Theoretical Backgrounds of Audio & Graphics

Pitch / Wavetable synth / Subtractive Synthesis

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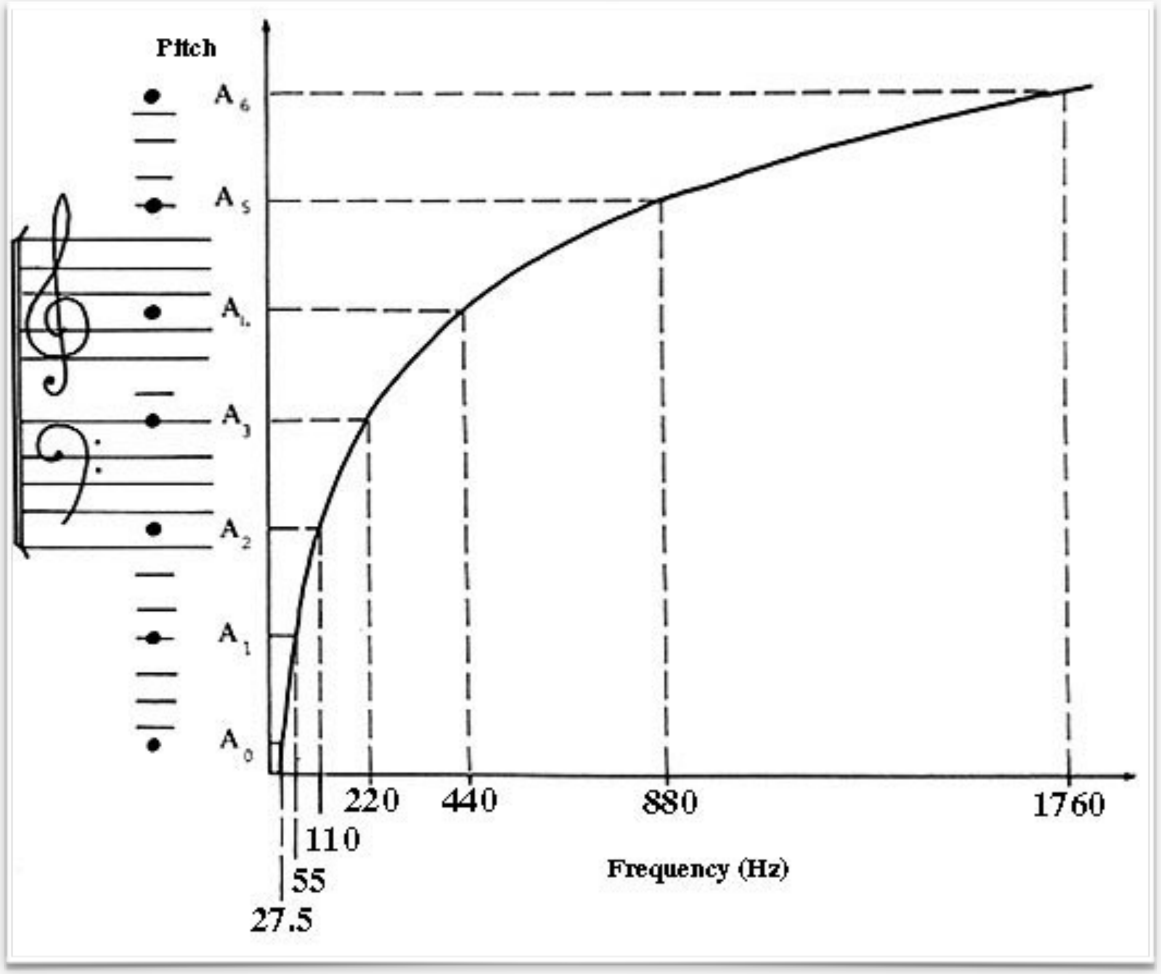
Frequencies, Pitch & Musical Tones

Terminology

- **Frequency** — Physical measure of vibrations per second
- **Pitch** — A corresponding perceptual experience of frequency as well as a musical sound
- **Tone** — A discrete musical sound of a musical system
- **Interval** — The ratio of two frequencies
- **Scale** — An ordered subset of theoretically infinite num of pitches
- **Note** — Something that is notated in a score of music

General

- Frequency f is perceived as pitch
- The higher the frequency, the higher the perceived pitch & vice versa
- Human hearing ranges between ~20Hz to ~20kHz



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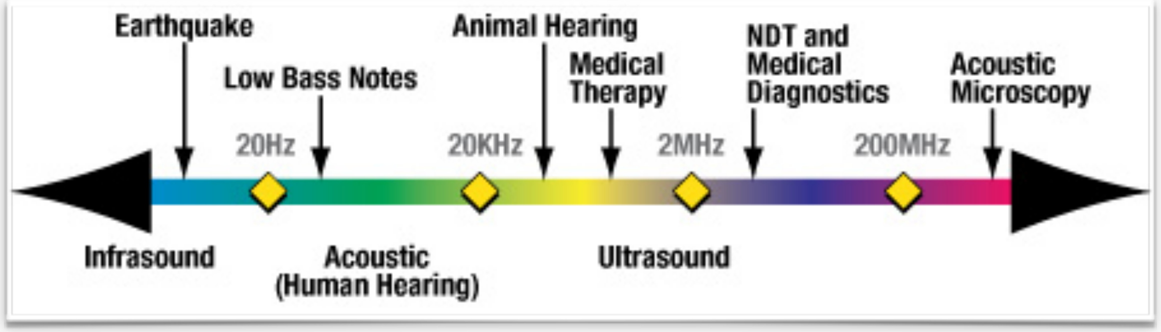

Image credit: https://www.olympus-ims.com/en/ndt-tutorials/thickness-gage/introduction/operation/

Frequency & Pitch

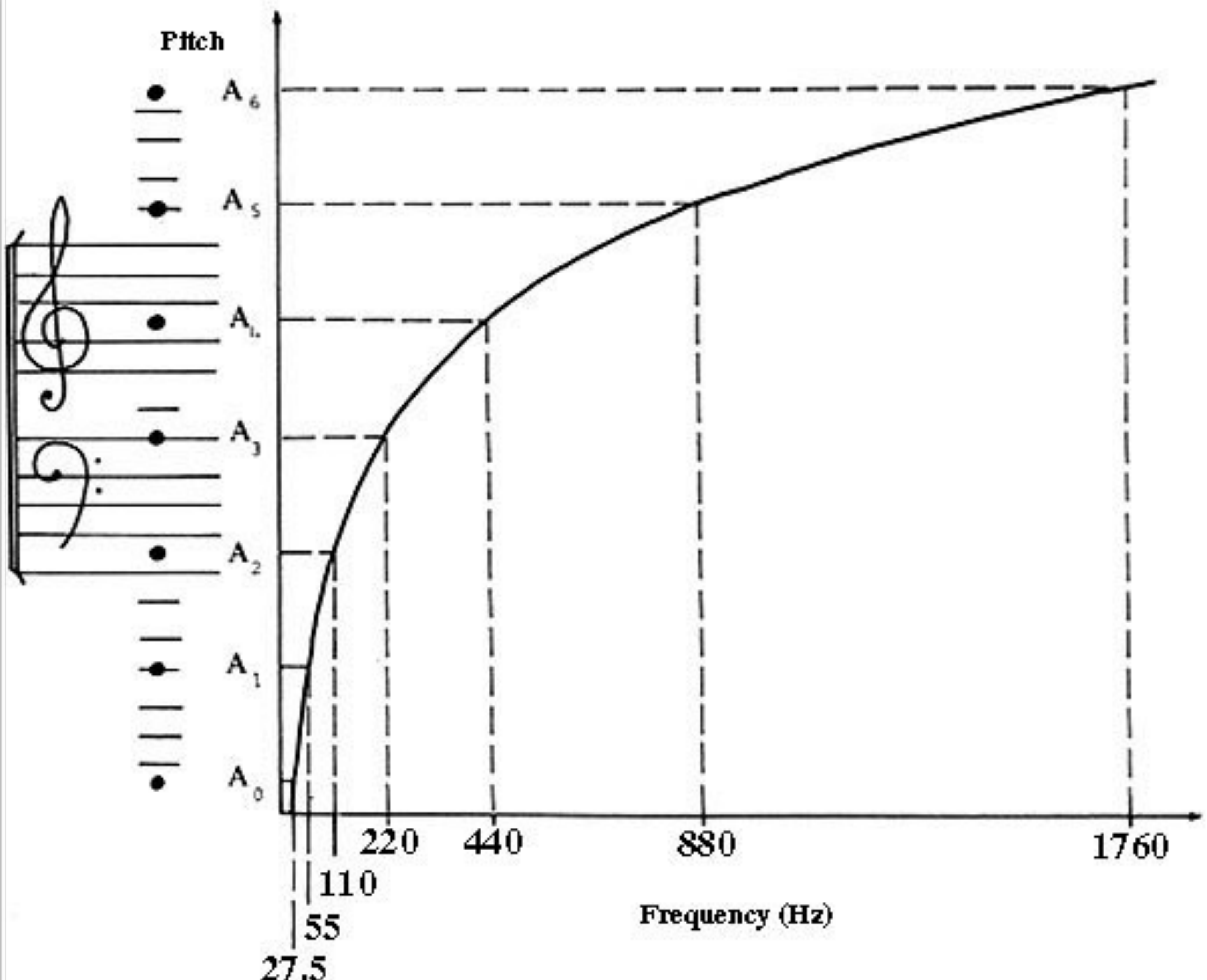

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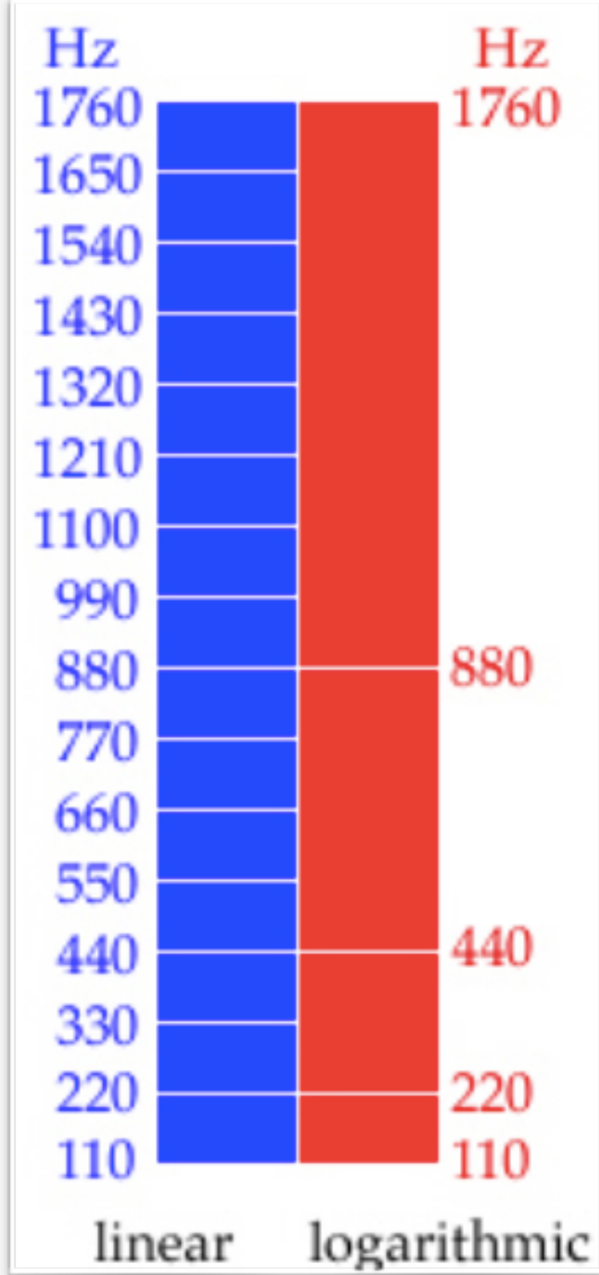
Image credit: http://cmc.music.columbia.edu/MusicAndComputers/chapter1/01_03.php



Frequency & Pitch

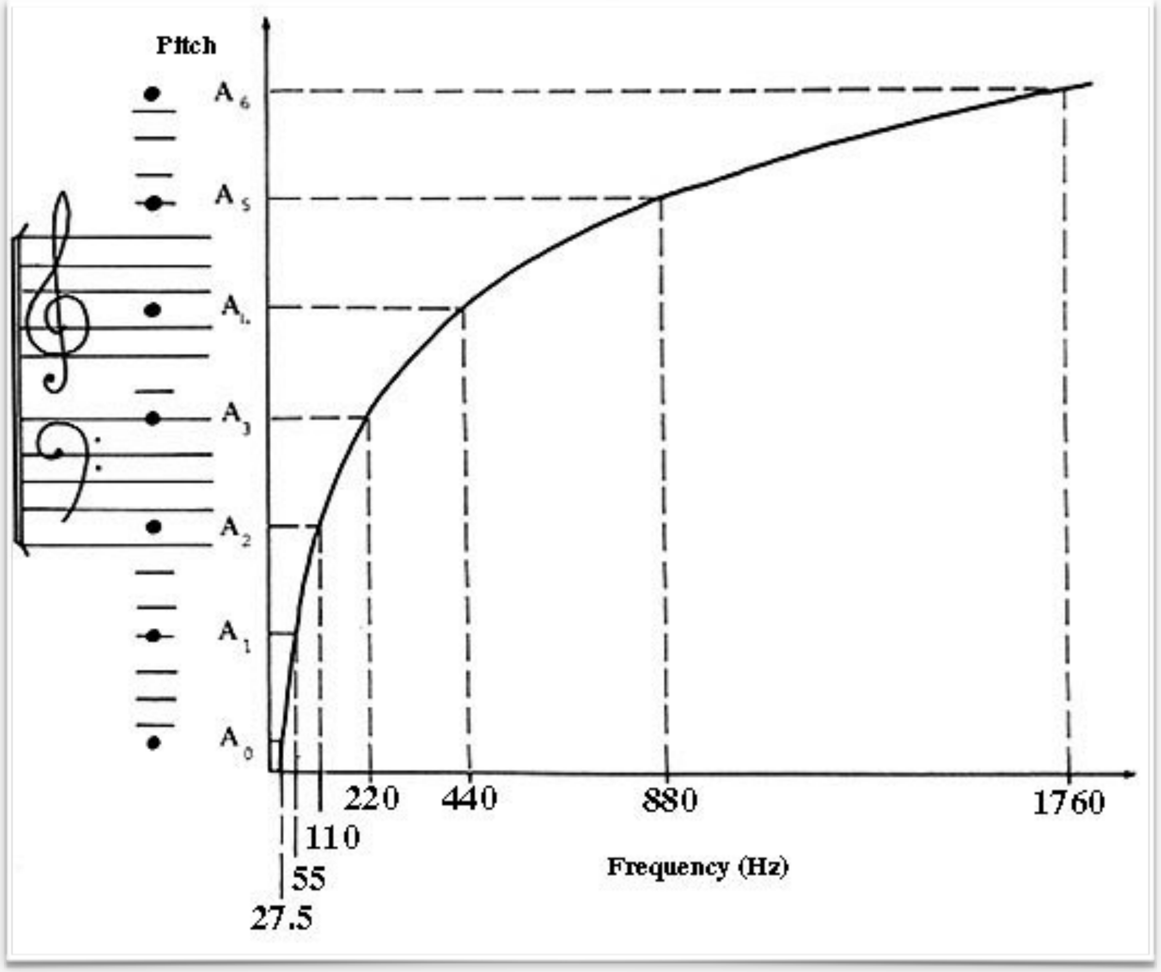
- Pitch is perceived logarithmically—this means, it takes more of a change in frequency to produce the same perceived change in pitch or, phrased differently, to go from pitch to pitch you cannot simply add a constant number to frequency; rather, you have to multiply the frequency by a constant number instead

- For example, when a sound at a certain frequency f (i.e., 440Hz) is being played back at $2*f$, $4*f$, $8*f$ (i.e., 2*440Hz, 2*2*440Hz, 2*2*2*440Hz, etc.) it is being perceived as the same pitch that increases linearly—the frequency, however, increase exponentially

Frequency & Pitch

- Human sense of pitch is proportional to frequency and influenced by
  - frequency range
  - presence of higher and lower frequencies

- Listeners associate a **range of frequencies** with one pitch



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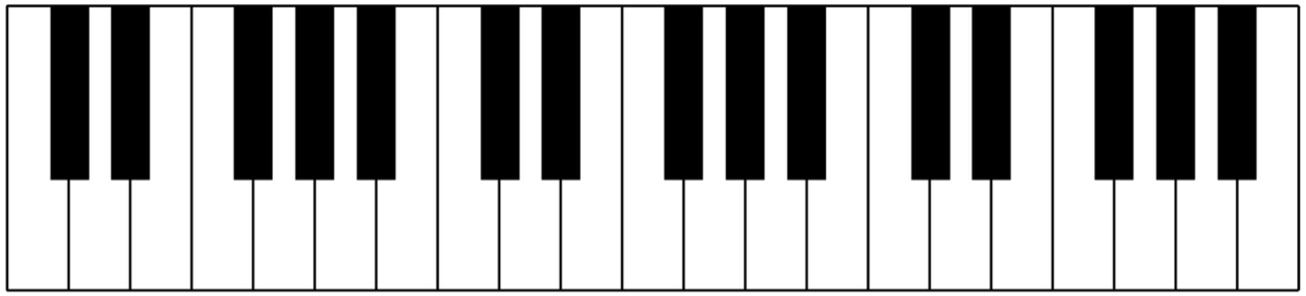


