Description of Gauss Elimination Method:

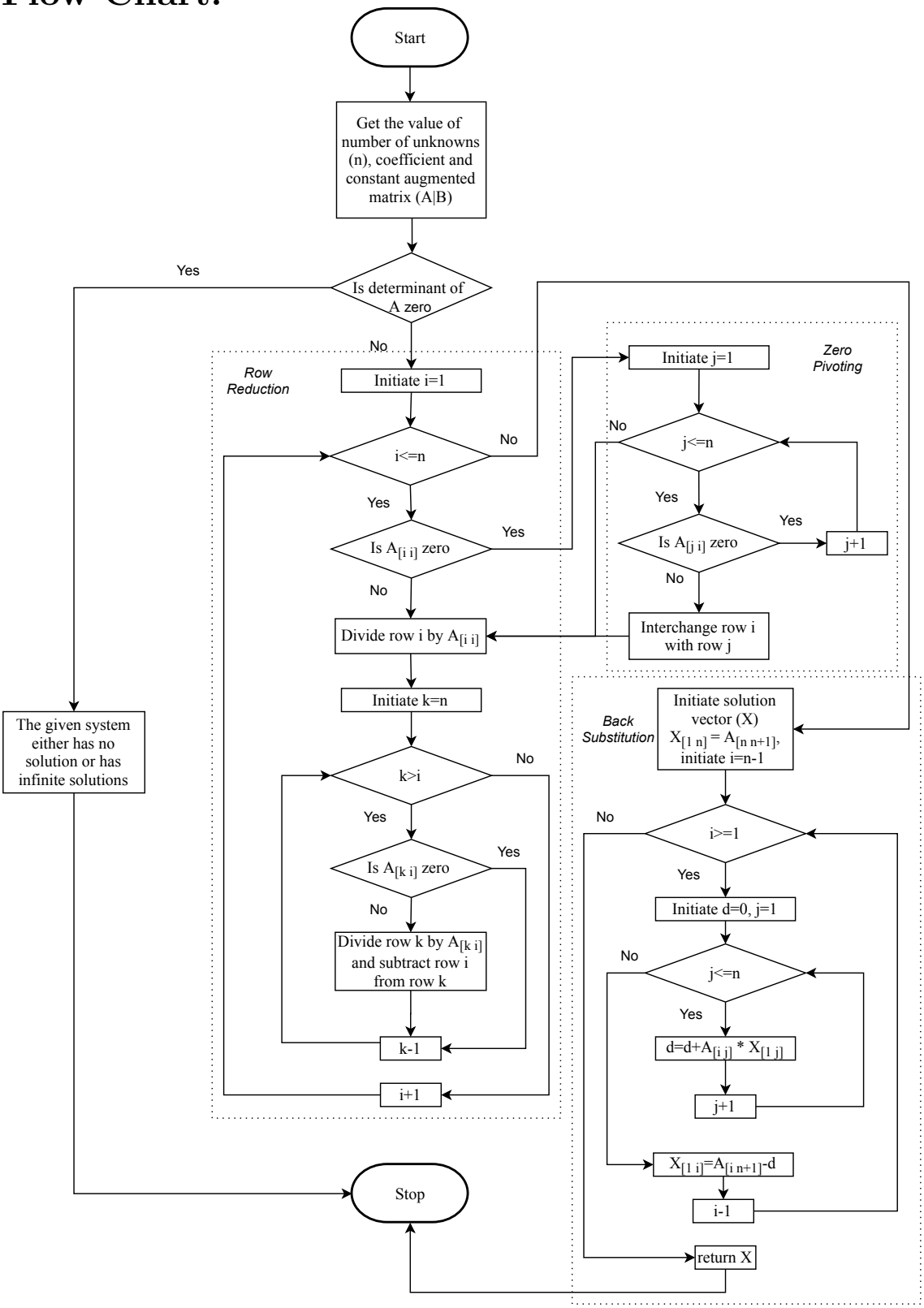
Gauss elimination can be divided into three parts

1. Partial pivoting:
   Partial pivoting is a crucial step to avoid division by zero. The diagonal elements are checked for zero of a row and replaced by non zero element by row interchange operation.

