Algorithm Design & Analysis (CS3383)[1]

Unit 3 : Greedy Algorithms

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[1]Thanks to Dr. Ptricia Evans and Dr. David Bremner at UNB, Dr. Erik Demaine at MIT for sharing the teaching stuffs

Outline[2]

Greedy

- Example
- Huffman Coding
- MST

[2] Reading:

- Main textbook (DPV), Greedy algorithms, Chapter 5.
- Algorithms(Cormen): Chapter 16 (mainly 16.1).
- Chapters 4 from Jeff Ericson's Algorithm page http://jeffe.cs.illinois.edu/teaching/algorithms/book/04-greedy.pdf

Contents

Example of Greedy Algorithms

Discussion: In a given graph, computing the

- shortest simple path can be done by a greedy algorithm. (e.g Dijkstra)
- Longest simple path is an NP-Hard problem?(Discussion on differences)

Contents

Prefix codes

Char	Freq	Symbol
A	70	0
B	3	001
C	20	01
D	37	11

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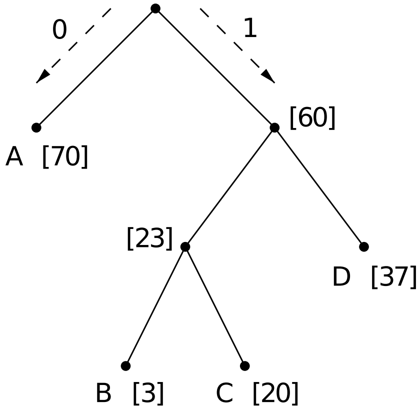
- variable length symbols
- avoiding ambiguous bitstreams: what is 001?
- no symbol should be a prefix of another.
- if A is 0, what is D?

Huffman Coding

- The goal is to compress data in an efficient way without loss of information
- The Huffman Coding algorithm uses the greedy approach to consider minimum possible length for data with high frequency
- Data with lowest frequencies are deepest in the Huffman tree
- Example (board)

Huffman coding (another example:)

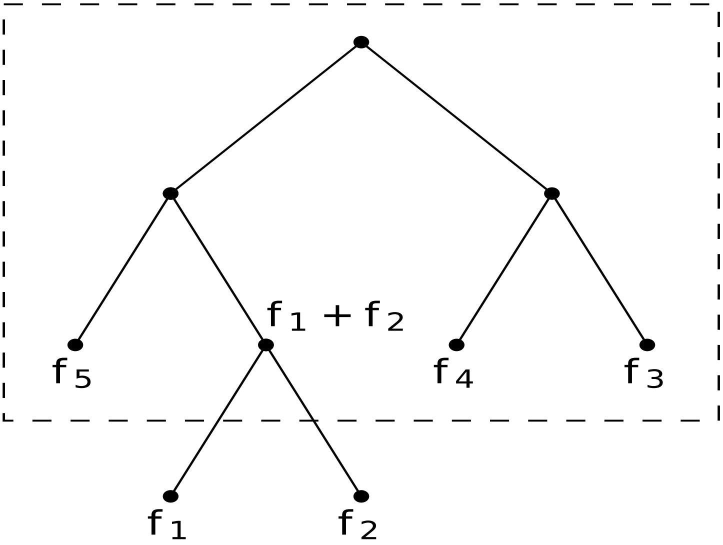
Symbol	Codeword
A	0
B	100
C	101
D	11



$$\text{cost}(T) = \sum_{i=1}^{n} f_i \text{depth}_i$$

(Avg cost)

Lightest leaves are deepest



- proof by swapping

Huffman Algorithm

```
Huffman(f[1..n])
H = priority queue of ind., by freq.
for i = 1 to n do
    H.insert(i)
for k = n+1 to 2n−1 do
    i = H.deletemin()
    j = H.deletemin()
    f[k] = f[i] + f[j]
    H.insert(k)
    children[k] = (i,j)
```

Correctness

Huffman yields an optimal prefix code (induction)

