# Lab: The Hand Warmer Design Challenge

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## 1 Objective

Given several ionic compounds that are soluble in water, to calculate the heat of solution ($\Delta H_{solution}$) of each and determine which one would be the best choice for a hand warmer and cold pack.

## 2 Materials

Styrofoam cups, thermometer, stirring rods, analytical balance, top loading scales, weighing dishes, measuring scoops, 50 mL beaker, distilled water, stopwatch, and salts for experimentation:

- Sodium acetate
- Sodium hydroxide
- Calcium chloride
- Magnesium chloride
- Ammonium chloride

## 3 Procedure

1. Measure 50 milliliters of tap water using a graduated cylinder.
2. Pour the water into a clean, dry Styrofoam cup, which is in turn placed inside a beaker. The cup-beaker acts as a calorimeter.
3. After stirring with the thermometer for a few minutes, record the initial temperature of the water. Keep the thermometer in the water even after measurement for future use,
4. Mass out 5 grams of one the compounds.
5. Prepare a stopwatch. Transfer the salt into the calorimeter and begin the timer immediately. Stir the solution.
6. Record the temperature of the solution every thirty seconds. After the temperature has returned close to pre-salt levels, rinse out the cup well.
7. Repeat the process for the next salt.

## 4 Data and Analysis

### 4.1 Raw Data Table

| Solid | $T_{initial}$°C | $T_{final}$°C | $\Delta T$ |
|---|---|---|---|
| $(NH_4)(Cl)$ | 22 | 15 | -7 |
| $CaCl_2$ | 30 | 36 | 6 |
| $(Na)(OH)$ | 23 | 44.2 | 21.2 |
| $(Na)(C_2H_3O_2)$ | 18 | 17 | -1 |
| $(Mg)(Cl_2)$ | 22.5 | 24.4 | 1.9 |

For all solids, the mass of the water is 50 milliliters = 50 grams and the mass of the solid is 5 grams, making the total mass of the solution 55 grams.

### 4.2 Calculating Experimental $\Delta H$ for each solution

To calculate the change in heat, $\Delta H_{solution}$ for each salt dissolved in water, we use the equation

$$\Delta H = mc\Delta T$$

where $m$ is the total mass of the solution, $c$ is the specific heat of the solution (in this case, it is equal to the specific heat of water $= 4.18\frac{\mathrm{J}}{\mathrm{g\,^\circ C}}$. We show the calculations for each solution:

$$(NH_4)(Cl) : \Delta H = (55\,\mathrm{g})(4.18\frac{J}{\mathrm{g\,^\circ C}})(-7\,^\circ\mathrm{C}) = 1609.3\,\mathrm{J}$$

$$CaCl_2 : \Delta H = (55\,\mathrm{g})(4.18\frac{J}{\mathrm{g\,^\circ C}})(6\,^\circ\mathrm{C}) = -1379.4\,\mathrm{J}$$

$$(Na)(OH) : \Delta H = (55\,\mathrm{g})(4.18\frac{J}{\mathrm{g\,^\circ C}})(21.2\,^\circ\mathrm{C}) = -4873.88\,\mathrm{J}$$

$$(Na)(C_2H_3O_2) : \Delta H = (55\,\mathrm{g})(4.18\frac{J}{\mathrm{g\,^\circ C}})(-1\,^\circ\mathrm{C}) = 229.9\,\mathrm{J}$$

$$(Mg)(Cl_2) : \Delta H = (55\,\mathrm{g})(4.18\frac{J}{\mathrm{g\,^\circ C}})(1.9\,^\circ\mathrm{C}) = -436.81\,\mathrm{J}$$

Note that we negated all the calculated values because the above calculations use the change in temperature of the thermometer, but in this case, this reflects the inverse of what occurred in the solution: If the thermometer's temperature rose, it is because the solution released heat to the surroundings and thus had a temperature drop. Furthermore, processes with a negative $\Delta H$ (after negation) indicate an exothermic reaction, because heat it released from the solution to the surroundings (including the thermometer), causing the recorded temperature to rise. In other words, the thermometer receives a heat transfer from the solution. The vice versa is true if the process has a positive $\Delta H$: The process is endothermic, meaning it absorbs heat from its surroundings. This causes the recorded temperature of the thermometer to drop.

