CARLINGFORD HIGH SCHOOL

Year 11 Mathematics Standard

Preliminary Yearly Exam 2019

Time allowed: 2 hours (+ Reading Time)

Student Number: SOLUTIONS

General Instructions:

- Write using black pen
- Approved calculators may be used
- A reference sheet is provided
- In Questions in Section 2, show relevant mathematical reasoning and/or calculations

Marks:

Section 1 (Multiple Choice) – 10 marks

- Allow about 15 minutes to complete and check this section

Section 2 (Short Response) – 70 marks

- Allow about 1 hour and 45 minutes to complete and check this section

Total Marks = 80

Marking Grid

	Multiple Choice	Questions 11-16	Questions 17-21	Questions 22-25	Questions 26-30	Mark
Algebra	1, 6, 8 /3	11 /3	17 /4	22, 25 /7	29 /4	/21
Financial Mathematics	3, 5, 10 /3	14 /3	20 /3	23 /5	26, 30 /6	/20
Measurement	2, 4 /2	12, 15 /6	19, 21 /7		27 /4	/19
Statistical Analysis	7, 9 /2	13, 16 /6	18 /4	24 /4	28 /4	/20
Total	/10	/18	/18	/16	/18	/80

2019 Preliminary Final Examination

Mathematics Standard

REFERENCE SHEET

Measurement

Limits of accuracy

Absolute error = $\frac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement − absolute error

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

$$A \approx \frac{h}{2}(d_f + d_l)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV(1 + r)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

Statistical Analysis

An outlier is a score

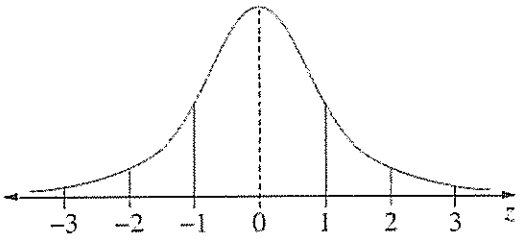
less than $Q_1 - 1.5 \times IQR$

or

more than $Q_3 + 1.5 \times IQR$

$$z = \frac{x - \bar{x}}{s}$$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

2019 Preliminary Final Examination

Mathematics Standard

Student Number SOLUTIONS

Section I – Multiple Choice Answer Sheet

Allow about 25 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

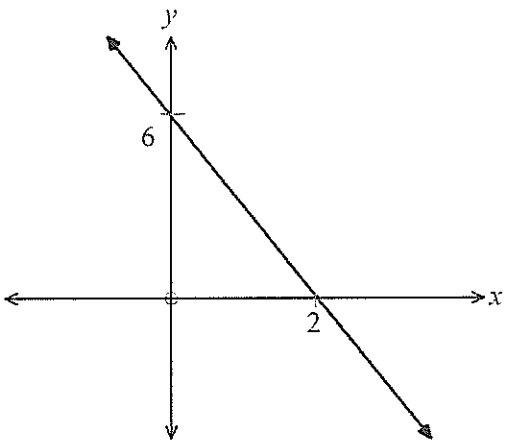
A ⊗ B ⊗ (correct ↓) C ○ D ○

A	1.	A ●	B ○	C ○	D ○
C	2.	A ○	B ○	C ●	D ○
D	3.	A ○	B ○	C ○	D ●
B	4.	A ○	B ●	C ○	D ○
C	5.	A ○	B ○	C ●	D ○
C	6.	A ○	B ○	C ●	D ○
C	7.	A ○	B ○	C ●	D ○
B	8.	A ○	B ●	C ○	D ○
B	9.	A ○	B ●	C ○	D ○
D	10.	A ○	B ○	C ○	D ●

Section 1

Use the multiple-choice answer sheet for Questions 1 – 10

1. What is the equation that describes this graph?



(A) $y = -3x + 6$

(B) $y = -2x + 6$

(C) $y = 3x + 6$

(D) $y = 6x + 2$

2. Which statement is correct for the number 2.408×10^{-2} ?

(A) It is a number less than zero, rounded correct to 3 significant figures

(B) It is a number less than one, rounded correct to 3 significant figures

(C) It is a number less than one rounded correct to 4 significant figures

(D) It is a number greater than one rounded correct to 4 significant figures

3. A retailer adds a 60% mark-up to the cost price of the clothing it buys and then adds a 10% GST to this price to determine the selling price.
What would be the selling price of a pair of shorts which had a cost price of $32.00?

