2024

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Plan 2024: A Hydrological Evaluation, Simulation, and Regulation Model

Summary

The Great Lakes, the largest group of freshwater lakes globally, hold significant ecological and economic importance for North America. Due to the complexity of the Great Lakes Basin, water level control faces numerous challenges. This paper aims to establish a water level control model for the Great Lakes, providing guidance on dam flow control schemes to maintain water levels within a reasonable range, balancing and maximizing the interests of stakeholders.

Firstly, we establish a flow network model for the Great Lakes, depicting the relationship between lake levels and environmental conditions. We consider factors such as inflow, outflow, precipitation, evaporation, runoff, and human activities, ensuring the model closely reflects reality.

Secondly, we quantitatively assess the interests of stakeholders using the Kuznets curve from economics, weighting stakeholders' interests reasonably to derive an optimal water level model that meets stakeholders' needs at different times.

Subsequently, we develop a water level control model for the Great Lakes, transforming water level control into a single-objective multi-variable optimization problem. We provide dam flow control schemes corresponding to different environmental conditions to maintain water levels close to optimal levels. In our control model, we simplify and accelerate model solving through global sensitivity methods and construct static and dynamic dam flow constraints to ensure the generated dam flow control schemes align with reality.

Lastly, we validate and compare our water level control model using historical data. Results show our model effectively controls water levels within a range closer to the optimal level, outperforming historical water level data. Additionally, focusing on Ontario and nearby areas, we simulate water level control for 2017 data, finding our model superior to historical water levels under Plan 2014 regulation. Furthermore, based on our model's dynamic constraints, we achieve Montreal Harbor water levels well within the safety range.

Based on our research, we have written a memo to the IJC, highlighting the shortcomings of the existing Plan 2014 and presenting the advantages of our model, explaining how it better maintains Great Lakes water levels. While our model performs well in simulation applications, it has some limitations, such as a lack of consideration for small tributaries in the Great Lakes Basin. We hope to further refine our model in future research to better maintain Great Lakes water levels and even extend it to water level control in other basins, better serving sustainable human development.

Keywords: Great Lakes, Single-objective optimization, Water levels, Dam control.

Contents

1 Introduction

1.1 Background and Problem

The Great Lakes are the largest group of freshwater lakes in the world. The water is used for many purposes (fishing, recreation, electricity generation, shipping, animal and plant habitat, human consumption, etc.). As a result, the management of water entering and leaving the lake is of great concern to numerous stakeholders. The water level in each lake is determined by how much water enters and leaves the lake. These levels are the result of complex interactions - because of interdependencies, complex requirements, and inherent uncertainty. In the case of lake problems, we are faced with changing dynamics and conflicting stakeholder interests. We are expected to assist with management and control models for the two dams that directly influence water levels in the Great Lakes flow network, specifically:

- Build a network model for the Great Lakes and connecting river flows from Lake Superior to the Atlantic Ocean.
- Determine the optimal water levels of the Great Lakes at any time of the year, considering the various stakeholders' costs and benefits.
- Establish algorithms to maintain optimal water levels in the Great Lakes from inflow and outflow data for the lakes.
- Test the sensitivity of the control algorithms for the outflow of the two control dams. And compare new control results with the actual recorded water levels of 2017 for the various stakeholders.
- Evaluate the sensitivity of the algorithm to changes in environmental conditions (e.g., precipitation, winter snowpack, ice jams).
- Focus on extensive analysis of ONLY the stakeholders and factors influencing Lake Ontario.
- Control the water levels in Lake Ontario and flows in Montreal Harbor.

1.2 Our Work

Our work is shown in Figure.1

2 Assumptions and Notations

2.1 Assumptions

- **Assumption 1:** In this problem, we only consider the Great Lakes and Lake St. Clair, along with the major rivers connecting them, including St. Mary's River, St. Clair River, Detroit River, Niagara River, St. Lawrence River, and Ottawa River.

  Justification: The focus of this study is the flow network of the Great Lakes; hence, we only consider the major rivers connecting the Great Lakes and the lakes.

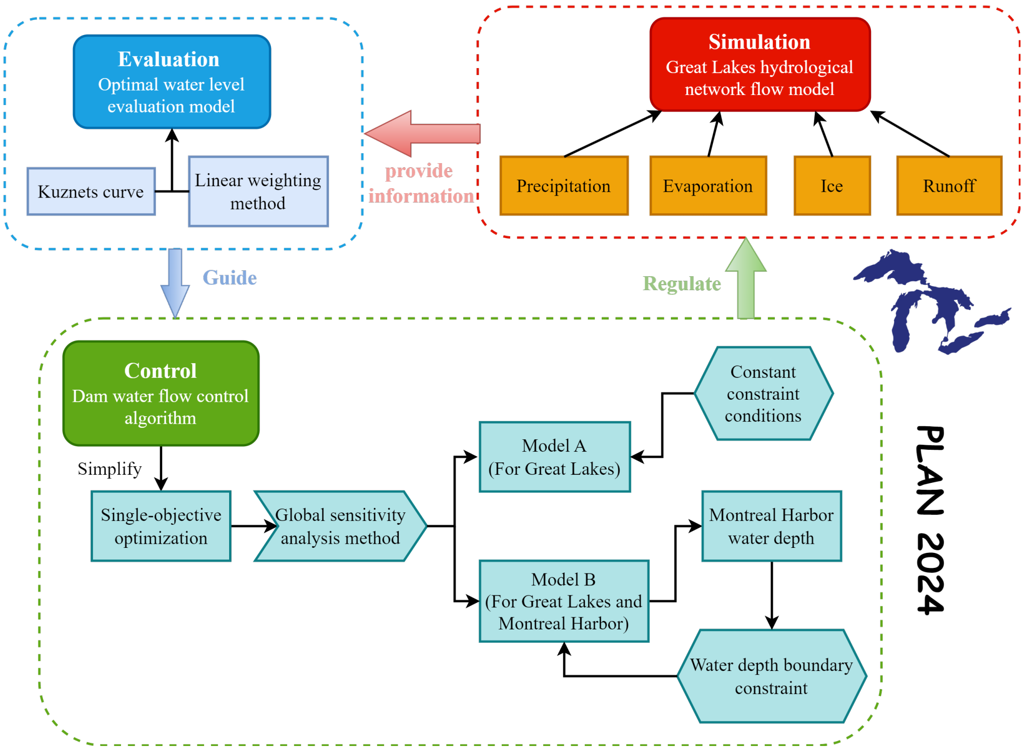


Figure 1: Our work

- **Assumption 2:** Lake water level changes are only influenced by factors such as inflow, outflow, precipitation, evaporation, snowmelt, and human activities (e.g., domestic water withdrawal, irrigation), excluding other factors like and seepage.

  Justification: The purpose of this study is to model the regulation of water levels in the Great Lakes. Therefore, we only consider factors that have a significant impact on lake water levels.

- **Assumption 3:** Assuming the lake's surface area is constant, variations in the lake's water volume will only result in changes in water level.

  Justification: Based on historical data, the annual fluctuations in water levels of the five Great Lakes are relatively small. Therefore, it can be assumed that changes in the lake's water volume will only result in variations in water level, without significantly affecting the lake's surface area.

- **Assumption 4:** The means of artificially controlling river flow only include regulating flow through dams. Specifically, it encompasses only Compensating Works and Moses-Saunders Dam.

  Justification: While there are various methods for artificially controlling river flow, in the Great Lakes basin, dams stand out as the primary means of dynamic regulation. The Compensating Works and the Moses-Saunders Dam are two major dam systems in the Great Lakes basin.

2.2 Notations

Table 1: Notations and Definition

Symbols	Description	units
t	Current time	
h_{lake}	Lake water level depth	m
h_{river}	River water level depth	m
A_{lake}	lake area	m^2
Q_{in}	River flow into the lake	m^3/s
Q_{out}	River flow out of lake	m^3/s
E	lake water evaporation rate	m/s
P	lake water precipitation rate	m/s
Q_{snow}	The flow of snowmelt water from land into lakes	m^3/s
Q_{human}	The flow of water from human activities into lakes	m^3/s
L_{river}	length of river	m
W_{river}	width of river	m
F_{river}	River flow	m^3/s
n	River bed roughness ratio	

3 Great Lakes Flow Network Model

In this section, we will construct a flow network model for the Great Lakes, depicting the flow of water and changes in lake water levels under external conditions. First, based on **Assumption 1**, we abstract and simplify the flow network of the Great Lakes. Next, we model the water level changes in individual lakes. Following that, we integrate river flow calculations with the lake water level model to build the flow network model for the Great Lakes. Finally, we validate the model based on historical data.

