The Hodgkin-Huxley formalism

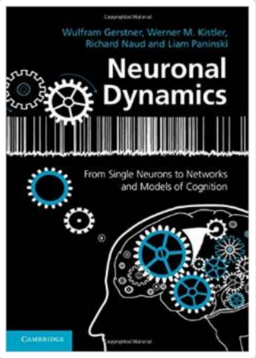
Alexey Brazhe

- http://neuronaldynamics.epfl.ch/
- http://neuronaldynamics-exercises.readthedocs.io/en/latest/

Neuronal Dynamics

From single neurons to networks and models of cognition

Wulfram Gerstner, Werner M. Kistler, Richard Naud and Liam Paninski



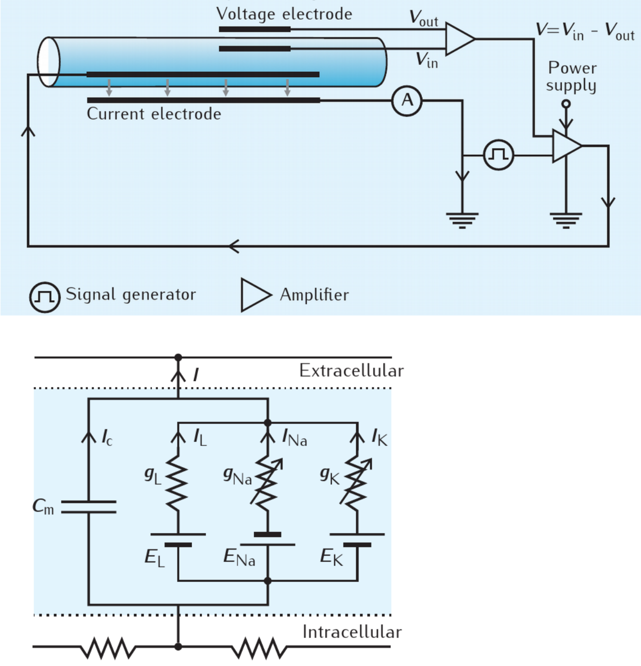
What happens in our brain when we make a decision? What triggers a neuron to send out a signal? What is the neural code? This textbook for advanced undergraduate and beginning graduate students provides a thorough and up-to-date introduction to the fields of computational and theoretical neuroscience.

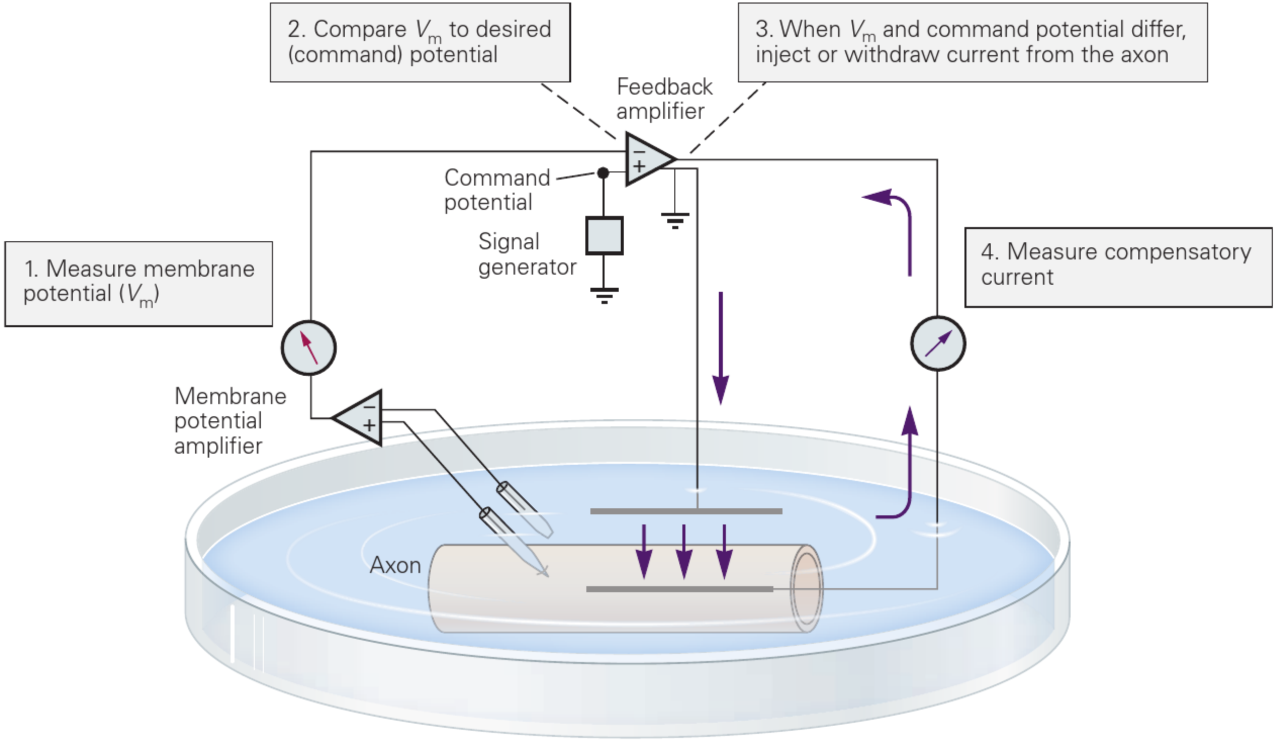
Read the online version of the book » | Order the book » | First-edition Errata »

Python Exercises »

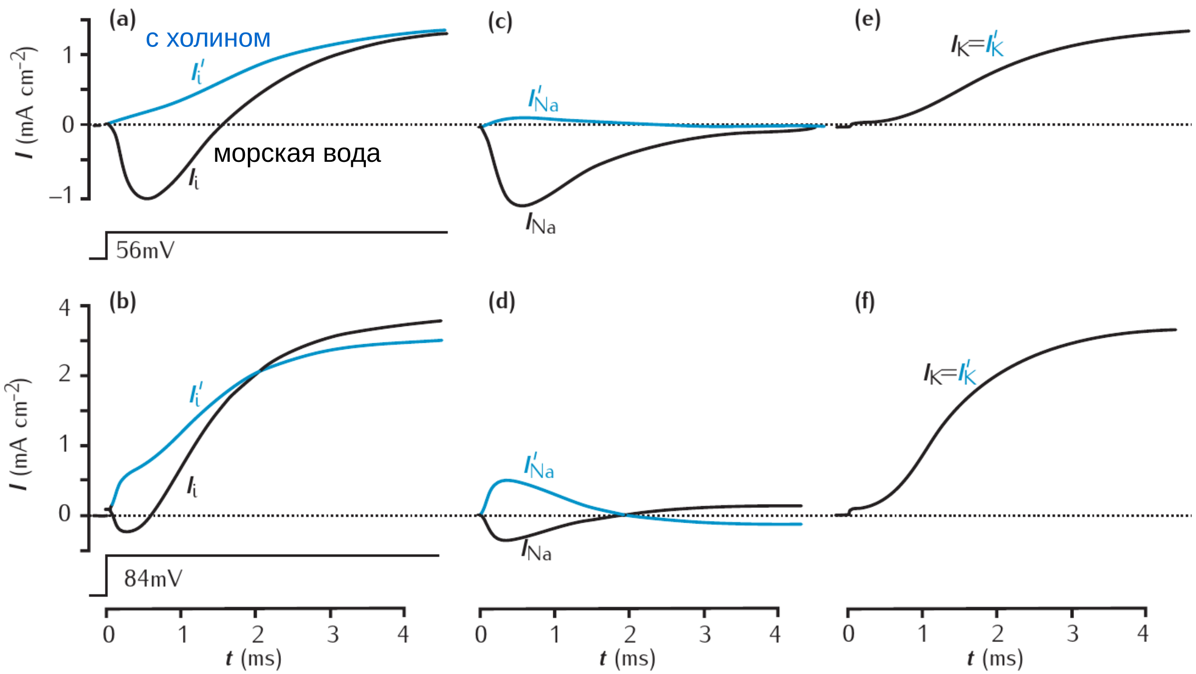
- http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html
- http://neuronaldynamics.epfl.ch/lectures.html
- https://senselab.med.yale.edu/ModelDB/ModelList.cshtml?id=113733&allsimu=true
-

