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EE 381
Lab 4
11-6-20

1. Simulate continuous random variables with selected distributions

1.1 Simulate a Uniform Random Variable.

a) Create a random variable X with a uniform distribution. Use the Python function "numpy.random.uniform(a,b,n)" to generate n values of the R.V. X of with uniform probability distribution in the open interval $[a,b)$.

b) Use the histogram function to plot a bargraph of the experimental values of the R.V. X. On the same graph plot the probability density function for the R.V. X, given by

$$f(x) = \begin{cases} \frac{1}{(b-a)}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$ and compare to the bargraph plot.

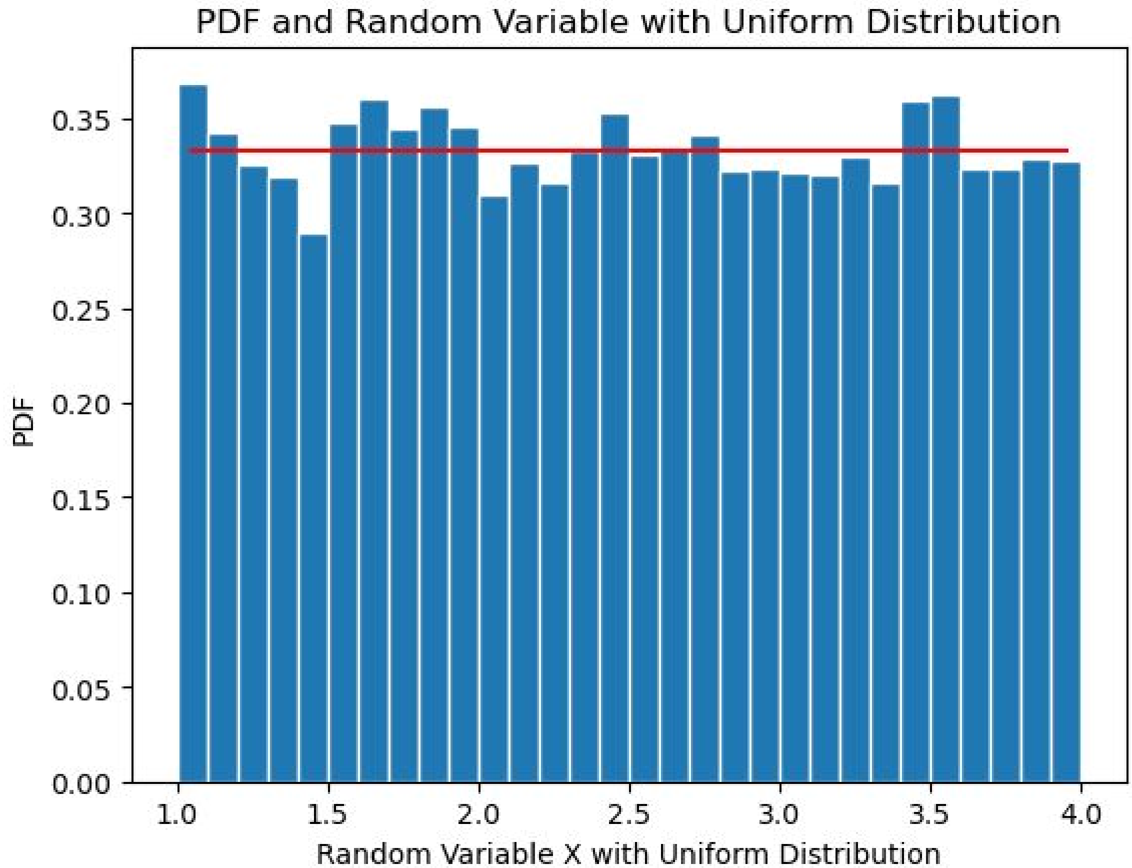
c) Calculate the expectation and standard deviation of the R.V. X, using the Python functions numpy.mean and numpy.std . Compare to the theoretical values given by $\mu_X = \frac{a+b}{2}$; $\sigma_X^2 = \frac{(b-a)^2}{12}$

d) Your report should contain the graph require in (b) and the values required in (c) tabulated in the following table. The graph should be properly labeled.

e) A sample code for (a-c) is given below.

