CS 331: Theory of Computing

Problem Set 1

Iowa State University
Computer Science Department
January 23, 2013

Due by the midnight of January 31, 2013

Problem 1 (15 points)

Use **Proof By Contradiction** to show that for any $k \geq 2$, $\sqrt[k]{2}$ is an irrational number.

Problem 2 (15 points)

The **depth** (or **height**) of a tree is the number of edges along the path from the root node to the deepest leaf node. A **full binary tree** is a tree in which every node other than the leaves has two children. A full binary tree is called **perfect** if all leaves are at the same depth. Note that the single root is a perfect binary tree with depth 0.

Use **Proof By Induction** to show that for every $n \geq 0$, a depth n perfect binary tree has $2^{n+1} - 1$ nodes.

Problem 3 (20 points)

A **truth assignment** M is a function that maps propositional variables to $\{0, 1\}$ (1 for true and 0 for false). We write $M \models \varphi$ if φ is true under M. We define a partial order $\leq$ on truth assignments such that $M \leq M'$ if $M(p) \leq M'(p)$ for every propositional variable p.

A propositional formula is **positive** if it only contains connectives $\wedge$ (and) and $\vee$ (or) (i.e., no negation $\neg$ or implication $\rightarrow$).

Use **Proof By Induction** to show that for any truth assignments M and M' such that $M \leq M'$, and any positive propositional formula φ, if $M \models \varphi$, then $M' \models \varphi$.

Problem 4 (10 points)

Let $\Sigma = \{0, 1\}$. What language is defined by the following regular expression? Describe that in one or two sentences.

1. (5 points) $\Sigma^* \ 0 \ \Sigma^* \ 1 \ \Sigma^*$.
2. (5 points) $0 \ 0^* \ 1^*$.

Problem 5 (20 points)

Let $\Sigma = \{0, 1, e, E, +, -\}$. Construct a deterministic finite automaton that recognizes words of the following form

$$(0 \cup 1)(0 \cup 1)^*(e \cup E)(+ \cup -)(0 \cup 1)(0 \cup 1)^*$$

Note that "(" and ")" are only for readability; they are not part of the regular expression.

Problem 6 (20 points)

Let $\mathcal{A} = \langle Q, \Sigma, \delta, q_0, F \rangle$ and $\mathcal{B} = \langle Q, \Sigma, \delta, q_0, F' \rangle$ be two **nondeterministic** finite automata such that $F' = Q \setminus F$. In other words, the only difference between $\mathcal{A}$ and $\mathcal{B}$ is the final state set; a state is final in $\mathcal{B}$ if and only if it is not in $\mathcal{A}$.

Prove or disprove: The language $\mathscr{L}(\mathcal{B})$ is the complement of the language $\mathscr{L}(\mathcal{A})$, i.e., $\mathscr{L}(\mathcal{B}) = \Sigma^* \setminus \mathscr{L}(\mathcal{A})$.

Note that if the statement is true, you must provide a formal proof, and if the statement is false, then all you need to do is describe a counterexample.