*Chapter* **5**

# Single Phase Series Circuits

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## 5.1 INTRODUCTION

In a dc circuit the relationship between the applied voltage V and current flowing through the circuit I is a simple one and is given by the expression I = V/R but in an a c circuit this simple relationship does not hold good. Variations in current and applied voltage set up magnetic and electrostatic effects respectively and these must be taken into account with the resistance of the circuit while determining the quantitative relations between current and applied voltage. With comparatively low-voltage, heavy-current circuits magnetic effects may be very large, but electrostatic effects are usually negligible. On the other hand with high-voltage circuits electrostatic effects may be of appreciable magnitude, and magnetic effects are also present.

In this chapter we will discuss how do the magnetic effects due to variations in current and electrostatic effects due to variations in the applied voltage affect the relationship between the applied voltage and current.

## 5.2 PURE RESISTIVE CIRCUITS

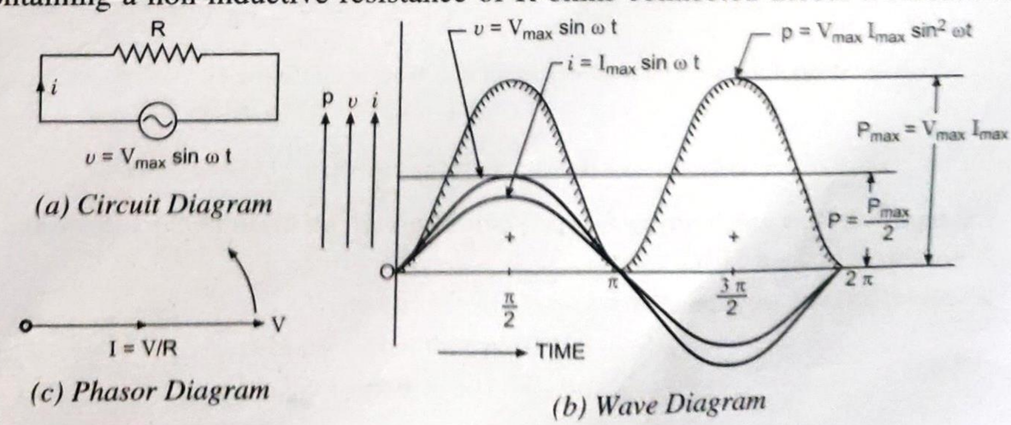
A pure resistive or a non-inductive circuit is a circuit which has inductance so small that at normal frequency its reactance is negligible as compared to its resistance. Ordinary filament lamps, water resistances etc. are the examples of non-inductive resistances. If the circuit is non-inductive, no reactance emf (*i.e.* self- induced or back emf) is set up and whole of the applied voltage is utilised in overcoming the ohmic resistance of the circuit.

Consider an ac circuit containing a non-inductive resistance of R ohms connected across a sinusoidal voltage represented by $v = V_{max} \sin \omega t$, as shown in Fig. 5.1 (*a*).

As already said, when the current flowing through a pure resistance changes, no back emf is set up, therefore, applied voltage has to overcome the ohmic drop of $iR$ only

*i.e.* $iR = v$

or $i = \dfrac{v}{R} = \dfrac{V_{max}}{R} \sin \omega t$




*(a) Circuit Diagram*

*(c) Phasor Diagram*

*(b) Wave Diagram*

**Fig. 5.1** *Pure Resistive Circuit*

$$\text{Current will be maximum when } \omega t = \frac{\pi}{2} \text{ or } \sin \omega t = 1$$

$$\therefore \quad I_{max} = \frac{V_{max}}{R}$$

and instantaneous current may be expressed as

$$i = I_{max} \sin \omega t$$

From the expressions of instantaneous applied voltage and instantaneous current, it is evident that in a pure resistive circuit, the applied voltage and current are in phase with each other, as shown by wave and phasor diagrams in Figs. 5.1 (*b*) and (*c*) respectively.

**5.2.1. Power in Pure Resistive Circuit.** The instantaneous power delivered to the circuit in question is the product of the instantaneous values of applied voltage and current.

$$\text{i.e. } p = v\, i = V_{max} \sin \omega t\, I_{max} \sin \omega t = V_{max}\, I_{max} \sin^2 \omega t$$

$$\text{or } p = \frac{V_{max}\, I_{max}}{2}(1 - \cos 2\,\omega t) \quad \text{Since } \sin^2 \omega\, t = \frac{1 - \cos 2\,\omega t}{2}$$

$$= \frac{V_{max}\, I_{max}}{2} - \frac{V_{max}\, I_{max}}{2} \cos 2\,\omega t$$

$$\text{Average power, } P = \text{Average of } \frac{V_{max}\, I_{max}}{2} - \textit{average of } \frac{V_{max}\, I_{max}}{2} \cos 2\,\omega t$$

Since average of $\frac{V_{max}\, I_{max}}{2} \cos 2\,\omega t$ over a complete cycle is zero,

$$P = \frac{V_{max}\, I_{max}}{2} = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = VI \text{ watts}$$

where V and I are the rms values of applied voltage and current respectively.

Thus, for pure resistive circuits, the expression for power is the same as for d c circuits. From the power curve for a pure resistive circuit shown in Fig. 5.1 (*b*) it is evident that power consumed in a pure resistive circuit is not constant, it is fluctuating. However, it is always positive. This is so because the instantaneous values of voltage and current are always either both positive or negative and, therefore, the product is always positive. This means that the voltage source constantly delivers power to the circuit and the circuit consumes it.

**Example 5.1. The element of 500 W electric press is designed for use on 200 V, 50 Hz supply. What value of resistance is needed to be connected in series in order that press can be operated at 230 V ac supply?**

[*Pb. Technical Univ. Basic Electrical and Electronics Engineering December-2004*]

**Solution:** Current drawn by electric press when operated on rated voltage.

$$I = \frac{P}{V} = \frac{500}{200} = 2.5 \text{ A}$$

Voltage drop across resistance R connected in series with the electric press,

$$V_R = 230 - 200 = 30 \text{ V}$$

$$\text{Resistance, } R = \frac{V_R}{I} = \frac{30}{2.5} = 12\ \Omega \quad \textbf{Ans.}$$

**Example 5.2. How much power is represented by a circuit in which the voltage and current equations are $e = 160 \sin 314t$ and $i = 42.5 \sin 314t$?**

[*G.G.S.I.P. Univ. Delhi Electrical Science May-2008*]

**Solution:** Instantaneous power represented by the circuit

$$p = ei = 160 \sin 314t \times 42.5 \sin 314t$$

$$= 6{,}800 \sin^2 314t = 3{,}400(1 - \cos 2 \times 314t) = 3{,}400(1 - \cos 628t) \quad \textbf{Ans.}$$

$$\because \sin^2 \omega t = \frac{1 - \cos 2\,\omega t}{2}$$

Average power of circuit = 3,400 W **Ans.**

**Example 5.3. A 250 V (rms), 50 Hz voltage is applied across a circuit consisting of a pure (non-inductive) resistance of 20 Ω. Determine (*i*) the current flowing through the circuit and (*ii*) power absorbed by the circuit. Give the expressions for the voltage and current.**

**Solution:** (*i*) Current flowing through the circuit, $I = \frac{V}{R} = \frac{250}{20} = 12.5$ A **Ans.**

(*ii*) Power absorbed, $P = VI = 250 \times 12.5 = 3{,}125$ W or 3.125 kW **Ans.**

$$\text{Peak value of applied voltage, } V_{max} = \sqrt{2}\, V_{rms} = \sqrt{2} \times 250 = 353.6 \text{ V}$$

$$\text{Peak value of current, } I_{max} = \sqrt{2}\, I_{rms} = \sqrt{2} \times 12.5 = 17.68 \text{ A}$$

$$\text{Expression for voltage, } v = V_{max} \sin 2\pi f t$$

Assuming that at $t = 0$, $v = 0$ and is increasing in positive direction

$$\text{so } v = 353.6 \sin 2\pi \times 50\, t = 353.6 \sin 314\, t \textbf{ Ans.}$$

$$\text{Expression for current, } i = I_{max} \sin 2\pi f t = 17.68 \sin 314\, t \textbf{ Ans.}$$

**Example 5.4. A 100 ohm resistance is carrying a sinusoidal current given by 3 cos ω*t*. Determine: (*i*) instantaneous power taken by resistance, (*ii*) average power.**

[*U.P. Technical Univ. Electrical Engineering First Semester 2003-04*]

**Solution:** (*i*) Instantaneous power taken by the resistance,

$$p = v\, i = i\, R \times i = 100 \times 3 \cos \omega t \times 3 \cos \omega t = 900 \cos^2 \omega t = 450\,(1 + \cos 2\omega t) \textbf{ Ans.}$$

(*ii*) Average power = 450 watts **Ans.**

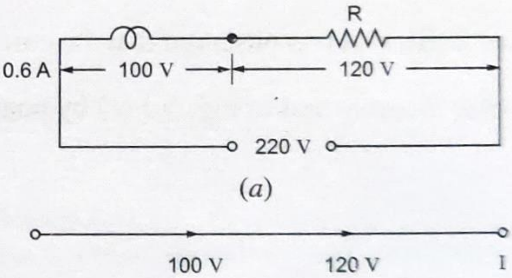
**Example 5.5. A 100 V, 60 watt bulb is to be operated from 220 V supply. What resistance must be connected in series with the bulb to glow normally?** [*Mahamaya Technical Univ. Electrical Engineering First Semester 2011-12*]

**Solution:** Compared with the resistance of the filament of a lamp its inductance is negligible. As such a lamp is considered to be a non-inductive resistance.

Rated power of lamp, P = 60 W

Rated voltage of lamp, V = 100 V

$$\text{Current drawn by lamp, } I = \frac{P}{V} = \frac{60}{100} = 0.6 \text{ A}$$




(*a*)

(*b*)

**Fig. 5.2**

Let a non-inductive resistance of R Ω be connected in series with the lamp, as shown in Fig. 5.2 (*a*).

Since in this case voltage across the lamp and that across the resistance, $V_R$ are in phase, their arithmetic sum is equal to applied voltage i.e.,

$$V_R + 100 = 220$$

$$\text{or } V_R = 220 - 100 = 120 \text{ V}$$

$$\text{Resistance, } R = \frac{V_R}{I} = \frac{120}{0.6} = 200\ \Omega \textbf{ Ans.}$$

## 5.3 PURE INDUCTIVE CIRCUITS

An inductive circuit is a coil with or without an iron core having negligible resistance. Practically pure inductance can never be had as the inductive coil has always small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance and is known as a *choke coil.**

When an alternating voltage is applied to a pure inductive coil, an emf, known as self-induced emf, is induced in the coil which opposes the applied voltage. Since coil has no resistance, at every instant applied voltage has to overcome this self-induced emf only.

Let the applied voltage $v = V_{max} \sin \omega t$

and self-inductance of coil = L henry

* Inductors or choke coils are made of iron core because large valued flux densities can be produced in iron cores and so inductances of large values can be had. Air-cored inductors become too much bulky to give an inductance of a given value.