Ion currents in the squid giant axon

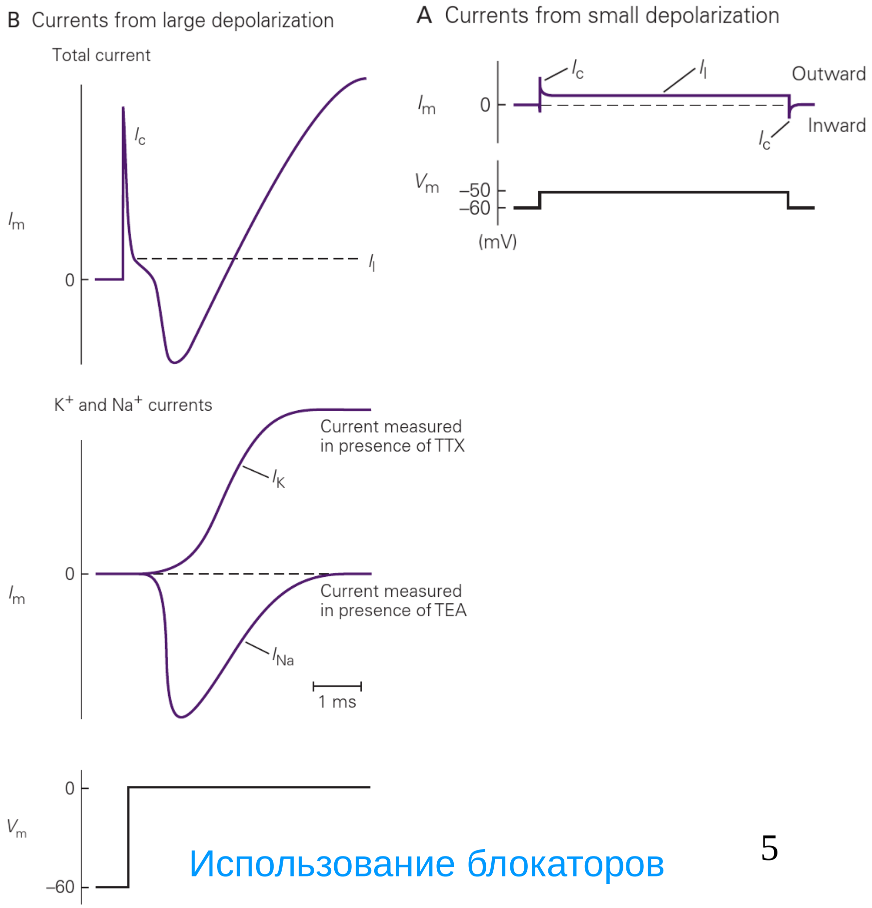




Ходжкин-Хаксли: разделение токов на I_{Na} и I_K



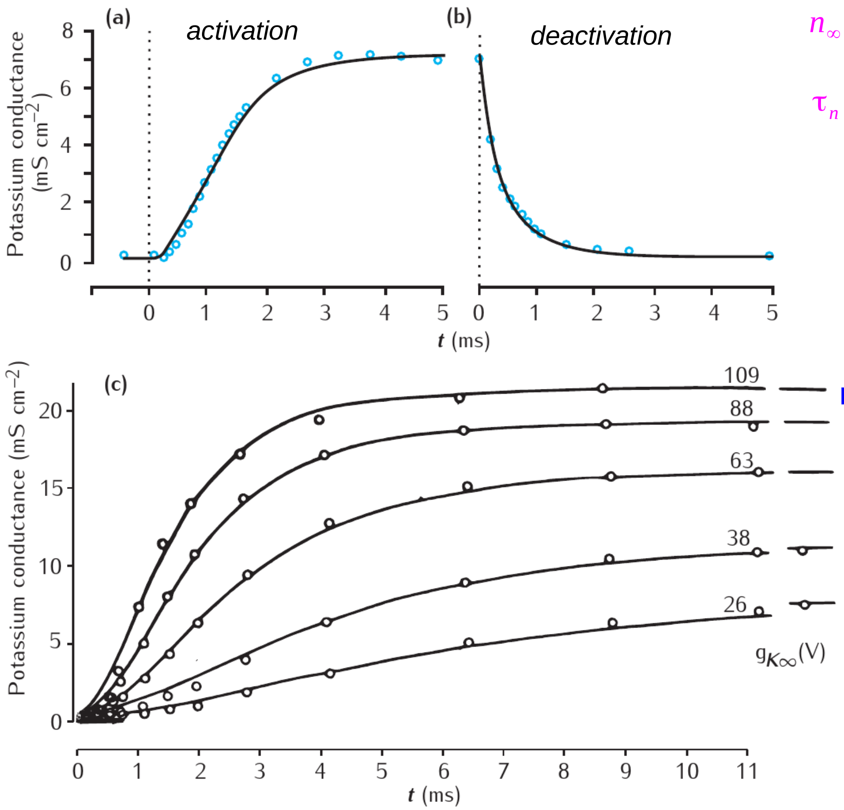
Замена ионного состава среды



Использование блокаторов

Potassium current in the HH model

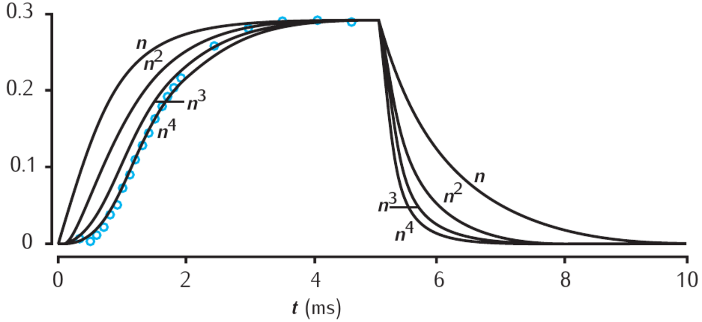
Dependence of K conductance on potential

