AMATH 582: FINDING THE SUBMARINE

ABHISHEK SHARMA

Mechanical Engineering Department, University of Washington, Seattle, WA
as711@uw.edu

Abstract. In this work, we design an algorithm to track a moving submarine, by detecting the signature acoustic signal it emits. The signature frequency is unknown, thus the algorithm first determines that. Then, the uncharacteristic frequencies are filtered out from the signal and the filtered signal is used to detect the location of the submarine.

1. Introduction and Overview

Submarines can be an effective means of stealthily delivering weapons. Thus, they pose a serious security threat and their detection is crucial for threat prevention. Submarine detection techniques can be broadly categorized into acoustic and non-acoustic techniques [1]. Acoustics based techniques use sound emitted by the vehicle or from its interaction with the environment to detect their location, while non-acoustic techniques rely on other sources of information e.g. satellite imagery, contaminant detection etc. Acoustic techniques remain popular due to their robustness to changes in the environment like lighting conditions and broad applicability regardless of the vehicle design.

In this work, we aim to locate a submarine in the Puget Sound using noisy acoustic data. We do not know much about this submarine as it is a new technology that emits an unknown acoustic frequency. This submarine is also moving and needs to be tracked.

Broad spectrum recording of acoustics data from the submarine, obtained over 24 hours in half-hour increments is available. The data is available in a matrix with 49 columns of data corresponding to the measurements of acoustic pressure taken over 24 hours. The measurements are taken on a uniform spatial grid of size 64 $\times$ 64 $\times$ 64. We assume that the submarine has distinct frequency signature, that no other acoustic source is present and the noise in the signal is white noise with zero mean.

We first extract the frequency signature of the acoustic signal from the submarine. Then, we design a filter to remove the non-characteristic frequencies and use the cleaned signal to determine the location of the submarine.

2. Theoretical Background

The spatial frequency content of an acoustic signal, f can be calculated using the Fourier Transform. The Fourier Transform of 3D signal is calculated as shown in eq (1).

$$\hat{f}(k_x, k_y, k_z) = \frac{1}{\sqrt{(2\pi)^3}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y, z) e^{i(k_x x + k_y y + k_z z)} dx dy dz \tag{1}$$

For evenly-spaced 3-dimensional discrete data, comprising N^3 points measured on a compact interval $x, y, z \in [-L, L]$, this integral can be approximated using Riemann sum along the dimensions x, y, z. This output fourier coefficients $\hat{f}$ corresponding to N^3 equidistant points in the frequency

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domain measured over the interval $k_x, k_y, k_z \in [-\frac{2\pi n}{2L}, \frac{2\pi n}{2L}]$. Points with large values of $\hat{f}$, are the dominant frequencies in the signal.

Presence of noise in the signal f, distorts frequency content of the signal. However, if the noise is small and the original characteristic frequency can be determined, extraneous frequencies can be filtered to recover the original signal with appropriate filter design.

Commonly, the noise in the signal can be assumed to be white noise with zero mean. Adding white noise with zero mean to the signal f is equivalent to adding white noise with zero mean to the Fourier domain representation of the signal $\hat{f}$. Thus, if several snapshots of the Fourier domain signal are averaged, the frequency content due to white noise would reduce and tend to zero as the number of snapshots tend to infinity. Averaging a finite number of snapshots can still reduce the contribution of the white noise, and help determine dominant characteristic frequencies in the signal. Assuming that the source of the signal f travels at a much slower speed compared to the acoustic wave, the motion of the source would not affect the characteristic frequency of the signal. Thus, averaging the signal in the fourier domain preserves the characteristic frequency. We use this to design a filter for our acoustic signal. Filtered signal can then be used to determine the position of the submarine, because it is safe to assume that the pressure signal attenuates with distance from the source and thus maximum pressure must occur at the source i.e. the submarine.