When you play 12 successive keys (also counting black ones) from C to B on a piano keyboard, you're playing a chromatic scale. Image: Tobias R.

https://teropa.info/blog/2016/08/10/frequency-and-pitch.html

Musical Tones

- Every note name/pitch represents a tone at a different frequency & the periodic pattern is defined by frequency ratios or **intervals**
- An **interval** is the difference from one tone to another and is defined by the **ratio of 2 frequencies**, i.e., 880Hz / 440Hz = 2/1

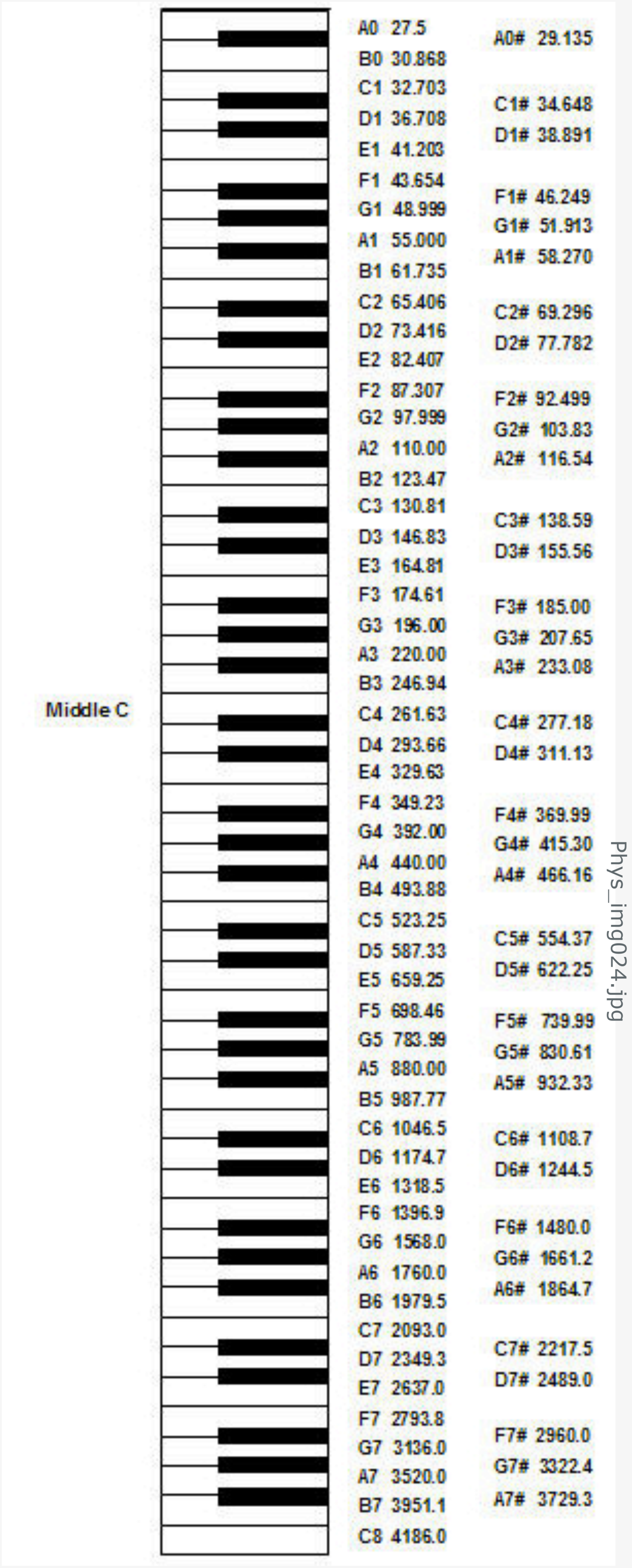
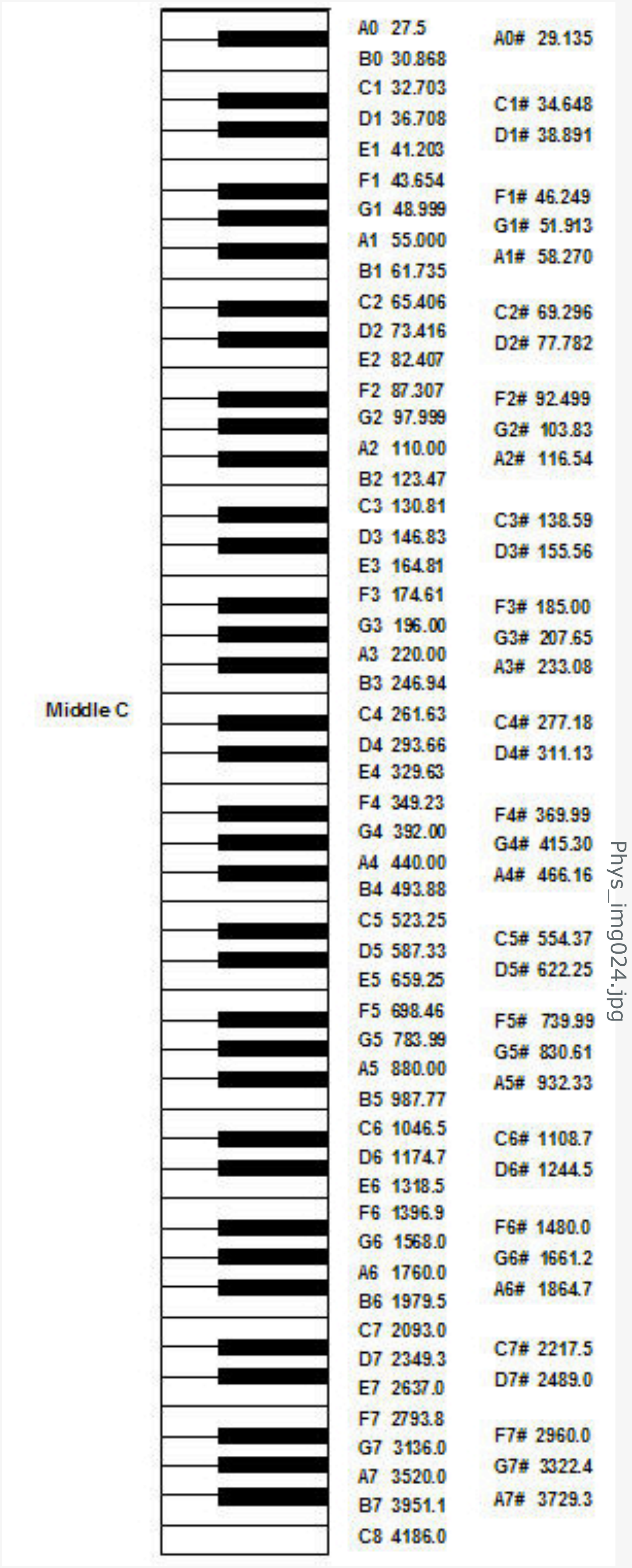


Image credit: https://www.sciencebuddies.org/Files/3436/5/Phys_img024.jpg

Musical Tones

- The fundamental interval that forms the basis for generally any musical system is the **octave**
- The octave relates two frequencies that have a ratio of 2/1 or 1/2, i.e., 880Hz/440Hz or 220Hz/440Hz, etc.



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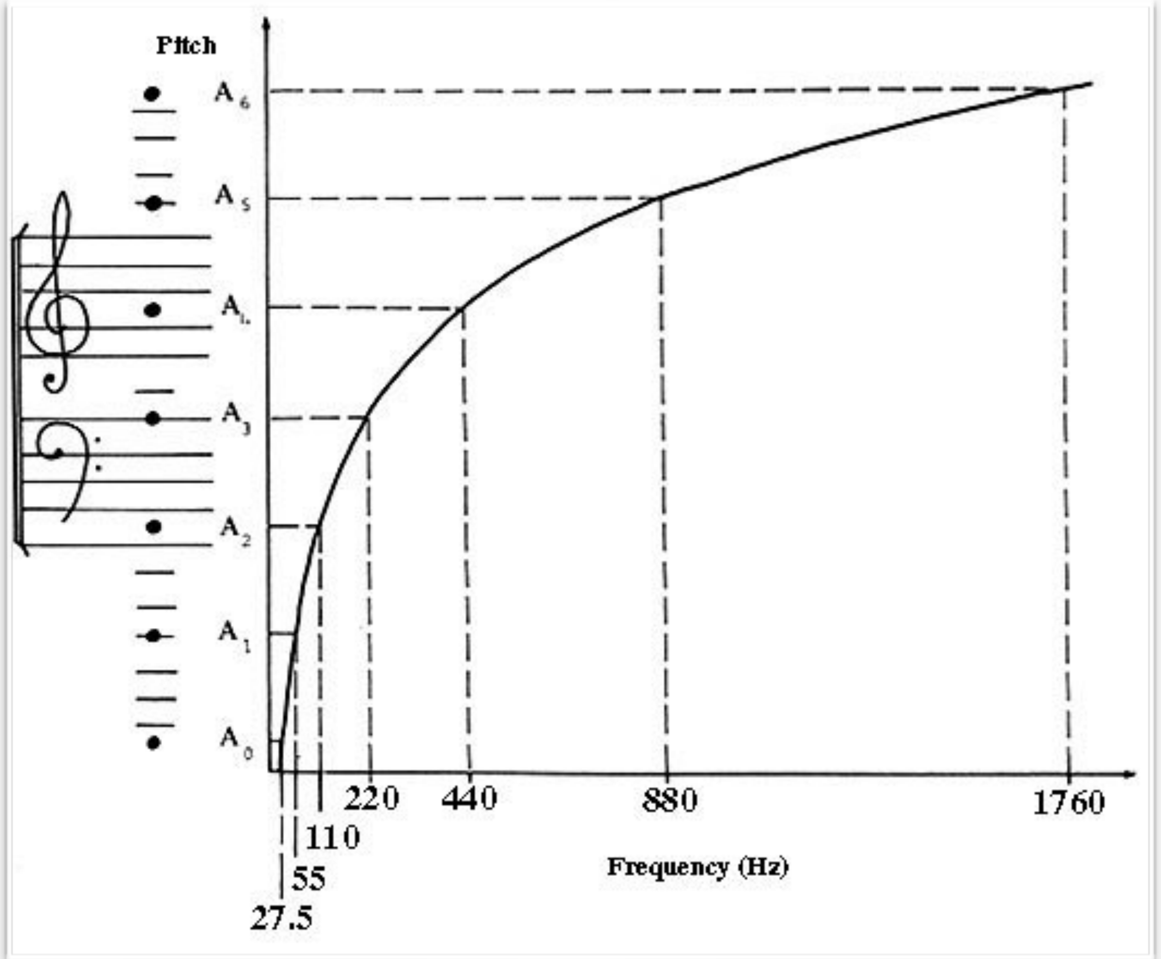
Intervals

- Mathematically, an octave can be represented as

$$f_x = f_R \cdot 2^x, \qquad x \in I$$

f_r — reference frequency

f_x — the frequency of any **octave** x of f_r



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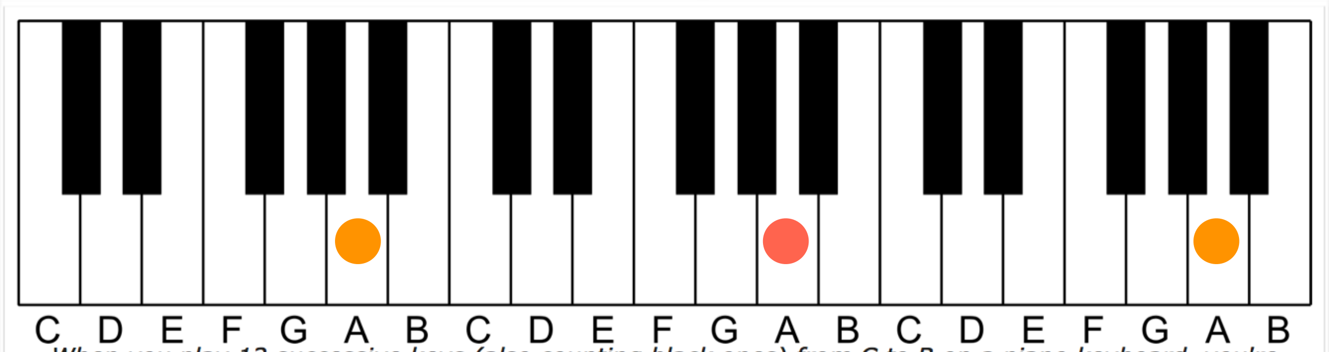

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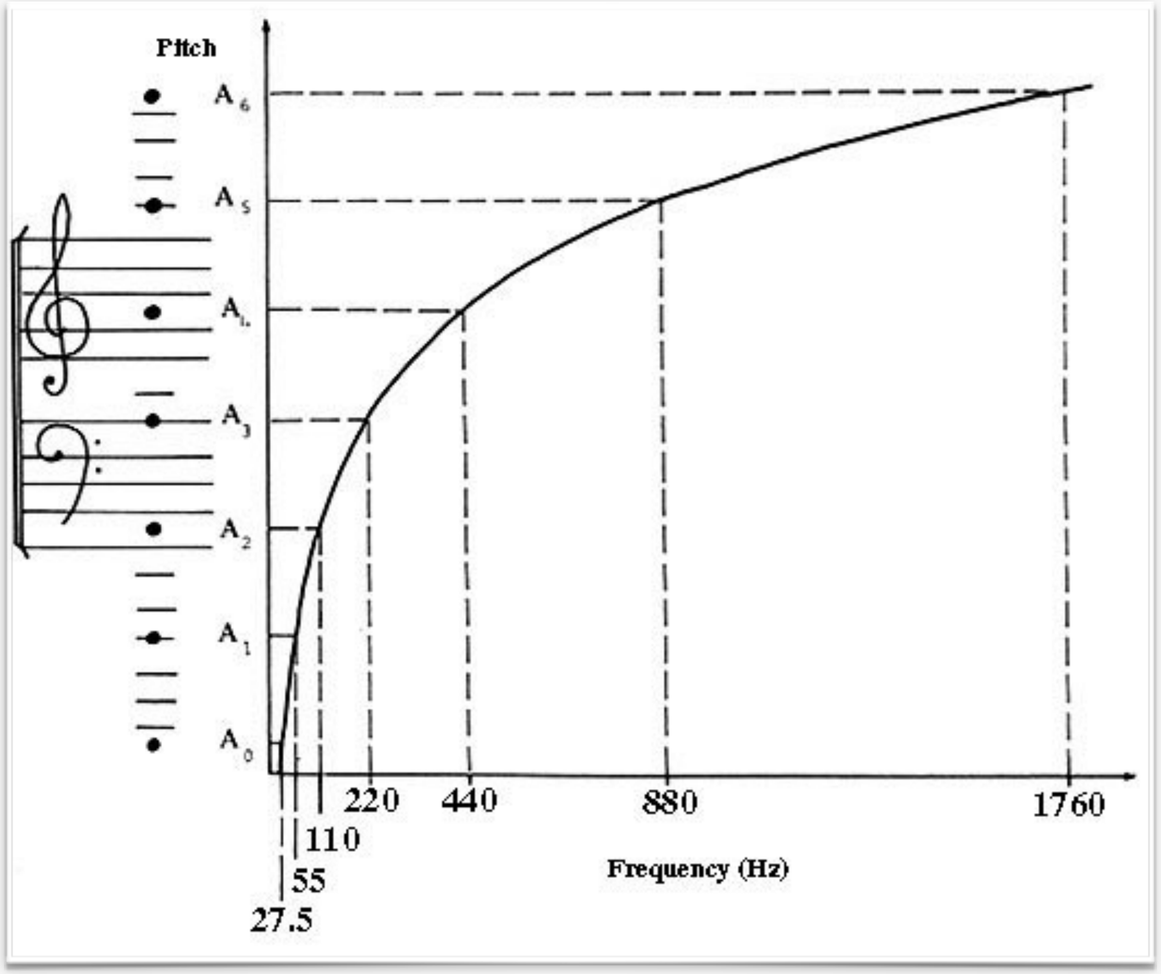
Intervals

- Mathematically, an octave can be represented as

$$f_x = f_R \cdot 2^x, \quad x \in I$$

For example, for f_r = 440Hz, x = -1, 0, 1 the following f_x and intervals are calculated:

440 * 2^-1 = 220 <=> 220/440 = 1/2 => **octave lower**
440 * 2^0 = 440 <=> 440/440 = 1/1 => **unison**
440 * 2^1 = 880 <=> 440/220 = 2/1 => **octave higher**



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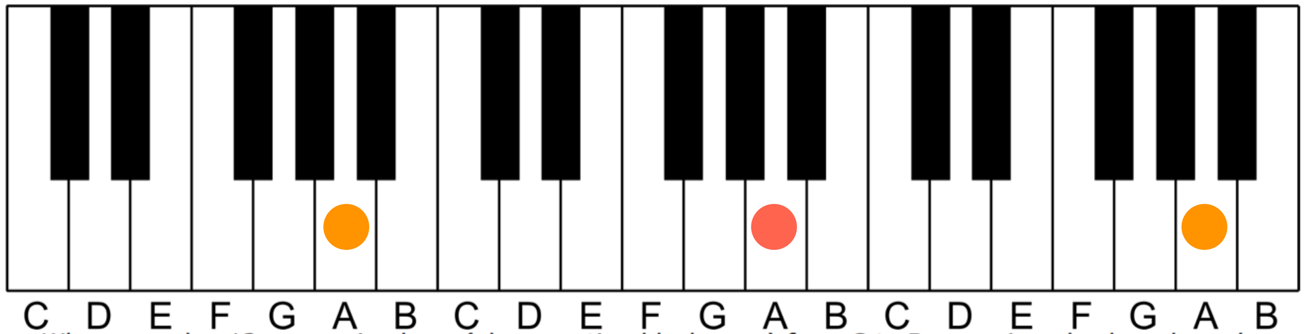

