Ryerson University

Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

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Instructor:	Dr. Dimitri Androutsos (TA: Mahdi Shamsi)

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*By signing above you attest that you have contributed to this written lab report and confirm that all work you have contributed to this lab report is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a "0" on the work, an "F" in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: http://www.ryerson.ca/senate/current/pol60.pdf

Contents

Abstract

In part A, overall, discrete-time Fourier series coefficients, D_r of various types of periodic signals were calculated based on the equation: $D_r = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x[n]e^{-jr\Omega_0 n}$, and then plotted as a function of r.In part A 1, and A 2, Fourier series coefficients of a signal composed of two sinusoids, were calculated upon manual calculations of the signal's fundamental period, and frequency. In part A 3, a discrete triangular train time-domain signal was analyzed to calculate its Fourier series coefficients in a similar manner.

For part B1, $X[r]$ is set for one period (from 0 to 31) as shown in figure 2 in the lab manual, and $x[n]$ is determined using the ifft function and then plotted. In part B2, the time-shifting property of the inverse discrete-time Fourier series was analyzed, that is, a signal $x[n]$ can be time shifted by a specific amount k ,if we multiply the Fourier series spectrum of $x[n]$, $X[r]$ by a complex exponential, to put simply, $X[r]e^{-jk\Omega_0 r}$ = $x[n-k]$.

For part C1, $H[r]$ is set for one period (from 0 to 31). Then ω is set from -2π to 2π in intervals of Ω. $H[r]$ is plotted from -2π to 2π to cover two periods so as to include the axes shown in figure 3. For parts C2 and C3, $x_1[n]$ and $x_2[n]$ variables were created over one period of $h[n]$ (32 values), then their Fourier series were determined, then the Fourier series of $y_1[n]$ and $y_2[n]$ were determined by multiplying the Fourier series of $x_1[n]$ and $x_2[n]$ with $H[r]$, and finally $y_1[n]$ and $y_2[n]$ were plotted. Following that, C4 was answered (comparing $y_1[n]$ and $y_2[n]$) and more plots were created ($X_1[r], X_2[r], H[r], x_1[n], x_2[n], h[n]$) to help understand C4.

Note: according to page 889 of *Linear Systems and Signals 3rd edition*, X = fft(x)/N_0 and x = real(ifft(X)*N_0), and only for plotting do we use real(X). This was how fft and ifft were used in this report. If we had set X = real(fft(x)/N_0), we would get different results for part C3.

Part A: Discrete-Time Fourier Series

Part 1

Here a given discrete-time signal is: $x[n] = 4\cos(2.4\pi n) + 2\sin(3.2\pi n)$, To fins this signal's fundamental period, N_0 and fundamental frequency, Ω_0 we first need to show that the signal $x[n]$ is periodic. Let $x_1[n] = 4\cos(2.4\pi n)$ and $x_2[n] = 2\sin(3.2\pi n)$. We observe that: $\frac{\Omega_{0\,1}}{2\pi} = \frac{2.4\pi}{2\pi} = \frac{6}{5} = \frac{m_{0\,1}}{N_{0\,1}}$ =rational number, and $\frac{\Omega_{0\,2}}{2\pi} = \frac{3.2\pi}{2\pi} = \frac{8}{5} = \frac{m_{0\,2}}{N_{0\,2}}$ = also a rational number. So, $x_1[n]$ and $x_2[n]$ will be periodic signals. Now, we observe $\frac{N_{0\,1}}{N_{0\,2}} = \frac{5}{5}$ = a rational number too. So, $x[n]$ must also be periodic. **So, the fundamental period of $x[n]$, N_o = LCM ($N_{0\,1}$, $N_{0\,2}$) = 5. Therefore, fundamental frequency of $x[n]$, $\Omega_0 = \frac{2\pi}{5}$**

Part 2

We know the discrete time Fourier series coefficients, D_r of any discrete-time signal, $x[n]$ can be calculated in Matlab by using the equation as follows (equation 9.4 in the textbook):

$$D_r = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x[n]e^{-jr\Omega_0 n} \quad (1)$$

For $x[n] = 4\cos(2.4\pi n) + 2\sin(3.2\pi n)$, its fundamental frequency, $\Omega_0 = \frac{2\pi}{5}$ (previously calculated in part 1 of part A). **Figure 1** shows the graph of the signal $x[n]$, while **Figure 2** shows its magnitude $|D_r|$ and phase $\angle D_r$ spectra with respect to r. **Please note that since D_r repeat themselves in every N_0 interval, we only need to show the D_r within the N_0 interval (from 0 to $N_0 - 1$).** Please refer to the attached .m file named "partA_first_signal.m" if necessary.

