Mechanics

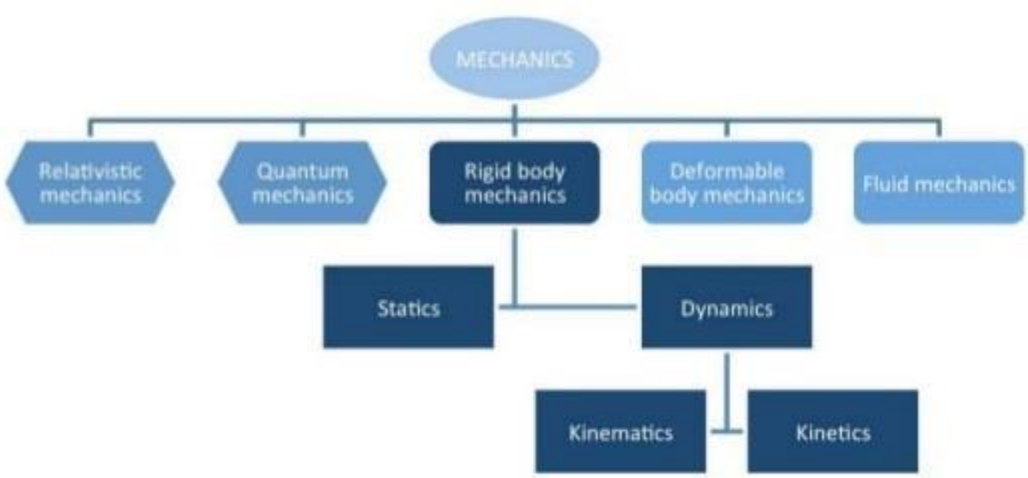
Key Ideas:

1. Mechanics
2. Classification Mechanics
3. Motion
4. Projectile & Projectile Motion
5. Circular Motion
6. Newton's Laws of Motion (1st, 2nd, 3rd)
7. Momentum
8. Friction
9. Work and Energy (Kinetic, Potential)

Mechanics

Mechanics is the branch of Physics dealing with the study of motion. No matter what your interest in science or engineering, mechanics will be important for you - motion is a fundamental idea in all of science.

Classification of Mechanics



Statics: dealing with describing bodies with no motion or rest.

Kinematics: dealing with describing motions, and

Dynamics: dealing with the causes of motion.

In physics, classical mechanics and quantum mechanics are the two major sub-fields of mechanics. Classical mechanics is concerned with the set of physical laws describing the motion of bodies under the action of a system of forces. The study of the motion of bodies is an ancient one, making classical mechanics one of the oldest and largest subjects in science, engineering and technology. It is also widely known as **Newtonian mechanics**.

Classical mechanics describes the motion of macroscopic objects, from projectiles to parts of machinery, as well as astronomical objects, such as **spacecraft, planets, stars, and galaxies**. Besides this, many specializations within the subject deal with solids, liquids and gases and other

Specific sub-topics. **Classical mechanics also provides extremely accurate results as long as the domain of study is restricted to large objects and the speeds involved do not approach the speed of light.**

When the objects being dealt with become **sufficiently small**, it becomes necessary to introduce the other major sub-field of mechanics, **quantum mechanics**, which reconciles the macroscopic laws of physics with the atomic nature of matter and handles the wave–particle duality of atoms and molecules.

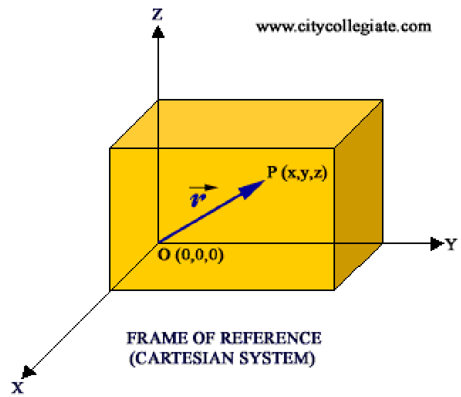
Motion

Motion is defined as change in position of a body over time with respect to the surrounding.

Reference Frame

For describing the motion of a body the co-ordinate system used in specific three dimensional space and in respect of which the motion of the body is called reference frame.

