Chapter 11

Transformers

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11.1 INTRODUCTION

Electrical energy is generated at places where it is easier to get water head, oil or coal for hydroelectric, diesel or thermal power stations respectively. Then energy is to be transmitted at considerable distances for use in villages, towns and cities located at distant places. As transmission of electrical energy at high voltages is economical, some means are required for stepping up the voltage at generating stations and stepping down the same at the places where it is to be used. Electric machine used for this purpose is "*transformer*". In our country the electrical energy is usually generated at 6.6, 11 or 33 kV, stepped up to 132, 220, 400, or 765 kV with the help of step-up transformers for transmission and then stepped down to 66 kV or 33 kV at grid substations for feeding various substations, which further step down the voltage to 11 kV for feeding distributing transformers stepping down the voltage further to 433/250 volts for the consumer uses.

Transformer is an ac machine that (*i*) transfers electrical energy from one electric circuit to another (*ii*) does so without a change of frequency (*iii*) does so by the principle of electromagnetic induction and (*iv*) has electric circuits that are linked by a common magnetic circuit. The energy transfer usually takes place with a change of voltage, although this is not always necessary. When the transformer raises the voltage *i.e.* when the output voltage of a transformer is higher than its input voltage, it is called the *step-up transformer* and when it lowers the voltage it is called the *step-down transformer*.

Since its basic construction requires no moving parts, it is often called the '*static transformer*' and it is very rugged machine requiring the minimum amount of repair and maintenance. Owing to the lack of rotating parts there are no friction or windage losses. Further, the other losses are relatively low, so that the efficiency of a transformer is high. Typical transformer efficiencies at full load lie between 96% and 97% and with extremely large capacity transformers the efficiencies are as high as 99%. The cost per kVA output of transformers is quite low as compared with other electrical machines.

11.2 OPERATING PRINCIPLE

An elementary transformer consists of a soft iron or silicon steel core and two windings placed on it. The windings are insulated from both the core and each other. The core is built up of thin soft iron or silicon steel laminations to provide a path of low reluctance to the magnetic flux. The winding connected to the supply main is called the *primary* and the winding connected to the load circuit is called the *secondary*. The winding connected to higher voltage circuit is called the *high-voltage (hv) winding* while that connected to the lower voltage circuit is called the *low-voltage (lv) winding*. In case of a step-up transformer, low-voltage winding is the primary and high-voltage winding is the secondary while in case of a step-down transformer the high-voltage winding is the primary and low-voltage winding is the secondary.

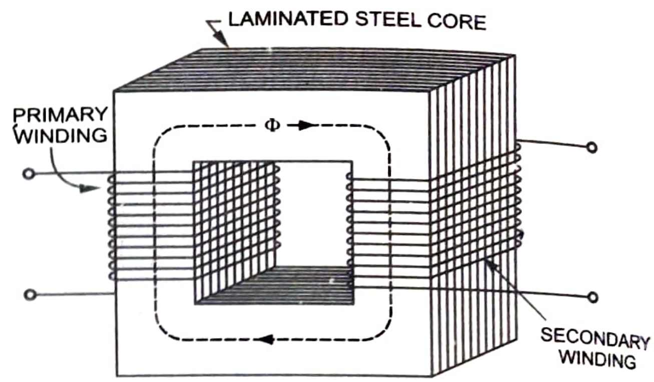


Fig. 11.1 *Simple Transformer*

The action of a transformer is based on the principle that energy may be efficiently transferred by induction from one set of coils to another by means of a varying magnetic flux, provided that both the sets of coils are on a common magnetic circuit. In a transformer, the coils and magnetic circuit are all stationary with respect to one another. The emfs are induced by the variation in the magnitude of flux with time, as illustrated in Fig. 11.3.

Although in the actual construction the two windings are usually wound one over the other, for the sake of simplicity, the figures for analysing transformer theory show the windings on opposite sides of the core as in Fig. 11.1.

When the primary winding is connected to an ac supply mains, a current flows through it. Since this winding links with an iron core, current flowing through this winding produces an alternating flux Φ in the core. Since this flux is alternating and links with the secondary winding also, induces an emf in the secondary winding. The frequency of induced emf in secondary winding is the same as that of the flux or that of the supply voltage. The induced emf in the secondary winding enables it to deliver current to an external load connected across it. Thus the energy is transformed from primary winding to the secondary winding by means of electromagnetic induction without any change in frequency. The flux Φ of the iron core links not only with the secondary winding but also with the primary winding, so produces self-induced emf in the primary winding. This induced emf in the primary winding opposes the applied voltage and, therefore, sometimes it is known as *back emf* of the primary. In fact the induced emf in the primary winding limits the primary current in much the same way as the back emf in a dc motor limits the armature current.

11.3 TRANSFORMER ON DC

A transformer cannot operate on dc supply and never be connected to a dc source. If a rated dc voltage is applied to the primary of a transformer, the flux produced in the transformer core will not vary but remain constant in magnitude and, therefore, no emf will be induced in the secondary winding except at the moment of switching *on*. Thus the transformer is not capable of raising or lowering the dc voltage. Also there will be no self-induced emf in the primary winding, which is only possible with varying flux linkage, to oppose the applied voltage and since the resistance of primary winding is quite low, a heavy current will flow through the primary winding which may result in the burning out of the primary winding. This is reason that *dc is never applied to a transformer*.

11.4 IDEAL TRANSFORMER

For a better understanding and an easier explanation of a practical transformer, certain idealizing assumptions are made which are close approximations for a practical transformer. A transformer having these ideal properties is hypothetical (has no real existence) and is referred to as the *ideal transformer*. It possesses certain essential features of a real transformer but some details of minor significance are ignored which will be introduced step by step while analysing a transformer. The idealizing assumptions made are as follows:

(i) *No winding resistance i.e.* the primary and secondary windings have zero resistance. It means that there is no ohmic power loss and no resistive voltage drop in an ideal transformer.

(ii) *No magnetic leakage i.e.* there is no leakage flux and all the flux set up is confined to the core and links both the windings.

(iii) *No iron loss i.e.* hysteresis and eddy current losses in transformer core are zero.

(iv) *Zero magnetizing current i.e.* the core has infinite permeability and zero reluctance so that zero magnetizing current is required for establishing the requisite amount of flux in the core.

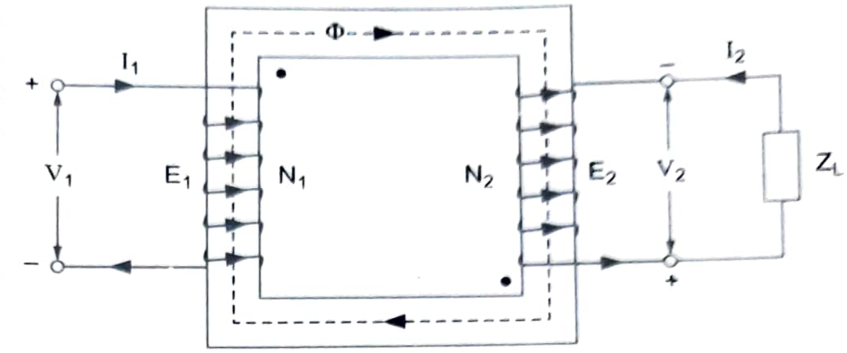


Fig. 11.2 *Ideal Iron-Core Transformer*

From the above discussion an ideal transformer is supposed to consists of two pure inductive coils wound on a loss-free core [Fig. 11.2].

11.5 EMF EQUATION

When an alternating (sinusoidal) voltage is applied to the primary winding of a transformer, an alternating (sinusoidal) flux, as shown is Fig. 11.3., is set up in the iron core which links both the windings (primary and secondary windings).

Let Φ_{max} = Maximum value of flux in webers

and f = Supply frequency in hertz.

As illustrated in Fig. 11.3, the magnetic flux increases from zero to its maximum value Φ_{max} in one-fourth of a cycle *i.e.* in $\frac{1}{4f}$ second.

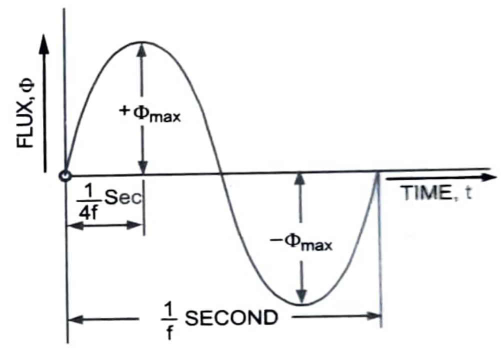


Fig. 11.3 *Sinusoidal Variation of Flux With Time*

So average rate of change of flux, $\frac{d\phi}{dt} = \frac{\Phi_{max}}{1/4f} = 4f\Phi_{max}$

Since average emf induced per turn in volts is equal to the average rate of change of flux,

average emf induced per turn $= 4f\Phi_{max}$ volts

Since flux Φ varies sinusoidally, emf induced will be sinusoidal and form factor for sinusoidal wave is 1.11 *i.e.* the rms or effective value is 1.11 times the average value.

$\therefore$ RMS value of emf induced per turn $= 1.11 \times 4f\Phi_{max}$ volts ...(11.1)

If the number of turns on primary and secondary windings are N_1 and N_2 respectively, then

RMS value of emf induced in primary,

$$E_1 = \text{EMF induced per turn} \times \text{number of primary turns}$$

$$= 4.44 f\Phi_{max} \times N_1 = 4.44 f N_1 \Phi_{max} \text{ volts} \quad ...(11.2)$$

Similarly rms value of emf induced in secondary,

$$E_2 = 4.44 f\Phi_{max} \times N_2 \text{ volts} \quad ...(11.3)$$

The above relations for emf induced in primary and secondary windings can be derived alternatively as below

The instantaneous value of sinusoidally varying flux may be given as

$$\Phi = \Phi_{max} \sin \omega t$$

$\therefore$ Instantaneous value of emf induced per turn

$$= \frac{-d\phi}{dt} \text{ volts} = -\omega\Phi_{max} \cos \omega t = \omega\Phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \text{ volts}$$

It is clear from the above equation that the maximum value of emf induced per turn

$$= \omega\, \Phi_{max} = 2\pi f \Phi_{max} \text{ volts} \qquad \because \omega = 2\pi f$$

and rms value of emf induced per turn $= \dfrac{1}{\sqrt{2}} \times 2\,\pi\, f\, \Phi_{max} = 4.44\, f\, \Phi_{max}$ volts

Hence rms value of emf induced in primary,

$$E_1 = 4.44\, f\, N_1\, \Phi_{max} \text{ volts}$$

and rms value of emf induced in secondary,

$$E_2 = 4.44\, f\, N_2\, \Phi_{max} \text{ volts}$$

In an ideal transformer the voltage drops in primary and secondary windings are negligible, so

EMF induced in primary winding, E = Applied voltage to primary, V_1

and terminal voltage, V_2 = EMF induced in secondary, E_2

Note: If B_{max} is the maximum allowable flux density in Wb/m^2 (or T) and a is the area of x-section of iron core in square metres, then in Eqs. (11.1), (11.2) and (11.3), Φ_{max} is given as

$$\Phi_{max} = B_{max}\, a \text{ webers}$$

11.6 VOLTAGE AND CURRENT TRANSFORMATION RATIOS

Referring to Eq. (11.1), it is clear that the *volts per turn* is exactly the same for both the primary and secondary windings *i.e.* in any transformer, the secondary and primary induced emfs are related to each other by the ratio of the number of secondary and primary turns. Thus,

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K \qquad \text{...(11.4)}$$

The same relationship can be derived by dividing Eq. (11.3) by Eq. (11.2).

The constant K in Eq. (11.4) is called the *voltage transformation ratio.*

For step-up transformer, $V_2 > V_1$ or voltage transformation ratio, $K > 1$.

For step-down transformer, $V_2 < V_1$ or voltage transformation ratio, $K < 1$.

In an ideal transformer, the losses are negligible, so the volt-ampere input to the primary and volt-ampere output from secondary can be approximately equated *i.e.*

$$\text{Output VA} = \text{Input VA}$$

$$\text{or } V_2 I_2 = V_1 I_1$$

$$\text{or } \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{K} \qquad \text{...(11.5)}$$

i.e. Primary and secondary currents are inversely proportional to their respective turns.

Note: For a transformer the true ratio of transformation (or turn ratio) is constant, while the voltage ratio $\left(\dfrac{V_2}{V_1}\right)$ varies about 1 to 8 per cent depending upon the load and its power factor.

Example 11.1. It is desired to have a 4.13 mWb maximum core flux in a transformer at 110 V and 50 Hz. Determine the required number of turns in the primary. [*Pb. Technical Univ. Electrical Engineering, December-2000*]

Solution: EMF induced in primary, $E_1 = 110$ V

Supply frequency, $f = 50$ Hz

Maximum core flux, $\Phi_{max} = 4.13 \text{ mWb} = 4.13 \times 10^{-3}$ Wb

Required number of turns on primary, $N_1 = \dfrac{E_1}{4.44\, f\, \Phi_{max}}$ ...Refer to Eq. (11.2)

$$= \frac{110}{4.44 \times 50 \times 4.13 \times 10^{-3}} = 120 \quad \textbf{Ans.}$$

Example 11.2. The emf per turn of a single-phase 10 kVA, 2200/220 V, 50 Hz transformer is 10 V. Calculate (i) the number of primary and secondary turns, (ii) the net cross-sectional area of core for a maximum flux density of 1.5 T.

