ME8135 — Assignment 1.1 Solution

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1. Part (f): Refer to GitHub repo (A1/Part_F.ipynb directory) for the script that produces the results documented in this report.

In part (d) we analytically computed the covariance of $\mathbf{y}$, Σ_y, to be:

$$\Sigma_y = \mathrm{J}\Sigma\mathrm{J}^T = \begin{bmatrix} cos\theta & -\rho sin\theta \\ sin\theta & \rho cos\theta \end{bmatrix} \begin{bmatrix} \sigma^2_{\rho\rho} & \sigma^2_{\rho\theta} \\ \sigma^2_{\rho\theta} & \sigma^2_{\theta\theta} \end{bmatrix} \begin{bmatrix} cos\theta & sin\theta \\ -\rho sin\theta & \rho cos\theta \end{bmatrix} \tag{1}$$

Now given 4 different scenarios, with varying Σ, we wish to simulate our model using the Monte Carlo simulation and describe our observations.

Our scenarios are defined by the following equations:

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5^\circ \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.005 \end{bmatrix} \tag{2}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5^\circ \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{3}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5^\circ \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{4}$$

$$\mathrm{x} = \begin{bmatrix} 1\mathrm{m} \\ 0.5^\circ \end{bmatrix}, \Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

In the above scenarios, we are only varying $\sigma^2_{\theta\theta}$ element of Σ. Initially, we use $\sigma^2_{\theta\theta} = 0.005$ to produce the results shown below:

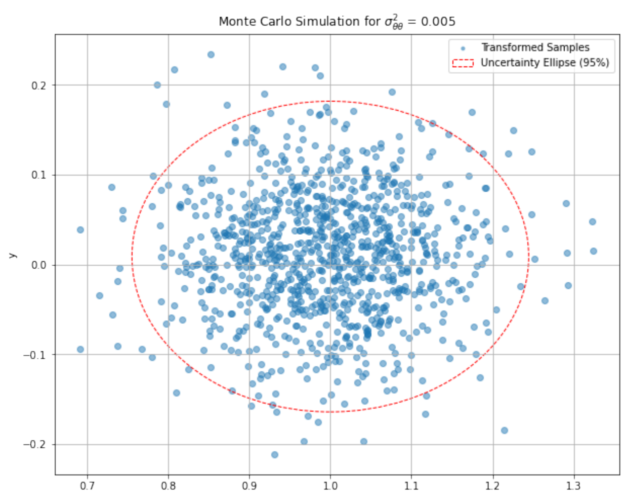

Figure 1: Monte Carlo simulation results for 2.

The simulation results in Figure 1 can be interpreted as follows:

- **Spread of samples in the y direction**: The transformed samples in the y direction, ($\rho\sin\theta$), have a relatively small spread. Since $\sigma^2_{\theta\theta}$ represents the variance of θ, a small value indicates less variability in the angular direction. Consequently, the samples in the y direction cover a narrow range ($\approx$ between -0.2 and 0.2).

- **Shape of the uncertainty ellipse**: The uncertainty ellipse, representing the 95% confidence region, will have a relatively small vertical dimension compared to the horizontal dimension. This indicates a lower uncertainty in the y direction.

- **Spread of samples in the x direction**: The transformed samples in the x direction, ($\rho\cos\theta$), are influenced by $\sigma^2_{\rho\rho}$ and independent of the value of $\sigma^2_{\theta\theta}$. The samples in the x direction cover a wider range compared to the ones in the y direction($\approx$ between 0.7 and 1.4).

In the second scenario we set $\sigma^2_{\theta\theta} = 0.1$ to produce the results shown below:

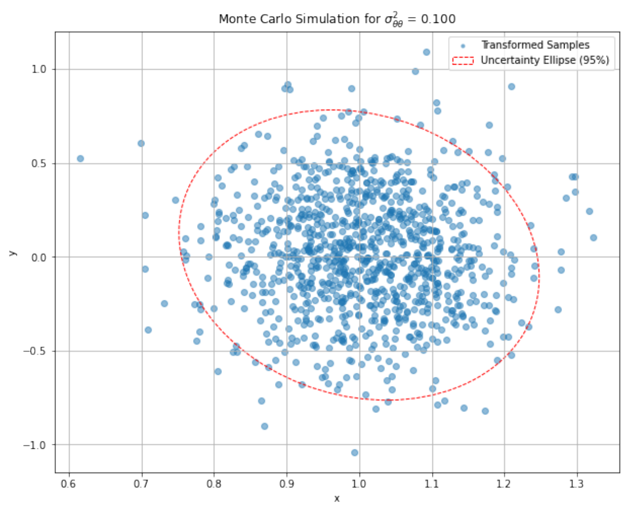

Figure 2: Monte Carlo simulation results for 3.

