* If c is a constant,

$$\boxed{V(X+c) = V(X)}$$

Proof:

$$V(X+c) = E(X+c)^2 - [E(X+c)]^2$$

$$= E(X^2) + c^2 + 2cE(X) - (E(X) + c)^2$$

$$= E(X^2) + \cancel{c^2} + \cancel{2cE(X)} - (E(X))^2 - \cancel{c^2} - \cancel{2cE(X)}$$

$$= E(X^2) - (E(X))^2$$

$$= V(X)$$

$$\boxed{V(X) = E(X^2) - (E(X))^2}$$

Dr. HARINAKSHI KARKERA

* $V(cX) = c^2 V(X)$

$V(X) = E(X^2) - (E(X))^2$

$$V(cX) = E(cX)^2 - [E(cX)]^2$$

$$= c^2 E(X^2) - c^2 (E(X))^2$$

$$= c^2 \{ E(X^2) - [E(X)]^2 \}$$

$$= c^2 V(X)$$

Dr. HARINAKSHI KARKERA

c.d.f

Let X be a c.r.v.

$$F(x) = P[X \le x]$$

$$= \int_{-\infty}^{x} f(t)\,dt = \int_{-\infty}^{\infty} f(t)\,dt - \int_{x}^{\infty} f(t)\,dt$$

$$\boxed{F(x) = 1 - \int_{x}^{\infty} f(t)\,dt}$$

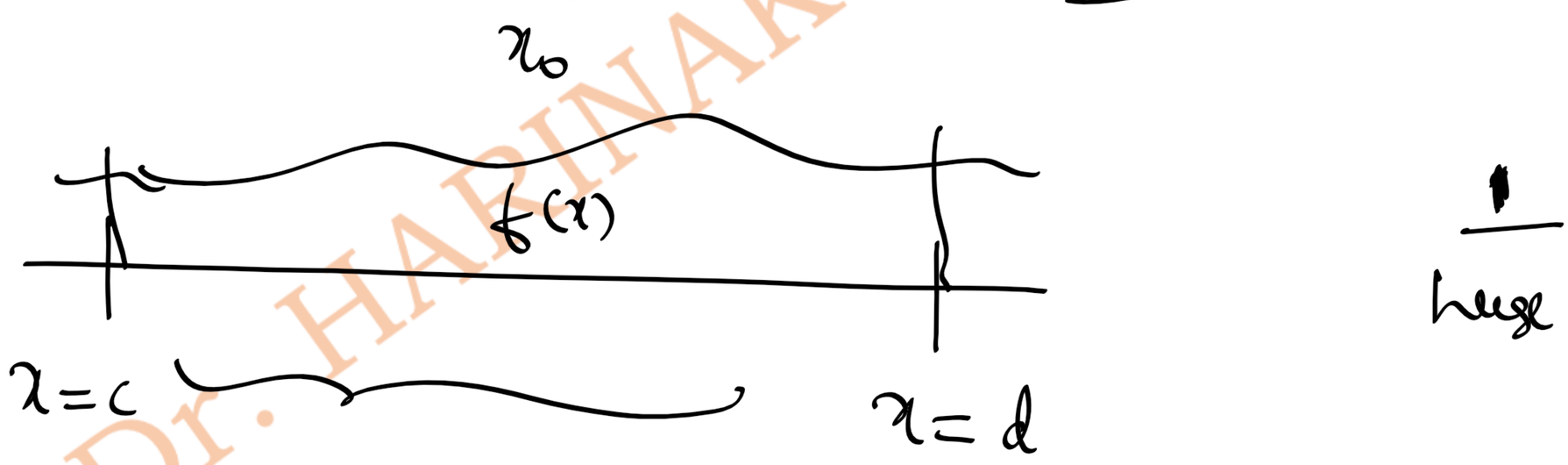
Dr. HARINAKSHI KARKERA

Note:

If X is c.r.v

(i) Probability of X for any specified value say x_0, is zero.

$$P[X = x_0] = \int_{x_0}^{x_0} f(x)\,dx = \underline{\underline{0}}$$



(ii) $P(c \le X \le d) = P(c \le X < d)$

$= P(c < X \le d) = P(c < X < d)$

Dr. HARINAKSHI KARKERA

* In a box containing 8 mobiles, 3 are found to be defective. Two mobiles are randomly purchased. Find the probability distribution for the no. of defective mobiles.

