Probability . . . . . .

* If A & B are independent events, occurrence of B will be independent of occurrence of A

∴ the conditional & unconditional probabilities are equal. ie $P(B/A) = P(B)$

$$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B)}$$

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Multiplication theorem

Let A & B be two events with respective probabilities P(A) & P(B).

Let P(B|A) be the conditional probability of event B given that A has already occurred.

Then the probability of simultaneous occurrence of A & B is

$$\boxed{P(A \cap B) = P(A) \cdot P(B|A)}, \text{ provided } P(A) > 0$$

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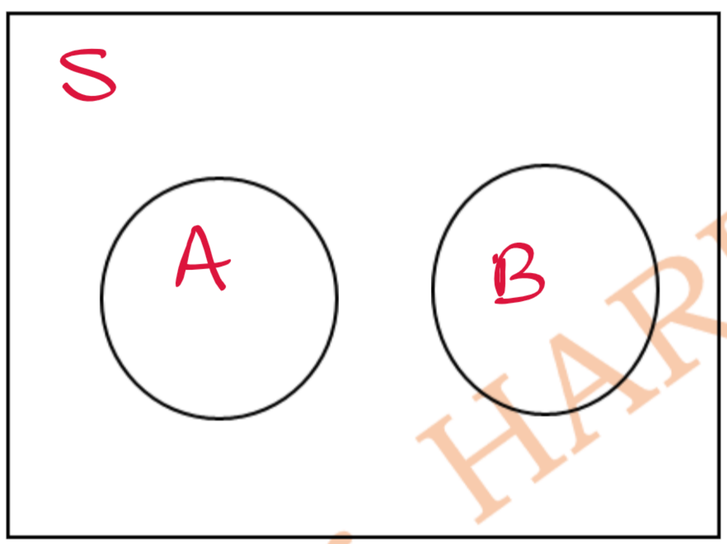
Generalized multiplication theorem

$$P[A_1 \cap A_2 \cap \dots \cap A_n] = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1, A_2) \cdots P(A_n|A_1, A_2, \dots, A_{n-1})$$

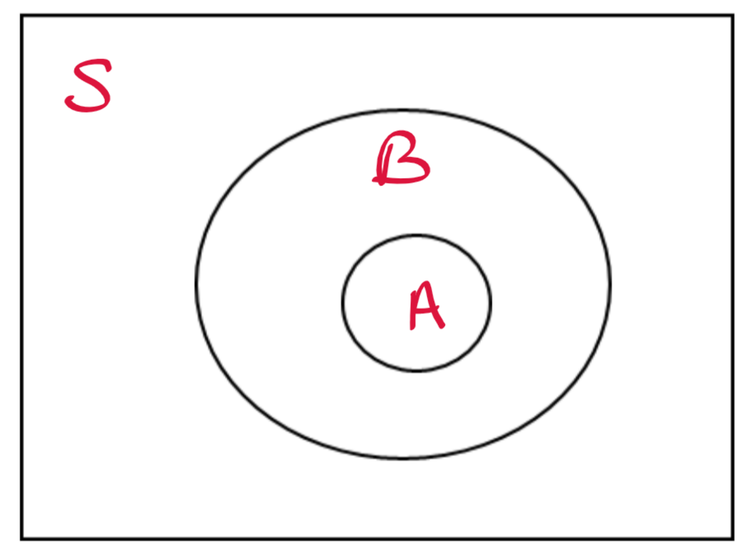
Relation between $P(A|B)$ & $P(A)$

Case 1: If $A \cap B = \phi$



$$\boxed{P(A|B) = 0 \leq P(A)}$$

Case 2:



