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Statistical Control of Multiple-Stream Processes: A Shewhart Control Chart for Each Stream

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ABSTRACT Shewhart control charts for multiple stream processes (MSP) are studied in this article, including control limits, correlation influence, and average run lengths. Graphs are presented to aid in the design of control charts. Subgroups of one unit per stream or partial samples were taken. An example based on industry data is presented and analyzed to show the application and a comparison with other well-known methodologies for statistical control of MSP.

KEYWORDS ARL, correlation, multiple stream processes, partial sampling, Shewhart control charts

INTRODUCTION

In industry or service activities, the occurrence of multiple stream processes is rather common. A multiple stream process (MSP) is a process with several individual sources or streams, each one producing nominally identical units of product (Montgomery, 2001). MSP can be found in many industrial activities, including the production of food, plastics, cosmetics and pharmaceuticals, etc. Service activities involving MSP are also frequent (e.g., linked cashiers at a supermarket, etc.)

Control charts are useful tools for MSP statistical control, and have been thoroughly discussed by Montgomery (2001). The three approaches mentioned are separate control charts for each stream reduced to a single chart when streams are highly correlated. Group control charts based on the detection of recurring extreme values (smallest, largest) are proposed by Boyd (1950) and monitoring of streams range and overall mean to detect shifts in individual means or in all streams are proposed by Mortell and Runger (1995).

When applying these methodologies, some practical problems arise: sampling, number of determinations and centering of streams. Sampling may become more difficult as stream number increases. High speed and sample identification are factors that contribute to difficulty in sampling. The number of products to be tested increase with the increase in the number of streams. Though streams are nominally centered to the same

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value, there are often small differences in the mean of products produced by each stream. It may be difficult to eliminate the differences.

Several theoretical and practical advantages and disadvantages are:

One Chart for Each Stream

One chart for each stream has the obvious disadvantage, as the number of streams increase more charts are required. This complexity, however, has been eased by current computing resources. In practice, sampling and identification of samples are often bigger problems than running several charts. If cross correlation is not taken into account, the rate of false alarms (type I error) increases. Control chart point patterns give insight in process performance when using a chart for each stream, because it uses all sample information available. Differences in centering streams can be easily accounted for.

Group Chart

ARL can take only some values, which in some cases are widely separated. Process insight is poor, and differences in stream means are not easy to account for. Detection capability for those shifts may decrease if more than one stream changes at the same time.

Mortell and Runger Method

Insight into the process is poor, and once an out-of-control signal is detected, a new data analysis could be needed for identification of stream/s with a special cause. Differences in stream means are not easy to account for. High capability to detect special causes in one stream is obtained when there exists a high correlation among streams.

The purpose of this article is to provide useful information for the statistical design of a system using one chart for each stream. This study will discuss control limits calculation for a type I risk of 0.0027 for different levels of cross correlation and different number of streams, ARL calculation for a shift in a single stream or in all streams and the effects of partial sampling on control limits and ARL.

MULTIPLE STREAM PROCESS MODEL

The following MSP model based on the model proposed by Mortell and Runger (1995) was used:

$$y_n = \mu + N_i(\mu_i, \sigma_w^2) + N_j(\mu_j, \sigma_B^2), \tag{1}$$

with

$$n = i + js, \tag{2}$$

where:

μ is the overall mean, y_n is the nth measurement, i identifies the stream, j identifies the time when one product of each stream is obtained simultaneously or "turn" when one product of each stream is obtained consecutively. Examples are simultaneous output of multiple cavity molds and tabletting in a rotary press machine, respectively.

s is the total number of streams.

$N_i(\mu_i, \sigma_w^2)$ is a random variable, which is normally distributed, with mean μ_i and standard deviation σ_w. The value of this variable changes when i changes (in each product). When the process is in the in-control condition, μ_i is taken as zero.

$N_i(\mu_j, \sigma_B^2)$ is a random variable, which is distributed normally, with mean μ_j and standard deviation σ_B. The value of this variable changes when j changes (in each turn). When the process is in the in-control condition, μ_j is taken as zero.

Frequently, in MSP, it is neither possible nor economically feasible to set each stream mean to exactly the same value. Further on in this article, an example with real data taken from the pharmaceutical industry will be shown to illustrate this situation. In these cases the model previously presented in Eq. (1) should be replaced by:

$$y_n = \mu + \Delta\mu_i + N_i(\mu_i, \sigma_w^2) + N_j(\mu_j, \sigma_B^2) \tag{3}$$

The meaning of the symbols remains as previously explained, while the new term, $\Delta\mu_i$ describes the difference between the mean of stream i and the overall in-control mean. Finally, the sum of these terms is zero.

$$\sum_{i=1}^{i=s} \Delta\mu_i = 0 \tag{4}$$

The $\Delta\mu_i$ term affects the statistical properties of methods used for controlling multiple streams processes in different ways in the three methods.

