Problems related to Normal distribution

Suppose that the life lengths of two electric devices, say D_1 and D_2 , have distributions $N(40, 36)$ and $N(45, 9)$ respectively. (i) If the electric device is to be used for a 45 hours period, which device is to be preferred? (ii) If it is to be used for a 48 hours period, which device is to be preferred?

(i) Let X be the life length of device D_1

Let Y " " " " " D_2

$N(\mu, \sigma^2, x)$ ✓

$N(\mu, \sigma^2)$ ✓

$\mu_1 = 40, \quad \sigma_1^2 = 36 \quad] \; D_1$

$\mu_2 = 45, \quad \sigma_2^2 = 9 \quad] \; D_2$

$$\therefore \; P[X \geq 45] = 1 - P[X < 45]$$

$$= 1 - P[\sigma_1 Z_1 + \mu_1 < 45]$$

$$= 1 - P[6 Z_1 + 40 < 45]$$

$$Z_1 = \frac{X - \mu_1}{\sigma_1}$$

$$\boxed{X = \sigma_1 Z_1 + \mu_1}$$

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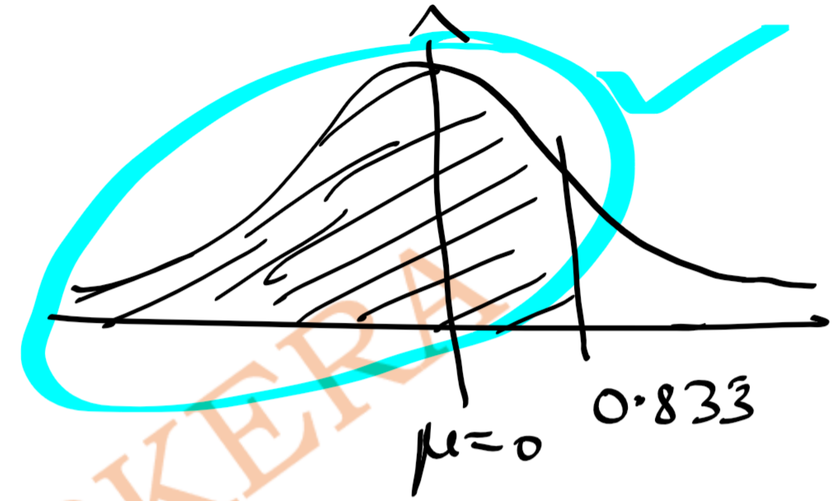
$\therefore P[X \geq 45] = 1 - P[Z_1 < \frac{5}{6}]$

$= 1 - P[Z_1 < 0.833]$

$= 1 - 0.7967$



$$\boxed{P[X \geq 45] = \underline{\underline{0.2033}}}$$

$$\phi(z) = \int_{-\infty}^{z} N(0,1)\, dz$$

$\therefore D_2$ is preferred

$$z_2 = \frac{Y - \mu_2}{\sigma_2}$$

$$\Rightarrow Y = \sigma_2 z_2 + \mu_2$$

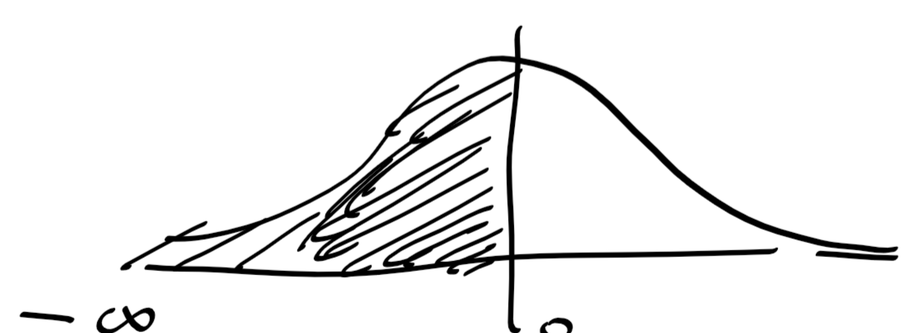
$$\boxed{P[Y \geq 45]} = 1 - P[Y < 45]$$

$= 1 - P[\sigma_2 z_2 + \mu_2 < 45]$

$= 1 - P[3 z_2 + 45 < 45]$

$= 1 - P[z_2 < 0]$

$= 1 - 0.5 \boxed{= \underline{\underline{0.5}}}$



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x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5369
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9131	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

May 2nd question

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The diameter of an electric cable is normally distributed with mean 0.8 and variance 0.0004. (i) What is the probability that the diameter will exceed 0.81 inch? (ii) Suppose that the cable is considered defective if the diameter differs from its mean by more than 0.025, what is the probability of obtaining a defective cable?

