Quantum computing for beginners

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1. Complex numbers

1.1. Basics

Quantum mechanics oftenly makes use of **complex numbers**. Complex numbers are elements of a number system that extends real numbers with a specific element denoted as i, called **imaginary unit**, which is a solution to $x^2 = -1$ ($i = \sqrt{-1}$).

Every complex number can be expressed in the form $a + bi$ where $a, b \in \mathbb{R}$. $\mathbb{R}$ here is a set of all real numbers. Set of all complex numbers is denoted as $\mathbb{C}$. In the given example, a is called the **real part** of the complex number and b is called the **imaginary part**.

Complex numbers can be added and multiplied.

Example: Let $a = 2 + 3i$ and $b = 1 + 4i$. Find $a + b$, $a - b$ and $a \times b$:

To compute the sum of two complex numbers, we just need to manually add real and imaginary parts separately: $a + b$ = $2 + 3i + 1 + 4i$ = $3 + 7i$.

Same applies to subtraction: $a - b = 2 + 3i - 1 - 4i = 1 - i$.

Multiplication is a bit different. To multiply complex numbers, we need to multiply each term of the first complex number with each term of the second one: $a \times b = (2 + 3i) \times (1 + 4i) = 2 + 6i + 8i + 12i^2$. From the definition of imaginary unit i, which is: $i = \sqrt{-1}$ follows: $i^2 = -1$. So $12i^2$ can be simplified to just -12. So the total value of $a \times b$ is $-10 + 14i$.

Exercise 1.1.1: Find $\frac{a}{b}$, given a and b from the previous example.

Exercise 1.1.2: Let $a = 4 - 3i$ and $b = 2 + 4i$. Find $a + 2b$, $3a - b$, $4a \times (-2b)$ and $\frac{10a}{3b}$.

Exercise 1.1.3: Prove that addition and multiplication over complex numbers are **commutative** and **associative** operations.

Note: binary operation is considered **commutative** if changing the order of the operands doesn't change the result. **Associativity property** is a property of some binary operations, which means that rearranging the parentheses in an expression won't affect the result, example: $(2 + 3) + 4 = 2 + (3 + 4)$.

1.2. Specific operations with complex numbers

Real numbers have a unary operation, the absolute value, given by:

$|x| = \sqrt{x^2}$

There is a generalization of this operation for complex numbers, which is also often referred as **modulus**. It is defined as:

$|c| = |a + bi| = \sqrt{a^2 + b^2}$

Exercise 1.2.1: Find the modulus of $3 + 4i$.

Exercise 1.2.2: Proof that $|a||b| = |ab|$, for all $a, b \in \mathbb{C}$.

Exercise 1.2.3: Proof that $|a + b| \leq |a| + |b|$, for all $a, b \in \mathbb{C}$.

We've shown, that a set of complex numbers $\mathbb{C}$ satisfies certain properties:

- Addition and multiplication are commutative.
- Addition and multiplication are associative.
- Addition operation has identity: 0.
- Multiplication operation has identity: 1.

- Subtraction is defined everywhere.
- Division is defined everywhere except when the value of divisor is 0.
- Multiplication distributes with respect to addition.

A set with operations satisfying all these properties is a **field**. $\mathbb{C}$ is a field of complex numbers. $\mathbb{R}$ is a field of real number.

2. Answers

1.1.1: $\frac{14}{17} - \frac{5}{17}i$; **1.1.2**: $8 + 5i$; $10 - 13i$; $-160 - 80i$; $-\frac{2}{3} - \frac{11}{3}i$; **1.2.1**: 5;