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Mathematical Contest In Modeling®

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Protect Biodiversity, Protect The Earth

Summary

Environmental stresses such as drought can have a negative impact on plants, and different species are different in their sensitivity to drought. Some phenomena suggest that the number of species in a plant community has an impact on the drought adaptability of the community. In this paper, we establishes a model to predict the changes of plant communities under drought conditions of different severity and frequencies, and studies the effects of factors such as community species number and species type on drought adaptability through models.

Firstly, we used the biomass of plant communities to represent the changes of communities and established a plant biomass prediction model. Our model is based on the Logistic model to reflect the competitive relationship between plants and improve it step by step. Considering the impact of drought on environmental capacity and on the rate of change of biomass, we added rain-heat influence factors to the model. We then use species numbers to represent interactions between species and study their impact on drought adaptability.

Secondly, we used the biomass prediction model described above for investigation. By comparing the biomass change curves of different species under the same conditions, we find that the increase of species within a certain range can enhance drought adaptability. By applying drought conditions with different frequencies and severity in the model, we found that the beneficial effects of interspecies interactions varied under different degrees of drought stress.

At the same time, we apply our model to other factors besides drought. For example, we found that both pollution and habitat loss cause functional damage to plant communities, which makes us more aware of the need to take steps to protect the environment.

Next, we applied the above model to the Site-based Neighborhood Model to study the impact of different types of species on the above conclusions through model simulation. We found that communities rich in plant species had greater resistance stability and resilience stability.

Finally, based on our research, we believe that there are some measures that can be taken to guarantee the long-term survival of the community. For example, we can improve the stability of a community by increasing the biodiversity in it. We also discussed our model further. In ecological engineering, for example, we use our models to predict the long-term effects of ecological measures to guide the implementation of farmland restoration.

Keywords: Logistic Model; Site-based Neighborhood Model; Species interactions; Droughts;

Contents

1 Introduction

1.1 Problem Background

Droughts are a frequent natural phenomenon that has amplified globally in the 21st century and are projected to become more common and extreme in the future.

Drought has great harm to human society and nature.For plants,droughts of different frequencies and intensities will have different effects on plants,and different plants have different sensitivities to drought.For example,grasslands are very sensitive to drought.Some experimental phenomena show that the local biodiversity of plant communities affects the ability of plants to adapt to drought cycles. The community with more plant species can adapt to drought better than the community with less plant species.Cooperative and mutually beneficial symbiotic relationships, diverse biotic and abiotic factors in ecosystems, have no clear theory to explain.

1.2 Restatement of the Problem

Considering background information, natural laws, we need to build a mathematical model to predict the changes of plant communities and explore the following questions:

- Consider the interactions between different species during drought cycles, explore how a plant community changes over time as it is exposed to various irregular weather cycles, including periods when precipitation should be abundant.
- Apply the above model to find out what is the minimum number of species required for a plant community to benefit from local biodiversity and predict what happens as the number of species grows.
- Explore how the type of plants in the community and other factors such as pollution,habitat reduction affects the results obtained.
- Predict the effects on plant communities of wider drought variation and changes in drought frequency in future weather cycles.
- Give feasible and effective measures to ensure the long-term viability of plant communities and analyze their impact on the larger environment.

1.3 Literature Review

Drought is a recurring natural disaster worldwide. In recent years, due to factors such as climate change, it has become more extreme and unpredictable. Drought has a profound impact on the environment, economy, and society. Sayed Shah Jan Sadiqi et al. gave a conceptual framework of ecosystem integration to assess drought hazards [1].

As early as more than 30 years ago, David Tilman and John A. Downing studied the impact of community species richness on the relative change rate of plant community biomass on grasslands under drought conditions. The research results support the diversitystability hypothesis[2]. Stress Gradient Hypothesis (SGH),which is widely circulated believes that competition is moderated in a high stress environment, and the strength of mutual benefit increases with the increase of environmental pressure[3]. However, in recent years, more and more studies have proved that SGH is not applicable to all relationships between plant-plant interactions and abiotic stress[4][5].

1.4 Our Work

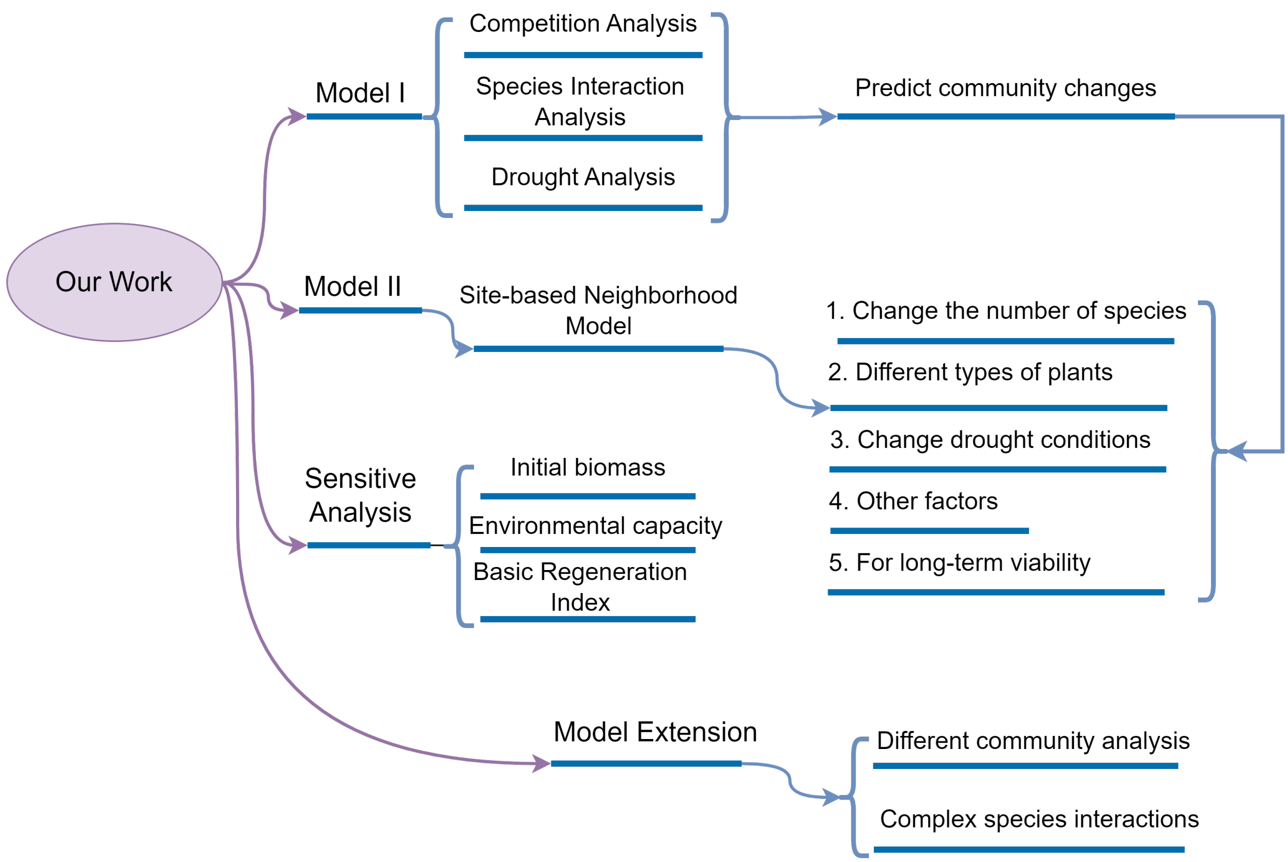

Figure 1: Our Work

2 Assumptions and Justifications

- **Assumption 1:** The change of total plant biomass in a community conforms to the logistic model if the environment is ideal.

  Justification: The Logistic model is a biomass change model derived by considering the competitive relationship under the condition of limited resources . When the biomass approaches the environmental capacity, resource competition intensifies, the growth rate of biomass slows down or almost stops completely, and finally stabilizes at the environmental capacity.