3.1 Graph Representation of the Great Lakes Flow Network

Based on **Assumption 1**, we abstract the Great Lakes network into a directed graph, comprising a set of vertices representing lakes and directed edges representing rivers.
The set of vertices is as follows.

$$Vertex = \{\text{Superior}, \text{Huron}, \text{Erie}, \text{Ontario}, \text{St. Clair}\} \tag{1}$$

Specifically, due to the consistent long-term water surface level between Lake Michigan and Lake Huron [1], we treat Lake Michigan and Lake Huron as a single lake. In other words, Lake Michigan and Lake Huron are merged into one point, referred to as Lake Huron.
The set of edges is as follows.

$$\begin{aligned} Edge = \{&(Superior,\ Michigan,\ St.Marys),\\ &(Huron,\ St.ClairLake,\ St.Clair),\\ &(St.ClairLake,\ Eire,\ Detroit),\\ &(Eire,\ Ontario,\ Niagara),\\ &(Ontario,\ AtlanticOcean,\ St.Lawrence)\} \end{aligned} \tag{2}$$

Here, $(start, end, name)$ represents a directed edge representing a river, where $start$ indicates the outflowing lake, end indicates the inflowing lake. Additionally, the St. Lawrence River flows into the Atlantic Ocean, and the Ottawa River, which flows into the St. Lawrence River, is not explicitly described in the above-directed graph but will be accounted for in the model.

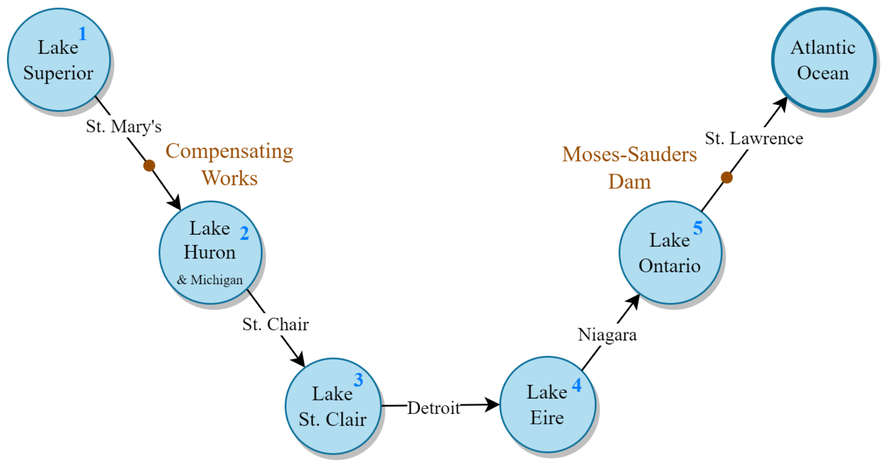


Figure 2: Great Lakes water flow network

3.2 Construction of Individual Lake Water Level Models

We begin by considering the water level changes in individual lakes, establishing a lake water level model to depict the flow of water and the variations in lake water levels under external conditions. In accordance with **Assumption 3**, we assume that the lake area is constant, and changes in the lake volume only result in water level variations. Therefore, we can relate the rate of change of lake volume to the rate of change of water level, as follows

$$\frac{dV}{dt} = A \cdot \frac{dh}{dt} \tag{3}$$

Here, V represents the lake volume, A is the lake area, and h denotes the lake water level. In accordance with **Assumption 2**, changes in lake volume are influenced solely by factors such as inflow, outflow, precipitation, evaporation, snowmelt, and human activities. Therefore, the rate of change of lake volume can be expressed as

$$\frac{dV}{dt} = Q_{in} + P \cdot A + Q_{snow} - (Q_{out} + E \cdot A + Q_{human}) \tag{4}$$

Here, Q_{in} represents the river inflow into the lake, $P \cdot A$ is the precipitation, Q_{snow} is the flow of melted snow entering the lake, Q_{out} is the outflow from the lake through rivers, $E \cdot A$ denotes the lake surface evaporation, and Q_{human} is the outflow from the lake due to human activities. The rate of change of lake water level concerning the factors mentioned can be derived from Eq.3 and Eq.4 as follows:

$$A \cdot \frac{dh}{dt} = Q_{in} + P \cdot A + Q_{snow} - (Q_{out} + E \cdot A + Q_{human}) \tag{5}$$

Based on the above formulas, we can characterize the flow of water and the changes in lake water levels under external conditions.

3.3 Calculation of River Flow between Connected Lakes

Observing the expression for the rate of change of lake water level in Eq. 5, we can see that factors such as precipitation, evaporation, snowmelt, and human activities can be estimated based on historical data. Therefore, our key task is to calculate the flow rates of rivers entering the lake (Q_{in}) and exiting the lake (Q_{out}), i.e., calculating river flow.

Estimating flow based on the Manning formula

The Manning-Strickler formula [2] can be used to estimate river flow. The Manning-Strickler formula is an empirical equation used for calculating river flow. Its expression is given by:

$$F_{river} = \frac{A}{n} \cdot R^{2/3} \cdot S^{1/2} \tag{6}$$

Here, F_{river} is the river flow, A is the cross-sectional area of the river, n is the Manning roughness coefficient of the riverbed, R is the wetted perimeter of the river cross-section, and S is the slope of the river. Below is a detailed explanation of these parameters.

a. Parameter A for Cross-sectional Area of the River
Considering the river channel as an equivalent rectangular cross-section, the cross-sectional area A of the river is given by:

$$A = h_{river} \cdot W_{river} \tag{7}$$

Here, h_{river} is the average depth of the river, and W_{river} is the average width of the river.

b. Parameter R for Wetted Perimeter of the River Cross-section
The wetted perimeter R of the river cross-section, which is the ratio of the cross-sectional area to the wetted perimeter, can be expressed as:

$$R = \frac{A}{P} \tag{8}$$

Here, P is the wetted perimeter of the river cross-section.
When the depth of the river channel is much smaller than the width of the channel, we can approximate the wetted perimeter P of the river channel cross-section as equal to the width of the channel W_{river}, $R = \frac{A}{W_{river}}$. As the depth of the rivers in the Great Lakes region is relatively small, we use the approximation formula to calculate the wetted perimeter parameter R:

$$R = \frac{h_{river} \cdot W_{river}}{W_{river}} = h_{river} \tag{9}$$

c. Slope of the River S
The slope of the river, S, which represents the ratio of the change in water surface elevation to the horizontal distance, can be expressed as:

$$S = \frac{h_{delta}}{L_{river}} \tag{10}$$

where h_{delta} is the change in river water surface elevation, and L_{river} is the length of the river.
Therefore, we can obtain an approximate expression for the river slope S as follows:

$$S = \frac{h_{upper-lake} - h_{down-lake}}{L_{river}} \tag{11}$$

d. Manning roughness coefficient n

The Manning roughness coefficient n is an empirical parameter used to describe the roughness of the riverbed. A smaller value indicates a smoother riverbed, while a larger value indicates a rougher riverbed. It typically falls within the range of 0.01 to 0.05.

Since we cannot directly obtain the roughness coefficient n for rivers in the Great Lakes basin, we combine the roughness and width W_{river} into a single constant C, referred to as the equivalent width parameter for the river. This is expressed as:

$$C = \frac{1}{n} \cdot W_{river} \tag{12}$$

By substituting Eq. 6 and Eq. 12 into Eq. 6, we can rewrite the Manning-Strickler formula in terms of C as follows:

$$\begin{aligned} F_{river} &= \frac{A}{n} \cdot R^{2/3} \cdot S^{1/2} \\ &= \frac{h_{river} \cdot W_{river}}{n} \cdot R^{2/3} \cdot S^{1/2} \\ &= C \cdot R^{2/3} \cdot S^{1/2} \end{aligned} \tag{13}$$

Solve for the constant C, that is,

$$C = \frac{F_{river}}{R^{2/3} \cdot S^{1/2}} \tag{14}$$

Substituting historical data, we obtained the equivalent width C for each river in the Great Lakes watershed.

Substituting the parameters calculated above, Eq.7, Eq.9, Eq.11, and Eq.12 into the Manning-Strickler formula, Eq. 6, we can compute the river flow F_{river}

$$F_{river} = C \cdot h_{river}^{2/3} \cdot \left(\frac{h_{upper-lake} - h_{down-lake}}{L_{river}} \right)^{1/2} \tag{15}$$

After obtaining the river flow F_{river}, for the lake water level model, we can substitute the inflow and outflow river flows into the parameters Q_{in} and Q_{out} in Eq. 5.

At this point, we have completed the modeling and parameter calculation for the water level of a single lake.

Flow calculation based on neural networks

We validated the river calculation method based on the Manning Formula in Section 3.3 using historical data and found significant discrepancies between the calculated and actual flows for the Niagara River and St. Lawrence River. The reason for this discrepancy lies in the substantial elevation drops of these two rivers, which are $99m$ and $76m$ respectively The immense elevation drops in these rivers make it impractical to estimate the flow using the Manning Formula.

To accurately calculate the flow of the aforementioned rivers with significant elevation drops, we employed a fully connected neural network as shown in Figure 3. The neural network consists of three hidden layers, each containing 10 neurons. The input layer has 5 features, as shown in Figure 3. The output layer represents the river flow. We trained the neural network using historical data[3] from 2000 to 2020. After validation, the trained neural network successfully calculates the flow of the two rivers within an acceptable margin of error.

