GWAB Method to Hospital-Resident Matching with Couples

Lydia Guarino, Matt Wey, Eric Addison, Ari Bruck

Abstract— **The Hospital-Resident Problem is a ranking and matching problem that is solved every year to match residents and hospitals as best as possible according to their preferences. Although at face value this is a simple problem to solve using Gale-Shapley algorithm, issues have arisen, such as matching resident-couples (partners) to hospitals, that add a non-trivial element to the matching problem. There is currently a method in use[1] that takes couples into account though there are increased burdens placed on the residents using this method. We propose an alternative solution to the Hospital-Resident Problem with Couples that decreases the number of non-matched residents and increases optimality among resident-partners.**

I. Introduction

Part of the training and certification required to become a doctor of medicine (MD) internationally, but especially in the United States, is the completion of a medical residency. These programs are typically available to graduates of medical school that wish to apply to specific hospitals to continue their medical education in a specific field of medicine. The National Resident Matching Program (NRMP), or simply the Match, oversees the matching of these residents to participating hospitals. Applicants rate the hospitals that they prefer from most to least and the hospitals would do the same for the interviewing applicants. The Match would then match an applicant (resident) to a hospital using these preferences as a guide. In the early 1950s, when this program was formally implemented on a national scale, there were concerns of that the matching process favored the hospitals over the applicants. Such bias, in turn, incentivized applicants to misrepresent their preferences. The Gale-Shapley Matching algorithm, published in 1962, showed that in fact there was an applicant-optimal and a hospital-optimal matching, and they are not necessarily the same. After many debates whether this algorithm should be used for the Match, it was adopted in 1997. The Match also allows couples (i.e. romantic, friendships, spouses, etc.) to apply together in hopes of matching at nearby hospitals. The algorithm used by the Match to handle couples has certain drawbacks and issues, and this paper puts forth a new and novel algorithm to handle matching the residents and hospitals while taking couples into account. Specifically, couples may put forth an arbitrary proximity function they wish to be considered during the matchings. We have termed this algorithm the GWAB-HRPP (Guarino-Wey-Addison-Bruck Hospital-Resident-Partner-Problem – named after its creators) algorithm, which is a mutation of the Gale-Shapley matching algorithm used by the Match. While, Marilda Sotomayor in 1983, Alvin Roth in 1984, and Klaus et al. in 2007, found that when couples are allowed to match together, there may exist no stable matching [2], we find that GWAB-HRPP reduces the onus placed on resident-couples attempting to match and is a more flexible algorithm than the one currently employed by the Match.

[1] http://www.nrmp.org/couples-in-the-match/

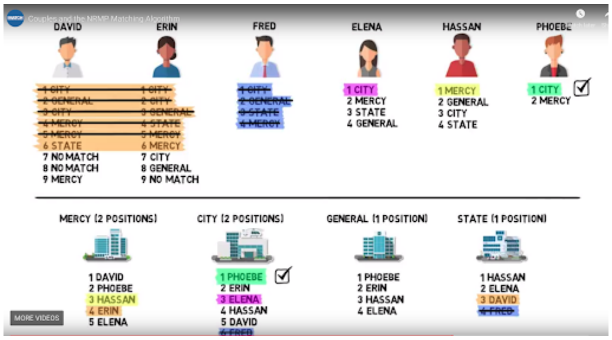

Fig. 1. Example Couples Matching Graphic

II. NRMP: The Match Algorithm

A. Hospital-Resident Problem (HRP)

The Match Algorithm used by the NRMP is shown in Fig. 1 [3]. This algorithm starts with an empty matching and walks through every resident so long as they are unassigned and have not depleted their hospital preference list. For each of the residents fitting this criteria, the algorithm first tentatively matches the resident to the next hospital on the residents preference list. If, due to this matching, the hospital is over its resident capacity, then the hospital removes its worst-resident (not necessarily the current resident under consideration) and the matching reflects this unassignment. As a performance enhancement, if the hospital is full, then all the residents ranked lower than the lowest-assigned resident on the hospitals preference list are removed from the hospitals list as well as the hospital from the residents list. This is because the hospital will not accept anybody worse than the worst resident if the hospital is already full. Our implementation in the code extends this algorithm slightly, by passing a non-empty matching as well as a list of the free residents. For implementation reasons we also have all residents ranked by all hospitals. This is not feasible in reality as a hospital cannot possibly interview all possible candidates.