Solution:

EMF per turn = 10 V

Primary induced emf, $E_1 = V_1 = 2{,}200$ V

Secondary induced emf, $E_2 = V_2 = 220$ V

Supply frequency, $f = 50$ Hz

Maximum flux density, $B_{max} = 1.5$ T

(*i*) Number of primary turns, $N_1 = \dfrac{E_1}{\text{EMF per turn}} = \dfrac{2{,}200}{10} = 220$ **Ans.**

Number of secondary turns, $N_2 = \dfrac{E_2}{\text{EMF per turn}} = \dfrac{220}{10} = 22$ **Ans.**

Maximum value of flux, $\Phi_{max} = \dfrac{\text{EMF per turn}}{4.44 f} = \dfrac{10}{4.44 \times 50} = 0.045$ Wb ...refer to Eq. (11.1)

(*ii*) Net cross-sectional area of core, $a = \dfrac{\Phi_{max}}{B_{max}} = \dfrac{0.045}{1.5} = 0.03\ \text{m}^2$ **Ans.**

Example 11.3. A 25 kVA loss-less transformer has 500 turns on the primary and 40 turns on the secondary winding. The primary is connected to 3,000 V, 50 Hz mains determine

(*i*) primary and secondary currents in at full load;

(*ii*) the secondary emf and

(*iii*) the maximum flux in the core. No-load current can be neglected. [*Pb. Technical Univ., May-2012*]

Solution: Primary induced emf, $E_1 = V_1 = 3{,}000$ V

Primary turns, $N_1 = 500$

Secondary turns, $N_2 = 40$

(*ii*) Secondary emf, $E_2 = E_1 \times \dfrac{N_2}{N_1} = 3{,}000 \times \dfrac{40}{500} = 240$ V

(*i*) Primary current, $I_1 = \dfrac{\text{Rated kVA} \times 1{,}000}{E_1} = \dfrac{\text{Rated kVA} \times 1{,}000}{V_1}$

$$= \frac{25 \times 1{,}000}{3{,}000} = 8.333\ \text{A} \quad \textbf{Ans.}$$

Secondary current, $I_2 = \dfrac{25 \times 1{,}000}{240} = 104.167$ A **Ans.**

(*iii*) Maximum flux in the core, $\Phi_{max} = \dfrac{E_1}{4.44 f N_1} = \dfrac{3{,}000}{4.44 \times 50 \times 500} = 0.027$ Wb or 27 mWb **Ans.**

Example 11.4. A single-phase, 50 Hz, core type transformer has square cores of 20 cm side, permissible maximum flux density is 1 Wb/m². Calculate the number of turns per limb on the high- and low-voltage sides for a 3,000/220 V ratio.

Solution: Core area = $20 \times 20 = 400\ \text{cm}^2 = 0.04\ \text{m}^2$

Permissible flux density, $B_{max} = 1.0\ \text{Wb/m}^2$

Maximum value of permissible flux, $\Phi_{max} = B_{max} \times a = 1 \times 0.04 = 0.04$ Wb

Number of turns on low-voltage (secondary) winding,

$$N_2 = \frac{E_2}{4.44 f \Phi_{max}} = \frac{220}{4.44 \times 50 \times 0.04} = 24.77;\ 26\ (\text{say})$$

Number of turns on high-voltage (primary) winding,

$$N_1 = \frac{E_1}{E_2} \times N_2 = \frac{3{,}000}{220} \times 26 = 354$$

$$\text{Number of } hv \text{ turns on each limb} = \frac{354}{2} = 177 \quad \textbf{Ans.}$$

$$\text{Number of } lv \text{ turns on each limb} = \frac{26}{2} = 13 \quad \textbf{Ans.}$$

Note: LV turns are calculated first, rounded one figure off to the next higher even number in order that maximum flux density does not exceed the permissible maximum flux density. Then *hv* turns are calculated with corrected number of turns by transformation ratio. Further there are two limbs and each limb accommodates half of the *lv* and half of the *hv* turns from the view point of reducing leakage reactance.

Example 11.5. A single-phase transformer has a core whose cross–sectional area is 150 cm²; operates at a maximum flux density of 1.1 Wb/m² from a 50 Hz supply. If the secondary winding has 66 turns, determine the output in kVA when connected to a load of 4 Ω impedance. Neglect any voltage drop in the transformer.

Solution: Peak value of flux density, $B_{max} = 1.1 \text{ Wb/m}^2$

Cross-sectional area of core, $a = 150 \text{ cm}^2 = 0.015 \text{ m}^2$

Maximum value of flux in the core, $\Phi_{max} = B_{max} \times a = 1.1 \times 0.015 = 0.0165$ Wb

Number of turns on secondary, $N_2 = 66$

Supply frequency, $f = 50$ Hz

RMS value of induced emf in the secondary winding,

$$E_2 = 4.44 f \Phi_{max} N_2 = 4.44 \times 50 \times 0.0165 \times 66 = 240 \text{ volts}$$

Secondary voltage, $V_2 = E_2 = 240$ V neglecting voltage drop in secondary winding

$$\text{Secondary current, } I_2 = \frac{V_2}{\text{Load impedance}} = \frac{240}{4} = 60 \text{ A}$$

$$\text{Output in kVA} = \frac{V_2 I_2}{1{,}000} = \frac{240 \times 60}{1{,}000} = 14.4 \ \textbf{Ans.}$$

Example 11.6. A single-phase transformer is rated 25 kVA, 600/200 V, 50 Hz. Calculate the impedance of load in ohms to fully load the transformer when connected to (a) 600 V side (b) 240 V side.

[*Mahamaya Technical Univ. Electrical Engineering Second Semester*, 2011-12]

Solution: (*a*) When load is connected to 600 V side

$$\text{Load current, } I_1 = \frac{\text{Rated kVA} \times 1{,}000}{V_1} = \frac{25 \times 1{,}000}{600} = 41.6667 \text{ A}$$

$$\text{Load impedance, } Z_{L_1} = \frac{V_1}{I_1} = \frac{600}{41.6667} = 14.4 \ \Omega \ \textbf{Ans.}$$

(*b*) When load is connected to 200 V side

$$\text{Load current, } I_2 = \frac{25 \times 1{,}000}{200} = 125 \text{ A}$$

$$\text{Load impedance, } Z_{L_2} = \frac{200}{125} = 1.6 \ \Omega \ \textbf{Ans.}$$

Example 11.7. A 6,600/440 V, single-phase 600 kVA transformer has 1,200 primary turns. Find (*i*) Transformation ratio, (*ii*) Secondary turns, (*iii*) Voltage per turn, (*iv*) Secondary current when it supplies a load of 400 kW at 0.8 power factor lagging.

[*R.G.T.U. Basic Elec. Engineering Jan./Feb.-2007*]

Solution: Primary voltage, $V_1 = 6{,}600$ V

Secondary voltage, $V_2 = 440$ V

Primary turns, $N_1 = 1{,}200$

Load supplied, $P = 400$ kW

Power factor of load, $\cos\phi = 0.8$ (lagging)

$$(i) \text{ Transformation ratio, } K = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{440}{6{,}600} = \frac{1}{15} \ \textbf{Ans.}$$

Example 11.9. A 1 kVA, 220/110 V, 400 Hz transformer is desired to be used at a frequency of 60 Hz. What will be the kVA rating of the transformer at reduced frequency?

Solution: From Eq. (11.2)

$$V_1 = 4.44\ \Phi_{max}\ N_1 f = 4.44\ B_{max}\ a\ N_1 f$$

Assuming flux density in the core remaining unchanged, we have

$$V_1 \propto f$$

$$\text{or} \quad \frac{V_1'}{V_1} = \frac{f'}{f}$$

$$\text{or} \quad V_1' = V_1 \times \frac{f'}{f} = 220 \times \frac{60}{400} = 33 \text{ volts}$$

KVA rating, being proportional to the voltage, will change to $1 \times \frac{33}{220}$ or 0.15 kVA or 150 VA **Ans.**

11.8 TRANSFORMER ON NO LOAD

When the primary of a transformer is connected to the source of ac supply and the secondary is open, the transformer is said to be at no load (there is no load on secondary).

Consider an ideal transformer whose secondary side is open and the primary winding is connected to a sinusoidal alternating voltage V_1. The alternating voltage applied to the primary winding will cause flow of alternating current in the primary winding. Since the primary coil is pure inductive and there is no output (secondary being open), the primary draws the magnetising current I_m only. The function of this current is merely to magnetize the core. If the transformer is truely ideal, the magnitude of I_m should be zero by virtue of assumption (*iv*) made in Art 11.4. Since the reluctance of the magnetic circuit is never zero, I_m has definite magnitude. The magnetising current, I_m is small in magnitude and lags behind supply voltage V_1 by 90°. This magnetising current I_m produces an alternating flux Φ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, in phase with it.

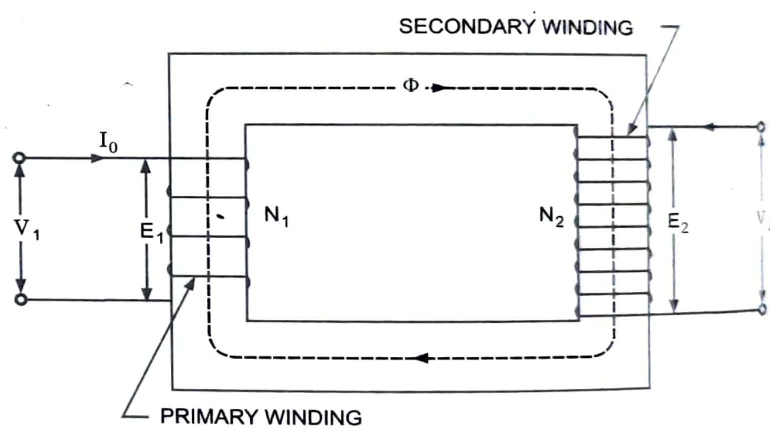


Fig. 11.4 *Transformer on No Load*

Let the instantaneous linking flux be given as

$$\phi = \Phi_{max} \sin \omega t. \quad \text{...(11.6)}$$

The varying flux is linked with both of the windings (primary and secondary) and so induces emfs in the primary and secondary windings. The instantaneous values of induced emfs in the primary and secondary windings will be

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\Phi_{max} \sin \omega t) = -N_1\ \omega\ \Phi_{max} \cos \omega t$$

$$= N_1\ \omega\ \Phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{...(11.7)}$$

$$\text{Similarly, } e_2 = N_2\ \omega\ \Phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{...(11.8)}$$

Since primary winding has no ohmic resistance, (as assumed), applied voltage to primary winding is to only oppose the induced emf in the primary winding, hence instantaneous applied voltage to primary will be given by

$$v_1 = -e_1 = -N_1 \omega \Phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(11.9)$$

Comparing Eqs. (11.6), (11.7), (11.8), and (11.9) we conclude that

(*i*) Induced emfs in primary and secondary windings, E_1 and E_2 lag behind the main flux Φ by $\frac{\pi}{2}$, so these emfs (E_1 and E_2) are in phase with each other, as shown in Fig. 11.6 vectorially.

(*ii*) Applied voltage to the primary winding leads the main flux by $\frac{\pi}{2}$ and is in phase opposition to the induced emf in the primary winding, as shown in Fig. 11.5 vectorially.

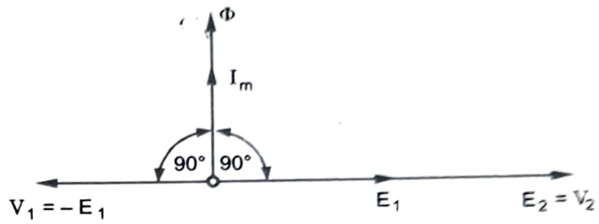


Fig. 11.5 *No-Load Phasor Diagram For an Ideal Transformer*

(*iii*) Secondary voltage $V_2 = E_2$ as there is no voltage drop in secondary.
The instantaneous value of applied voltage, induced emfs, flux and magnetising current, in case of an ideal transformer, are illustrated by sinusoidal waves in Fig. 11.6.

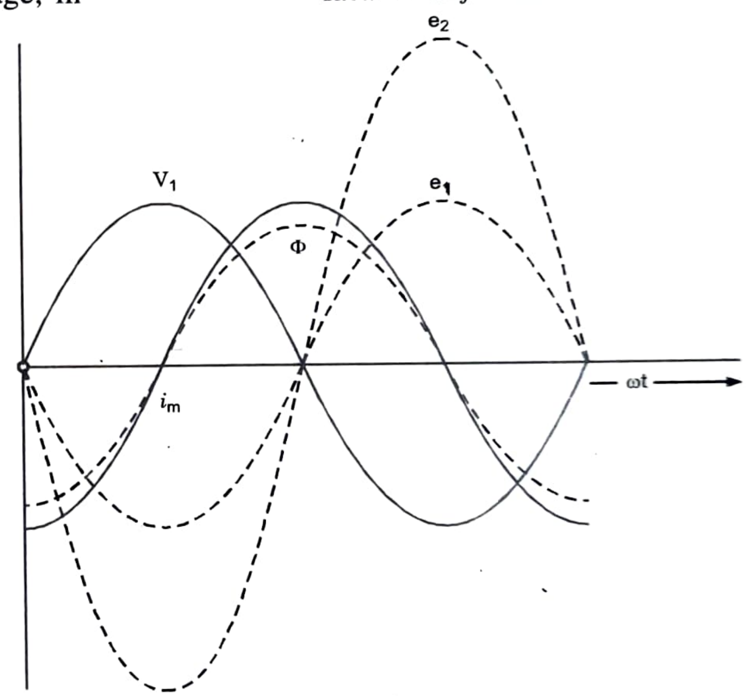


Fig. 11.6

However, when a varying flux is set up in magnetic material, there will be power loss, called the *iron or core loss*. So the input current to the primary under no-load condition has also to supply the hysteresis and eddy current losses (iron losses) occuring in the core in addition to small amount of copper loss occuring in primary winding (no copper loss occurs in secondary winding on open circuit or on no load). Hence, the no-load primary current I_0 does not lag behind applied voltage V_1 by 90 ° but lags behind V_1 by angle $\phi_0 < 90$ °.

Input power on no load, $P_0 = V_1 I_0 \cos \phi_0$ where $\cos \phi_0$ is the primary power factor under no-load condition.