One Control Chart for Each Stream

Errors due to different average values for each stream may be avoided using the average value of each stream for the calculation of control limits. In the case where charts are not built or only one chart is used for all streams, a difference control variable may be used to remove the influence of centering differences:

$$z_n = y_n - (\mu + \Delta\mu_i) \tag{5}$$

Group Method

In statistical control, if a stream is set at the highest or lowest value, the probability of obtaining a product with the highest or lowest value will increase and type I error probability will also increase. An example is presented further on.

Mortell and Runger Method

In statistical control, if a stream is set at a higher or lower value than the others, due to differences in centering, the average control chart will not be affected. On the other hand, the range among streams increases its value because there are streams centered at values higher and lower than the average. This increases type I error probability for the range control chart, and there are more false alarms. An example of this will also be presented later on.

CROSS CORRELATION IN MSP

The cross correlation between any pair of streams is given by:

$$\rho = \frac{\sigma_B^2}{\sigma_B^2 + \sigma_w^2} \tag{6}$$

Additionally, we have kept total variability equal to 1 for all our simulations:

$$\sigma_T^2 = \sigma_B^2 + \sigma_w^2 = 1 \tag{7}$$

Parameter ρ characterizes the MSP and determines its statistical behavior.

In manufacturing and service activities, a broad diversity of MSP can be found with low cross correlation (ρ near zero), high cross correlation (ρ near 1) and intermediate cross correlation. In this article, all these situations have been considered.

Two types of special causes are considered. First, changes affecting all streams were modeled as a sustained shift in the process mean that changes μ_j from zero to a new value (Δ). Examples of these situations may be the change in the viscosity of a liquid in a filling MSP, or a change in the moisture of a granulated in a tabletting MSP.

In the second, changes affecting one stream were modeled as a sustained shift in the i stream mean that changes μ_i from zero to a new value (δ). An example of this situation may be when a punch becomes entangled along a tabletting MSP.

When Shewhart control charts are used for each stream, the probability of a type I (α) error for the set of all charts usually increases. Consequently, the probability of false alarm increases. If a situation with zero correlation ($\rho = 0$) is considered, each stream behaves in an independent way and the probability of a false alarm will be roughly:

$$p = 1 - (1 - \alpha)^s \approx 1 - 1 + s\alpha = s\alpha \tag{8}$$

If type I error probability for each stream is 0.0027 ($\alpha = 0.0027$), calculated risks are shown in Table 1. Probability of type I error for the MSP may reach unacceptable levels, so a change in control limits is needed to keep such type I risks low.

On the other hand, when streams are highly correlated (ρ approaches 1), the variability is produced between turns ($\sigma_B = 1$), and the output of all streams from a turn is quite similar ($\sigma_W = 0$). The risk of type I error for the MSP is the same as that chosen for one stream.

When observed correlation shows an intermediate value, the probability of type I error for the MSP will be between 0.0027 and the values presented in Figure 1, if α was defined at 0.0027 for each stream.

TABLE 1 Probability of Type I Error when $\rho = 0$

Number of streams s	Probability of type I error[a] α
1	0.0027
2	0.0054
5	0.0134
10	0.0267
20	0.0526
50	0.1264
100	0.2369

[a] For the set of MSP Shewhart control charts (one for each stream), when $\pm 3\sigma_T$ control limits for each stream are used, and there is no correlation between them.

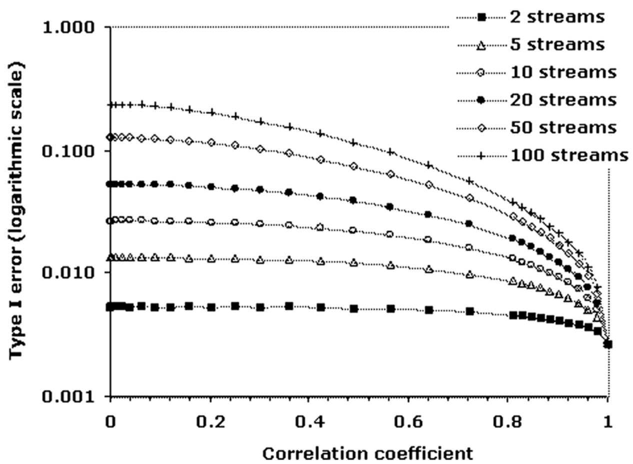

FIGURE 1 Type I error probability (α) as a function of correlation coefficient for different number of streams.

