EE381 Homework 1

$\boxed{R_i}$ $\boxed{W_j}$

1.3.2)

a) $P[R_3 W_2] \rightarrow P\left[\frac{1}{6} \cdot \frac{1}{6}\right] = \boxed{\frac{1}{36}}$

b) $P[S_9] \rightarrow \frac{4}{36} = \frac{1}{9}$

1.3.3) $P[D_3]$ is absolute = 36 total outcomes (sample space)

possible outcome = (1,4), (4,1), (2,5), (5,2), (3,6), (6,3) = 6 outcomes

$P(D_3) = \frac{6}{36} = \frac{1}{6}$

1.3.5) B > 150 lines, L ≤ 150 lines

F < 0.1 sec, W at least 0.1 sec

$P(LF) = 0.5$, $P(BF) = 0.2$, $P(BW) = 0.2$

$P(W)$, $P(B)$, $P(W \cup B)$?

- $P(S) \stackrel{!}{=} 1.0$

  $P(BF) + P(BW) + P(LF) + P(LW) = 1.0$

  $0.2 + 0.2 + 0.5 + P(LW) = 1.0$

  $P(LW) = 1.0 - 0.2 - 0.2 - 0.5$

  $= \boxed{0.1}$

- $P(B \cap W) + P(L \cap W) = P(W)$

  $P(W) = P(BW) + P(LW)$

  $P(W) = 0.2 + 0.1 = 0.3$

- $P(B) = P(B \cap F) + P(B \cap W)$

  $P(B) = P(BF) + P(BW)$

  $P(B) = 0.2 + 0.2 = \boxed{0.4}$

- $P(W \cup B) = P(W) + P(B) - P(W \cap B)$

  $P(W \cup B) = P(W) + P(B) - (BW)$

  $P(W \cup B) = 0.3 + 0.4 - 0.2 = 0.7 - 0.2$

  $= \boxed{0.5}$

1.4.1)

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

a) Brief call (B) only

$P(B) = P(BH_0) + P(BH_1) + P(BH_2)$

$P(B) = 0.4 + 0.1 + 0.1$

$P(B) = 0.6$

A brief call have no handoffs is BH_0

$P(H_0 | B) = \frac{P(BH_0)}{P(B)} = \frac{0.4}{0.6} = \boxed{0.67}$

b) Performing one handoff (H_1)

Since, a call can be long or brief $\{B, L\}$ is a partition

$P(H_1) = P(H_1B) + P(H_1L)$

$P(H_1) = 0.1 + 0.1$

$P(H_1) = 0.2$

A call with one handoff is long is H_1L

$P(L|H_1) = \frac{P(H_1L)}{P(H_1)} = \frac{0.1}{0.2} = \boxed{0.5}$

c) since call is Long, sample space we have to work on is reduced to Long call (L) only

Since $\{H_0, H_1, H_2\}$ is a partition

$P(L) = P(LH_0) + P(LH_1) + P(LH_2)$

$P(L) = 0.1 + 0.1 + 0.2$

$P(L) = 0.4$

A long call will have one or more Handoffs is LH_1 or LH_2

call is Long

so, $P((H_1 \text{ or } H_2) | L) = \frac{P(LH_1) + P(LH_2)}{P(L)}$

$= \frac{0.2 + 0.1}{0.4} = \boxed{0.75}$

a)

1.4.2) P(rolling any number) = $\frac{1}{6}$

G_1 is sample case in this case

$G_1 = \{2,3,4,5,6\}$

$P(G_1) = P(R_2) + P(R_3) + P(R_4) + P(R_5) + P(R_6)$

$P(G_1) = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 5/6$

and $R_3 = \{3\}$, $G_1 \cap R_3 = \{3\}$

$P(G_1 \cap R_3) = P(R_3) = \frac{1}{6}$

$$P(R_3 | G_1) = \frac{P(G_1 \cap R_3)}{P(G_1)} = \frac{1/6}{5/6} = 0.2$$

b) $G_3 = \{4,5,6\}$

$P(G_3) = P(R_4) + P(R_5) + P(R_6)$

$= 1/6 + 1/6 + 1/6 = 3/6$

and $R_6 = \{6\}$

$G_3 \cap R_6 = \{6\} = R_6$, $P(G_3 \cap R_6) = P(R_6) = \frac{1}{3}$

$$P(R_6 | G_3) = \frac{P(G_3 \cap R_6)}{P(G_3)} = \frac{1/6}{3/6} = 0.33$$

c) E(Roll is even)

