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### EE 381 HW 3

3.2.1)  $P_N(n) = \begin{cases} c(1/2)^n & n=0,1,2 \\ 0 & \text{otherwise} \end{cases}$

a)

$\therefore \sum_{n=0}^{\infty} P_N(n) = c + c/2 + c/4 = 1$ , so  $c = 4/7$

b)  $N \leq 1$  is:  $P[N \leq 1] = P[N=0] + P[N=1] = 4/7 + 2/7 = 6/7$

3.2.2) Random Variable  $V$  is PMF

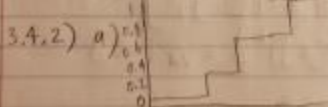
$P_V(v) = \begin{cases} cv^2 & v=1,2,3,4 \\ 0 & \text{otherwise} \end{cases}$

a)  $\sum_{v=1}^4 P_V(v) = c(1^2 + 2^2 + 3^2 + 4^2) = 30c = 1$   
Hence  $c = 1/30$

b)  $U = \{u^2 | u = 1, 2, \dots, 3\}$   
 $P[V \in U] = P_V(1) + P_V(4) = \frac{1}{30} + \frac{16}{30} = \frac{17}{30}$

c)  $V$  is even = Probability  $P_V(2) + P_V(4) = \frac{4}{30} + \frac{16}{30} = \frac{2}{3}$

d)  $V > 2$  Prob of  
 $P_V(3) + P_V(4) = \frac{9}{30} + \frac{16}{30} = \frac{5}{6}$



b)  $P_X(x) = \begin{cases} 0.2, & x = -1 \\ 0.5, & x = 0 \\ 0.3, & x = 1 \\ 0, & \text{otherwise} \end{cases}$

3.5.2) a) PMF of  $X$  is  $P_X(X) = \begin{cases} 0.6 & X=20 \\ 0.4 & X=30 \end{cases}$

b)  $E[C] = 20(0.6) + 30(0.4) = 24 \text{ cents}$

3.7.5)  $E[D] = \sum_{d=1}^4 d \cdot P_D(d) = 1(0.2) + 2(0.4) + 3(0.3) + 4(0.1) = 2.3$

b) Deviation from waiting time is

$$E[D - \mu_D] = E[D] - E[\mu_D] = \mu_D - \mu_D = 0$$

c)  $C$  expressed as function of  $D$

$$C(D) = \begin{cases} 90 & D=1 \\ 70 & D=2 \\ 40 & D=3 \\ 40 & D=4 \end{cases}$$

3.8.7) By theorem 3.12;  $E[aX+b] = aE[X] + b$   
we get

$$\begin{aligned} \text{Var}[Y] &= E[(aX+b - (aE[X]+b))^2] \\ &= E[a^2(X-E[X])^2] \end{aligned}$$

$$\text{Since } E[(X-E[X])^2] = \text{Var}[X]$$

$$3.62) \quad E[(X-1)^2] = 10$$

$$E[X^2 - 2X + 1] = 10$$

$$E[X^2] - 2E[X] + 1 = 10 \quad (1)$$

$$E[X^2] - 2E[X] = 10 - 1$$

$$E[X^2] - 2E[X] = 9$$

$$E[(X-2)^2] = 6$$

$$E[X^2 - 4X + 4] = 6$$

$$E[X^2] - 4E[X] + 4 = 6 \quad (2)$$

$$E[X^2] - 4E[X] = 2$$

Solve for land 2:

$$a) E[X] = 7/2, E[X^2] = 16$$

$$b) \text{Var}(X) = E[X^2] - (E[X])^2$$

$$= 16 - (7/2)^2$$

$$= 16 - 49/4 \Rightarrow 15/4 = 3.75$$

$$c) \sigma_X = \sqrt{\sigma^2 X} = \sqrt{3.75} = 1.94$$

$$3.79) \text{Covariance}(X, Y) = E(XY) - E(X)E(Y)$$

$$= 10 - 2 \cdot 3$$

$$= 10 - 6 = 4$$

$$\frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)} \sqrt{\text{Var}(Y)}}$$

$$= \frac{4}{\sqrt{5} \sqrt{7}} = \frac{4}{\sqrt{35}} = 0.6761$$

$$\text{Var}(X) = E(X^2) - E(X)^2 = 9 - 4 = 5$$

$$\text{Var}(Y) = 16 - 9 = 7$$