Evaluating the probability distribution of Y using probability distribution of X

(2) Continuous random variable

Let $X$ be a continuous r.v with pdf $f$ & $H$ is a continuous function. Then $Y=H(X)$ is a c.r.v.

Procedure to obtain pdf of $Y$, $g$

i) Obtain $G$, the cdf of $Y$, where $G(Y) = P(Y \leq y)$, by finding the event $A$ (in the range space of $X$) which is equivalent to the event $\{Y \leq y\}$.

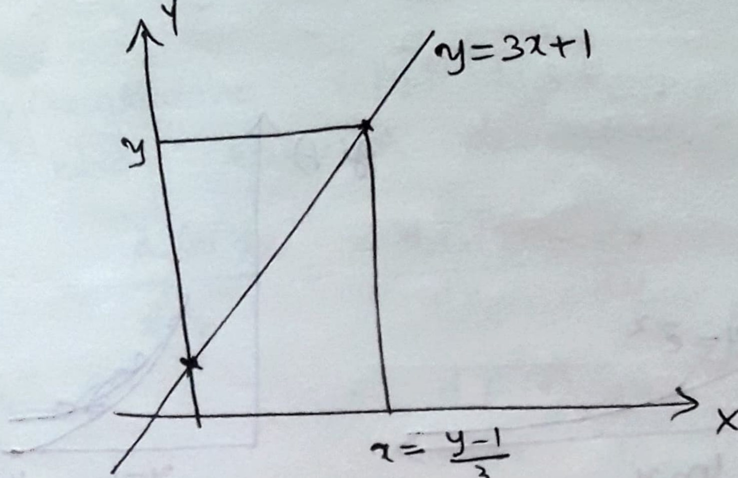
ii) Differentiate $G(y)$ w.r.t $y$ in order to obtain $g(y)$

iii) Determine those values of $y$ in the range space of $Y$ for which $g(y) > 0$.

* Suppose that $X$ has pdf

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Let $H(x) = 3x+1$. Find the pdf of $Y = H(X)$.




1) $G(y) = P(Y \le y) = P(3X+1 \le y)$

$$= P\left(X \le \frac{(y-1)}{3}\right)$$

$$= \int_0^{(y-1)/3} 2x\,dx = \left[\frac{(y-1)}{3}\right]^2$$

ii) $g(y) = G'(y) = \frac{2}{9}(y-1)$

iii) $\because f(x) > 0$ for $0 < x < 1$, $g(y) > 0$ for $1 < y < 4$.

Note: $G(Y) = P(Y \le y) = P\left(X \le \frac{y-1}{3}\right) = F\left(\frac{y-1}{3}\right)$

where $F$ is the cdf of $X$

ie $F(x) = P(X \le x)$

(ii) Let $H(x) = e^{-x}$. Find the pdf of $Y = H(X)$

$$G(y) = P(Y \le y) = P(e^{-X} \le y)$$

$$= P(X \ge -\log y) = \int_{-\log y}^{1} 2x\,dx$$

$$= 1 - (-\log y)^2$$

$\therefore\ g(y) = G'(y) = -\dfrac{2\log y}{y}$.

$\because\ f(x) > 0 \quad$ for $0 < x < 1$

$\Rightarrow g(y) > 0 \quad$ for $\frac{1}{e} < y < 1$.