Measures of Central Tendency…….

- *Topics covered:*

✓ Properities of Mean
✓ Median for a grouped data
✓ Mode
✓ Percentile
✓ Measures of dispersion

Dr. HARINAKSHI KARKERA

1. The following is the frequency distribution of the number of telephone calls received in 245 successive one-minute intervals at an exchange. Obtain the mean number of calls per minute

x_i	No. of calls	0	1	2	3	4	5	6	7	
f_i	Frequncy	14	21	25	43	51	40	39	12	
$f_i x_i$	$f_i x_i$	0	21	50	129	204	200	234	84	$= 922$ $\sum f_i x_i$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{922}{245} = \underline{\underline{3.763}}$$

x_i ——
f_i ——
} large

Dr. HARINAKSHI

Step-deviation method

x_i, f_i } large numbers

$x_1, x_2, \ldots, x_n$
$f_1, f_2, \ldots, f_n$

$$\boxed{d_i = x_i - A}$$

A — any arbitrary pt

$$f_i d_i = f_i x_i - f_i A$$

$\sum f_i = N$

$$\sum_{i=1}^{n} f_i d_i = \sum_{i=1}^{n} f_i x_i - A \sum_{i=1}^{n} f_i$$

$$'' \quad = \sum f_i x_i - AN$$

$\div N$ $\Downarrow$

$$\frac{1}{N} \sum f_i d_i = \frac{1}{N} \sum f_i x_i - A$$

$$= \bar{x} - A$$

$$\Rightarrow \boxed{\bar{x} = A + \frac{1}{N} \sum f_i d_i}$$

Dr. HARINAKSHI KARKERA

$$\boxed{\bar{x} = A + \frac{1}{N} \sum f_i d_i}$$

C.I $\Rightarrow$ $d_i' = \dfrac{x_i' - A}{h}$

$$\boxed{\bar{x} = A + \frac{h}{N} \sum f_i d_i}$$

Dr. HARINAKSHI KARKERA

C. I	No. of persons	Mid value (x_i)	d_i=(x_i-47.5)/5	f_i*d_i
40-45	125	42.5	-1	-125
45-50	105	47.5	0	0
50-55	70	52.5	1	70
	N = 300			

$\sum f_i d_i = -55$

Step-deviation method

$A = 47.5$

$$\bar{x} = A + \frac{h}{N} \sum f_i d_i$$

$$= 47.5 + \frac{5}{300} \times (-55)$$

$$= 46.5833 \approx 47$$

Dr. HARINAKSHI KA

Properties of AM

1) The sum of the deviations of the given set of observations from their arithmetic mean is zero.

$$\sum (x_i - \bar{x}) = ?$$

freq. distn $\quad \sum f_i (x_i - \bar{x}) = ?$

$$= \sum f_i x_i - \sum f_i \bar{x}$$

$$= N\bar{x} - \bar{x} N$$

$$= \underline{\underline{0}}$$

Dr. HARINAKSHI KARKERA

2) The sum of squares of deviations of a set of values is minimum when taken about mean.

$$S = \sum_{i=1}^{n} f_i (x_i - a)^2$$

S is minimum $\left.\begin{array}{l} \dfrac{dS}{da} = 0 \\ \dfrac{d^2S}{da^2} > 0 \end{array}\right\}$ $a = \bar{x}$

Dr. HARINAKSHI KARKERA

3) Mean of the combined distⁿ

Let $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k$ be the arithmetic means of k distributions with respective frequencies $n_1, n_2, \ldots, n_k$, then the mean $\bar{x}$ of the whole distⁿ with frequency $N = \sum_{i=1}^{k} n_i$ is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \cdots + n_k\bar{x}_k}{n_1 + n_2 + \cdots + n_k}$$

$$= \frac{1}{N}\sum_{i=1}^{k} n_i\bar{x}_i$$

Dr. HARINAKSHI KARKERA

Median :