2. Row reduction:
   The coefficient matrix is transformed into an upper triangular matrix by row operations. A simple approach is to divide a row by it's diagonal element, divide the succeeding row by element corresponding to the diagonal element of the former row and subtract the former row from the later row. If the succeeding row has zero at the place corresponding to the diagonal element, skip the row. Repeating this for all the rows and their diagonal elements will give an upper triangular matrix.

3. Back substitution:
   The coefficient matrix thus can be used to find the unknowns in reverse order by substituting the values.

Flow Chart:


Start
Get the value of number of unknowns (n), coefficient and constant augmented matrix (A|B)
Is determinant of A zero
Yes
No
The given system either has no solution or has infinite solutions
Row Reduction
Initiate i=1
i<=n
No
Yes
Is A[i i] zero
Yes
No
Divide row i by A[i i]
Initiate k=n
k>i
No
Yes
Is A[k i] zero
Yes
No
Divide row k by A[k i] and subtract row i from row k
k-1
i+1
Zero Pivoting
Initiate j=1
No
j<=n
Yes
Is A[j i] zero
Yes
j+1
No
Interchange row i with row j
Back Substitution
Initiate solution vector (X) X[1 n] = A[n n+1], initiate i=n-1
No
i>=1
Yes
Initiate d=0, j=1
No
j<=n
Yes
d=d+A[i j] * X[1 j]
j+1
X[1 i]=A[i n+1]-d
i-1
return X
Stop

Description of Jacobi's Method:

The Jacobi's method is an iterative technique for solving a system of n linear equations with unknown X:

$$\text{AX=B}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix}$$

The solution is then obtained by iteration via

$$x_{in}^{k+1} = \frac{1}{a_{ii}} \left(b_{in} - \sum_{j \neq i} a_{ij} x_{jn}^{k} \right), \; i = 1, 2, \ldots, n.$$

where X^k is the kth approximation of X and X^{k+1} is the next or $k+1$ iteration of X. The computation of x_i^{k+1} requires each element in x^k except itself. Unlike Gauss Seidel method, we can't overwrite x_i^k with x_i^{k+1}, as the value will be needed by the rest of the computation. The minimum amount of storage is two vectors of size n.

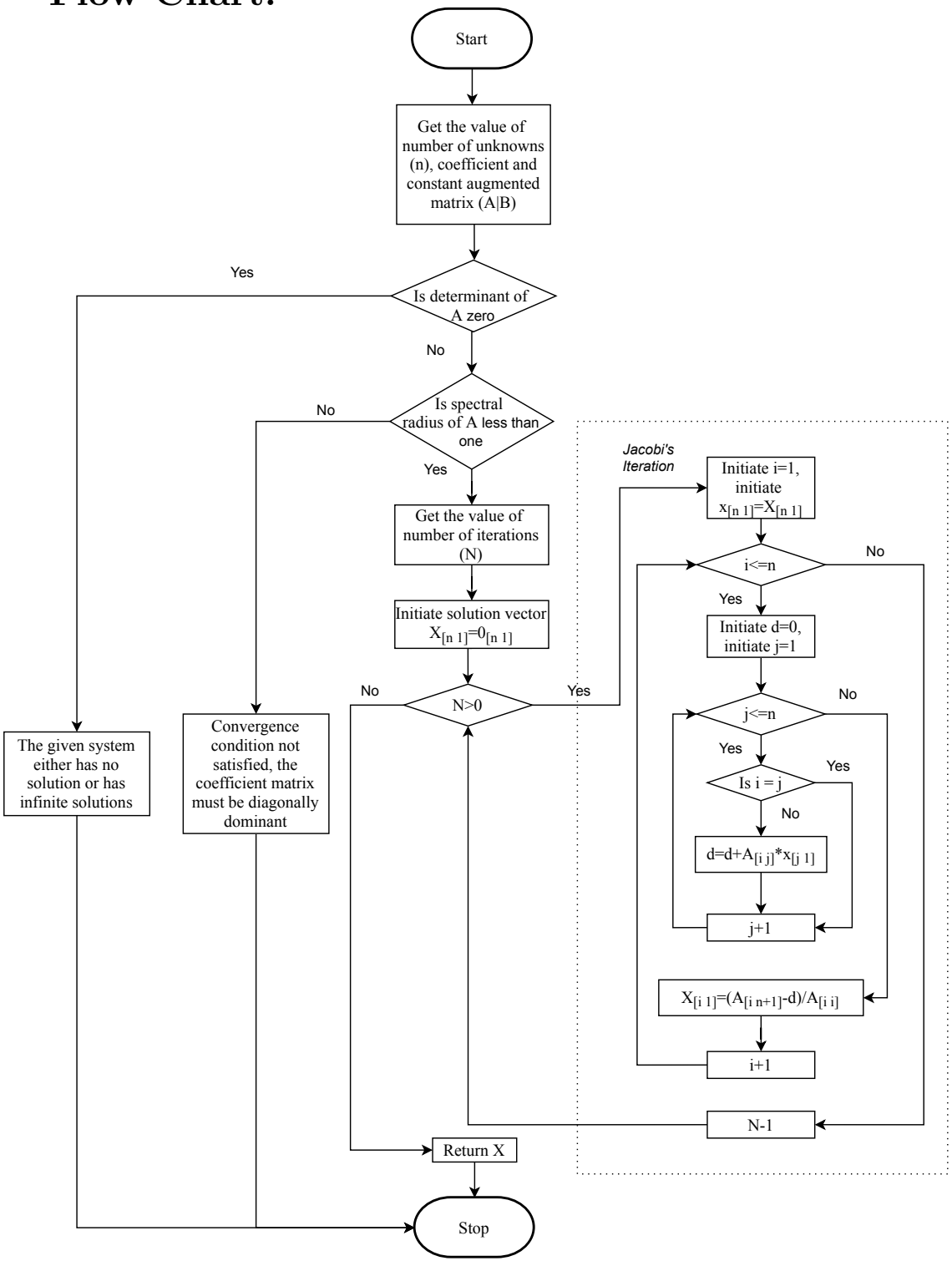
The standard convergence condition for any iterative method is the spectral radius of the iteration matrix should be less than 1:

