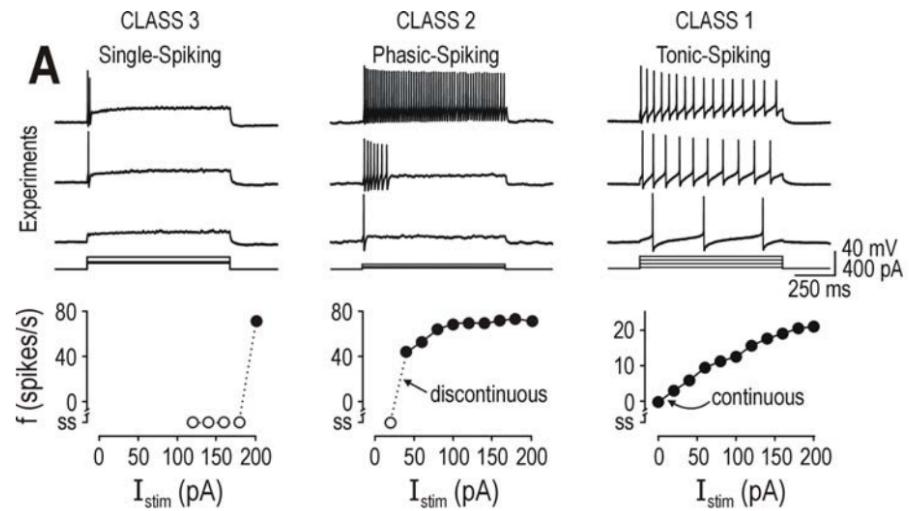
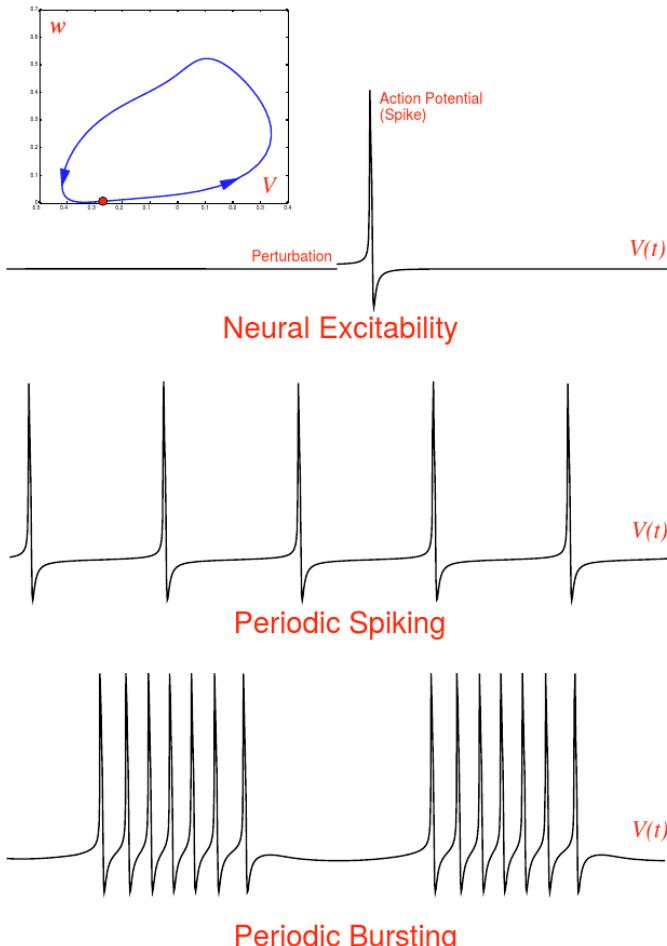


# **Bifurcations and excitability in a simple conductance-based model**

# Excitability, spiking and bursting



Prescott SA, De Koninck Y, Sejnowski TJ (2008)  
*PLoS Comput Biol* 4(10): e1000198.  
doi:10.1371/journal.pcbi.1000198

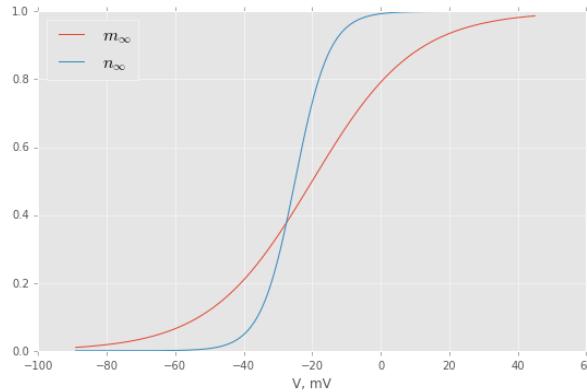
# Instantaneous persistent sodium and potassium current model ( $I_{Na,p} + I_K$ )

System:

$$C\dot{V} = I - \bar{g}_K n(V - E_K) - \bar{g}_{Na} m_\infty(V)(V - E_{Na}) - g_l(V - E_l)$$

$$\tau_n \dot{n} = (n_\infty(V) - n)$$

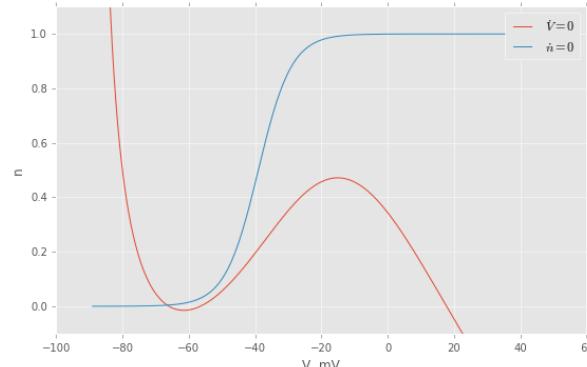
$$x_\infty = \frac{1}{1 + \exp(\frac{V_x^{0.5} - V}{k_x})}$$



Nullclines:

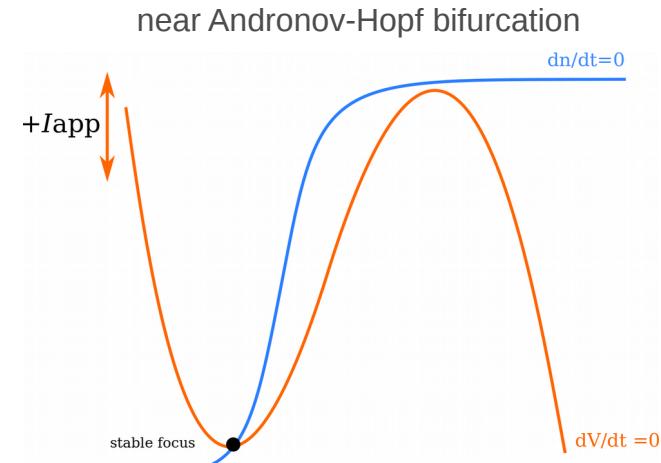
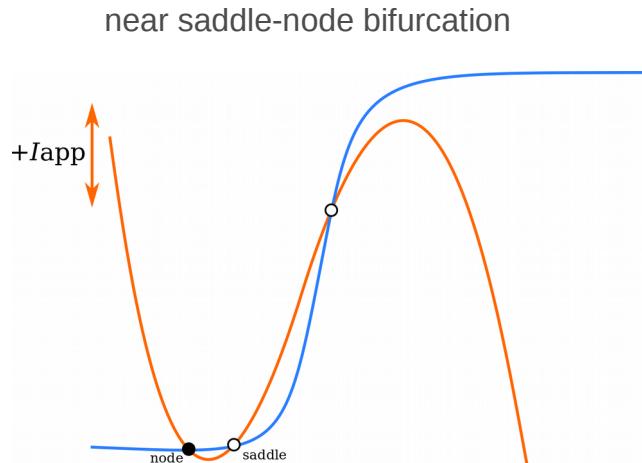
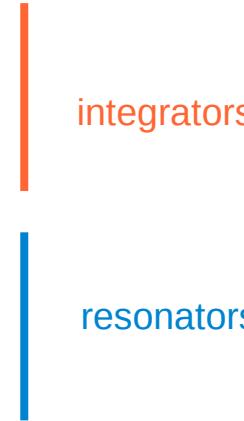
$$\dot{V} = 0 \rightarrow n(V) = \frac{I - \bar{g}_{Na} m_\infty(V - E_{Na}) - g_l(V - E_l)}{\bar{g}_k(V - E_k)}$$

$$\dot{n} = 0 \rightarrow n(V) = n_\infty(V)$$



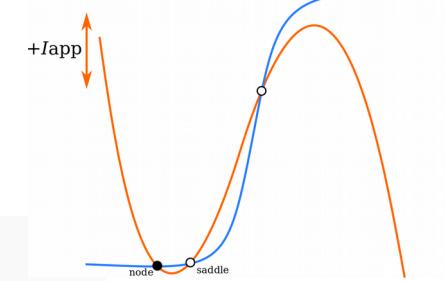
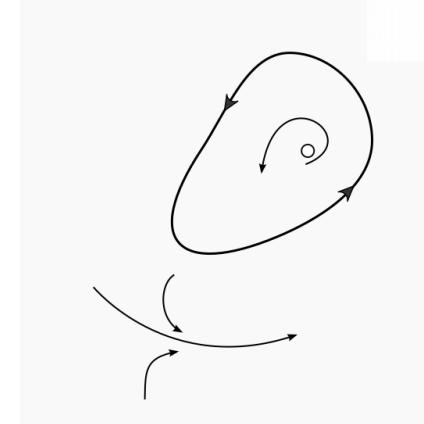
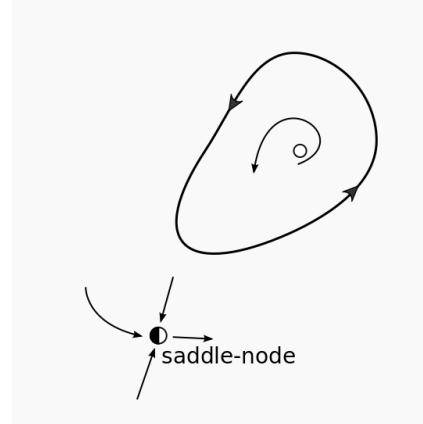
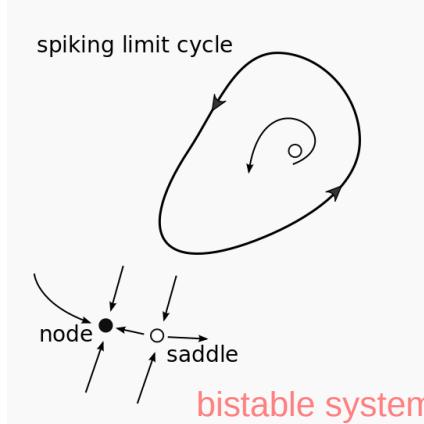
# Bifurcations from the stable/quiescent state

- Saddle-node
- Saddle-node on invariant circle
- Supercritical Andronov-Hopf
- Subcritical Andronov-Hopf

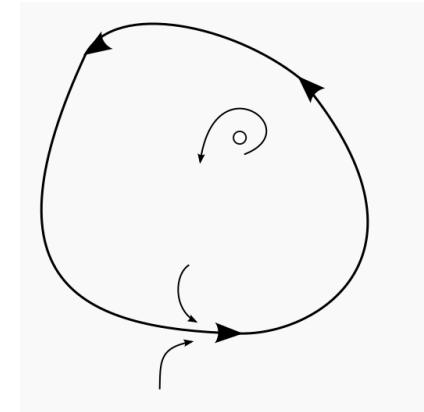
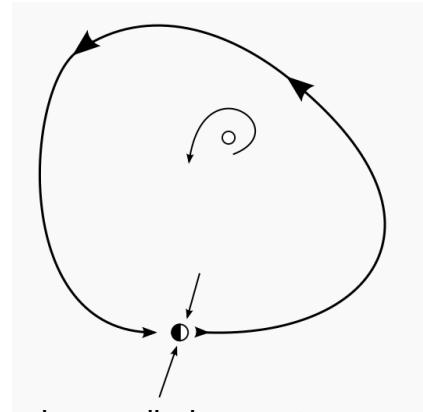
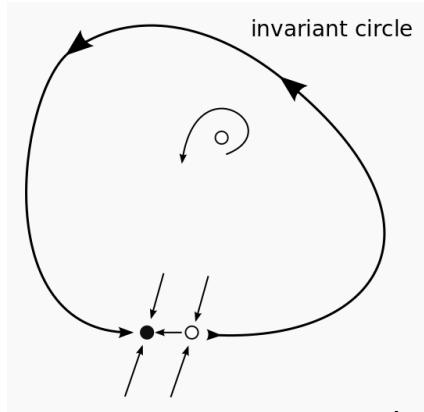


