Description of Bisection Method:

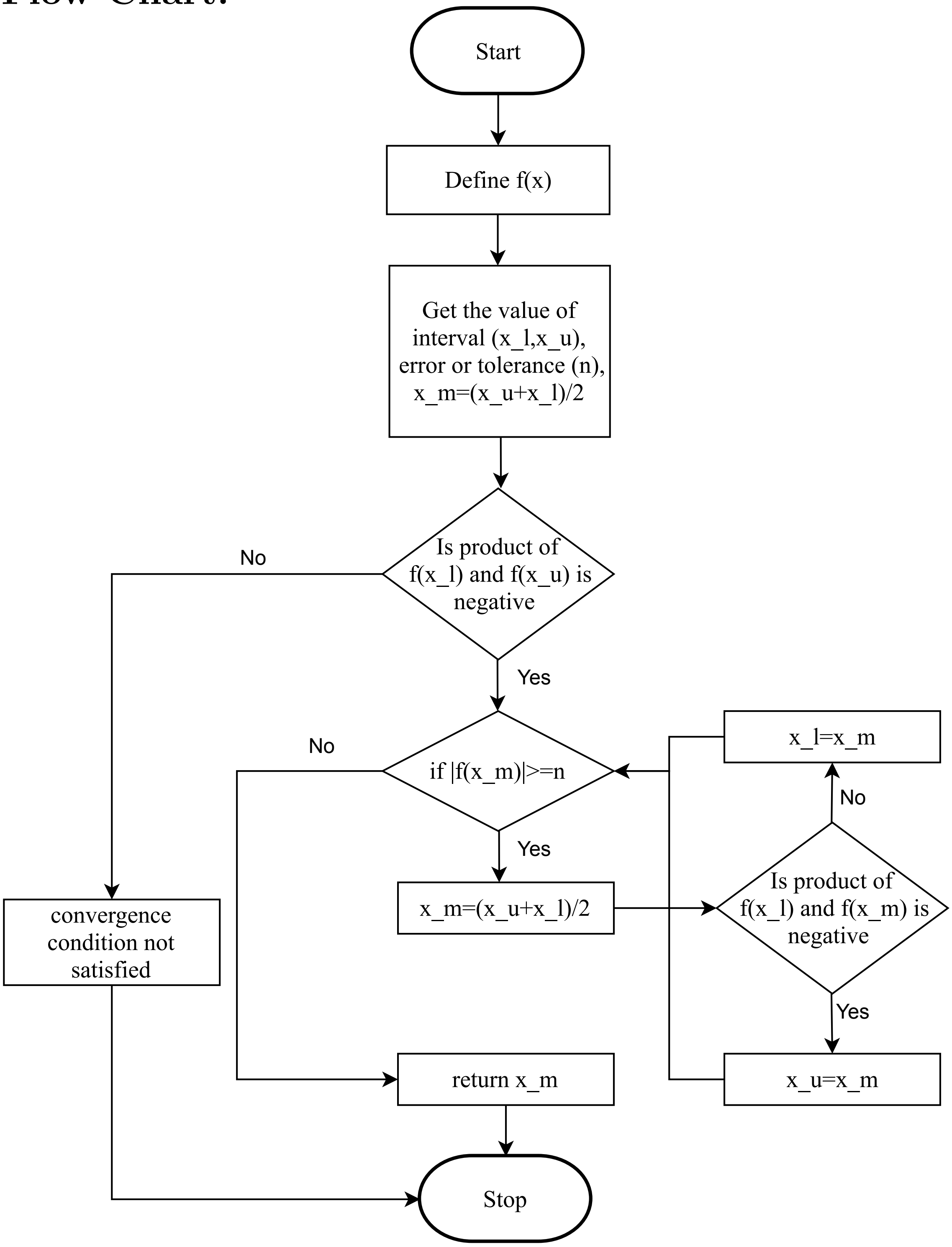
Let $f(x)$ be a function continuous on interval $[a, b]$ and $f(a)$ and $f(b)$ have opposite signs and ϵ be the tolerance or error in the root, each iteration performs following steps.

1. Calculate mean of the given interval $c = \frac{a+b}{2}$
2. Calculate $f(c)$
3. If $|f(c)| \leq \epsilon$,return c and stop iterating
4. Else examine sign of $f(c)$ and replace either $(a, f(a))$ or $(b, f(b))$ with $(c, f(c))$ so that there is a zero crossing within the new interval.

