Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

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Assignment/Lab Title:	Time-Domain Analysis of Discrete-Time Systems - Part 2

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Executive Summary

In Part A of this experiment, the superior efficiency and similar correctness of the Matlab *filter* function compared to hand-calculations in calculating discrete time systems' impulse response was demonstrated. In Part B, the use of Matlab *filtic* function to define systems' initial conditions, and how this command can be used along with *filter* command to calculate systems' zero-input response were presented. Next, in Part C the system's zero-state response was calculated when the system is subject to a given input $x[n]$ using *filter* command.

In part D, a, b, z_i and $y_{zero\ input}$ were taken from part B and $x[n]$ and $y_{zero\ state}$ were taken from part C. For D1, the total response was calculated using the filter function with the appropriate a, b, z_i and x obtained previously. For D2, the sum was calculated using $y_{zero\ input}$ and $y_{zero\ state}$ obtained previously. Then, these two were plotted on different subgraphs of the same figure to compare them.

In part E, a and b were taken from part B and $x[n]$ was taken from part C. $h[n]$ was calculated using the filter function with b, a and $\delta[n]$. $y[n]$ was calculated by taking the convolution of $x[n]$ and $h[n]$. Then, $y[n]$ was plotted. Then, it was verified whether $y[n]$ is the same as observed in part C and whether it is asymptotically stable.

In part F, first the constant coefficient equation was determined using two methods. Then, a simple function was constructed to generate the filter vectors based on a number N. Then, this function was used to generate and plot $y[n]$ for $N = 4$, $N = 8$ and $N = 12$ for 45 points.

Part A

Part 1

In this part, impulse response $h[n]$ for the given systems was found by using Matlab's *filter* command, and their graphs were generated using Matlab as well. Please refer to the files attached namely **partA_part1_i.m** ,and **partA_part1_ii.m** for the codes for this part. **Figure 1**, and **2** below show the $h[n]$ graphs of the system $y[n]+\frac{1}{6}y[n-1]-\frac{1}{6}y[n-2]=\frac{1}{3}x[n]$, and the system $y[n]+\frac{1}{4}y[n-2]=x[n]$, respectively. **Please note that in these figures a data-tip at n = 3 has been added to later verify the theoretical calculations of $h[n]$ of these two systems**

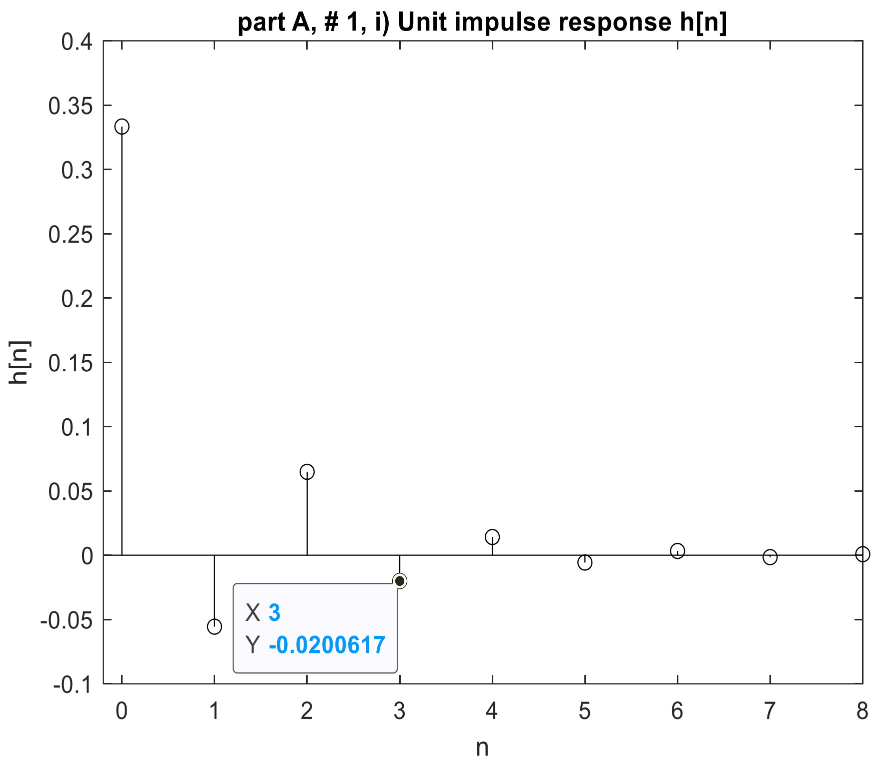

Figure 1: Impulse response of $y[n]+\frac{1}{6}y[n-1]-\frac{1}{6}y[n-2]=\frac{1}{3}x[n]$

