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Assignment - 2

MC-303 Stochastic Processes

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Q1 Ans. $p = \frac{1}{2}$ $q = \frac{1}{3}$ $1-p-q = \frac{1}{6}$

Here let $r_1 \rightarrow$ no. of +ve steps
$r_2 \rightarrow$ no. of -ve steps
$n - r_1 - r_2 \rightarrow$ no. of 0 length steps.
$\rightarrow 4 - r_1 - r_2$
$r_1 - r_2 = 1.$

(r_1, r_2) can be $(1, 0)$ & $(2, 1)$.

So, $P(X(4) = +1) = \frac{4!}{(1)!\,0!\,3!} \cdot \left(\frac{1}{2}\right)^1 \cdot \left(\frac{1}{3}\right)^0 \left(\frac{1}{6}\right)^3$

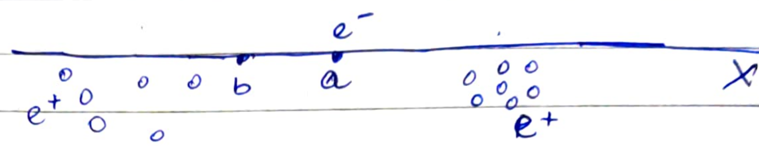
$$+ \frac{4!}{2!\,1!\,1!} \left(\frac{1}{2}\right)^2 \left(\frac{1}{3}\right)\left(\frac{1}{6}\right)$$

$$= \frac{\cancel{4}}{3^2 \times 2\cancel{6} \times \cancel{2}} + \frac{\cancel{4} \times \cancel{3}}{\cancel{2^2} \times \cancel{3} \times 6}$$

$$= \frac{1}{18} + \frac{1}{12} = \frac{19}{108}$$

Q2 Ans. i) non-homogeneous markovian chain

An example of non-homogeneous Markovian chain would be the motion or random walk exhibited by an electron which can move in a step of $\pm 1 \mu m$ in a linear direction in a pool of protons whose density varies throughout the pool.



Consider the figure, here the random walk by the electron has a probability towards moving left or moving right.

In the figure, if the electron is at 'a', i.e. equidistant from both clusters of protons, the probability of moving towards right would be higher as the cluster is more dense there.

But if it is at 'b', then the probability of moving towards left would be higher as the cluster is closer there.

~~So~~ Also, since the motion probability / transition probability is only dependant on its location, the probabilities are dependant upon present only.

So, this is an example of non-homogeneous Markov chain.

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ii) Homogeneous Markovian Chain.

An example of Homogeneous Markovian chain would be unrestricted random walk. Here, throughout the length of the process, the transition probabilities of +1 step & -1 step remains the same.

Q. $P[X_0 = 1] = P[X_0 = 2] = P[X_0 = 0] = 1/3$

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.75 & 0.25 & 0 \\ 1 & 0.25 & 0.50 & 0.25 \\ 2 & 0 & 0.75 & 0.25 \end{array}$$

Transition Probabilities matrix

$$P[X_3=1, X_2=2, X_0=2] = P[X_3=1, X_2=2, X_1=0, X_0=2) + P[X_3=1, X_2=2, X_1=1, X_0=2] + P[X_3=1, X_2=2, X_1=2, X_0=2)$$

$$= P(X_3=1 | X_2=2) . P(X_2=2 | X_1=0) . P(X_1=0 | X_0=2) . P(X_0=2)$$

$$+ P(X_3=1 | X_2=2) . P(X_2=2 | X_1=1) . P(X_1=1 | X_0=2) . P(X_0=2)$$

$$+ P(X_3=1 | X_2=2) . P(X_2=2 | X_1=2) . P(X_1=2 | X_0=2) . P(X_0=2)$$

$$= P(X_3=1 | X_2=2) . P(X_0=0) \left(\sum_{i=0}^{2} P(X_2=2 | X_1=i) . P(X_i=i | X_0=2) \right)$$

$$= 0.75 \times 0.33 \left(0 \times 0 \; \cancel{\times 0.33} + 0.25 \times 0.75 + 0.25 \times 0.25 \right)$$

$$= \frac{\cancel{3}}{4} \times \frac{1}{\cancel{3}} \left(\frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \right) = \frac{4}{64} = \frac{1}{16}$$

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Q5 Ans)

$$T = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \end{array}$$

$$T^2 = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

$$T^3 = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

$\vdots$

$$T^5 = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

a) The required probability $P(X_5 = 0 \mid X_0 = 0)$

$$= P_{00}^{5}$$

$$= \frac{1}{2}$$

b) The required probability is $P(X_5 = 0)$

$$= P(X_5 = 0 \mid X_0 = 0) \cdot P(X_0 = 0) + P(X_5 = 0 \mid X_0 = 1) \cdot P(X_0 = 1)$$

Also, given that $P(X_0 = 0) = P(X_0 = 1) = 1/2$.

$$\Rightarrow P(X_5 = 0) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Q6 Ans) Let R → Raining & NR → Not Raining

And let ~~a~~ describe the state ~~be~~ to be the weather of two consecutive days.

