Relationship between the Poisson and Exponential Distributions

The Poisson distribution provides an appropriate description of the number of occurrences per interval

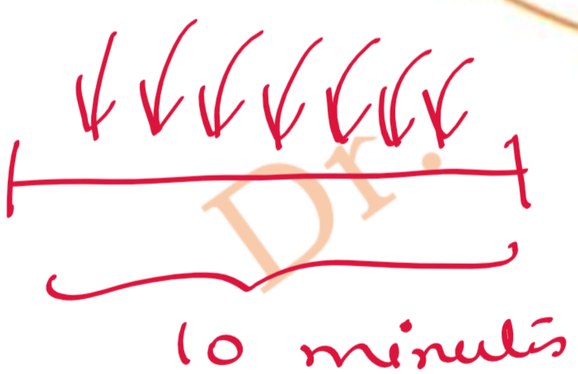
The exponential distribution provides an appropriate description of the length of the interval between occurrences

$r = 1$

Gamma

$r = 3$

$= 4$



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Mean : Poisson & exponential distribution

$E(X) = \lambda$ (or α) $\qquad E(X) = \frac{1}{\alpha}$

Ex: 10 cars 1 hour

✓ Poisson, mean $= \alpha = 10$

✓ Exponential, mean $= \frac{1}{\alpha} = \frac{1 \text{ hrs}}{10 \text{ cars}} = 0.1$ hours/car

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Problem of Gamma distribution

The daily consumption of electric power is in million kw hour in a certain city is a random variable x having the pdf

$$f(x) = \begin{cases} \frac{1}{9} x e^{-x/3} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that the power supply is inadequate on any given date if the daily capacity of the power plant is 12 million kw hours. Also find the mean & variance.

Gamma distribution

$$f(x) = \frac{\alpha}{\Gamma\lambda} (\alpha x)^{\lambda - 1} e^{-\alpha x} \quad ; x > 0$$

$\alpha = \frac{1}{3}$, $\lambda = 2$

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$$\therefore P(X>12) = \int_{12}^{\infty} f(x)\,dx = \int_{12}^{\infty} \frac{1}{9}\underbrace{x}_{\mathrm{I}}\,\underbrace{e^{-x/3}}_{\mathrm{II}}\,dx$$

$$= \frac{1}{9}\left[x \cdot \frac{e^{-x/3}}{-1/3} - (1)\cdot \frac{e^{-x/3}}{1/9} \right]\Bigg|_{x=12}^{\infty}$$

$$\boxed{= 5e^{-4}}$$

Find the mean & variance

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Problem of Geometric distribution

X: no. of failures preceding the first success

$$P(X=x) = \begin{cases} q^x p & ; x = 0,1,2,\ldots \\ 0 & \text{otherwise} \end{cases}$$

Problem: If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try ?

X: no. of trials required to pass the road test

$p = 0.75$ Using GD

$$P(X=x) = q^x p \quad ; x = 0,1,2,\ldots$$

$P[\text{Applicant will pass the test on } 4^{th} \text{ try}] = q^3 p = (0.25)^3 (0.75)$
$= 0.0117$

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Negative binomial distribution (Pascal Distribution)

