# ME8135 — Assignment 1.1 Solution

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**1. Part (f)**: Please visit this link for the script that produces the results documented in this report.

In part (d) we analytically computed the covariance of $\mathbf{y}$, $\Sigma_y$, to be:

$$\Sigma_y = \mathrm{J}\Sigma\mathrm{J}^T = \begin{bmatrix} cos\theta & -\rho sin\theta \\ sin\theta & \rho cos\theta \end{bmatrix} \begin{bmatrix} \sigma^2_{\rho\rho} & \sigma^2_{\rho\theta} \\ \sigma^2_{\rho\theta} & \sigma^2_{\theta\theta} \end{bmatrix} \begin{bmatrix} cos\theta & sin\theta \\ -\rho sin\theta & \rho cos\theta \end{bmatrix} \tag{1}$$

Now given 4 different scenarios, with varying $\Sigma$, we wish to simulate our model using the Monte Carlo simulation and describe our observations.

Our scenarios are defined by the following equations:

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.005 \end{bmatrix} = \begin{bmatrix} \sigma^2_{\rho\rho} & \sigma^2_{\rho\theta} \\ \sigma^2_{\rho\theta} & \sigma^2_{\theta\theta} \end{bmatrix} \tag{2}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{3}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{4}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5° \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

In the above scenarios, we are only varying $\sigma^2_{\theta\theta}$ element of $\Sigma$. Initially, we use $\sigma^2_{\theta\theta} = 0.005$ to produce the results shown below:

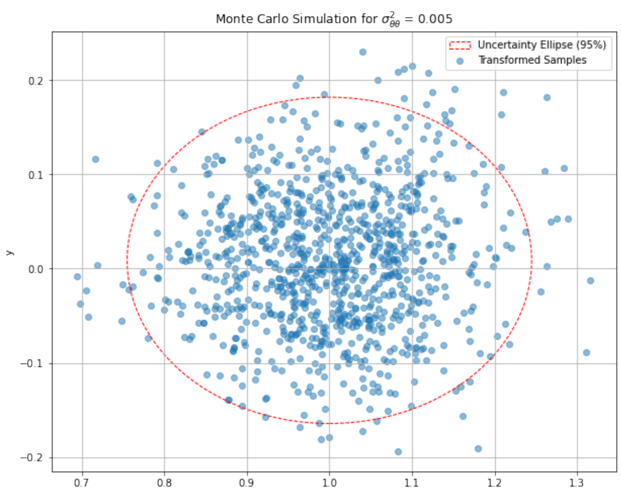


Figure 1: Monte Carlo simulation results for 2.

The simulation results in Figure 1 can be interpreted as follows:

- **Spread of samples in the $y$ direction**: The transformed samples in the $y$ direction, ($\rho \sin\theta$), have a relatively small spread. Since $\sigma^2_{\theta\theta}$ represents the variance of $\theta$, a small value indicates less variability in the angular direction. Consequently, the samples in the $y$ direction cover a narrow range ($\approx$ between -0.2 and 0.2).
- **Shape of the uncertainty ellipse**: The uncertainty ellipse, representing the 95% confidence region, has a smaller vertical dimension compared to the horizontal dimension. This indicates a lower uncertainty in the $y$ direction.
- **Spread of samples in the $x$ direction**: The transformed samples in the $x$ direction, ($\rho \cos\theta$), are influenced by $\sigma^2_{\rho\rho}$ and independent of the value of $\sigma^2_{\theta\theta}$. The samples in the $x$ direction cover a wider range compared to the ones in the $y$ direction($\approx$ between 0.7 and 1.3).

In the second scenario we set $\sigma^2_{\theta\theta} = 0.1$ to produce the results shown below:

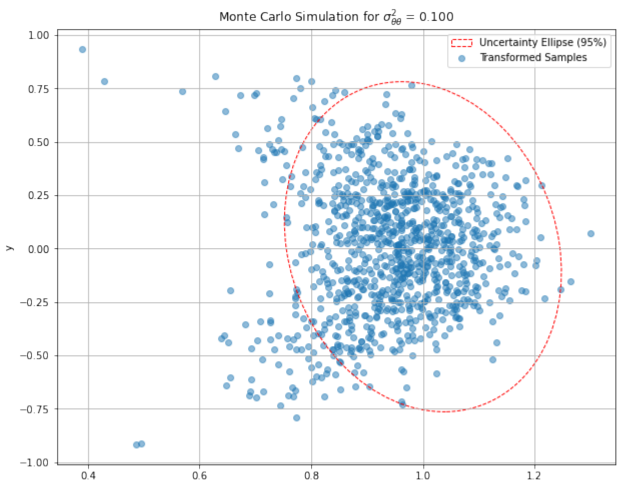


Figure 2: Monte Carlo simulation results for 3.

