Motivational example for conditional probability

Health spa

	(M) Males	(F) Females
(O) overweight	38	62
(A) Average weight	27	41
(U) underweight	6	15

$\#(O \cap M) = 38$

$\#(O \cap F) = 62$

$\#M = 38 + 27 + 6 = 71$

$\#F = 62 + 41 + 15 = 118$

$\#S = 71 + 118 = \underline{\underline{189}}$

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$$P(O \cap M) = \frac{\#(O \cap M)}{\# S} = \frac{38}{189}$$

$$P(M) = \frac{\# M}{\# S} = \frac{71}{189}$$

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* restricting the selection to "males"

Probability the selected man will be overweighted

$$P(O/M) = \frac{\# O \cap M}{\# M} = \frac{38}{71} = \frac{\frac{38}{189}}{\frac{71}{189}}$$

$$\boxed{P(O|M) = \frac{P(O \cap M)}{P(M)}}$$

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* restricting the selection to overweight people probability that the selected person is a male

$$P[M|O] = \frac{\#\, O \cap M}{\#\, O} = \frac{38}{100}$$

$$= \frac{\left(\frac{38}{189}\right)}{\left(\frac{100}{189}\right)}$$

$$\boxed{P(M|O) = \frac{P(O \cap M)}{P(O)}}$$

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Probability Problems

A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn, there is atleast one ball of each colour.

$\underbrace{6R, \quad 4W, \quad 5B}_{4 \text{ balls}}$

RWBB ✓
RWBW ✓
RWBR ✓

P[atleast one ball of each colour]

$$= \frac{{}^6C_1 \times {}^4C_1 \times {}^5C_2 + {}^6C_1 \times {}^4C_2 \times {}^5C_1 + {}^6C_2 \times {}^4C_1 \times {}^5C_1}{{}^{15}C_4}$$

$= 0.5275$

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A shipment of 100 washers contains 20 defective and 80 nondefective items. Two washers are chosen at random (a) with replacement (b) without replacement. What is the probability that second chosen item is defective.

$$\left.\begin{array}{l} 20\ D \\ 80\ ND \end{array}\right] 2$$

(a) With replacement:

P[second chosen item is defective]

$$= \frac{100 \times 20}{100 \times 100} = \frac{1}{5}$$

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(b) Without replacement

Conditional probility

P[second chosen item is defective]

$$= \frac{{}^{80}C_1 \times {}^{20}C_1 + {}^{20}C_1 \times {}^{19}C_1}{{}^{100}C_2}$$

?

?

Think again

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Independent events

A speaks truth in 75% cases, B speaks in 80%. In what percentage of they are likely to contradict.

A: A speaks the truth

B: B speaks the truth

$P(A) = \frac{75}{100}$, $P(\bar{A}) = 0.25$

$P(B) = \frac{80}{100}$, $P(\bar{B}) = 0.2$

P[they are likely contradict]

$= P[A \cap \bar{B}] + P[\bar{A} \cap B]$

$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B) = 0.75 \times 0.2 + 0.25 \times 0.8$

$= \underline{\underline{0.35}}$

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A certain item is manufactured by three factories, say 1, 2 and 3. It is known that 1 turns out twice as many items as 2, and that 2 and 3 turn out the same number of items (during a specified production period). It is also known that 2 percent of the items produced by 1 and 2 are defective, while 4 percent of those manufactured by 3 are defective. All the items produced are put into one stockpile, and then one item is chosen at random. What is the probability that this item is defective?

$A = \{$ the item is defective $\}$

$B_1 = \{$ the item came from factory 1 $\}$

$B_2 = \{$ " " 2 $\}$

$B_3 = \{$ " " " 3 $\}$

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$$P(A) = P(A|B_1)\cdot P(B_1) + P(A|B_2)\,P(B_2) + P(A|B_3)\cdot P(B_3)$$

$$= \underline{\underline{0.025}}$$

$P(A|B_1) = 0.02$

$P(A|B_2) = 0.02$

$P(A|B_3) = 0.04$

$P(B_1) = \frac{1}{2}$

$P(B_2) = P(B_3) = \frac{1}{4}$

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A laboratory blood test is 99% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1% of the healthy persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he or she has the disease). If 0.5% of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?

D : the tested person has the disease

A : his test result is positive

$$P[D/A] = \frac{P(D \cap A)}{P(A)} = \frac{P(A|D) \cdot P(D)}{P(A|D) \cdot P(D) + P(A|D')P(D')}$$

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$$P[D/A] = \frac{P(D \cap A)}{P(A)} = \frac{P(A|D) \cdot P(D)}{P(A|D) \cdot P(D) + P(A|D')P(D')}$$

$$= \frac{(0.99) \times (0.005)}{(0.99) \times (0.005) + (0.01) \times (0.995)}$$

$$= \underline{\underline{0.3322}}$$

$P(A|D) = 0.99$

$P(A|D') = 0.01$

$P(D) = 0.005$

$P(D') = 0.995$

$\frac{0.5}{100}$

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