Probability

B. TECH. DATA SCIENCE & ENGINEERING – MATHEMATICS PAPERS

THIRD SEMESTER

MAT 2157: MATHEMATICAL FOUNDATIONS FOR DATA SCIENCE-I [3 0 0 3]

Statistics: Measures of central tendency – mean, median, mode, measures of dispersion – mean deviation, standard deviation, quartile deviation, skewness and kurtosis. Correlation coefficient, regression, least squares principles of curve fitting.

Probability: Introduction, finite sample spaces, conditional probability and independence, Baye's theorem, one dimensional random variable, mean, variance. Two and higher dimensional random variables: mean, variance, correlation coefficient.

L9	Basic concepts of probability	CO2
T2	Tutorials	CO2

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MIT/GEN/F-01/R2

L10	Conditional probability, concept of independence	CO2
L11	Bayes theorem and applications	CO2
L12	Random variables-descriptive measures expectation, mgf	CO2

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Probability

— numerical measure which indicates the chance of occurrence.

* Head, Tail

$$\frac{1}{2} = 0.5$$

50%

Random experiment:

— An experiment which may not result in same outcome when repeated under the same condition.

* Deterministic model
* Probabilistic model (Stochastic model)

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Sample space:

S : Set of all possible outcomes of a random experiment

Event: Subset of S

A, B, C

Null event: ϕ

Simple event: Contains only one outcome

Sure event: " all the outcomes

Complement of event A $\bar{A}$

$\bar{A} =$

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Union of events

A & B

$A \cup B$

Intersection of events

$A \cap B$

R. Experiment: Tossing a coin

A : Head occurs

B : Tail occurs

A & B

Equally likely events

Two or more events are equally likely if they have equal chance of occurrence

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Mutually exclusive events

Events with no common outcome if only one of them occur at a time

A & B $\quad A \cap B = \phi$, $P(A \cap B) = 0$

Exhaustive events:

A set of events is exhaustive if one or the other of the events in the set occur whenever the experiment is conducted.

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{x \in S \mid x \geq 2\}$

$= \{2, 3, 4, 5, 6\}$

$B = \{x \in S \mid x$ is divisible by $2\}$

$= \{2, 4, 6\}$

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$$P(A) = \frac{\text{No. of favourable outcomes to event } A}{\text{Total no. of possible outcomes}}$$

Kolmogorov's axioms

(1) $0 \leq P(A) \leq 1$

(2) $P(S) = 1$

(3) A & B are mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

$A_1, A_2, \ldots, A_n$ pairwise mutually exclusive events

$$P\left(\bigcup_{i=1}^{n} A_i\right) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

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* $P(\phi) = 0$

$$A = A \cup \phi$$

$$P(A) = P(A \cup \phi)$$

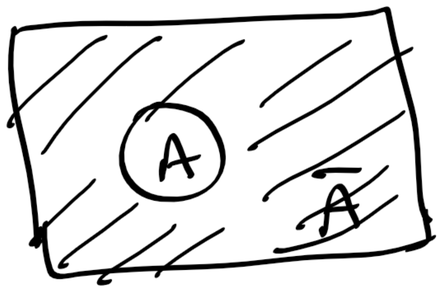
$$P(A) = P(A) + P(\phi)$$

$$\Rightarrow \boxed{P(\phi) = 0}$$

A & ϕ are mutually exclusive events

* $P(\bar{A}) = 1 - P(A)$

A & $\bar{A}$



$$S = A \cup \bar{A}$$

$$P(S) = P(A \cup \bar{A})$$

$$1 = P(A) + P(\bar{A}) \Rightarrow \boxed{P(\bar{A}) = 1 - P(A)}$$

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* If $A \subset B$, then $P(A) \leq P(B)$

* For any two events A & B

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

HW

Probability that atleast one of the two events A or B occurs

A, B, C

$$\boxed{\begin{aligned} P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) - P(A \cap B) \\ & - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \end{aligned}}$$

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$A_1, A_2, \ldots, A_n$ are n events