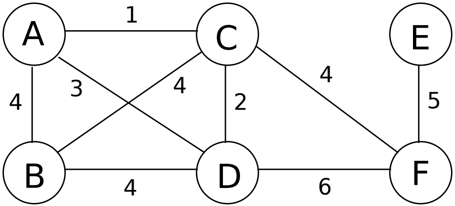
Contents

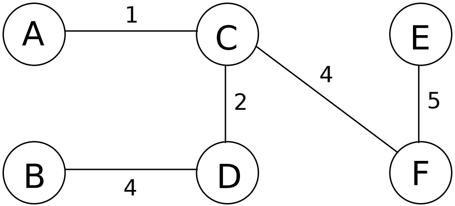
Minimum spanning tree

Definition (Minimum Spanning Tree)

Given $G = (V, E)$, $w : E \to \mathbb{R}$, a minimum spanning tree T is a spanning tree (i.e. connecting all vertices) that minimizes $\mathrm{cost}(T) = \sum_{e \in T} w(e)$

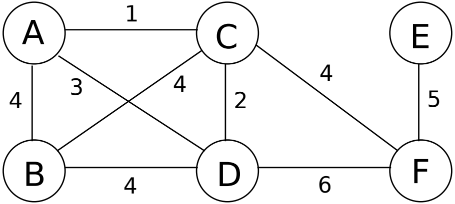
Minimum Spanning trees

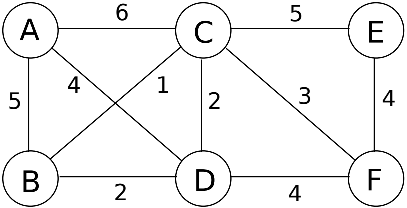


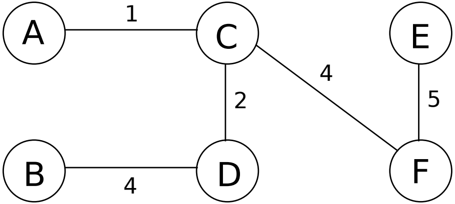


Is this solution unique?

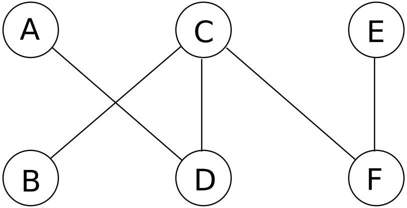
Minimum Spanning trees





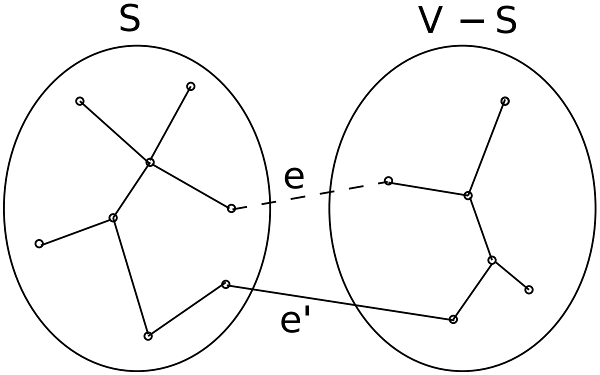


Is this solution unique?



How about this one?

Cut Property

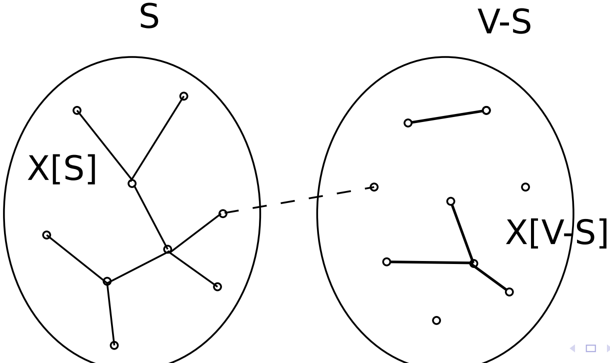


Lemma (Board)

Let T be a minimum spanning tree, $X \subset T$ s.t. X does not connect $(S, V - S)$. Let e be the lightest edge from S to $V - S$. $X \cup e$ is part of some MST.