- **Assumption 2:** We believe that the plant biomass change rate and environmental capacity are positively correlated with the suitability of natural conditions such as rain and heat.

  Justification: Considering the growth of plants within a year cycle, plants grow faster in the rainy season (when the rainy and hot conditions are suitable), but grow slowly in the dry season (when the rainy and hot conditions are not suitable), which is in line with the natural growth law of plants.

- **Assumption 3:** The intensification of environmental stresses such as drought, pollution, and habitat reduction will reduce the growth rate of plant biomass.

  Justification: Environmental stresses negatively affect a range of conditions that plants need for growth. For example, drought leads to insufficient water supply to plants, pollution may poison cells, and habitat loss increases competition, all of which hinder normal plant growth and development processes.

- **Assumption 4:** The number of species in a community is related to the resistance and adaptability of the community to environmental stress.

  Justification: The number of species in a community is the localized biodiversity. Appropriate biodiversity will make species more fully utilize environmental resources and enhance mutual beneficial effects among species, thus changing the adaptability of the community to environmental pressure.

3 Model Preparation

3.1 Notations and Definition

Symbols	Description	units
t	Current time	month
$n(t)$	Total plant biomass	t/ha
F_1	Nature growth term	
F_2	Biodiversity related term	
F_3	Drought related term	
N	The number of species	
n_0	Initial biomass	
n_{max}	Environment capacity	
r_0	Basic regeneration index	
$P(t)$	Historical mean precipitation	mm
$\overline{P}$	Average annual precipitation	mm
$p(t)$	precipitation	mm
$c(t)$	Rain and heat condition	
$Z(t)$	Z-score of precipitation(severity of drought)	
n_{min}	The minimum possible biomass in the change	t/ha
A	Species interaction coefficient	

B	Biodiversity index	
C	Species interaction coefficient matrix	
δ	Species interaction matrix	
POL	Pollution index	
H_0	Original habitat area	ha
H_{eff}	Effective habitat area	ha
H_{loss}	Lost habitat area	ha
HL	Habitat residual coefficient	

3.2 Data Collection

We mainly collected data from National Drought Mitigation Center, U.S. Deparment oF Agriculture, GCB bioenergy.We extracted the data and unified the units.

Table 2: Data source

Database	Websites
National Drought Mitigation Center	https://drought.unl.edu/
U.S. Department oF Agriculture	https://www.fs.usda.gov/
GCB bioenergy	https://onlinelibrary.wiley.com/

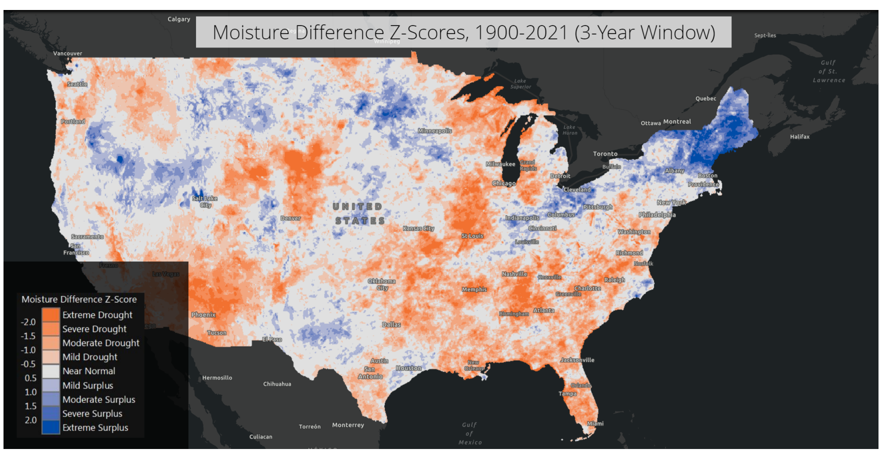

Figure 2: GIS map of drought conditions in the United States[6]

We collected nearly 30 years of precipitation and temperature data (Fig.2) in Urbana, Illinois, and total plant biomass data collected five times per year from 2006 to 2008. Due to the large amount of data, we used SPSS to analyze and visualize the data, and obtained the monthly average precipitation histogram and temperature change curve(Fig.3) of the 30-year history, as well as the monthly precipitation and temperature data from 2006 to

2008. The reason why we collected the data of Illinois is because Illinois is known as the "Prairie State", so its data can best represent the grassland.And the community structure of the grassland is relatively simple and the grassland is very sensitive to drought, so it is the best place we study grasslands and get results.

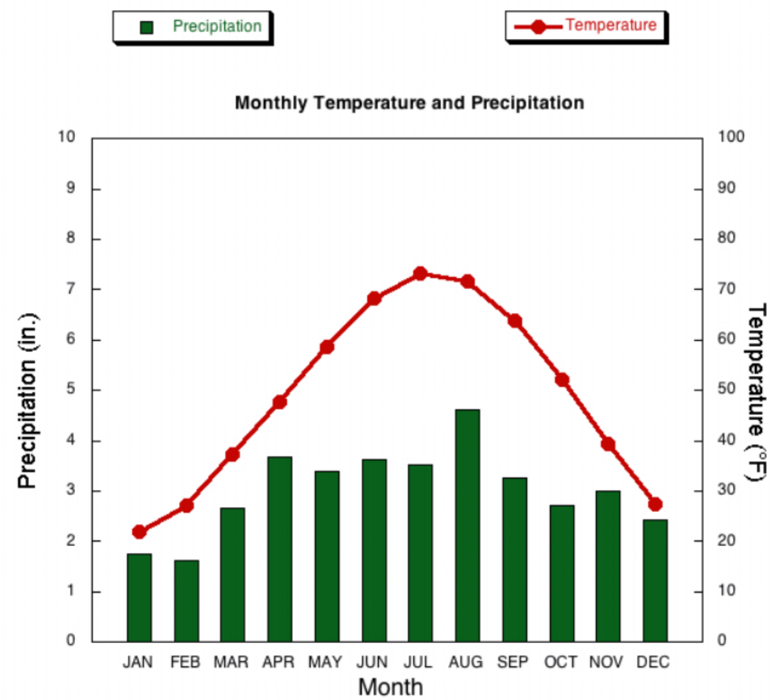

Figure 3: the monthly average precipitation histogram and temperature change curve[7]

We can get the annual total precipitation by adding up the precipitation in each month, and divide it by the number of months to get the monthly average precipitation $\overline{P}$. Because the actual data are discrete data points, in order to obtain continuous $P(t)$ and $p(t)$, we perform cubic spline interpolation fitting on the data.

4 Biomass Change Model

4.1 Model Construction

In order to study the effect of the number of species in the plant community on the drought adaptability, and predict the change of the plant community over time; we use the total plant biomass in the community to reflect the growth of the plant community, and to evaluate the drought adaptability. Considering the natural growth law of plants, climatic conditions such as rain and heat, and the influence of species number on the drought adaptability of the community, we established the following model framework to reflect the biomass change rate:

$$\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}t} = F_1 + F_2 \times F_3 \tag{1}$$

Some explanations:

1. The left side of the equation $\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}t}$ represents the relative change rate of plant biomass($\frac{\mathrm{d}n}{\mathrm{d}t}$ is the change rate of plant biomass);
2. F_1 is the natural biomass change rate of the plant community under certain rain and heat conditions without considering the drought condition;
3. F_2 is the effect of the number of species in a plant community on the rate of biomass change;
4. F_3 measures the degree of drought in the community environment($F_3 < 0$ in the period of drought);
5. $F_2 \times F_3$ represents the effect of species number in the community on the ability to adapt to drought.

