HPSC 101 — Lecture 3

This lecture:

- computing square roots
- Python demo
- git demo

Computing square roots

Hardware arithmetic units can add, subtract, multiply, divide.

Other mathematical functions usually take some software.

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Other mathematical functions usually take some software.

Example: Compute $\sqrt{2} \approx 1.4142135623730951$

In most languages, sqrt(2) computes this.

```
>>> from numpy import sqrt
>>> sqrt(2.)
```

One possible algorithm to approximate $s = \sqrt{x}$

```
s = 1.     # or some better initial guess
for k in range(kmax):
    s = 0.5 * (s + x/s)
```

where `kmax` is some maximum number of iterations.

Note: In Python, range(N) is $[0, 1, 2, , \ldots, N-1]$.

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Why this works...

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In fact this is Newton's method to find root of $s^2 - x = 0$.

Newton's method

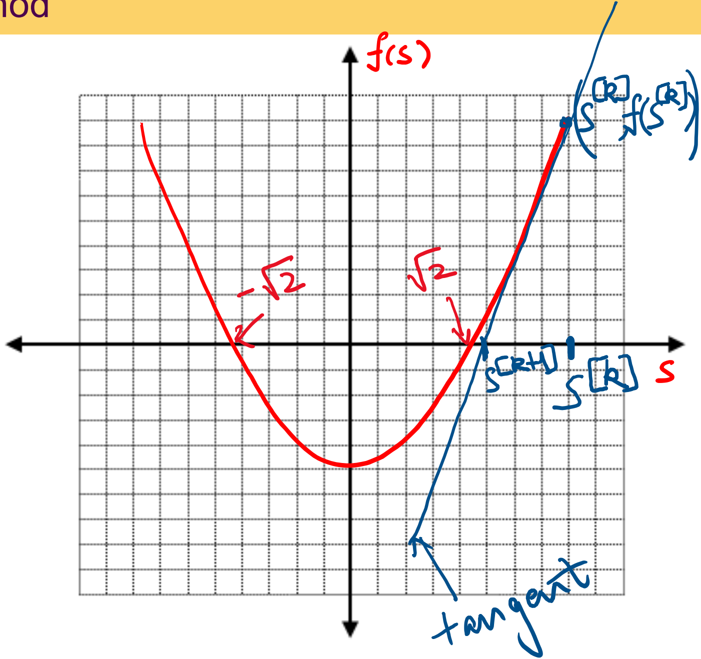
Problem: Find a solution of $f(s) = 0$ (zero or root of f)

Idea: Given approximation $s^{[k]}$,
approximate $f(s)$ by a linear function,
the tangent line at $(s^{[k]}, f(s^{[k]}))$.

Find unique zero of this function and use as $s^{[k+1]}$.

Newton's method

$f(s) = s^2 - x$



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Updating formula:

$$s^{[k+1]} = s^{[k]} - \frac{f(s^{[k]})}{f'(s^{[k]})}$$

$$s = s - \frac{s^2 - x}{2s} = \frac{1}{2}\left(s + \frac{x}{s}\right)$$

Demo...

Goals:

- Develop our own version of `sqrt` function.
- Start simple and add complexity in stages.
- Illustrate some Python programming.
- Illustrate use of git to track our development