Generic MST

$X \leftarrow \{\}$
while $|X| < |V| - 1$ **do**
  Choose S s.t. X does not connect $(S, V - S)$
  Add the lightest crossing edge to X
end while



Greedy Algorithms in General

Discrete Optimization Problems

- solution defined by a sequence of choices
- solutions are ranked from best to worst

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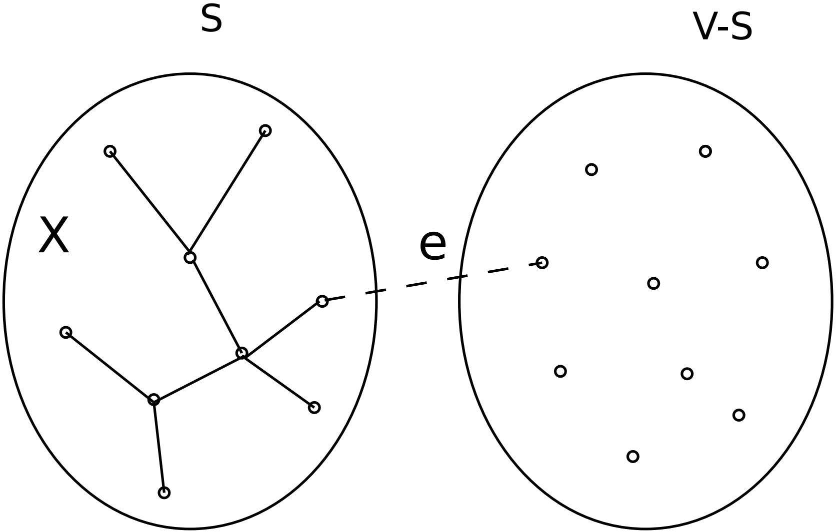
Greedy Design Strategy

- Each choice leaves one smaller subproblem
- Prove that $\exists$ an optimal solution that makes the greedy choice
- Show that the greedy choice, combined with an optimal solution to the subproblem, yields an optimal solution to the original problem.

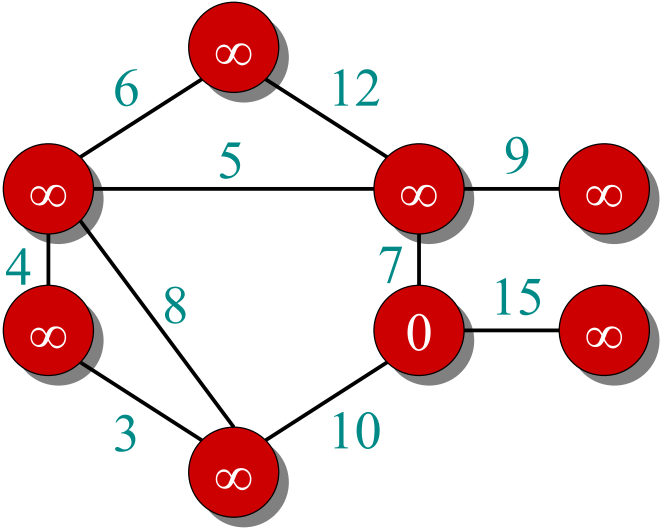
Prim's Algorithm



$S =$ nodes reached so far

Example of Prim's algorithm

- $\in A$
- $\in V - A$



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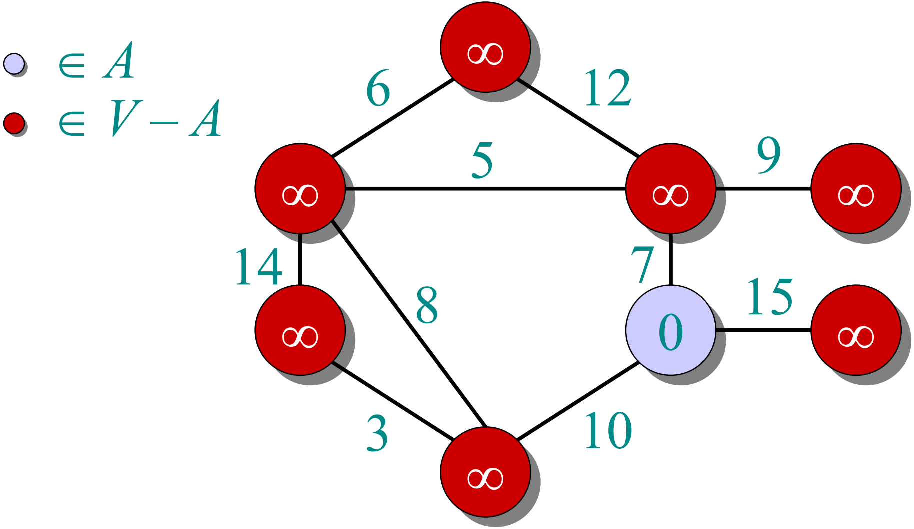