4.1.1 Natural growth of plant biomass

If we only consider the individual competition of the plant community under the condition of limited resources, we can describe the biomass change of the plant community by the following equation:

$$\frac{1}{n(t)}\frac{\mathrm{d}n}{\mathrm{d}t} = r_0\left(1 - \frac{n(t)}{n_{max}}\right) \tag{2}$$

Based on Assumption 2, we believe that the environmental capacity at a certain moment should be related to the suitability of the current rain and heat conditions, so we define the suitability of environmental rain and heat conditions $c(t)$:

$$c(t) = \left(\frac{P(t)}{\bar{P}}\right)^2$$

$$\frac{1}{n(t)}\frac{\mathrm{d}n}{\mathrm{d}t} = r_0\left(1 - \frac{n(t)}{c(t)n_{max}}\right) \tag{3}$$

Based on Assumption 2, we believe that the change rate of biomass is related to the rain and heat conditions, so we add the rain and heat factor on the left side of **(2)**, The above completes the derivation of F_1 in **(1)**:

$$F_1 = r_0 \times \left(c(t) - \frac{n(t)}{n_{max}}\right) \tag{4}$$

4.1.2 Biodiversity and Drought Impacts

According to Assumption 3, the relative change rate of biomass is negatively correlated with the degree of drought. We use the Z-score of precipitation to measure the degree of drought. Generally, the Z-score will be between [-2,0] when drought occurs. More detailed See fig for the classification. For the effect of drought degree on the relative change rate of biomass, we can take a simple linear relationship,

$$F_2 = AZ(t) \tag{5}$$

We use the number of species N to measure biodiversity, and assume that there is competition and mutual assistance among different species. We record the impact factor matrix of the competition and mutual assistance among different species on the relative change rate of each species' biomass as C, where the element C_{ij} represents the impact factor of the interaction between species i and j on species i, where $C_{ii} = 1$.
We assume that during the interaction between species i and j, the relative change in biomass of species i is

$$\delta_{ij} = C_{ij} e^{-1/B} \tag{6}$$

In this formula, B is a parameter related to the minimum biomass required to maintain a stable community. When B becomes larger, the absolute value of **(6)** becomes larger, indicating that the environmental pressure of the community is greater and more species are needed to maintain community stability. It also shows that the interaction between species is enhanced, which is conducive to maintaining community stability.

The relative change rate of biomass of species i is affected by species 1 to N respectively as $\delta_{i,1}$, $\delta_{i,2} \cdots \delta_{i,N}$, the specific expressions refer to **(6)**, because the interaction between species i and other species is not independent of each other, but mutually coupled, we think that species i is affected by the total impact of all species interactions

$$\Delta_i = \prod_{j=1}^{n} \delta_{ij} = \prod_{j=1}^{n} (C_{ij} exp^{-1/B}) = (\prod_{j=1}^{n} C_{ij}) exp^{-N/B} \overset{def}{=} A_i exp^{-N/B} \tag{7}$$

Here, $A_i \equiv \prod_{j=1}^{n} C_{ij}$

During the interaction process, the relative change rate of biomass of all species is proportional to exp-N/B, and the difference is only the difference of A_i. Therefore, the relative change rate of the total biomass is also proportional to $exp^{-N/B}$, so we can denote the proportional coefficient of the relative change rate of the total species biomass as A.

To sum up, we use the impact factor matrix to get

$$F_3 = A exp^{-N/B} \tag{8}$$

The interactions between species can be affected by environmental factors (drought, etc.),According to Hypothesis 4, the interaction between species will enhance the resistance and adaptability of the community to environmental pressure, and weaken the impact of adverse environmental factors on the community, that is, F_2 and F_3 restrict each other, and jointly affect the relative change rate of biomass. Therefore, it is reasonable to use $F_2 \times F_3$ as a term in the expression of the model equation.

4.1.3 Biomass Change Model Considering Biodiversity and Drought Effects

After the above modeling process, we obtained the model for predicting the change of plant population biomass over time as follows:

$$\frac{1}{n(t)} \frac{\mathrm{d}n}{\mathrm{d}t} = r_0 \times \left(c(t) - \frac{n(t)}{n_{max}} \right) + Ae^{-\frac{N}{B}} \times Z(t) \tag{9}$$

4.2 Model solving

We used the 2006-2008 rain-thermal data (Fig. 4(a)), historical average RA-thermal data (Fig. 5), and biomass statistics (Fig. 4(b)) to calculate the parameters in the model. We use Matlab and calculate the parameters as follows:

$$
\begin{aligned}
r_0 &= 0.2183 \\
n_{max} &= 60.8797 \\
A &= 3.1011 \\
B &= 46.2066
\end{aligned}
$$

From the above parameter, we find that $A > 0, B > 0$, that is, when the number of species N increases, the effect of drought on the reduction rate of biomass change tends to weaken, which can explain the influence of the number of different species in the plant community on the drought adaptation capacity.

4.3 Model prediction

According to the above model, we can apply drought conditions with different frequency, duration, severity by setting the rainfall function $p(t)$, and obtain the biomass change prediction curve under the corresponding drought conditions, so as to predict the change of plant community over time. At the same time, by adjusting the parameter N of the number of species in the community, we can observe the difference in the change of biomass under different parameters N, so as to study the influence of the number of species on the ability to adapt to drought.

We brought the 2006 rainfall data $p(t)$ and the number of species N into the model for validation, and predicted the biomass change curve in 2006, and the results are shown in (Fig.4); it can be found that the predicted curve fits well with the real data.

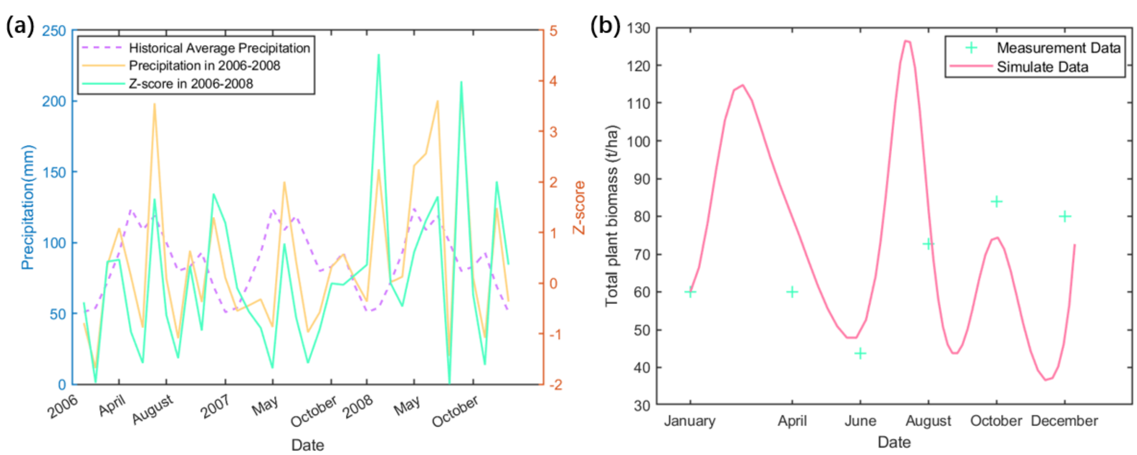

Figure 4: (a)The data we use.
(b)Model result and experimental data

5 Model Improvement and Application

This part mainly uses the model established above to solve the following problems.

5.1 Change the number of species to make predictions

In order to explore the minimum number of species required for the community to benefit and to study the effect of continued increase in the number of species, we bring the parameter N = 1-200 into the model for prediction while keeping the rainfall function $p(t)$ unchanged. By comparing the differences in biomass changes, we studied the impact of species number on drought adaptability.

