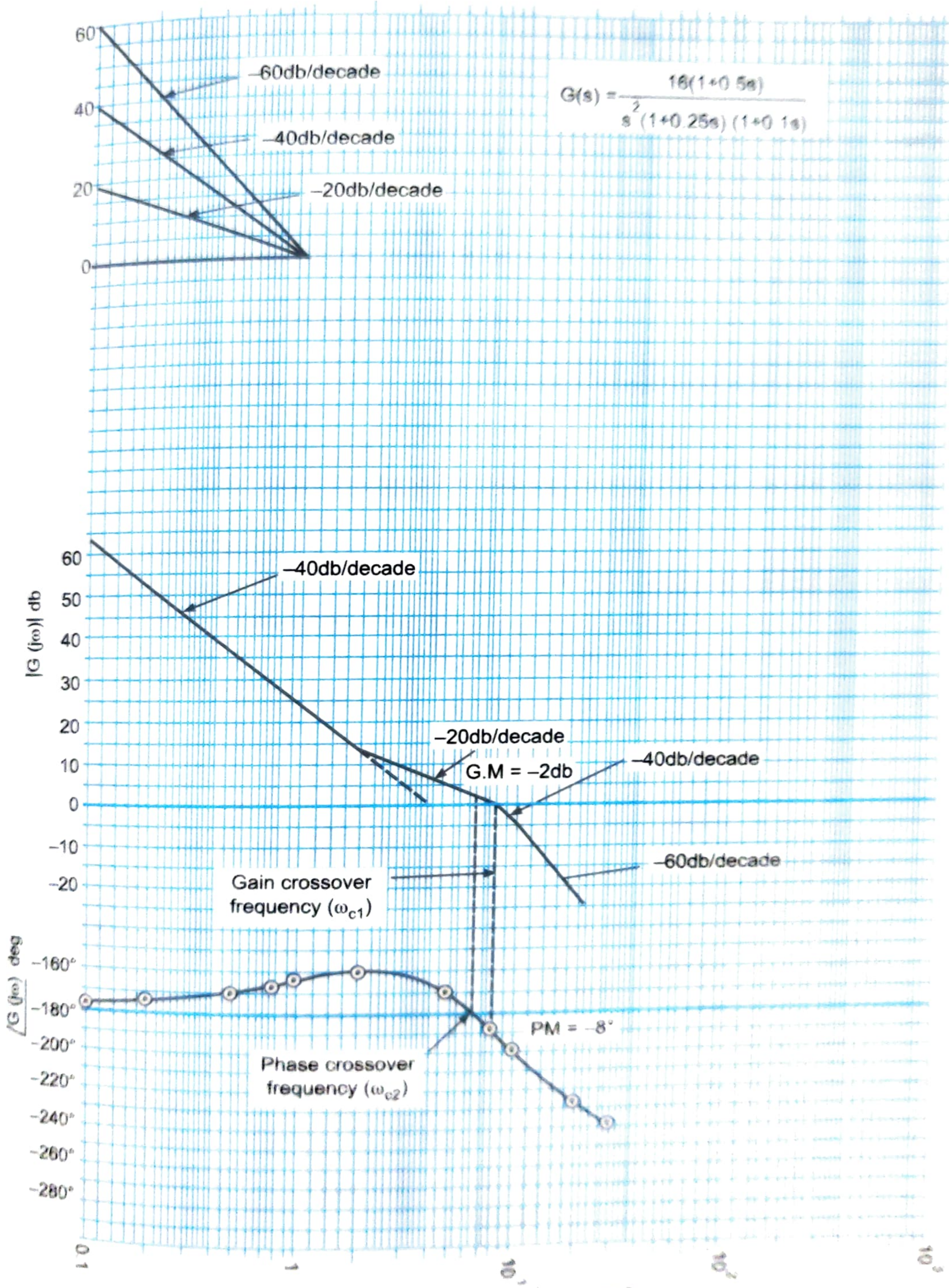

$G(s) = \frac{16(1+0.5s)}{s^2(1+0.25s)(1+0.1s)}$
–60db/decade
–40db/decade
–20db/decade
60
40
20
0
–40db/decade
–20db/decade
G.M = –2db
–40db/decade
–60db/decade
Gain crossover frequency (ω_{c1})
Phase crossover frequency (ω_{c2})
PM = –8°
$|G(j\omega)|$ db
60
50
40
30
20
10
0
–10
–20
$\angle G(j\omega)$ deg
–160°
–180°
–200°
–220°
–240°
–260°
–280°
0.1
1
10^1
10^2
10^3
ω (rad/sec)

Fig. 4.32

EXAMPLE 4.12. Draw the Bode plot for the transfer function

$$G(s) = \frac{50}{s(1+0.25s)(1+0.1s)}$$

From the graph determine:

1. **Gain crossover frequency**
2. **Phase crossover frequency**
3. **G.M and P.M.**
4. **Stability of the system.**

Solution : Since this is type one system, the initial slope of the line will be – 20 db/decade. Mark the point on ω-axis at 50 rad/sec and draw a line with slope –20 db/dec. from the point at 50, this line will meet the y-axis. This is the starting point.

