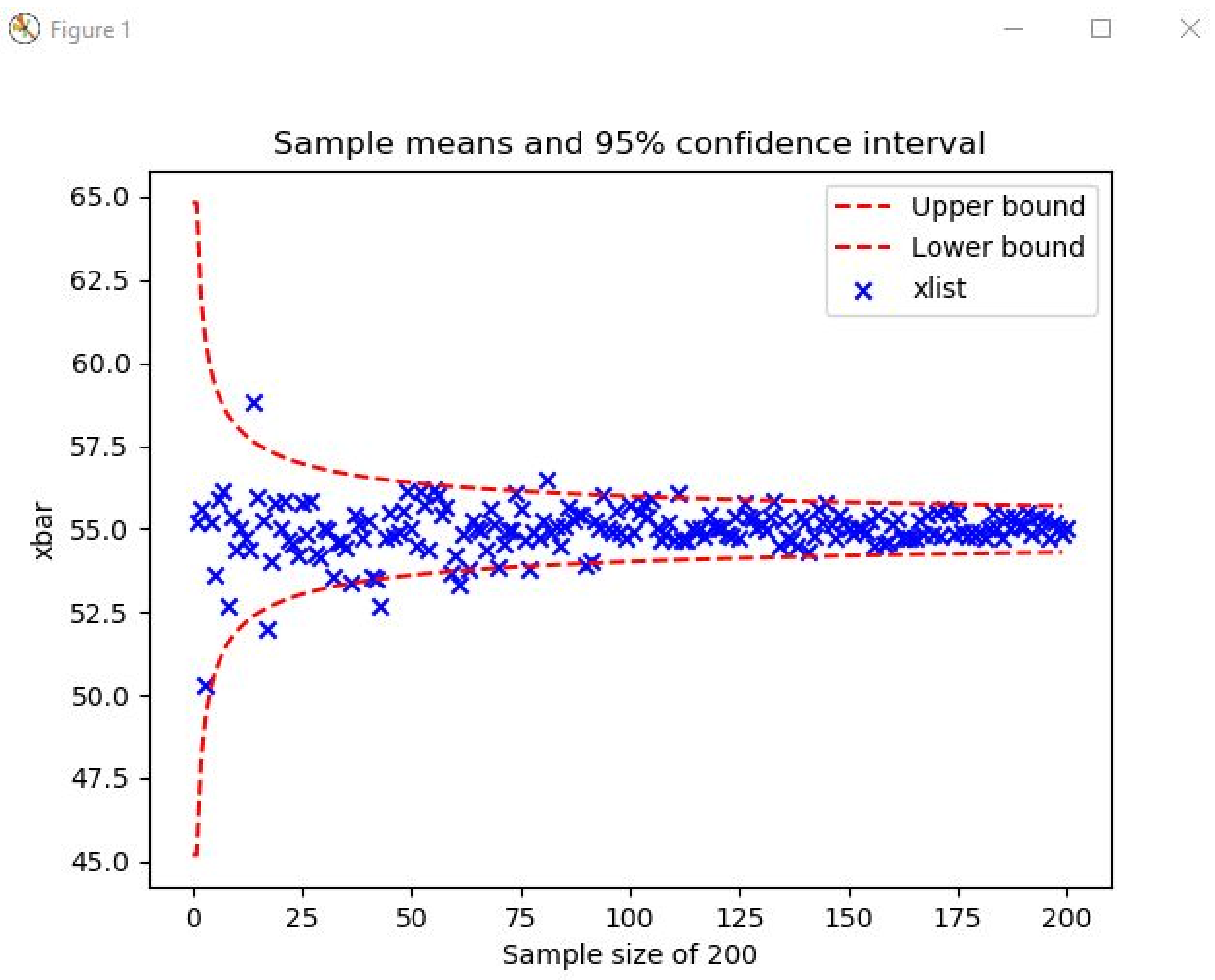
Sam Chen
EE 381
013502214
11/20/20
Lab 5

1. Effect of sample size on confidence intervals

Create two plots as in Figure 1a and Figure 1b, showing the effect on sample size on the confidence intervals.

The values of the following parameters have been provided to you:

- Total number of bearings: N ;
- Population mean: μ (grams) ;
- Population standard deviation: σ (grams) ;
- Sample sizes: $n = 1, 2, \cdots 200$