X : the no. of defective mobiles

x_i : 0, 1, 2

x_i	0	1	2
$p(x_i)$	0.3571	0.5357	0.1071

$$p(0) = P[X=0] = \frac{{}^5C_2}{{}^8C_2} = 0.3571$$

$$p(1) = P[X=1] = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2}$$

$$p(2) = P[X=2] = \frac{{}^3C_2}{{}^8C_2}$$

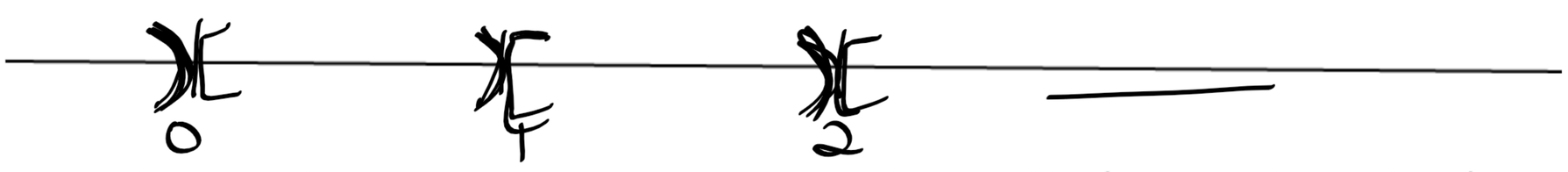
Dr. HARINAKSHI KARKERA

* Suppose that r.v. X assumes three values 0, 1 & 2 with probabilities $1/3, 1/6, 1/2$ respectively. Obtain c.d.f of X.

x_i	0	1	2
$p(x_i)$	$1/3$	$1/6$	$1/2$

$= 1$

$$F(x) = P[X \le x] = \begin{cases} 0 & x < 0 \\ 1/3 & 0 \le x < 1 \\ 1/2 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$



Dr. HARINAKSHI KARKERA

* The probability distribution $P(X)$ of a random variable X is given by the following table

x_i	0	1	2	3	4	5	6
$p(x_i)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

For what value of k does this represent a valid probability dist^n? Also find

i) $P[X<4]$ ii) $P[3<X\leq 6]$

$$\sum p(x_i) = 1$$

$$\Rightarrow k = \frac{1}{49}$$

(i) $P[X<4] = \sum_{x_i<4} p(x_i) = 16/49$

(ii) $P[3<X\leq 6] = p(4)+p(5)+p(6) = 33/49$.

Dr. HARINAKSHI KARKERA

* Let X be a c.r.v with pdf given by

$$f(x) = \begin{cases} ax & 0 \le x < 1 \\ a & 1 \le x < 2 \\ -ax+3a & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

(i) determine the constant 'a'

(ii) determine the cdf $F(x)$.

(a) $f(x) \geq 0 \Rightarrow a \geq 0$

(b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

DR. HARINAKSHI KARKERA

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

$$a\cdot\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^2 - a\frac{x^2}{2}\Big|_2^3 + 3a\cdot x\Big|_2^3 = 1$$