Corner frequencies are $\omega_1 = \frac{1}{0.25} = 4$ rad/sec.

$$\omega_2 = \frac{1}{0.01} = 10 \text{ rad/sec.}$$

Slope of the line from starting point to I corner freqency = – 20 db/dec.

Slope of the line from I corner frequency to II = –20 + (–20)

= – 40 db/dec.

After second corner frequency the slope will be – 40 + (–20) = – 60 db/dec.

Phase Plot : For phase plot calculate the phases at different frequencies.

Table 4.7.

ω	jω	$-\tan^{-1} 0.1\omega$	$-\tan^{-1} 0.25\omega$	Resultant
1	–90°	– 5.71°	– 14.03°	– 109.74°
2	–90°	– 11.3°	– 26.56°	– 127.86°
5	–90°	– 26.56°	– 51.34°	– 168°
10	–90°	– 45°	– 68.19°	– 203.19°
20	–90°	– 63.43°	– 78.69°	– 232.12°
40	–90°	– 76°	– 84.28°	– 250.28°
60	–90°	– 80.53°	– 86.18°	– 256.71°
80	–90°	– 82.87°	– 87.13°	– 260°
100	–90°	– 84.29°	– 87.7°	– 262°

Gain crossover frequency (ω_{c_1}) = 13 rad/sec.

Phase crossover frequency (ω_{c_2}) = 6.5 rad/sec.

P.M = –36°

G.M = – 13 db

Since, both phase margin and gain margin are negative and gain crossover frequency > phase crossover frequency, the system is unstable.