B. Hospital-Resident Partner Problem (HRPP)

The Match allows couples to apply as a couple, which changes the manner in which the algorithm attempts their match. This is termed the Match with couples in the NRMP.

1) Current HRPP Solution: Couples have the onus of coming up with their preference list together to ensure that they would be matched in programs that fit their goals (ideally this is distance i.e. hospitals in same city). The Match treats a couple as a unit. This means that using the HRP algorithm described above, the first person in a couple is tentatively matched. If they are not able to be matched, then the preference list of the couple is incremented. If they are matched, then the other person in the couple is attempted to be matched. If successful, then the next individual/couple in the matching is considered. However, if the second person in the couple fails to match, the original (first) person that matched is unmatched and their preference list is incremented. Refer to the figure 1 [4]where David and Erin are a couple. The figure shows the final result of the HRPP matching.

2) Shortcomings of Current HRPP Solution: The main shortcoming of the HRPP algorithm is that it places the onus of possible acceptable matchings for that couple on the couple and they must enumerate all acceptable permutations of their preferences. A strategy that the Match offers is for couples to add a NO MATCH preference to one of the members of the couple to increase the chance that they will both match. If they reach the end of their list, then neither will match. This is rather risky for a couple. GWAB HRPP addresses this by supplying a proximity function on top of their respective preference lists such that the enumeration of the couples permutation is not needed.

[2]Klaus B, Klijn F, Mass J.: "Some things couples always wanted to know about stable matchings (but were afraid to ask)". Review of Economic Design 2007; 11:175-184.

[3]https://pdfs.semanticscholar.org/77d9/f84082674888ca90ca662847983381b23338.pdf?_ga=2.216999441.622816506.1541285838-70410261.1541285838

[4]http://www.nrmp.org/couples-match-videos/

```
matching = result of initial HRP execution
unmatchedQ = queue of unmatched residents
violationQ = queue of non-dominant residents with proximity violations
prefProgress = map of indices indication the progress of a resident through its preference list

while violationQ not empty:
    ndResident = next non-dominant resident in violationQ
    unassign ndResident from current match and add to unmatchedQ
    restrict ndResident preferences to hospitals in partner's matched location
    run classical HRP with existing matching
    reset ndResident preferences to previous state

    if ndResident still not matched:
        unassign both ndResident and partner, add to unmatchedQ
        increment the value of prefProgress[partner]
        run classical HRP with existing matching

    update violationQ
    add all unmatched residents to unmatchedQ
    give matched single residents a chance to improve
```

Fig. 2. The GWAB algorithm in pseudo-code.

III. Implementation

This section explores the details of the GWAB HRPP algorithm. The GWAB algorithm allows all residents to participate in the matching by making individual preference lists, but with extra importance given to matching with a hospital in the same location as their partner. Before expanding the details of the GWAB algorithm, a few concepts are needed:

- Each hospital has an associated *Location Id*, used to determine whether coupled residents are close enough together.
- A *Proximity Violation* occurs when a resident R_1 and its partner R_2 have been matched to hospitals in different locations.
- The *Non-Dominant Resident* is the resident in a violating couple who is either not matched, or is matched to a less preferable hospital.
- *Preference Progress* refers to a resident's progress through its preference list. It can be thought of as an index which indicates the head of the working preference list.

