Solutions – Junior Division

2015

1. $2015 + 201.5 = 2216.5$,

hence (D).

2. The three angles add to $180°$, so $x° = 110°$,

hence (C).

3. Since 23 minutes before 9:05 am is 18 minutes before 9 am, the latest I can leave home is 8:42 am,

hence (D).

4. *Alternative 1*
100 times 1 c is \$1, so 100 times 20 c is \$20,

hence (A).

Alternative 2
There are 5 twenty-cent coins for every dollar, and $100 \div 5 = 20$,

hence (A).

5. (Also I1)
The area is $\frac{1}{2} \times 12 \times 2 = 12\,\text{cm}^2$,

hence (B).

6. Since there were 3 teachers, there would be 12 students in all. Six of these were there on time, so 6 arrived after the bell,

hence (C).

7. (Also I2)
The movie lasts for $120 + 20 = 140$ minutes, so each half is $140 \div 2 = 70$ minutes,
hence (B).

8. Counting unit lengths, (A) has perimeter 10, (B) has perimeter 10, and (D) has perimeter 8. The perimeter of (C) is $8 + 4 \times \frac{1}{2} = 10$ and the perimeter of (E) is $10 + 2 \times \frac{1}{2} = 11$,

hence (E).

9. Bryce is $186 - 14 = 172$ cm tall.
Cy is $172 + 6 = 178$ cm tall.
Eric is $178 + 11 = 189$ cm tall,

hence (D).

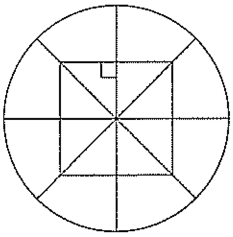
10. (Also I6)
The table below shows that Ben leaves after calling out the number 7, Eve leaves after calling out the number 14, Ana leaves after calling out the number 21, and Con leaves after calling out the number 28. Therefore, Dan is the last person remaining at the table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	B	C	D	E	A	**B**	C	D	E	A	C	D	**E**
15	16	17	18	19	20	21	22	23	24	25	26	27	28
A	C	D	A	C	D	**A**	C	D	C	D	C	D	**C**

hence (D).

11. $\frac{1}{19}$ of 38 is $38 \div 19 = 2$, so that $\frac{5}{19}$ of 38 is 10,

hence (E).

12. 

Drawing lines from the centre through the corners and parallel to the sides divides the circle into 8 equal sectors.

Five of these are shaded, which is $\frac{5}{8}$ of the circle,

hence (A).

13. In the hundreds column, z must be 1. Then the units column must have $7+4=11$, so $x=7$. Finally $47+74=121$ and so $y=2$. Then $x+y+z=7+2+1=10$,

hence (C).

14. (Also UP20, I13)
Faces C and T must be opposite, ruling out (B) and (C). With A upright, the face to the right is M, ruling out (D). With C upright, the face above is A, ruling out (E). (A) is possible with H opposite face A,

hence (A).

15. Call the tallest A and the shortest D and E, and the others B and C. The only possible arrangements are DBACE, DCABE, EBACD, ECABD,

hence (E).

16. The smallest total of 6 different positive integers is $1+2+3+4+5+6=21$. However, half of the total is boys' ages and half is girls' ages, so 21 is not possible.

A total of $22 = 1+2+3+4+5+7$ is possible with boys' ages $1+3+7=11$ and girls' ages $2+4+5=11$,

hence (A).

17. (Also I11, S6)
Alternative 1
Jenna must leave out a longer side and Dylan a shorter side, where the longer side is 8 cm longer than the shorter side. So the sides are x cm and $(x+8)$ cm. Then Jenna's measurement is $2x+x+8=80$, so that $3x=72$ and $x=24$. The rectangle is 24 cm by 32 cm. The perimeter is then $2\times 24+2\times 32=112$ cm,

hence (A).

Alternative 2

Suppose the rectangle has width w and height h. Dylan's and Jenna's measurements are $2w + h$ and $2h + w$. Adding these, $3w + 3h = 80 + 88 = 168$ and so $w + h = 168 \div 3 = 56$. Then the perimeter is $2(w + h) = 112\,\text{cm}$,

hence (A).

18. Jim has completed $3\frac{3}{4} = \frac{15}{4}$ out of $5 = \frac{20}{4}$ laps, which is $\frac{15}{20} = \frac{3}{4}$ of his run,

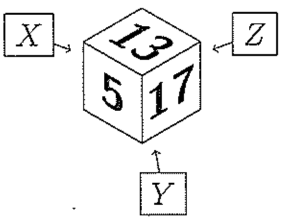
hence (C).

