Moments about origin

The rth moment of a variable x

$$\boxed{\mu_r' = \frac{1}{N}\sum_i f_i x_i^r}$$ — raw moment

* Find the second, third and fourth central moments of the frequency distribution given below. Hence find the measure of skewness (γ_1).

class limits	Frequency
110.0 – 114.9	5
115.0 – 119.9	15
120.0 – 124.9	20
125.0 – 129.9	35
130.0 – 134.9	10
135.0 – 139.9	10
140.0 – 144.9	5

$\mu_2 = 54$?

$\boxed{\mu_3 = 100.3}$?

$\mu_4 = 7872$?

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* Age distribution of hundred life insurance policy holders is as follows: Calculate the mean deviation from median age.

Age as on nearest birthday	Number
17 – 19.5	9
20 – 25.5	16
26 – 35.5	12
36 – 40.5	26
41 – 50.5	14
51 – 55.5	12
56 – 60.5	6
61 – 70.5	5
	100

Median = 38.25

M.D = 3.56

?

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Moment coeff. of skewness

$$\gamma_1 = \sqrt{\beta_1} \qquad \beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\boxed{\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}}$$

$\gamma_1 < 0$, −vely skewed

$\gamma_1 > 0$, +vely skewed

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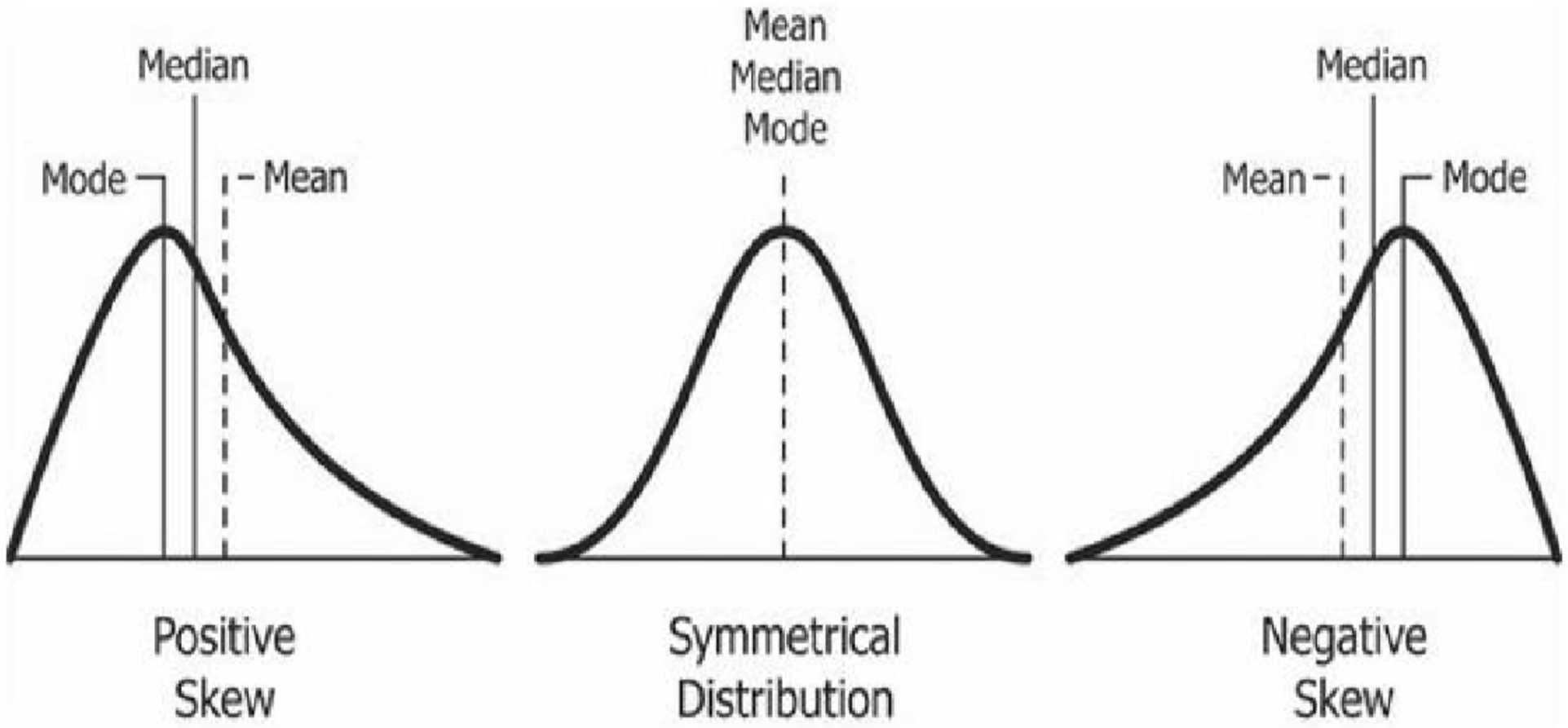
Skewness