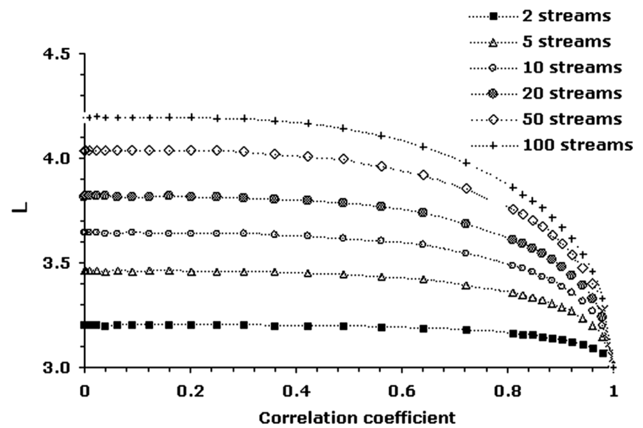

FIGURE 2 Control limits half width in σ_T units (L) as a function of correlation coefficient for different number of streams in order to keep type I error probability at 0.0027.

Then, to keep the probability of false alarms within the desired values for an MSP, for example, 0.0027, it will be necessary to widen control limits, except when ρ approaches 1. This change will alter the probability of detecting special causes.

The risk of a type II error for the MSP (β) depends on type and magnitude of special causes, the number of streams and the correlation coefficient, ρ.

METHODOLOGY

Statistical properties of control charts were studied using the Monte Carlo simulation method. The software used was validated by comparison with numerical calculations on multivariate normal distributions (with two and three streams) (Montgomery & Runger, 1996; MAS 368, 2006).

Enough simulations were run in order to ensure 1% confidence intervals with a confidence level of 95%.

TYPE I ERROR PROBABILITY

Probabilities were obtained through Monte Carlo simulations carried out in conditions I (Table 2).

Probability of false alarms was studied as a function of the number of streams and the correlation coefficients. The results obtained are shown in Figure 1.

It can be noted that when there is total correlation ($\rho = 1$), all curves intercept at $\alpha = 0.0027$ and when correlation approaches zero, type I error probability is proportional to the number of streams.

CONTROL LIMITS

To keep type I error probability in the desired value ($\alpha = 0.0027$), it is necessary to change control limits. Control limits are defined as:

$$UCL_i = \mu + L\sigma_T \tag{9}$$

$$CL_i = \mu \tag{10}$$

$$LCL_i = \mu - L\sigma_T \tag{11}$$

Corresponding L values for different correlation degrees are presented in Figure 2 to keep the probability of type I error equal to 0.0027, as a function of the number of streams and the correlation coefficient, ρ.

TABLE 2

Simulation conditions	I	II	III
Process was modeled by	Eq. (1)	Eq. (1)	Eq. (1)
Total variability	$\sigma_T = 1$	$\sigma_T = 1$	$\sigma_T = 1$
Special cause applied	No, $\mu_i = 0$; $\mu_j = 0$	Yes, $\mu_i = 0$, $\mu_j = \Delta$	Yes, $\mu_i = \delta$, $\mu_j = 0$
Control charts limits	$\pm 3\sigma_T$	$\pm L\sigma_T$, L from Table 3	$\pm L\sigma_T$, L from Table 3
Subgroup size	1	1	1
Shewhart control chart	1 chart/stream	1 chart/stream	1 chart/stream

Process is out of control when one or more charts showed a point outside control limits.

TABLE 3

Number of streams s	Control chart half width[b] L
1	3.00
2	3.20
5	3.46
10	3.64
20	3.82
50	4.04
100	4.20

[b] In σ_T units, calculated limits for MSP Shewhart control charts (one for each stream), in order to maintain type I error probability at 0.0027, when $\rho = 0$.

New control limit values were obtained by simulation in the same conditions mentioned before, except that control limits were changed until type I error was equal to 0.0027. In Figure 2, the graph shows that when ρ approaches 1, all curves approximate to $L = 3$. For a low correlation coefficient (up to 0.5), the values of control limits are practically constant for each stream value and every value is similar to that calculated (Table 3) for independent streams if $\alpha = 0.0027$.