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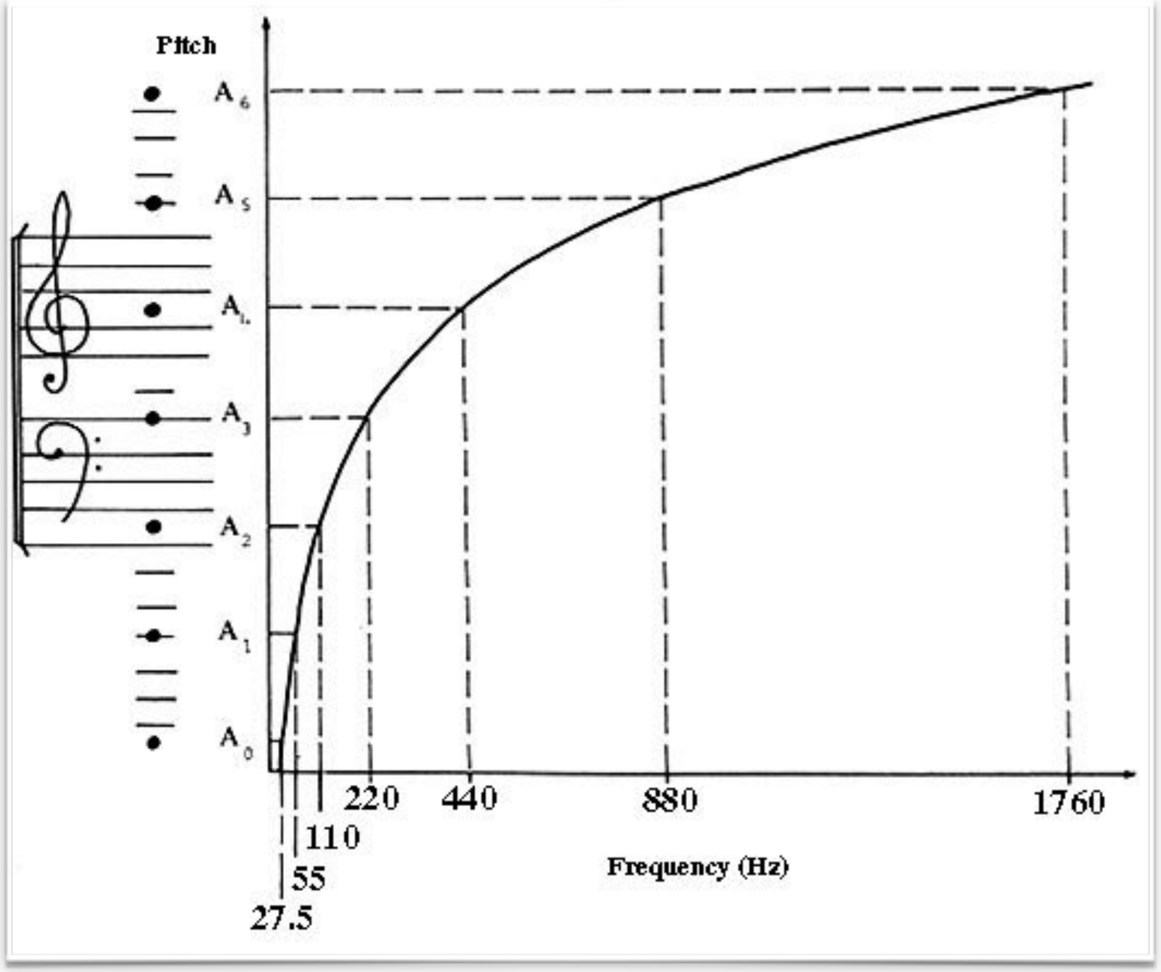
Intervals

- Mathematically, any interval can be represented as ***x*** becomes element of ***R***

$$f_x = f_R \cdot 2^x, \qquad x \in R$$

f_r — reference frequency

f_x — the frequency of any **arbitrary interval** 0<=x<1 of f_r



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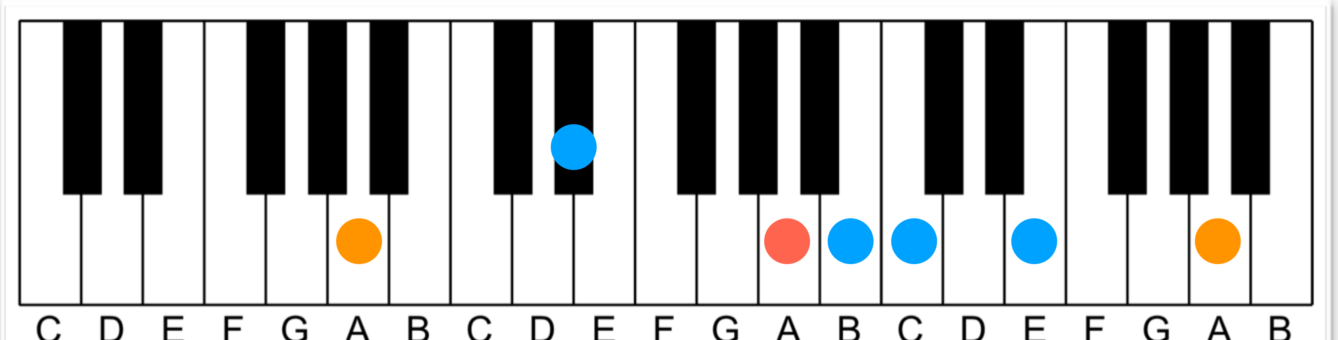

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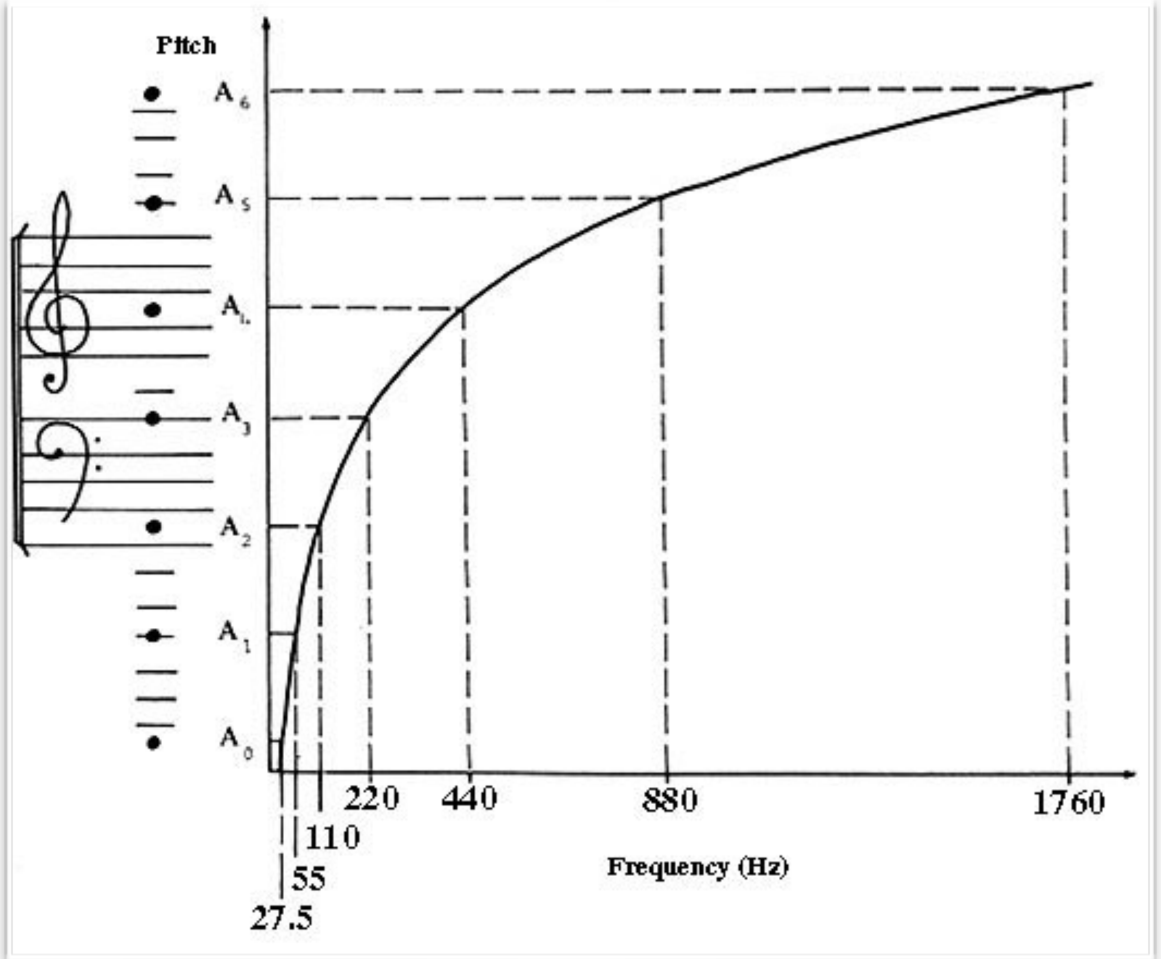
Intervals

- Fundamental intervals in nearly all musical cultures
  - **Unison:** 1/1
  - **Octave up**: 2/1
  - **Octave down**: 1/2



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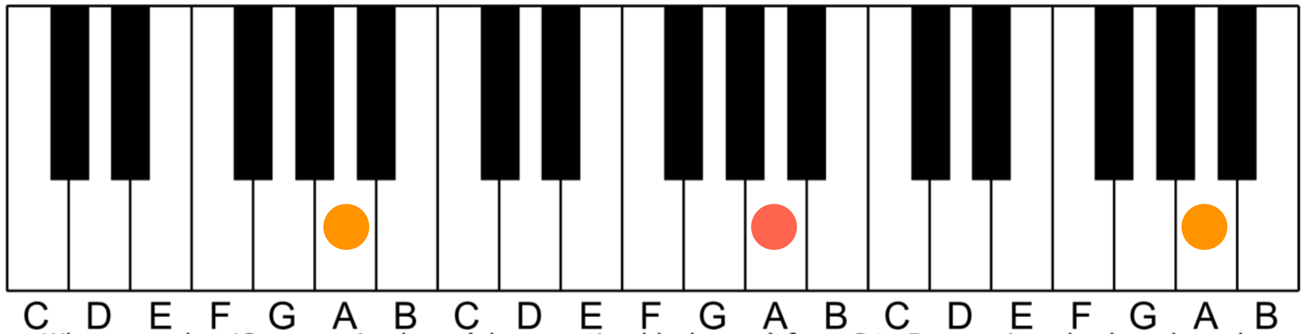

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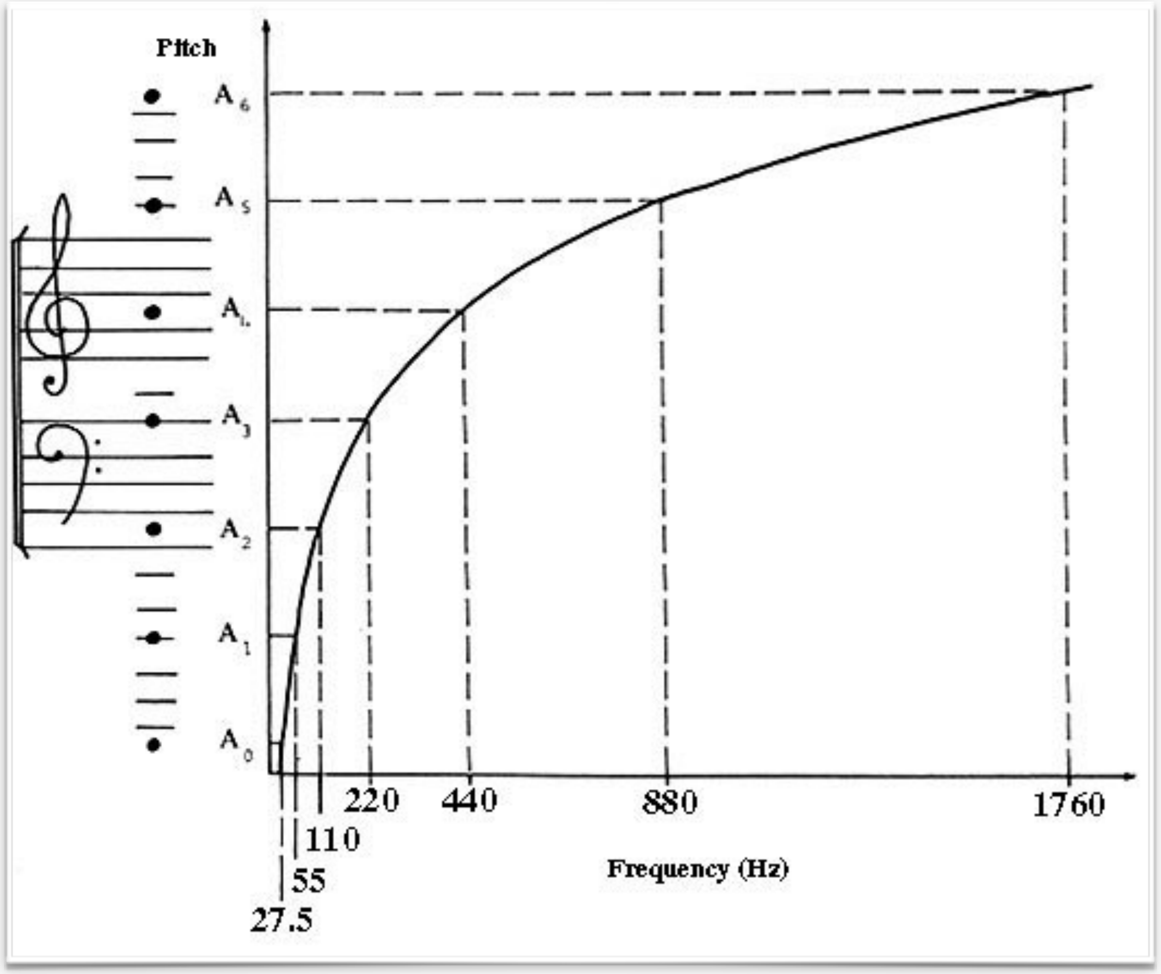
Intervals

- Unison's musical quality is **identity**
- Octave's musical quality is **equivalence**
- Whereas any other interval's quality is referred to as **individuality**



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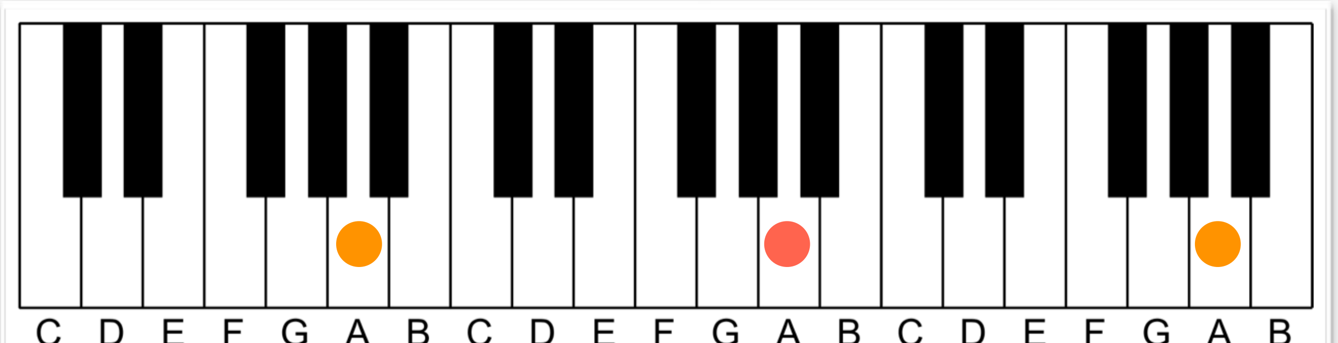

