Excitable cells as dynamical systems

Part 1: bifurcations in a simplified model of a neuron

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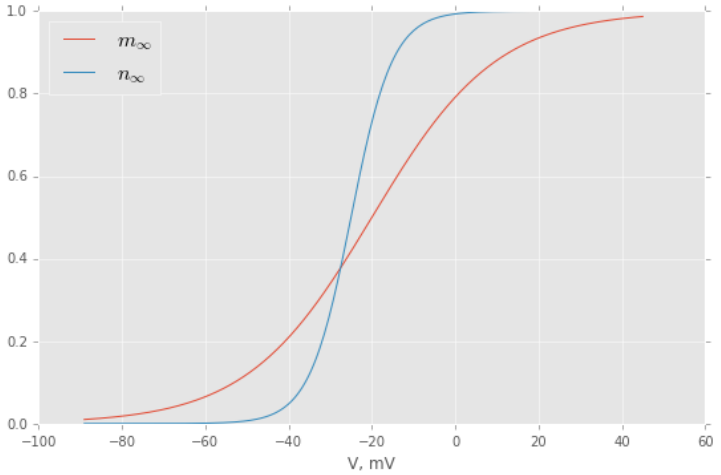
$I_{Na,p}+I_K$ model

System:

$$C\dot{V} = I - \bar{g}_K n(V - E_K) - \bar{g}_{Na} m_\infty(V)(V - E_{Na}) - g_l(V - E_l)$$

$$\tau_n \dot{n} = (n_\infty(V) - n)$$

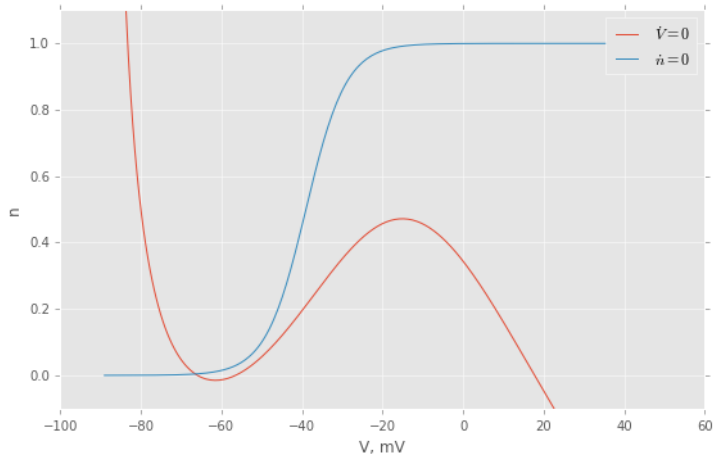
$$x_\infty = \frac{1}{1 + \exp(\frac{V_x^{0.5} - V}{k_x})}$$



Nullclines:

$$\dot{V} = 0 \rightarrow n(V) = \frac{I - \bar{g}_{Na} m_\infty(V - E_{Na}) - g_l(V - E_l)}{\bar{g}_k(V - E_k)}$$

$$\dot{n} = 0 \rightarrow n(V) = n_\infty(V)$$



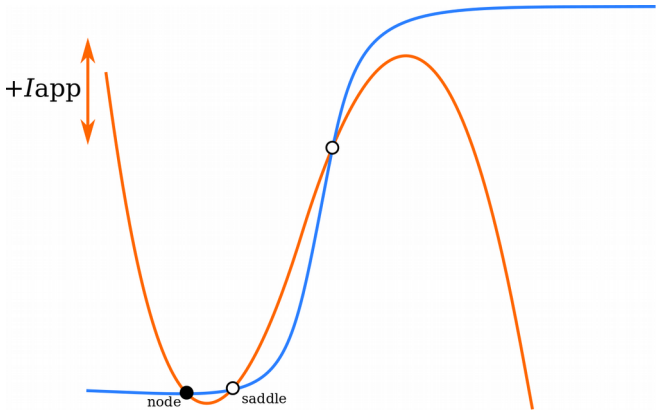
Bifurcations from the stable/quiescent state

- Saddle-node
- Saddle-node on invariant circle
- Supercritical Andronov-Hopf
- Subcritical Andronov-Hopf