$$\rho(D^{-1}(L+U)) < 1$$

where A is decomposed into it's diagonal component D, a lower triangular part L and an upper triangular part U. D^{-1} is inverse of D.

The spectral radius of a square matrix is the largest absolute value of its eigenvalues. It is denoted by $\rho(\cdot)$.

Flow Chart:


Start
Get the value of number of unknowns (n), coefficient and constant augmented matrix (A|B)
Is determinant of A zero
Yes
No
Is spectral radius of A less than one
No
Yes
The given system either has no solution or has infinite solutions
Convergence condition not satisfied, the coefficient matrix must be diagonally dominant
Get the value of number of iterations (N)
Initiate solution vector X[n 1]=0[n 1]
N>0
No
Yes
Jacobi's Iteration
Initiate i=1, initiate x[n 1]=X[n 1]
i<=n
No
Yes
Initiate d=0, initiate j=1
j<=n
No
Yes
Is i = j
Yes
No
d=d+A[i j]*x[j 1]
j+1
X[i 1]=(A[i n+1]-d)/A[i i]
i+1
N-1
Return X
Stop

Description of Gauss Seidel Method:

The Gauss Seidel method is an iterative technique for solving a system of n linear equations with unknown X:

$$\text{AX=B}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix}$$

The solution is then obtained by iteration via

$$x_{in}^{k+1} = \frac{1}{a_{ii}} \left(b_{in} - \sum_{j \neq i} a_{ij} x_{jn}^{k} \right), \; i = 1, 2, \ldots, n.$$

where X^k is the kth approximation of X and X^{k+1} is the next or $k+1$ iteration of X. The computation of X^{k+1} uses the elements of X^{k+1} that have already been computed and elements of x^k that have not been computed in the $k+1$ iteration. Thus unlike the Jacobi's method, only one storage vector is required as elements can be overwritten as they are computed.