As seen from phasor diagram shown in Fig. 11.7, input current to the primary I_0, called the *exciting current,* has two components (*i*) in-phase, active or energy component, I_e used to meet the iron loss in addition to small amount of copper loss occurring in the primary winding and (*ii*) quadrature component or wattless component called the magnetizing component, I_m used to create the alternating flux in the core.

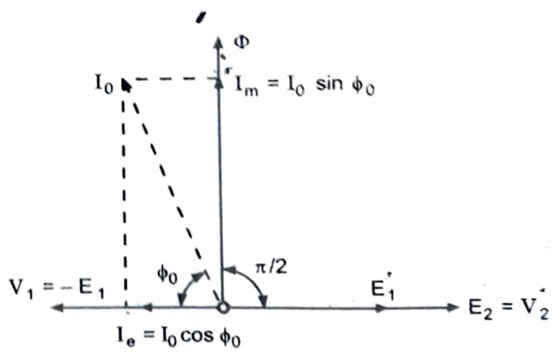


Fig. 11.7

$$\text{Thus, } I_e = I_0 \cos \phi_0$$

$$\text{and } I_m = I_0 \sin \phi_0$$

$$\text{and } \sqrt{I_e^2 + I_m^2} = I_0$$

$$\text{Angle of lag, } \phi_0 = \text{Tan}^{-1} \frac{I_m}{I_e}$$

The equivalent circuit of a transformer on no load is shown in Fig. 11.8. The actual transformer is replaced by an ideal transformer with a resistance R_0 and an inductive reactance X_0 in parallel with primary.

The two components no-load current I_0 *i.e.* I_e and I_m are represented by currents drawn by R_0 and X_0 respectively. Both these currents are drawn at induced emf $E_1 = V_1$ for resistanceless, no-leakage primary coil, even otherwise $E_1 = V_1$.

From the equivalent circuit shown in Fig. 11.8.

$$\text{Energy component of no-load current, } I_e = \frac{V_1}{R_0}$$

$$\text{Magnetising component of no-load current, } I_m = \frac{V_1}{X_0}.$$

$$\text{Core or iron loss} = I_e^2 R_0 = \frac{V_1^2}{R_0}$$

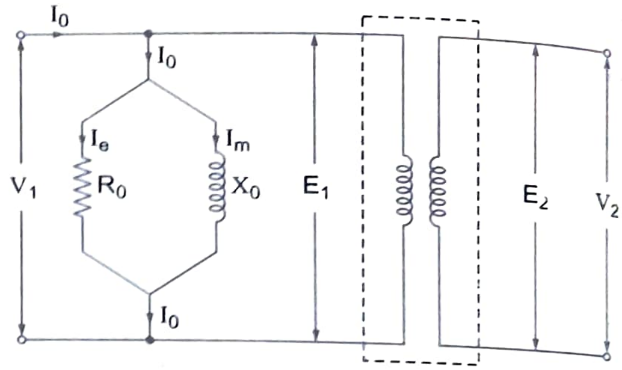


Fig. 11.8 *No-Load Equivalent Circuit For an Actual Transformer*

The noteworthy points are given below:

1. The no-load primary current I_0, called the *exciting current*, is very small in comparison to the full-load primary current. It ranges from 2 to 5 per cent of full-load primary current.
2. The exciting or no-load current I_0 is made up of a relatively large quadrature or magnetizing component I_m and a comparatively small in-phase or energy component I_e, so the power factor of a transformer on no load is very small (usually varies between 0.1 and 0.2 lag). The phase angle between I_0 and V_1 is about 78 ° to 87 °.
3. No-load primary copper loss *i.e.* $I_0^2 R_1$ is very small and may be neglected. Thus the no-load primary input power is practically equal to the iron loss occuring in the core of the transformer.

Example 11.10. A 11 kV/400 V distribution transformer takes a no-load primary current of 1 amp at a power factor of 0.24 lagging. Find (*i*) The core loss current (*ii*) The magnetizing current (*iii*) The iron loss.

[*R.G. Technical Univ.. June-2012*]

Solution: Exciting or no-load current, $I_0 = 1$ A

Primary voltage, $V_1 = 11$ kV $= 11{,}000$ V

(*i*) Core loss current, $I_e = I_0 \cos\phi_0 = 1 \times 0.24 = 0.24$ A **Ans.**

(*ii*) Magnetising current, $I_m = \sqrt{I_0^2 - I_e^2} = \sqrt{1^2 - 0.24^2} = 0.971$ A **Ans.**

(*iii*) Iron loss $= V_1 I_0 \cos\phi_0 = 11{,}000 \times 1 \times 0.24 = 2{,}640$ W or 2.64 kW.

Example 11.11. A single-phase transformer of 3,300/220 V, 50 Hz takes a no-load current of 0.8 A and 500 W. Calculate (*i*) the active and magnetising current (*ii*) its pf

[*R.G.T.U. Basic Elec. Engineering, December-2006*]

Solution: Exciting or no-load current, $I_0 = 0.8$ A

Primary voltage, $V_1 = 3{,}300$ V

Iron loss $= V_1 I_0 \cos\phi_0 = 500$ W

(*i*) Iron loss or active component of no-load current,

$$I_e = I_0 \cos\phi_0 = \frac{500}{V_1} = \frac{500}{3300} = 0.152 \text{ A } \textbf{Ans.}$$

$$\text{Magnetising component, } I_m = \sqrt{I_0^2 - I_e^2} = \sqrt{0.8^2 - 0.152^2} = 0.786 \text{ A } \textbf{Ans.}$$

$$(ii) \text{ Power factor, } \cos\phi_0 = \frac{500}{V_1 I_0} = \frac{500}{3{,}300 \times 0.8} = 0.1894 \text{ (lagging) } \textbf{Ans.}$$

Example 11.12. A voltage $v = 200 \sin 314\,t$ is applied to the transformer winding in a no-load test. The resulting current is found to be

$$i = 3 \sin (314\,t - 60°)$$

Determine the core loss and the parameters of no-load approximate equivalent circuit.

[U.P. Technical Univ. Electrical Machines 2002-03; Mahamaya Technical Univ. Electrical Engineering Odd Semester 2011-12]

Solution:

$$\text{No-load current, } I_0 = \frac{3}{\sqrt{2}} = 2.12 \text{ A}$$

$$\text{Core loss} = \text{Input at no load} = V\, I_0 \cos\phi_0 = \frac{200}{\sqrt{2}} \times \frac{3}{\sqrt{2}} \cos 60^\circ = 150 \text{ W } \textbf{Ans.}$$

$$\text{Energy component of no-load current, } I_e = I_0 \cos\phi_0 = \frac{3}{\sqrt{2}} \cos 60^\circ = 1.06 \text{ A}$$

$$\text{Magnetising component of no-load current, } I_m = \sqrt{I_0^2 - I_e^2} = \sqrt{(2.12)^2 - (1.06)^2} = 1.836 \text{ A}$$

$$\text{No-load resistance, } R_0 = \frac{V}{I_e} = \frac{200/\sqrt{2}}{1.06} = 133.42\ \Omega \ \textbf{Ans.}$$

$$\text{No-load reactance, } X_0 = \frac{V}{I_m} = \frac{200/\sqrt{2}}{1.836} = 77\ \Omega \ \textbf{Ans.}$$

11.9 TRANSFORMER ON LOAD

When the secondary circuit of a transformer is completed through an impedance or load, the transformer is said to be loaded and current flows through the secondary and the load. The magnitude and phase of secondary current I_2 with respect to secondary terminal voltage V_2 will depend upon the characteristic of load *i.e.* current I_2 will be in phase, lag behind and lead the terminal voltage V_2 respectively when the load is non-inductive, inductive and capacitive.

When the transformer is on no load, as shown in Fig. 11.4, it draws no-load current I_0 from the supply mains. The no-load current I_0 sets up an mmf N_1I_0 which produces flux Φ in the core. When an impedance is connected across the secondary terminals, as shown in Fig. 11.9, current I_2 flows through the secondary winding. The secondary current I_2 sets up its own mmf and hence creates a secondary flux Φ_2. The secondary flux Φ_2 opposes the main flux Φ set up by the exciting current I_0 according to Lenz's law. The opposing secondary flux Φ_2 weakens the main flux Φ momentarily, so primary back emf E_1 tends to be reduced. So difference of applied voltage V_1 and back emf E_1 increases, therefore, more current is drawn from the source of supply flowing through the primary winding until the original value of flux Φ is obtained. It again causes increase in back emf E_1 and it adjusts itself as such that there is a balance between applied voltage V_1 and back emf E_1. Let the additional primary current be I_1'. The current I'_1 is in phase opposition with secondary current I_2 and is called the *counterbalancing current.* The additional current I'_1 sets up an mmf $N_1I'_1$ producing flux Φ_1' in the same direction as that of main flux Φ and cancels the flux Φ_2 produced by secondary mmf $N_2 I_2$ being equal in magnitude.

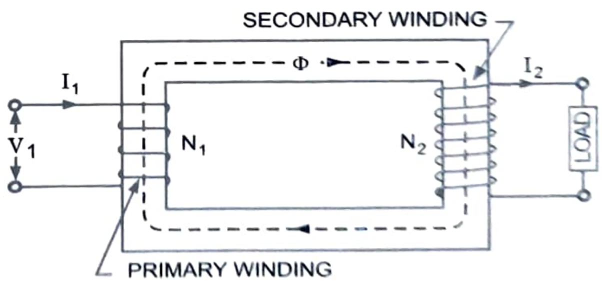


Fig. 11.9 *An Ideal Transformer On Load*

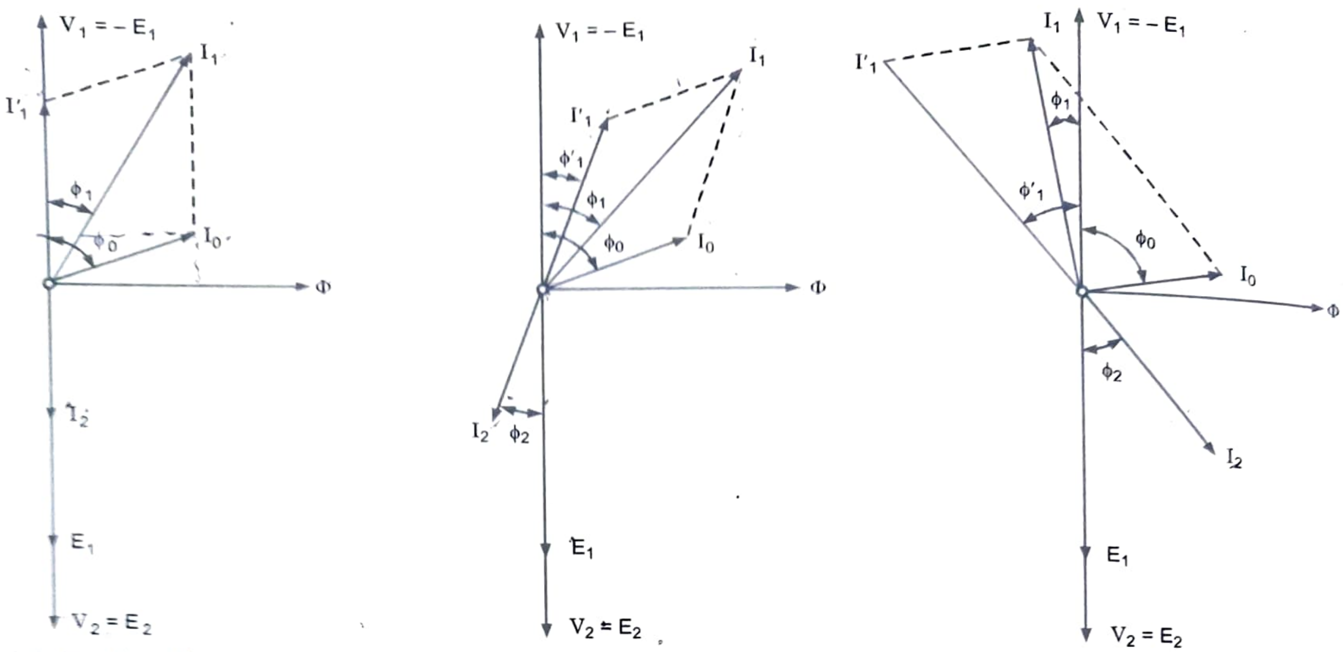
$$\text{So } N_1 I_1' = N_2 I_2$$

$$\text{or } I_1' = \frac{N_2}{N_1} I_2$$

The total primary current I_1 is, therefore, phasor sum of primary counterbalancing current I_1' and no-load current I_0, which will be approximately equal to I'_1 as I_0 is usually very small in comparison to I_1'.

$$\therefore\ I_1 = I_1' = \frac{N_2}{N_1} I_2$$

$$\text{or } \frac{I_1}{I_2} = \frac{N_2}{N_1} = K \qquad \text{(transformation ratio)}$$



(a) For Pure Resistive Load (b) For Inductive Load (c) For Capacitive Load

Fig. 11.10 *Phasor Diagram For an Ideal Transformer On Load*

Hence primary and secondary currents are inversely proportional to their respective turns.

Since the secondary flux Φ_2 produced by secondary mmf $N_2 I_2$ is neutralized by the flux Φ'_1 produced by mmf N_1I_1' set up by counterbalancing primary current I'_1, so the flux in the transformer core remains almost constant from no load to full load.

The phasor diagrams for transformer on non-inductive, inductive and capacitive loads are shown in Figs. 11.10(*a*), (*b*) and (*c*) respectively.

Since the voltage drops in both of the windings of the transformer are assumed to be negligible, therefore

$$\mathbf{V_2 = E_2 \text{ and } V_1 = -E_1}$$

The secondary current I_2 is in phase, lags behind and leads the secondary terminal voltage V_2 by an angle ϕ_2 for pure resistive, inductive and capacitive loads respectively.

