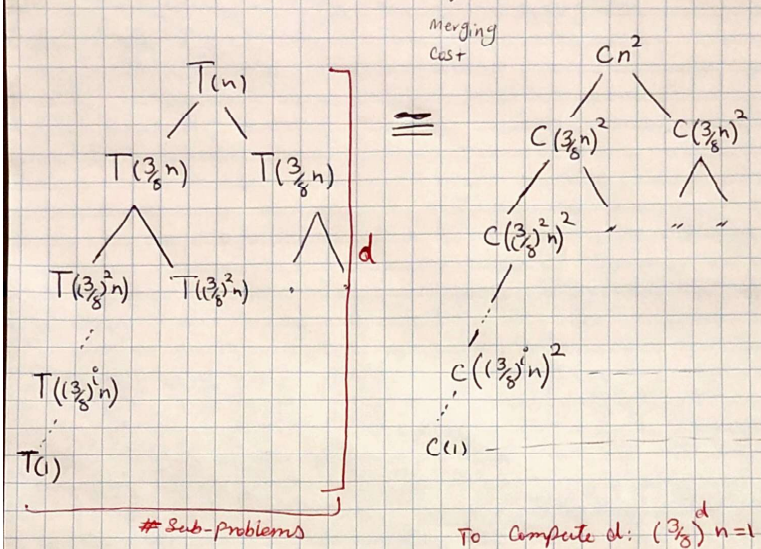
Devil is hidden in the details !!!

Q: Solve $T(n) = 2T(\frac{3}{8}n) + n^2$ using the recursion tree method.



Rules and needed info:

① $\log_b^a = \log_c^a / \log_c^b$

② $b^{\log_b^a} = a$ ② $\log_a b = 1 / \log_b a$

③ $\log_{\frac{9}{32}}^{8/3} = -0.773$

④ $\frac{1}{-0.773} \cong -1.29$

⑤ G.S. $\sum_{i=0}^{n} a^i = \frac{1-a^{n+1}}{1-a}$

⑥ $a^{\log_b^x} = x^{\log_b^a}$

To Compute d: $(\frac{3}{8})^d n = 1 \Rightarrow n = (\frac{8}{3})^d \Rightarrow d = \log_{8/3}^n$

Sub-problems in Level $i = 2^i$

$$\Rightarrow T(n) = \sum_{i=0}^{d} c\, 2^i \left((\tfrac{3}{8})^i n\right)^2$$

$$\Rightarrow T(n) = \sum_{i=0}^{\log_{8/3}^n} c\, 2^i \left((\tfrac{3}{8})^i\right)^2 n^2 = cn^2 \sum_{i=0}^{\log_{8/3}^n} 2^i \left((\tfrac{3}{8})^2\right)^i = cn^2 \sum_{i=0}^{\log_{8/3}^n} 2^i \left(\tfrac{9}{64}\right)^i =$$

$$= cn^2 \sum_{i=0}^{\log_{8/3}^n} \left(\tfrac{9}{32}\right)^i \xrightarrow[\text{Rule ⑤}]{\text{G.S.}} = cn^2 \frac{1-\left(\frac{9}{32}\right)^{\log_{8/3}^n + 1}}{1-\frac{9}{32}} =$$

$$= cn^2 \frac{1-\left(\frac{9}{32}\right)^{\log_{8/3}^n} \cdot \left(\frac{9}{32}\right)^1}{\frac{23}{32}} = \frac{32}{23} cn^2 - \frac{32}{23} cn^2 \left(\frac{9}{32}\right)^{\log_{8/3}^n} \left(\frac{9}{32}\right)^1$$

Shortcut ? ⑥, ⑦ ?

$$= \frac{32}{23} cn^2 - \frac{9}{23} cn^2 \left(\frac{9}{32}\right)^{\log_{8/3}^n} \xrightarrow{\text{Rule ①}} \frac{32}{23} cn^2 - \frac{9}{23} cn^2 \left(\frac{9}{32}\right)^{\frac{\log_{9/32}^n}{\log_{9/32}^{8/3}}} =$$

$$= \frac{32}{23} cn^2 - \frac{9}{23} cn^2 \left(\left(\frac{9}{32}\right)^{\log_{9/32}^n}\right)^{\frac{1}{\log_{9/32}^{8/3}}} \xrightarrow[\text{Rule ③}]{\text{Rule ②}} \frac{32}{23} cn^2 - \frac{9}{23} cn^2 (n)^{\frac{1}{-0.773}}$$

$$= \frac{32}{23} cn^2 - \frac{9}{23} cn^2 (n)^{-1.29} = \frac{32}{23} cn^2 - \frac{9}{23} c * n^{2-1.29} =$$

$$= \frac{32}{23} cn^2 - \frac{9}{23} c\, n^{0.71} \xrightarrow{0.71 < 2} = \Theta(n^2)$$

Lesson: Kill all devils before the exam !