Proof:

Given a valid flow in a network G with capacities and V $\in$ V in G, the flow still satisfies the conservation constraints on this network.

Let A be a subset of vertices in G. We want to show that the total flow out of A is equal to the total flow into A.

By summing over all edges in G:

$$\sum_{e} f(e) = \sum_{e} f(e) \tag{1}$$

Consider an edge e = (a, b) in G.

We rearrange the equation:

$$\sum_{e\in\delta^+(A)} f(e) = \sum_{e\in\delta^-(A)} f(e) \tag{2}$$

Where $\delta^+(A)$ represents the set of edges with one end in A, and $\delta^-(A)$ represents the set of edges with one end in A.

Now, let's consider the different cases for the edge e in terms of its endpoints:

- Case 1: Entirely in A, so both the start and end are in A.
- Case 2: Leaving A, so it starts at a, where a $\in$ A, and goes towards b, where b $\in VnA$.
- Case 3: Entering A, so it starts at b, where b $\in VnA$, and comes towards a, where a $\in$ A.
- Case 4: Entirely in VnA, so it never appears in the sum.

From observation, if edge e = (a, b) is in Case 1, then $f(e)$ appears on the left-hand side when $v = b$, and $f(e)$ also appears on the right-hand side when $v = a$.

Thus, we have:

$$\sum_{e\in\delta^+(A)} f(e) + \sum_{e\in\delta^-(A)} f(e) = \sum_{e\in\delta^+(A)} f(e) \tag{3}$$

This implies that the total flow out of A equals the total flow into A, which completes the proof.