The induced primary current I_1', also known as *counterbalancing current,* is always in opposition to secondary current I_2 and since no-load current I_0 is very small, the total primary current I_1 is almost opposite in phase to I_2 and K times the secondary current I_2, where K is transformation ratio.

Note. In phasor diagrams shown in Figs. 11.10(*a*), 11.10(*b*) and 11.10(*c*) no-load current has been drawn on exaggerated scale for sake of clarity.

Example 11.13. A single-phase transformer with a ratio of 440/110 V takes a no-load current of 5 A at 0.2 power factor lag. If the secondary supplies a current of 120 A at a pf of 0.8 lagging, calculate the current taken by the primary.

[Electrical Machines-I; Rajasthan Univ. 2004; M.D. Univ. May-2009; Electromechanical Energy Conversion, M.D. Univ., December-2012]

Solution:

Primary voltage, $V_1 = 440$ V

Secondary voltage, $V_2 = 110$ V

No-load power factor, $\cos \phi_0 = 0.2$ (lag)

Secondary current, $I_2 = 120$ A

Load power factor, $\cos \phi_2 = 0.8$ (lag)

$$\text{Transformation ratio, } K = \frac{V_2}{V_1} = \frac{110}{440} = 0.25$$

Primary counterbalancing current, $I'_1 = K\,I_2 = 0.25 \times 120 = 30$ A lagging behind the supply voltage V_1 by an angle ϕ'_1

where $\phi_1 = \phi_2 = \text{Cos}^{-1}\, 0.8 = 36.87^\circ$

$$X_1 = \frac{E_{L1}}{I_1} = \frac{2\pi f L_1 I_1}{I_1} = 2\pi f L_1$$

Similarly secondary leakage flux Φ_{L2} is set up by secondary ampere-turns and is proportional to secondary current I_2. On no load there is no current in secondary winding and, therefore, no leakage flux exists across the secondary winding on no load. On load leakage flux Φ_{L2} in phase with secondary current I_2 and produces self-induced emf $E_{L2} = 2\pi f L_2 I_2$ in the secondary winding where L_2 is self-inductance of secondary winding due to leakage flux Φ_{L2}. This is also a reactance voltage, and the component that balances it leads the secondary current by 90°. The secondary reactance X_2 opposes the current flowing out of the transformer and can be obtained by dividing self-induced emf in secondary winding, E_{L2} by the secondary current I_2 *i.e.*

$$X_2 = \frac{E_{L2}}{I_2} = \frac{2\pi f L_2 I_2}{I_2} = 2\pi f L_2$$

The effect of magnetic leakage is, thus to produce in their respective windings emfs of self-inductance which are proportional to the current, and are, therefore, equivalent in effect to the addition of an inductive coil in series with each winding, the reactance of which is called the leakage reactance.

A transformer with magnetic leakage and winding resistance is equivalent to an ideal transformer (having no resistance and leakage reactance) having inductive and resistive coils connected in series with each winding as shown in Fig. 11.12.

Few important points to be kept in mind are given below:

1. The leakage flux links one or the other winding but not both, hence it in no way contributes to the transfer of energy from the primary winding to the secondary winding.
2. The applied voltage to the primary winding, V_1 will have to meet the reactive drop $I_1 X_1$ in addition to $I_1 R_1$. Similarly induced emf in the secondary winding E_2 will have to meet the resistive and reactive drops $I_2 R_2$ and $I_2 X_2$ respectively.
3. Transformation ratio is reduced* due to resistance and magnetic leakage, since these reduce the secondary terminal voltage V_2 on load for a given primary applied voltage V_1.
4. The main useful flux Φ decreases slightly with the increase in load but the leakage fluxes are practically proportional to the currents in the respective winding.
5. In actual transformers, the primary and secondary windings are not placed on separate legs, as shown in Fig. 11.1, as due to their being widely separate, large primary and secondary leakage fluxes would result. The leakage fluxes are reduced to a minimum by sectionalizing and interleaving the primary and secondary windings.

11.11 PHASOR DIAGRAM OF ACTUAL TRANSFORMER ON LOAD

Consider a transformer shown in Fig. 11.12 having primary and secondary windings of resistances R_1 and R_2 and reactances X_1 and X_2 respectively. The impedance of primary winding is given by $\mathbf{Z}_1 = R_1 + jX_1$ and impedance of secondary winding is given by $\mathbf{Z}_2 = R_2 + jX_2$.

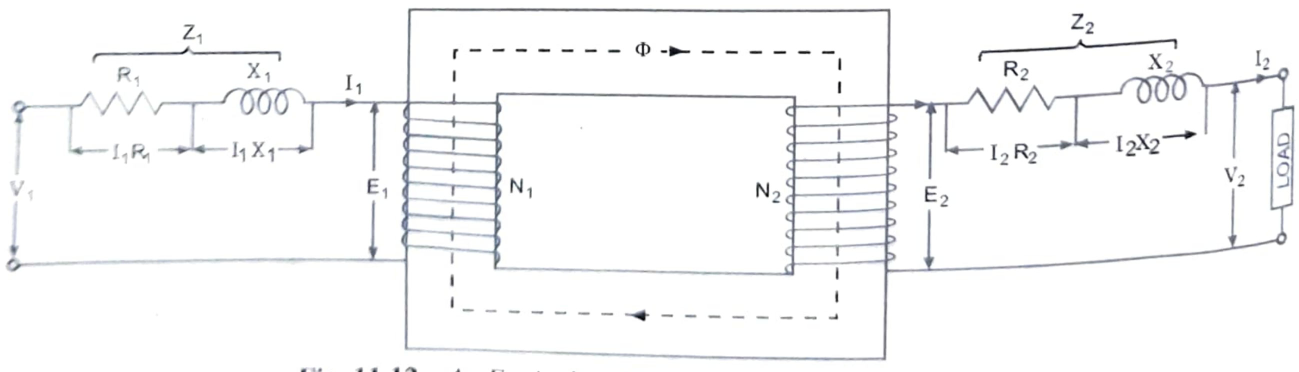


Fig. 11.12 *An Equivalent Diagram of Actual Transformer*

* Assuming lagging power factor

The phasor diagrams of above transformer on (*i*) pure resistive, (*ii*) resistive-inductive, and (*iii*) resistive-capacitive loads are shown in Fig. 11.13 (*a*), (*b*) and (*c*) respectively.

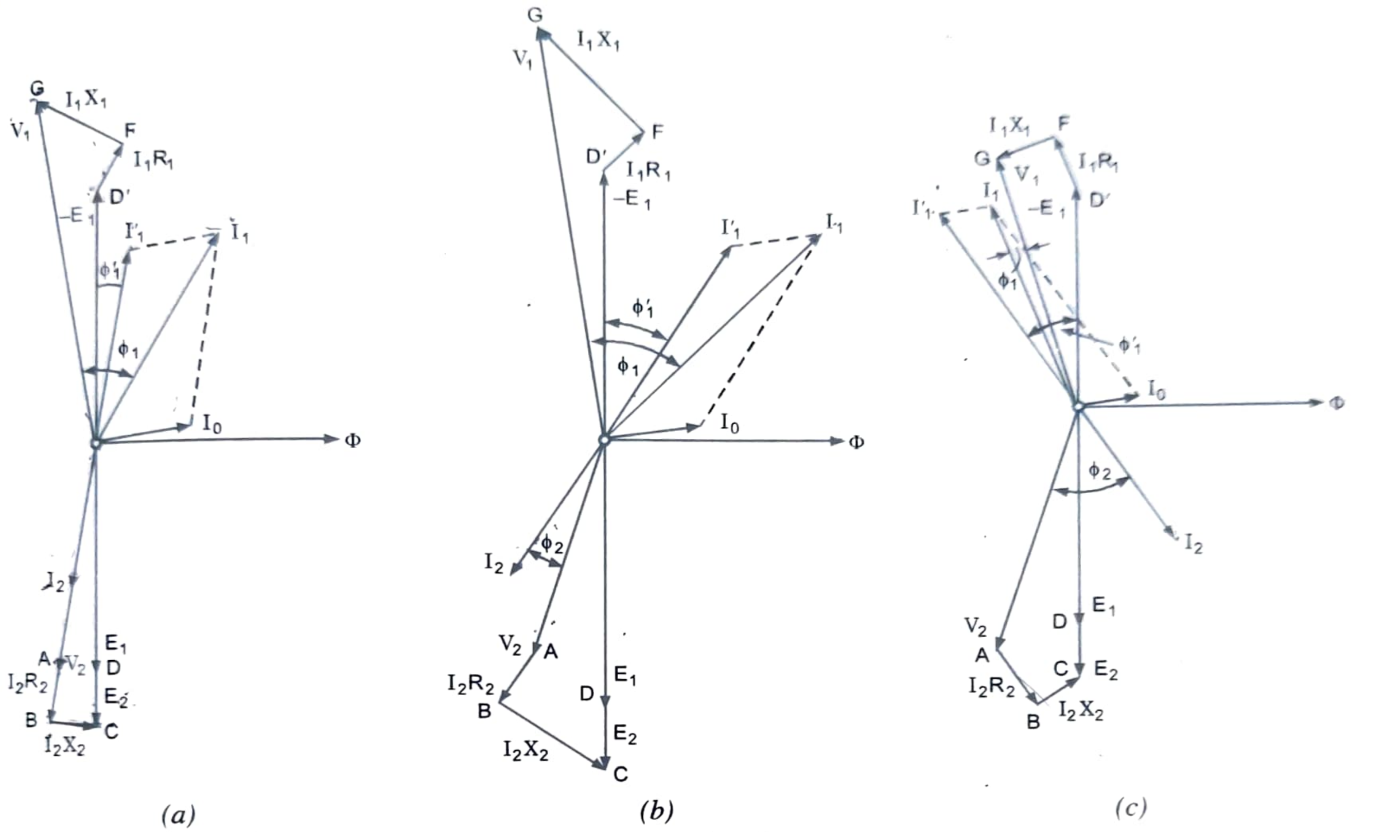


Fig. 11.13 *Phasor Diagram of an Actual Transformer*

Draw *OA* representing secondary terminal voltage V_2 and OI_2 representing secondary current I_2 in phase as well as magnitude. Since voltage drops due to secondary winding resistance and reactance are $I_2\ R_2$ in phase with current I_2 and $I_2\ X_2$ leading current I_2 by $\pi/2$ respectively, so draw AB parallel to OI_2 and equal to $I_2\ R_2$ in magnitude representing resistive drop in secondary winding and draw BC perpendicular to AB and equal to $I_2\ X_2$ in magnitude representing reactive drop of secondary winding. Since phasor sum of terminal voltage V_2, secondary resistive drop $I_2\ R_2$ and secondary reactive drop $I_2\ X_2$ is equal to induced emf E_2 in secondary winding, phasor OC represents secondary induced emf E_2. Hence we have

$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2\ (\mathrm{R}_2 + j\ \mathrm{X}_2) = \mathbf{V}_2 + \mathbf{I}_2\ \mathbf{Z}_2 \qquad ...(11.10)$$

The induced emf E_1 in primary winding is in phase with E_2 and equal to $\frac{N_1}{N_2} E_2$ in magnitude, so take OD $= \frac{N_1}{N_2}$ OC representing E_1. Produce DO to D′ taking OD′ = OD hence representing $(-\mathbf{E}_1)$.

The induced primary current I'_1 is equal to $-I_2 \frac{N_2}{N_1}$ so draw O I'_1 equal to $OI_2 \times \frac{N_2}{N_1}$ by producing line I_2 O. Draw line OI_0 representing no-load current in magnitude as well as in phase. The phasor sum of induced primary current I'_1 and no-load current I_0 gives primary current represented by phasor OI_1 in Fig. 11.13.

Since voltage drop due to primary winding resistance and reactance are I_1R_1 in phase with primary current I_1 and I_1X_1 leading current I_1 by $\frac{\pi}{2}$ respectively, draw D′ F parallel to OI_1 and equal to I_1R_1 in magnitude representing resistive drop in primary winding and draw FG perpendicular to D′ F equal to I_1X_1 in magnitude representing reactive drop in primary winding. As the phasor sum of $(-E_1)$, primary resistive drop and primary reactive drop gives the applied voltage V_1 to primary winding, hence phasor OG represents the applied voltage V_1 in magnitude as well as in phase.

$$i.e.\ \mathbf{V_1} = -\mathbf{E_1} + \mathbf{I_1}(R_1 + jX_1) = -\mathbf{E_1} + \mathbf{I_1}\mathbf{Z_1} \qquad ...(11.11)$$

The phase angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.

Since no-load current I_0, resistive drops $I_1 R_1$ and $I_2 R_2$ and reactive drops, $I_1 X_1$ and $I_2 X_2$ are very small, neglecting these we have $\phi_2 = \phi_1' = \phi_1 = \phi$, the phase angle of the load. In Fig. 11.13, no-load current, resistive drops and reactive drops are shown, for clarity, on exaggerated scales.