X: the diameter of an electric cable

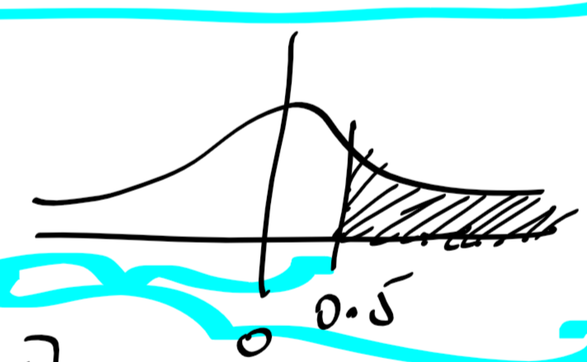
$\mu = 0.8$, $\sigma^2 = 0.0004$

$$Z = \frac{X-\mu}{\sigma}$$

$$X = \sigma Z + \mu$$

$\therefore P[X > 0.81] = P[\sigma Z + \mu > 0.81]$

$= P[0.02Z + 0.8 > 0.81]$



$= P\left[Z > \frac{0.01}{0.02}\right] = P[Z > 0.5]$

$= 1 - P[Z < 0.5]$

$= 1 - 0.6915 = \boxed{0.3085}$

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* The cable is considered to be defective if $|X-\mu| > 0.025$

$\therefore$ P[cable is defective]

$= P[|X-\mu| > 0.025] = 1 - P[|X-\mu| \le 0.025]$

$= 1 - P[|X-0.8| \le 0.025]$

$= 1 - P[|0.02Z + \cancel{0.8} - \cancel{0.8}| \le 0.025]$

$= 1 - P[|0.02Z| \le 0.025]$

$= 1 - P[-0.025 \le 0.02Z \le 0.025]$

$= 1 - P[-1.25 \le Z \le 1.25]$

$Z = \dfrac{X-\mu}{\sigma}$

$X = \sigma Z + \mu$

$|x| < 2$

$\Rightarrow -2 < x < 2$

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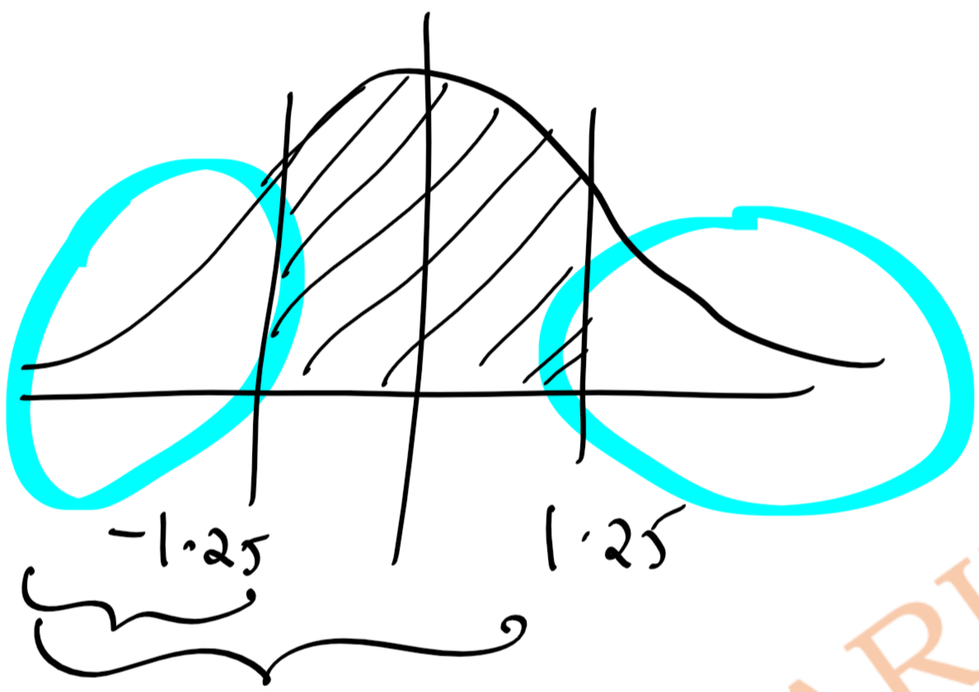
$\therefore P[|X-\mu| > 0.025] = 1 - \{P[Z<1.25] - P[Z<-1.25]\}$



$= 1 - [P[Z<1.25] - \{1 - P[Z<1.25]\}]$

$= 1 - P[Z<1.25] + 1 - P[Z<1.25]$

$= 2 - 2P[Z<1.25]$

$= 2 - 2 \times 0.8944$

$= 2(1 - 0.8944)$

$\boxed{= 0.2112}$

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In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

X : no. of items

μ, σ

$P[X<45] = 0.31$

$z = \frac{x-\mu}{\sigma}$

$P[X>64] = 0.08$

$P[\sigma z+\mu < 45] = 0.31$

$P\left[z < \frac{45-\mu}{\sigma}\right] = 0.31$

$\phi\left(\frac{45-\mu}{\sigma}\right) = 0.31$

① eqn σ & μ

2nd eqn σ & μ

solve

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Geometric distribution

Binomial — independent trials ← success, failure

X: no. of successes in n trials.