There are two types of Skewness: Positive and Negative



- If the skewness is between -0.5 and 0.5, the data are fairly symmetrical.

- If the skewness is between -1 and -0.5(negatively skewed) or between 0.5 and 1(positively skewed), the data are moderately skewed.

- If the skewness is less than -1(negatively skewed) or greater than 1(positively skewed), the data are highly skewed.

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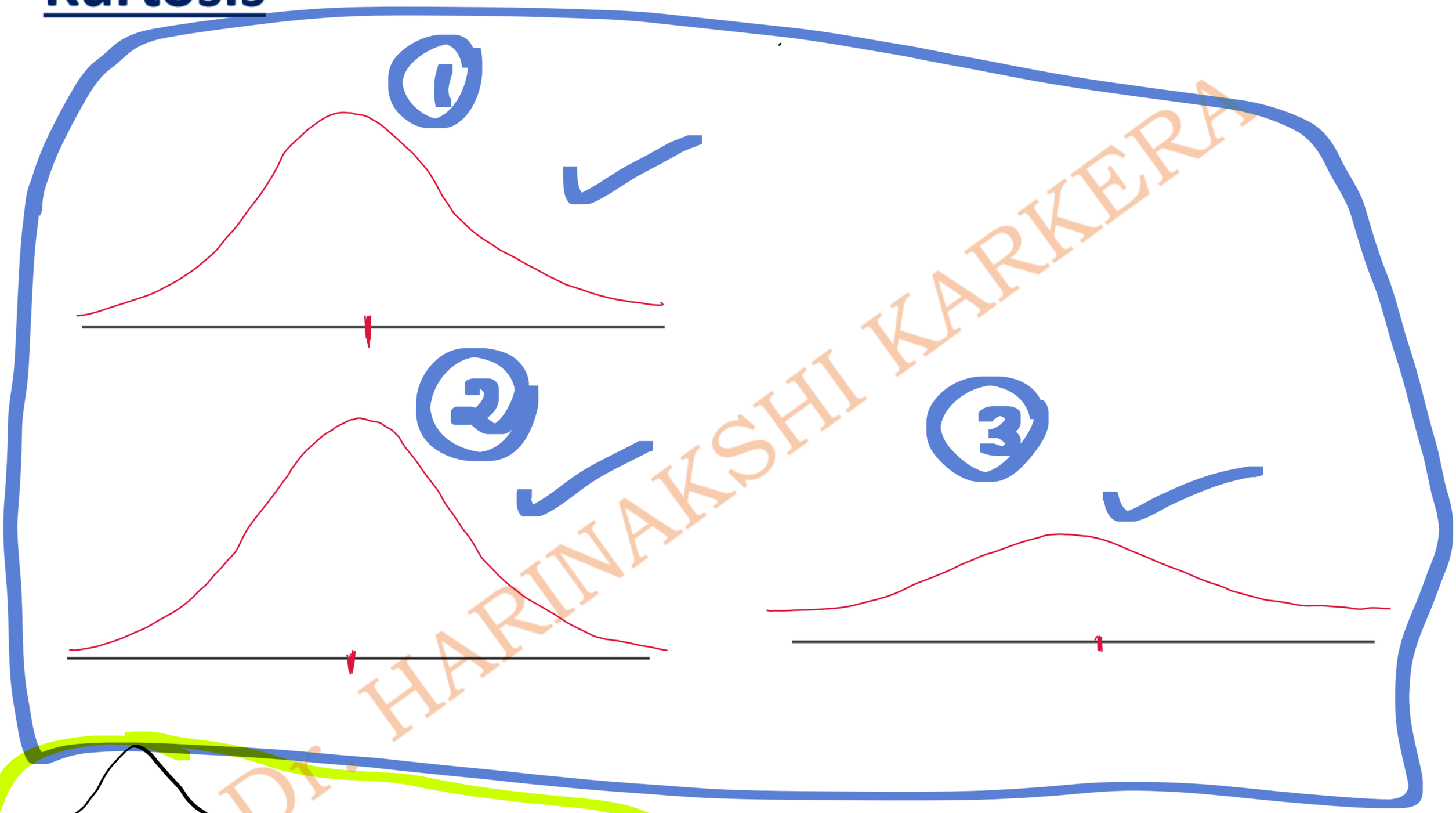
Kurtosis



