An Integer Programming Model for Branching Cable Layouts in Offshore Wind Farms

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Abstract. An integer programming model for minimizing the cabling costs of offshore wind farms which allows for branching of the cables is developed. Model features include upper bounds on the number of cable branches made at any wind turbine, upper bounds on the cable loads, and exclusion of crossing cable segments. The resulting model resembles the capacitated minimum spanning tree problem, with the addition of degree and planarity constraints. Numerical experiments with realistic wind farm data indicate that the benefit from branching is small when using only one cable type, but is up to 13% if allowing for two different cable types.

Keywords: offshore wind farms, cable routes, integer programming, constrained minimum spanning tree.

1 Introduction

Offshore wind energy is becoming an increasingly more important energy source. Up to now, the main development is taking place in Northern Europe, with 6562 MW out of the global installed capacity of 7045 MW installed in Europe at the end of 2013 [6]. The by far most important countries in this respect are the United Kingdom and Denmark, with PR China, Belgium, Germany, Netherlands and Sweden following. The installed capacity in other countries is negligible.

Starting with a yearly annual installed capacity of only 4 MW in 2000, the industry has been steadily growing to 1567 MW being newly installed in 2013 [4]. With an estimated total installed offshore wind capacity of 23500 MW in 2020, which is almost four times the capacity installed at the end of 2013, the industry is expected to continue its quick growth [5].

During the planning and construction phase of an offshore wind farm, the decision on how to choose the cabling routes has a significant influence on the total cost of the cabling, as the cable as well as the trenching in the seabed cost per meter are considerably higher offshore than onshore. There are usually

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one or more substations, and each turbine has to be connected to at least one of these. However, multiple turbines can be connected in series. The maximum number of turbines on one series circuit is determined by the cable type, and the maximum power which can be transported by it. Usual numbers of turbines per cable are between 4 and 8.

This opens an optimization problem of finding cable routes between turbines and substations with minimum total cable length. If taking into account different cable types with respectively different power capacities, the objective changes to minimizing cable cost.

Optimization of cable routes in offshore wind farms was recently addressed by Bauer and Lysgaard [1], who suggested a model with hop-indexed variables, resembling a planar open vehicle routing problem. Reflecting the fact that cable lines are not allowed to cross each other, planarity constraints apply to the model. For a given cable capacity, as well as fixed turbine and substation locations, the objective of the model in [1] is to find the cable routes of minimum total length.

An important assumption of [1] is that the turbines are connected to substations along paths. That is, with the exceptions of the turbine closest to and most remotely from the substation, all turbines have a direct link with exactly two other turbines. In practice, however, it is in some cases possible to branch the power cables at the turbine locations without significant additional effort or cost, which opens the possibility of a further reduction of the total required cable length. This has been done for example in the Walney 1 offshore wind farm [2], which is located on the Northwestern English coast. The branching option is not captured in [1].

We present a new optimization model incorporating all features of [1]. In addition, our model allows for branching of the power cables at the wind turbine locations. Reflecting practical limitations, our model accepts an upper bound on the number of branches that can be made at any turbine location, which is dependent on the cable setup and connection possibilities at a turbine. We will refer to this bound as the branching capacity. We assume that no additional cost is connected to branching within the branching capacity. Following [1], there is an upper bound, referred to as the cable capacity, on the number of turbines to be connected by one cable (to one substation), and no two cable lines may cross because of the applied cable trenching methods.

The problem, which is presented in detail in section 2, can be defined in terms of a graph where the node set represents turbines and substations, in addition to an imaginary node referred to as the root, representing the electrical grid to which the turbines will supply power. We use the root node for setting the problem into context with existing literature, but not in our model formulation, as we do not optimize the grid connection. The edges represent possible connections between turbines, between turbines and substations, and between substations and the grid. Each node but the root is associated with a point in the plane, and each edge but those incident to the root have a weight equal to the Euclidean distance between its end nodes (the weights of all edges incident to the root are negative with sufficiently large absolute value). In this graph, we are asking for a spanning tree of minimum weight, satisfying the following constraints:

– The number of nodes in each subtree below the root does not exceed the cable capacity,
– The degree of each turbine node is no more than the branching capacity plus one,
– When embedding the edges as straight lines between its end nodes, intersections occur only between edges incident to the same node.