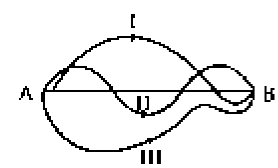
Examples: A long rod, a long thin thread, a hanging etc. (**One dimensional**), A moving football in the field, Thin Paper, Thin metal sheet etc. (**two**), Table, chair, brick, stone etc. (**three**)



Different quantities related to motion

1. Displacement : Change of position of a body with respect to time in a particular direction is called displacement of the body. Displacement is a vector quantity. So, it has both magnitude and direction.

Although this particle moves in different paths but displacement in each case = AB. Length of the straight line and direction of displacement is from A to B. That means, the displacement of the particle is path-independent, depends only on initial and final position.



Unit and dimension of displacement : In S.I. method unit of displacement is metre (m) and dimension is [L].

2. Speed : Distance travelled by a moving body in one second is called speed of that body.

If a particle travels distance s in t seconds, then speed of that particle, $u = \frac{s}{t}$.

Speed is a scalar quantity. It has magnitude but no direction.

Unit and dimension of speed : In S. I. method unit of speed is metre/second (ms^{-1}). Its dimension $= \frac{[L]}{[T]} = \boxed{[LT^{-1}]}$

3. Average speed : Division of total distance travelled by a body to the total expended time is called average speed.

Explanation : Suppose a moving body travels total distance s in total time t.

$$\therefore \quad \text{Average speed, } \bar{v} = \frac{\text{total distance traversed}}{\text{total expended time}} = \frac{s}{t}$$

If a body in each unit time travels equal distance i.e., the body is in uniform speed, then speed and average speed of the body become same.

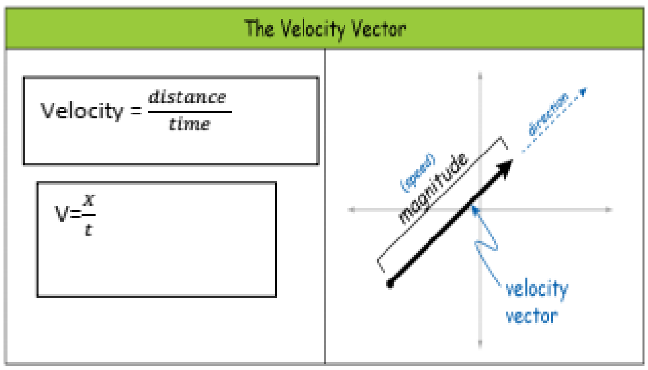
4. Instantaneous speed : If time intervel tends to zero, then rate of change of distance traversed is called instantaneous speed or speed. According to the rule of calculus, if a body travels distance Δs in time Δt, then instantaneous speed will be,

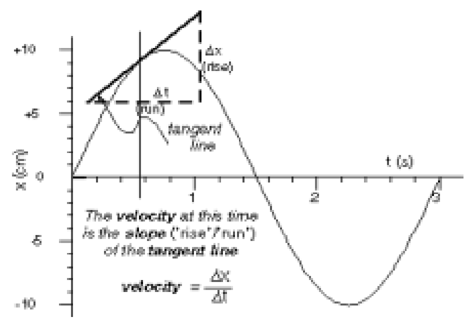
$$v = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

If the body is in uniform speed, then in each instance instantaneous speed of the body becomes same.

Velocity

Velocity is defined as the rate of which displacement changes over time. The higher the velocity, the faster a body is moving. It is a vector quantity





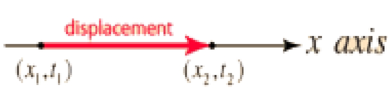
In terms of mathematics, the most general definition of velocity is, $v = \frac{dx}{dt}$. What this means is that velocity is the derivative of displacement (x) with respect to time.

Average Velocity

The average speed of an object is defined as the distance traveled divided by the time elapsed.