A. Details of GWAB Solution

The GWAB algorithm generates an initial `matching` by solving the standard HRP problem with all residents participating. GWAB then proceeds to re-solve smaller instances of the HRP problem, attempting to resolve proximity violations. The subsequent HRP executions are run with the existing matching already known. For a given violating pair, this is done twice. The first attempt is run after (temporarily) restricting the non-dominant resident's preferences such that only hospitals that are location compatible with the partner's match are listed. If this does not succeed, i.e. if the non-dominant resident still does not have a location-compatible match, then both residents in the couple are unassigned from their current hospitals, the preference progress of the dominant resident is incremented, and HRP is run again with both members of the violating couple participating as unmatched residents. Once two attempts have been made to resolve the current proximity violation, it is possible that previously matched residents have been unmatched. All unmatched residents are added back to the unmatched queue to participate in the next execution of HRP. Running HRP could cause new proximity violations if coupled residents were displaced or reassigned, so violations are re-computed. Finally, it is possible that through the process of matching and unmatching of coupled residents due to partner location, a position in a hospital more preferred by a non-coupled resident could open up, or that non-coupled resident could now be more preferred by that hospital than its worst assigned resident, so all single residents are given the chance to unassign and participate in the next execution of HRP as an unmatched resident if they could do better. This process is repeated until there are no more proximity violations. Progress in the algorithm is guaranteed, since after each iteration either a proximity violation has been resolved in the first run of HRP, and the hospitals' matches have increased in preference, or the preference progress of a resident has progressed.

The GWAB algorithm presented in pseudo code in figure 2. Lines `10-25` set up the initial variables, including the initial `matching` generated with HRP. Line `35` polls `violationQ` to get the next non-dominant resident of a violating couple, which is then unassinged from its current assigned hospital and added to the `unmatchedQ` in line `40`. The restriction of the non-dominant resident's preference list to only hospitals that are colocated with the partner's match is done on line `45`. `HRP` is run again, using the existing `matching` and `unmatchedQ`, after which the preferences of the non-dominant resident are restored in line `55`. If the non-dominant resident is still not matched to the same location as its partner, as tested on line `60`, both residents in the violating couple are unassigned and added to `unmatchedQ` on line `65`. The preference progress of the dominant partner is incremented on line `70`, so that it will begin attempting to match further down its preference list in the next run of `HRP` on line `75`. Finally, lines `80-90` perform the loop cleanup of finding new proximity violations, adding any unmatched residents to the `unmatchedQ`, and giving singles a chance to re-enter the `unmatchedQ` if they have the chance to be matched with a preferable hospital who also prefers them.

B. Stability Criteria for HRPP

An extended definition of stability is needed to accommodate the inclusion of couples. We define a stable matching with couples to be a matching such that:

- There are no blocking pairs that consist of single residents. If two coupled residents R_1 and R_2 are matched to different hospitals H_1 and H_2, then H_1 and H_2 are in the same location.
- If two coupled residents R_1 and R_2 are part of blocking pairs (R_1, H_1) and (R_2, H_2), then H_1 and H_2 are in different locations.

C. Differences between GWAB and Current HRPP Solution

The primary qualitative difference between GWAB and the current HRPP solution is in how residents create their preference lists. In the current solution, residents and their couples must (should) work together closely while constructing their lists to make sure that they accept the possible pairs of matches. In GWAB, each resident is free to compile their own preference list, and the algorithm will attempt to match them to the location that maximizes the happiness of one of the residents. In the current solution, residents must use multiple slots on their list to represent different pairings of hospitals in the same location. For example, if two coupled residents wish to be matched with any of three hospitals in the same location, the current solution would require them to enter preference pairs $[(H_1, H_1), (H_1, H_2), (H_1, H_3), (H_2, H_1), ...]$, while the GWAB algorithm only requires that each resident includes $\{H_1, H_2, H_3\}$ on their lists. This is a benefit to residents, both because it simplifies the preference process, and because there is a limit to the number of entries allowed on a preference list.

