AI1103 Assignment 7

Nagubandi Krishna Sai
MS20BTECH11014

Download LaTex file from below link :

https://github.com/KRISHNASAI1105/demo/blob/main/Assignment_7/LaTex/Assignment_7.tex

***Problem number CSIR UGC NET* 2014 *Q*.106**

Consider a Markov chain with state space 1,2,....,100. Suppose states 2i and 2j communicate with each other and states 2i-1 and 2j-1 communicate with each other for every i,j = 1,2,...,50. Further suppose that $p^{(2)}_{3,3} > 0, p^{(3)}_{4,4} > 0$ and $p^{(7)}_{2,5} > 0$. Then

1) The Markov chain is irreducible.
2) The Markov chain is aperiodic.
3) State 8 is recurrent.
4) State 9 is recurrent.

Solution

Definition 1. *We say that Markov chain is **irreducible** if and only if all states belong to one communication class and all states communicate with each other.*

Definition 2. *In an **irreducible chain** all states belong to a single communicating class. This means that, if one of the states in an irreducible Markov chain is **aperiodic**. Then, all the remaining states are also aperiodic.*

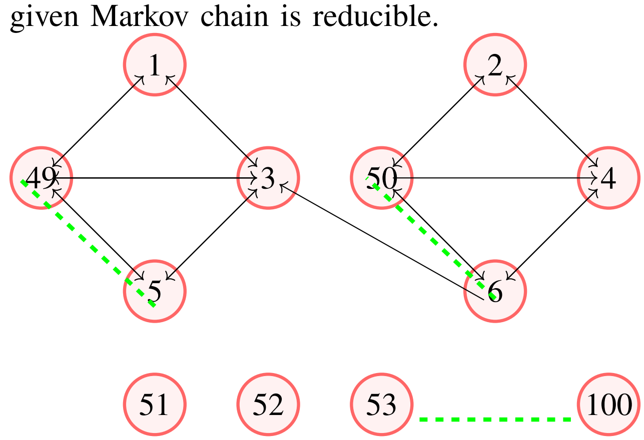
1) If $p^{(n)}_{i,j} > 0$, for $n \geq 0$, then we say that the state j is accessible from state i. Communication means state j is accessible from state i and state i is accessible from state j.
2) According to the question, all even and odd positioned states communicate with each other.

Consider, the communication classes of the given Markov chain as follows :

Communication class	set of elements
$C_1(1)$	{1,3,5,7,....,49}
$C_1(2)$	{2,4,6,8,....,50}
$C_1(51)$	{51}
$C_1(52)$	{52}
$\vdots$	$\vdots$
$C_1(100)$	{100}

TABLE 2: Communication class

$\therefore$ As there are 52 communication classes, the given Markov chain is reducible.



Regarding periodicity,

$$d(K) = gcd(m \geq 1 : P^m_{k,k} > 0).$$

If d(K) = 1, then the state K is aperiodic.
If d(K) = 0, then the state K is periodic.

1) Hence, by the above figure, states from 1 to 50 are recurrent. Because as it is given in question $p^{(7)}_{2,5} > 0$, this means that state 5 is accessible from state 2. As all even states communicate with each other, from state 2 the process continues with state 5. As all odd states communicate with each other, this means all the odd states return to their respective states again.

Periodicity of elements	set of elements
d(1)	{1,3,5,7,....,49}
d(2)	{2,4,6,8,....,50}

TABLE 1: Periodicity of some of elements of set S

$$d(1) = d(2) = \gcd\{2,3,....,50\} = 1.$$

$\therefore$ The states 1 to 50 are **Aperiodic**.

$$d(51) = d(52) = ... = d(100) = 0.$$

$\therefore$ The states 51 to 100 are **Periodic**.
Hence, we can concluded that {1,3,5,7,...,49,51,52,53,....,100} are **recurrent** states. And {2,4,6,8,....,50} are **transient** states. Hence, The given **Markov chain** is **reducible** and not a **aperiodic** chain.

$\therefore$ $\boxed{\textbf{Option 4} \text{ is a } \textbf{correct} \text{ answer}}$