A C E
EXAM
PAPER

Student name: _____

PAPER 1

YEAR 12
YEARLY
EXAMINATION

Mathematics Standard 2

General Instructions

- Working time - 150 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 15 marks

- Attempt Questions 1-15
- Allow about 25 minutes for this section

Section II – 85 marks

- Attempt all questions in Section II
- Allow about 2 hours and 5 minutes for this section

Section I

15 marks
Attempt questions 1 - 15
Allow about 25 minutes for this section

Use the multiple-choice answer sheet for questions 1-15

1. Sarah and Nathan wish to buy a block of land and are considering these two options.

Town A
Block of land
18 m by 30 m
$243 000

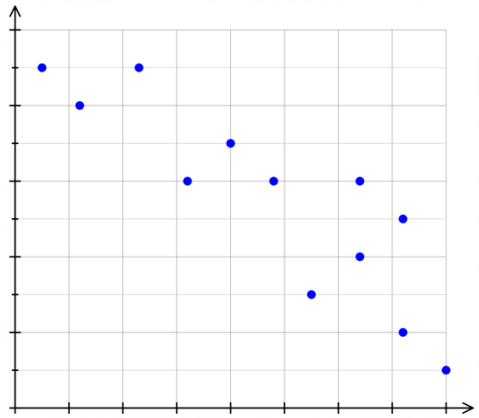
Town A
Block of land
20 m by 40 m
$340 000

What is the difference between the two options in cost per square metre of land?

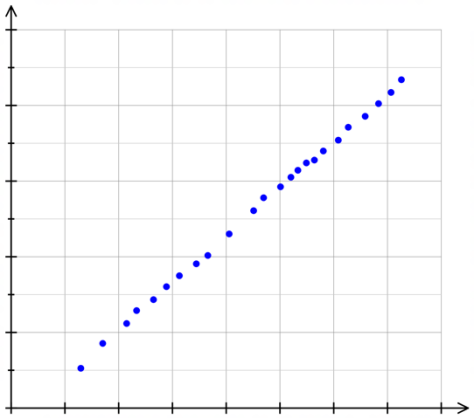
(A) $25
(B) $260
(C) $1000
(D) $97 000

2. The correlation coefficient for two quantities was –0.5. Which scatterplot could represent this result?

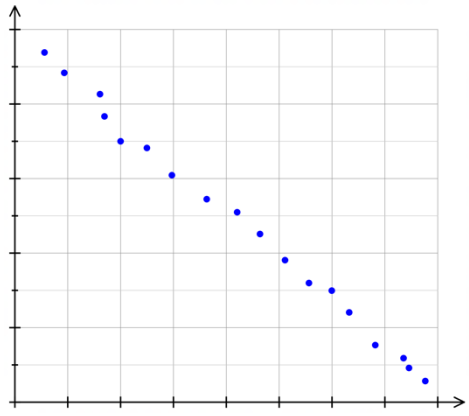
(A)



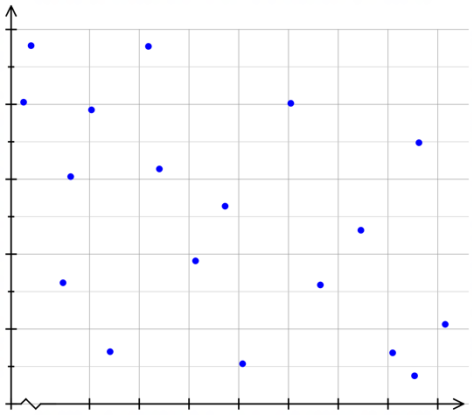
(B)



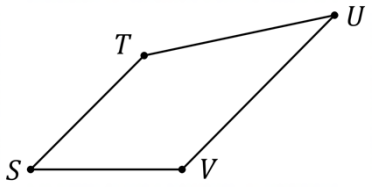
(C)



(D)



3.



Which of the following walks is a path in the above network diagram?