Self-induced emf in the coil, $e_L = -L \dfrac{di}{dt}$

Since applied voltage at every instant is equal and opposite to the self-induced emf *i.e.* $v = -e_L$

$$\therefore\ V_{max} \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

$$\text{or} \quad di = \frac{V_{max}}{L} \sin \omega t\, dt$$

Integrating both sides we get, $i = \dfrac{V_{max}}{L} \displaystyle\int \sin \omega t\, dt = \dfrac{V_{max}}{\omega L}(-\cos \omega t) + A$

where A is a constant of integration, which is found to be zero from initial conditions

$$i.e. \quad i = \frac{-V_{max}}{\omega L} \cos \omega t = \frac{V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

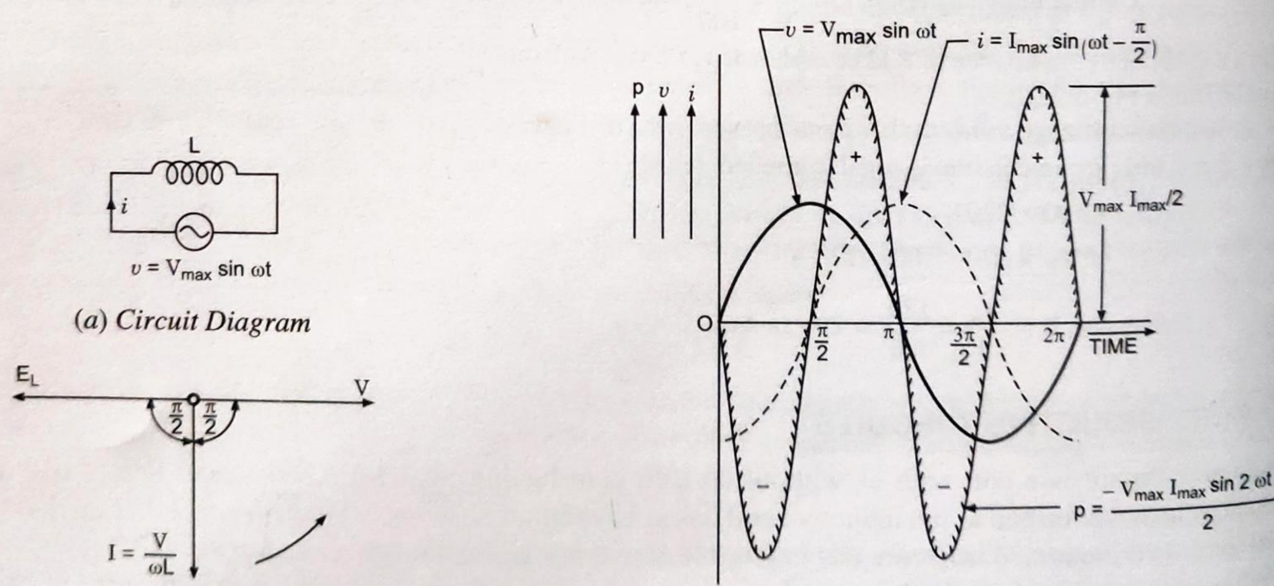
Current will be maximum when $\sin\left(\omega t - \dfrac{\pi}{2}\right) = 1$, hence, maximum value of current,

$$I_{max} = \frac{V_{max}}{\omega L}$$

and instantaneous current may be expressed as

$$i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

From the expressions of instantaneous applied voltage and instantaneous current flowing through a pure inductive coil it is observed that the current lags behind the applied voltage by $\dfrac{\pi}{2}$, as shown in Fig. 5.3 (*b*) by wave diagram and in Fig. 5.3 (*c*) by phasor diagram.




(*a*) *Circuit Diagram*

(*c*) *Phasor Diagram*

(*b*) *Wave Diagram*

**Fig. 5.3** *Pure Inductive Circuit*

**5.3.1. Inductive Reactance.** $\omega L$ in the expression $I_{max} = \dfrac{V_{max}}{\omega L}$ is known as *inductive reactance* and is denoted by $X_L$ *i.e.* $X_L = \omega L$

If L is in henry and $\omega$ is in radians per second then $X_L$ will be in ohms.

### 5.3.2. Power in Pure Inductive Circuit

Instantaneous power, $p = v \times i = V_{max} \sin \omega t \, I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$

or $p = -V_{max} I_{max} \sin \omega t \cos \omega t = -\frac{V_{max} I_{max}}{2} \sin 2\omega t$

The power measured by wattmeter is the average value of $p$ which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. *Hence, in a pure inductive circuit power absorbed is zero.*

Physically the above fact can be explained as below:

During the second quarter of a cycle the current and the magnetic flux of the coil increases and the coil draws power from the supply source to build up the magnetic field (the power drawn is positive and the energy drawn by the coil from the supply source is represented by the area between the curve $p$ and the time axis). The energy stored in the magnetic field during build up is given as $W_{max} = \frac{1}{2} LI^2_{max}$.

In the next quarter the current decreases. The emf of self-induction will, however, tend to oppose its decrease. The coil acts as a generator of electrical energy, returning the stored energy in the magnetic field to the supply source (now the power drawn by the coil is negative and the curve $p$ lies below the time axis).

The chain of events repeats itself during the next half cycles. Thus, a proportion of power is continually exchanged between the field and the inductive circuit and the power consumed by a pure inductive coil is zero.

**Example 5.6. A coil of inductance 0.05 H is connected to a supply of 220 V, 50 Hz. Calculate the current in the coil.**

*[Pb. Technical Univ. Basic Electrical and Electronics Engineering May 2007]*

**Solution:** Inductive reactance of the coil $X_L = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$

Current in coil, $I = \frac{V}{X_L} = \frac{220}{15.7} = 14.0$ A **Ans.**

**Example 5.7. The voltage and current through a circuit element are**

**$v = 100 \sin (314\,t + 45°)$ volts**

**$i = 10 \sin (314\,t + 315°)$ amperes.**

**(*i*) Identify the circuit element (*ii*) Find the value (*iii*) Obtain expression for power.**

*[U.P. Technical Univ. Electrical Engineering September 2001]*

**Solution :** $v = 100 (\sin 314\,t + 45°)$ volts

$i = 10 \sin (314\,t + 315°) = 10 \sin (314\,t + 315° - 360°) = 10 \sin (314\,t - 45°)$ amperes

(*i*) From the equation for voltage and current it is observed that the circuit current $i$ lags behind the applied voltage $v$ by 90°. It means the circuit element is an *inductor*. **Ans.**

(*ii*) Inductive reactance of the inductor, $X_L = \frac{V_{rms}}{I_{rms}} = \frac{100/\sqrt{2}}{10/\sqrt{2}} = 10\ \Omega$ **Ans.**

and $L = \frac{X_L}{\omega} = \frac{10}{314} = 0.0318$ H or 31.8 mH **Ans.**

(*iii*) Expression for power is given as, $p = \frac{-V_{max} I_{max}}{2} \sin 2\omega t = \frac{-100 \times 10}{2} \sin (2 \times 314\,t)$

$= -500 \sin 628\,t$ **Ans.**

## 5.4 PURE CAPACITIVE CIRCUITS

When a dc voltage is impressed across the plates of a perfect condenser, it will become charged to full voltage almost instantaneously. The charging current will flow only during the period of "build up" and will cease to flow as soon as the capacitor has attained the steady voltage of the source. This implies that for a direct current, a capacitor is a break in the circuit or an infinitely high resistance.

In Fig. 5.4 a sinusoidal voltage is applied to a capacitor. During the first quarter-cycle, the applied voltage increases to the peak value, and the capacitor is charged to that value. The current is maximum in the beginning of the cycle and becomes zero at the maximum value of the applied voltage, so there is a phase difference of 90° between the applied voltage and current. During the first quarter-cycle the current flows in the normal direction through the circuit; hence the current is positive.

In the second quarter-cycle, the voltage applied across the capacitor falls, the capacitor loses its charge and current flows through it against the applied voltage because the capacitor discharges into the circuit. Thus, the current is negative during the second quarter-cycle and attains a maximum value when the applied voltage is zero.

The third and fourth quarter-cycles repeat the events of the first and second, respectively, with the difference that the polarity of the applied voltage is reversed, and there are corresponding current changes. In other words, an alternating current flows in the circuit because of the charging and discharging of the capacitor. As illustrated in Figs. 5.4 (*b*) and (*c*) the current begins its cycle 90 degrees ahead of the voltage, so the current in a capacitor leads the applied voltage by 90 degrees – the opposite of the inductance current-voltage relationship.

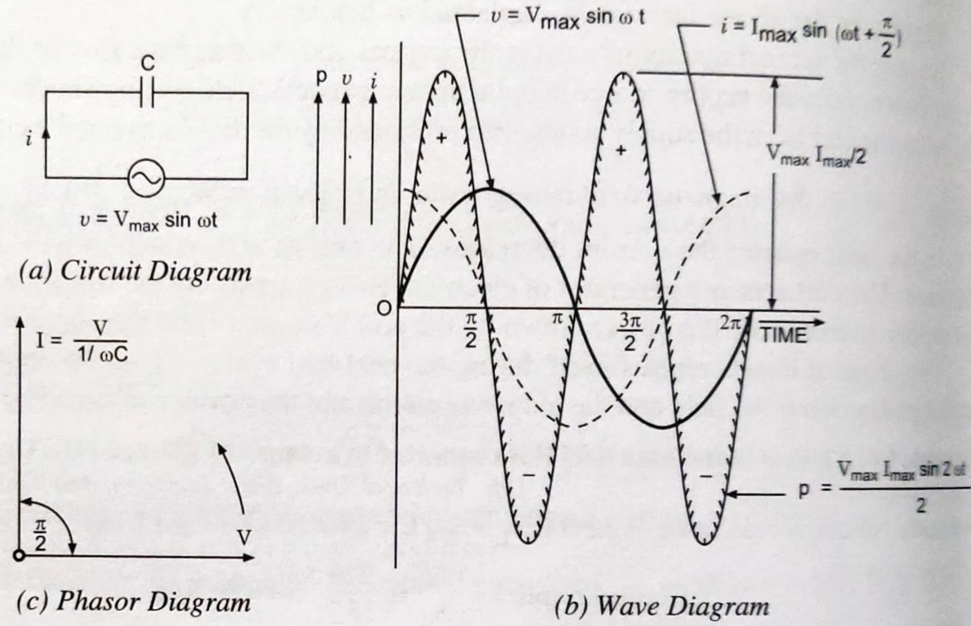



**Fig. 5.4** *Pure Capacitive Circuit*

Let an alternating voltage represented by $v = V_{max} \sin \omega t$ be applied across a capacitor of capacitance C farads. The expression for instantaneous charge is given as

$$q = C\,V_{max} \sin \omega t$$

Since the capacitor current is equal to the rate of change of charge, the capacitor current may be obtained by differentiating the above equation

$$i = \frac{dq}{dt} = \frac{d}{dt}[C\,V_{max} \sin \omega t] = \omega\, CV_{max} \cos \omega t = \frac{V_{max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Current is maximum when $t = 0$

$$\therefore\ I_{max} = \frac{V_{max}}{1/\omega C}$$

Substituting $\frac{V_{max}}{1/\omega C} = I_{max}$ in the above equation for instantaneous current, we get

$$i = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the equations of instantaneous applied voltage and instantaneous current flowing through capacitance, it is observed that the current leads the applied voltage by $\frac{\pi}{2}$, as shown in Figs. 5.4 (*b*) and (*c*) by wave and phasor diagrams respectively.

**5.4.1. Capacitive Reactance.** $\frac{1}{\omega C}$ in the expression $I_{max} = \frac{V_{max}}{1/\omega C}$ is known as *capacitive reactance* and is denoted by $X_C$ *i.e.* $X_C = \frac{1}{\omega C}$

If C is in farads and $\omega$ is in radians/s, then $X_C$ will be in ohms.

**5.4.2. Power in Pure Capacitive Circuit**

Instantaneous power,

$$p = vi = V_{max} \sin \omega t . I_{max} \sin\left(\omega t + \frac{\pi}{2}\right) = V_{max} I_{max} \sin \omega t \cos \omega t = \frac{V_{max} I_{max}}{2} \sin 2\omega t$$

Average power, $P = \frac{V_{max} I_{max}}{2} \times$ average of sin 2 $\omega t$ over a complete cycle = 0.

Hence, power absorbed in a pure capacitive circuit is zero. The same is shown graphically in Fig. 5.4(*b*). The energy taken from the supply circuit is stored in the capacitor during the first quarter-cycle and returned during the next.

The energy stored by a capacitor at maximum voltage across its plates is given by the expression

$$W_C = \frac{1}{2} CV^2_{max}$$

This can be realized when it is recalled that no heat is produced and no work is done while current is flowing through a capacitor. As a matter of fact, in commercial capacitors, there is a slight energy loss in the dielectric in addition to a minute $I^2R$ loss due to flow of current over the plates having definite ohmic resistance. The power curve is a sine wave of double the supply frequency. Although it raises the power factor from zero to 0.002 or even a little more, but for ordinary purposes the power factor is taken to be zero. Obviously the phase angle due to dielectric and ohmic losses decreases slightly.