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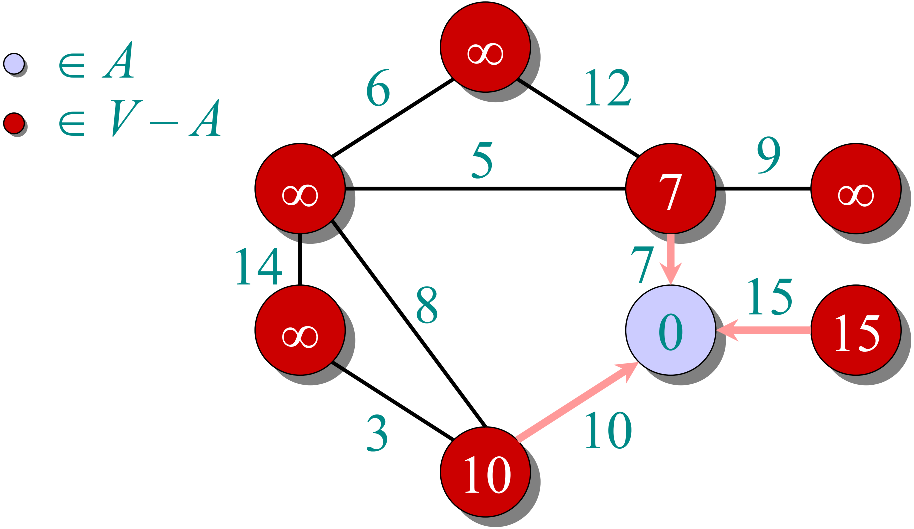

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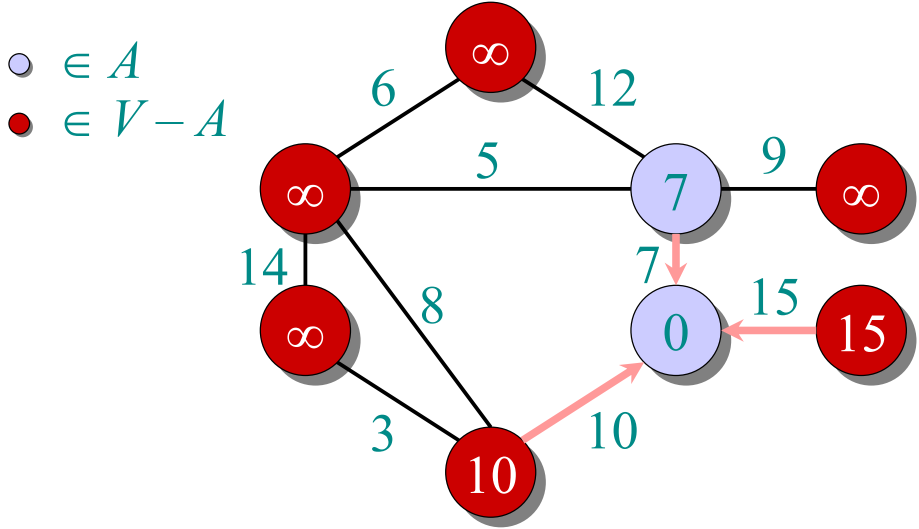

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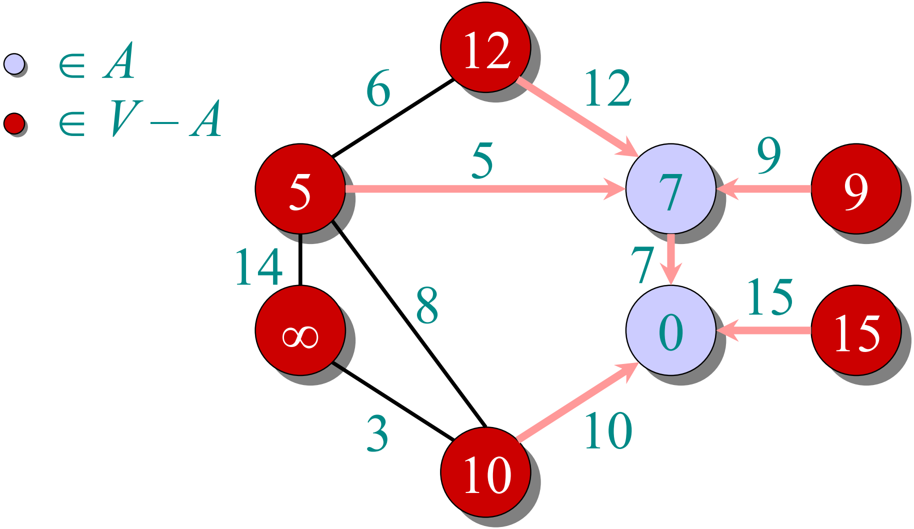