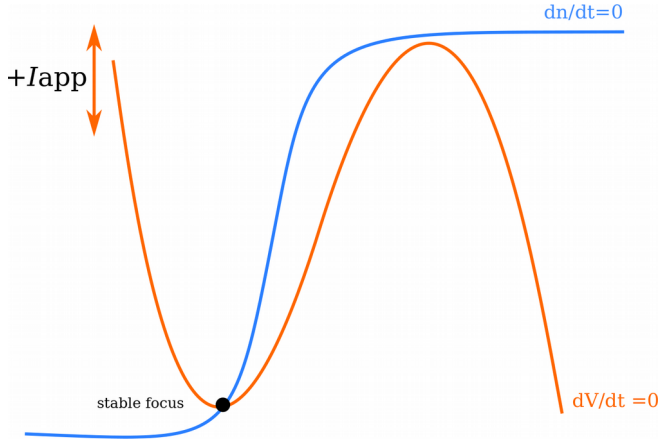
integrators

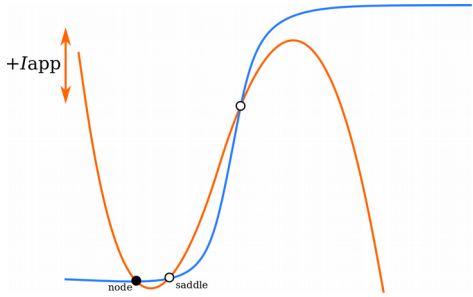
resonators

near saddle-node bifurcation



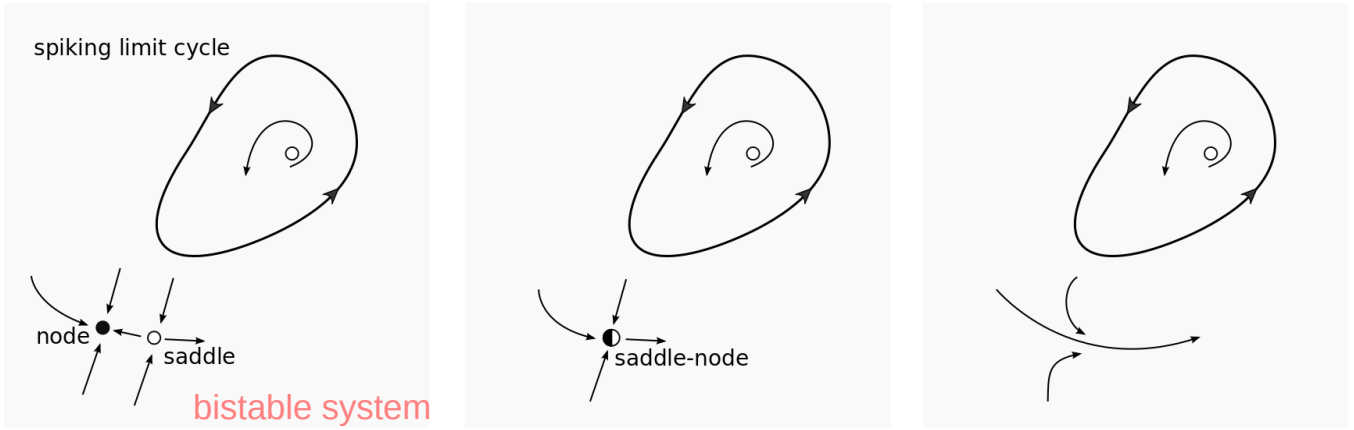
near Andronov-Hopf bifurcation



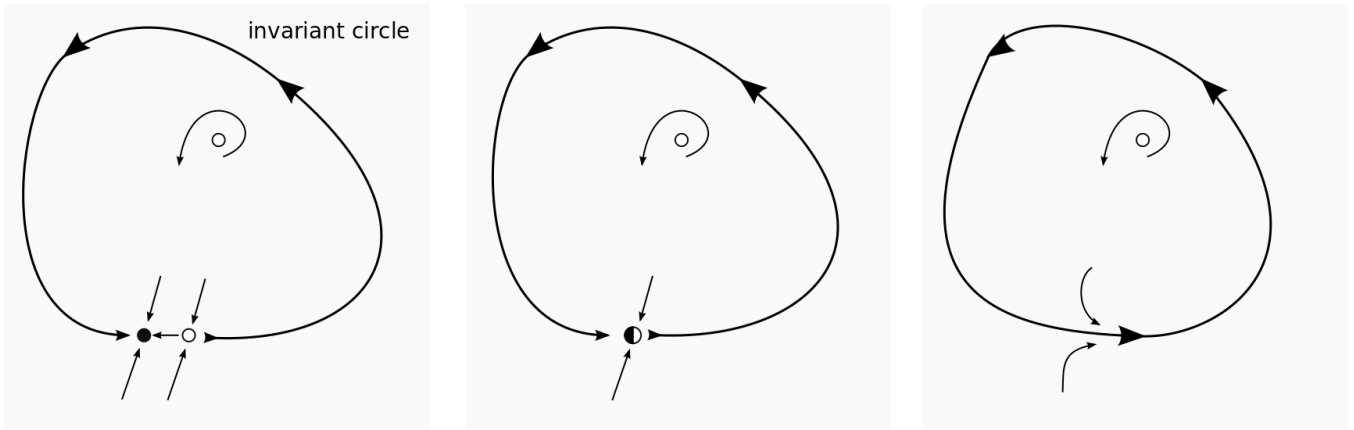


Saddle-node bifurcations

Saddle-node



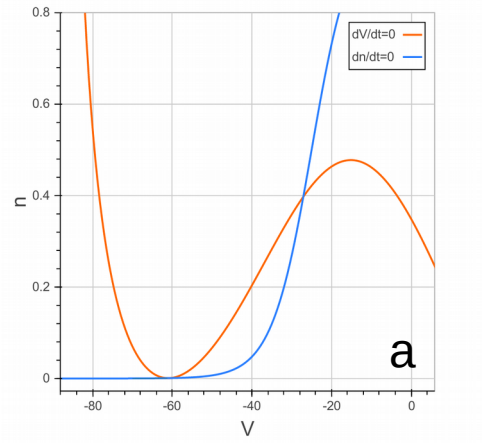
SNIC



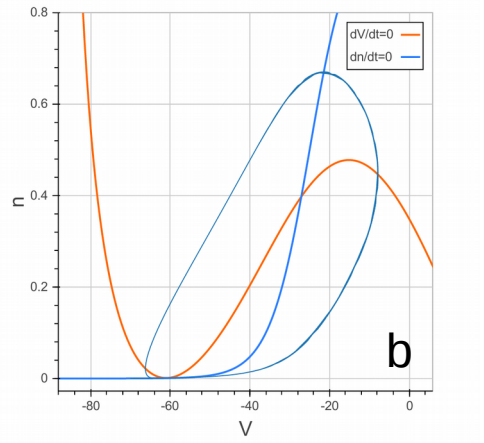
Increasing applied current

Example of SNIC in $I_{Na,p}+I_K$ model

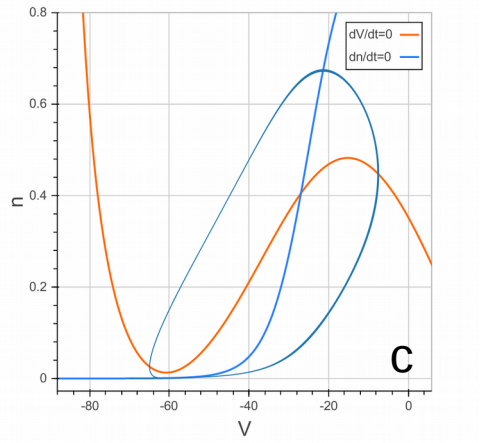