The problem under study combines the features of several well-studied graph optimization problems. In particular, this applies to the *capacitated* minimum spanning tree problem [10], where bounds on the subtree sizes are introduced. In the special case where the bounds are equal for all nodes, which we assume, the problem version is referred to as the *unitary demand* version. The *degree-constrained* minimum spanning tree problem [9] addresses the issue of branching capacities. Adding the degree constraint to the minimum spanning tree problem renders the problem NP-hard [11], at least for branching capacity no larger than 3, proving that also our problem is NP-hard.

The remainder of this text is organized as follows: In the next section, we develop an integer programming model based on a set of variables suggested by Gouveia [8]. After introducing model and notation, we continue in section 3 with a presentation of the numerical results obtained from an implementation of our model. We give the optimal solutions for different cable capacities, and by comparing to results from the model allowing only linear cabling [1], we determine cost savings obtainable by allowing branches.

2 An Integer Programming Model for Minimizing Cable Lengths

In this section, we develop an integer programming model for the cabling problem outlined in Section 1. We start with a description of the model parameters and variables and their meaning in the offshore wind farm context of the problem. We also relate them to their respective equivalents in the capacitated minimum spanning tree problem. After the introduction of the variables we continue using the wind energy context in further discussions.

Consider a graph with nodes set $V = V_c \cup V_d$, where V_c represents a given set of wind turbines, and V_d represents power substations. The edge set $E \subseteq V^2$ of the graph represents the possible connections between a turbine and a substation or another turbine. The corresponding arc set is denoted $A_E = \{(i,j) : \{i,j\} \in E\}$. Each edge and arc has an associated cost $c_{ij} \forall (i,j) \in A_E$. We assume that the edge cost is proportional to the Euclidean distance between the locations of the end nodes of the edge, which implies that the costs are symmetric, i.e., $c_{ij} = c_{ji}$. However, validity of the model below does not depend on this assumption.

The maximum cable capacity $C \in \mathbb{N}^+$ is the maximum number of turbines which can be connected by the chosen cable type. It is dependent on the type of

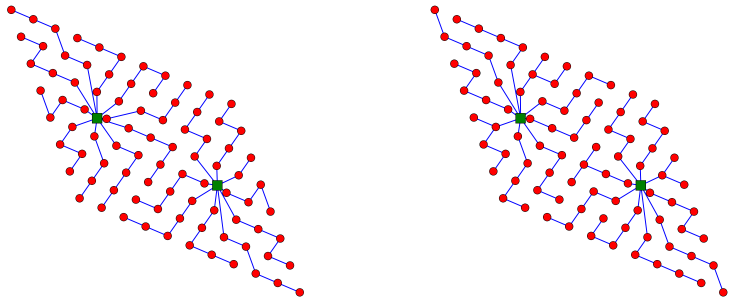

Fig. 1. Sheringham Shoal layout for C=5; left WM1, right WMB1; circles are turbine locations, squares are substation locations

cable and determined by a combination of the power output of a single turbine, the current carrying capacity of the cable, as well as different kinds of power losses which occur in cables.

The maximum number of cable branches at a turbine location, including all incoming and outgoing cables, is defined by the branching capacity $m \in \mathbb{N}^+$.

The set of crossing routes $\chi \subset E^2$ is defined such that $\{\{i,j\},\{u,v\}\} \in \chi$ if edges $\{i,j\}$ and $\{u,v\}$ cross each other.

Gavish [7] suggested a single-commodity flow model for the capacitated minimum spanning tree problem, which later was proved to be equally strong as the model by Gouveia [8]. It can be argued that the latter model is more appealing since the number of constraints is smaller, and that it exclusively has binary variables. It is proved [8] that by use of binary variables indexed by arcs $(i,j) \in A_E$ and feasible subtree sizes $h = 1,\ldots,C$, no continuous variables are needed in the formulation. Following the idea of [8], the integer programming model below contains only binary decision variables.

As our main focus is to analyze what cost reductions can be obtained when branched cable routes are allowed, rather than finding the strongest possible model formulation, we do not incorporate valid inequalities such as those proposed in [8]. For the same reason, we neither make any attempt to integrate in our model any of the more recent contributions to strong formulations for the capacitated minimum spanning tree problem, which have emerged since [8]. Interested readers are referred to e.g. the thesis by Ruiz y Ruiz [12].