Table 1: Statistics for a Uniform Distribution

Expectation		Standard Deviation	
Theoretical Calculation	Experimental Measurement	Theoretical Calculation	Experimental Measurement
2.50	2.496	0.750	0.856



PDF and Random Variable with Uniform Distribution
PDF
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Random Variable X with Uniform Distribution

1.2 Simulate an Exponentially distributed Random Variable.

a) Create a random variable T with an exponential distribution. Use the Python function "`numpy.random.exponential(beta,n)`" to generate n values of the R.V. T of with exponential probability distribution.

b) Use the histogram function to plot a bargraph of the experimental values of the R.V. T. On the same graph plot the probability density function for the R.V. T, given by

$$f_T(t\ ;\ \beta) = \begin{cases} \frac{1}{\beta}\, exp(-\frac{1}{\beta}t), & t \geq 0 \\ 0, & t < 0 \end{cases}$$ and compare to the bargraph plot.

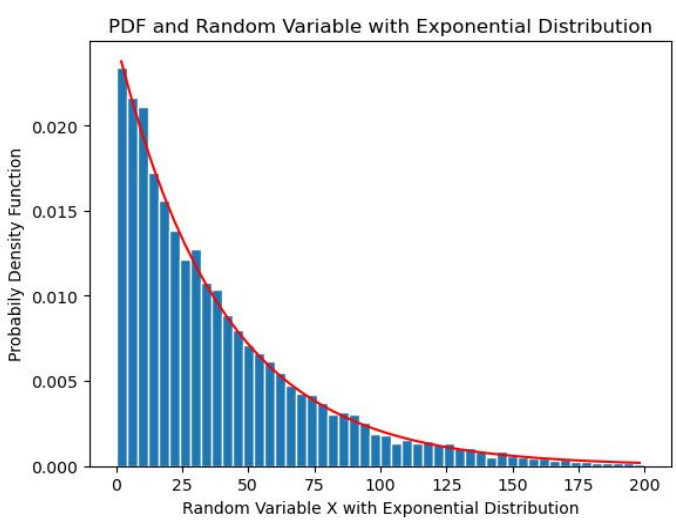
c) Calculate the expectation and standard deviation of the R.V. X, using the Python functions numpy.mean and numpy.std . Compare to the theoretical values given by $\mu_T = \beta \quad ; \quad \sigma_T = \beta$

d) Your report should contain the graph require in (b) and the values required in (c) tabulated in the following table. The graph should be properly labeled.

e) Modify the sample code for (a-c) given previously.

Table 2: Statistics for Exponential Distribution

Expectation		Standard Deviation	
Theoretical Calculation	Experimental Measurement	Theoretical Calculation	Experimental Measurement
39.482	40.000	39.171	40.000



1.3 Simulate a Normal Random Variable.

a) Create a random variable X with a normal distribution. Use the Python function "`numpy.random.normal(mu,sigma,n)`" to generate n values of the R.V. X of with normal probability distribution.

b) Use the histogram function to plot a bargraph of the experimental values of the R.V. X. On the same graph plot the probability density function for the R.V. X, given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}$$

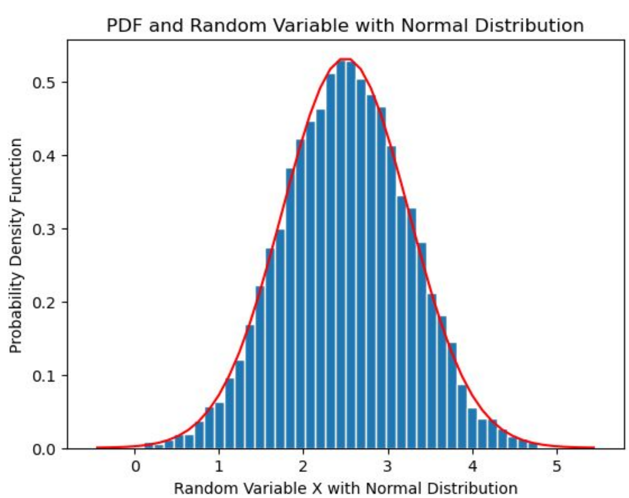
and compare to the bargraph plot.

c) Calculate the expectation and standard deviation of the R.V. X, using the Python functions numpy.mean and numpy.std . Compare to the theoretical values given by $\mu_X = \mu \quad ; \quad \sigma_X = \sigma$

d) Your report should contain the graph required in (b) and the values required in (c) tabulated in the following table. The graph should be properly labeled.

e) Modify the sample code for (a-c) given previously.

Table 3: Statistics for Normal Distribution			
Expectation		Standard Deviation	
Theoretical Calculation	Experimental Measurement	Theoretical Calculation	Experimental Measurement
2.500	2.507	0.750	0.744



2. The Central Limit Theorem

Central Limit Theorem.