# Saddle-node bifurcations

Saddle-node



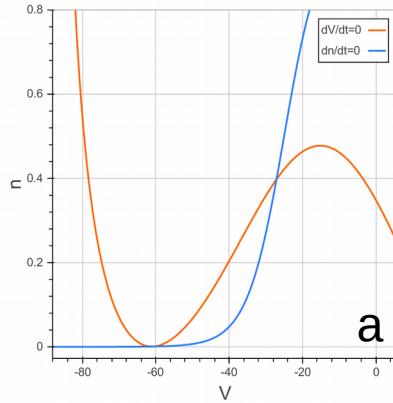
SNIC



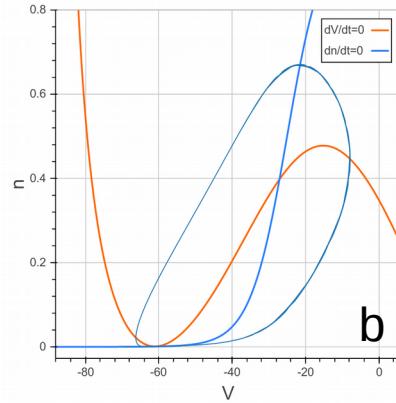
Increasing applied current

# Example of SNIC in $I_{Na,p} + I_K$ model

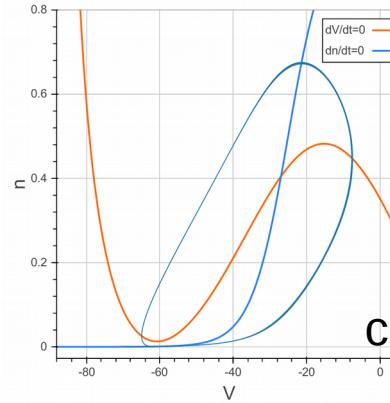
just before bifurcation



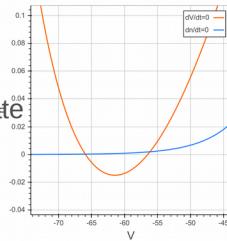
just after bifurcation



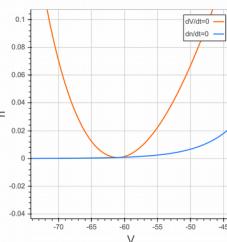
large current



at resting state



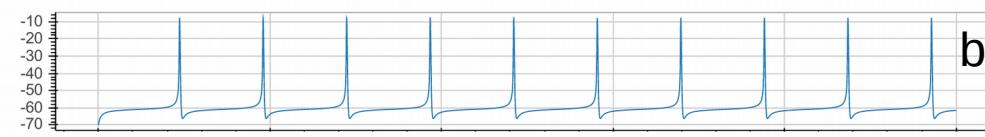
at bifurcation



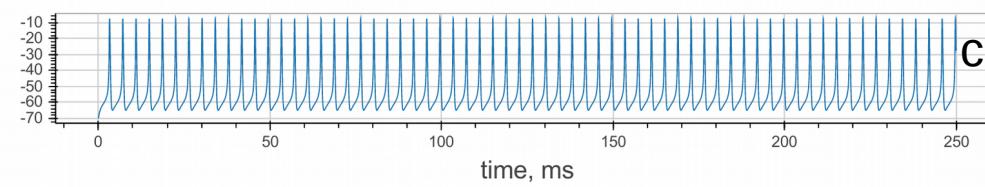
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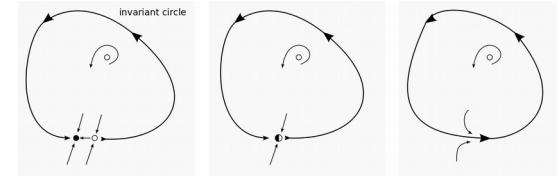
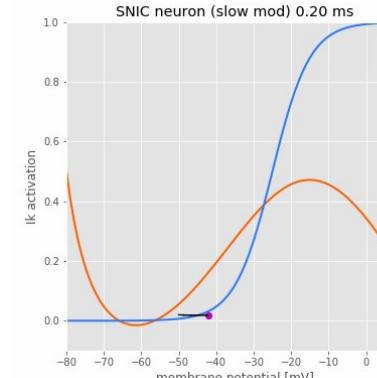
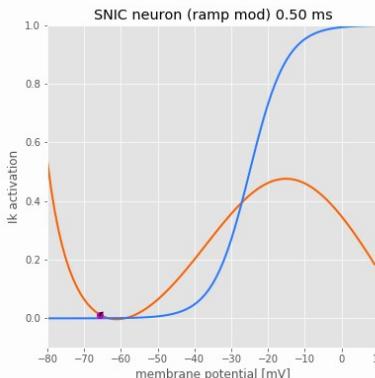
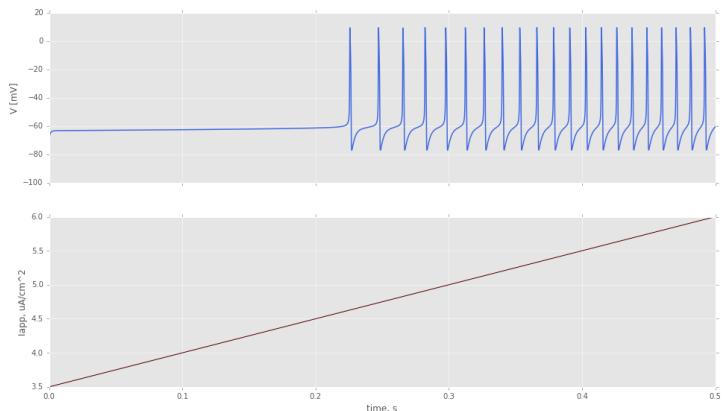
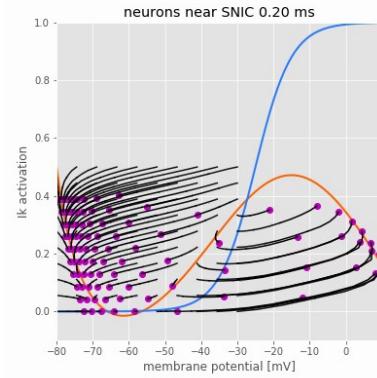
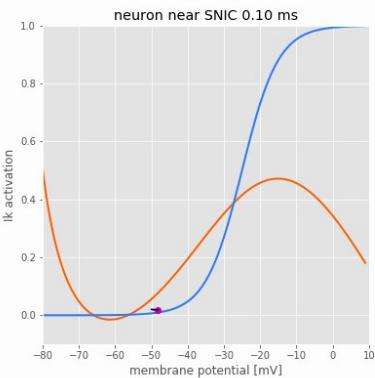
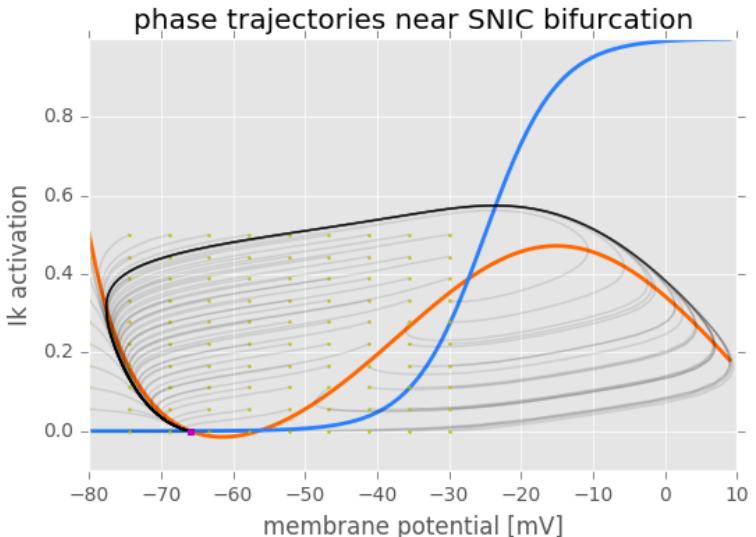
b



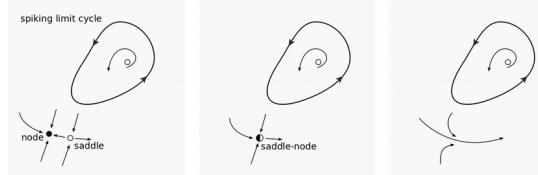
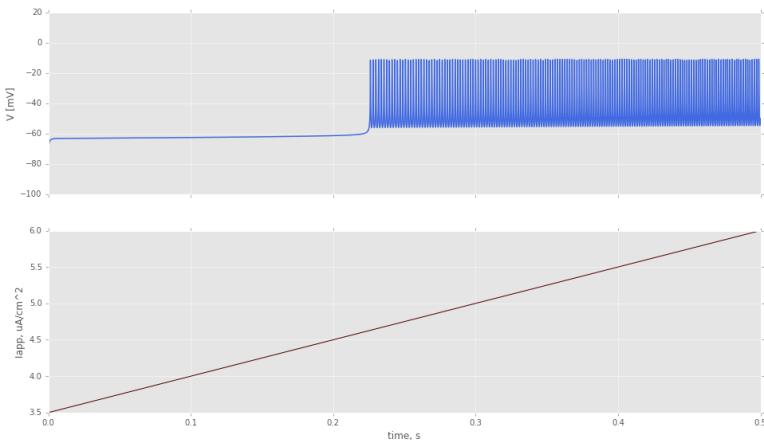
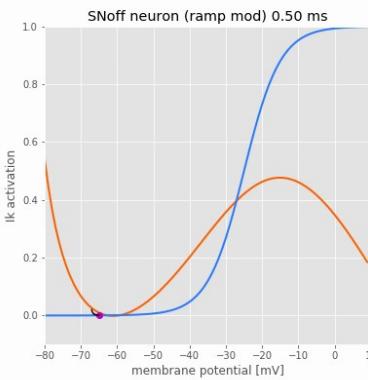
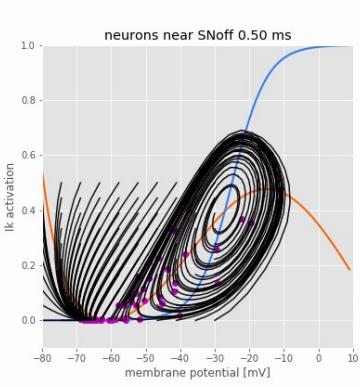
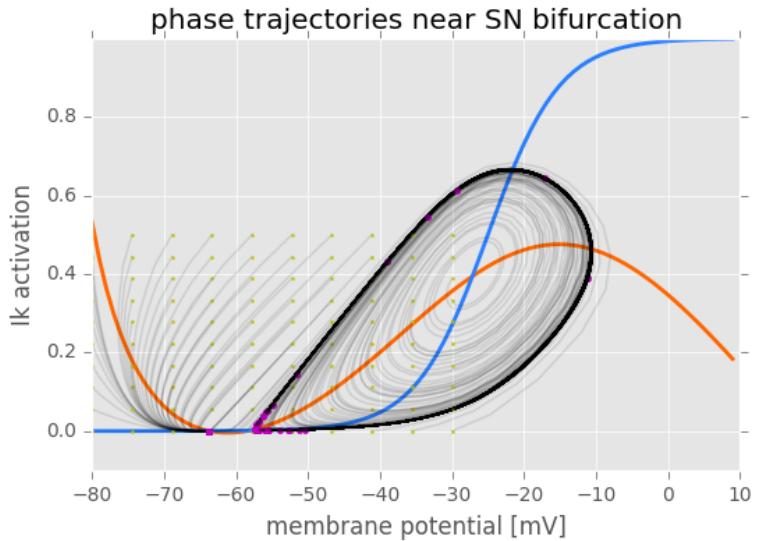
c



