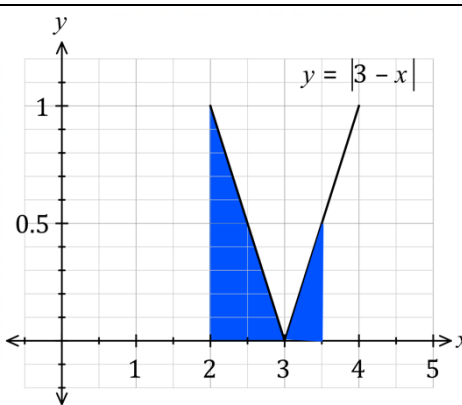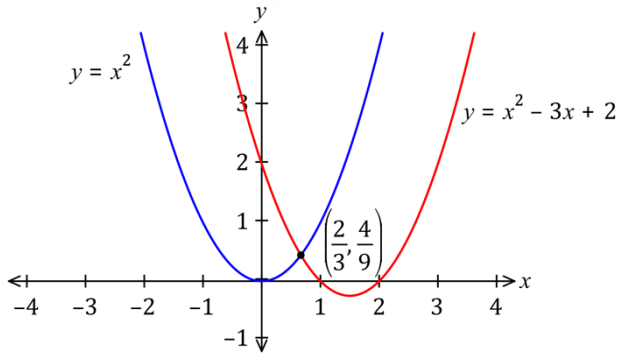**ACE Examination Paper 1**
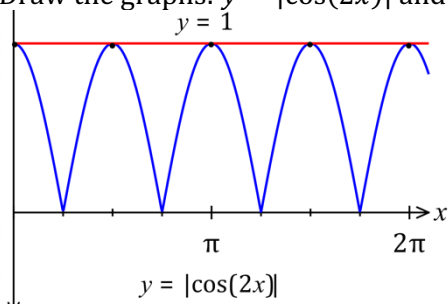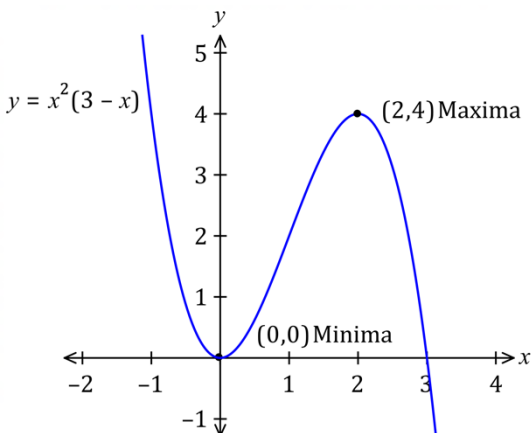**Year 12 Mathematics Advanced Yearly Examination**
Worked solutions and Marking guidelines
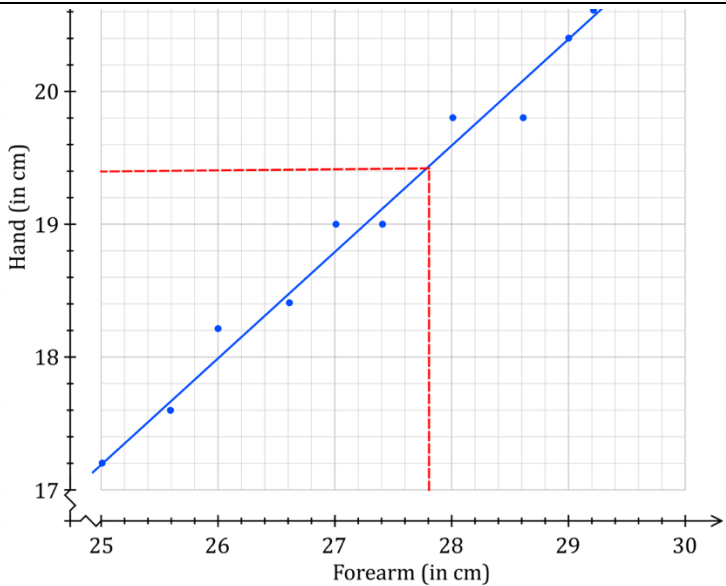
Section I		
	Solution	**Criteria**
1.	$2\cos^2 x - 1 = 0$ $$\cos^2 x = \frac{1}{2} \text{ or } \cos x = \pm\frac{1}{\sqrt{2}}$$ $$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$	1 Mark: D
2.	$f'(x) > 0$ (increasing) $f''(x) < 0$ (concave down)	1 Mark: A
3.	$$z = \frac{x - \bar{x}}{s}$$ $$= \frac{1000 - 900}{50}$$ $$= 2$$ 95% of scores have a z-score between −2 and 2 $\therefore$ 2.5% have a z-score greater than 2.	1 Mark: B
4.	$$\int_0^1 (e^{3x} + 1)dx = \left[\frac{1}{3}e^{3x} + x\right]_0^1$$ $$= \left(\frac{1}{3}e^3 + 1\right) - \frac{1}{3}$$ $$= \frac{1}{3}(e^3 + 2)$$	1 Mark: D