$\binom{n}{r} p^r q^{n-r}$

no. of success
n Be. trials

BD mean > variance

PD mean = variance

variance > mean

Generalization of geometric distn

Y: no. of repetitions needed in order to have an event A occur exactly r times

$$P(Y=k) = p\binom{k-1}{r-1} p^{r-1} q^{k-r} \quad ; k = r, r+1, \ldots$$

$$= \binom{k-1}{r-1} p^{r} q^{k-r} \quad ; k = r, r+1, \ldots$$

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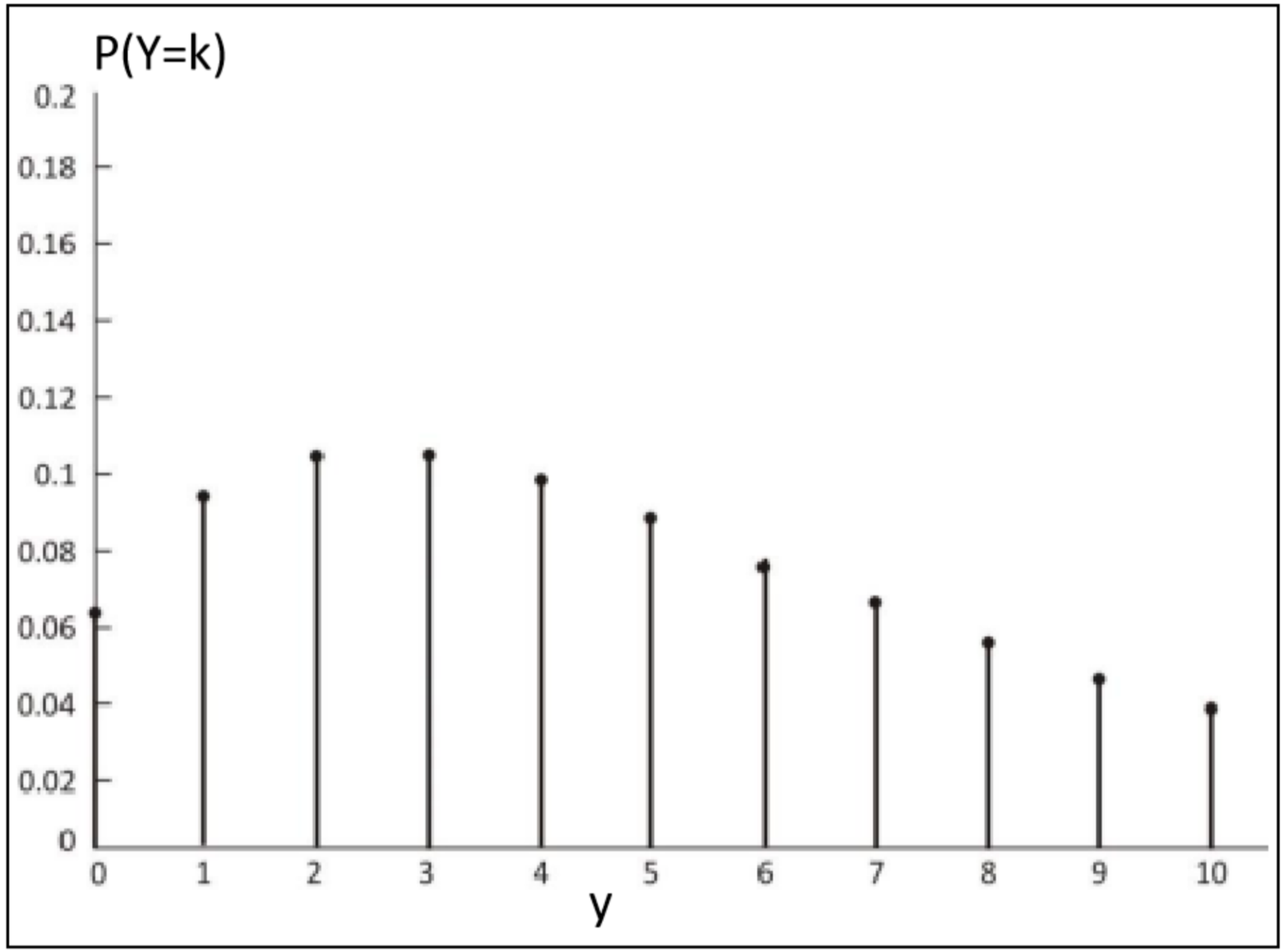

Figure 1.4. Plot of p.m.f. of NB $(2, \frac{1}{4})$

$$\sum_{k=r}^{\infty} P(Y=k) = 1$$

Mean and Variance of Pascal distribution

$$E(Y) = \frac{r}{p}$$

$$V(Y) = \frac{rq}{p^2}$$

$r = 1$

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Relationship between binomial and negative binomial distributions

Let X have a binomial distribution with parameters n & p

ie X : no. of successes in n Bernoulli trials with $P(\text{success}) = p$

Let Y have a Pascal distribution with parameters r & p

ie Y : no. of Bernoulli trials required to obtain r successes with $P(\text{success}) = p$

Then the following relationships hold:

(a) $P(Y \leq n) = P(X \geq r)$

(b) $P(Y > n) = P(X < r)$

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Problem: An expert sharpshooter misses a target 5 percent of the time. Find the probability that she will miss the target for the second time on the fifteenth shot.

Y: target is missed for the second time

$r = 2$, $k = 15$, $p = \frac{5}{100} = 0.05$

Pascal dist^n

$$P[Y=k] = \binom{k-1}{r-1} p^r q^{k-r} \quad ; k = r, r+1, \ldots$$

$$P[Y=15] = \binom{14}{1} (0.05)^2 (0.95)^{13}$$

$$= \underline{\underline{0.017966}}$$

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Problem: If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it?