From phasor diagrams we have

(*a*) *For pure resistive load [phasor diagram 11.13(a)]*

$$E_2 = \sqrt{(V_2 + I_2 R_2)^2 + (I_2 X_2)^2} \approx V_2 + I_2 R_2 \qquad ...(11.12)$$

$$E_1 = \frac{E_2}{K}$$

$$\text{and } V_1 = \sqrt{(E_1 + I_1 R_1)^2 + (I_1 X_1)^2} \approx E_1 + I_1 R_1 \qquad ...(11.13)$$

(*b*) *For resistive-inductive load [phasor diagram 11.13(b)]*

$$E_2 = \sqrt{(V_2 + I_2 R_2 \cos\phi + I_2 X_2 \sin\phi)^2 + (I_2 X_2 \cos\phi - I_2 R_2 \sin\phi)^2}$$

$$\approx V_2 + I_2 R_2 \cos\phi + I_2 X_2 \sin\phi \qquad ...(11.14)$$

$$E_1 = \frac{E_2}{K} \text{ and}$$

$$V_1 = \sqrt{(E_1 + I_1 R_1 \cos\phi + I_1 X_1 \sin\phi)^2 + (I_1 X_1 \cos\phi - I_1 R_1 \sin\phi)^2}$$

$$\approx E_1 + I_1 R_1 \cos\phi + I_1 X_1 \sin\phi \qquad ...(11.15)$$

(*c*) *For resistive-capacitive load [phasor diagam 11.13(c)]*

$$E_2 = \sqrt{(V_2 + I_2 R_2 \cos\phi - I_2 X_2 \sin\phi)^2 + (I_2 X_2 \cos\phi + I_2 R_2 \sin\phi)^2}$$

$$\approx V_2 + I_2 R_2 \cos\phi - I_2 X_2 \sin\phi \qquad ...(11.16)$$

$$E_1 = \frac{E_2}{K} \text{ and}$$

$$V_1 = \sqrt{(E_1 + I_1 R_1 \cos\phi - I_1 X_1 \sin\phi)^2 + (I_1 X_1 \cos\phi + I_1 R_1 \sin\phi)^2}$$

$$= E_1 + I_1 R_1 \cos\phi - I_1 X_1 \sin\phi \qquad ...(11.17)$$

Example 11.14. A 230/460 V transformer has a primary resistance of 0.2 Ω and a reactance of 0.5 Ω and the corresponding values for the secondary are 0.75 Ω and 1.8 Ω respectively. Find the secondary terminal voltage when supplying (*a*) 10 A at 0.8 pf lagging (*b*) 10 A at pf 0.8 leading.

Solution: Transformation ratio, $K = \frac{V_2}{V_1} = \frac{460}{230} = 2$

(*i*) **When Load is 10 A at 0.8 pf lagging** *i.e.* when

$$I_2 = 10 \text{ A and } \cos\phi = 0.8 \text{ lagging.}$$

Primary current $I_1 = K I_2 = 2 \times 10 = 20$ A

From Eq. (11.17)

- Primary induced emf, $E_1 = V_1 - (I_1 R_1 \cos\phi + I_1 X_1 \sin\phi) = 230 - (20 \times 0.2 \times 0.8 + 20 \times 0.5 \times 0.6) = 220.8$ V

Secondary induced emf, $E_2 = KE_1 = 2 \times 220.8 = 441.6$ V

From Eq. (11.14)

Secondary terminal voltage, $V_2 = E_2 - (I_2 R_2 \cos\phi + I_2 X_2 \sin\phi) = 441.6 - (10 \times 0.75 \times 0.8 + 10 \times 1.8 \times 0.6)$
$= 441.6 - (6 + 10.8) = 424.8$ V **Ans**

(*ii*) When load is 10 A at 0.8 pf leading

From Eq. (11.17)

Primary induced emf, $E_1 = V_1 - (I_1 R_1 \cos\phi - I_1 X_1 \sin\phi) = 230 - (20 \times 0.2 \times 0.8 - 20 \times 0.5 \times 0.6) = 232.8$ V

and from Eq. (11.16)

Secondary terminal voltage,

$$V_2 = KE_1 - (I_2 R_2 \cos\phi - I_2 X_2 \sin\phi) = 2 \times 232.8 - (10 \times 0.75 \times 0.8 - 10 \times 1.8 \times 0.6)$$
$$= 465.6 - (6 - 10.8) = 470.4 \text{ V} \quad \textbf{Ans.}$$

11.12 EQUIVALENT RESISTANCE AND REACTANCE

The two independent circuits of a transformer can be resolved into an equivalent circuit to make the calculations simple.

Let resistances and reactances of primary and secondary windings be R_1 and R_2 and X_1 and X_2 ohms respectively and let transformation ratio be K.

Resistive drop in secondary winding $= I_2 R_2$
Reactive drop in secondary winding $= I_2 X_2$
Resistive drop in primary winding $= I_1 R_1$
Reactive drop in primary winding $= I_1 X_1$

Referred To Secondary Side. Since transformation ratio is K, primary resistive and reactive drops as referred to secondary will be K times, *i.e.* KI_1R_1 and KI_1X_1 respectively. If I_1 is substituted equal to KI_2 then we have primary resistive and reactive drops referred to secondary equal to $K^2I_2R_1$ and $K^2I_2X_1$ respectively.

Total resistive drop in a transformer $= K^2 I_2 R_1 + I_2 R_2 = I_2 (K^2 R_1 + R_2) = I_2 R_{02}$

Total reactive drop in a transformer $= K^2 I_2 X_1 + I_2 X_2 = I_2 (K^2 X_1 + X_2) = I_2 X_{02}$

The term $(K^2 R_1 + R_2)$ and $(K^2 X_1 + X_2)$ represent the equivalent resistance and reactance respectively of the transformer referred to secondary and let these be represented by R_{02} and X_{02} respectively. Equivalent circuit referred to secondary has been shown in Fig. 11.14(*a*).

From phasor diagram [Fig. 11.14(*b*)]

$$KV_1 = \sqrt{(V_2 + I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi)^2 + (I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)^2}$$

where V_2 is secondary terminal voltage, I_2 is secondary current lagging behind the terminal voltage V_2 by ϕ.

Since term $(I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)$ is very small as compared to the term $(V_2 + I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi)$, neglecting the former we have

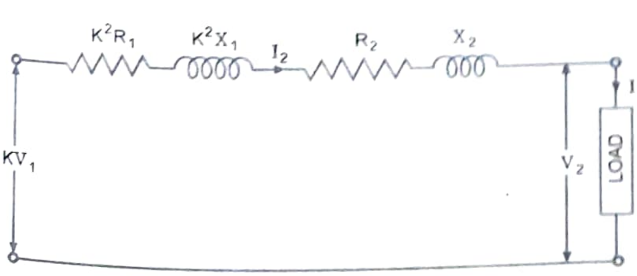


Fig. 11.14(*a*) *Equivalent Circuit Of a Transformer Referred To Secondary*

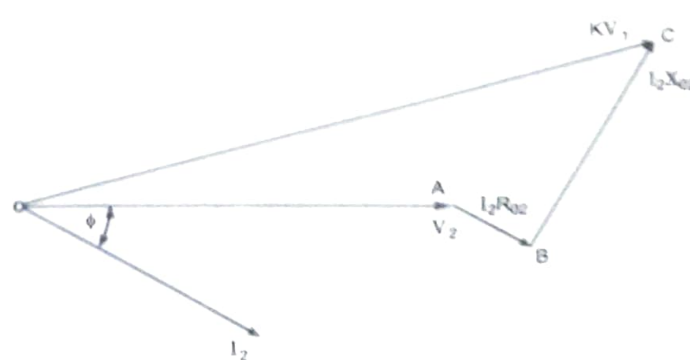


Fig. 11.14(*b*) *Phasor Diagram*

$$K V_1 = V_2 + I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$$

$$\text{or } V_2 = KV_1 - I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi \quad ...(11.18)$$

where V_1 is applied voltage to primary winding.

If load is pure resistive, $\phi = 0$ and $V_2 = KV_1 - I_2 R_{02}$ ...(11.19)

If load is capacitive then ϕ should be taken as –ve hence we have

$$V_2 = KV_1 - I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi \quad ...(11.20)$$

Referred To Primary Side. Secondary resistive drop referred to primary

$$= \frac{I_2 R_2}{K} = \frac{I_1 R_2}{K^2} \qquad \text{since } I_2 = \frac{I_1}{K}$$

Secondary reactive drop referred to primary

$$= \frac{I_2 X_2}{K} = \frac{I_1 X_2}{K^2}$$

Total resistive drop in the transformer referred to primary

$$= I_1 R_1 + \frac{I_1 R_2}{K^2} = I_1 \left(R_1 + \frac{R_2}{K^2}\right) = I_1 R_{01}$$

Total reactive drop in the transformer referred to primary

$$= I_1 X_1 + \frac{I_1 X_2}{K^2} = I_1 \left(X_1 + \frac{X_2}{K^2}\right) = I_1 X_{01}$$

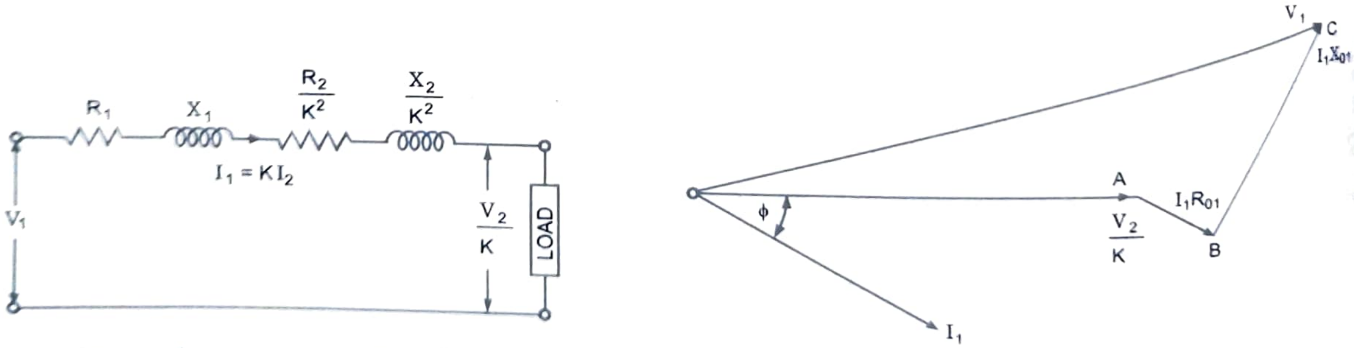
The terms $\left(R_1 + \frac{R_2}{K^2}\right)$ and $\left(X_1 + \frac{X_2}{K^2}\right)$ represent the total resistance and reactance of the transformer referred to primary respectively. Let these be represented by R_{01} and X_{01} respectively.

Equivalent circuit referred to primary is shown in Fig. 11.15(*a*)

From phasor diagram shown in Fig. 11.15(*b*) we have

$$V_1 = \sqrt{\left(\frac{V_2}{K} + I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi\right)^2 + (I_1 X_{01} \cos\phi - I_1 R_{01} \sin\phi)^2}$$

Since term $(I_1 X_{01} \cos\phi - I_1 R_{01} \sin\phi)$ is very small as compared to the term $\left(\frac{V_2}{K} + I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi\right)$, neglecting the former we have



(a) Equivalent Circuit of a Transformer Referred To Primary

(b) Phasor Diagram

Fig. 11.15

$$V_1 = \frac{V_2}{K} + I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi \quad ...(11.21)$$

If load is pure resistive *i.e.* $\phi = 0$ then

$$V_1 = \frac{V_2}{K} + I_1 R_{01} \quad ...(11.22)$$

If load is capacitive then ϕ should be taken as –ve so we have

$$V_1 = \frac{V_2}{K} + I_1 R_{01} \cos\phi - I_1 X_{01} \sin\phi \qquad \text{... (11.23)}$$

Example 11.15. A 25 kVA, 2,200/220 V, 50-Hz, 1-phase transformer has following parameters:

$$\mathbf{R_1 = 1.75\ \Omega,\ R_2 = 0.0045\ \Omega,\ X_1 = 2.6\ \Omega,\ X_2 = 0.0075\ \Omega.}$$

Calculate: (*i*) Equivalent resistance referred to primary and secondary. (*ii*) Equivalent reactance referred to primary and secondary. [*U.P. Technical Univ. Electrical Engineering Odd Semester, 2013-14*]

Solution:

$$\text{Transformation ratio, } K = \frac{V_2}{V_1} = \frac{220}{2{,}200} = 0.1$$

(*i*) Equivalent resistance referred to primary, $R_{01} = R_1 + \frac{R_2}{K^2} = 1.75 + \frac{0.0045}{(0.1)^2} = 2.2\ \Omega$ **Ans.**

Equivalent resistance referred to secondary, $R_{02} = K^2R_1 + R_2 = (0.1)^2 \times 1.75 + 0.0045 = 0.022\ \Omega$ **Ans.**

(*ii*) Equivalent reactance referred to primary, $X_{01} = X_1 + \frac{X_2}{K^2} = 2.6 + \frac{0.0075}{(0.1)^2} = 3.35\ \Omega$ **Ans.**

Equivalent reactance referred to secondary, $X_{02} = K^2X_1 + X_2 = (0.1)^2 \times 2.6 + 0.0075 = 0.0335\ \Omega$ **Ans.**

Example 11.16. A transformer has a primary winding of 600 turns and a secondary winding of 150 turns. When the load current on the secondary is 60 A at 0.8 power factor lagging, the primary current is 20 A at 0.707 power factor lagging. Determine the no-load current of the transformer and its phase with respect to the voltage.