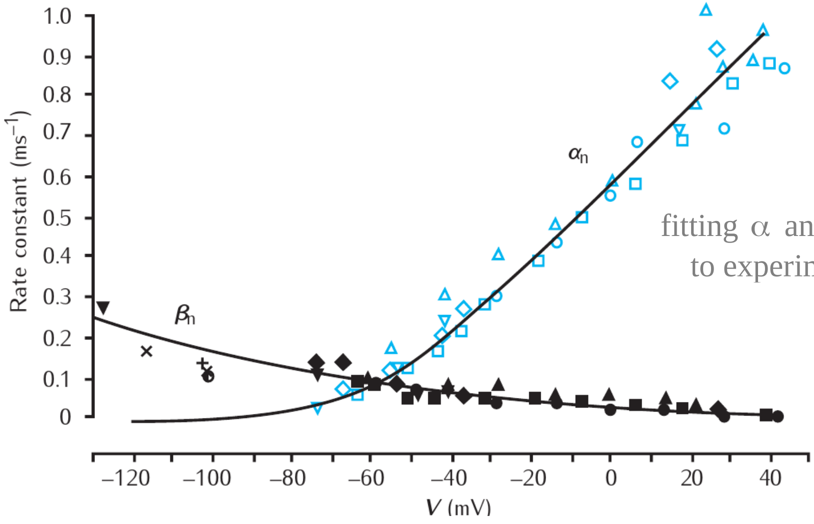


max g_K

$$n_\infty = \frac{\alpha_n}{\alpha_n+\beta_n}$$

$$\tau_n = \frac{1}{\alpha_n+\beta_n}$$

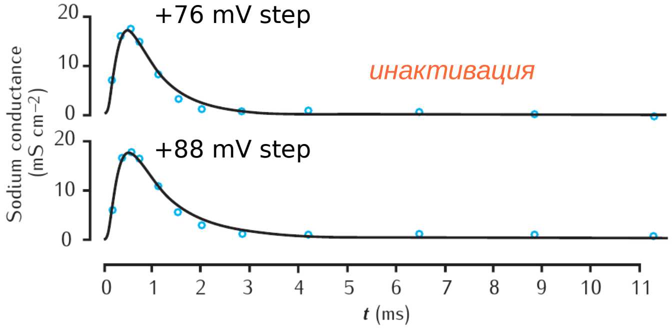




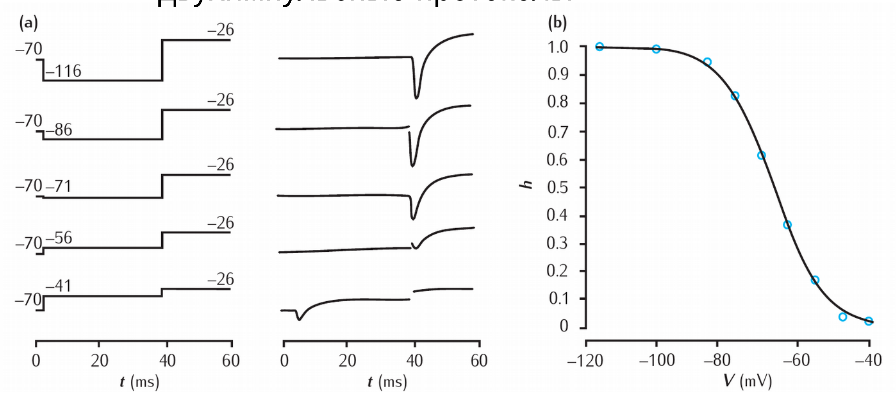
fitting α and β parameters to experimental curves

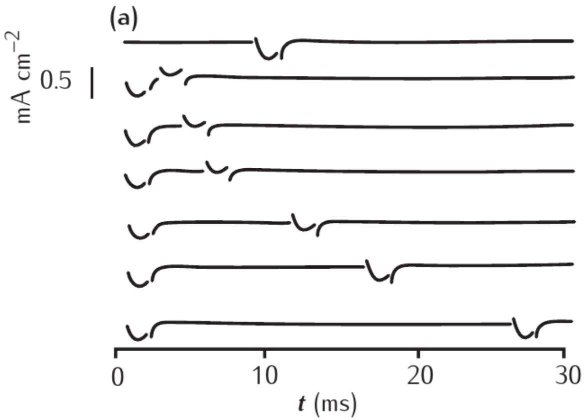
Sodium current in the HH model

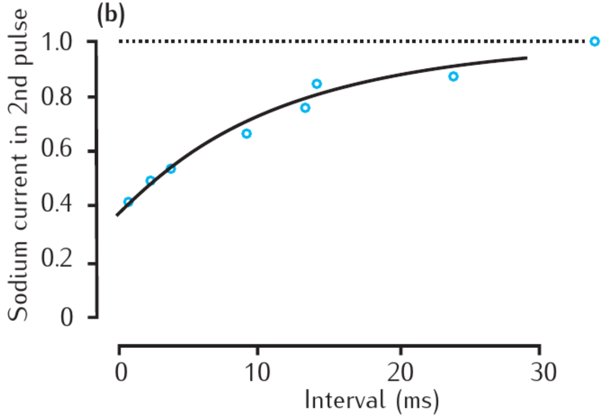
$$I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na})$$



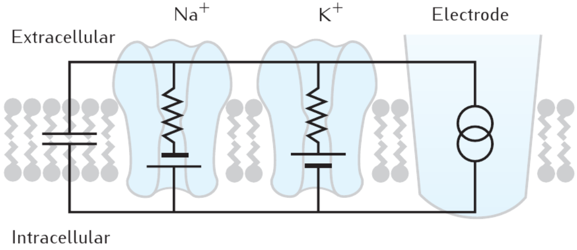
двухимпульсные протоколы







The Hodgkin-Huxley formalism



Membrane potential dynamics is governed by an ODE:

$$C_m \frac{dV}{dt} = I_{electrode} - \bar{g}_L (V - E_L) - \bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_K n^4 (V - E_K)$$

General summary

1) $I_i = g_i (V - E_{i\,Nernst})$ Instantaneous I-V relationship is linear

2) $g_i = \bar{g}_i w^{\gamma} v^{\delta}$ Instantaneous conductance is maximal conductance weighted by fraction of open channels (2 types of gates)

3) $\frac{dw}{dt} = \frac{1}{\tau_w}(w_\infty - w) \equiv \alpha_w (1 - w) - \beta_w w$ Linear gate kinetics

4) $\alpha_w = f_1(V), \beta_w = f_2(V)$ Gate kintetic rates are nonlinear functions of Vm

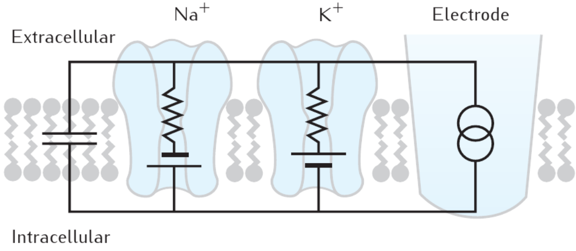
$I_k = \bar{g}_K n^4 (V - E_K)$ K-current (through Kv channels)

$I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na})$ Na-current (through Nv-channels)

$I_L = \bar{g}_L (V - E_L)$ Non potential-sensitive leak

$$\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n$$

The Hodgkin-Huxley formalism



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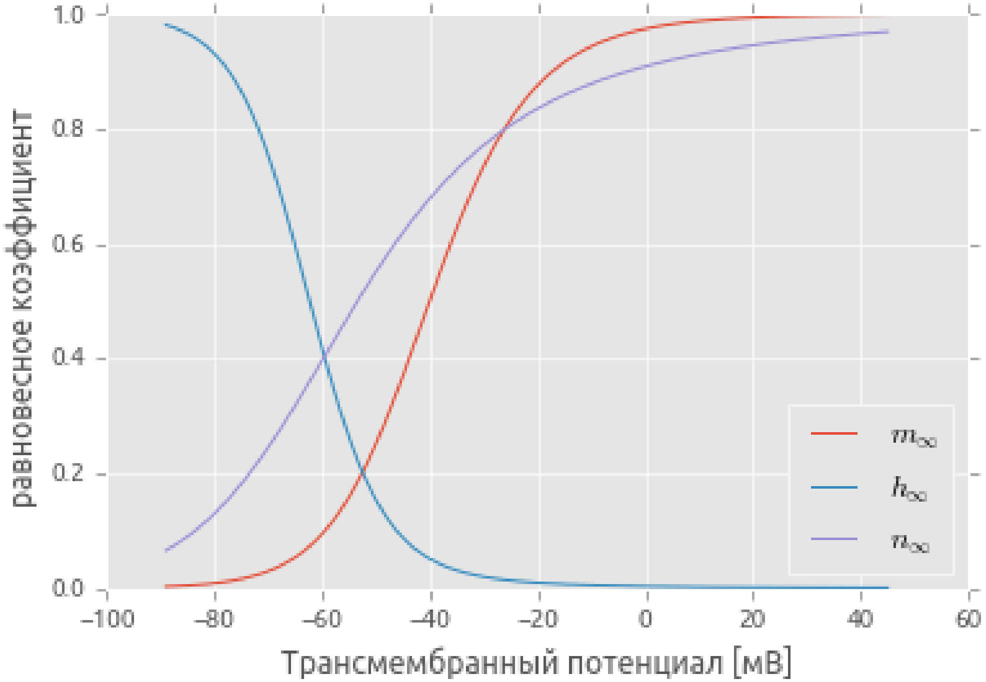


