

Assignment 2. Stochastic Process (MC-303). B. Tech-V Semester

1. Calculate the probability that a particle undergoing unrestricted random walk is at position +1 at the 4th step where P is equal to 1/2 and Q is equal to 1/3 .

2. Design an example for a (i) non homogeneous markovian chain (ii) homogeneous markovian chain

3. Consider a Bernoulli process where X_n is the outcome of the nth trial and $X_n = k$ ($k=0,1,2,3,\dots,n$) denotes a 'run' of k successes. Design the transition probability matrix if a ladder has 10 steps use the model described to find the probability that you can reach the 10th step by the 50th jump.

4. Let $\{X_n, n \geq 0\}$ be a three state 0, 1, 2 Markov chain with transition probability matrix

$$\begin{pmatrix} 0.75 & 0.25 & 0 \\ 0.25 & 0.50 & 0.25 \\ 0 & 0.75 & 0.25 \end{pmatrix}$$

with initial distribution $p_i = P[X_0=i] = 1/3$; $i=0,1,2$. Find $P[X_3=1, X_2=2, X_0=2]$.

5. Use the following transition probability matrix to find:

$$\begin{matrix} & 0 & 1 \\ 0 & 1/2 & 1/2 \\ 1 & 1/2 & 1/2 \end{matrix}$$

a) The probability that 0 transmitted is received as 0 at the 5th stage.

b) The probability that 0 is received at the 5th stages when at initial stage both digits are to be passed with equally likely probability.

6. Suppose that whether it rains today depends on previous weather conditions only from the last two days and let:

$P[\text{If it has rained for the past two days, then it will rain tomorrow}] = 0.7$

$P[\text{If it has rained today but not yesterday, then it will rain tomorrow}] = 0.5$

$P[\text{If it rained yesterday but not today then it will rain tomorrow}] = 0.4$

$P[\text{if it has not rained for the past two days then it will rain tomorrow}] = 0.2$

Assuming the system to be homogeneous, write it as a Markov chain. Let it has rained on Monday but not on Tuesday. What is the probability that it will rain on Thursday?

7. Prove that the relation communication is an equivalence relation.

8. Prove that if $i \leftrightarrow j$, then i and j have the same period.

