CARLINGFORD HIGH SCHOOL

DEPARTMENT OF MATHEMATICS

Year 11 Extension 1 Mathematics Examination

Term 1 Week 11B 2019

Time allowed : ***50 Minutes***

Student Number : ______________________________

Instructions

- Start each question on a **new Booklet**
- Board approved calculators may be used
- Show all necessary working by using blue / black pen except graphs / diagrams
- Marks may be deducted for untidy setting out

Topics	Question 1	Question 2	Question 3	Total
Further Functions & Inverse Functions	/ 14			**/ 14**
Polynomials		/ 14		**/ 14**
Graphing Functions			/ 12	**/ 12**
Total	**/ 14**	**/ 14**	**/ 12**	**/40**

QUESTION 1 **(14 marks)** **- START A NEW BOOKLET -**

a). Solve the inequality $|2x + 5| \leq 1$ and graph your solutions on a number line. [2]

b). Solve the inequality $x^2 - 3x - 10 > 0$ and graph your solutions on a number line. [3]

c). Solve the inequality $\frac{x-2}{2-3x} \geq -\frac{2}{3}$ and graph your solutions on a number line. [3]

d). Given the function $f(x) = x^2 - 2x$.

i). Explain why, without a restricted domain the inverse would not be a function. [1]

ii). What is the largest possible domain (including $x = 3$) that $f(x)$ can be restricted to, so that its inverse is a function? [1]

iii). Find the equation of the inverse function $f^{-1}(x)$. [2]

iv). Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same graph, show important features. [2]

QUESTION 2 **(14 marks)** **- START A NEW BOOKLET –**

a). By using the division process show that the first polynomial is exactly divisible by the second polynomial.

i). $(x^3 + 6x^2 + 4x - 16)$, $(x + 4)$. [2]

ii). $(x^5 + x^3 - 2x)$, $(x - 1)$. [2]

b). Given $f(x) = ax^3 + bx^2 - 3$, where a and b are constants.

i). $f(x)$ is divisible by $(x - 1)$. When $f(x)$ is divided by $(x + 1)$, a remainder is -2. Find the values of a and b. [2]

ii). What is the maximum number of roots that $f(x)$ could have ? and why ? [2]

c). Use the factor theorem to fully factorise the polynomial $x^3 - 4x^2 + x + 6$. [3]

d). If α, β and γ are the roots of $2x^3 + 8x - 1 = 0$ find the value of $\alpha^2 + \beta^2 + \gamma^2$. [3]

QUESTION 3 **(12 marks)** **- START A NEW BOOKLET –**

a). Find the Cartesian equation of the curve whose parametric equations are $x = t + 1$ and $y = 2t^2 - 3$. **[2]**

b). **Detach the last page and sketch your graphs for part b) i), ii), iii) and part c) i) on the given graphs.**

The graph of $f(x)$ is given on the next page,
sketch the following graphs on top of $f(x)$.

i). $y = \dfrac{1}{f(x)}$ **[2]**

ii). $y^2 = f(x)$ **[2]**

iii). $y = |f(x)|$ **[2]**

c). The graph of $f(x)$ and $c(x)$ are given on the next page.

