Carlingford High School

Mathematics

Year 10 Term 3 Test

2017

Name: _____

Teacher: Mr Cheng Mrs Strilakos Mrs Lego

Time allowed: ***55 minutes***

- Calculators allowed.
- Show all necessary working.
- Complete the examination in blue or black pen.
- Attempt all questions.
- Extension questions are marked with an asterisk.

	Trigonometry	Coordinate Geometry	Total
Questions	/20	/17	/37
Extension	/3	/4	/7
Total	/23	/21	/44

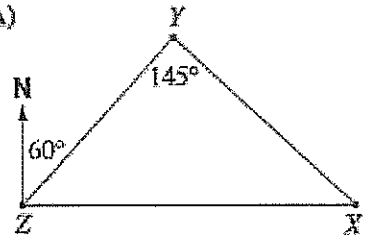
Trigonometry: 23 marks

1. The following information is given about the locations of three towns X, Y and Z:

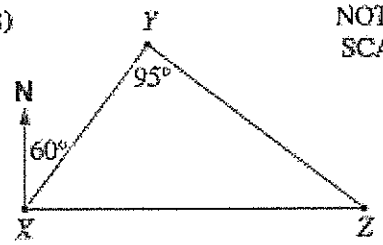
   - X is due east of Z
   - X is on a bearing of 145° from Y
   - Y is on a bearing of 060° from Z.

   Which diagram below best represents this information?

(A)

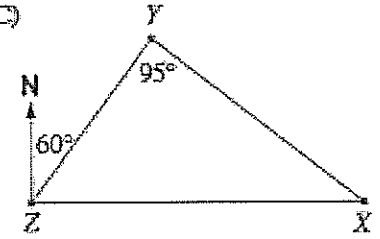


(B)

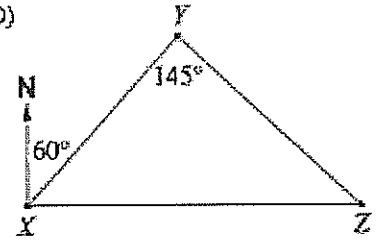


NOT TO SCALE

(C)



(D)



2. If θ is acute, find the value of θ if:
   $\tan 140° = -\tan \theta$

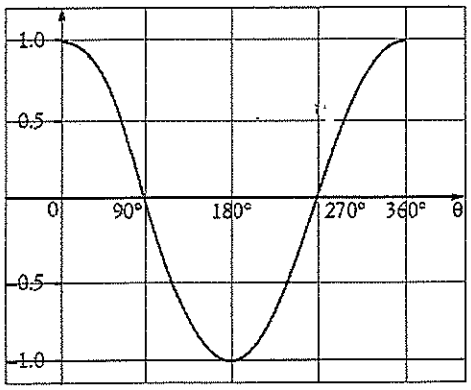
3. Find the exact value of $\sin Y$ if $\cos X = \frac{2}{3}$ and X and Y are complementary angles. (2 marks)

4. For each of the following, find all possible values for θ, where $0° < \theta < 180°$. Answer correct to the nearest degree. (3 marks)

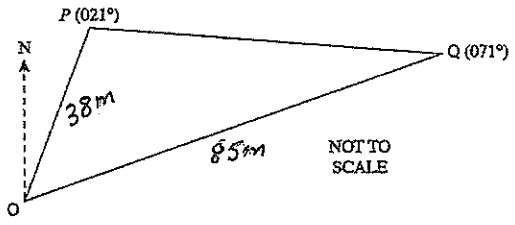
   a) $\sin \theta = 0.2924$

   b) $\cos \theta = -0.8511$

5. Name the curve drawn below.

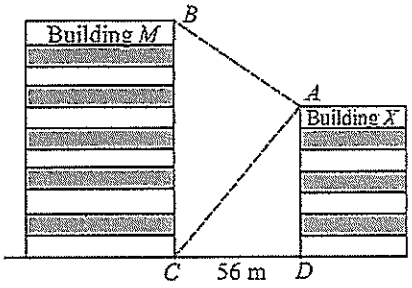


6. The diagram below shows a section of land proposed for a park. Point P from O has a bearing of 021° and point Q from O has a bearing of 071°.



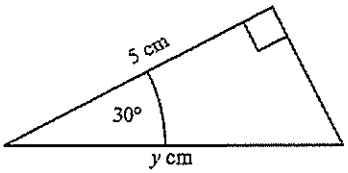
What is the area of land POQ? Give your answer correct to the nearest square metre. (2 marks)

7. Two buildings, called M and X, are situated 56 m apart on level ground. From point C. the angle of elevation of point A is 65°. From point B, the angle of depression of point A is 38°.