D. GWAB Repository

All code, input, and output files associated with this project can be found at https://github.com/abruck1/Social_Computing_Project

IV. Evaluation

A. GWAB HRPP Results

1) Test Data Generation: In order to evaluate the effectiveness of the GWAB HRPP algorithm, we first required reasonable test data. To achieve this, we wrote a CSV generator to produce test inputs for hospitals and residents. This allowed us to compare the performance of our algorithm against the current HRPP solution algorithm, using the same input. The test files were produced in the following format, based on input parameters of hospital count, location count, resident count, and number of couples desired for the test case:

Residents:

- resident id: a unique identifier for the resident
- hospital preferences list: a rank-ordered list of hospital ids, randomly generated from the available list of hospitals. For partners, one partner was selected and half their list was modified to match their partners list, then randomly sorted to more closely mimic coordinated preference lists among couples.
- partner id: the unique identifier for the residents partner in a couple, if applicable

Hospitals:

- hospital id: a unique identifier for the hospital
- location id: a unique identifier for the hospitals location, used in our algorithm to determine proximity-based matches.
- capacity: a random value between 1 and 5 representing the open vacancies for residents
- resident preferences list: a rank-ordered list of resident ids, generated as a sublist of all residents that list the hospital as a preferred match. The sublist is then sorted randomly to produce variation in preference order. This was done to mimic the application and interview process of the real matching process, where residents and hospitals can only match with an individual or institution where an interview has taken place.

2) Evaluation Methodology: To evaluate the performance of our algorithm, we produced 5 test cases and ran them against the current HRPP solution algorithm and our GWAB HRPP algorithm, producing CSV exports of the resulting matchings. In addition, we wrote validators to detect capacity, proximity, and stability violations (blocking pairs) in the final matchings. We detail the evaluation metrics and results below.

3) Primary Evaluation Metrics:

Percentage Matched - After the algorithm terminates, we evaluate the percentage of residents that successfully matched with a hospital.

Resident Happiness - For the evaluation of the results we define happiness as the numerical preference rank the resident has assigned to the hospital they match with. If a resident is unassigned, we consider the happiness value to be the highest rank in their list + 1. A smaller happiness value is representative of a better outcome (lower rank = more preferred). So a resident assigned to a hospital in the bottom quartile of their preference list is happier than if they were assigned to a hospital in the top quartile of their preference list.

Hospital Happiness - Similarly, a hospital has a happiness in a particular matching. A hospital is considered to be happier if they are assigned more residents ranked lower in their preference list than if they are assigned residents higher in their preference list. A rank of 0 represents the hospitals first choice.

Percentage of Utilized Hospital Capacity - A hospital must also consider the number of assigned residents relative to its capacity, the closer to full capacity, the happier the hospital is. We evaluate the percentage of filled vacancies for hospitals as defined by assignment count/capacity.

4) Performance Results:

Percentage matched. Of the 5 test cases we evaluated, the least performant case was when there were 50 hospitals in 50 different locations which resulted in approximately an 83% resident match. For this scenario, it is only possible to achieve a maximum percentage matched of 97% as there were only 147 spots available for 150 residents applying. This is not the case in other scenarios but does align with how the actual matching takes place. The remaining cases resulted in 100% residents being matched. In many cases residents were placed in hospitals that had not ranked the resident. We consider this acceptable for two reasons: this replaces the need for the hospital having to rank all possible applicants and it is accounted for in our hospital happiness metrics.

Resident Happiness. The table in Fig. 3 details the results of our resident happiness evaluation, broken down by test case. The overall average happiness value for all residents

Test Case	Res Count	Avg Happiness	No Match Count	1st Choice Count	Percent Unmatched	Percent First Choice
5-2-16-3	16	0.56	0	11	0.00%	68.75%
50-10-100-50	100	2.73	0	48	0.00%	48.00%
50-50-150-20	150	5.33	26	41	17.33%	27.33%
100-10-200-50	200	1.19	0	126	0.00%	63.00%
300-50-500-100	500	1.41	0	327	0.00%	65.40%
Grand Total	**966**	**2.10**	**26**	**553**	**2.69%**	**57.25%**

