Lab: Projectile Motion

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1 Objective

Shoot a marble through two rings spaced a distance apart: one vertical, and one horizontal. Base the location and height of each ring on the initial angle and speed of the marble, and conduct mathematical analysis thereof.

2 Materials

- Marble Launcher with protractor
- Meterstick
- Lab stands and Rings

3 Design & Procedure

1 Test for the initial velocity v_o of the marble when projected 90° upwards. Use a meter stick to measure the height of the marble's peak Δy and record. Using the third kinematics equation ($v^2 = v_o^2 + 2a\Delta x$) as a base equation, derive and solve for v_o.

2 Next, we chose 30° as the angle of elevation for the trajectory of the marble. Record the horizontal distance of the marbles trajectory, Δx. Compute y_{max}, which will be the height of the first stand that the marble travels through mid-air.

4 Derivations

To compute the initial velocity v_o of the trajectile when projected directly upwards at 90°, we use the second kinematics equation, where

$$y = y_o + v_o t + \frac{1}{2} g t^2 \tag{1}$$

y_o is 0 meters, and we are given

From the third kinematics equation, expressed in the vertical dimension, that

$$v_y = v_{o_y}^2 + 2g\Delta y \tag{2}$$

Additionally, v can be partitioned into vertical and horizontal components where

$$v_y = v \sin\theta \tag{3}$$

$$v_x = v \cos\theta$$

on an instantaneous basis. We can thus substitute for for $v_{o_y}^2$ and isolate Δy, and $v\prime = 0$ because at the peak of its trajectory, the vertical velocity of the marble is 0. From a mathematical point of view, this can be understood as

$$f'[x \mid \arg\max_x f(x)] = 0$$

for an arbitrary function that is convex with a global maximum. In other words, the derivative of any parabola with negative end behavior (a negative derivative) as x approaches infinity, is 0 at is maximum, and this holds true for the maximum if the parabola has positive end behavior. In our case, the derivative of vertical displacement, which is vertical velocity, is 0 at the vertical maximum.

Moving forward, we perform the aforementioned simplifications. The maximum height of the projectile

$$\Delta y = \frac{-v_{o_y}^2}{2g} = \frac{-(v_o \sin\theta)^2}{-2g} = \frac{v_o^2 \sin^2\theta}{2g} \tag{4}$$

where v_o is the computed velocity for the trajectile when projected 90° upwards. We use this because in that case, the vertical component of the velocity was equal to the net velocity since the horizontal component was 0 meters per second. The magnitude of the net velocity never changes, but its division between vertical and horizontal components does change. We thus attain its vertical component in this case through multiplying by $\sin\theta$.

Substituting for the variables yields:

$$\Delta y = \frac{(3.25)^2[sin^2(30°)]}{(2)(9.8)} = .135\ m.$$

Note that this is the difference in height between the top of the launcher (when tilted at 30°) and the maximum height. Since the height of the launcher in this case is .19 $m.$, the actual maximum height was $.135 + .19 = .325\ m.$

Finally, we also compute the maximum distance to be traveled by the projectile. To do this, we begin by deriving $\max t = t_{y=0} = \arg\min_t f(t)$ where $f(t) = v_{o_y} t + \frac{1}{2}gt^2$ based on the second kinematics equation, and where $f(0) = 0$ and $f(\max t) = 0$. At $y = 0$,

$$t(v_o \sin\theta - \frac{1}{2}gt) = 0 \tag{5}$$

By factoring, we find that

$$\max t = \frac{2v_o \sin\theta}{g} \tag{6}$$

Moving forward, the second kinematics equation also states that in the x-dimension,

$$x = x_o + v_o t\cos\theta + \frac{1}{2}at^2 \tag{7}$$

where the first and third terms are both equal to 0. So we are left with

$$x = v_o \cos\theta t \tag{8}$$

and substituting for t

$$\max x = v_o \cos\theta(\frac{2v_o \sin\theta}{g}) = \frac{v_o^2 2\sin\theta\cos\theta}{g} \tag{9}$$

Finally, applying a trigonometric addition formula stating that for any arbitrary θ, $2\sin\theta\cos\theta = \sin 2\theta$, we find that

$$\max x = \frac{v_o^2 \sin 2\theta}{g} = \frac{(3.25)^2\sin(2\cdot 30°)}{9.8} = .93\ m. \tag{10}$$

5 Conclusion

We determined that the initial velocity of the marble in this launcher was $3.25m/s$ by shooting it directly upright. Using this, we determined that at a 30° angle, the marble went through a vertical ring at m and a horizontal ring at m.

6 Errors

It is possible that we were imprecise in our measurements. We were also slow in the derivations, and should have simply used the given variables instead of trying to solve with separate, isolated equations. This was due to misunderstanding.

7 Future Experimentation

Future experimentation can include projectile motion in a case with non-zero horizontal acceleration and a non-g value for vertical acceleration, complicating computation. Additionally, it would be interesting to conduct a similar experiment but integrate a collision between two objects into the experiment. In other words, the projectile collides with another object mid-air, which is in turn launched as a projectile.