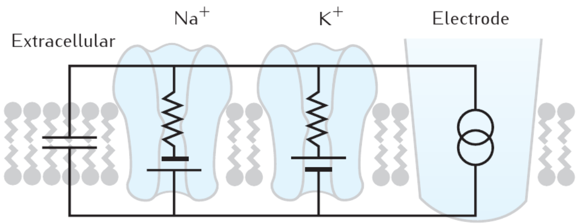
;)

α_n

β_n

80 18 10

The Hodgkin-Huxley formalism



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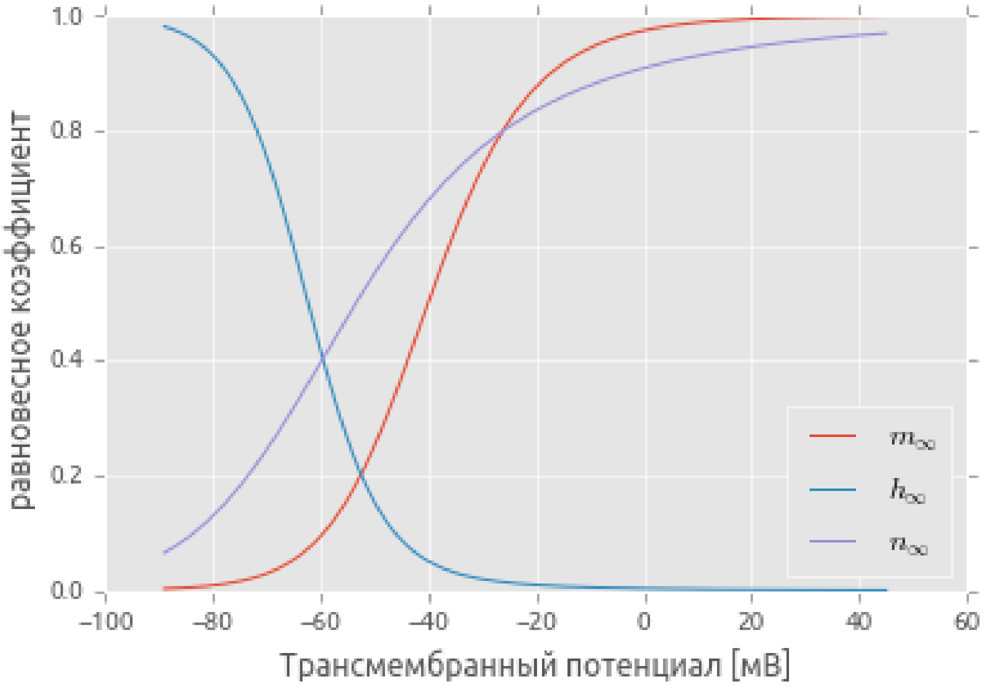
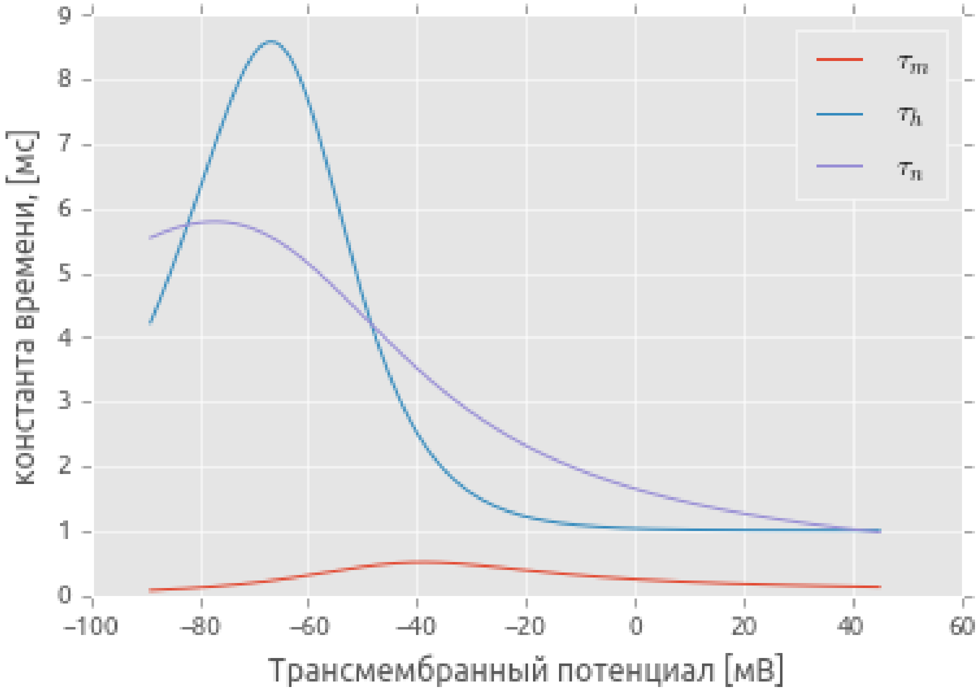
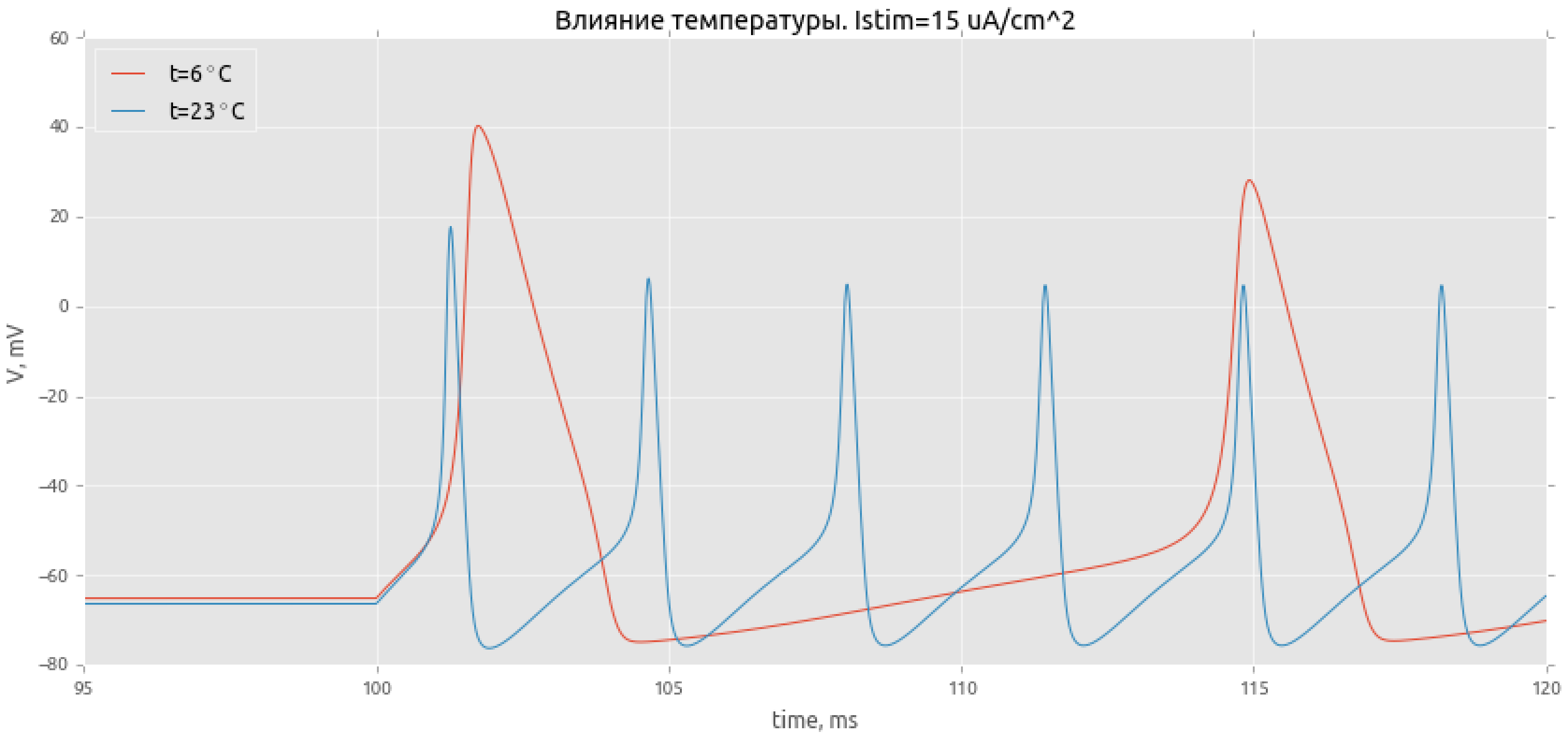