median's position $\left(\frac{n+1}{2}\right)$

$$\frac{20+1}{2} = \frac{21}{2} = \underline{\underline{10.5}}$$

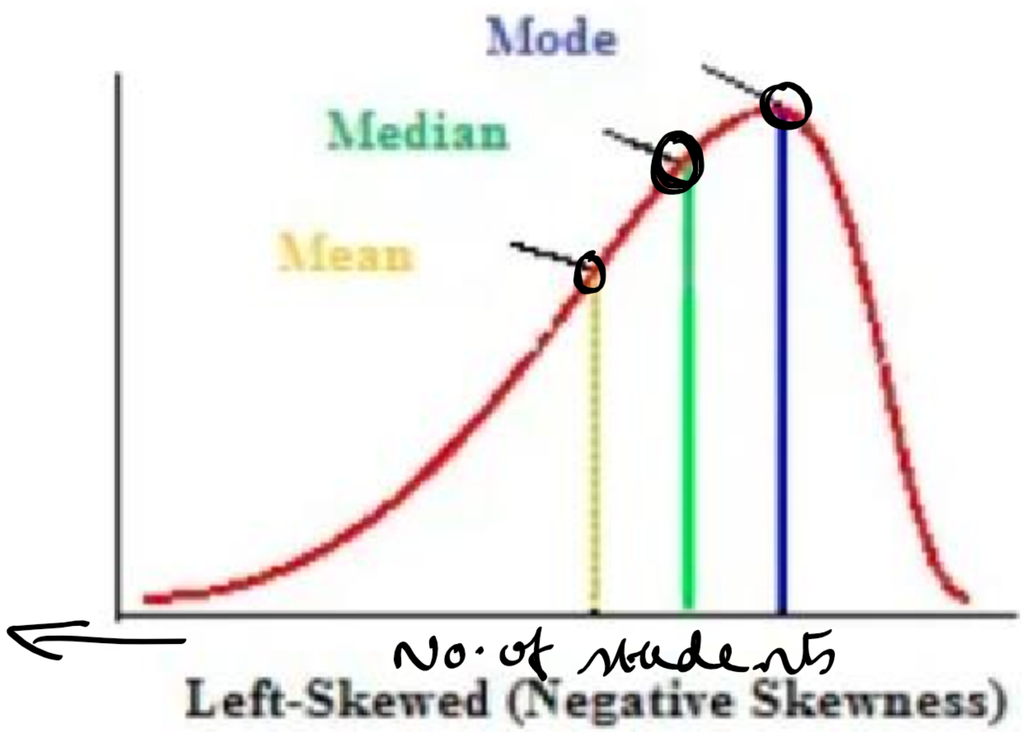
$$\underbrace{10^{th} \quad 11^{th}}_{\text{mean}}$$

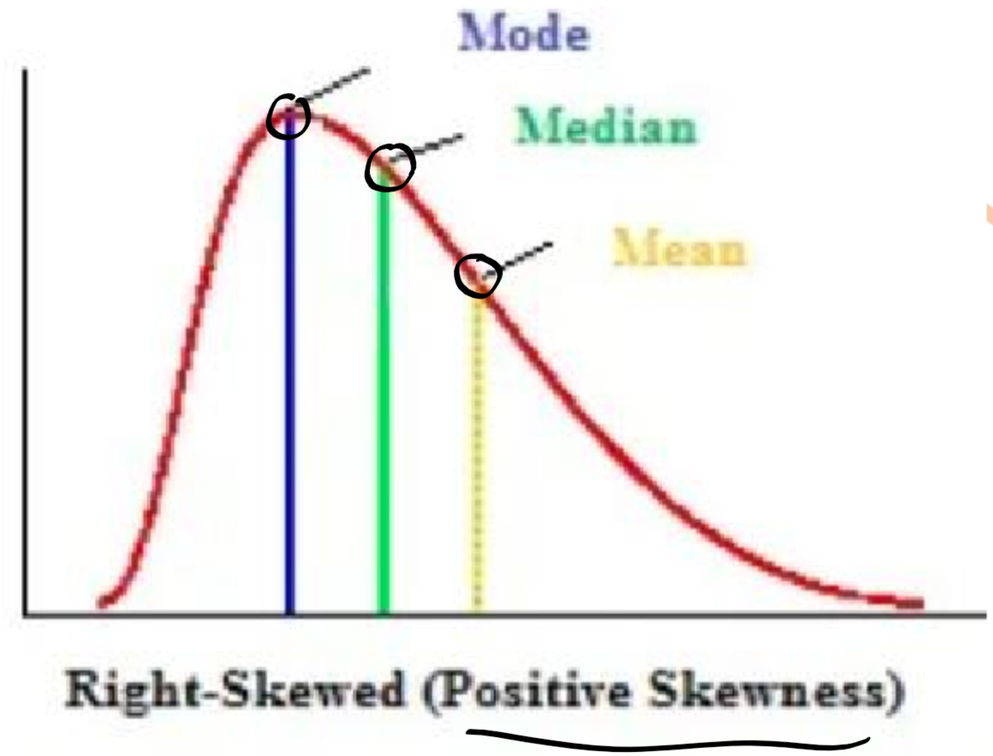
No. of calls	0	1	2	3	4	5	6	7
Frequncy	14	21	25	43	51	40	39	12
C.f	14	35	60	103	154	194	233	245

1) C.F
2) $\frac{N}{2}$
3) Identify C.F just greater than $N/2$
4) The corresponding value of the variable is median.