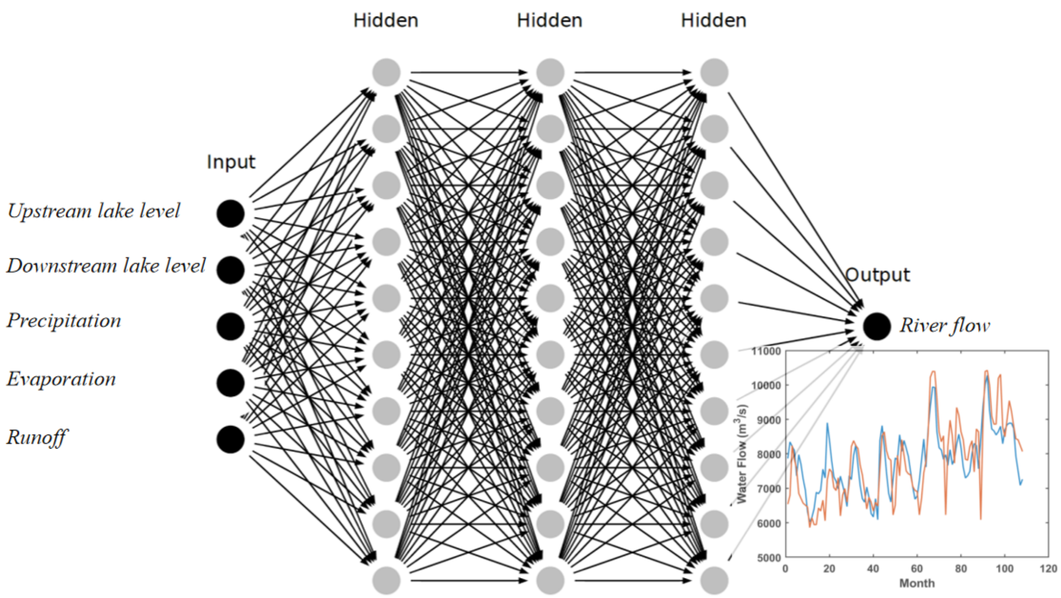


Figure 3: Neural Network for River Flow Calculation.The illustration shows the real flow rate of the St. Lawrence River (in orange) alongside the neural network predicted flow rate (in blue).

3.4 Construction of the Comprehensive Great Lakes Flow Network Model

Observing the water level model for a single lake in Eq. 5, it is evident that it includes the river flow rates Q_{in} and Q_{out} connecting the two lakes. In other words, the river flow rates link the water level models of various lakes, forming the flow network of the Great Lakes. In this section, we discuss two scenarios of river flow rates (natural flow and regulated flow), and substitute them into the lake water level model to construct the network model for the Great Lakes.

Model Establishment

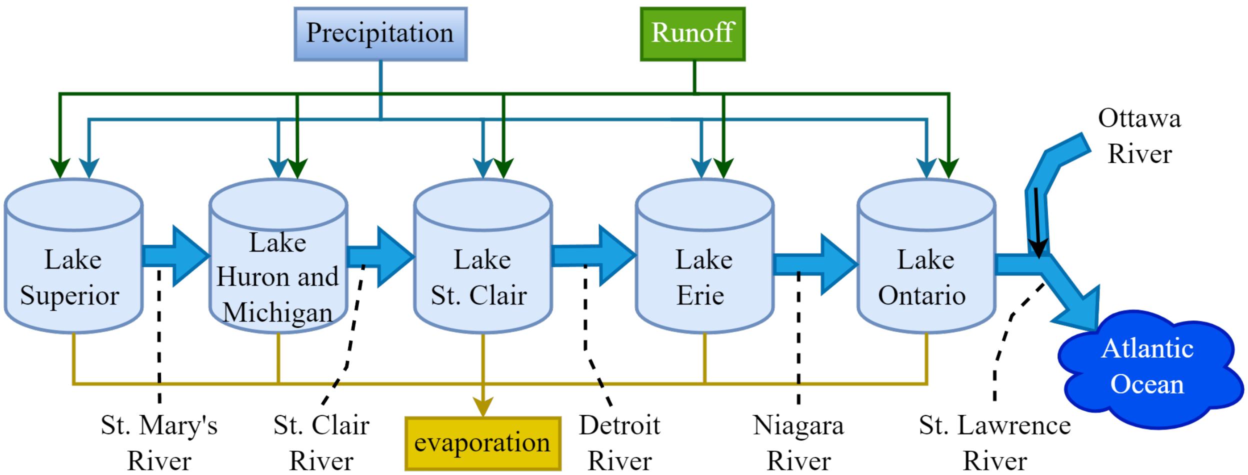


Figure 4: Great Lakes Flow Network

As shown in Figure 4, we abstract the flow network of the Great Lakes as a directed graph, where river flows connect the water levels of the lakes. Combining the lake water level model in Eq. 5 with the river flow rates F_{river}, we sequentially list the water level change equations for Lake Superior, Lake Huron, Lake St. Clair, Lake Erie, and Lake Ontario, deriving the network model for the Great Lakes.

Lake Superior: For *Lake Sup*, there is no river inflow ($Q_{\text{in, sup}} = 0$), only outflow through the St. Marys River ($Q_{\text{out, Sup}} = F_{\text{Stm}}$). Therefore, the water level change equation for *Lake Sup* is:

$$\begin{aligned} A_{Sup} \cdot \frac{dh_{Sup}}{dt} =& P_{Sup} \cdot A_{Sup} + Q_{snow,Sup} \\ & - (F_{Stm} + E_{Sup} \cdot A_{Sup} + Q_{human,Sup}) \end{aligned} \tag{16}$$

Where A_{Sup} is the area of the lake (Superior), h_{Sup} is the lake water level, P_{Sup} is the precipitation, $Q_{\text{snow, Sup}}$ is the flow of melted snow into the lake, F_{Stm} is the flow out of the lake through the St. Marys River, E_{Sup} is the flow of evaporation from the lake surface, and $Q_{\text{human, Sup}}$ is the flow out of the lake due to human activities.

Lake Huron: For *Lake Hur*, there is inflow from the St. Marys River ($Q_{\text{in, Hur}} = F_{\text{Stm}}$) and outflow through the St. Clair River ($Q_{\text{out, Hur}} = F_{\text{Stc}}$). Therefore, the water level change equation for *Lake Hur* is:

$$\begin{aligned} A_{Hur} \cdot \frac{dh_{Hur}}{dt} =& P_{Hur} \cdot A_{Hur} + Q_{snow,Hur} + F_{Stm} \\ & - (F_{Stc} + E_{Hur} \cdot A_{Hur} + Q_{human,Hur}) \end{aligned} \tag{17}$$

The water level fluctuation equations for **Lake St. Clair, Lake Erie, and Lake Ontario** follow the same principles.

The Great Lakes Flow Network Model

Summarizing the above lake water level change equations, we obtain the Great Lakes Flow Network Model as follows.

$$\begin{cases} A_{Sup} \cdot \frac{dh_{Sup}}{dt} = Q_{prec,Sup} + Q_{snow,Sup} - (F_{Stm} + Q_{evap,Sup} + Q_{human,Sup}) \\ A_{Hur} \cdot \frac{dh_{Hur}}{dt} = Q_{prec,Hur} + Q_{snow,Hur} + F_{Stm} - (F_{Stc} + Q_{evap,Hur} + Q_{human,Hur}) \\ A_{Stc} \cdot \frac{dh_{Stc}}{dt} = Q_{prec,Stc} + Q_{snow,Stc} + F_{Stc} - (F_{Det} + Q_{evap,Stc} + Q_{human,Stc}) \\ A_{Eri} \cdot \frac{dh_{Eri}}{dt} = Q_{prec,Eri} + Q_{snow,Eri} + F_{Det} - (F_{Nia} + Q_{evap,Eri} + Q_{human,Eri}) \\ A_{Ont} \cdot \frac{dh_{Ont}}{dt} = Q_{prec,Ont} + Q_{snow,Ont} + F_{Nia} - (F_{Stl} + Q_{evap,Ont} + Q_{human,Ont}) \end{cases} \tag{18}$$

$$Q_{prec} = P \cdot A, \quad Q_{evap} = E \cdot A \tag{19}$$

Where A_{lake} is the lake area, h_{lake} is the lake water level, Q_{prec} is the flow of precipitation into the lake, Q_{snow} is the flow of melted snow into the lake, F_{river} is the river flow, Q_{evap} is the flow of evaporation from the lake surface, and Q_{human} is the flow into the lake due to human activities.

Validation of the Great Lakes Flow Network Model

In this section, to demonstrate the effectiveness of the Great Lakes Flow Network Model in calculating the water levels of individual lakes, we conducted model validation using historical data.

Utilizing water level data for each lake in the Great Lakes watershed from 2000 to 2020, we compared the lake water levels calculated by the model with the historical data. Specifically, for parameters such as precipitation, melted snow, evaporation, and human activities involved in the model Eq. 18, we used historical data. For river flow F_{river}, we estimated it using the natural flow method Eq. 15.

