A shipment of 100 washers contains 20 defective and 80 nondefective items. Two washers are chosen at random (a) with replacement (b) without replacement. What is the probability that second chosen item is defective.

(b) without replacement

A: the first chosen item is defective

B: the second " " " "

$P(A) = \frac{20}{100} = \frac{1}{5}$, $P(\bar{A}) = 1 - \frac{1}{5} = \frac{4}{5}$

$P(B) = P(B|A) \cdot P(A) + P(B|\bar{A})\, P(\bar{A})$

$= \frac{19}{99} \times \frac{1}{5} + \frac{20}{99} \times \frac{4}{5} = \frac{1}{5}$

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Conditional probability

Ex: $P(A|B)$ — straight vertical line

Don't write $P(A/B)$

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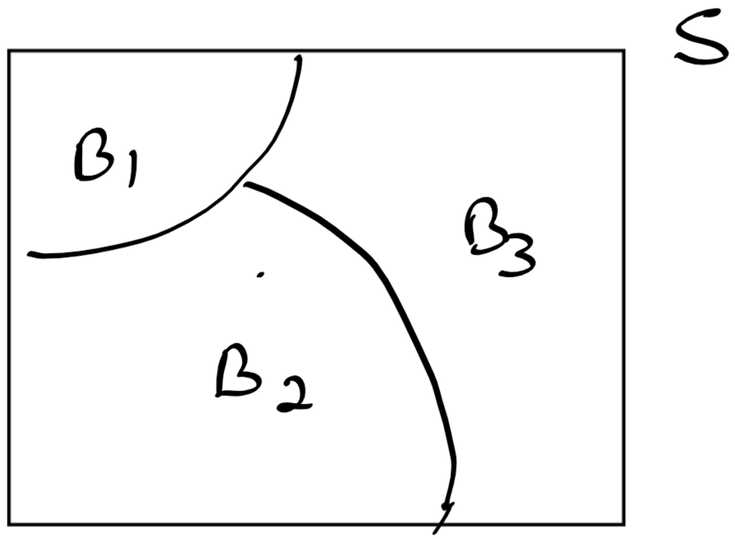
Of the three men, chances that a politician, a business man and an academician will be appointed as VC of a University are 0.5, 0.3 and 0.2 respectively. Probability that research is promoted by these people if they are appointed as VC are 0.3, 0.7 and 0.8 respectively. If research is promoted in a University, what is the probability that VC is an academician?

A: Appointed VC is a politician

B: " " a BM.

C: " " an academician



$P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$

D: Research is promoted

$P(D|A) = 0.3$, $P(D|C) = 0.8$

$P(D|B) = 0.7$

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$$P[C|D] = \frac{P(C) \cdot P(D|C)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)}$$

$$= \underline{\underline{0.3077}}$$

$P(D|A) = 0.3$, $P(D|C) = 0.8$

$P(D|B) = 0.7$

$P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$

Find $P[A|D] =$?

$P[B|D] =$?

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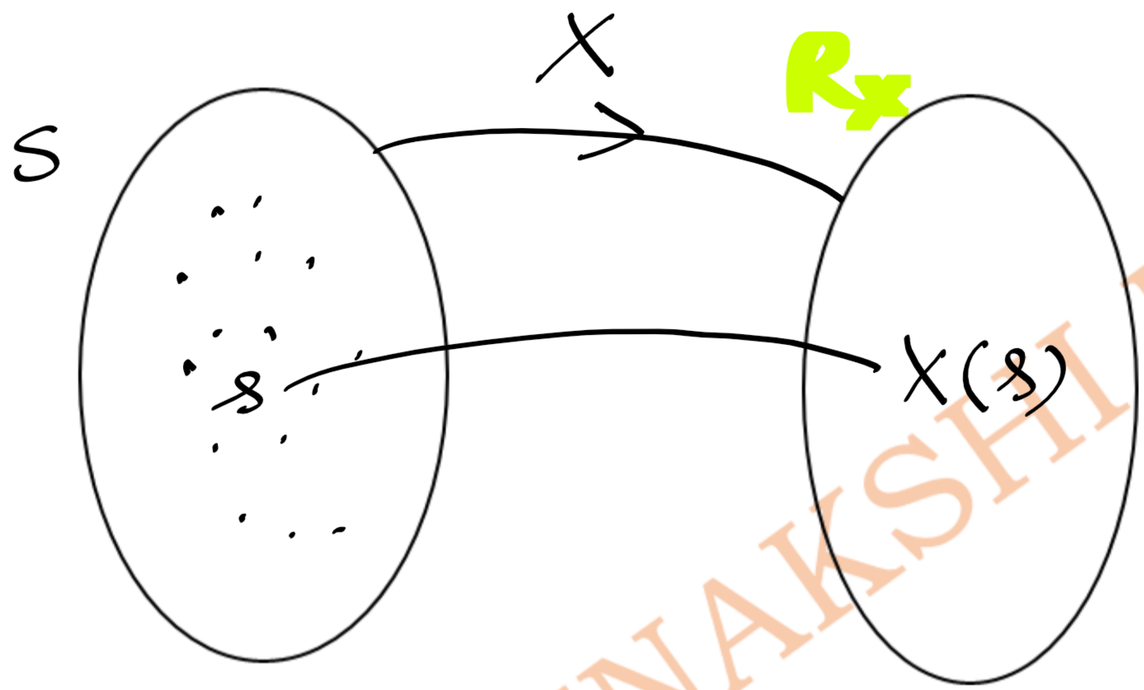
Random variable