just before bifurcation



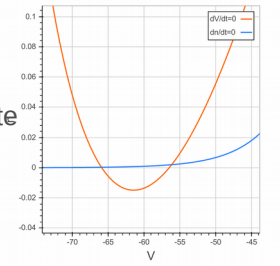
just after bifurcation



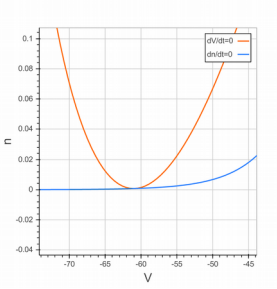
large current

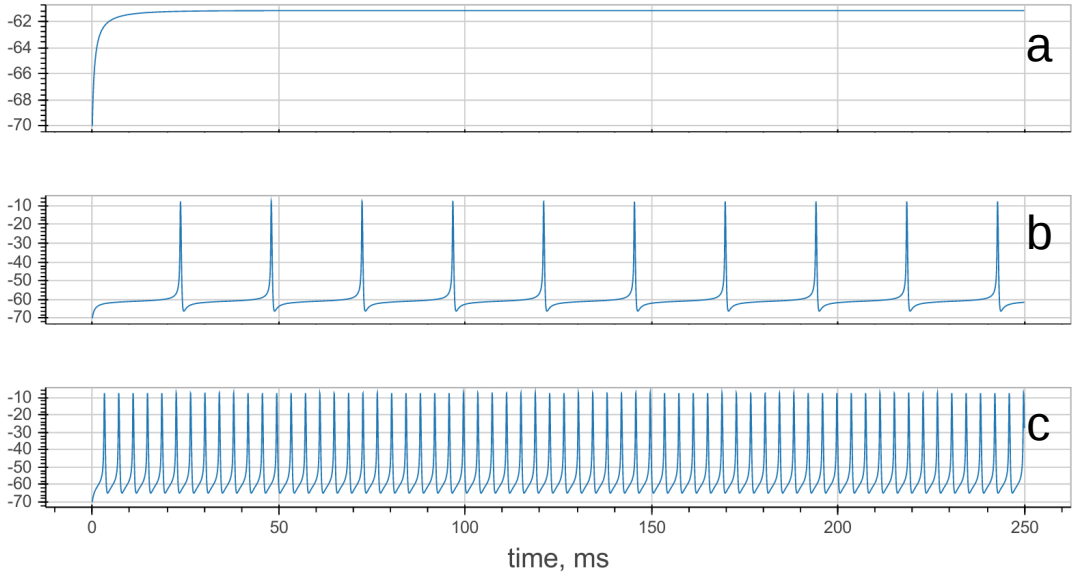


at resting state

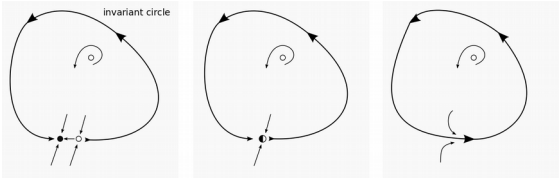


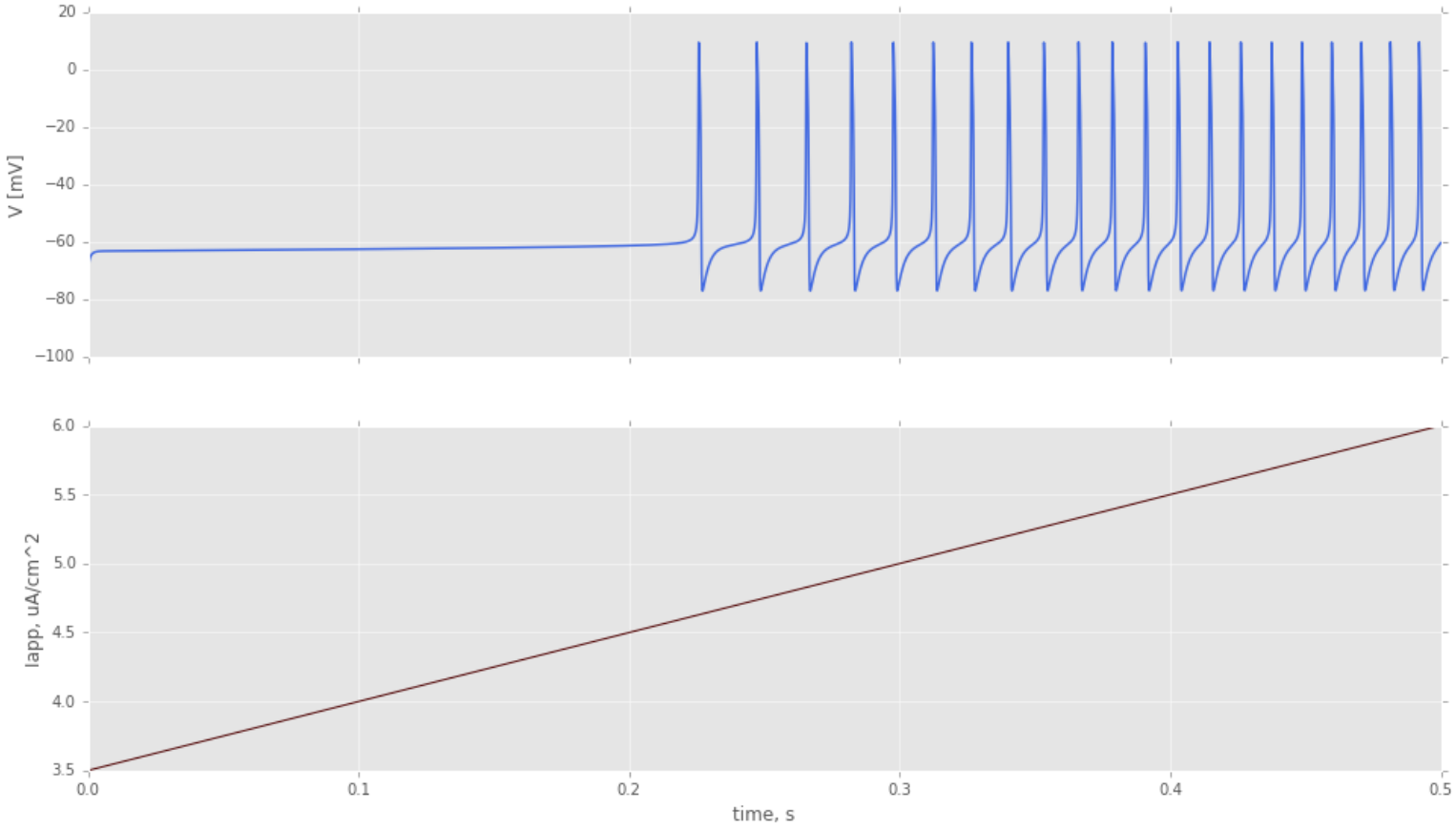
at bifurcation



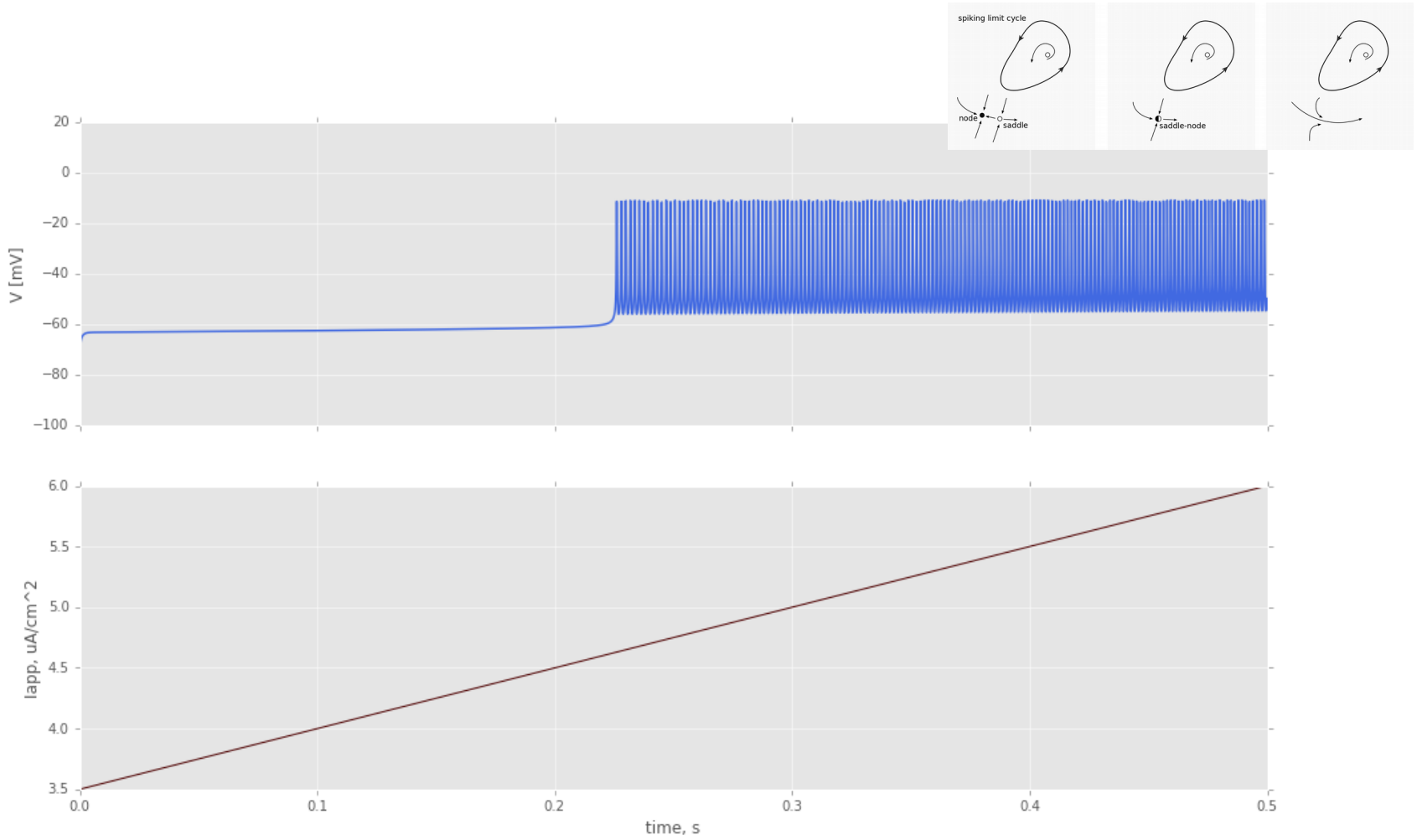


Resting → spiking via SNIC

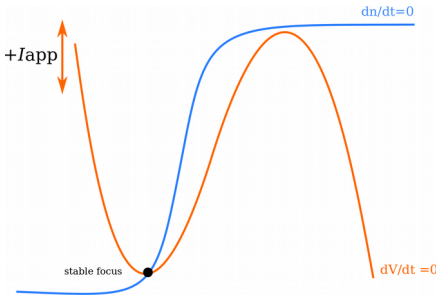




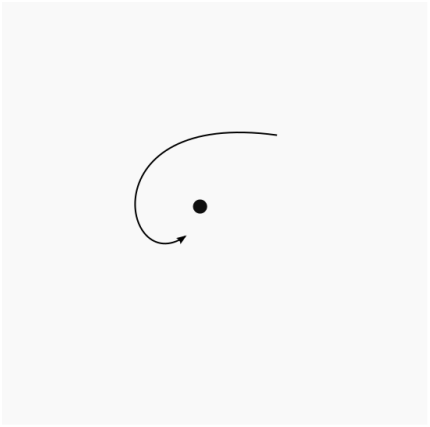
Resting → spiking via saddle-node off limit cycle

