NOTE:

The conditional probability for two-dimensional random variable is already discussed in L12. Kindly refer the class Notes of L12.

We follow the same notations while defining conditional expectation.

Conditional Expectation

(1) If (X, Y) is a two-dimensional discrete random variable. We define the conditional expectation of X for given $Y = y_j$ is

$$E(X|y_j) = \sum_{i=1}^{\infty} x_i \, p(x_i|y_j)$$ where $p(x_i|y_j)$ – conditional probability of X for given $Y = y_j$

$||^{lly}$ conditional expectation of Y for given $x = x_i$

$$E(Y|x_i) = \sum_{j=1}^{\infty} y_j \, q(y_j|x_i)$$ where $q(y_j|x_i)$ – conditional probability of Y for given $X = x_i$

(2) If (X,Y) is a two-dimensional continuous random variable. We define the conditional expectation of X for given $Y=y$ is

$$\underbrace{E(X|y)} = \int_{-\infty}^{\infty} x\, \underbrace{g(x|y)}\, dx$$

where $g(x|y)$ - conditional pdf of X for given $Y=y$

IIIly we can define $E(Y|x) = \int_{-\infty}^{\infty} y\, h(y|x)\, dy$

$\therefore$ $E[X|y]$ – is the expectation of X conditioned on the event $\{Y=y\}$

Note: $E(X|Y)$ – is a function of Y & hence it is a random variable.

$|||^{ly}$ $E(Y|X)$ – is a function of X & is also a random variable.

$$E[E(X|Y)] = E(X)$$

Theorem: 1) $E[E(X|Y)] = E(X)$

2) $E[E(Y|X)] = E(Y)$

If X & Y are independent random variable.
Then
$$E(X|Y) = E(X) \quad \& \quad E(Y|X) = E(Y)$$

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* Suppose that shipments involving a varying no. of parts arrive each day. If N is the no. of items in the shipment, the probability distribution of the random variable N is given as follows.

n	:	10	11	12	13	14	15
$P(N=n)$	:	0.05	0.10	0.10	0.20	0.35	0.20

The probability that any particular part is defective is the same for all parts and equals to 0.10.

If X is the no. of defective parts arriving each day, what is the expected value of X? For a given $N=n$, X has a binomial distribution.

$$E(X) = E[E(X|N)]$$

$$E(X|N) = 0.10\,N$$

$$\therefore E(X) = E[0.10\,N]$$

$$= 0.10\,E(N)$$

$$= 0.10\,[(10 \times 0.05) + (11 \times 0.10) + (12 \times 0.10) + (13 \times 0.20) + (14 \times 0.35) + (15 \times 0.20)]$$

$$\boxed{= 1.33}$$