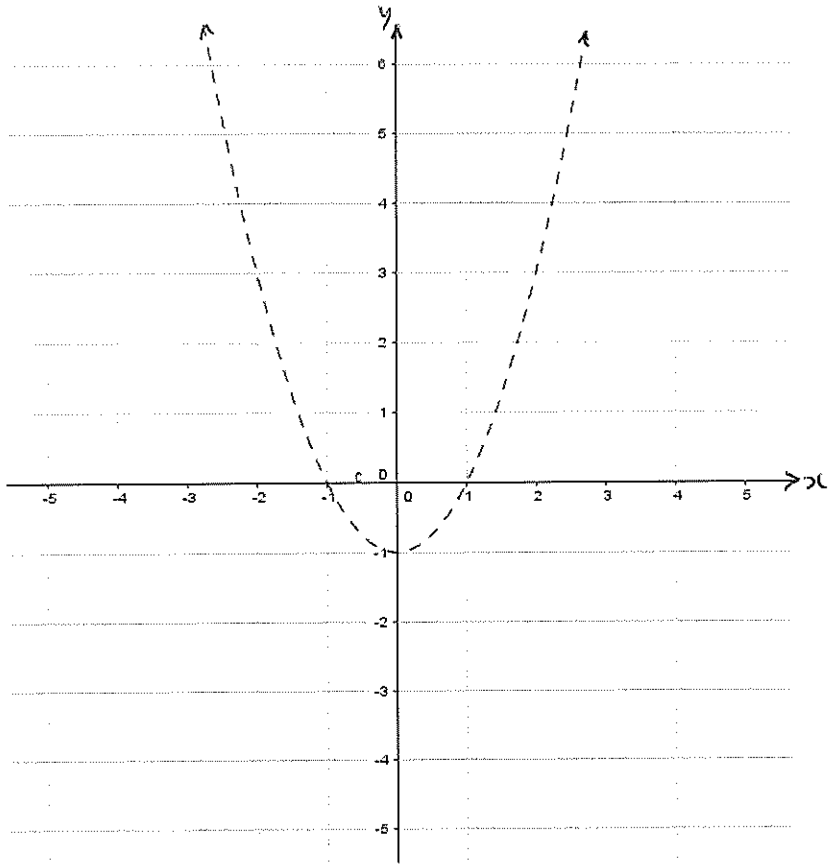
i). Sketch the graph of $g(x) = f(x) \times c(x)$ on the same diagram. **[2]**

ii). Why must $g(x)$ pass through the origin ? **[1]**

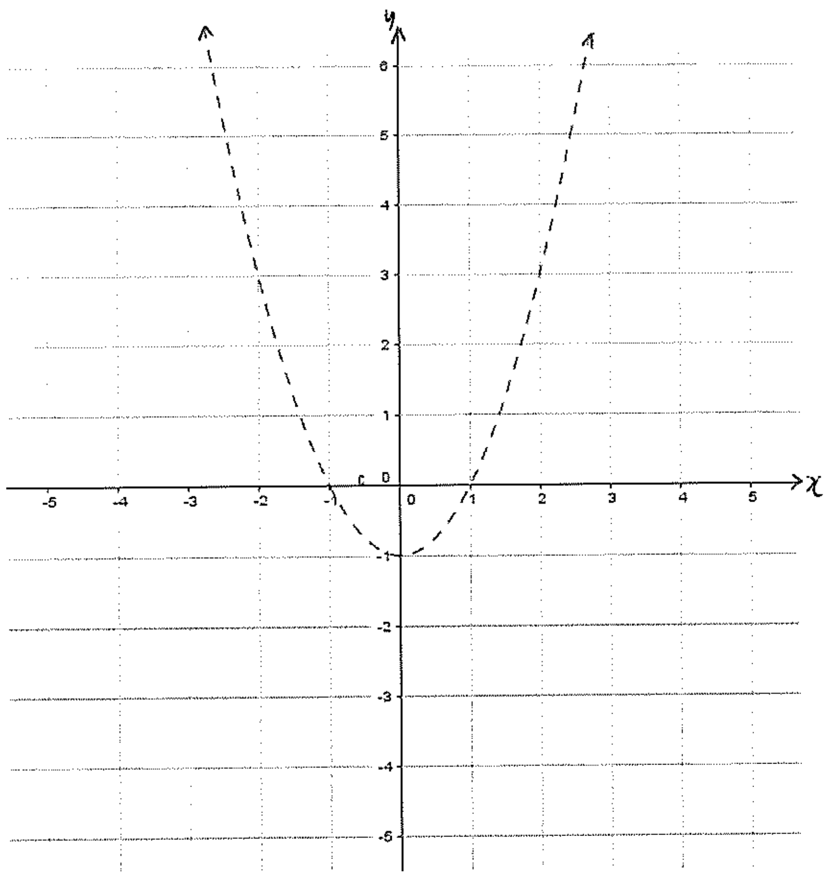
iii). What would be the degree of $g(x)$? **[1]**