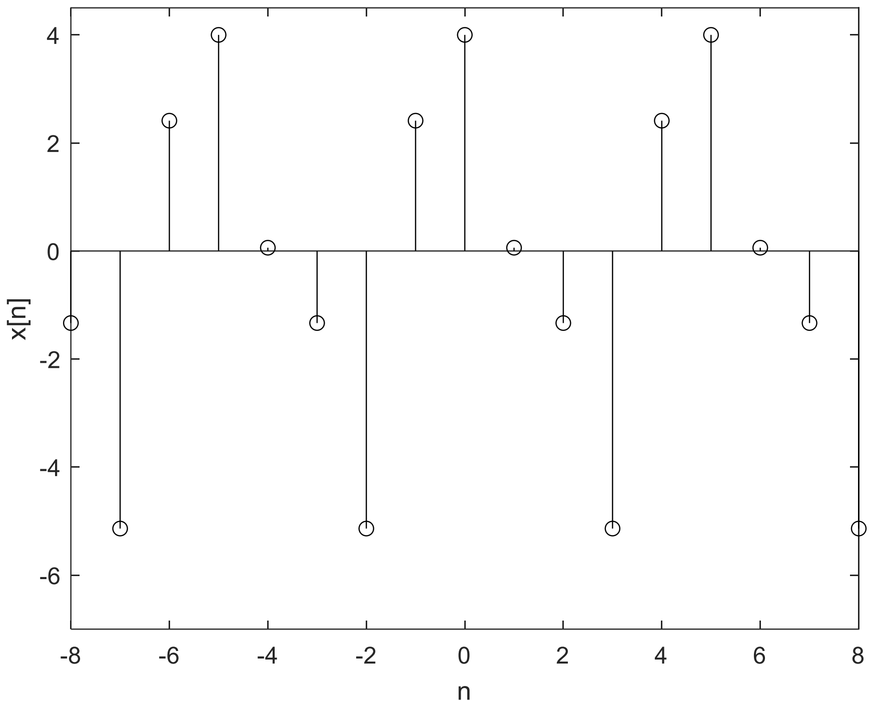

Figure 1: Matlab plot of the signal $x[n] = 4\,cos(2.4\pi n) + 2sin\,(3.2\pi n)$.

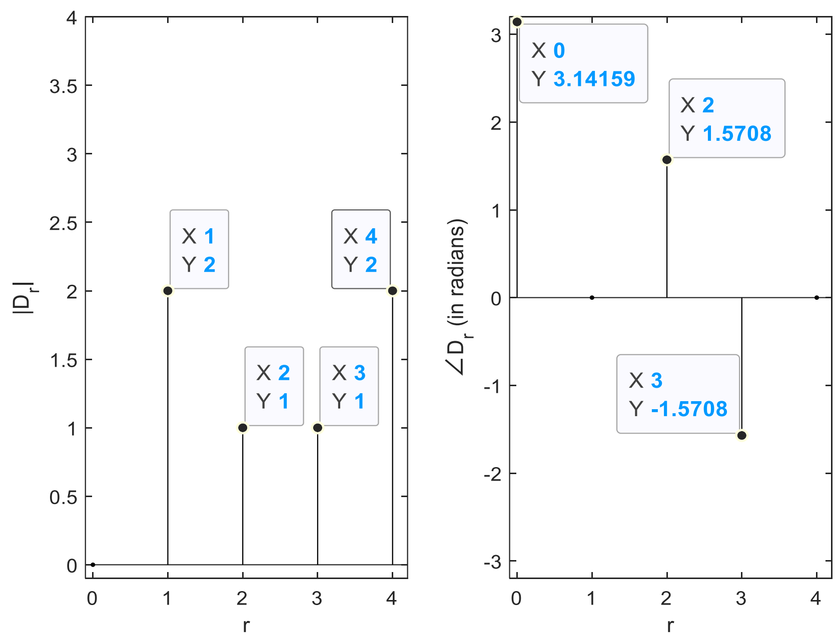

Figure 2: Magnitude and phase plot showing each DTFS coefficient, D_r over N_0 interval (0 to $N_0 - 1$).