**Example 5.8. A capacitor of 100 μF is connected across a 200 V, 50 Hz single-phase supply. Calculate (*i*) the reactance of the capacitor (*ii*) rms value of the current and (*iii*) maximum current.**

[*Gujarat Technological Univ. Elements of Electrical Engineering, January-2011*]

**Solution:** Capacitance, C = 100 μF = $1 \times 10^{-4}$F

Supply voltage, V = 200 V

Supply frequency, $f$ = 50 Hz

(*i*) Reactance of capacitor, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 1 \times 10^{-4}} = 31.83\ \Omega$ **Ans.**

(*ii*) RMS value of current, $I = \frac{V(\text{rms value})}{X_C} = \frac{200}{31.83} = 6.283$ A **Ans.**

(*iii*) Maximum value of current, $I_{max} = \sqrt{2} \times I_{rms} = \sqrt{2} \times 6.283 = 8.886$ A **Ans.**

**Example 5.9. A 50-Hz voltage of 230 V effective value is impressed on a capacitance of 26.5 μF. (*a*) Write the time equations for the voltage and the resulting current. Let the zero axis of the voltage wave be at $t$ = 0. (*b*) Show the voltage and current on a time diagram. (*c*) Show the voltage and current on a phasor diagram. (*d*) Find the maximum energy stored in the capacitor. Find the relative heating effects of two current waves of equal peak value, the one sinusoidal and the other rectangular in waveform.**

**Solution:** (*a*) The time equation for the applied voltage is given by

$$v(t) = V_{max} \sin(2\pi f t + \theta) = 230\sqrt{2} \sin(2\pi \times 50 t + 0°) = 325 \sin 314 t \ \textbf{Ans.}$$

$$\because V_{max} = 230\sqrt{2}\ \text{V}; f = 50 \text{ Hz and } \theta = 0°$$

Capacitive reactance, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 26.5 \times 10^{-6}}\ \Omega = 120\ \Omega$

Maximum value of current flowing through the capacitor,

$$I_{max} = \frac{V_{max}}{X_C} = \frac{325}{120} = 2.71 \text{ A}$$

Phase angle $\phi = 90°$ (lead)

So the time equation for the resulting current is given by

$$i\,(t) = 2.71 \sin (314\,t + 90°) = 2.71 \cos 314\,t \quad \textbf{Ans.}$$

(*b*) Voltage and current on a time diagram are shown in Fig. 5.4(*b*).

(*c*) Voltage and current on a phasor diagram are shown in Fig. 5.4(*c*).

(*d*) Maximum energy stored in the capacitor,

$$W_C = \frac{1}{2} C\,(V_{max})^2 = \frac{1}{2} \times 26.5 \times 10^{-6} \times \left(230\sqrt{2}\right)^2 = 1.4 \text{ J} \quad \textbf{Ans.}$$

For equal peak value, say $I_{max}$,

RMS value of current in case of rectangular waveform, $I_{rms1} = I_{max}$

RMS value of current in case of sinusoidal waveform, $I_{rms2} = \frac{I_{max}}{\sqrt{2}}$

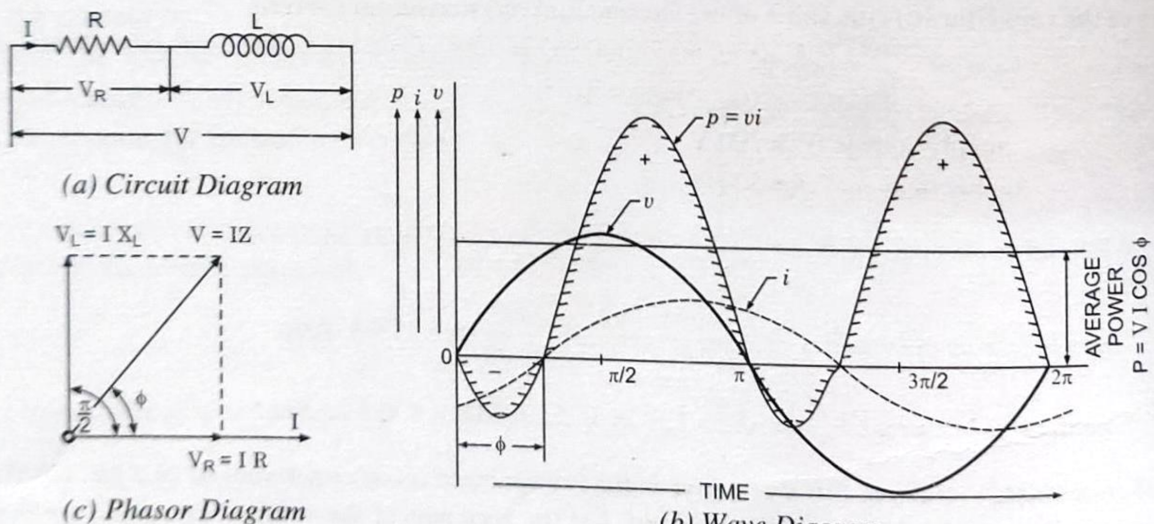
The ratio of heating effects of two current waves is given by

$$\frac{H_{rectangular}}{H_{sinusoidal}} = \frac{(I_{rms_1})^2 \times R}{(I_{rms_2})^2 \times R} = \frac{I_{max}^2\ R}{\left(\frac{I_{max}}{\sqrt{2}}\right)^2 R} = 2 \quad \textbf{Ans.}$$

## 5.5 RESISTANCE-INDUCTANCE (R-L) SERIES CIRCUIT

This is the case most generally met within practice, nearly all circuits contain both resistance and inductance.

Consider an ac circuit consisting of resistance of R ohms and inductance of L henrys connected in series, as shown in Fig. 5.5(*a*).




(*a*) *Circuit Diagram*

(*c*) *Phasor Diagram*

(*b*) *Wave Diagram*

**Fig. 5.5** *Resistance and Inductance in Series*

Let the supply frequency be *f* and current flowing through the circuit be of I amperes (rms value).

Now voltage drop across resistance, $V_R = I\,R$ in phase with the current.

Voltage drop across inductance,

$$V_L = I\,X_L = I\omega L \text{ leading I by } \pi/2 \text{ radians, as shown in Fig 5.5 } (c)$$

The applied voltage, being equal to the phasor sum of $V_R$ and $V_L$, will be given by the diagonal of the parallelogram.

$$\text{Applied voltage, } V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2} = IZ$$

where $X_L = \omega L = 2\pi f L$

Quantity $\sqrt{R^2 + X_L^2}$ is known as *impedance*, denoted by Z and is expressed in ohms.

From phasor diagram it is also evident that the current lags behind the applied voltage V by angle $\phi$, which is given by

$$\text{Tan}\,\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\text{or} \quad \phi = \text{Tan}^{-1}\frac{X_L}{R}$$

Since $X_L$ and R are known, the value of phase angle $\phi$ can be computed. If the applied voltage

$v = V_{max} \sin \omega t$, then expression for the circuit current will be

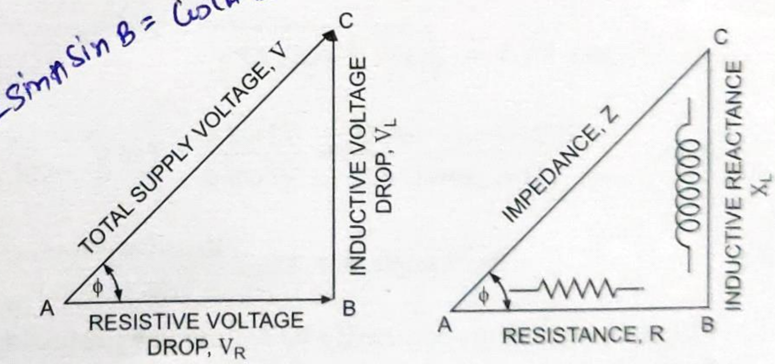
$i = I_{max} \sin (\omega t - \phi)$

where $I_{max} = \frac{V_{max}}{Z}$ and $\phi = \text{Tan}^{-1}\frac{X_L}{R}$.

**5.5.1. Impedance Triangle.** If a triangle ABC is drawn so that $AB = \frac{V_R}{I} = R$, $BC = \frac{V_L}{I} = X_L$ and $AC = \frac{V}{I} = Z$, it is a triangle similar to the voltage triangle shown in Fig. 5.6 (*a*). Such a triangle is called an *impedance triangle*, which is most useful in letting one see at a glance how R, X, and Z are related to each other. The angle between Z and R sides of the impedance triangle is known as *phase angle of the circuit* and cos of this angle is known as *power factor of the circuit*.

$$\text{Power factor} = \text{Cos}\,\phi = \frac{R}{Z}$$

2 sin A sin B = Cos(A−B) − Cos(A+B)




(*a*) *Voltage Triangle* (*b*) *Impedance Triangle*

**Fig. 5.6**

**5.5.2. Power in Resistance-Inductance (R-L) Circuit.**

Instantaneous power,

$$p = vi = V_{max} \sin \omega t \times I_{max} \sin (\omega t - \phi)$$

$$= \frac{1}{2} V_{max} I_{max} [2 \sin \omega t \,.\, \sin (\omega t - \phi)]$$

$$= \frac{1}{2} V_{max} I_{max} [\cos \phi - \cos (2\omega t - \phi)]$$

$$= \frac{1}{2} V_{max} I_{max} \cos \phi - \frac{1}{2} V_{max} I_{max} \cos (2\omega t - \phi)$$

Since average value of pulsating component $\frac{1}{2} V_{max} I_{max} \cos (2\omega t - \phi)$ over a complete cycle is zero,

average power of the circuit, P = Constant component, $\frac{1}{2} V_{max} I_{max} \cos \phi = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos \phi = VI \cos \phi$

where V and I are the rms values of voltage and current and $\phi$ is the phase angle between applied voltage V and circuit current I.

**Alternatively** power consumed by the circuit,

P = Power consumed by resistance $\qquad \because$ Power consumed by inductance is zero

$= I^2 R = I (IR) = \frac{V}{Z} . IR = VI \frac{R}{Z} = VI \cos \phi \qquad$ Since from impedance triangle $\cos \phi = \frac{R}{Z}$

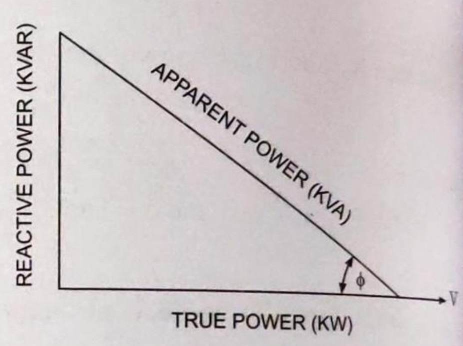
*So the power in an a c circuit is given by the product of rms values of current and voltage and cosine of the phase angle between voltage and current.* Cosine of the phase angle between the voltage and current, $\cos\phi$ is known as the *power factor* of the circuit, and is equal to $\frac{R}{Z}$ which is obvious from impedance triangle [Fig. 5.6 (*b*)].

## 5.6 APPARENT POWER, TRUE POWER, REACTIVE POWER AND POWER FACTOR

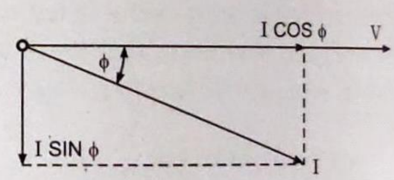
The product of rms values of current and voltage, VI is called the *apparent power* and is measured in volt-amperes or kilovolt amperes (kVA).

The *true power* in an a c circuit is obtained by multiplying the apparent power by the power factor and is expressed in watts or kilowatts (kW).

The product of apparent power, VI and the sine of the angle between voltage and current, $\sin\phi$ is called the *reactive power*. This is also known as *wattless power* and is expressed in reactive volt-amperes or kilovolt amperes reactive (kVAR).




(*a*) *Power Triangle*




(*b*)

**Fig. 5.7**

*i.e.* $$\text{Apparent power, } S = VI \text{ volt-amperes or } \frac{VI}{1{,}000} \text{ kVA}$$

$$\text{True power, } P = VI\cos\phi \text{ watts or } \frac{VI\cos\phi}{1{,}000} \text{ kW}$$

$$\text{Reactive power, } Q = VI\sin\phi \text{ VAR or } \frac{VI\sin\phi}{1{,}000} \text{ kVAR}$$

$$\text{and kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2}$$

Also, $$\frac{\text{Reactive power, Q}}{\text{True power, P}} = \frac{VI\sin\phi}{VI\cos\phi} = \text{Tan}\,\phi$$

and $$\text{Phase angle, } \phi = \text{Tan}^{-1}\frac{\text{Reactive power, Q}}{\text{True power, P}}$$

The above relations can easily be followed by referring to the power diagram shown in Fig. 5.7 (*a*).