If $A \subset B$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

$$\boxed{P(A|B) \geq P(A)} \quad (\because P(A \cap B) = P(A))$$

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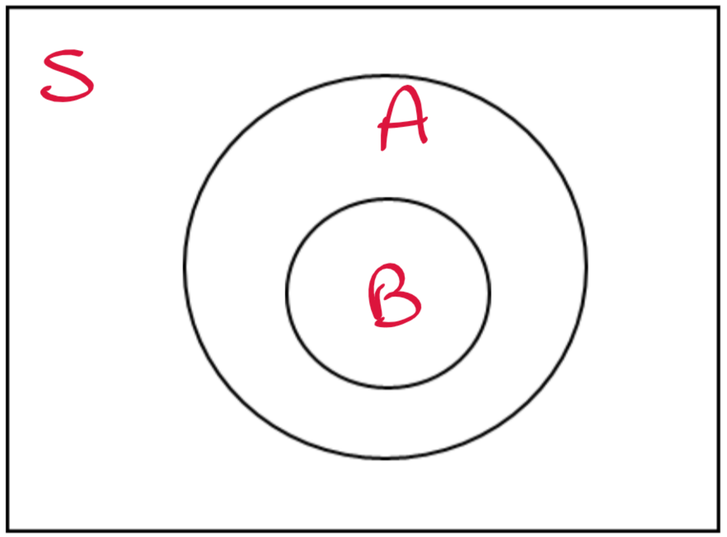
Relation between $P(A|B)$ & $P(A)$

Case (3) :



If $B \subset A$

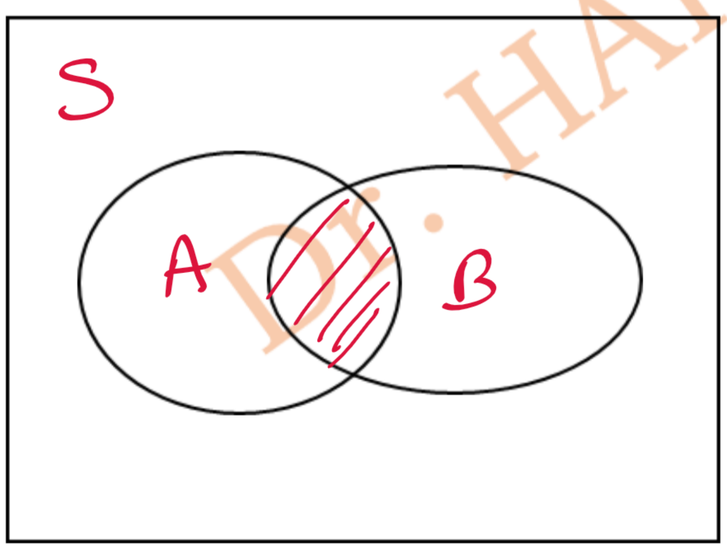
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$\boxed{\therefore P(A|B) \geq P(A)}$$

$$P(A|B) = 1 \geq P(A)$$

Case (4) :



We cannot compare

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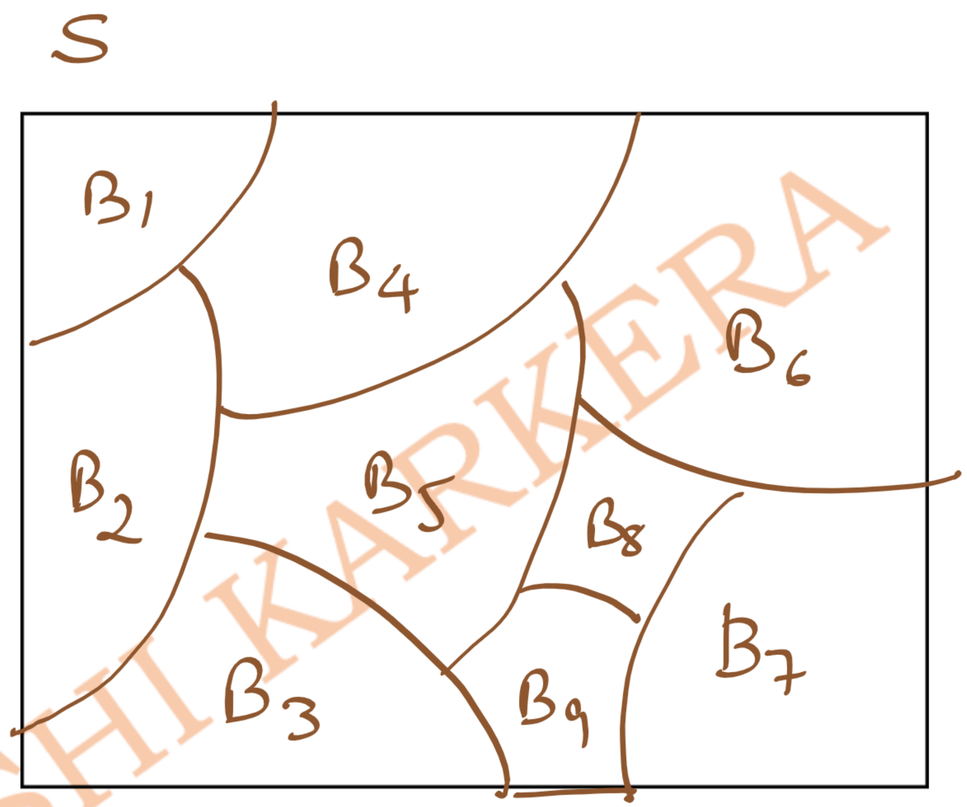
Partition of the sample space S



