Theoretical Backgrounds of Audio & Graphics

Matrices

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Winter term 2020/2021

Matrices

- Mathematically, a matrix is a **n x m tuple** of scalar values

- The columns of a matrix **M** can be interpreted as **column vectors v**
- The components of a vector **v** (or of a matrix **M**) are scalar values

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} m_{ij} \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Jarosz, W., Computer Graphics, Fall 2016, Dartmouth College

Matrices

- Geometrically, the columns of a matrix can be interpreted as **basis vectors** of a specific vector space

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Matrices

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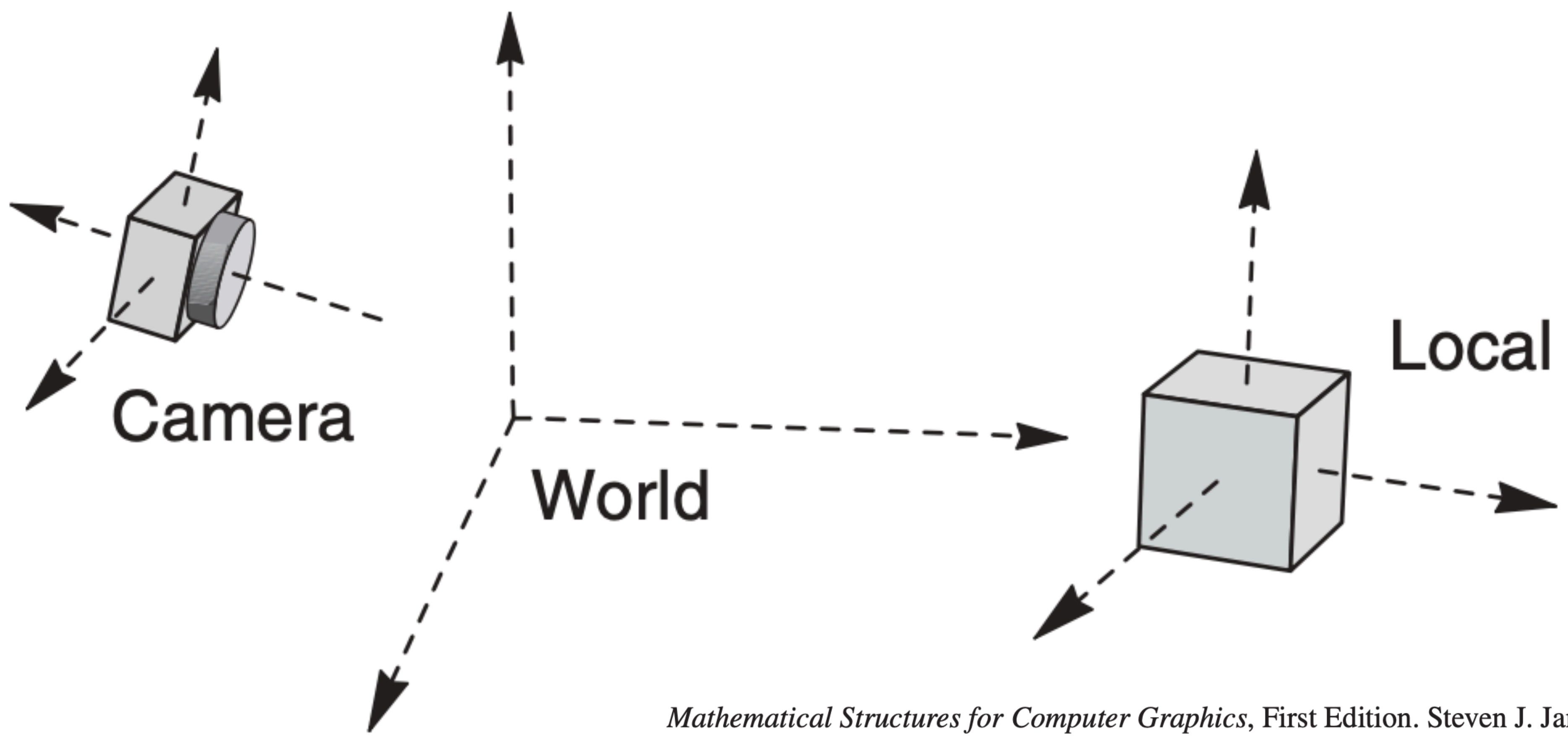

