Algorithm Design & Analysis (CS3383)[1]

Unit 5 : Dynamic Multithreded Algorithms

Rasoul Shahsavarifar

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[1] Thanks to Dr. Ptricia Evans and Dr. David Bremner at UNB for sharing the teaching stuffs

Outline

Dynamic Multithreaded Algorithms
- Fork-Join Model

Using More than One Processor
- Capabilities
- Span, Work, And Parallelism
- Parallel Loops
- Scheduling
- Race Conditions

Contents

Introduction to Parallel Algorithms

Dynamic Multithreading

- Also known as the *fork-join* model
- Shared memory, *multicore*
- Cormen et. al 3rd edition, Chapter 27

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Multithreading

So far we've been looking at traditional *serial algorithmics*, designing algorithms to run on a single processor and analyzing the single-processor running time.

However, it's quite common for computers to have multiprocessors, and for code to be multithreaded.

At a larger parallel scale, processors may have access to many processors.

So, how does this change what we do for algorithm design and analysis?

Contents

Concurrency Capabilities

We examine a shared-memory framework, so multiple processors can access the same memory. This is consistent with many architectures, and also eliminates passing data between processors.

The model we use is often referred to as the *fork-join* model. The structures we seek to parallelize are the two fundamental code structures: the *branch* and the *loop*. While loops don't parallelize well due to their structure, so the loops are limited to iterative *for* loops with a counter.

New keywords:

parallel	run the loop potentially concurrently
spawn	run the called routine potentially concurrently
sync	wait for all spawned children to complete

These keywords can be added to serial code, and can also be removed from parallel code to serialize it.

Writing parallel (pseudo)-code

Keywords

parallel	Run the loop (potentially) concurrently
spawn	Run the procedure (potentially) concurrently
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Serialization

- remove keywords from parallel code yields correct serial code
- Adding parallel keywords to correct serial code might break it
  - missing sync
  - loop iterations not independent

Fibonacci Example

function $\textsc{Fib}(n)$
    if $n \leq 1$ **then**
        return n
    else
        $x = \mathrm{Fib}(n-1)$
        $y = \mathrm{Fib}(n-2)$

        return $x + y$
    end if
end function

Fibonacci Example

function $\textsc{Fib}(n)$
  if $n \leq 1$ **then**
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  else
    $x =$ spawn $\mathrm{Fib}(n-1)$
    $y = \mathrm{Fib}(n-2)$
    sync
    return $x + y$
  end if
end function

- Code in C, Java, Clojure and Racket available from `http://www.cs.unb.ca/~bremner/teaching/cs3383/examples`

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Computation DAG

Strands

Seq. inst. with no *parallel*, *spawn*, return from *spawn*, or *sync*.

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  if $n \leq 1$ **then** $\triangleright$
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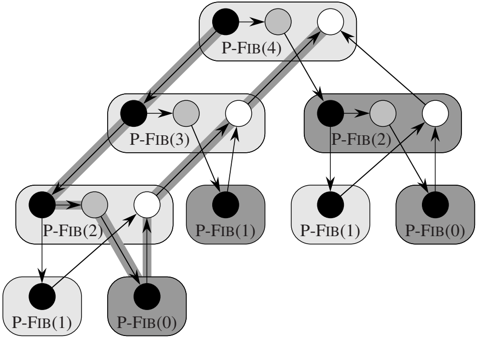
    sync
    return $x + y$ $\triangleright$
  end if
end function



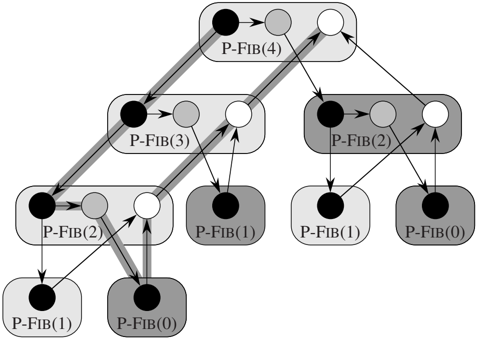
Computation DAG

Strands

Seq. inst. with no *parallel*, *spawn*, return from *spawn*, or *sync*.