Velocity is a vector quantity, and **average velocity** can be defined as the displacement divided by the time. For the special case of straight line motion in the x direction,

The average velocity takes the form:

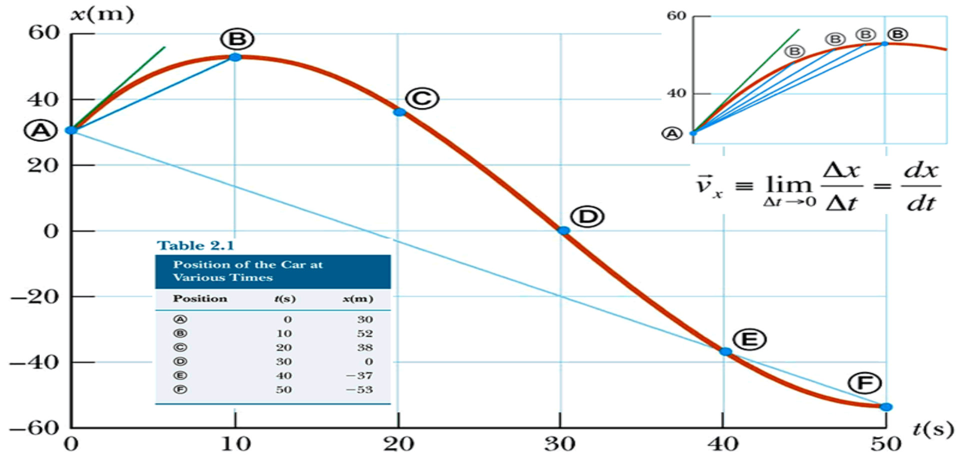


$$v_{average} = \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

The units for velocity can be implied from the definition to be meters/second or in general any distance unit over any time unit.

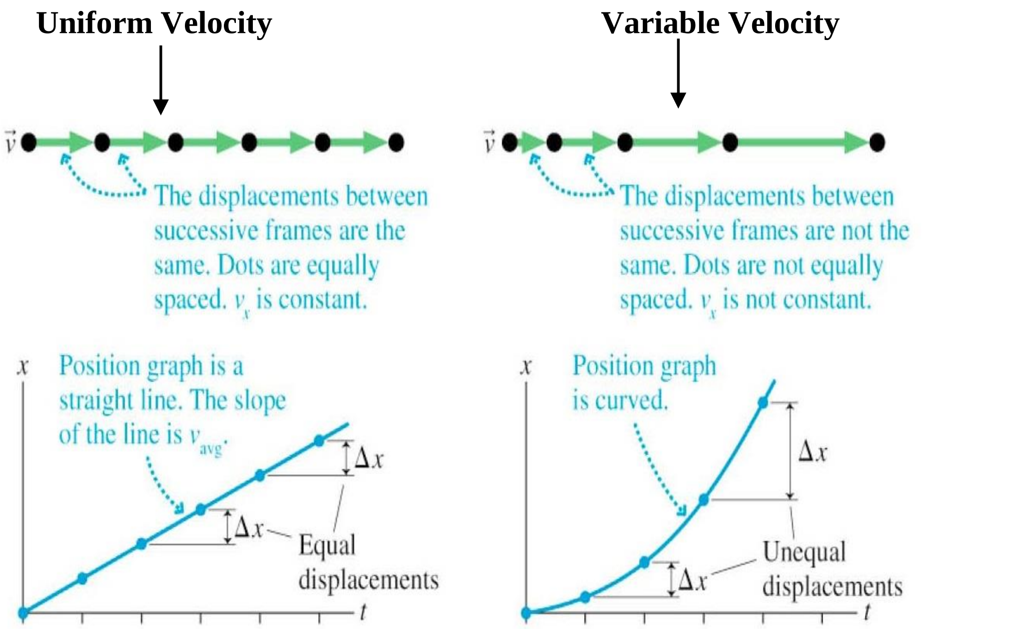
You can approach an expression for the **instantaneous velocity** at any point on the path by taking the limit as the time interval gets smaller and smaller. Such a **limiting process** is called a **derivative** and the instantaneous velocity can be defined as

$$v_{\text{instantaneous}} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



Instantaneous Velocity

Uniform Velocity and variable Velocity



Acceleration

Acceleration is the rate of change of velocity with time. A body with a positive acceleration is gaining velocity over time. A body with a negative acceleration is losing velocity over time.