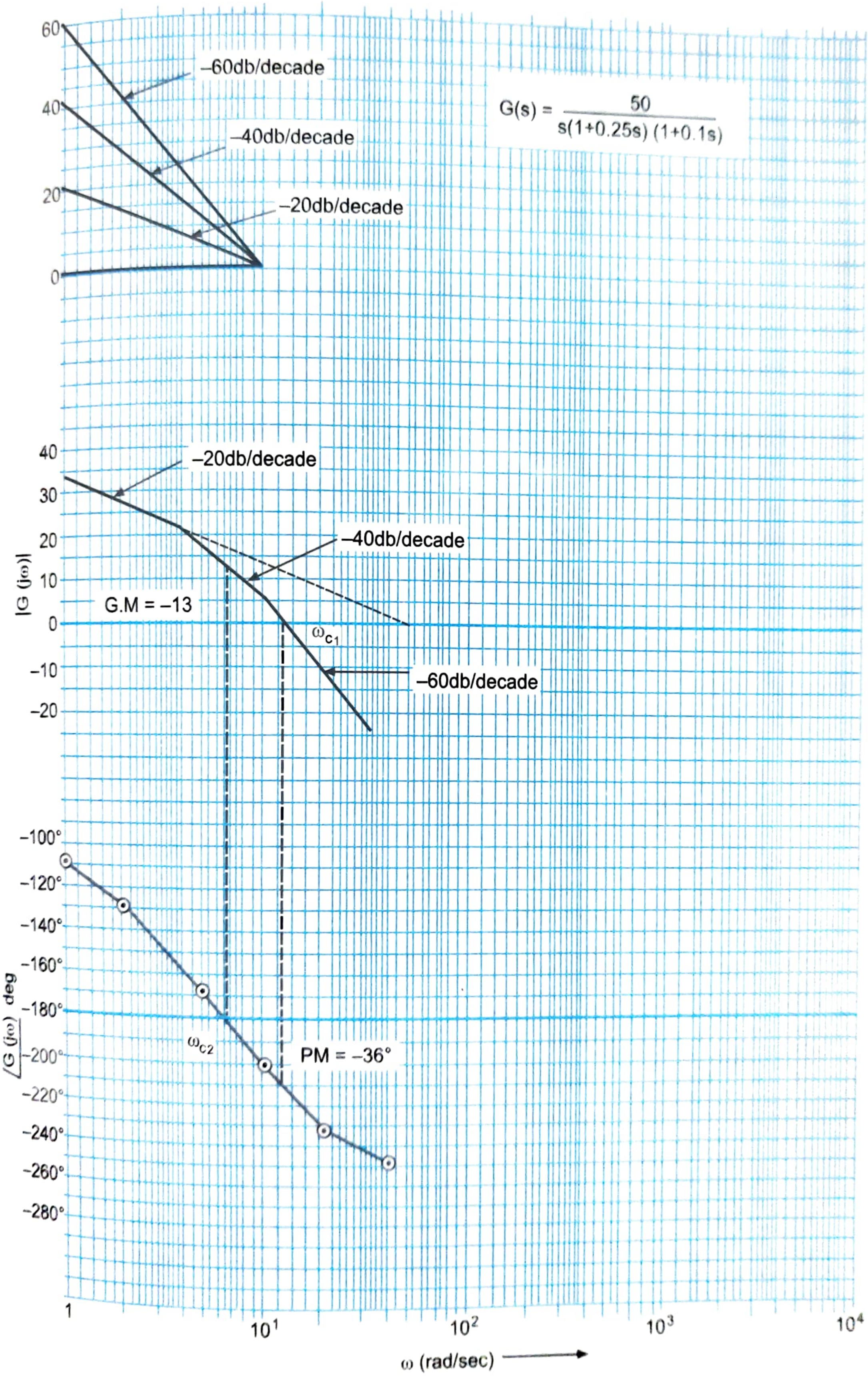

60
40
20
0
–60db/decade
–40db/decade
–20db/decade
$G(s) = \frac{50}{s(1+0.25s)(1+0.1s)}$
40
30
20
10
0
–10
–20
|G (jω)|
–20db/decade
–40db/decade
G.M = –13
ω_{c1}
–60db/decade
–100°
–120°
–140°
–160°
–180°
–200°
–220°
–240°
–260°
–280°
∠G (jω) deg
ω_{c2}
PM = –36°
1
10^1
10^2
10^3
10^4
ω (rad/sec)

Fig. 4.33.

EXAMPLE 4.13. **Draw the Bode plot for**

$$G(s) = \frac{23.7(1+j\omega)(1+j0.2\omega)}{(j\omega)(1+j3\omega)(1+j0.5\omega)(1+j0.1\omega)}$$

From the plot find G.M and P.M

Solution :

Step 1 : On ω-axis mark the point at 23.7 rad/sec. Since in denominator (jω) term is having power one, from 23.7 draw a line of slope – 20 db/decade to meet y-axis. This will be the starting point.

From the starting point to the I corner frequency (0.33) the slope of the line is – 20 db/decade.

From I corner frequency (0.33) to II corner frequency (1) the slope of the line will be 20 + (–20) = – 40 db/decade.

From II corner frequency (1) to III corner frequency (2) the slope of the line – 40 + (+ 20) = – 20 db/decade.

From III corner frequency to IV corner frequency, the slope of the line will be – 20 + (–20) = – 40 db/decade.

From IV corner frequency (5) to V corner frequency the slope will be – 40 + (+20) = – 20 db/decade.

After V corner frequency, the slope will be – 20 + (–20) = – 40 db/dec.

Step 2 : Draw the phase plot.

Values of ϕ at different frequencies are tabulated.

Step 3 : From graph

P.M = + 34°

G.M = ∞

Table 4.8.

ω	$-\tan^{-1} j\omega$	$-\tan^{-1} 3\omega$	$-\tan^{-1} 0.5\omega$	$-\tan^{-1} 0.1\omega$	$\tan^{-1}\omega$	$\tan^{-1} 0.2\omega$	Resultant
0.1	–90°	–16.7°	–2.86°	–0.57°	+ 5.71°	+1.14°	–103°
0.2	–90°	–31°	–5.71°	–1.14°	+ 11.3°	+2.3°	–114.25°
0.5	–90°	–56.3°	–14.03°	–2.86°	+ 26.56°	+5.71°	–130.92°
0.8	–90°	–67.4°	–21.8°	–4.57°	+ 38.65°	+9.09°	–136.03°
1.0	–90°	–71.56°	–26.56°	–5.71°	+ 45°	+11.3°	–137.5°
2.0	–90°	–80.54°	–45°	–11.3°	+ 63.43°	+ 21.8°	– 141.61°
5.0	–90°	–86.18°	– 68.19°	–26.56°	+ 78.7°	+ 45°	– 147.23°
8.0	–90°	–87.61°	– 76°	– 38.65°	+ 82.87°	+ 58°	– 151.39°
10.0	–90°	– 88°	– 78.7°	– 45°	+ 84.3°	+ 63.4°	– 154.0°
20.0	–90°	– 89°	– 84.3°	– 63.43°	+ 87.13°	+ 76°	– 163.6°