END OF EXAM

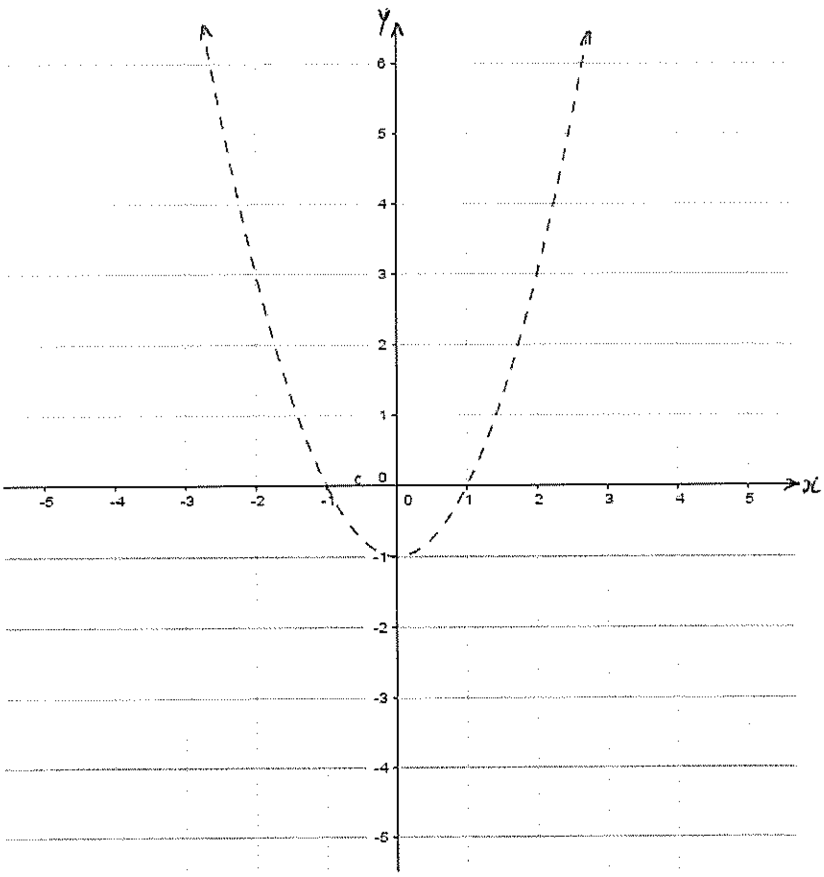
b).
i).



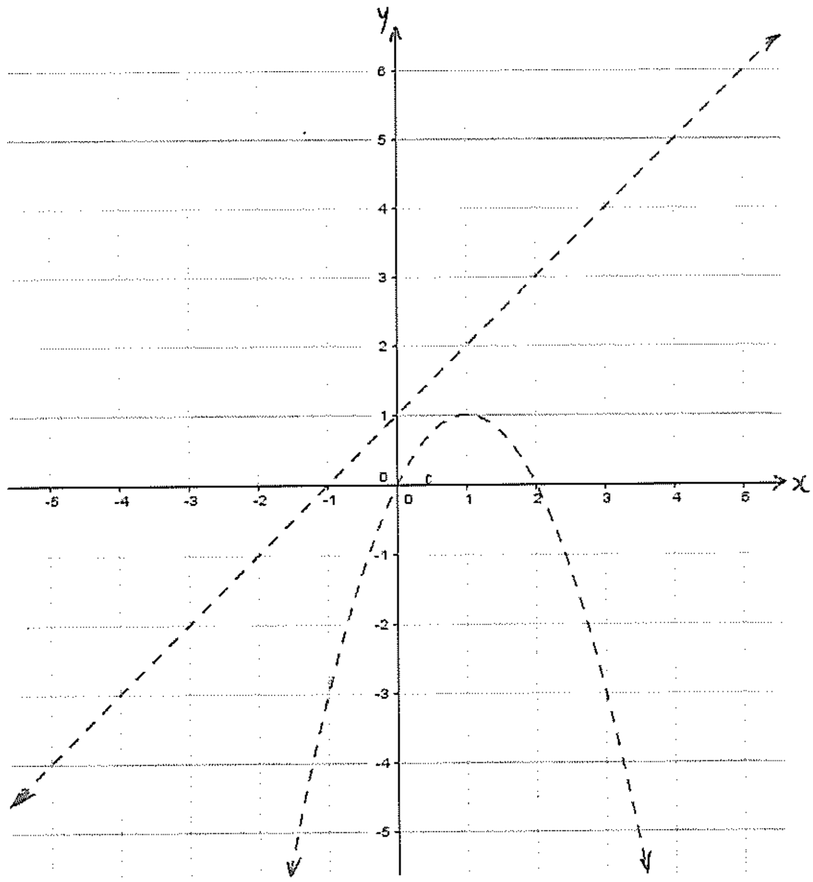
ii).



b).
iii).



c).
i).