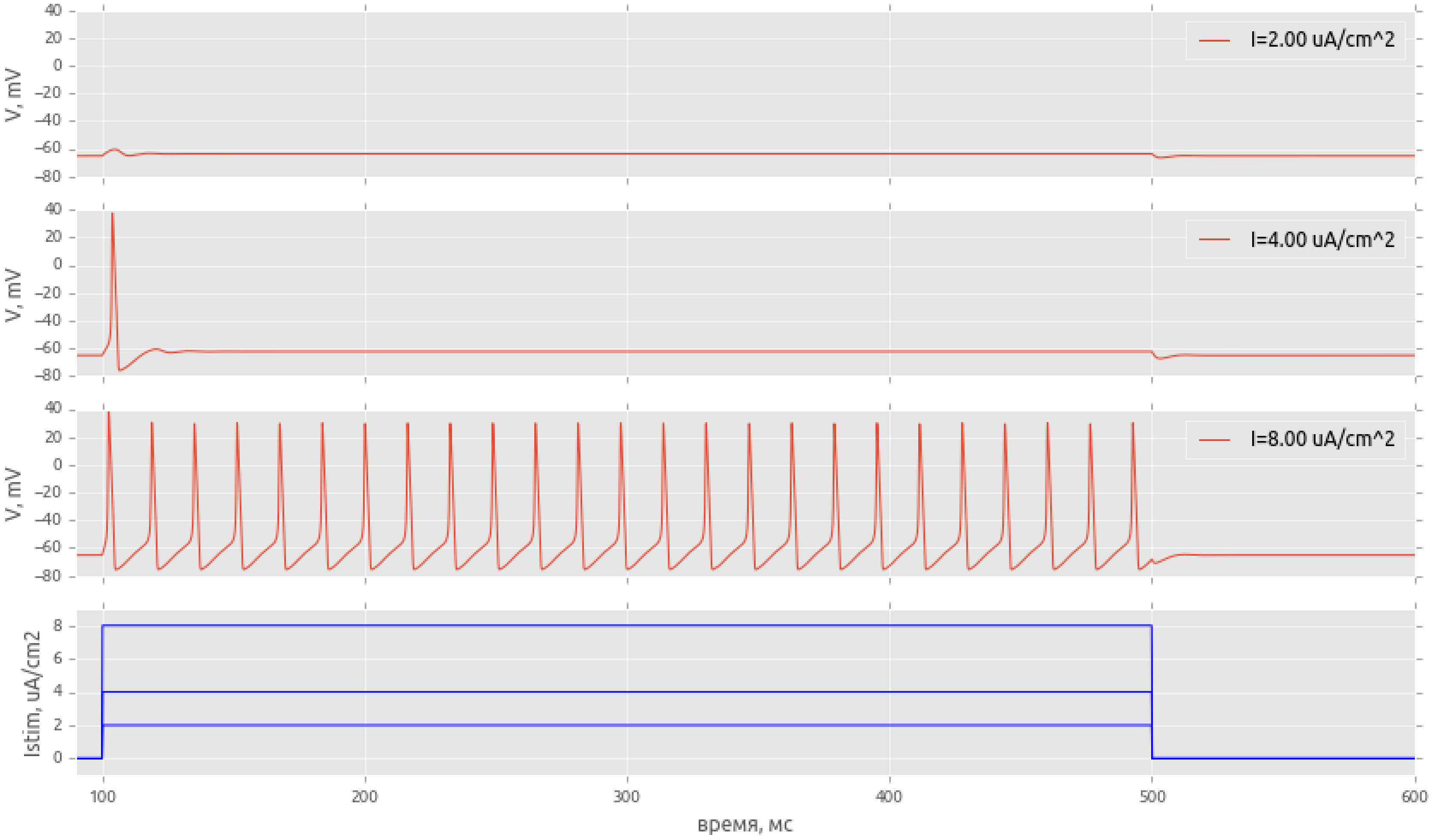


Effect of temperature

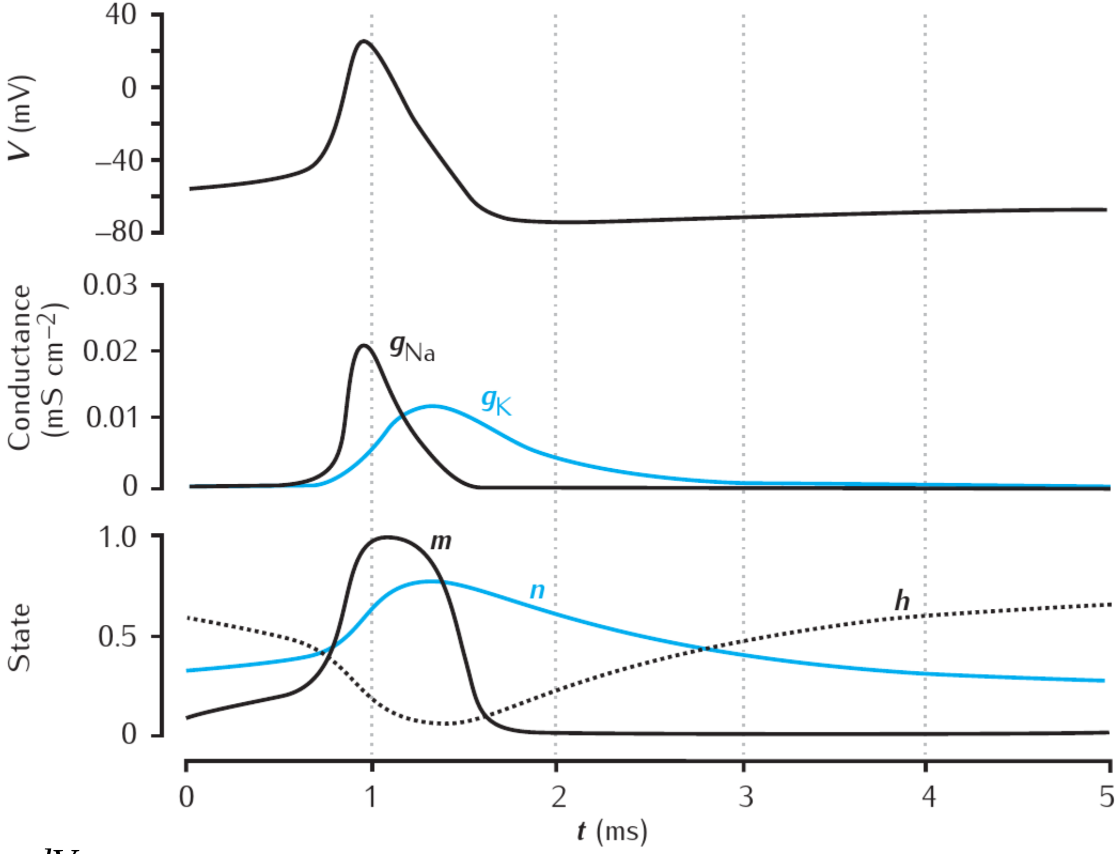
$$\alpha(T)=\alpha(T_0)\,Q_{10}^{\frac{T-T_0}{10}}$$

