A C E
EXAM
PAPER

Student name: _____

PAPER 2

YEAR 12
YEARLY
EXAMINATION

Mathematics Advanced

General Instructions

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

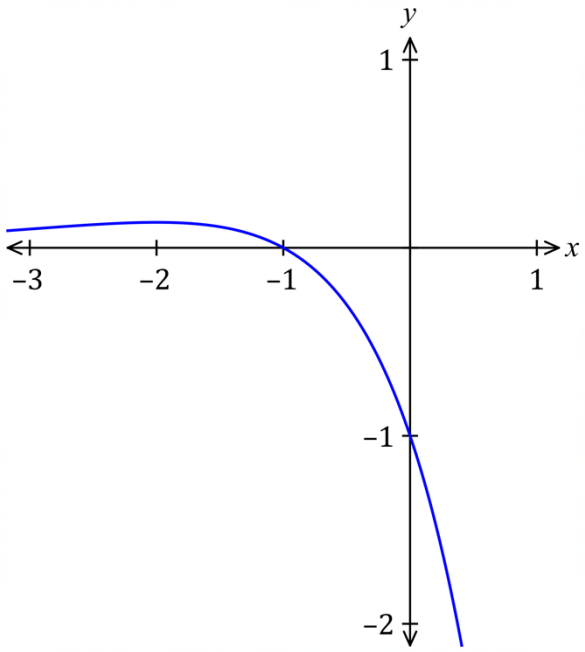
10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. What is the value of $\int_0^1 (6x^2 - 4)dx$?

   (A) -2
   (B) -1
   (C) 0
   (D) 1

2. The diagram shows the graph of $y = e^x(x - 1)$.



   How many solutions are there to the equation $e^x(x - 1) = x^2 - 1$?

   (A) 0
   (B) 1
   (C) 2
   (D) 3

3. An infinite geometric series has a first term of 3 and a limiting sum of 1.8.
   What is the common ratio?

   (A) $-0.\dot{3}$
   (B) $-0.\dot{6}$
   (C) -1.5
   (D) -3.75

4. What is the derivative of e^{x^3}?

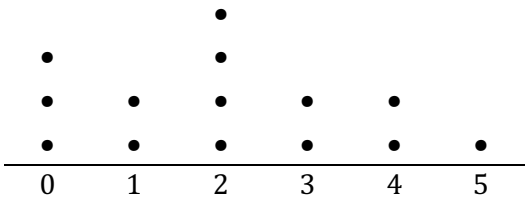
(A) $3x^2e^{x^3}$

(B) $3xe^{x^3}$

(C) $3x^2e^{3x^2}$

(D) $x^3e^{x^3-1}$

5. A sample of 14 people were asked to indicate the time (in hours) they had spent watching television on the previous night. The results are displayed in the dot plot below.



What is the mean and sample standard deviation of these times? Give your answers correct to one decimal place.

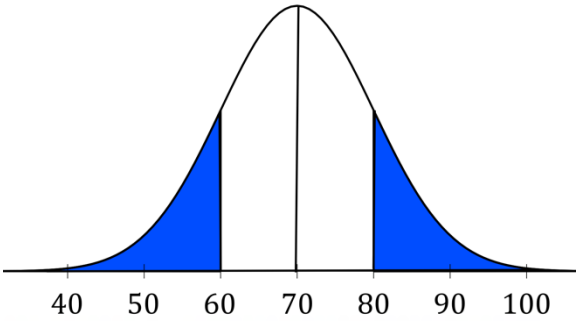
(A) $\bar{x} = 2.0$ and $s = 1.5$

(B) $\bar{x} = 2.1$ and $s = 1.5$

(C) $\bar{x} = 2.1$ and $s = 1.6$

(D) $\bar{x} = 2.6$ and $s = 1.2$

6. The normal distribution shows the results of a mathematics assessment task. It has a mean of 70 and a standard deviation of 10



What percentage of results lie in the shaded region?