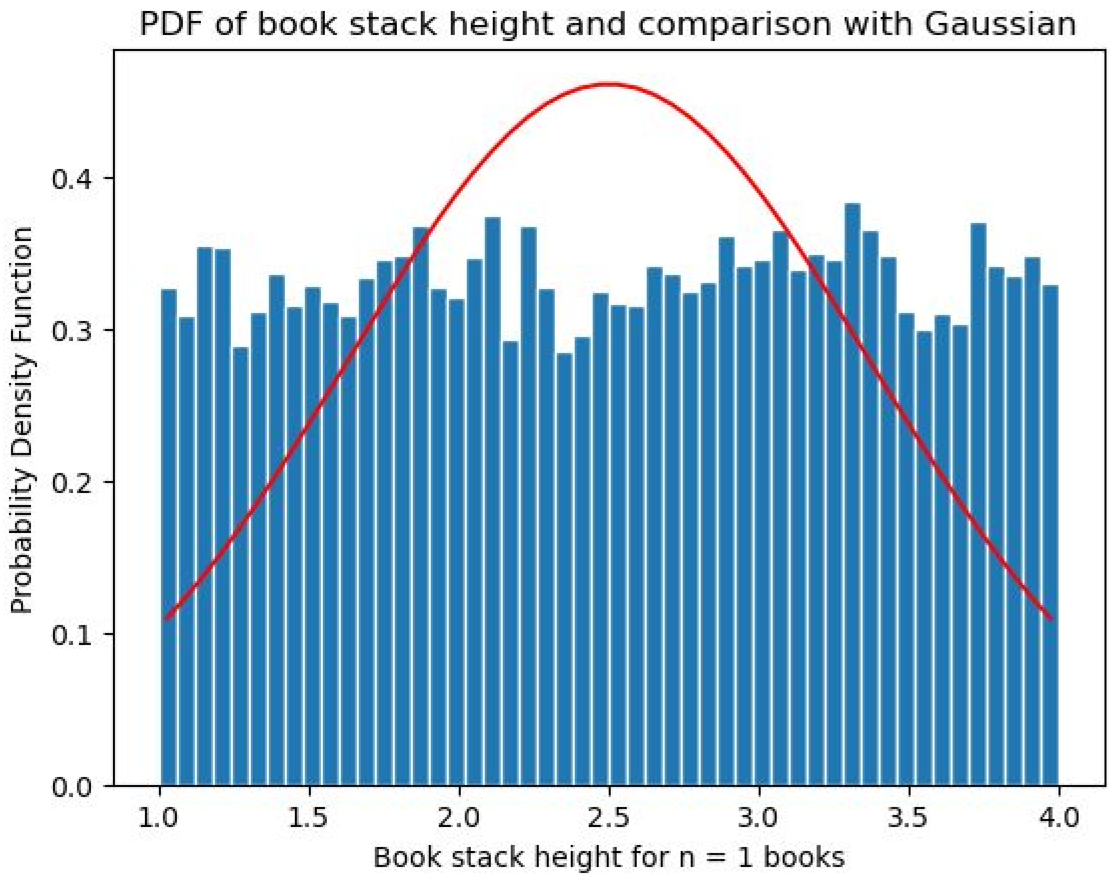
Consider a collection of books, each of which has thickness W. The thickness W is a RV, uniformly distributed between a minimum of a and a maximum of b cm. Use the values of a and b that were provide to you, and calculate the mean and standard deviation of the thickness. Use the following table to report the results. Points will be taken off if you do not use the table to report .