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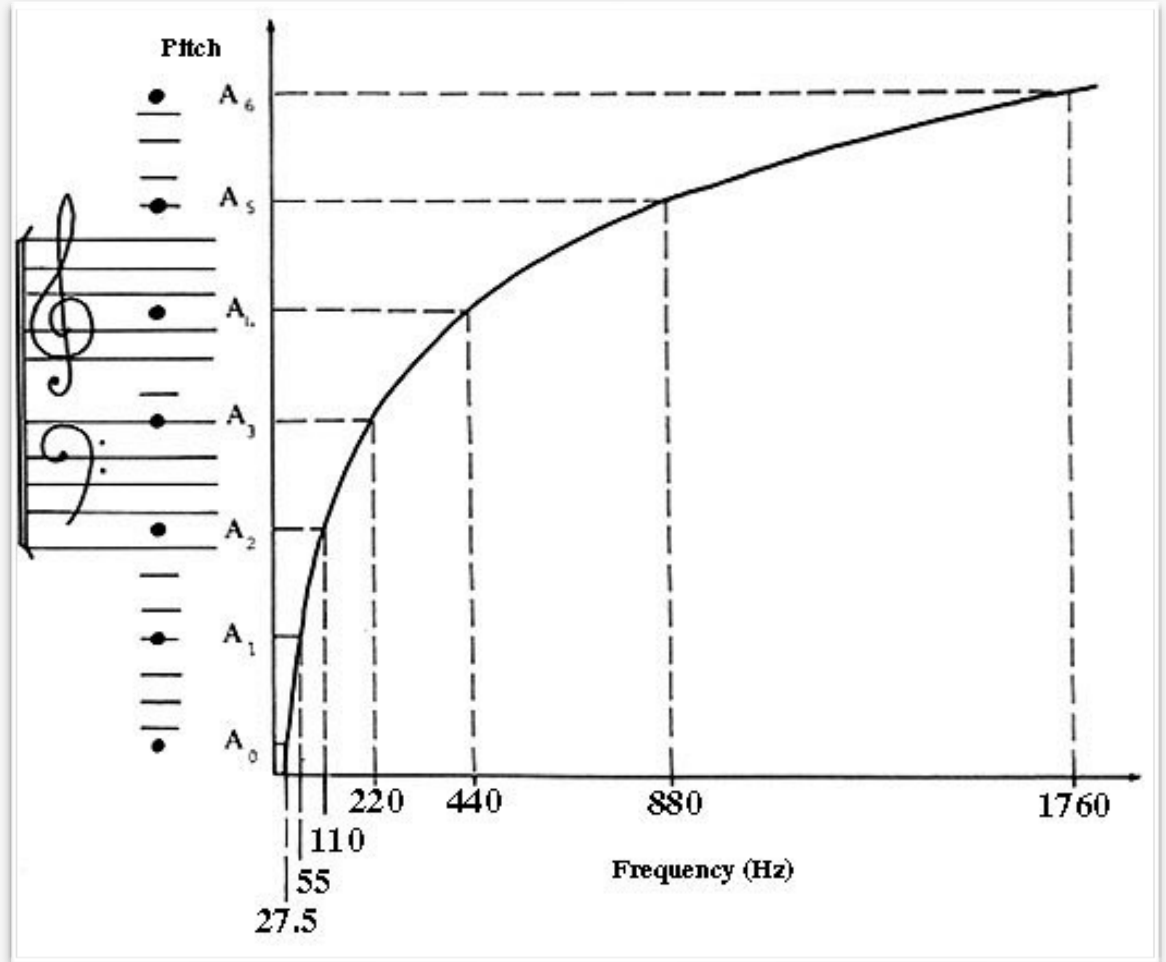
Intervals

- Octaves occur naturally and form the basis for most musical systems
- All other tone intervals that occur within the range of an octave form the basis for what is known as **scales**



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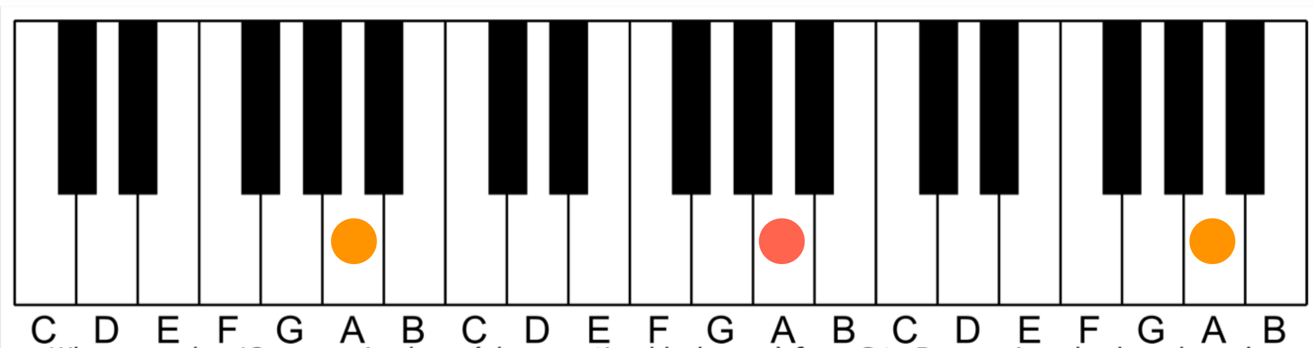

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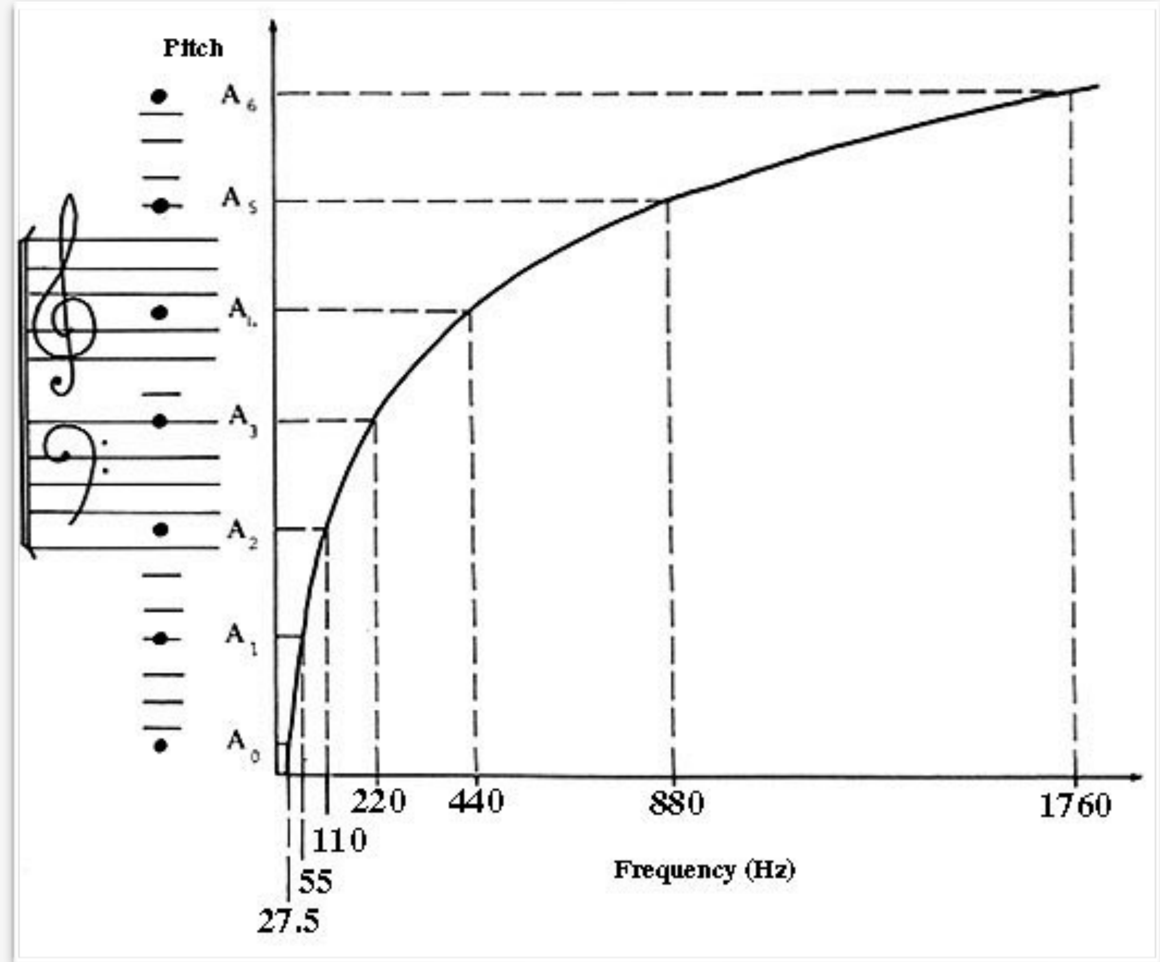
Intervals — Example

Note name (for example)	**Interval**	**Frequency ratio**
A4	Unison	A4/A4
B4	Second (minor, major)	B4/A4
C4	Third (minor, major)	C4/A4
D4	Fourth	D4/A4
E4	Fifth	E4/A4
F4	Sixth (minor, major)	F4/A4
G4	Seventh (minor, major)	G4/A4
A4	Octave	A5/A4

Scales

- A **scale** is an **ordered set of pitches** together with a pattern that specifies the individual frequencies
- The first note of a scale determines its **tonal center**
- All other notes of that scale relate to the tonal center



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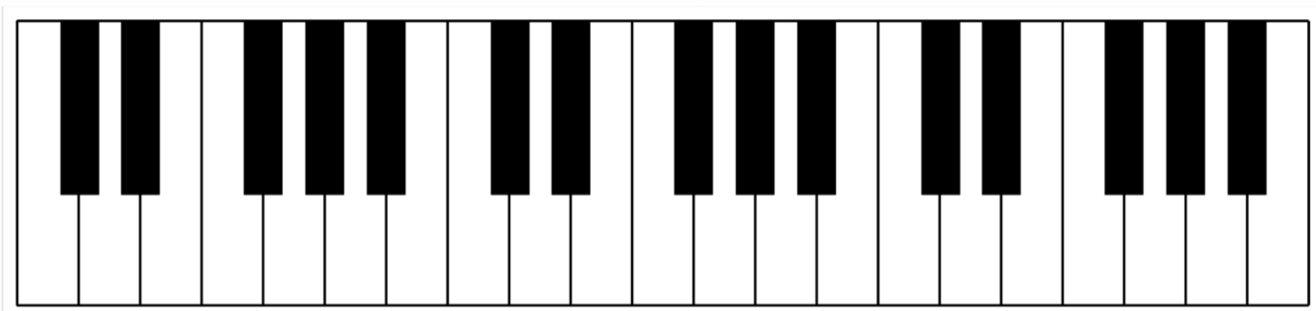

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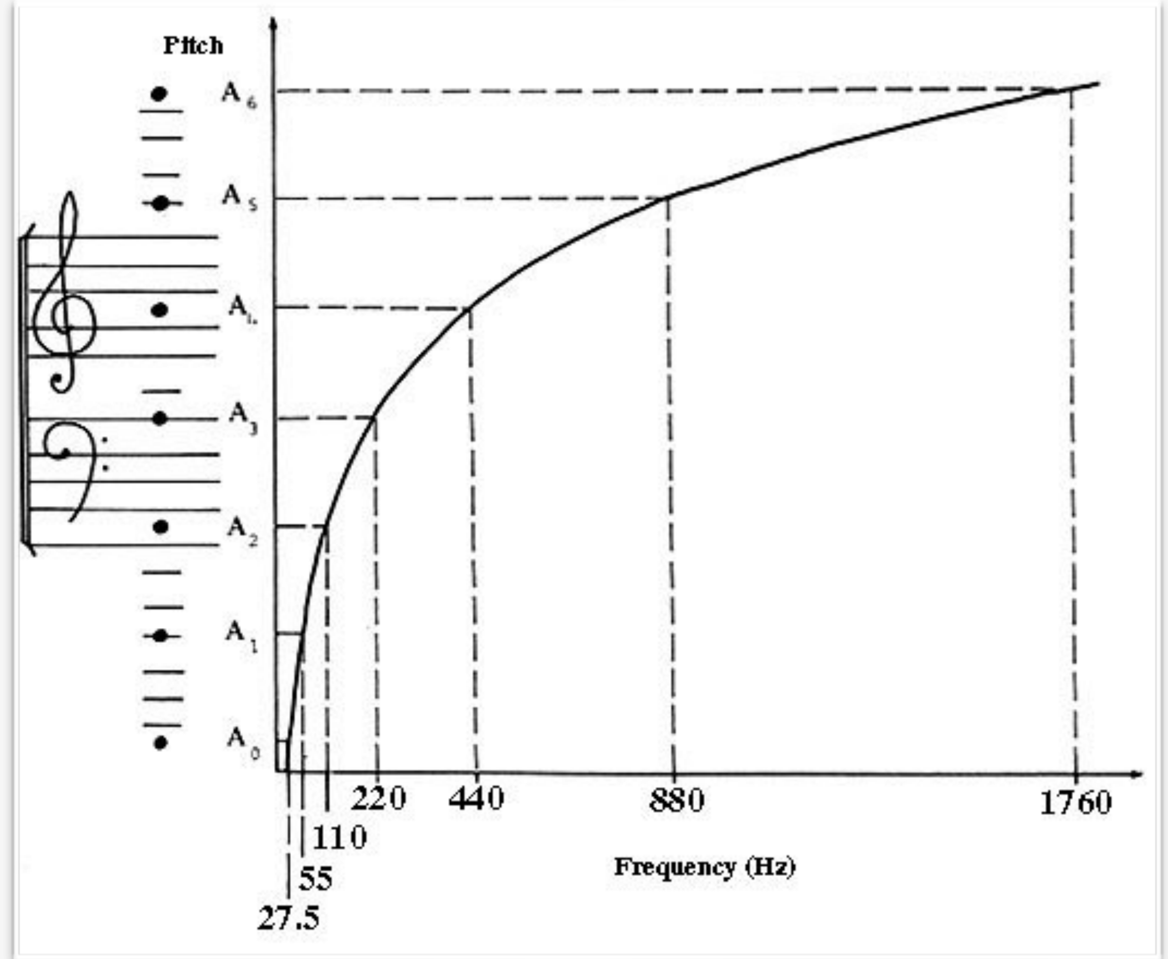
Scales

- Prototypical scale in Western music is the **diatonic scale** (also referred to as Major scale)
  - 7 pitches per octave
  - C, D, E, F, G, A, B
- Intervals between adjacent tones are defined by
  - whole tone/whole step (w)
  - semitone/half step (h)

  corresponding to major and minor second



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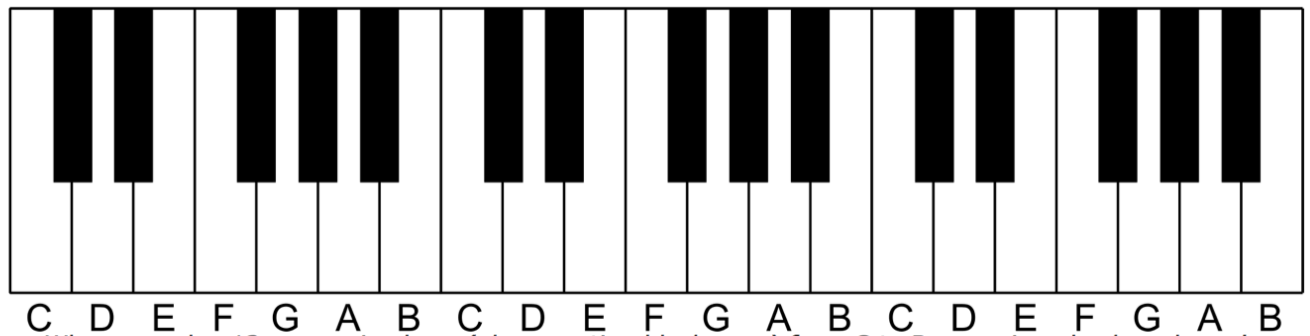

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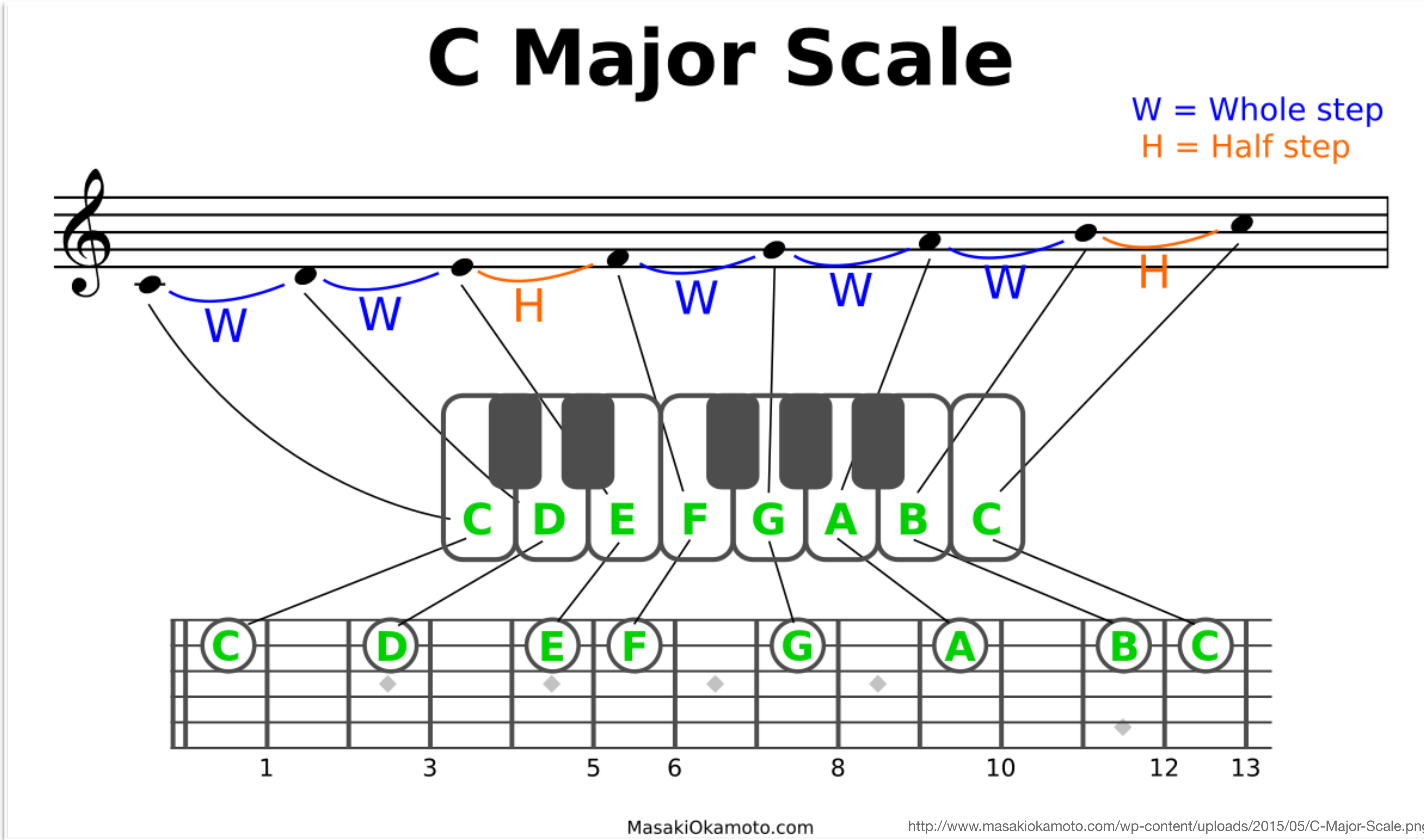
Scales