AN EXAMPLE FROM INDUSTRY

Data were obtained from the production process of a pharmaceutical tablet using a rotary machine with 14 punches, and 53 samples were obtained every 15 min, each sample consisting of one unit from each stream. The process was set to produce a 0.700 g tablet. The process ran without disruptions throughout production. From the collected data (phase I of this study), process parameters were calculated and they are shown in Table 4.

Figures 3–5, show the design of phase I studies, meriting the following comments:

TABLE 4 **Estimated Parameters of the Compression MSP with 14 Streams[a]**

Total average	0,705 g
Maximum average of all streams[b]	0,718 g
Minimum average of all each streams[c]	0,693 g
Standard deviation over all streams (s_B)	0,0060 g
Standard deviation for each stream (s_w)	0,0097 g
Total standard deviation (s_T)	0,0115 g
Correlation coefficient	0,2782

[a] The estimated values were calculated from 53 units for each stream. The machine was set to a target value of 0.700 g.
[b] Stream #13.
[c] Stream #5.

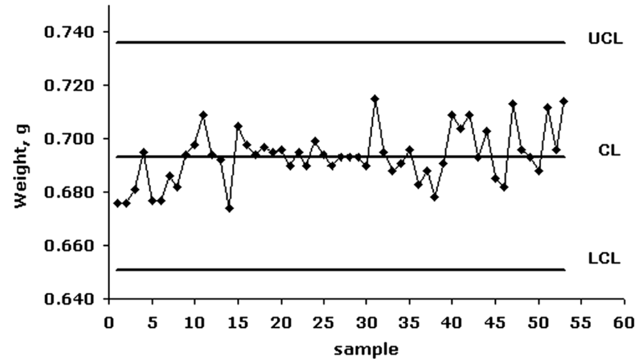

FIGURE 3 **Industry example: Control chart of punch #5 showing statistical control (control limits set to L value = 3.72).**

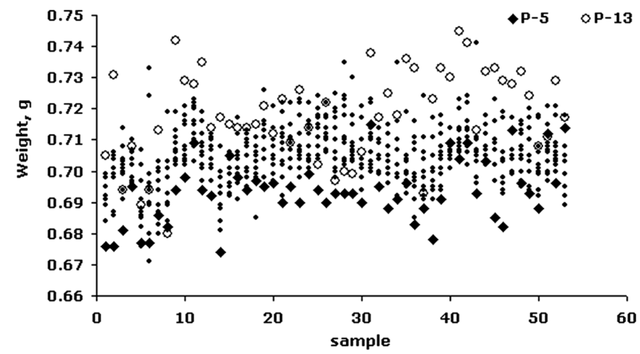

FIGURE 4 **Industry example: Graph for group method showing out-of-control signals caused by punches #5 and #13.**

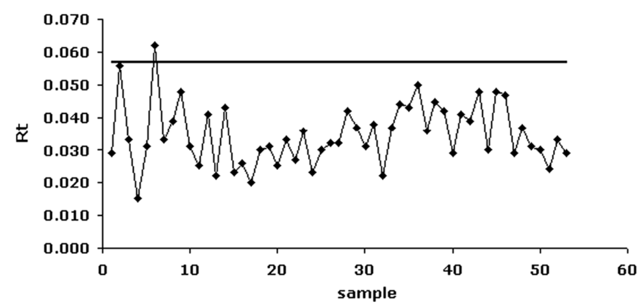

FIGURE 5 **Industry example: *Rt* chart (Mortell and Runger) showing out-of-control signal in sample 6.**

In-Control (Only Common Causes Present)

One Control Chart for Each Stream

Control limits were calculated (Figure 2). A value of $L = 3.72$ was obtained and the following equations were applied:

$$UCL_i = \mu + \Delta\mu_i + L\sigma_T \tag{12}$$

$$CL_i = \mu + \Delta\mu_i \tag{13}$$

$$LCL_i = \mu + \Delta\mu_i - L\sigma_T \tag{14}$$

No point outside control limits was found. Control chart for punch #5 is shown as an example in Figure 3.

Group Method

If r is the number of consecutive times that a particular stream becomes the largest or smallest value, the one-sided in-control ARL_o is taken from Nelson (1986) as:

$$ARL_o = \frac{(s^r - 1)}{(s - 1)} \tag{15}$$

The calculated ARL_o values for 14 streams are shown in Table 5. The $r = 3$ value was selected to give an ARL_o roughly consistent with $\alpha = 0.0027$. For the given industrial example (14 streams), more than ten out-of-control signals were obtained from the 53 samples analyzed. Figure 4 shows all out-of-control signals for punches 5 or 13 with extreme means (punches #5 and #13 are shown with larger symbols).