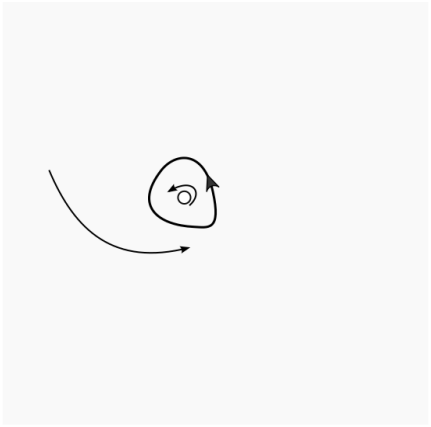


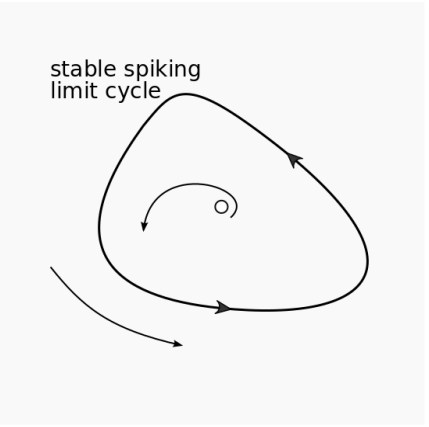
Andronov-Hopf bifurcations



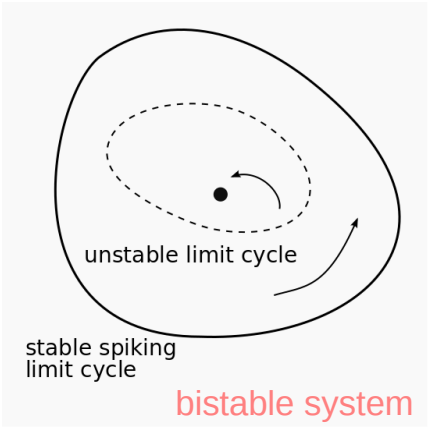
supercritical

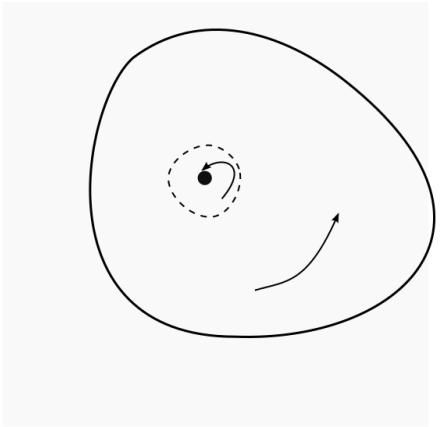


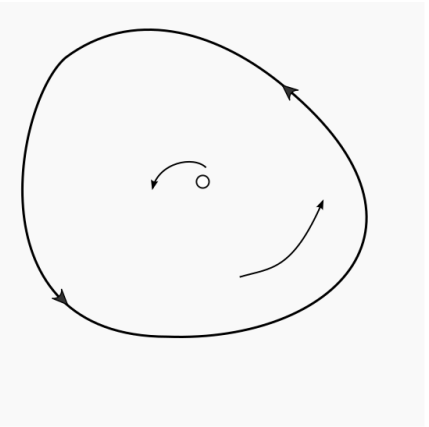




subcritical





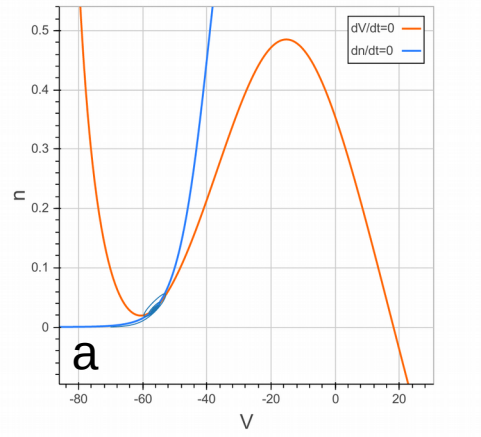


Increasing applied current

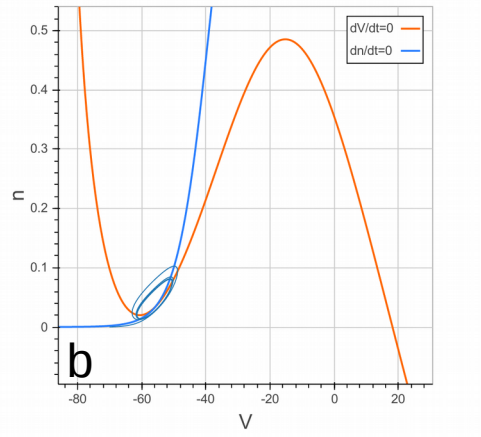
Supercritical AH bifurcation in $I_{Na,p}+I_K$ model

(low-threshold K current)