The events $B_1, B_2, \ldots, B_k$ represents a partion of the sample spaces if

(a) $B_i \cap B_j = \phi$ for all $i \neq j$

(b) $\bigcup_{i=1}^{k} B_i = S$

(c) $P(B_i) > 0$ for all i

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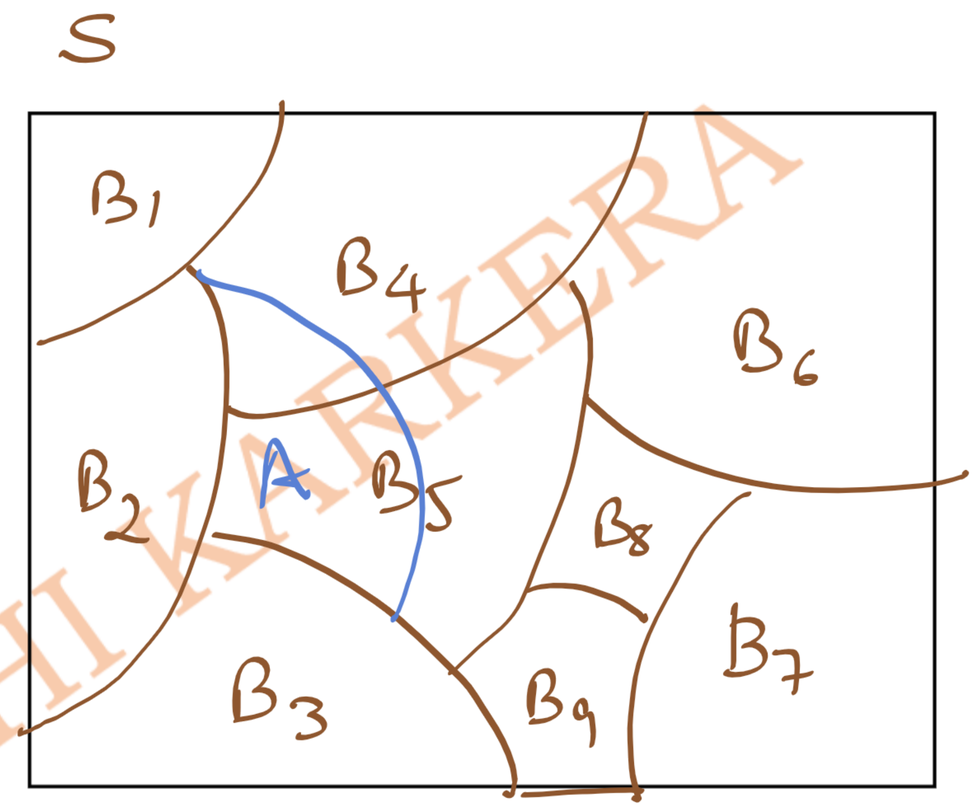
Total Probability

Let S be the sample space associated with an experiment E. Let A be some event w.r.t to S. and let $B_1, B_2, \ldots, B_K$ be a partition of S



$$A = (A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_K)$$

$(A \cap B_1), (A \cap B_2), \ldots, (A \cap B_K)$ are pairwise mutually exclusive

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_K)$$

$$\boxed{P(A) = P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2) + \cdots + P(A|B_K) \cdot P(B_K)}$$

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Bayes' theorem

Let $B_1, B_2, \ldots, B_K$ be a partition of the sample space S and let A be an event associated with S.