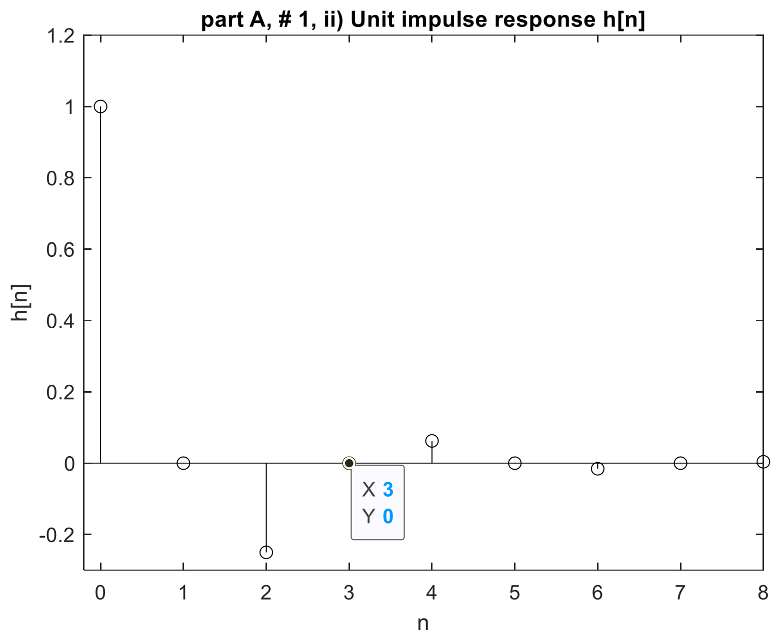

Figure 2: Impulse response of $y[n] + \frac{1}{4}y[n-2] = x[n]$

Part 2

Hand calculation of $h[n]$ of the first given system

$$y[n] + \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = \frac{1}{3}x[n]$$

First, we write the difference equation in advance form:

$$y[n+2] + \tfrac{1}{6}y[n+1] - \tfrac{1}{6}y[n] = \tfrac{1}{3}x[n+2] \qquad \textbf{(1)}$$

$$\text{Thus, } \left(E^2 + \tfrac{1}{6}E - \tfrac{1}{6}\right)y[n] = \tfrac{1}{3}E^2x[n] \qquad \textbf{(2)}$$

Now we know $h[n]$ has this form:

$$h[n] = A_0\delta[n] + y_0[n]u[n] \qquad \textbf{(3)}$$

In equation **(3)**, $y_0[n]$ is the system's zero-input response, $A_0 = 0$

From equation **(2)** the system's characteristic equation:

$$\gamma^2 + \tfrac{1}{6}\gamma - \tfrac{1}{6} = 0 \qquad \textbf{(4)}$$

The solutions to the equation **(4)** are: $\gamma_1 = \frac{1}{3}$ and $\gamma_2 = -0.5$

Thus,

$$h[n] = \left(c_1\left(\tfrac{1}{3}\right)^n + c_2\left(-\tfrac{1}{2}\right)^n\right)u[n] \qquad \textbf{(5)}$$

Now we rewrite equation **(1)** as follows:

$$h[n+2] = \tfrac{1}{3}\delta[n+2] + \tfrac{1}{6}h[n] - \tfrac{1}{6}h[n+1] \qquad \textbf{(6)}$$

when n = -2, equation **(6)** becomes,

$$h[0] = \frac{1}{3}\delta[0] + \frac{1}{6}h[-2] - \frac{1}{6}h[-1]$$

$$= \tfrac{1}{3}$$

$$= c_1 + c_2 \qquad \textbf{(7)}$$

Similarly, for n = -1,

$$h[1] = \frac{1}{3}\delta[1] + \frac{1}{6}h[-1] - \frac{1}{6}h[0]$$

$$= -\tfrac{1}{18}$$

$$= c_1\left(\tfrac{1}{3}\right) + c_2\left(-\tfrac{1}{2}\right) \qquad \textbf{(8)}$$

By solving equation **(7)**, and **(8)**, $c_1 = \frac{2}{15}$, and $c_2 = \frac{1}{5}$