# Behavior near SNIC

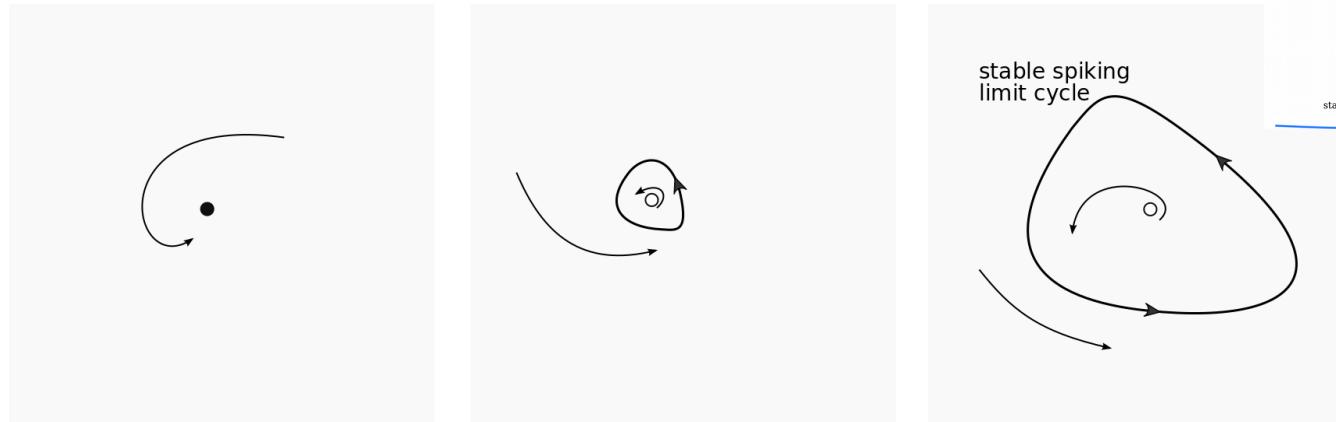


# Behavior near S<sub>N</sub>off

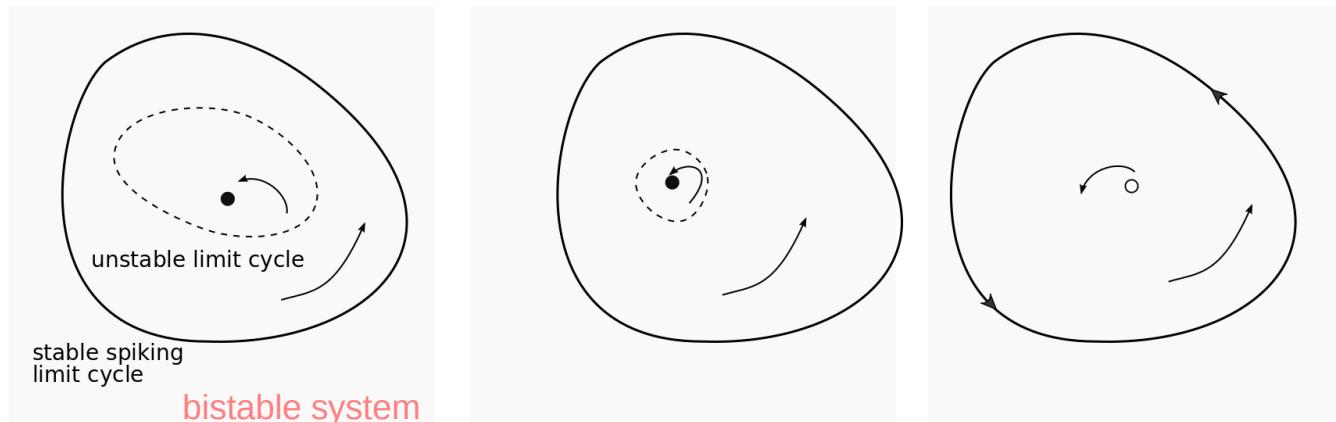


# Andronov-Hopf bifurcations

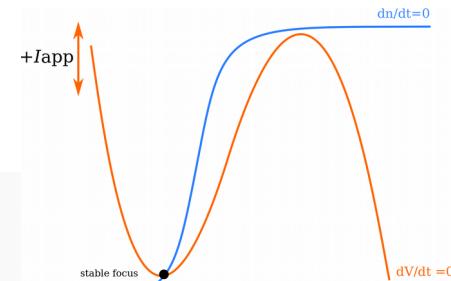
supercritical



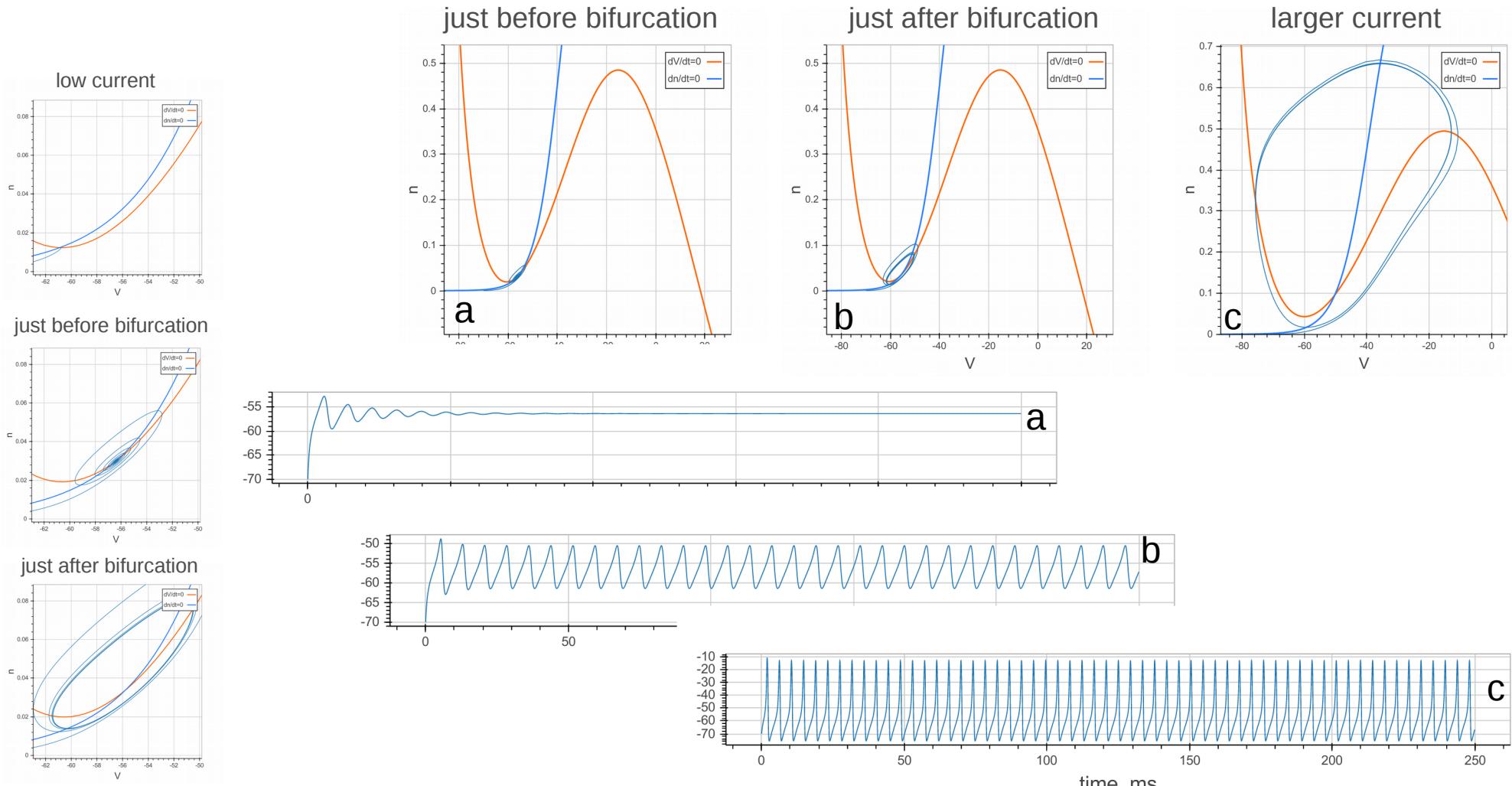
subcritical



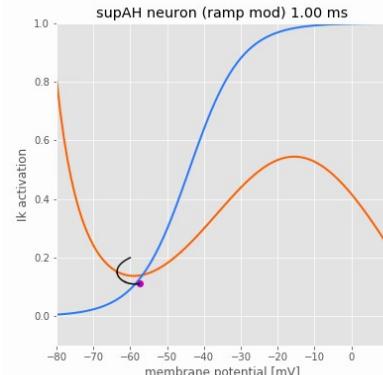
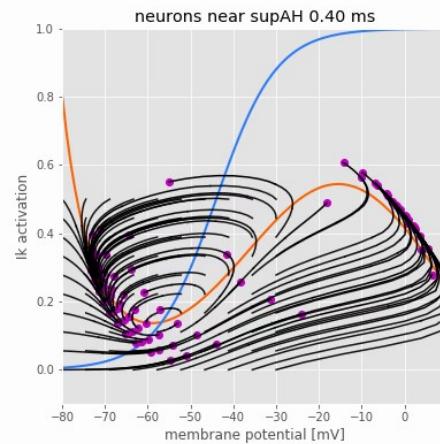
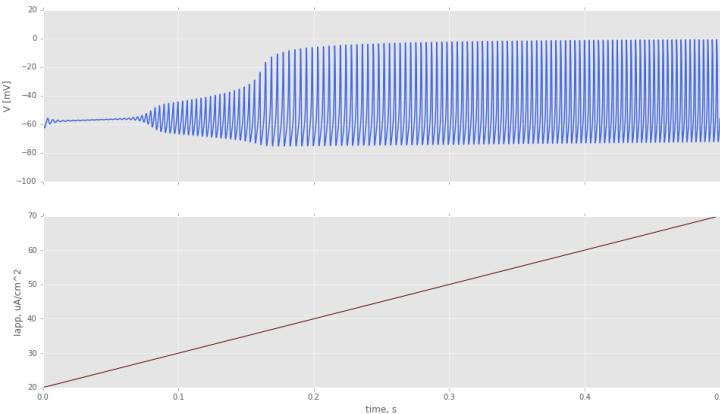
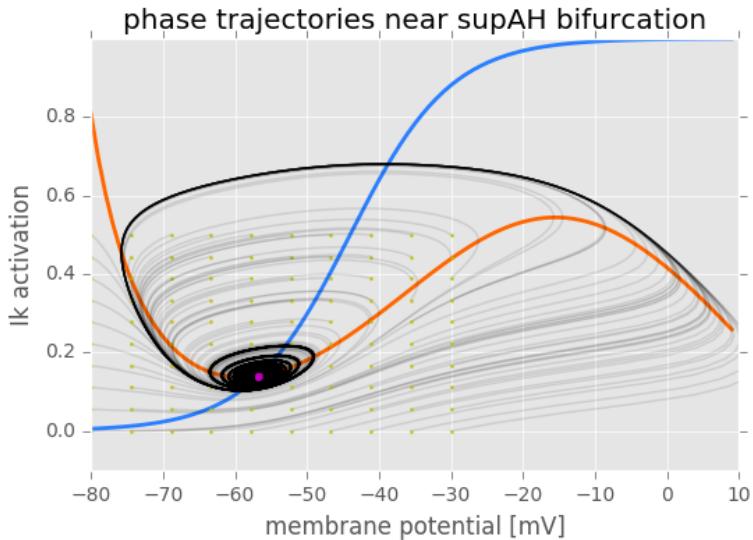
Increasing applied current



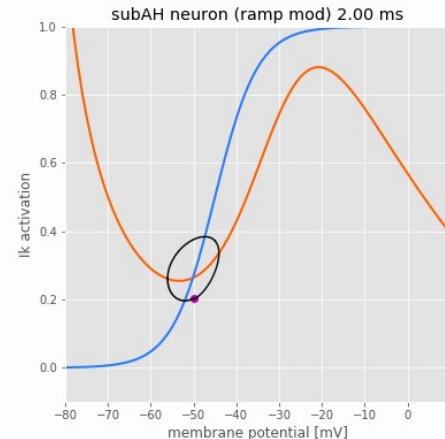
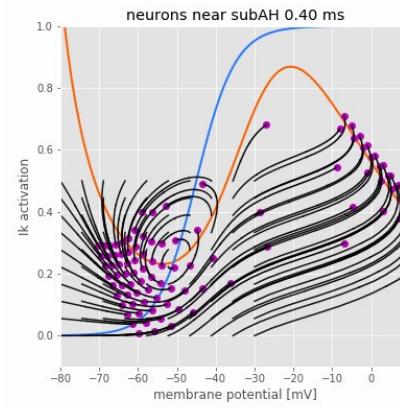
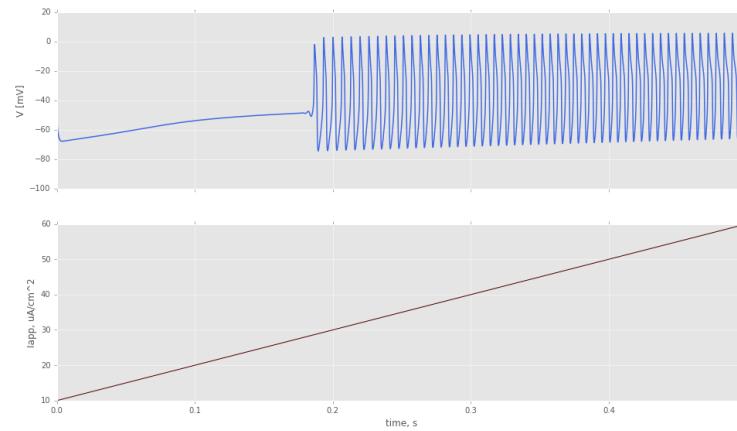
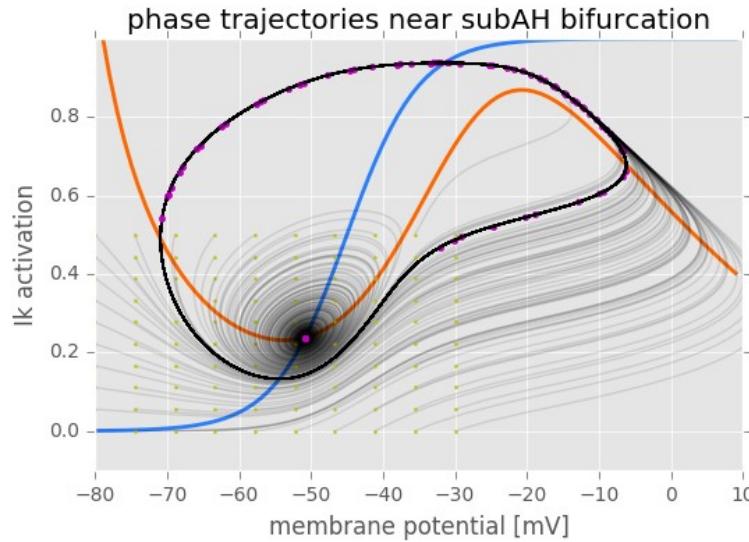
# Supercritical AH bifurcation in $I_{Na,p} + I_K$ model (low-threshold K current)