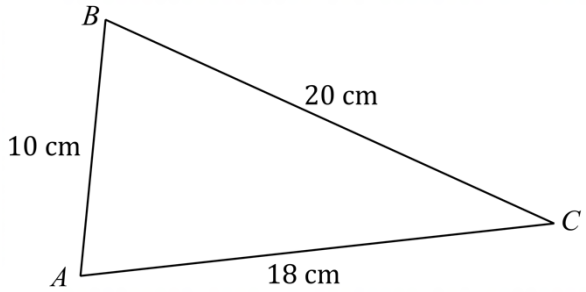
(A) S–T–S–V

(B) S–T–U–V

(C) S–T–V–S

(D) S–T–U–V–S

4.



Not to scale

What is the size of $\angle BAC$ to the nearest degree?

(A) 30°

(B) 64°

(C) 80°

(D) 86°

5. The scale on an aerial photograph is given as 1 mm = 180 m. If the length of land is 240 m, what is the map length between these points?

(A) 1.33 mm

(B) 2.67 mm

(C) 3.20 mm

(D) 60 mm

6. A loan is modelled by the recurrence relation $V_{n+1} = V_n \times 1.004 - 360$ where V_n is the balance of the loan after n payments and $V_0 = 72\,000$. What is the balance of the loan after three payments? Answer correct to the nearest whole number.

(A) $71 710

(B) $71 783

(C) $71 855

(D) $71 928

7. In a normally distributed set of scores, the mean is 60 and the standard deviation is 8. Approximately, what percentage of the scores will lie between 44 and 76?

   (A) 34%

   (B) 68%

   (C) 95%

   (D) 99.7%

8. Michael invests \$3125 at 6% per annum compounding quarterly.
   How much will he have after 4 years? Answer to the nearest dollar.

   (A) \$3317

   (B) \$3945

   (C) \$3966

   (D) \$7939

9. What is the point of intersection of the lines $y = x + 4$ and $y = -x + 4$?

   (A) $(0, 0)$

   (B) $(0, 4)$

   (C) $(4, 0)$

   (D) $(3, 4)$

10. Andrew was driving at a speed of 70 km/h and reaction time of 0.50 seconds. What is the stopping distance using the formula below?

    $$d = \frac{5vt}{18} + \frac{v^2}{170}$$

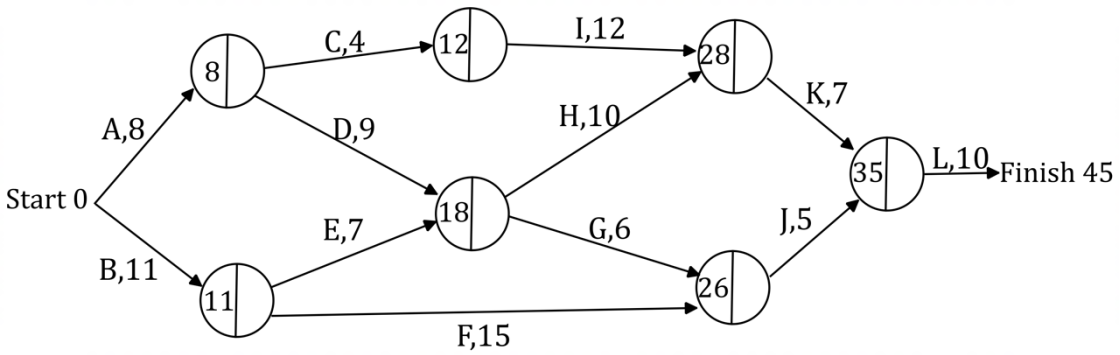
    (A) 12 m

    (B) 24 m

    (C) 39 m

    (D) 44 m

11.



What is the float time of activity *I*?