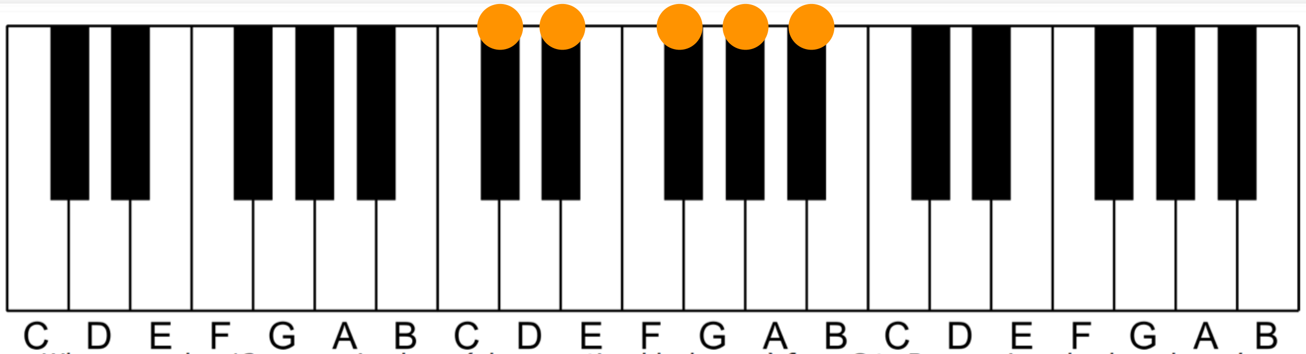
http://www.masakiokamoto.com/wp-content/uploads/2015/05/C-Major-Scale.png

Scales

- The **chromatic scale** breaks up whole tones into half tones & adds semitones between all whole tones of the **diatonic scale**

- It divides an octave into **12 semitones**
  - **#** raises degree by semitone
  - ***b*** lowers a degree by a semitone

https://www.quora.com/Why-does-the-chromatic-scale-have-both-sharps-and-flats



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Scales

- Chromatic scale allows to play many other (mostly Western music) scales like, i.e.,
  - harmonic minor
  - melodic minor
  - whole tone scale
  - pentatonic
  - …

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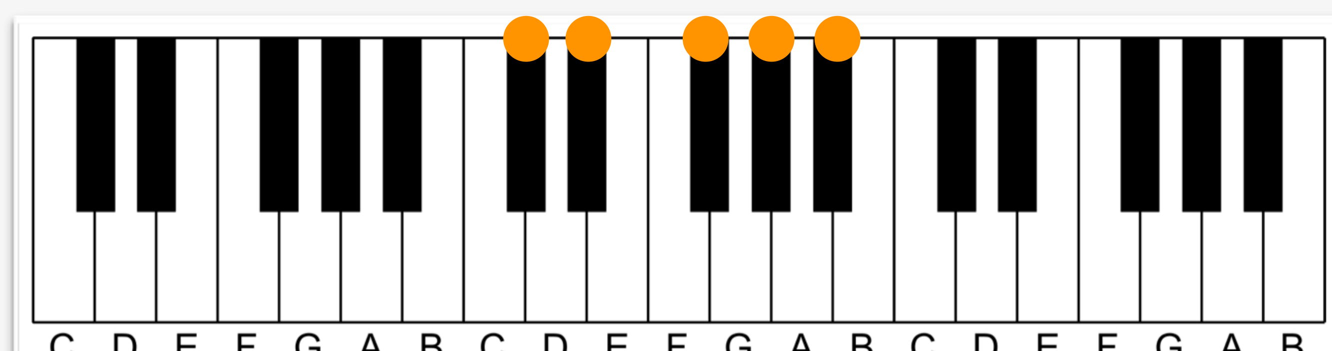

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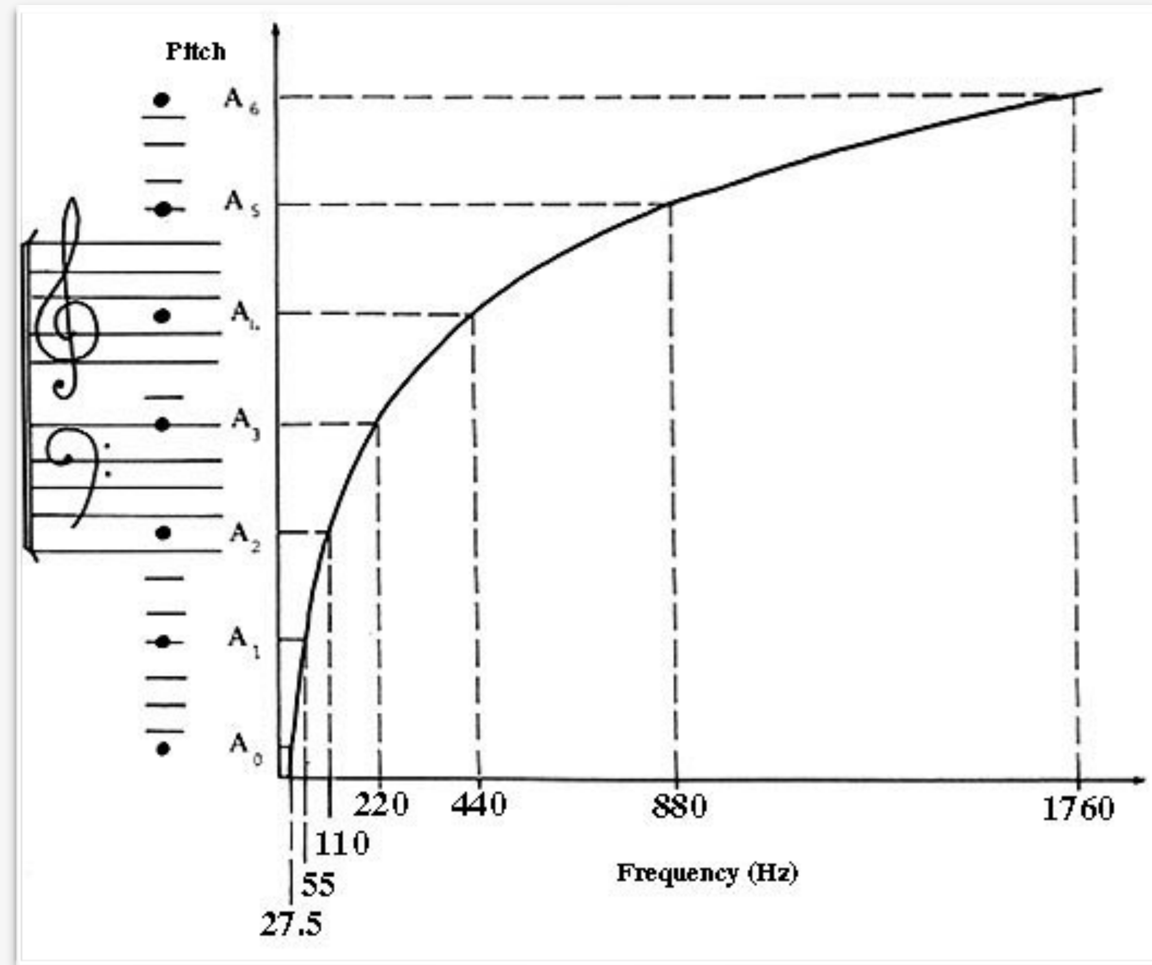
Tuning

- A4 is used in Western music as **reference pitch** to tune all instruments
- All other musical tones are related to the reference pitch
- Reference pitch became standardized at 440Hz in the early 20th century



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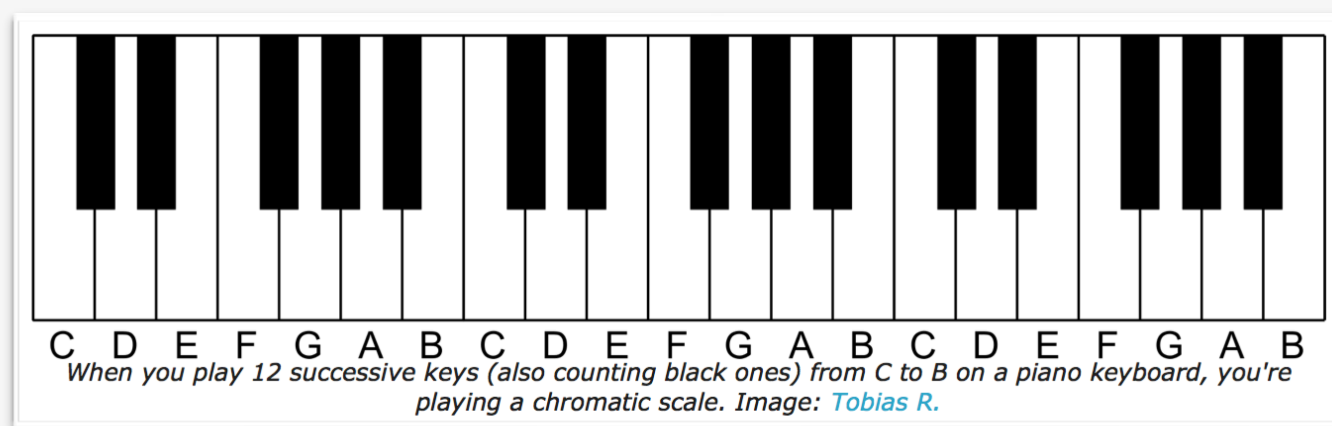

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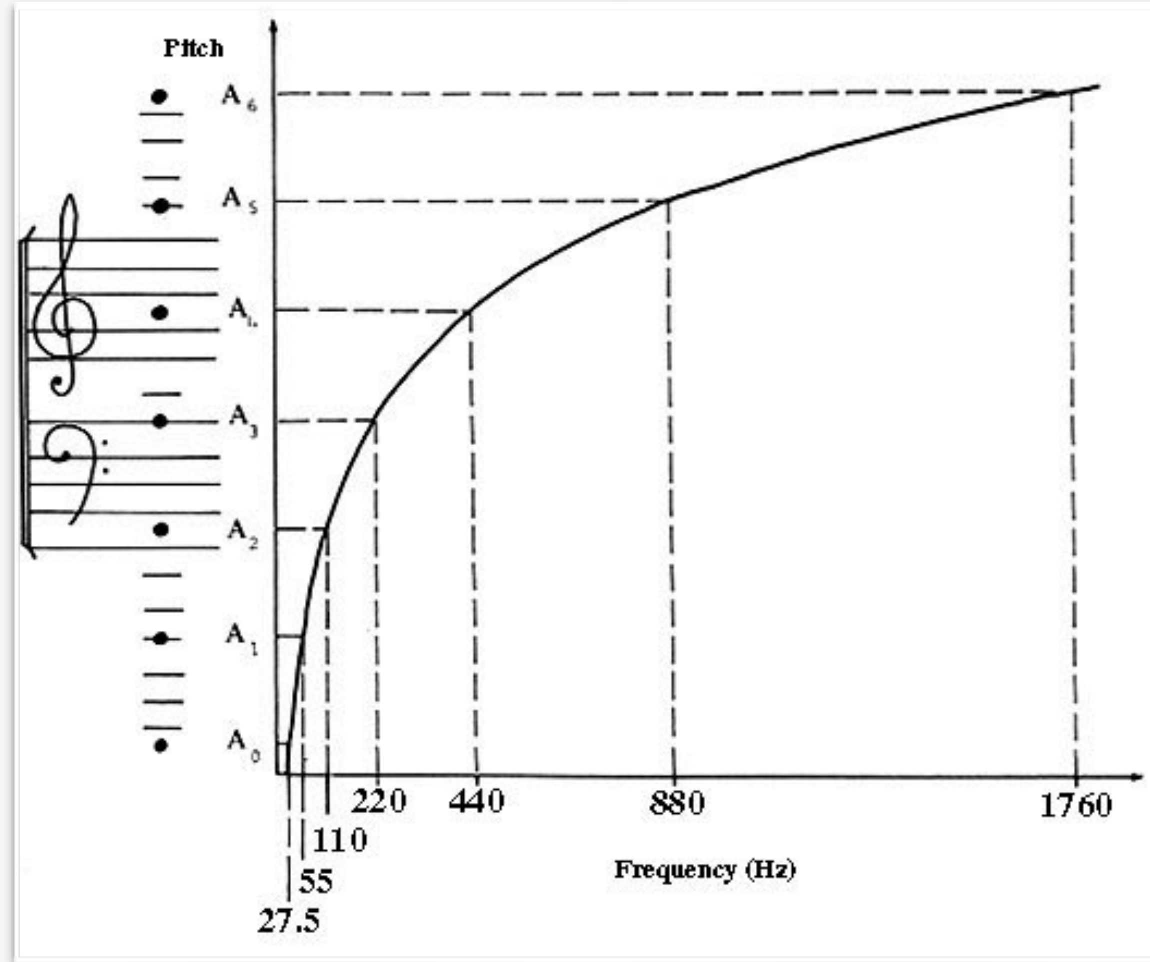
Tuning

- Western music is based on **equal tempered tuning** (artificial tuning, not natural)
- An octave is divided into 12 semitones of **relatively equal size**



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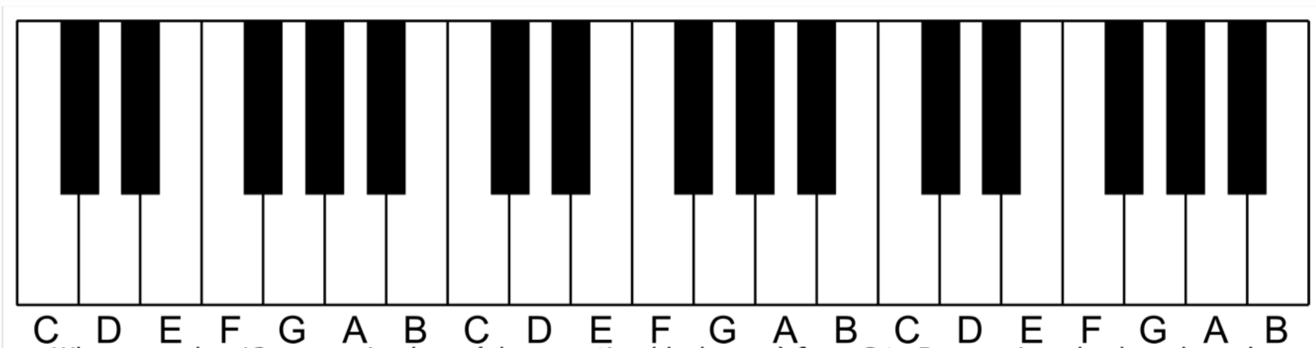