# Behavior near supAH



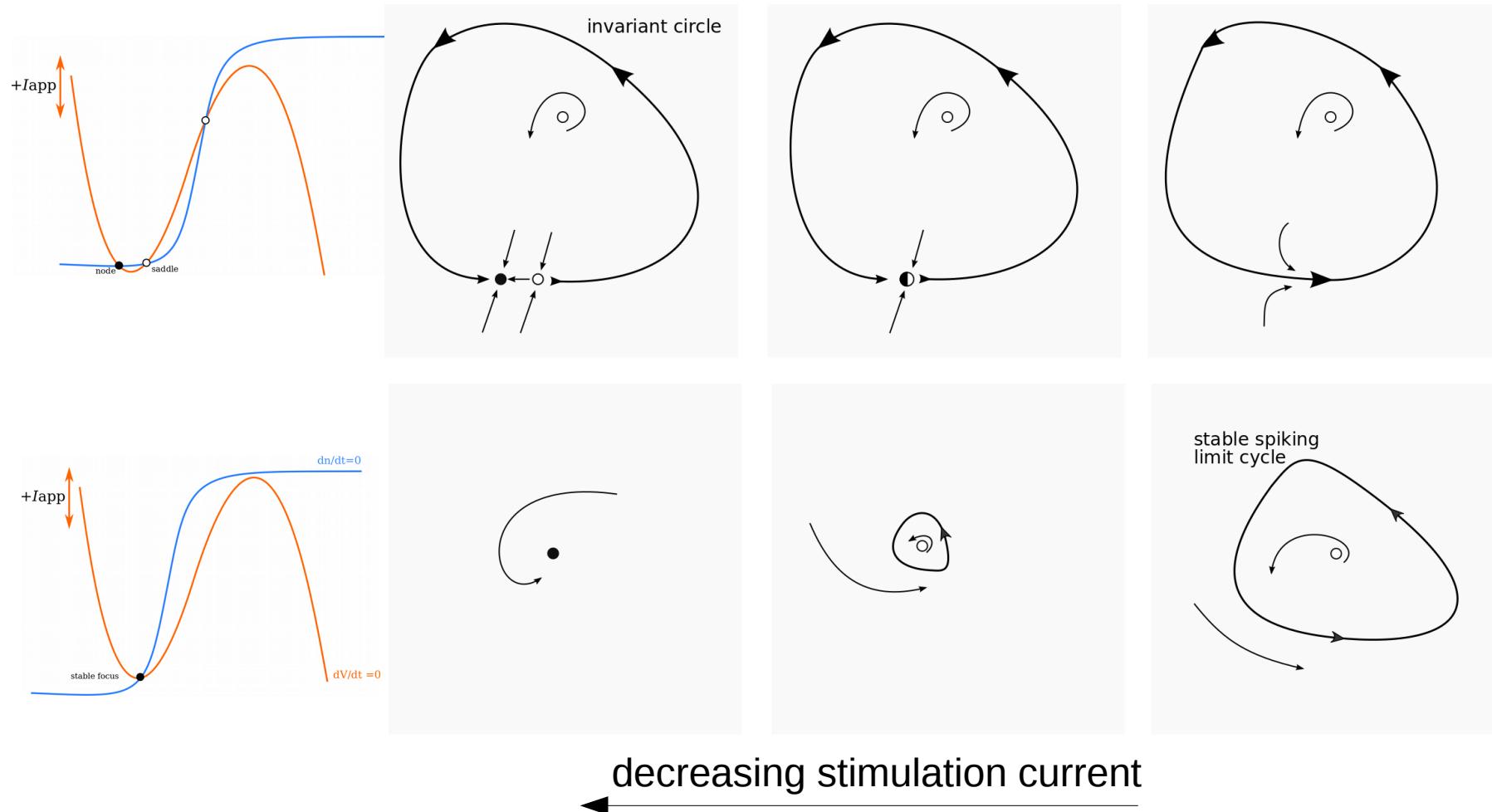
# Behavior near subAH



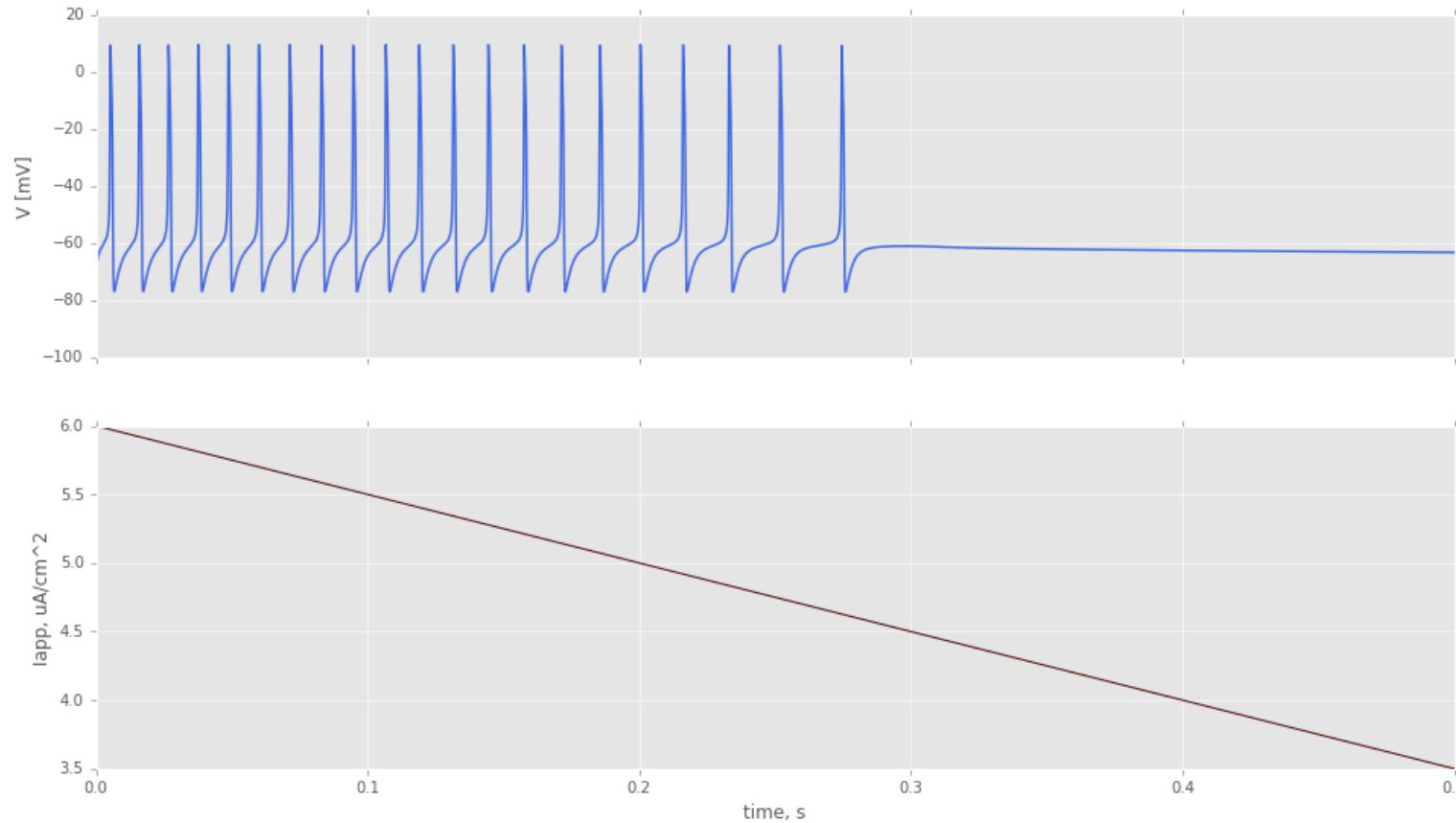
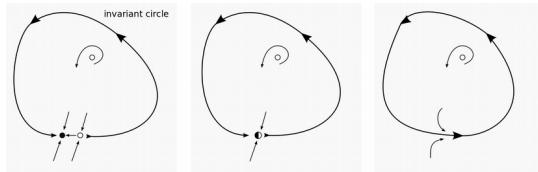
# Bifurcations from the spiking state

- Saddle-node on invariant circle
- Supercritical Andronov-Hopf
- Fold limit cycle
- Saddle homoclinic orbit

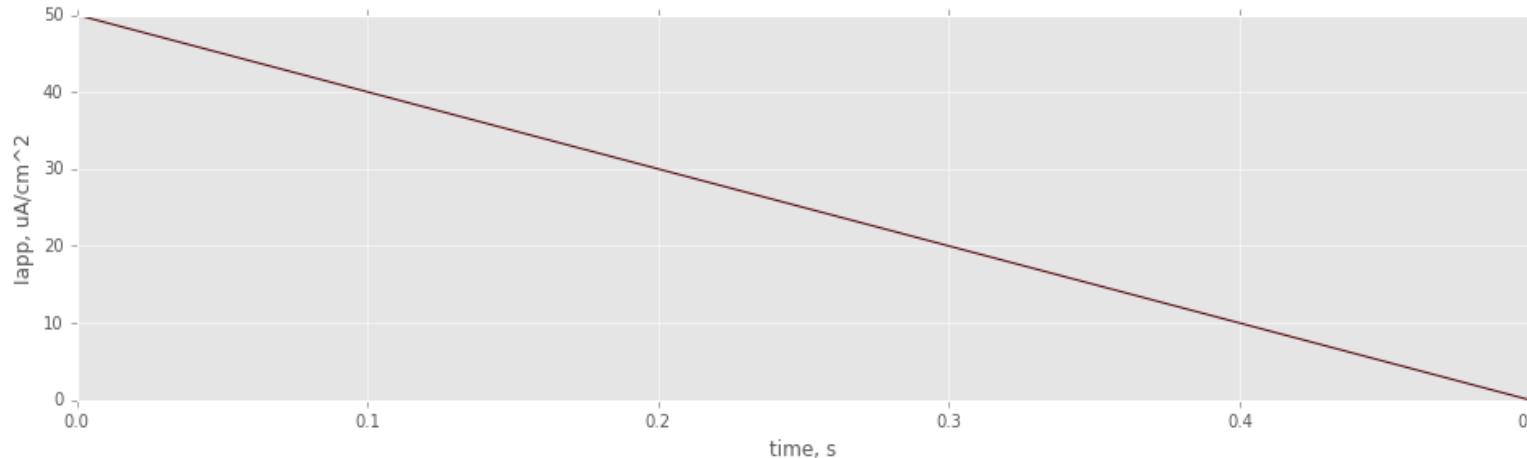
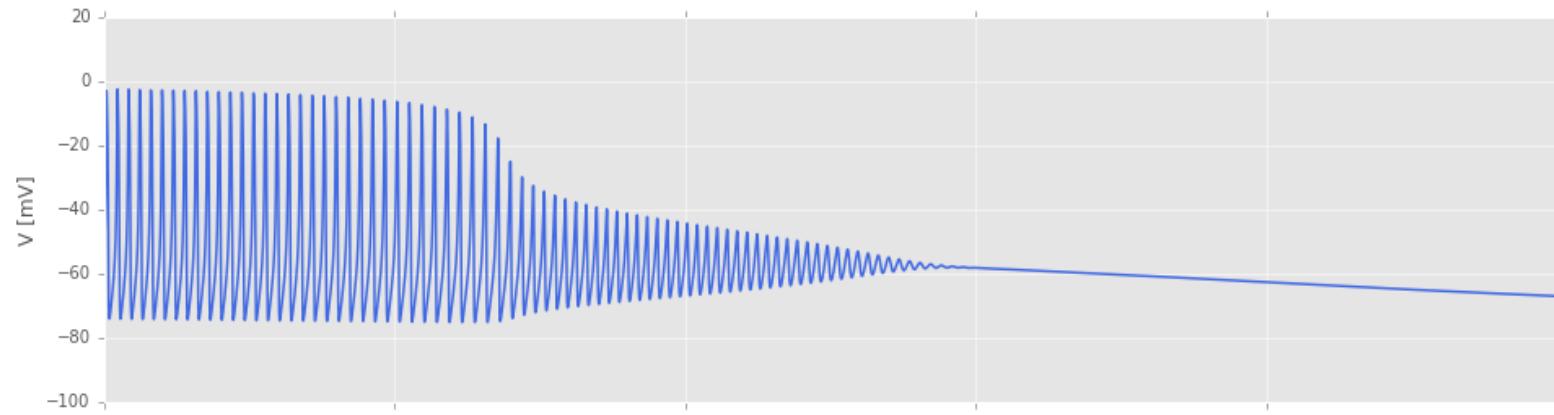
# “Mirrored” bifurcations: SNIC and supercritical AH