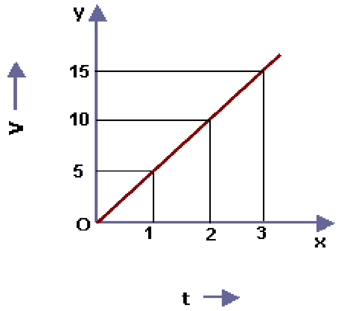
It is denoted by a.

$$a = \frac{Change\ in\ Velocity}{Time\ taken} = \frac{dv}{dt}$$

Uniform Acceleration

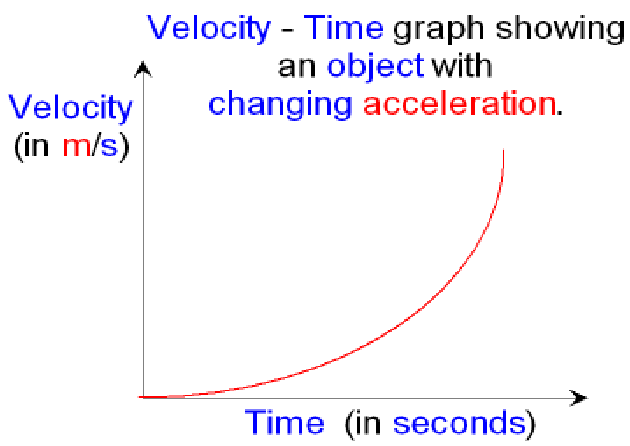
If the acceleration remains always constant, then that acceleration is called uniform acceleration

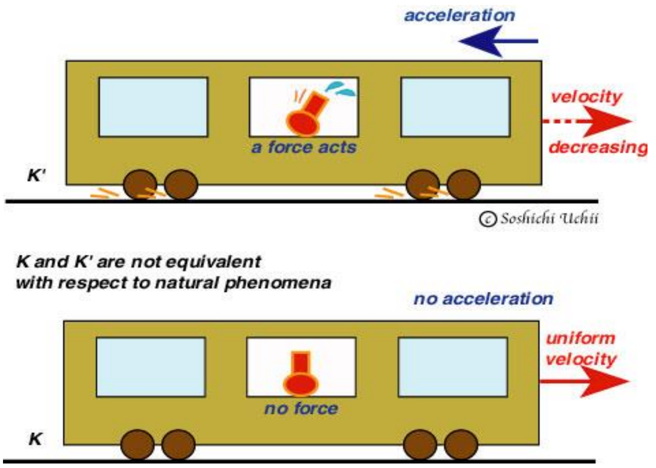
The uniform acceleration of a body is 10 ms^{-2} means that the velocity of the body changes in each second by 10 ms^{-1} in the same direction.



Variable Acceleration

When the acceleration of a body changes with time, the acceleration is called variable acceleration. The acceleration of bus, train, car etc is examples of variable acceleration.





Average acceleration

Average acceleration is determined over a "long" time interval. The word long in this context means finite — something with a beginning and an end. The velocity at the beginning of this interval is called the initial velocity, represented by the symbol $\mathbf{v_0}$, and the velocity at the end is called the final velocity, represented by the symbol **v** .

Average acceleration is a quantity calculated from two velocity measurements.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{\Delta t}$$

Instantaneous acceleration

Instantaneous acceleration is measured over a "short" time interval. The word short in this context means infinitely small or infinitesimal — having no duration or extent whatsoever. It's a mathematical ideal that can only be realized as a limit.

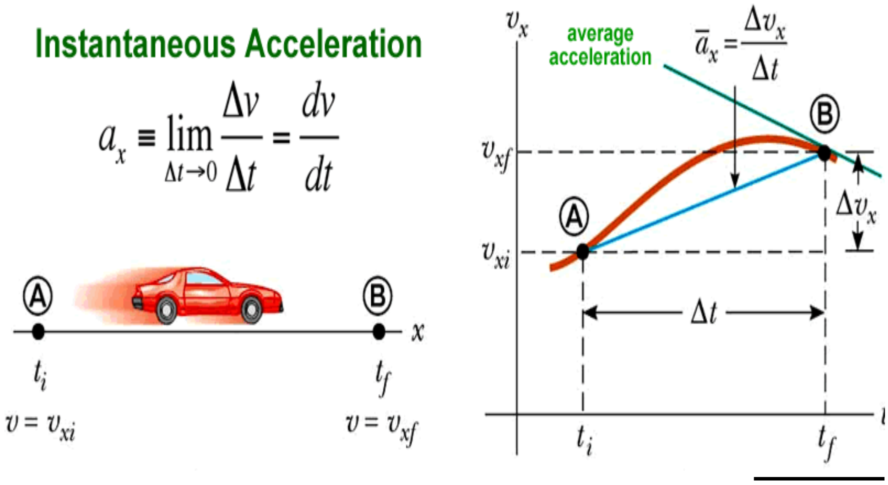
The limit of a rate as the denominator approaches zero is called a derivative.