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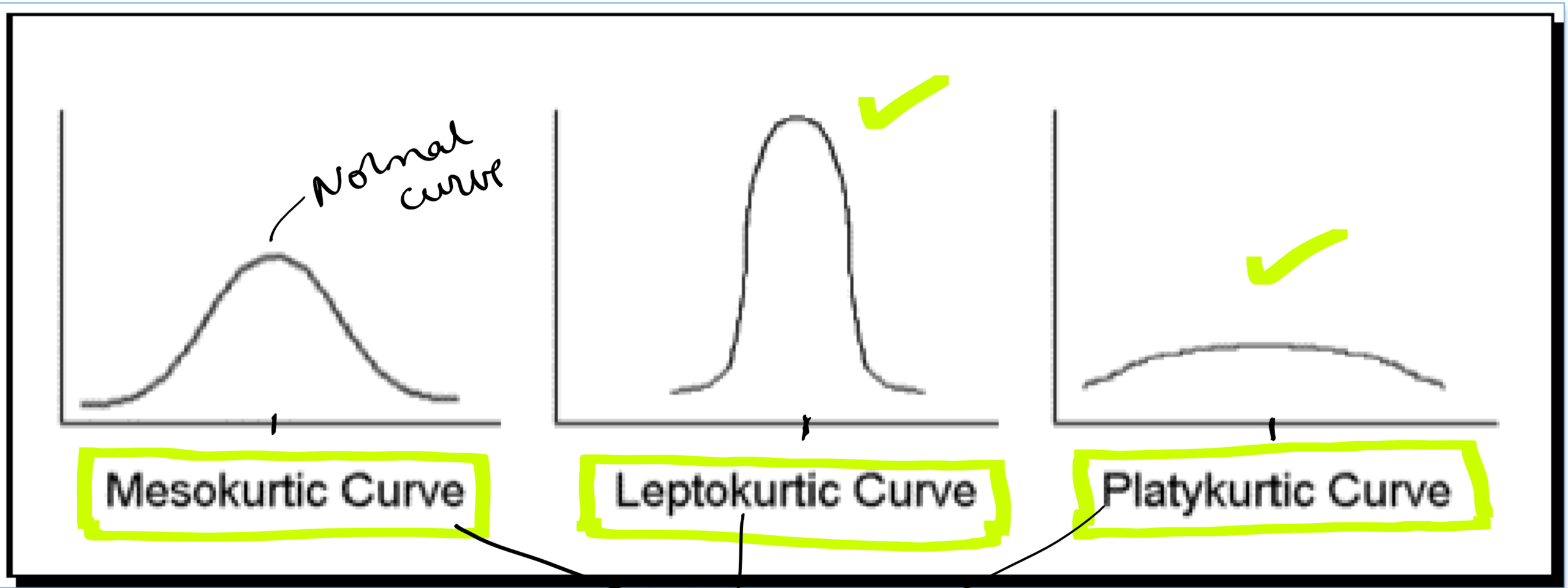