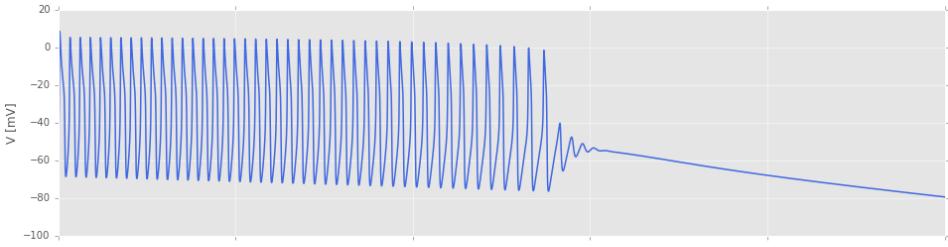
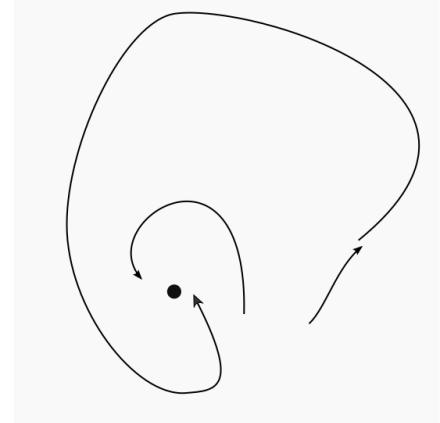
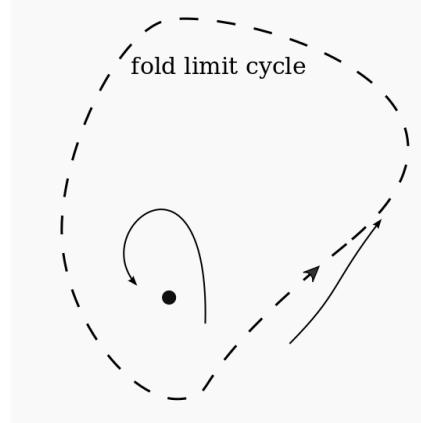
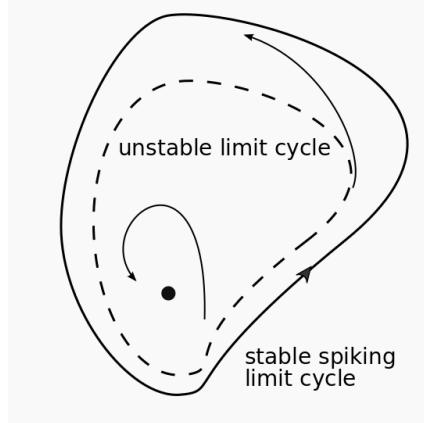
# From spiking to rest via SNIC



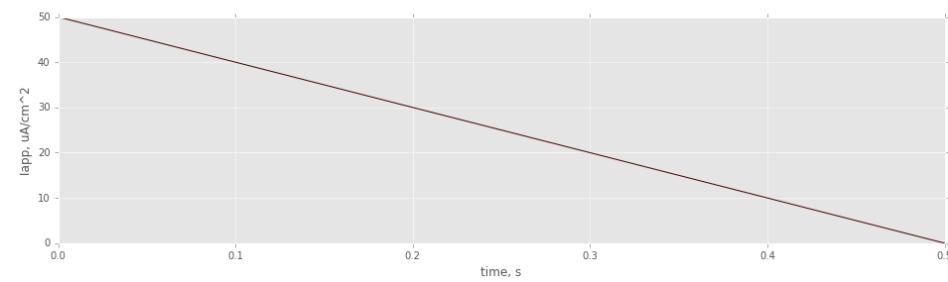
# From spiking to rest via supercrititical AH



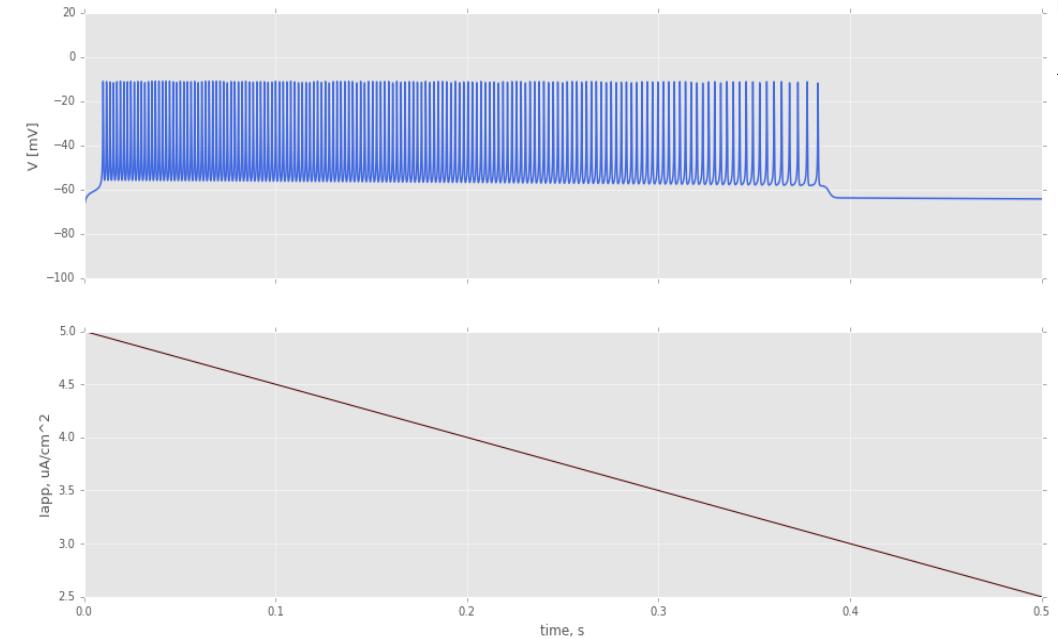
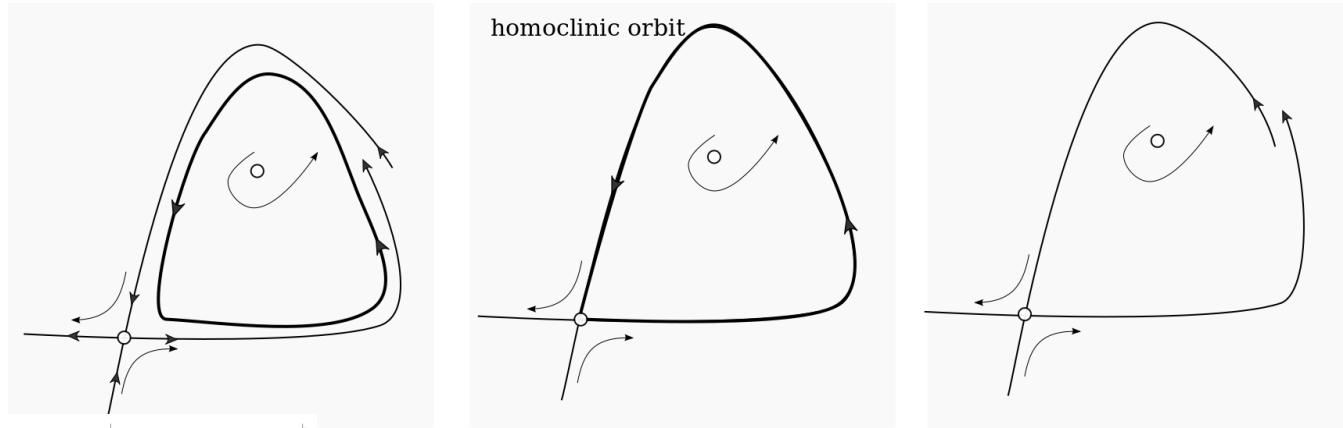
# Fold limit cycle



decreasing stimulation current



# Homoclinic

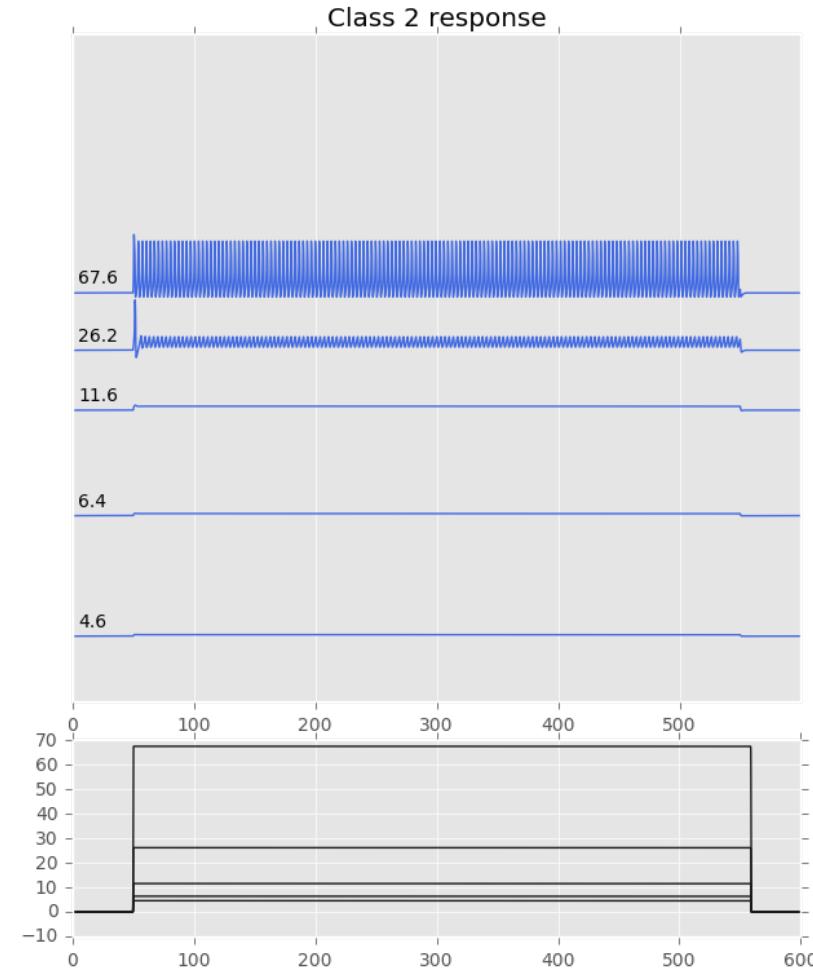
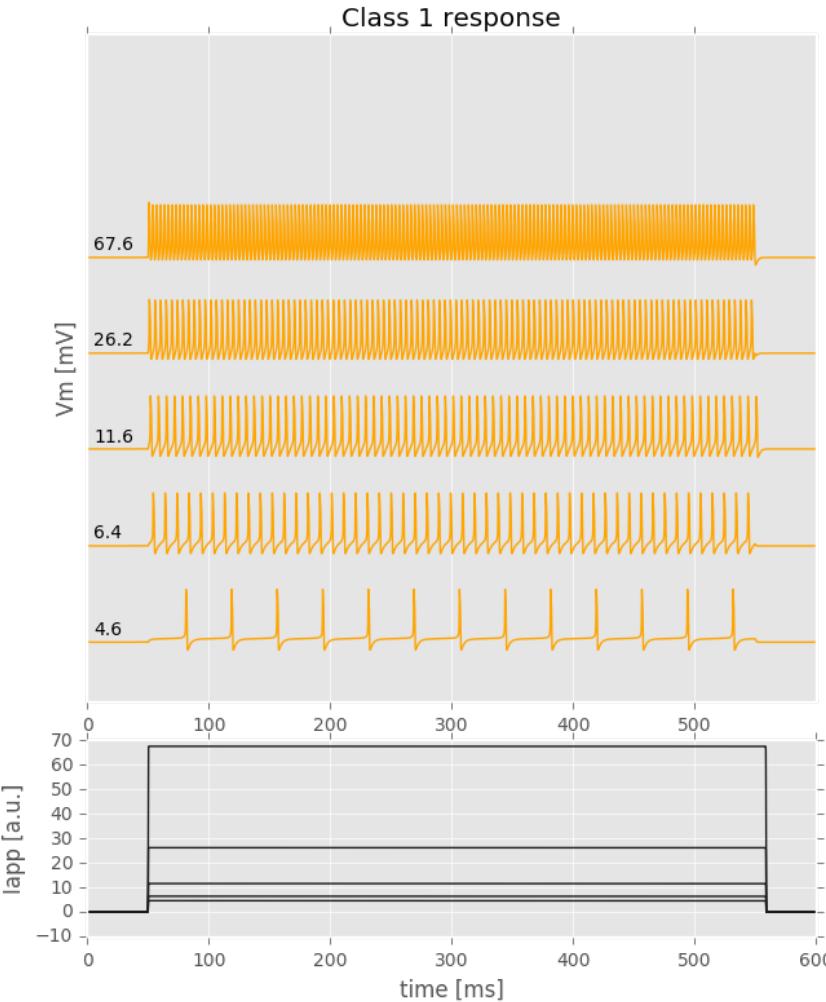