19. The sum of the two digits is at least 1 and at most 18. As a perfect square, the sum is one of 1, 4, 9 or 16. We consider the two digit numbers with each of these digit sums:

Sum	Numbers	Count
1	10	1
4	13, 22, 31, 40	4
9	18, 27, 36, 45, 54, 63, 72, 81, 90	9
16	79, 88, 97	3
		17

hence (E).

20. Suppose the hidden prime numbers are X, Y and Z as shown below. Then $17 + X = 13 + Y = 5 + Z$. The smallest total $X + Y + Z$ will occur when the prime X is as small as possible. The table shows possible values for X, eliminating composite values.



X	2	3	5	7
$17 + X$	19	20	22	24
Y	~~6~~	7	~~9~~	11
Z	~~14~~	~~15~~	17	19

Then the smallest possible sum of hidden faces is $7 + 11 + 19 = 37$,

hence (E).

21. *Alternative 1*

For a given quantity of sugar, the recipe requires twice as much butter and three times as much flour. So to use 9 kg sugar, you would need $2 \times 9 = 18$ kg butter and $3 \times 9 = 27$ kg flour. Although there is enough flour to do this, there is not enough butter — so the quantity of butter is the limiting factor. Given that the recipe requires 4 kg butter to make $2 \times 4 = 8$ cakes, you can use 17 kg butter to make $2 \times 17 = 34$ cakes,

hence (B).

Alternative 2

Each cake requires $\frac{1}{4}$ kg sugar, $\frac{1}{2}$ kg butter and $\frac{3}{4}$ kg flour.

Then there is enough sugar for $9 \times 4 = 36$ cakes, but there is only enough butter for $17 \times 2 = 34$ cakes.

These 34 cakes require $34 \times \frac{3}{4} = \frac{102}{4} = 25\frac{1}{2}$ kg flour, which is available,

hence (B).

22. (Also I14, S7)

The score is a multiple of 6 if one of the dice is 6 or if one of the dice is even (2, 4 or 6) and the other is 3. We tabulate these possibilities amongst the 36 equally likely rolls.

Second dice

First dice	1	2	3	4	5	6
1						✓
2			✓			✓
3		✓		✓		✓
4			✓			✓
5						✓
6	✓	✓	✓	✓	✓	✓

Then the probability that the score is a multiple of 6 is $\frac{15}{36} = \frac{5}{12}$,

hence (B).

23. When Jill turns, she has travelled twice as far as Jack, so her speed is twice Jack's. So when they pass, Jill has run twice the distance that Jack has walked, which means that Jack has covered $\frac{1}{3}$ of their combined distance of 2 beach lengths.

That is, Jack has walked $\frac{2}{3}$ of the beach,

hence (A).

24. (Also I19)

We claim that it is possible to pay exactly for any amount up to 200 cents with the following 10 coins: one 1c coin, one 2c coin, two 4c coins, one 10c coin, one 20c coin, and four 40c coins.

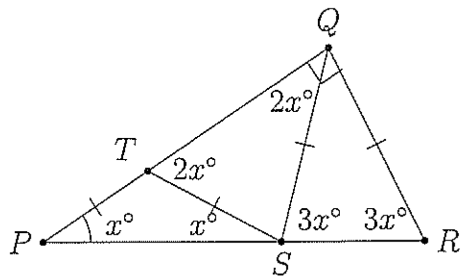
It is easy to check that the three smaller denominations can be used to pay for any amount up to 10c. It is also easy to check that the three larger denominations can be used to pay for any multiple of 10c up to 190c. Combining these two facts, we see that it is possible to pay exactly for any amount up to 200c.

To see that it is not possible with just 9 coins, consider 199c. The smallest number of coins required to make 199c is 9, with $40+40+40+40+20+10+4+4+1 = 199$. However with these 9 coins, there are several amounts that are not possible, such as 2c or 6c. So 10 coins are required,

hence (B).

25. *Alternative 1*

Using equal angles on the isosceles triangles and the fact that the sum of two angles on a triangle is equal to the external angle on the other corner, we can fill in the angles shown. Then in $\triangle PQR$, $90 + x + 3x = 180$, so that $x = 90 \div 4 = 22.5$,



hence (D).