The number of iterations needed to converge to a root within a certain tolerance ϵ, n is

$$n = \log_2\left(\frac{\epsilon_0}{\epsilon}\right) \text{ where } \epsilon_0 = b - a$$

Flow Chart:


Start
Define f(x)
Get the value of interval (x_l,x_u), error or tolerance (n), x_m=(x_u+x_l)/2
Is product of f(x_l) and f(x_u) is negative
No
Yes
if |f(x_m)|>=n
No
Yes
x_m=(x_u+x_l)/2
Is product of f(x_l) and f(x_m) is negative
No
Yes
x_l=x_m
x_u=x_m
convergence condition not satisfied
return x_m
Stop

Description of Newton Raphson's Method:

Let $f(x)$ have a zero at α i.e.$f(\alpha) = 0$ and $f(x)$ is differentiable in a neighbourhood of α.

If $f(x)$ is continuously differentiable and its derivative and second derivatives are non zero at α then the convergence is quadratic. If the derivative is zero at α, then the convergence is usually only linear.

Let x_0 be initial guess for a root of the $f(x)$. If the function satisfies sufficient assumptions and the initial guess is close, then

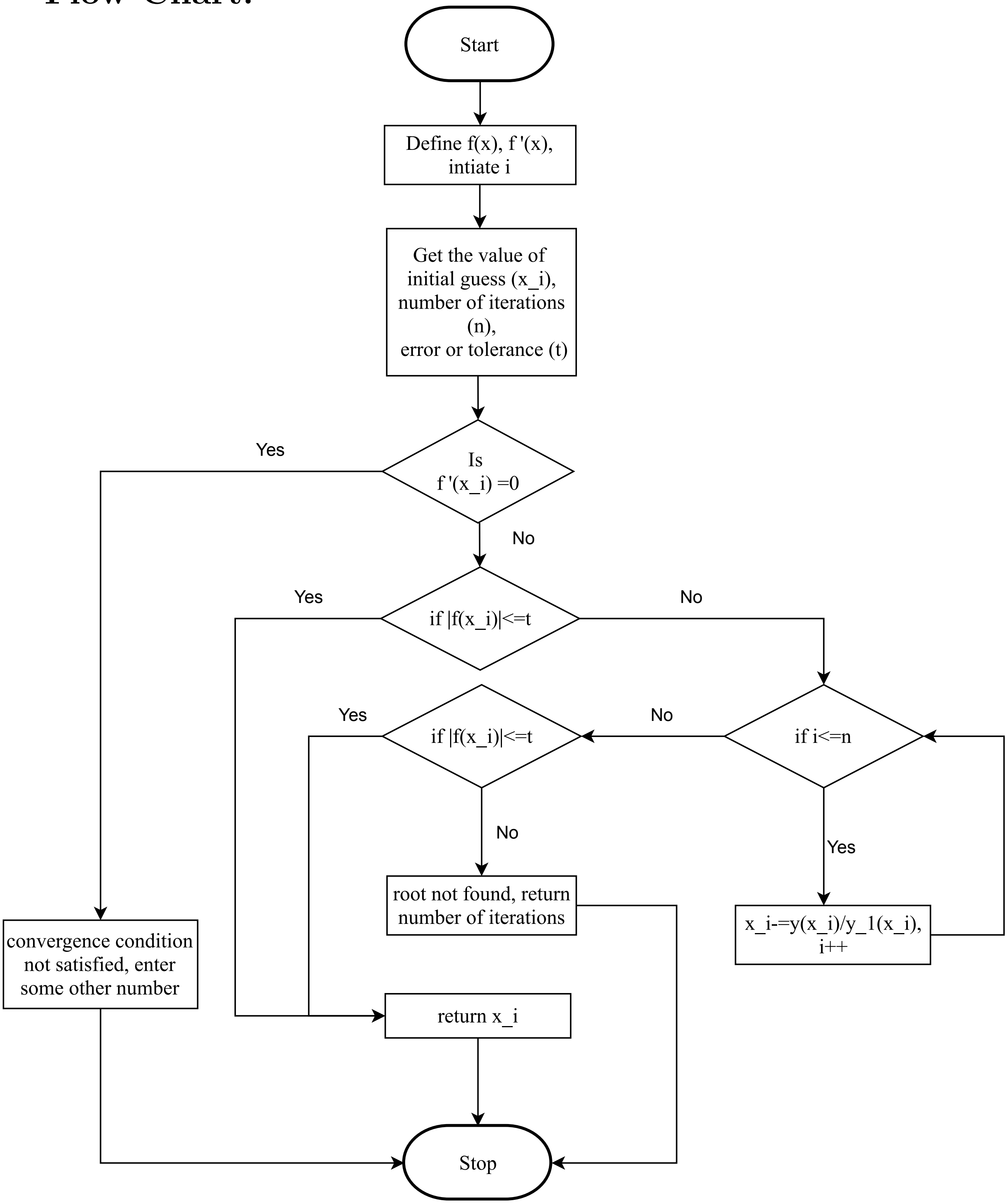
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