Instantaneous acceleration is then the limit of average acceleration as the time interval approaches zero — or alternatively, acceleration is the derivative of velocity.

$$a = lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Acceleration is the derivative of velocity with time, but velocity is itself the derivative of displacement with time. The derivative is a mathematical operation that can be applied multiple times to a pair of changing quantities. Doing it once gives you a first derivative. Doing it twice (the derivative of a derivative) gives you a second derivative. That makes acceleration the first derivative of velocity with time and the second derivative of displacement with time.

$$a = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$$



Equations of motion

(i) $\mathbf{v = v_o + at}$

Consider a particles moving with uniform acceleration a. From definition

$a = \frac{dv}{dt}$

$dv = adt$ ……………… (1)

Integrating both sides

$\int dv = a \int dt$

$v = at + k$ ………….. (2)

Let v_o be the initial velocity at time t = 0 from equation (2), we get

$v_o = k$

Submitting this value in equation number (2)

$v = v_o + at$

(ii) $\mathbf{s = v_o t + \frac{1}{2} at^2}$

Suppose at any instant of time, the velocity of the particle is v. The distance covered by the particle in an interval of time is dt, then

$ds = v\, dt$ ……………. (1)

But, $v = v_o + at$

$ds = (v_o + at)dt$

$ds = v_o dt + at\, dt$

Integrating this equation

$\int ds = v_o \int dt + at \int dt$

$s = v_o t + \frac{1}{2} at^2 + k$

When, t = 0, s = 0, we get k = 0

$s = v_o t + \frac{1}{2} at^2$

(iii) $\mathbf{v^2 = v_o^2 + 2as}$

We know that

$v = v_o + at$

$at = v - v_o$ ………… (1)

The distance travelled

$s = \frac{v + v_o}{2} t$

$\frac{2s}{t} = v + v_o$ …………… (2)

Multiplying (1) and (2)

$at \times \frac{2s}{t} = (v - v_o)(v + v_o)$

$2as = v^2 - v_o^2$

$v^2 = v_o^2 + 2as$

In case of free falling bodies

$v = v_o + gt$

$v^2 = v_o^2 + 2gh$

$h = v_o t + \frac{1}{2} g t^2$

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t \qquad v^2 = u^2 + 2as$$

a = acceleration
v = final velocity
u = initial velocity
t = time taken
s = displacement

Newton's Laws of Motion

Newton's laws of motion are three physical laws that, together, laid the foundation for classical mechanics. They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces.

Newton's Laws of Motion

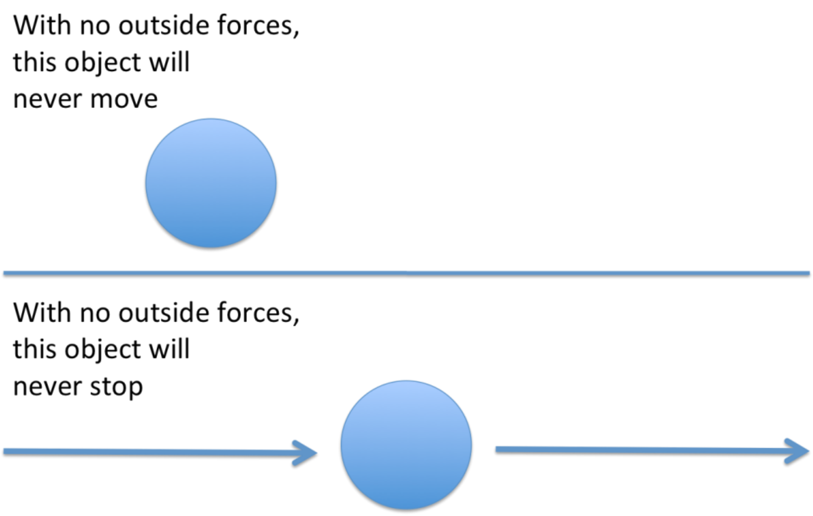
Isaac Newton was an English Scientist

In 1667, he developed 3 laws of motion that described movement of objects in terms of forces