Fig. 3. GWAB Resident happiness for test cases.

testcase	Res Count	Avg Happiness	No Match Count	First Choice Count	Percent Unmatched	Percent First Choice
5_2_16_3	16	4.00	4	12	25.00%	75.00%
50_10_100_50	100	7.70	26	20	26.00%	20.00%
50_50_150_20	150	6.35	34	38	22.67%	25.33%
100_10_200_50	200	4.79	28	50	14.00%	25.00%
300_50_500_100	500	3.31	46	177	9.20%	35.40%
Grand Total	**966**	**4.55**	**138**	**297**	**14.29%**	**30.75%**

Fig. 4. Current HRPP Resident happiness for test cases.

evaluated in our study for GWAB HRPP was 2.10, indicating that the average resident received a match in their top 4 choices.

Fig. 4 outlines the same results for the current HRPP solution. GWAB performs considerably better on all metrics, with the current HRPP solution obtaining an overall happiness average of 4.55 with considerably higher unmatched rates at 14.29% vs 2.69% for GWAB and a much lower percentage of residents receiving matches with their first choice at 30.75% vs 57.25% for GWAB.

We evaluated these same metrics when broken down by resident applicant type - Dominant (the anchor resident in our proximity calculations, Non-dominant (the location-restricted resident or partners where both partners match with an equally ranked hospital), or Single (residents that have applied without a partner).

As shown in Fig. 5, we find that overall, dominant partners fared the best in our algorithm, with an average happiness value of 0.14 across all test cases, while non-dominant or

Resident Type	Count	Average Happiness	Unassigned Count	First Choice Count	Percent Unassigned	Percent First Choice
Dominant	185	0.14	0	167	0.00%	90.27%
Non-dominant	261	5.72	24	44	9.20%	16.86%
Single	520	0.97	2	342	0.38%	65.77%
Grand Total	**966**	**2.10**	**26**	**553**	**2.69%**	**57.25%**

Fig. 5. GWAB Resident happiness by resident type.

Resident Type	Res count	Avg Happiness	Unassigned Count	First Choice Count	Percent Unassigned	Percent First Choice
Non-dominant	446	7.88	134	66	30.04%	14.80%
Single	520	1.70	4	231	0.77%	44.42%
Grand Total	**966**	**4.55**	**138**	**297**	**14.29%**	**30.75%**

Fig. 6. Current HRPP Resident happiness by resident type.

testcase	Resident Type	Count	Average Happiness	Unassigned Count	First Choice Count
100-10-200-50	Dominant	39	0.03	0	38
	Non-dominant	61	3.33	0	16
	Single	100	0.33	0	72
100-10-200-50 Total		200	1.19	0	126
300-50-500-100	Dominant	90	0.11	0	82
	Non-dominant	110	5.41	0	18
	Single	300	0.33	0	227
300-50-500-100 Total		500	1.41	0	327
5-2-16-3	Dominant	3	0.33	0	2
	Non-dominant	3	2.00	0	0
	Single	10	0.20	0	9
5-2-16-3 Total		16	0.56	0	11
50-10-100-50	Dominant	47	0.13	0	42
	Non-dominant	53	5.04	0	6
50-10-100-50 Total		100	2.73	0	48
50-50-150-20	Dominant	6	1.17	0	3
	Non-dominant	34	12.44	24	4
	Single	110	3.36	2	34
50-50-150-20 Total		150	5.33	26	41
Grand Total		**966**	**2.10**	**26**	**553**