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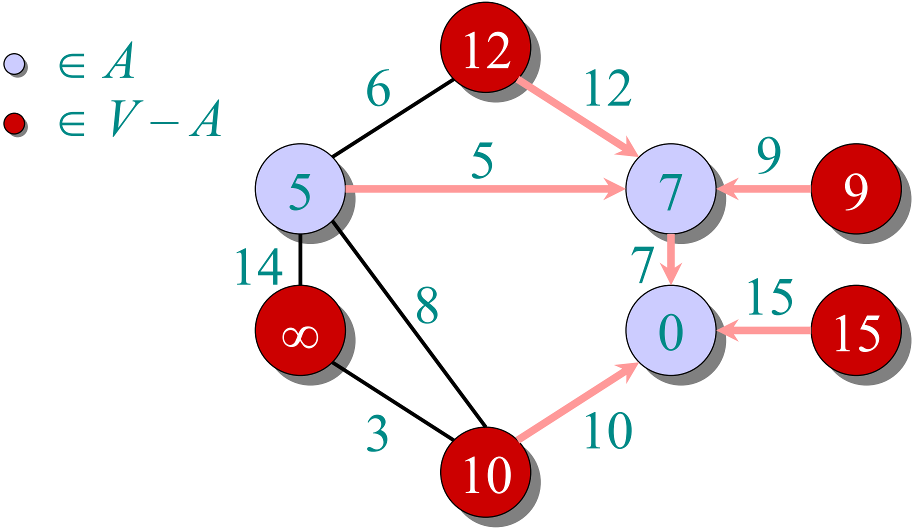

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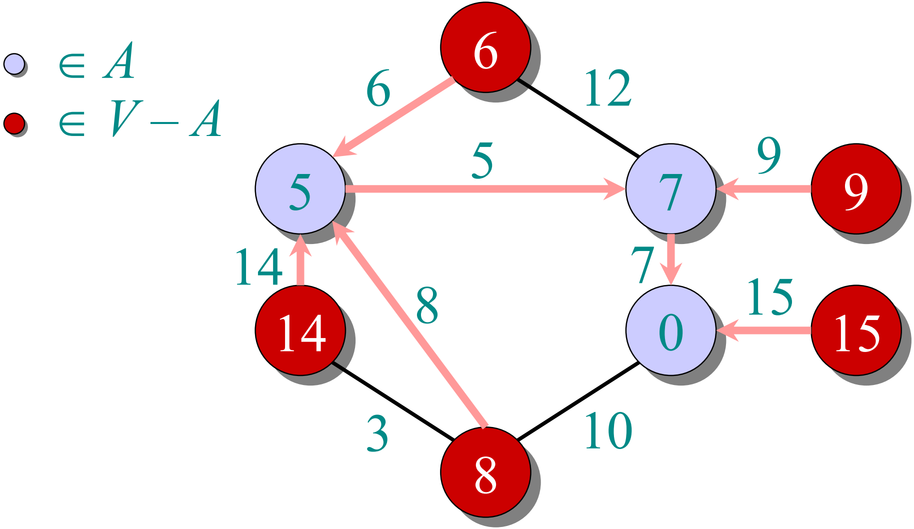