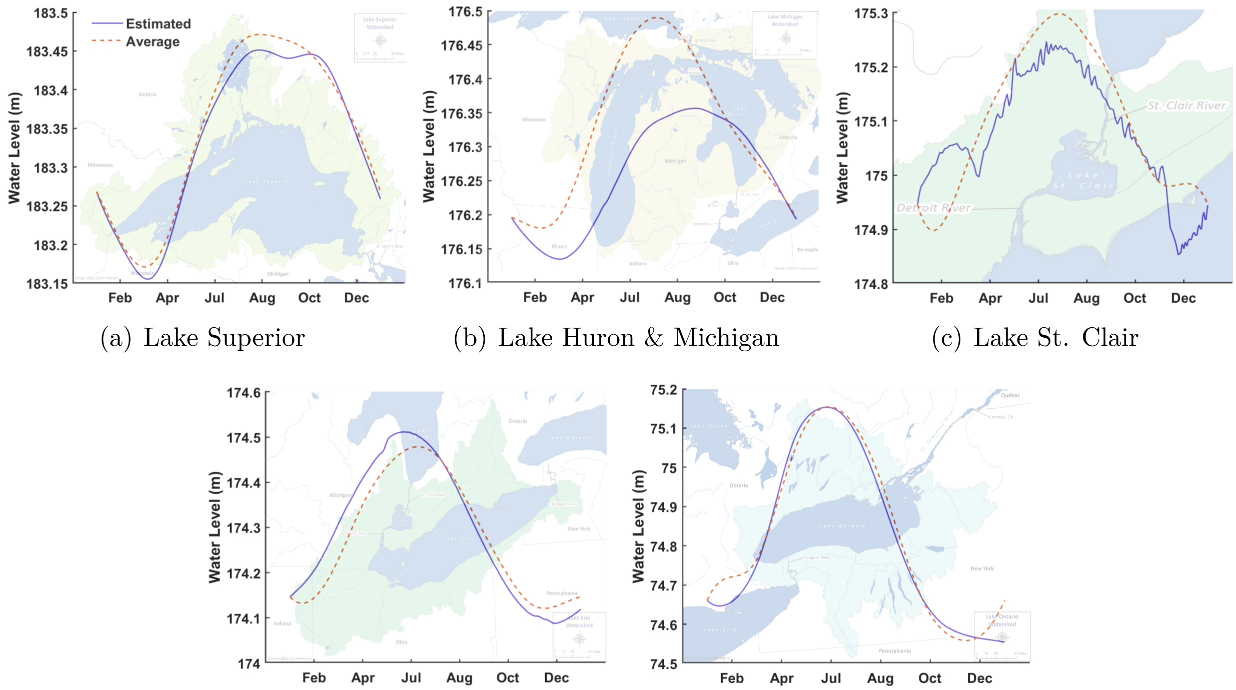
By substituting all parameters into the model Eq. 18 and solving using MATLAB, we obtained the calculated water levels for each lake in the Great Lakes watershed throughout the years 2000 to 2020. (shown in Figure 5)

As shown in Figure 5, we compared the lake water levels calculated by the model with historical data and found a good agreement between the model results and the historical data. This validates the effectiveness of the Great Lakes Flow Network Model.

4 Optimal Water Level evaluation model

To optimize the regulation of water levels in the Great Lakes, it is crucial to comprehensively consider the demands of various stakeholders. Calculating the expected optimal water level for each moment throughout the year is necessary. This water level will serve as the subsequent control target.

In this section, we will first discuss the requirements of various stakeholders in the Great Lakes basin, establishing the benefit-water level functions for each stakeholder. Then, we will weight the functions of each stakeholder to obtain the comprehensive benefit-water level



(a) Lake Superior (b) Lake Huron & Michigan (c) Lake St. Clair

(d) Lake Erie (e) Lake Ontario

Figure 5: Flow Network Model Verification

function at each moment. Finally, we will determine the maximum value of the comprehensive benefit-water level function, representing the optimal water level for that moment by considering the demands of all stakeholders.

4.1 Analysis of Stakeholders' Interests

In this part, we first identified six categories of stakeholders related to lake water levels and qualitatively analyzed their interests from two dimensions. Following that, we constructed a benifit-water level function to quantitatively evaluate the preferences of different stakeholders regarding water levels.

Qualitative Analysis of Stakeholders' Interests

We have categorized stakeholders in the Great Lakes basin into six groups, as illustrated in Table 2. We have considered their requirements in terms of water level expectations and the stability of water levels.

For the two aspects of requirements mentioned in Table 2, we need to quantify them into evaluation metrics for subsequent calculations.

Definition of Profit Functions

To describe the relationship between the benefits of various stakeholders and the current water level, we introduce the Kuznets Curve from economics[4], where the relationship between benefits and water level exhibits an inverted "U-shape".

Table 2: Stakeholder Needs

No.	Stakeholder Group	Desired Water Level	Stability Required
1	Shipping Companies	High	Yes
2	Dock Managers/Locals near Harbor	Low	No
3	Environmentalists	Seasonal Variation	Yes
4	Lakefront Property Owners	Mid	Yes
5	Recreational Boaters/Fishing Boats	Mid	Yes
6	Hydro-power Generation Companies	High	No

The benefit function for stakeholder No. k is defined as $f_k(t,h)$, where h represents the water level, t is the moment in time, and k denotes the stakeholder. We define the benefit function $f_k(t,h)$ as follows:

$$f_k(t,h) = -(h - m_k - r_k) \cdot (h - m_k + r_k) \tag{20}$$

Here, $m_k(t)$ represents the optimal water level for stakeholder k, where their benefit is maximized. $r_k(t)$ denotes the profit radius for stakeholder k, meaning that the benefit function is positive when the water level is within the interval $(m_k - r_k, m_k + r_k)$.
The parameter $m_k(t)$ is employed to characterize stakeholder k's expectations regarding the water level, which can be categorized into high, medium, and low classes. On the other hand, the parameter $r_k(t)$ is utilized to describe stakeholder k's stability requirements concerning the water level. A smaller value of r_k indicates a higher demand for stability in the water level as perceived by the stakeholders.

Determination of Profit Function Parameters

In this part, we determine the parameters $m_k(t)$ and $r_k(t)$ in the benefit function Eq. 20. As the understanding of the concept of ”high water level” varies during different seasons throughout the year, these two parameters are functions of time.

We define $m_k(t)$ and $r_k(t)$ based on historical data for different periods. Specifically, we first define the historical high water level as $h_{high'}(t)$ and the historical low water level as $h_{low'}(t)$ for the month t. That is,

$$h_{high'}(t) = \frac{1}{n}\sum_{i=1}^{n} h_{max,i}(t), \quad h_{low'}(t) = \frac{1}{n}\sum_{i=1}^{n} h_{min,i}(t) \tag{21}$$

Here, $h_{max,i}(t)$ denote the maximum water level for the i-th year in the t-th month, $h_{min,i}(t)$ denotes the minimum water level for the i-th year in the t-th month, and n is the number of years in the historical data. We will use historical data from the years 2000 to 2020 for these calculations.

Continuing, within the interval $[h_{low'}(t), h_{high'}(t)]$, we define the stakeholders' high water level h_{high}, mid-water level h_{mid}, and low water level h_{low} as follows:

$$h_{high}(t) = h_{high'}(t) + \frac{1}{3} \cdot \delta h, \tag{22}$$

$$h_{mid}(t) = h_{high'}(t) + \frac{1}{2} \cdot \delta h, \tag{23}$$

$$h_{low}(t) = h_{high'}(t) + \frac{2}{3} \cdot \delta h \tag{24}$$

hereδh represents the difference between the historical high water level and low water level, i.e.,

$$\delta h = h_{high'}(t) - h_{low'}(t) \tag{25}$$

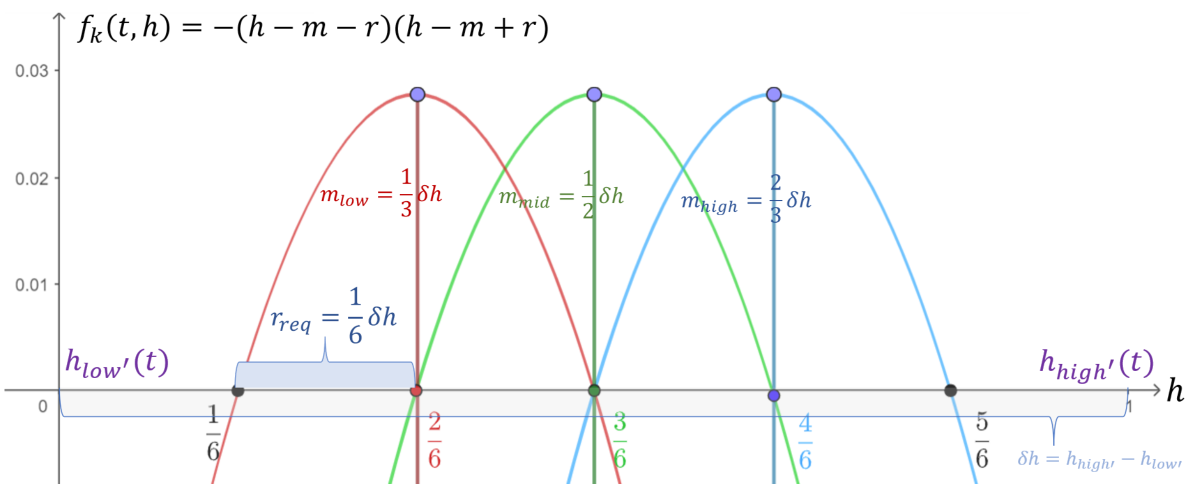


Figure 6: Profitability-Water Level Function under Different m_k Parameters

As shown in Figure 6, the three categories of optimal water levels are defined at positions $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$. We consider the water levels at the ends of the historical range $[h_{low'}(t), h_{high'}(t)]$ as extreme levels. Therefore, we did not define the optimal h_{low} and h_{high} at the ends of the range. For the mid-level h_{mid}, positioned in the middle of the range, we view it as a balanced water level. Hence, we defined it at the midpoint of the range.
Specifically, for environmentalists, whose demand for water levels varies seasonally, we set the parameter $m_k(t)$ in their profit function to be the average water level for each month based on historical data. This is expressed as:

$$m_k(t) = h_{avg'}(t) \tag{26}$$

With the definitions of high, mid, and low water levels, as well as the seasonally varying water levels, we can set the m_k parameters for each stakeholder according to the needs described in Table 2.