These laws of motion still hold true today

First law

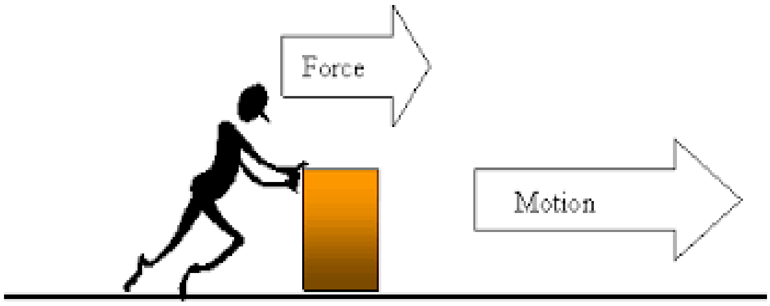
"Each body, in this universe continues to be in its state of rest or uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it."



Second law

"The rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the force."

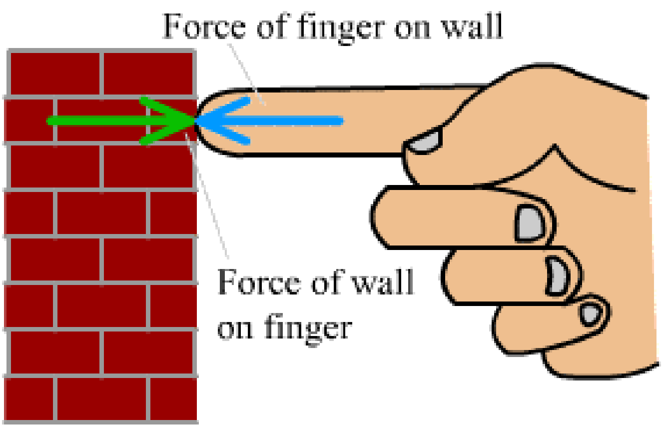
The vector sum of the external forces F on an object is equal to the mass m of that object multiplied by the acceleration vector a of the object: F = ma.

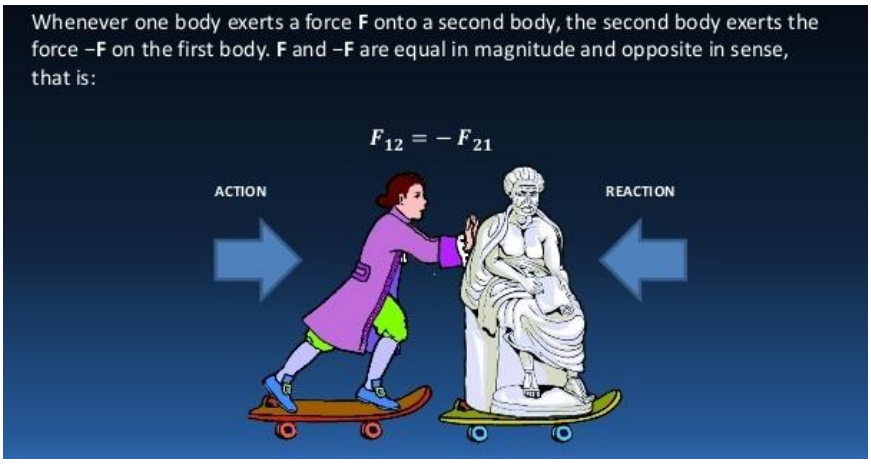


Third law

"To every action there is always an equal and opposite reaction."

When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.





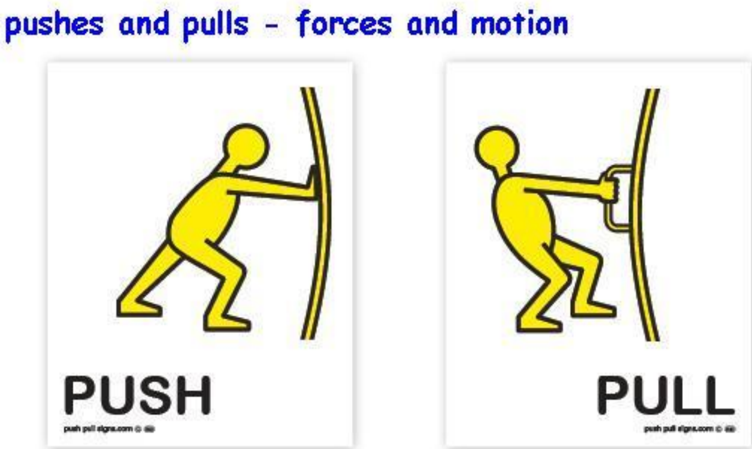
The three laws of motion were first compiled by Isaac Newton in his Philosophiæ Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy), first published in 1687. Newton used them to explain and investigate the motion of many physical objects and systems.