Mortell and Runger Method

A range (Rt) chart was built with the 53 samples analyzed. An out-of-control point was observed in point 6, despite the fact that the process is in statistical control and a false alarm appears. The out-of-control signal is related to the difference between a high mean punch and a low mean punch (Figure 5).

Out-of-Control (Special Causes Present)

Using parameters calculated during phase I, additional data were simulated in phase II in which two other conditions occurred.

First, a change of $-2\,\sigma_T$ in the mean is applied to punch #1 (e.g., due to loss of volume caused by some material stuck on the faces of the punch #1). Shift begins in sample 54, which is the first simulated sample. Figure 6 shows the first out-of-control point in sample 63 in the control chart.

TABLE 5 **ARLo Calculated for Group Method with Nelson Formula**

Consecutive extreme sample, r	2	3	4	5
ARLo values for 14 streams	15	211	2955	41371

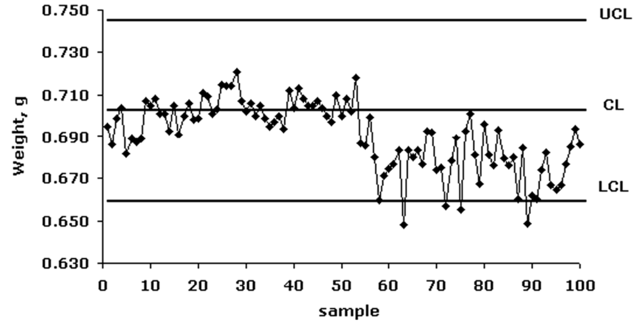

FIGURE 6 **Industry example: Control chart of punch #1, with additional simulated data, showing out-of-control condition caused by a shift of $-2\sigma_T$ in the mean applied to punch #1 (control limits with L value = 3.72).**

For comparison, Figure 7 shows behavior of products from punch #2; no out-of-control points are observed in the control chart. The charts for all other punches showed statistical control.

In the second condition, a change of $2\sigma_T$ in the mean was applied to all streams (e.g., caused by a change in the particle size of the powder). Change begins in sample 54, which is the first simulated sample. The first out-of-control point appears in sample 58 in the control chart. Figure 8 shows the response of one control chart (punch #1). Similarly, out-of-control signals appear in all 14 charts.

This example shows that in comparison, the performance of the three methods shown for a MSP in-control condition, the influence of centering differences, which causes an increase in false alarm rates for Group and Mortell and Runger methods, does not affect individual charts. The response to special causes, as individual charts provide insight into the process and different types of special causes (either in one or all streams) are clearly

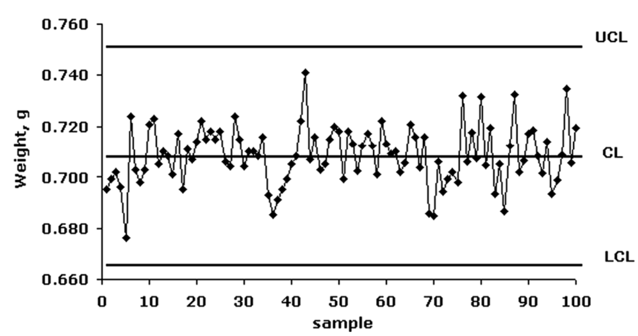

FIGURE 7 **Industry example: Control chart of punch #2, with additional simulated data, showing in-control condition of punch #2, (a shift of $-2\sigma_T$ in the mean that is applied only to punch #1, control limits with L value = 3.72).**

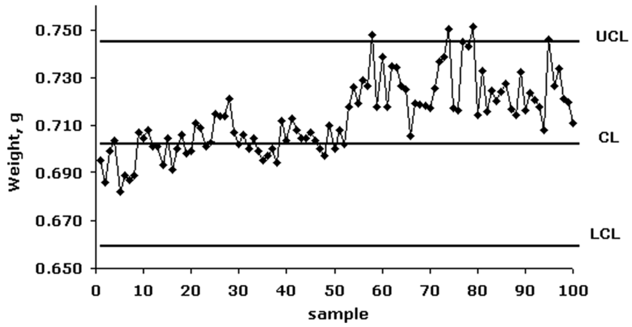

FIGURE 8 **Industry example: Control chart of punch #1, with additional simulated data, showing out-of-control condition of punch #1, caused by a shift of $+2\sigma_T$ that is applied to all punches (control limits with L value $= 3.72$). Out-of-control signals appear in all 14 charts.**