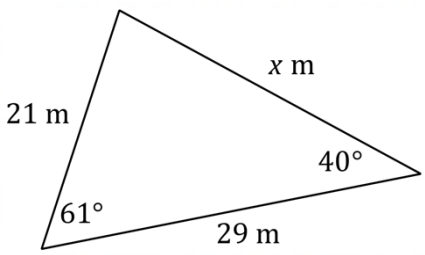
(A) 4

(B) 6

(C) 8

(D) 12

12.



Not to scale

What is the correct expression for x in triangle ABC?

(A) $x = \dfrac{29\sin 79°}{\sin 61°}$

(B) $x = \dfrac{21\sin 40°}{\sin 61°}$

(C) $x = \dfrac{29\sin 61°}{\sin 79°}$

(D) $x = \dfrac{21\sin 40°}{\sin 79°}$

13. Olive obtained a personal loan of \$30 000. She made a deposit of \$2200 and agreed to payments of \$820 per month for 4 years. What is the total amount paid for the loan?

(A) \$9360

(B) \$11 560

(C) \$39 360

(D) \$41 560

14. The heights of a group of friends are normally distributed with a mean of 160 cm and a standard deviation of 15 cm. What percentage of the group are more than 190 cm tall?

(A) 1%

(B) 2.5%

(C) 5%

(D) 95%

15. The number of people in a town is given by $N = 1000(2.5^t)$ where N is the number of people and t is the time in years. What is the population after 2 years?

(A) 1581

(B) 2500

(C) 5000

(D) 6250

Section II

85 marks
Attempt all questions
Allow about 2 hours and 5 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 16 (3 marks) **Marks**

An energy company charges for gas over a 3-month period are shown below.

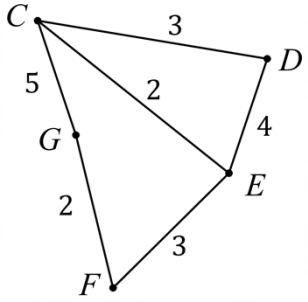
Usage charge	First 2000 MJ	\$0.02580 per MJ
	Additional MJ over 2000	\$0.01620 per MJ

(a) Savannah used 5000 MJ of gas in this period. What is the charge? **1**

(b) Gabriel has decided to reduce his energy bills. He has a target of \$80 for gas. What is the maximum number of MJ he is allowed in this period? Answer correct to the nearest megajoule. **2**

Question 17 (2 marks)

The network diagram below shows the possible paths (in km) for laying gas pipes between five locations. **2**



What is the minimum length of pipes required to provide gas to all locations?

Question 18 (4 marks) **Marks**

The table below shows the present value of a $1 annuity.

Present value of $1						
Period	2%	4%	6%	8%	10%	12%
1	0.98	0.96	0.94	0.93	0.91	0.89
2	1.94	1.89	1.83	1.78	1.74	1.69
3	2.88	2.78	2.67	2.58	2.49	2.40
4	3.81	3.63	3.47	3.31	3.17	3.04

(a) What would be the present value of a $6 000 per year annuity at 4% per annum for 2 years, with interest compounding yearly? **1**

(b) What is the value of an annuity that would provide a present value of $47 988 after 3 years at 8% per annum compound interest? **2**

(b) An annuity of $1000 each six months is invested at 12% per annum, compounded biannually for 2 years. What is the present value of the annuity? **1**

Question 19 (2 marks)

Jack scored 66% in the first assessment task for which the mean was 82% and the standard deviation was 8. In the second assessment task the mean was 71% and standard deviation was 10. Jack scored 61%. **2**

Did Jack improve? Justify your answer.

