Hedging with Futures

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Introduction

Short Hedge: a hedge that involves a short position in futures contracts. A short hedge is appropriate when the hedger already owns as asset and expects to sell it at some time in the future.

Long Hedge: a hedge that involves a long position in futures contracts. A long hedge is appropriate when a hedger knows it will have to purchase a certain asset in the future and wants to lock in the price now.

The Minimum-Variance Hedge

The minimum-variance hedge is given by the following:

$$h^* = \rho \frac{\sigma_s}{\sigma_f}$$

where ρ is the correlation between the spot and futures prices.

This can be most efficiently estimated via OLS regression, since:

$$\begin{aligned} \hat{\beta} &= \frac{Cov(s,f)}{Var(f)} \\ &= \frac{\rho \sigma_s \sigma_f}{\sigma_f \sigma_f} \\ &= \rho \frac{\sigma_s}{\sigma_f} \end{aligned}$$

The Simple Fixed 1-for-1 Hedge

The simplest hedge is the fixed one-for-one hedge ratio that was used by MGRM. This simple hedge sets $h = 1$. That is for every unit of the underlying asset the risk manager hedges with futures in exactly the same amount of units (either short or long).

MGRM's Hedging Strategy

Let's first estimate the risk-minimizing hedge ratio as a benchmark.

Let's start with the month that the program began in December, 1991.

```
basePath <- "/home/brough/USU/Research/Projects/local/MGRM"
srcDir <- paste(basePath, "/src/R", sep="")
datDir <- paste(basePath, "/data/December/", sep="")
setwd(srcDir)
```

```
## Read in the data for heating oil
infile1 <- paste(datDir, "heatingoil-spot.csv", sep="")
ho.s.raw <- read.csv(infile1, sep=",", header=T)
names(ho.s.raw) <- c("Date", "Spot")
ho.s.raw$Date <- as.Date(ho.s.raw$Date, "%Y-%m-%d")

infile2 <- paste(datDir, "heatingoil-futures.csv", sep="")
ho.f.raw <- read.csv(infile2, sep=",", header=T)
names(ho.f.raw) <- c("Date", "Futures")
ho.f.raw$Date <- as.Date(ho.f.raw$Date, "%Y-%m-%d")

## Merge the datasets
oil.raw <- merge(ho.s.raw, ho.f.raw, by="Date")
oil.raw <- oil.raw[order(oil.raw$Date), ]
oil.raw$Basis <- log(oil.raw$Futures) - log(oil.raw$Spot)
head(oil.raw)
```

```
##         Date  Spot Futures       Basis
## 1 1986-06-02 0.402   0.378 -0.06155789
## 2 1986-06-03 0.393   0.380 -0.03363836
## 3 1986-06-04 0.378   0.358 -0.05436121
## 4 1986-06-05 0.390   0.374 -0.04189094
## 5 1986-06-06 0.385   0.372 -0.03434948
## 6 1986-06-09 0.373   0.366 -0.01894509
```

```
tail(oil.raw)
```

```
##            Date  Spot Futures      Basis
## 7867 2017-11-03 1.791   1.887 0.05221414
## 7868 2017-11-06 1.831   1.942 0.05885610
## 7869 2017-11-07 1.827   1.922 0.05069103
## 7870 2017-11-08 1.808   1.922 0.06114505
## 7871 2017-11-09 1.833   1.947 0.06033576
## 7872 2017-11-13 1.830   1.932 0.05423977
```

We will now subset the data and perform the statistical analysis.

```
## Subset the data
begDate <- as.Date("1990-11-30", "%Y-%m-%d")
endDate <- as.Date("1991-12-30", "%Y-%m-%d")
ind <- (oil.raw$Date >= begDate & oil.raw$Date <= endDate)
oil.sub <- oil.raw[ind, ]

## Take a Peak
head(oil.sub)
```

```
##            Date  Spot Futures        Basis
## 1127 1990-11-30 0.860   0.859 -0.001163467
## 1128 1990-12-03 0.854   0.847 -0.008230499
## 1129 1990-12-04 0.848   0.869  0.024462489
## 1130 1990-12-05 0.832   0.811 -0.025564387
## 1131 1990-12-06 0.818   0.797 -0.026007658
## 1132 1990-12-07 0.810   0.792 -0.022472856
```

```
tail(oil.sub)
```

```
##            Date  Spot Futures      Basis
```

```
## 1393 1991-12-20 0.489   0.511 0.04400710
## 1394 1991-12-23 0.494   0.518 0.04743973
## 1395 1991-12-24 0.497   0.518 0.04138522
## 1396 1991-12-26 0.477   0.503 0.05307368
## 1397 1991-12-27 0.479   0.504 0.05087567
## 1398 1991-12-30 0.457   0.481 0.05118388
```