Power factor may be defined as

(*i*) cosine of the phase angle between voltage and current or
(*ii*) the ratio of the resistance to impedance or
(*iii*) the ratio of true power to apparent power.

The power factor can never be greater than unity. The power factor is expressed either as fraction or as a percentage. It is usual practice to attach the word 'lagging' or 'leading' with the numerical value of power factor to signify whether the current lags behind or leads the voltage.

**5.6.1. Active Component of Current.** The current component which is in phase with circuit voltage (*i.e.* $I\cos\phi$) and contributes to active or true power of the circuit is called the *active* (*wattful* or *in-phase*) *component* of current.

**5.6.2. Reactive Component of Current.** The current component which is in quadrature (or 90° out of phase) to circuit voltage (*i.e.* $I\sin\phi$) and contributes to reactive power of the circuit, is called the *reactive* (or *wattless*) *component* of current.

## 5.7 Q-FACTOR OF A COIL

Reciprocal of power factor is known as Q-factor of the coil. It is also called the *quality factor* or *figure of merit* of a coil.

$$\text{Mathematically Q-factor} = \frac{1}{\text{Power factor}} = \frac{1}{\text{Cos }\phi} = \frac{Z}{R}$$

If R is very small in comparison to inductive reactance $X_L$, the

$$\text{Q-factor} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\text{Also Q} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

**Example 5.10. An alternating current of 1.5 A flows in a circuit when applied voltage is 300 V. The power consumed is 225 W. Find resistance and reactance of the circuit.**

*[U.P. Technical Univ. Electrical Engineering First Semester 2004-05]*

**Solution:**

$$\text{Circuit resistance, R} = \frac{\text{Power consumed}}{I^2} = \frac{225}{(1.5)^2} = 100\ \Omega \quad \textbf{Ans.}$$

$$\text{Circuit impedance, Z} = \frac{V}{I} = \frac{300}{1.5} = 200\ \Omega$$

$$\text{Circuit reactance, X} = \sqrt{Z^2 - R^2} = \sqrt{(200)^2 - (100)^2} = 173.2\ \Omega \textbf{ Ans.}$$

**Example 5.11. V = $\sqrt{2} \times 200$ cos 500*t*, $P_{av}$ = 250 W, power factor = 0.7 lagging. Calculate the reactive power of the system.**

*[G.B. Technical Univ. Electrical Engineering Even Semester 2012-13]*

**Solution:**

$$\text{RMS value of supply voltage, V} = \frac{V_{max}}{\sqrt{2}} = \frac{\sqrt{2} \times 200}{\sqrt{2}} = 200\text{ V}$$

$$\text{Power factor} = 0.7 \text{ lagging}$$

$$\text{Phase angle, } \phi = \text{Cos}^{-1}\ 0.7 = 45.573^\circ$$

$$\text{Sin }\phi = \text{Sin } 45.573^\circ = 0.714$$

$$\text{Average power, P} = 250\text{ W}$$

$$\text{Current drawn, I} = \frac{P}{V \cos \phi} = \frac{250}{200 \times 0.7} = 1.7857\text{ A}$$

$$\text{Reactive power of the system, Q} = VI \sin \phi = 200 \times 1.7857 \times 0.714 = 255 \text{ VAR (inductive)} \quad \textbf{Ans.}$$

**Example 5.12. A resistance and inductance are connected in series across a voltage *v* = 283 sin 314*t*. An expression of current is found to be *i* = 4 sin (314*t* – 45°). Find the values of resistance, inductance and power factor.**

*[G.B. Technical Univ. Electrical Engineering Even Semester 2012-13; U.P. Technical Univ. Basic Electrical Engineering Second Semester 2013-14]*

**Solution:**

$$\text{Supply frequency, } f = \frac{\text{Coefficient of time, } t}{2\pi} = \frac{314}{2\pi} = 50\text{ Hz}$$

$$\text{RMS value of applied voltage, V} = \frac{V_{max}}{\sqrt{2}} = \frac{283}{\sqrt{2}} = 200.11$$

$$\text{RMS value of current, I} = \frac{I_{max}}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828\text{ A}$$

$$\text{Phase angle, } \phi = 45^\circ \text{ (lagging)}$$

$$\text{Impedance of the circuit, Z} = \frac{V}{I} = \frac{200.11}{2.828} = 70.75\ \Omega$$

$$\text{Resistance of the circuit, R} = Z \cos \phi = 70.75 \cos 45^\circ = 50\ \Omega \quad \textbf{Ans.}$$

Reactance of the circuit, $X_L = Z \sin\phi = 70.75 \sin 45° = 50\ \Omega$

$$\text{Inductance, } L = \frac{X_L}{2\pi f} = \frac{50}{2\pi \times 50} = 0.159 \text{ H} \quad \textbf{Ans.}$$

Power factor = Cos $\phi$ = Cos 45° = 0.707 (lagging) **Ans.**

**Example 5.13. If load draws a current of 10 A at 0.8 pf lagging when connected to 100 V supply, calculate the value of real, reactive and apparent powers. Also find out the resistance of the load.**

*[G.B. Technical Univ. Electrical Engineering Second Semester 2011-12]*

**Solution:**

Supply voltage, V = 100 V

Current, I = 10 A

Power factor, cos $\phi$ = 0.8 (lagging)

Phase angle, $\phi = \text{Cos}^{-1}\ 0.8 = 36.87°$ (lagging)

Real power, $P = VI \cos\phi = 100 \times 10 \times 0.8 = 800$ W **Ans.**

Reactive power, $Q = VI \sin\phi = 100 \times 10 \sin(-36.87°) = 100 \times 10 \times 0.6 = 600$ VAR (lagging) **Ans.**

$$\text{Apparent power, } S = \sqrt{P^2 + Q^2} = \sqrt{800^2 + 600^2} = 1{,}000 \text{ VA} \quad \textbf{Ans.}$$

$$\text{Load impedance, } Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$$

Resistance of load, $R = Z \cos\phi = 10 \times 0.8 = 8\ \Omega$ **Ans.**

**Example 5.14. A voltage $v(t) = 170 \sin(377t + 10°)$ is applied to a circuit. It causes a steady state current to flow which is described by $i(t) = 14.14 \sin(377t - 20°)$. Determine the variation of instantaneous power. Also find the average power delivered to circuit and power factor.**

*[Pb. Technical Univ. Basic Electrical and Electronics Engineering, May-2014]*

**Solution:**

$v(t) = 170 \sin(377t + 10°)$

$i(t) = 14.14 \sin(377t - 20°)$

$V_{max} = 170$ V

$I_{max} = 14.14$ A

Phase angle, $\phi = 10 - (-20°) = 30°$ (lagging)

$$\text{Variation of instantaneous power} = \frac{1}{2} V_{max} I_{max} \cos(2\omega t - \phi) = \frac{1}{2} \times 170 \times 14.14 \cos(2 \times 377t - 30°)$$

$$= 1{,}202 \cos(754t - 30°) \quad \textbf{Ans.}$$

$$\text{Average power delivered, } P = V_{rms} \times I_{rms} \times \cos\phi = \frac{170}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \cos 30° = 1{,}041 \text{ W} \quad \textbf{Ans.}$$

Power factor = Cos 30° = 0.866 (lagging)

**Example 5.15. A coil of resistance 1,000 Ω and inductive reactance 200 Ω is connected across supply voltage of 230 V. Find the supply current.** *[Him. Technical Univ. Basic Electrical and Electronics Engineering, June-2014]*

**Solution:**

Supply voltage, V = 230 V

Resistance of coil, R = 1,000 Ω

Inductive reactance of coil, $X_L = 200\ \Omega$

$$\text{Impedance of coil, } Z = \sqrt{R^2 + X_L^2} = \sqrt{1{,}000^2 + 200^2} = 1{,}019.8\ \Omega$$

$$\text{Supply current, } I = \frac{V}{Z} = \frac{230}{1{,}019.8} = 0.2255 \text{ A} \quad \textbf{Ans.}$$

**Example 5.16. How will you obtain the current through single phase load of 2 kW with PF = 0.8 at 230 volts. Find active power, reactive power and apparent power for the given load.**

*[R.G. Technical Univ. Basic Electrical and Electronics Engineering, June-2014]*

**Solution:**

Load, $P = 2$ kW

Power factor, $\cos\phi = 0.8$

Supply voltage, $V = 230$ V

$$\sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-0.8^2} = 0.6$$

$$\text{Load current, } I = \frac{P}{V\cos\phi} = \frac{2\times 1{,}000}{230\times 0.8} = 10.87 \text{ A} \quad \textbf{Ans.}$$

Active power, $P$ = Power supplied to load in watts or kW = 2 kW **Ans.**

$$\text{Reactive power, } Q = P\tan\phi = 2\times\frac{0.6}{0.8} = 1.5 \text{ kVAR} \quad \textbf{Ans.}$$

$$\text{Apparent power, } S = \sqrt{P^2+Q^2} = \sqrt{2^2+1.5^2} = 2.5 \text{ kVA} \quad \textbf{Ans.}$$

**Example 5.17. A voltage, $e(t) = 150\sin(2\pi f)t$, 50 Hz, is applied to series circuit consisting of 10 Ω resistance, 0.0318 henry inductance. Determine: (*i*) expression for current $i(t)$, (*ii*) phase angle between voltage and current, (*iii*) power factor, (*iv*) active power consumed, (*v*) maximum value pulsating energy.**

[*Pb. Technical Univ. Basic Electrical and Electronics Engineering, May-2012*]

**Solution:**

$$e(t) = 150\sin(2\pi f)t = 150\sin(2\pi\times 50)t = 150\sin 100\pi t$$

Resistance, $R = 10\ \Omega$

Inductance, $L = 0.0318$ H

Inductive reactance, $X_L = 2\pi fL = 2\pi\times 50\times 0.0318 = 10\ \Omega$

$$\text{Phase angle, } \phi = \text{Tan}^{-1}\frac{X_L}{R} = \text{Tan}^{-1}\frac{10}{10} = \frac{\pi}{4} \text{ (lagging)}$$

$$\text{Circuit impedance, } Z = \sqrt{R^2+X_L^2} = \sqrt{10^2+10^2} = 14.142\ \Omega$$

$$\text{Maximum value of current, } I_{max} = \frac{E_{max}}{Z} = \frac{150}{14.142} = 10.6 \text{ A}$$

(*i*) So expression for current $i(t)$ is, $i(t) = I_{max}\sin(100\pi t - \phi) = 10.6\sin\left(100\pi t - \frac{\pi}{4}\right)$ **Ans.**

(*ii*) Phase angle between voltage and current, $\phi = \frac{\pi}{4} = 45°$ (lagging) **Ans.**

(*iii*) Power factor = $\text{Cos}\,\phi = \text{Cos}\,45° = 0.707$ (lagging) **Ans.**

(*iv*) Active power consumed $= EI\cos\phi = \frac{E_{max}}{\sqrt{2}}\times\frac{I_{max}}{\sqrt{2}}\times 0.707$

$$= \frac{150}{\sqrt{2}}\times\frac{10.6}{\sqrt{2}}\times 0.707 = 562 \text{ W} \quad \textbf{Ans.}$$

(*v*) Maximum value of pulsating energy $= \frac{1}{2}E_{max}I_{max} = \frac{1}{2}\times 150\times 10.6 = 795$ VA **Ans.**

**Example 5.18. A 50 Hz sinusoidal voltage = 311 sin ωt is applied to R-L series circuit. If the magnitude of resistance is 5 Ω and that of inductance is 0.02 H,**

**(*i*) Calculate the effective value of steady current and relative phase angle.**

**(*ii*) Obtain expression for instantaneous current.** [*Rajasthan Technical Univ. 2008*]

**Solution:**

Peak value of applied voltage, $V_{max} = 311$ V

$$\text{RMS value of applied voltage, } V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{311}{\sqrt{2}} = 219.91 \text{ V}$$