(A) 16%

(B) 32%

(C) 34%

(D) 68%

7. The acceleration of a particle moving in a straight line is given by the formula $a = 12t + 6$. Initially the particle is at $x = 5$ metres and the initial velocity of the particle is −36 m/s. When is the particle at rest?

(A) $t = 0$

(B) $t = 1$

(C) $t = 2$

(D) $t = 3$

8. The equation of least-squares line of best fit is given by $y = mx + c$ where

$$m = r\frac{S_y}{S_x} \text{ and } c = \bar{y} - m\bar{x}$$

What is the gradient of the least-squares line of best fit given $r = 0.561$, $S_x = 1.987$ and $S_y = 4.579$?

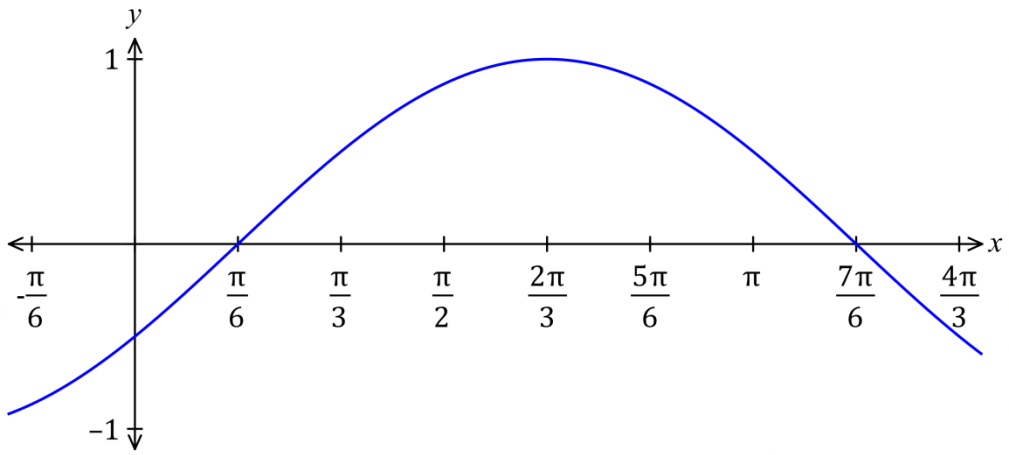
(A) 0.24

(B) 1.29

(C) 7.13

(D) 16.21

9.



A possible equation for the graph shown above is :

(A) $y = \cos\left(x + \frac{\pi}{6}\right)$

(B) $y = \sin\left(x - \frac{\pi}{6}\right)$

(C) $y = \sin\left(x + \frac{\pi}{6}\right)$

(D) $y = -\sin\left(x - \frac{\pi}{6}\right)$

10. An area is bounded by the curve $y = \frac{2}{3}\sqrt{9 - x^2}$ the coordinate axes and line $x = 2$.

What is an approximation for this area using the trapezoidal rule and three function values?

(A) 1.82

(B) 2.69

(C) 3.26

(D) 3.63

Section II

90 marks
Attempt questions 11 - 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

Simplify $\frac{y}{y^2-4}-\frac{2}{y-2}$ **2**

Question 12 (2 marks)

(a) Express $\sin\theta\cos\theta+\frac{\cos^3\theta}{\sin\theta}$ as a single trigonometric ratio. **1**

(b) Hence solve $\sin\theta\cos\theta+\frac{\cos^3\theta}{\sin\theta}=1$ for $0\leq\theta\leq 2\pi$. **1**

Question 13 (3 marks) **Marks**

Differentiate

(a) $\tan 5x$ **1**

(b) $\frac{\ln x}{x}$ **1**

(c) $x\cos x$ **1**

Question 14 (3 marks)

The second term of an arithmetic series is 39 and the sixth term is 19. **3**
What is the sum of the first ten terms?

Question 15 (2 marks)

Find the anti-derivative of $4 - x^{-3}$. **2**

Question 16 (5 marks)

Marks

(a) Sketch the graphs of $y = 4 - x^2$ and $y = 3$ on the same number plane. **2**

(b) The graph of $y = 3$ cuts the parabola at P and Q. **1**
What are the coordinates of P and Q?

