ME8135 — Assignment 1.1 Solution

Student: Arash Basirat Tabrizi
Submitted to: Dr. Sajad Saeedi
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1. Part (f): Refer to GitHub repo (A1/Part_F.ipynb directory) for the script that produces the results documented in this report.

In part (d) we analytically computed the covariance of $\mathbf{y}$, Σ_y, to be:

$$\Sigma_y = \mathrm{J}\Sigma\mathrm{J}^T = \begin{bmatrix} cos\theta & -\rho sin\theta \\ sin\theta & \rho cos\theta \end{bmatrix} \begin{bmatrix} \sigma_{\rho\rho}^2 & \sigma_{\rho\theta}^2 \\ \sigma_{\rho\theta}^2 & \sigma_{\theta\theta}^2 \end{bmatrix} \begin{bmatrix} cos\theta & sin\theta \\ -\rho sin\theta & \rho cos\theta \end{bmatrix} \tag{1}$$

Now given 4 different scenarios, with varying Σ, we wish to simulate our model using the Monte Carlo simulation and describe our observations.

Our scenarios are defined by the following equations:

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.005 \end{bmatrix} \tag{2}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{3}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{4}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

In the above scenarios, we are only varying $\sigma_{\theta\theta}^2$ element of Σ. Initially, we use $\sigma_{\theta\theta}^2 = 0.005$ to produce the results shown below:

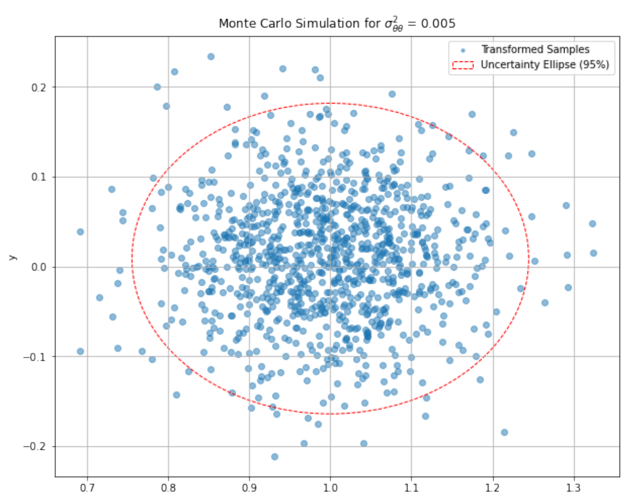

Figure 1: Monte Carlo simulation results for 2.

The simulation results in Figure 1 can be interpreted as follows:

- **Spread of samples in the y direction**: The transformed samples in the y direction, ($\rho \sin\theta$), have a relatively small spread. Since $\sigma^2_{\theta\theta}$ represents the variance of θ, a small value indicates less variability in the angular direction. Consequently, the samples in the y direction cover a narrow range ($\approx$ between -0.2 and 0.2).
- **Shape of the uncertainty ellipse**: The uncertainty ellipse, representing the 95% confidence region, has a smaller vertical dimension compared to the horizontal dimension. This indicates a lower uncertainty in the y direction.
- **Spread of samples in the x direction**: The transformed samples in the x direction, ($\rho \cos\theta$), are influenced by $\sigma^2_{\rho\rho}$ and independent of the value of $\sigma^2_{\theta\theta}$. The samples in the x direction cover a wider range compared to the ones in the y direction($\approx$ between 0.7 and 1.4).

In the second scenario we set $\sigma^2_{\theta\theta} = 0.1$ to produce the results shown below:

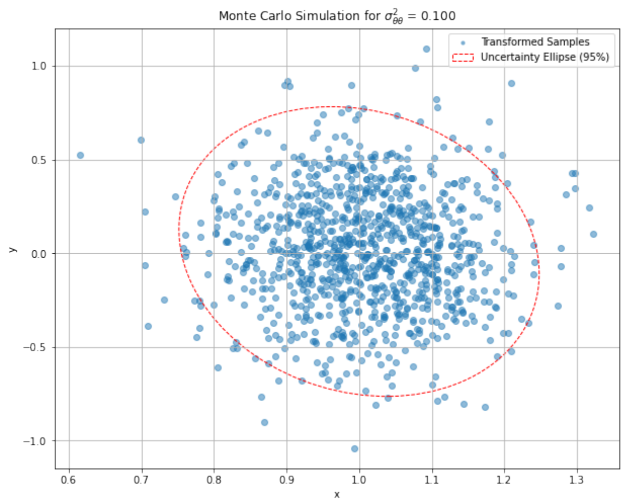

Figure 2: Monte Carlo simulation results for 3.