Question 20 (4 marks) **Marks**

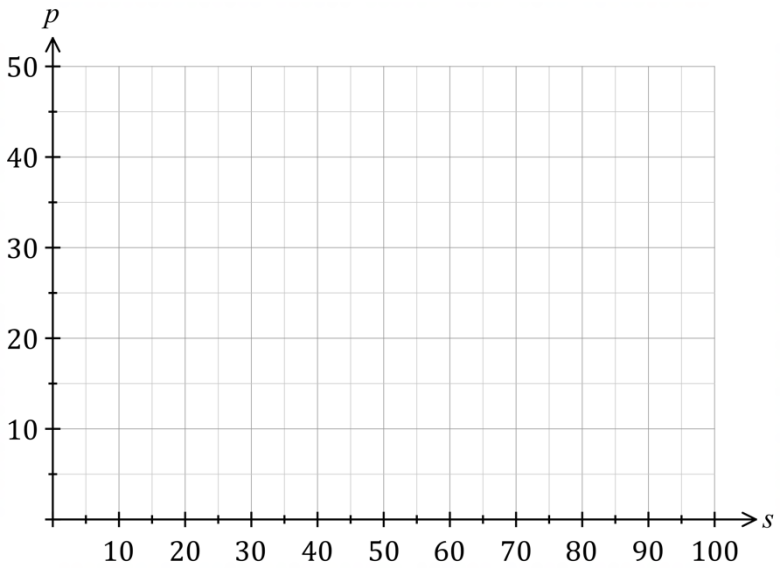
The petrol consumption (p litres per 100 km) and the speed of a car (s km/h) are modelled by the formula:

$p = 0.01s^2 - s + 33$

(a) Complete the following table of values. **1**

s	0	20	40	50	60	80	100
p							

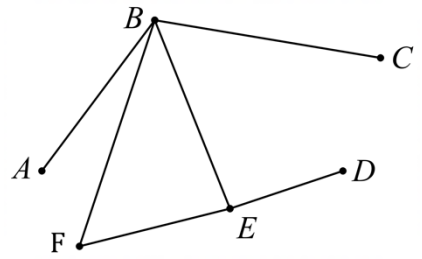
(b) Draw the graph of $p = 0.01s^2 - s + 33$ using the number plane below. **1**



(c) A car was driven at 30 km/h for 40 km. How many litres of petrol did it use? **1**

(d) Why is the formula $p = 0.01s^2 - s + 33$ not a good model if $s = 0$? **1**

Question 21 (2 marks) **Marks**



(a) Complete the table of vertex degrees for the network diagram. **1**

Vertex	*A*	*B*	*C*	*D*	*E*	*F*
Degree						

(b) Is there a path in the network that visits every edge exactly once? Give a reason for your answer. **1**

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Question 22 (3 marks)

Mitchell purchased a used car for \$16 000. It depreciated by 20% per annum and is expected to be used for 10 years.

(a) What is the salvage value of the car after two years? **1**

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(b) How many years will it take for the salvage value of the car be less than \$4 000? Answer to the nearest whole year. **2**

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Question 23 (2 marks)

The cost per passenger of hiring a bus is inversely proportional to the number of passengers on the bus. If there are twenty passengers, the cost per passenger is \$14. What is the cost per passenger when there are fifteen passengers? **2**

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Question 24 (2 marks) **Marks**

Claire gained a standardised score (z-score) of 2.5 for a class test out of 100.

(a) Describe Claire's result in terms of mean and standard deviation of the class test. **1**

(b) The class test has a mean of 56% and a standard deviation of 9.5. **1**

What is the actual mark scored by Claire?

Question 25 (3 marks)

Edward borrows $220 000 over 7 years at an interest rate of 9.5% p.a. reducible. He pays $1910 per fortnight.

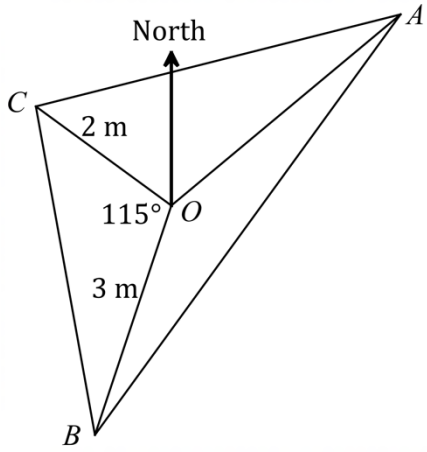
(a) How much will Edward pay back altogether? **1**

(b) What is the interest paid for this loan? **1**

(c) What is the equivalent flat interest rate charged per annum on this loan? Answer correct to 1 decimal place. **1**

Question 26 (5marks) **Marks**

The diagram below shows the position of A, B and C relative to O.
In the diagram A is NE of O, C is NW of O, ΔCOB is 115°, CO is 2 m and BO is 3m.