Mathematical Structures for Computer Graphics, First Edition. Steven J. Janke.
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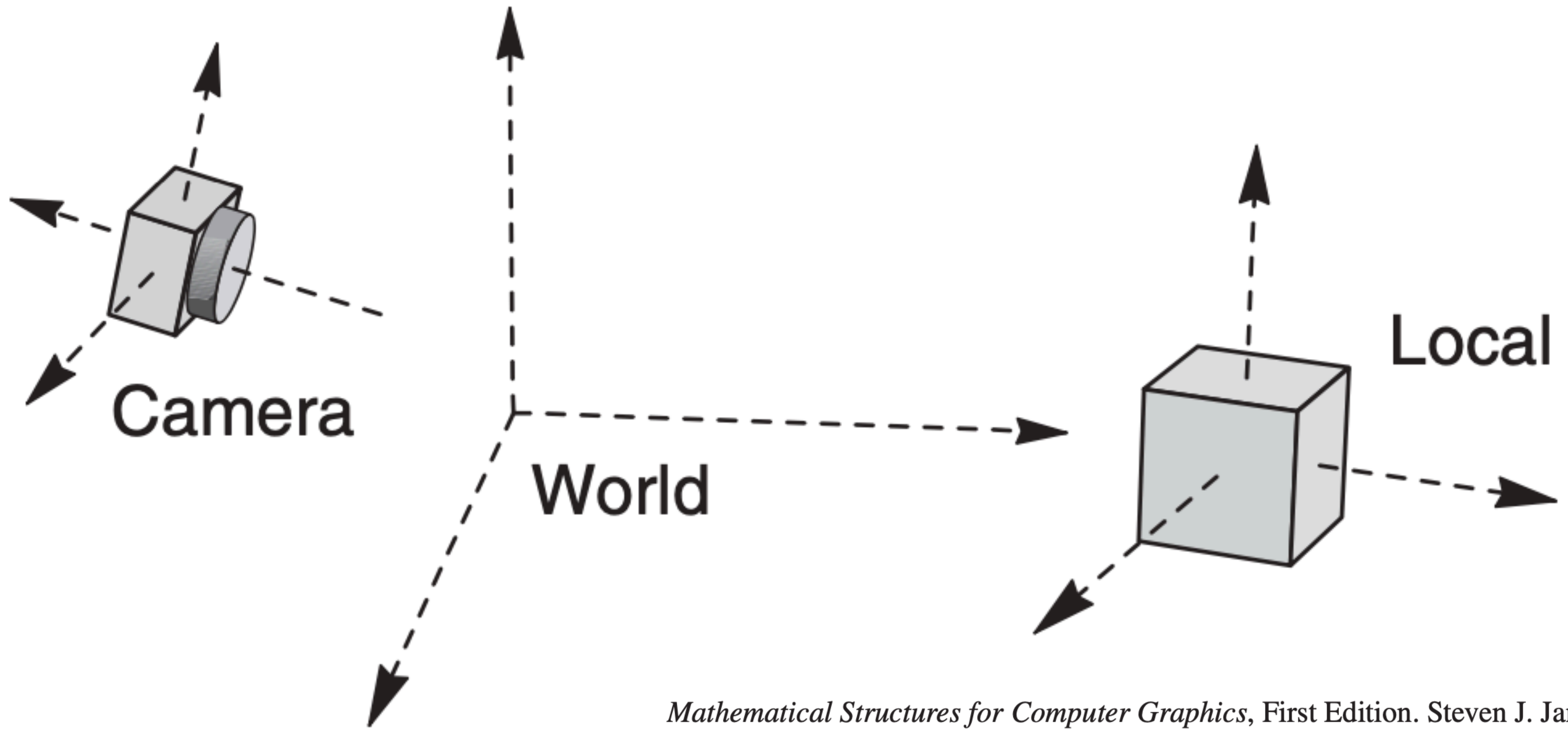
Matrices