5.	$y = (x - a)(x^2 - 1)$ $\quad = x^3 - ax^2 - x + a$ $\frac{dy}{dx} = 3x^2 - 2ax - 1$ Gradient at the point when $x = -2$ $m = 3 \times (-2)^2 - 2a \times (-2) - 1$ $\quad = 4a + 11$	1 Mark: C
6.	Correlation between 0.5 and 0.74. $\therefore$ Moderate positive.	1 Mark: D
7.	Period $= \frac{\pi}{n} = \frac{\pi}{2}$ $\therefore n = 2$ Also a translation of $\frac{\pi}{4}$ in the positive x-direction. $f(x) = \tan\left(2\left(x - \frac{\pi}{4}\right)\right)$	1 Mark: B
8.	Increasing function $f'(x) > 0$ $\{x : -3 < x < 2\}$	1 Mark: C
9.	Intersection value is 5.4172 (3% and 6 years) $PV = 5.4172 \times 12\,000$ $\quad = \$65\,006.40$	1 Mark: C
10.	Fundamental property of a probability density is that for any value of x, the value of $f(x)$ is non-negative. $\therefore$ Graph (A) has $f(x) < 0$	1 Mark: A

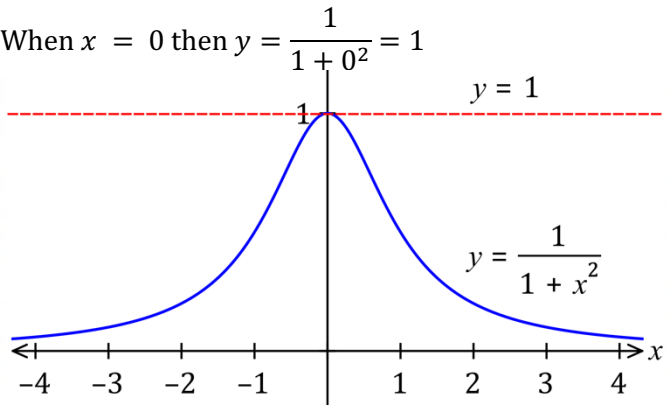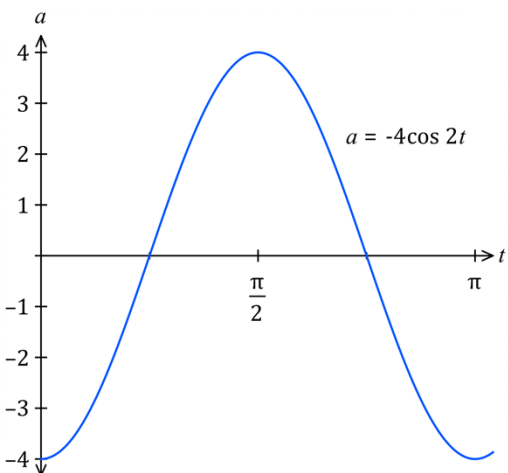| Section II		
11(a)	$f(x) = (\sin x + x^2)$ $f'(x) = \cos x + 2x$	1 Mark: Correct answer.
11(b)	$f(x) = \ln(x^2 + 1)$ $f'(x) = \dfrac{2x}{x^2 + 1}$	1 Mark: Correct answer.