We define the minimum biomass as

$$n_{min} = \min_{t>0} n(t) \tag{10}$$

which is the minimum value that $n(t)$ can reach in the $n(t)$ change curve predicted by adding a certain species number N into the above model. Our results are shown in the

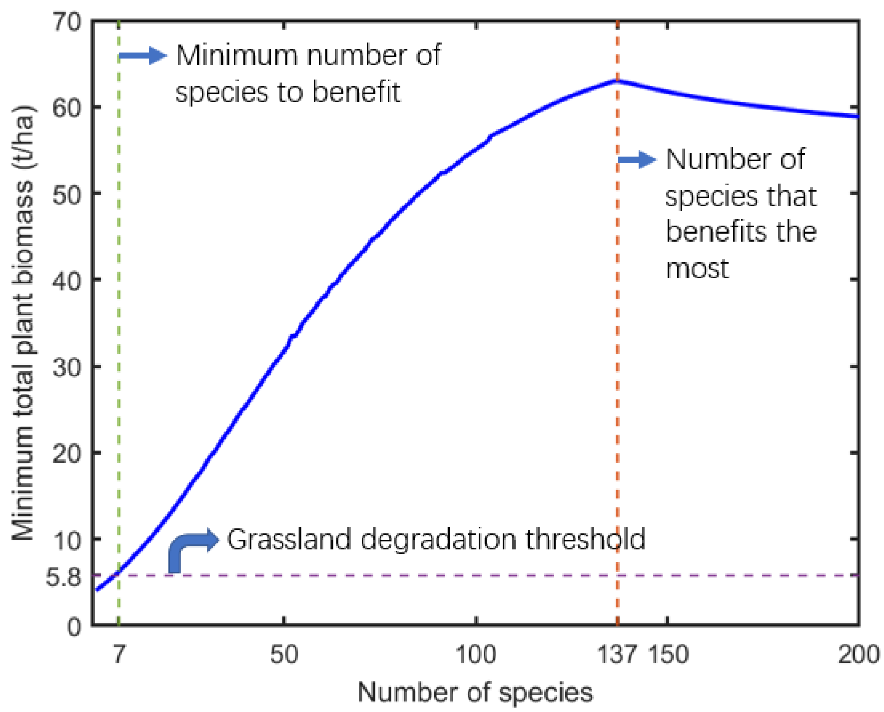

Figure 5: Plots of minimum biomass with number of specie

Fig.5 We found that in the process of N gradually increasing, n_{min} presents a change that first increases rapidly and then decreases slightly.

When the number of species N increases from 1 to the turning point N=137 (marked by the orange line), the increase in the number of species can benefit the plant community in adapting to drought; when the number of species exceeds the turning point N=137, the increase of species number reduces the benefit.

We have studied that the minimum biomass per unit area required for a typical plant community to ensure no degradation is 5.8 t/ha, and in our curve, the number of species corresponding is $N = 7$ (marked by the green line in Fig.5). That is, from the analysis of the prediction results obtained from our model, at least the number of species $N = 7$ is required so that the benefits of "enhanced drought adaptability" are sufficient to ensure that the plant community does not degradation.

5.2 Consider the impact of different types of species

Different types of species have different competitive abilities and different adaptations to drought. In order to study the effect of different types of plants on the above results, we need to improve the above model. However, because we do not have enough relevant data to solve more complex equations, we cannot add parameters to the above model that reflect the differences between different types of species. Therefore, we used an individual-based spatially-explicit model similar to that of Xiao et [7]up to investigate the effect of different types of species on our results by simulating them for individuals under certain rules.

We make the following model improvements, and use NetLogo for simulation.

5.2.1 Improvement based on individual simulation

We define a two-dimensional grid space, each cell can be empty or different types of plants, and each grid can only be occupied by one individual plant.

• The z-score reflecting the drought degree in the above model is normalized to define the environmental harshness S($0 \leq S \leq 1$).

$$S = 1 - \frac{(Z - Z_{min})}{(Z_{max} - Z_{min})}$$

That is, by introducing variables that respond to drought degree, our simulations can explore the drought resilience of plants.

• **Defines the competitiveness of Class i species as** $p_i \ \ (0 \leq p_i \leq 1)$

The probability of a class I species invading a cell of a class j species depends on $p_i - p_j$, It is impossible to invade when $p_i \leq p_j$.

We assign values through the competition of different types of plants in practice and their ability to withstand harsh environments. Competitive plants p are larger, while stress-tolerance plants p are smaller.

• **Define the value-added rate of group I plants r_i.**

When $r_i > 0$, plants are able to proliferate and spread. When $r_i \leq 0$, the plant cannot reproduce.

When r_i is less than a certain negative critical value, the plant individual dies.

• **At each step of simulation**. All plant individuals have a certain probability of death, and the grid becomes an empty space after death. And if alive, try to proliferate.

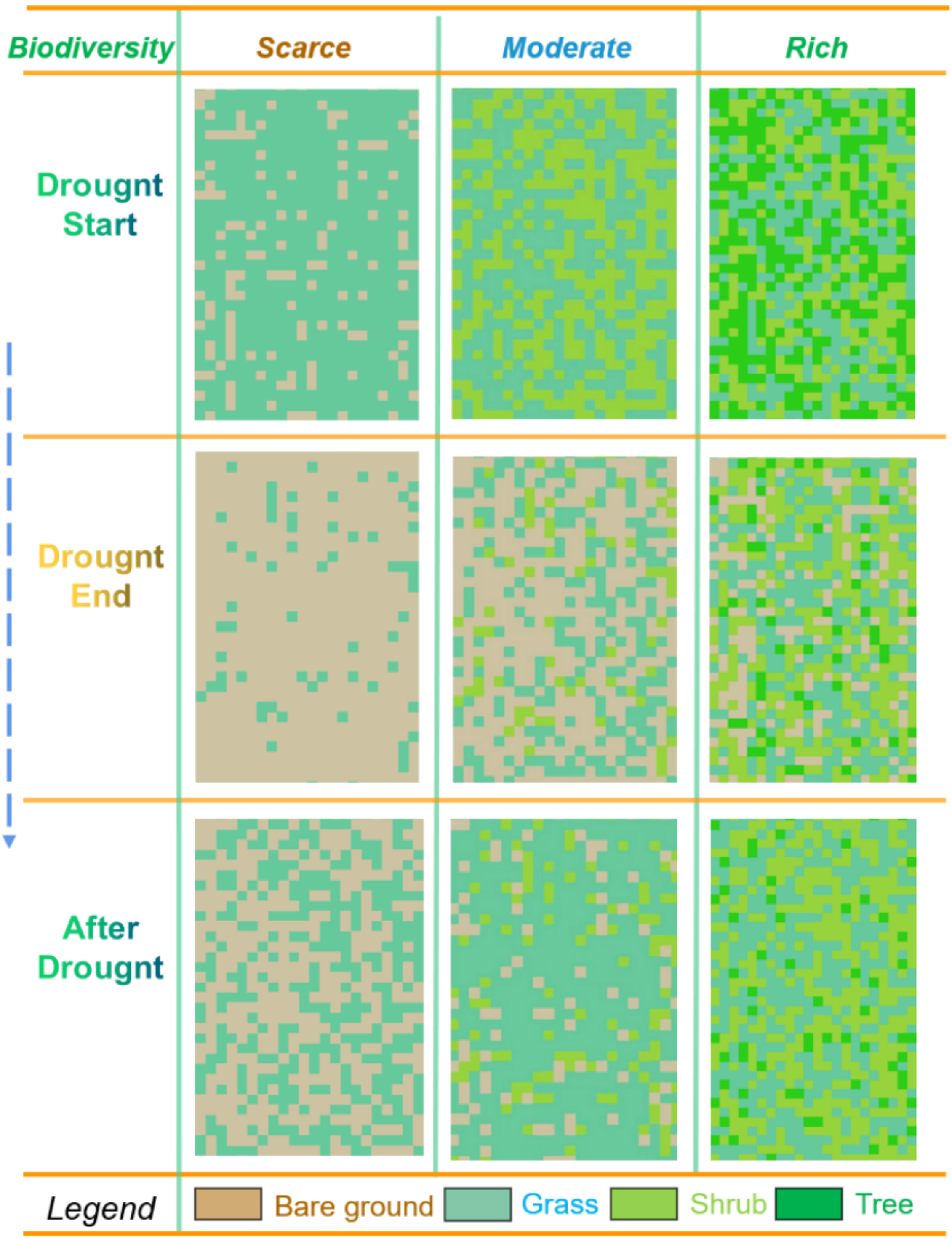

Figure 6: Simulation results of communities under the number of different species types

5.2.2 Model Simulation

We gradually increased the number of different types of plants and conducted the following three simulations under the same drought conditions to obtain the simulation results in Fig.6.

