Math 351: Exam 1B (Take Home)

Write your name in the top right

Due: February 25, 2019

Problem 1. (5+5+10 = 20 points)

Let Y_1 be a random sample from Binomial $(3, 1-\theta)$ and Y_2 be a random sample from Binomial $(4, \theta)$, where θ is unknown. Also, Y_1 and Y_2 are independent.

- (a) Use moment generating function to find the distribution of $3 - Y_1$.
- (b) Use moment generating function to find the distribution of $3 - Y_1 + Y_2$
- (c) Find an **unbiased estimator** of θ^2 based on Y_1 and Y_2 (*The unbiased estimator should involve both* Y_1 *and* Y_2)

Problem 2. (10 points)

X and Y have the joint probability density function given by $f_{X,Y}(x,y) = 4y(x-y)e^{-x-y}$ if $0 \leq x < \infty$, $0 \leq y \leq x$ and $f_{X,Y}(x,y) = 0$ otherwise.

- (a) Find the conditional density of $Y|X = x$
- (b) Compute $E(Y|X = x)$

Problem 3. (20 points)

X and Y are independently and uniformly distributed over the interval $(0,1)$ i.e. X and Y are i.i.d $U(0,1)$

- (a) Find $P(X \leq x, Y \leq y)$ for $0 < x < 1, 0 < y < 1$
- (b) Find $P(|X - Y| \leq 0.25)$
- (c) Find $P(\min\{X, Y\} \leq 0.25)$
- (d) Find $P(\max\{X, Y\} \leq 0.25)$