Resistance of circuit, $R = 5\ \Omega$

Inductive reactance of circuit, $X_L = 2\pi fL = 2\pi\times 50\times 0.02 = 6.28\ \Omega$

$$\text{Impedance of the circuit, } Z = \sqrt{R^2+X_L^2} = \sqrt{5^2+6.28^2} = 8.03\ \Omega$$

(i) Effective value of steady current, $I_{rms} = \frac{V_{rms}}{Z} = \frac{219.91}{8.03} = 27.387$ A **Ans.**

Phase angle, $\phi = \text{Tan}^{-1}\frac{X}{R} = \text{Tan}^{-1}\frac{6.28}{5} = 51.474°$ (lagging)

Since in R-L circuit current lags behind the applied voltage

(ii) Instantaneous current is given by, $i = I_{max}\sin(\omega t - \phi) = 27.387\sqrt{2}\sin(\omega t - 51.474°)$

$= 38.73\sin(\omega t - 51.474°)$ **Ans.**

**Example 5.19. A coil of resistance 10 Ω and inductance 0.02 H is connected in series with another coil of resistance 6 Ω and inductance 15 mH across a 230 V, 50 Hz supply. Calculate**

**(i) Impedance of the circuit.**

**(ii) Voltage drop across each coil.**

**(iii) The total power consumed by the circuit.**

*[Rajasthan Technical Univ. June 2010]*

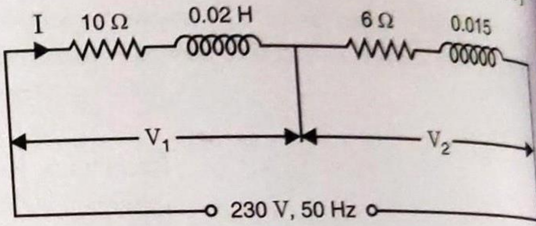



**Fig. 5.8**

**Solution:** Impedance of coil 1

$\mathbf{Z}_1 = 10 + j2\pi \times 50 \times 0.02$

$= (10 + j6.283)\ \Omega = 11.81\ \angle 32.14°\ \Omega$

$\mathbf{Z}_2 = 6 + j2\pi \times 50 \times 0.015$

$= (6 + j4.712)\ \Omega = 7.629\ \angle 38.14°\ \Omega$

(i) Impedance of whole circuit, $\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2$

$= (10 + j6.283) + (6 + j4.712) = (16 + j10.995)\Omega = 19.414\ \angle 34.5°\ \Omega$ **Ans.**

Current through the circuit, $I = \frac{V}{Z} = \frac{230}{19.414} = 11.847$ A

(ii) Voltage drop across coil 1, $V_1 = IZ_1 = 11.847 \times 11.81 = 139.9$ V **Ans.**

Voltage drop across coil 2, $V_2 = IZ_2 = 11.847 \times 7.629 = 90.38$ V **Ans.**

(iii) Total power consumed by the circuit,

$P = I^2(R_1 + R_2) = 11.847^2 \times (10 + 6) = 2{,}246$ W or 2.246 kW **Ans.**

Total power consumed can also be determined from the relation

$P = VI\cos\phi = 230 \times 11.847 \times \cos 34.5°$

$= 2{,}246$ W or 2.246 kW, the same as before.

**Example 5.20. Two coils A and B are connected in series across 240 V, 50 Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR. Find the inductance of A and the resistance of B. Calculate the voltage across each coil.**

*[Rajasthan Univ. 2002; MNIT 2002; Hyderabad Univ. 1991; Rajasthan Technical Univ. December-2013]*

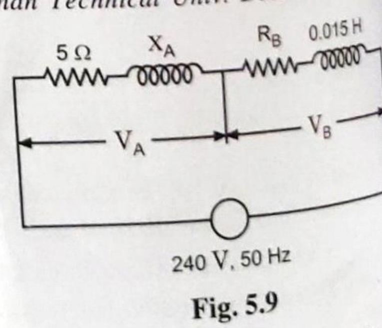



**Fig. 5.9**

**Solution:** Input true power, P = 3 kW

Input reactive power, Q = 2 kVAR

Input apparent power, $S = \sqrt{P^2 + Q^2}$

$= \sqrt{3^2 + 2^2} = 3.606$ kVA

Circuit current, $I = \frac{S \times 1{,}000}{V} = \frac{3.606 \times 1{,}000}{240} = 15.025$ A

Total resistance of the circuit, $R_T = R_A + R_B = \frac{P}{I^2} = \frac{3 \times 1{,}000}{15.025^2} = 13.29\ \Omega$ $\because P = I^2R$

Resistance of coil B, $R_B = R_T - R_A = 13.29 - 5 = 8.29\ \Omega$ **Ans.**

Impedance of the circuit, $Z = \frac{V}{I} = \frac{240}{15.025} = 15.97\ \Omega$

Total inductive reactance of the circuit, $X_{LT} = \sqrt{Z^2 - R_T^2} = \sqrt{(15.97)^2 - (13.29)^2} = 8.86\ \Omega$

Inductive reactance of coil A, $X_A = X_{LT} - X_B = 8.86 - 2\pi \times 50 \times 0.015 = 8.86 - 4.71 = 4.15\ \Omega$

Inductance of coil A, $L_A = \frac{X_A}{2\pi f} = \frac{4.15}{2\pi \times 50} = 0.0132$ H **Ans.**

Voltage across coil A, $V_A = I \times Z_A = I \times \sqrt{R_A^2 + X_A^2} = 15.025 \times \sqrt{5^2 + 4.15^2} = 97.63$ V **Ans.**

Voltage across coil B, $V_B = I \times Z_B = I \times \sqrt{R_B^2 + X_B^2} = 15.025 \times \sqrt{8.29^2 + 4.71^2} = 143.25$ V **Ans.**

**Example 5.21. For an ac circuit, the voltage and current are given by:**

$$v = 200 \sin 377\,t \text{ volts, and}$$

$$i = 8 \sin (377\,t - 30°) \text{ amperes.}$$

**Determine the power factor, true power, apparent power and reactive power of the circuit. Also cross verify from power triangle.** [*Mahamaya Technical Univ. Electrical Engineering First Semester 2011-12*]

**Solution:**

$$v = 200 \sin 377t \text{ V}$$

$$i = 8 \sin (377t - 30°) \text{ A}$$

Phase angle, $\phi = 30°$ (lagging)

RMS value of applied voltage, $V = \frac{V_{max}}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.42$ V

RMS value of current, $I = \frac{I_{max}}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 5.657$ A

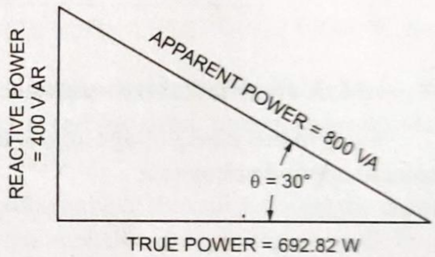



**Fig. 5.10**

Power factor = $\cos \phi = \cos 30° = 0.866$ (lagging) **Ans.**

True power, $P = VI \cos \phi = 141.42 \times 5.657 \times 0.866 = 692.82$ W **Ans.**

Apparent power, $S = VI = 141.42 \times 5.657 = 800$ VA **Ans.**

Reactive power, $Q = VI \sin \phi = 141.42 \times 5.657 \times \sin 30° = 400$ VAR (lagging) **Ans.**

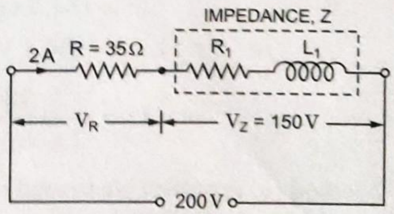
Power triangle is shown in Fig. 5.10.

From power triangle $S = \sqrt{P^2 + Q^2} = \sqrt{(692.82)^2 + 400^2} = 800$ VA, same value as determined above

Phase angle, $\phi = \tan^{-1} \frac{Q}{P} = \tan^{-1} \frac{400}{692.8} = 30°$

**Example 5.22. A circuit consists of R = 35 Ω in series with unknown impedance Z, for a sinusoid current of 2 A, the observed voltages are 200 V across R and Z together and 150 V across impedance Z. Find the impedance Z.**

[*G.G.S.I.P. Univ. Delhi Electrical Science May/June-2009*]




(*a*)

**Solution:** Voltage across resistance R,

$$V_R = IR = 2 \times 35 = 70 \text{ V}$$

Voltage across impedance, Z,

$$V_Z = 150 \text{ V}$$

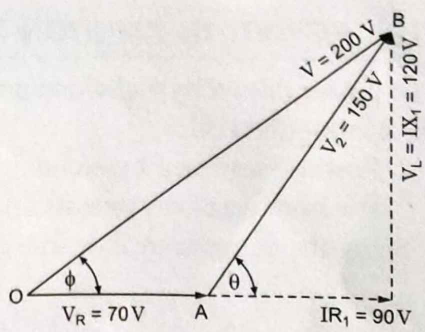
Supply voltage, V = 200 V

From Δ OAB shown in Fig. 5.11(*b*), we have

$$OB^2 = OA^2 + AB^2 + 2 \times OA \times AB \times \cos \theta$$

$$\text{or } \cos \theta = \frac{OB^2 - OA^2 - AB^2}{2 \times OA \times AB} = \frac{200^2 - 70^2 - 150^2}{2 \times 70 \times 150}$$

$$= 0.6$$




(*b*)

**Fig. 5.11**

Impedance, $Z = \frac{V_Z}{I} = \frac{150}{2} = 75\ \Omega$

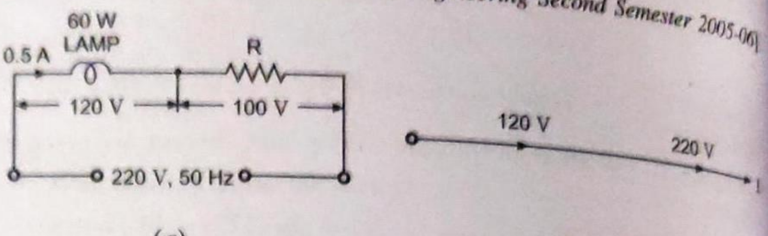
Resistance of impedance Z, $R_1 = Z \cos \theta = 75 \times 0.6 = 45\ \Omega$ **Ans.**

Reactance of impedance, $X_1 = \sqrt{Z^2 - R_1^2} = \sqrt{75^2 - 45^2} = 60\ \Omega$ **Ans.**

**Example 5.23. A 120 V, 60 W lamp is to be operated on 220 V, 50 Hz supply mains. In order that lamp should operate on correct voltage, calculate value of—**

**(a) non-inductive resistance, (b) pure inductance.** [*U.P. Technical Univ. Elec. Engineering Second Semester 2005-06*]

**Solution:** Compared with the resistance of the filament of a lamp its inductance is negligible. As such a lamp is considered to be a non-inductive resistance.