The simulation results in Figure 2 can be interpreted as follows:

- **Spread of samples in the $y$ direction**: Increasing $\sigma^2_{\theta\theta}$ to 0.1 introduces more variance in the $\theta$ direction. As a result, the transformed samples in the $y$ direction, which depends on $\rho \sin\theta$, appear to have a larger spread compared to before. Consequently, the samples in the $y$ direction cover a wider range than in the previous scenario ($\approx$ between -1.0 and 1.0).
- **Shape of the uncertainty ellipse**: The uncertainty ellipse is observed to be rotated counter-clockwise. The general shape of the ellipse is also growing in the vertical direction to account for the spread of samples in the $y$ direction.
- **Spread of samples in the $x$ direction**: The spread of the samples in the the $x$ direction, which depends on $\rho \cos\theta$, has reduced. The spread in the $x$ direction covers a narrower range compared to previous scenario ($\approx$ between -0.4 and 1.2).

In the third scenario we set $\sigma^2_{\theta\theta} = 0.5$ to produce the results shown in the next page:

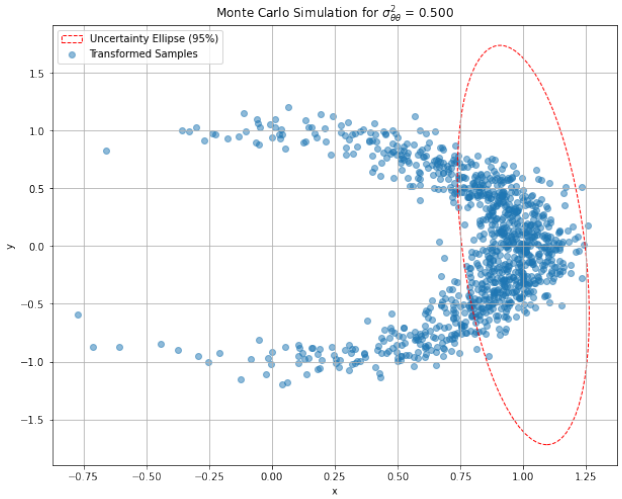


Figure 3: Monte Carlo simulation results for 4.

The simulation results in Figure 3 can be interpreted as follows:

- **Spread of samples in the $y$ direction**: Increasing $\sigma^2_{\theta\theta}$ to 0.5 introduces even more variance in the $\theta$ direction. As a result, the transformed samples in the $y$ direction demonstrate another spread. Consequently, the samples in the $y$ direction now cover a wider range than in the previous scenario ($\approx$ between -1.1 and 1.1).
- **Shape of the uncertainty ellipse**: The uncertainty ellipse appears to be rotated and narrowed. The general shape of the ellipse is also changing to account for the spread of samples in the $y$ direction which makes it to be appeared more stretched.
- **Spread of samples in the $x$ direction**: The spread of the samples in the the $x$ direction appear to have grown to mimic the spread of the first scenario ($\approx$ between -0.75 and 1.25).

In the last scenario we set $\sigma^2_{\theta\theta} = 1$ to produce the results shown below:

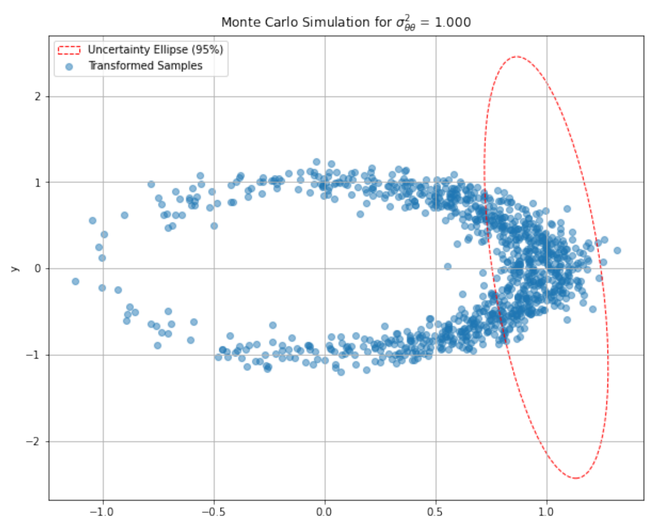


Figure 4: Monte Carlo simulation results for 5.

The simulation results in Figure 4 can be interpreted as follows:

- **Spread of samples in the $y$ direction**: Increasing $\sigma^2_{\theta\theta}$ to 1.0 shows that the transformed samples in the $y$ direction have more or less stopped spreading. Consequently, the samples in the $y$ direction still cover the same range than in the previous scenario ($\approx$ between -1.2 and 1.2).

- **Shape of the uncertainty ellipse**: The uncertainty ellipse appears to be rotated counter-clockwise and stretched when compared to the previous scenario.

- **Spread of samples in the $x$ direction**: The spread of the samples in the the $x$ direction appear to have grown, now covering an even larger range than the previous scenario ($\approx$ between -1.2 and 1.25).

In conclusion, smaller values of $\sigma^2_{\theta\theta}$ correspond to smaller angular uncertainties. Increasing $\sigma^2_{\theta\theta}$ has the reverse effect. The orientation of the uncertainty ellipse is dictated by the `angle` parameter in the script, and this angle was determined from the eigenvectors of the covariance matrix of $\mathbf{y}$, and the covariance matrix of $\mathbf{y}$ depends on the covariance matrix of $\mathbf{x}$ and the Jacobian matrix $\mathbf{J}$. Since $\sigma^2_{\theta\theta}$ is stored in the covariance matrix of $\mathbf{x}$, changing this value affects the orientation of the uncertainty ellipse. This concept is observable from the generated simulation plots.