Rotational Motion Lab

Nathan Clune, Stefan Gruneis, Jesse Kindberg, Abraham Horowitz

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1 Objective

Find the moment of inertia of a pulley wheel in a modified atwood machine.

2 Materials

- Low friction dynamics cart and track (516.7 gram cart)
- Hanging mass set (200 gram mass)
- String
- Pulley
- Meter stick
- Stopwatch

3 Procedure

1. Set up the low friction dynamics cart and track on top of a table. Put the pulley at the end of the track
2. Attach a string to a hanging mass through the pulley and then attach it to the cart. Measure how high the hanging mass is and the amount of time it takes for it to hit the ground. Be careful with the equipment when letting the car and mass accelerate

4 Derivations with Data and Conclusion

We begin with Newton's second law, which will serve as a foundation for all derivations:

$$\sum \vec{F} = m\vec{a} \tag{1}$$

In the y-component of the cart, denoted m_1, connected with the axle on the horizontal axis, we establish that

$$\vec{F_N} - m_1\vec{g} = m_1\vec{a} = 0 \tag{2}$$

where F_N is the normal force. So

$$\vec{F_N} = m_1\vec{g}$$

This will turn out to be irrelevant for our purposes. In the x-component

$$\vec{T_H} = m_1\vec{a} \tag{3}$$

where T_H is the tension force in the horizontal section of the rope. Note that the cart's movement is essentially frictionless and we thus consider kinetic friction to be negligible. Continuing with the description of the weight, denoted m_2, which connects with the axle on the vertical axis,

$$m_2\vec{g} - \vec{T_V} = m_2\vec{a} \tag{4}$$

so

$$\vec{T_V} = m_2\vec{g} - m_2\vec{a}$$

with there existing no significant forces applied on its x-component.

We continue with the axle. There are no translation forces applied, but the tension forces on the horizontal and vertical axis both create non-zero torque. Beginning again with Newton's second law,

$$\sum \vec{\tau} = I\vec{\alpha} \tag{5}$$

We are, to reiterate, seeking to calculate the moment of inertia of the axle, I. Furthermore

$$R \cdot \vec{T_V} - R \cdot \vec{T_H} = I\vec{\alpha} \tag{6}$$

where R is the radius of the axle, measured to be .038 meters. Substituting

$$R \cdot (m_2\vec{g} - m_2\vec{a}) - R \cdot (m_1\vec{a}) = I(\frac{\vec{a}}{R}) \tag{7}$$

Finally isolating I

$$I = \frac{R^2 \cdot [m_2(\vec{g} - \vec{a}) - m_1\vec{a}]}{\vec{a}} \tag{8}$$

However, we cannot compute I without the linear/tangential acceleration of the system $\vec{a}$. To find this, we will use kinematics. We conducted three experimental tests where we released the system from rest with the weight initially directly below the axle. The distance to the floor was measured to be .809 meters, and we recorded and averaged 3 times for the system to travel this distance: $\{.87\ s, 1.0\ s, .94\ s\}$, where the mean time $\bar{t} = .939\ s$ Using the second kinematics equation

$$\vec{x} = \vec{x_o} + v_o(t) + \frac{1}{2}\vec{a}t^2 \tag{9}$$

but in our case the first two terms are 0. So

$$\vec{a} = \frac{2\vec{x}}{\bar{t}^2} = \frac{2 \cdot .809}{(.939)^2} = 1.835\ \frac{m}{s^2} \tag{10}$$

Substituting the value of $\vec{a}$ back into Equation 7 yields

$$I = \frac{(.038\ m.)^2[.2\ kg(9.8\ \frac{m}{s^2} - 1.835\ \frac{m}{s^2}) - (.5167\ kg \cdot 1.835\ \frac{m}{s^2})]}{1.835\ \frac{m}{s^2}} = 5.1 \cdot 10^{-4}\ kg \cdot m^2$$

Thus, the moment of inertia of the axle is equal to $5.1 \cdot 10^{-4}\ kg \cdot m^2$.

5 Analysis, Errors, and Future Experimentation

- Because the axle has significant mass, we could not assume the string had the same tension value as in Honors Physics.We had to solve for the different tension values and multiply them to the radius of the pulley to find the net torques value. This lab would not have been as effective if we did not take into account torque.
- Errors that might have affected our data was we did not get a uniform time for every occasion the hanging object hit the ground. The height might have been off on some of the trials as well
- A further experiment we could do is find the inertia of the pulley but do that when taking friction into account. We could also find the inertia of the pulley that is attached to an object with another object on top of it, complication derivations