Not to scale

(a) What is the true bearing of A from O? **1**

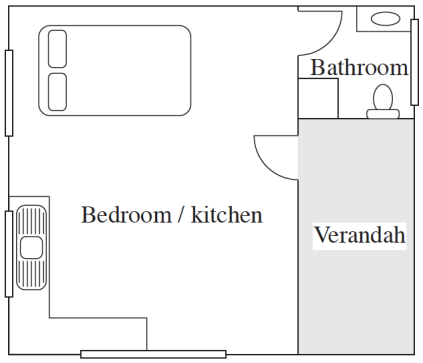
(b) What is the true bearing of B from O? **1**

(c) What is the distance from B to C? Answer correct to two decimal places. **2**

(d) What is the area of triangle BOC? Answer correct to two decimal places. **1**

Question 27 (3 marks) **Marks**

A building plan for an extension is shown below. It uses a scale of 1:100.



(a) What is the symbol used for the door? **1**

(b) What are the dimensions of the verandah? **1**
Answer correct to one decimal place.

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(c) Calculate the area of the extension. **1**
Answer correct to the nearest square metre.

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Question 28 (3 marks)

Audrey owns a credit card that has no annual fee and charges 16.3% p.a. interest on all purchases. The interest is charged on the date of purchase and the date of payment.

(a) Show that the daily interest rate is 0.04466%. **1**

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(b) On the 27th of January, Audrey bought a TV for $1029 using her credit card. Audrey paid her credit card account on the 7th of February. What was the total amount she paid for the TV, including interest? Answer correct to the nearest cent. **2**

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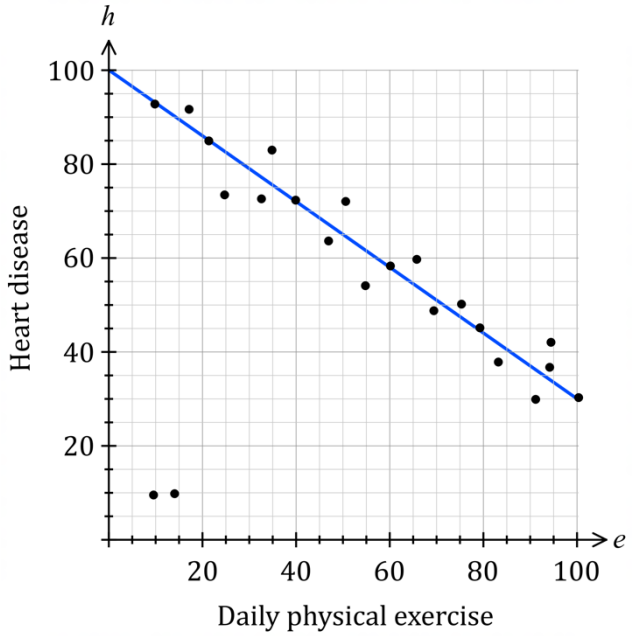
Question 29 (3 marks) **Marks**

The scatterplot shows daily physical exercise (e) versus heart disease (h).

