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EE 381 HW 4

4.2.1) $F_X(x) = \begin{cases} 0 & x < -1 \\ (x+1)/2 & -1 \le x < 1 \\ 1 & x \ge 1 \end{cases}$

a) $P[X > 1/2] = 1 - P[X \le 1/2]$
$= 1 - F_X(1/2) = 1 - 3/4 = 1/4$

b) $P[-1/2 \le X\ 3/4] = P[-1/2 < X \le 3/4] + P[X = -1/2] - P[X = 3/4]$

$P[-1/2 \le X < 3/4] = P[-1/2 < X \le 3/4]$
$= F_X(3/4) - F_X(-1/2) = 5/8$

c) $P[|X| < 1/2] = P[X \le 1/2] - P[X < -1/2]$
$= 3/4 - 1/4 = 1/2$

D) Since $F_X(1) = 1$, we must have $a \le 1$
we get $P[X \le a] = F_X(a) = \frac{a+1}{2} = 0.8$, $a = 0.6$

4.4.1) a) $E[X] = 1$ and $Var[X] = \frac{(3+1)^2}{12} = 4/3$

b) New random variable Y is defined as $Y = h(X) = X^2$
$h(E[X]) = h(1) = 1$
and $E[h(X)] = E[X^2] = Var[X] + E[X]^2 = 4/3 + 1 = 7/3$

c) $Var[Y] = E[X^4] - E[X^2]^2$
$= \int_{-1}^{3} \frac{x^4}{4} dx - \frac{49}{9} = \frac{61}{5} - \frac{49}{9}$

4.6.3) a) $P[V > 4] = 1 - P[V \le 4] = 1 - P\left[\frac{V-0}{\sigma} \le \frac{4-0}{\sigma}\right]$

$= 1 - \phi(4/\sigma) = 1 - \phi(2) = 0.023$

b) $P[W \le 2] = P\left[\frac{W-2}{5} \le \frac{2-2}{5}\right] = \phi(0) = \frac{1}{2}$

c) $P[X \le \mu + 1] = P[X - \mu \le 1]$

$= P\left[\frac{X-\mu}{\sigma} < \frac{1}{\sigma}\right] = \phi(1/\sigma) =$

$\phi(0.5) =$

0.692

D) $P[Y > 65] = 1 - P[Y \le 65]$

$= 1 - P\left[\frac{Y-50}{10} \le \frac{65-50}{10}\right]$

$= 1 - \phi(1.5) = 1 - 0.933 = 0.067$

5.1.1) a) $P[X \le 2, Y \le 3] = F_{XY}(2,3) - F_{XY}(0,0)$

$= F_{XY}(2,3)$

$= (1-e^{-2})(1-e^{-3}) = \frac{(e^{2}-1)(e^{3}-1)}{e^{6}}$

$P[X \le 2, Y \le 3] = \frac{(e^{2}-1)(e^{3}-1)}{e^{5}} = 0.8216$

b) Marginal CDF of x is $F_X(x)$ is

$$F_{XY}(x,y)\Big|_{y\geq 0} = (1-e^{-x})(1-e^{-y})_{y\geq 0}$$
$$=(1-e^{-x})$$

c) Marginal CDF of y is $F_Y(y)$ is

$$F_{XY}(x,y)\Big|_{x\geq 0} = (1-e^{-x})(1-e^{-y})_{x\geq 0}$$
$$=(1-e^{-y})$$

5.6.2)	Factory Q	Factory R
small order	0.3	0.2
medium order	0.1	0.2
large order	0.1	0.1

(a)

PMF for B and M

$P_{B,M}(b,m)$	m=60	m=180
b=1	0.3	0.2
b=2	0.1	0.2
b=3	0.1	0.1

b) $P_{B,M}(b,m)$	m=60	m=180	$P_B(b)$
b=1	0.3	0.2	0.5
b=2	0.1	0.2	0.3
b=3	0.1	0.1	0.2
$P_M(m)$	0.5	0.5	

Expected boxes $= \sum_b b P_B(b) = 1(0.5)+2(0.3)+3(0.2) = 1.7$

c) B and M are not independent because
$P_{B,M}(1,60) \neq P_B(1)\, P_M(m) 60$

5.6.3) $X \sim \text{Binomial}(n=75,\ p=0.75)$

$P(X=x) = \binom{n}{x} p^x q^{n-x},\ x = 0,1,2 \ldots n$

$= \binom{75}{x}(0.5)^x(0.5)^{75-x},\ x = 0,1,2 \ldots 75$

$P(Y=y) = \binom{n}{y} p^y q^{n-y},\ y = 0,1,2 \ldots n$

$= \binom{25}{y}(0.5)^y(0.5)^{25-y},\ x = 0,1,2 \ldots 25$

$\therefore$ jpmf of (x,y) is

$P(X=x, Y=y) = P(X=x)P(Y=y)$

$= \binom{75}{x}(0.5)^x(0.5)^{75-x}\binom{25}{y}(0.5)^y(0.5)^{25-y}$

$= \binom{75}{x}\binom{25}{y}(0.5)^{100}$

5.7.3) X, Y with the sum $W = X+Y$

$P_{X,Y}$	$y=1$	$y=2$	$y=3$	$y=4$
$x=5$	0.05 $(W=6)$	0.1 $(W=7)$	0.2 $(W=8)$	0.05 $(W=9)$
$x=6$	0.1 $(W=7)$	0.1 $(W=8)$	0.3 $(W=9)$	0.1 $(W=10)$

$E[x+y] = \sum_{x,y} (x+y)\, P_{x,y}(x,y)$

$= 6(0.05) + 7(0.2) + 8(0.3) + 9(0.35) + 10(0.1) = 8.25$

$E[(x+y)^2] = \sum_{x,y} (x+y)^2 P_{x,y}(x,y)$

$6^2(0.05) + 7^2(0.2) + 8^2(0.3) + 9^2(0.35) + 10^2(0.1)$
$= 69.15$

$Var[x+y] = E[(x+y)^2] - (E[x+y])^2$
$= 69.15 - (8.25)^2 = 1.0875$

Expected:

5.7.6) $E[W] = E[2X+2Y] = 2E[X] + 2E[Y] = 2$

Variance:

$Var[W] = Var[2(X+Y)] = 4\, Var[X+Y]$
$= 4(Var[X] + Var[Y] + 2\, Cov[X,Y])$
$= 4(3^2 + 4^2 + 2(-3)) = 76$

4.66) $P(9) = 4/36$

$P(Y=1),\ n=3,\ p=0.111111$

$Y \sim Binomial(n=3, p=0.111111)$

$P(Y=1) = \binom{n}{y} p^y (1-p)^{n-y} = \binom{3}{1}(0.111111)^1(1-0.111111)^{3-1}$

$= 3 \cdot (0.111111)^1 (0.0263374) \approx 0.263$

# of ways to pick 4 red and 4 white / # of ways to pick any 8

4.98) a) P(4 is red) = 5C4 X 10C4 / 15C8

= 5 x 210 / 6435 = 0.1632

b) P(all is white) = # of ways to pick 8 white / # of ways to pick any 8

= 10C8 / 15C8

= 45 / 6435 = 0.0070

c) P(at least red) = 1 - P(all white)

1 - 0.0070 = 0.9930