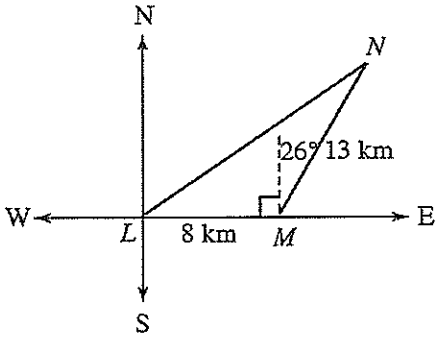


Calculate the height of building M, correct to one decimal place. (3 marks)

8. Find the exact value (in simplest form) of y in the diagram below. (2 marks)



9. A bushwalker walks 8 km due east from L to M, then proceeds to walk a further 13 km to N on a bearing of 026°.

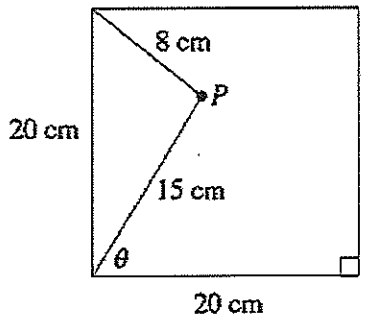


Calculate his distance from L, correct to the nearest kilometre. (2 marks)

10. In ΔABC, $\angle BAC = 25°$, $BC = 9$ cm and $AB = 20$ cm.

Find all possible values for $\angle BCA$.
Answer correct to the nearest degree.
(3 marks)

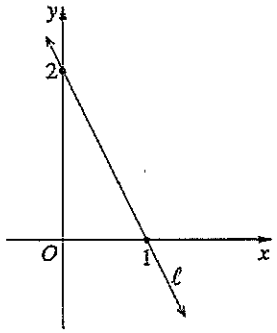
*11. Square tiles of side length 20 cm are being used to tile a bathroom. The tiler needs to drill a hole in one of the tiles at a point P which is 8 cm from one corner and 15 cm from an adjacent corner.
To locate the point P the tiler needs to know the size of the angle θ shown in the diagram below.



Find the size of the angle θ, correct to the nearest degree. (3 marks)

Coordinate Geometry (21)

1. What is the equation of the line l ?



(A) $y = -2x + 2$

(B) $y = 2x + 2$

(C) $y = \frac{-x}{2} + 2$

(D) $y = \frac{x}{2} + 2$

2. Find, correct to the nearest degree, the angle of inclination of a line with a gradient of -4. (2 marks)

3. Find the distance between the points $(4, 5)$ and $(-6, 3)$. Leave your answer in simplified surd form. (2 marks)

4. Find the equation of the line which is parallel to $y = 4 - 3x$ and passes through the point $(3, -1)$. Express your answer in gradient - Intercept form. (2 marks)

5. a) Find the coordinates of S, the point where the line $3x - 2y + 14 = 0$ cuts the y −axis.

b) Hence, find the equation of the line which passes through S and is perpendicular to the line $5x - 4y - 1 = 0$ Give your answer in general form. (3 marks)

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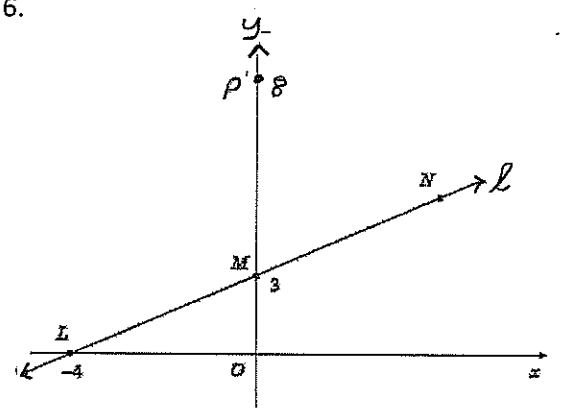

The line l cuts the x axis at $L(-4, 0)$ and the y axis at $M(0, 3)$ as shown. N is a point on the line l, and P is the point $(0, 8)$.

a) Find the equation of the line l. (2 marks)

b) By considering the lengths of ML and MP, show that ΔLMP is isosceles. (2 marks)

c) Show that the point $(16, 15)$ lies on the line l.

d) Calculate the gradient of the line PL.