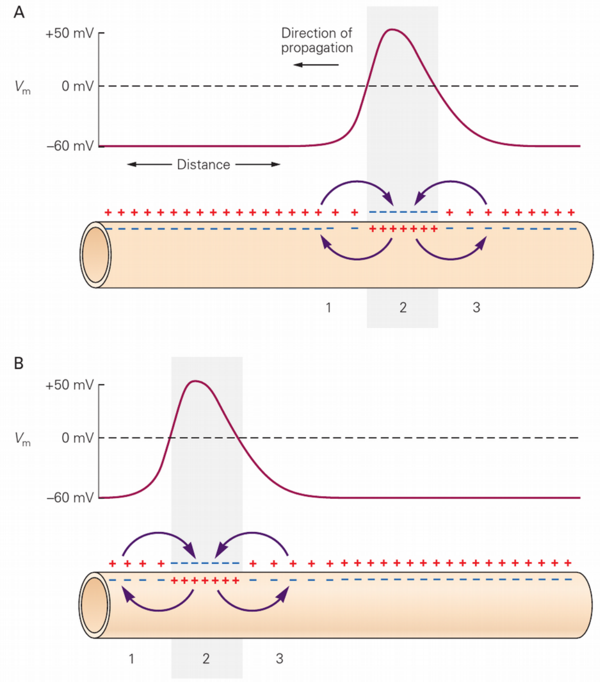


Response of the HH model to current pulses: Class 2

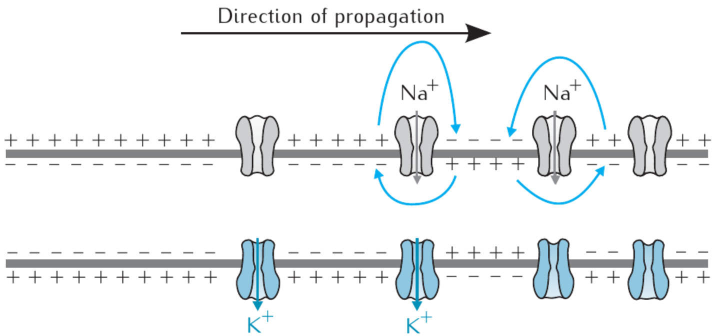


Generation and conduction of spike in the squid axon



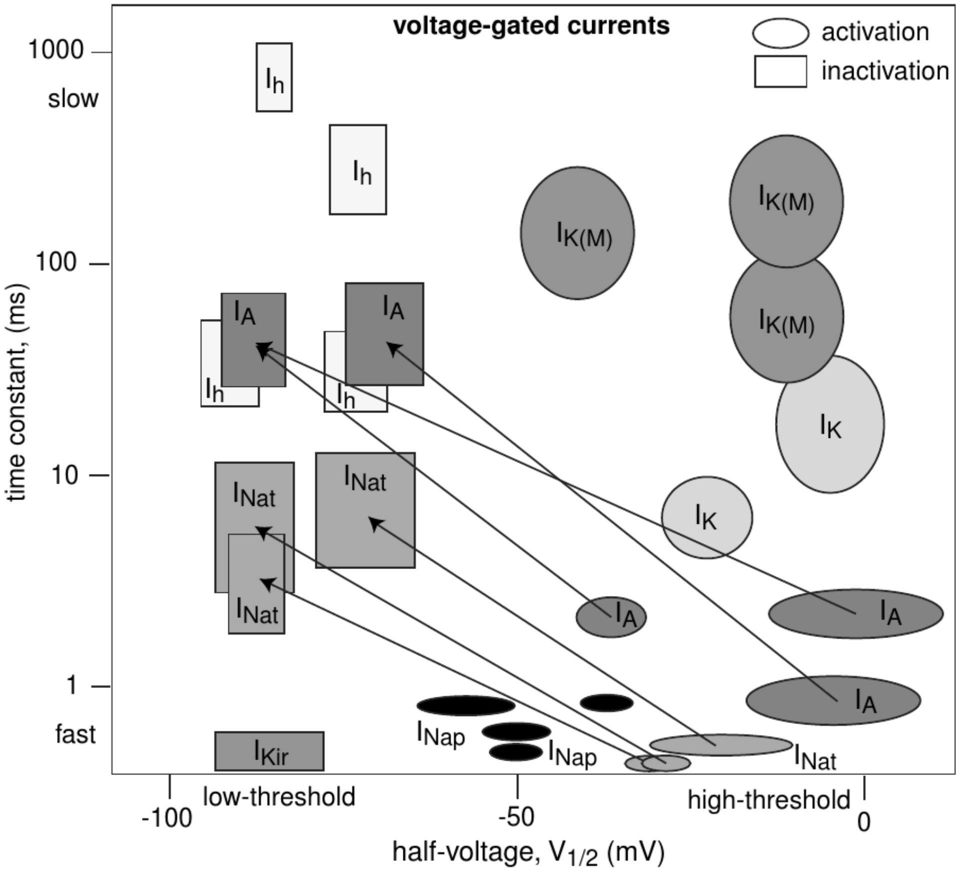


(e)



$$C_m \frac{dV}{dt} = I_{electrode} - \bar{g}_L (V - E_L) - \bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_K n^4 (V - E_K)$$