nodes strands

down edges spawn



Computation DAG

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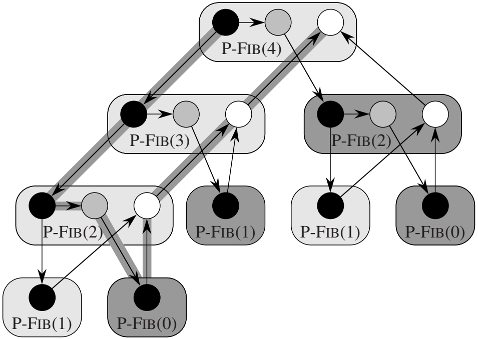
nodes	strands
down edges	spawn
up edges	return



Computation DAG

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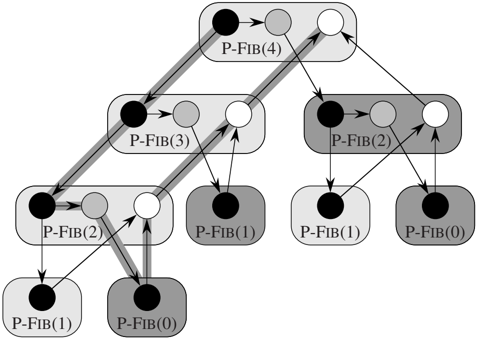
nodes	strands
down edges	spawn
up edges	return
horizontal edges	sequential



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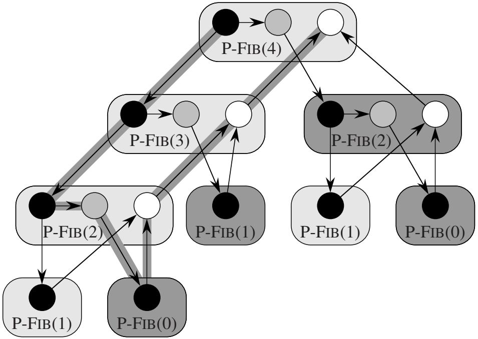
critical path	longest path in DAG



Computation DAG

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Seq. inst. with no *parallel*, *spawn*, return from *spawn*, or *sync*.

nodes	strands
down edges	spawn
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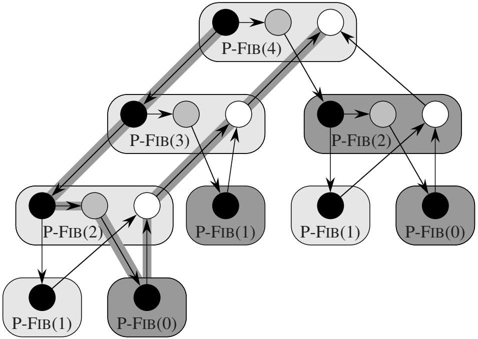
horizontal edges	sequential
critical path	longest path in DAG
span	weighted length of critical path ≡ lower bound on time



Work and Speedup

T_1 *Work*, sequential time.

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T_p Time on p processors.

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Work Law

$$T_p \geq T_1/p$$

$$\text{speedup} := T_1/T_p \leq p$$

Parallelism

T_p Time on p processors.

Parallelism

We could idle processors:

$$T_p \geq T_\infty \tag{1}$$

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T_∞ *Span*, time given unlimited processors.

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We could idle processors:

$$T_p \geq T_\infty \tag{1}$$

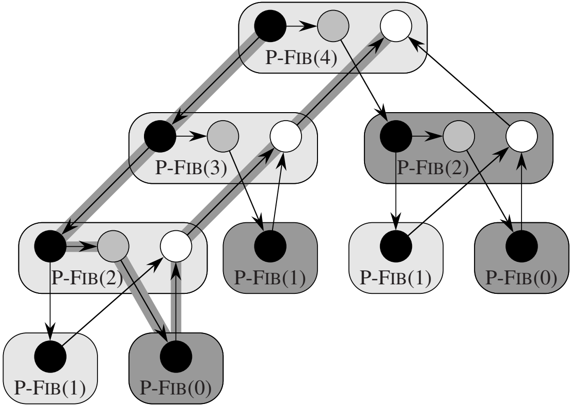
Best possible speedup:

$$\begin{aligned} \textcolor{red}{\text{parallelism}} &= T_1/T_\infty \\ &\geq T_1/T_p = \text{speedup} \end{aligned}$$

Span and Parallelism Example

Assume strands are unit cost.