Thus,

$$\boldsymbol{h[n]} = \left(\left(\tfrac{\mathbf{2}}{\mathbf{15}}\right)\left(\tfrac{\mathbf{1}}{\mathbf{3}}\right)^{\boldsymbol{n}} + \left(\tfrac{\mathbf{1}}{\mathbf{5}}\right)\left(-\tfrac{\mathbf{1}}{\mathbf{2}}\right)^{\boldsymbol{n}}\right)\boldsymbol{u[n]} \qquad \textbf{(9)}$$

Hand calculation of $h[n]$ of the second given system

$$y[n] + \tfrac{1}{4}y[n-2] = x[n]$$

First, we re-write the difference equation in advance form, and in terms of $h[n]$:

$$h[n+2] + \tfrac{1}{4}h[n] = \delta[n+2] \qquad \textbf{(10)}$$

Thus, $\left(E^2 + \frac{1}{4}\right)h[n] = E^2\delta[n]$ **(11)**

Now we know $h[n]$ has this form:

$$h[n] = A_0\delta[n] + y_0[n]u[n] \qquad \textbf{(12)}$$

In equation **(12)**, $y_0[n]$ is the system's zero-input response, $A_0 = 0$

From equation **(11)** the system's characteristic equation:

$$\gamma^2 + \tfrac{1}{4} = 0 \qquad \textbf{(13)}$$

The solutions to the equation **(13)** are: $\gamma_1 = \frac{1}{2}j$ and $\gamma_2 = -\frac{1}{2}j$

Thus,

$$h[n] = \left(c_1\left(\frac{1}{2}j\right)^n + c_2\left(-\frac{1}{2}j\right)^n\right)u[n] \qquad \textbf{(14)}$$

From equation **(10)**,

Upon setting n = -2,

$$h[0] = \delta[0] - \frac{1}{4}h[-2]$$

$$= 1$$

$$= c_1 + c_2 \qquad \textbf{(15)}$$

Upon setting n = -1,

$$h[1] = \delta[1] - \frac{1}{4}h[-1]$$

$$= 0$$

$$= c_1\left(\frac{1}{2}j\right) + c_2\left(-\frac{1}{2}j\right) \qquad \textbf{(16)}$$

From equation **(15)**,

$$c_1 = 1 - c_2 \qquad \textbf{(17)}$$

Using equation **(15)**, we rewrite equation **(16)**:

$$(1 - c_2)\left(\frac{1}{2}j\right) + c_2\left(-\frac{1}{2}j\right) = 0 \qquad \textbf{(18)}$$

Using equation **(18)**, we solve $c_2 = \frac{1}{2}$, and later using equation **(15)**, $c_1 = \frac{1}{2}$

Now from equation **(14)**,

$$h[n] = \left(\left(\frac{1}{2}\right)\left(\frac{1}{2}j\right)^n + \left(\frac{1}{2}\right)\left(-\frac{1}{2}j\right)^n\right)u[n]$$

$$= \frac{2^{-n}}{2}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right)u[n]$$

$$= 2^{-n}cos\left(\frac{\pi}{2}n\right)u[n] \qquad \textbf{(19)}$$

Part 3

Equation **(9)** evaluated at n = 3, is exactly the same as the data-tip value presented in **figure 1**, and equation **(19)** evaluated at n = 3, is also exactly the same as the data-tip value shown in **figure 2**. Thus, our hand calculations support our experimental Matlab procedures.

Part B

Here a given system $y[n] - \frac{3}{10}y[n-1] - \frac{1}{10}y[n-2] = 2x[n]$ zero-input response was found using Matlab's *filtic* (to define initial conditions), and *filter* commands. **Please note that,** $\boldsymbol{y[-1] = 1}$**, and** $\boldsymbol{y[-2] = 2}$. **Figure 3** shows the zero-input response of the system. Please see the attached Matlab code file named **partB.m** if required.

