Q1 a) $\int \left(5x^2 - \frac{1}{x^2}\right)dx = \int \left(5x^2 - x^{-2}\right)dx$

GF

$= \frac{5x^3}{3} + x^{-1} + C$

$= \frac{5}{3}x^3 + \frac{1}{x} + C$

b). $\int 4\sqrt{5x+7}\,dx = 4\int (5x+7)^{\frac{1}{2}}\,dx$

GF

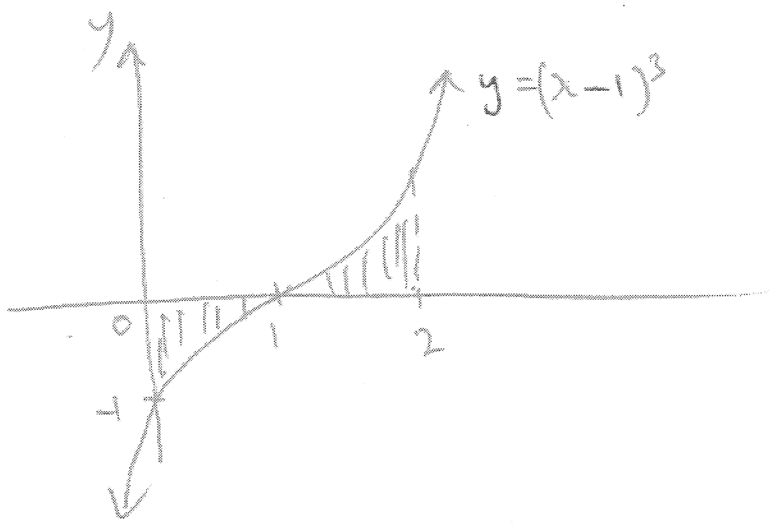
$= 4\left[\frac{(5x+7)^{\frac{3}{2}}}{5 \times \frac{3}{2}}\right] + C$

$= 4\left[\frac{2(5x+7)^{\frac{3}{2}}}{15}\right] + C$

$= \frac{8}{15}\sqrt{(5x+7)^3} + C$

c).

TL



$A = -\int_0^1 (x-1)^3\,dx + \int_1^2 (x-1)^3\,dx$

$= -\left[\frac{(x-1)^4}{4}\right]_0^1 + \left[\frac{(x-1)^4}{4}\right]_1^2$

$= -\frac{1}{4}\left[(x-1)^4\right]_0^1 + \frac{1}{4}\left[(x-1)^4\right]_1^2$

$= -\frac{1}{4}\left[(1-1)^4 - (0-1)^4\right] + \frac{1}{4}\left[(2-1)^4 - (1-1)^4\right]$

$= -\frac{1}{4} \times -1 + \frac{1}{4} \times 1$

$\therefore A = \frac{1}{2}\ u^2.$

d) i) Let $y = x+1$ —— ①

TL   & $y = x^2 - 2x + 1$ —— ②

Sub ① into ② get

$x^2 - 2x + 1 = x + 1$ ✓

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ or $x = 3$.

$y = 1$ or $y = 4$.

∴ The coordinate of point A is $(3,4)$. ✓

ii) $A = \int_0^3 \left[x+1 - (x^2 - 2x + 1)\right] dx$

TL

$= \int_0^3 (x + 1 - x^2 + 2x - 1)\, dx$ ✓

$= \int_0^3 (3x - x^2)\, dx$

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$

$= \frac{3}{2}(3)^2 - \frac{1}{3}(3)^3 - 0$

$= \frac{27}{2} - \frac{27}{3}$

∴ $A = \frac{9}{2}$ or $4\frac{1}{2}$ u^2 ✓

e). i) AG $y=\sqrt{4-x^2}$

x	0	0.5	1	1.5	2
y	2	$\frac{\sqrt{15}}{2}$	$\sqrt{3}$	$\frac{\sqrt{7}}{2}$	0
	y_0	y_1	y_2	y_3	y_4