12(a)	$a = 4$ and $d = 5$ for 4, 9, 14, 19, ... $T_n = a + (n-1)d$ $\quad = 4 + (n-1) \times 5$ $\quad = 5n - 1$	1 Mark: Correct answer.
12(b)	$T_{25} = 5 \times 25 - 1$ $\quad\ = 124$	1 Mark: Correct answer.
12(c)	$S_n = \dfrac{n}{2}[2a + (n-1)d]$ $\quad = \dfrac{100}{2}[2 \times 4 + (100-1) \times 5]$ $\quad = 25\,150$	1 Mark: Correct answer.
13(a)	 $P(2 \le X \le 3.5) = \dfrac{1}{2} \times 1 \times 1 + \dfrac{1}{2} \times 0.5 \times 0.5$ $\qquad\qquad\qquad = 0.625$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
13(b)	$P(2 \le X \le 2.5) = \dfrac{1}{2} \times 0.5 \times (1 + 0.5)$ $\qquad\qquad\qquad = 0.375$	2 Marks: Correct answer. 1 Mark: Shows understanding.
14(a)	$\dfrac{d}{dx}(2e^x \cos x) = 2e^x(-\sin x) + \cos x\, 2e^x$ $\qquad\qquad\quad = 2e^x(\cos x - \sin x)$	2 Marks: Correct answer. 1 Mark: Applies the product rule.
14(b)	$\dfrac{d}{dx}\left(\dfrac{\tan x}{x}\right) = \dfrac{x \times \sec^2 x - \tan x \times 1}{x^2}$ $\qquad\qquad\quad = \dfrac{x\sec^2 x - \tan x}{x^2}$	2 Marks: Correct answer. 1 Mark: Applies the quotient rule.
15	$\displaystyle\int (2x+3)^{10}\, dx = \dfrac{(2x+3)^{11}}{11 \times 2} + C$ $\qquad\qquad\qquad\quad = \dfrac{(2x+3)^{11}}{22} + C$	1 Mark: Correct answer.

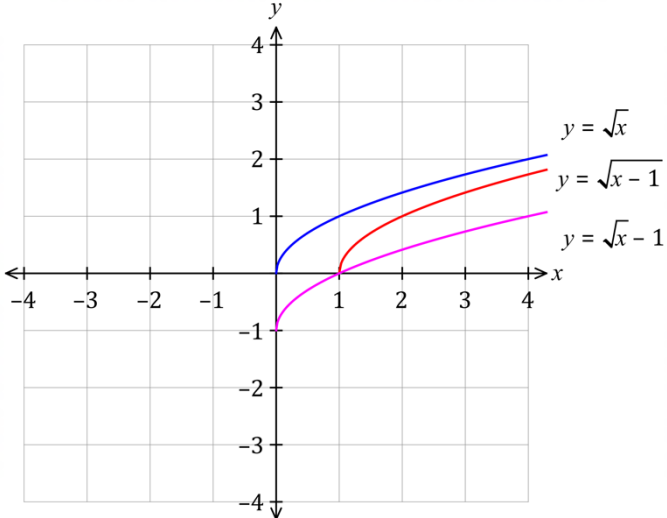| 16(a)	$z = \dfrac{x - \bar{x}}{s}$ $-1 = \dfrac{x - 12.5}{0.5}$ $x = (-1 \times 0.5) + 12.5$ $= 12$ $\therefore$ Minimum weight to be accepted is 12 kg.	1 Mark: Correct answer.
16(b)	$z = \dfrac{x - \bar{x}}{s}$ $2 = \dfrac{x - 12.5}{0.5}$ $x = (2 \times 0.5) + 12.5$ $= 13.5$ $\therefore$ Maximum weight to be accepted is 13.5 kg.	1 Mark: Correct answer.