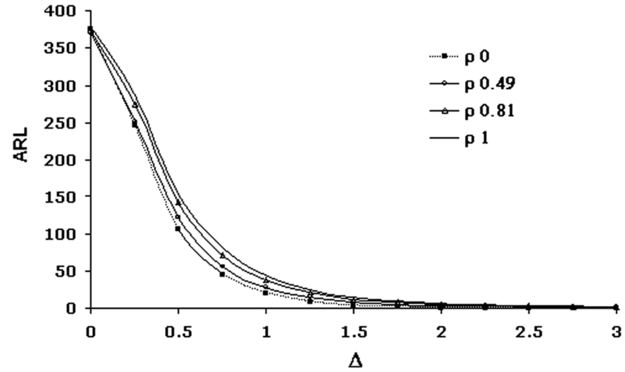

FIGURE 10 **ARL, as a function of μ_j shifts (Δ), for a 20 streams process; a special cause appears in all streams.**

distinguished. Moreover, patterns can provide information about the operation of the process itself. When one control chart for each stream is used, out of control chart(s) identifies stream(s) with special cause.

SPECIAL CAUSE APPLIED TO ALL STREAMS

Special cause detection probabilities were obtained through simulation in conditions II of Table 2; μ_j changes from zero to the value shown in x-axis, Δ. Figure 9 shows the results obtained from simulations for a correlation coefficient of 0.49. The curves of streams 2, 5, 10, 20, 50 and 100 are very close to each other. For the sake of clarity only streams 2, 5 and 100 are displayed.

ARL is defined as the average number of subgroups required for detecting a statistical out of control state. For $\rho < 0.49$, the curves are more separated, while for a larger ρ, ARL curves are closer. Figure 10 shows the results obtained from simulations for 20 streams with different degrees of correlation. It shows that when ρ increases ARL also increases.

The behavior for a significant shift in all streams ($\Delta = 2\sigma_T$) is shown in Figure 11. With such magnitude of special cause, Shewhart control charts are usually preferred.

It can be noted that ARL depends on the number of streams and the correlation coefficient. When there is total correlation of the streams ($\rho = 1$), all curves intercept at the same ARL, as in the traditional Shewhart control chart for a single stream; ARL $= 6.303$ for a shift of 2 standard deviations in the mean. When correlation approaches zero, the ARL decreases because each chart is an independent opportunity for detecting an out-of-control

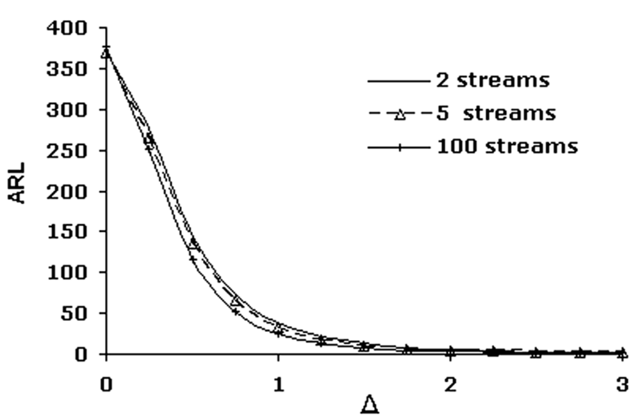

FIGURE 9 **ARL as a function of μ_j shifts (Δ) when the correlation coefficient is 0.49, with special cause in all streams.**

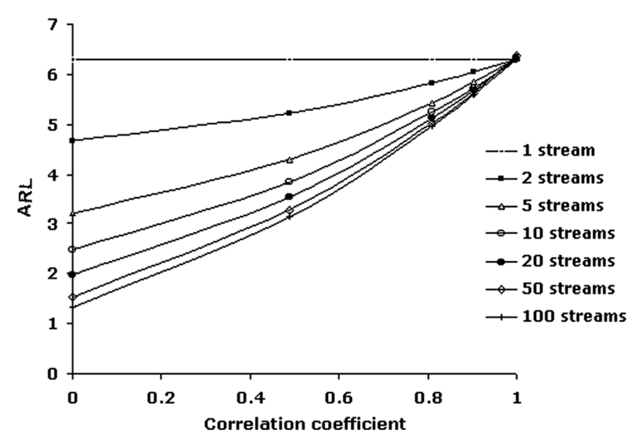

FIGURE 11 **ARL, when a shift in the mean of all streams ($\Delta = 2\sigma_T$) is produced, as a function of correlation coefficient for different number of streams.**

condition. ARL is always lower than or equal to the traditional Shewhart control chart for a single stream.