(a) (b)

**Fig. 5.12**

Rated power of lamp, P = 60 W

Rated voltage of lamp, V = 120 V

Current drawn by the lamp when operated on rated voltage

$$I = \frac{P}{V} = \frac{60}{120} = 0.5 \text{ A}$$

(a) Let a non-inductive resistance of R ohms be connected in series with the lamp, as shown in Fig. 5.12(a)

Since in this case voltage across the lamp and that across the resistance $V_R$, both are in phase, their arithmetic sum is equal to applied voltage *i.e.*

$$V_R + 120 = 220$$

$$\text{or } V_R = 220 - 120 = 100 \text{ V}$$

$$\text{Resistance, } R = \frac{V_R}{I} = \frac{100}{0.5} = 200\ \Omega \textbf{ Ans.}$$

(b) Let pure inductance of L henry be connected in series with the lamp, as shown in Fig. 5.13(a)

Since in this case voltage drop across the inductance leads the current by 90° or $\pi/2$ radians while that across the lamp will be in phase with it, therefore, from phasor diagram shown in Fig. 5.13(b) we have

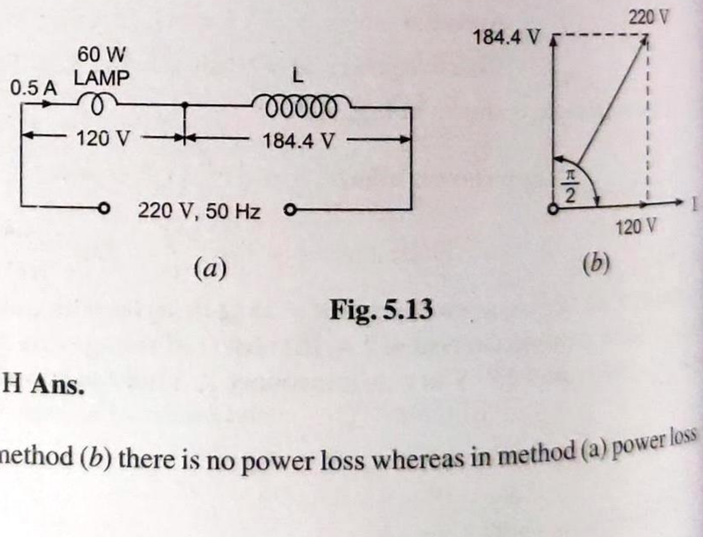
$$120^2 + V_L^2 = 220^2$$

$$\text{or } V_L = \sqrt{220^2 - 120^2}$$

$$= 184.4 \text{ volts}$$

$$\text{or } 2\pi f L I = 184.4 \text{ V}$$

$$\text{or } L = \frac{184.4}{2\pi \times 50 \times 0.5} = 1.174 \text{ H } \textbf{Ans.}$$




(a) (b)

**Fig. 5.13**

Method (b) is preferable to method (a) because in method (b) there is no power loss whereas in method (a) power loss comes out to be $0.5^2 \times 200$ *i.e.* 50 watts.

## 5.8 POWER IN AN IRON-CORED CHOKING COIL

The power drawn by a choking coil is used up to supply the iron losses in the core in addition to that used up for heating the coil.

Power consumed by a choking coil,

P = Iron losses (hysteresis and eddy current loss) $P_i$ + power loss in ohmic resistance R *i.e.* $I^2 R$

R being the dc resistance or true resistance of the coil

$$= P_i + I^2 R$$

$$\text{or } \frac{P}{I^2} = \frac{P_i}{I^2} + R$$

$\frac{P}{I^2}$ is called the *effective* resistance of the choking and it is equal to the true resistance plus resistance representing iron loss $\frac{P_i}{I^2}$

$$\therefore \quad R_{eff} = \frac{P}{I^2} = R + \frac{P_i}{I^2}$$

**Example 5.24. A direct current of 1 A passes through a series circuit of 1 Ω resistor and 1 mH inductor. What is the voltage across the circuit ? If the current was a 50 Hz current, th n what would be the voltage ?**

*[Pb. Technical Univ. Electrical Engineering May 2002]*

**Solution: For direct current**

Voltage across the circuit, $V = IR = 1 \times 1 = 1$ V **Ans.**

$\because$ in a dc circuit the flow of current is opposed by only resistance.

**For AC With 50 Hz Frequency**

Voltage across the circuit, $V = IZ = I\sqrt{R^2 + (2\pi + fL)^2} = 1 \times \sqrt{1^2 + (2\pi \times 50 \times 1 \times 10^{-3})^2} = 1.048$ V **Ans.**

**Example 5.25. A coil takes 4 A when connected to 24 V, dc supply. If this coil is connected to 40 V, 50 Hz ac supply then same amount of power is consumed. Explain why are different voltages required for same power consumption. Calculate inductance of the coil and the phase angle between voltage and current?**

**Solution:** When dc voltage is applied to the coil, there is no induced emf due to self-inductance of the coil because the current is steady and frequency is zero. Hence the applied voltage is to overcome only the resistive drop $I \cdot R$, where R is the resistance of the coil. But when ac voltage is applied, the coil offers resistance as well as reactance, the combination of the two *i.e.* impedance of the coil is certainly greater in magnitude as compared to resistance. Hence to cause the same current to flow through the coil greater voltage is required to overcome the impedance of the coil.

*When dc voltage is applied*

Applied voltage, $V_{dc} = 24$ V

Current through coil, $I_{dc} = 4$ A

Resistance of coil, $R = \frac{V_{dc}}{I_{dc}} = \frac{24}{4} = 6\ \Omega$

*When ac voltage is applied*

Applied voltage, $V_{ac} = 40$ V

Resistance of coil, R = The same as when dc is applied *i.e.* 6 Ω

Current, $I_{ac} = 4$ A

Coil impedance, $Z = \frac{V_{ac}}{I_{ac}} = \frac{40}{4} = 10\ \Omega$

Inductive reactance of coil,

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(10)^2 - (6)^2} = 8\ \Omega$$

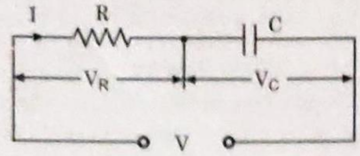
Inductance of the coil, $L = \frac{X_L}{2\pi f} = \frac{8}{2\pi \times 50} = 0.0255$ H **Ans.**

The angle between the applied pd and current,

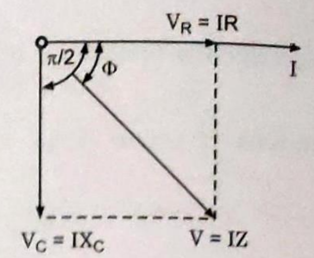
$$\phi = \text{Tan}^{-1}\frac{X_L}{R} = \text{Tan}^{-1}\frac{8}{6} = 53.13^\circ \text{ (lag)} \quad \textbf{Ans.}$$

## 5.9 RESISTANCE-CAPACITANCE (R-C) SERIES CIRCUIT

Consider an ac circuit consisting of resistance of R ohms and capacitance of C farads connected in series, as shown in Fig. 5.14(*a*).




(a) *Circuit Diagram*




(b) *Phasor Diagram*

**Fig. 5.14**

Let the supply frequency be of $f$ Hz and current flowing through the circuit be of I amperes (rms value).

Voltage drop across resistance,

$V_R = I\,R$ in phase with the current.

Voltage drop across capacitance,

$V_C = I\,X_C$ lagging behind I by $\frac{\pi}{2}$ radians or 90°, as shown in Fig. 5.14(*b*).

The applied voltage, being equal to phasor sum of $V_R$ and $V_C$, is given in magnitude by

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2} = I\,Z \qquad \text{where } Z^2 = R^2 + X_C^2$$

The applied voltage lags behind the current by an angle $\phi$

where $\tan\phi = \frac{V_C}{V_R} = \frac{I\,X_C}{I\,R} = \frac{X_C}{R} = \frac{1}{\omega RC}$

or $\phi = \text{Tan}^{-1}\frac{1}{R\omega C}$

Power factor, $\cos\phi = \frac{R}{Z}$

If instantaneous voltage is represented by

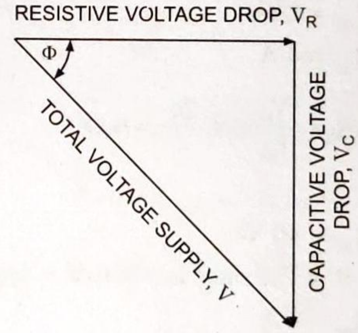
$v = V_{max}\sin\omega t$

then instantaneous current will be expressed as

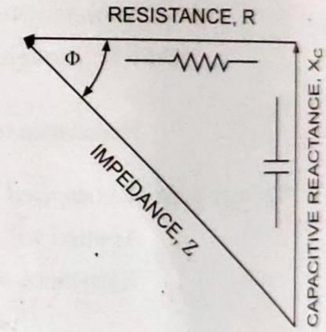
$i = I_{max}\sin(\omega t + \phi)$

and power consumed by the circuit is given by

$P = VI\cos\phi$




(a) *Voltage Triangle*




(b) *Impedance Triangle*

**Fig. 5.15**

Voltage triangle and impedance triangle are shown in Figs. 5.15(*a*) and 5.15(*b*) respectively.

**Example 5.26. In a series RC circuit, the values of R = 10 Ω and C = 25 nanofarad. A sinusoidal voltage of 1 MHz is applied and the maximum voltage across the capacitance is 2.5 volts. Find the maximum voltage across the series combination.**

[*Rajasthan Technical Univ. March 2009*]

**Solution:** Supply frequency, $f$ = 1 MHz = $10^6$ Hz

Capacitance of capacitor, $C = 25 \times 10^{-9}$ F

Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 10^6 \times 25 \times 10^{-9}} = 6.3662\ \Omega$

Impedance of the circuit, $Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 6.3662^2} = 11.85\ \Omega$

Maximum value of current through combination,

$$I_{max} = \text{Maximum value of current through capacitor}$$

$$= \frac{\text{Maximum value of voltage across capacitor}}{X_C} = \frac{2.5}{6.3662} = 0.3927 \text{ A}$$

Maximum voltage across the combination,

$$V_{max} = I_{max} Z = 0.3927 \times 11.85 = 4.65 \text{ V} \textbf{ Ans.}$$

**Example 5.27. A voltage $v = 100 \sin 314t$ is applied to a circuit consisting of a 25 Ω resistor and an 80 μF capacitor in series. Determine (*i*) peak value of current (*ii*) power factor and (*iii*) total power consumed by the circuit.**

*[V.T.U. Basic Electrical Engineering First Semester, 2014-15]*

**Solution:** Maximum value of applied voltage, $V_{max}$ = Coefficient of the sine of the time angle = 100 V

$$\text{Supply frequency}, f = \frac{\text{Coefficient of time, } t}{2\pi} = \frac{314}{2\pi} = 50 \text{ Hz}$$

$$\text{Capacitive reactance of the circuit, } X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.789 \ \Omega$$

$$\text{Impedance of circuit, } Z = \sqrt{R^2 + X_C^2} = \sqrt{25^2 + 39.789^2} = 47 \ \Omega$$

$$(i) \text{ Peak value of current, } I_{max} = \frac{V_{max}}{Z} = \frac{100}{47} = 2.128 \text{ A} \textbf{ Ans.}$$

$$(ii) \text{ Power factor of the circuit, } \cos\phi = \frac{R}{Z} = \frac{25}{47} = 0.5319 \text{ (lagging)} \textbf{ Ans.}$$

$$(iii) \text{ Total power consumed by the circuit, } P = VI\cos\phi = \frac{V_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} \times \cos\phi$$

$$= \frac{100}{\sqrt{2}} \times \frac{2.128}{\sqrt{2}} \times 0.5319 = 56.6 \text{ W} \textbf{ Ans.}$$

**Example 5.28. The voltage applied to a circuit is $v = 100 \sin(\omega t + 30°)$ and current flowing in the circuit is $i = 20 \sin(\omega t + 60°)$. Determine the impedance, resistance, reactance, power and power factor of the circuit.**

*[G.B. Technical Univ. Electrical Engineering First Semester 2011-12]*

**Solution:**

Voltage applied to a circuit, $v = 100 \sin(\omega t + 30°)$

Current flowing in the circuit, $i = 20 \sin(\omega t + 60°)$

Current leads the applied voltage by (60° – 30°) i.e. 30°

$$\text{Circuit impedance, } Z = \frac{V_{max}}{I_{max}} = \frac{100}{20} = 5 \ \Omega \textbf{ Ans.}$$

$$\text{Circuit resistance, } R = Z\cos\phi = 5\cos 30° = 4.33 \ \Omega \textbf{ Ans.}$$

$$\text{Circuit reactance, } X = Z\sin\phi = 5\sin 30° = 2.5 \ \Omega \text{ (capacitive)} \textbf{ Ans.}$$

∵ phase angle $\phi$ is leading one

$$\text{Power factor of the circuit} = \text{Cos}\,\phi = \text{Cos } 30° = 0.866 \text{ (leading)} \textbf{ Ans.}$$

$$\text{Power of the circuit, } P = V_{rms} \times I_{rms} \cos\phi = \frac{V_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} \times \cos 30°$$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times 0.866 = 866 \text{ watts} \textbf{ Ans.}$$

**Example 5.29. A supply of 400 V, 50 Hz is applied to a series R-C circuit. Find the value of C if the power absorbed by the resistor be 500 W at 150 V. What is the energy stored in capacitor?**