17	Solving the two equations simultaneously. $x^2 + 1 = 3x + 1$ $x^2 - 3x = 0$ $x(x - 3) = 0$ $\therefore$ Point of intersection occurs when $x = 0$ and $x = 3$. $A = \displaystyle\int_0^3 (3x + 1) - (x^2 + 1)\,dx$ $= \displaystyle\int_0^3 (3x - x^2)\,dx = \left[\dfrac{3x^2}{2} - \dfrac{x^3}{3}\right]_0^3$ $= \left[\left(\dfrac{3 \times 3^2}{2} - \dfrac{3^3}{3}\right) - \left(\dfrac{3 \times 0^2}{2} - \dfrac{0^3}{2}\right)\right]$ $= \dfrac{9}{2}$ square units	2 Marks: Correct answer. 1 Mark: Finds the points of intersection or shows some understanding of the problem.
18	$y = e^{kx}\sin x$ $\dfrac{dy}{dx} = e^{kx} \times \cos x + \sin x \times ke^{kx}$ $\quad = e^{kx}(\cos x + k\sin x)$ $\qquad\qquad \dfrac{dy}{dx} - 3y = e^{kx}\cos x$ $e^{kx}(\cos x + k\sin x) - 3e^{kx}\sin x = e^{kx}\cos x$ $\qquad ke^{kx}\sin x - 3e^{kx}\sin x = 0$ $\qquad\qquad e^{kx}\sin x(k - 3) = 0$ $\qquad\qquad\qquad\qquad k = 3$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds the derivative.
19(a)	$A = \dfrac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$ $= \dfrac{1.5}{2}[2 + 0 + 2(4.5 + 5.1 + 3.6)]$ $= 21.3 \text{ m}^2$ $\therefore$ Area of the native garden is approximately 21.3 m².	2 Marks: Correct answer. 1 Mark: Uses trapezoidal rule.
19(b)	Now 25 mm = 0.025 m $V = Ah$ $= 21.3 \times 0.025$ $= 0.5325 \text{ m}^3 = 532.5 \text{ L}$ $\therefore 532.5$ L of water fell in the native garden.	1 Mark: Correct answer.

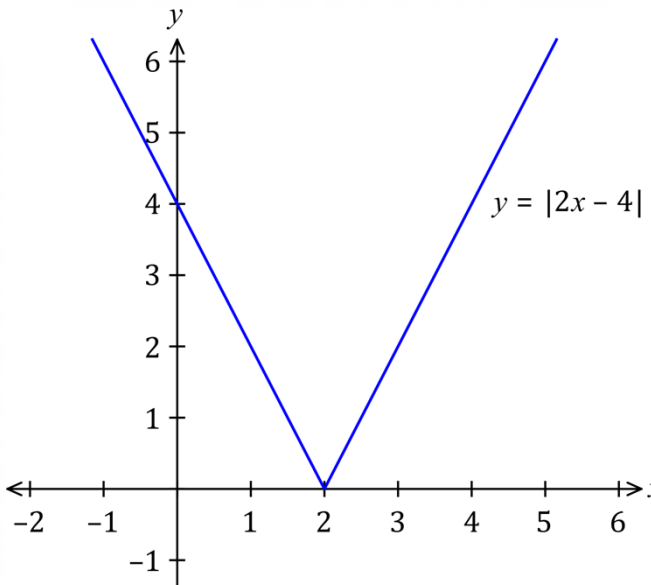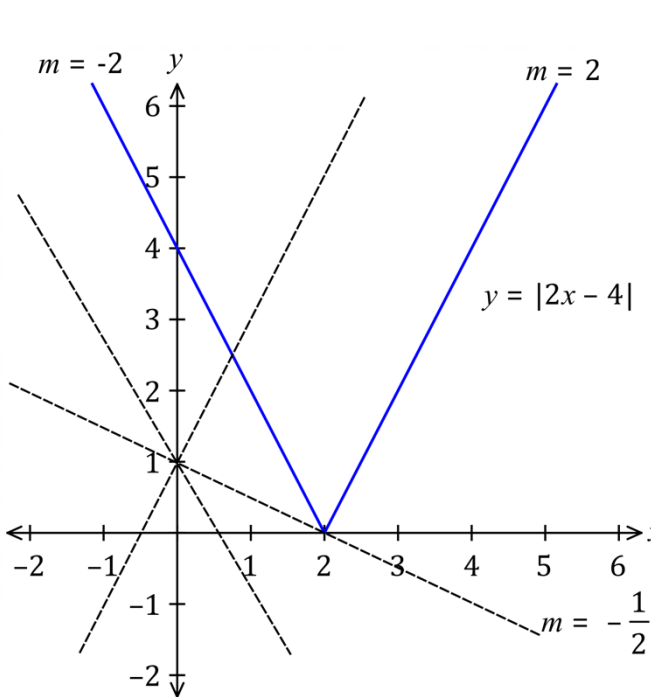| 20(a)	$y = x^2 - 3x + 2 = (x-1)(x-2)$	2 Marks: Correct answer. 1 Mark: One graph drawn correctly.