Geometric —

X: no. of repetitions required upto and including the first occurrence of Event A (success)

$$P[X=x] = q^{x-1} p, \quad x = 1, 2, \ldots$$

$$P[X=x] = q^{x} p, \quad x = 0, 1, 2, \ldots$$

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pmf of geometric distribution

$$P[X=x] = \begin{cases} q^x p & ; x = 0,1,2,\ldots \\ 0 & \text{otherwise} \end{cases} \qquad 0 < p \leq 1$$

(i) $P[X=x] \geq 0$ ✓

(ii) $\sum_{k=1}^{\infty} P[X=k] = 1$ ✓

Mean $E(X) = \frac{1}{p}$

$M_X(t) = E(e^{tX})$

using mgf $M'(0) = E(X)$

$V(X) = M''(0) - (M'(0))^2$

$\therefore V(X) = q/p^2$

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Memory less or lack of memory

$$P[X \geq s+t \mid X > s] = P(X > t)$$

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* If the probability that a certain test yields a "positive" reaction equals 0.4, what is the probability than fewer than 5 "negative" reactions occur before the first positive one?

X: no. of "negative" reactions before the first positive one

Using geometric distribution

$$P[X = x] = q^x p = (0.6)^x (0.4)$$

$p = 0.4$

$q = 0.6$

$x = 0, 1, 2, \ldots$

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$$\therefore P[X<5] = \sum_{x=0}^{4} (0.6)^x (0.4) = 0.4\left[(0.6)^0 + (0.6)^1 + (0.6)^2 + (0.6)^3 + (0.6)^4\right]$$

$$= \underline{\underline{0.92224}}$$

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* In a certain locality the probability that a thunderstorm will occur on any given day during the summer (say July & August) equals 0.1. Assuming independence from day to day, what is the probability that the first thunderstorm of the summer season occurs on August 3?

Meyer.

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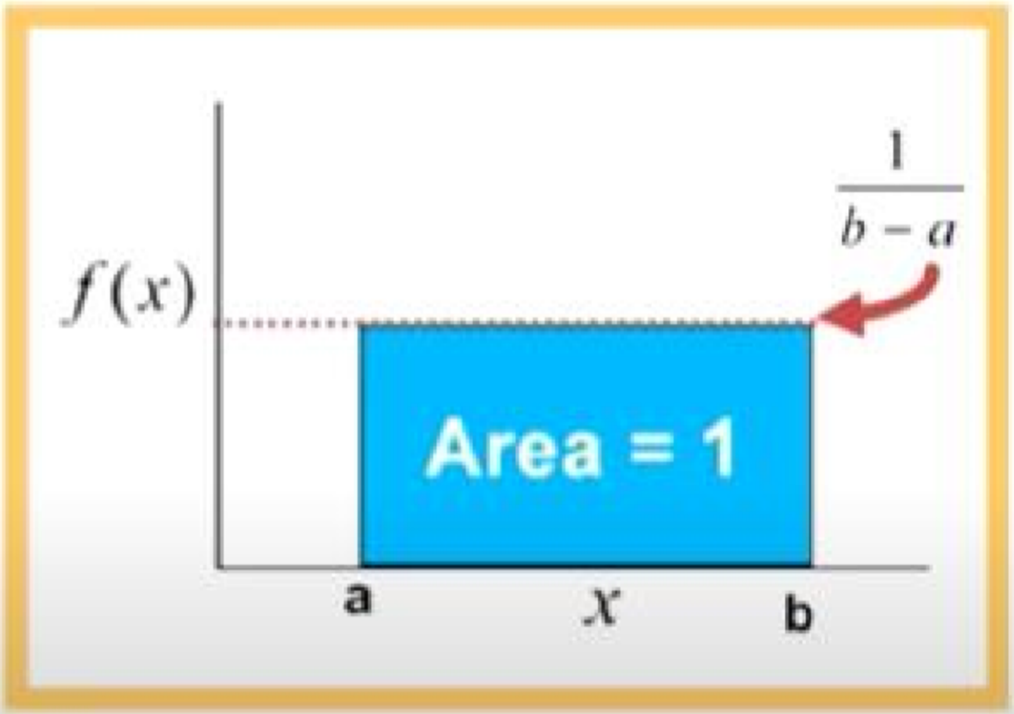
Uniform distribution on an interval

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for} \quad a \le x \le b \\ 0 & \text{for} \quad \text{all other values} \end{cases}$$