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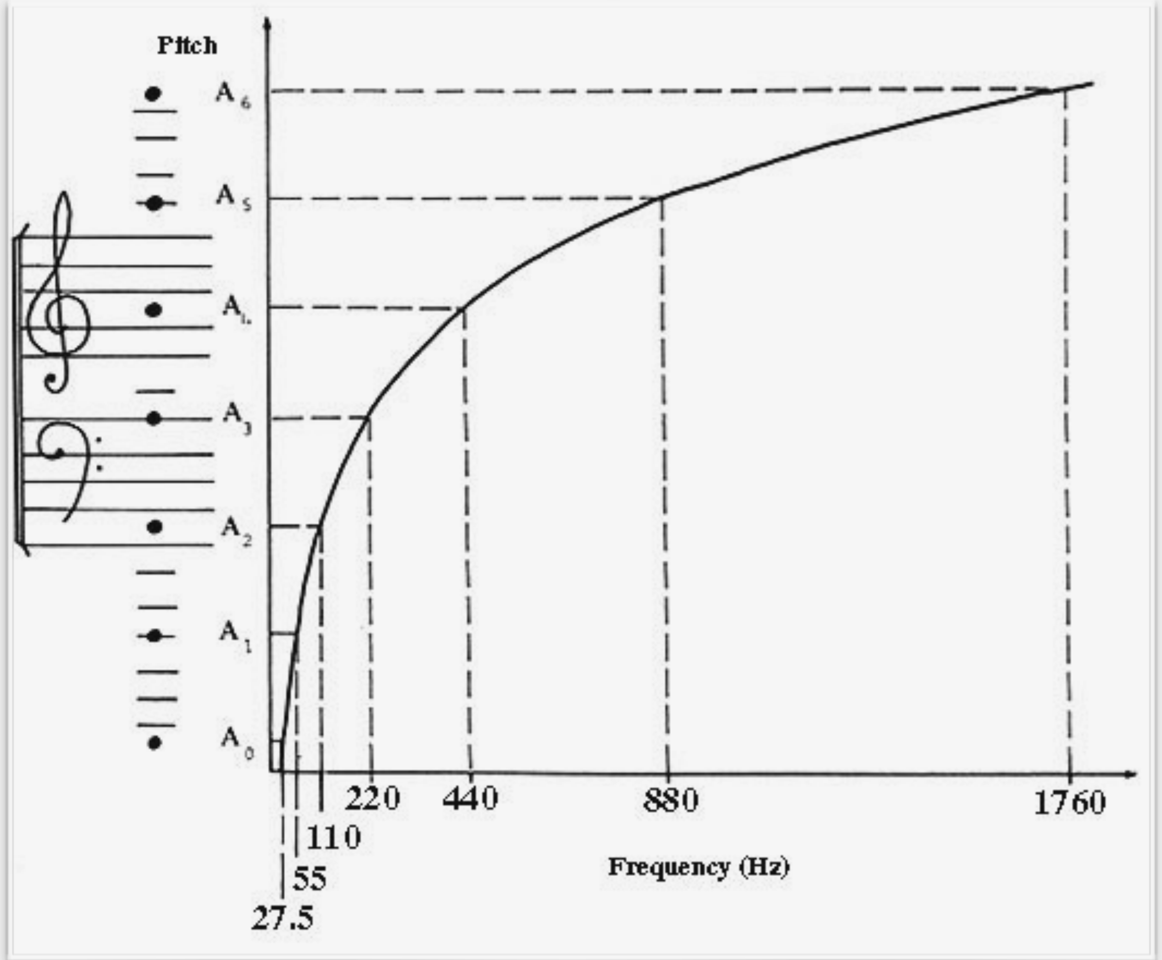
Tuning

- How can we adapt the equation to compute frequencies corresponding to equally tempered tuning?

$$f_x = f_R \cdot 2^x, \qquad x \in R$$

f_r — reference frequency

f_x — the frequency of any **arbitrary interval** 0<=x<1 of f_r



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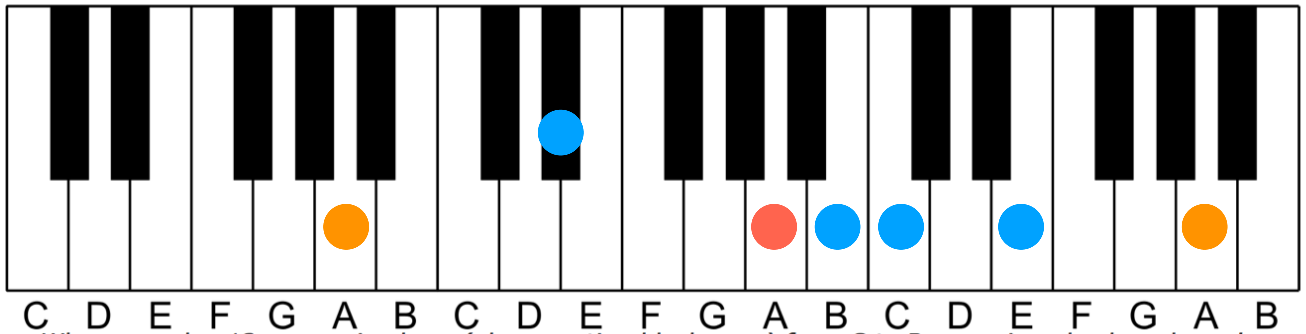

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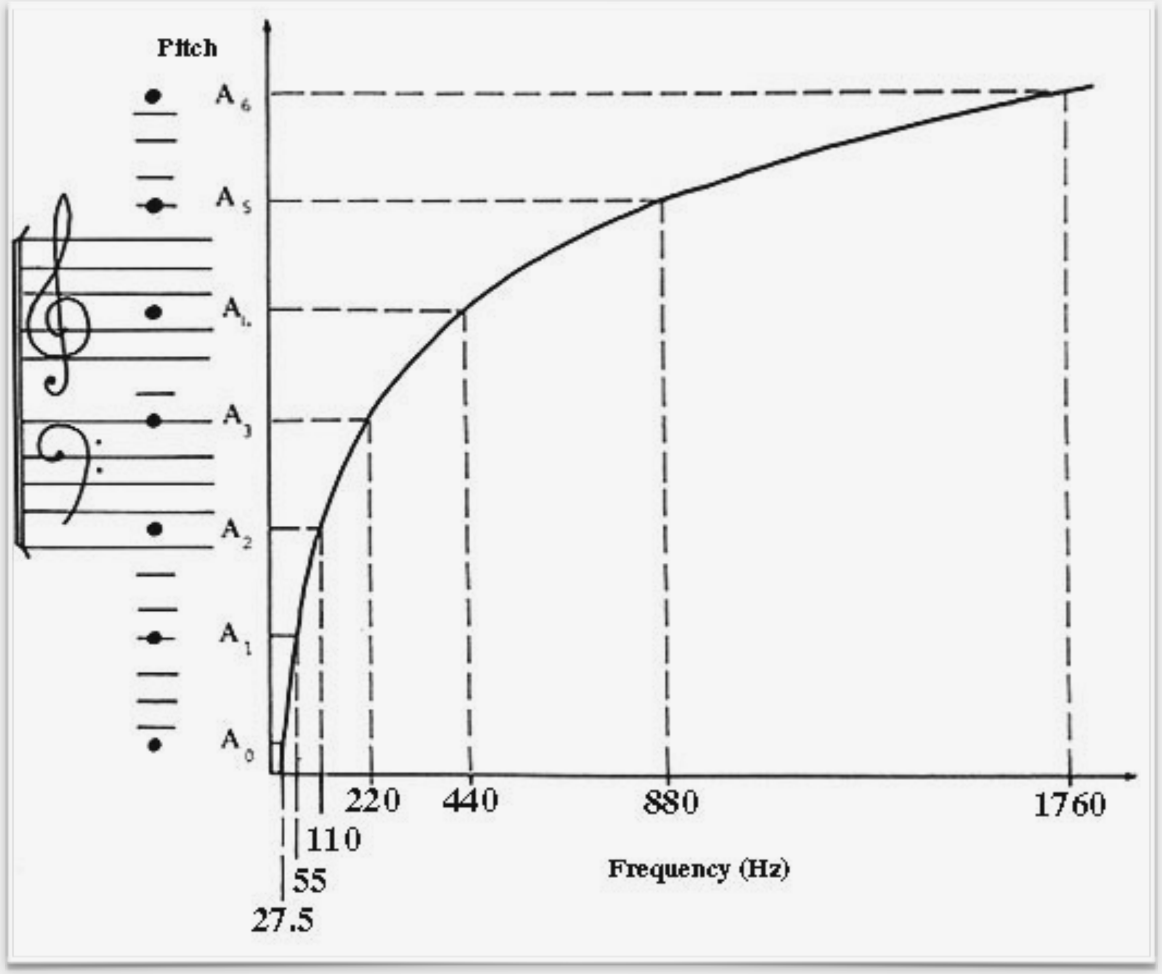
Tuning

- The exponent **x** needs to be divided by **12**

$$f_k = f_R \cdot 2^{k/12}, \qquad k=0,1,2,\ldots,11$$

f_r — reference frequency

f_k — the frequency of any **arbitrary interval** k within **the first** octave



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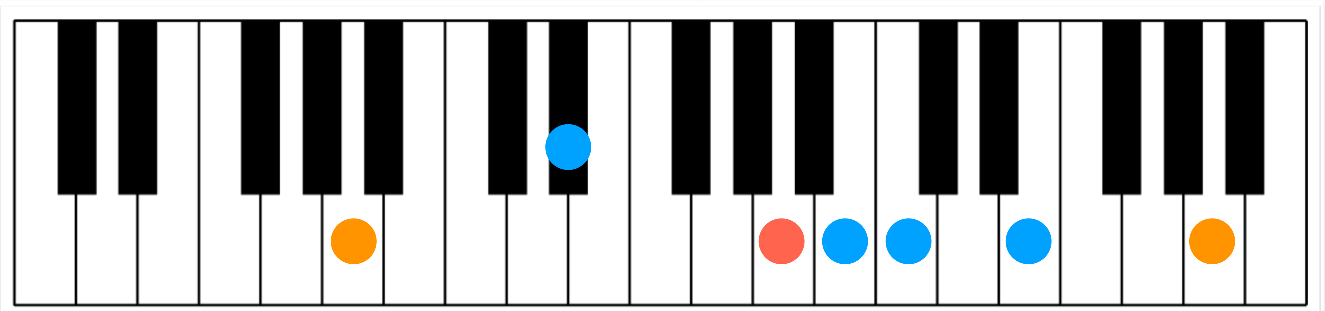

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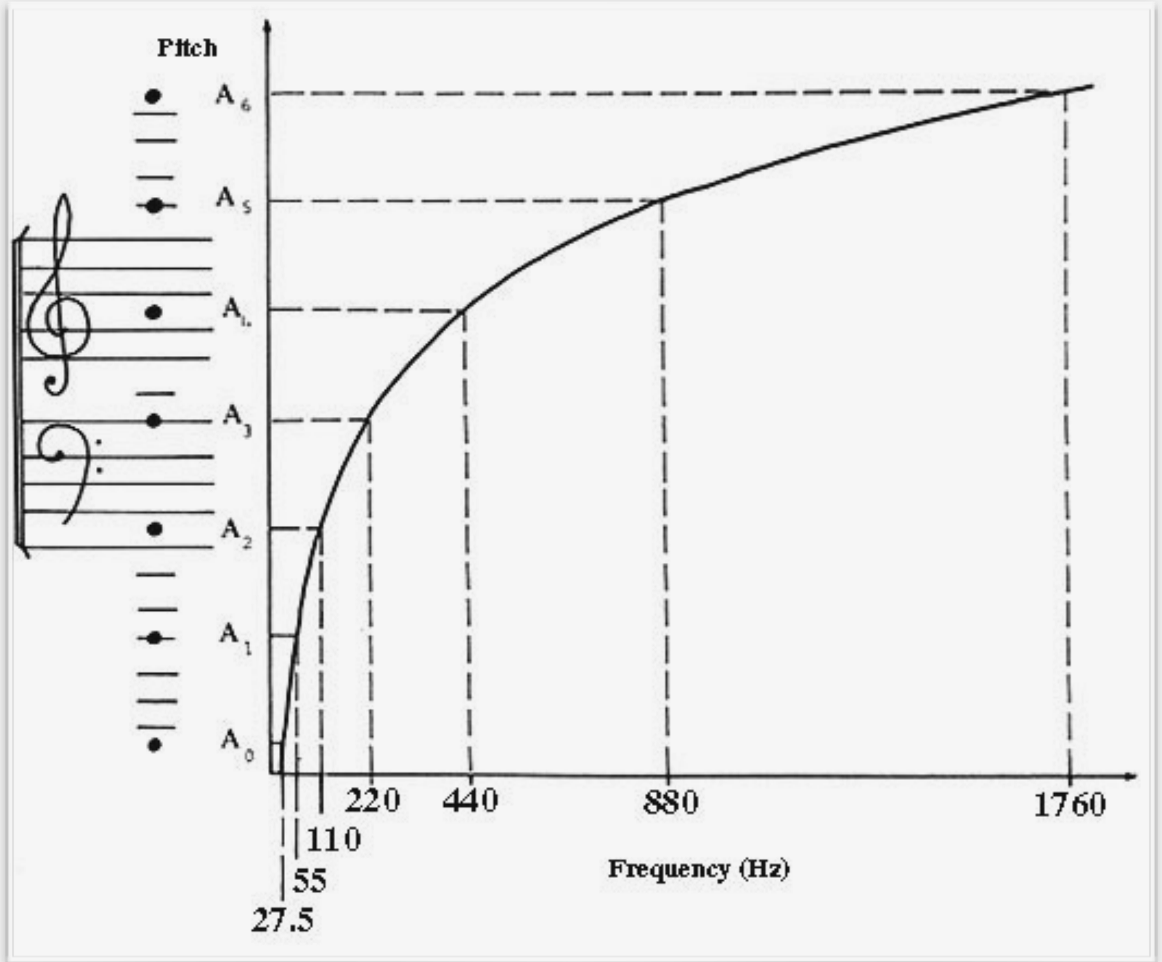
Tuning

- The exponent **x** needs to be divided by **12**

$$f_k = f_R \cdot 2^{k/12}, \qquad k=0,1,2,\ldots,11$$

f_r = A4 (440Hz), f_k = C5 (???)

f_k = 440Hz * 2^(3/12) = 523.25Hz



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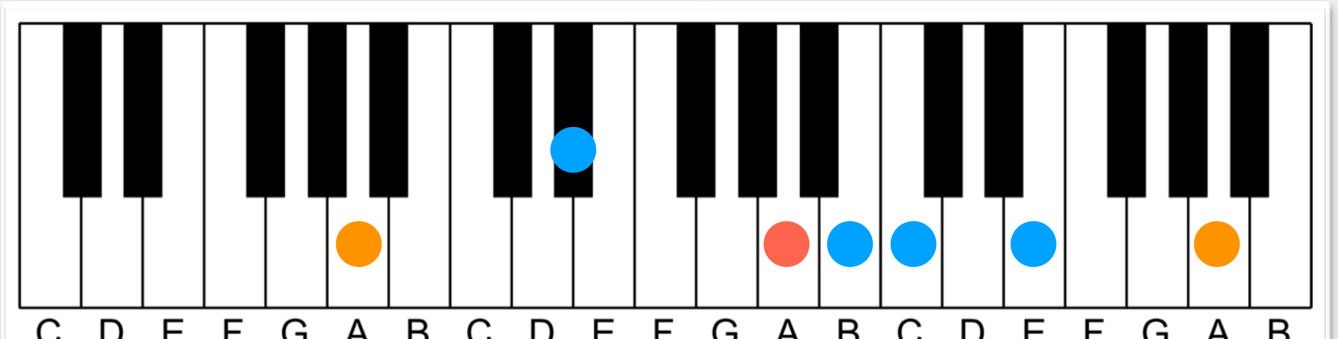

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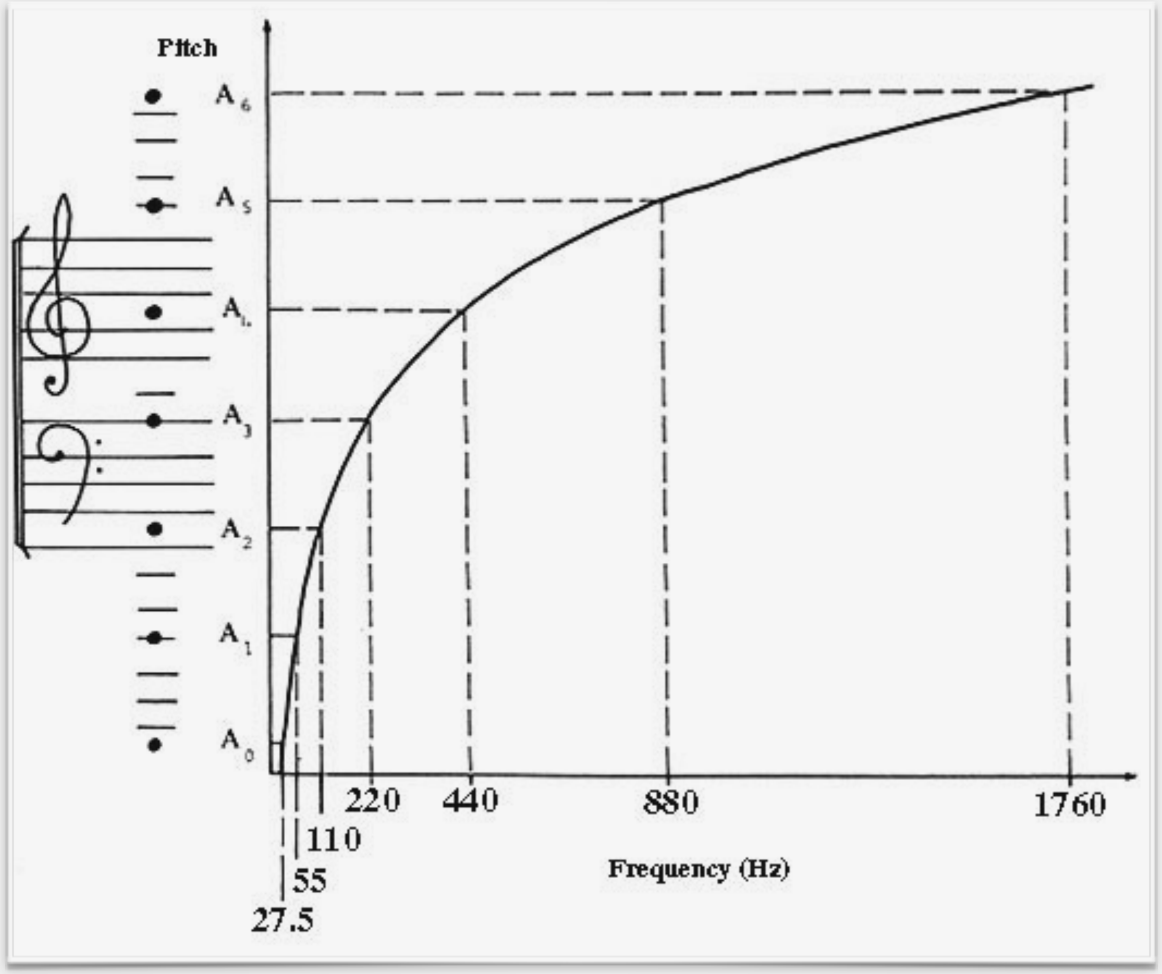
Tuning