[*G.B. Technical Univ. Electrical Engineering Even Semester, 2009-10*]

Solution:

$$\text{Turn ratio } K = \frac{N_2}{N_1} = \frac{150}{600} = \frac{1}{4}$$

$$\text{Secondary current, } \mathbf{I_2} = 60 \angle \cos^{-1} 0.8 = 60 \angle -36.87^\circ \text{ A}$$

$$\text{Load current referred to primary, } \mathbf{I'_1} = KI_2 = \frac{1}{4} \times 60 \angle -36.87^\circ = 15 \angle -36.87^\circ \text{ A}$$

$$\text{Primary current, } \mathbf{I_1} = 20 \angle \cos^{-1} 0.707 = 20 \angle -45^\circ \text{ A}$$

$$\text{No-load current, } \mathbf{I_0} = \mathbf{I_1} - \mathbf{I'_1} \qquad \because\ I_1 = I_0 + I'_1$$

$$= 20 \angle -45^\circ - 15 \angle -36.87^\circ = (14.14 - j\,14.14) - (12 - j\,9)$$

$$= (2.2 - j\,5.14) \text{ A} = 5.59 \angle -66.83^\circ \text{ A} \quad \textbf{Ans.}$$

Example 11.17. A 10 kVA, single-phase transformer for 2,000/400 V at no load has $R_1 = 5.5\ \Omega$, $R_2 = 0.2\ \Omega$, $X_1 = 12\ \Omega$, $X_2 = 0.45\ \Omega$. Determine the approximate value of the secondary voltage at full load, 0.8 pf (lagging), when the primary applied voltage is 2,000 V. [*G.G.S.I.P. Univ. Delhi, May-June-2007*]

Solution:

$$\text{Transformation ratio, } K = \frac{400}{2{,}000} = 0.2$$

Equivalent resistance referred to secondary, $R_{02} = K^2R_1 + R_2 = 0.2^2 \times 5.5 + 0.2 = 0.42\ \Omega$

Equivalent reactance referred to secondary, $X_{02} = K^2X_1 + X_2 = 0.2^2 \times 12 + 0.45 = 0.93\ \Omega$

$$\text{Secondary full-load current, } I_2 = \frac{\text{KVA} \times 1{,}000}{V_2} = \frac{10 \times 1{,}000}{400} = 25 \text{ A}$$

$$\text{Power factor, } \cos\phi = 0.8 \text{ (lagging)}$$

$$\sin\phi = \sin\cos^{-1} 0.8 = 0.6$$

$$\text{Secondary terminal voltage at full load, } V_2 = KV_1 - I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi$$

$$= 0.2 \times 2{,}000 - 25 \times 0.42 \times 0.8 - 25 \times 0.93 \times 0.6$$

$$= 400 - 8.4 - 13.95 = 377.65 \text{ V} \quad \textbf{Ans.}$$

11.13 EQUIVALENT CIRCUIT OF A TRANSFORMER

The equivalent circuit of any device can be quite helpful in predetermination of the behaviour of the device under various conditions of operation and it can be drawn if the equations describing its behaviour are known. If any electrical device is to be analysed and investigated further for suitable modifications, its appropriate equivalent circuit is necessary. The equivalent circuit for electromagnetic devices consists of a combination of resistances, inductances, capacitances, voltage etc. Such an equivalent circuit (or circuit model) can, therefore, be analysed and studied easily by the direct application of electric circuit theory.

As stated above, equivalent circuit is simply a circuit representation of the equations describing the performance of the device. Eqs. (11.10) and (11.11) describe the behaviour of the transformer under load and are helpful in arriving at the transformer equivalent circuit.

Equivalent circuit of a transformer having transformation ratio $K = E_2/E_1$ is shown in Fig. 11.16.

The induced emf in primary winding E_1 is primary applied voltage V_1 less primary voltage drop. This voltage causes iron loss current I_e and magnetising current I_m and we can, therefore, represent these two components of no-load current by the current drawn by a non-inductive resistance R_0 and pure reactance X_0 having the voltage E_1 or (V_1— primary voltage drop) applied across them, as shown in Fig. 11.16.

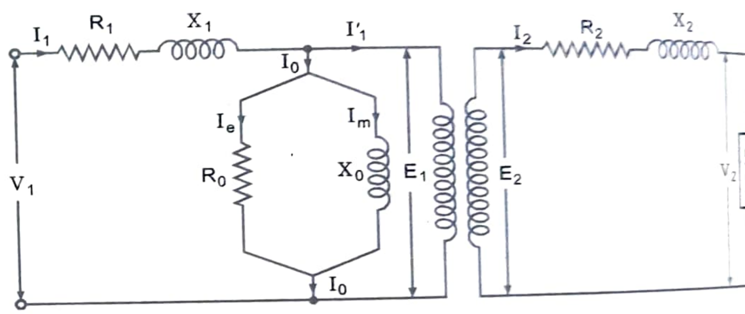


Fig. 11.16 *Equivalent Circuit of a Transformer*

Secondary current, $I_2 = \frac{I'_1}{K} = \frac{I_1 - I_0}{K}$

Terminal voltage V_2 across load is induced emf E_2 in secondary winding less voltage drop in secondary winding.

The equivalent circuit can be simplified by transferring the voltage, current and impedance to the primary side. After transferring the secondary voltage, current and impedance to primary side equivalent circuit is reduced to that shown in Fig. 11.17.

The equivalent circuit diagram can further be simplified by transferring the resistance R_0 and reactance X_0 towards left end, as shown in Fig. 11.18. The error introduced by doing so is very small and can be neglected.

No-load current I_0 is hardly 3 to 5 per cent of the full-load rated current, the parallel branch consisting of resistance R_0 and reactance X_0 can be omitted without introducing any appreciable error in the behaviour of the transformer under loaded condition. Such a circuit is shown in Fig. 11.15(*a*). The equivalent circuit referred to secondary side (neglecting no-load current I_0) is illustrated in Fig. 11.14(*a*).

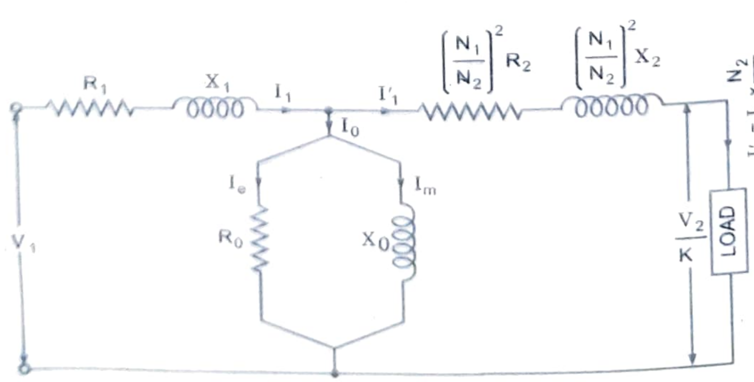


Fig. 11.17 *Equivalent Circuit of a Transformer With All Secondary Impedances Transferred To Primary Side*

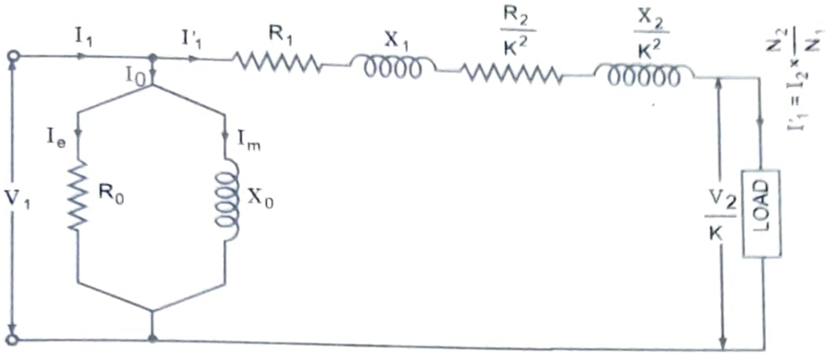


Fig. 11.18 *Approximate Equivalent Circuit of a Transformer Referred To Primary Side*

Example 11.18. The ohmic values of the circuit parameters of a transformer, having a turns ratio of 5, are $R_1 = 0.5\ \Omega$, $R_2 = 0.021\Omega$, $X_1 = 3.2\ \Omega$, $X_2 = 0.12\ \Omega$, $R_c = 350\ \Omega$, referred to the primary and $X_m = 98\ \Omega$ referred to primary. Draw the approximate equivalent circuit of the transformer referred to secondary. Show the numerical values of the circuit parameters. *[U.P. Technical Univ. Electrical Engineering First Semester 2008-09]*

Solution: $R_1 = 0.5\ \Omega$; $R_2 = 021\ \Omega$; $X_1 = 3.2\ \Omega$ and $X_2 = 0.12\ \Omega$

$$R_c = 350\ \Omega \text{ referred to primary}$$

$$X_m = 98\ \Omega \text{ referred to primary}$$

Turn ratio, $\frac{N_1}{N_2} = 5$

Transformation ratio,

$$K = \frac{N_2}{N_1} = \frac{1}{5} = 0.2$$

Resistance of primary winding referred to secondary

$$= K^2 R_1 = 0.2^2 \times 0.5 = 0.02\ \Omega$$

Reactance of primary winding referred to secondary

$$= K^2 X_1 = 0.2^2 \times 3.2 = 0.128\ \Omega$$

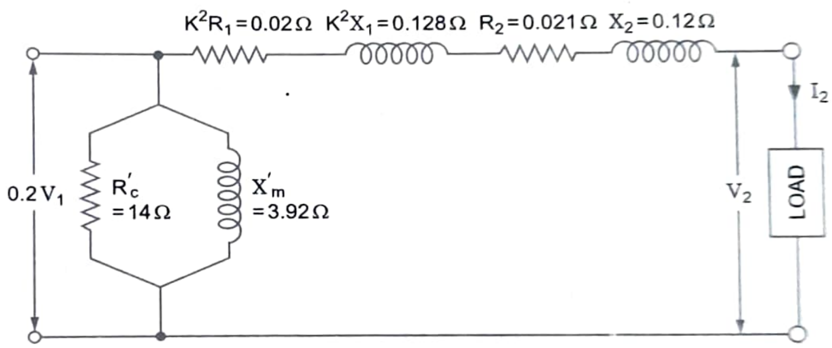


Fig. 11.19

R_c referred to secondary $= K^2R_c = 0.2^2 \times 350 = 14\ \Omega$

X_m referred to secondary $= 0.2^2 \times 98 = 3.92\ \Omega$

Approximate equivalent circuit of transformer referred to secondary is shown in Fig. 11.19.

11.14 VOLTAGE REGULATION

The way in which the secondary terminal voltage varies with the load depends upon load current, the internal impedance and the load power factor. The change in secondary terminal voltage from no load to full load, with primary voltage and frequency held constant, is termed the *inherent regulation*.

The figure of merit which determines the voltage characteristic of a transformer is termed as *voltage regulation*. It is usually expressed as a percentage of the rated terminal (secondary) voltage. It is defined as the change in magnitude of terminal (secondary) voltage when full load (rated load) of specified power factor supplied at rated voltage is thrown off (reduced to no load) with primary voltage and frequency held constant, as percentage of the rated terminal voltage.

If V_2 is the secondary (terminal) voltage at full load at specified power factor and E_2 is the secondary (terminal) voltage at no load, then percentage regulation is given as

$$\text{Voltage regulation} = \frac{E_2 - V_2}{\text{Secondary rated voltage}} \times 100$$

As per IS, the secondary rated voltage of a transformer is equal to the secondary voltage at no load *i.e.* E_2

So percentage regulation

$$= \frac{E_2 - V_2}{E_2} \times 100$$

$$= \frac{\text{Voltage drop in transformer at full load}}{\text{No-load rated secondary voltage, } E_2} \times 100$$

$$= \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{E_2} \times 100 \qquad ...(11.24)$$

When the power factor is leading, the percentage regulation is given as

$$\text{Percentage regulation} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{E_2} \times 100 \qquad ...(11.25)$$

The percentage regulation can be given as

$$\% \text{ regulation} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{E_2} 100$$

+ ve sign for lagging power factor and – ve sign for leading power factor

$$= \left(\frac{I_2 R_{02}}{E_2} \cos\phi \pm \frac{I_2 X_{02}}{E_2} \sin\phi\right) \times 100$$

$$= (\text{PU resistance} \times \cos\phi \pm \text{PU reactance} \sin\phi) \times 100 \qquad ...(11.26)$$

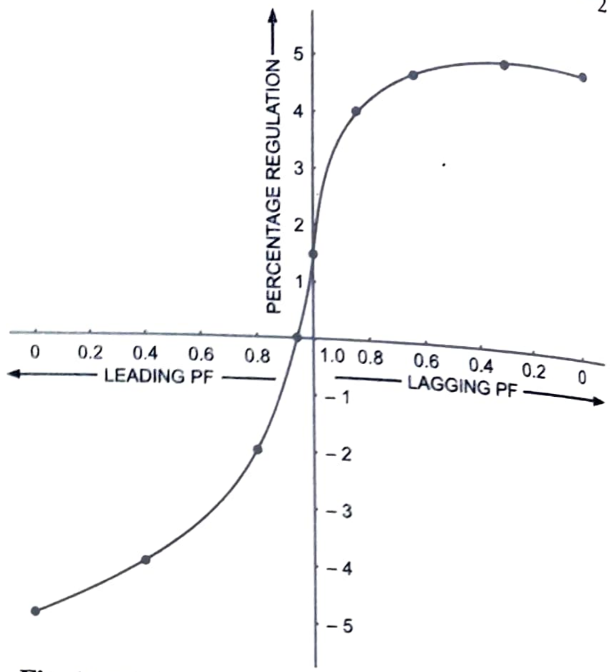


Fig. 11.20 *Percentage Regulation Vs Power Factor (R = 0.02 PU, X = 0.06 PU)*

Voltage regulation of a transformer, on an average, is about 4 per cent.

From the consumer's viewpoint, voltage regulation due to variations in load is undesirable and should be kept smallest possible.

11.14.1. Condition For Zero Regulation.

$$\text{Regulation} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{E_2}$$

Regulation will be zero if the numerator will be equal to zero

$$\text{or } I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi = 0$$

$$\text{or } \tan\phi = \frac{-R_{02}}{X_{02}} \qquad ...(11.27)$$

The –ve sign indicates that zero regulation occurs at a leading power factor.