Let E be an experiment & S be the sample space associated with E. A function X assigning to every element $s \in S$, a real number $X(s)$, is called a $\boxed{\text{random variable}}$



R_X — range space.

$$H \begin{cases} H \\ T \end{cases} \begin{matrix} HH \\ HT \end{matrix} \qquad T \begin{cases} H \\ T \end{cases} \begin{matrix} TH \\ TT \end{matrix}$$

E: tossing two coins

$S = \{\underbrace{HH, HT, TH, TT}\}$.

X: no. of heads occurring

$X(S) = \{2, 1, 1, 0\}$

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Discrete random variable

$X(S) = \{$ finite or countable infinite $\}$

Continuous random variable

$X(S)$ will be given in terms of interval

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Probability Distributions

The probability distribution function or probability density function (PDF) of a random variable X means the values taken by that random variable and their associated probabilities.

$X = \{x_1, x_2, \ldots, x_n\}$

$P[X = x_1] = \checkmark$ $\quad P[X = x_2] = \checkmark$

X	x_1	x_2	$\ldots$	x_n
$P(X)$	✓	✓	$\ldots$	✓

Discrete probability distributions

probability mass function (pmf) $\boxed{(x_i, p(x_i))}$

$p(x_i) = P(X = x_i)$ — probability of x_i

(i) $p(x_i) \geq 0$ for all i

(ii) $\sum_i p(x_i) = 1$

x_i	•	•	•	•	•	•	•
$p(x_i)$	✓	✓	✓	✓	✓	✓	✓

$(x_i, p(x_i))$ — pmf

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Example

E: Tossing two coins

$S = \{HH, HT, TH, TT\}$

X: no. of heads occurring

$X(S) = \{0, 1, 2\}$

$p(x_i) = P[X = x_i]$

$p(0) = P[X = 0]$

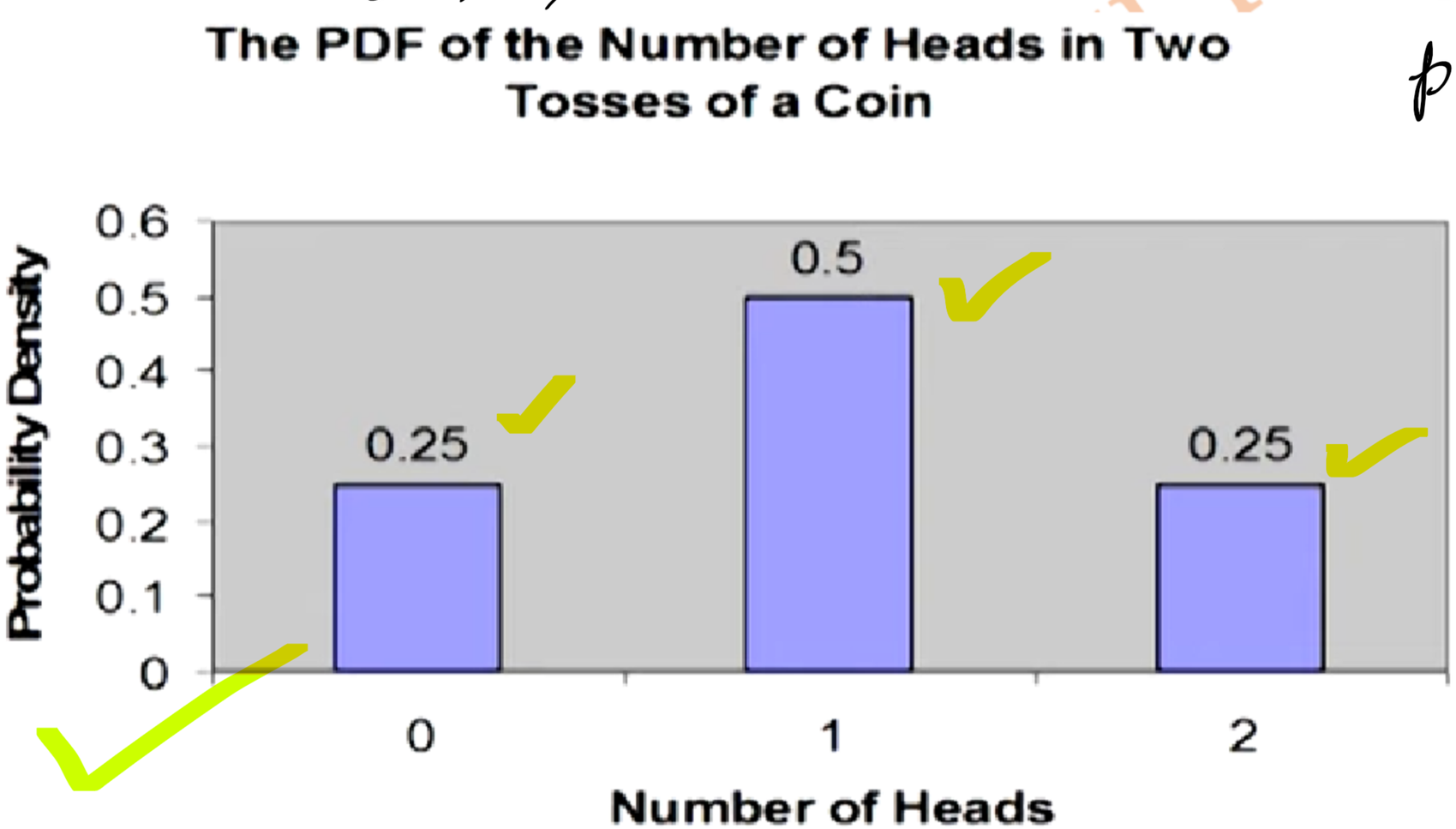
$= \frac{1}{4} = 0.25$

$p(1) = P[X = 1]$

$= \frac{2}{4} = 0.5$

$p(2) = P[X = 2]$

$= \frac{1}{4} = 0.25$



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x_i	0	1	2
$p(x_i)$	0.25	0.5	0.25

$\sum p(x_i) = 1$

pdf: X — continuous random variable

$f(x)$ — pdf if

(a) $f(x) \geq 0$ for all x

(b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(c) For any a, b, $-\infty < a < b < \infty$

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx$$