(c) Calculate the area bounded by the graphs of $y = 4 - x^2$ and $y = 3$. **2**

Question 17 (2 marks)

Riley's class achieved a 72% mean and 8% standard deviation for their project work. What was Riley's mark if he achieved a z-score of –2.5? **2**

Question 18 (4 marks) **Marks**

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

Present value of \$1				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

Jessica borrows \$16 000 for a car. She arranges to repay the loan with monthly repayments over 4 years. She is charged 8.4% per annum interest.

(a) Find Jessica's monthly repayment. Answer to the nearest cent. **2**

(b) Calculate the amount of interest Jessica will pay over the term of the loan. Answer to the nearest dollar. **2**

Question 19 (2 marks)

Evaluate $\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x)\,dx$. Write your answer correct to three decimal places. **2**

Question 20 (3 marks) **Marks**

Sketch the graph of $f(x) = 3 - \dfrac{2}{x+1}$. **3**

Label all axis intercepts. Label each asymptote with its equation.

Question 21 (2 marks)

The variables profit made and amount spent on advertising are strongly correlated with a correlation coefficient r = 0.9. What conclusions can you draw from this information? **2**

Question 22 (3 marks) **Marks**

The mean for a class test is 64% and the standard deviation is 12.5.

(a) In a class test Marcus has a z-score of 2. What does that mean? **1**

(b) Fletcher has a mark of 51.5%. What is his z-score? **1**

(c) Ayla said she had a z-score of 3 but Hannah is unconvinced. Why? **1**

Question 23 (3 marks)

The displacement of an object at time (t) seconds is given by: **3**

$x = 3e^{-2t} + 10e^{-t} + 4t$

Find the time the object comes to rest.

Question 24 (5 marks) **Marks**

Lara borrows \$50,000 to purchase furniture for her small business. The interest is calculated monthly at a rate of 2% per month. She intends to repay the loan with interest in two annual instalments of \$$M$ at the end of the first and second years.

(a) Write an expression involving M for the total amount owed by Lara after 12 months, just after the first instalment of \$$M$ has been paid. **1**

(b) Show that $M = \dfrac{\$50\,000 \times 1.02^{24}}{1.02^{12} + 1}$ **2**

(c) What will be the total amount of interest paid on this loan? **2**

Question 25 (2 marks)

The probability density function for the continuous random variable X is: **2**

$$f(x) = \begin{cases} x^3 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of $\mathrm{E}(X)$?

Question 26 (5 marks)

Marks

(a) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \leq \theta \leq \pi$. **2**

(b) Find the values of x for which $\sin x = 1 - \cos x$ in the domain $0 \leq \theta \leq \pi$. **1**

(c) Find the area between $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \leq \theta \leq \pi$. **2**

Question 27 (2 marks)

What is the equation of the normal to the curve $y = x^2 - 4x$ at the point (1,–3)? **2**

Question 28 (7 marks) **Marks**

A function $f(x)$ is defined by $f(x) = 7 + 4x^3 - 3x^4$.

(a) Find the coordinates of the stationary points for the curve $y = f(x)$. **2**

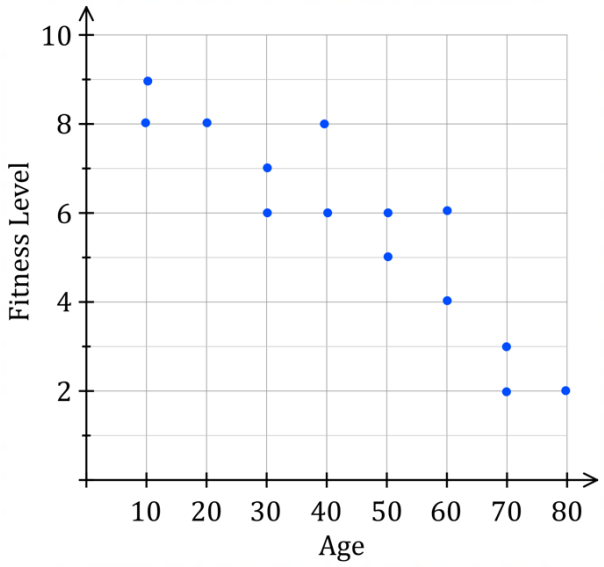
(b) Find all values of x for which $f''(x) = 0$. **1**

(c) Determine the nature of the stationary points. **2**

(d) Sketch the graph of $y = f(x)$ for the domain $-1 \leq x \leq 2$. **2**

Question 29 (5 marks) **Marks**

The scatterplot below shows the relationship between age and fitness level.



