

Welcome again to the **Foundation of applied machine learning!**

## Problem 1: Python programming

This is a problem to check your coding skills! So **do not use** fancy modules and scripts!

### Production of Fibonacci Sequence!

I have been assigned to write a function that takes an integer value  $n$ , where  $n \in [0, 1, 2, 3, \dots]$  and return the  $n^{\text{th}}$  value in Fibonacci sequence. Since, I am very lazy I just wrote the function **recursively** as you can see below. Another person, who was assigned to do the same thing, wrote the function with FOR LOOPS and claims his code is much faster!!

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#### Algorithm 1: Recursive Fibonacci Function

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```
def Fib_rec(n=0):  
    if n==0:  
        return 1  
    elif n==1:  
        return 1  
    else:  
        return Fib_rec(n-1)+Fib_rec(n-2)
```

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1. **Part 1:** Rewrite the function with for loop.
2. **Part 2:** Which function is actually faster? (Explain without running the codes)
3. **Part 3:** Write a code to time the **average** time for  $k$  times function call. A function that takes three arguments (function to time (Fib\_rec), input of the function ( $n$ ), number of runs( $k$ )) and run the Fib\_rec function  $k$  times for the input of  $n$  and returns the average time. **Tip:** You can use the time module in the python:

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#### Algorithm 2: importing time modules

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```
import time  
# if you run this, the current time in (s) will be recorded in x  
x=time.time()
```

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4. **Part 4:** Make a plot in which the  $x$ -axis is the value of the input function  $n$  and the  $y$ -axis is the average time (output of the previous function), for both recursive and non-recursive Fibonacci. (Both in the same plot; also use matplotlib package for making the plots)

**Problem 2: Linear Algebra**

Given the Matrix below answer the questions:

$$M = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}$$

**Part 1:** Find the determinant, transpose, inverse(if exist) for  $M$ .

**Part 2:** Find the eigenvalues and eigenvectors for  $M$ .

**Part 3:** Find the Gradient if the  $\nabla_A f(A)$  for the following:

$$A = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$

$$f(A) = x_{11}^2 x_{22} x_{23} + x_{11} x_{12} x_{13} x_{31} - x_{33}^2 x_{32} x_{21}$$

**Part 4:** Find the Hessian Matrix for:

$$g(x, y, z) = x^3 y + yz \sin(x) + xy^2 z^5$$

**Problem 3: Machine Learning**

**Part 1:** Explain the difference between validation and test samples.

**Part 2:** Explain the difference between supervised and unsupervised learning algorithm.