- For calculating the frequency in any octave we add ***v***

$$f_k = f_R \cdot 2^{v+k/12}, \quad v \epsilon I, \; k=0,1,2,...,11$$

f_r — reference frequency

f_k — the frequency of any **arbitrary interval** k within **any** octave



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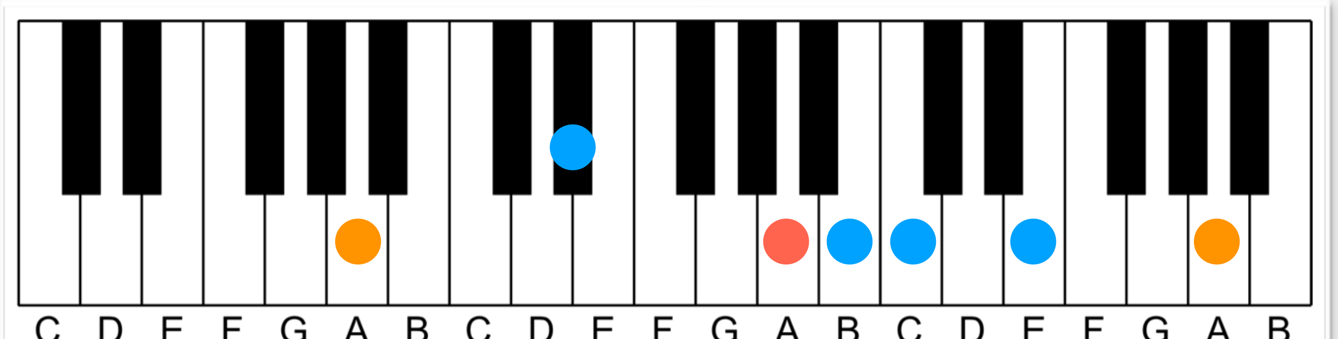

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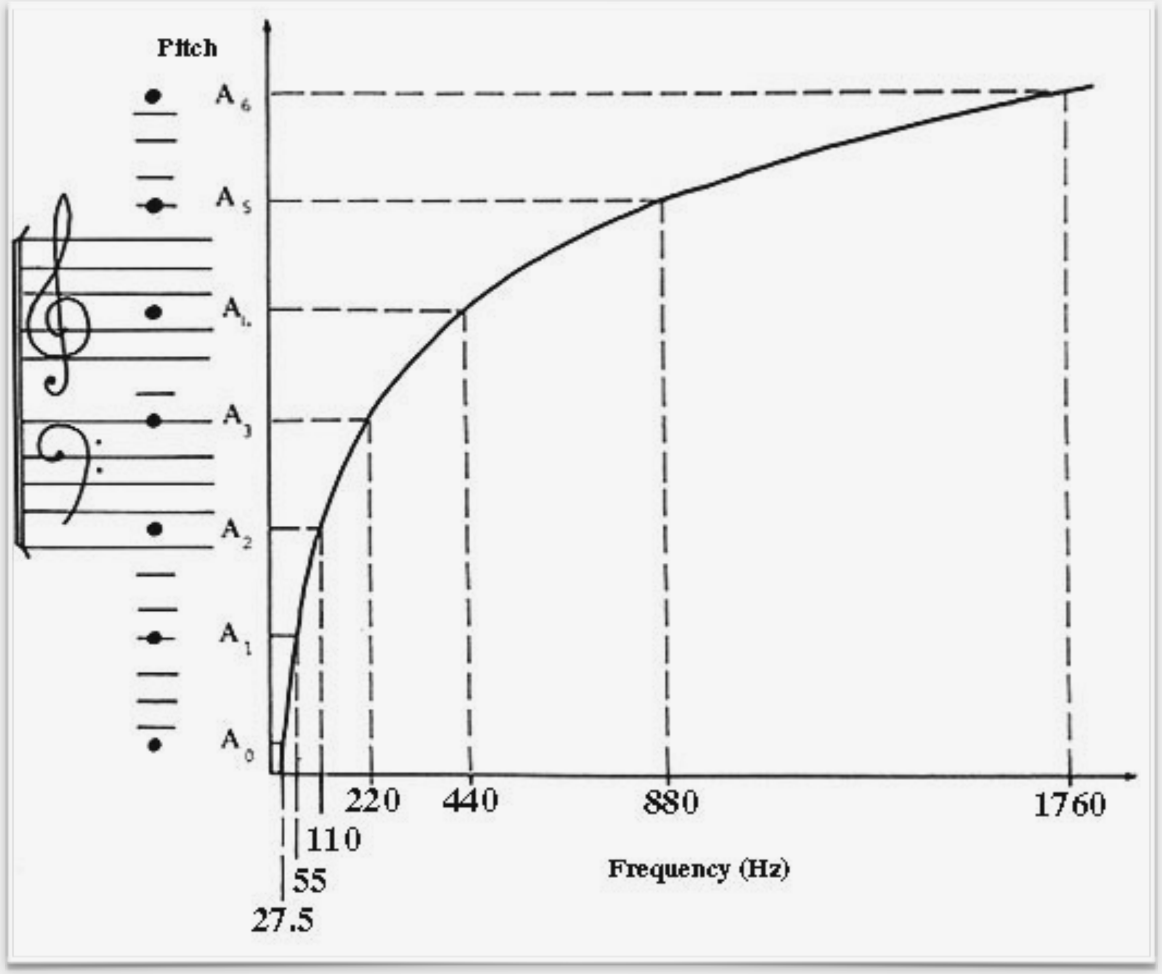
Tuning

- For calculating the frequency in any octave we add ***v***

$$f_k = f_R \cdot 2^{v+k/12}, \quad v \epsilon I, \; k=0,1,2,\ldots,11$$

f_r = A4 (440Hz), f_k = C8 (???)

f_k = 440Hz * 2^((3)+(3/12)) = 4186,01Hz



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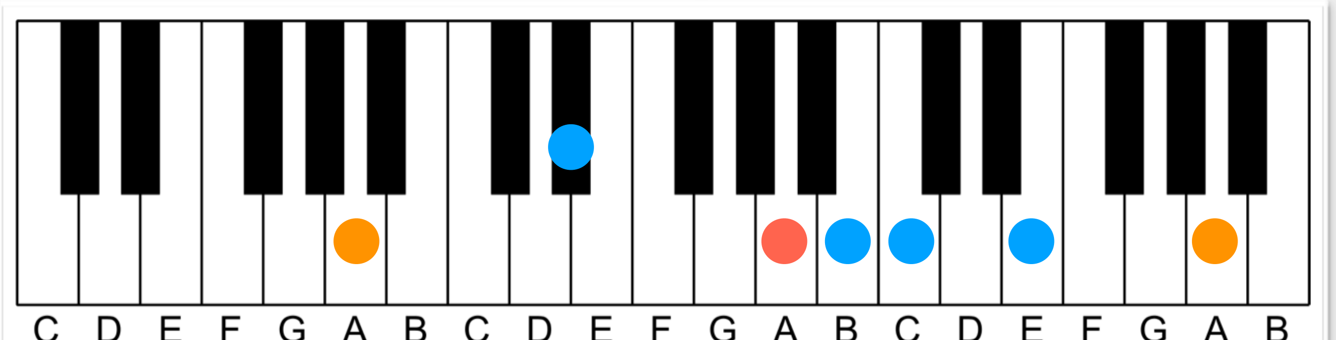

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Tuning

- To conform with Western practice of naming octaves after their order of appearance on a standard 88-key piano, adapt

$$f_k = f_R \cdot 2^{(v-4)+k/12}, \quad v \epsilon I, \quad k=0,1,2,\ldots,11$$

f_r — reference frequency

f_k — the frequency of any **arbitrary interval** k within **any** octave conform to 88-key practice

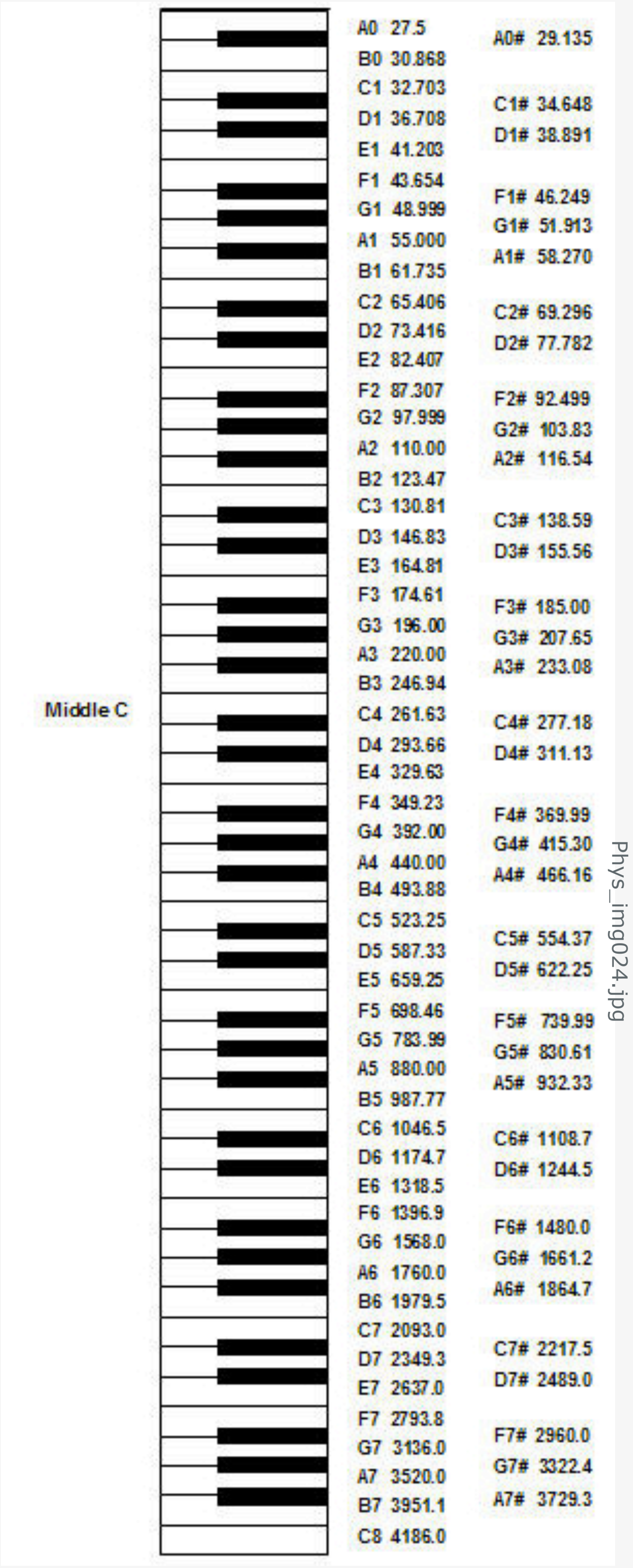

Image credit: https://www.sciencebuddies.org/Files/3436/5/Phys_img024.jpg

Tuning

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$$f_k = f_R \cdot 2^{(v-4)+k/12}, \quad v \epsilon I, \quad k=0,1,2,\ldots,11$$

f_r = A4 (440Hz), f_k = B5 (???)

f_k = 440Hz * 2^((5-4)+(2/12)) = 987.77Hz

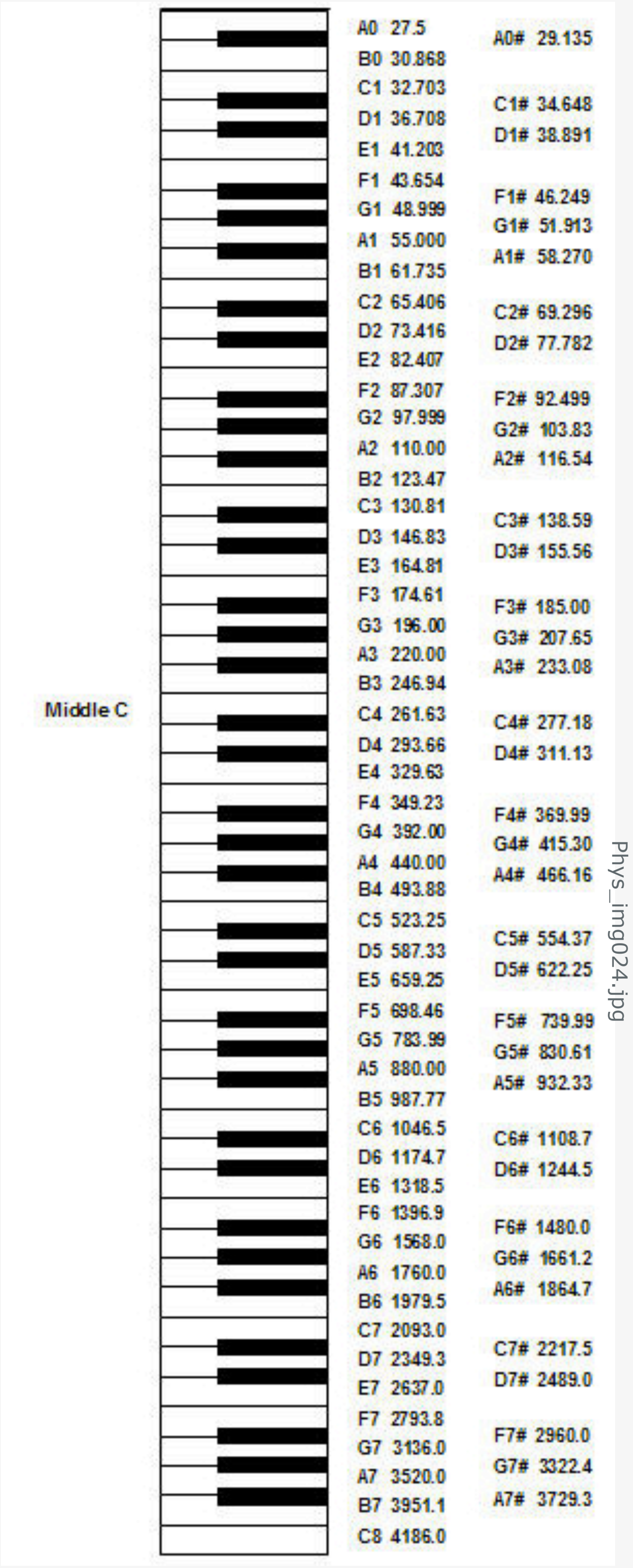
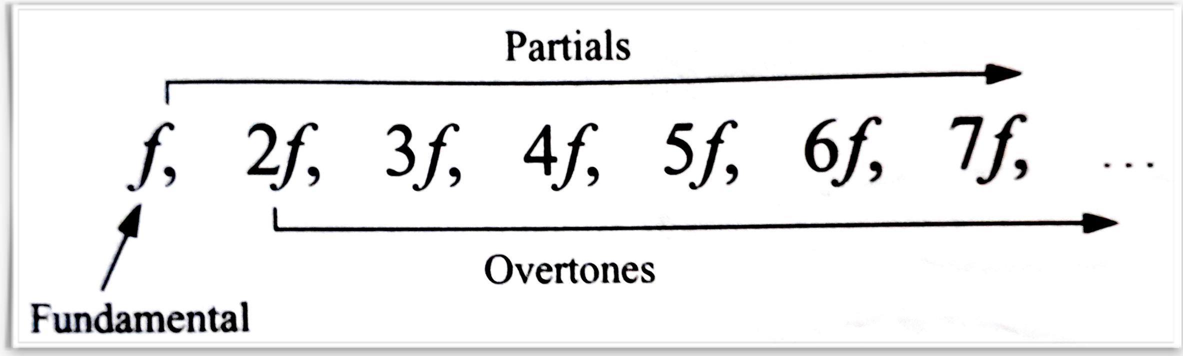


Image credit: https://www.sciencebuddies.org/Files/3436/5/Phys_img024.jpg

Overview

- Any musical sound (natural, periodic vibration) is composited out of several frequencies starting with a **fundamental frequency** that makes the pitch of the sound & weaker frequencies at integer multiples of that fundamental frequency: f, 2f, 3f, 4f, …
- The additional frequencies are called **overtones** of the fundamental and — together with the fundamental — form the **partials** or **harmonics** of the musical sound



Overview

- Musical sounds consist of a fundamental tone as well as a (theoretically) infinite number of **overtones**

Frequency	Order	Name 1	Name 2	Name 3
$1 \cdot f = 440$ Hz	$n = 1$	fundamental tone	1st harmonic	1st partial
$2 \cdot f = 880$ Hz	$n = 2$	1st overtone	2nd harmonic	2nd partial
$3 \cdot f = 1320$ Hz	$n = 3$	2nd overtone	3rd harmonic	3rd partial
$4 \cdot f = 1760$ Hz	$n = 4$	3rd overtone	4th harmonic	4th partial

https://en.wikipedia.org/wiki/Overtone

Overview

- **Partials** are individual sinusoids that collectively make up an instrumental tone — they need not be multiple integers of the fundamental tone