(A) $35.86

(B) $51.20

(C) $54.40

(D) $56.32

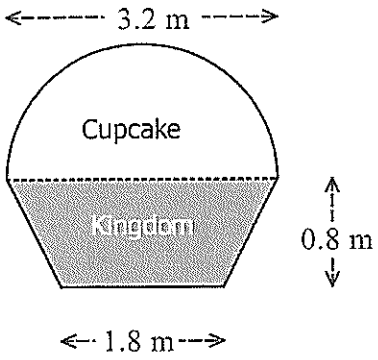
4. A sign for a cupcake store is to be made to the dimensions shown.
What is the area of the sign? (Give your answer correct to 2 significant figures)

(A) $5.5\ m^2$

(B) $6.0\ m^2$

(C) $9.5\ m^2$

(D) $10.0\ m^2$



5. Nate is a real-estate agent who is paid a retainer of $420.00 per week plus 1.5% on all sales.
In the last four weeks he sold one property for $860 000.
What was his average weekly income for this period?

(A) $1 680.00

(B) $3 330.00

(C) $3 645.00

(D) $14 580.00

6. Which of the following is the solution for $4x + 3 = 12 - x$?

(A) 5

(B) $1\frac{4}{7}$

(C) $1\frac{4}{5}$

(D) 3

7. Marjorie rolled a fair die 12 times and the number ***six*** showed 5 times.
If she rolled the same die again, what would be the probability of rolling a ***six***?

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{5}{12}$

8. The formula $BAC_{Male} = \frac{10N - 7.5H}{6.8M}$ can be used to find the blood alcohol content of a male,

where N is the number of standard drinks consumed, H is the number of hours of drinking, and M is the person's weight in kilograms.

Keiran, who weighs 90 kg, consumed five bottles of beer in two hours.

Each bottle contained 1.2 standard drinks.

What was his BAC after the last drink?

(A) 0.06

(B) 0.07

(C) 0.08

(D) 0.09

9. Which of the following is an example of *quantitative discrete* data?

(A) Manufacturers' energy ratings on home appliances

(B) The number of home appliances sold at different locations on a particular day.

(C) The type of finish applied to home appliances, e.g. anti-fingerprint stainless steel

(D) A list of the manufacturers of home appliances

10. A plumber quoted $300, excluding GST, to complete a job.

A GST of 10% is added to the price. What is the full price for the job?

(A) $30

(B) $303

(C) $310

(D) $330

Section 2

Answer questions 11-30 in spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the booklet.
If you use this space, clearly indicate which question you are answering.

Question 11 (3 marks) (ALGEBRA)

The formula $V_R = \frac{S_r \times V_a}{S_a}$ is used to find the required volume of a liquid medication.

(a) Find the value of V_R when $S_r = 500, S_a = 200$ and $V_a = 60$. [1]

$$V_R = \frac{500 \times 60}{200}$$

$$= 150 \quad \checkmark$$

(b) Find the value of V_a when $S_r = 300, S_a = 400$ and $V_R = 600$. [2]

$$600 = \frac{300 \times V_a}{400}$$

$$240000 = 300V_a \quad \checkmark$$

$$V_a = \frac{240000}{300} = 800 \quad \checkmark$$

Question 12 (2 marks) (MEASUREMENT)

Mike measured the mass of his puppy to be 3.6 kg, using a bathroom scale which measures to the nearest tenth of a kilogram. Find the:

(a) The absolute error in the measurement [1]

± 0.05 ✓

(b) Lower and upper bounds of the true measurement. [1]

$3.6 - 0.05 = 3.55$
$3.6 + 0.05 = 3.65$ } ✓

Question 13 (3 marks) (STATISTICS)

