CS 331: Theory of Computing

Problem Set 6

Iowa State University
Computer Science Department
March 7, 2013

Due by the midnight of March 14, 2013

Problem 1 (20 points)

Let $\Sigma = \{0, 1, \#\}$. Consider the following language over Σ:

$$\mathscr{L} = \{\, w_1 \# w_2 \ \mid\ w_1, w_2 \in \{0,1\}^* \textit{ and } w_1 < w_2 \,\}.$$

Note: $w_1 < w_2$ means w_1 is less than w_2 when they are both viewed as integers in binary. For example, $1110\#11100 \in \mathscr{L}$ while $0011\#011 \notin \mathscr{L}$.

Design a Turing machine (in pseudo-code) that recognizes $\mathscr{L}$.

Problem 2 (20 points)

A 2-dimensional Turing machine is a Turing machine with a 2-dimensional tape that is an unbounded grid of tape squares over which the head can move in 4 directions: left (L), right (R), up (U) and down (D). The tape space is denoted by $\mathbb{N} \times \mathbb{N}$. The head starts at $(0, 0)$, and is governed by two restrictions:

- The head never moves down when it is on the bottom row, i.e., in positions $(i, 0)$ for $i \in \mathbb{N}$,
- The head never moves left when it is on the leftmost column, i.e., in positions $(0, i)$ for $i \in \mathbb{N}$.

Prove that 2-dimensional Turning machines are **no** more powerful than the standard Turing machines.

Note: Your proof need not be completely formal, but it should have sufficient details on how a 2-dimensional Turing machine can simulated by a standard Turing machine step by step.

Problem 3 (20 points)

Prove that the class of Turing-recognizable languages are closed under the following operations.

- (5 points) Union: $\mathscr{L} = \mathscr{L}_1 \cup \mathscr{L}_2$.
- (5 points) Intersection: $\mathscr{L} = \mathscr{L}_1 \cap \mathscr{L}_2$.
- (10 points) Concatenation: $\mathscr{L} = \mathscr{L}_1 \cdot \mathscr{L}_2$.

Note: Your proof should be constructive. That is, given Turing machines TM_1 and TM_2 that recognize $\mathscr{L}_1$ and $\mathscr{L}_2$ respectively, your proof should show how to construct a Turing machine TM that recognize $\mathscr{L}$ for each operation.

Problem 4 (20 points)

Prove the following languages are **not** Turing-decidable.

- (10 points) $\mathscr{L}_B = \{\, \langle M \rangle \ \mid\ M \textit{ will write “V” somewhere on the tape } \}$.
- (10 points) $\mathscr{L}_U = \{\, \langle M \rangle \ \mid\ M \textit{ halts on all words except } \textbf{\textit{one}} \ \}$.

Problem 5 (20 points)

We say that a Turing machine is n-bound if its head visits at most n squares on the tape.

Is the following language Turing decidable? Prove or disprove it.

$$\mathscr{L}_S = \{ \langle M \rangle \mid M \text{ is } |w| * |w|\text{-bound on any input } w. \}.$$

Note: $|w|$ denotes the length of w.