3. Algorithm Implementation and Development

All the algorithms were implemented in Python [2]. Numpy package [3] was used for data array manipulations, Fast Fourier Transform(fft) [4] and inverse fft computations. All the visualizations and figures were generated using plotly [5]. Algorithm 1 is sued to determine the characteristic frequencies of the signal. While Algorithm 2 uses the characteristic frequencies to design a filter and determine the trajectory of the submarine.

Algorithm 1: Determining the characteristic frequency of the acoustic signal

Input : Pressure data (f) for all time steps; avg_fft = 0

Output: Characteristic frequencies (k_{1x}, k_{1y}, k_{1z}) and (k_{2x}, k_{2y}, k_{2z}) of the acoustic signal

for *each measurement f in time* **do**
  avg_fft = avg_fft + fftshift(fftn(f));
end
avg_fft = avg_fft/timesteps;
return the frequency coordinates (k_{1x}, k_{1y}, k_{1z}) and (k_{2x}, k_{2y}, k_{2z}) corresponding to the max value of abs(avg_fft)

Using the characteristic frequency (k_{1x}, k_{1y}, k_{1z}) and (k_{2x}, k_{2y}, k_{2z}) determined by Algorithm 1, we design a filter given by eq (2). s determines the extent to which frequencies are decimated. Smaller values allow fewer frequencies to remain in the filtered signal. The value of s is determined experimentally by visualizing the acoustic signal in the time domain for remaining noise and finally set to $s = 1$.

$$g = e^{(-(x-k_{1x})^2-(y-k_{1y})^2-(z-k_{1z})^2)/2s^2} + e^{(-(x-k_{2x})^2-(y-k_{2y})^2-(z-k_{2z})^2)/2s^2} \tag{2}$$

Algorithm 2: Filtering the acoustic signal and finding the submarine coordinates

Input : Pressure data (f) from all time steps; Characteristic frequencies of the acoustic signal (k_{1x}, k_{1y}, k_{1z}) and (k_{2x}, k_{2y}, k_{2z})

Output: Submarine Coordinates $[x, y, z]$ for all time steps

filter = $e^{(-(x-k_{1x})^2-(y-k_{1y})^2-(z-k_{1z})^2)/2s^2} + e^{(-(x-k_{2x})^2-(y-k_{2y})^2-(z-k_{2z})^2)/2s^2}$

for *each measurement f in time* **do**

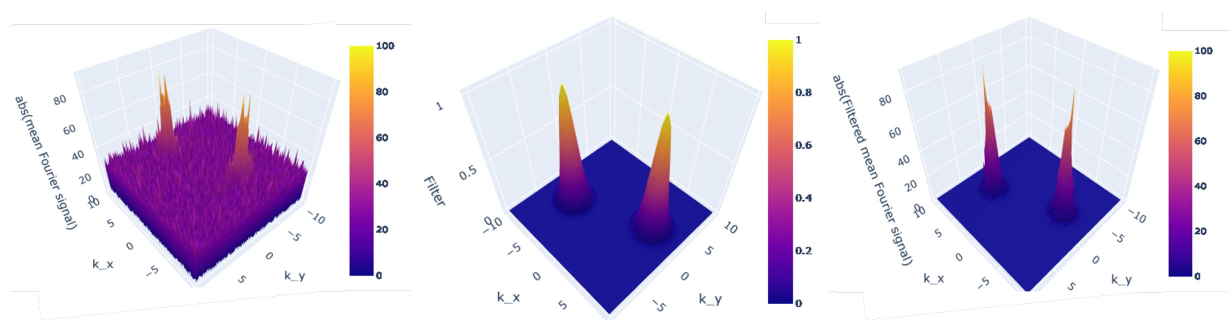
  filt_fFT = filter*fftshift(fftn(f)))) (* is an element-wise multiplication);

  filtered_f = real(ifftn(ifftshift(filt_fFT));

  $[x, y, z]$ = index corresponding to max(abs(filtered_f))

end

return $[x, y, z]$ for all time steps;



(A) Original signal (B) The Filter (C) Filtered signal

FIGURE 1. Overlayed surface plots vs x,y coordinate of the (A) absolute value of the mean of the original signal in the Fourier domain, (B) The Filter, and (C) absolute value of the mean of the filtered signal in the Fourier domain.The plots are overlayed while iterating over the z-coordinate. The x and y coordinate corresponding to the peak are determined by visual inspection in the plotly interactive figure.