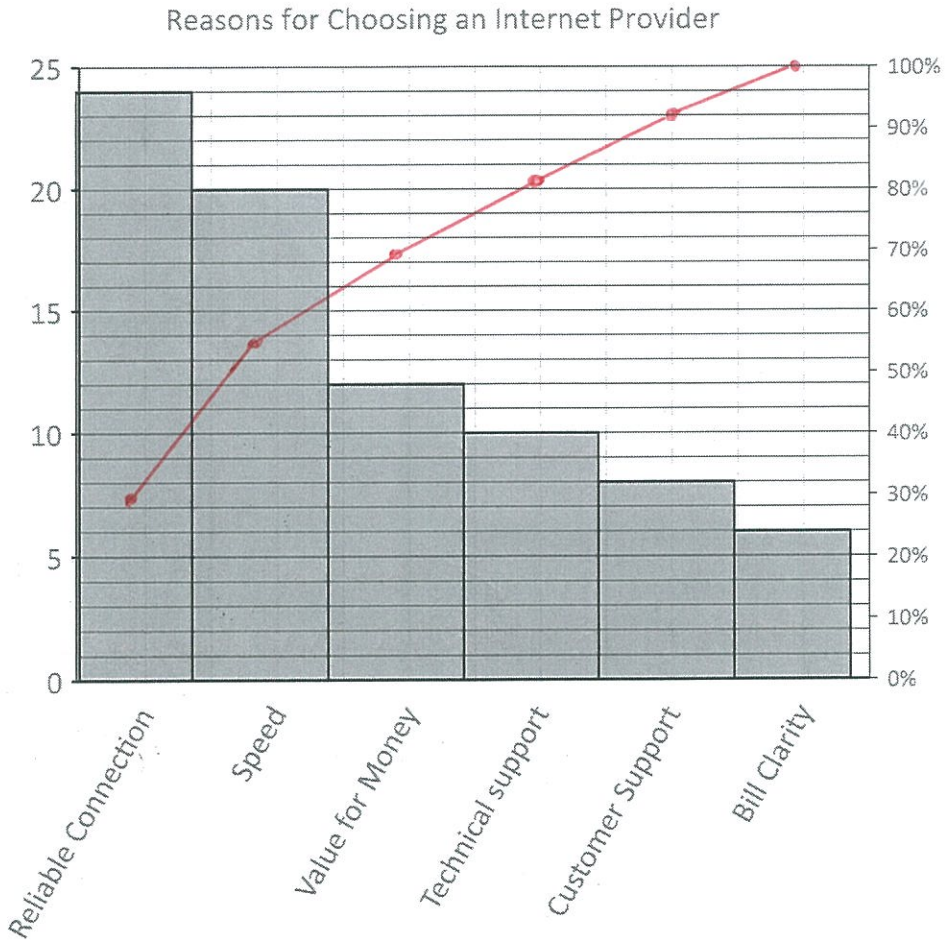
A survey of 80 internet users asked their main reason for choosing their internet provider.

The results are shown in the partially completed table and bar graph below.

(a) Complete the missing values in the table. [2]

Reason	Frequency	Cumulative Frequency	Percentage Cumulative Frequency
Reliable Connection	24	24	30
Speed	20	44	55
Value for Money	12	56	70
Technical support	10	66	82.5
Customer Support	8	74	92.5 ✓
Bill Clarity	6	80 ✓	100

(b) Draw a Pareto line on the chart. [1]



✓

Question 14 (3 marks) (FINANCIAL)

In Australia the GST is 10% and in New Zealand it is 15%.

A product sells for A$137.50 (including GST).