The simulation results in Figure 2 can be interpreted as follows:

- **Spread of samples in the y direction**: Increasing $\sigma^2_{\theta\theta}$ to 0.1 introduces more variance in the θ direction. As a result, the transformed samples in the y direction, which depends on $\rho\sin\theta$, will have a larger spread. Consequently, the samples in the y direction cover a wider range than in the previous scenario ($\approx$ between -1.0 and 1.0).

- **Shape of the uncertainty ellipse**: The uncertainty ellipse is observed to be more elongated in the vertical direction (y-axis) since it corresponds to the θ direction. This is an indication of higher uncertainty in the y direction.

- **Spread of samples in the x direction**: The spread of the samples in the the x direction, which depends on $\rho\cos\theta$, appear unaffected. The spread in the x direction appears to remain the same as in the previous scenario.

In the third scenario we set $\sigma^2_{\theta\theta} = 0.5$ to produce the results shown below:

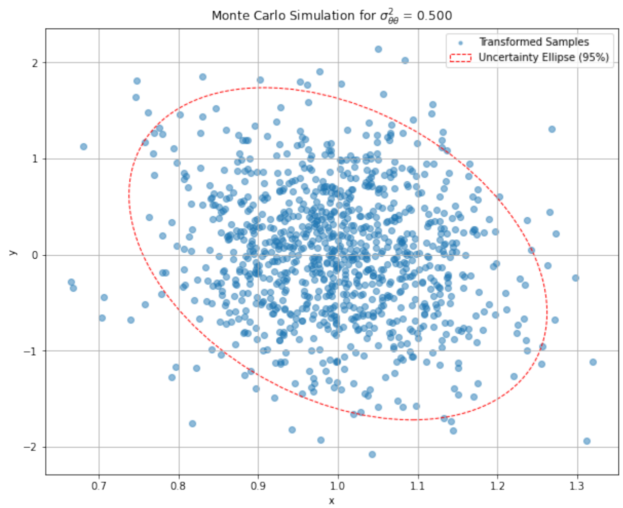

Figure 3: Monte Carlo simulation results for 4.

The simulation results in Figure 3 can be interpreted as follows:

- **Spread of samples in the y direction**: Increasing $\sigma^2_{\theta\theta}$ to 0.5 introduces even more variance in the θ direction than before. As a result, the transformed samples in the y direction will continue to spread. Consequently, the samples in the y direction cover a wider range than in the previous scenario ($\approx$ between -1.2 and 1.2). However, the increase in range is not as significant as the one observed going from scenario 1 to 2. It appears that the spread in the y direction is about to halt.
- **Shape of the uncertainty ellipse**: The uncertainty ellipse is observed to be even more elongated in the vertical direction (y-axis) than previous scenarios.
- **Spread of samples in the x direction**: The spread of the samples in the the x direction appears to be unaffected but rather shifted further to the left. The samples in the x direction appear to cover a wider range compared to the previous two scenarios ($\approx$ between -0.5 and 1.0).

In the last scenario we set $\sigma^2_{\theta\theta} = 1$ to produce the results shown below:

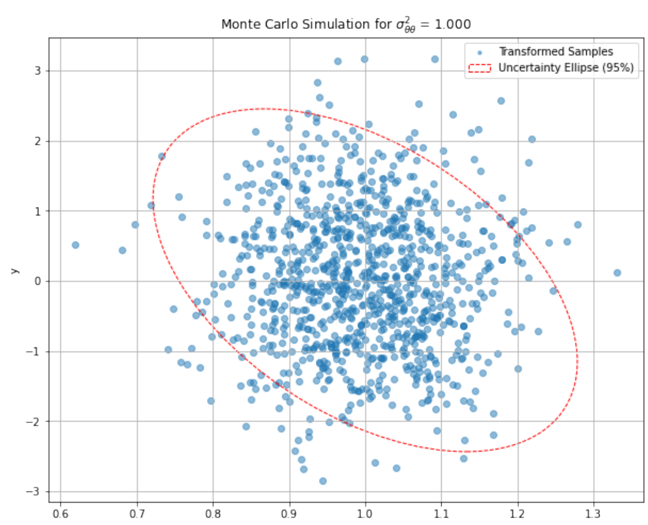

Figure 4: Monte Carlo simulation results for 5.

The simulation results in Figure 4 can be interpreted as follows:

- **Spread of samples in the y direction**: Increasing $\sigma^2_{\theta\theta}$ to 0.1 introduces more variance in the θ direction. As a result, the transformed samples in the y direction, which depends on $\rho\sin\theta$, will have a larger spread. Consequently, the samples in the y direction cover a wider range than in the previous scenario ($\approx$ between -1.0 and 1.0).
- **Shape of the uncertainty ellipse**: the uncertainty ellipse is observed to be more elongated in the vertical direction (y-axis) since it corresponds to the θ direction. This is an indication of higher uncertainty in the y direction.
- **Spread of samples in the x direction**: The spread of the samples in the the x direction, which depends on $\rho\cos\theta$, appear unaffected. The spread in the x direction appears to remain the same as in the previous scenario.