Frequency	Order	Name 1	Name 2	Name 3
$1 \cdot f = 440$ Hz	$n = 1$	fundamental tone	1st harmonic	1st partial
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https://en.wikipedia.org/wiki/Overtone

Overview

- **Harmonics** or harmonic partials denote partials of harmonic instruments that consist of a fundamental frequency and positive integer multiples of the fundamental tone

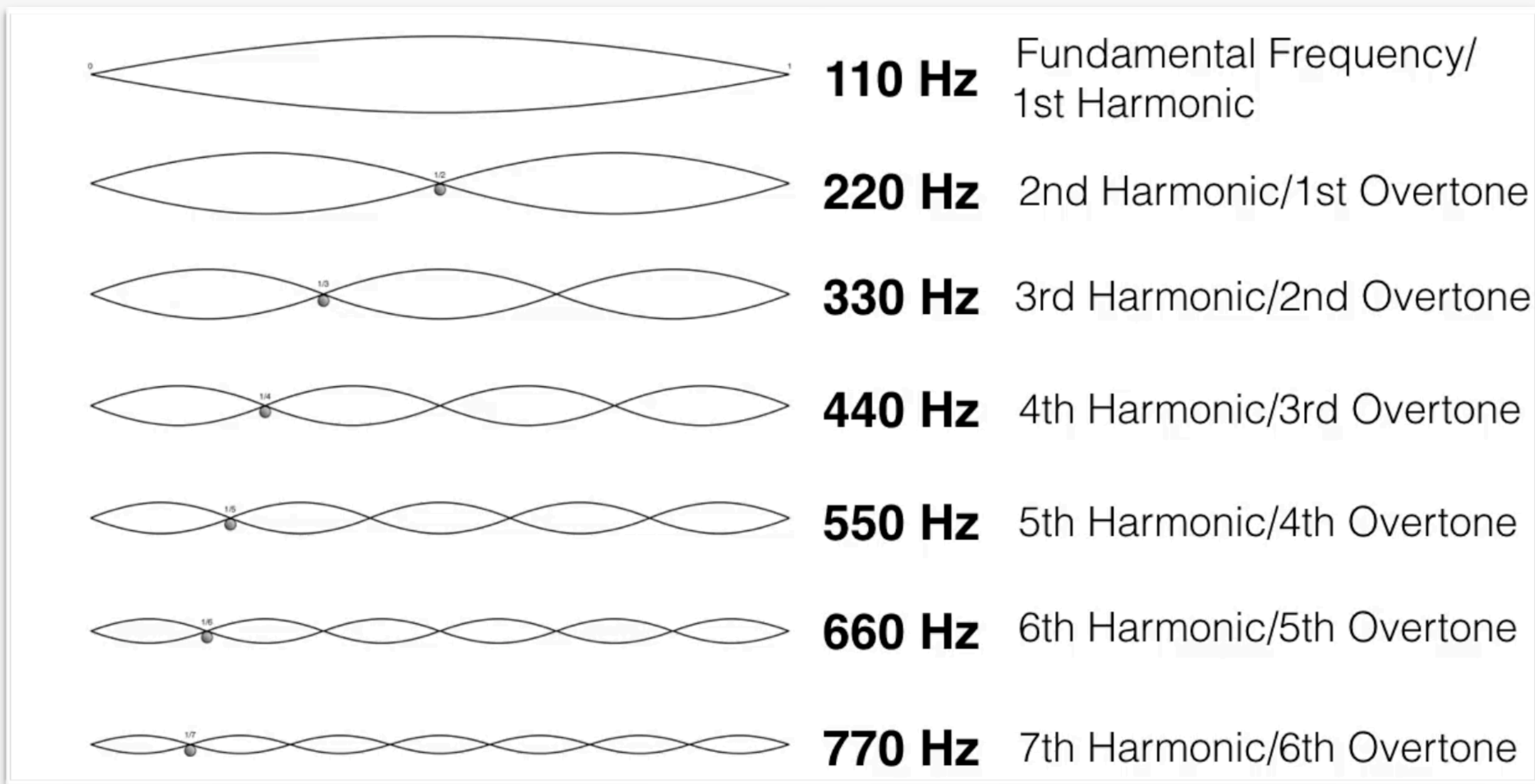
Frequency	Order	Name 1	Name 2	Name 3
$1 \cdot f = 440$ Hz	$n = 1$	fundamental tone	1st harmonic	1st partial
$2 \cdot f = 880$ Hz	$n = 2$	1st overtone	2nd harmonic	2nd partial
$3 \cdot f = 1320$ Hz	$n = 3$	2nd overtone	3rd harmonic	3rd partial
$4 \cdot f = 1760$ Hz	$n = 4$	3rd overtone	4th harmonic	4th partial

https://en.wikipedia.org/wiki/Overtone

Harmonic Series

- The harmonic series of the fundamental frequency at 110Hz (A2)
- This series is a natural phenomenon occurring as a result of a vibrating string or air column of any musical string or wind instrument

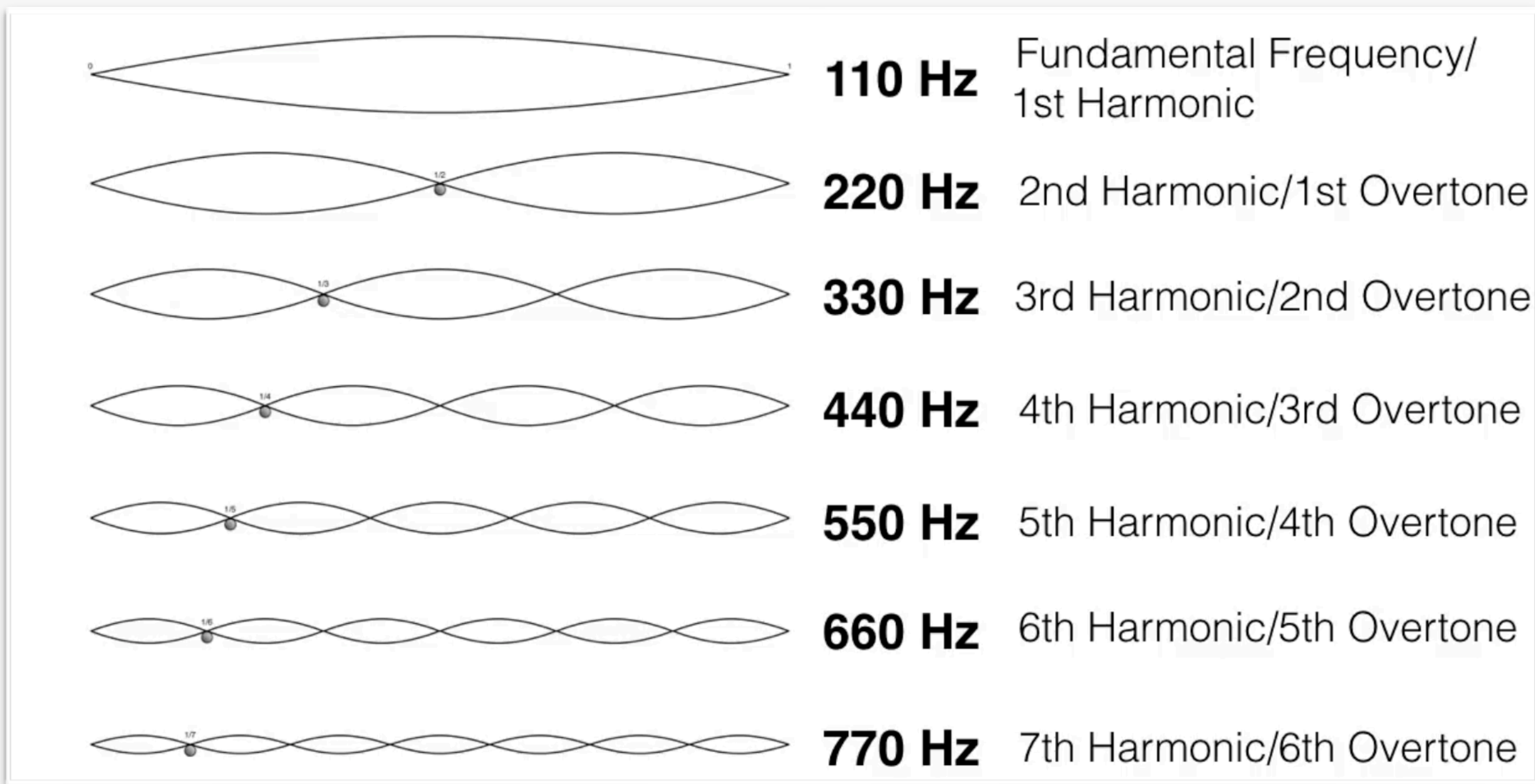
https://www.oberton.org/en/overtone-singing/harmonic-series/



https://www.youtube.com/watch?v=XPbLYD9KFAo

Harmonic Series

- There is a direct antiproportional relationship between the length of a string (or air column) & the resulting frequency at which it vibrates—the greater the frequency, the shorter the string length



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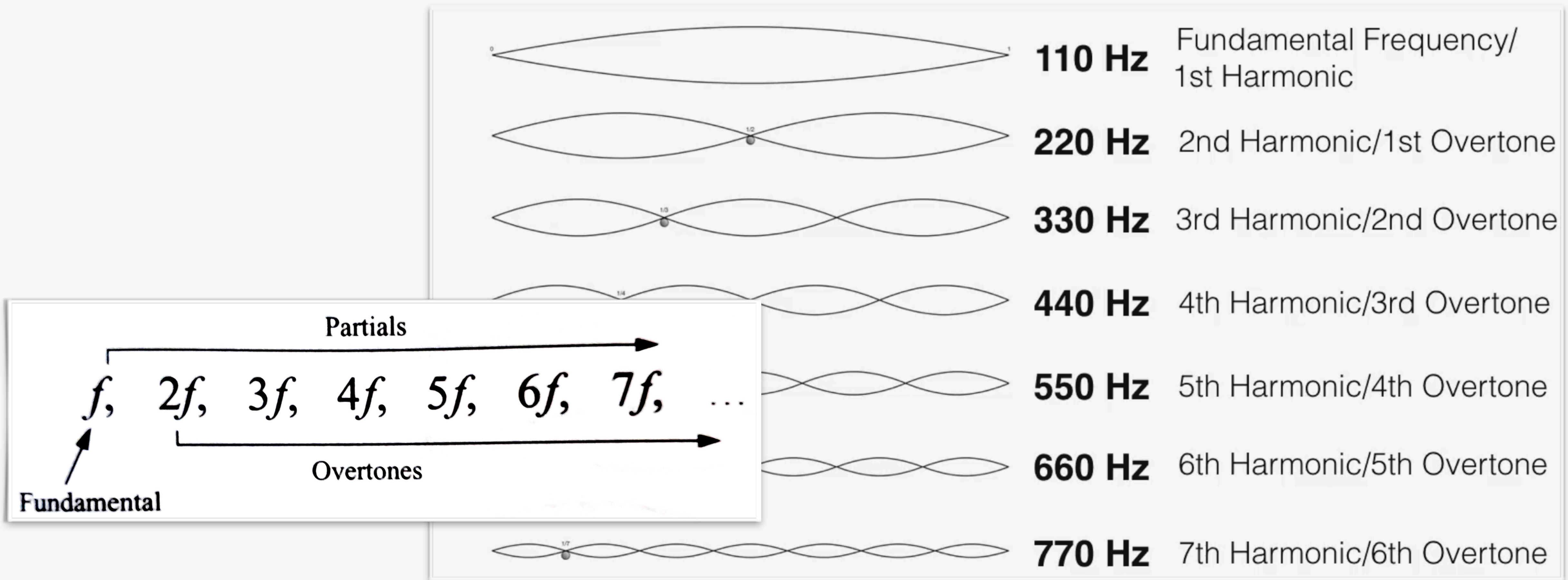
Harmonic Series

- The overtones of a fundamental tone are positive integer multiples of the fundamental frequency



https://www.youtube.com/watch?v=XPbLYD9KFAo

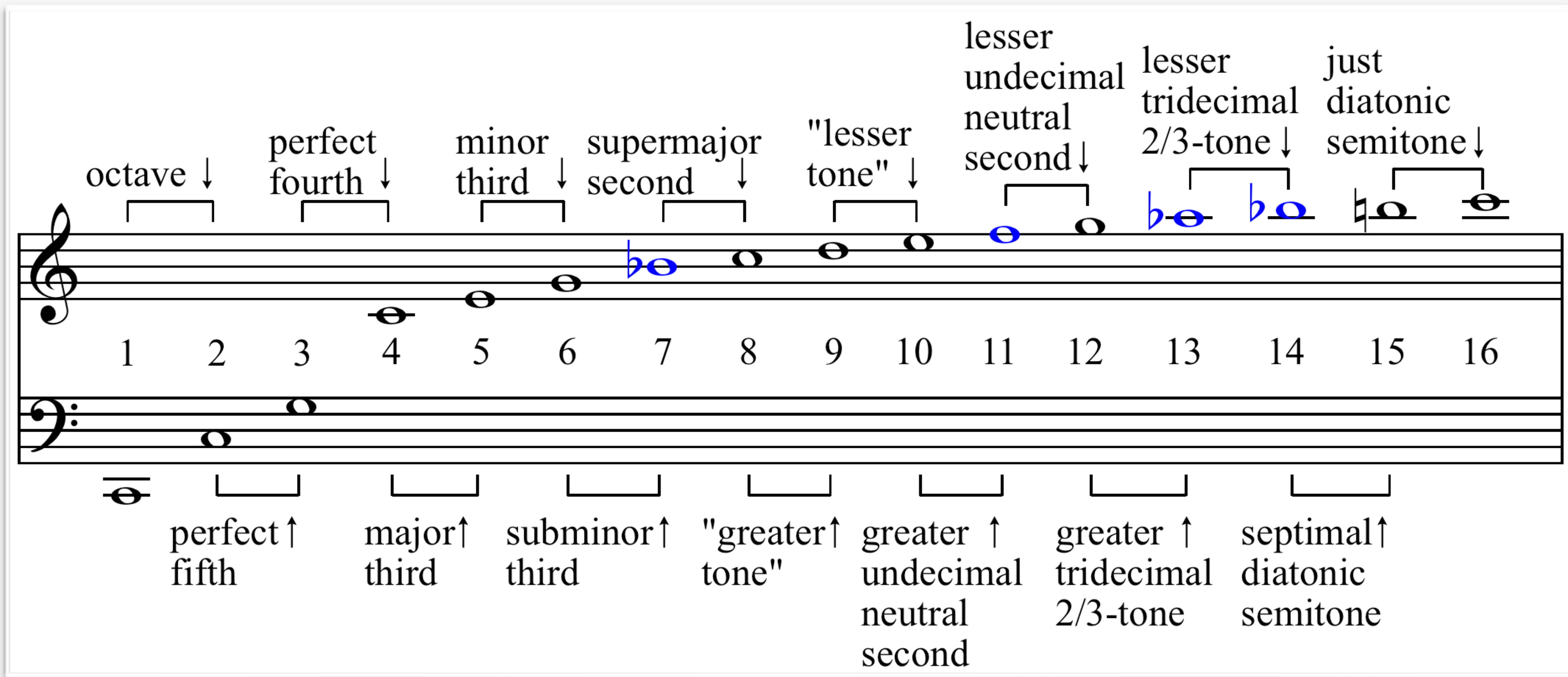
Harmonic Series

- The fundamental frequency **determines the pitch** of the sound
- The overtones make it more **complex**, they are not perceived as separate notes but as part of the timbre of the sound

- The first **eight harmonics** can be perceived clearly
  the intensity of the rest of the harmonics diminishes
- Lower tones sound usually more complex than higher tones;
  this is due to higher overtones reinforcing lower tones

Harmonic Series

- Each harmonic equals an particular musical tone
- The depicted intervals refer to adjacent pitches
- With increasing harmonics, the tones & intervals deviate from the tones of the (artificially) equal tempered tuning system



https://upload.wikimedia.org/wikipedia/commons/e/e8/Harmonic_series_intervals.png

Harmonic Series

- The first partials of the harmonic series and corresponding intervals (left)
- The frequency ratios of the harmonic series with corresponding musical intervals (right)

Harmonic	Frequency	Ratio	Note
Fundamental	110 Hz	NA	A_2
1st Overtone	220 Hz	220:110 = 2:1	A_3
2nd Overtone	330 Hz	330:220 = 3:2	E_4
3rd Overtone	440 Hz	440:220 = 2:1	A_4
4th Overtone	550 Hz	550:440 = 5:4	$C\#_5$
5th Overtone	660 Hz	660:440 = 3:2	E_5

Interval	Ratio
Perfect 8av	2:1
Perfect 5th	3:2
Perfect 4th	4:3
Major 6th	5:3
Major 3rd	5:4
Minor 3rd	6:5
Minor 6th	8:5
Minor 7th	9:5
Major 2nd	9:8
Major 7th	15:8
Minor 2nd	16:15
Tritone	45:32

https://www.youtube.com/watch?v=0lmS5lQ5MSU

Harmonic Series

- The harmonic or overtone series is a natural phenomenon that occurs for any musical instrument and determines the relationship of the individual harmonics of a musical sound
- Overtones, partials and harmonics refer to the partial sound waves that form a complex musical tone
- The harmonic or overtone series does not correspond to the equal temperament tuning used in Western music however it has critical influence on the evolvement of scales, tuning and composition as well as on our perception of musical tones

References & Further Reading

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