$$m_k(t) = \begin{cases} h_{high}(t) = h_{high'}(t) + \frac{1}{3} \cdot \delta h, & \text{if } k = 6, \\ h_{mid}(t) = h_{high'}(t) + \frac{1}{2} \cdot \delta h, & \text{if } k = 4 \text{ or } k = 5, \\ h_{low}(t) = h_{high'}(t) + \frac{2}{3} \cdot \delta h, & \text{if } k = 2, \\ h_{avg'}(t) & \text{if } k = 3, \end{cases} \tag{27}$$

The parameter k denotes the six categories of stakeholders, as illustrated in Table 2.

For the profit radius $r_k(t)$, we similarly utilize $h_{high'}$ and $h_{low'}$ for its definition, establishing two types of benefit radii corresponding to stakeholders who require and do not require water level stability. Specifically, we define:

$$r_k(t) = \begin{cases} \frac{1}{3} \cdot \delta h, & \text{if } k = 1, 4, 5, 6, \\ \frac{1}{2} \cdot \delta h, & \text{if } k = 2, 3. \end{cases} \tag{28}$$

Here, δh represents the difference between the historical high water level and the low water level. When water level stability is required, the profit radius $r_k(t)$ is smaller; when water level stability is not required, the profit radius $r_k(t)$ is larger. The parameter k denotes the six categories of stakeholders as well.

So far, we have defined the benefit functions $f_k(t, h)$ for various stakeholders regarding water levels based on the Kuznets Curve concept. Additionally, we have determined the parameters $m_k(t)$ and $r_k(t)$ that affect these benefit functions.

4.2 Construction of the Optimal Water Level Model

In this section, we will provide the weights for each stakeholder. Subsequently, we will perform a weighted sum of the benefit functions $f_k(t, h)$ for each stakeholder to obtain the comprehensive benefit function $f(t, h)$. Then, by maximizing the comprehensive benefit function, we will determine the optimal water level for each moment t.

Weighting Stakeholders' Profit Functions

Integrated with our considerations, the stakeholders' demands presented in Plan 2014[5], and taking into account recent facts and public opinions, the weights used in our model are listed inTable 3：

Note: Our model possesses strong flexibility and adjustability. Proper adjustments to the weights can better meet the interests of certain stakeholders (though it may reduce the interests of others).
We strongly recommend that the IJC continuously updates the weights based on comprehensive surveys to achieve optimal results.

Comprehensive Profit Function

Following the weights provided in Table 3, we calculate the weighted sum of benefit functions for various stakeholders to obtain the comprehensive benefit function $f(t, h)$. Particularly, during the annual ice cover from December to March, when there is non-navigation, we won't

Table 3: Stakeholder Weights (Percentage)

Stakeholder Group	Weight (%)
Shipping Companies	25.00
Dock Managers/Locals near Montreal Harbor	15.67
Environmentalists	24.00
Lakefront Property Owners	14.33
Recreational Boaters/Fishing Boats	8.33
Hydro-power Generation Companies	12.67

consider the benefits of Shipping Companies and Recreational Boaters/Fishing Boats during this period. Specifically, the comprehensive benefit function $f(t,h)$ is defined as:

$$f(t,h) = \begin{cases} \sum_{k=1}^{6} w_k \cdot \dfrac{f_k(t,h)}{\max f_k(t,h)}, & \text{if } 4 \le t \le 11, \\ \sum_{k \in D} w_k \cdot \dfrac{f_k(t,h)}{\max f_k(t,h)}, & \text{if } t = 12 \text{ or } 1 \le t \le 3, \\ & D = \{2,\ 3,\ 4,\ 6\} \end{cases} \tag{29}$$

where w_k represents the weight for stakeholder k, $f_k(t,h)$ is the benefit function for stakeholder k, and $\max f_k(t,h)$ is the maximum value of the benefit function for stakeholder k, used for normalization. Since f_k is a quadratic function with respect to h, $\max f_k(t,h)$ corresponds to the vertex value of f_k. Further details are not reiterated here.

Optimal Water Level Function

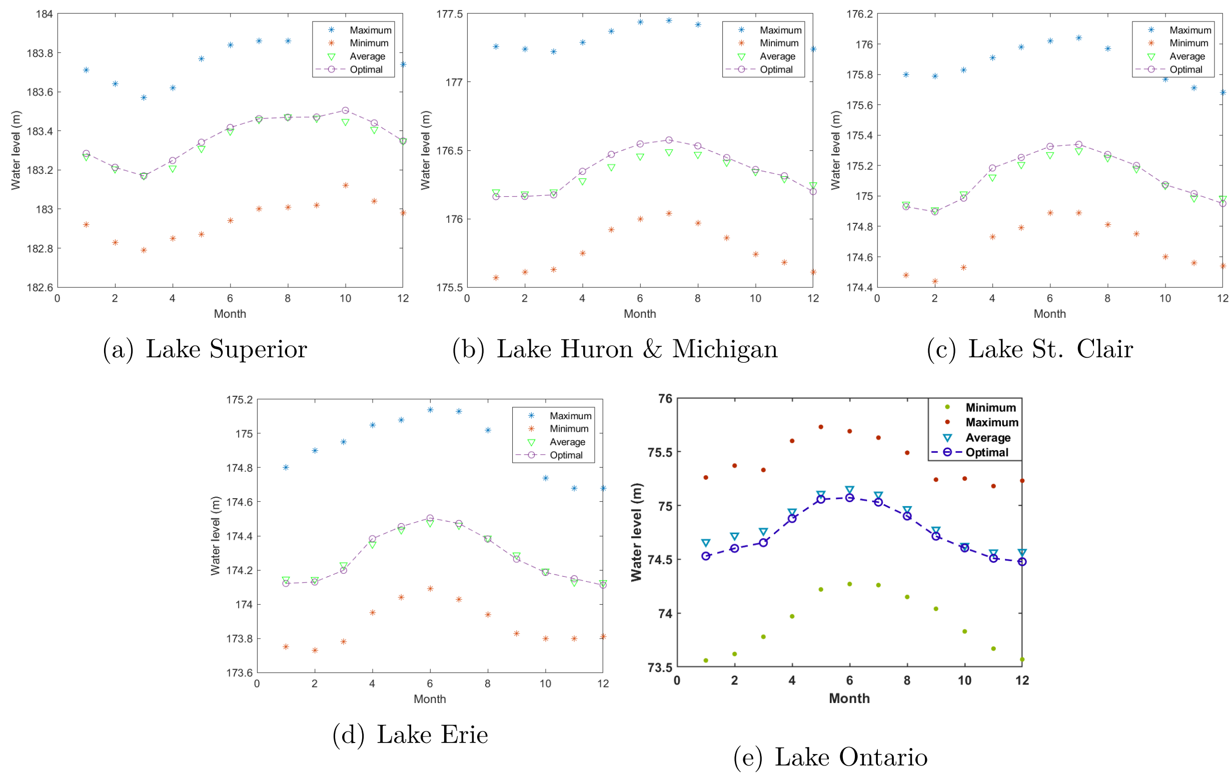
The optimal water level model requires determining the optimal water level $h_{opt}(t)$ at each moment. We define the water level corresponding to the maximum value of the comprehensive benefit function $f(t,h)$ as the optimal water level $h_{opt}(t)$ at time t.

$$h_{\text{opt}}(t) = \text{argmax}_h\, f(t,h) \tag{30}$$

where $h_{\text{opt}}(t)$ represents the optimal water level at time t, and $\text{argmax}_h\, f(t,h)$ corresponds to the water level at which $f(t,h)$ attains its maximum value at time t.

We utilized MATLAB to solve the model Eq. 30, obtaining the comprehensive optimal water levels $h_{\text{opt}}(t)$ for each lake at every moment throughout the year.See Figure 7.