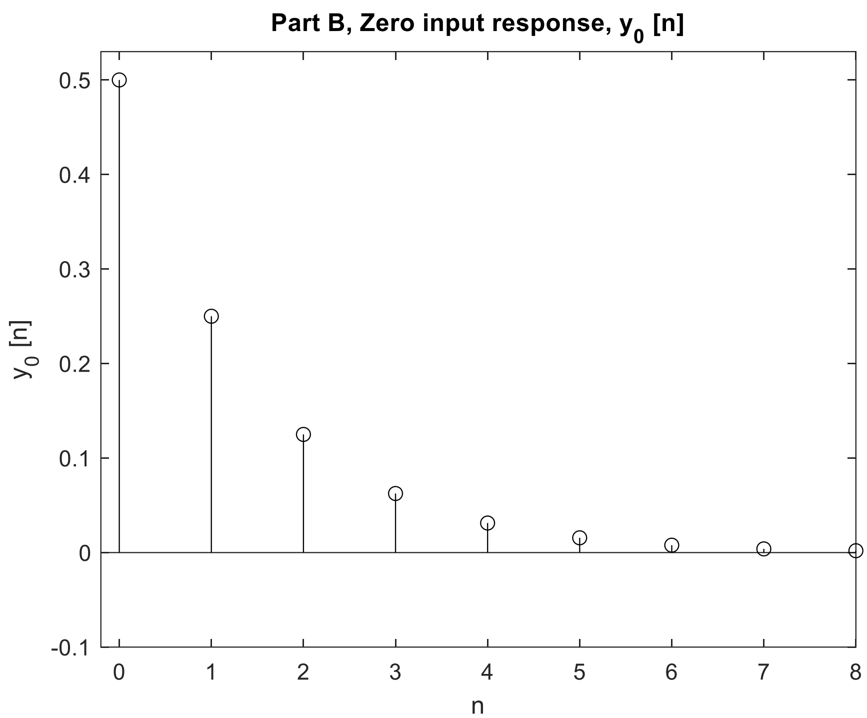

Figure 3: Zero-input response plot of the system $y[n] - \frac{3}{10}y[n-1] - \frac{1}{10}y[n-2] = 2x[n]$

Part C

Matlab was used to determine the zero-state response of the system in **Part B**, to the input $x[n] = 2cos\left(\frac{2\pi n}{6}\right)(u[n] - u[n-10])$. Please see the attached Matlab code file named **partC.m** if needed. **Figure 4** shows the zero-state response graph.

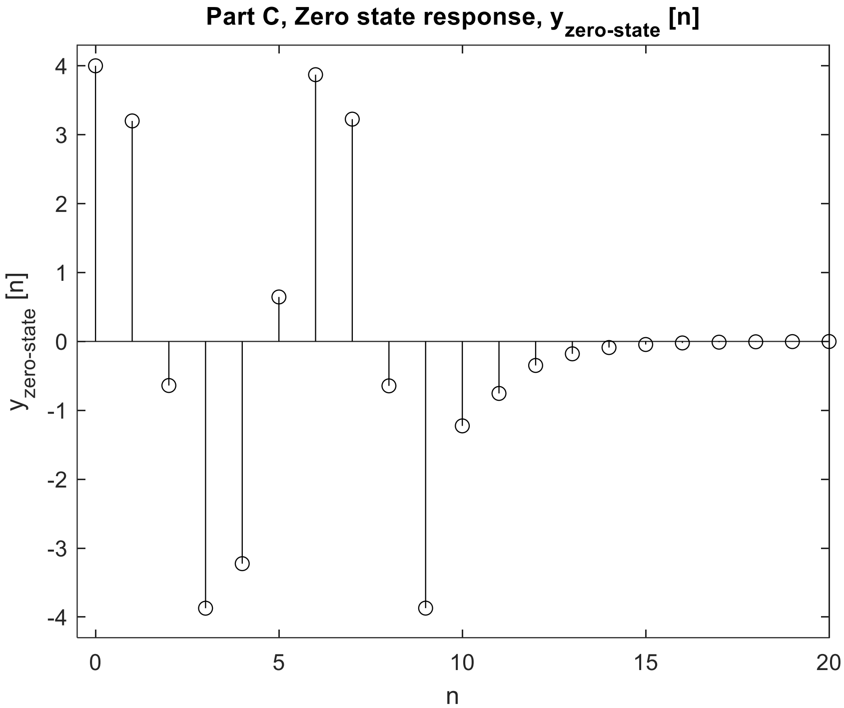

Figure 4: Zero-state response graph of **Part C**

Part D

Part 1

The MATLAB script is attached. Both parts 1 and 2 are shown in the diagram below.

