# An Introduction to Nonparametric Statistics

Dr Austin R Brown

Kennesaw State University

# Introduction

- Suppose I hypothesize that the mean weight of high school football players in Georgia is greater than 163 pounds.
- To test this hypothesis, suppose I collect a random sample of football players and find their mean weight and their sample standard deviation.
- Generally speaking, we learned in our introductory statistics courses that we would use a one-population $t$-test to test this hypothesis.

# Introduction

- However, one thing that we did not discuss in our introductory statistics courses is that the $t$-test assumes that the data is normally distributed or that the sample size is large enough for the Central Limit Theorem to apply.

- What if neither of these assumptions hold? Can we just ignore them and move on?

- Let's see why that might not be a good idea.

# Introduction

- Remember from earlier in the semester that the main way we compare competing process control charts is by calculating ARL, which is a function of statistical power.
- In words, statistical power is the probability of a hypothesis test (which a control chart is just an application of hypothesis tests) rejecting the null hypothesis for a given (read, specific) alternative hypothesis.

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- So for a given test statistic, $T$, and a given significance level, $\alpha$, the power of a hypothesis test is given by:

$$\text{Power} = P(T > t_\alpha | H_1 \text{is true})$$

- where $t_\alpha$ is the critical value of the test statistic for a given significance level, $\alpha$.

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- Let's do a little simulation to estimate the power of a one-population $t$-test when the data is and is not normally distributed.

# Introduction

- As we can see, the power of the $t$-test can be severely impacted when the data is not normally distributed.
- But now we run into a new problem! If the data are non-normally distributed or if I am unable to reasonably assume that the data is normally distributed, how do I perform hypothesis tests?
- *Nonparametric Statistics* to the rescue!

# Alternatives to One-Population $t$-Test

- One alternative nonparametric test to the one-population $t$-test is called the *Sign Test*.

- First, the main difference with most every nonparametric method is that rather than testing the mean, we test the median.

$$H_0 : \tilde{\mu}_0 = C \quad \text{vs.} \quad H_1 : \tilde{\mu}_0 \neq C$$

# Alternatives to One-Population $t$-Test

- Suppose we take a random sample of size $n$. The test statistic for the Sign Test is given by:

$$T = \sum_{i=1}^{n} I(X_i > C)$$

- where:

$$I(X_i > C) = \begin{cases} 1 & \text{if} X_i > C \\ 0 & \text{if} X_i \leq C \end{cases}$$

# Alternatives to One-Population $t$-Test

- If $H_0$ is true and $\tilde{\mu} = C$, then we would expect $T$ to be approximately $n/2$.
  - Half of the observations greater than $C$ and half less than $C$.
- This suggests that if $H_0$ is true, then:

$$T \sim \text{Binomial}(n, 0.5)$$

# Alternatives to One-Population $t$-Test

- To calculate the P-Value for the Sign Test, we would calculate:

$$P = P(|T \geq t_\alpha||H_0 \text{is true})$$

# Alternatives to One-Population $t$-Test

- Let's look at an example with the Sign Test, as well as its power, using R.

# Alternatives to One-Population $t$-Test

- While the Sign Test is a good alternative to the one-population $t$-test, it is not the only alternative.
- Another (more famous and widely-used) alternative is the *Wilcoxon Signed-Rank Test*.
- Like the Sign Test, the WSRT tests the median of a population.

# Alternatives to One-Population $t$-Test

- To calculate the test statistic for the WSRT, we would:
  1. Rank the absolute values of the differences between the sample values and the hypothesized median.
  2. Sum the ranks of the positive differences.
  3. Calculate the test statistic as the sum of the ranks of the positive differences.
- Unlike the Sign Test, however, the WSRT's null distribution is more complicated and is based on the total possible number of combinations of observed signed ranks.

# Alternatives to One-Population $t$-Test

- What do I mean by this? Let's look at how the null distribution of the WSRT statistic would be calculated and why we usually use an approximation as the sample size increases.