Alternative 2

Since $\triangle PTS$ is isosceles, $\angle PST = x°$, $\angle PTS = 180° - 2x°$ and $\angle STQ = 2x°$.
Since $\triangle STQ$ is isosceles, $\angle SQT = 2x°$ and $\angle TSQ = 180° - 4x°$.
Clearly $\angle PRQ = 90° - x°$, and since $\triangle SQR$ is isosceles, $\angle QSR = 90° - x°$.
Then the three angles at S are $x°$, $180° - 4x°$ and $90° - x°$, so that

$$\begin{aligned} x + 180 - 4x + 90 - x &= 180 \\ 90 - 4x &= 0 \\ x &= 22.5 \end{aligned}$$

hence (D).

26. Let the three numbers be a, b, c, where $a > b > c > 0$. Then $a + b + c = 96$.
Now, c is a divisor of $a+b$, so c is a divisor of 96, and also $c < 96$. Likewise for a and b. So possible values for a, b and c are $\{48, 32, 24, 16, 12, 8, 6, 4, 3, 2, 1\}$, the proper divisors of 96.
If $a = 48$, then $b + c = 48$, which is only possible with $b = 32$, $c = 16$.
If $a < 48$, then the largest possible values are $a = 32$, $b = 24$, $c = 16$ and $a+b+c = 72$, so there is no solution.
So $a = 48, b = 24, c = 16$ is the only possible solution.
Checking, $a = 48$ divides $b + c = 48$, $b = 32$ divides $a + c = 64$, and $c = 16$ divides $a+b = 80$, so that there is exactly one possibility $\{16, 32, 48\}$ and the largest number is 48,

hence (48).

27. *Alternative 1*

Originally the ratio of Reds : Blues is 1 : 2 or 2 : 4. When 345 Blues leave, the ratio changes to 2 : 1, therefore 345 represents a reduction in Blues by 3 parts (the Reds remain unchanged at 2 parts). Hence, one part is 115 spectators and the Reds is 2 parts or 230 spectators,

hence (230).

Alternative 2

Let r be the number of Reds, so that the original total number of spectators is $3r$ and the number of Blues is $2r$. After 345 Blues leave the ratio of Blues to total is

$$\begin{aligned} \frac{1}{3} &= \frac{2r - 345}{3r - 345} \\ \Longrightarrow \quad 3r - 345 &= 6r - 3 \times 345 \\ \Longrightarrow \quad 3r &= 2 \times 345 = 690 \\ \Longrightarrow \quad r &= 230 \end{aligned}$$

hence (230).

Alternative 3

For every 2 Reds, there are 4 Blues before half-time, and 1 Blue who stays after half-time, so there are 3 Blues who leave. Since $345 = 3 \times 115$ Blues leave, there are $2 \times 115 = 230$ Reds,

hence (230).

28. (Also MP30, UP29)

Suppose the top square next to the flagpole is green. Then there may be 0, 2, 3 or 4 blue squares. Here are the possibilities in each case.

Blue squares	Possible flags				
0	GGG GGG				
2	GGG BBG	GBB GGG	GBG GBG	GGG GBB	GGB GGB
3	GGG BBB	GGB GBB	GBB GGB		
4	GGB BBB	GBB GBB			

Along with these 11 possibilities, there are 11 where the top square next to the flagpole is blue, making 22 ways in all,

hence (22).

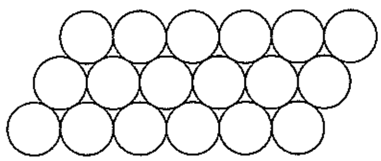
29. Observe that if a number is larger than 333 and not a multiple of 3, then it cannot be on the list.

On the other hand, it is possible for all of the remaining whole numbers larger than 0 but smaller than 1000 to be on the list.

Consequently Zoltan's list is largest when it contains all of the whole numbers from 1 to 333 as well as all of the multiples of 3 from 334 to 999. There are 333 numbers of the first type and 222 numbers of the second type, making 555 numbers in total,

hence (555).

30. Thinking of a stack as a trapezium, two copies of it form a parallelogram as shown.



Note that the number of rows must be less than the number of logs in each row of this parallelogram. Hence finding the possible stacks amounts to finding non-trivial factorisations of $2 \times 2015 = 2 \times 5 \times 13 \times 31$ into two numbers.

There are 7 such factorisations of 4030:

$$2 \times 2015 = 5 \times 806 = 10 \times 403 = 13 \times 310 = 26 \times 155 = 31 \times 130 = 62 \times 65$$

The height of the stack is the smaller factor, so 62×65 is the candidate for the tallest stack. Checking, the logs can be stacked in 62 rows of lengths $2, 3, 4, \ldots, 63$,

hence (62).