AUTOMATION OF SPUR GEAR DESIGN AND CAD MODELLING: TECHNICAL SUMMARY FOR DESIGN

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Principal geometrical symbols[1,2]

a centre distance mm
b face width mm
s tooth thickness mm
e space thickness mm
c tip and root clearance mm
m module mm
h_a addendum mm
h_f dedendum mm
n speed of rotation rpm
c clearance mm
ρ_f fillet radius mm
d pitch circle diameter mm
z number of teeth
p pitch, lead mm
u gear ratio
α pressure angle $^\circ$
ϕ tooth thickness half angle $^\circ$
P pinion
G gear

Formulae:[2]

1. $a = \frac{m(z_P+z_G)}{2}$
2. $d = mz$
3. $p = \pi m$
4. $u = \frac{n_P}{n_G} = \frac{z_G}{zP}$
5. $\rho_f = \frac{c}{1-\sin(\alpha)} = \frac{\pi\frac{m}{4}-(h_f\tan(\alpha))}{\tan\left(\frac{90^\circ-\alpha}{2}\right)}$
6. $s = e = \frac{p}{2} = \frac{\pi m}{2}$

Specified values of modules:[3]

Series	
I	II
1	
	1.125
1.25	
	1.375
1.5	
	1.75
2	
	2.25
2.5	
	2.75
3	
	3.5
4	
	4.5
5	
	5.5
6	
	6.5
	7
8	
	9
10	
	11
12	
	14
16	
	18
20	
	22
25	
	28
32	
	36
40	
	45
50	

Design procedure:[4]

I. Standard system of gear tooth:

Table 1: Proportions of standard involute teeth in terms of module

	Full depth		Stub
α	$14.5°$	$20°$	$20°$
h_a	m	m	$0.8m$
h_f	$1.157m$	$1.25m$	m
c	$0.157m$	$0.25m$	$0.2m$
s	$1.5708m$	$1.5708m$	$1.5708m$

II. Interference and undercutting:

The minimum number of teeth to avoid interference is given by

$$z_{min} = \frac{2}{\sin^2 \alpha}$$

Table 2: Minimum number of teeth to avoid interference and undercutting

Involute full depth 14.5°	32
Involute full depth 20°	17
Involute stub 20°	14

III. Force analysis:

Assumptions

i. As the point of contact moves, the magnitude of the resultant force changes. This effect is neglected.
ii. Only one pair of teeth takes the entire load.
iii. Effect of dynamic force is neglected.

Then, $P\,kW$ is power transmitted by the gear,

$$M_t = \frac{60 \times 10^6 \times P}{2\pi n}\, N \cdot mm \tag{1}$$

where M_t is torque transmitted by gears.

$$P_t = \frac{2M_t}{d}\, N \tag{2}$$

where P_t is tangential load at the pitch circle radius.

$$P_r = P_t \tan\alpha\, N \tag{3}$$

where P_r is radial load acting towards the centre.

$$P_N = \frac{P_t}{\cos\alpha}\, N \tag{4}$$

where P_N is resultant load.

Beam strength of gear tooth.[5]

Assumptions

i. The effect of radial load P_r which induces compressive stresses, is neglected.
ii. The tangential load P_t is uniformly distributed over the face and width of the gear.
iii. The effect of stress concentration is neglected.
iv. Only one pair of teeth is in contact and takes the total load at any given time.

$$S_b = mb\sigma_b Y \; N \tag{5}$$

where,
S_b is beam strength of gear tooth
$b = 10m$ is face width in mm
$\sigma_b = \frac{1}{3} S_{ut}$ is permissible bending stress in $N \cdot mm^{-2}$ and S_{ut} is ultimate tensile strength[6] in $N \cdot mm^{-2}$
$Y = y\pi$ is Lewis form factor.