(a) Calculate the GST paid in Australia. [1]

$GST = \frac{137.50}{11}$

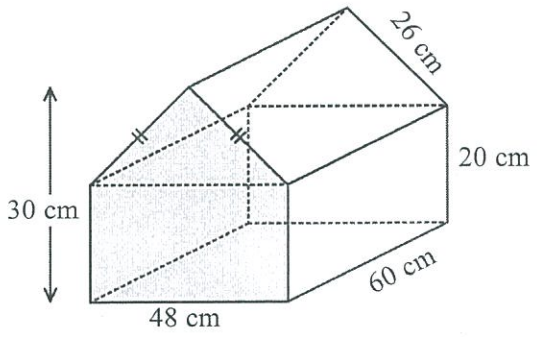
$= \$12.50$ ✓

(b) What would be the selling price of this product in New Zealand (in Australian dollars, including GST)? [2]

Accept: $15\% \times 137.50 = 20.625$ ✓ or $137.50 - 12.50 = \$125$.
$137.50 + 20.625 = \$158.13$ ✓ $\$18.75$ ✓ and $\$143.75$ ✓
(or $1.15 \times 137.50 = \$158.13$) or $137.50 - 13.75 = 123.75$.
$\$18.56$ ✓ and $\$142.31$ ✓

Question 15 (4 marks) (MEASUREMENT)

(a) Find the area of the shaded front side. [1]



Area $= (48 \times 20) + (\frac{1}{2} \times 48 \times 10)$
$= 960 + 240$
$= 1200\ cm^2$ ✓

(b) Find the volume of the solid. [1]

Volume $= 1200 \times 60$
$= 72000\ cm^3$ ✓

(c) Find the surface area of the solid. [2]

from (a)

Surface Area $= (2 \times 1200) + (48 \times 60) + (2 \times 60 \times 20) + (2 \times 60 \times 26)$
$= 2400 + 2880 + 2400 + 3120$
$= 10800\ cm^2$ ✓

(-1 mark for each mistake)

Question 16 (3 marks) (STATISTICS)

A cricket club holds a raffle and Tad buys 15 tickets and Carmel buys 20 tickets.

The winner is chosen by drawing one ticket from a bucket that contains the 200 tickets that were sold in the raffle.

(a) What is the probability that Tad wins the raffle? [1]

$\frac{15}{200}$ or $\frac{3}{40}$ ✓

(b) What is the probability that neither Tad nor Carmel win the raffle? [1]

$\frac{200-35}{200} = \frac{165}{200}$ or $\frac{33}{40}$ ✓

(c) What is the probability that Tad or Carmel win the raffle? [1]

$\frac{35}{200}$ or $\frac{7}{40}$ ✓

Question 17 (4 marks) (ALGEBRA)

A company produces wire rope in its factory.

The equation $C = 25L + 50$ gives the cost of producing the wire rope.

L = length of rope produced in metres

C = Cost of producing the rope in $.

If the graph representing this equation was drawn,

(a) The vertical intercept would be 50. What is its meaning in this context? [1]

Fixed initial cost.

(b) Find the gradient of the graph. [1]

25

(c) What length of rope could be produced for $550? [2]

$550 = 25L + 50$

$500 = 25L$

$L = \frac{500}{25} = 20\text{ m}$

Question 18 (4 marks)

Nutritionists tested the fat content of beef sausages and chicken sausages. Each sausage tested was given a 'fat content' score – the higher the score, the higher the fat content of the sausage.

The results of the tests are shown below in the stem-and-leaf plot.

Beef									*Chicken*					
								0	3	4	7			
								1	0	1	1	3		
				9	7	6	5	2	5	6	7	8		
9	8	7	5	5	4	4	0	3	1	2	2	3	6	
						8	7	4	5					
			5	5	5	3	2	1	5					

(a) How many sausages were tested? [1]

20+17 = 37

(b) What percentage of chicken sausages had a fat content score less than 30? [2]

(Write your answer correct to 1 decimal place)

$\frac{11}{17} \times 100 = 64.7\%$

(c) What was the most common score? [1]

55

Question 19 (4 marks)

A full bottle of Bilpin apple juice has a mass of 850 g.

It is packed in crates which have a mass of 2.4 kg and hold 24 bottles.

(a) Calculate the combined mass (in kilograms) of a crate packed full of bottles. [2]

$2.4 + (0.85 \times 24) = 2.4 + 20.4$

$= 22.8$ kg

(b) A truck is used to take a load of full crates of apple juice to a warehouse. [2]

The maximum load the truck can carry is 1.82 tonnes.