Part 3

Please note that in this part $y[n]$ was assumed to be a discrete version of a triangular train signal (each side being linear). This assumption was made because of the missing data values for $y[n]$ in the lab manual (only peak values of 3 were given, but other values were missing).

Figure 3 shows the graph of the $y[n]$. From **figure 3**, we observe that the fundamental period, ***N_0* = 6**, thus, the fundamental frequency, $\mathbf{\Omega_0} = \frac{\pi}{3}$.

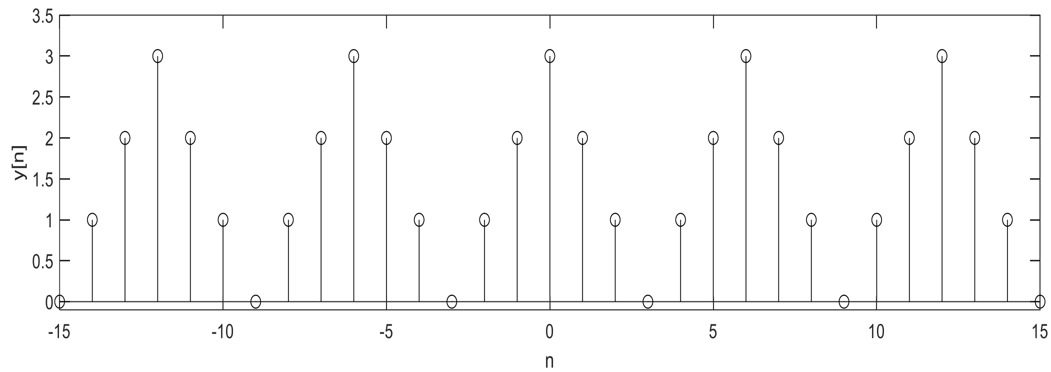

Figure 3: Matlab time-domain plot of $y[n]$.

Figure 4 shows $y[n]$'s magnitude $|D_r|$ and phase $\angle D_r$ spectra with respect to r. **Please note that since D_r repeat themselves in every *N_0* interval, we only need to show the D_r within the *N_0* interval (from 0 to *N_0* – 1).** Please refer to the attached .m file named "partA_second_signal.m" if necessary.

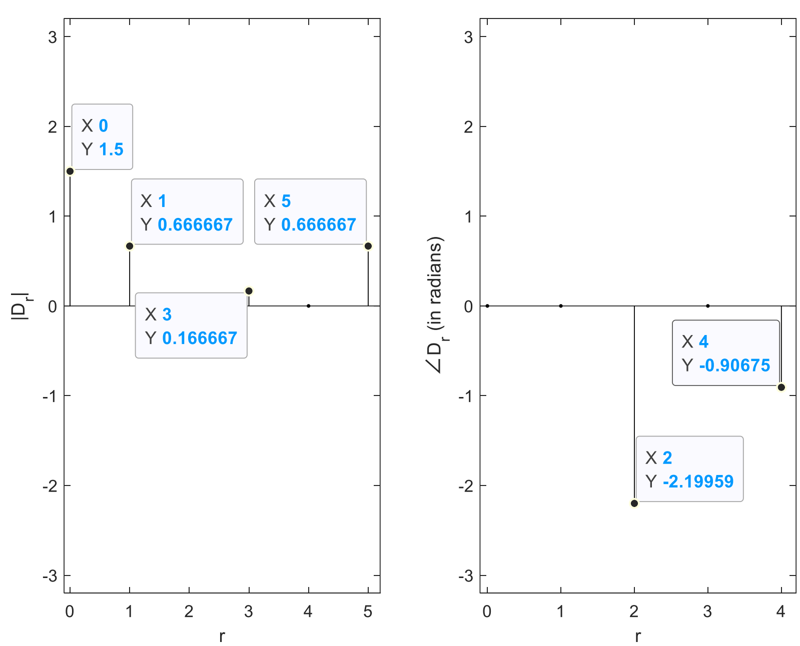

Figure 4: Magnitude and phase plot of *y[n]*, showing each DTFS coefficient, D_r over *N_0* interval (0 to *N_0* – 1).