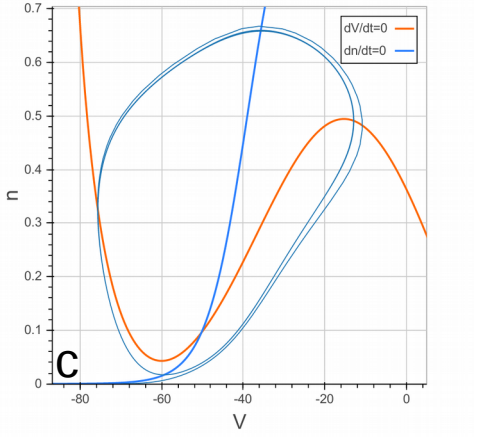
just before bifurcation



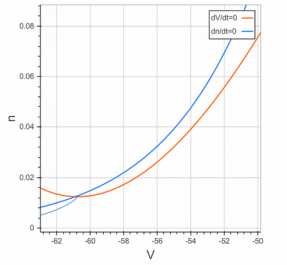
just after bifurcation



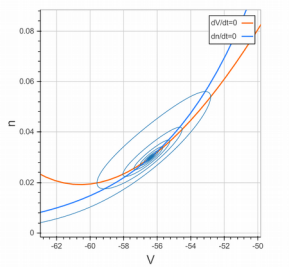
larger current



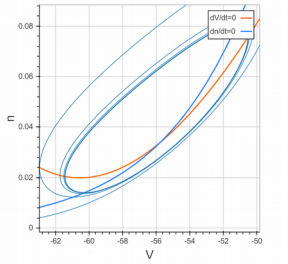
low current

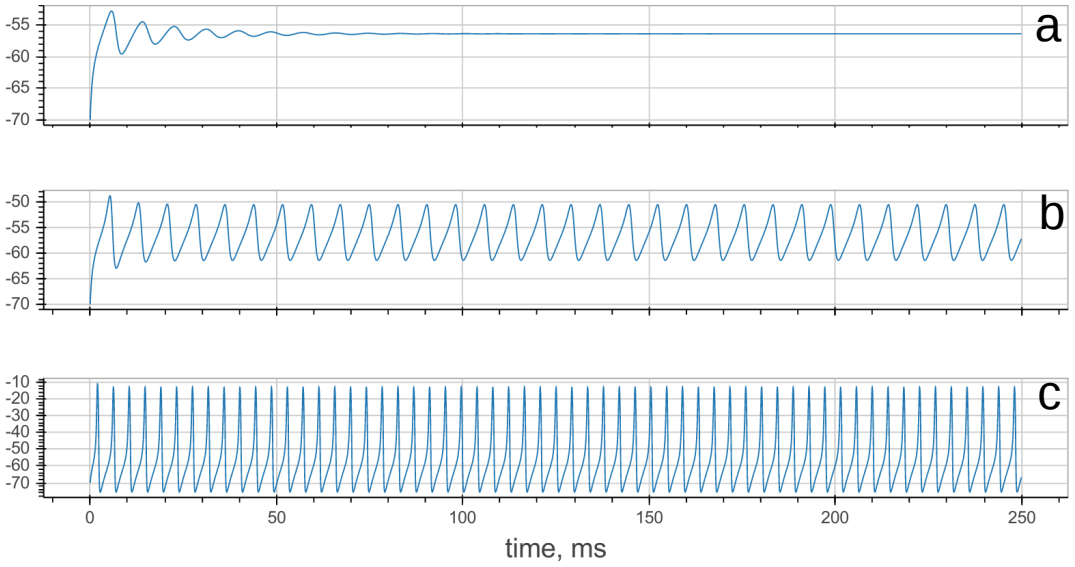


just before bifurcation

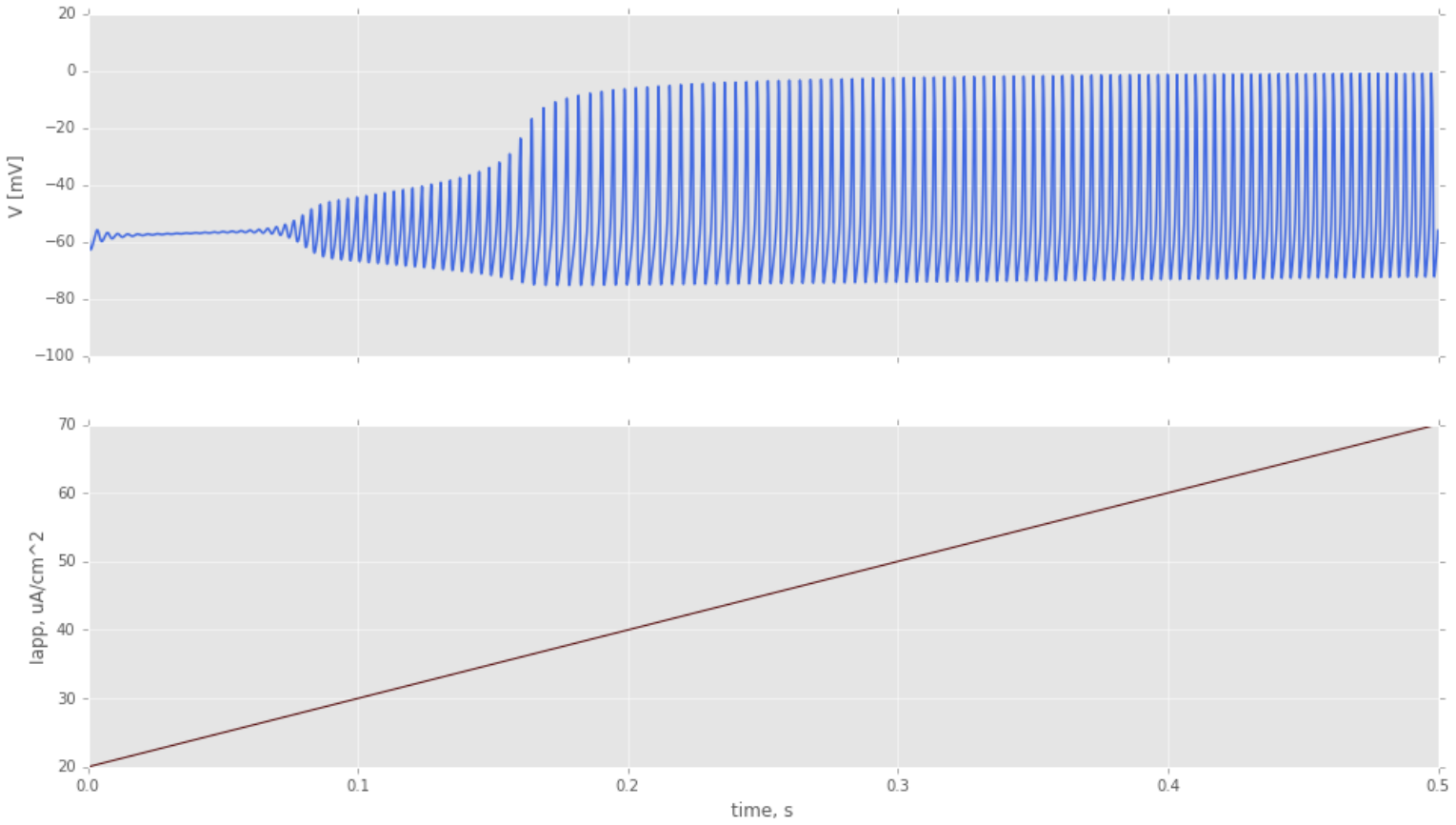


just after bifurcation