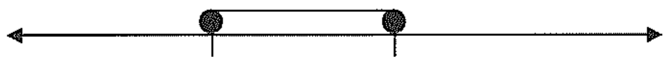
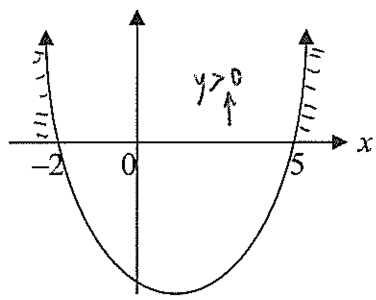
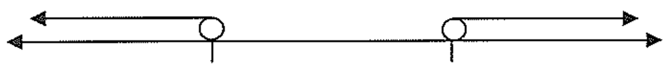
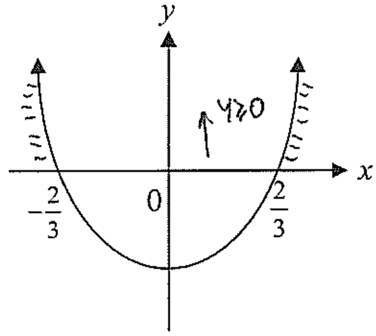
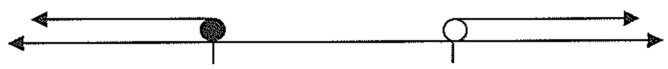
CARLINGFORD HIGH SCHOOL