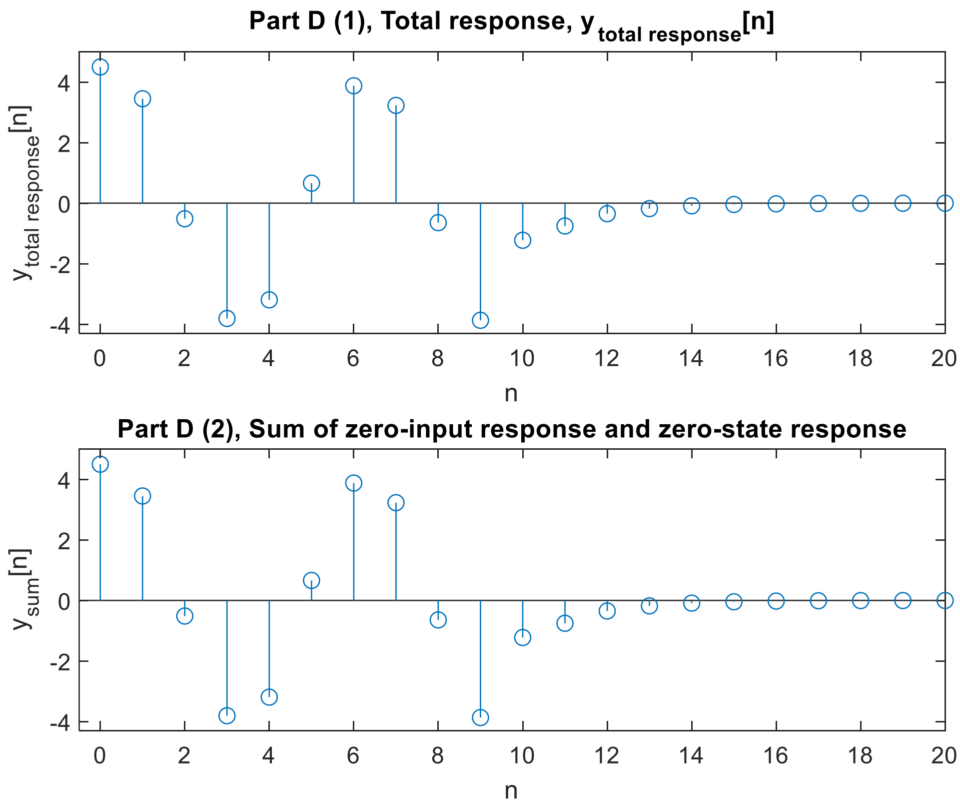

Figure 5 Part D

Part 2

Yes, the total response can be found by adding the zero-input response and zero-state response, because as seen in the preceding diagram, both subgraphs are the same.

Part E

Part 1

The MATLAB script is attached. The graph is shown below.