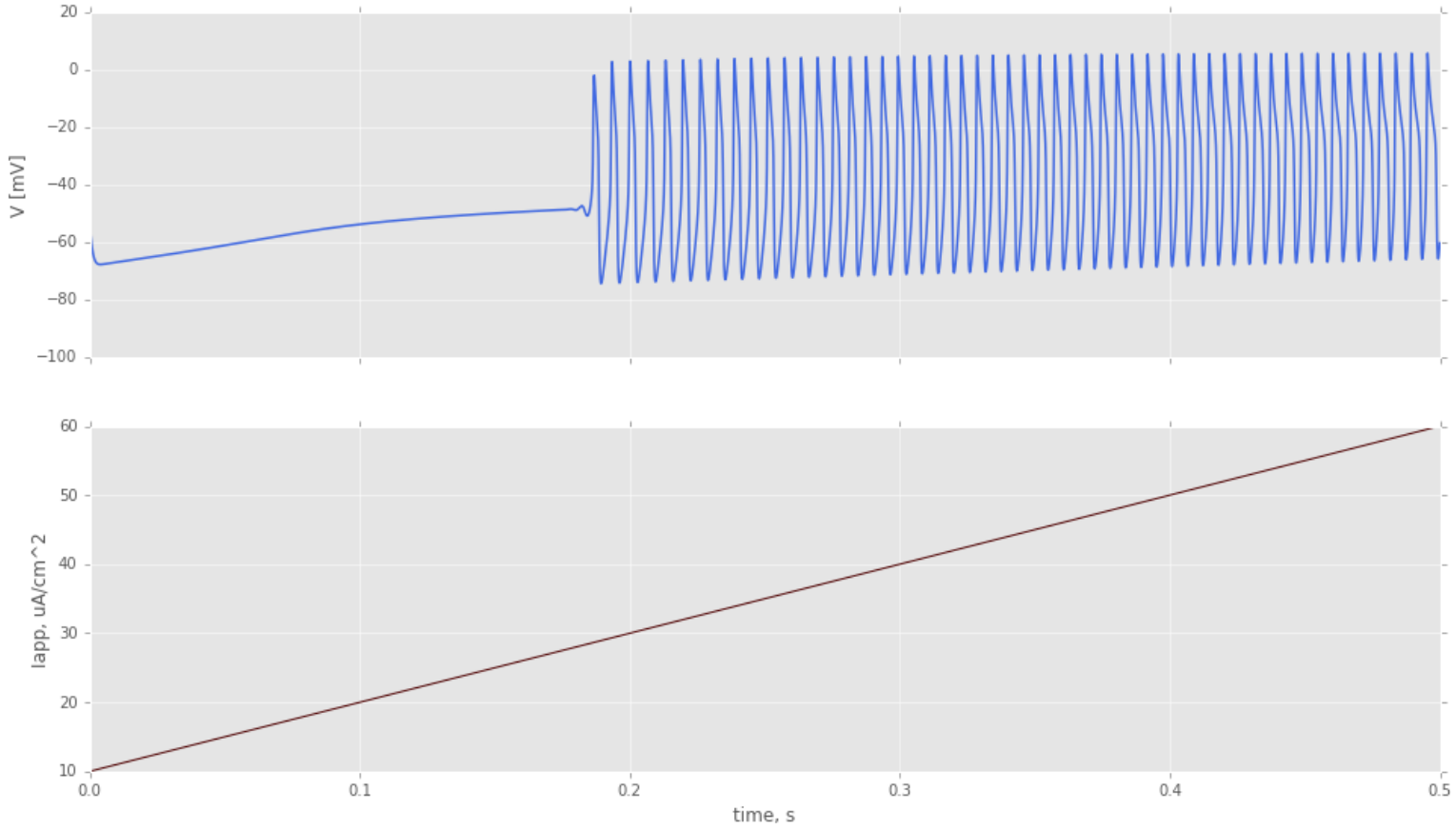




Supercritical AH bifurcation in $I_{Na,p}+I_K$ model (ramp stim)



Subcritical AH bifurcation in $I_{Na,p}+I_K$ model (ramp stim)

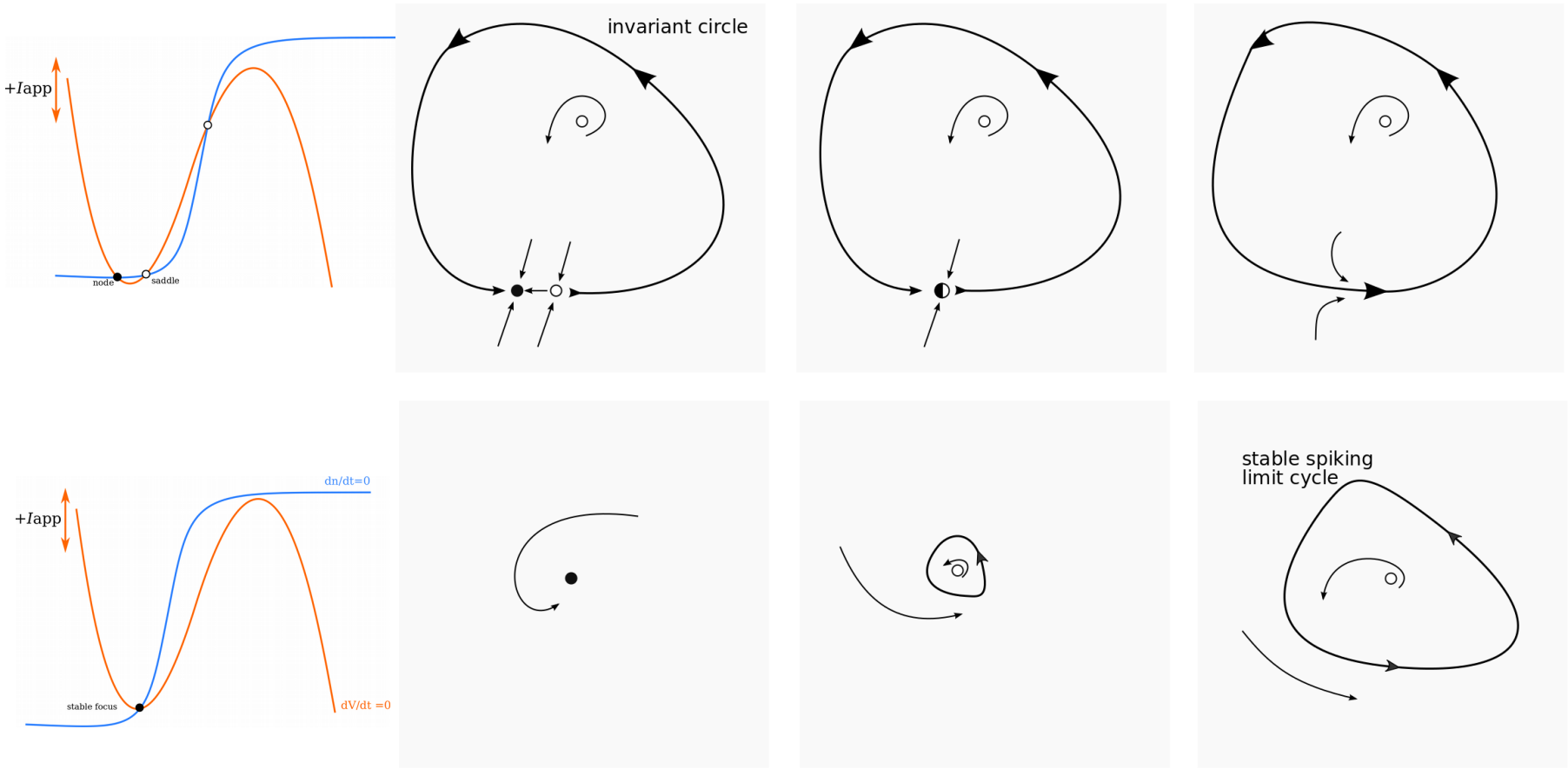


Bifurcations from the spiking state

- Saddle-node on invariant circle
- Supercritical Andronov-Hopf

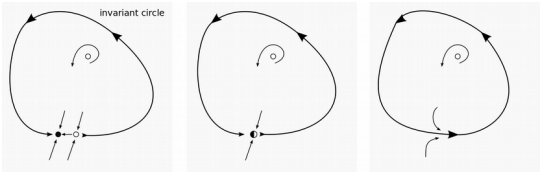
- Fold limit cycle
- Saddle homoclinic orbit

