CS 331: Theory of Computing

Problem Set 2

Iowa State University
Computer Science Department
January 31, 2013

Due by the midnight of February 7, 2013

Problem 1 (20 points)

Let $\Sigma = \{0, 1\}$. Construct a DFA $\mathcal{A}$ that recognizes the language that consists of all binary numbers that can be divided by 5. For example, $\mathcal{A}$ should accept 101, 1010, 1111 etc, and should reject 010, 111, 1110 etc. **Justify your answer**.

Note: Your DFA should have no more than 5 states.

Problem 2 (20 points)

Let $w = \sigma_1 \cdots \sigma_n$ be a word on an alphabet Σ. By $w^{\mathcal{R}}$ we mean the word $\sigma_n \cdots \sigma_1$. Define $\mathscr{L}^{\mathcal{R}}$ such that

$$\mathscr{L}^{\mathcal{R}} = \{ w \in \Sigma^* \mid w^{\mathcal{R}} \in \mathscr{L} \}.$$

Prove that if $\mathscr{L}$ is regular, then so is $\mathscr{L}^{\mathcal{R}}$.

Problem 3 (20 points)

Let $\Sigma = \{0, 1\}$. We use $||w||_{w'}$ to denote the number of occurrences of the subwords w' in w. Construct a DFA that recognizes the following language:

$$\{ w \in \Sigma^* \mid ||w||_{10} = ||w||_{01} \} .$$

Problem 4 (20 points)

For language $\mathscr{L}_1$ and $\mathscr{L}_2$, let the **imperfect shuffle** of $\mathscr{L}_1$ and $\mathscr{L}_2$ be the language

$$\{ w \in \Sigma^* \mid w = w_1 v_1 \cdots w_k v_k, \\ w_1 \cdots w_k \in \mathscr{L}_1, v_1 \cdots v_k \in \mathscr{L}_2, w_i, v_i \in \Sigma^* \}.$$

Prove that the class of regular languages is closed under imperfect shuffle.

Problem 5 (20 points)

An NFA $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ is called **co-deterministic** (CDFA) if for any $a \in \Sigma$ and any two distinct states q and q',

$$\delta(q, a) \cap \delta(q', a) = \emptyset.$$

Prove or disprove the following statements.

- Every CDFA is a DFA. (5 points)
- Every DFA is a CDFA. (5 points)
- Every NFA can be converted to an equivalent CDFA. (10 points)

Hint: A CDFA can be viewed as a DFA as if the word is read from the end to the beginning. Your solution to Problem 2 may help.