Example 11.19. A 75 kVA transformer has 500 turns on primary and 100 turns on secondary. The primary and secondary resistances are 0.4 Ω and 0.02 Ω respectively and the corresponding leakage reactances are 1.5 Ω and 0.045 Ω respectively. The supply voltage is 2,200 V. Calculate (a) The equivalent impedance referred to primary (b) The voltage regulation at pf of 0.8 lagging. [*West Bengal Univ. of Technology Basic Electrical Engineering, 2012-13*]

Solution: Primary voltage, $V_1 = 2{,}200$ V

Transformation ratio, $K = \text{Turn ratio} = \frac{100}{500} = 0.2$

Primary resistance, $R_1 = 0.4\ \Omega$

Secondary resistance, $R_2 = 0.02\ \Omega$

Primary reactance, $X_1 = 1.5\ \Omega$

Secondary reactance, $X_2 = 0.045\ \Omega$

Equivalent resistance referred to primary,

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.4 + \frac{0.02}{(0.2)^2} = 0.9\ \Omega$$

Equivalent reactance referred to primary,

$$X_{01} = X_1 + \frac{X_2}{K^2} = 1.5 + \frac{0.045}{(0.2)^2} = 2.625\ \Omega$$

(*a*) Equivalent impedance referred to primary,

$$Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2} = \sqrt{0.9^2 + 2.625^2} = 2.775\ \Omega \quad \textbf{Ans.}$$

$$\text{(b) Full-load primary current, } I_1 = \frac{\text{Rated kVA} \times 1{,}000}{V_1} = \frac{75 \times 1{,}000}{2{,}200} = 34.091\text{ A}$$

$$\cos\phi = 0.8 \text{ (lagging)}$$

$$\sin\phi = 0.6$$

$$\text{Voltage regulation} = \frac{I_1 R_{01}\cos\phi + I_1 X_{01}\sin\phi}{V_1} = \frac{34.091 \times 0.9 \times 0.8 + 34.091 \times 2.625 \times 0.6}{2{,}200} \times 100$$

$$= 3.556\% \quad \textbf{Ans.}$$

Example 11.20. Determine voltage regulation of transformer whose ohmic drop is 1.5% and leakage reactance drop is 5% at 0.8 pf lagging. [*G.B. Technical Univ. Electromechanical Energy Conversion-I, 2011-12*]

Solution: Percentage regulation $= \dfrac{I_2 R_{02}\cos\phi \times I_2 X_{02}\sin\phi}{E_2} \times 100$

$$= \frac{I_2 R_{02}}{E_2} \times 100\cos\phi + \frac{I_2 X_{02}}{E_2} \times 100\sin\phi$$

$$= \%R\cos\phi + \%X\sin\phi = 1.5 \times 0.8 + 5 \times 0.6 = 4.2\% \quad \textbf{Ans.}$$

$$\because \quad \cos\phi = 8 \text{ and } \sin\phi = \sqrt{1 - 0.8^2} = 0.6$$

Example 11.21. Calculate the regulation of a transformer in which ohmic losses are 1% of output and reactance drop 5% of the voltage when the pf is (*i*) 0.8 lagging (*ii*) 0.8 leading and (*iii*) unity. [*M.D. Univ. Electromechanical Energy Conversion, May-2007*]

Solution: Per unit resistance $= \dfrac{\text{Percentage copper loss}}{100} = \dfrac{1}{100} = 0.01$

Per unit reactance $=$ Per unit reactance drop $= \dfrac{5}{100} = 0.05$

(*i*) When pf, $\cos\phi = 0.8$ lagging

$$\sin\phi = 0.6$$

Percentage voltage regulation = (PU resistance × cos ϕ + PU reactance × sin ϕ) × 100

$$= (0.01 \times 0.8 + 0.05 \times 0.6) \times 100 = 3.8\% \quad \textbf{Ans.}$$

(*ii*) When pf, $\cos\phi = 0.8$ leading

$$\sin\phi = -0.6$$

Percentage voltage regulation $= [0.01 \times 0.8 + 0.05 \times (-0.6)] \times 100 = -2.2\%$ **Ans.**

(*iii*) When pf is unity, $\cos\phi = 1.0$

$$\sin\phi = 0$$

Percentage voltage regulation $= (0.01 \times 1.0 + 0.05 \times 0) \times 100 = 1\%$ **Ans.**

Example 11.22. Define voltage regulation of transformer. A 100 kVA, 2,200/220 V transformer has leakage reactance drop of 8% and resistance drop of 2%. Find its voltage regulation at full load and 0.8 pf lagging. Find also the power factor at which the regulation will be zero.

Solution: Regulation = PU resistance × 0.8 + PU reactance × 0.6

$$= 0.02 \times 0.8 + 0.08 \times 0.6 = 0.064 \text{ or } 6.4\% \quad \textbf{Ans.}$$

Regulation will be zero when power factor

$$\cos\phi = \cos\tan^{-1}\frac{-R_{02}}{X_{02}} = \cos\tan^{-1}\frac{-I_2 R_{02}/E_2}{I_2 X_{02}/E_2} = \cos\tan^{-1}\frac{-0.02}{0.08} = 0.97 \text{ (lead)} \quad \textbf{Ans.}$$

11.15 TRANSFORMER LOSSES

The transformer is a static machine and, therefore, there are no friction or windage losses. The various power losses occuring in a transformer are enumerated below:

1. Iron or Core Losses. Iron loss is caused by the alternating flux in the core and consists of hysteresis and eddy current losses.

(a) Hysteresis Loss. The core of a transformer is subjected to an alternating magnetizing force and for each cycle of emf a hysteresis loop is traced out. The hysteresis loss per second is given by the equation

Hysteresis loss, $P_h = \eta'(B_{max})^x f\, v$ joules per second or watts ...(11.28)

where f is the supply frequency in Hz, v is the volume of core in cubic metres, η' is the hysteresis coefficient, B_{max} is peak value of flux density in the core and x lies between 1.5 and 2.5 depending upon the material and is often taken as 1.6.

(b) Eddy Current Loss. If the magnetic circuit is made up of iron and if the flux in the circuit is variable, currents will be induced by induction in the iron circuit itself. All such currents are known as *eddy currents.*

Eddy currents result in a loss of power, with consequent beating of the material.

The eddy current loss is given by equation

$P_e = K_e\,(B_{max})^2 f^2 t^2$ v watts or joules per second ...(11.29)

These losses are determined from the open-circuit test (refer to Art. 11.16.1).

The input to the transformer with rated voltage applied to the primary and secondary open circuited is equal to the core loss.

2. Copper or Ohmic Losses. These losses occur due to ohmic resistance of the transformer windings. If I_1 and I_2 are the primary and secondary currents respectively and R_1 and R_2 are the respective resistances of primary and secondary windings then copper losses occuring in primary and secondary windings will be $I_1^2 R_1$ and $I_2^2 R_2$ respectively. So total copper losses will be $(I_1^2 R_1 + I_2^2 R_2)$. These losses vary as the square of the load current or kVA. For example if the copper losses at full load are P_c then copper losses at one-half or one-third of full load will be $\left(\frac{1}{2}\right)^2 P_c$ or $\left(\frac{1}{3}\right)^2 P_c$ *i.e.* $\frac{P_c}{4}$ or $\frac{P_c}{9}$ respectively.

Copper losses are determined on the basis of constant equivalent resistance R_{eq} determined from the short-circuit test (refer to Art. 11.16.2) and then corrected to 75 °C (since the standard operating temperature of electrical machines is taken 75 °C).

Example 11.23. A transformer is connected to a 1000 V, 50 Hz supply. The total core loss is 1000 W of which 700 W are hysteresis and 300 W are eddy current loss. If the applied voltage is raised to 2000 V and the frequency to 100 Hz, find the new core loss.

Solution: At 1,000 V, 50 Hz

Hysteresis loss, $P_{h_1} = 700$ W

Eddy current loss, $P_{e_1} = 300$ W

In a transformer, since induced emf, which is nearly equal to applied voltage is given by

$$E = 4.44\,\Phi_{max} f N$$

When applied voltage (*i.e.* E) and supply frequency, f both are doubled (2000 V and 100 Hz respectively), B_{max} will remain unchanged and

$$\text{Hysteresis loss, } P_{h_2} = P_{h_1}\frac{f_2}{f_1} = 700 \times 2 = 1{,}400 \text{ W}$$

$\because P_h \propto f$, B_{max} remaining constant

$$\text{Eddy current loss, } P_{e_2} = P_{e_1} \times \left(\frac{f_2}{f_1}\right)^2 = 300 \times 2^2 = 1{,}200 \text{ W}$$

$\because P_e \propto f^2$, B_{max} remaining constant

$$\text{Total core loss} = P_{h_2} + P_{e_2} = 1{,}400 + 1{,}200 = 2{,}600 \text{ watts}$$ **Ans.**

11.16 TESTING OF TRANSFORMERS

The main difficulties encountered in testing of large power transformers by direct loading are (*i*) wastage of large amount of energy and (*ii*) a stupendous (impossible for large transformers) task of arranging a load large enough for direct loading. The performance characteristics of a transformer can be conveniently computed from the knowledge of its equivalent circuit parameters which, in turn, may be determined by conducting simple tests called the *open-circuit* or *no-load test* and *short-circuit* or *impedance test* involving very little power cosumption (power needed to supply the losses incurred).

11.16.1. Open-Circuit Test (or No-Load Test). The purpose of this test is to determine the core (or iron or excitation) loss, P_i and no-load current I_0 and thereby the shunt branch parameters R_0 and X_0 of the equivalent circuit.

In this test, one of the windings (usually high-voltage winding) is kept open circuited and the rated voltage at rated frequency is applied to the other winding, as shown Fig. 11.21. No doubt, the core loss will be the same whether the measurements are made on *lv* winding or hv winding so long as the rated voltage of that winding is applied to it but in case the measurements are made on *hv* winding, the voltage required to be applied would often be inconveniently large while the current I_0 would be inconveniently small.

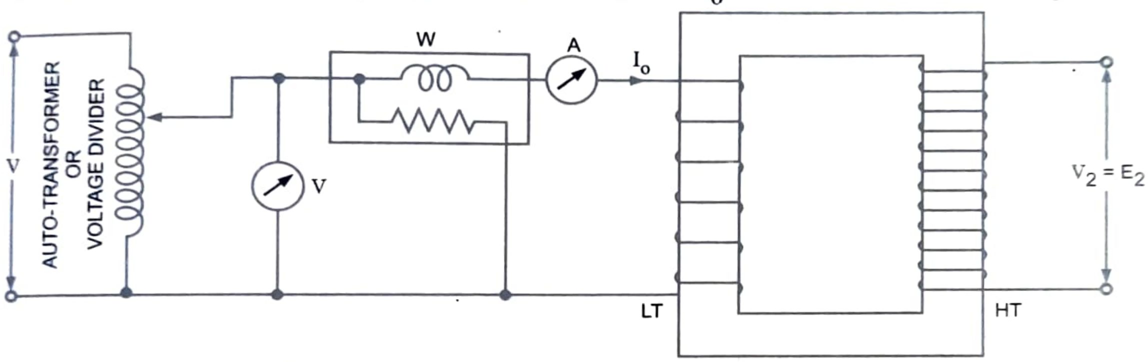


Fig. 11.21 *Circuit Diagram For Open-Circuit Test*

Either an auto-transformer or a voltage divider (VD) is used for varying the voltage applied to the low-voltage winding. Ammeter A and wattmeter W are connected to measure no-load current I_0 and input power W_0. Voltmeter V is connected to measure the applied voltage.

Since no current flows in the open-circuited secondary, the current in the primary will be merely that necessary to magnetize the core at normal voltage. Moreover, this magnetising current is a very small fraction of the full-load current (usually 3 to 10% of full-load current) and may be neglected as far as the copper loss is concerned consequently, the test gives core loss alone practically.

With normal voltage applied to the primary, normal flux will be set up in the core and, therefore, normal iron (or core) loss will occur which are recorded by a wattmeter W.

The open-circuit test gives enough data to compute the equivalent circuit constants R_0, X_0, no-load power factor cos ϕ_0, no-load current I_0 and no-load power loss (iron loss) of a transformer.

Iron loss, P_i = Input power on no load = W_0 watts (say)

No-load current = I_0 amperes

Applied voltage to primary = V_1 volts

$$\text{Angle of lag, } \phi_0 = \text{Cos}^{-1}\frac{W_0}{V_1 I_0} \qquad \text{...(11.30)}$$

$$\text{No-load current energy component, } I_e = I_0 \cos\phi_0 = \frac{W_0}{V_1} \qquad \text{...(11.31)}$$

$$\text{No-load current magnetizing component, } I_m = \sqrt{I_0^2 - I_e^2} \qquad \text{...(11.32)}$$

$$\text{Equivalent circuit parameter, } R_0 = \frac{V_1}{I_e} = \frac{V_1^2}{W_0} \qquad \text{...(11.33)}$$

$$\text{Equivalent circuit parameter, } X_0 = \frac{V_1}{I_m} = \frac{V_1}{\sqrt{I_0^2 - I_e^2}} \qquad \text{...(11.34)}$$

Note. 1. Since no-load current I_0 is very small, pressure coils of wattmeter and the voltmeter should be connected such that the currents drawn by them do not flow through the current coils of the wattmeter and ammeter.

2. Since power factor at no load is quite low (in the range of 0.1–0.2 lag), a low power factor wattmeter should be used to ensure accurate measurements.
3. The error due to power loss in ammeter can be eliminated by short circuiting the ammeter while reading wattmeter.
4. Sometimes a high resistance voltmeter is connected across the secondary to indicate the emf induced in the secondary (*hv* winding). This helps in determination of transformation ratio K.
5. It must, however, be remembered that in making this test, hv side is hot and, therefore, its terminals must be properly insulated.

11.16.2. Short-Circuit Test (or Impedance Test). The purpose of this test is to determine full-load copper loss and equivalent resistance and equivalent reactance referred to metering side.

In this test, the terminals of secondary winding (usually of low-voltage winding*) are short circuited by a thick wire or strip or through an ammeter (which may serve the additional purpose of indicating secondary rated load current) and variable low voltage is applied to the primary through an auto-transformer or potential divider, as shown in Fig. 11.22. The transformer now becomes equivalent to a coil having an impedance equal to impedance of both the windings.