Part B: Inverse DTFS and time shifting property

Part 1

B1 (described in abstract, code attached)

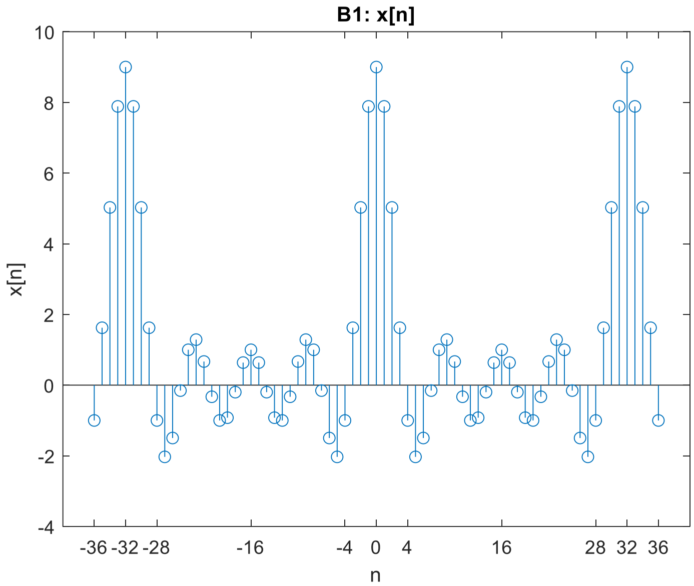

Figure 5: B1

Part 2

In this part, we multiply the $X[r]$ (refer to the **figure 2** in the **lab manual** for the plot of $X[r]$) by $e^{-j5\Omega_0 r}$, and find the inverse DTFS of this product. **Figure 6** shows the inverse DTFS plot, that is, the time domain plot of $X[r]e^{-j5\Omega_0 r}$. **In figure 6, we see an expected time shift by 5, that is $\boldsymbol{x[n-5]}$. This shows the time shifting property of discrete-time Fourier series, that is, $\boldsymbol{X[r]e^{-jk\Omega_0 r}}$ = $\boldsymbol{x[n-k]}$.** Please refer to the .m file called “partB_part2.m” for the code, if necessary.