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Cumulative distribution function (cdf)

X — random variable discrete or continuous

Cdf $F(x) = P(X \leq x)$

1) If X is a d.r.v

$$F(x) = P(X \leq x)$$

$$= \sum_{\substack{i \\ x_i \leq x}} p(x_i)$$

2) If X is a c.r.v

$$F(x) = P[X \leq x] = \int_{-\infty}^{x} f(t)\,dt$$

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Expected value of a random variable X (Mean)

Let X be a d.r.v with possible values

$x_1, x_2, \ldots, x_n, \ldots$

$p(x_1), p(x_2), \ldots, p(x_n), \ldots$

$$\boxed{E(X) = \sum_{i=1}^{\infty} x_i p(x_i)}$$ if the series converges absolutely, i.e. $\sum_{i=1}^{\infty} x_i p(x_i) < \infty$

Let X be a c.r.v with pdf $f(x)$

$$\boxed{E(X) = \int_{-\infty}^{\infty} x f(x)\,dx}$$ if $\int_{-\infty}^{\infty} |x| f(x)\,dx$ is finite

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Variance of a random variable X

Let X be a random variable (discrete or continuous)

V(X) or σ_X^2

$$\boxed{V(X) = E(X - E(X))^2}$$

Standard deviation

$$\boxed{\sigma_X = \sqrt{V(X)}}$$

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Properties of E(X)

(i) $E(CX) = C\,E(X)$ $\quad$ C is a constant

X — c.r.v

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\therefore\ E(CX) = \int_{-\infty}^{\infty} cx\, f(x)\, dx$$

$$= c \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\boxed{E(CX) = C\,E(X)}$$

(ii) If $X = C$ $\quad$ C is a constant

$E(X) = ?$

$$\left[\ E(X) = \int_{-\infty}^{\infty} C\, f(x)\, dx\right.$$

$$= C \underbrace{\int_{-\infty}^{\infty} f(x)\, dx}_{=1}$$

$$E(X) = C$$

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$$* \quad V(X) = E(X^2) - (E(X))^2$$

$$V(X) = E(X - E(X))^2$$

$$= E(X^2 - 2X\,E(X) + (E(X))^2)$$

$\because E(X)$ is a const-ant

$$= E(X^2) - 2E(X)\cdot E(X) + (E(X))^2$$

$$\boxed{V(X) = E(X^2) - (E(X))^2}$$

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* If c is a constant

1) $V(X+c) = ?$

2) $V(cX) = ?$

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