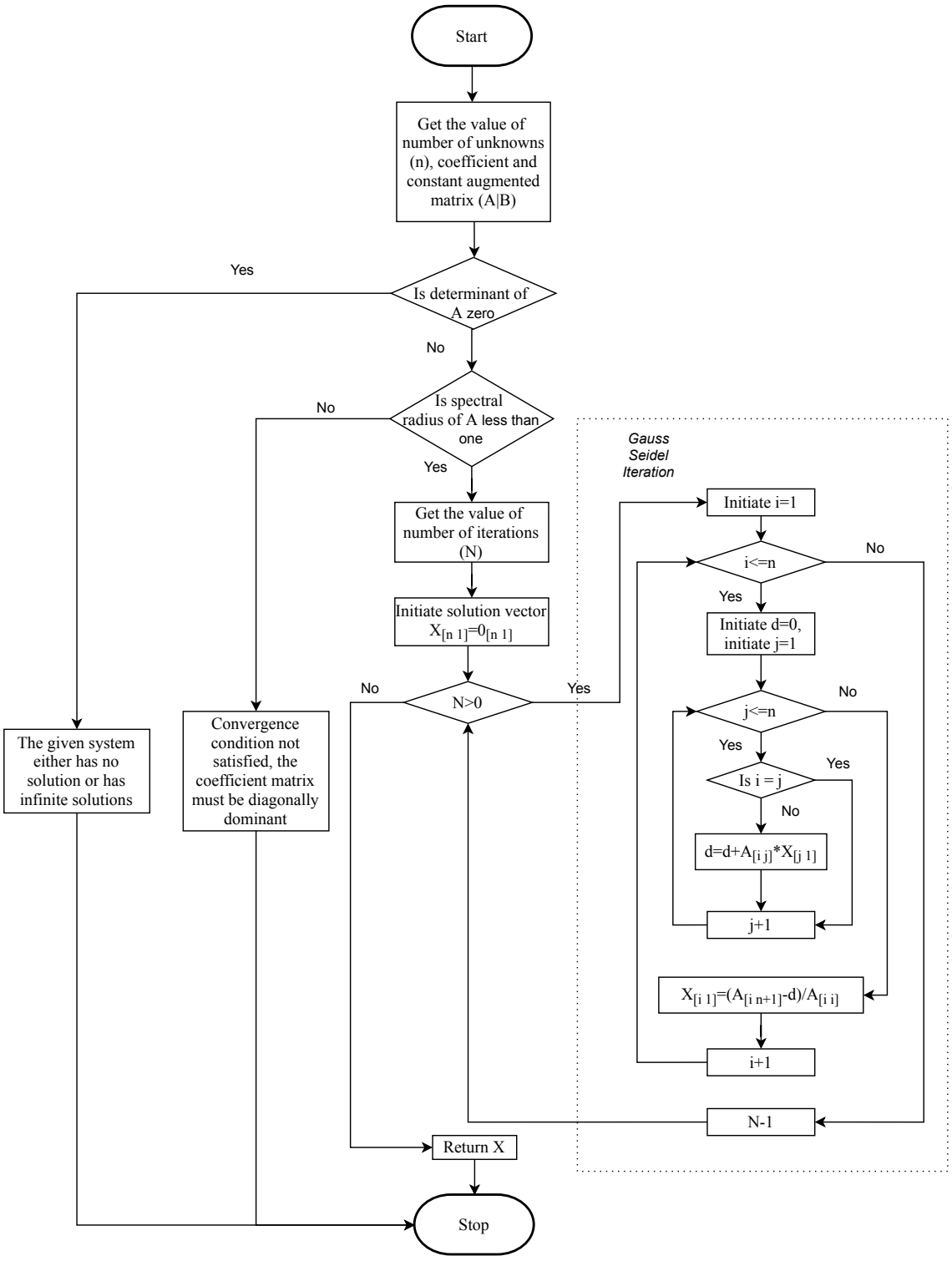
The standard convergence condition for any iterative method is the spectral radius of the iteration matrix should be less than 1:

$$\rho(D^{-1}(L+U)) < 1$$

where A is decomposed into it's diagonal component D, a lower triangular part L and an upper triangular part U. D^{-1} is inverse of D.

The spectral radius of a square matrix is the largest absolute value of its eigenvalues. It is denoted by $\rho(\cdot)$.

Flow Chart:


Start
Get the value of number of unknowns (n), coefficient and constant augmented matrix (A|B)
Is determinant of A zero
Yes
No
Is spectral radius of A less than one
No
Yes
The given system either has no solution or has infinite solutions
Convergence condition not satisfied, the coefficient matrix must be diagonally dominant
Get the value of number of iterations (N)
Initiate solution vector X[n 1]=0[n 1]
N>0
No
Yes
Return X
Stop
Gauss Seidel Iteration
Initiate i=1
i<=n
No
Yes
Initiate d=0, initiate j=1
j<=n
No
Yes
Is i = j
Yes
No
d=d+A[i j]*X[j 1]
j+1
X[i 1]=(A[i n+1]-d)/A[i i]
i+1
N-1