For example, in the third volume of the text, Newton showed that these laws of motion, combined with his law of universal gravitation, explained Kepler's laws of planetary motion.

The first law gives the definition of force, the second law gives a measure of the force and third law specifies the property of force.

Force

Force is defined as that, external agency that changes or tends to change the state of rest or uniform motion of a body in a straight line. The first law of motion is also called the “law of inertia.”



Momentum

The momentum of a particle is defined as the product of its mass times its velocity. It is a vector quantity. The momentum of a system is the vector sum of the momenta of the objects which make up the system.

Like velocity, linear momentum is a vector quantity, possessing a direction as well as a magnitude:

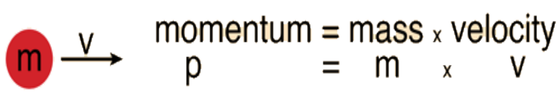
$$\mathbf{p} = m\mathbf{v},$$

Where p is the three-dimensional vector stating the object's momentum in the three directions of three-dimensional space, v is the three-dimensional velocity vector giving the object's rate of movement in each direction, and *m* is the object's mass.

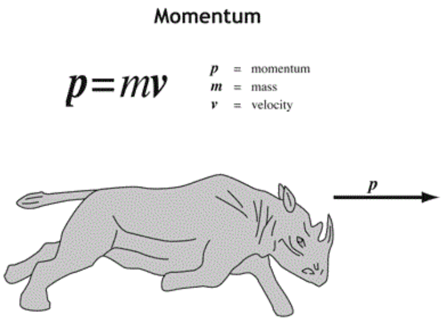
If the system is an isolated system, then the momentum of the system is a constant of the motion and subject to the principle of conservation of momentum.

The basic definition of momentum applies even at relativistic velocities but then the mass is taken to be the relativistic mass.

The most common symbol for momentum is p. The SI unit for momentum is kg m/s.



Linear momentum is also a *conserved* quantity, meaning that if a closed system is not affected by external forces, its total linear momentum cannot change.



The charging rhinoceros has a great deal of momentum because of its large mass and high velocity.

Impulse

The product of average force and the time it is exerted is called the impulse of force. From Newton's second law

$$F_{average} = ma_{average} = m\frac{\Delta v}{\Delta t}$$

Newton's Second Law can be rearranged to define the impulse, *J*, delivered by a constant force, *F*.

Impulse is a vector quantity defined as the product of the force acting on a body and the time interval during which the force is exerted. If the force changes during the time interval, *F* is the average net force over that time interval. The impulse caused by a force during a specific time interval is equal to the body's change of momentum during that time interval: impulse, effectively, is a measure of change in momentum.

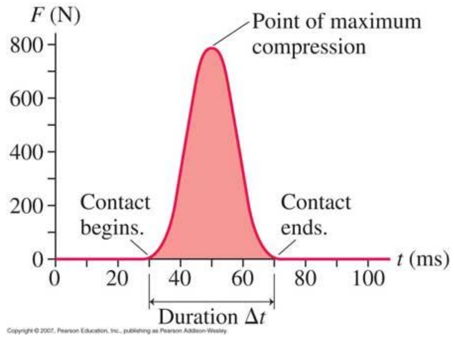
$$\boldsymbol{J} = \boldsymbol{F}\Delta t = \Delta \boldsymbol{p}$$

a small force applied for a long time produces the same change in momentum—the same impulse—as a larger force applied briefly.

$$J = F_{average}(t_2 - t_1)$$

The impulse is the integral of the resultant force (*F*) with respect to time:

$$J = \int F dt$$



Impulse

Newton's Form of Second Law:

$$\vec{F} = \frac{d\,\vec{p}}{d\,t} = \frac{\Delta\vec{p}}{\Delta t}$$

Impulse Form:

