CS 331: Theory of Computing

Problem Set 8

Iowa State University
Computer Science Department
April 4, 2013

Due by the midnight of April 11, 2013

Problem 1 (20 points)

Let $A \leq_L B$ mean that $A \leq_T B$ with the additional condition that the oracle Turing machine M^B that solves A queries the oracle for B only once, at the very last step.

Prove that $A \leq_L B$ if and only if $A \leq_m B$ (10 points for each direction).

Problem 2 (20 points)

Describe two **different** Turing machines, M_1 and M_2, such that, when started on any input, M_1 outputs $\langle M_2 \rangle$ and M_2 outputs $\langle M_1 \rangle$.

Hint: Take a look at program SELF.

Problem 3 (20 points)

For notation simplicity, in this problem, we identify a Turing machine with its encoding. A computable function $f : \Sigma^* \to \Sigma^*$ is said to be a **universal corruptor** if for any Turing machine M, $f(M)$ is a Turing machine that behaves differently from M. Formally, $\mathscr{L}(M) \neq \mathscr{L}(f(M))$.

Prove that no universal corruptor exists. You need to show that for every purported corruptor f, there exists a Turing machine *UNTOUCHABLE* such that $f(\mathit{UNTOUCHABLE})$ and *UNTOUCHABLE* are equivalent.

Problem 4 (20 points)

Let $SELF_{TM} = \{\langle M \rangle \mid \mathscr{L}(M) = \{\langle M \rangle\}\}$. Prove that neither $SELF_{TM}$ nor $\overline{SELF_{TM}}$ is Turing-recognizable (10 points for each).

Hint: Diagonalization with the help of Recursion Theorem.

Problem 5 (20 points)

Prove that the class P is closed under union and complementation (10 points for each).