Define the decision variable $x_{ij}^h \in \{0,1\} \quad \forall (i,j) \in A_E, h = 1,\ldots,C$ such that it takes the value 1 if the solution contains the edge $\{i,j\}$, with j closer than i to some substation in the tree, and if h turbines (including i) are connected to the substation via i. If not all conditions are met, $x_{ij}^h = 0$. That is, $x_{ij}^h = 1$ indicates that (i,j) is an arc in the spanning tree pointing towards the root, and the subtree rooted at i contains h nodes. Note that in the original capacitated minimum spanning tree formulation [8], the decision variables are defined such that arcs point from the root towards the leaves. Our definition is however adopted for the purpose of consistency with [1].

$$\min \quad \sum_{(i,j)\in A_E} \sum_{h=1}^{C} c_{ij} x_{ij}^h \tag{1}$$

$$\text{s.t.} \quad \sum_{(i,j)\in A_E} \sum_{h=1}^{C} x_{ij}^h = 1 \qquad \forall\, i \in V_c \tag{2}$$

$$\sum_{(i,j)\in A_E} \sum_{h=1}^{C-1} h x_{ij}^h - \sum_{(j,k)\in A_E} \sum_{h=1}^{C} h x_{jk}^h = -1 \qquad \forall\, j \in V_c \tag{3}$$

$$\sum_{(i,j)\in A_E} \sum_{h=1}^{C} x_{ij}^h \leq m \qquad \forall\, j \in V_c \tag{4}$$

$$\sum_{h=1}^{C} \left(x_{ij}^h + x_{ji}^h + x_{uv}^h + x_{vu}^h \right) \leq 1 \qquad \forall\, \{\{i,j\},\{u,v\}\} \in \chi \tag{5}$$

$$x_{ij}^h \in \{0,1\} \quad \forall\, (i,j) \in A_E,\, h = 1,\ldots,C \tag{6}$$

$$x_{ij}^C = 0 \qquad \forall\, (i,j) \in A_E \cap \{V \times V_c\} \tag{7}$$

We minimize the total cost or distance over all used routes in the objective function (1). Constraint (2) assures that each turbine has exactly one outgoing cable directed towards some substation. By equation (3), the load of the cable outgoing from turbine j equals the sum of the cables entering j, plus the load 1 of turbine j. The cable load is defined as the number of turbines that connect to some substation via the cable.

A maximum number of branches per turbine location is defined in (4). The planarity constraints are defined in (5) and assure that no cables cross each other.

As it is also of interest to investigate optimal cable routes with two different cable types available, each of these with different cost and capacity, we introduce a second model. Assume that if the load of any cable $(i,j) \in A_E$ is no more than $Q \in \mathbb{N}^+$, where $Q < C$, the connection cost is $q_{ij} \forall (i,j) \in A_E$. A reasonable assumption is that $q_{ij} < c_{ij}$, but validity of our model does not require this to hold.

The resulting formulation of the alternative objective function is given by:

$$\min \quad \sum_{(i,j)\in A_E} \left[\sum_{h=1}^{Q} q_{ij} x_{ij}^h + \sum_{h=Q+1}^{C} c_{ij} x_{ij}^h \right] \tag{8}$$

s.t. (2) – (7). In the computational experiments reported in the next section, we also apply this objective function to the model from [1] for comparing our results.

3 Numerical Experiments

The models suggested in Section 2, as well as the model from [1], are implemented in Python using the Python CPLEX library with CPLEX 12.6.1.0 as the integer programming solver. Default options with multithreading disabled are used.

The non-crossing constraints of the models are implemented via a lazy constraint callback, which only adds the corresponding non-crossing constraint if the solution contains the respectively crossing routes. This is a necessity resulting from the large number of constraints, increasing with $O(|V|^4)$.

All computational experiments are performed on an i7-4600U CPU with 8GB of RAM.

We choose four different real wind farm layouts as the data base for our numerical experiments. In addition to Barrow, Sheringham Shoal and Walney 1, which have also been used in [1], we also use the data from Walney 2 [3]. With the turbine and substation locations of the respective farm layouts, the distance between the turbines are computed and subsequently taken as the edge cost c_{ij}. We allow all possible connections in all wind farms, i.e. $E = V^2$. The branching capacity is set to $m = 3$ in all tests.