SPECIAL CAUSE APPLIED TO A SINGLE STREAM

Special cause detection probabilities were obtained through simulation in conditions III of Table 2; μ_i changes from zero to the value shown in x-axis, δ. The results obtained from simulations for a correlation coefficient of 0.49 are shown in Figure 12. Again, ARL refers to the number of turns needed to detect shifts.

Figure 13 shows the results obtained from simulations for 20 streams with different degrees of correlation. It shows that when ρ increases, ARL decreases.

The behavior for a significant shift in one stream ($\delta = 2\sigma_T$) where Shewhart control charts are usually preferred is shown in Figure 14.

It can be noted that ARL depends on the number of streams and the correlation coefficient. When there is total correlation of the streams ($\rho = 1$), all curves intercept at the same ARL, as in the traditional Shewhart control chart for a single stream (ARL = 6.303 for a shift of 2 standard deviations in the mean). For other values of ρ, the ARL are higher than those corresponding to a process with only one stream. For values of ρ larger than 0.5, the ARL strongly depends on ρ. ARL is higher than or equal to the traditional Shewhart control chart for a single stream, unlike what occurs when the special cause is applied to all streams.

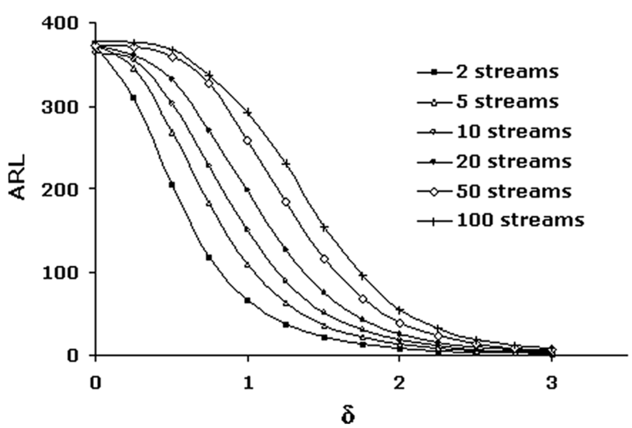

FIGURE 12 ARL, as a function of μ_i shift (δ), when the correlation coefficient is 0.49; a special cause appears in one stream.

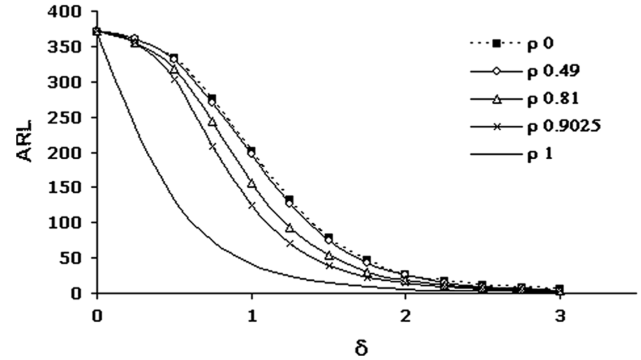

FIGURE 13 ARL as a function of μ_i shift (δ), for a 20 streams process; a special cause appears in one stream.

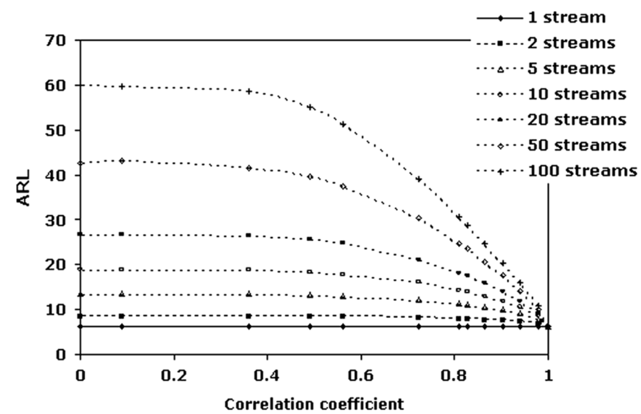

FIGURE 14 ARL, when a shift in the mean of one stream ($\delta = 2\sigma_T$) is produced, as a function of correlation coefficient for different number of streams.