Now calculate the minimum-variance hedge ratio.

```
delS <- diff(oil.sub$Spot)
delF <- diff(oil.sub$Futures)
fit <- lm(delS ~ delF)
summary(fit)
```

```
##
## Call:
## lm(formula = delS ~ delF)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.033855 -0.002605  0.000158  0.002087  0.063850
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0001583  0.0005277   -0.30    0.764
## delF         0.9526565  0.0202164   47.12   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.008674 on 269 degrees of freedom
## Multiple R-squared:  0.8919, Adjusted R-squared:  0.8915
## F-statistic:  2221 on 1 and 269 DF,  p-value: < 2.2e-16
```

What if we use log-differences instead of price level differences?

```
delS <- diff(log(oil.sub$Spot))
delF <- diff(log(oil.sub$Futures))
fit <- lm(delS ~ delF)
summary(fit)
```

```
##
## Call:
## lm(formula = delS ~ delF)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.050850 -0.004256  0.000481  0.003654  0.094901
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0003996  0.0007855  -0.509    0.611
## delF         0.9035155  0.0215311  41.963   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01291 on 269 degrees of freedom
```

```
## Multiple R-squared:  0.8675, Adjusted R-squared:  0.867
## F-statistic:  1761 on 1 and 269 DF,  p-value: < 2.2e-16
```

Let's see how the minimum-variance hedge ratio changes over time.

```
## Make a function to take in data of appropriate period and output the mv hedge ratio
minvarHedgeRatio <- function(dat)
{
  delS <- diff(dat$Spot)
  delF <- diff(dat$Futures)
  fit <- lm(delS ~ delF)
  hr <- coef(fit)[2]
  return(hr)
}

addMonth <- function(date, n)
{
  return(seq(from=date, by=paste(n, "months"), length=2)[2])
}

lastDate <- as.Date("2001-11-30", "%Y-%m-%d")
ind <- (oil.raw$Date >= begDate & oil.raw$Date <= lastDate)
oil.full <- oil.raw[ind, ]
head(oil.full)
```

```
##            Date  Spot Futures        Basis
## 1127 1990-11-30 0.860   0.859 -0.001163467
## 1128 1990-12-03 0.854   0.847 -0.008230499
## 1129 1990-12-04 0.848   0.869  0.024462489
## 1130 1990-12-05 0.832   0.811 -0.025564387
## 1131 1990-12-06 0.818   0.797 -0.026007658
## 1132 1990-12-07 0.810   0.792 -0.022472856
```

```
tail(oil.full)
```

```
##            Date  Spot Futures        Basis
## 3877 2001-11-21 0.531   0.534  0.005633818
## 3878 2001-11-26 0.509   0.522  0.025219571
## 3879 2001-11-27 0.541   0.539 -0.003703708
## 3880 2001-11-28 0.530   0.531  0.001885015
## 3881 2001-11-29 0.507   0.519  0.023392880
## 3882 2001-11-30 0.531   0.532  0.001881468
```

```
nrow(oil.full)
```

```
## [1] 2756
```

Okay. Down to business.