- Matrix representations allow us to let us move from one coordinate system / vector space representation into another through **linear transformations**



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Matrices

- The transpose **T** of a matrix or vector is an operator which switches row and column indices

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} m_{ij} \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$$

Jarosz, W., Computer Graphics, Fall 2016, Dartmouth College

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Matrix Operations

- Element-wise addition

Matrix Operations

- Scalar multiplication

Matrix Operations

- Matrix multiplication

Matrix Operations

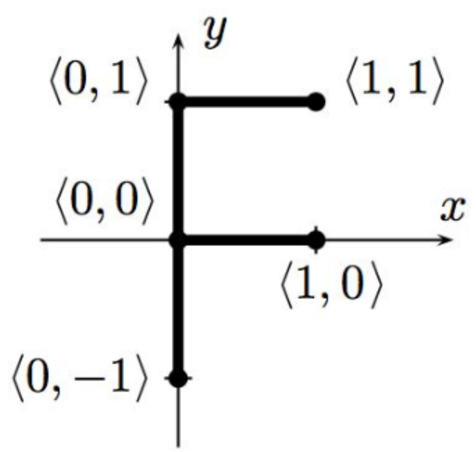
- Matrix multiplication

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1p} \\ B_{21} & B_{22} & \cdots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mp} \end{pmatrix}$$

n x m $\qquad$ m x p $\qquad$ = n x p

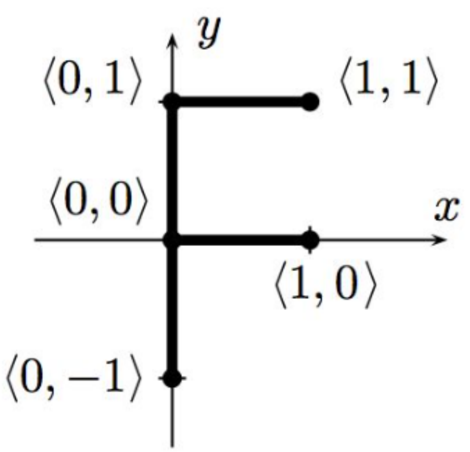
Matrix Transformation by Example

- In this example, we are looking for matrix **M** which describes the following transformation:
  - M: (x, y) -› (x, 2y)



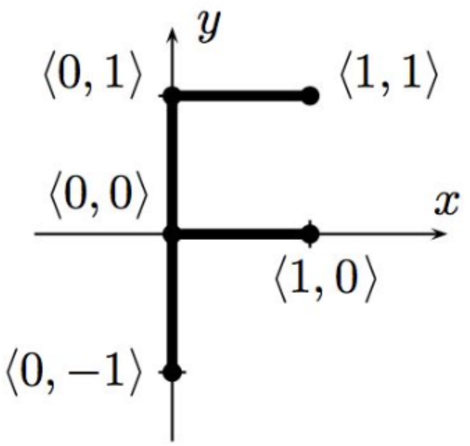
Matrix Transformation by Example

- In this example, we are looking for matrix **M** which describes the following transformation:
  - M: (x, y) -› (x+y, y)



Matrix Transformation by Example

- In this example, we are looking for matrix **M** which describes the following transformation:
  - M: (x, y) -› (x, x+y)



References

- Buss, S. (2003): **3D Computer Graphics—A Mathematical introduction with OpenGL**. Cambridge University Press, New York, NY, USA.
- Angel, E. and Shreiner, D. (2012): **Interactive Computer Graphics—A Top-down Approach with Shader-based OpenGL**. Addison-Wesley, USA.
- Shiffman, D. (2012): **Nature of Code.** https://natureofcode.com/book/

- Lecture slides
  - Gieseke, L. Mathematics for Audio & Graphics, 2017/18, Film University
    - **Note:** *If not indicated differently, all images, equations, and calculations have been taken from Lena Gieseke's slides.*
  - Jarosz, W., Computer Graphics, Fall 2016, Dartmouth College
  - Marschner S., Computer Graphics, Spring 2018, Cornell University
  - Schlechtweg, S. (2010/11), „Computergraphik Grundlagen", Hochschule Anhalt (FH) Köthen.