(a) Draw a line of best fit on the scatterplot. Find the gradient of this line. **2**

(b) Lachlan is 30 years old. What is his expected fitness level? **1**

(c) Calculate the value of the Pearson's correlation coefficient. Answer correct to two decimal places. **2**

Question 30 (2 marks)

State the domain and range of $f(x) = \sqrt{1 - x^2}$. **2**

Question 31 (2 marks) **Marks**

The probability density function for the continuous random variable X is: **2**

$$f(x) = \begin{cases} \frac{1}{9}(4x - x^2) & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find value of $P(X \leq 2)$.

Question 32 (3 marks)

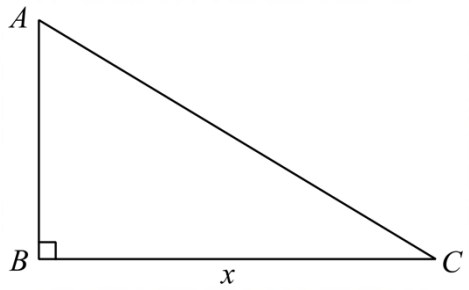
Determine the equation of a curve given by $\frac{d^2y}{dx^2} = 12x + 6$ and (1, –2) as a stationary point on the curve. **3**

Question 33 (2 marks)

Simplify $\lim_{x \to 0} \frac{\sin 6x}{x}$ **2**

Question 34 (7 marks) **Marks**

A cable of length 3 metres is to be bent to form the hypotenuse and base of a right-angled triangle *ABC*. Let the length of the base *BC* is *x* metres.



(a) What is the length of the hypotenuse *AC* in terms of *x*? **1**

(b) Show that the area of the triangle *ABC* is $0.5x\sqrt{9-6x}$. **2**

(c) What value of *x* gives the maximum possible area of the triangle? **3**

(d) Find the maximum possible area of the triangle. **1**

Question 35 (3 marks) **Marks**

How many solutions exist for x of the equation $e^x + x + 2 = 0$? **3**
Hint: Draw graphs.

Question 36 (3 marks)

The number of students absent from year 12 for the past nine days was as follows:
15, 18, 14, 17, 18, 13, 12, 29, 20

(a) What is the mean? Answer correct to one decimal place. **1**

(b) Find the interquartile range? **1**

(c) Is 29 an outlier for this set of data? Justify your answer with calculations. **1**

Question 37 (6 marks) **Marks**

Consider the function $f(x) = (2x - 3)^4$.

(a) Find the value of $f'(1)$. **2**

(b) Find equation of the tangent at the point (1, 1) to the curve $y = (2x - 3)^4$. **1**

(b) The equation of the tangent cuts the x-axis at A and the y-axis at B. **3**
Find the area of ΔOAB, where O is the origin.

End of paper

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

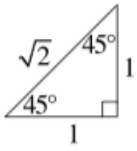
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

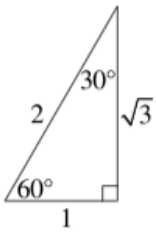
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

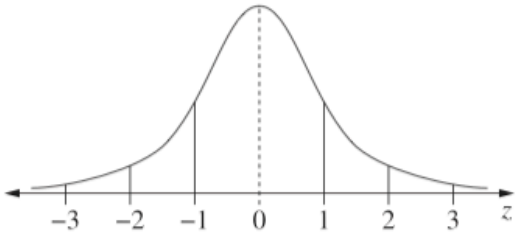
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$