How many crates of juice can the truck carry?

$\frac{1.82 \times 1000}{22.8} = 79.82$

$\therefore$ 79 crates

Question 20 (3 marks)

Lucy purchased a $17 000 car and intends to keep it for 15 years. At the end of this time its value will be $3500.

(a) What is the annual amount of depreciation (correct to the nearest dollar), if the amount of depreciation is constant? [1]

$$\frac{17000 - 3500}{15} = \frac{13500}{15} = \$900 \text{ p.a}$$

(b) Calculate the total depreciation as a percentage of the original price. [2]

$$\frac{13500}{17000} \times 100 = 79.41\%$$

$$= 79.4\% \text{ or } 79\%$$

Question 21 (3 marks)

Andy has a desktop computer and monitor which together are rated at 400 watts.

He uses them for an average of 5 hours per day but leaves them turned on 24/7.

The cost of electricity is 30 cents per kWh.

(a) How much does it cost Andy to run the computer and monitor for a year? [1]

$$\text{Cost} = 0.400 \times 24 \times 365 \times 0.3$$

$$= \$1051.20$$

(b) How much could he save, ***over a year,*** if he only had the computer and monitor turned on when it was being used? [2]

$$\text{Cost} = 0.4 \times 5 \times 365 \times 0.3$$

$$= \$219$$

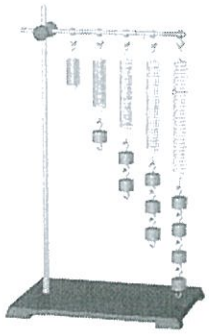
$$\text{Saving} = 1051.20 - 219$$

$$= \$832.20$$

Question 22 (4 marks)

A spring is 16cm long when it has no weight attached.

When a weight is attached, the length of the spring stretches by 5cm for each kilogram added.

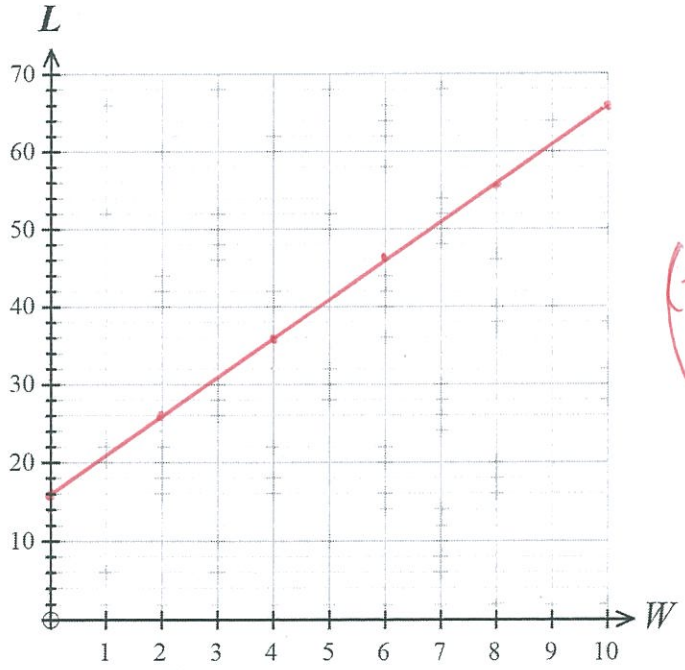


(a) Complete the table of values for this situation. [1]

Weight (W)	0	2	4	6	8	10
Length (L)	16	26	36	46	56	66

✓

(b) Plot each point on the axes below and join them with a straight line. [2]



✓✓

(-1 mark for one incorrect point)

(c) The equation $L =$ 5 ✓ $W + 16$ could be used to model this situation. [1]

What is the missing number?

gradient = $\frac{10}{2} = 5$

Question 23 (~~8~~ 5 marks)

Matilda is a public servant whose annual salary is $96 200 and who receives her net pay deposited to her account in fortnightly instalments.

She has 22% of her wage deducted for PAYG tax instalments and she also has $160.00 deducted for health fund membership and $25.00 for union fees.

