A Nonstandard Approach to Nonstandard Analysis

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Chapter 1

Introduction

Nonstandard analysis is a mathematical framework that was introduced by Abraham Robinson in the 1960s as a way of rigorously dealing with infinitesimals and infinite numbers. The basic idea behind nonstandard analysis is to extend the usual real number system to include infinitesimal and infinite elements, which allows for a more intuitive and flexible approach to calculus and analysis.

However, traditional approaches to nonstandard analysis, such as Robinson's original framework or Nelson's internal set theory, can be quite technical and difficult to work with. In this book, we will take a nonstandard approach to nonstandard analysis that is based on category theory and universal algebra. This approach provides a more conceptual and unified perspective on nonstandard analysis, and allows for the development of new and interesting applications of the theory.

Throughout the book, we will assume a basic knowledge of calculus and analysis, as well as some familiarity with category theory and universal algebra. However, we will provide a brief review of these topics in the first chapter, and we will strive to make the book accessible to a wide audience.

In the following chapters, we will explore various aspects of nonstandard analysis from a nonstandard perspective. We will discuss the construction of nonstandard models of the real numbers, the development of nonstandard calculus and analysis, and the application of nonstandard analysis to problems in physics, geometry, and other areas of mathematics.

We hope that this book will serve as an introduction to a new and exciting approach to nonstandard analysis, and that it will inspire readers to explore this fascinating field further.