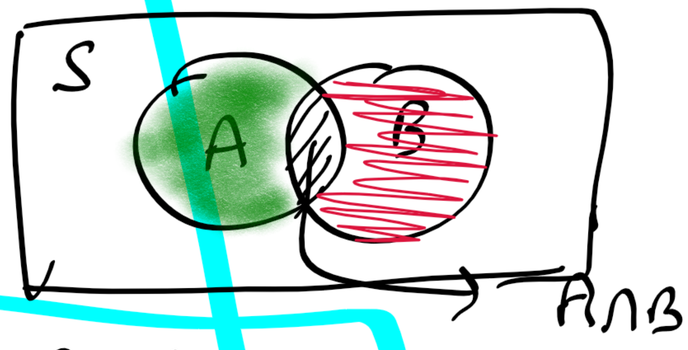
HW

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P\left(\bigcup_{i=1}^{n} A_i\right)$$

$$= \sum_{i=1}^{n} P(A_i) - \sum_{i<j=2}^{n} P(A_i \cap A_j)$$

$$+ \sum_{i<j<k=3}^{n} P(A_i \cap A_j \cap A_k) \cdots + (-1)^{n-1} P(A_1 \cap A_2 \cap \cdots \cap A_n)$$

Probability that exactly one of the events A or B occurs



$$P\left[(A \cap B') \cup (B \cap A')\right] = P(A) + P(B) - 2P(A \cap B)$$

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Conditional Probability

$\longrightarrow$ A & B

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{x \in S \mid x \geq 2\}$

$= \{2, 3, 4, 5, 6\}$

$B = \{x \in S \mid x \text{ is divisible by } 2\}$

$= \{2, 4, 6\}$

$P(A) = \frac{5}{6}$

$P(B) = \frac{3}{6}$

Probability of even A when event B is already occurred.

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}, \quad P(B) \neq 0$$

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A & B $\qquad P(B|A) = \dfrac{P(A \cap B)}{P(A)}$ $\qquad P(A) > 0$

(1) $0 \leq P(B|A) \leq 1$

(2) $P(S|A) = 1$

(3) $P(B_1 \cup B_2 | A) = P(B_1|A) + P(B_2|A)$ $\quad$ if $B_1 \cap B_2 = \phi$

(4) $P(B_1 \cup B_2 \cup \dots | A) = P(B_1|A) + P(B_2|A) + \dots$

if $B_i \cap B_j = \phi$, $i \neq j$

(5) If $A = S$, $P(B|S) = \dfrac{P(B \cap S)}{P(S)}$

$$\boxed{P(B|S) = P(B)}$$

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* A lot contains 10 good articles, 4 with minor defects & 2 with major defects. Two articles are chosen from the lot at random.

Find the probability that

i) at least one is good

ii) at most one is good

iii) exactly one is good

$$\left.\begin{array}{l} 10\,G \\ 4\,MD \\ 2\,D \end{array}\right\} 2$$

No. of possible outcomes $= {}^{16}C_2$

(i) P[at least one article is good]

$$= \frac{{}^{10}C_2 + {}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2}$$

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$$\left.\begin{array}{l}10\,G\\4\,MD\\2\,D\end{array}\right\}2$$

No. of possible outcomes $= {}^{16}C_2$

(i) P[atleast one article is good]

$$= \frac{{}^{10}C_2 + {}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2}$$

(ii) P[atmost one article is good]

$$= \frac{{}^{10}C_0 \times {}^{6}C_2 + {}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2}$$

(iii) P[exactly one article is good]

$$= \frac{{}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2}$$

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Independent Events

- Occurrence of one event does not affect the occurrence or nonoccurrence of the other event
- The conditional probability of X given Y is equal to the marginal probability of X.
- The conditional probability of Y given X is equal to the marginal probability of Y.

$$\boxed{P(X|Y) = P(X) \; and \; P(Y|X) = P(Y)}$$

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- Two events are **independent** if and only if:

$$P(A \mid B) = P(A)$$

- Events A and B are independent when the probability of one event is not affected by the other event

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