- $T_1 = 17$



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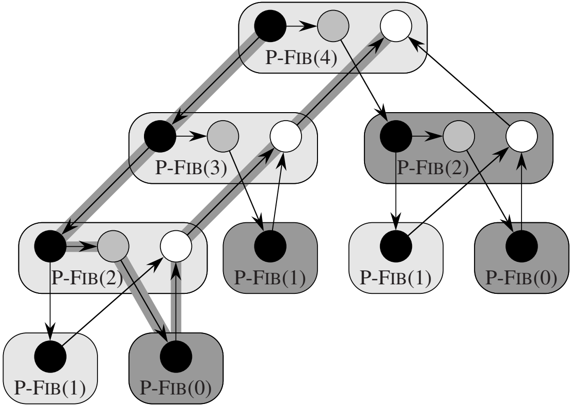
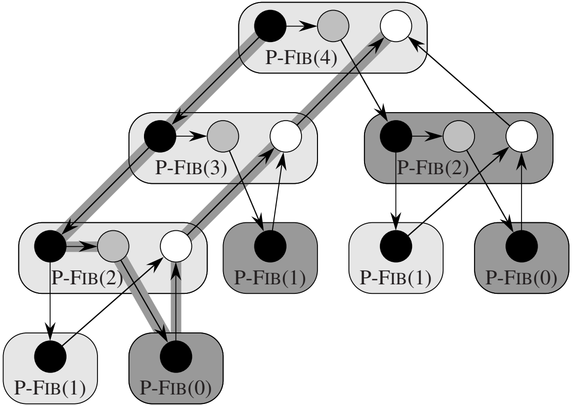
- $T_\infty = 8$



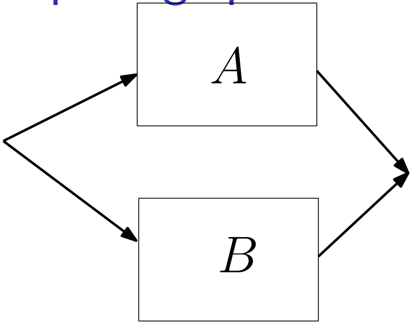
Span and Parallelism Example

Assume strands are unit cost.

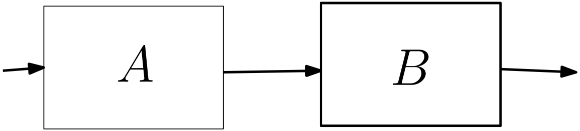
- $T_1 = 17$
- $T_\infty = 8$
- Parallelism $= 2.125$ for this input size.



Composing span and work



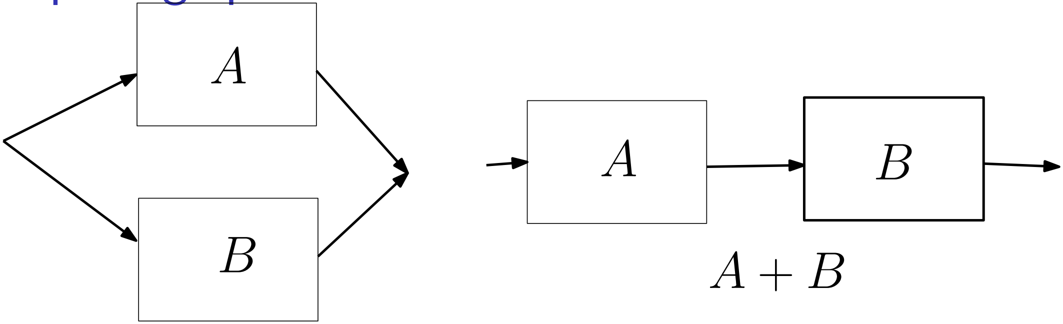
$A \| B$



$A + B$

series $T_\infty(A + B) = T_\infty(A) + T_\infty(B)$

Composing span and work



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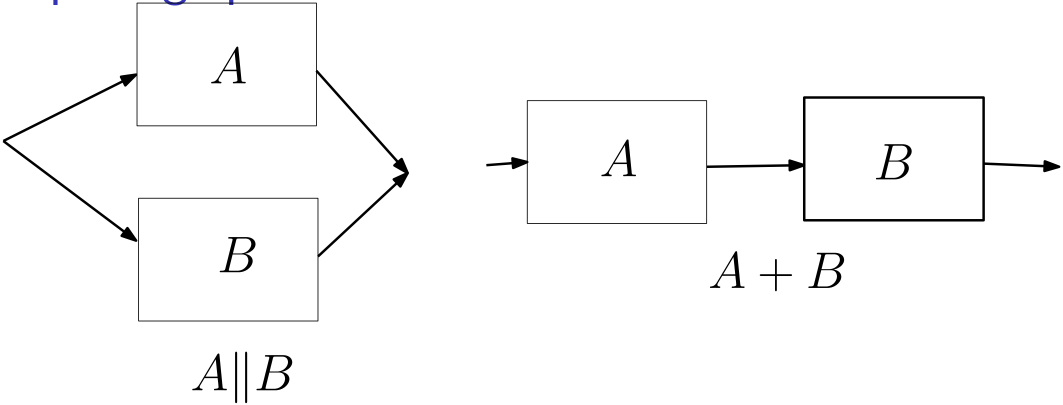
Composing span and work



