CS 331: Theory of Computing

Problem Set 9

Iowa State University
Computer Science Department
April 11, 2013

Due by the midnight of April 18, 2013

Problem 1 (20 points)

(a) (10 points) Prove that $n! \in O(n^n)$.

(b) (10 points) Which of the following relations is true and which is false?

(b1) (2 points) $n \in O((\lg n)^3)$
(b2) (2 points) $(\lg n)^3 \in o(n)$
(b3) (2 points) $n^{\lg n} \in O(2^{n \lg n})$
(b4) (2 points) $n^4 \in o(100n^4)$
(b5) (2 points) $(\lg n)^n \in O(\sqrt{2^n})$

Problem 2 (20 points)

Prove that any language in P is **polynomial reducible** to any language in P which is not $\emptyset$ or Σ^*.

Hint: Follow the definitions.

Problem 3 (20 points)

Let $\mathscr{L} = \{0^i 1^j \mid i > j\}$. Show that $\mathscr{L} \in TIME(n \lg n)$.

Hint: Pages 279-280 (3rd Edition) or Pages 251-252 (2nd Edition).

Problem 4 (20 points)

Prove that Graph Isomorphism is in *NP*. That is,

$$GI = \{\langle G, H\rangle \mid G, H \textit{ are isomorphic } \} \in NP.$$

Two graphs $G = \langle V_G, E_G\rangle$ and $H = \langle V_H, E_H\rangle$ are isomorphic iff there is a bijection $f : V_G \rightarrow V_H$ such that $\langle v, v'\rangle \in E_G$ if and only if $\langle f(v), f(v')\rangle \in E_H$.

Problem 5 (20 points)

Prove that Double Satisfaction Problem, defined as

$$SAT2 = \{\langle \varphi \rangle \mid \varphi \textit{ is a } 3NF\textit{-formula with at least two solutions}\}$$

is *NP*-complete.

Hint: Reduce 3SAT to it.

Problem 6 (20 points)

Prove that the class P is closed under union and complementation (10 points for each).