# Vector & Matrix Worksheet

Marco Braune

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## 1 Vectors

- What is a column vector vs a row vector?

Different notation:

- Column Vector: $\begin{pmatrix} x \\ y \end{pmatrix}$

- Row Vector: $\begin{pmatrix} x & y \end{pmatrix}$

- How do you calculate the dot or inner product?

$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = x \cdot a + y \cdot b$

Alternative: $\vec{v_1} \cdot \vec{v_2} = ||\vec{v_1}|| \cdot ||\vec{v_2}|| \cdot cos(\alpha)$

- What does it tell you if the dot product is 0, $<0$ or $>0$?

$=$0: angle between vectors equals 90°
$<$0: angle between vectors is $>$90° (obtuse angle)
$>$0: angle between vectors is $<$90° (acute angle)

- What is a normalized vector and how do you calculate it?

- resizing a vector to length $= 1$
- Calculation: $\frac{\vec{v}}{||\vec{v}||}$

- What is a normal vector?

- a vector that is perpendicular to a surface

# 2 Matrices

- What is a matrix generally speaking and what does it consist of

- contains multiple values
- is a tuple of scalar vectors

- What does a matrix represent geometrically?

- the columns represent the basis vectors of a vector space

- What is a transposed matrix?

- different notation
- the vectors that build the matrix are written as row vectors instead of columns vectors

- How do you multiply two matrices?

- column $\cdot$ row

Example:

$$\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \cdot \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} = \begin{pmatrix} x_1 \cdot a_1 + x_2 \cdot b_1 & x_1 \cdot a_2 + x_2 \cdot b_2 \\ y_1 \cdot a_1 + y_2 \cdot b_1 & y_1 \cdot a_2 + y_2 \cdot b_2 \end{pmatrix}$$

- Why is order important when multiplying several matrices?

- matrix multiplication is not commutative

# 3 Transformation

## 3.1 Transformation 1

- What matrix operations are involved?

- Translation along the x-axis

- Derive the transformation matrices.

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## 3.2 Transformation 2

- What matrix operations are involved?

- scaling and translation

- Derive the transformation matrices.

1. Scaling: $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Translation: $\begin{bmatrix} 1 & 0 & 3,5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

## 3.3 Transformation 3

- What matrix operations are involved?

- scaling, shearing, translation

- Derive the transformation matrices.

1. Scaling: $\begin{bmatrix} 0,25 & 0 & 0 \\ 0 & 0,5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Shearing: $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Translation: $\begin{bmatrix} 1 & 0 & 6,25 \\ 0 & 1 & 0,25 \\ 0 & 0 & 1 \end{bmatrix}$

### 3.4 Transformation 4

- Is this an affine transformation?

- yes, it's a composition of linear functions adding translation, the origin does not remain fixed

- What kinds of transformation are involved and how have they been applied?

- rotation, shearing, scaling, translation

- Derive the transformation matrices.

1. Rotation: $\begin{bmatrix} cos(\frac{\pi}{2}) & -sin(\frac{\pi}{2}) & 0 \\ sin(\frac{\pi}{2}) & cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Shearing: $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Scaling: $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Translation: $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$