

CS 331: Theory of Computing

Problem Set 4

Iowa State University
Computer Science Department
February 21, 2013

Due by the midnight of February 28, 2013

Problem 1 (20 points)

Let $\Sigma = \{a, b, c\}$. Define a Context-free Language (CFL)

$$\mathcal{L} = \{a^i b^j c^i \mid i, j \geq 0\}.$$

- (10 points) Find a Context-free Grammar (CFG) to describe \mathcal{L} .
- (10 points) Construct a PDA that recognizes \mathcal{L} . Draw the state diagram like Figure 2.15 in the textbook.



Problem 2 (20 points)

- (10 points) Prove that if \mathcal{L} is a context-free language and \mathcal{L}' is a regular language, then $\mathcal{L} \cap \mathcal{L}'$ is context-free too.
- (10 points) Let $\Sigma = \{a, b, c\}$ and

$$\mathcal{L} = \{ w \in \Sigma^* \mid w \text{ contains equal numbers of } a\text{'s, } b\text{'s, and } c\text{'s} \}.$$

Use the first part to prove that \mathcal{L} is **not** a context-free language.



Problem 3 (20 points)

A **Right Linear Grammar** (RLG) is a context-free grammar such that in each production rule, at most one variable can appear in the right-hand side and such an occurrence can only take place at the right end. For example, the following is an RLG.

$$S \rightarrow abS \mid abS \mid \epsilon$$

Prove that RLGs recognize exactly the class of regular languages. Your proof should consist of two parts: (1) any regular language can be described by an RLG (10 points), and (2) any language described by an RLG is regular (10 points).

Hint: Correspondence between the variables in an RLG and the states of a DFA.



Problem 4 (20 points)

Use the pumping lemma to show that the following language is not context-free.

$$\{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i > j \text{ and } j > k \}$$



Problem 5 (20 points)

Let $G = \langle \{S\}, \{a, b\}, R, S \rangle$ be a context-free grammar where R consists of the following production rules:

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

- (5 points) Prove that G is ambiguous.
- (15 points) Give an unambiguous grammar that generates the same language as G does.

Hint: Use precedence as in Example 2.4 in the textbook.