series $T_\infty(A+B) = T_\infty(A) + T_\infty(B)$

parallel $T_\infty(A\|B) = \max(T_\infty(A), T_\infty(B))$

series or parallel $T_1 = T_1(A) + T_1(B)$. Why?

Work of Parallel Fibonacci

Write $T(n)$ for T_1 on input n.

$$T(n) = T(n-1) + T(n-2) + \Theta(1) \qquad \text{(I.H.)}$$

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Write $T(n)$ for T_1 on input n.

$$T(n) = T(n-1) + T(n-2) + \Theta(1) \quad T(n) \leq a\phi^n - b \qquad \text{(I.H.)}$$

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$$\begin{aligned} T(n) &\leq a(\phi^{n-1} + \phi^{n-2}) - 2b + \Theta(1) \\ &= a\frac{\phi + 1}{\phi^2}\phi^n - b + (\Theta(1) - b) \end{aligned}$$

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Span and Parallelism of Fibonacci

$$
\begin{aligned}
T_\infty(n) &= \max(T_\infty(n-1), T_\infty(n-2)) + \Theta(1) \\
&= T_\infty(n-1) + \Theta(1)
\end{aligned}
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- So an inefficient way to compute Fibonacci, but very parallel

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parallel for $i = 1$ to n **do**
    statement...
    statement...
end for

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- Can be implemented with spawn and sync
- Span

$$T_\infty(n) = \Theta(\log n) + \max_i T_\infty(\text{iteration i})$$

Parallel Matrix-Vector product

To compute $y = Ax$, in parallel

$$y_i = \sum_{j=1}^{n} a_{ij} x_j$$

Parallel Matrix-Vector product

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function ROWMULT(A,x,y,i)
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function $\textsc{Mat-Vec}(A, x, y)$
  Let $n =$ rows(A)
  parallel for $i = 1$ to n **do**
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$T_1(n) \in \Theta(n^2)$ (serialization)

$$T_\infty(n) = \underbrace{\Theta(\log(n))}_{\text{parallel for}} + \underbrace{\Theta(n)}_{\text{RowMult}}$$

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function MAT-VEC(A, x, y)
  Let $n =$ rows(A)
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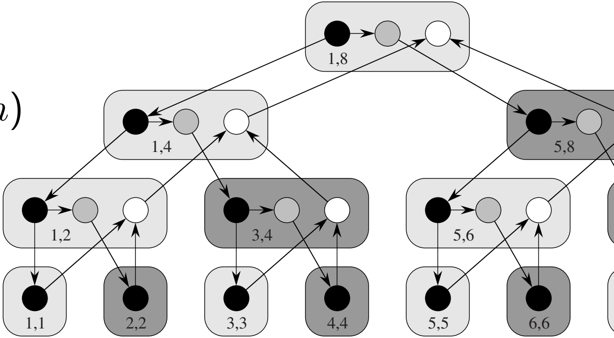
    RowMult(A,x,y,i)
  end for
end function

- Why is RowMult not using parallel for?

Divide and Conquer Matrix-Vector product

function $\mathrm{MVDC}(A, x, y, f, t)$
  if $f == t$ **then**
    RowMult(A,x,y,f)
  else
    $m = \lfloor (f + t)/2 \rfloor$
    spawn MVDC(A, x, y, f, m)
    MVDC$(A, x, y, m + 1, t)$
    sync
  end if
end function



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- $\Theta(n)$ interior nodes (binary tree)

