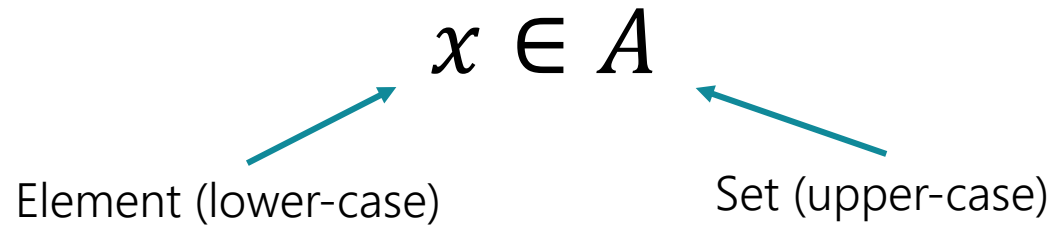# Bayesian Notation

A **set** is a collection of elements, which hold certain values. Additionally, every event has a set of outcomes that satisfy it.

The **null-set** (or **empty set**), denoted ∅, is an set which contain no values.

$$x \in A$$

Element (lower-case)   Set (upper-case)

Notation: Interpretation: Example:
$x \in A$ "Element x is a part of set A." $2 \in All\ even\ numbers$
$A \ni x$ "Set A contains element x." $All\ even\ numbers \ni 2$
$x \notin A$ "Element x is NOT a part of set A." $1 \notin All\ even\ numbers$
$\forall x:$ "For all/any x such that…" $\forall x: x \in All\ even\ numbers$
$A \subseteq B$ "A is a subset of B" $Even\ numbers \subseteq Integers$

Remember! Every set has at least 2 subsets.
- $A \subseteq A$
- $\emptyset \subseteq A$

365√DataScience

Multiple Events

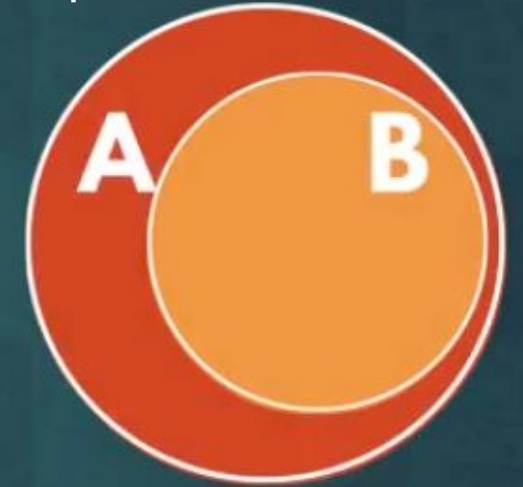The sets of outcomes that satisfy two events A and B can interact in one of the following **3** ways.

Not touch at all.

Intersect (Partially Overlap)

One completely overlaps the other.