• **Scarce:** There is only one type of plant, which we labeled in the figure as Grass species.

• **Moderate:** There are two types of plants, each with several species, which we have labeled in the figure as Grass and Shrub.

• **Rich:** There are three types of plants, each with several species, which we have labeled in the figure as Grass, Shrub and Tree.

From the above figure, we found that at the end of the drought, the plants in all three groups of tests were heavily reduced, while the amount of plants recovered some time after the drought.

As the number of plant types (biodiversity) increases from left to right, the remaining biomass of the community increases at the end of the drought; and the biomass of the community rich in species types recovers faster after the same time as the end of the drought (in the most species-rich test, the community recovers almost completely after the drought). The following figure (Fig.7) shows the parameter settings and biomass changes for the three simulations.

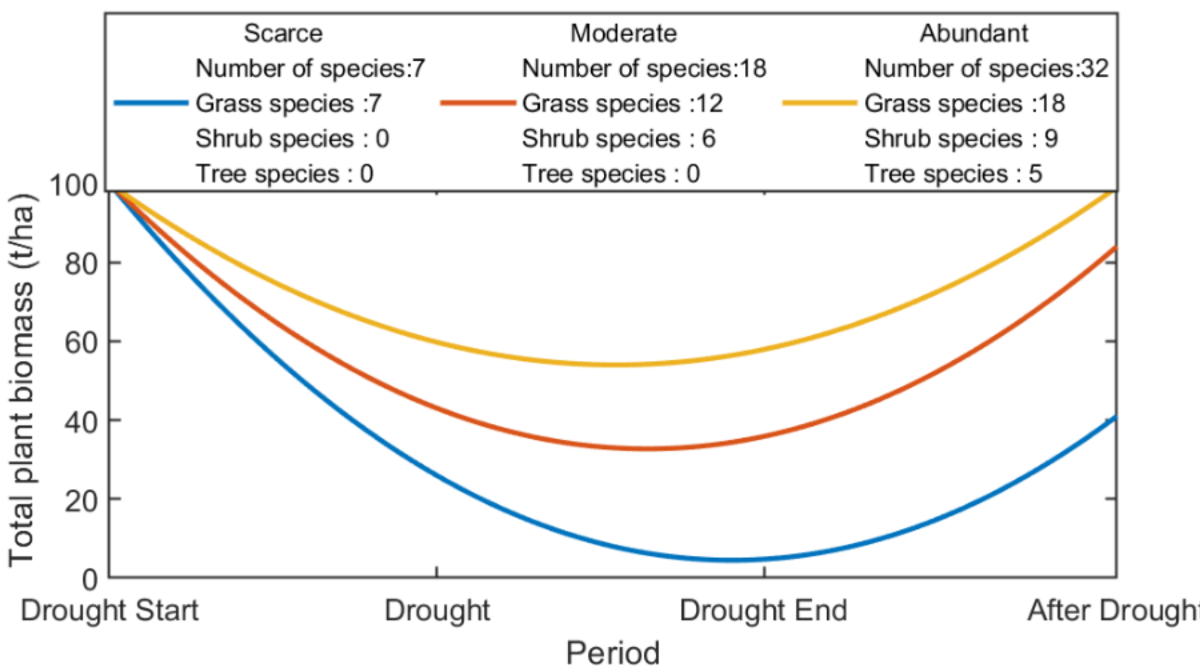

Figure 7: Model test parameters and biomass change curves

Results: Abundant species types can enhance the resistance stability and resilience stability of the community.

5.3 Explore results under different frequency and wider variation of droughts

Our model imposes different frequencies and severity of droughts by changing the actual rainfall function $p(t)$. We regard the historical average rainfall $P(t)$ for the same period as normal rainfall.And $P(t)$ is subtracted by the basis function $bias(t)$,because we know this will cause drought; that is, the actual rainfall function $p(t)$ considering the drought $bias(t)$ is:

$$p(t) = P(t) - bias(t) \tag{11}$$

We adjust $p(t)$ by adjusting the form of the $bias(t)$ function, and then adjust the drought degree $Z(t)$, so as to control the frequency, intensity, occurrence period and length of the drought period.

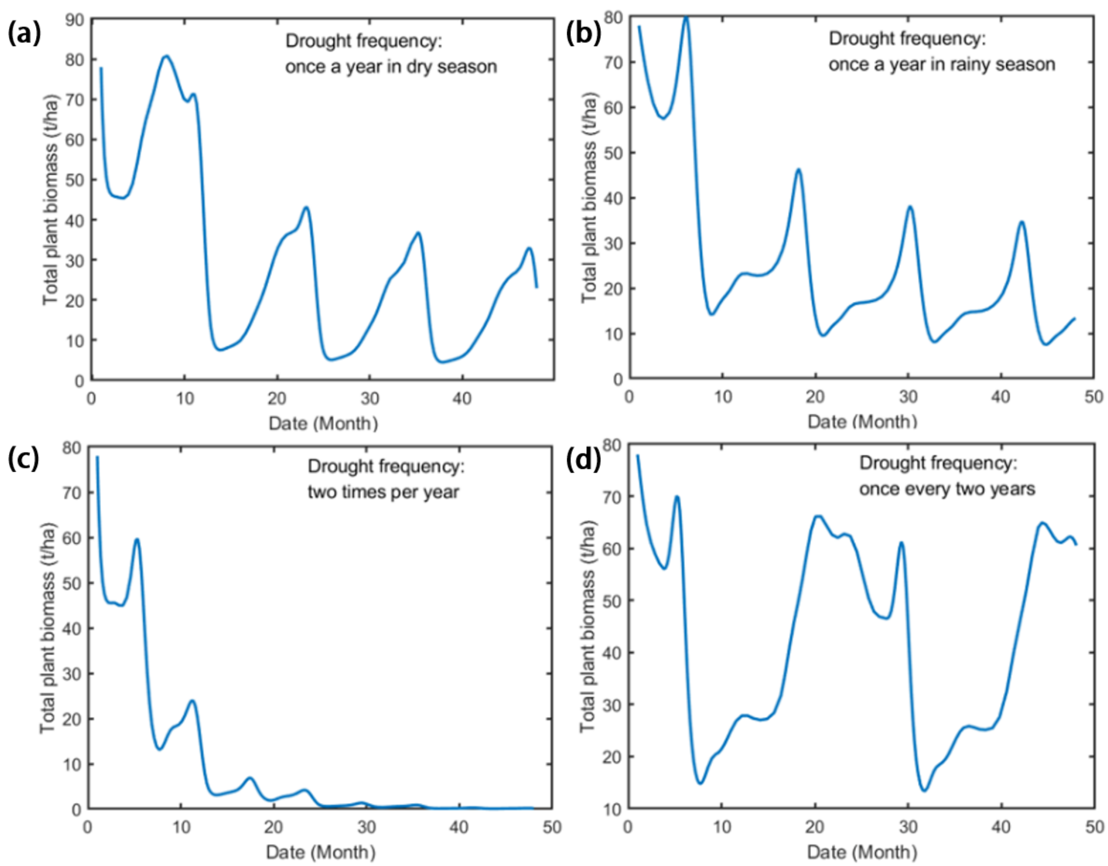

Figure 8: The relationship between biomass and drought frequency and occurrence time

We examine a community with normal species richness (N=50), first set the length of the drought period as 2 months, set the rainfall in the drought period as 50% of the usual (moderate drought), and consider the different frequencies of drought, for example Once a year, twice a year, and once every two years, predict the long-term change of biomass

(Fig.8); among them, for the frequency of once a year, we also consider that the drought occurs in the rainy season and the dry season respectively.