Table 1. General information on test case wind farms

Wind farm	Number of turbines	Number of substations
Barrow	30	1
Sheringham Shoal	88	2
Walney 1	51	1
Walney 2	51	1

In the following, we refer to the original model formulation without branching from [1] as WM1, and to our model formulation from equation (1) – (7), which allows for branching, as WMB1. Both of these models use the distances between turbines c_{ij} as the cost in the objective function.

The computational results in Table 2 are computed within 15 minutes each. Values for Sheringham Shoal for capacities $C \geq 6$ are not reported, as no optimal solution is found after 1.5 hours of calculation for model WMB1. Computations for $C \geq 8$ are also not possible for a part of the other wind farms within one hour and less than 8 GB of memory consumption and thus not included.

The optimality gap in the root node g_{WM1} and g_{WMB1} does not follow a systematic pattern, while the number of processed nodes n_{WM1} and n_{WM1} is generally higher for the WMB1 model which includes branching.

Table 2. Optimal values for models with one cable type

	Walney 1						
C	WM1	g_{WM1}	n_{WM1}	WMB1	g_{WMB1}	n_{WMB1}	ρ
4	47802	1.67	1248	47654	59.84	4757	0.31
5	43539	0.04	1	43421	8.24	2676	0.27
6	41587	26.39	483	41420	0.85	554	0.40
7	40789	19.27	1221	40620	62.76	963566	0.42

	Walney 2						
C	WM1	g_{WM1}	n_{WM1}	WMB1	g_{WMB1}	n_{WMB1}	ρ
4	62233	0.86	75	62061	62.04	1418	0.28
5	56572	4.74	8620	56258	3.32	69440	0.56
6	52228	2.15	327	51943	1.70	1887	0.55
7	49788	4.84	666	49568	69.89	422671	0.44

	Barrow						
C	WM1	g_{WM1}	n_{WM1}	WMB1	g_{WMB1}	n_{WMB1}	ρ
4	23568	1.00	62	23568	5.90	406	0.00
5	20739	6.26	114	20738	3.85	326	0.00
6	18375	0.00	1	18374	0.00	1	0.01
7	17781	4.67	564	17781	6.76	7116	0.00

	Sheringham Shoal						
C	WM1	g_{WM1}	n_{WM1}	WMB1	g_{WMB1}	n_{WMB1}	ρ
4	69222	0.00	1	68937	32.19	1087	0.41
5	64828	60.50	6530	64365	60.93	211005	0.72
6	62031	62.18	5585				
7	60667	17.42	8152				

C is the cable capacity. The columns WM1 and WMB1 are the optimal values given in m. The relative improvement ρ of the objective value is calculated by $\rho = \frac{\mathrm{WMB1}}{\mathrm{WM1}} - 1$ with the WM1 and WMB1 values of the respective row and given in %. g_{WM1} and g_{WMB1} are the optimality gaps at the root nodes of the respective model, given in %, and n_{WM1} and n_{WMB1} the corresponding number of nodes processed for the solution.

For the investigated wind farms and cable capacities the relative savings in cable length are below 1% in all cases. It is thus only marginally useful to apply branching in a wind farm if $E = V^2$, and there is only one cable type available.

The Barrow offshore wind farm turns out to be particularly unsuitable for branching. The reason for this is that the turbines are located in several rows, with a significant larger spacing tangential to this row. This favors connecting the turbines sequentially without branching into another row.

For the investigation of the models allowing for two different cable types, we set the cost of the cable with lesser capacity q_{ij} to the distance between the turbines, and increase the cost for the cable type with larger capacity by a factor f, such that $c_{ij} = f q_{ij}$.