A   B

A   B

A   B

Examples:
A -> Diamonds
B -> Hearts

Diamonds
Queens

Red Cards
Diamond

365√DataScience
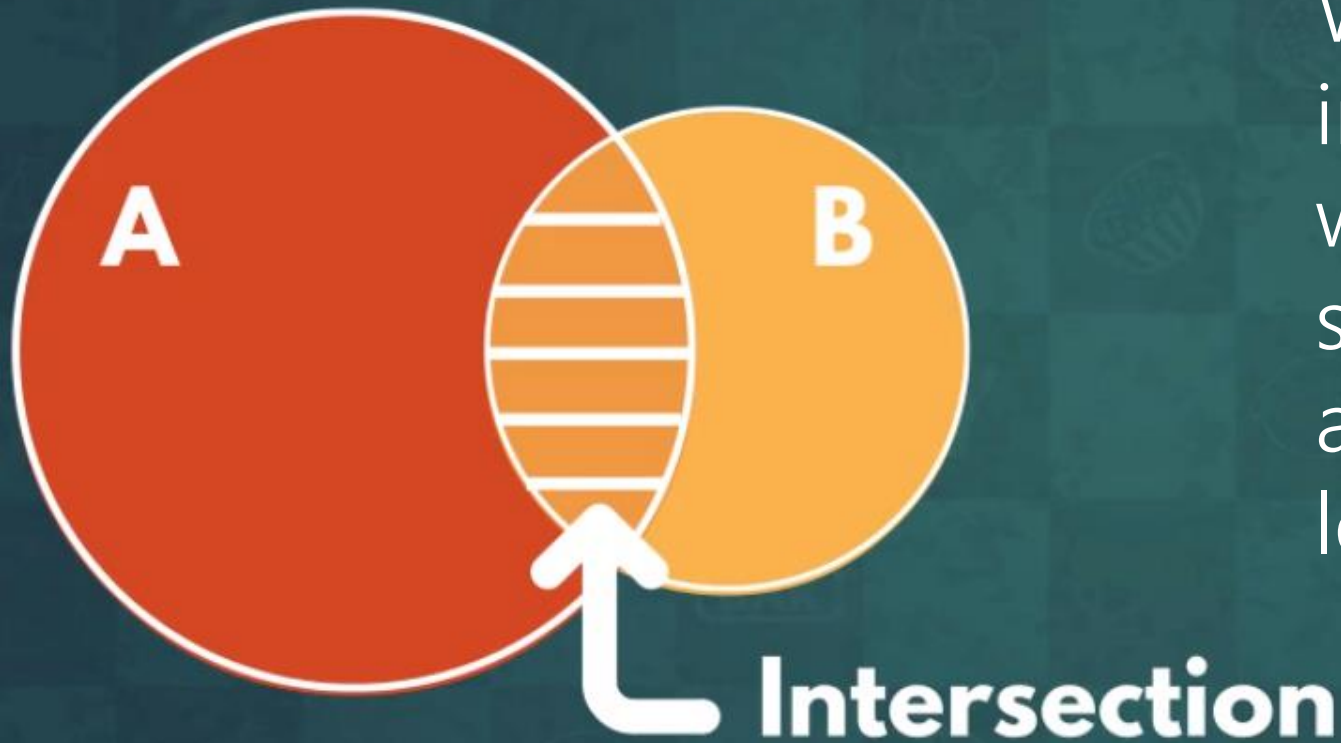
Intersection

The **intersection** of two or more events expresses the set of outcomes that satisfy all the events simultaneously. Graphically, this is the area where the sets intersect.
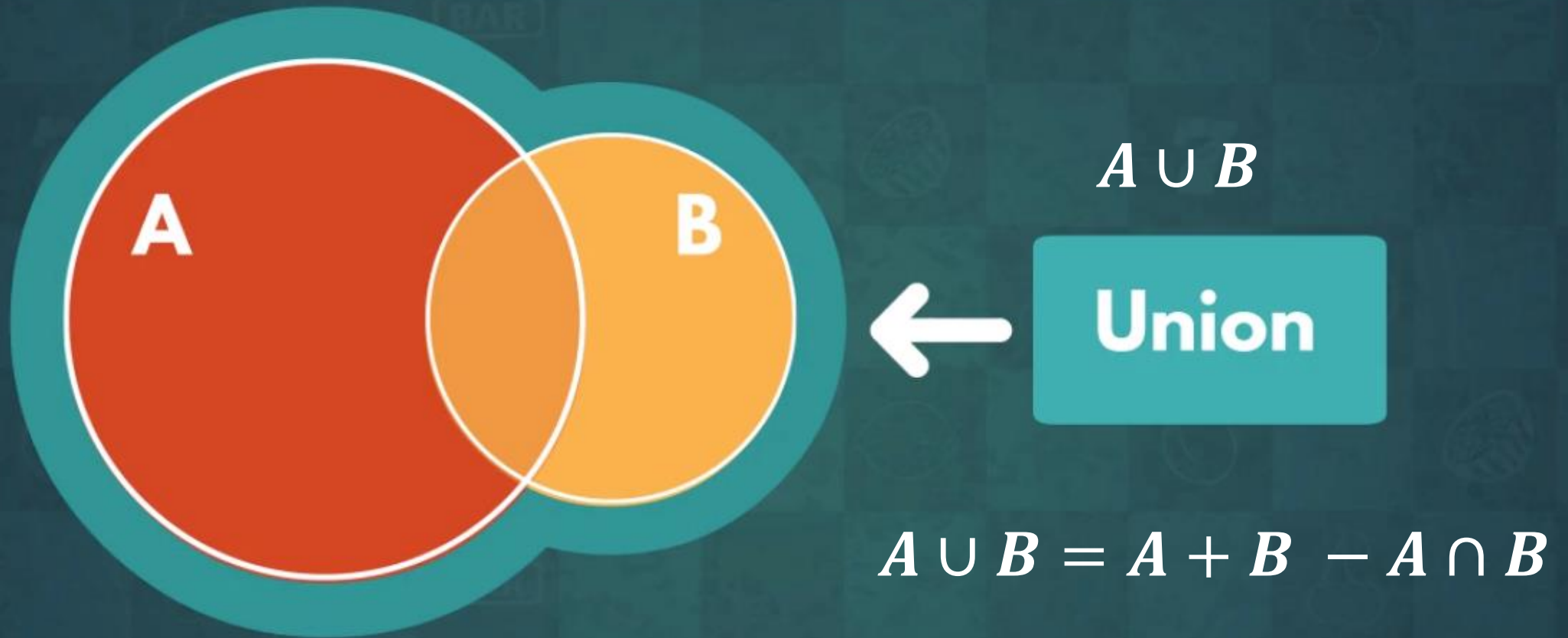

