Task 7.1 (15min)

Manually compute the dot and cross product of vectors `a(0,0,1)` and `b(2,0,0)` . Show your calculation steps, the result alone is not enough. Explain the results briefly.

Dot product:

$a \cdot b$

$= (0,0,1) \cdot (2,0,0)$

$= a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$

$= 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 2$

$=0 + 0 + 0$

$=0$

Conversely, the only way the dot product can be zero is if the angle between the two vectors is 90 degrees. Two non-zero vectors have dot product zero if and only if they are orthogonal.

TASK 7.1

Dot Product:

$\vec{a} = (0, 0, 1)$

$\vec{b} = (2, 0, 0)$

$\vec{a} \cdot \vec{b} = 0 \cdot 2 + 0 \cdot 0 + 1 \cdot 0 =$

0

- Because the DOT product is zero, that means $\vec{a}$ and $\vec{b}$ are at right angles (perpendicular) to each other

Cross product:

$$a \times b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= \mathbf{i}\,(a_y \cdot b_z - a_z \cdot b_y) - \mathbf{j}\,(a_x \cdot b_z - a_z \cdot b_x) + \mathbf{k}\,(a_x \cdot b_y - a_y \cdot b_x)$$

$$= \mathbf{i}\,(0 \cdot 2 - 0 \cdot 0) - \mathbf{j}\,(1 \cdot 2 - 0 \cdot 0) + \mathbf{k}\,(1 \cdot 0 - 0 \cdot 0)$$

$$= \mathbf{i}\,(0 - 0) - \mathbf{j}\,(2 - 0) + \mathbf{k}\,(0 - 0)$$

$$= \{0, -2, 0\}$$

The result is in a right angle to a and b and ?anticommutative?. The magnitude (length) of the cross product equals the area of a parallelogram with vectors a and b for sides.

TASK 7.1

CROSS Product:

$\vec{a}\,(0, 0, 1)$

$\vec{b}\,(2, 0, 0)$

$$\vec{a} \times \vec{b} = \begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 & 0 & 0 \end{array} =$$

$$= \left(\underset{0}{0-0},\ \underset{2}{2-0},\ \underset{0}{0-0} \right) =$$

$$(0, 2, 0)$$

$$\vec{c} = (0, 2, 0)$$

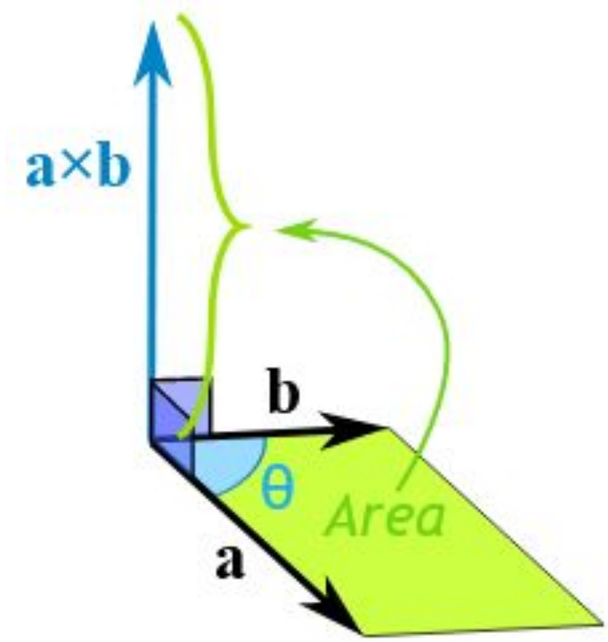
- $\vec{c}$ is at right angles to $\vec{a}$ and $\vec{b}$
- the magnitude of $\vec{c}$ equals the area of a parallelogram with $\vec{a}$ and $\vec{a}$ for sides

Task 7.2 (20min)

You learned about the dot product and the cross product. What functions in processing (/p5.js) implement the two operations if any? Find an application scenario (processing sketch, online) that makes use of the dot product. In which way has the dot product been used? Can you think of any other applications?

Dot product:

```
let v1 = createVector(1, 2, 3);
let v2 = createVector(3, 2, 1);
print(p5.Vector.dot(v1, v2));
```



The dot product can be used to find the length of a vector or the angle between two vectors.

Example:

https://p5js.org/examples/motion-non-orthogonal-reflection.html

Here the dot product is used to check if the object is colliding with the base and the reflection vector, that is used to assign the direction that the object is moving after a collision.

Cross product:

```
let v1 = createVector(1, 0, 0);
let v2 = createVector(0, 1, 0);
let crossProduct =
p5.Vector.cross(v1, v2);
```

The cross product is used to find a vector which is perpendicular to the plane spanned by two vectors. It has many applications in physics when dealing with the rotating bodies.