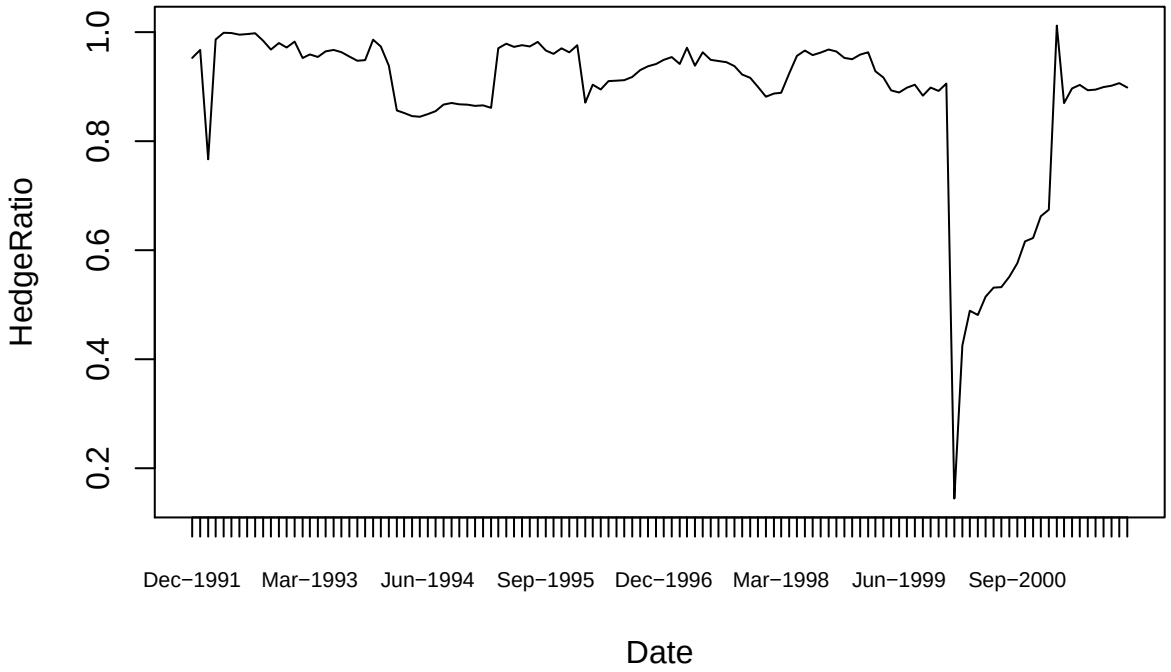
```
dates <- seq(from=endDate, to=lastDate, by="month")
nper <- length(dates)

indBeg <- begDate
indEnd <- endDate
hr <- rep(0, nper)

for(i in 1:nper)
{
```

```
  ind <- (oil.full$Date >= indBeg & oil.full$Date <= indEnd)
  oil.tmp <- oil.full[ind, ]
  hr[i] <- minvarHedgeRatio(oil.tmp)
  indBeg <- addMonth(indBeg, 1)
  indEnd <- addMonth(indEnd, 1)
}
summary(hr)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.1445  0.8830  0.9293  0.8864  0.9636  1.0120
```

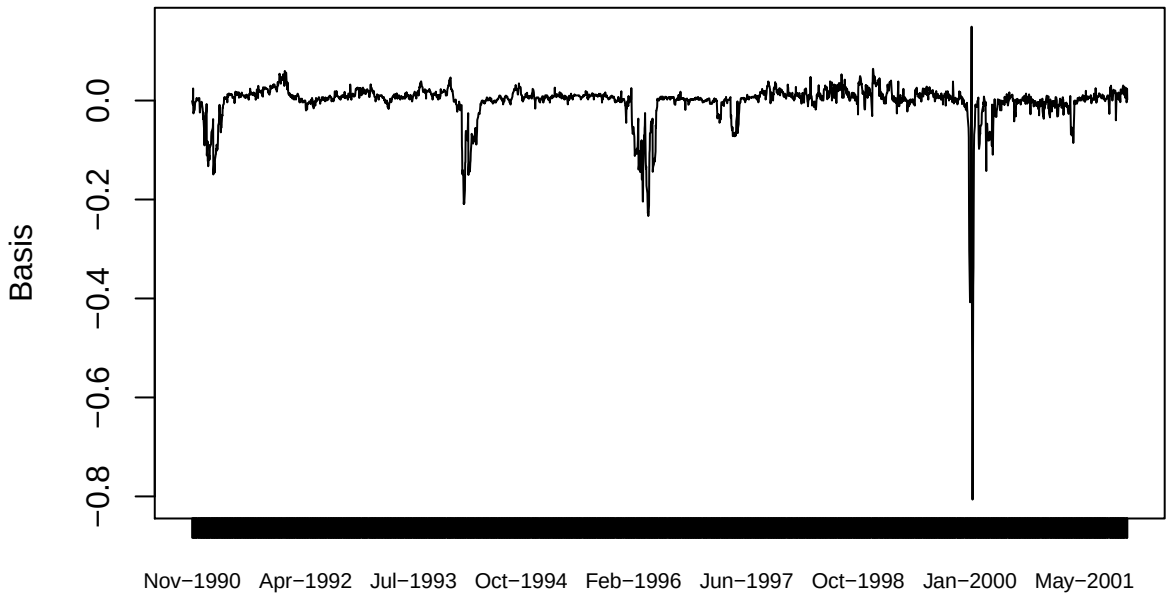
Let's plot the time series of hedge ratios.

```
dm <- data.frame(Date=dates, HedgeRatio=hr)
plot(HedgeRatio ~ Date, dm, xaxt="n", type="l")
axis(1, dm$Date, format(dm$Date, "%b-%Y"), cex.axis = .7)
```