is better approximation at the initial point. The process is repeated as,

$$x_{i+1} = x_i - \frac{f(xi)}{f'(x_0)}$$

until sufficient precision is reached.

Flow Chart:



Start
Define f(x), f '(x), intiate i
Get the value of initial guess (x_i), number of iterations (n), error or tolerance (t)
Is f '(x_i) =0
Yes
No
if |f(x_i)|<=t
Yes
No
if i<=n
No
Yes
if |f(x_i)|<=t
Yes
No
root not found, return number of iterations
x_i-=y(x_i)/y_1(x_i), i++
convergence condition not satisfied, enter some other number
return x_i
Stop

Description of Successive Approximation or Fixed Point Iteration Method:

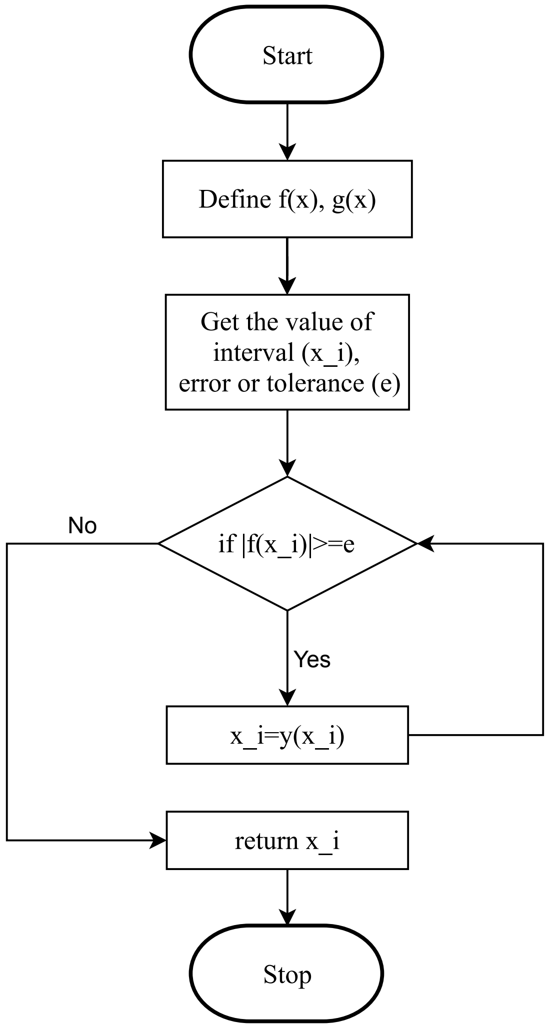
Let $f(x)$ be a function continuous, rewrite the equation $f(x) = 0$ in the form $x = g(x)$ in such a way that any solution of $x = g(x)$, which is a fixed point of $g(x)$, is a solution of $f(x)$.

Start from any point x_0 and consider the recursive process

$$x_{n+1} = g(x_n)$$

If $f(x_n)$ converges to some point c then it is clear that c is a fixed point of $g(x)$ and hence it is a solution of $f(x)$. Moreover x_n (for a large n) can be considered as an approximate solution of $f(x)$.

Flow Chart:


Start
Define f(x), g(x)
Get the value of
interval (x_i),
error or tolerance (e)
if |f(x_i)|>=e
No
Yes
x_i=y(x_i)
return x_i
Stop