Sam Chen

EE381 HW 6

11.1.1) If we assume coin is fair, we can choose a region R that $\alpha = P[R] = 0.05$. Then pick another rejection region $R = \{L > r\}$. The remaining r will be $P[R] = 0.05$. So, $P[L > l] = (1/2)^l$.
Therefore, $P[R] = P[L > r] = 2^{-r} = 0.05$
We get $r = -\log_2(0.05) = \log_2(20) = 4.32$.
Because of this we cancel the hypothesis of $L \geq 5$
This test's significance is $\alpha = P[L > 4] = 2^{-4} = 0.0625$
which is close but not 0.05.
Therefore our test cannot fairly judge a biased coin flip. We will always have to accept the hypothesis the coin is fair.

11.1.3) we disagree with the null hypothesis when rate M is too high.
For a larger T we use central limit theorem.

$$P[R] = P[M \geq m_0] = P\left[\frac{M - 2.5}{\sigma M} \geq \frac{m_0 - 2.5}{\sigma M}\right]$$

$$= 1 - \Phi\left(\frac{m_0 - 2.5}{\sqrt{\frac{2.5}{T}}}\right) = 0.05$$

$$\frac{m_0 - 2.5}{\sqrt{2.5/T}} = 1.65 \rightarrow m_0 = 2.5 + \frac{1.65\sqrt{2.5}}{\sqrt{T}} = 2.5 + \frac{2.6}{\sqrt{T}}$$

We reject the hypothesis when

$$M \geq 2.5 + \frac{2.6}{\sqrt{T}},$$

As T gets larger $E[M] = 2.5$ are sufficient to reject it

11.1.5) To start we test n peacemakers then reject null hypothesis if any of them fails the test. Failures will be X, $(n, q_0 = 10^{-4})$ as binomial

$\alpha = P[X>0] = 1 - P[X=0] = 1-(1-q_0)^n$

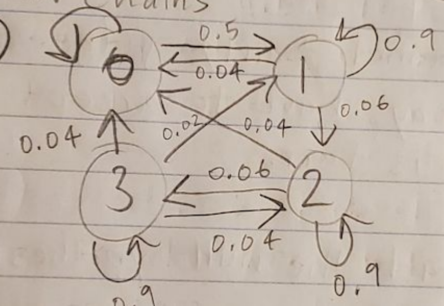
For the significance level $\alpha = 0.01$,

$$n = \frac{\ln(1-\alpha)}{\ln(1-q_0)} = 100.5$$

So, one percent of failure in a normal factory

Markov Chains

1.2)

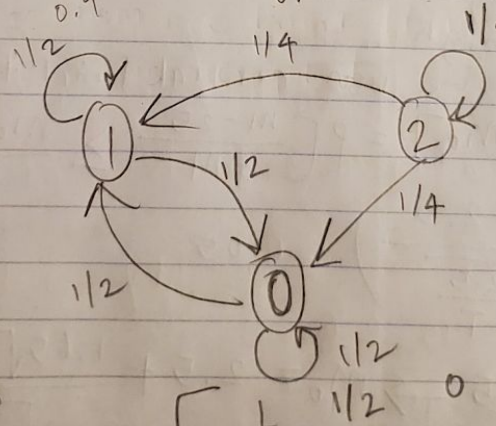


$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.04 & 0.9 & 0 \\ 0.04 & 0 & 0.6 \\ 0.04 & 0.02 & 0.9 \end{bmatrix}$$

Next state = trans matrix · curr state

$X_1 = P \cdot X$

1.3)



Transition Matrix $= P = \begin{bmatrix} \frac{1}{2} & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$

7.63) a) This is a type I error. standard deviation is of 64 samples $\sqrt{npq} = \sqrt{64 \times 0.5 \times 0.5} = 4$. Probability distribution is 36.5. Both marbles symmetric around mean of 32. $z = \frac{x - \mu}{\sigma} = \frac{36.5 - 32}{4} = 1.125$

b) Cumulative probability of 1.125 is normalized to 0.369.

Tail probability is $0.5 - 0.369 = 0.131$

Probability of rejecting the hypothesis is $0.131 \cdot 2 = 0.262$

7.64) a) Type I error. The 0.01 level of significance can be translated to 0.005 tail probability in a normalized Gaussian distribution.

Normalized distance is 2.58. So we know mean of 64 samples is 32, standard deviation is 4.

Threshold is $32 \pm 2.58 \times 4 \approx 22$ and 42

Between 22 and 42 is the only accepted hypothesis

b) since its 0.01 confidence level, then its 99% confidence

c) If level of significance is 0.05, normalized distance is 1.96. Threshold is $32 \pm 1.96 \times 4 \approx$ 40 and 24. we can only accept hypothesis if its between 40 and 24

7.66) a) Probability is $6/36 = 1/6$ of two dices
Standard deviation is $\sqrt{pq} = \sqrt{\frac{1}{6} \cdot \frac{5}{6}} = 0.3727$

In the two tailed test, the 0.05 level of sig. is 1.96 normalized, 100 tosses is $100 \cdot 1/6 = 16.67$
The thresholds we get are $16.67 \pm 1.96 \times \sqrt{100 \times 0.3727}$
We get 23.97 and 9.37.
We only accept hypothesis if 7 appears more than 24 times or 9 times

b) In the one tailed test, Threshold becomes
$16.67 \pm 1.645 \times \sqrt{100 \times 0.3727} \approx 22.8$ and 10.54.
So this hypothesis is rejected.

7.67) If its 0.01 level of significance, the two points for two/one tails are 2.58 and 2.33. Both of them are larger than 1.96 so this hypothesis is accepted.