$$v_{rr} = \frac{1}{\sqrt{2}}v_x - \frac{1}{\sqrt{2}}v_y - \frac{1}{\sqrt{2}}\omega(r_{rr_x} + r_{rr_y})$$

Now we'll factor them out into matrices.

$$\begin{bmatrix} v_{fl} \\ v_{fr} \\ v_{rl} \\ v_{rr} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}(r_{fl_x} + r_{fl_y}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}(r_{fr_x} - r_{fr_y}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}(r_{rl_x} - r_{rl_y}) \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}(r_{rr_x} + r_{rr_y}) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} v_{fl} \\ v_{fr} \\ v_{rl} \\ v_{rr} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & -(r_{fl_x} + r_{fl_y}) \\ 1 & 1 & (r_{fr_x} - r_{fr_y}) \\ 1 & 1 & (r_{rl_x} - r_{rl_y}) \\ 1 & -1 & -(r_{rr_x} + r_{rr_y}) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} \tag{11.12}$$

11.6.2 Forward kinematics

Let $\mathbf{M}$ be the 4×3 inverse kinematics matrix above including the $\frac{1}{\sqrt{2}}$ factor. The forward kinematics are

$$\begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = \mathbf{M}^+ \begin{bmatrix} v_{fl} \\ v_{fr} \\ v_{rl} \\ v_{rr} \end{bmatrix} \tag{11.13}$$

where $\mathbf{M}^+$ is the pseudoinverse of $\mathbf{M}$.

11.7 Swerve drive kinematics

A swerve drive has an arbitrary number of wheels which can rotate in place independent of the chassis. The forces they generate are shown in figure 11.4.

11.7.1 Inverse kinematics

$$\begin{aligned} \vec{v}_{wheel1} &= \vec{v}_{robot} + \vec{\omega}_{robot} \times \vec{r}_{robot2wheel1} \\ \vec{v}_{wheel2} &= \vec{v}_{robot} + \vec{\omega}_{robot} \times \vec{r}_{robot2wheel2} \\ \vec{v}_{wheel3} &= \vec{v}_{robot} + \vec{\omega}_{robot} \times \vec{r}_{robot2wheel3} \\ \vec{v}_{wheel4} &= \vec{v}_{robot} + \vec{\omega}_{robot} \times \vec{r}_{robot2wheel4} \end{aligned}$$

where $\vec{v}_{wheel}$ is the wheel velocity vector, $\vec{v}_{robot}$ is the robot velocity vector, $\vec{\omega}_{robot}$ is the robot angular velocity vector, $\vec{r}_{robot2wheel}$ is the displacement vector from the

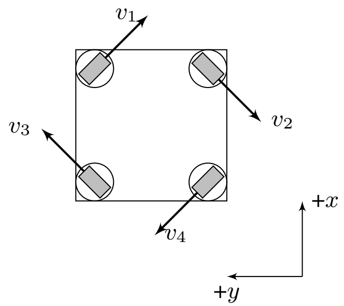

Figure 11.4: Swerve drive free body diagram

robot's center of rotation to the wheel,[1] $\vec{v}_{robot} = v_x\hat{i} + v_y\hat{j}$, and $\vec{r}_{robot2wheel} = r_x\hat{i} + r_y\hat{j}$. The number suffixes denote a specific wheel in figure 11.4.

$$\vec{v}_1 = v_x\hat{i} + v_y\hat{j} + \omega\hat{k} \times (r_{1x}\hat{i} + r_{1y}\hat{j})$$
$$\vec{v}_2 = v_x\hat{i} + v_y\hat{j} + \omega\hat{k} \times (r_{2x}\hat{i} + r_{2y}\hat{j})$$
$$\vec{v}_3 = v_x\hat{i} + v_y\hat{j} + \omega\hat{k} \times (r_{3x}\hat{i} + r_{3y}\hat{j})$$
$$\vec{v}_4 = v_x\hat{i} + v_y\hat{j} + \omega\hat{k} \times (r_{4x}\hat{i} + r_{4y}\hat{j})$$

$$\vec{v}_1 = v_x\hat{i} + v_y\hat{j} + (\omega r_{1x}\hat{j} - \omega r_{1y}\hat{i})$$
$$\vec{v}_2 = v_x\hat{i} + v_y\hat{j} + (\omega r_{2x}\hat{j} - \omega r_{2y}\hat{i})$$
$$\vec{v}_3 = v_x\hat{i} + v_y\hat{j} + (\omega r_{3x}\hat{j} - \omega r_{3y}\hat{i})$$
$$\vec{v}_4 = v_x\hat{i} + v_y\hat{j} + (\omega r_{4x}\hat{j} - \omega r_{4y}\hat{i})$$