It can be seen that under the same drought severity, when the frequency of drought increases, the biomass shows a downward trend, and the minimum value of biomass becomes smaller, the population becomes more vulnerable, and grassland degradation is prone to occur. When the frequency increases to twice a year, after two years of evolution, the ecosystem is close to collapse, the organisms are close to extinction, and the grassland is seriously desertified. We can also see that the impact of drought on the community in the dry season is greater than that in the rainy season. It is speculated that the base rainfall in the rainy season is large, so even if there is a drought in the rainy season, there is still some rainwater for plant growth.

We then analyzed the effect of different drought severity and drought durations on biomass. We define the degree of drought severity as follows:

$$\bullet Normal : p(t) > 0.8P(t)$$
$$\bullet Mild\ drought : 0.8P(t) > p(t) > 0.6P(t)$$
$$\bullet Moderate\ drought : 0.6P(t) > p(t) > 0.3P(t)$$
$$\bullet Severe\ drought : p(t) < 0.3P(t)$$

In the analysis of drought duration (Fig.9), we uniformly set $p(t) = 0.5P(t)$ (moderate drought).

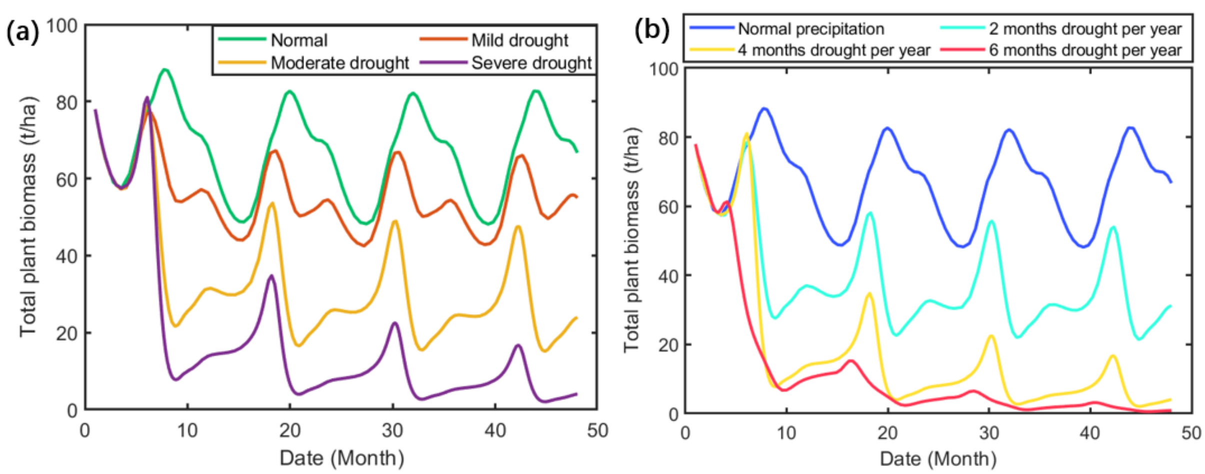

Figure 9: The relation between biomass and drought severity and drought durations

As the severity of drought increased and the duration of drought increased, community biomass showed a downward trend, and prolonged drought would lead to loss of community resilience and stability. Due to the resistance stability of the community, short-term severe drought will not lead to the loss of community function, and it can still recover to the normal biomass level at a relatively fast speed after the drought ends. However, long-term drought will lead to the loss of community function, structural collapse, and it is difficult to recover in a short time.

We also simulated the effects of different drought frequencies over a wider time scale (Fig.10).

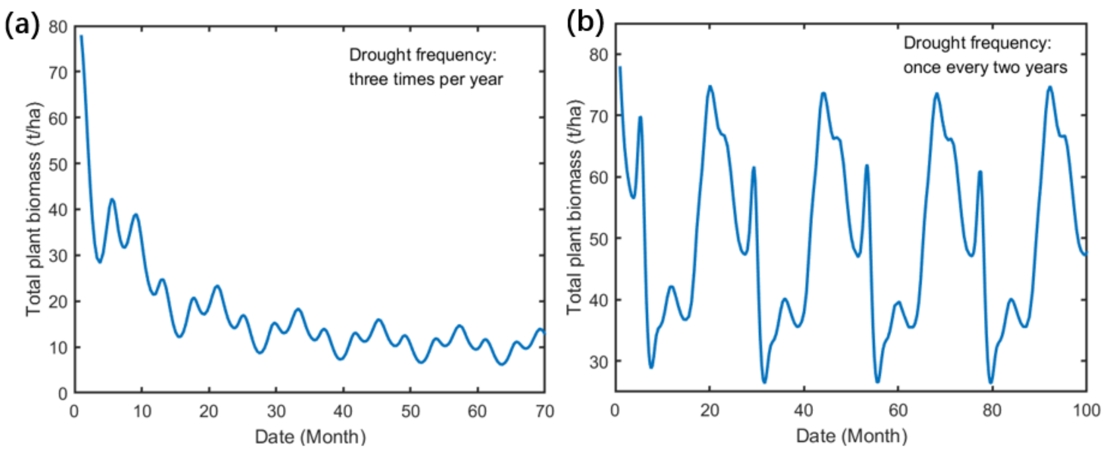

Figure 10: Simulation of long-term biomass change at different drought frequencies

It can be seen that the long-term high-frequency drought will lead to the memory and adaptability of the community to drought, and the biomass of the community is always maintained at a low level but relatively stable, which can be explained by the reduction of the environmental capacity.

But less frequent droughts did not have similar effects on community. The low-frequency drought is not enough to make the community produce drought memory and adaptability, so the biomass decreases sharply every time a drought occurs, and the biomass increases sharply when the drought ends, which is manifested as a huge fluctuation of the biomass curve.

5.4 The impact of pollution and habitat reduction

We define the pollution index POL(varies between 0 and 1) as the proportion of habitat function loss. For example, POL = 0.2 represents a 20 % loss of habitat function and a remaining 80%. We can use POL to measure the degree of pollution.

We define effective habitat area as:

$$H_{eff} = (H_0 - H_{loss}) \times POL \tag{12}$$

Here,H_0 is the original habitat area and H_{loss} is the habitat area lost due to human activities or other reasons.

We define habitat residual coefficient as:

$$HL = H_{eff}/H_0, \quad 0 \leq HL \leq 1 \tag{13}$$

So,the effects of pollution and habitat reduction on changes in population biomass can be characterized simultaneously by HL.

Both pollution and habitat reduction lead to a decrease in n_{max} and r_0. We assume that n_{max} and r_0 change after being affected as follows:

$$n'_{max} = n_{max} \times HL$$

$$r'_0 = r_0 \times HL$$

Substitute into the control **(10)**, simplify to get:

$$\frac{1}{n(t)}\frac{\mathrm{d}n}{\mathrm{d}t} = r_0 \times \left(c(t)HL - \frac{n(t)}{n_{max}}\right) + Ae^{-\frac{N}{B}} \times Z(t) \tag{14}$$

We set different HLs to predict the community biomass changes, and the results are shown in Fig.11.