decreasing stimulation current

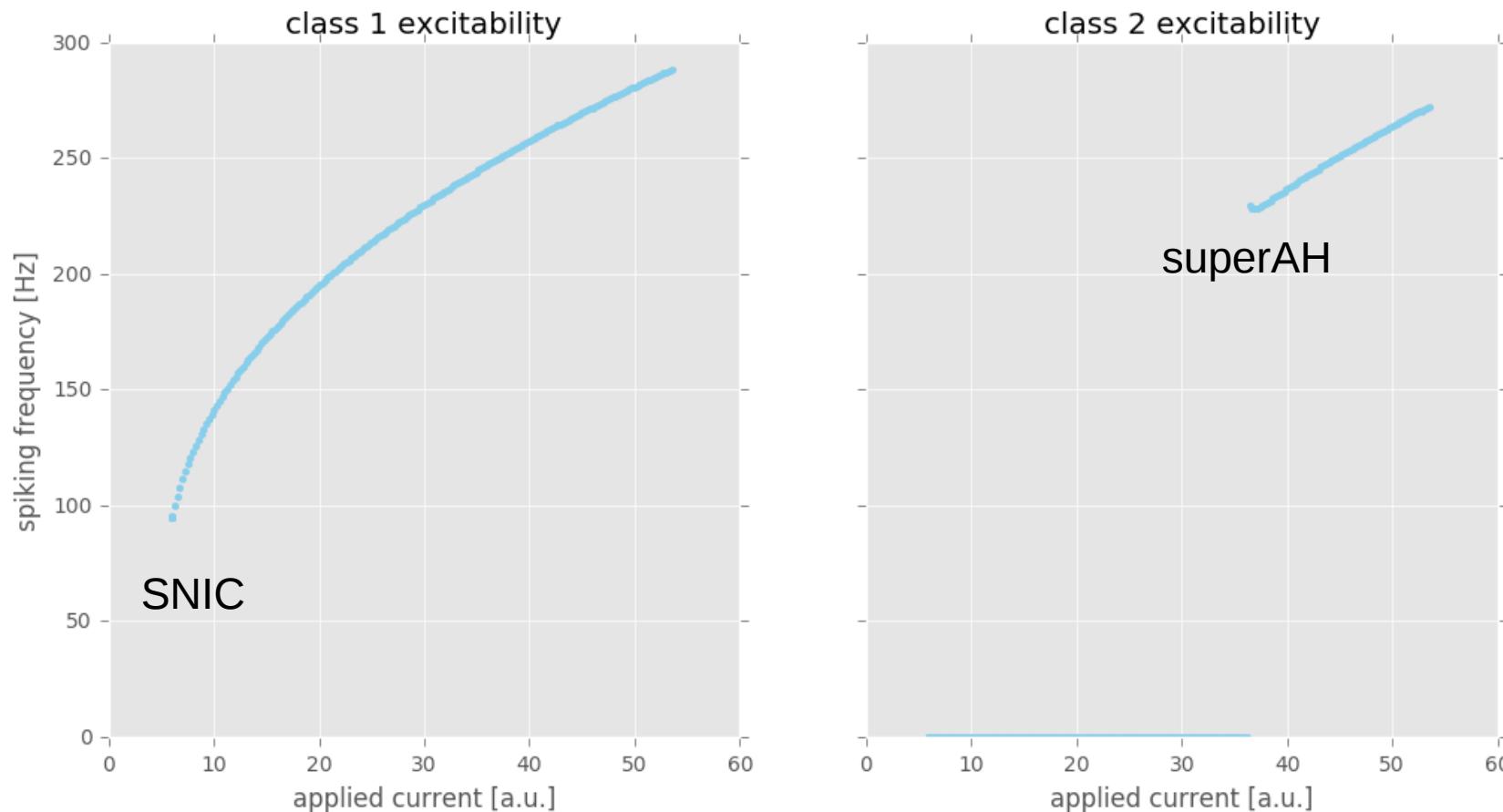


- Izhikevich E. *Dynamical Systems in Neuroscience: the Geometry of Excitability and Bursting*. MIT Press 2007
- Izhikevich E. Neural excitability, spiking and bursting. International journal of bifurcations and chaos. 2000; **10**:6, 1171 —1266
- Prescott SA, De Koninck Y, Sejnowski TJ Biophysical Basis for Three Distinct Dynamical Mechanisms of Action Potential Initiation. *PLoS Comput Biol* 2008 **4**(10): e1000198.
- Rinzel J, Huguet G. Nonlinear dynamics of neuronal excitability, oscillations and coincidence detection. *Communications on Pure and Applied Mathematics*, Vol. LXVI, 1464–1494 (2013)

# Hodgkin classification of excitability ( $I_{Na,p}+I_K$ model)

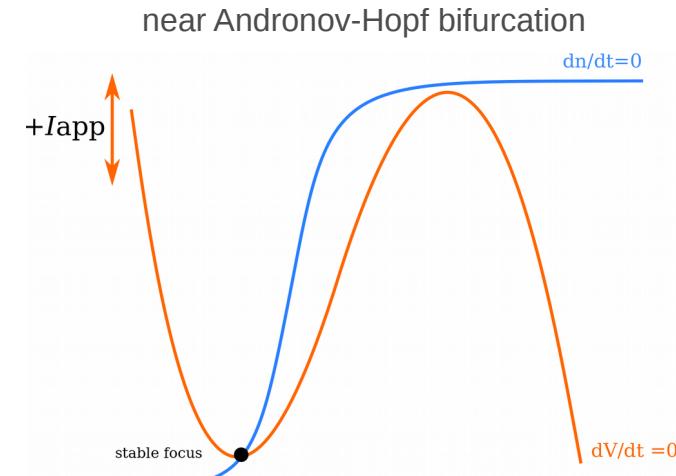
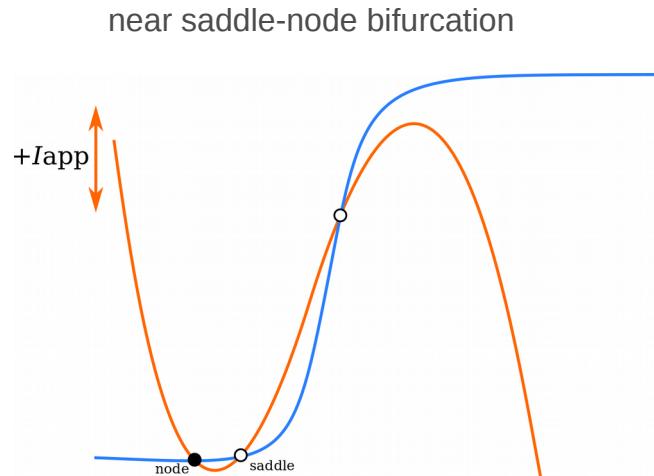
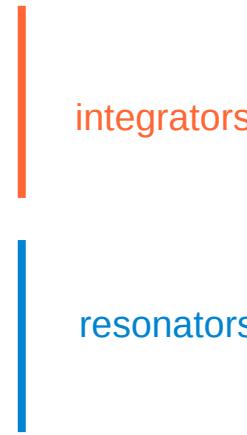


# Hodgkin classification of excitability ( $I_{Na,p} + I_K$ model)

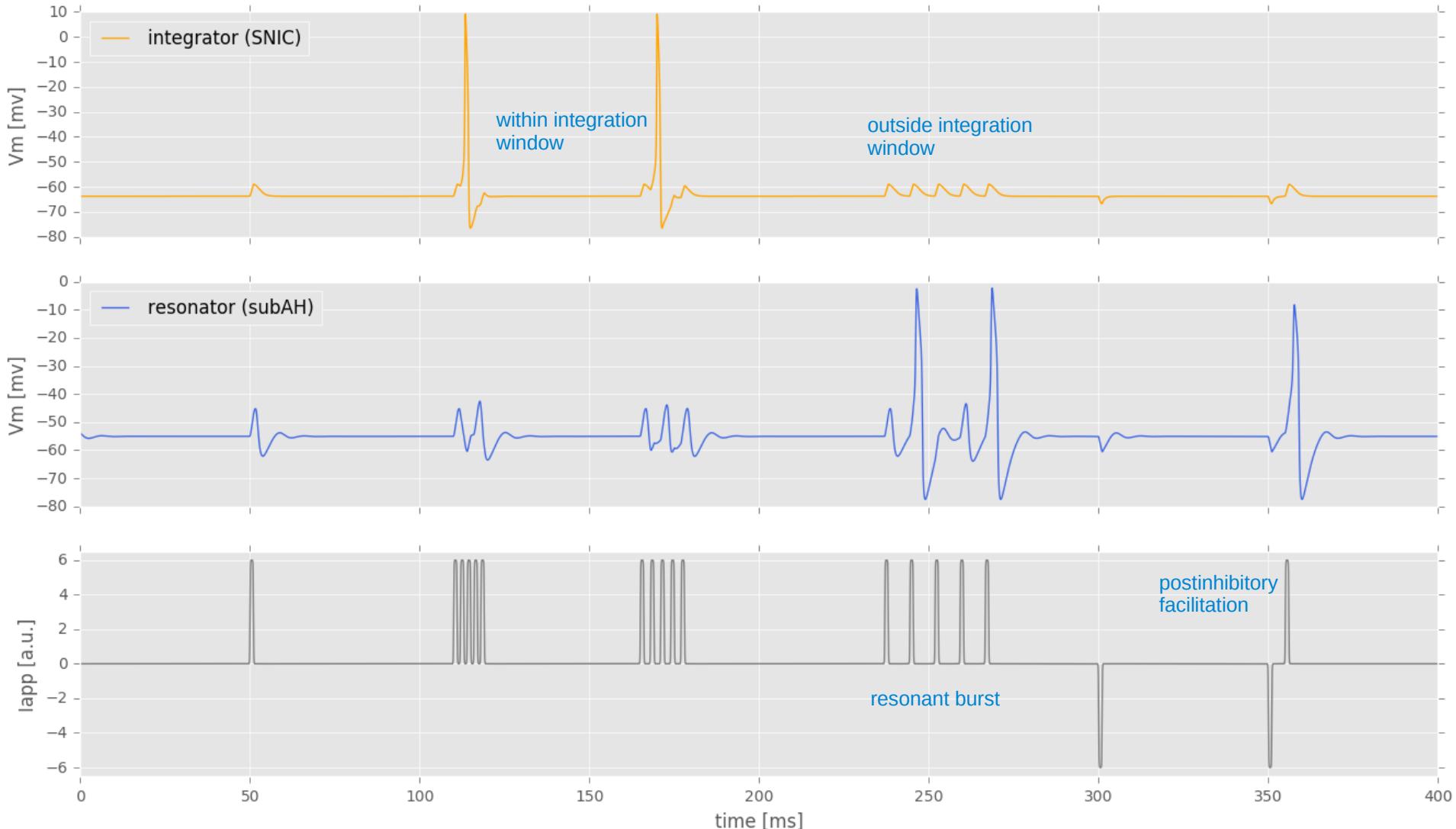


# Bifurcations from the stable/quiescent state

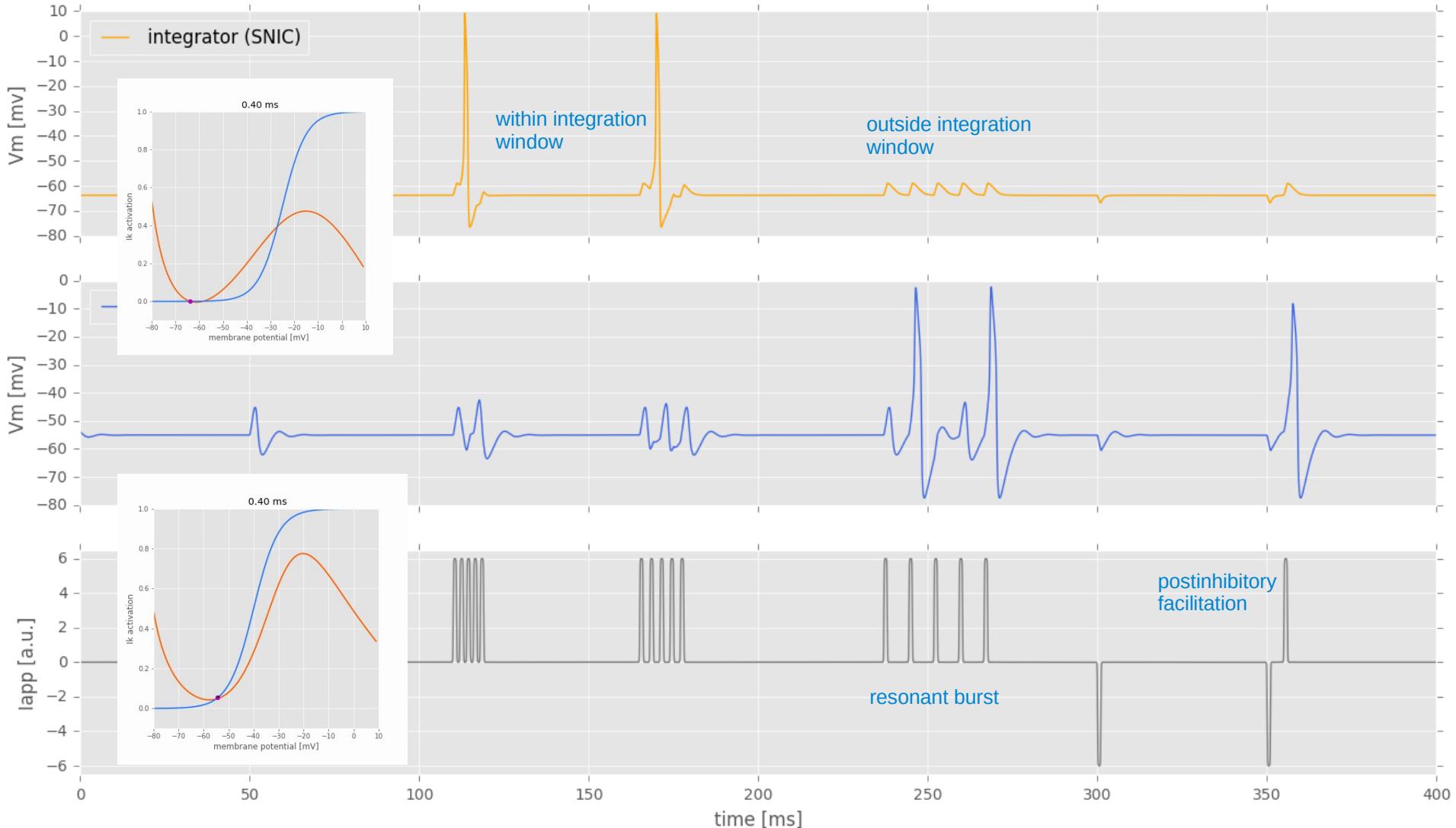
- Saddle-node
- Saddle-node on invariant circle
- Supercritical Andronov-Hopf
- Subcritical Andronov-Hopf