* e) M is the midpoint of the interval LN. Find the coordinates of the point N. (2 marks)

* f) Show that $\angle NPL$ is a right angle. (2 marks)

KC p 1/2
SS p 3/4
VL p 5/6

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   Which diagram below best represents this information?

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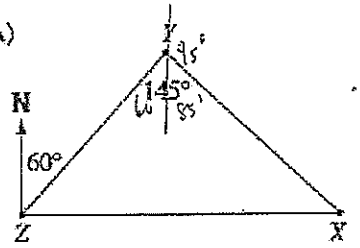

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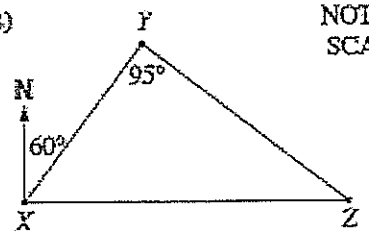

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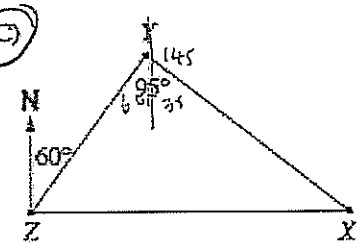

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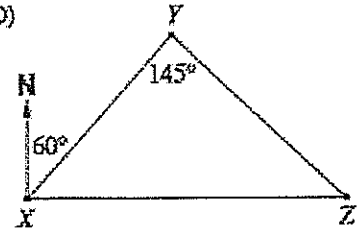
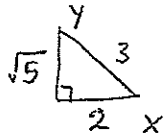


2. If θ is acute, find the value of θ if:
   $\tan 140° = -\tan\theta$

$\theta = 180 - 140$
$= 40°$ (1)

$\therefore \tan 140° = -\tan 40°$

3. Find the exact value of $\sin Y$ if $\cos X = \frac{2}{3}$ and X and Y are complementary angles. (2 marks)



$\therefore \sin Y = \frac{2}{3}$

4. For each of the following, find all possible values for θ, where $0° < \theta < 180°$. Answer correct to the nearest degree. (3 marks)

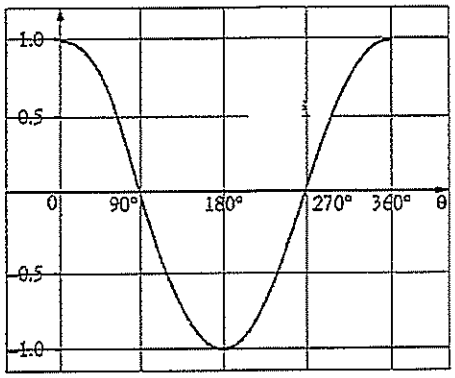
a) $\sin\theta = 0.2924$

$\theta = 17°, 163°$
(1) (1)

b) $\cos\theta = -0.8511$

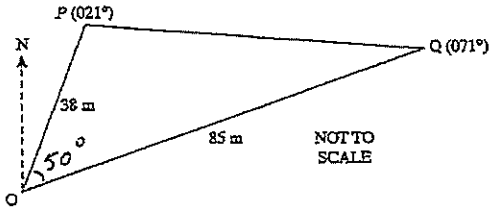
$\theta = 148°$

5. Name the curve drawn below.



$y = \cos\theta$

6. The diagram below shows a section of land proposed for a park. Point P from O has a bearing of 021° and point Q from O has a bearing of 071°.



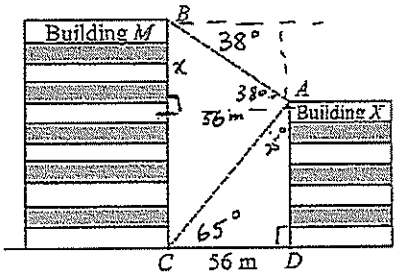
What is the area of land POQ? Give your answer correct to the nearest square metre. (2 marks)

$A = \frac{1}{2} \times 38 \times 85 \times \sin 50^\circ$

$= 1237.161\ldots$

$= 1237\ m^2$

7. Two buildings, called M and X, are situated 56 m apart on level ground. From point C. the angle of elevation of point A is 65°. From point B, the angle of depression of point A is 38°.