1 wrong −1
2 wrong 0

ii) AG $\int_0^2 \sqrt{4-x^2}\,dx \doteqdot \frac{1}{6}\left[2+4\left(\frac{\sqrt{15}}{2}+\frac{\sqrt{7}}{2}\right)+2(\sqrt{3})+0\right]$ ✓

$\doteqdot 3.084\,u^2$ (3 dec pls). ✓

Q2.a) i) SS $\log_a\left(\frac{4x}{3}\right)=\log_a(x+4)$ ✓

so $\frac{4x}{3}=x+4$

$4x=3x+12$

$\therefore x=12.$ ✓

ii) SS $\frac{d}{dx}\left[\ln(5x-1)\right]=\frac{1}{5x-1}\times 5$

$=\frac{5}{5x-1}$ ✓

bi. SS $\int e^{5-2x}dx$

$=\frac{-1}{2}e^{5-2x}+C$ ✓

ii) SS $\int_0^1 \frac{6x}{x^2+1}dx = 3\int_0^1 \frac{2x}{x^2+1}dx$

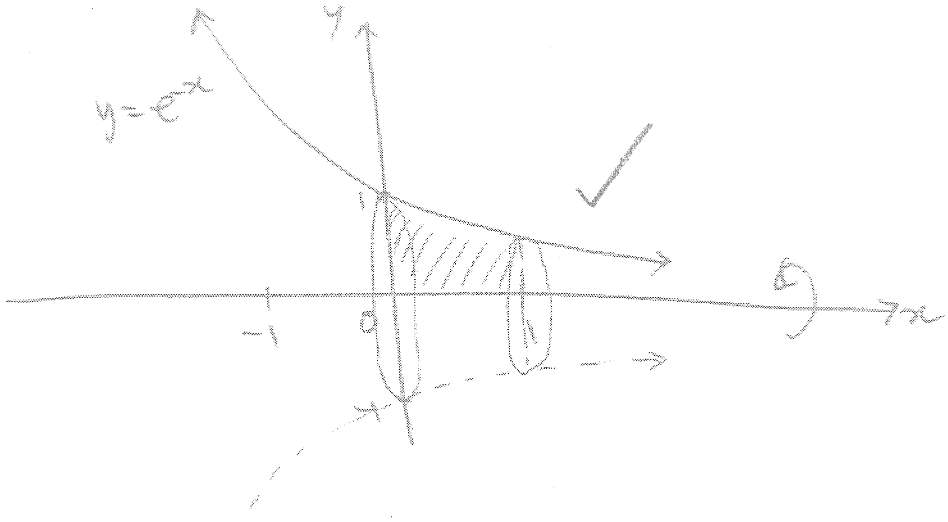
$=3\left[\ln(x^2+1)\right]_0^1$ ✓

$=3\left[\ln 2-\ln 1\right]$

$=3\ln 2$ or $\ln 8$ ✓

c). i)

PW



ii) $V = \pi \int_0^1 (e^{-x})^2\, dx$ ✓

PW

$= \pi \int_0^1 e^{-2x}\, dx$

$= -\frac{\pi}{2}\left[e^{-2x}\right]_0^1$ ✓

$\therefore V = -\frac{\pi}{2}\left[e^{-2} - 1\right] u^3$ or $\frac{\pi}{2}(1 - e^{-2})\, u^3$ ✓

d. $y = e^{2x} + e^{4x}$

PW

$$\left.\begin{aligned} \frac{dy}{dx} &= 2e^{2x} + 4e^{4x} \\ \frac{d^2y}{dx^2} &= 4e^{2x} + 16e^{4x} \end{aligned}\right] ✓$$

Now LHS $= \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y$

$= 4e^{2x} + 16e^{4x} - 6(2e^{2x} + 4e^{4x}) + 8(e^{2x} + e^{4x})$ ✓

$= 4e^{2x} + 16e^{4x} - 12e^{2x} - 24e^{4x} + 8e^{2x} + 8e^{4x}$

$= 4e^{2x} - 12e^{2x} + 8e^{2x} + 16e^{4x} - 24e^{4x} + 8e^{4x}$

$\therefore$ LHS $= 0 =$ RHS. ✓

So $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$ #.