The simulation results in Figure 2 can be interpreted as follows:

- **Spread of samples in the y direction**: Increasing $\sigma^2_{\theta\theta}$ to 0.1 introduces more variance in the θ direction. As a result, the transformed samples in the y direction, which depends on $\rho \sin\theta$, appear to have a larger spread compared to before. Consequently, the samples in the y direction cover a wider range than in the previous scenario ($\approx$ between -1.0 and 1.0).
- **Shape of the uncertainty ellipse**: The uncertainty ellipse is observed to be rotated clockwise. The general shape of the ellipse is also growing in the vertical direction to account for the spread of samples in the y direction.
- **Spread of samples in the x direction**: The spread of the samples in the the x direction, which depends on $\rho \cos\theta$, appear unaffected. The spread in the x direction appears to remain the same as in the previous scenario.

In the third scenario we set $\sigma^2_{\theta\theta} = 0.5$ to produce the results shown below:

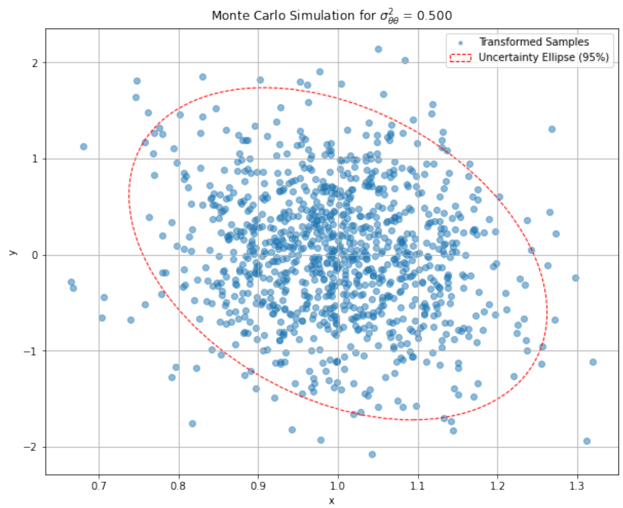

Figure 3: Monte Carlo simulation results for 4.

The simulation results in Figure 3 can be interpreted as follows:

- **Spread of samples in the y direction**: Increasing $\sigma^2_{\theta\theta}$ to 0.5 introduces even more variance in the θ direction. As a result, the transformed samples in the y direction demonstrate another spread. Consequently, the samples in the y direction now cover a wider range than in the previous scenario ($\approx$ between -2.0 and 2.0).
- **Shape of the uncertainty ellipse**: The uncertainty ellipse appears to be rotated clockwise again. The general shape of the ellipse is also growing in the vertical direction to account for the spread of samples in the y direction.
- **Spread of samples in the x direction**: The spread of the samples in the the x direction appears to be unaffected. The samples in the x direction still cover the same range when compared to the previous two scenarios ($\approx$ between 0.7 and 1.4).

In the last scenario we set $\sigma^2_{\theta\theta} = 1$ to produce the results shown below:

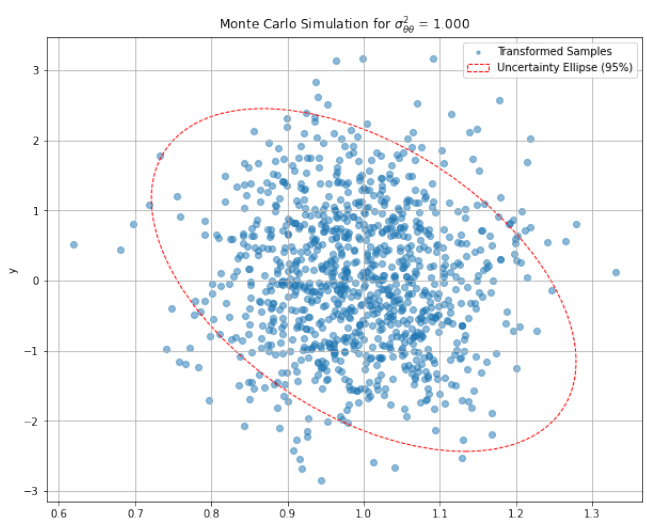

Figure 4: Monte Carlo simulation results for 5.

The simulation results in Figure 4 can be interpreted as follows:

- **Spread of samples in the y direction**: Increasing $\sigma^2_{\theta\theta}$ to 1.0 introduces more variance, as expected, in the θ direction. As a result, the transformed samples in the y direction demonstrate another spread. Consequently, the samples in the y direction now cover a wider range than in the previous three scenario ($\approx$ between -3.0 and 3.0).
- **Shape of the uncertainty ellipse**: The uncertainty ellipse appears to be rotated clockwise again. The general shape of the ellipse is also growing in the vertical direction to account for the spread of samples in the y direction.
- **Spread of samples in the x direction**: The spread of the samples in the the x direction appears to be unaffected.