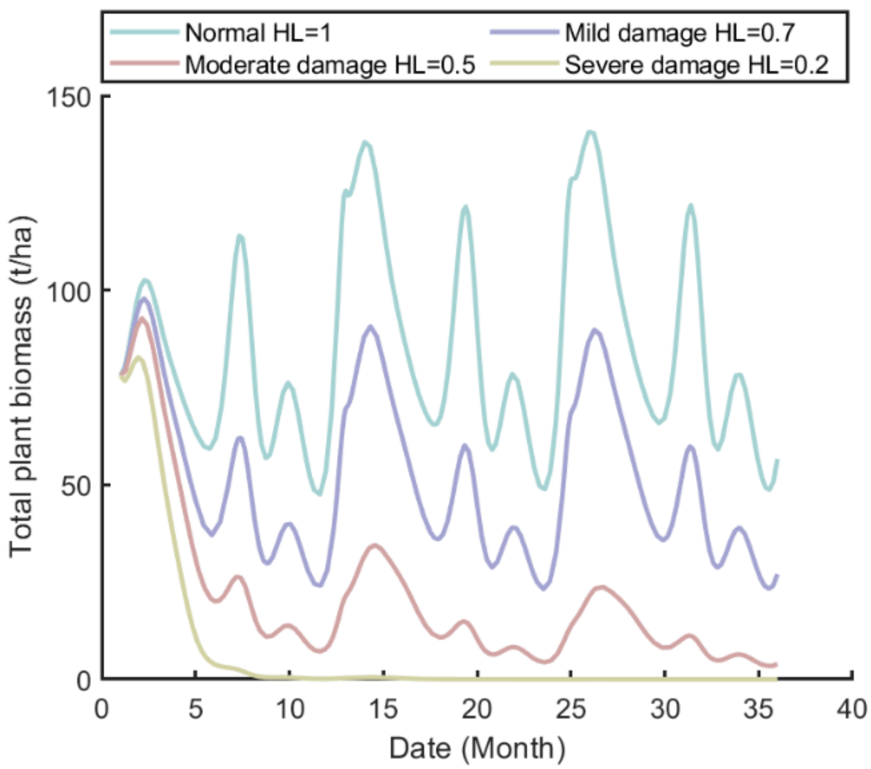

Figure 11: The impact of effective habitual area on biomass

It can be seen that pollution and habitat destruction can cause functional damage to the community, making the stability of the community's resistance to drought and the stability of its resilience much weaker. Moderate pollution damage makes it difficult for the population to recover after suffering from drought, and multiple droughts can lead to severe degradation of the grassland. Severe pollution damage can cause the community to lose stability,so the community collapses rapidly when droughts strike.

5.5 The long-term viability and the impact of the larger environment

Based on our model, the intensification of survival pressure is a heavy blow to the plant community. Properly increasing the number of species will improve the ability of the plant community to resist drought and recover quickly from drought. Therefore, in order to ensure the long-term viability of plant communities, we need to consider the three aspects of enhancing community resistance, improving resilience, and reducing plant survival pressure, There are many protective measures that humans can take. Here we summarize some of the most effective measures and express them in the form of a framework (Fig.12).

Since the plant community is the producer of nature, it connects the inorganic matter and the organic matter; at the same time, plants also play an important role in regulating humidity, conserving water, etc. Therefore, measures to improve the long-term viability of the plant community are always conducive to improving the natural environment. diversity and stability. We summarize some positive effects and draw them in the Fig.12 as well.

Figure 12: Measures and Impacts

After our model is combined with more detailed data or algorithms, it can exert greater value in real-world scenarios, such as ecological engineering, policy guidance for fallow and rotational grazing, which we will discuss in the Further Discussion.

6 Sensitivity Analysis

6.1 Rationality of initial biomass versus long-term prediction

We first analyze the effect of initial biomass on the predictions of the model. Set a different initial biomass n_{max} in the above model, and then use the model to predict the change of biomass over time, the prediction result is shown in Fig. 13.

It can be seen that the model prediction results are more sensitive to the change of initial biomass in the short term, but the long-term prediction results after one year are not sensitive to the initial biomass, which also justifies the use of the model for long-term biomass prediction.

From the figure, we can also see that community stability is not sensitive to initial biomass, because after being affected by drought, biomass fluctuation patterns are independent of initial biomass.

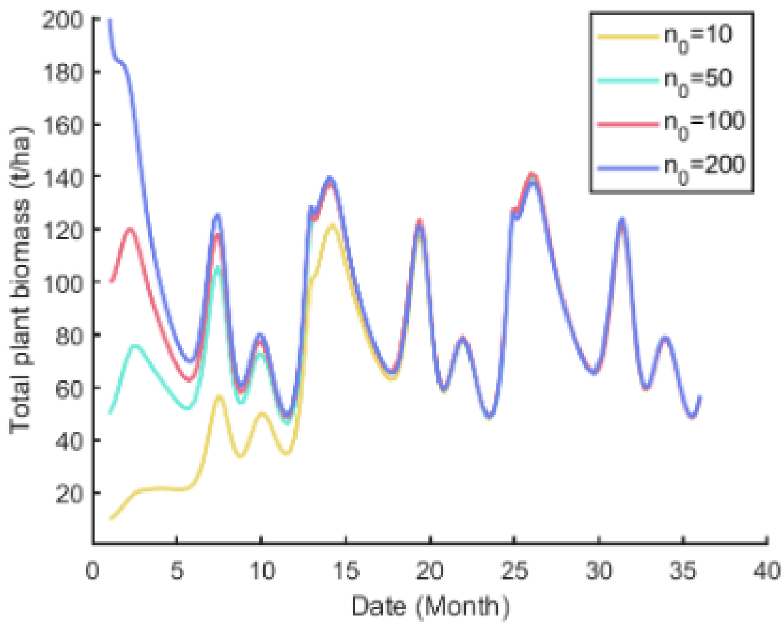

Figure 13: Graph of biomass over time at different initial biomasses

6.2 Analysis of environmental capacity and basic regeneration index

Environmental holding capacity and basic regeneration index are two important factors affecting community stability. We made long-term predictions for different n_{max} (Fig.14.a)and basic regeneration index r_0(Fig.14.b), and we can see that the model prediction results are more sensitive to n_{max} and r_0, where the curve fluctuation pattern is less sensitive to n_{max} but more influenced by r_0; the curve overall biomass trend is more sensitive to n_{max}, but less sensitive to r_0.

We can also see from the figure that the larger the n_{max} and the larger the r_0, the more resistant the community is to drought and the more stable the community is, which is also consistent with the findings of related essays[8][9].

7 Model Evaluation and Further Discussion

7.1 Strengths

- **Strong Universality.** Our model is based on a logistic model and is improved by taking into account the effect of biodiversity on drought resilience. Thus, our model is able to respond to the natural variability of plant populations in the community. By simply evaluating and determining the intensity of drought and adjusting the parameters appropriately, we are able to predict changes in different plant communities under different frequencies and intensities of drought.

- **Precise Interpretability.** The results show that our model can make good long-term predictions, and the conclusions drawn are consistent with the actual phenomenon. The plant interaction theory in some studies can also explain our results reasonably well.

- **Extensibility.** Our model uses a few simple, comprehensive, and universal parameters to describe the state of the population and use them as a basis for prediction. Thus, the effects of other complex factors on the community can be indirectly reflected in the effects on the parameters we selected. For example, we studied pollution and habitat destruction, which can be indirectly reflected in the effects on n_{max} and r_0. In practical applications, we can use hierarchical analysis (AHP) to analyze the relationship between complex factors and simple parameters, and thus incorporate complex factors into the equation.

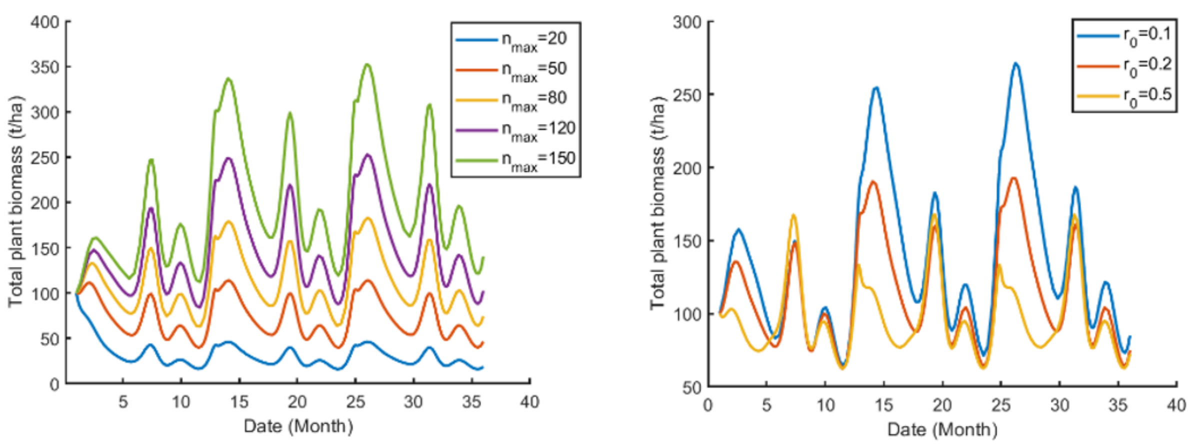

Figure 14: Variation of biomass with n_{max} and r_0

7.2 Weakness

- **The distinction between plant types is relatively simple**. We know that different types of plants differ in their ability to tolerate drought, their ability to compete, and the resources they need to survive, but we only divided the species types into three categories for analysis, and did not make a more detailed division.