Table 3. Optimal values for models with two cable type

f	C	Q	Walney 1 WM2	Walney 1 WMB2	Walney 1 ρ	Walney 2 WM2	Walney 2 WMB2	Walney 2 ρ	Sheringham Shoal WM2	Sheringham Shoal WMB2	Sheringham Shoal ρ
1.5	5	2	57055	54958	3.82	74547	72863	2.31	83598	80264	4.15
1.5	5	3	52416	52108	0.59	68770	68165	0.89	75882	75164	0.96
1.5	5	4	47040	46874	0.35	61481	61309	0.28	69182	68897	0.41
1.5	6	2	55460	52027	6.60	70421	67578	4.21			
1.5	6	3	51434	49887	3.10	65874	64436	2.23			
1.5	6	4	46846	46641	0.44	60483	60312	0.28			
1.5	6	5	43400	43282	0.27	55875	55663	0.38			
1.7	5	2	62372	59028	5.67	81520	78184	4.27	89857	84685	6.11
1.7	5	3	54178	54095	0.15	72018	71354	0.93	77896	77591	0.39
1.7	5	4	47410	47243	0.35	62129	61957	0.28	69222	68937	0.41
1.7	6	2	60754	55589	9.29	77197	72839	5.98			
1.7	6	3	53831	52660	2.22	70668	68644	2.95			
1.7	6	4	47410	47198	0.45	61600	61417	0.30			
1.7	6	5	43491	43374	0.27	56341	56080	0.47			
1.7	7	2	60205	52873	13.87	74572	[1]68828	8.35			
1.7	7	3	53647	49354	8.70	53646	49354	8.70	77391	[3]72486	6.51
1.7	7	4	47410	47066	0.73	61409	61001	0.67			
1.7	7	5	43491	43374	0.27	56276	[2]56075	0.36			
1.7	7	6	41587	41420	0.40	52170	51943	0.44			

f is the cost multiplier and C and Q are the cable capacities. The columns WM2 and WMB2 are the optimal values given in m. The relative improvement ρ is calculated by $\rho = \frac{\text{WMB2}}{\text{WM2}} - 1$ with the WM2 and WMB2 values of the respective row and wind farm and given in %. The values with footnotes were not solved to optimality, but with the following optimality gaps: 1) 1.85%, 2) 1.40%, 3) 0.33%

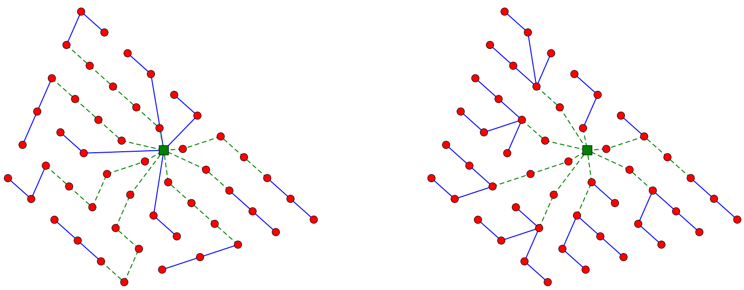

Fig. 2. Walney 1 layout for $C = 7$, $Q = 2$, $f = 1.7$; left WM2, right WMB2; circles are turbine locations, squares are substation locations, dashed lines are cables with higher capacity

In practice, the actual choice of f depends on the wind farm in question, as well as the cable capacities C and Q. It is indicated in [1] that a factor of about $f = 1.7$ is suitable for $Q = 8$ and $C = 5$. We choose this value as well as a slightly lower value of $f = 1.5$ for our numerical experiments, to also investigate the influence of the relative cost factor f.

All numerical results in Table 3 are computed within one hour of computation time each. The computations for WMB2 are more expensive than for WM2, and are thus the limiting factor. Most calculations with WM2 were completed within of less than 10 seconds. The computations for Sheringham Shoal are limited to $C =$ because of the computation time limit, except for the sample with $C = 7$, $Q = 3$. The optimality gap at the root node and the number of processed nodes are not included for the sake of brevity, as they are similar to the results in Table 2.

The numerical results show that relative savings increase in average with an increasing difference $d = C - Q$ in cable capacities. Only for $d \geq 2$ and $Q \leq 3$ relative savings of more than 1% are achieved by applying branching. The highest computed saving of 13.87% by using branching in the two cable type formulation makes a significant difference in the total cabling cost (see fig. 2).

A selection of Q, C values is computed for $f = 1.5$ to investigate the influence of this parameter on the relative improvement. There is no systematic difference observable between $f = 1.5$ and $f = 1.7$. The optimal values are in the same order of magnitude, but fluctuating in both directions.

4 Conclusion and Further Work

The results in this article show that it is of advantage to consider branching cable layouts for offshore wind farms utilizing two or possibly more different kinds of cable types. As the relative cost improvements are below 1% for farms with only one cable type, branching is not a necessity in these cases.

Possible further work on this model includes a more detailed cost modeling of the cost parameters c_{ij} and q_{ij}. This can take into account the physical location of cables, as well as length, and non-straight routes, for example because of seabed topography. In addition, the investigation of layouts with forbidden cable routes, i.e. $E \subset V^2$, is of interest for the modeling of real offshore wind farms.

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