Table 3: Lewis form factor[5]

Number of Teeth	y	Number of Teeth	y
12	0.078	27	0.111
13	0.083	30	0.114
14	0.088	34	0.118
15	0.092	38	0.122
16	0.094	43	0.126
17	0.096	50	0.130
18	0.098	60	0.134
19	0.100	75	0.138
20	0.102	100	0.142
21	0.104	150	0.146
23	0.106	300	0.150
25	0.108		

Approximation of Lewis form factor:[7]

$$\begin{aligned} y &= 0.124 - \frac{0.684}{z} \text{ for involute full depth } 14.5^\circ \\ &= 0.154 - \frac{0.912}{z} \text{ for invloute full depth } 20^\circ \\ &= 0.175 - \frac{0.950}{z} \text{ for involute stub } 20^\circ \end{aligned} \tag{6}$$

Wear strength of gear tooth:[6]

$$S_w = bQd_PK\,N \tag{7}$$

where,
S_w is wear strength of the gear tooth.
Q is ratio factor defined as

$$\begin{aligned} Q &= \frac{2z_G}{z_G + z_P} \text{ for external gearing} \\ &= \frac{2z_G}{z_G - z_P} \text{ for internal gearing} \end{aligned} \tag{8}$$

K is load stress factor defined as

$$K = \frac{\sigma_c^2 \sin\alpha \cos\alpha \left(\frac{1}{E_P} + \frac{1}{E_G}\right)}{1.4} \tag{9}$$

According to Gustav Niemann,[8]

$$\begin{aligned} \sigma_c &= 0.27(BHN)\,kgf \cdot mm^{-2} \\ &= 2.65(BHN)\,N \cdot mm^{-2} \end{aligned} \tag{10}$$

Effective load on gear tooth:

The service factor is defined as $C_s = \frac{maximum\,torque}{rated\,torque}$.

Table 4: Service factor for speed reduction gearbox

Workin characteristics of driving machine	Working characteristics of driven machine		
	Uniform	Moderate	Heavy
Uniform	1.00	1.25	1.75
Light	1.25	1.50	2.00
Medium	1.50	1.75	2.25

Table 5: Examples of driving machines with different working characteristics

Characteristic of operation	Driving machines
Uniform	Electric motor, steam turbine, gas turbine
Light	Multi-cylinder internal combustion engine
Medium	Single-cylinder internal combustion engine

Table 6: Examples of driven machines with different working characteristics

Characteristic of operation	Driven machines
Uniform	Generator, belt conveyor, platform conveyor, light elevator, electric hoist, feed gears of machine tools, ventilators, turbo-blower, mixer for constant density material
Moderate	Main drive to machine tool, heavy elevator, turning gear of crane, mine ventilator, mixer for variable densoty material, multi-cylinder piston pump, feed pump
Heavy	Press, shear, rubber dough mill, rolling mill drive, power shovel, heavy centrifuge, heavy feed pump, rotary drilling apparatus, briquette press, pug mill

In the final stage of gear design, when gear dimensions are known, errors specified and the quality of gears determined, the dynamic load is calculated by equations derived by Earle Buckingham.[6]

$$P_{eff} = C_s P_t + P_d\, N \tag{11}$$

where,

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}\, N \tag{12}$$

P_d is dynamic load.
v is pitch line velocity in $m \cdot s^{-1}$
$e = e_P + e_G$ is sum of errors between two meshing teeth in mm
b is face width of tooth in mm
P_t is tangential force due to rated torque in N
The deformation factor C depends upon the modulii of elasticity of materials for pinion and gear and the form factor of tooth or pressure angle. It is given by,

$$C = \frac{k}{\left(\frac{1}{E_P} + \frac{1}{E_G}\right)} \tag{13}$$

where,
k is constant depending upon the form of tooth.
E is modulus of elasticity of materials in $N \cdot mm^{-2}$

Table 7: The values of k for various tooth forms

Involute full depth 14.5	0.107
Involute full depth 20	0.111
Involute stub 20	0.115

The tolerances are calculated by using the following basic equation

$$\phi = m + 0.25\sqrt{d} \tag{14}$$

where, ϕ is tolerance factor.

Table 8: Tolerances on the adjacent pitch

Grade	$e\,\mu m$
1	$0.80 + 0.06\phi$
2	$1.25 + 0.10\phi$
3	$2.00 + 0.16\phi$
4	$3.20 + 0.25\phi$
5	$5.00 + 0.40\phi$
6	$8.00 + 0.63\phi$
7	$11.00 + 0.90\phi$
8	$16.00 + 1.25\phi$
9	$22.00 + 1.80\phi$
10	$32.00 + 2.50\phi$
11	$45.00 + 3.55\phi$
12	$63.00 + 5.00\phi$

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