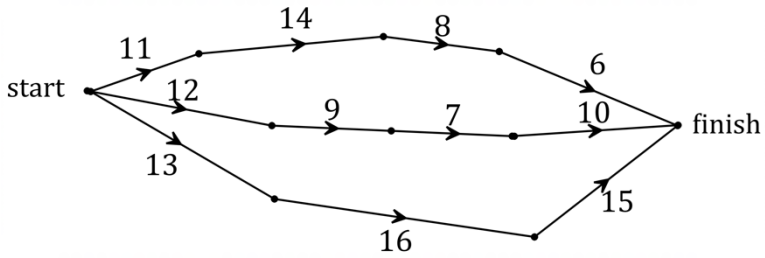


(a) Calculate the gradient of the line. **1**

(b) What is the equation of the line of best fit drawn? **1**

(c) Estimate the value of the correlation coefficient. **1**

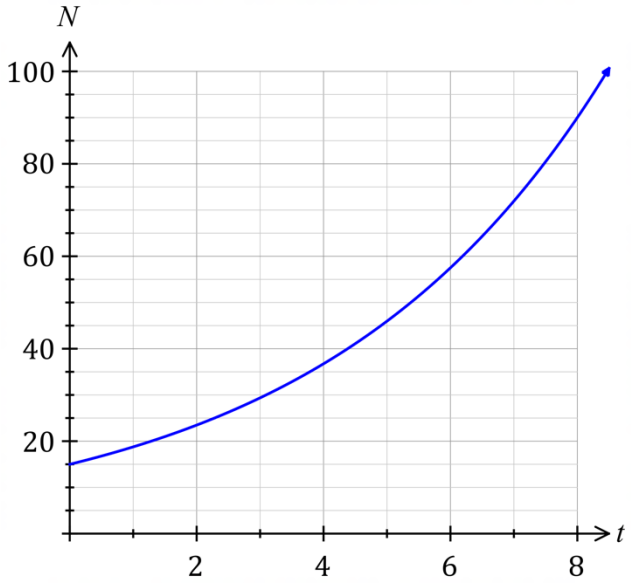
Question 30 (3 marks)



What is the maximum flow for this network? **3**

Question 31 (3 marks) **Marks**

The graph below shows the exponential increase in bacteria where N is the number of bacteria in thousands after t hours.



(a) What is the initial number of bacteria? **1**

(b) Estimate the time taken for the number of bacteria to reach 45 000. **1**

(c) Estimate the time taken for the number of bacteria to double its initial size. **1**

Question 32 (2 marks)

A class compared their assessment results to their head circumference. The correlation coefficient for these quantities was 0.2. What is the meaning of this correlation? **2**

Question 33 (2 marks) **Marks**

A machine creates metal disks with mean diameter of 4.50 cm and a standard deviation of 0.03 cm. The diameters of these metal disks are normally distributed.

(a) State the interval where the mean diameter of the metal disks will almost certainly lie? **1**

(b) A metal disk is produced at random with a diameter of 4.62 cm. **1**

Why is the manager concerned?

Question 34 (2 marks)

The table below shows the future value of an annuity with a contribution of $1

Future value of $1				
Period	1%	4%	8%	12%
1	1.0000	1.0000	1.0000	1.0000
3	3.0301	3.1216	3.2464	3.3744
5	5.1010	5.4163	5.8666	6.3528

(a) Find the future value of $16 000 invested at the end of each year for 3 years at 8% p.a. compounding annually? Answer correct to the nearest whole number. **1**

(b) Find the future value of $2100 invested at the end of each month for 5 months at 12% p.a. compounding monthly? Answer correct to the nearest whole number. **1**

Question 35 (5 marks) **Marks**

Items with a different mass (m in kg) are attached to a spring. The length of the spring (L in cm) is measured for each item. The results are shown below.

m	2	5	8	11	14	17
L	41.2	55.0	68.8	82.6	96.4	110.2

(a) A linear model in the form $L = km + 32$ describes this situation. What is the value of k? **1**

(b) What is the length of the spring when no item is attached? **2**

(c) Calculate the mass of an item that will make the spring 78 cm long? **2**

Question 36 (2 marks)