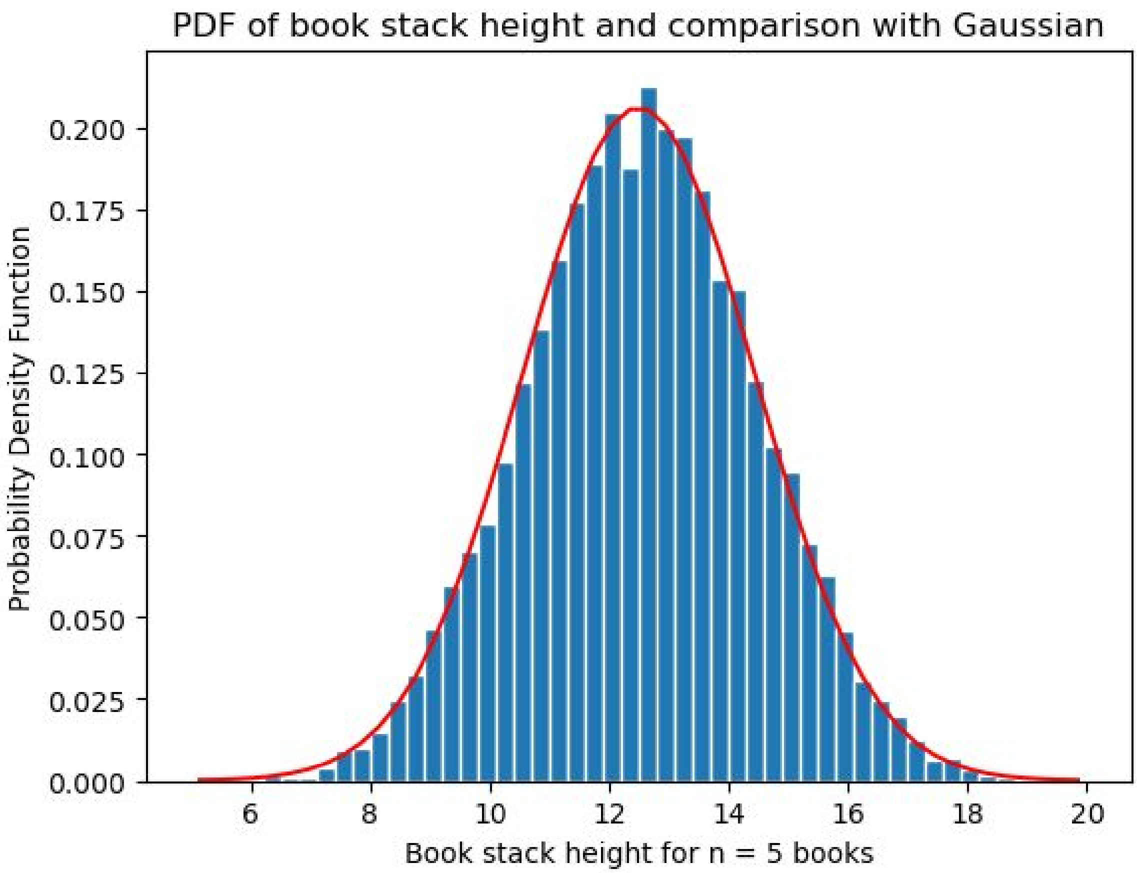
Mean thickness of a single book (cm)	Standard deviation of thickness (cm)
$\mu_w =$	$\sigma_w =$

The books are piled in stacks of $n = 1, 5, 10, \text{or } 15$ books. The width S_n of a stack of n books is a RV (the sum of the widths of the n books). This RV has a mean $\mu_{S_n} = n\mu_w$ and a standard deviation of $\sigma_{S_n} = \sigma_w\sqrt{n}$.

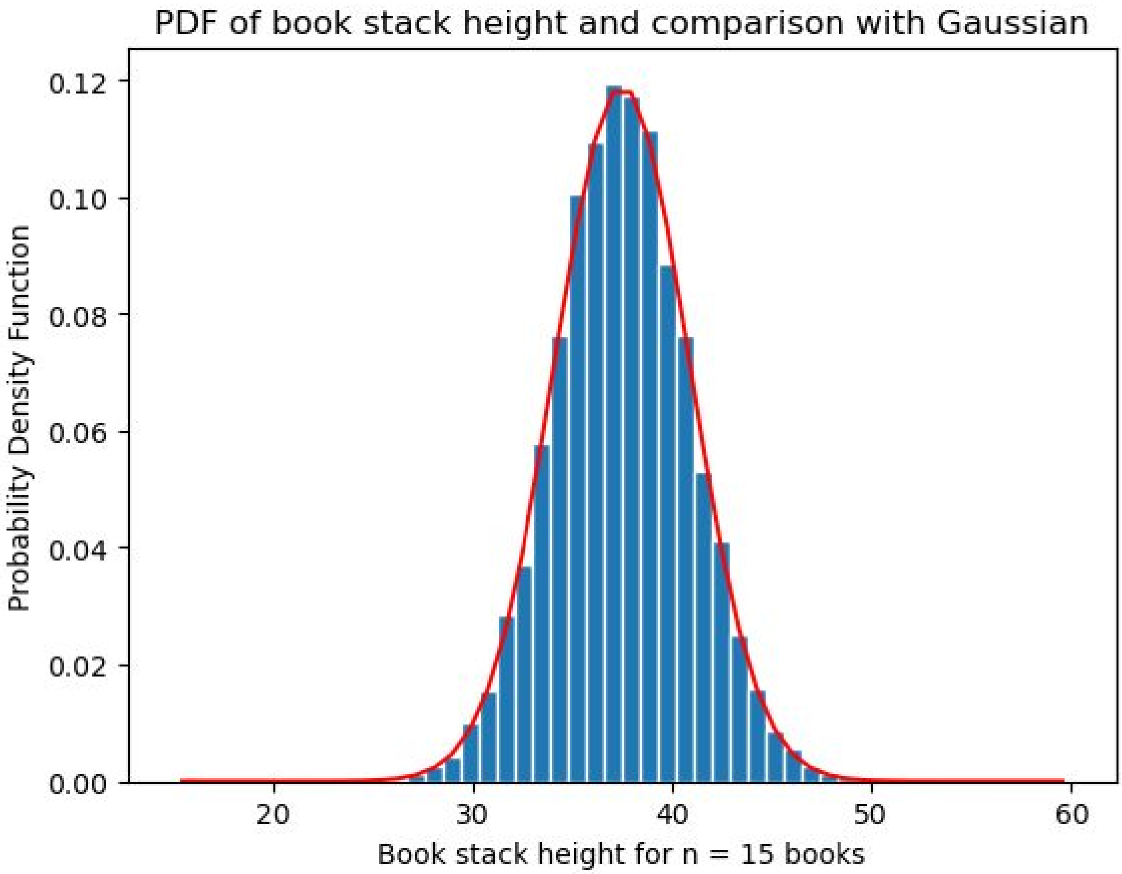
Calculate the mean and standard deviation of the stacked books, for the different values of $n = 1, 5, 10, \text{or } 15$. Use the following table to report the results. Points will be taken off if you do not use the table to report.