ie $a\left[\frac{1}{2}\right] + a - \frac{a}{2}(9-4) + 3a = 1$

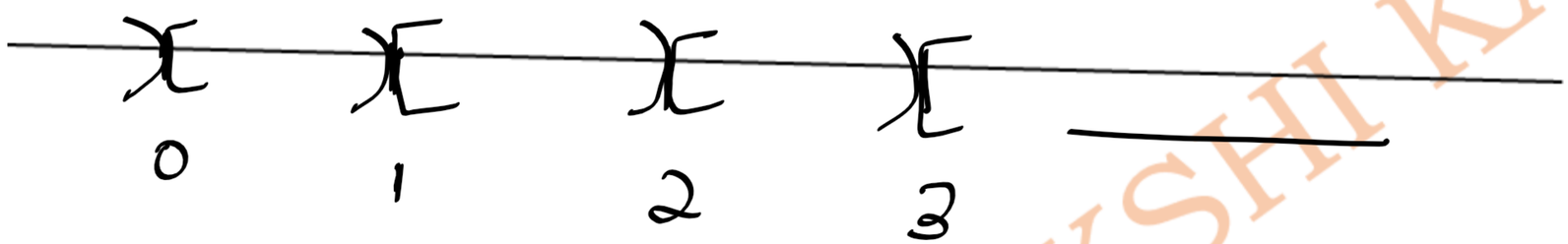
$$\frac{a+2a-5a+6a}{2} = 1$$

$$\Rightarrow \boxed{a = 1/2}$$

Dr. HARINAKSHI KARKERA

(ii) finding cdf

$$f(x) = \begin{cases} ax & 0 \le x < 1 \\ a & 1 \le x < 2 \\ -ax+3a & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$



$$F(x) = P[X \le x] = \int_{-\infty}^{x} f(x)\,dx$$

For $-\infty < x < 0$,

$$F(x) = \int_{-\infty}^{x} 0\,dx = 0$$

Dr. HARINAKSHI KARKERA

For $0 \le x \le 1$

$$f(x) = \begin{cases} ax & 0 \le x < 1 \\ a & 1 \le x < 2 \\ -ax+3a & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx = 0 + \int_{0}^{x} ax \cdot dx$$

$$= a \cdot \frac{x^2}{2}\Big|_{0}^{x} = \underline{\underline{\frac{x^2}{4}}}$$

For $1 \le x < 2$

$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} ax \cdot dx + \int_{1}^{x} a \cdot dx$$

Dr. HARINAKSHI KARKERA

For $1 \le x < 2$

$$F(x) = \int_{-\infty}^{0} 0 \, dx + \int_{0}^{1} ax \cdot dx + \int_{1}^{x} a \cdot dx$$

$$= 0 + \frac{x^2}{4}\Big|_0^1 + \frac{1}{2} x \Big|_1^x$$

$$= \frac{1}{4} + \frac{1}{2}(x-1)$$

$$F(x) = \frac{x}{2} - \frac{1}{4}$$

Dr. HARINAKSHI KARKERA

For $2 \le x < 3$

$$F(x) = \int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{1} ax \cdot dx + \int_{1}^{2} a\, dx + \int_{2}^{x} (-ax + 3a)\, dx$$

$$= 0 + \frac{1}{4} + \frac{1}{2} - \frac{a}{2} x^2 \Big|_{2}^{x} + 3ax \Big|_{2}^{x}$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{4}(x^2 - 4) + \frac{3}{2}(x - 2)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{x^2}{4} + \frac{3x}{2} - 2$$

$$F(x) = \frac{-5 - x^2 + 6x}{4}$$

Dr. HARINAKSHI KARKERA

For $x \geq 3$,

$$F(x) = \int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{3} (-ax+3a)\,dx + \int_{3}^{x} 0 \cdot dx$$

$$= 0 + \frac{1}{4} + \frac{1}{2} - \frac{1}{4}(9-4) + \frac{3}{2}(3-2) + 0$$

$$= \frac{1+2-5+6}{4} = \frac{4}{4} = 1$$

Dr. HARINAKSHI KARKERA

$$\therefore\ F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^2}{4} & 0 \le x < 1 \\ \dfrac{x}{2} - \dfrac{1}{4} & 1 \le x < 2 \\ \dfrac{-5 - x^2 + 6x}{4} & 2 \le x < 3 \\ 1 & x \geq 3 \end{cases}$$

Dr. HARINAKSHI KARKERA

* Let X be the life length of an electric device (measured in hrs). Suppose that X is a c.r.v with $f(x) = \frac{k}{x^n}$;

$2000 \le x \le 10{,}000$,

i) determine k

ii) What is the probability that the device will fail before 5000 hrs have elapsed?

$P(X < 5000)$

HW

Dr. HARINAKSHI KARKERA

* A random variable X has the density function

$$f(x) = \begin{cases} kx^2 & -3 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

Evaluate the constant k. Find

(i) $P(1 \le x \le 2)$ (ii) $P(x > 1)$ (iii) $P(x \le 2)$

HW

Dr. HARINAKSHI KARKERA