4. Computational Results

Fig.1 shows the frequency domain representation of the (A) original signal, (B) the filter and (C) the Filtered signal, as a function of k_x and k_y. In Fig 1(A) the surface plots for all values of k_z are overlayed on the same plot. These plots show peaks in frequency corresponding to the signature frequency in x and y direction of the submarine. Similarly, we determine the characteristic frequency in z direction by iterating over the k_y coordinate. The peaks occur at $(k_x = -5.341, k_y = -2.199, k_z = 6.911)$ and $(k_x = 5.341, k_y = 2.199, k_z = -6.911)$. We use these frequencies to design a filter given by eq (2). Fig.2 shows the 3D trajectory of the submarine over a period of 24 hours in half hour increments, while Fig.3 shows how the position coordinates (x, y, z) vary with time.

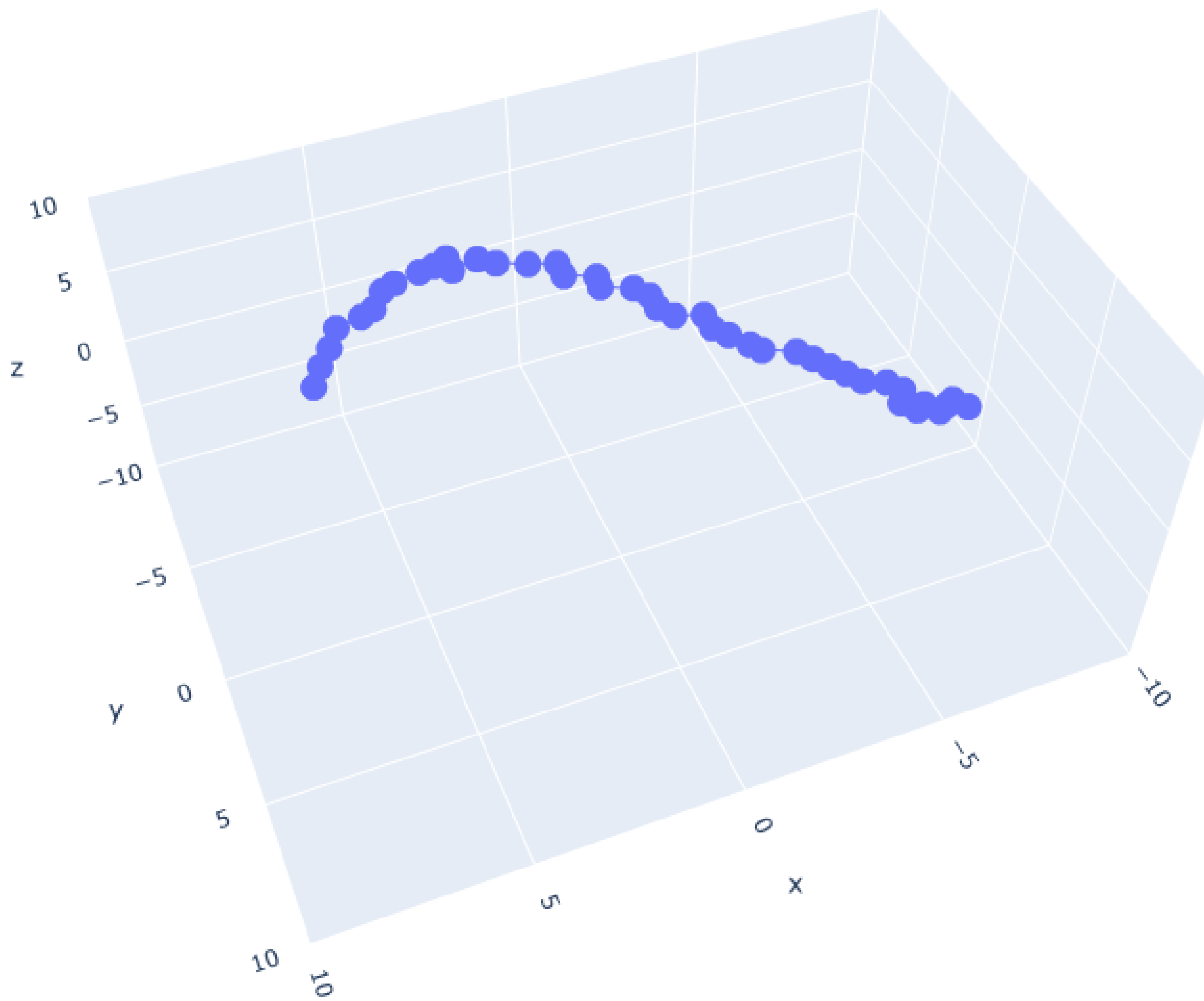


FIGURE 2. The 3-dimensional trajectory of the submarine over a period of 24 hours.

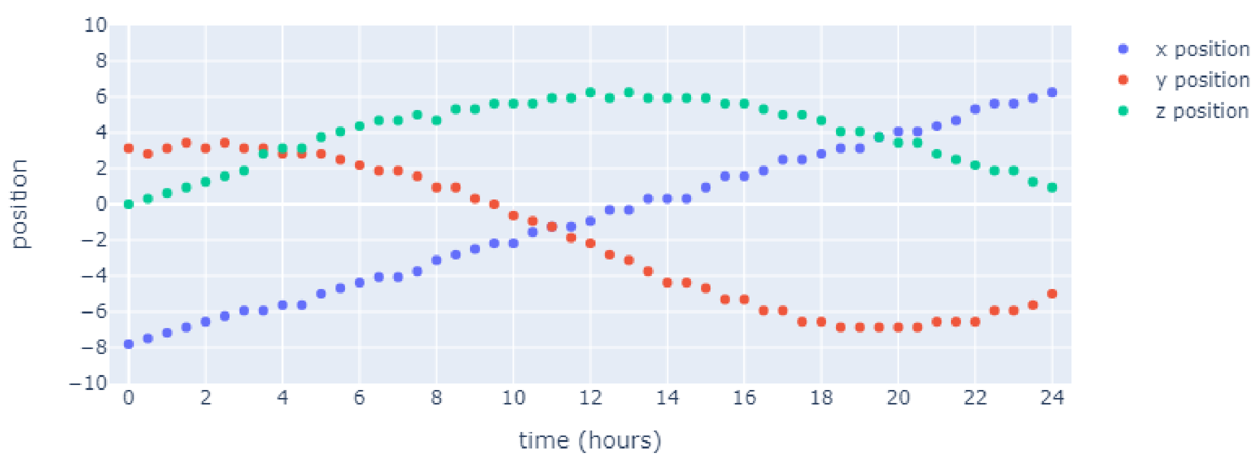


FIGURE 3. The x, y and z coordinates of the submarine over the 24 hour period.

5. Summary and Conclusions

In this report we used Fourier transform for determining the signature frequency of the acoustic signal from a submarine, use it to design a noise filter and determined the location of a submarine over a 24 hour time period. This can be use to deploy a submarine tracker.

The strength of this approach is that since we now know the signature frequency, we can determine the location even if there is a second source of acoustic signal in the vicinity, provided its frequency signature doesn't coincide with that of the submarine.

The caveats of the method are, that if the given data was captured in the presence of another source the method would fail. The method would also not work if noise present in the signal is not white noise with zero mean. The summation of Fourier domain data (Algorithm 1) in that case can exacerbate the noise and lead to failure in determining the signature frequency of the submarine.

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