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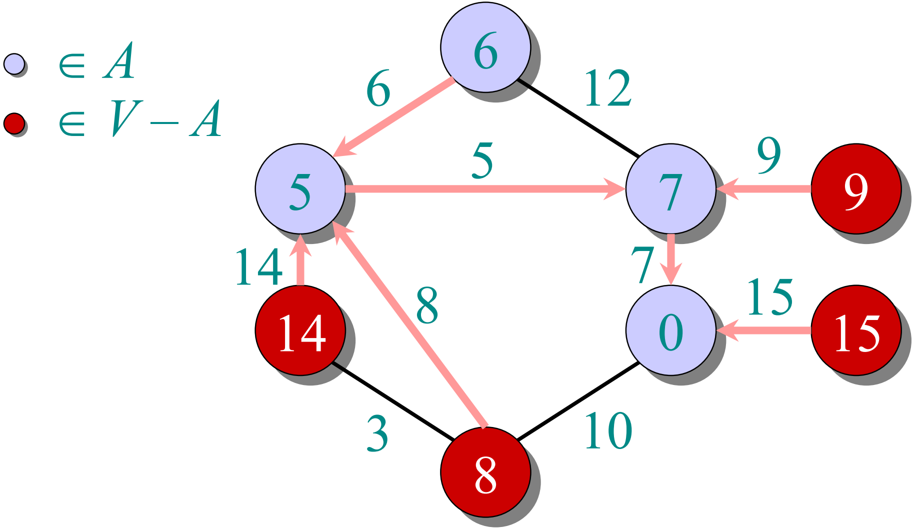

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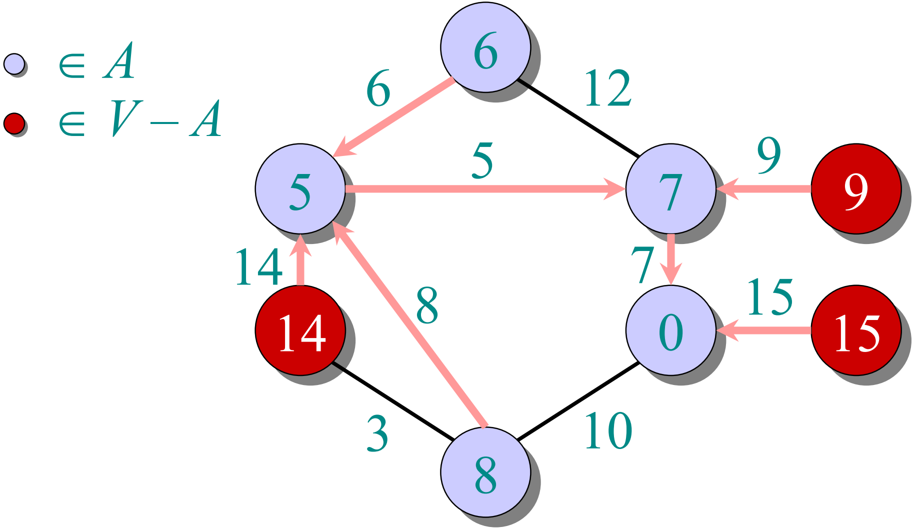

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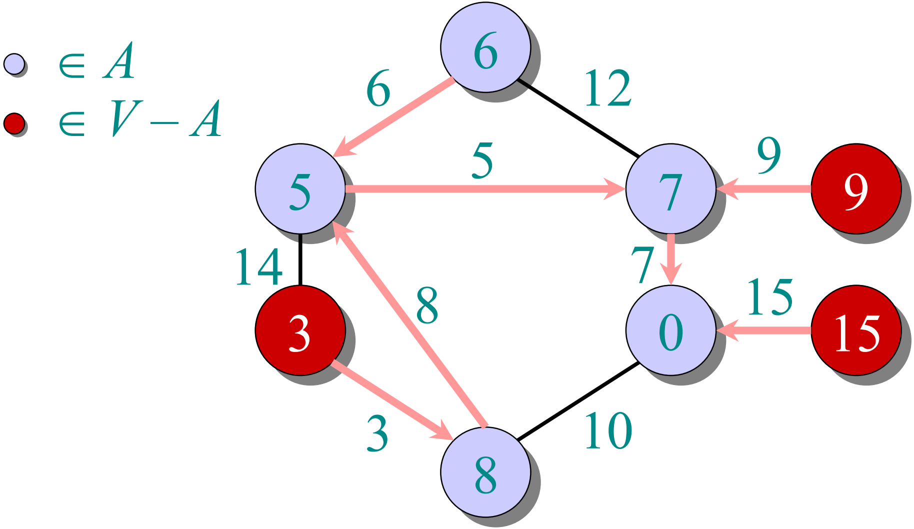