Calculate the height of building M, correct to one decimal place. (3 marks)

AD: $\tan 65^\circ = \frac{AD}{56}$

$AD = 56 \times \tan 65^\circ$

$= 120.092\ldots$

Top of M to Top of X:

$\tan 38^\circ = \frac{x}{56}$

$x = 56 \times \tan 38^\circ$

$= 43.751\ldots$

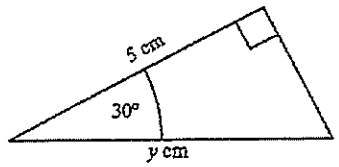
∴ height of building is

$120.092\ldots + 43.751\ldots$

$= 163.843\ldots$

$= 163.8\ m.$

8. Find the exact value (in simplest form) of y in the diagram below. (2 marks)

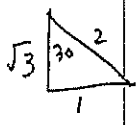


$$\cos 30 = \frac{5}{y}$$

$$y = \frac{5}{\cos 30}$$

$$= \frac{5}{\frac{\sqrt{3}}{2}}$$

$$= \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$$



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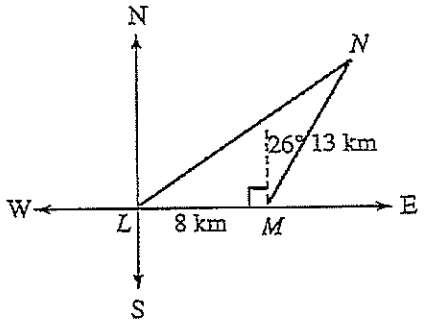


Calculate his distance from L, correct to the nearest kilometre. (2 marks)

$$NL^2 = 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos 116°$$

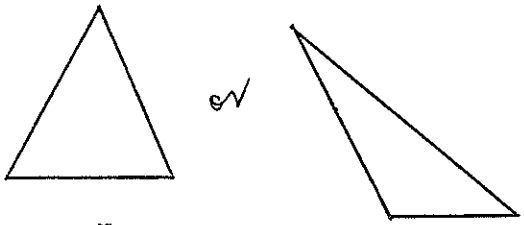
$$= 324.181\ldots$$

$$NL = \sqrt{\quad}$$

$$= 18.005\ldots$$

$$= 18 \text{ km}.$$

10. In ΔABC, $\angle BAC = 25°$, $BC = 9$ cm and $AB = 20$ cm.



Find all possible values for $\angle BCA$.
Answer correct to the nearest degree.
(3 marks)

$$\frac{\sin\theta}{20} = \frac{\sin 25}{9}$$

$$\sin\theta = \frac{\sin 25}{9} \times 20$$

$$\theta = 69° 54'$$

$$\theta = 70°$$

$$\text{obtuse} = 180 - 70°$$

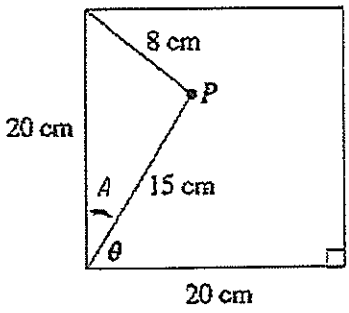
$$= 110°$$

$$\text{test}: 180 - 25 - 110°$$

$$= 45°$$

$\therefore \theta = 70°$ or $110°$.

*11. Square tiles of side length 20 cm are being used to tile a bathroom. The tiler needs to drill a hole in one of the tiles at a point P which is 8 cm from one corner and 15 cm from an adjacent corner.
To locate the point P the tiler needs to know the size of the angle θ shown in the diagram below.



Find the size of the angle θ, correct to the nearest degree. (3 marks)

$$\cos A = \frac{20^2 + 15^2 - 8^2}{2 \times 20 \times 15}$$

$A = 20^\circ 46' \ldots$

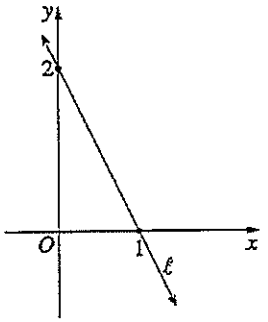
$A = 21^\circ$

$\therefore \quad \theta = 90^\circ - 21^\circ$

$= 69^\circ$

Coordinate Geometry (21)

1. What is the equation of the line l?



(A) $y = -2x + 2$

(B) $y = 2x + 2$

(C) $y = \frac{-x}{2} + 2$

(D) $y = \frac{x}{2} + 2$

2. Find, correct to the nearest degree, the angle of inclination of a line with a gradient of -4. (2 marks)

$m = \tan\theta$

$= -4$

$\tan\theta = -4$

$\theta = 104^\circ.$

3. Find the distance between the points $(4, 5)$ and $(-6, 3)$. Leave your answer in simplified surd form. (2 marks)

$$d = \sqrt{(-6-4)^2 + (3-5)^2}$$

$= \sqrt{100 + 4}$

$= \sqrt{104}$

$= 2\sqrt{26}$ units

4. Find the equation of the line which is parallel to $y = 4 - 3x$ and passes through the point $(3, -1)$. Express your answer in gradient - intercept form. (2 marks)