20(b)	Solve simultaneously to find the point of intersection $x^2 = x^2 - 3x + 2$ $3x = 2$ $x = \dfrac{2}{3}$ Therefore $x^2 > x^2 - 3x + 2$ when $x > \dfrac{2}{3}$	1 Mark: Correct answer.
21	Amplitude = 3 Period $= \dfrac{2\pi}{\dfrac{\pi}{2}} = 4$	2 Marks: Correct answer. 1 Mark: Finds either amplitude or the period.
22(a)	Students with a z-score of –2 is two standard deviations below the mean $(60 - (2 \times 10) = 40$. $\therefore$ A score of 40 has a z-score of –2 .	1 Mark: Correct answer.
22(b)	z-score for 65 $z = \dfrac{x - \bar{x}}{s}$ $\quad = \dfrac{65 - 60}{10}$ $\quad = 0.5$ $\therefore z$-score is 0.5	1 Mark: Correct answer.
23	$\displaystyle\int_0^{\frac{\pi}{8}} (\sec^2 2x)\,dx = \left[\dfrac{1}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$ $= \dfrac{1}{2}\left(\tan\dfrac{\pi}{4} - \tan 0\right)$ $= \dfrac{1}{2}$	2 Marks: Correct answer. 1 Mark: Finds the primitive function or shows some understanding.
24	Draw the graphs: $y = |\cos(2x)|$ and $y = 1$ $\therefore$ There are 5 solutions.	2 Marks: Correct answer. 1 Mark: Shows some understanding.

25(a)	$f(x) = x^2(3 - x) = 3x^2 - x^3$ Stationary points $f'(x) = 0$ $f'(x) = 6x - 3x^2$ $3x(2 - x) = 0$ $x = 0, x = 2$ $\therefore$ Stationary points are $(0, 0)$ and $(2, 4)$ $f''(x) = 6 - 6x$ At $(0, 0), f''(0) = 6 > 0$ Minima At $(2, 4), f''(2) = -6 < 0$ Maxima	2 Marks: Correct answer. 1 Mark: Finds one of the stationary points or recognises $6x - 3x^2 = 0$.
25(b)	x-intercepts $(y = 0)$ $x^2(3 - x) = 0$ $x = 0, x = 3$ 	2 Marks: Correct answer. 1 Mark: Makes some progress towards sketching the curve.
25(c)	$f'(x) = 6x - 3x^2$ Gradient of the tangent at the point $P(1,2)$ $m = 6 \times 1 - 3 \times 1^2 = 3$ $y - y_1 = m(x - x_1)$ $y - 2 = 3(x - 1)$ $y = 3x - 1$ or $3x - y - 1 = 0$	1 Mark: Correct answer.
26	$r = \dfrac{0.06}{12} = 0.005$ $D = 810$ and $V_0 = 58\,000$ Recurrence relation $V_{n+1} = V_n \times (1 + r) - D$ $= V_n \times 1.005 - 810$	2 Marks: Correct answer. 1 Mark: Substitutes one correct value into the recurrence relation.