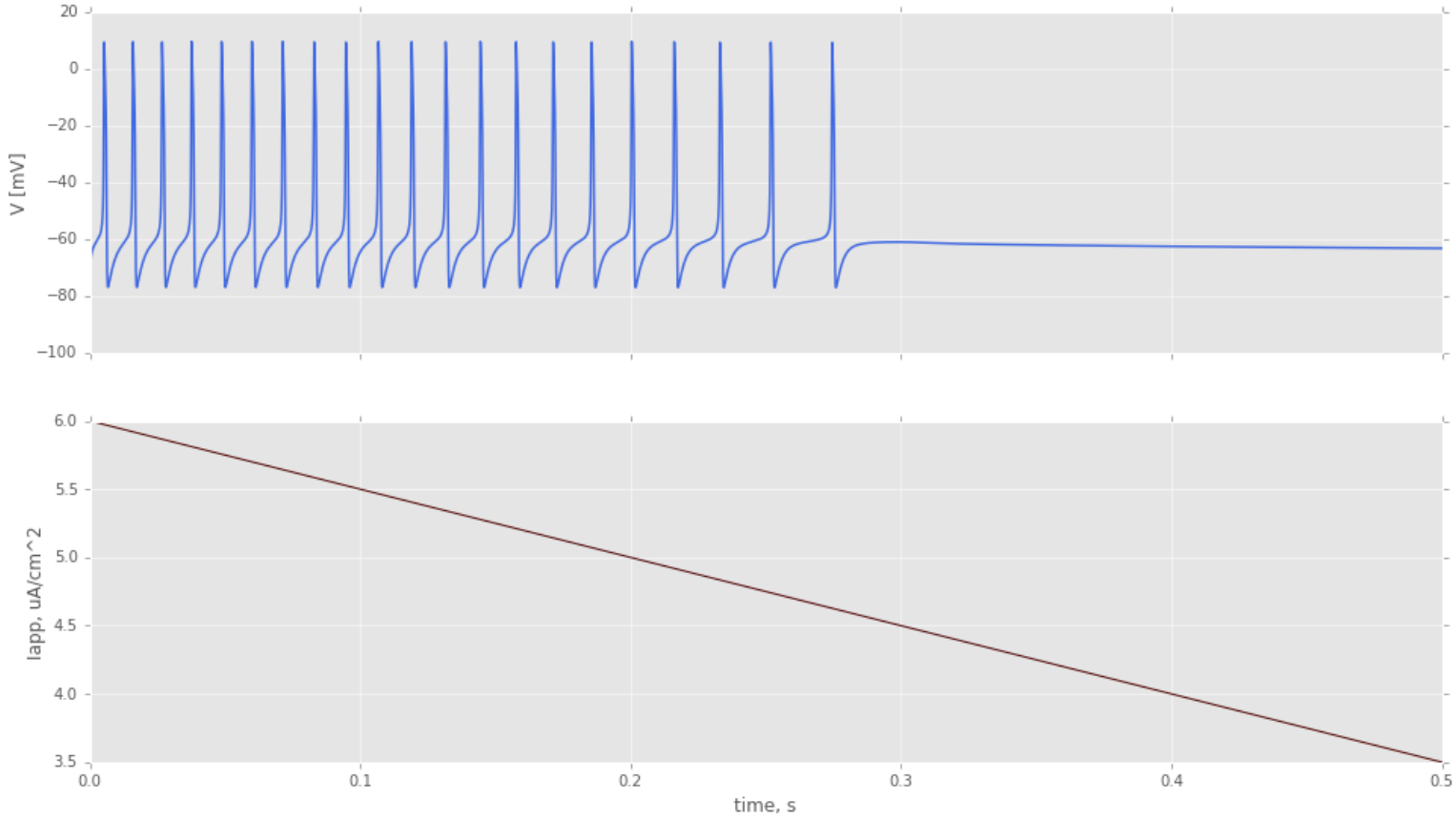
“Mirrored” bifurcations: SNIC and supercritical AH



