* The pdf of a r.v X is given by

$$f(x) = \begin{cases} \frac{x^2}{81} & -3 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the pdf of $U = \frac{1}{3}(12 - X)$

$$u = \frac{1}{3}(12 - x)$$

$$\Rightarrow x = 12 - 3u = \psi(u)$$

$-3 < x < 6$

$x = -3, \quad u = 5$

$x = 6, \quad u = 2$

$$\underline{\underline{2 < u < 5}}$$

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$\therefore$ The pdf of U is

$$g(u) = f(x(u)) \cdot |x'(u)|$$

$$x'(u) = \frac{dx}{du} = -3$$

$$|x'(u)| = 3$$

$$\therefore g(u) = \begin{cases} \frac{(12-3u)^2}{81} \cdot 3 & , \quad 2 < u < 5 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore g(u) = \begin{cases} \frac{(12-3u)^2}{27} & , \quad 2 < u < 5 \\ 0 & \text{otherwise} \end{cases}$$

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* If the random variables X & Y has joint pdf

$$f(x,y) = \begin{cases} \frac{xy}{96}, & 0<x<4,\ 1<y<5 \\ 0 & \text{otherwise} \end{cases}$$

Find the pdf of $V = X + 2Y$

$u = x + 2y$, $v = x$

$f(x,y) \longrightarrow g(u,v)$ } joint pdf of (u,v)

marginal pdf of V

$x = v$, $y = \frac{1}{2}(u - x) = \frac{1}{2}(u - v)$

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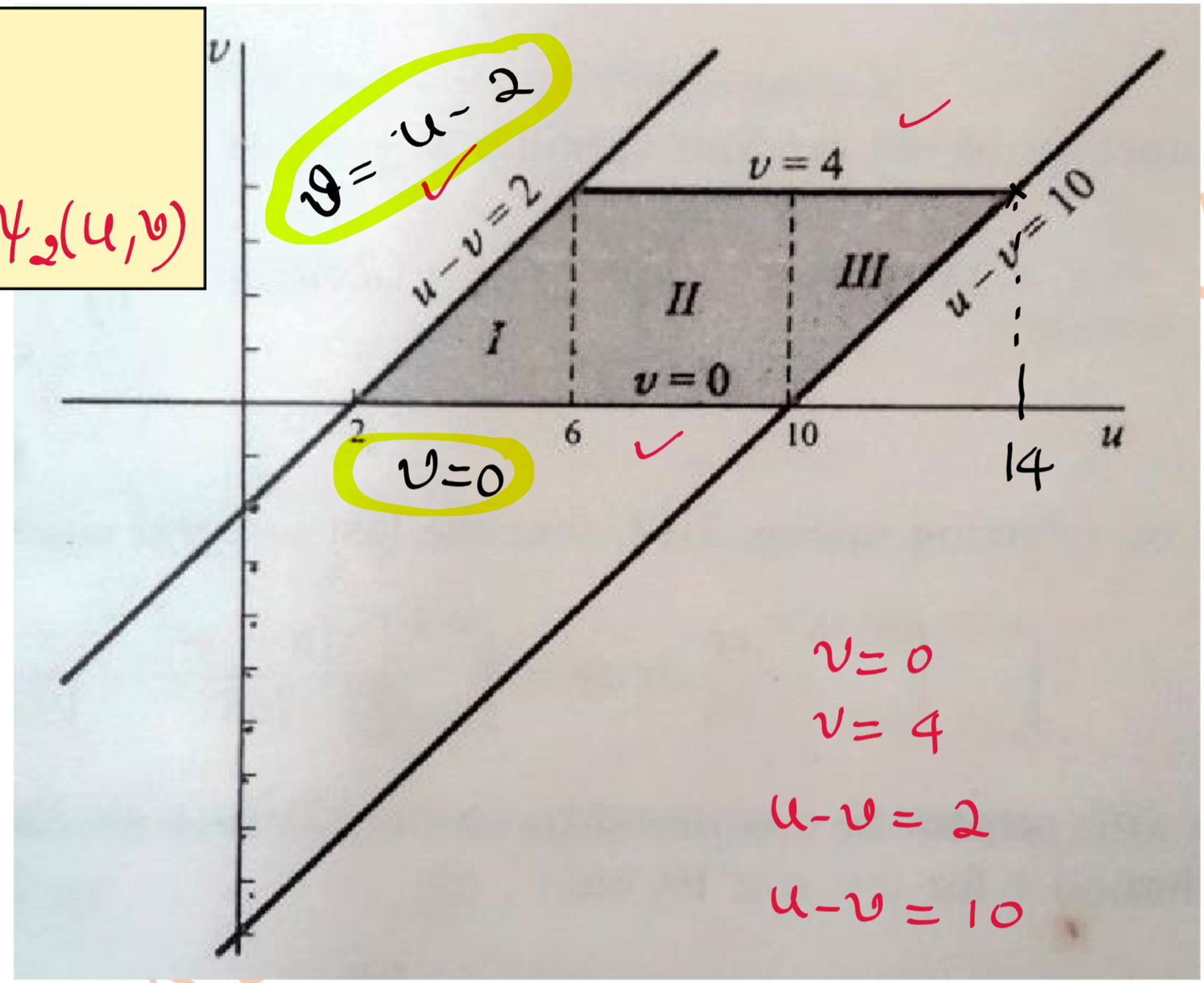
$$x = v = \psi_1(u,v)$$
$$y = \frac{1}{2}(u-v) = \psi_2(u,v)$$

$$0 < v < 4$$
$$2 < u - v < 10$$



$$0 < x < 4$$
$$1 < y < 5$$

$$2 < 2y < 10$$

$$2 < u - v < 10$$

∴ the joint pdf of U, V, $g(u,v) = f(\psi_1(u,v), \psi_2(u,v))\,|J|$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2}$$