27(a)	$A = 10e^{-kt}$ $3.6 = 10e^{-k \times 5}$ $e^{-5k} = 0.36$ $-5k\ln e = \ln 0.36$ $k = \dfrac{\ln 0.36}{-5}$ $= 0.2043\dots$ ≈ 0.204	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution

27(b)	$A = 10e^{-kt}$ $1 = 10e^{-0\cdot204\ldots\times t}$ $e^{-0\cdot204\ldots\times t} = 0.1$ $-0.204 \times t \times \ln e = \ln 0.1$ $t = \dfrac{\ln 0.1}{-0.204\ldots}$ $= 11.2689\ldots$ $\approx 11.3 \text{ hours}$ $\therefore$ One kilogram of chlorine dissolves after 11.3 hours.	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution
28	$T_n = ar^{n-1}$ $T_3 = ar^2 = 1.25$ ① $T_7 = ar^6 = 20$ ② Dividing the two equations $\dfrac{ar^6}{ar^2} = \dfrac{20}{1.25}$ $r^4 = 16$ $r = \pm 2$ $T_7 = a \times (\pm 2)^6 = 20$ $a = \dfrac{20}{64} = \dfrac{5}{16}$ $\therefore$ First term is $\dfrac{5}{16}$	2 Marks: Correct answer. 1 Mark: Finds two equations using the nth term of a GP or shows some understanding.
29(a)		1 Mark: Correct answer.
29(b)	See line of best fit on the above scatterplot.	1 Mark: Correct answer.
29(c)	When forearm length = 27.8 then hand length = 19.4 cm (from the scatterplot) $\therefore$ Charlotte's hand length should be 19.4 cm.	1 Mark: Correct answer.
29(d)	Use the calculator to find Pearson's correlation coefficient. $r = 0.990691\ldots$ ≈ 0.9907	2 Marks: Correct answer. 1 Mark: Finds a value of r close to 0.99.

30(a)	$FV = PV(1 + r)^n$	1 Mark: Correct answer.
	$\quad = 1000(1 + 0.05)^{100}$	
	$\quad = 131\,501.257\dots$	
	$\quad \approx \$131\,501$	
	$\therefore$ World vision will receive $\$131\,501$	
30(b)	$A_{100} = 1000(1.05)^{100} + 1000(1.05)^{99} + \cdots + 1000(1.05)^1$	2 Marks: Correct answer.
	GP with $a = 1000(1.05)$, $r = 1.05$ and $n = 100$	
	$A_{100} = \dfrac{1000(1.05)[1.05^{100} - 1]}{1.05 - 1}$	1 Mark: Identifies a G.P. with 100 terms.
	$\quad = 2740526.41\dots$	
	$\quad \approx \$2\,740\,526$	
	$\therefore$ World vision will receive $\$2\,740\,526$ after 100 years.	
31(a)	$\displaystyle\int_{-1}^{2} x^2 + 1\,dx = \left[\dfrac{x^3}{3} + x\right]_{-1}^{2}$	1 Mark: Correct answer.
	$\qquad\qquad\qquad = \left[\left(\dfrac{2^3}{3} + 2\right) - \left(\dfrac{-1^3}{3} + (-1)\right)\right]$	
	$\qquad\qquad\qquad = 6$	
31(b)	$\displaystyle\int_{-1}^{4} \sqrt{3x + 4}\,dx = \left[\dfrac{2}{9}(3x + 4)^{\frac{3}{2}}\right]_{-1}^{4}$	2 Marks: Correct answer.
	$= \dfrac{2}{9} \times \left[\left((3 \times 4 + 4)^{\frac{3}{2}}\right) - \left((3 \times (-1) + 4)^{\frac{3}{2}}\right)\right]$	1 Mark: Finds the primitive function.
	$= 14$	
32(a)	Intersection value is 4.51 (8% and 4 years)	1 Mark: Correct answer.
	$FV = 4.51 \times 32\,000$	
	$\quad = \$144\,320$	
32(b)	Intersection value is 4.25 (4% and 4 years)	1 Mark: Correct answer.
	$FV = 4.25 \times 6300$	
	$\quad = \$26\,775$	
33(a)	$f(x) = \dfrac{1}{1 + x^2} = (1 + x^2)^{-1}$	2 Marks: Correct answer.
	$f'(x) = -(1 + x^2)^{-2} \times 2x$	
	$\quad\quad = \dfrac{-2x}{(1 + x^2)^2}$	1 Mark: Shows some understanding.

33(b)	The tangent has the same gradient as the x-axis (parallel) The x-axis has a gradient of 0 (horizontal line) $$f'(x) = \frac{-2x}{(1+x^2)^2} = 0$$ $$-2x = 0$$ $$x = 0$$ When $x = 0$ then $y = \dfrac{1}{1+0^2} = 1$ $\therefore$ Point is $(0, 1)$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent or makes some progress.