Fig. 7. GWAB Resident happiness by test and resident type.

```
GWAB                                           Current HRPP Solution
100 Hospitals, 10 Locations, 200 Residents, 50 100 Hospitals, 10 Locations, 200 Residents, 50
Couples:                                       Couples:
Average Happiness: 56.002                      Average Happiness: 39.765
Standard Deviation: 35.403                     Standard Deviation: 33.323
average filled percentage: 0.709               average filled percentage: 0.636

300 Hospitals, 50 Locations, 500 Residents, 100 300 Hospitals, 50 Locations, 500 Residents, 100
Couples:                                       Couples:
Average Happiness: 164.543                     Average Happiness: 122.428
Standard Deviation: 111.517                    Standard Deviation: 108.131
average filled percentage: 0.614               average filled percentage: 0.603

50 Hospitals, 50 Locations, 100 Residents, 50  50 Hospitals, 50 Locations, 100 Residents, 50
Couples:                                       Couples:
Average Happiness: 28.156                      Average Happiness: 26.216
Standard Deviation: 13.913                     Standard Deviation: 13.925
average filled percentage: 0.702               average filled percentage: 0.533

50 Hospitals, 10 Locations, 150 Residents, 20  50 Hospitals, 10 Locations, 150 Residents, 20
Couples:                                       Couples:
Average Happiness: 13.888                      Average Happiness: 11.738
Standard Deviation: 13.396                     Standard Deviation: 11.698
average filled percentage: 0.859               average filled percentage: 0.838

5 Hospitals, 2 Locations, 16 Residents, 3      5 Hospitals, 2 Locations, 16 Residents, 3
Couples:                                       Couples:
Average Happiness: 7.037                       Average Happiness: 7.4
Standard Deviation: 2.841                      Standard Deviation: 2.671
average filled percentage: 0.920               average filled percentage: 0.723
```

Fig. 8. Hospital happiness for test cases.

tied partners fared the worst with an average happiness value of 5.72. This statistic is inflated by a number of unmatched residents in one test case which reported an overall happiness average among non-dominant partners of 12.44, though drilling down (as seen in Fig. 7) into individual test cases shows that the general distribution of happiness between dominant and non-dominant partners is fairly consistent with the overall average. Also of note is the fact that dominant partners fare better in our algorithm than single residents, who have an average happiness value of 0.97.

Additionally, we find that couples have much better outcomes with GWAB than with the current HRPP solution. Fig. 6 highlights the drastic difference between happiness rates for couples vs single participants in the current HRPP model. GWAB provides a more balanced result by resident type.

Hospital Happiness. When a hospital is assigned residents in the matching, it receives some number of residents up to its capacity. Each hospital may have a different capacity so in order to compare the hospital happiness we devised the following formula to determine their happiness.

$$happiness = \sum_{n=1}^{assigned} \frac{rank_i}{capacity}$$

Where the rank is how highly the assigned resident was ranked in the hospitals preference list. If the hospital was not at capacity or was assigned someone it did not rank then a penalty of the length of that hospitals preference list was added to the happiness score. With this information we can say that the lower happiness number the hospital receives, the happier the hospital is with the assignment. The results can be seen in Fig. 8.

Although happiness cannot be compared across test cases, it is normalized across hospitals so it is fair to compare a hospitals happiness within a test case.

When comparing the hospital results of GWAB and the current HRPP solution we can see they both exceed each other in different categories. Average filled percentage is higher in all 5 test scenarios for the GWAB method but the current HRPP has a better (lower) happiness level in 4 of the 5 test scenarios.

V. Future Work

We would like to allow residents to define an allowable proximity function that is per couple. This will allow GWAB more flexibility and would allow for matches that would currently result in a proximity violation. For example, a resident r could provide a function such that the couple must be within 75 miles *unless* r gets into their top choice on the east coast and r's partner gets accepted to their top choice on the West Coast. This would allow a minimally coordinated preference list that GWAB allows but still provide the flexibility of the current HRPP solution.

VI. Conclusion

The current HRPP solution the is used by the Match to match hospitals and residents. We have identified shortcomings with the process they use and have proposed a new algorithm, GWAB, to be used in its place. GWAB simplifies the application process for resident couples by requiring less coordination on preferences between the residents in a couple. Although there is decreased hospital happiness with a matching, GWAB provides an increase in resident happiness, residents matched percentage, and hospital fill percentage.