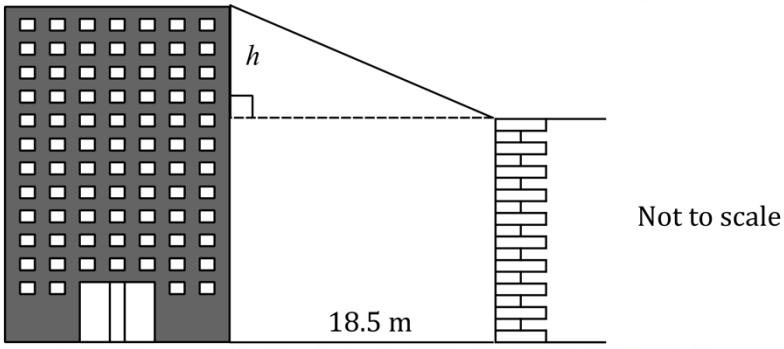
2

	W	X	Y	Z
W	–	3	9	8
X	3	–	6	1
Y	9	6	–	2
Z	8	1	2	–

Represent the table shown above as a weighted network.

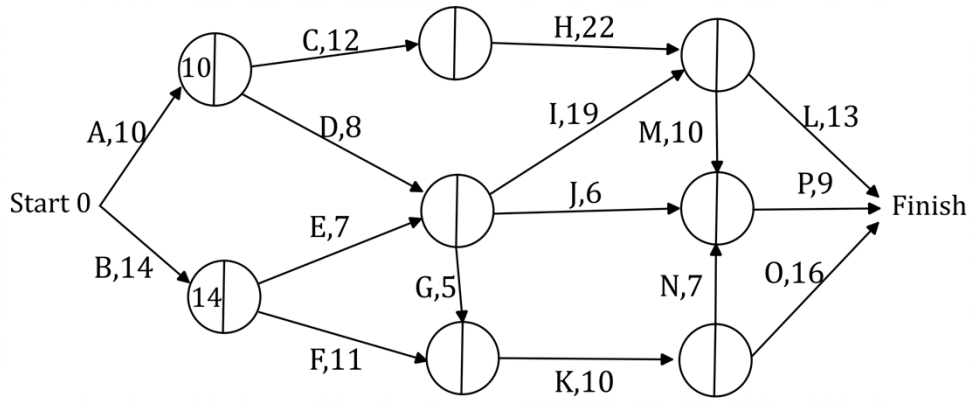
Question 37 (2 marks) **Marks**

The two buildings below are standing on level ground. The horizontal distance between the buildings is 18.5 metres and the angle of elevation between the buildings is 32˚. **2**



What the difference in height (*h)* between the buildings? Answer correct to one decimal place.

Question 38 (4 marks)

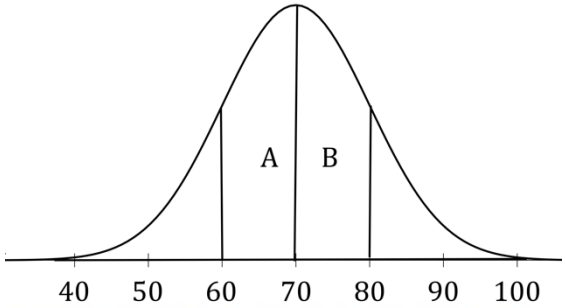


(a) Write the earliest starting times (EST) and latest staring times (LST) on the above network diagram. **3**

(b) What is the critical path? **1**

Question 39 (4 marks) **Marks**

The normal distribution below represents the mass of 400 students. It has a standard deviation of 10 kg. All measurements are in kilograms.



(a) What is the weight of a student with a z-score of –2? **1**

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(b) How many students have a mass in the region marked with an A? **1**

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(c) How many students will have a mass less than 100 kg? **2**