34(a)	$$x = \int (1 - 2\sin 2t)\,dt$$ $$= t + \cos 2t + C$$ Initially $t = 0$ and $x = 0$ $$0 = 0 + \cos(2 \times 0) + C$$ $$C = -1$$ $$\therefore x = t + \cos 2t - 1$$	2 Marks: Correct answer. 1 Mark: Integrates the velocity function.
34(b)	When $t = \dfrac{\pi}{3}$ then $$x = \frac{\pi}{3} + \cos\left(2 \times \frac{\pi}{3}\right) - 1$$ $$= \frac{\pi}{3} - \frac{1}{2} - 1 = \frac{\pi}{3} - \frac{3}{2}$$	1 Mark: Correct answer.
34(c)	$$a = \frac{d}{dt}(1 - 2\sin 2t)$$ $$= -4\cos 2t$$	1 Mark: Correct answer.
34(d)	$a = -4\cos 2t$ for $0 \le t \le \pi$.	2 Marks: Correct answer. 1 Mark: Draws the general shape of the curve.
34(e)	$-1 \le \cos 2t \le 1$ $-4 \le -4\cos 2t \le 4$ (or from the graph) $\therefore$ Maximum acceleration is 4 ms^{-2}	1 Mark: Correct answer.

35		3 Marks: Correct answer. 2 Marks: Draws two of the graphs correctly 1 Mark: Shows some understanding.
36	Class A total number of marks $75.5 \times 24 = 1812$. Class B total number of marks $80.5 \times 28 = 2254$ $\text{Mean} = \dfrac{1812 + 2254}{24 + 28}$ $\qquad = 78.1923 \dots \%$ $\qquad \approx 78.2\%$ $\therefore$ Mean mark for both classes is 78.2%	2 Marks: Correct answer. 1 Mark: Makes so
37(a)	$P = 2x + \dfrac{1}{2} \times \pi \times y + \dfrac{1}{2} \times \pi \times 2y$ $4.8 = 2x + \dfrac{1}{2}\pi \times 3y$ $9.6 = 4x + 3\pi y$ $y = \dfrac{9.6 - 4x}{3\pi}$	2 Marks: Correct answer. 1 Mark: Finds an expression for the perimeter.
37(b)	Express the speed in terms of x $S = 200 - \left(\dfrac{x^3}{27} + \dfrac{\pi}{6}y\right)$ $\quad = 200 - \left(\dfrac{x^3}{27} + \dfrac{\pi}{6} \times \dfrac{9.6 - 4x}{3\pi}\right)$ $\quad = 200 - \dfrac{x^3}{27} - \dfrac{9.6 - 4x}{18}$ $\dfrac{dS}{dx} = -\dfrac{3x^2}{27} + \dfrac{4}{18}$ Maximum length of the straight occurs when $\dfrac{dS}{dx} = 0$ $-\dfrac{3x^2}{27} + \dfrac{4}{18} = 0$ $3x^2 = 6$ $x = \sqrt{2}$ km Check When $x = \sqrt{2}$ km then $\dfrac{d^2S}{dx^2} = -\dfrac{6x}{27} = -\dfrac{6 \times \sqrt{2}}{27} < 0$ (Maxima)	3 Marks: Correct answer. 2 Marks: Finds the length of the straight for maximum speed. 1 Mark: Differentiates the S formula with respect to x.

38(a)		2 Marks: Correct answer. 1 Mark: Draws the general shape or shows some understanding.
38(b)	 From the graph $m < -2$ or $m \geq 2$ or $m = -\dfrac{1}{2}$	2 Marks: Correct answer. 1 Mark: Finds one of the solutions.
39	Assessment results increase as height increases. Low positive correlation. Not a strong relationship.	2 Marks: Correct answer. 1 Mark: Shows understanding
40	$z = \dfrac{x - \bar{x}}{s}$ $ = \dfrac{179 - 167}{12}$ $ = 1$ 68% of scores have a z-score between –1 and 1. $\therefore 32\% \div 2 = 16\%$ have a z-score greater than 1.	2 Marks: Correct answer. 1 Mark: Finds the z-score.