The zoo of ion currents

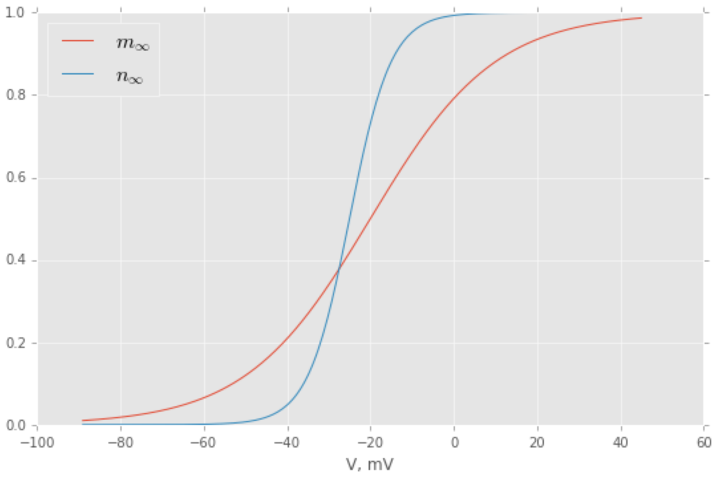


$I_{Na,p}$+I_K-model:

$$C\dot{V} = I - \bar{g}_K n(V - E_K) - \bar{g}_{Na} m_\infty(V)(V - E_{Na}) - g_l(V - E_l)$$

$$\tau_n \dot{n} = (n_\infty(V) - n)$$

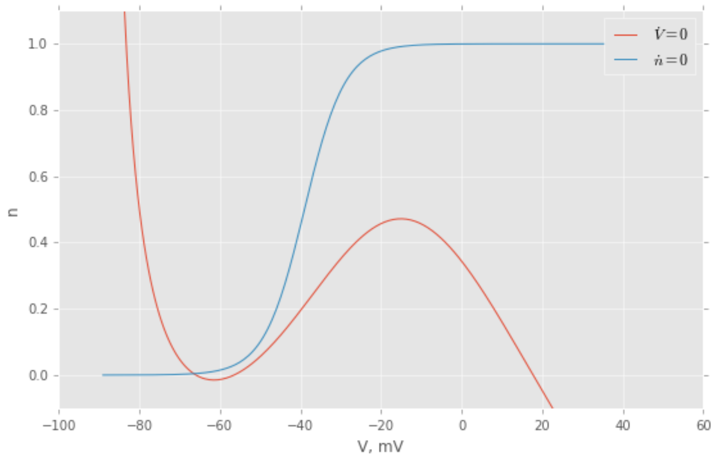
$$x_\infty = \frac{1}{1 + \exp(\frac{V_x^{0.5} - V}{k_x})}$$



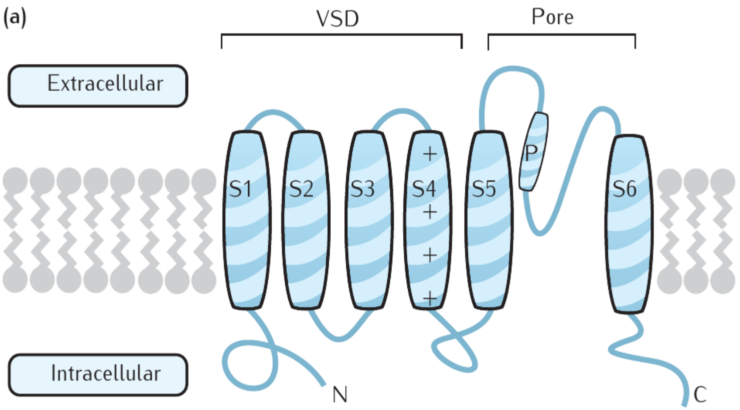
Nullclines:

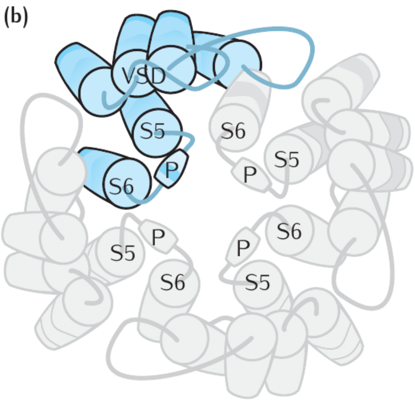
$$\dot{V} = 0 \rightarrow n(V) = \frac{I - \bar{g}_{Na} m_\infty(V - E_{Na}) - g_l(V - E_l)}{\bar{g}_k(V - E_k)}$$

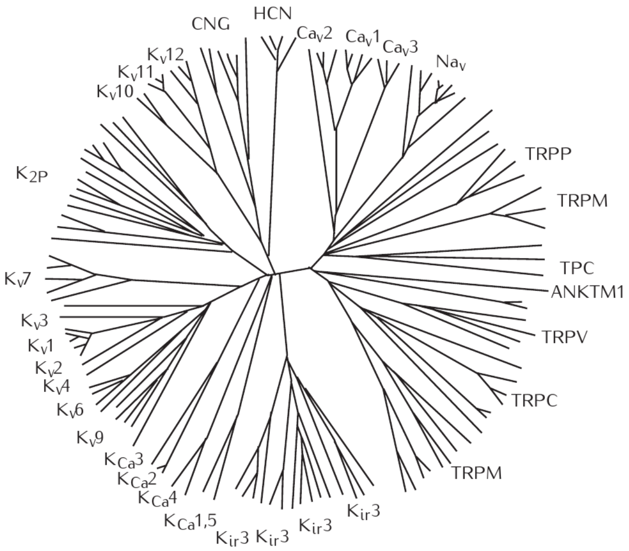
$$\dot{n} = 0 \rightarrow n(V) = n_\infty(V)$$

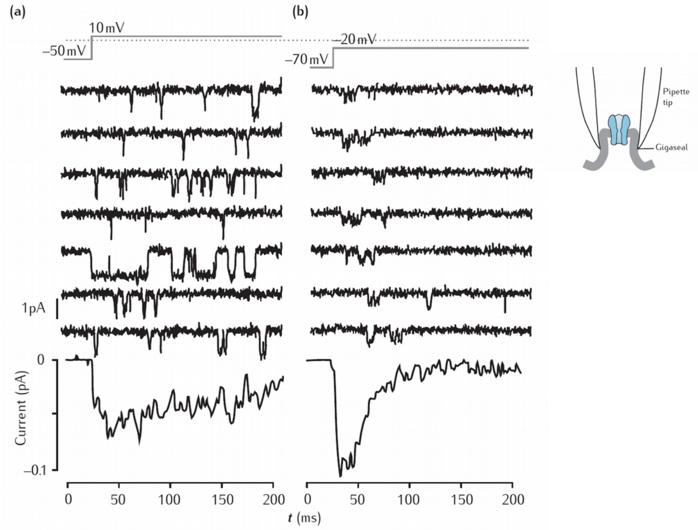


More detailed channel models



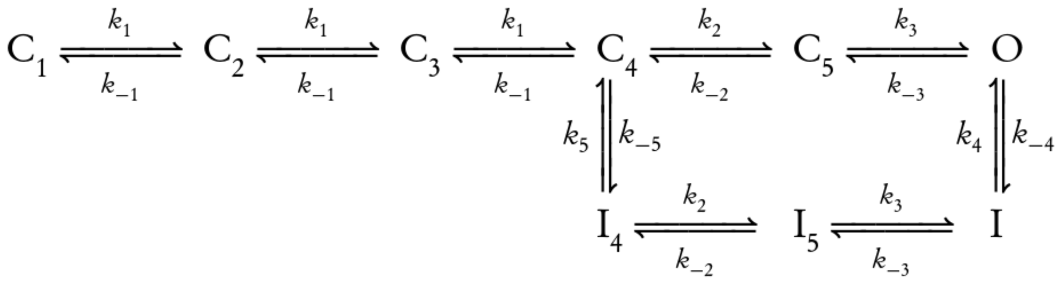






Markov models and kinetic schemes

Na_v-channel (Vandenberg & Bezanilla 1991):



Why does everyone still uses HH formalism and independent gating particles?