We denote the intersection of two sets with the "intersect" sign, which resembles an upside-down capital letter U:
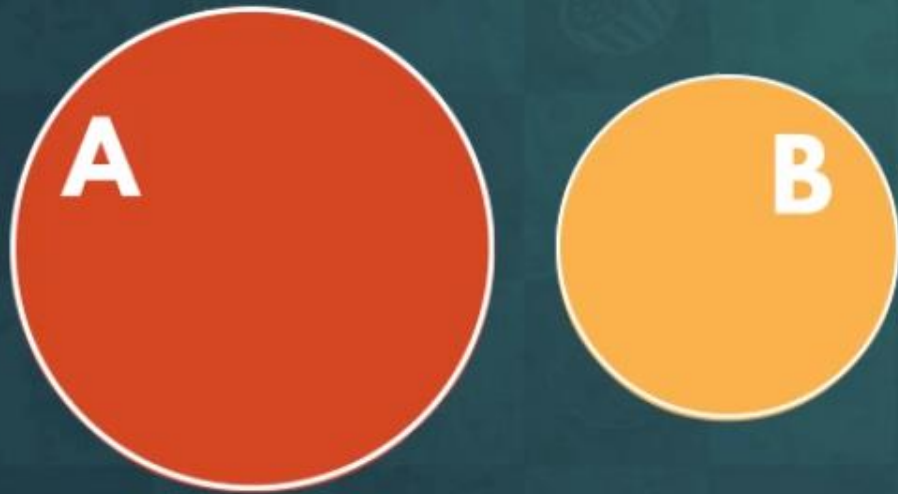
$$A \cap B$$

Union

The **union** of two or more events expresses the set of outcomes that satisfy at least one of the events. Graphically, this is the area that includes both sets.

$$A \cup B$$

Union

$$A \cup B = A + B - A \cap B$$

Mutually Exclusive Sets

Sets with no overlapping elements are called **mutually exclusive**. Graphically, their circles never touch.

If $A \cap B = \emptyset$, then the two sets are mutually exclusive.

Remember:
All complements are mutually exclusive, but not all mutually exclusive sets are complements.

Example:
Dogs and Cats are mutually exclusive sets, since no species is simultaneously a feline and a canine, but the two are not complements, since there exist other types of animals as well.

365 DataScience

If the likelihood of event A occurring (P(A)) is affected event B occurring, then we say that A and B are **dependent** events.

Alternatively, if it isn't – the two events are **independent**.

We express the probability of event A occurring, given event B has occurred the following way $\boldsymbol{P(A|B)}$.
We call this the conditional probability.

Independent:

- All the probabilities we have examined so far.

- The outcome of A does not depend on the outcome of B.

- $P(A|B) = P(A)$

Dependent

- New concept.

- The outcome of A depends on the outcome of B.

- $P(A|B) \neq P(A)$

Example

- A -> Hearts

- B -> Jacks

Example

- A -> Hearts

- B -> Red

Conditional Probability

For any two events A and B, such that the likelihood of B occurring is greater than 0 ($P(B) > 0$), the conditional probability formula states the following.

Probability of A, given B has occurred

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of the intersection.

Probability of event B

Intuition behind the formula:

- Only interested in the outcomes where B is satisfied.

- Only the elements in the intersection would satisfy A as well.

- Parallel to the "favoured over all" formula:

  - Intersection = "preferred outcomes"

  - B = "sample space"

Remember:

- Unlike the union or the intersection, changing the order of A and B in the conditional probability alters its meaning.