The applied voltage, V_s to the primary is gradually increased till the ammeter A indicates the full-load (rated) current of the metering side. Since applied voltage is very low (5–8% of the rated voltage), flux linking with the core is very small and, therefore, iron losses are so small that these can be neglected. Thus the power input (reading of wattmeter W) gives total copper loss at rated load, output being nil. Let the readings of voltmeter, ammeter and wattmeter be V_s, I_s and W_s respectively.

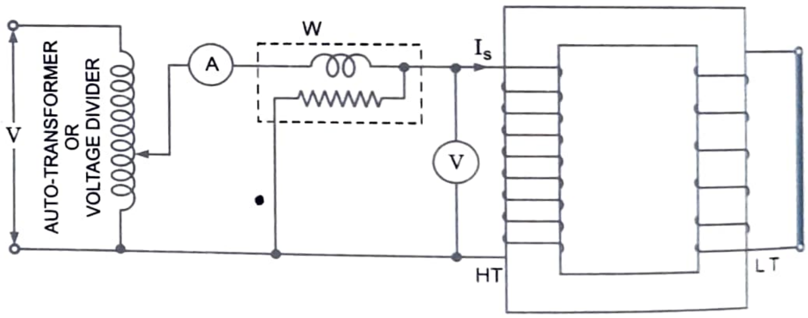


Fig. 11.22 *Circuit Diagram For Short-Circuit Test*

$$\text{Full-load copper loss, } P_c = I_s^2 R_{eq} = W_s \qquad \text{...(11.35)}$$

$$\text{Equivalent resistance, } R_{eq} = \frac{W_s}{I_s^2} \qquad \text{...(11.36)}$$

* Voltage required for the short-circuit test is about 5 per cent of the rated value. For a 200 kVA, 2200/220 V transformer, test on high voltage side would need the voltage (to be applied) of $2{,}200 \times \frac{5}{100}$ *i.e.* 110 V (which is standard voltage for instrument coils) and a current of $\frac{200 \times 1{,}000}{2{,}200} = 91\text{A}$.

If the test is conducted on low voltage side of the above transformer, the voltage needed would be $220 \times \frac{5}{100} = 11$ V (very small) and the current would be $\frac{200 \times 1{,}000}{220} = 910$ A (very high). At this low voltage, high precision would not be readily obtainable with ordinary instruments.

Thus we see that the if the measurements were made on low-voltage side, the voltage needed would be inconveniently low, while the current would often be inconveniently large.

Example 11.27. A short-circuit test when performed on the *hv* side of a 10 kVA 2,000/400 V single-phase transformer, gave the following data — 60 V, 4 A, 100 W.

***If the lv* side is delivering full-load current at 0.8 pf lag and at 400 V, find the applied voltage to hv side.**

Solution: Equivalent impedance referred to primary,

$$Z_{01} = \frac{V_s}{I_s} = \frac{60}{4} = 15\,\Omega \quad \because \text{ short-circuit test has been conducted on the hv (primary) side}$$

Equivalent resistance referred to primary,

$$R_{01} = \frac{P_s}{I_s^2} = \frac{100}{4^2} = 6.25\,\Omega$$

Equivalent reactance referred to primary,

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(15)^2 - (6.25)^2} = 13.64\,\Omega$$

$$\text{Full-load primary current, } I_1 = \frac{\text{kVA (rated)} \times 1{,}000}{V_1} = \frac{10 \times 1{,}000}{2{,}000} = 5 \text{ A}$$

Total voltage drop in transformer winding as referred to primary

$$= I_1 (R_{01} \cos\phi + X_{01} \sin\phi) = 5(6.25 \times 0.8 + 13.64 \times 0.6) = 65.9 \text{ V}$$

Applied voltage to the hv side = Load voltage + voltage drop = 2,000 + 65.9 = 2,065.9 V **Ans.**

11.18 TRANSFORMER EFFICIENCY

The rated capacity of a transformer is defined as the product of rated voltage and full-load (rated) current on the output side. The power output depends upon the power factor of the load.

The efficiency (η) of a transformer, like that of any other apparatus, is defined as the ratio of useful power output to the input power, the two being measured in same units (either in watts or killowatts).

i.e. Transformer efficiency,

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + losses}} = \frac{\text{Output}}{\text{Output + iron loss + copper loss}}$$

$$= 1 - \frac{\text{iron loss + copper loss}}{\text{Output + iron loss + copper loss}}$$

Now power output = $V_2 I_2 \cos\phi$

where V_2 is the secondary terminal voltage on load, I_2 is the secondary current at load and $\cos\phi$ is the power factor of the load.

Iron loss, P_i = Hysteresis loss + eddy current loss

Copper loss = $I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$

11.18.1. Determination of Transformer Efficiency. The ordinary transformer has a very high efficiency (in the range of 96-99%). Hence the transformer efficiency cannot be determined with high precision by direct measurement of output and input, since the losses are of the order of only 1–4%. The difference between the readings of output and input instruments is then so small that an instrument error as low as 0.5% would cause an error of the order of 15% in the losses. Further, it is inconvenient and costly to have the necessary loading devices of the correct current and voltage ratings and power factor to load the transformer. There is also a wastage of large amcunt of power (equal to that of power output + losses) and no information is available from such a test about the proportion of copper and iron losses.

The best and accurate method of determining of efficiency of a transformer would be to compute losses from open-circuit and short-circuit tests and determine efficiency as follows :

Iron loss, $P_i = W_0$ or P_0, determined from open-circuit test

Copper loss at full load, $P_c = W_s$ or P_s, determined from short-circuit test

Copper loss at a load x times full load

Since the distribution transformer does not supply the rated load for the whole day so the all-day efficiency of such a transformer will be lesser than ordinary or commercial efficiency.

For determination of all-day efficiency of a transformer, it is necessary, of course, to know how the load varies from hour to hour during the day.

Higher energy efficiencies are achieved by designing distribution transformers to yield maximum commercial efficiency at less than full load (usually 50–75 per cent of full load). This is achieved by restricting the core flux density to the lower values by using a relatively larger cross section. Thus the ratio of iron loss to copper loss is reduced.

Example 11.34. What do you mean by all-day Efficiency ? A 5 kVA transformer has 34 W core loss and 40 W copper loss at full load. It operates at rated kVA and 0.8 power factor lagging for 6 hours, one-half rated kVA and 0.5 power factor lagging for 12 hours and no load for 6 hours. What is its all day efficiency?

[*Chhattisgarh Vivekanand Technical Univ. 2006-07*]

Solution:

$$\text{Full-load output} = 5 \text{ kVA}$$

$$\text{Full-load copper loss, } P_c = 40 \text{ W}$$

$$\text{Iron loss, } P_i = 34 \text{ W}$$

$$\text{All-day output} = 5 \times 0.8 \times 6 + \frac{1}{2} \times 5 \times 0.5 \times 12 = 39 \text{ kWh}$$

$$\text{Copper loss for 24 hours} = 40 \times 6 + \left(\frac{1}{2}\right)^2 \times 40 \times 12 = 360 \text{ Wh} = 0.36 \text{ kWh}$$

$$\text{Iron loss for 24 hours} = 24 \times 34 = 816 \text{ Wh} = 0.816 \text{ kWh}$$

$$\text{All-day input} = 39 + 0.36 + 0.816 = 40.176 \text{ kWh}$$

$$\text{All-day efficiency, } \eta = \frac{\text{All-day output}}{\text{All-day input}} \times 100 = \frac{39}{40.176} \times 100 = 97.07\% \ \textbf{Ans.}$$

Example 11.35. The daily variation of load on a 100 kVA transformer is as follows:

8.00 AM to 1.00 PM — 65 kW, 35 kVAR
1.00 PM to 6.00 PM — 80 kW, 50 kVAR
6.00 PM to 1.00 AM — 30 kW, 30 kVAR
1.00 AM to 8.00 AM — No-load

This transformer has no-load core loss of 270 watts and a full–load ohmic loss of 1,200 watts. Determine the all-day efficiency of the transformer. [*J.N. Technological Univ. Electrical Machines-II, January-2014*]

Solution: $\text{All-day output} = 65 \times 5 + 80 \times 5 + 30 \times 7 = 935 \text{ kWh}$

$$\text{Iron loss for 24 hours} = \frac{270 \times 24}{1{,}000} = 6.48 \text{ kWh}$$

$$\text{Load in kVA from 8.00 AM to 1.00 PM} = \sqrt{\text{kW}^2 + \text{kVAR}^2} = \sqrt{65^2 + 35^2} = 73.824 \text{ kVA}$$

$$\text{Load in kVA from 1.00 PM to 6.00 PM} = \sqrt{80^2 + 50^2} = 94.34 \text{ kVA}$$

$$\text{Load in kVA from 6.00 PM to 1.00 AM} = \sqrt{30^2 + 30^2} = 42.426 \text{ kVA}$$

$$\text{Copper losses for 24 hours} = \left(\frac{73.824}{100}\right)^2 \times 1{,}200 \times 5 + \left(\frac{94.34}{100}\right)^2 \times 1{,}200 \times 5 + \left(\frac{42.426}{100}\right)^2 \times 1{,}200 \times 7$$

$$= 10{,}122 \text{ Wh or } 10.122 \text{ kWh}$$

$$\text{All-day efficiency, } \eta = \frac{935}{935 + 6.48 + 10.122} \times 100 = 98.26\% \ \textbf{Ans.}$$

11.20 TRANSFORMER CONSTRUCTION

The transformer is very simple in construction and consists of *magnetic circuit* linking with two windings known as *primary* and *secondary windings*. Besides magnetic circuit and windings it consists of a suitable container for the assembled core and windings, such as a *tank*, a suitable medium for insulating the core and windings from its container such as *transformer oil*, suitable *bushings* (either of porcelain, oil filled or condenser type) for insulating and bringing out terminals of the windings from the container, *temperature gauge* for measurement of temperature of hot oil or hottest spot temperature, and *oil gauge* to indicate the oil level inside the tank. Some transformers are also provided with *conservator tank* in order to slow down deterioration of oil and keep the main tank full of oil, emergency vent to relieve the pressure inside the tank in case the pressure inside the transformer rises to a danger point and *gas operated relay* (*Buchholz relay*) in order to give alarm to indicate the presence of gas in case of some minor fault and take out the transformer out of circuit in case of serious fault.

Magnetic circuit consists of an iron core. Since core is magnetic link between the two systems in a transformer and it itself contains a lot a energy, it is not by any means the passive component it would first appear.

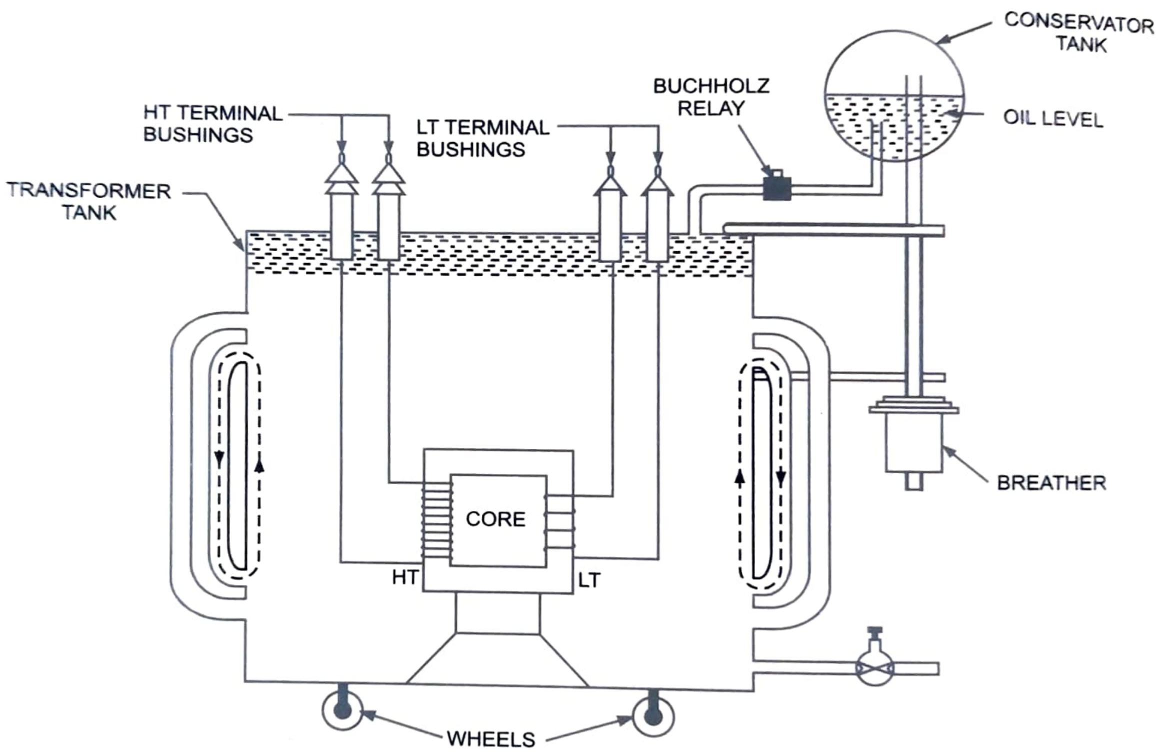


Fig. 11.26

Ideally the steel, of which the transformer core is made, is simple in chemical composition, the hot rolled steel has 4-5% silicon and the cold rolled has 3%. Nowadays cold rolled steel is used almost exclusively inspite of its high price, because flux density 30% higher can be used. This is because of high permeability and lower losses at given flux density. Since flux is an alternating one, in order to reduce eddy current losses the core is laminated. The thickness of laminations or stampings varies from 0.35 mm to 0.5 mm. The laminations are insulated from each other by coating them with a thin coat of varnish. The iron core is rectangular in cross section for small size transformers and circular for large size ones to reduce the quantity of copper required.

As already mentioned, the electric circuit consists essentially of two windings, each being split up into two equal number of coils and the coils of two windings are arranged on each limb of iron core to reduce the magnetic leakage. In order to minimise the amount of high voltage insulation, low voltage coils are placed adjacent to the iron.