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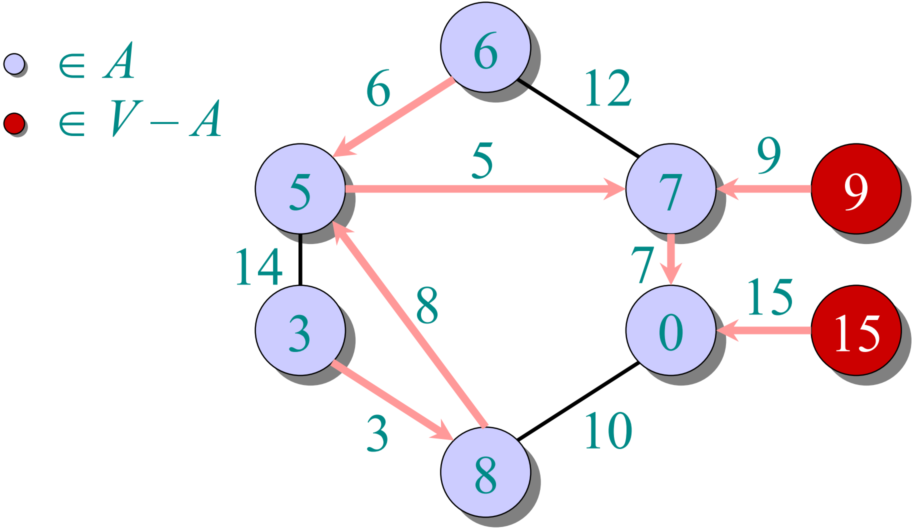

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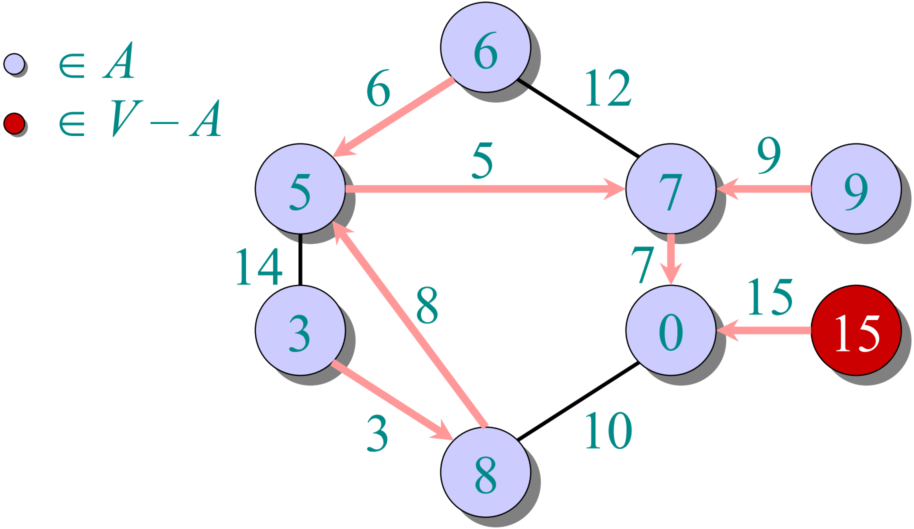
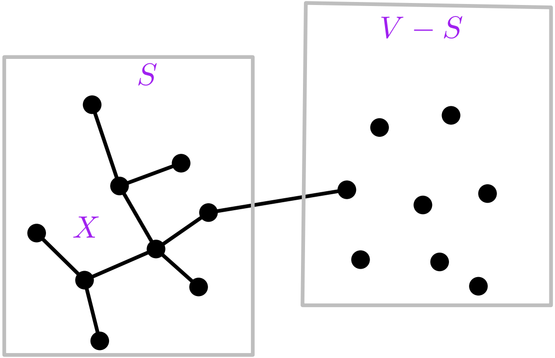


Prim's Algorithm

$u_0 =$ arbitrary vertex
cost$(u_0) = 0$; cost$(v) = \infty$, $v \neq u_0$
for $v \in V$: enq(H, v)
while H is not empty **do**
    $v =$ deletemin(H)
    for $e = \{v, z\}$, $e \in E$, $z \in H$ **do**
        if cost$(z) > w(v, z)$ **then**
            cost$(z) = w(v, z)$
            prev$(z) = v$
            decreasekey(H,z)
        end if
    end for
end while



Analysis of Prim's Algorithm (board)

- Correctness follows from the cut property, induction
- Closely connected with the Djikstra's Shortest path algorithm; only two lines change
- Tree can be read back from prev
- Cost is dominated by priority queue operations