*[R.G.P.V. Bhopal Basic Electrical Engineering June 2005]*

**Solution:**

$$\text{Supply voltage, } V = 400 \text{ V}$$

$$\text{Power absorbed, } P = 500 \text{ W}$$

$$\text{Voltage across resistor, } V_R = 150 \text{ V}$$

$$\text{Current through R-C series circuit, } I = \frac{P}{V_R} = \frac{500}{150} = 3.333 \text{ A}$$

$$\text{Voltage drop across capacitor, } V_C = \sqrt{400^2 - 150^2} = 370.8\text{V}$$

$$\text{Capacitive reactance of the circuit, } X_C = \frac{V_C}{I} = \frac{370.8}{3.333} = 111.25\ \Omega$$

$$\text{Capacitance of circuit, } C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 111.25} = 28.6\ \mu\text{F} \textbf{ Ans.} \qquad \because\ X_C = \frac{1}{2\pi f C}$$

$$\text{Energy stored in capacitor} = \frac{1}{2} C \times V_{C\,max}^2 = \frac{1}{2} \times 28.6 \times 10^{-6} \times (370.8\sqrt{2})^2 = 3.934 \text{ J } \textbf{Ans.}$$

**Example 5.30. A metal-filament lamp, rated at 750 W, 100 V is to be connected in series with a capacitance across a 230 V, 50 Hz supply. Calculate the value of capacitance required. Draw phasor diagram.**

*[U.P. Technical Univ. Electrical Engineering July 2002; G.B. Technical Univ. First Semester 2011-12; U.P. Technical Univ. Basic Electrical Engineering First Semester 2013-14]*

**Solution:** Let a pure capacitance of C farads be connected in series with the lamp, as shown in Fig. 5.16(*a*).

Since in this case voltage drop across the capacitor lags behind the current by 90° or $\frac{\pi}{2}$ radians while that across the lamp will be in phase with it, from phasor diagram shown in Fig. 5.16(*b*), we have

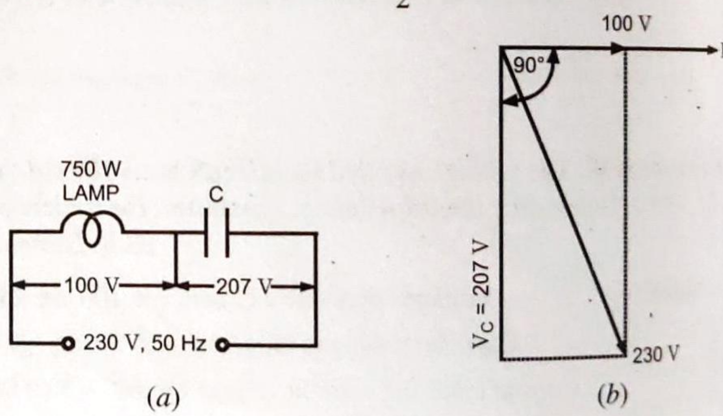
$$V_c = \sqrt{(230)^2 - (100)^2} = 207 \text{ V}$$

$$I = \frac{P}{V} = \frac{750}{100} = 7.5\text{A}$$

$$X_c = \frac{V_C}{I} = \frac{207}{7.5} = 27.6\ \Omega$$

$$\text{and } C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 27.6}$$

$$= 115.33\ \mu\text{F } \textbf{Ans.}$$




(*a*) (*b*)

**Fig. 5.16**

## 5.10 RESISTANCE-INDUCTANCE-CAPACITANCE (R-L-C) SERIES CIRCUIT

Consider an ac circuit containing resistance of R ohms, inductance of L henries and capacitance of C farads connected in series, as shown in Fig. 5.17(*a*).

Let the current flowing through the circuit be of I amperes and supply frequency be *f* Hz.

Voltage drop across resistance,

$V_R$ = IR in phase with I

Voltage drop across inductance,

$V_L = I\omega L$ leading I by $\pi/2$ radians or 90°

Voltage drop across capacitance,

$V_C = \frac{I}{\omega C}$ or $IX_C$ lagging behind I by $\pi/2$ radians or 90°

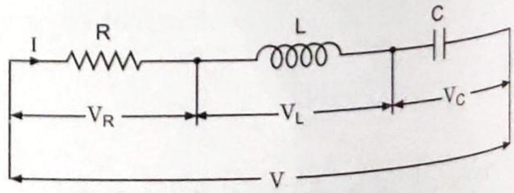



**Fig. 5.17** (*a*) *Circuit Diagram*

$V_L$ and $V_C$ are 180° out of phase with each other (or reverse in phase), therefore, when combined by parallelogram they cancel each other. The circuit can either be effectively inductive or capacitive depending upon which voltage drop ($V_L$ or $V_C$) is predominant. Let us consider the case when $V_L$ is greater than $V_C$.

The applied voltage $V$, being equal to the phasor sum of $V_R$, $V_L$ and $V_C$ is given in magnitude by

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$
$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

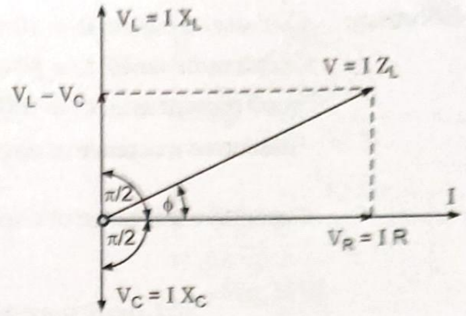



**Fig. 5.17** *(b) Phasor Diagram*

The term $\sqrt{R^2 + (X_L - X_C)^2}$ is known as *impedance* of the circuit and is represented by Z. Its unit is ohm.

Phase angle $\phi$ between voltage and current is given by

$$\phi = \text{Tan}^{-1}\frac{V_L - V_C}{V_R} = \text{Tan}^{-1}\frac{IX_L - IX_C}{IR} = \text{Tan}^{-1}\frac{X_L - X_C}{R} = \text{Tan}^{-1}\frac{X}{R}$$

$\phi$ will be + ve *i.e.* applied voltage will lead the current if $X_L > X_C$ and $\phi$ will be – ve *i.e.* applied voltage will be behind the current if $X_L < X_C$.

Power factor of the circuit is given by

$$\text{Cos}\,\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Power consumed in the circuit,

$$P = I^2 R \text{ or } V I \cos\phi.$$

**5.10.1. Reactance.** Inductive reactance, $X_L$ is directly proportional to frequency being equal to $\omega L$ or $2\pi f L$ and capacitive reactance, $X_C$ is inversely proportional to frequency being equal to $\frac{1}{\omega C}$ or $\frac{1}{2\pi f C}$.

Inductive reactance causes the current to lag behind the applied voltage, while the capacitive reactance causes the current to lead the voltage. So when inductance and capacitance are connected in series, their effects neutralize each other and their combined effect is then their difference. The combined effect of inductive reactance and capacitive reactance is called the reactance and is found by subtracting the capacitive reactance from the inductive reactance or according to equation

$$X = X_L - X_C$$

When $X_L > X_C$ *i.e.* $X_L - X_C$ is positive, the circuit is inductive and phase angle $\phi$ is positive.

When $X_L < X_C$ *i.e.* $X_L - X_C$ is negative, the circuit is capacitive and phase angle $\phi$ is negative.

When $X_L = X_C$ *i.e.* $X_L - X_C = 0$, the circuit is purely resistive and phase angle $\phi$ is zero.

If the expression for applied voltage is taken as

$$v = V_{max} \sin \omega t$$

then expression for the current will be

$$i = I_{max} \sin(\omega t \pm \phi) \qquad \text{where } I_{max} = \frac{V_{max}}{\text{Circuit impedance, Z}}$$

+ve sign is used when current leads the applied voltage V *i.e.* when $X_C > X_L$ and –ve sign is used when current lags behind the applied voltage V *i.e.* when $X_L > X_C$.

**Example 5.31. A series circuit has R = 10 Ω; L = 50 mH, and C = 100 μF and is supplied with 200 V, 50 Hz. Find (*i*) impedance (*ii*) current (*iii*) power (*iv*) power factor (*v*) voltage drop across each element.**

[*Him. Technical Univ. Basic Electrical and Electronics Engineering, June-2014*]

**Solution:** Circuit resistance, R = 10 Ω

Circuit inductance, L = 50 mH = 0.05 H

Circuit capacitance, C = 100 μF = $100 \times 10^{-6}$ F

Inductive reactance of circuit, $X_L = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$

Capacitive reactance of circuit, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$

(*i*) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (15.7 - 31.83)^2} = 18.979\ \Omega$ **Ans.**

(*ii*) Circuit current, $I = \dfrac{\text{Supply voltage, V}}{Z} = \dfrac{200}{18.979} = 10.538$ A **Ans.**

Phase angle of circuit, $\phi = \text{Tan}^{-1}\dfrac{X_L - X_C}{R} = \text{Tan}^{-1}\dfrac{15.7 - 31.83}{10} = -58.2°$

(*iii*) Power of circuit, $P = V \times I \cos\phi = 200 \times 10.538 \times \cos(-58.2°) = 1{,}110$ W or 1.11 kW **Ans.**

(*iv*) Power factor of circuit, $pf = \cos\phi = \cos(-58.2°) = 0.5269$ (leading) **Ans.**

(*v*) Voltage drop across resistance R, $V_R = IR = 10.538 \times 10 = 105.38$ V **Ans.**

Voltage drop across inductance L, $V_L = IX_L = 10 \times 15.7 = 157$ V **Ans.**

Voltage drop across capacitance C, $V_C = IX_C = 10 \times 31.83 = 318.3$ V **Ans.**

**Example 5.32. A voltage $e(t) = 150 \sin 1{,}000t$ is applied across a series R-L-C circuit where R = 40 Ω, L = 0.13 H and C = 10 μF.**

**(i) Compute the rms value of the S.S. current. (ii) Find the rms voltage across the inductor (iii) Find the rms voltage across the capacitor (iv) Draw the complete phasor diagram showing all voltage components (v) Determine the reactive power supplied by the source.**

**Solution:** Maximum value of supply voltage, $V_{max}$ = Coefficient of the sine of time angle = 150 V

ω = Coefficient of time, $t$ = 1,000 radians per second

RMS value of supply voltage, $V_{rms} = \dfrac{V_{max}}{\sqrt{2}} = \dfrac{150}{\sqrt{2}} = 106$ V

Circuit resistance, R = 40 Ω

Inductive reactance of circuit, $X_L = \omega L = 1{,}000 \times 0.13 = 130\ \Omega$

Capacitive reactance of circuit, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{1{,}000 \times 10 \times 10^{-6}} = 100\ \Omega$

Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (130 - 100)^2} = 50\ \Omega$

(*i*) RMS value of the steady state (SS) current, $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{106}{50} = 2.12$ A

Phase angle, $\phi = \text{Tan}^{-1}\dfrac{X_L - X_C}{R} = \text{Tan}^{-1}\dfrac{130 - 100}{40} = 36.87°$ (lagging)

Voltage drop across resistor, $V_R = I_{rms} \times R$

$= 2.12 \times 40$

$= 84.8$ V in phase with I

(*ii*) Voltage drop across inductor L, $V_L = I_{rms} \times X_L$

$= 2.12 \times 130$
$= 275.6$ V leading current I by 90° **Ans.**

(*iii*) Voltage drop across capacitor C, $V_C = I_{rms} \times X_C$
$= 2.12 \times 100$
$= 212$ V lagging current I by 90° **Ans.**

(*iv*) Phasor diagram is shown in Fig. 5.18.

