three exits. What is the expected time taken for the mouse to leave the maze?

3. Let X be a discrete random variable with PGF $G_X(s) = \frac{s}{5}(2+3s^2)$. Find the distribution of X.

4. Let $X \sim Poisson(\lambda)$. The PGF of X is $G_X(s) = \exp^{\lambda(s-1)}$. Find $E(X)$ and $Var(X)$.

5. Let X and Y are independent random variable with $X \sim Poisson(\lambda)$ and $Y \sim Poisson(\mu)$. Find the distribution of random variable $Z = X + Y$.

6. Describe the Brownian Motion and elaborate its applications by taking some numerical illustration.

7. Describe Gaussian Process and give its applications.

2. What is time homogeneous continuous Markov chain? Give one example each of time homogeneous and time non-homogeneous Markov chain. Derive the holding time distribution in state i for a time homogeneous Markov chain.

3. Explain matrix transition probability function in reference to a continuous time Markov chain. What's the concept parallel to this in case discrete time Markov chain? If $P(t)$ is the matrix transition probability function, then show that $P(t + s) = P(t) + P(s)$.

material from the net or book will lead to the cancellation of the exam of concern students.

1. In a classical loss model M/M/c/c, if the call arrival rate is 2 per minute and on the average if a subscriber occupies a line for 3 minutes, then in case of a 7 lines telephone exchange find
   (i) probability of customer loss,
   (ii) mean number of busy channels.
2. Find the distribution of the waiting time in the queue in case of M/M/1 model. In case arrival and service rate is equal find the mean and variance time in the queue in steady state.
3. In case of M/M/1/N model, where N is the maximum capacity of the system if arrival rate is 2 units per minutes, the average service time is 2 minutes and N = 5, then find
   (i) the probability that a customer is not allowed to join the system,
   (ii) mean number of customers in the system,
   (iii) if the mean service time reduces to 0.5 minutes, then what's the probability distribution of the customers in the system ?
4. Design a problem of which results in a specific queuing model. Check for the existence of its steady state solutions. If the system is in working mode from 9 AM to 5 PM, find
   (i) the expected number of idle periods and the mean duration of each idle period,
   (ii) the expected number of customers served between 9 AM to 5 PM.

1. Design a problem which models a Markov chain. Modified this so that Markov property no more holds. Show how this modified problem can be viewed as a Markov chain.
2. Give example of two Markov chains in which
   (i) Every state communicate with every other state.
   (ii) No state communicate with any other state.

   Give their state diagrams. Further prove that relation "*communication*" is transitive.

3. A system is assumed to be Markov chain with transition matrix

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0 & 0.2 \end{bmatrix} \end{array}$$

   The profit that the system makes per unit time in State 1 and 2 are Rs. 1000 and Rs. 600 respectively, whereas, when in State 3, the system generates a loss of Rs. 100 per unit time. Find the expected profit per unit time after sufficiently long time.
4. Define absorbing Markov chain and fundamental matrix. Illustrate the application of fundamental matrix by considering a suitable example.

1. In a classical loss model M/M/c/c, if the call arrival rate is 2 per minute and on the average if a subscriber occupies a line for 3 minutes, then in case of a 7 lines telephone exchange find
   3 (i) probability of customer loss, $c=7,\ \lambda=2,\ \mu=\frac{1}{3}\quad \lambda/\mu=6$
   2 (ii) mean number of busy channels. $P_c=\dfrac{(6)^7/7!}{1+6+\frac{(6)^2}{2!}+\dots\frac{(6)^7}{7!}}=0.185$

$\rho(1-P_7)=6(1-0.185)=4.89$

2. Find the distribution of the waiting time in the queue in case of M/M/1 model. In case arrival and service rate is equal find the mean and variance time in the queue in steady state. $\lambda=\mu=1$

3 $P\{w_q=0\}=1-\rho,\quad f_q(w_q)=\rho\mu(1-\rho)e^{-(1-\rho)\mu w_q}$

2 Steady state solution does not exist.

3. In case of M/M/1/N model, where N is the maximum capacity of the system if arrival rate is 2 units per minutes, the average service time is 2 minutes and N = 5, then find $\lambda=2\ \ \mu=1/2\ \ N=5$, $\lambda/\mu=4$
   2 (i) the probability that a customer is not allowed to join the system, $P_5=(1-4)(4)^5/[1-(4)^6]=(-3)(1024)/(-4095)=0.75$
   2 (ii) mean number of customers in the system, $\sum_{n=1}^{5} nP_n=4.46$
   1 (iii) if the mean service time reduces to 0.5 minutes, then what's the probability distribution of the customers in the system ?

$\lambda=\mu=2\quad \rho=1$ uniform $P_n=\frac{1}{6},\ n=0,1,2,3,4,5$

4. Design a problem ~~of~~ which results in a specific queuing model. Check for the existence of its steady state solutions. If the system is in working mode from 9 AM to 5 PM, find
   2+1 (i) the expected number of idle periods and the mean duration of each idle period,
   (ii) the expected number of customers served between 9 AM to 5 PM.

$T\lambda(1-\rho)\qquad \dfrac{T(1-\rho)}{T\lambda(1-\rho)}=1/\lambda$

$\dfrac{1}{(1-\rho)}\times T\rho.$

$\dfrac{\lambda}{\mu}\cdot\mu\left(1-\dfrac{\lambda}{\mu}\right)$

$\lambda\left(\dfrac{\mu-\lambda}{\mu}\right)$