Measure of kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\gamma_2 = \beta_2 - 3$$

$\gamma_2 = 0 \quad (\beta_2 = 3)$

$\gamma_2 > 0, \quad (\beta_2 > 3)$

$\gamma_2 < 0, \quad \beta_2 < 3$

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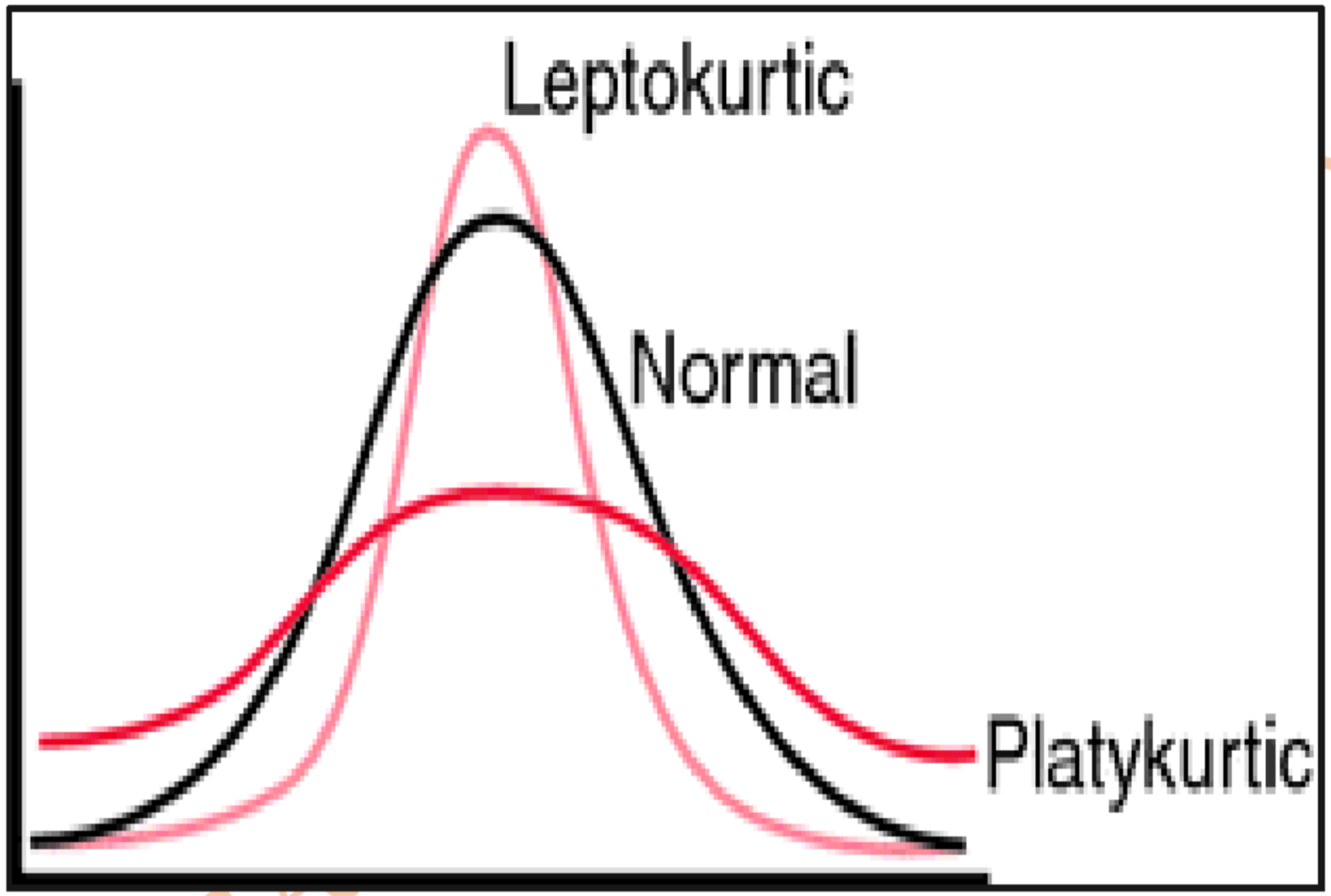