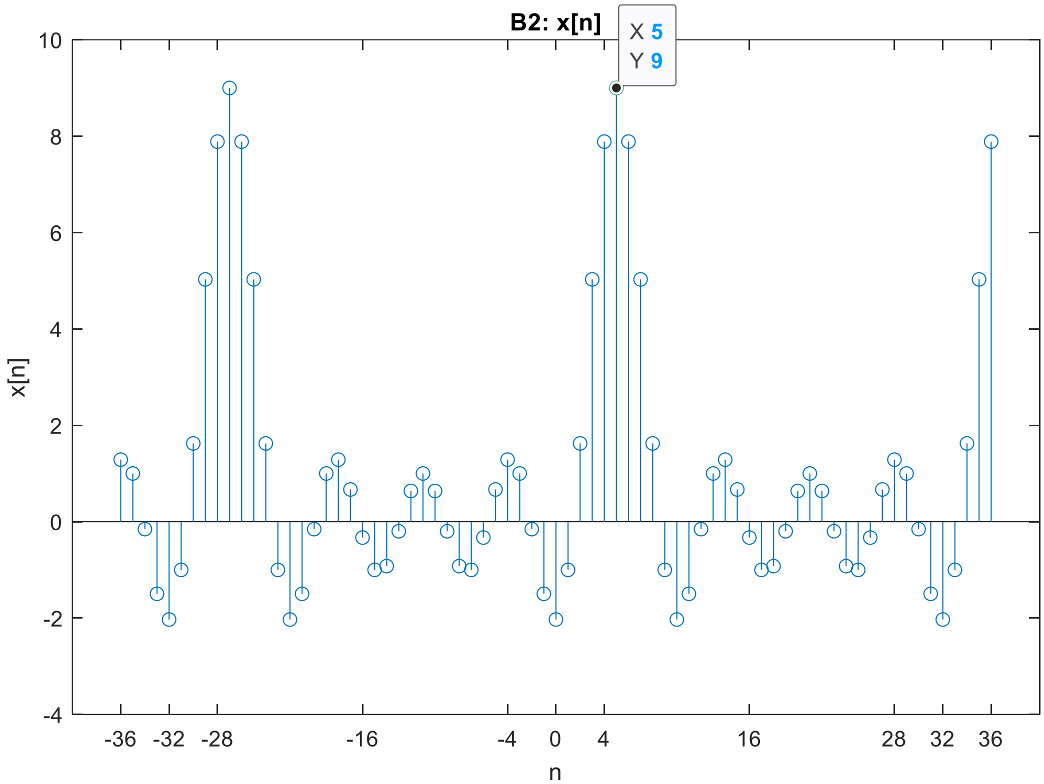

Figure 6: Time domain plot of $X[r]e^{-j5\Omega_0 r}$

Part C System Response

The process for obtaining C1, C2 and C3 is described in the abstract and their Matlab file is attached.

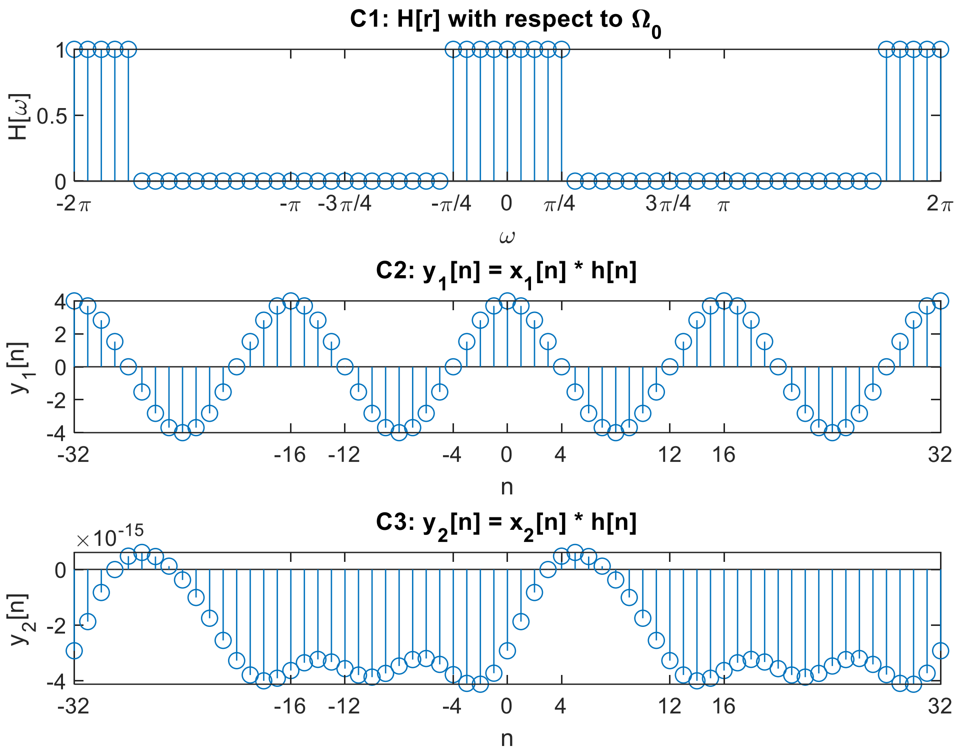

Figure 7 C1, C2, C3

C4

They are different graphs because $x_1[n]$ and $x_2[n]$ are different functions with **different fundamental frequencies**. See the following graphs which are for one period of $H[r]$ and $h[n]$. $x_1[n]$ has a low enough frequency for its discretized graph to look sinusoidal, whereas $x_2[n]$'s frequency is too high and due to the sampling rate it doesn't look sinusoidal. Furthermore as you can see when comparing the real parts of the Fourier series, $X_1[r]$ is high when $H[r]$ is high whereas $X_2[r]$ is not. For a signal like $x_2[n]$ the frequency is too high so in discrete time insufficient data is transferred.