Plot the time series of basis

```
plot(Basis ~ Date, oil.full, xaxt="n", type="l")
axis(1, oil.full$Date, format(oil.full$Date, "%b-%Y"), cex.axis = .7)
```



Something weird is happening between Jun-1999 and Sep-2000. Let's check it out.

Note: turns out there were extremely cold temperatures in the north east and there was a resulting heating oil shortfall during Jan - Feb, 2000.

```
beg <- as.Date("2000-01-01", "%Y-%m-%d")
end <- as.Date("2000-03-01", "%Y-%m-%d")
ind <- (oil.full$Date >= beg & oil.full$Date <= end)
oil.eh <- oil.full[ind, ]
oil.eh
```

```
##            Date  Spot Futures        Basis
## 3406 2000-01-04 0.687   0.678 -0.013187004
## 3407 2000-01-05 0.671   0.666 -0.007479466
## 3408 2000-01-06 0.675   0.663 -0.017937701
## 3409 2000-01-07 0.660   0.648 -0.018349139
## 3410 2000-01-10 0.660   0.647 -0.019893541
## 3411 2000-01-11 0.671   0.668 -0.004480963
## 3412 2000-01-12 0.697   0.685 -0.017366572
## 3413 2000-01-13 0.709   0.693 -0.022825527
## 3414 2000-01-14 0.762   0.738 -0.032002731
## 3415 2000-01-18 0.814   0.770 -0.055569851
## 3416 2000-01-19 0.862   0.800 -0.074643543
## 3417 2000-01-20 1.080   0.865 -0.221986813
## 3418 2000-01-21 1.265   0.935 -0.302280872
## 3419 2000-01-24 1.288   0.864 -0.399273138
## 3420 2000-01-25 1.359   0.904 -0.407675054
## 3421 2000-01-26 1.308   0.921 -0.350794496
## 3422 2000-01-27 1.212   0.912 -0.284387177
## 3423 2000-01-28 1.185   0.925 -0.247704316
## 3424 2000-01-31 0.820   0.952  0.149260695
## 3425 2000-02-01 1.030   0.772 -0.288329531
## 3426 2000-02-02 1.231   0.755 -0.488864377
## 3427 2000-02-03 1.403   0.779 -0.588357034
```

```
## 3428 2000-02-04 1.765   0.788 -0.806407880
## 3429 2000-02-07 1.103   0.758 -0.375105634
## 3430 2000-02-08 1.020   0.728 -0.337256858
## 3431 2000-02-09 0.926   0.746 -0.216148634
## 3432 2000-02-10 0.825   0.745 -0.101999168
## 3433 2000-02-11 0.795   0.742 -0.068992871
## 3434 2000-02-14 0.797   0.757 -0.051491425
## 3435 2000-02-15 0.773   0.752 -0.027542725
## 3436 2000-02-16 0.773   0.762 -0.014332493
## 3437 2000-02-17 0.757   0.751 -0.007957602
## 3438 2000-02-18 0.765   0.755 -0.013158085
## 3439 2000-02-22 0.757   0.749 -0.010624270
## 3440 2000-02-23 0.771   0.761 -0.013055016
## 3441 2000-02-24 0.789   0.780 -0.011472401
## 3442 2000-02-25 0.830   0.829 -0.001205546
## 3443 2000-02-28 0.816   0.817  0.001224740
## 3444 2000-02-29 0.820   0.825  0.006079046
## 3445 2000-03-01 0.849   0.797 -0.063204508
```