Number of books n	Mean thickness of a stack of n books (cm)	Standard deviation of the thickness for n books
n = 1	2.5	0.8660
n = 5	12.5	2.739
n = 15	37.5	3.354


PDF of book stack height and comparison with Gaussian
Probability Density Function
0.0
0.1
0.2
0.3
0.4
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Book stack height for n = 1 books



PDF of book stack height and comparison with Gaussian
Probability Density Function
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
6
8
10
12
14
16
18
20
Book stack height for n = 5 books



PDF of book stack height and comparison with Gaussian
Probability Density Function
0.00
0.02
0.04
0.06
0.08
0.10
0.12
20
30
40
50
60
Book stack height for n = 15 books

3. Distribution of the Sum of Exponential RVs

This problem involves a battery-operated critical medical monitor. The lifetime (T) of the battery is a random variable with an exponentially distributed lifetime. A battery lasts an average of β days, which has been provided to you. Under these conditions, the PDF of the battery lifetime is given by:

$$f_T(t\,;\beta) = \begin{cases} \dfrac{1}{\beta}\exp(-\dfrac{1}{\beta}t), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

As mentioned before, the mean and variance of the random variable T are:

$$\mu_T = \beta \quad ; \quad \sigma_T = \beta$$

When a battery fails it is replaced immediately by a new one. Batteries are purchased in a carton of 24. The objective is to simulate the RV representing the lifetime of a carton of 24 batteries, and create a histogram. To do this, follow the steps below.

a) Create a vector of 24 elements that represents a carton. Each one of the 24 elements in the vector is an exponentially distributed random variable (T) as shown above, with mean lifetime equal to β. Use the same procedure as in the previous problem to generate the exponentially distributed random variable T.

b) The sum of the elements of this vector is a random variable (C), representing the life of the carton, i.e.

$$C = T_1 + T_2 + \cdots T_{24}$$

where each T_j, $j = 1, 2, \cdots 24$ is an exponentially distributed R.V. Create the R.V. C, i.e. simulate one carton of batteries. This is considered one experiment.

c) Repeat this experiment for a total of N=10,000 times, i.e. for N cartons. Use the values from the N=10,000 experiments to create the experimental PDF of the lifetime of a carton, $f(c)$.

d) According to the Central Limit Theorem the PDF for one carton of 24 batteries can be approximated by a normal distribution with mean and standard deviation given by:

$$\mu_C = 24\mu_T = 24\beta \quad ; \quad \sigma_C = \sigma_T\sqrt{24} = \beta\sqrt{24}$$

Plot the graph of a normal distribution with

mean = μ_C and (standard deviation) = σ_C,

over plot of the experimental PDF on the same figure, and compare the results.

e) Create and plot the CDF of the lifetime of a carton, $F(c)$. To do this use the Python "`numpy.cumsum`" function on the values you calculated for the experimental PDF. Since the CDF is the integral of the PDF, you must multiply the PDF values by the `barwidth` to calculate the areas, i.e. the integral of the PDF.

If your code is correct the CDF should be a nondecreasing graph, starting at 0.0 and ending at 1.0.

QUESTION	**Ans.**
1. Probability that the carton will last longer than three years	1 - 0.777 = 0.223
2. Probability that the carton will last between 2.0 and 2.5 years	0.414 - 0.139 = 0.275