$$\frac{N}{2} = \frac{245}{2} = \underline{\underline{122.5}}$$

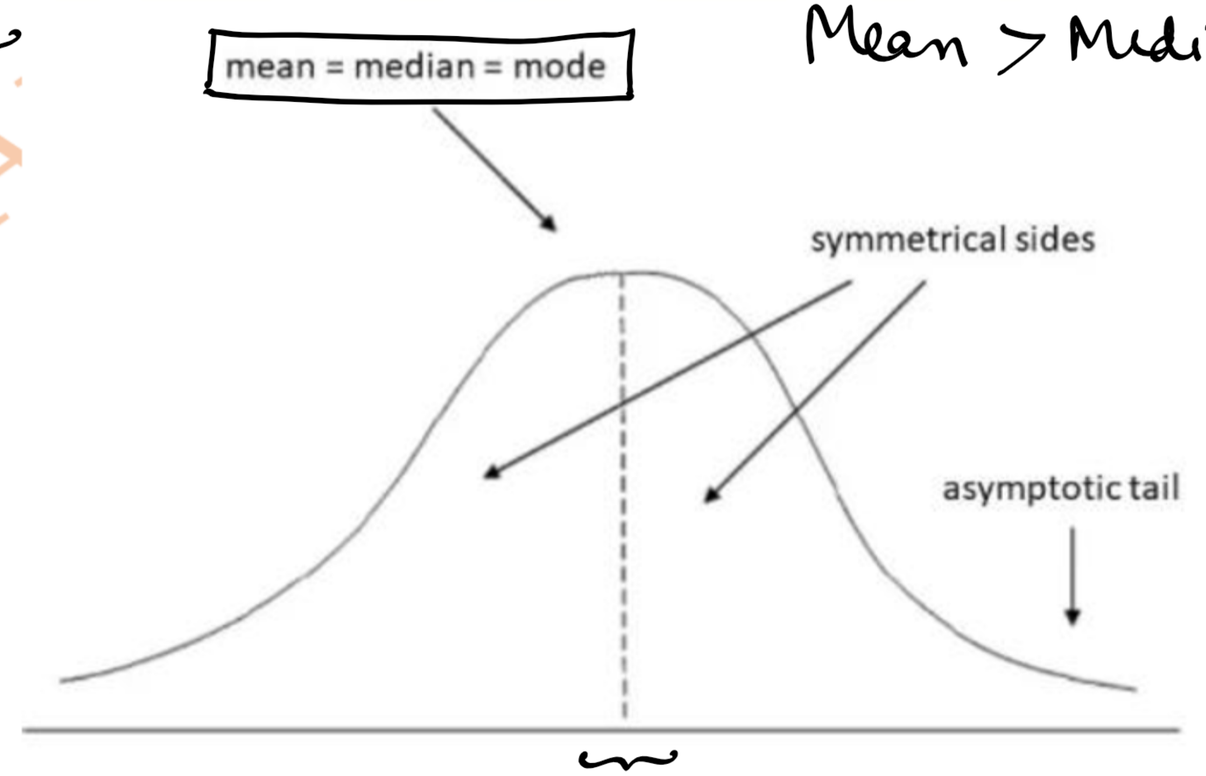
Dr. HARINAKSHI KARKERA





Mean > Median > Mode

Mode > Median > Mean



Dr. HARINA

Median (grouped data) (C.I)

1) C.F
2) Find $N/2$
3) see c.f just greater than $N/2$
4) The corresponding class contains the median value

$$\boxed{\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)}$$

l — the lower limit of the median class

h — width of the median class

f — frequency of the median class

N — total frequencies

c — c.f of the class preceding the median class

Dr. HARINAKSHI KARKERA

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

Class interval	Frequency	C. F
20-30	6	6
30-40	18	24
40-50	11	35
50-60	11	46
60-70	3	49
70-80	1	50

$N = 50$

$\frac{N}{2} = 25$

$l = 40$

$h = 10$

$f = 11$

$c = 24$

$$\text{Median} = 40 + \frac{10}{11}[25 - 24]$$

$$= 40.91$$

Dr. HARINA

2. The following table shows the age distribution of persons in a particular region. Find the median age. Why is the median a more suitable measure of central tendency than the mean in this case?

Age (years)	No. of persons (in thousands)
Below 10	2
Below 20	5
Below 30	9
Below 40	12
Below 50	14
Below 60	15
Below 70	15.5
70 and over	15.6

C.I	No. of persons (1000's)	C.F
0–10	2	2
10–20	5 − 2 = 3	5
20–30	9 − 5 = 4	9
30–40	= 3	12
40–50	2	14
50–60	1	15
60–70	0.5	15.5
70 and over	15.6 − 15.5 = 0.1	15.6

$$\frac{N}{2} = \frac{15.6}{2} = 7.8$$

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$l = 20$, $h = 10$, $f = 4$, $c = 5$

$$\text{Median} = 20 + \frac{10}{4}[7.8 - 5]$$

$= 27$ years

Dr. HA