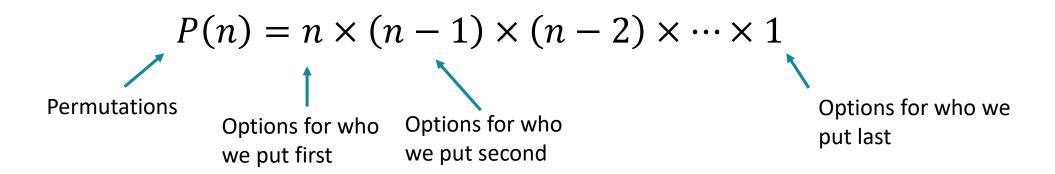
## **Permutations**

Permutations represent the number of different possible ways we can arrange a number of elements.



#### Characteristics of Permutations:

- Arranging all elements within the sample space.
- No repetition.
- $P(n) = n \times (n-1) \times (n-2) \times \cdots \times 1 = n!$  (Called "n factorial")

#### **Example:**

If we need to arrange 5 people, we would have P(5) = 120 ways of doing so.

# **Factorials**

Factorials express the **product** of all integers from 1 to n and we denote them with the "!" symbol.

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 1$$

### Key Values:

- 0! = 1.
- If n<0, n! does not exist.</li>

Rules for factorial multiplication. (For n>0 and n>k)

• 
$$(n+k)! = n! \times (n+1) \times \cdots \times (n+k)$$

• 
$$(n-k)! = \frac{n!}{(n-k+1)\times\cdots\times(n-k+k)} = \frac{n!}{(n-k+1)\times\cdots\times n}$$

• 
$$\frac{n!}{k!} = \frac{k! \times (k+1) \times \cdots \times n}{k!} = (k+1) \times \cdots \times n$$

Examples: n = 7, k = 4

• 
$$(7+4)! = 11! = 7! \times 8 \times 9 \times 10 \times 11$$

• 
$$(7-4)! = 3! = \frac{7!}{4 \times 5 \times 6 \times 7}$$

• 
$$\frac{7!}{4!} = 5 \times 6 \times 7$$

## **Variations**

Variations represent the number of different possible ways we can pick and arrange a number of elements.

### Intuition behind the formula. (With Repetition)

- We have n-many options for the first element.
- We **still have n-many options** for the second element because repetition is allowed.
- We have n-many options for each of the p-many elements.
- $n \times n \times n \dots n = n^p$

### Intuition behind the formula. (Without Repetition)

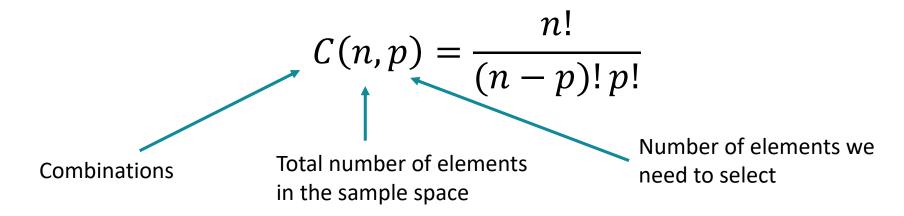
- We have n-many options for the first element.
- We **only have (n-1)-many options** for the second element because we cannot repeat the value for we chose to start with.
- We have less options left for each additional element.

• 
$$n \times (n-1) \times (n-2) \dots (n-p+1) = \frac{n!}{(n-p)!}$$



## **Combinations**

Combinations represent the number of different possible ways we can **pick** a number of elements.



#### **Characteristics of Combinations:**

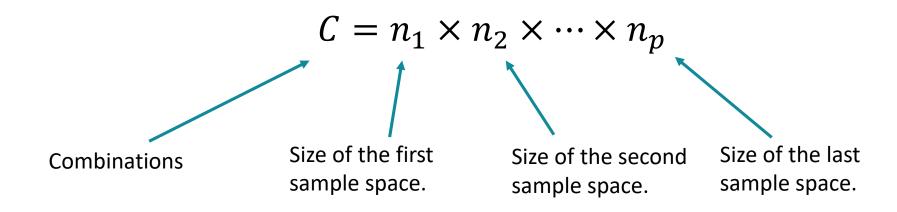
- Takes into account double-counting. (Selecting Johny, Kate and Marie is the same as selecting Marie, Kate and Johny)
- All the different permutations of a single combination are different variations.

• 
$$C = \frac{V}{P} = \frac{n!/(n-p)!}{p!} = \frac{n!}{(n-p)!p!}$$

• Combinations are symmetric, so  $C_p^n = C_{n-p}^n$ , since selecting p elements is the same as omitting n-p elements.

# **Combinations with separate sample spaces**

Combinations represent the number of different possible ways we can **pick** a number of elements.



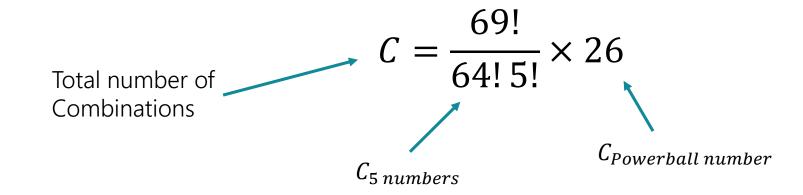
## Characteristics of Combinations with separate sample spaces:

- The option we choose for any element does not affect the number of options for the other elements.
- The order in which we pick the individual elements is arbitrary.
- We need to know the size of the sample space for each individual element.  $(n_1, n_2 ... n_p)$

# Winning the Lottery

To win the lottery, you need to satisfy two distinct independent events:

- Correctly guess the "Powerball" number. (From 1 to 26)
- Correctly guess the 5 regular numbers. (From 1 to 69)



#### Intuition behind the formula:

- We consider the two distinct events as a combination of two elements with different sample sizes.
  - One event has a sample size of 26, the other has a sample size of  $C_5^{69}$ .
- Using the "favoured over all" formula, we find the probability of any single ticket winning equals  $1/(\frac{69!}{64!5!} \times 26)$ .