As shown in Figure 7, it illustrates the optimal water levels $h_{opt}(t)$ for each lake every month throughout the year. We observe a clear seasonal pattern in the optimal water levels for the Great Lakes. This is attributed to varying stakeholder demands for water levels in different seasons, leading to corresponding fluctuations in optimal water levels. Additionally, an interesting observation is that the optimal water level curves in Figure 7$(a) \sim (d)$ are slightly below the historical average water levels, while the curve for Lake Ontario in Figure 7(e) is slightly above the historical average water level. This discrepancy can be explained by the higher population density, economic activities, and shipping near Lake Ontario, resulting in



(a) Lake Superior (b) Lake Huron & Michigan (c) Lake St. Clair

(d) Lake Erie (e) Lake Ontario

Figure 7: Optimal Water Level of Five Great Lakes

a higher demand for water level stability. In contrast, the lower population density near other lakes allows for a more significant natural seasonal variation in water levels to preserve the ecological environment. This observation aligns with the rationality of our method for quantifying stakeholder demands.

5 Outflow control algorithm for maintaining optimal water level

Our goal is to establish an inflow and outflow control algorithm that, by manipulating the flow rates of Compensating Works and Moses-Saunders Dam, optimally regulates the water levels of the Great Lakes. In Section 7, we have further achieved simultaneous regulation of water levels near the Great Lakes and Montreal Harbor.
To address this issue, we first simplified the problem into a single-objective optimization problem. We then employed the method of global sensitivity analysis to express the specific form of the optimization objective function. Subsequently, we utilized the Interior-Point Method to obtain the optimal flow regulation plans for the two dams.

5.1 Single-objective optimization model

We aim to bring the water levels of the Great Lakes as close as possible to the optimal levels. This is a complex multi-objective optimization problem, which we address by transforming it into a single optimization objective. Firstly, in the temporal dimension, we employ a greedy strategy, calculating adjustments each day that optimize the water levels for the following day. Then, for the five different lake levels, we use the mean square error between the current lake levels and the optimal levels as the optimization objective. Therefore, the problem is transformed into:

$$\begin{aligned}
&\min_{F_{dam1},F_{dam2}} \quad G(F_{dam1},F_{dam2}) = \frac{1}{5}\sum_{k=1}^{5}(h_k - h_{k\,opt})^2 \\
&s.t. \quad \begin{cases} F_{\mathrm{dam}i} < S_i(t) \\ F_{\mathrm{dam}i} > I_i(t) \\ h_{k\,min} > h_k > h_{k\,max} \end{cases}
\end{aligned} \tag{31}$$

In the above equations, G represents the objective function of the optimization, $F_{\mathrm{dam}i}, i = 1, 2$ denotes the discharge of the two dams, h_k and $h_{k\,opt}$ represent the current water level and optimal water level of each lake, and $S_i(t)$ and $I_i(t)$ represent the upper and lower bounds of the discharge (typically functions of time t).
In this model, we consider $S_i(t)$ and $I_i(t)$ as constants independent of time (while in the model presented in Section 7, they are functions dependent on time). According to Plan 2014, the upper and lower limits of discharge for the two dams are as shown in Table 4:

Table 4: Limits of the two dams

	Minimum flow $I_i\,(m^3/s)$	Maximum flow $S_i\,(m^3/s)$
Compensating Works (dam_1)	1200	4000
Moses-Saunders Dam (dam_2)	5800	10500

The upper and lower bounds of h_k, denoted as $h_{k,max}$ and $h_{k,min}$, can be calculated based on historical data[?]. Throughout the adjustment process, none of the five lakes experienced any cases of exceeding these boundaries.

5.2 Determine the optimization objective function form

To establish the specific relationship between G and $F_{\mathrm{dam1}}, F_{\mathrm{dam2}}$, we employ the method of global sensitivity analysis to assess the impact of $F_{\mathrm{dam1}}, F_{\mathrm{dam2}}$ on h_k. Taking the first-order total differential of the bivariate function $h_k = H_k(F_{\mathrm{dam1}}, F_{\mathrm{dam2}})$, we obtain:

$$\mathrm{d}h_k = \frac{\partial H}{\partial F_{\mathrm{dam1}}}\mathrm{d}F_{\mathrm{dam1}} + \frac{\partial H}{\partial F_{\mathrm{dam2}}}\mathrm{d}F_{\mathrm{dam2}} = \mu_{1k}\mathrm{d}F_{\mathrm{dam1}} + \mu_{2k}\mathrm{d}F_{\mathrm{dam2}} \tag{32}$$

We refer to μ_{ik} as the impact factor of the ith dam on the kth lake. To determine the specific values of μ_{ik}, we can set $\mathrm{d}F_{\mathrm{dam1}} = 0$ or $\mathrm{d}F_{\mathrm{dam2}} = 0$, thus obtaining:

$$\mu_{ik} = \frac{\mathrm{d}h_k}{\mathrm{d}F_{\mathrm{dam}i}}\ , i = 1, 2; k = 1, 2, 3, 4, 5 \tag{33}$$

Specifically, our approach is as follows: we perturb the outflow of Compensating Works, Moses-Saunders, and Montreal, denoted as $\mathrm{d}F_{\mathrm{dam}i}$, separately. We then substitute these perturbations into the model to calculate the water levels h_k' for each lake one day later. Subsequently, we compute the impact factors $\mu_{ik}(t)$ based on these results.

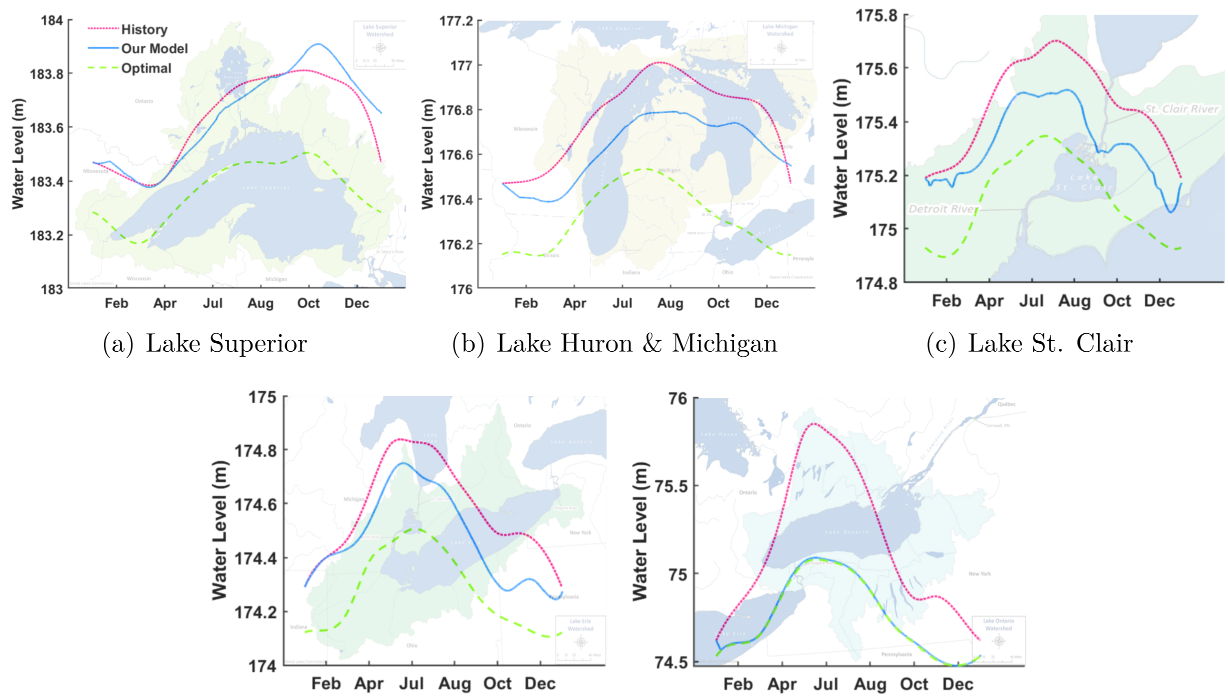
$$\mu_{ik}(t) = \frac{\mathrm{d}h_k}{\mathrm{d}F_{\mathrm{dam}i}} = \frac{h_k' - h_k}{\mathrm{d}F_{\mathrm{dam}i}} \tag{34}$$

The significance of the impact factor is that when the flow rate of dam i changes by $1, m^3/s$, the water level of lake k will change by μ_{ik}, m one day later. Therefore, one day later, the water level of the lakes becomes $h_k' = h_k + \mu_{1k}\mathrm{d}F_{\mathrm{dam1}} + \mu_{2k}\mathrm{d}F_{\mathrm{dam2}}$. Hence, the optimization objective is:

$$\min_{F_{dam1}, F_{dam2}} G(F_{dam1}, F_{dam2}) = \frac{1}{5}\sum_{k=1}^{5}(h_k + \mu_{1k}\mathrm{d}F_{\mathrm{dam1}} + \mu_{2k}\mathrm{d}F_{\mathrm{dam2}} - h_{k\,opt})^2 \tag{35}$$

This is a specific function that can be solved, and the optimal solution $(\mathrm{d}F_{\mathrm{dam1}}, \mathrm{d}F_{\mathrm{dam2}})$ can be obtained using the Interior-Point Method (Matlab fmincon function). This represents the optimal adjustment plan for dam flow rates, where the flow for dam i should increase or decrease by $\mathrm{d}F_{\mathrm{dam}i}$.