Then
$$\boxed{P(B_i|A) = \frac{P(A|B_i)\cdot P(B_i)}{\sum_{j=1}^{K} P(A|B_j)\cdot P(B_j)}}, \quad i=1,2,\ldots,K$$

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Probability Problems

From six positive and eight negative numbers, four numbers are chosen at random & multiplied. What is the probability that product is +ve number.

6 +ve no's
8 −ve no's] 4 no's multiplied.

i) + + + +

$$P[\text{product is +ve number}] = \frac{{}^{6}C_4}{{}^{14}C_4}$$

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ii) $- - - -$ $\quad P[\text{product is a +ve number}]$

$$= \frac{{}^{8}C_{4}}{{}^{14}C_{4}}$$

iii) $- - + +$ $\quad P[\text{product is a +ve number}]$

$$= \frac{{}^{6}C_{2} \times {}^{8}C_{2}}{{}^{14}C_{4}}$$

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Show that for any two events A_1 & A_2, $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$. Deduce the result for n events.

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A box contains bags marked $1, 2, 3, \ldots, n$. Two bags are chosen at random. Find the probability that the number on the bags will be consecutive integer

i) if bags are chosen with replacement

ii) if bags are chosen without replacement.

(i) <u>with replacement</u>

$$\left.\begin{array}{l}(1,2), (2,3), (3,4), \ldots, (n-1, n) \\ (2,1), (3,2), (4,3), \ldots, (n, n-1)\end{array}\right]$$

Total no. of favourable outcomes

$(n-1) + (n-1) = 2(n-1)$

The no. of possible outcomes $= n \times n = n^2$

$\therefore P[\text{the no. on the bags will be consecutive integers}]$

$$= \frac{2(n-1)}{n^2}$$

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(ii) <u>Without</u> <u>replacement</u>

The no. of possible outcomes $= {}^{n}C_{2}$

Favourable outcomes

$(1,2), (2,3), (3,4), \ldots, (n-1, n)$

The no. of favourable outcomes $= n-1$

$\therefore$ P[the no. on the tags will be consecutive integers]

$$= \frac{n-1}{{}^{n}C_{2}} = \frac{n-1}{\left(\frac{n(n-1)}{2}\right)} = \frac{2(n-1)}{n(n-1)}$$

$$= \frac{2}{n}$$

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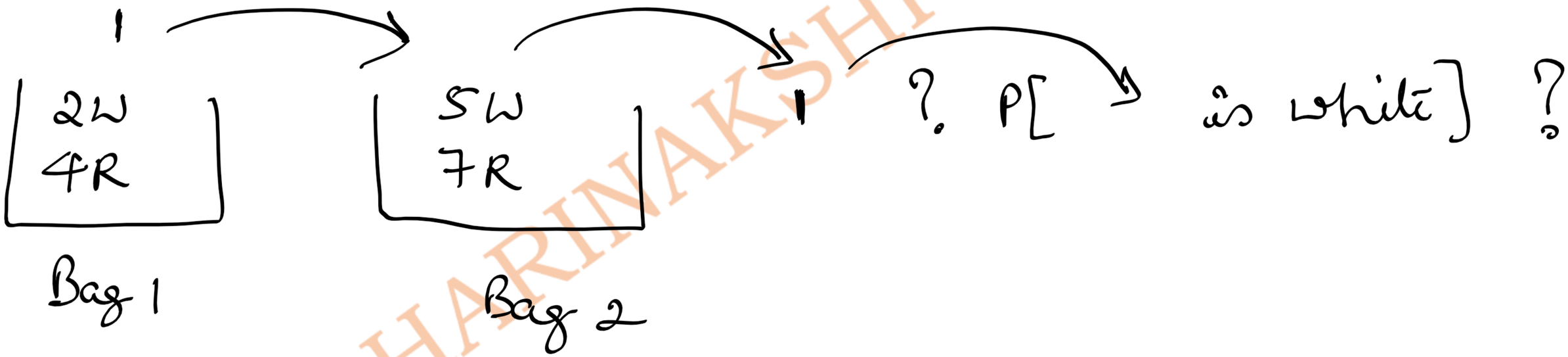
Bag 1 contains 2 white and 4 red marbles. Bag 2 contains 5 white and 7 red marbles. A marble is chosen at random from Bag 1 and put into Bag 2 and then a marble is chosen at random from Bag 2. What is the probability that this marble is white.



0.84 ?

0.41

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