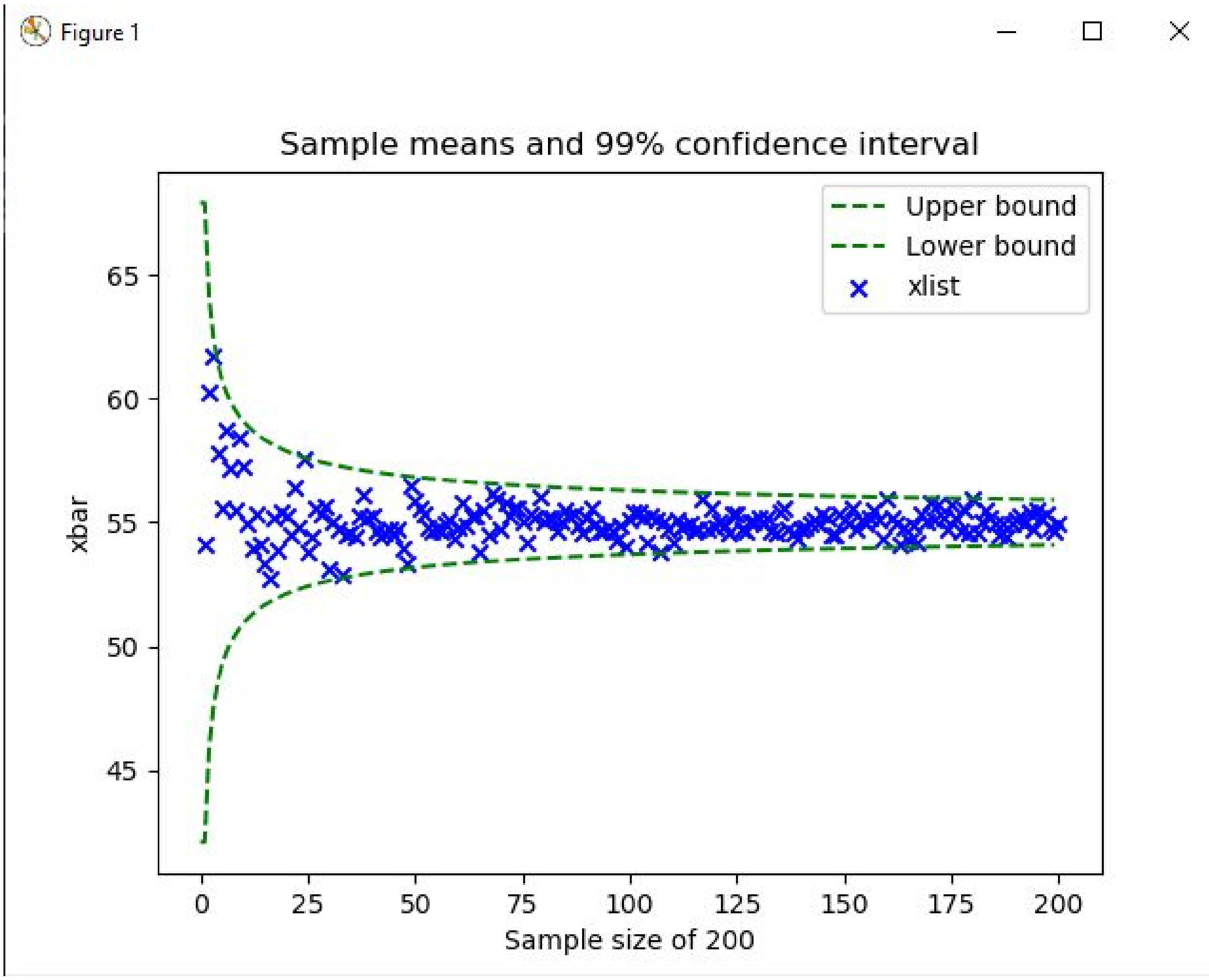


Figure 1
Sample means and 99% confidence interval
Upper bound
Lower bound
xlist
xbar
65
60
55
50
45
0
25
50
75
100
125
150
175
200
Sample size of 200

2. Using the sample mean to estimate the population mean

(A) Perform the following simulation experiment. Use Table 1 to tabulate the results.

1. Choose a random sample of $n=5$ bearings from the N bearings you created in the previous problem. Calculate the sample mean and the sample standard deviation:

$$\bar{X}=\frac{1}{n}\sum_{j=1}^{n}X_j \quad \text{and} \quad \hat{S}=\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(X_j-\bar{X})^2}$$

2. Create the 95% confidence interval using the normal distribution to fill in the first two entries in the top row. You realize, however, that this is not an appropriate distribution to use because you have a small sample $n=5<30$

$$[\mu_{\text{Lower}},\mu_{\text{Upper}}]=[\bar{X}-1.96\frac{\hat{S}}{\sqrt{n}},\bar{X}+1.96\frac{\hat{S}}{\sqrt{n}}]$$

3. Check if the confidence interval includes the actual mean μ of the population of N bearings. If it does, then Step 2 is considered a success.
4. The appropriate distribution for small samples ($n\le 30$) is the t-distribution. Create the 95% confidence interval using the t-distribution with $\nu=n-1=4$

$$[\mu_{\text{Lower}},\mu_{\text{Upper}}]=[\bar{X}-t_{0.975}\frac{\hat{S}}{\sqrt{n}},\bar{X}+t_{0.975}\frac{\hat{S}}{\sqrt{n}}]$$

At the 95% confidence level with $\nu=4$ degrees of freedom, the value of $t_{0.975}$ can be found from the tables, and it is seen to be: $t_{0.975}=2.78$. This is the value that will be used to determine the 95% confidence interval:

$[\mu_{\text{Lower}},\mu_{\text{Upper}}]=[\bar{X}-2.78\frac{\hat{S}}{\sqrt{n}},\bar{X}+2.78\frac{\hat{S}}{\sqrt{n}}]$. For a different sample size, the values of $t_{0.975}$ will be different than the ones above. You should find these values from the t-distribution tables, and you should modify the confidence intervals accordingly.

Sample Size(n)	**95% Confidence** (Using Normal Distribution)	**99% Confidence** (Using Normal Distribution)	**95% Confidence** (Using Student's t distribution)	**99% Confidence** (Using Student's t distribution)
5	88.14	95.02	94.93	98.92
40	94.26	99.1	95.12	99.01
120	94.51	99.34	95.19	98.98