$E = \{2,4,6\}$, $P(E) = P(R_2) + P(R_4) + P(R_6)$

$P(E) = 1/6 + 1/6 + 1/6 = 3/6$

$G_3 = \{4,5,6\}$, $G_3 \cap E = \{4,6\} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

$$P(G_3 | E) = \frac{P(G_3 \cap E)}{P(E)} = \frac{2/6}{3/6} = 0.67$$

d) $G_3 = \{4,5,6\}$

$P(G_3) = P(R_4) + P(R_5) + P(R_6)$

$P(G_3) = 1/6 + 1/6 + 1/6 = 3/6$

$E = \{2,4,6\}$, $G_3 \cap E = \{4,6\}$

$P(G_3 \cap E) = P(R_4) + P(R_6) = 1/6 + 1/6 = 2/6$

$$P(E | G_3) = \frac{P(G_3 \cap E)}{P(G_3)} = \frac{2/6}{3/6} = 0.67$$

1.4.3) a) $S = \{2, 3, 4\}$

$P(2) = P(3) = P(4) = 1/3$

$E = \{2, 4\}$

$P(E) = 1/3 + 1/3 = 2/3$

$C_2 = 2$

$E \cap C_2 = 2$

So, $P(E \cap C_2) = P(2) = 1/3$

$$P(C_2 | E) = \frac{P(E \cap C_2)}{P(E)} = \frac{1/3}{2/3} = \boxed{0.5}$$

Samplespace is

b) $C_2 = 2$

$P(C_2) = 1/3$

$E = \{2, 4\}$

$E \cap C_2 = \{2\}$

Hence, $P(E \cap C_2) = P(2) = \frac{1}{3}$

$$P(E | C_2) = \frac{P(E \cap C_2)}{P(C_2)} = \frac{1/3}{1/3} = \boxed{1.0}$$

1.6.2) $P(A) = P(B)$. Mutually exclusive

$P(A \text{ and } B) = 0$; Because independent events, $P(A \text{ and } B) =$

$P(A) * P(B)$. Since, $P(A) * P(B) = P(A)^2 = P(B)^2$

$= 0$

we get $P(A) = P(B) = 0$

1.6.7) a) If A and B are mutually exclusive

$P(A \cup B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{5}{8} = \frac{3}{8} + P(B) - 0 \Rightarrow \boxed{P(B) = \frac{2}{8} = \frac{1}{4}}$$

$$P(A \cap B^c) = P(A) - P(A \cap B)$$
$$= \frac{3}{8} - 0 = \frac{3}{8}$$

$$P(A \cup B^c) = 1 - P(B) + P(A \cap B)$$
$$= 1 - 1/4 + 0$$
$$\Rightarrow P(A \cup B^c) = 3/4$$

b) No because $P(A \cup B) = 0$

$P(A) \cdot P(B) = 3/8 \times 1/4 = 3/32$

$P(A \cup B) \neq P(A) \cdot P(B)$

1.6.8) a) $P(C \cap D) = P(C) \cdot P(D)$

$$\frac{1}{3} = \frac{1}{2} \cdot P(D)$$
$$\Rightarrow P(D) = 2/3$$

$$P(C \cap D^c) = P(C) - P(C \cap D)$$
$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$P(C^c \cap D^c) = 1 - P(C \cup D)$$
$$= 1 - [P(C) + P(D) - P(C \cap D)]$$
$$= 1 - \left[\frac{1}{2} + \frac{2}{3} - \frac{1}{3}\right] = \frac{1}{6}$$

b) $P(C \cup D^c) = P(C) + P(D) - P(C \cap D)$

$= 1/2 + 2/3 - 1/3$

$P(C \cup D) = 5/6$

$P(C \cup D^c) = 1 - P(D) + P(C \cap D)$

$= 1 - 2/3 + 1/3 = 2/3$

$P(D^c) = 1 - P(D) = 1 - 2/3 = 1/3$

$P(C \cap D^c) = \frac{1}{6}$ $\quad P(C) \cdot P(D^c) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$=$

C and D^c are independent

1.5.8) Urn I = 2 white + 3 blacks

Urn II = 4 white + 1 black

Urn III = 3 white + 4 black $= \frac{14}{57}$

$$P(A_k|A) = \frac{P(A_k)\,P(A|A_k)}{P(A_j)\,P(A|A_j)}$$

$$= \frac{2/15}{2/15 \times 4/15 \times 3/21}$$

$P(\text{Urn I}) = 1/3$

$P(\text{Urn}_1 \cdot \text{white}) = \frac{1}{3} \cdot \frac{2}{5} = \frac{2}{15}$

$P(\text{Urn}_2 \cdot \text{white}) = \frac{1}{3} \cdot \frac{4}{5} = 4/15$

$P(\text{Urn}_3 \cdot \text{white}) = \frac{1}{3} \cdot \frac{3}{7} = 3/21$

$P(\text{Urn}_1 \cdot \text{white}) \,/\, P(\text{Urn}_1 \cdot \text{white}) + P(\text{Urn}_2\,\text{white}) + P(\text{Urn}_3 \cdot \text{white})$