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$\therefore$ The joint pdf $g(u,v) = \begin{cases} \dfrac{v(u-v)}{2\times 96}\cdot\left(\dfrac{1}{2}\right), & 2<u-v<10,\ 0<v<4 \\ 0 & \text{otherwise} \end{cases}$

$$\boxed{g(u,v) = \begin{cases} \dfrac{v(u-v)}{384}, & 2<u-v<10,\ 0<v<4 \\ 0 & \text{otherwise} \end{cases}}$$

The marginal pdf of v is

$$g_1(u) = \begin{cases} \displaystyle\int_{v=0}^{u-2} g(u,v)\,dv, & 2<u<6 \\ \displaystyle\int_{v=0}^{4} g(u,v)\,dv, & 6<u<10 \\ \displaystyle\int_{v=u-10}^{4} g(u,v)\,dv, & 10<u<14 \end{cases}$$

HW

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* Suppose that X has pdf

$$f(x) = \begin{cases} 2x & , 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the pdf of $Y = 3X + 1$

HW

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Bernoulli Trials

$\in$ – random experiment } n trials

2 possible outcomes

success

failure

n – independent trials

* probability of success denoted by p
* probability of failure " q, $q = 1 - p$

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ϵ — r. experiment repeated n times

The no. of successes in n Bernoulli trails

coin 10 times

p, q

p, q

⋮

Binomial random variable

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Binomial distribution

Bernoulli trail — success probability p

failure probability $q = 1-p$

X: the no. of successes in n Bernoulli trails

$$P[X=x] = b(n, p, x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

$$\boxed{b(n, p, x) = \binom{n}{x} p^{x} q^{n-x}, \quad x = 0, 1, 2, \ldots, n}$$

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Note:

1) $b(n, p, x_i) = P[X = x_i] \geq 0$

for $x_i = 0, 1, 2 \ldots n$

2) $$\sum_{x_i=0}^{n} b(n, p, x_i) = \binom{n}{0} p^0 q^{n-0} + \binom{n}{1} p^1 q^{n-1} + \ldots + \binom{n}{n} p^n q^{n-n}$$

$$= (p+q)^n$$

$$\boxed{= 1}$$

p

$q = 1 - p$

$$\boxed{p + q = 1}$$

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Mean and Variance

$$\mu = E(X) = \sum_{x_i=0}^{n} x_i \, b(n, p, x_i)$$

$$= \sum_{x_i=0}^{n} x_i \binom{n}{x_i} p^{x_i} q^{n-x_i}$$

$$= 0 + \binom{n}{1} p q^{n-1} + 2\binom{n}{2} p^2 q^{n-2} + 3\binom{n}{3} p^3 q^{n-3} + \cdots + n\binom{n}{n} p^n q^0$$

$$= n p q^{n-1} + 2\,\frac{n(n-1)}{2} p^2 q^{n-2} + 3\,\frac{n(n-1)(n-2)}{3!} p^3 q^{n-3} + \cdots + n p^n$$

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$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!}p^2q^{n-3} + \cdots + p^{n-1}\right]$$

$$= np\left[p+q\right]^{n-1}$$

Binomial expansion of $(p+q)^{n-1}$

$$\boxed{E(X) = np}$$

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Variance $V(X) = E(X^2) - [E(X)]^2$

$$E(X^2) = \sum_{x_i=0}^{n} x_i^2 \, b(n, p, x_i)$$

$$= \sum_{x_i=0}^{n} \{x_i(x_i-1) + x_i\} \, b(n, p, x_i)$$

$$= \sum_{x_i=0}^{n} x_i(x_i-1) \, b(n, p, x_i) + \underbrace{\sum_{x_i=0}^{n} x_i \, b(n, p, x_i)}_{E(X)}$$

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$$E(X^2) - E(X) = \sum_{x_i=0}^{n} x_i(x_i-1)\binom{n}{x_i} p^{x_i} q^{n-x_i}$$

$$= 2\binom{n}{2} p^2 q^{n-2} + 3\cdot 2\binom{n}{3} p^3 q^{n-3}$$

$$+ \cdots + n(n-1)p^n$$

$$= 2\,\frac{n(n-1)}{2!}\, p^2 q^{n-2} + 3\cdot 2\cdot \frac{n(n-1)(n-2)}{3!}\, p^3 q^{n-3}$$

$$+ \cdots + n(n-1)p^n$$

$$= n(n-1)p^2\left[\underbrace{q^{n-2} + (n-2)q^{n-3}\cdot p + \cdots}\; + p^{n-2}\right]$$

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$$E(X^2) - E(X) = n(n-1)p^2\underbrace{(p+q)}_{1}{}^{n-2}$$

$$= n(n-1)p^2$$

$$\therefore E(X^2) = n(n-1)p^2 + E(X)$$

$$E(X^2) = n(n-1)p^2 + np$$

$$\therefore V(X) = E(X^2) - (E(X))^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$\therefore V(X) = np(1-p)$$

$$\boxed{V(X) = npq}$$

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Poisson Distribution

Let X be a discrete random variable assuming the possible values $0, 1, 2, \ldots, \ldots$

If

$$P[X=x] = p(\lambda, x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \ldots$$

We say that X has a Poisson distribution with parameter $\lambda > 0$

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