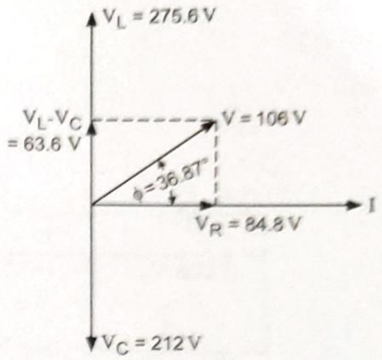


**Fig. 5.18**

(*v*) Reactive power supplied by the source,

$$Q = V_{rms} I_{rms} \sin\phi$$
$$= 106 \times 2.12 \times \sin 36.87°$$
$$= 134.832 \text{ VAR } \textbf{Ans.}$$

**Example 5.33. A 20 Ω resistor is connected in series with an inductor, a capacitor and ammeter across a 25 V variable frequency supply. When the frequency is 400 Hz, the current is at its maximum value of 0.5 A and potential difference across the capacitor is 150 V. Calculate (*i*) The capacitance of the capacitor. (*ii*) The resistance and inductance of the inductor.**

[*Pb. Technical Univ. Basic Electrical and Electronics Engineering, June-2013*]

**Solution:** Voltage drop across capacitor C, $V_C = 150$ V

Current through capacitor, I = Supply current = 0.5 A

$$(i) \text{ Capacitance, } C = \frac{I}{2\pi f V_C} = \frac{0.5}{2\pi \times 400 \times 150} = 1.3263\ \mu\text{F} \quad \textbf{Ans.}$$

Since current is at its maximum value, reactance is zero i.e.,

(*ii*) Inductive reactance of the circuit = Capacitive reactance of the circuit

$$\text{or } X_L = X_C$$

$$\text{or } 2\pi f L = \frac{1}{2\pi f C}$$

$$\text{or inductor inductance, } L = \frac{1}{(2\pi f)^2 C} = \frac{1}{(2\pi \times 400)^2 \times 1.3263 \times 10^{-6}} = 0.11937 \text{ H } \textbf{Ans.}$$

$$\text{Total resistance of circuit, } R = \frac{V}{I} = \frac{25}{0.5} = 50\ \Omega$$

So resistance of inductor, $R' = 50 - 20 = 30\ \Omega$ **Ans.**

**Example 5.34. An alternating voltage of 250 V is applied to a series circuit consisting of a resistor, iron-cored coil and a capacitor. The voltage measured across resistance, coil, and capacitance are 212.5 V, 187.5 V and 125 V respectively. Determine the power factor of the coil and power factor of series circuit if the current drawn by the circuit is 5 A.**

[*M.D. Univ. Electrical Technology December 2007*]

**Solution:** Given series circuit is shown in Fig. 5.19(*a*).

Voltage drop across resistor, $V_R = 212.5$ V in phase with supply voltage V

Voltage drop across coil, $V_{COIL} = 187.5$ V leading $V_R$ by an angle θ (say)

Voltage drop across capacitor, $V_C = 125$ V lagging behind $V_R$ by 90°.

Phasor diagram is shown in Fig. 5.19(*b*)

Resolving $V_R$, $V_{COIL}$ and $V_C$ along X-axis and Y-axis we have

Algebraic sum of X-components $= V_R + V_{COIL} \cos\theta = 212.5 + 187.5 \cos\theta$

Algebraic sum of Y-components $= V_{COIL} \sin\theta - V_C = 187.5 \sin\theta - 125$

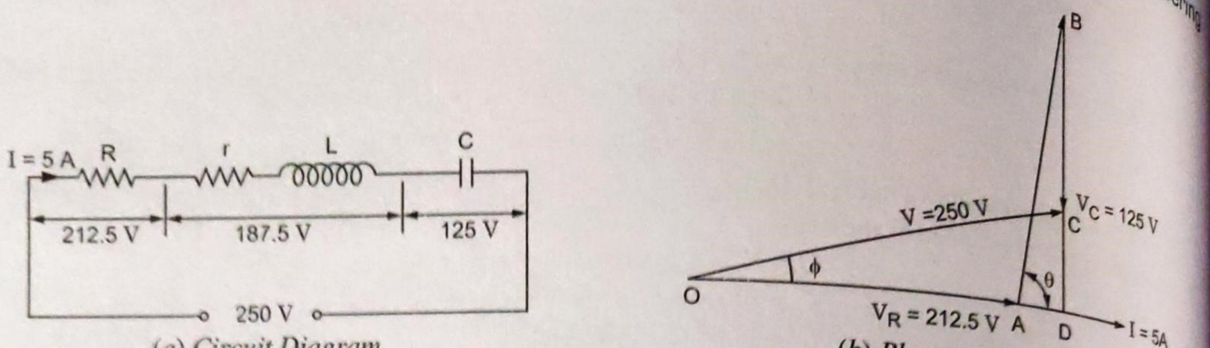
Supply voltage,

$$V = \sqrt{(\text{X-components})^2 + (\text{Y-components})^2}$$

$$\text{or } 250 = \sqrt{(212.5 + 187.5\cos\theta)^2 + (187.5\sin\theta - 125)^2}$$

$$\text{or } 250^2 = (212.5 + 187.5\cos\theta)^2 + (187.5\sin\theta - 125)^2$$

$$\text{or } 250^2 = (212.5)^2 + 2 \times 212.5 \times 187.5\cos\theta + (187.5\cos\theta)^2 + (187.5\sin\theta)^2 - 2 \times 187.5 \times 125\sin\theta + 125^2$$




(*a*) *Circuit Diagram* (*b*) *Phasor Diagram*

**Fig. 5.19**

or $46{,}875 \sin\theta - 79{,}687.5 \cos\theta = 212.5^2 + 125^2 + 187.5^2 (\cos^2\theta + \sin^2\theta) - 250^2$

$$= 45{,}156.25 + 15{,}625 + 35{,}156.25 - 62{,}500 = 33{,}437.5$$

Solving above equation for θ we have

$$\theta = 80.74°$$

So power factor of the coil = $\text{Cos}\,\theta = \text{Cos}\,80.74° = 0.161$ (lagging) **Ans.**

X-component of supply voltage, $V \cos\phi = 212.5 + 187.5 \cos 80.74° = 212.5 + 30.2 = 242.7$ V

Y-component of supply voltage, $V \sin\phi = 187.5 \sin 80.74° - 125 = 185 - 125 = 60$ V

$$\text{Power factor of the series circuit, } \cos\phi = \frac{V\cos\phi}{V}$$

$$= \frac{242.7}{250} = 0.9708 \text{ (lagging)} \quad \textbf{Ans.}$$

**Example 5.35. Find applied voltage and power loss in a circuit shown below in Fig. 5.20. The value of C is 20 μF, current I = 0.345 A.** [*U.P. Technical Univ. Electrical Engineering First Semester 2007-08*]

**Solution:** From circuit diagram we have

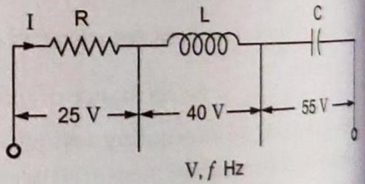



**Fig. 5.20**

$$R = \frac{\text{Voltage drop across R}}{\text{Current flowing through R}} = \frac{25\text{ V}}{0.345} = 72.46\ \Omega$$

Power loss in the circuit = Power loss in resistor

∵ power loss consumed in L and C is zero.

$$= I^2 R = (0.345)^2 \times 72.46 = 8.625 \text{ W} \quad \textbf{Ans.}$$

$$\text{Applied voltage, } V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{25^2 + (40-55)^2} = 29.155 \text{ V} \quad \textbf{Ans.}$$

**Example 5.36. Consider series R-L-C circuit as shown in Fig. 5.21(*a*). When $V_R$ = 8 V, $V_L$ = 18 V, $V_C$ = 12 V. Find V in the circuit.** [*U.P. Technical Univ. Electrical Engineering Odd Semester 2013-14*]

**Solution:** Phasor diagram is shown in Fig. 5.21(*b*).

Voltage across resistor R, $V_R = 8$ V

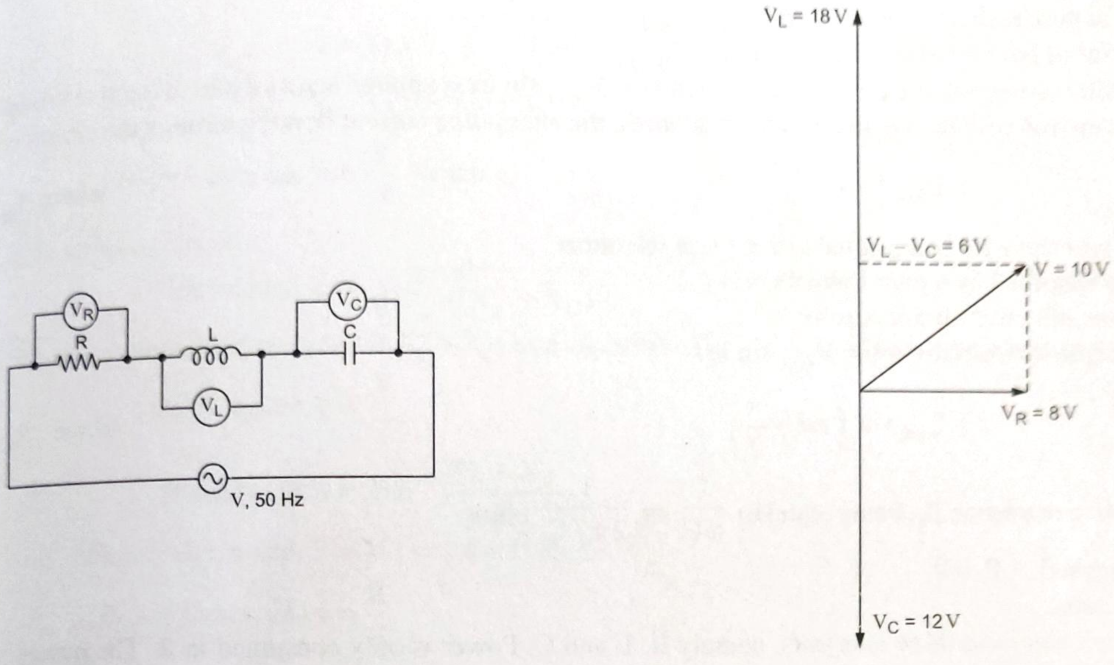
Voltage across inductor L, $V_L = 18$ V

Voltage across capacitor C, $V_C = 12$ V

From phasor diagram shown in Fig. 5.21(*b*).

$$\text{Applied voltage, } V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{8^2 + (18-12)^2} = 10 \text{ V} \quad \textbf{Ans.}$$




*(a) Circuit Diagram* *(b) Phasor Diagram*

**Fig. 5.21**

Example 5.37. **A capacitor (50 μF) is connected in series with a resistor (100 Ω) and the combination is supplied from a single phase ac source of value 230 V, 50 Hz. Calculate the impedance, current, power factor and power consumption in the circuit. Also find the value of a suitable inductor to be connected in series with a combination so as to maintain unity power factor.**

*[B.P. Univ. of Technology Basic Electrical Engineering, 2010]*

**Solution:**

$$\text{Resistor, R} = 100\ \Omega$$

$$\text{Capacitor, C} = 50\ \mu\text{F} = 50 \times 10^{-6}\ \text{F}$$

$$\text{Capacitive reactance, } X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.662\ \Omega$$

$$\text{Circuit impedance, Z} = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + 63.662^2} = 118.545\ \Omega\ \textbf{Ans.}$$

$$\text{Circuit current, I} = \frac{V}{Z} = \frac{230}{118.545} = 1.94\ \text{A}\ \textbf{Ans.}$$

$$\text{Power factor, } pf = \text{Cos}\ \phi = \frac{R}{Z} = \frac{100}{118.545} = 0.84356\ \text{(leading)}\ \textbf{Ans.}$$

$$\text{Power consumption, P} = \text{VI}\cos\phi = 230 \times 1.94 \times 0.84356 = 376.4\ \text{W}\ \textbf{Ans.}$$

Power factor will be maintained unity if the inductive reactance of the inductor connected in series with the combination is equal to capacitive reactance of the circuit i.e.,

$$X_L = X_C = 63.662\ \Omega$$

So inductance of suitable inductor to be connected in series with the combination,

$$L = \frac{X_L}{2\pi f} = \frac{63.662}{2\pi \times 50} = 0.2026\ \text{H}\ \textbf{Ans.}$$

## Highlights

1. The path of flow of alternating current is called the ac circuit.
2. When an alternating voltage given by expression $v = V_{max} \sin \omega t$ is applied across a pure resistive circuit, the alternating current flowing through the circuit is given as

$$i = I_{max} \sin \omega t \quad \text{where } I_{max} = \frac{V_{max}}{R}$$