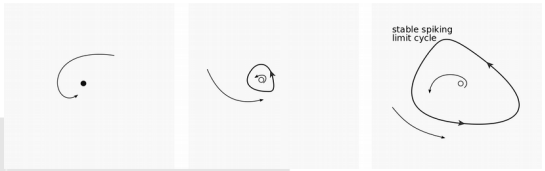
decreasing stimulation current

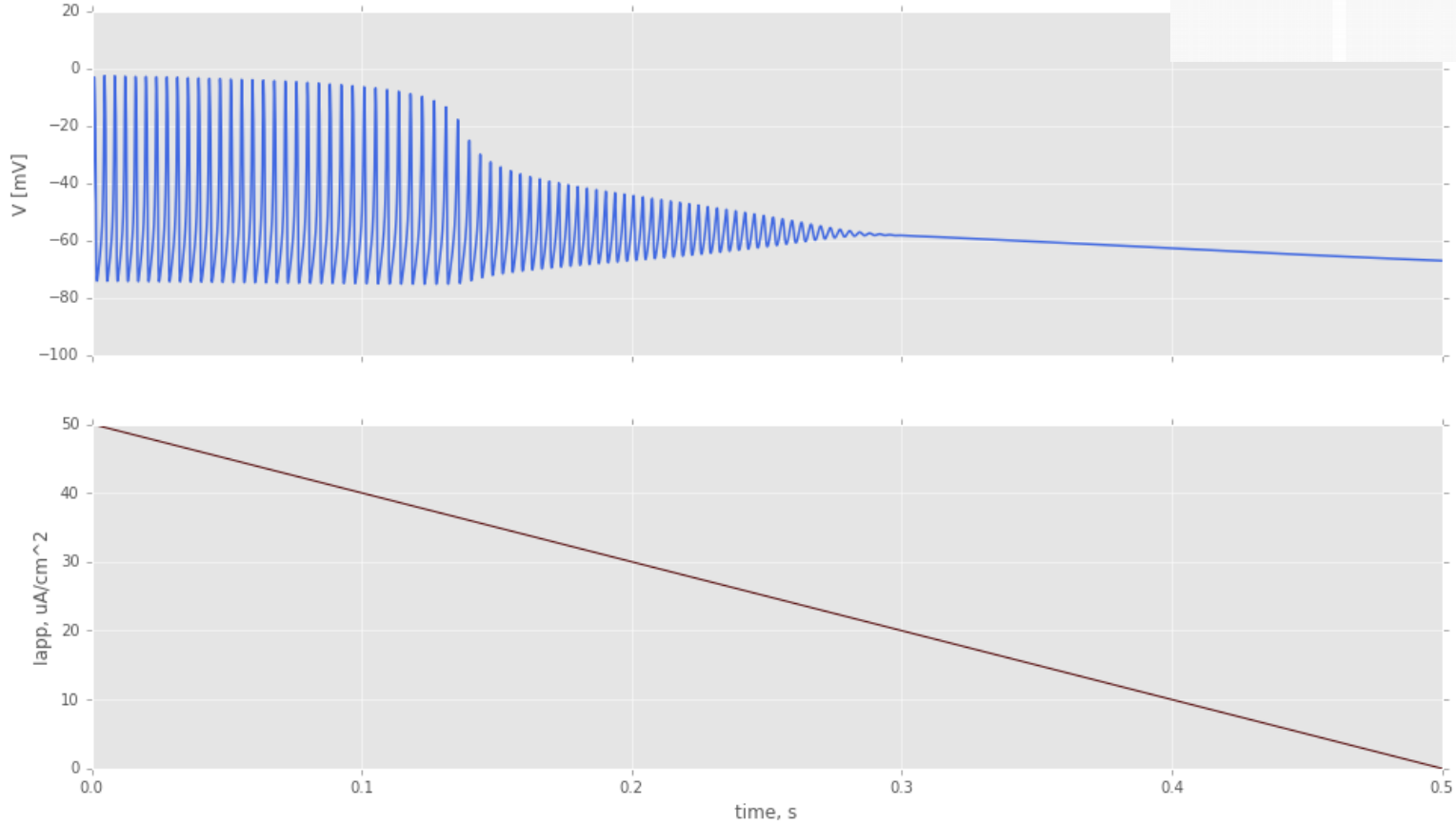
From spiking to rest via SNIC





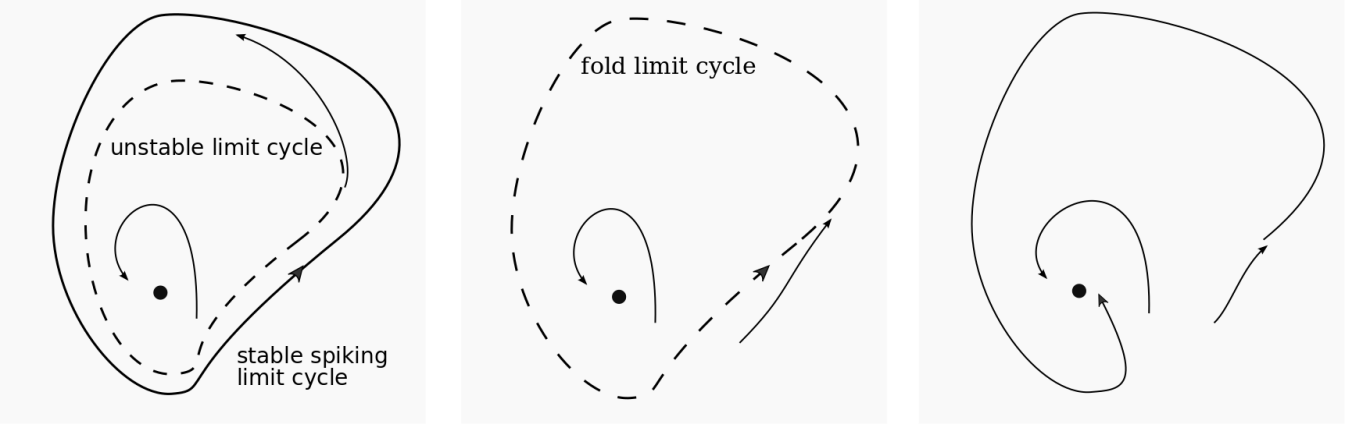
From spiking to rest via supercrititical AH

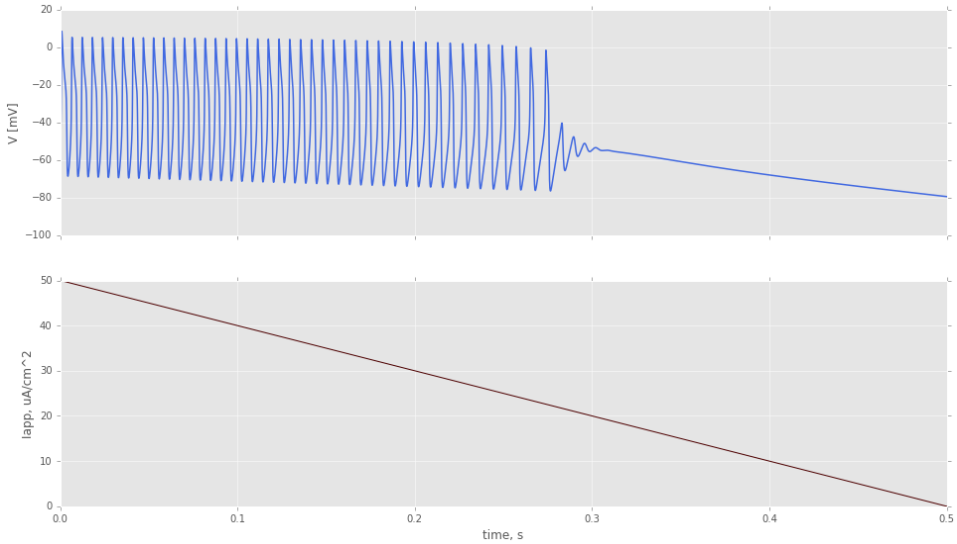




Fold limit cycle

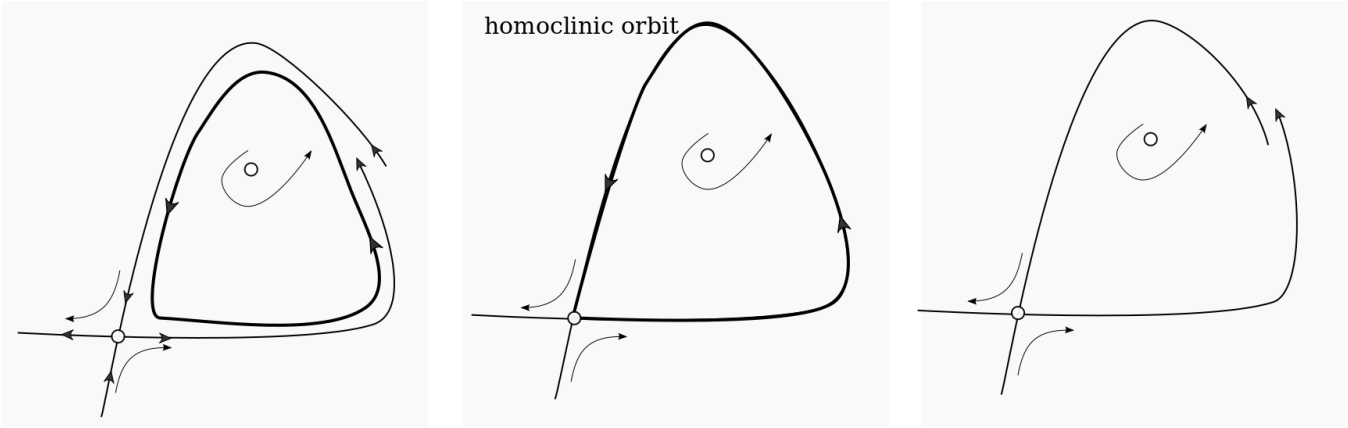
Stable and unstable limit cycles approach each other and annihilate



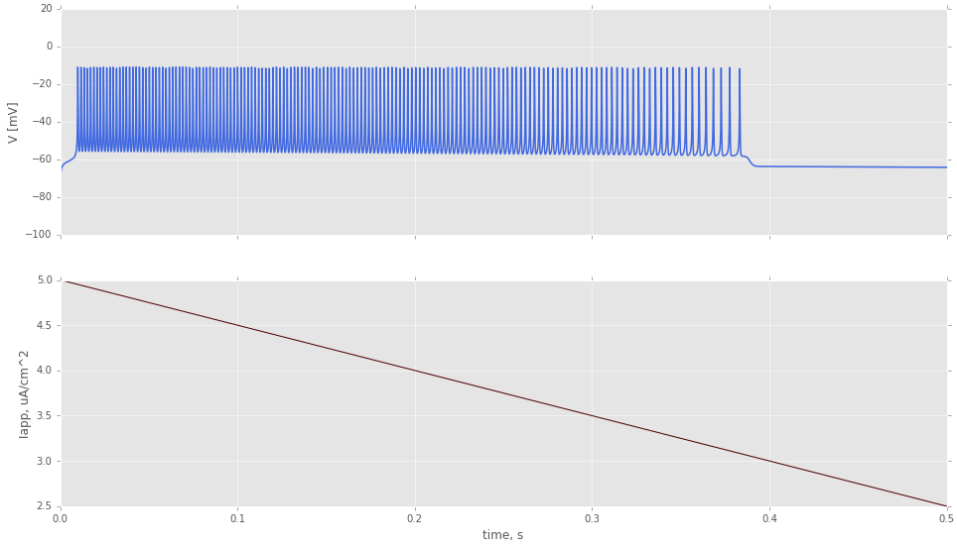


decreasing stimulation current

Homoclinic



decreasing stimulation current



Further reading

- Izhikevich E. *Dynamical Systems in Neuroscience: the Geometry of Excitability and Bursting.* MIT Press 2007
- Izhikevich E. Neural excitability, spiking and bursting. International journal of bifurcations and chaos. 2000; **10**:6, 1171 —1266
- Prescott SA, De Koninck Y, Sejnowski TJ Biophysical Basis for Three Distinct Dynamical Mechanisms of Action Potential Initiation. *PLoS Comput Biol* 2008 **4**(10): e1000198.
- Rinzel J, Huguet G. Nonlinear dynamics of neuronal excitability, oscillations and coincidence detection. *Communications on Pure and Applied Mathematics,* Vol. LXVI, 1464–1494 (2013)

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