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The Hypergeometric Distribution

without replacement

N items

r — defective

$N-r$ — Nondefective

Random selection

n items $(n \le N)$

X : no. of defectives found

$X = k$ 'k' defective items

$k \,] \xrightarrow{n} n-k$

$$P[X=k] = \frac{\binom{r}{k}\binom{N-r}{n-k}}{\binom{N}{n}} \; ; \quad k = 0, 1, 2, \ldots$$

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Theorem: Let X have a hypergeometric distribution as given by

$$P[X=k] = \frac{\binom{r}{k}\binom{N-r}{n-k}}{\binom{N}{n}} \quad ; k=0,1,2,\ldots$$

Let $p = \frac{r}{n}$, $q = 1-p$

then (a) $E(X) = np$

(b) $V(X) = npq\,\frac{N-n}{N-1}$

(c) $P(X=k) \simeq \binom{n}{k} p^k (1-p)^{n-k}$ for large N

Problem: Small electric motors are shipped in lots of 50. Before such a shipment is accepted, an inspector chooses 5 of these motors and inspects them. If none of these tested motors are defective, the lot is accepted. If one or more are found to be defective, the entire shipment is inspected. Suppose that there are, in fact, three defective motors in the lot. What is the probability that 100 percent inspection is required?

X: no. of defective motors found

Then 100% inspection will be required if $X \geq 1$

hypergeometric

$N = 50$

$n = 5$

$k = 1$

$r = 3$

$$P[X=k] = \frac{\binom{r}{k}\binom{N-r}{n-k}}{\binom{N}{n}}$$

$$\therefore P(X \geq 1) = 1 - P[X=0]$$

$$= 1 - \frac{\binom{3}{0}\binom{47}{5}}{\binom{50}{5}}$$

$$= 0.28$$

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Problem: As part of an air-pollution survey, an inspector decides to examine the exhaust of six of a company's 24 trucks. If four of the company's trucks emit excessive amounts of pollutants, what is the probability that none of them will be included in the inspector's sample?

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The Multinomial Distribution

BD 'n' Bernoulli trials

Each each < success, failure

Generalization of BD

'$\mathcal{E}$' Random experiment —— S = sample space

$A_1, A_2, \ldots, A_k$] partitions of S

'n'

$p_i = P(A_i)$ $\qquad \sum_{i=1}^{k} p_i = 1$

$X_1, X_2, \ldots, X_k$

X_i: no. of times A_i occurs among n repetitions

$$\sum_{i=1}^{k} n_i = n$$

$$P[X_1 = n_1, X_2 = n_2, \ldots, X_k = n_k] = \frac{n!}{n_1!\, n_2!\, n_3! \ldots n_k!} p_1^{n_1} \cdot p_2^{n_2} \cdots p_k^{n_k}$$

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Theorem: Suppose that $(X_1, X_2, \cdots, X_K)$ has a multinomial distribution (as given in previous slide)

then $E(X_i) = n p_i$

$V(X_i) = n p_i (1 - p_i)$; $i = 1, 2, \cdots, K$

* $(p_1 + p_2 + \cdots + p_K)^n$

* $K = 2$, $(p_1 + p_2)^n$ $\longleftarrow$ $p_2 = 1 - p_1$

$\binom{n}{k} p^k (1-p)^{n-k}$

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Problem: A certain city has three television stations. During prime time on Saturday nights, Channel 12 has 50 percent of the viewing audience, Channel 10 has 30 percent of the viewing audience, and Channel 3 has 20 percent of the viewing audience. Find the probability that among eight television viewers in that city, randomly chosen on a Saturday night, five will be watching Channel 12, two will be watching Channel 10, and one will be watching Channel 3.

channel 12	channel 10	channel 3
50%	30%	20%

A_1 : channel 12 viewing audience on Saturday night

A_2 : channel 10 " " "

A_3 : channel 3 " " " "

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$p_1 = P(A_1) = 0.5$

$p_2 = P(A_2) = 0.3$

$p_3 = P(A_3) = 0.2$

$$\sum_{k=1}^{3} p_k = 1$$

Multinomial distn

$$P[X_1 = n_1, X_2 = n_2, \ldots, X_k = n_k] = \frac{n!}{n_1!\, n_2! \ldots n_k!} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

X_i : no. of viewers for A_i

$$P[X_1 = 5, X_2 = 2, X_3 = 1] = \frac{8!}{5!\,2!\,1!}(0.5)^5(0.3)^2(0.2)^1$$

$$= \underline{\underline{0.0945}}$$

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