- $P(A|B)$ is not the same as $P(B|A)$, even if $P(A|B) = P(B|A)$ numerically.

- The two conditional probabilities possess **different meanings** even if they have equal values.

365√DataScience

Law of total probability

The **law of total probability** dictates that for any set A, which is a union of many mutually exclusive sets $B_1, B_2, \ldots, B_n$, its probability equals the following sum.

$$P(A) = P(A|B_1) \times P(B_1) + P(A|B_2) \times P(B_2) + \cdots + P(A|B_n) \times P(B_n)$$

Conditional Probability of A, given B_1 has occurred.

Probability of B_1 occurring.

Conditional Probability of A, given B_2 has occurred.

Probability of B_2 occurring.

Intuition behind the formula:

- Since P(A) is the union of mutually exclusive sets, so it equals the **sum of the individual sets**.

- The **intersection** of a union and one of its subsets is the entire subset.

- We can rewrite the conditional probability formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$ to get $P(A \cap B) = P(A|B) \times P(B)$.

- Another way to express the law of total probability is:

  - $P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)$

Additive Law

The additive law calculates the probability of the union based on the probability of the individual sets it accounts for.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability of the union

Probability of the intersection

Intuition behind the formula

- Recall the formula for finding the size of the Union using the size of the Intersection:

  - $A \cup B = A + B - A \cap B$

- The probability of each one is simply its size over the size of the sample space.

- This holds true for any events A and B.

365√DataScience

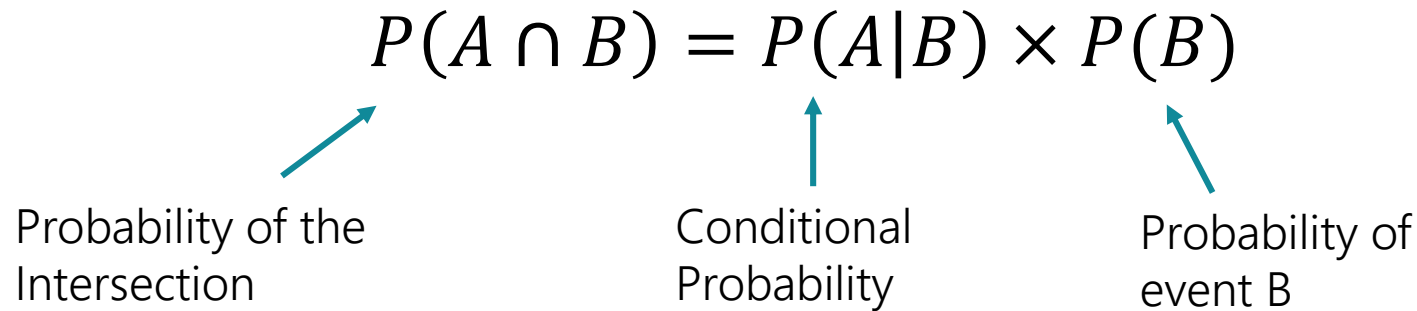The multiplication rule calculates the probability of the intersection based on the conditional probability.

$$P(A \cap B) = P(A|B) \times P(B)$$

Probability of the
Intersection

Conditional
Probability

Probability of
event B

Intuition behind the formula

- We can multiply both sides of the conditional probability formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$ by $P(B)$ to get $P(A \cap B) = P(A|B) \times P(B)$.

- If event B occurs in 40% of the time ($P(B) = 0.4$) and event A occurs in 50% of the time B occurs ($P(A|B) = 0.5$), then they would simultaneously occur in 20% of the time ($P(A|B) \times P(B) = 0.5 \times 0.4 = 0.2$).

Bayes' Law

Bayes' Law helps us understand the relationship between two events by computing the different conditional probabilities.

We also call it Bayes' Rule or Bayes' Theorem.

Conditional probability of B, given A.

Conditional probability of A, given B.

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Intuition behind the formula

- According to the multiplication rule $P(A \cap B) = P(A|B) \times P(B)$, so $P(B \cap A) = P(B|A) \times P(A)$.

- Since $P(A \cap B) = P(B \cap A)$, we plug in $P(B|A) \times P(A)$ for $P(A \cap B)$ in the conditional probability formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$

- Bayes' Law is often used in medical or business analysis to determine which of two symptoms affects the other one more.