5.3 New control results for 2017 *vs.* Actual recorded water levels



(a) Lake Superior (b) Lake Huron & Michigan (c) Lake St. Clair

(d) Lake Erie (e) Lake Ontario

Figure 8: Water Level in 2017: Under Our Controls vs. Historical Data

Next, we applied our control algorithm to adjust the water levels of the Great Lakes in 2017. We then compared these adjusted water levels with the actual recorded levels in 2017 under the regulation of PLAN 2014 to assess the control effectiveness of our algorithm.See Figure 8.

As depicted in Figure 8, the red line represents the actual recorded water levels for that year, the green line signifies the optimal water levels and the blue line illustrates the new results obtained from our algorithm. It is evident that under our control algorithm, the water level trends for each lake closely align with the optimal water levels. Although the results for Lake Superior show a slightly less effective outcome compared to the actual water levels, the effects for other lakes are notably better than the actual scenario. Particularly, the water level control for Lake Ontario in our results is very close to the optimal water levels. Overall, the performance of the water level control significantly surpasses that of Plan 2014.

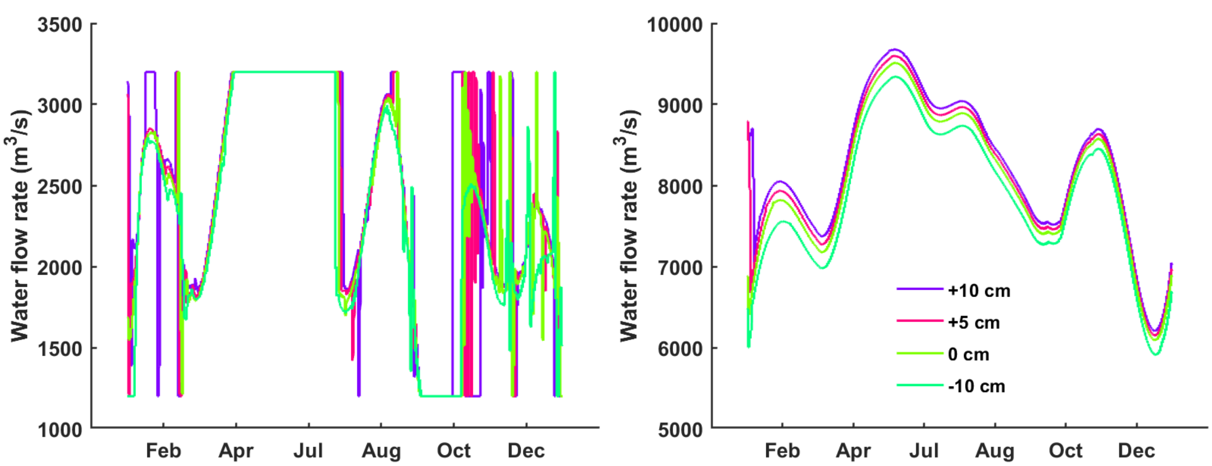
6 Sensitivity Analysis

6.1 Dam Outflow Control Sensitivity Analysis

To analyze the sensitivity of our algorithm to the deviation between actual water levels and optimal water levels, we set different initial water level values. Using our water level control algorithm, we simulated the water levels for one year. We then compared the results of multiple sets of dam flow control outcomes, analyzing their sensitivity to water level deviations.

Specifically, we used environmental condition data from the year 2017. The optimal water level for the month of January, determined by the optimal water level model (Eq. 30), was taken as the initial water level. Subsequently, the initial water level was increased by $0cm$, $5cm$, $10cm$, and decreased by $-10cm$ respectively. Our water level control algorithm was then applied to simulate the water levels over the course of a year. The results are depicted in Figure 9.

As shown in Figure 9——the outflow discharge curves for both the Compensating Works and the Moses-Saunders Dam under different initial water levels. It can be observed that, for the Moses-Saunders Dam, the algorithm-controlled drainage discharge remains relatively stable, and the outflow discharge curves are generally consistent across various initial water levels.



(a) Compensating Works (b) Moses-Saunders Dam

Figure 9: Dam Outflow Control Sensitivity Analysis

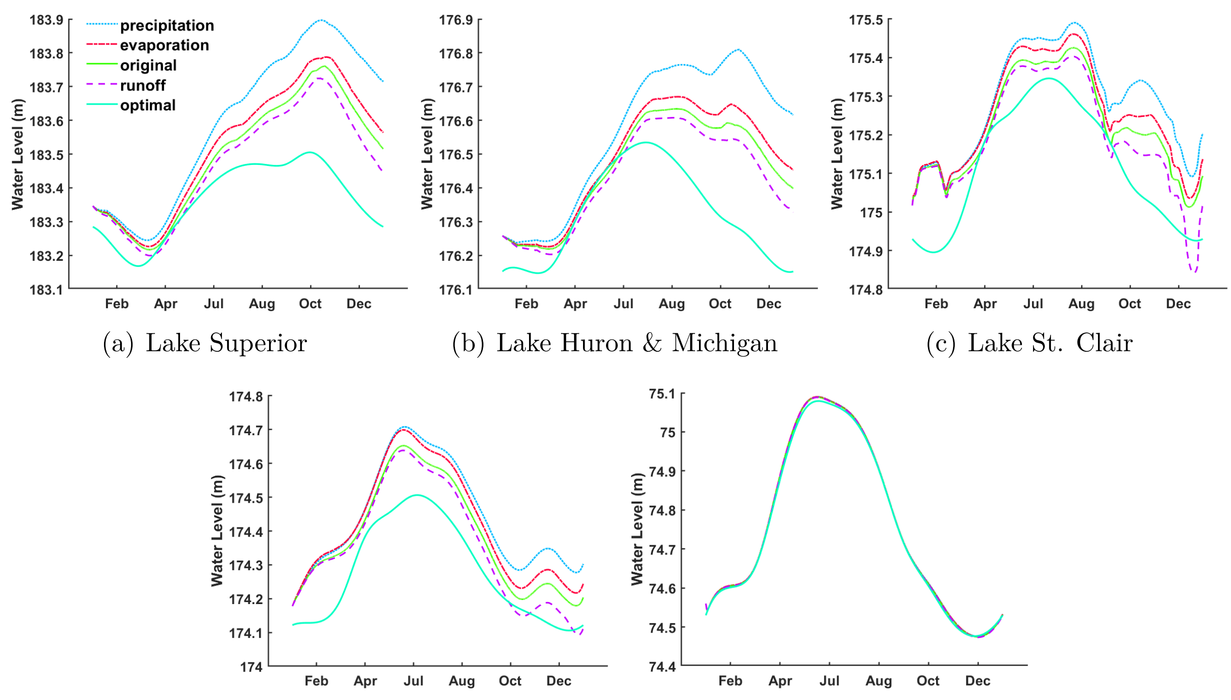
In contrast, for the Compensating Works, the dam's flow control is relatively stable during the summer and autumn seasons, while it exhibits significant fluctuations during the spring and winter seasons in response to changes in the initial water level. This is attributed to the Compensating Works being located upstream of the Great Lakes, where factors influencing the system are complex, leading to a more chaotic system. As a result, the dam's regulation of discharge is more sensitive to changes in the initial water level.

Note:We have used 2017 data for comparison of new results with actual recorded water levels. See Fig. 8.

6.2 Environmental Conditions Sensitivity Analysis

We selected historical data from the year 2017 and made minor changes to environmental conditions. By comparing the water level of each lake before and after these changes under our algorithm, we analyzed the sensitivity of our model to environmental conditions.

Specifically, we made individual increases of 10% to the precipitation, evaporation, and snowmelt on the basis of the historical data from the year 2017. Using our algorithm, we simulated the water levels for one year, comparing the changes in water levels for each lake before and after these adjustments, as shown in Figure 10.



(a) Lake Superior (b) Lake Huron & Michigan (c) Lake St. Clair

(d) Lake Erie (e) Lake Ontario

Figure 10: Environmental Sensitivity Analysis: Impact on Water Levels (2017)

As shown in Figure 10, illustrates the changes in water levels for the 5 lakes before and after minor changes to the environmental conditions in 2017. The curve labeled *original* represents the water level changes under our water level control algorithm based on the original environmental data for 2017. The curves labeled precipitation, evaporation, and runoff represent the water level changes after increasing precipitation, evaporation, and snowmelt by 10%, respectively.

We can observe that under our water level control algorithm, the water level curves for the lakes are relatively sensitive to minor adjustments in environmental conditions. Increases in precipitation and runoff from snowmelt tend to raise the overall water level curves for the lakes, while an increase in evaporation leads to an overall decline in the water level curves. This characteristic of water level curves changing with variations in environmental water input and loss aligns with our expectations for the lake water level control algorithm.

In particular, for Lake Ontario, under minor changes in various environmental conditions, its water level change curves show no significant variations, and the deviations from the optimal water level curves are small. We attribute this to the fact that Ontario is downstream among the Great Lakes, allowing it to be well-regulated by the two dams. As a result, it exhibits low sensitivity to minor adjustments in environmental conditions. This effective regulation aligns with the characteristics of Lake Ontario, which has a high population density and is characterized by active economic and shipping activities in the vicinity.