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Example:

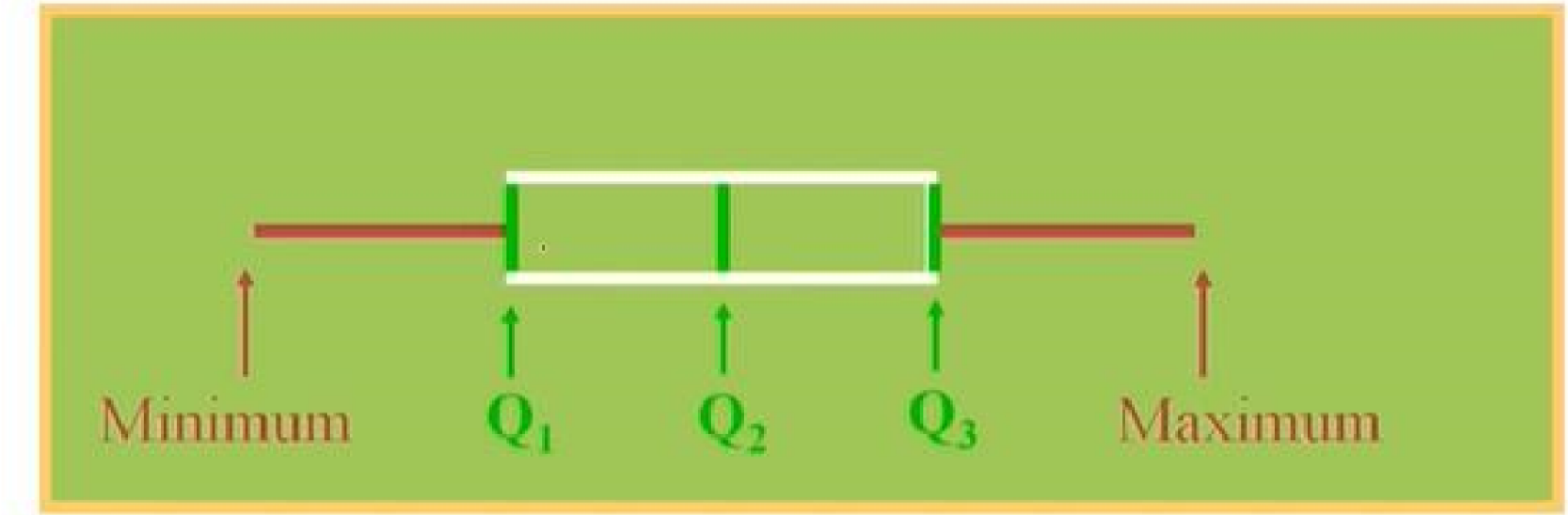
The variance of a distribution is 5. What must be the value of the fourth central moment in order that the distribution be (i) leptokurtic (ii) mesokurtic and (iii) platykurtic?

Solution:

Using the relation $\beta_2 = \frac{\mu_4}{\mu_2^2}$, given second central moment $\mu_2 = 5$, we get $\mu_4 > 75$ for the curve to be leptokurtic, $\mu_4 = 75$ for the curve to be mesokurtic and $\mu_4 < 75$ for the curve to be platykurtic.