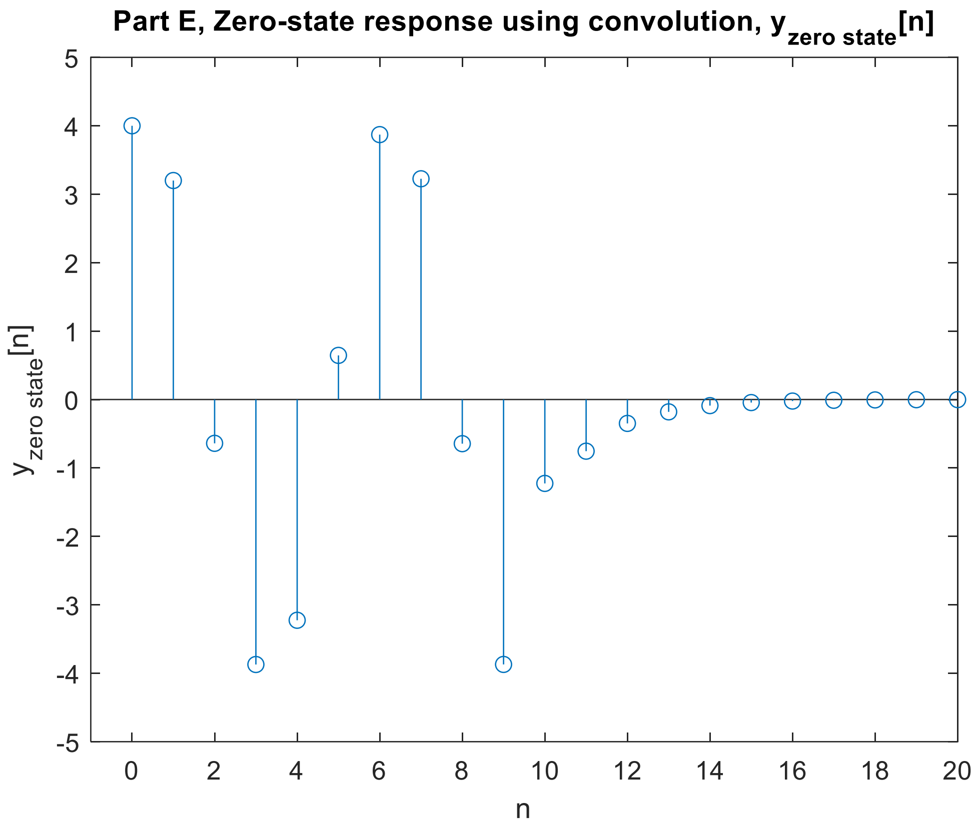

Figure 6 Part E

Part 2

Yes, the result is the same as that observed in part C, which is verified by comparing the graphs.

Part 3

Yes, the system is asymptotically stable.

Characteristic roots of $y[n]-\frac{3}{10}y[n-1]-\frac{1}{10}y[n-2]=2x[n]$:

$$\lambda^2-\frac{3}{10}\lambda-\frac{1}{10}=0$$

$$10\lambda^2-3\lambda-1=0$$

$$\lambda=\frac{-(-3)\pm\sqrt{(-3)^2-4*10(-1)}}{2*10}$$

$$\lambda=\frac{3\pm\sqrt{9+40}}{20}=\frac{3\pm 7}{20}=\frac{10}{20},-\frac{4}{20}=\frac{1}{2},-\frac{1}{5}$$

The magnitude of both characteristic roots is less than 1, hence they fall within the unit circle and the system is thus asymptotically stable.

In addition, the graph of the system is converging towards 0 as n approaches infinity, which confirms that it is asymptotically stable.

Part F

Part 1

$$Nh[n] = u[n] - u[n-N]$$

$Q[E]h[n] = P[E]\delta[n]$ has constant coefficient difference equation $Q[E]y[n] = P[E]x[n]$.

$$Q[E] = N, P[E] = 1$$

Note: $u[n] - u[n-1] = \delta[n]$

$$Nh[n] - u[n-1] + u[n-N-1] = u[n] - u[n-1] - (u[n-N] - u[n-N-1])$$
$$Nh[n] - u[n-1] + u[n-N-1] = \delta[n] - \delta[n-N]$$

Note: $Nh[n] = u[n] - u[n-N] \therefore u[n-1] - u[n-N-1] = Nh[n-1]$

$$\therefore N(h[n] - h[n-1]) = \delta[n] - \delta[n-N]$$

From this equation we can find the constant coefficient difference equation:

$$N(y[n] - y[n-1]) = x[n] - x[n-N]$$

This part was also solved using the Z-transform method from chapter 5 of Linear Systems and Signals

First, take the Z-transform of $h[n]$:

Z-transform of $u[n]$ is: (from table 5.1)

$$\frac{z}{z-1}$$

Z-transform of $u[n-N]$ is:

$$\sum_{n=0}^{\infty} \frac{u[n-N]}{z^n} = u[-N] + \frac{u[1-N]}{z} + \frac{u[2-N]}{z^2} + \frac{u[3-N]}{z^3} + \cdots + \frac{u[N-1-N]}{z^{N-1}} + \frac{1}{z^N} + \frac{1}{z^{N+1}} \ldots$$

All the terms before $1/z^N$ are 0

$$\frac{1}{z^N} + \frac{1}{z^{N+1}} \ldots = \left(\frac{1}{z^N}\right)\left(1 + \frac{1}{z} \ldots\right) = \frac{1}{z^N}\left(\frac{1}{1-\frac{1}{z}}\right) = \frac{1}{z^N}\left(\frac{z}{z-1}\right)$$

$$\therefore H(Z) = \left(\frac{1}{N}\right)\left(\frac{z}{z-1}\right)\left(1 - \frac{1}{z^N}\right) = \left(\frac{1}{N}\right)\left(\frac{z(1-z^{-N})}{z-1}\right) = \left(\frac{1}{N}\right)\left(\frac{z - z^{1-N}}{z-1}\right)$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{z - z^{1-N}}{N(z-1)}$$
$$X(Z)(z^1 - z^{1-N}) = Y(Z)N(z^1 - z^0)$$
$$x[n](\delta[n+1] - \delta[n+1-N]) = y[n]N(\delta[n+1] - \delta[n])$$

Thus,

$$Ny[n+1] - Ny[n] = x[n+1] - x[n+1+N]$$
$$Ny[n] - Ny[n-1] = x[n] - x[n-N]$$

Part 2

MATLAB function attached.

