ME8135 — Assignment 2 Solution

Student: Arash Basirat Tabrizi
Submitted to: Dr. Sajad Saeedi
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1. Problem Statement:

We wish to use PyGame to simulate a simplified 2D robot and perform state estimation using a Kalman Filter. The robot's Motion Model is defined as follows:

$$\dot{x} = \frac{r}{2}(u_r + u_l) + w_x, \dot{y} = \frac{r}{2}(u_r + u_l) + w_y \tag{1}$$

In equation (1), r = 0.1 m, is the radius of the wheel, u_r and u_l are control signals applied to the right and left wheels. Additionally, $w_x = N(0, 0.1)$ and $w_y = N(0, 0.15)$.

We wish to simulate the system such that the robot is driven 1 m to the right. We are to assume the speed of each wheel is fixed at $0.1\frac{m}{s}$. We use the following initial values:

$$x_0 = 0, y_0 = 0, P_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{(initial covariance matrix)} \tag{2}$$

Also, we are to assume that the motion model is computed 8 times a second. We assume every second a measurement is given:

$$z = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} r_x \\ r_y \end{bmatrix} \tag{3}$$

Where $r_x = N(0, 0.05)$ and $r_y = N(0, 0.075)$.

We wish to plot and animate the trajectory along with covariance ellipse for all motion models and measurement updates. We also aim to report what happens when a measurement arrives. We should note to discretize the system: $\dot{x} = \frac{(x_k - x_{k-1})}{T}$ where T is 1/8 sec. In the animation, we will show both ground-truth and estimates.

Solution Formulation: From the discretized system model we have:

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{r_x}{2} \\ \frac{r_y}{2} \end{bmatrix} \tag{4}$$

where $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$ is our *state transition matrix*. Since the discretized system is formulated as follows:

$$\begin{aligned} \frac{x_k - x_{k-1}}{T} &= \frac{r}{2}(u_{r,k} + u_{l,k}) + w_{x,k} \\ x_k &= x_{k-1} + \frac{r}{2}(u_{r,k} + u_{l,k})T + w_{x,k}T \\ \frac{y_k - y_{k-1}}{T} &= \frac{r}{2}(u_{r,k} + u_{l,k}) + w_{y,k} \\ y_k &= y_{k-1} + \frac{r}{2}(u_{r,k} + u_{l,k})T + w_{y,k}T \end{aligned} \tag{5}$$

Comparing this with the general state transition equation $\mathbf{x}_k = A\mathbf{x}_{k-1} + B\mathbf{u}_k + \mathbf{w}_k$, we identify the *control* *matrix* B to be:

$$B = \begin{bmatrix} \frac{r}{2}T & \frac{r}{2}T \\ \frac{r}{2}T & \frac{r}{2}T \end{bmatrix} \tag{6}$$

Note that in the state transition equation, $\mathbf{u}_k = \begin{bmatrix} u_{r,k} \\ u_{l,k} \end{bmatrix}$ is the control vector and $\mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \end{bmatrix}$ is the state vector. $\mathbf{w}_k$ is a Gaussian random vector that models the uncertainty introduced by the state transition.

To determine the *process noise covariance matrix*, Q, of the system, we need to consider the variances of the individual process noise components:

$$Q = \begin{bmatrix} \mathrm{Var}(w_x) & 0 \\ 0 & \mathrm{Var}(w_x) \end{bmatrix} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.15 \end{bmatrix} \tag{7}$$

Similarly, from the measurement/observation model's general form of $\mathbf{z} = C\mathbf{x} + \mathbf{n}$, the *measurement noise covariance matrix*, R, of the system is as follows:

$$R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.075 \end{bmatrix} \tag{8}$$

Additionally, we deduce the *observation matrix*, C, to be:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \tag{9}$$

Thus, the state transition model in matrix form is as follows:

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{r}{2}T & \frac{r}{2}T \\ \frac{r}{2}T & \frac{r}{2}T \end{bmatrix} \begin{bmatrix} u_{r,k} \\ u_{l,k} \end{bmatrix} + \begin{bmatrix} w_{x,k} \\ w_{y,k} \end{bmatrix} T \tag{10}$$

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