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EE 381 HW 5

5.49) a) Population mean is $(3+7+11+15)/4 = 9$

b) Popular standard deviation is

$$\frac{(3-9)^2+(7-9)+(11-9)^2+(15-9)^2}{4} = 20$$

so standard deviation is 4.47

c) Total of $4 \times 4 = 16$ samples:

(3,3) (3,7) (3,11) (3,15)
(7,3) (7,7) (7,11) (7,15)
(11,3) (11,7) (11,11) (11,15)
(15,3) (15,7) (15,11) (15,15)

Mean of each sample is 3, 5, 7, 9, 5, 7, 9, 11, 7, 9, 11, 13, 9, 11, 13, 15

Therefore, the mean of sampling distribution is of means is sum(3, 5, 7, 9, 5, 7, 9, 11, 7, 9, 11, 13, 9, 11, 13, 15)/16 = 9

d) The variance of the sample mean is

$$\frac{(3-9)^2+(5-9)^2+(7-9)^2+(9-9)^2}{16} + \frac{(5-9)^2+(7-9)^2+(9-9)^2+(11-9)^2}{16} + \frac{(7-9)^2+(9-9)^2+(11-9)^2+(13-9)^2}{16} + \frac{(9-9)^2+(11-9)^2+(13-9)^2+(15-9)^2}{16}$$

$= 10$, the popular standard deviation is 3.16

The result of c and d comes from a and b

5.54) standard deviation of 16 samples mean is $\frac{60}{\sqrt{16}} = 15$

a) Normalized distance is $\frac{790-800}{15} = -0.67$, $\frac{810-800}{15} = 0.67$, so probability between 790 and 810 hrs is $0.2486 \times 2 = 0.4972$

b) Normalized distance is $\frac{785-800}{15} = -1$, so probability is less than 785 hrs is 0.1587

c) $\frac{820-800}{15} = 1.33$, probability is more than 820 hrs is 0.0918

d) $\frac{770-800}{15} = -2$, $\frac{830-800}{15} = 2$, probability between 770 and 830 hrs is 0.9544

5.62) standard deviation of 100 samples mean is $\sqrt{npq} = \sqrt{100 \times 0.95 \times 0.05} = 2.18$

a) Normalized distance is $\frac{90.5 - 100 \times 0.95}{2.18} = -2.064$ so, probability of fewer than 90 good bulbs is 0.019. With 1000 slots, expected number is 19

b) Normalized distance is $\frac{97.5 - 100 \times 0.95}{2.18} = 1.15$

probability of more than 98 good bulbs is 0.125

with 1000 slots, expected number is 125

5.66) Vrn has 60 red marbles + 40 white marbles

Variance of 30 samples mean of red marble is $npq = 30 \times 0.6 \times 0.4 = 7.2$, so variance of difference of two sample mean is $7.2 + 7.2 = 14.4$, so standard deviation is 3.8.

Normalized distance is $\frac{8 - 0.5}{3.8} = 1.98$

Probability is 0.0482

5.71) Variance of each battery is 0.04.

4 samples summed together is $4 \times 0.04 = 0.16$

so standard deviation is 0.4. Normalized distance is $\frac{60.8 - 15 \times 4}{0.4} = 2$. Probability combined voltage is 60.8 or more is 0.0228

5.74) $n = 5$, $\sigma^2 = 15$

a) Variance threshold of 10, we have $\frac{ns^2}{\sigma^2} = \frac{5 \times 10}{15} = 3.33$

By theorem 5.6, $\chi^2 = \frac{ns^2}{\sigma^2}$ is chi square distribution with $n-1 = t$ degree of freedom we get probability is 0.496

b) with variance threshold of 20 we get $\frac{ns^2}{\sigma^2} = \frac{5 \times 20}{15} = 6.67$. Probability of variance larger than 20 is 0.155

c) For threshold of, $\frac{ns^2}{\sigma^2} = \frac{5 \times 5}{15} = 1.67$

Prob of variance smaller than 5 is 0.203.
Prob of variance between 5 and 10 is
$0.496 - 0.203 = 0.293$

6.32) $s^2 = \frac{n}{n-1} s^2 = \frac{60}{60-1} \times 0.533 = 0.542$

Confidence limits of 95% is
$11.09 \pm 1.96 \cdot \left(\frac{\sqrt{0.542}}{\sqrt{60}}\right) = 11.09 \pm 0.186$

Confidence limits of 99% is
$11.09 \pm 2.58 \cdot \left(\frac{\sqrt{0.542}}{\sqrt{60}}\right) = 11.09 \pm 0.245$