Part 3

MATLAB script attached.

The graph is shown below. As N grows larger, the average becomes smaller, which makes sense because half the values of $x[n]$ are above 0 and half are below 0 and as the average is taken over a great number of n, it becomes more accurate i.e. closer to 0.

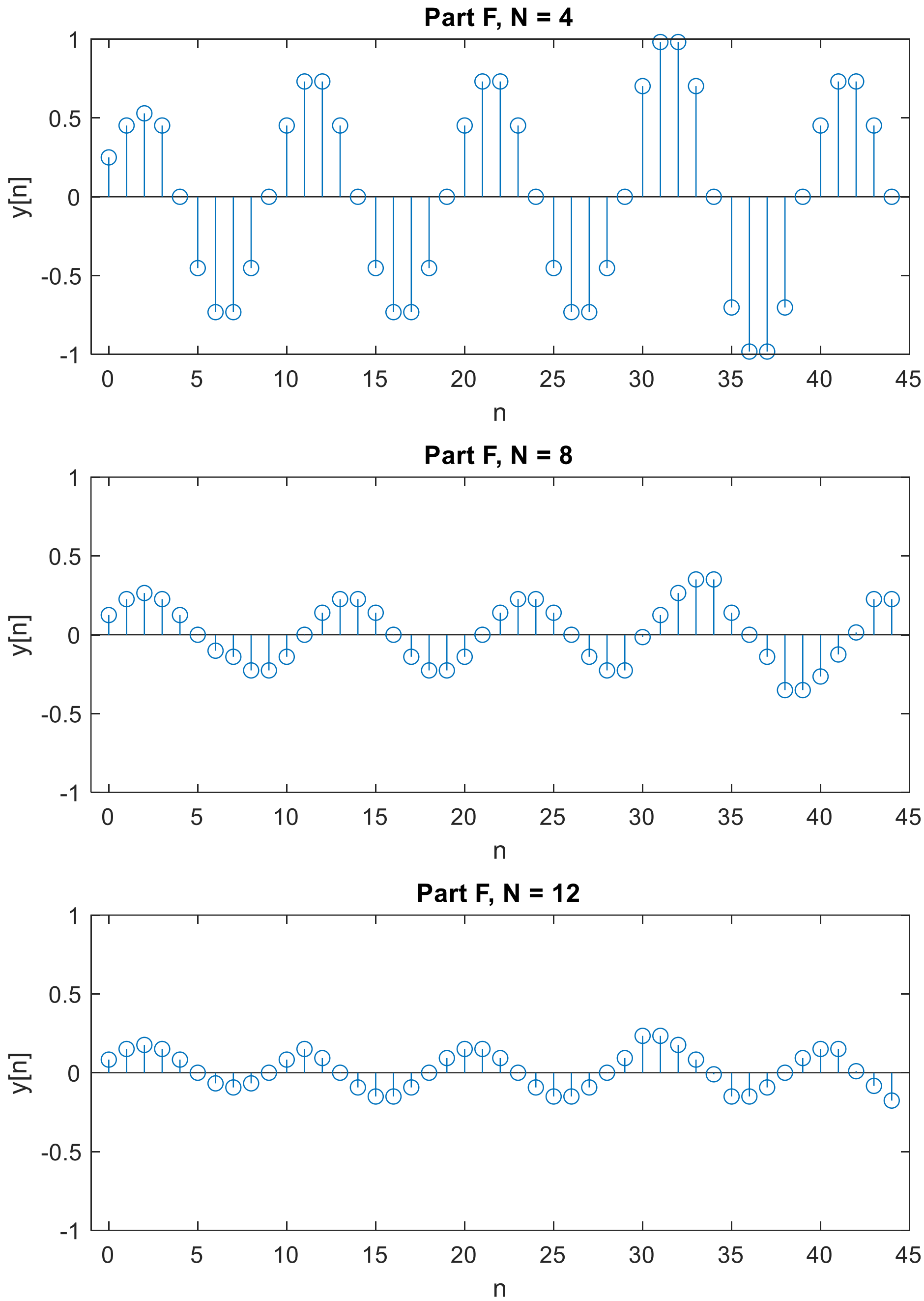

Figure 7 Part F