The tranation probability matrix is :

$$
T.P.M = \begin{array}{c} \\ (R,R) \\ (R,NR) \\ (NR,R) \\ (NR,NR) \end{array}
\begin{array}{c} \begin{array}{cccc} (R,R) & (R,NR) & (NR,R) & (NR,NR) \end{array} \\
\begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0\cancel{.4} & 0 & 0.4 & 0.6 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 \end{bmatrix} \end{array}
$$

The above matrix describes the transition probability of (Weather yesterday, Weather today) to (Weather today, Weather tomorrow) as in single step transition.

Now, for the required probability of (Rain Monday, No Rain Tuesday) to (No Rain Wednesday, Rain Thursday) & (Rain Wednesday, Rain Thursday) as in two steps.

So, the required probability $= P^{2}_{(R,NR)\to(NR,R)+(R,R)}$

$= 0.6 \times 0.2 + 0.4 \times 0.5$

$= 0.12 + 0.2 = 0.32.$

Q7A. Communication relation is the relation between two states of the random variable which are accessible to each other, i.e. :

$R : \mathcal{X} \longrightarrow \mathcal{X} \quad (x,y) \in R$ iff $x \leftrightarrow y$

To show it is equivalence, we have:

a) <u>symmetry</u>: Let $(x,y) \in R$, $\Rightarrow x \leftrightarrow y$.
So, we can go from x ~~$\to y$~~ to y $(x \to y)$
And we can go from y to x $(x \leftarrow y)$.

$$x \leftarrow y \Rightarrow y \to x \quad \& \quad x \to y \Rightarrow y \leftarrow x.$$

$$\Rightarrow y \to x \ \& \ y \leftarrow x \Rightarrow y \leftrightarrow x \Rightarrow (y,x) \in R.$$

So, $\forall (x,y) \in R$, $(y,x) \in R \Rightarrow R$ is symmetric

b) <u>Transitive</u>: Let $(x,y) \in R$ & $(y,z) \in R$.
$\Rightarrow x \leftrightarrow y$ & $y \leftrightarrow z$
$\Rightarrow p_{xy}^{n} > 0$ & $p_{yz}^{m} > 0$ for some $m, n \in$

$$\Rightarrow p_{xz}^{m+n} \geq p_{xy}^{n} \cdot p_{yz}^{m} > 0$$

$\Rightarrow x \to z$ (i.e. we can go from x to z)

Similarly, we'll get $p_{zx}^{a+b} \geq p_{zy}^{a} \cdot p_{yx}^{b} > 0$ for some $a, b \in$

$\Rightarrow z \to x$
So, $x \leftrightarrow z$
Hence, $(x,z) \in R$.
Therefore $\forall (x,y)$ & $(y,z) \in R$ we have $(x,z) \in R$.

R is transitive.

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c) Reflexivity:
We know from symmetry proof that if $(x,y) \in R$ then $(y,x) \in R$.

And from transitive proof that if (x,y) & $(y,z) \in R$. Then $(x,z) \in R$.

So, if we take $z = x$.
we have, (x,y) & $(y,x) \in R$.
And by transitivity, $(x,x) \in R$.

So, $(x,x) \in R \quad \forall x \in X$.

$\therefore$ R is ~~symmetric~~ reflexive.

So. Thus, R is an equivalence relation from X to X.

8Ans. Let the period of i be $d(i)$
& the period of j be $d(j)$.

Now, let $k > 0$ & $l > 0$ such that
$p_{ij}^{k} > 0$ & $p_{ji}^{l} > 0$

$\Rightarrow p_{ii}^{k+l} \geq p_{ij}^{k} \cdot p_{ji}^{l} > 0$

So, $d(i)$ divides $k+l$.

Now, let $d(j) = m > 0$ such that $p_{jj}^{m} > 0$.

So, $p_{ii}^{k+m+l} \geq p_{ij}^{k} \cdot p_{jj}^{m} \cdot p_{ji}^{l} > 0$.

$\Rightarrow p_{ii}^{k+m+l} > 0$

$\Rightarrow d(i)$ divides $k+m+l$ $\Rightarrow d(i)$ divides m.

So, $d(j) \geq d(i)$. ——— (I)

Similarly, exchanging the roles of i & j, we'd get

$$d(i) \geq d(j). \quad \text{——— (II)}$$

From (I) & (II): $d(i) = d(j)$.

Hence, proved.