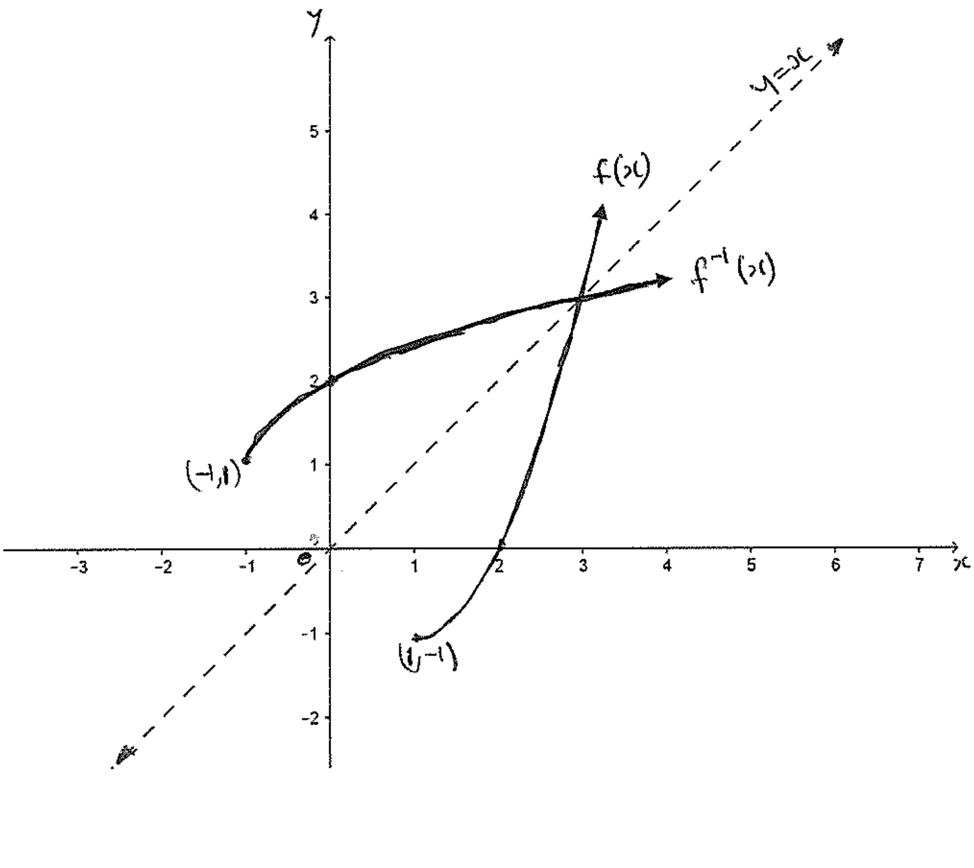
TERM 1 TEST

WEEK 11B 2019

Mathematics - Extension 1

SOLUTIONS

Question 1	Mathematics Extension 1		
Part	Solution	Marks	Comment
a).	Now $\lvert 2x+5\rvert \le 1$ can be solved as $-1 \le 2x+5 \le 1$ $-6 \le 2x \le -4$ $-3 \le x \le -2$ Graph the solutions on the number line. 	**2**	**1** mark for correct answer. **1** mark for correct number line.
b).	Now $x^2 - 3x - 10 > 0$ can be factorized as $(x-5)(x+2) > 0$ From the graph we have the solutions as $x < -2$ or $x > 5$  Graph the solutions on the number line. 	**3**	**1** mark for factorising. **1** mark for correct solutions from the graph. **1** mark for the correct number line.
c).	Now $\frac{x-2}{2-3x} \ge -\frac{2}{3}$ can be solved by multiplying both sides with $(2-3x)^2$ where $x \neq \frac{2}{3}$. i.e. $(2-3x)^2 \times \frac{x-2}{2-3x} \ge -\frac{2}{3} \times (2-3x)^2$ $3(2-3x)(x-2) \ge -2(2-3x)^2$ $3(2-3x)(x-2) + 2(2-3x)^2 \ge 0$ $(2-3x)[3(x-2) + 2(2-3x)] \ge 0$ $(2-3x)(-3x-2) \ge 0$ From the graph we have the solutions as $x \le -\frac{2}{3}$ or $x > \frac{2}{3}$  Graph the solutions on the number line. 	**3**	**1** mark for x both sides with $(2-3x)^2$. **1** mark for correct solutions from the graph. **1** mark for the correct number line.

d). i).	If the domain is not restricted then we will not have a **one-to-one** function.	**1**	**1** mark for the correct reason.
ii).	Now $f(x) = x^2 - 2x$ can be factorized as $f(x) = x(x-2)$. $\therefore$ the x-intercepts are 0 & 2, so the axis of symmetry is $x = 1$. $f(1) = 1^2 - 2(1)$ $= -1$ Thus the largest possible restricted domain is $x \geq 1$.	**1**	**1** mark for the correct domain.
iii).	f: Let $y = x^2 - 2x$ then f^{-1}: $x = y^2 - 2y$ $x + 1 = y^2 - 2y + 1$ $x + 1 = (y-1)^2$ $y - 1 = \pm\sqrt{x+1}$ $y = 1 \pm \sqrt{x+1}$ $\therefore f^{-1}(x) = 1 + \sqrt{x+1}$ (because of $x \geq 1$)	**2**	**1** mark for completing the square. **1** mark for the correct inverse.
iv).		**2**	**1** mark for $f(x)$. **1** for $f^{-1}(x)$

Question 2	Mathematics Extension 1			
Part	Solution		Marks	Comment
a). i).	x^2+2x-4 $x+4\overline{)x^3+6x^2+4x-16}$ $\underline{x^3+4x^2}$ $2x^2+4x$ $\underline{2x^2+8x}$ $-4x-16$ $\underline{-4x-16}$ 0	So $x^3+6x^2+4x-16$ is exactly divisible by $x+4$.	**2**	**1** mark for correct division. **1** mark for correct conclusion
ii).	$x^4+x^3+2x^2+2x$ $x-1\overline{)x^5+0x^4+x^3+0x^2-2x}$ $\underline{x^5-x^4}$ x^4+x^3 $\underline{x^4-x^3}$ $2x^3+0x^2$ $\underline{2x^3-2x^2}$ $2x^2-2x$ $\underline{2x^2-2x}$ 0	So x^5+x^3-2x is exactly divisible by $x-1$.	**2**	**1** mark for correct division. **1** mark for correct conclusion
b). i).	If $f(x)=ax^3+bx^2-3$ is divisible by $(x-1)$ then $f(1)=0$ i.e. $a(1)^3+b(1)^2-3=0$ So $\boldsymbol{a+b=3}$ …………… [1] If $f(x)=ax^3+bx^2-3$ is divided by $(x+1)$, the remainder is -2. i.e. $f(-1)=-2$ $a(-1)^3+b(-1)^2-3=-2$ So $\boldsymbol{-a+b=1}$ …………… [2] From [1] + [2] get $2b=4$ $\therefore$ $\boldsymbol{b=2}$ Sub into [1] get $a+2=3$ $\therefore$ $\boldsymbol{a=1}$		**2**	**1** mark for the 2 equations. **1** mark for the values of a & b.
ii).	Maximum number of roots is 3, because the degree of $f(x)$ is 3.		**2**	**1** mark for max roots. **1** mark for degree.