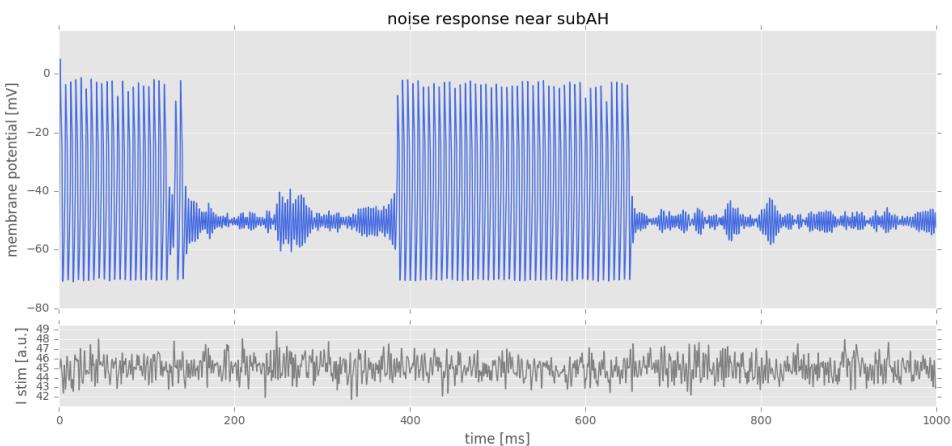
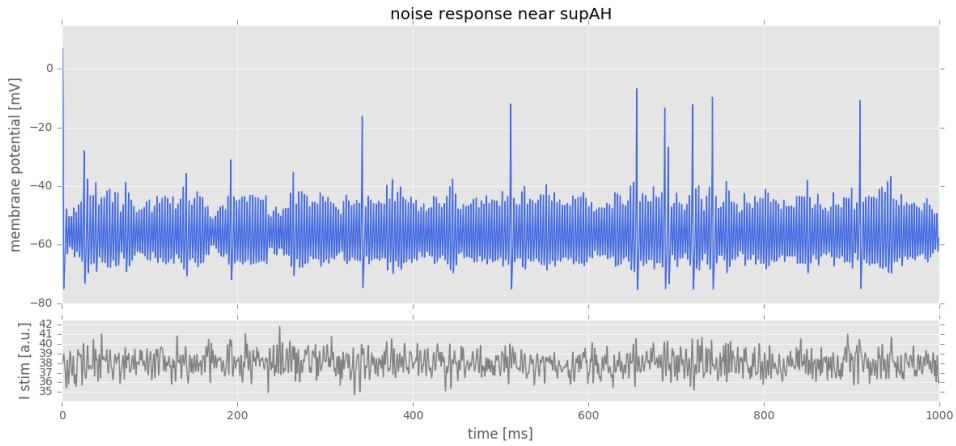
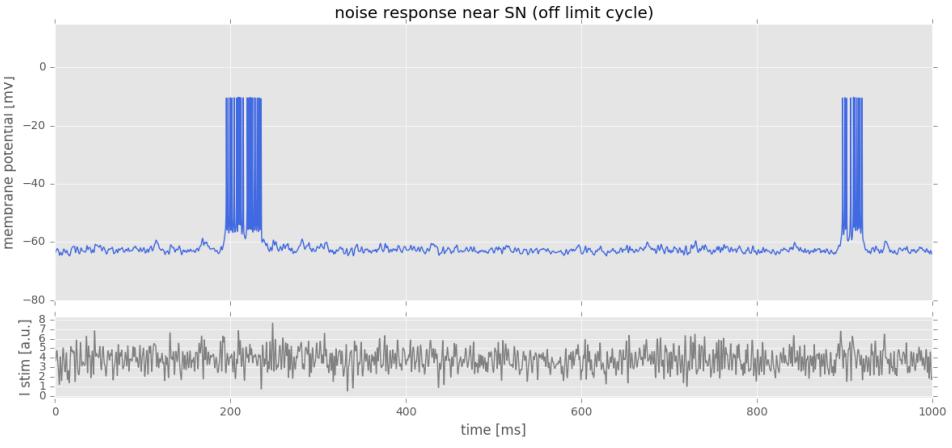
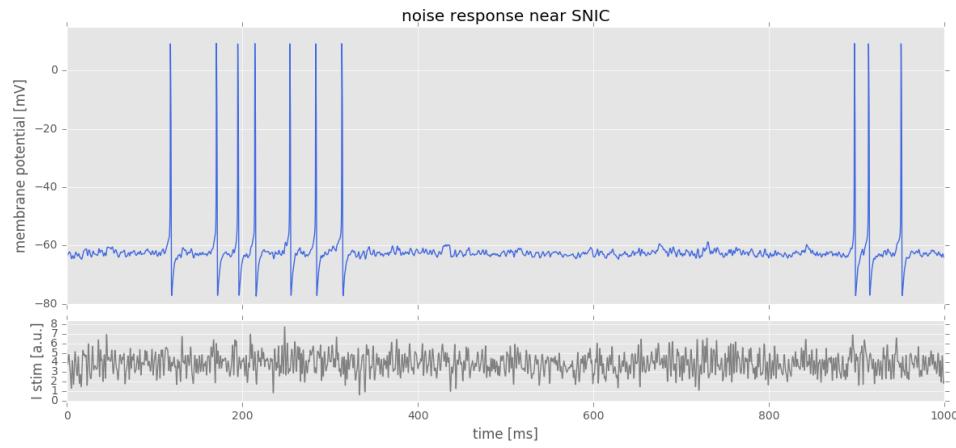
# Telling an integrator from a resonator: responses to short subthreshold pulses



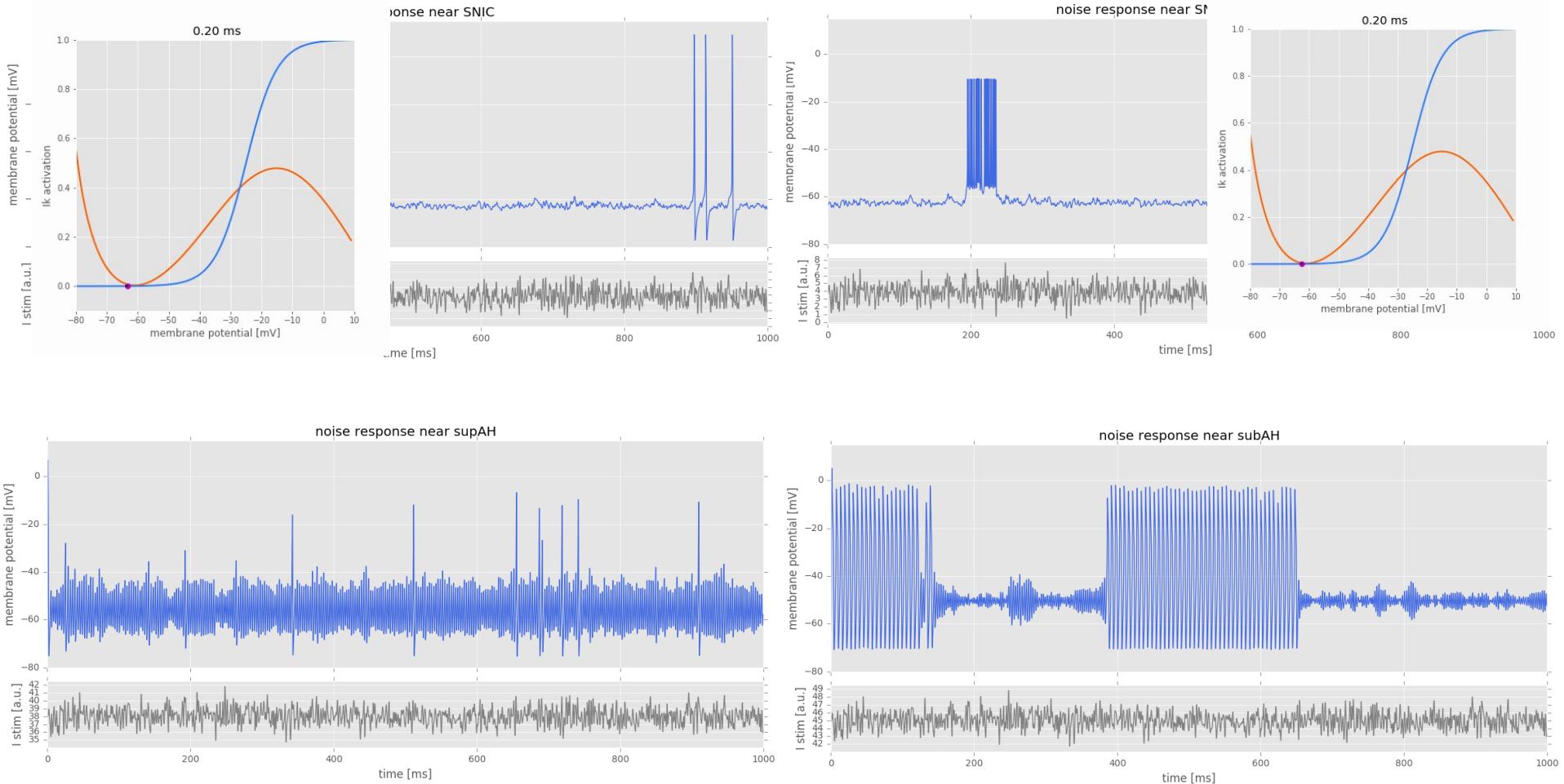
# Telling an integrator from a resonator: responses to short subthreshold pulses