$\beta_2 > 3$ $\quad \frac{\mu_4}{25} > 3 \Rightarrow \mu_4 > 75$

Box and Whisker plot



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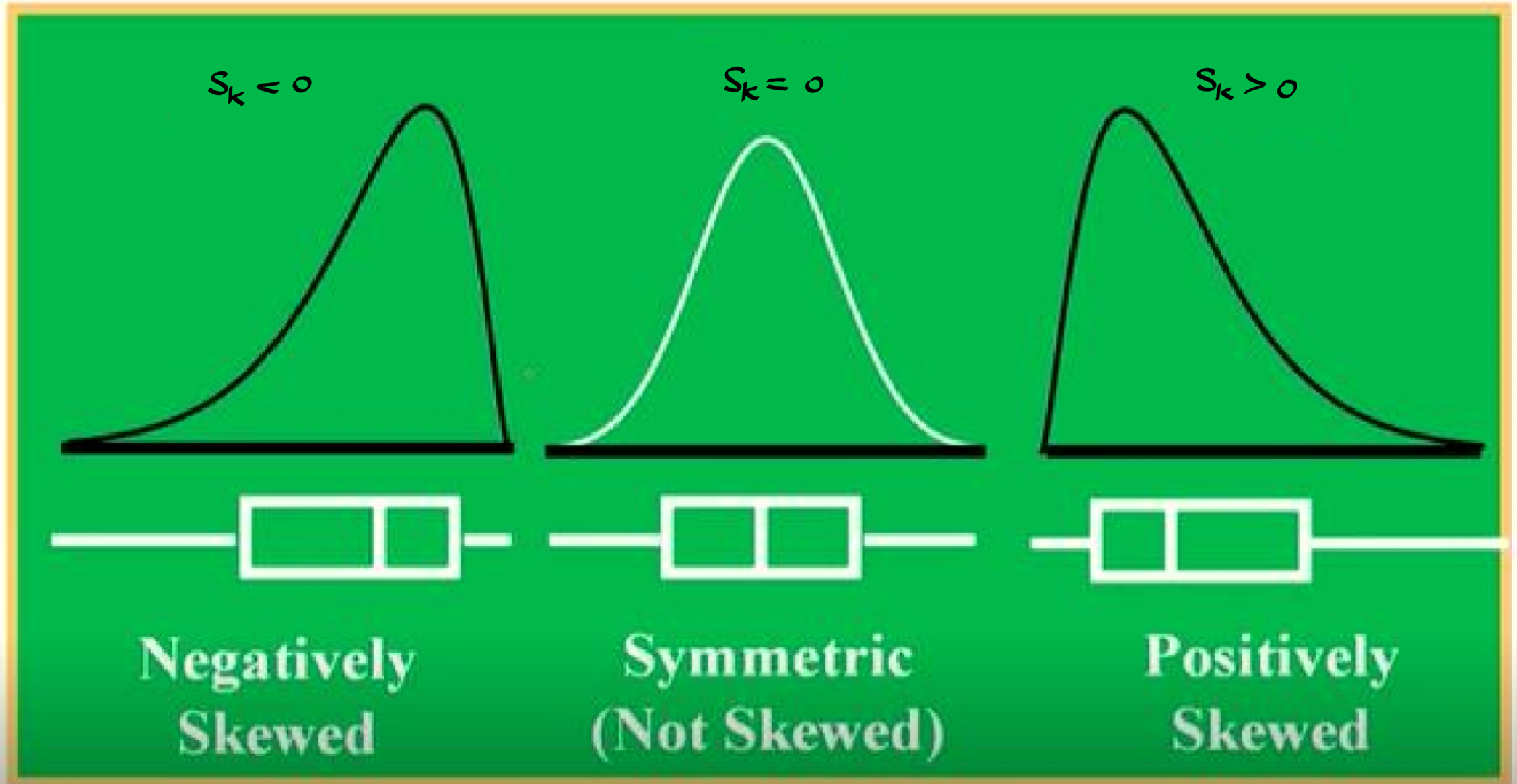
$S_k < 0$
$S_k = 0$
$S_k > 0$
Negatively Skewed
Symmetric (Not Skewed)
Positively Skewed