Algorithm Design and Analysis (CS3383)
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Verifying if a given instance is a solution for N-Queen Problem:

In class, we checked if a special instance (just with one diagonal considered) is a solution for N-Queen problem. Here is a general test to apply on every given instance and check if it can be a solution or not.

Question: Suppose that n queens are assigned to n elements of a $(n \times n)$ board. For any arbitrary configuration of the queens on the board, design a test to check if there is a queen under attack by other queens in the current configuration.

Test: Suppose that $Qij = 1$ if a queen is assigned to the element (i, j) of the board, and otherwise, $Qij = 0$. In order to make the test general, we define $Ones = \{Q_{ij} | Q_{ij} = 1\}$. The pseudocode of the Test is as follows:

$SumOverRows \leftarrow 0;$
$SumOverColumns \leftarrow 0;$
$SumOverAllDiagonals \leftarrow 0;$
$for\ \ each\ Q_{yz} \in Ones$
  $for\ \ i = 1\ to\ n$
    $SumOverRows \longleftarrow SumOverRows + Qiz$
    $SumOverColumns \longleftarrow SumOverColumns + Qyi$
    $SumOverAllDiagonals \longleftarrow SumOverAllDiagonals + Q(z-i)(y-i) + Q(z-i)(y+i)$
        $+ Q(z+i)(y-i) + Q(z+i)(y+i)$ $(*)$
$if\ \ (SumOverRows + SumOverColumns + SumOverAllDiagonals == 3)$
  $return\ \ True$
$else$
  $return\ \ False$

Note that in $(*)$, if any of the indices is out of bound, we assign the value of zero to its corresponding element. Figure 1 is an illustration for $(*)$.
Analysis: Since we have only n elements with the value of 1 (there are n queens on the board), the first loop iterates exactly n times. The second loop also iterates exactly n times. The rest of the algorithm requires some constant time. So, the running time is $\Theta(n^2)$. N-Queen problem is verified by an algorithm with a polynomial running time.

Note: For the exams and assignment questions, if you are asked to provide a verification test for an NP-complete problem, you should design a test that works for all possible instances.

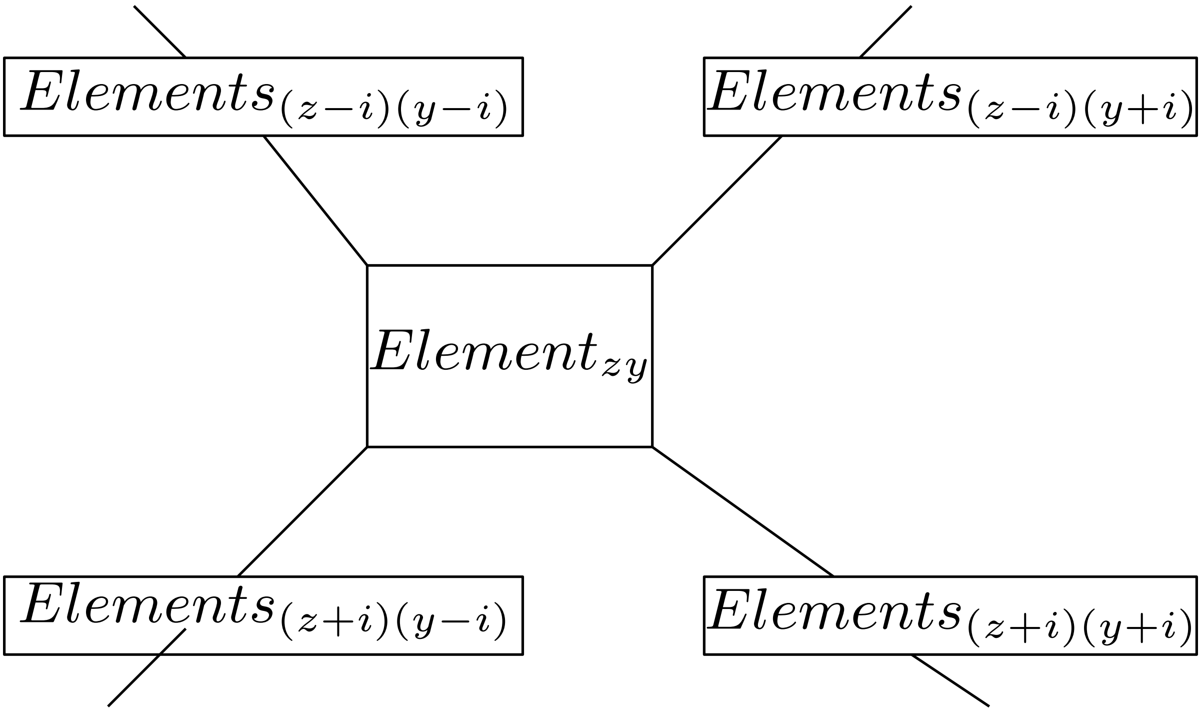

Figure 1: All possible diagonally attacks for the queen assigned to the element (z, y) of the board