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        sync
    end if
end function
```

- $T_\infty(n) = \Theta(\log n)$ (binary tree) $+$ $T_\infty(\text{RowMult})$
- $\Theta(n)$ leaves (one per row)
- $\Theta(n)$ interior nodes (binary tree)
- $T_1(n) = \Theta(n^2)$

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Scheduling

Scheduling Problem

Abstractly Mapping threads to processors

Pragmatically Mapping logical threads to a thread pool.

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On-Line No advance knowledge of when threads will spawn or complete.

Distributed No central controller.

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Pragmatically Mapping logical threads to a thread pool.

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Distributed No central controller.

- to simplify analysis, we relax the second condition

A greedy centralized scheduler

Maintain a *ready queue* of strands ready to run.

Scheduling Step

Complete Step If $\geq p$ (# processors) strands are ready, assign p strands to processors.

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Incomplete Step Otherwise, assign all waiting strands to processors

- To simplify analysis, split any non-unit strands into a chain of unit strands
- Therefore, after one time step, we schedule again.

Optimal and Approximate Scheduling

Recall

$$T_p \geq T_1/p \qquad \text{(work law)}$$
$$T_p \geq T_\infty \qquad \text{(span)}$$

Therefore

$$T_p \geq \max(T_1/p, T_\infty) = \text{opt}$$

Optimal and Approximate Scheduling

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Therefore

$$T_p \geq \max(T_1/p, T_\infty) = \text{opt}$$

With the greedy algorithm we can achieve

$$T_p \leq \frac{T_1}{p} + T_\infty \leq 2\max(T_1/p, T_\infty) = 2 \times \text{opt}$$

Counting Complete Steps

- Let k be the number of complete steps.

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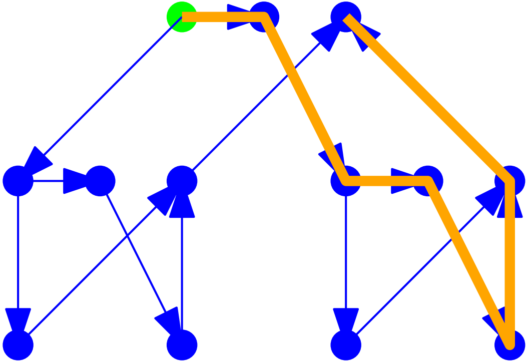
- At each complete step we do p units of work.
- Every unit of work corresponds to one step of the serialization, so $kp \leq T_1$.
- Therefore $k \leq T_1/p$

Counting Incomplete Steps



- Let G be the DAG of *remaining strands*.

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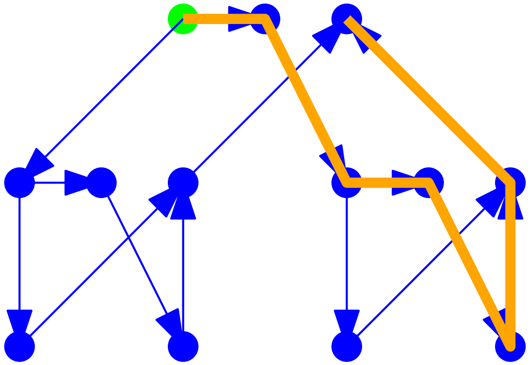

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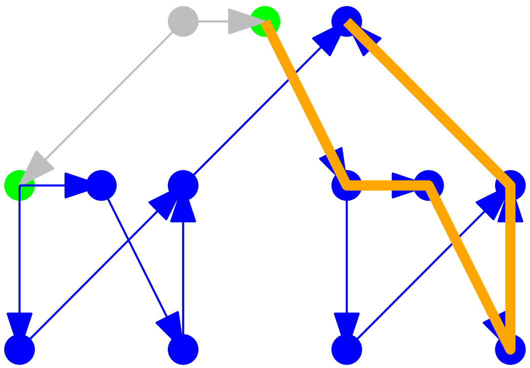

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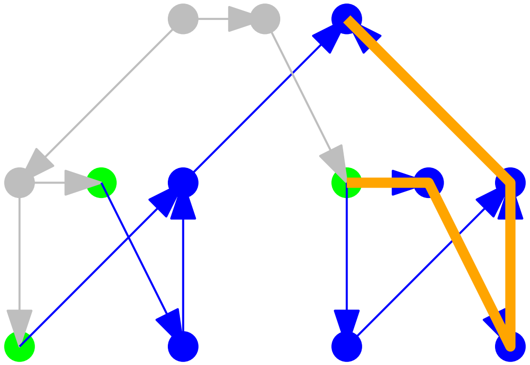

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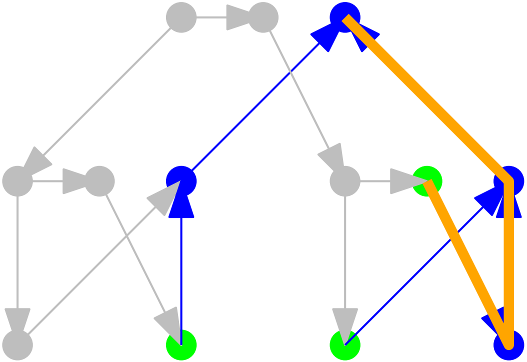

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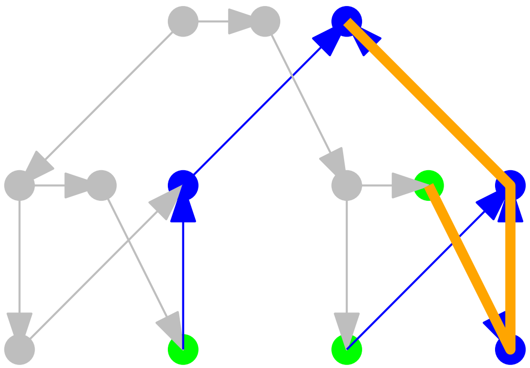

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- Every longest path starts at a source (otherwise, extend)
- After an incomplete step, length of longest path shrinks by 1
- There can be at most T_∞ steps.

Parallel Slackness

$$\text{parallel slackness} = \frac{\text{parallelism}}{p} = \frac{T_1}{pT_\infty}$$

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$$\text{speedup} = \frac{T_1}{T_p} \leq \frac{T_1}{T_\infty} = p \times \text{slackness}$$

- If slackness < 1, speedup $< p$
- If slackness ≥ 1, linear speedup achievable for given number of processors

Slackness and Scheduling

$$\textcolor{red}{\text{slackness}} := \frac{T_1}{p \times T_\infty}$$

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Substituting (2), we have

$$T_p \leq \frac{T_1}{p}\left(1 + \frac{1}{c}\right)$$

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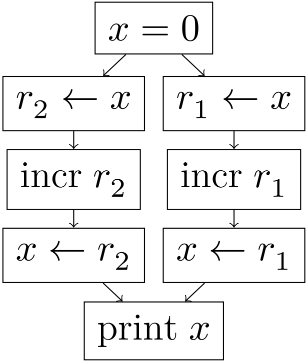
Example