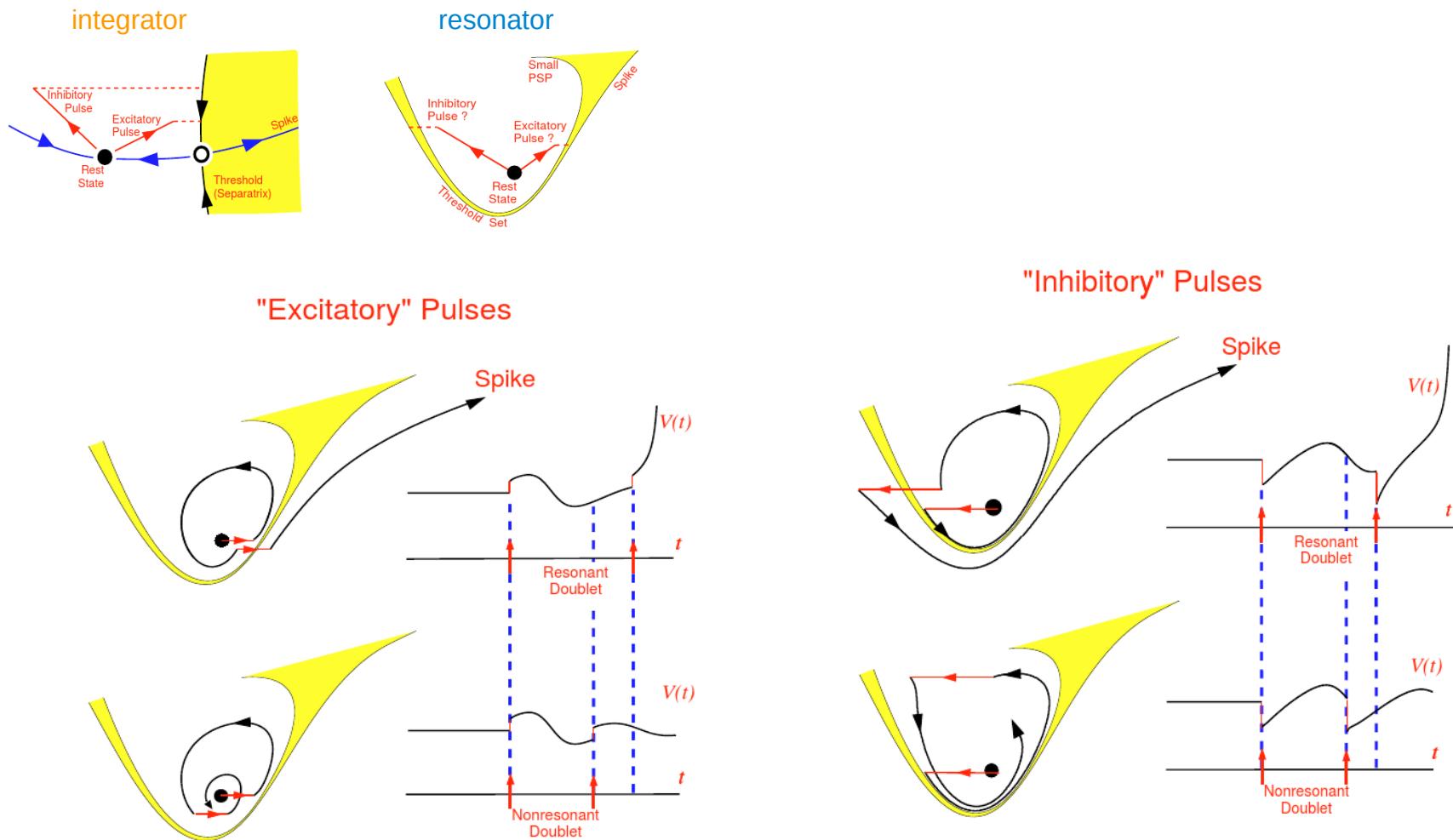
# Response to noise



# Response to noise



# Implications of subthreshold oscillations: excitation by hyperpolarization



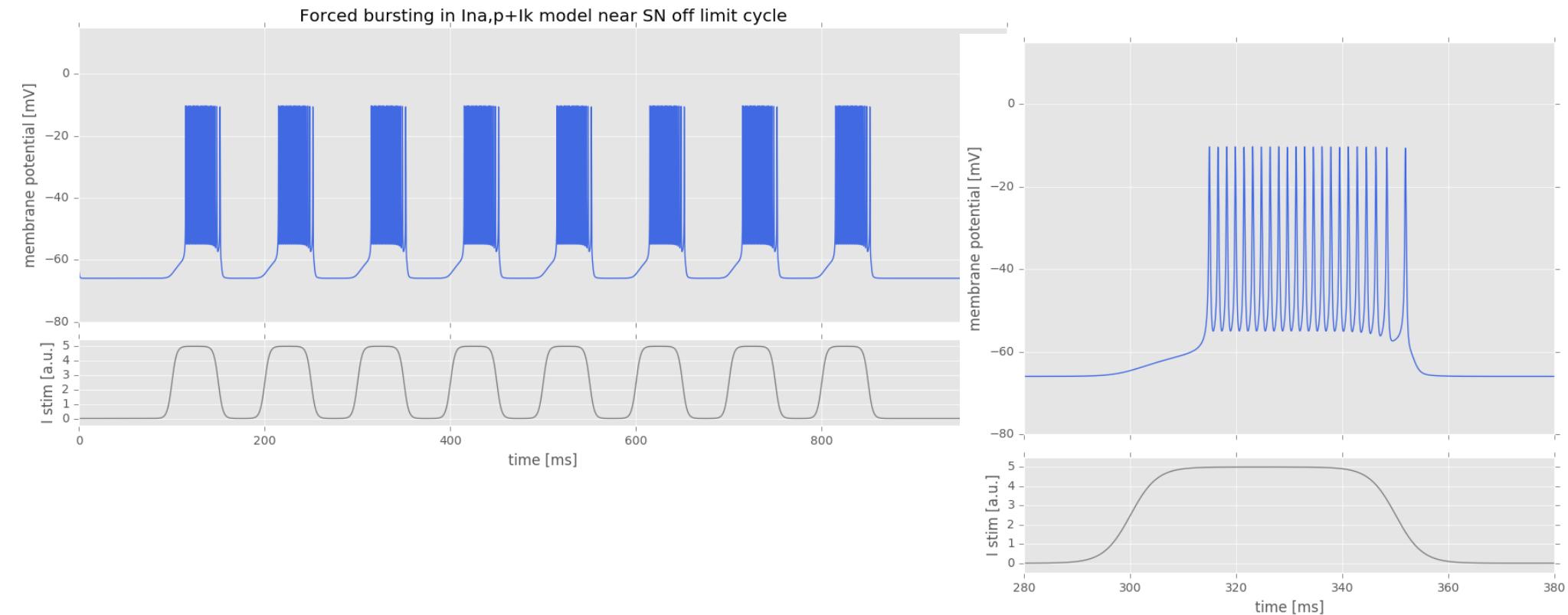
# Integrators

- Saddle-node bifurcations from resting state
- Arbitrary low-frequency firing
- Well-defined threshold manifold
- Distinguish between inhibitory and excitatory stimuli
- The higher the frequency of incoming EPSPs, the shorter the spike latency

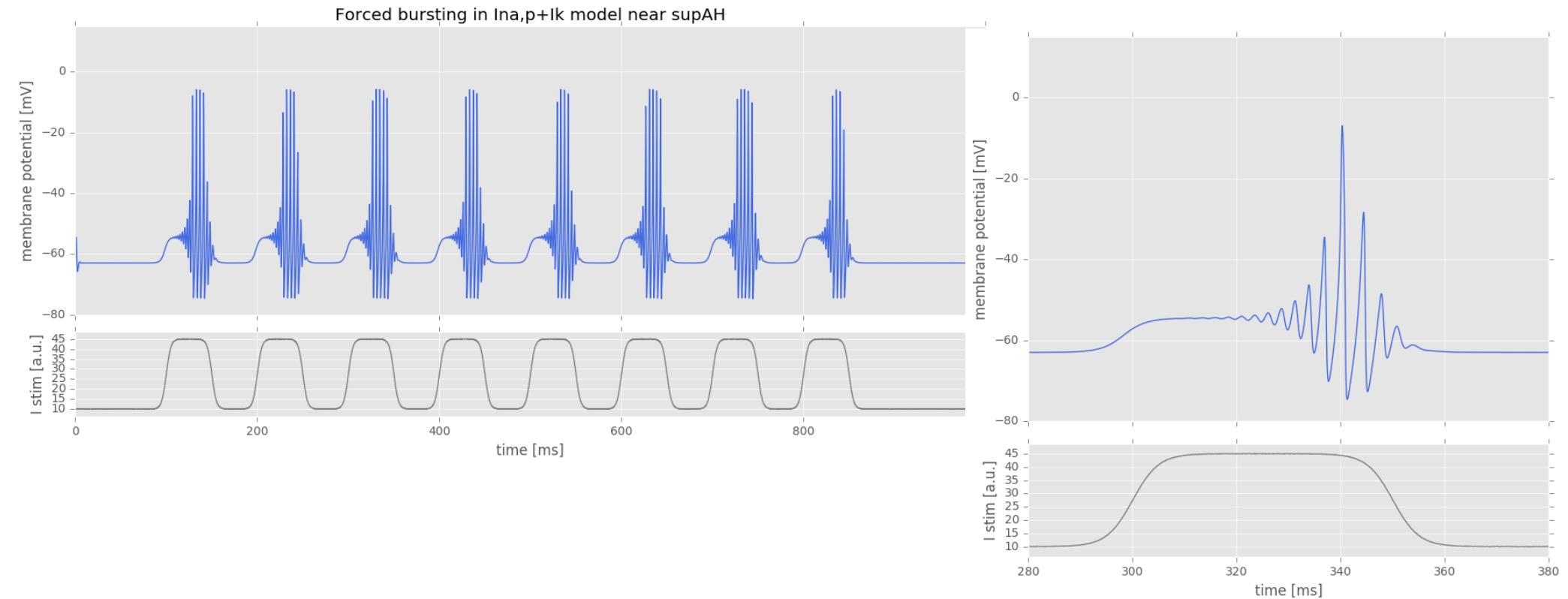
# Resonators

- AH bifurcation
- Fires within specific frequency range
- doesn't have all-or none spikes
- No well-defined threshold manifold
- Can fire in response to inhibitory pulse train
- Increased input frequency can delay spike generation

# Slow modulation: forced bursting



# Slow modulation: forced bursting (supercritical AH)



# Bursting

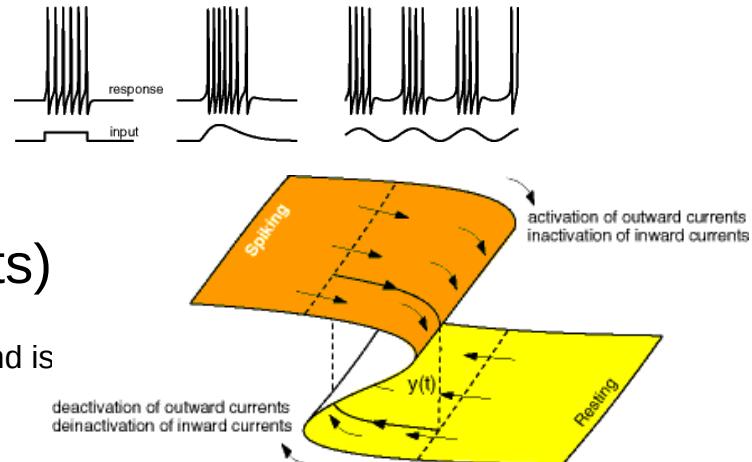
- Forced bursting
- Intrinsic bursting  
(additional slow membrane currents)

- **Slow wave:** slow subsystem is insensitive to fast and is in oscillatory regime (must be at least 2D)
- **Hysteresis loop:** *bistability* of resting and spiking in the fast subsystem

$$\dot{x} = f(x, y) \quad \text{fast subsystem} \quad (1)$$

$$\dot{y} = \mu g(x, y) \quad \text{slow modulation} \quad (2)$$

$$\mu \ll 1$$



## To stop a burst, either:

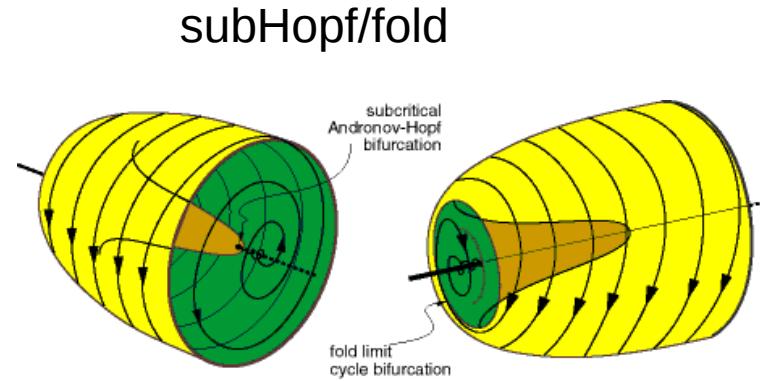
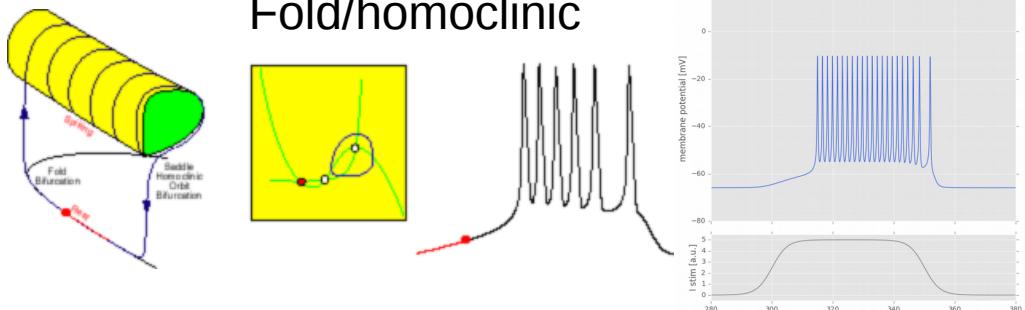
- activate outward current (e.g.  $K^+$  M-current)
- Inactivate inward current (e.g.  $Ca^{2+}$  T-current)

# Classification of bursters by bifurcations involved

bifurcations of equilibria

bifurcations of limit cycles

	saddle-node on invariant circle	saddle homoclinic orbit	supercritical Andronov-Hopf	<b>fold limit cycle</b>
<b>saddle-node (fold)</b>	fold/ circle	fold/ homoclinic	fold/ Hopf	fold/ fold cycle
saddle-node on invariant circle	circle/ circle	circle/ homoclinic	circle/ Hopf	circle/ fold cycle
supercritical Andronov-Hopf	Hopf/ circle	Hopf/ homoclinic	Hopf/ Hopf	Hopf/ fold cycle
<b>subcritical Andronov-Hopf</b>	subHopf/ circle	subHopf/ homoclinic	subHopf/ Hopf	subHopf/ fold cycle



# Real-life example: neurons of the ventral cochlear nucleus