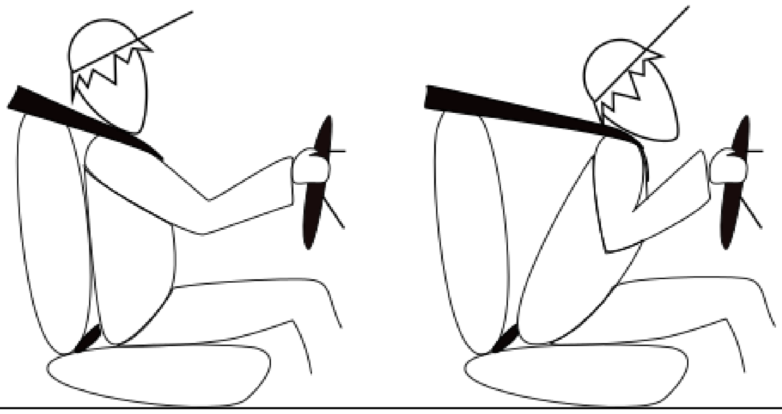
$$\Delta\vec{p} = \int \vec{F}\,dt$$

$$\Delta\vec{p} = F\,\Delta t$$

Inertia

Inertia is the resistance of any physical object to any change in its state of motion (this includes changes to its speed, direction or state of rest). It is the tendency of objects to keep moving in a straight line at constant velocity.

The principle of inertia is one of the fundamental principles of classical physics that are used to describe the motion of objects and how they are affected by applied forces. Inertia comes from the Latin word, iners, meaning idle, sluggish. Inertia is one of the primary manifestations of mass, which is a quantitative property of physical systems. Isaac Newton defined inertia as his first law in his Philosophiæ Naturalis Principia Mathematica.



Car is in motion	**Car Stopping**

Principle of Conservation of Linear Momentum

Consider two particles in an isolated system. These two particles interact with each other and no external forces are acting on the system. In such a case, the momentum of the system will remain constant but the momentum of each particle may change due to the interaction.

“The vector sum of the linear momentum of all the particles in an isolated system remains constant in the absence of any external forces.”

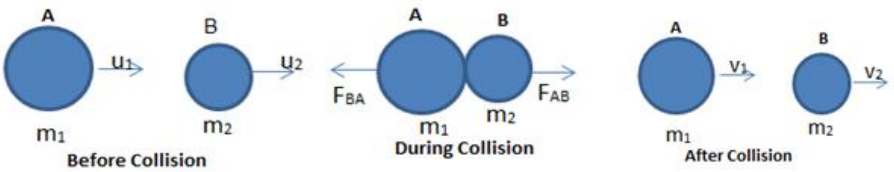
Proof

Let two particles A and B of mass m_1 and m_2 move in the same direction in straight line with velocity u_1 and u_2 respectively. Here ($u_1 > u_2$).

At one time the first particle (A) will hit the second particle (B) and then two particles will continue moving in the same direction and along the same line with velocities v_1 and v_2 respectively.

Let the time of action and reaction due to collision is t. Thus, the resultant of initial momentum of the two particles

$$P_1 = (m_1u_1 + m_2u_2)$$



The resultant of the final momentum of the particles

$$P_2 = m_1v_1 + m_2v_2$$

Rate of change of momentum of the first particle = $\frac{m_1v_1 - m_1u_1}{t}$

Rate of change of momentum of the second particle = $\frac{m_2v_2 - m_2u_2}{t}$

From Newton’s third law

$$\text{Applied Force} = -\text{ Reaction Force}$$

$$F_{AB} = -F_{BA}$$

$$\frac{dP_{AB}}{dt} = -\frac{dP_{BA}}{dt}$$

$$\frac{dP_{AB}}{dt} + \frac{dP_{BA}}{dt} = 0$$

$$\frac{dP}{dt} = 0$$

where, $P=P_{AB}+P_{BA}$

$$so,\ P=constant$$

that means initial momentum and final momentum of the particles are same

$$P_1=P_2$$

$$i.e.,\ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Summation or resultant of the initial momentum of the two particles = Summation or resultant of the final momentum of the particles. Hence the conservation principle is proved.