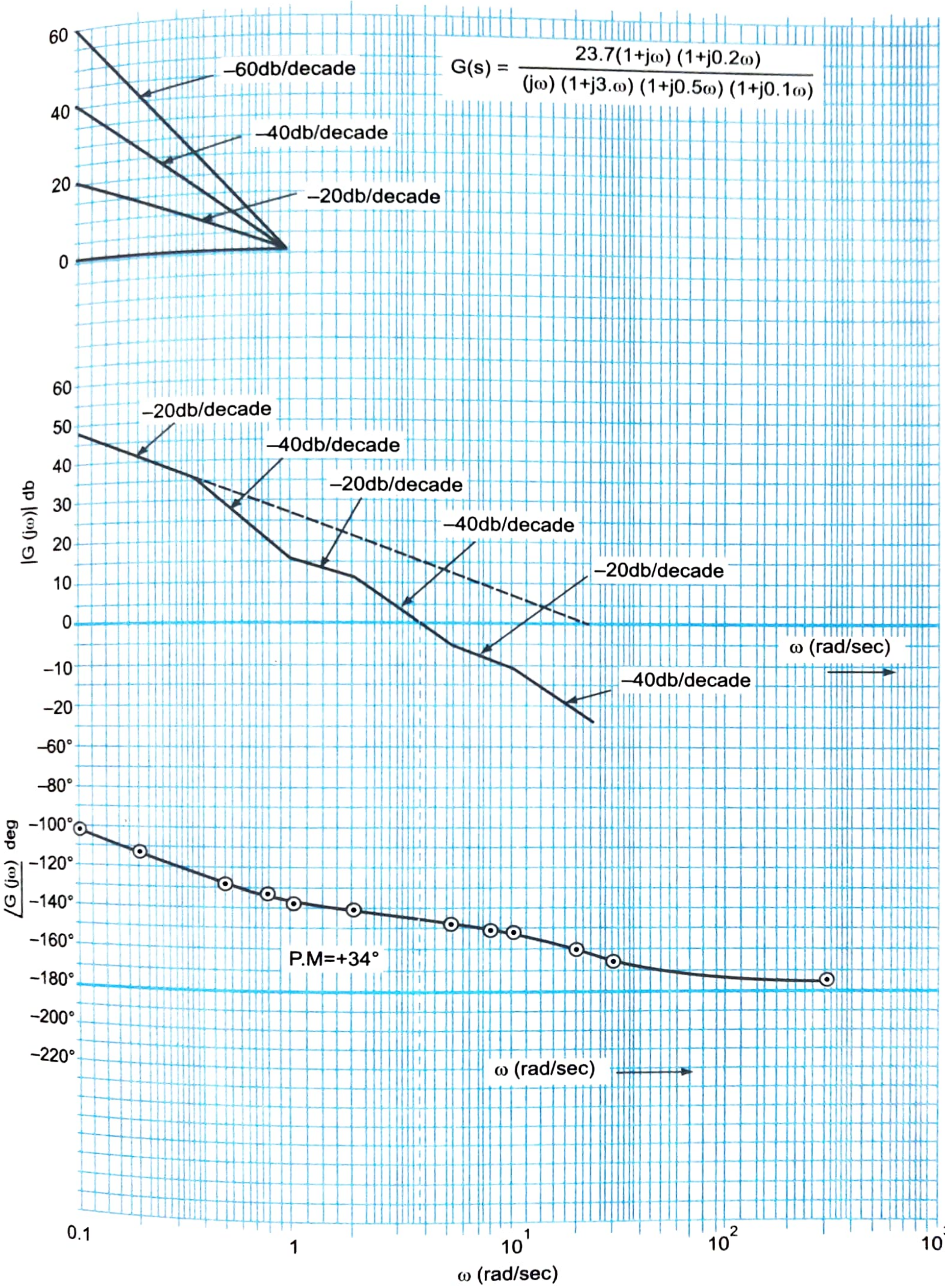

$G(s) = \frac{23.7(1+j\omega)(1+j0.2\omega)}{(j\omega)(1+j3.\omega)(1+j0.5\omega)(1+j0.1\omega)}$
–60db/decade
–40db/decade
–20db/decade
60
40
20
0
–20db/decade
–40db/decade
–20db/decade
–40db/decade
–20db/decade
–40db/decade
|G (jω)| db
60
50
40
30
20
10
0
–10
–20
ω (rad/sec)
–60°
–80°
–100°
–120°
–140°
–160°
–180°
–200°
–220°
/G (jω) deg
P.M=+34°
ω (rad/sec)
0.1
1
10^1
10^2
10^3
ω (rad/sec)

Fig. 4.34.