PDF of a continuous RV PDF: Probability Density Function

PDF formula: The PDF of continuous random var Y is the function $f(y)$, such that for interval $[a, b], a \leqslant b$, $P(a \leqslant Y \leqslant b) = \int_a^b f(y)dy$. That is, the probability that the continuous random variable is within an interval, is the area under the curve of the density function between a and b.

PDF Axioms:

1. The total area under the curve of $f(x)$, from $(-\infty, \infty) = 1$:
That is, $\int_{-\infty}^{\infty} f(x)dx = 1$.

Continuous variables have a "smooth curve" graph $f(x)$ that looks like the result of a histogram, or a result of Riemann sums.

This axiom is analogous to the discrete RV's having all probabilities sum to 1 discretely.

2. $f(x) \geqslant 0, \forall x$. All probabilities of the PDF function are positive.

Expected Value and Variance: Continuous RV

Mean or Expected Value of a continuous random variable: $E(Y) = \int_{-\infty}^{\infty} y * f(y)dy$. Similarly, for $h(y)$, a function of y, $E[h(Y)] = \int_{-\infty}^{\infty} h(y) * f(y)dy$

Variance of a continuous random variable with PDF $f(x)$: $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 * f(x)dx = E[(X-\mu)^2]$

CDFs

The CDF for a continuous random variable X is: $F(x) = P(X \leqslant x) = \int_{-\infty}^{x} f(y)dy$. For each x, $F(x)$ is the area under the density curve to the left of x.

Axioms:

- $P(Xa) = 1 - F(a)$, and
- $P(a \leqslant X \leqslant b) = F(b) - F(a)$.

Uniform Probability Distribution

PDF: PDF of uniform distributions is $f(y; A, B) = \dfrac{1}{B-A}$ between A, B; 0 otherwise.

In the uniform distribution, the probability over a subinterval is proportional to the length of that subinterval.

CDF: $F(x) = \dfrac{x-a}{b-a}$

$\mu = \dfrac{a+b}{2}$ and $\sigma^2 = \dfrac{(b-a)^2}{12}$

Normal Probability Distribution for continuous RV

PDF: $f(y; \mu, \sigma) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(y-\mu)}{2\sigma^2}}$.

Area under the normal density function from a to b: $\int_a^b \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(y-\mu)}{2\sigma^2}}$

Z Values: Distance in standard deviations from the mean $z = \dfrac{y-\mu}{\sigma}$

Standard Normal Distribution

This is the normal distribution, with param values $\mu = 0, \sigma = 1$.

PDF: $f(z; \mu = 0, \sigma = 1) = \dfrac{1}{\sqrt{2\pi}} e^{\dfrac{-z^2}{2}}$.

The "z -curve" is the standard normal curve. Z-scores: How many std dev from the mean a value is; areas under the curve. *68-95-99 rule:* 68% of the distribution is within one standard deviation; 95% within two; 99% within three.

Standardizing (nonstandard) distributions: $\mu = 1, \sigma = 1$. Recall distance from the mean in standard deviations was $z = \frac{y-\mu}{\sigma}$. This is similar; the "standardized variable Y" is $\frac{Y-\mu}{\sigma}$.

Standard normal distribution axioms:

- $P(a \leqslant X \leqslant b) = P(\frac{a-\mu}{\sigma} \leqslant Z \leqslant \frac{b-\mu}{\sigma})$
- $P(X \leqslant a) = \phi(\frac{a-\mu}{\sigma})$.
- $P(X \geqslant b) = 1 - \phi(\frac{b-\mu}{\sigma})$.

The CDF of Z: $\frac{X-\mu}{\sigma} = P(Z \leqslant z) = P(X \leqslant \sigma z + \mu) = \int_{-\infty}^{\sigma z+\mu} = f(x;\mu,\sigma)dx$.

Standard Normal Approximation of Binomial:

Use the Normal approximation: $\mu = np, \sigma = \sqrt{npq}$ like binomial, and $P(X \leqslant x) = \phi(\frac{x+0.5-\mu}{\sigma})$.

Gamma and Exponential Distributions

Exponential:

- $\mu = \frac{1}{\lambda}$, and $\sigma^2 = \frac{1}{\lambda^2}$
- PDF: $f(x,\lambda) = \lambda e^{-\lambda x}, x \geqslant 0$, else 0; CDF: $F(x,\lambda) = 1 - e^{-\lambda x}, x0$, else 0

Gamma:

With params α,,

- PDF: $f(y;\alpha,) = \frac{y^{\alpha-1}e^{-y/}}{^{\alpha}(\alpha)}$, Standard Gamma Distribution (= 1): $f(y;\alpha) = \frac{y^{\alpha-1}e^{-y}}{(\alpha)}$

where gamma function $(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy$;

- CDF: $F(y,\alpha) = \int_0^y \frac{y^{\alpha-1}e^{-y}}{(\alpha)}$
- $\mu = \alpha$ and $\sigma^2 = \alpha^2$