```
x = 0
parallel for i ← 1 to 2 do
    x ← x + 1
```

Racy execution



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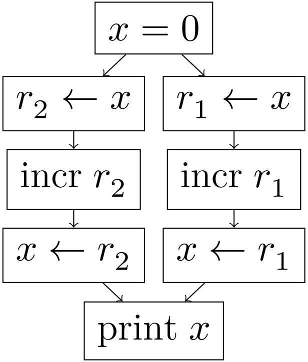

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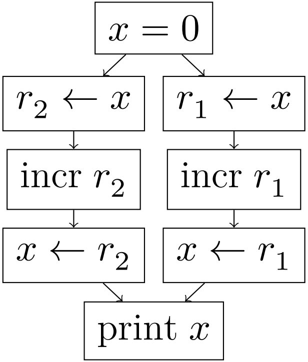

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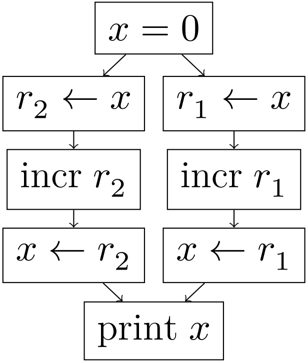

- all possible topological sorts are valid execution orders
- In particular it's not hard for both loads to complete before either store
- In practice there are various synchronization strategies (locks, etc...).
- Here we will insist that parallel strands are independent

We can write bad code with spawn too

```
sum(i, j)
  if (i>j)
    return;
  if (i==j)
    x++;
  else
    m=(i+j)/2;
    spawn sum(i,m);
    sum(m+1,j);
    sync;
```

- here we have the same non-deterministic interleaving of reading and writing x
- the style is a bit unnatural, in particular we are not using the return value of spawn at all.

Being more *functional* helps

```
sum(i, j)
  if (i>j) return 0;
  if (i==j) return 1;

  m ← (i+j)/2;

  left ← spawn sum(i,m);
  right ← sum(m+1,j);
  sync;
  return left + right;
```

- each strand writes into different variables

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- each strand writes into different variables
- sync is used as a barrier to serialize

Single Writer races

```
x ← spawn foo(x)
y ← foo(x)
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- arguments to spawned routines are evaluated in the parent context
- but this isn't enough to be race free.
- which value x is passed to the second call of 'foo' depends how long the first one takes.