### 4.3 Calculating $\Delta H$ per Mole of Solution

We now use the experimental $\Delta H$ to calculate the $\Delta H$ for one mole of the same solution. In order to do this, we must first convert the quantity of each substance used in this experiment from grams to moles. Mathematically, this is expressed as

$$n = \frac{m}{M}$$

where $n$ is the number of moles of the sample, $m$ is the mass of the sample, and $M$ is the molar mass of the sample substance. Using this equation, we calculate the number of moles of each substance used in this experiment given that the sample mass is 5 grams for each. These values are displayed in the below table.

| Solid | Molar Mass ($M$) in Grams | Number of Moles ($n$) in mol |
|---|---|---|
| $(NH_4)(Cl)$ | 53.5 | 1.03 |
| $CaCl_2$ | 75.53 | 0.73 |
| $(Na)(OH)$ | 40 | 1.38 |
| $(Na)(C_2H_3O_2)$ | 82.04 | 0.67 |
| $(Mg)(Cl_2)$ | 59.76 | 0.92 |

Using these quantities, we now create stoichiometric ratios to relate the $\Delta H_{solution}$ in our trials to the $\Delta H$ in processes with one mole of each substance. Mathematically,

$$\frac{n}{1\,\mathrm{mol}} = \frac{\Delta H_{solution}}{\Delta H_1}$$

where $n$ is the number of moles in the sample calculated above, $\Delta H_{solution}$ is the quantity of heat absorbed/released in the solution in our experiment (calculated in 4.2.1), and $\Delta H_1$ is the quantity of heat absorbed/released in the dissolving of one mole of each salt into water under the same conditions. Furthermore,

$$\Delta H_1 = \frac{\Delta H_{solution}}{n}$$

The results of these calculations are shown in the below table.

| Solid | $\Delta H_1$ (J) |
|---|---|
| $(NH_4)(Cl)$ | 1562.43 |
| $CaCl_2$ | -1889.59 |
| $(Na)(OH)$ | -3531.8 |
| $(Na)(C_2H_3O_2)$ | 343.13 |
| $(Mg)(Cl_2)$ | -474.79 |

### 4.4 Analysis

Solutions that had a larger $\Delta H$ were more endothermic and thus absorbed larger quantities of heat from its surroundings when put into water. Similarly, solutions that had a smaller (negative) $\Delta H$ were more exothermic and released larger quantities of heat to its surroundings when put into water. The solution of ammonium chloride was the most endothermic, and the solution of sodium hydroxide was the most exothermic.

### 4.5 Financial Calculations

A stronger hand-warmer would release more heat to its surroundings (e.g. consumer's hands), and a stronger hand-*cooler* would be more absorbent of heat. While it would thus initially seem that sodium hydroxide would be the greatest hand-warmer and ammonium chloride the greatest hand-cooler, we can only ascertain this by calculating the $\Delta H$ per one dollar purchases of the substance. Given the cost of kilogram per each substance:

| Solid | Cost per Kilogram (USD) |
|---|---|
| $(NH_4)(Cl)$ | 25.00 |
| $CaCl_2$ | 20.00 |
| $(Na)(OH)$ | 38.00 |
| $(Na)(C_2H_3O_2)$ | 28.00 |
| $(Mg)(Cl_2)$ | 39.00 |

We must determine the optimal compound to use as material for a hand-warmer. This is easy enough, since the 5 grams we used of each substance in our experiment is simply equal to .005 kilograms. We then establish the following ratio, where we first convert the $\Delta H$ for 5 grams of a substance to $\Delta H$ for one kilogram of it.

$$\frac{\Delta H_{1\,\text{kg}}}{\Delta H_{0.005\,\text{kg}}} = \frac{1\,\text{kg}}{0.005\,\text{kg}}$$

So then

$$\Delta H_{1\,\text{kilogram}} = \frac{\Delta H_{0.005\,\text{kg}}}{0.005\,\text{kilogram}}$$

We then divide these values by the costs per kilogram of the respective substances to attain the $\Delta H$ per dollar purchased of the substance. The results are below, and we elaborate in Section 5:

| Solid | $\frac{\Delta H}{\text{kg}}$ | $\Delta H$ per Dollar |
|---|---|---|
| $(NH_4)(Cl)$ | 312,486 | 12,499.44 |
| $CaCl_2$ | -377,918 | -18,895.90 |
| $(Na)(OH)$ | -706,360 | -18,588.42 |
| $(Na)(C_2H_3O_2)$ | 68,600 | 2,450 |
| $(Mg)(Cl_2)$ | -94,958 | -2,434.82 |

## 5 Conclusion

We see that in practice, calcium chloride acts as a better hand warmer than sodium hydroxide (as well as the rest of the tested substances), because even though the magnitude of its $\Delta H$ per kilogram is only about 54% of that sodium hydroxide, $(Na)(OH)$ is almost twice as expensive. After calculations, we see that calcium chloride releases 307.48 J more than sodium hydroxide per dollar purchased of it. The best hand warmer is thus calcium chloride.