X — continuous random variable. interval $I = [a, b]$

$a < b$

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for} \quad a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

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1) $f(x) \geq 0 \qquad a \leq x \leq b$

2) $$\int_{-\infty}^{\infty} f(x)\,dx = \int_{a}^{b} \frac{1}{b-a}\,dx = \frac{1}{b-a}\,x\Big|_{x=a}^{b} = 1$$

3) cdf $\quad F(x) = P[X \leq x]$

Finding cdf

HW

$$= \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } x > b \end{cases}$$

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Mean, $E(X) = \frac{a+b}{2}$

Variance, $V(X) = \frac{(b-a)^2}{12}$

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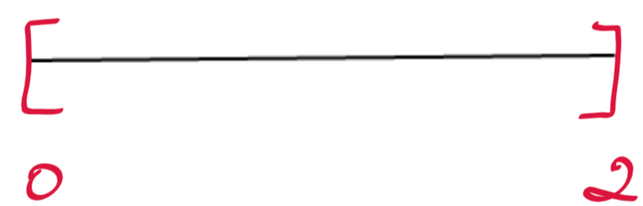
* A point is chosen at random on the line segment $[0, 2]$. What is the probability that the chosen point lies between 1 & $3/2$?

X: the coordinate of the chosen point

Using uniform dist^n,



$$f(x) = \frac{1}{2}, \quad 0 \le x \le 2$$

$$\therefore P[1 \le x \le 3/2] = \int_1^{3/2} f(x)\,dx = \int_1^{3/2} \frac{1}{2}\,dx = \frac{1}{2}x\Big|_{x=1}^{3/2} = \frac{1}{2}\left(\frac{3}{2}-1\right) = \boxed{\frac{1}{4}}$$

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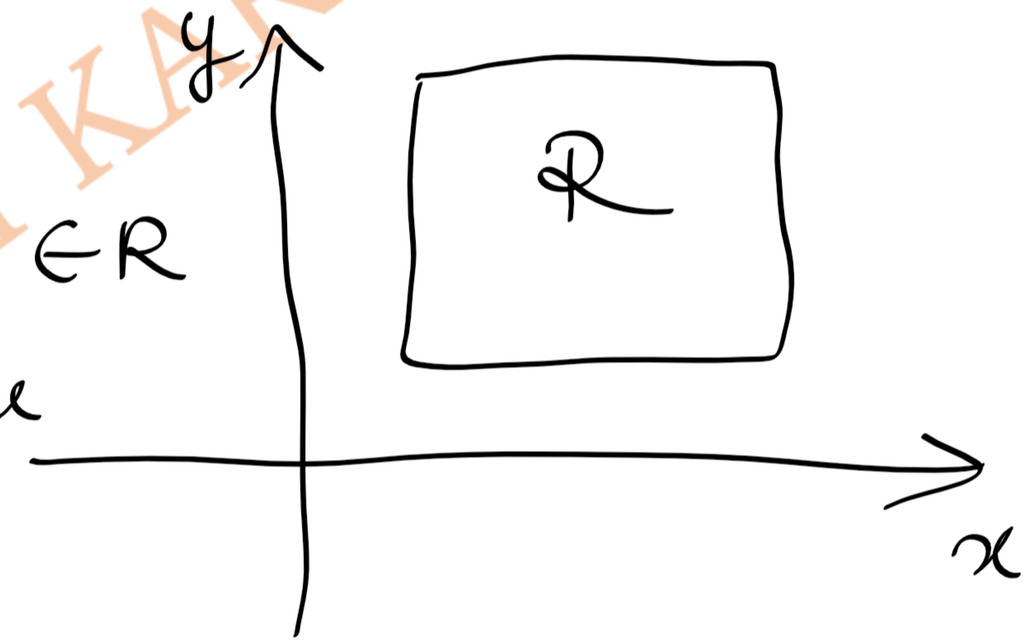
Uniform distribution in 2D

(X, Y) — two-dimensional continuous random variable

joint pdf

$$f(x,y) = \begin{cases} \text{const}, c & \text{for } (x,y) \in R \\ 0 & \text{otherwise} \end{cases}$$



Note: $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

$$\Rightarrow c\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy = 1 \quad \Rightarrow c \times \text{area of } R = 1$$

$$\Rightarrow \boxed{c = \frac{1}{\text{Area of } R}}$$

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∴ Uniform Distribution in 2D

$$f(x,y) = \begin{cases} \dfrac{1}{\text{Area of } R} & ; \text{ for } (x,y) \in R \\ 0 & \text{otherwise} \end{cases}$$

* Suppose that the two-dimensional r.v (X,Y) is uniformly distributed over the region R bounded between $y=x$ & $y=x^2$. Find the marginal pdf of X & Y.

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