<table>
<tr><td>Question 2</td><td colspan="3">Mathematics Extension 1</td></tr>
<tr><td>Part</td><td>Solution</td><td>Marks</td><td>Comment</td></tr>
<tr><td>c).</td><td>Let $f(x) = x^3 - 4x^2 + x + 6$

Try $f(-1) = (-1)^3 - 4(-1)^2 + (-1) + 6$

$\therefore \quad f(-1) = 0$

i.e. $(x + 1)$ is a factor of $f(x)$.

By long division we get

$x^3 - 4x^2 + x + 6 = (x + 1)(x^2 - 5x + 6)$
$= (x + 1)(x - 2)(x - 3)$

$$\begin{array}{r} x^2 - 5x + 6 \\ x+1 \overline{\big)\, x^3 - 4x^2 + x + 6} \\ \underline{x^3 + x^2 } \\ -5x^2 + x \\ \underline{-5x^2 - 5x } \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$</td><td>3</td><td>1 mark for first factor.

1 mark for division

1 mark for correct factors.</td></tr>
<tr><td>d).</td><td>Given the polynomial $2x^3 + 0x^2 + 8x - 1 = 0$

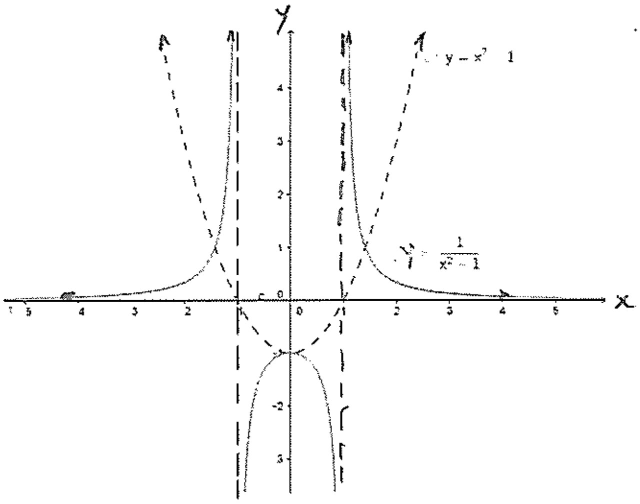
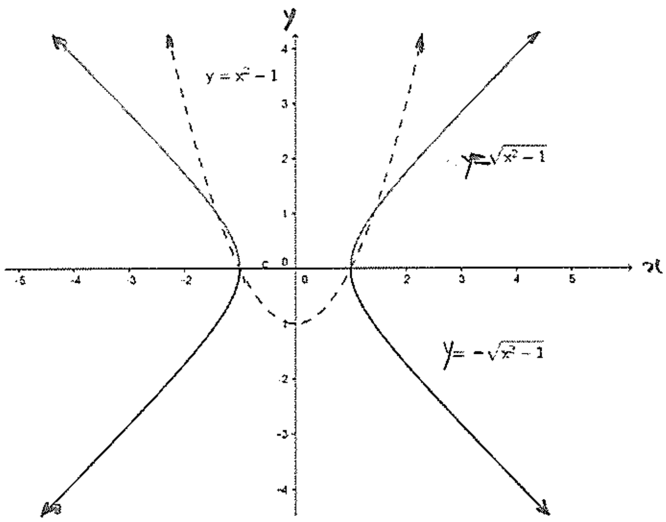
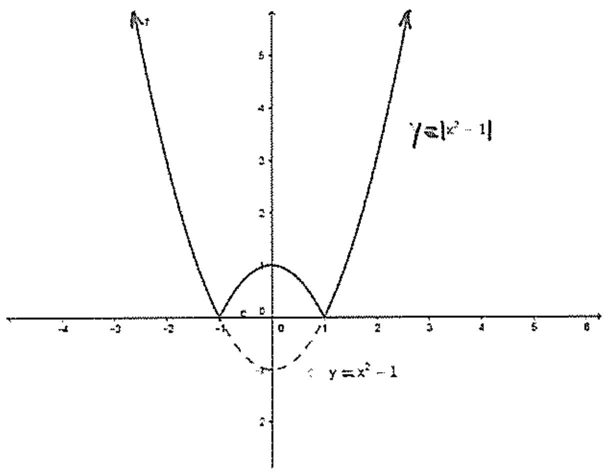
Now sum of roots one at a time: $\alpha + \beta + \gamma = -b/a = 0$