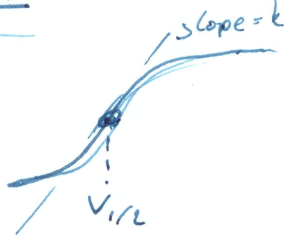
- less parameters to fit
- less equations
- accurate enough
- when sub-ms accuracy is not required
- for large ensembles of channels

Parameterization of gating kinetics

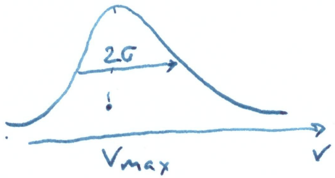
Simple parameterization

Empirical:

x_∞ : Boltzman function : $\dfrac{1}{1+\exp\left(\dfrac{V_{1/2}-V}{k}\right)}$



τ : gaussian:



we will use this parameterization due to its simplicity.

read more on thermodyn. models:

- Tsien & Noble 1969
- Destexhe, Huguenard 2000 JCN

or: Markov models (stochastic)

<cit.>

Borg-Graham 1989; Destexhe&Huguenard 2000

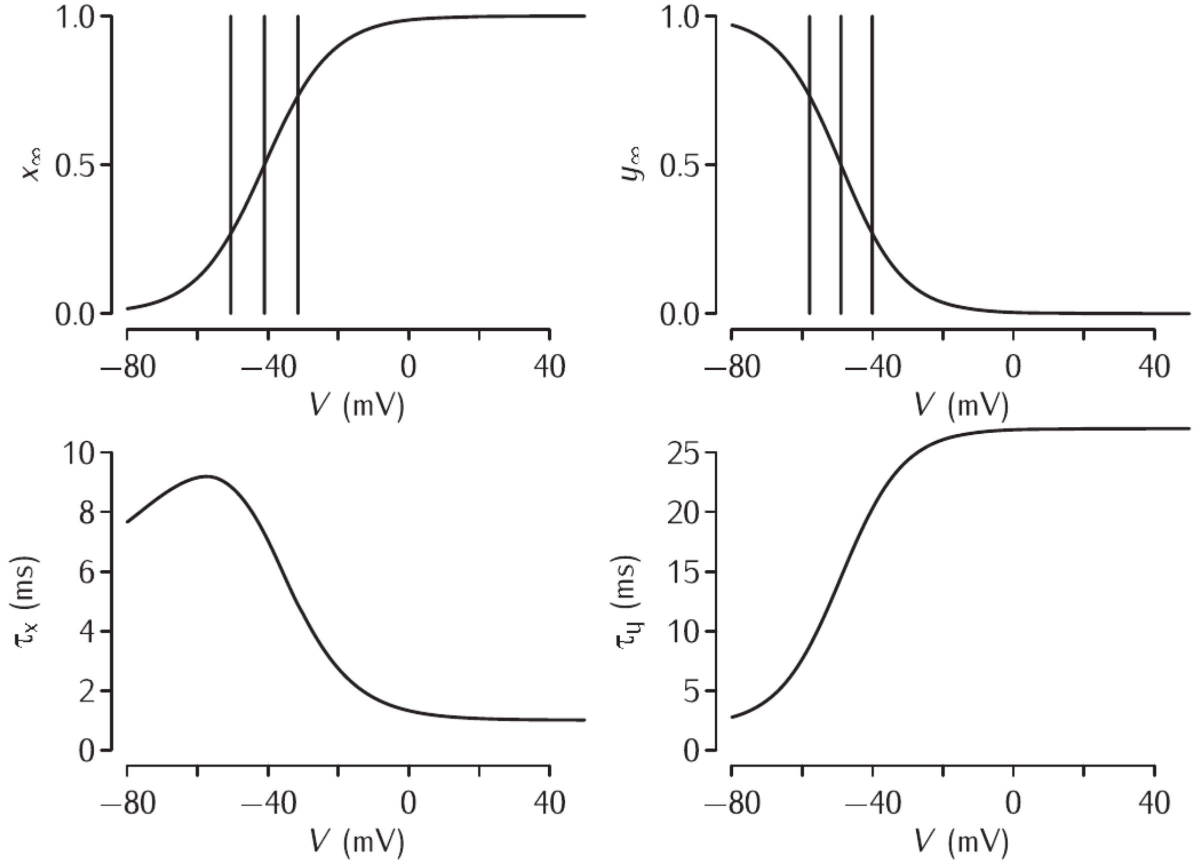
Thermodynamic-based models

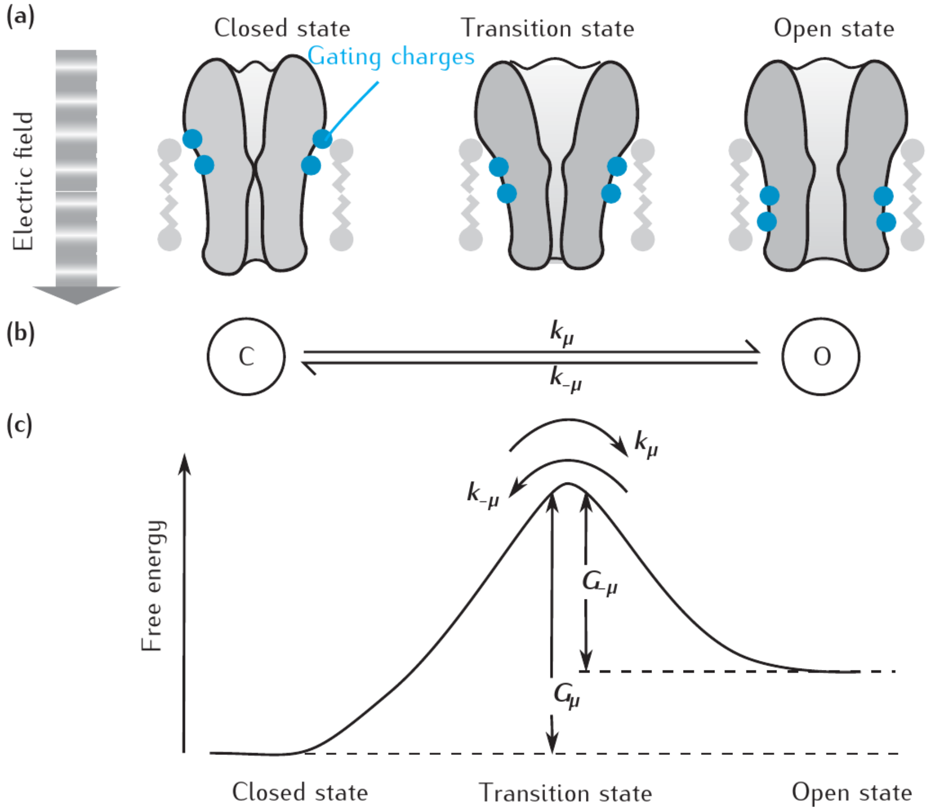
$$x_\infty = \frac{1}{1+\exp(-(V-V_{1/2})/\sigma)},$$

$$\tau_x = \frac{1}{\alpha'(V)+\beta'(V)} + \tau_0,$$

$$\alpha'_x(V) = K\exp\left(\frac{\delta(V-V_{1/2})}{\sigma}\right)$$

$$\beta'_x(V) = K\exp\left(\frac{-(1-\delta)(V-V_{1/2})}{\sigma}\right).$$





$$k_\mu = \frac{k_B T}{h} \exp\left(-\frac{\Delta G_\mu}{RT}\right) = \frac{k_B T}{h} \exp\left(\frac{\Delta S_\mu}{R}\right) \exp\left(-\frac{\Delta H_\mu}{RT}\right)$$

$$\Delta H_\mu(V) = \Delta H_\mu^{(0)} - \delta_\mu z_\mu F V,$$

$$\Delta H_{-\mu}(V) = \Delta H_{-\mu}^{(0)} + (1 - \delta_\mu) z_\mu F V.$$