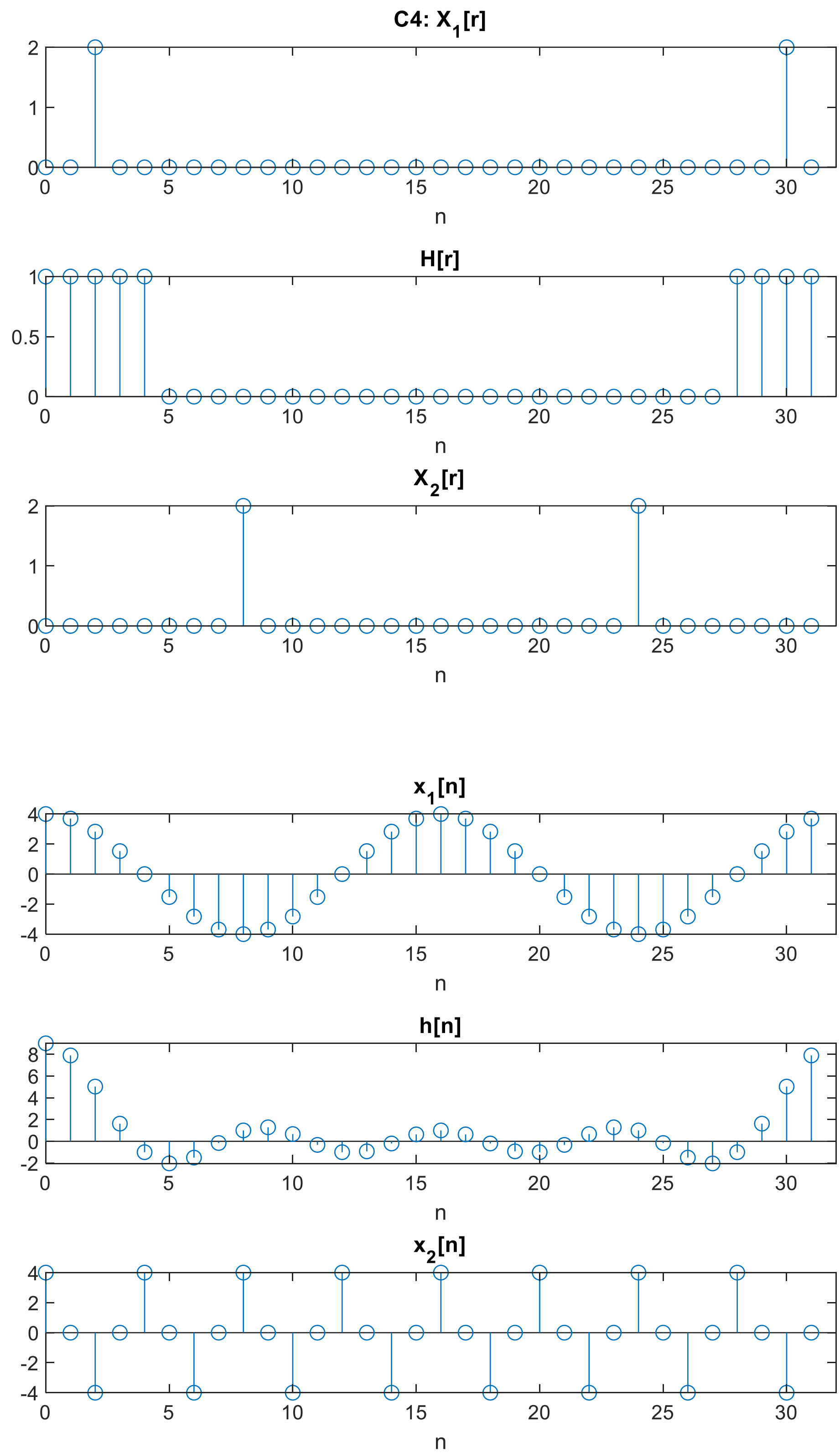

Figure 8 C4