$$\vec{v}_1 = v_x\hat{i} + v_y\hat{j} + \omega r_{1x}\hat{j} - \omega r_{1y}\hat{i}$$
$$\vec{v}_2 = v_x\hat{i} + v_y\hat{j} + \omega r_{2x}\hat{j} - \omega r_{2y}\hat{i}$$
$$\vec{v}_3 = v_x\hat{i} + v_y\hat{j} + \omega r_{3x}\hat{j} - \omega r_{3y}\hat{i}$$
$$\vec{v}_4 = v_x\hat{i} + v_y\hat{j} + \omega r_{4x}\hat{j} - \omega r_{4y}\hat{i}$$

$$\vec{v}_1 = v_x\hat{i} - \omega r_{1y}\hat{i} + v_y\hat{j} + \omega r_{1x}\hat{j}$$
$$\vec{v}_2 = v_x\hat{i} - \omega r_{2y}\hat{i} + v_y\hat{j} + \omega r_{2x}\hat{j}$$

[1]The robot's center of rotation need not coincide with the robot's geometric center.

$$\vec{v}_3 = v_x\hat{i} - \omega r_{3y}\hat{i} + v_y\hat{j} + \omega r_{3x}\hat{j}$$
$$\vec{v}_4 = v_x\hat{i} - \omega r_{4y}\hat{i} + v_y\hat{j} + \omega r_{4x}\hat{j}$$

$$\vec{v}_1 = (v_x - \omega r_{1y})\hat{i} + (v_y + \omega r_{1x})\hat{j}$$
$$\vec{v}_2 = (v_x - \omega r_{2y})\hat{i} + (v_y + \omega r_{2x})\hat{j}$$
$$\vec{v}_3 = (v_x - \omega r_{3y})\hat{i} + (v_y + \omega r_{3x})\hat{j}$$
$$\vec{v}_4 = (v_x - \omega r_{4y})\hat{i} + (v_y + \omega r_{4x})\hat{j}$$

Separate the i-hat components into independent equations.

$$v_{1x} = v_x - \omega r_{1y}$$
$$v_{2x} = v_x - \omega r_{2y}$$
$$v_{3x} = v_x - \omega r_{3y}$$
$$v_{4x} = v_x - \omega r_{4y}$$

Separate the j-hat components into independent equations.

$$v_{1y} = v_y + \omega r_{1x}$$
$$v_{2y} = v_y + \omega r_{2x}$$
$$v_{3y} = v_y + \omega r_{3x}$$
$$v_{4y} = v_y + \omega r_{4x}$$

Now we'll factor them out into matrices.

$$\begin{bmatrix} v_{1x} \\ v_{2x} \\ v_{3x} \\ v_{4x} \\ v_{1y} \\ v_{2y} \\ v_{3y} \\ v_{4y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -r_{1y} \\ 1 & 0 & -r_{2y} \\ 1 & 0 & -r_{3y} \\ 1 & 0 & -r_{4y} \\ 0 & 1 & r_{1x} \\ 0 & 1 & r_{2x} \\ 0 & 1 & r_{3x} \\ 0 & 1 & r_{4x} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

Rearrange the rows so the x and y components are in adjacent rows.

$$\begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \\ v_{3x} \\ v_{3y} \\ v_{4x} \\ v_{4y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -r_{1y} \\ 0 & 1 & r_{1x} \\ 1 & 0 & -r_{2y} \\ 0 & 1 & r_{2x} \\ 1 & 0 & -r_{3y} \\ 0 & 1 & r_{3x} \\ 1 & 0 & -r_{4y} \\ 0 & 1 & r_{4x} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} \tag{11.14}$$

To convert the swerve module x and y velocity components to a velocity and heading, use the Pythagorean theorem and arctangent respectively. Here's an example for module 1.

$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} \tag{11.15}$$

$$\theta_1 = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right) \tag{11.16}$$

11.7.2 Forward kinematics

Let $\mathbf{M}$ be the 8×3 inverse kinematics matrix from equation (11.14). The forward kinematics are

$$\begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = \mathbf{M}^+ \begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \\ v_{3x} \\ v_{3y} \\ v_{4x} \\ v_{4y} \end{bmatrix} \tag{11.17}$$

where $\mathbf{M}^+$ is the pseudoinverse of $\mathbf{M}$.