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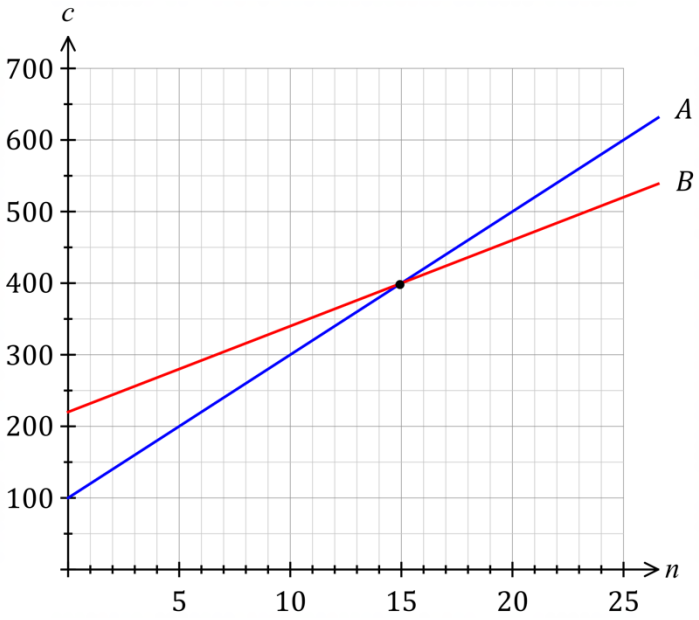
Question 40 (3 marks)

Activity	*Duration (min)*	*Immediate predecessors*
A	12	–
B	17	*A*
C	27	*A*
D	14	*B*
E	7	*C*
F	12	*D, E*

Construct a network diagram using the activity chart. Show the earliest staring times (EST) and latest staring times (LST). **3**

Question 41 (4 marks) **Marks**

The graph shows the cost charged by two different businesses to cater for a party. In each case the total cost ($\$c$) depends on the number of people attending (n).



(a) For what number of people attending do the two businesses charge the same amount? **1**

(b) If ten people attend the party, what business would you recommend? Justify you answer. **1**

(c) If 25 people are to attend the party, what is the difference in the cost per person between the two businesses? **2**

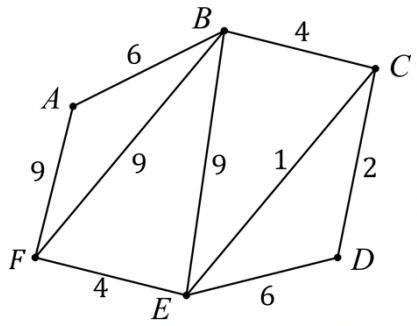
Question 42 (2 marks)

A scatterplot showing the profit made by a worker for different amounts of output. A line of best fit is drawn and its equation found to be $P = 0.5n + 4.5$, where P is the profit in dollars and n is the number of units produced. How much profit does the equation give for a worker producing one hundred million units? Do you think this an accurate prediction. Explain. **2**

Question 43 (2 marks) **Marks**

Fertiliser was added to a garden at a rate of 100 g/m^2. The fertiliser consists of nitrogen, phosphorus and potassium in the ratio of 7 : 6 : 12. How many grams of each element is that per square metre? **2**

Question 44 (4 marks)



(a) Find the length of the shortest path from A to E. **2**

(b) Find a walk that visits every edge of the network only once, starting at C. **2**

End of paper

NSW Education Standards Authority

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Standard 1
Mathematics Standard 2

REFERENCE SHEET

Measurement

Precision

Absolute error = $\frac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement – absolute error

Length, area, surface area and volume

$$l = \frac{\theta}{360} \times 2\pi r$$

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(x + y\right)$$

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV\left(1 + r\right)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0\left(1 - r\right)^n$$

Statistical Analysis

$$z = \frac{x - \bar{x}}{s}$$

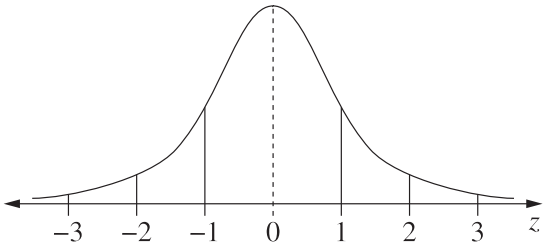
An outlier is a score

less than $Q_1 - 1.5 \times IQR$

or

more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3