$m = -3$ (1)

$y - -1 = -3(x - 3)$

$y + 1 = -3x + 9$

$y = -3x + 8$

5. a) Find the coordinates of S, the point where the line $3x - 2y + 14 = 0$ cuts the y –axis.

when $x = 0$

$3(0) - 2y + 14 = 0$

$-2y = -14$

$y = 7$

$\therefore (0, 7)$

b) Hence, find the equation of the line which passes through S and is perpendicular to the line $5x - 4y - 1 = 0$ Give your answer in general form. (3 marks)

$5x - 4y - 1 = 0$

$5x - 1 = 4y$

$\frac{5}{4}x - \frac{1}{4} = y$

$\therefore m = -\frac{4}{5}$ (1)

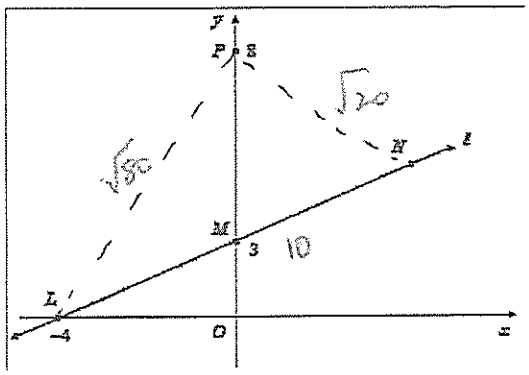
$y - 7 = -\frac{4}{5}(x - 0)$ (1)

$y - 7 = -\frac{4}{5}x$

$5y - 35 = -4x$

$4x + 5y - 35 = 0$

6.



The line l cuts the x axis at $L(-4, 0)$ and the y axis at $M(0, 3)$ as shown. N is a point on the line l, and P is the point $(0, 8)$.

a) Find the equation of the line l. (2 marks)

$m = \frac{3}{4}$ (1)

$\therefore y = \frac{3}{4}x + 3$ (1)

$4y = 3x + 12$

$3x - 4y + 12 = 0$

b) By considering the lengths of ML and MP, show that ΔLMP is isosceles. (2 marks)

$MP = 5$ $ML = 5$ must have both (1)

$\therefore ML = MP$

$\therefore \Delta LMP$ is isosceles. (1)

must have both

c) Show that the point (16, 15) lies on the line l.

$y = \frac{3}{4}x + 3$

sub (16, 15) $\quad 15 = \frac{3}{4}(16) + 3$

$15 = 12 + 3$

true

$\therefore$ (16, 15) lies on l.

d) Calculate the gradient of the line PL.

$P(0, 8) \qquad L(-4, 0)$

$m = \frac{0 - 8}{-4 - 0}$

$m = 2$

* e) M is the midpoint of the interval LN. Find the coordinates of the point N. (2 marks)

$L(-4, 0) \qquad M(0, 3)$

$0 = \frac{-4 + x}{2} \qquad 3 = \frac{0 + y}{2}$

$0 = -4 + x \qquad 6 = 0 + y$

$4 = x \qquad 6 = y$

$\therefore N(4, 6)$

* f) Show that $\angle NPL$ is a right angle. (2 marks)

m_1 of $PL = 2$

m_2 of $PN = \frac{6 - 8}{4 - 0}$

$= \frac{-2}{4}$

$= -\frac{1}{2}$

Since $m_1 \times m_2 = 2 \times -\frac{1}{2}$

$= -1$ (1)

$\therefore$ ($PL \perp PN$) perpendicular

$\angle NPL$ is a right angle (1)

$PL = \sqrt{(-4)^2 + (-8)^2}$ $(0,8)$ $(-4,0)$

$= \sqrt{16 + 64}$

$= \sqrt{80}$

$PN = \sqrt{(-4)^2 + (2)^2}$ $(4,6)$ $(0,8)$

$= \sqrt{16 + 4}$

$= \sqrt{20}$

$LN = \sqrt{(8)^2 + (6)^2}$ $(-4,0)$ $(4,6)$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

$= 10$

Since $LN^2 = PN^2 + PL^2$

$(10)^2 = (\sqrt{80})^2 + (\sqrt{20})^2$

$100 = 80 + 20$

$\therefore$ $\angle NPL$ is a right-angle.