CASES IN WHICH A PARTIAL RANDOM SAMPLE OF THE STREAMS IS TAKEN

When it is not possible to take samples from all streams, random partial samples may be taken. Statistical properties of these control charts have been studied by simulation in the same condition as before, except the subgroup size that was set to a random sample of size m (smaller than the number of streams, s). In these conditions, the simulations yielded the following results:

Type I Error and Control Limits

Type I error probability depends on the sample size. For example, if the MSP has 20 streams (s)

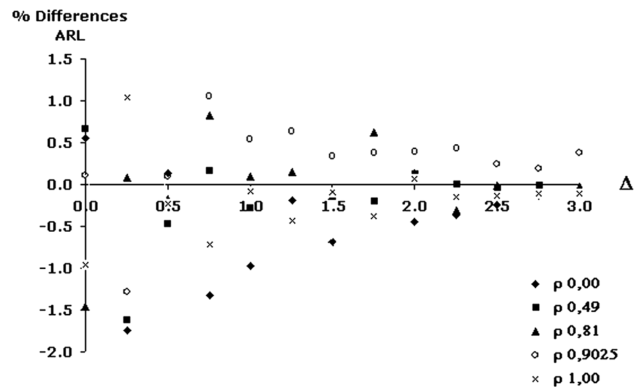

FIGURE 15 Differences [ARL 20 streams ($m = 10$) – ARL 10 streams ($n = 10$)]% as a function of μ_j shifts (Δ) for different correlation coefficients.

and the subgroup has 5 units (m) (units taken randomly from 5 streams of a turn), type I error probability and the control limits can be obtained from control charts for 5 streams (Figures 1 and 2).

Special Cause Applied to all Streams

When a special cause is applied to all streams, ARL is the same as the corresponding ARL for an MSP with a number of streams equal to the number of units of the subgroup (m):

$$ARL = ARL_m \tag{16}$$

Figure 15 shows the percentage differences between the ARL of an MSP with 20 streams and random partial samples (10 units) and the ARL of an MSP with 10 streams and random samples on all streams (10 units). Differences are lower than 2% throughout the studied range.

Special Cause Applied to One Stream

When a special cause is applied to a single stream in an MSP with s streams and random partial samples (m units) are taken on a turn of the process, then the ARL can be calculated as the product:

$$ARL = ARL_m.\frac{s}{m} \tag{17}$$

where s is the number of streams of the MSP and m is the number of the units that form the subgroup. If the MSP has 20 streams and the subgroup has 5 units, then the $ARL_{20} = 4 \cdot ARL_5$. The last two statements are applicable when the shifts are significant ($2\sigma_T$ or larger).

CONCLUSIONS

When designing a set of Shewhart control charts for a multiple stream process, the increase of type I error probability caused by the correlation among streams must be taken into consideration. The magnitudes of these increased values are shown in Figure 1.

To keep the risk of false alarms within usual values (0.0027), the control limits must be set taking into account the number of streams and the correlation coefficient among streams. The appropriate values are presented in Figure 2.

If the control limits are established following the preceding criteria, the resulting control charts exhibit different characteristic operating curves. When a shift of $2\sigma_T$ or greater, occurs in the mean of all the streams, an ARL lower than or equal to the ARL for a similar control chart applied to a single stream process, is produced. When a shift of $2\sigma_T$ or greater, occurs in the mean of a single stream, an ARL higher than the ARL for a similar control chart applied to a single stream process, is produced. When correlation approaches zero, the ARL decreases because each chart is an independent opportunity for detecting an out-of-control condition.

When a partial sampling scheme is used in an MSP and the control limits are defined according to the preceding criteria, the control charts have certain characteristics. Control limits are similar to those calculated for a process with a number of streams equal to the number of units sampled. The probability of detecting a shift of $2\sigma_T$ or greater in all streams is equal to the probability for a process with a number of streams equal to the number of units sampled. A shift of $2\sigma_T$ or greater in the mean of a single stream produces an ARL higher than the ARL for a similar control chart applied to a single stream process. This ARL can be calculated as shown in Eq. (17).

Control charts for each stream, in an out-of-control condition, give more information about special causes. They easily distinguish special causes in one stream from special causes in all streams. When the special cause is in one stream, the stream is identified. Graph patterns may be used for process understanding or for sensitizing using WECO rules.

Differences in centering at mean values of the different streams affect Group and Mortell-Runger methods, thereby increasing false alarm rates.

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