7.3 Further Discussion

Our model is instructive for ecological engineering. Ecological engineering is the regulation of numerous parameters in the present model, such as r_0, n_{max}, n_0, $p(t)$, A, B, HL, etc. Our model can predict the long-term changes of community biomass after parameter adjustment, and thus estimate the long-term effects of ecological engineering. If the costs and benefits of ecological engineering are considered, combined with the optimization algorithm, our model can give the implementation plan for ecological engineering to achieve the best cost performance.

Our model can also provide guidance for the implementation of the policy of returning farmland and rotational grazing. For example, for grasslands degraded by drought, the following results can be obtained using our model:

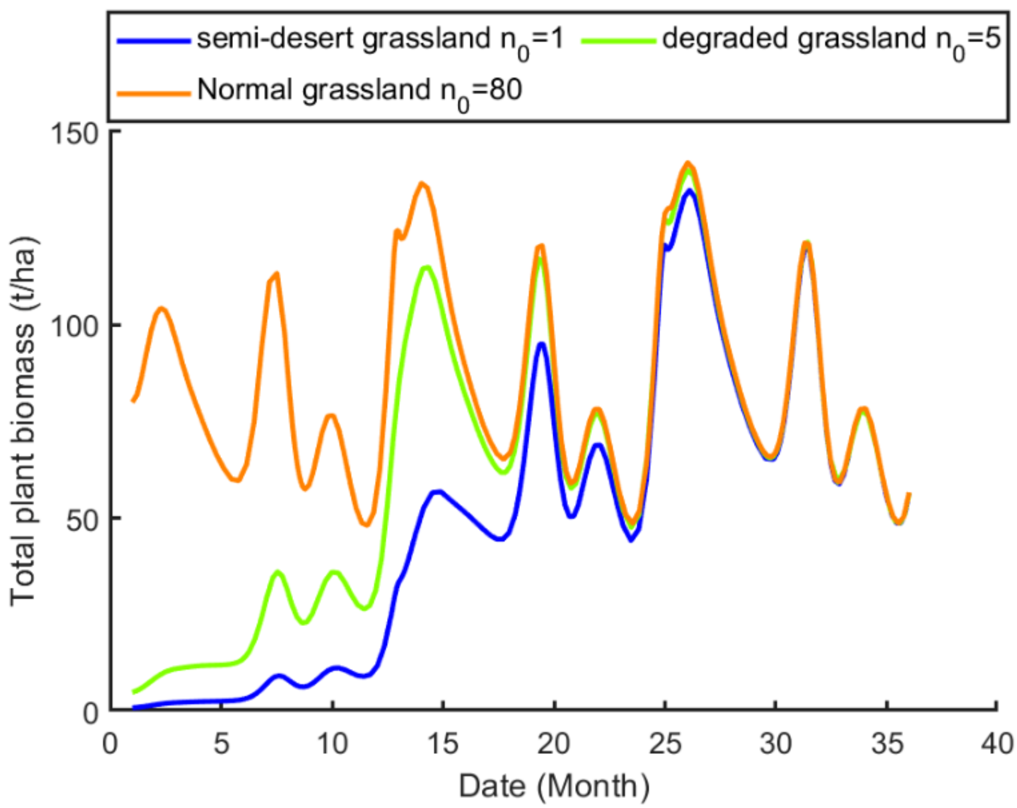

Figure 15: Farming Guidance

It can be seen from Fig.15 that for degraded pastures caused by overuse, we need at least one and a half years (18 months) of closed restoration to restore their functions. For grasslands that have been severely degraded and led to semi-desertification, they must be closed and restored for at least two years (24 months) to fully restore their functions. Most of the current fallow systems for rotational grazing set the rest period as one year (12 months). It can be seen from the model results that for communities with a low degree of damage, a one-year rest period is a more reasonable setting.

8 Conclusion

In this paper, we aim to predict the long-term variation of population biomass under the influence of drought, taking into consideration competition, cooperation, community biodiversity, and environmental factors such as pollution and habitat loss. We use the logistic model to describe the competition in the community, and add a term to describe the impact of drought. In order to consider the enhanced effect of biodiversity on drought resistance, we use biodiversity-related items to weaken the negative impact of drought. We conducted several tests and studies using the model, and came to the following valuable conclusions:

• The increase in the number of species can benefit the plant community in adapting to drought; but when the number of species exceeds a certain critical value, the increase in the number of species will reduce the benefit.

• At least N=7 species are needed to make the benefit of "enhancing drought adaptability" enough to ensure that the plant community does not degenerate.

The increase of species types can enhance the community's ability to adapt to drought. Species richness can enhance the resistance stability and resilience stability of the community.

• Under the same drought intensity, when the frequency of drought increased, the biomass showed a downward trend, and the minimum value of biomass became smaller, the population became more vulnerable, and grassland degradation and even community collapse were prone to occur.

• As the severity of drought increased and the duration of drought increased, community biomass showed a downward trend, and prolonged drought would lead to loss of community resilience and stability. Due to the resistance stability of the community, short-term severe drought will not lead to the loss of community function, and it can still recover to the normal biomass level at a relatively fast speed after the drought ends.

• Long-term and high-frequency drought will lead to memory and adaptability of the population to drought, and the biomass of the population is always maintained at a low level but relatively stable, which can be explained as the reduction of the environmental capacity. But low-frequency drought is not enough to make the population produce drought memory and adaptability, so low-frequency drought can cause greater fluctuations in biomass.

• The long-term trend of the biomass curve has nothing to do with the initial biomass.

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Appendix

```
%% Model solving Matlab code
%%
syms A B r0 nmax t n y
m=1:13;
for i=1:13
    if z(i,4)>0
        z(i,4)=0;
    end
end
plot(m,z(:,4))
ppz=spline(m,z(:,4));
ppp=spline(m,data(1:13,4));
    m=1:0.1:12;
    pps=ppval(ppp,m);
    plot(m,pps)
m2=[0 4 6 8 10 12];
bm=[60 47.66 43.7 72.664 91.464 82.218];
    ppbm=spline(m2,bm);
    bms=ppval(ppbm,m);
    plot([1 4 6 8 10 12],[60 60 43.7 72.664 84 80],'*')
t=[4,6,12];
an=fsolve(@(x) myfun(x,t,y,ppp,p_bar,ppz,N,ppbm),[100,100,0.4,80]);
an=vpa(an);A=an(1);B=an(2);
r0=an(3);nmax=an(4);
zf=@(t) (my_p(t,ppp,data)-ppval(ppp,mod(t-1,12)+1))...
/ppval(spline(1:13,u),mod(t-1,12)+1);
syms t y
N=7;
n0=70;
[tt,yy]=ode45(@(t,y) odef(t,y,r0,A,B,nmax,ppp,p_bar,zf,N),[1,12.3],n0);
hold on
for i=1:size(yy)
    if yy(i)<=5.8
        yy(i:size(yy))=0;
        break
    end
end
plot(tt,yy)
```