7 Sub-Problem: Lake Ontario Control and Montreal Harbor Protection

In this section, our primary focus is on the water level regulation of Lake Ontario, downstream of the Great Lakes, and the protection of Harbor Montreal downstream of Ontario. To safeguard Montreal Harbor, we initially determined a safe water level range for the St. Lawrence River, which flows through Montreal Harbor. Subsequently, based on this range, additional constraints were imposed on the flow rates at the Moses-Saunders Dam. Building upon these constraints, we enhanced our water level control model to ensure the absolute safety of Montreal Harbor. Finally, we applied the improved control model to historical data, comparing and validating its advantages in water level regulation for Lake Ontario over Plan 2014.

7.1 Montreal Harbor Safety Measures

Downstream of the Moses-Saunders Dam, the St. Lawrence River converges with the Ottawa River and flows through Montreal Harbor. We calculated the flow range of the St. Lawrence River corresponding to the safe water level for Montreal Harbor. Based on this range, we imposed constraints on the flow rates at the Moses-Saunders Dam, ensuring the safety of Montreal Harbor under the control of dam discharge.

We can obtain the safe water level range for Montreal Harbor, denoted as $[h_{mon,low}, h_{mon,high}]$, and acquire its historical mean water level $h_{mon,avg}$ along with the corresponding flow mean $F_{mon,avg}$ based on historical data. Assuming a linear relationship between the water level and flow of Montreal Harbor, we can express the flow F_{mon} as a function of the water level h_{mon}, i.e.,

$$F_{mon}(h_{mon}) = \frac{F_{mon,avg}}{h_{mon,avg}} \cdot (h_{mon} - h_{mon,avg}) + F_{mon,avg} \tag{36}$$

By substituting the safe water level range for Montreal, $[h_{mon,low}, h_{mon,high}]$, into Eq. 36, we obtained the safe flow range for Montreal Harbor, denoted as $[F_{mon,low}(t), F_{mon,high}(t)]$.

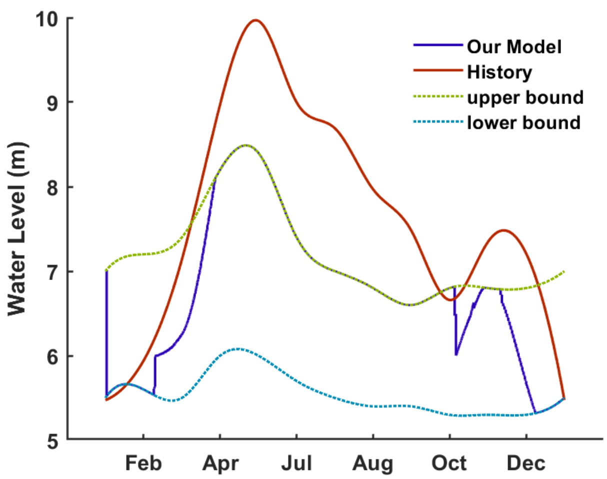
Due to the confluence of the St. Lawrence River and the Ottawa River, which then passes through Montreal Harbor, the flow of the St. Lawrence RiverF_{stl}

$$F_{stl}(t) = F_{mon}(t) - F_{ott}(t) \tag{37}$$

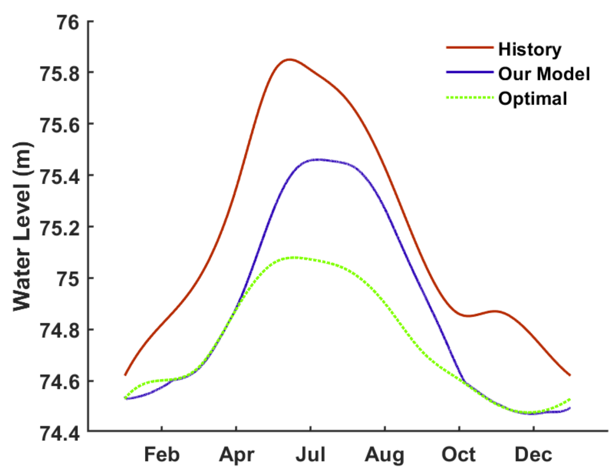
Consequently, we can derive the safe flow range for the St. Lawrence River, denoted as $[F_{stl,low}(t), F_{stl,high}(t)]$, where

$$F_{stl,low}(t) = F_{mon,low}(t) - F_{ott,high}(t), \quad F_{stl,high}(t) = F_{mon,high}(t) - F_{ott,low}(t) \tag{38}$$

Assuming no flow loss in the river during the flow process, the flow range $[F_{stl,low}(t), F_{stl,high}(t)]$ for the St. Lawrence River is considered as the output flow range for the Moses-Saunders Dam.We incorporate this range as an additional constraint on the flow control of the Moses-Saunders Dam, transforming the static constraint conditions of the dam's flow control algorithm in Section 5 into dynamic constraints.This enhancement is implemented to ensure the absolute safety of Montreal Harbor.



(a) Montreal Harbor Water Level Control Results



(b) Lake Ontario Water Level Control Results

Figure 11: Montreal Harbor Protection and Lake Ontario Control Results

We applied the improved water level control model to historical data from the year 2017, resulting in the water level control outcomes for Montreal Harbor as depicted in Figure 11(a). It is evident that the enhanced water level control model prevents the occurrence of extreme high water levels observed in the historical data. The water level variations consistently remain within the upper and lower bounds, ensuring the absolute safety of Montreal Harbor.

7.2 Ontario Water Level Control Model Improvement

In the improved water level control model, we have ensured the absolute safety of Montreal Harbor.We applied this model to the historical data of the Great Lakes in 2017 and compared the water level control results for Lake Ontario with Plan 2014, as illustrated in Figure 11(b).

From Figure 11(b), it can be observed that compared to the historical water levels under the regulation of Plan 2014 for Lake Ontario, our model results in a smoother water level variation. In both spring and winter, the regulated water levels are nearly identical to the optimal water levels, while in summer and autumn, the regulated water levels are also closer to the optimal levels. This indicates that our model exhibits significant advantages over Plan 2014 in the regulation of Lake Ontario's water levels, better meeting the needs of the six categories of stakeholders.

Furthermore, in comparison to Plan 2014, our model demonstrates superiority in considering a more comprehensive set of influencing factors. Plan 2014 falls short in adequately addressing the following factors:

1. The flow from the Ottawa River and its confluence with the St. Lawrence River is taken into account. In Eq. 37, we consider the factors related to the confluence of the Ottawa River with the St. Lawrence River and derive a flow range that varies with the seasons.

2. Snowmelt and evaporation are considered in the flow network water level model, as shown in Eq. 18. Q_{snow} and Q_{evap} account for the impact of snowmelt and evaporation on the lake water levels.

3. The impact of other lakes is also taken into account. Our enhanced water level control model is still based on the water level network model for the Great Lakes (Eq. 18), thus comprehensively considering the water levels of all five lakes. The model considers the influence of inflow from upstream lakes into Lake Ontario.

4. Temperature is also considered in our model. The model takes into account the effects of evaporation and snowmelt, both of which are temperature-dependent factors. Therefore, our model indirectly considers the influence of temperature on lake water levels.

Memorandum

To: International Joint Commission

From: Team 2423551

Subject: Plan 2024, Powerful Beyond Imagination!

Date: February 5,2024

Dear Sir/Madam,

Considering that Plan 2014 has become outdated and ineffective, the disasters of 2017 and 2019 have clearly demonstrated its inability to safeguard the interests and safety of the people downstream of Lake Ontario and the St. Lawrence River. In response, our company designs the more robust Plan 2024 Great Lakes hydrological control system.

Regarding what kind of data you want to know that we use, we are more interested in precipitation, evaporation, and snowmelt data, but we also place a high value on the impact of extreme data because precipitation and drought are more drastic due to climate change, and extremes tend to be costly.

Our system consists of three main components: a hydrological network flow model, an optimal water level evaluation model, and a dam water flow control algorithm. We did not rely on the threshold-triggered adjustment method of Plan 2014 and instead adopted a dynamic feedback control mechanism using a nonlinear constrained single-objective optimization algorithm. This allows for dynamic adjustment of the water level towards the optimal level, thereby avoiding the disasters caused by threshold-triggered delays in Plan 2014.

Our optimal water level evaluation method utilizes economic Kuznets curves and linear weighting, ensuring quantitative and objective evaluation that can adapt to changes in stakeholder interests, unlike the subjective evaluation of various factors in Plan 2014.

Our algorithm prioritizes safeguarding the interests of Montreal Harbor and the cities along the St. Lawrence River, focusing on stakeholders in Lake Ontario, while also considering the regulation of water levels in upstream lakes, an aspect lacking in Plan 2014.

Our system has demonstrated high stability and has been proven to operate rapidly and reliably under various disturbances. In simulated controls for the year 2017, we successfully maintained water levels in the Great Lakes and the St. Lawrence River within safe ranges, significantly outperforming Plan 2014.

In conclusion, Plan 2024 aims to replace Plan 2014 and protect the interests of the people surrounding the Great Lakes water system in the next decade.

Best regrads,
Team 2423551

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