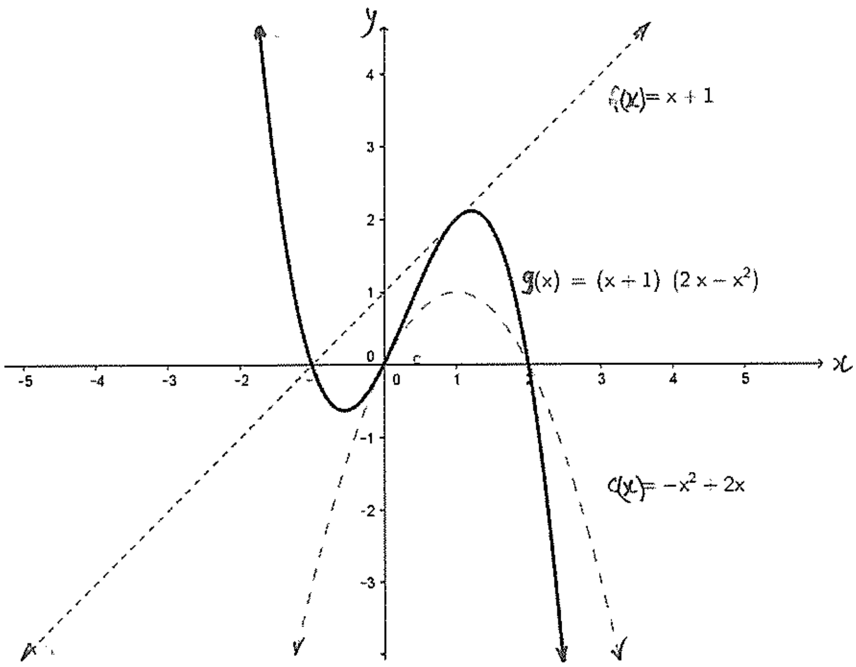
Now sum of roots two at a time: $\alpha\beta + \beta\gamma + \alpha\gamma = c/a = 8/2 = 4$

So $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$
$= 0 - 2 \times 4$
$= -8$</td><td>3</td><td>1 mark for $\alpha + \beta + \gamma$

1 mark for $\alpha\beta + \beta\gamma + \alpha\gamma$

1 mark for final answer.</td></tr>
</table>

Question 3	Mathematics Extension 1		
Part	Solution	Marks	Comment
a)	Given $x = t + 1$ .... [1] and $y = 2t^2 - 3$ .... [2] Make t the subject from [1] i.e. $t = x - 1$ .... [3] Now sub [3] into [2] get $y = 2(x-1)^2 - 3$ $= 2(x^2 - 2x + 1) - 3$ $= 2x^2 - 4x + 2 - 3$ $\therefore$ the Cartesian equation is $y = 2x^2 - 4x - 1$	**2**	**1** mark for sub t in [2]. **1** mark for final answer.
b) i).		**2**	**1** mark for all asymptotes. **1** mark for correct curve.
b). ii).		**2**	**1** mark for the 2 x-intercepts. **1** mark for correct curve.
b). iii).		**2**	**1** mark for the x-y-intercepts. **1** mark for correct curve.

Question 3	Mathematics Extension 1		
Part	Solution	Marks	Comment
c) i).		**2**	**1** mark for the 3 x-intercepts. **1** mark for correct curve.
ii).	Because $c(x)$ passes through the origin, hence when multiplying $f(x)$ by 0, this will result in a value of zero. Therefore $g(x)$ must also pass through the origin.	**1**	**1** mark for the correct statement.
iii).	The degree of $g(x)$ is 3.	**1**	**1** mark for the correct value.