(a) How much tax did she have deducted each fortnight? [2]

Gross fortnightly pay $= \frac{96200}{26} = \$3700$ ✓

Tax = 22% of 3700

= $814 ✓

(b) What amount will be deposited to her account each fortnight? [1]

3700 − 814 − 160 − 25 = $2701

(c) Matilda calculates her taxable income to be $90 450.

She uses the formula from the tax table:

Tax = $19 822 plus 37c for each dollar over $87 000

to calculate the amount of tax she should have paid.

How much is this amount? [2]

Tax = 19822 + 0.37 × (90450 − 87000)

= 19822 + 0.37 × 3450 ✓

= 19822 + 1276.50

= $21098.50 ✓

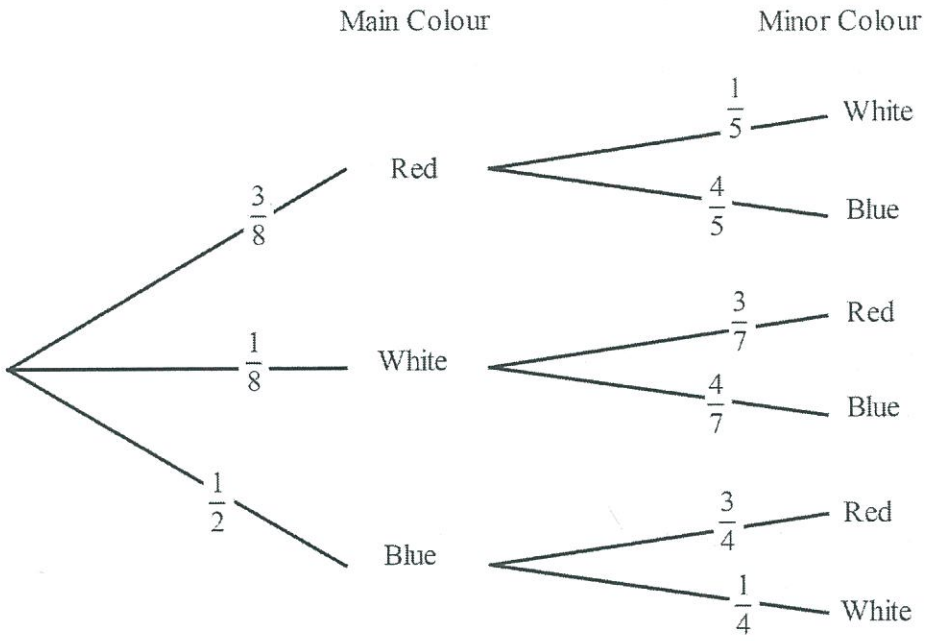
Question 24 (4 marks)

A sports team is to have a uniform which has one main colour and one minor colour, using the colours red, white and blue.

Each of the eight club committee members puts a marble into each tub with the colour they prefer.

The main colour is drawn, then the remaining marbles of that colour are removed before the minor colour is drawn.

The tree diagram shows the possible outcomes.



Calculate the probability that:

(a) The main colour is blue and the minor colour is white. [1]

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

(b) The two colours are blue and white in either order. [1]

$\frac{1}{8} + \left(\frac{1}{8} \times \frac{4}{7}\right) = \frac{11}{56}$

(c) Only one of the colours chosen is blue. [2]

RB WB B

$\frac{3}{8} \times \frac{4}{5} + \frac{1}{8} \times \frac{4}{7} + \frac{1}{2} = \frac{61}{70}$ ✓ ✓

(-1 for not having one correct component).

Question 25 (3 marks)

Clark's rule is used to calculate dosages of medicine for children. It is shown below.

$$\text{Dosage} = \frac{\text{Weight (kg)} \times \text{Adult dose}}{70}$$

Adam, who weighs 85kg, reads the label of a cough medicine to find that the adult dosage is 60ml.

(a) Adam's child, Kane, weighs 32kg. What dosage should he be taking? [1]
(Write your answer correct to the nearest mL)

$\text{Dosage} = \frac{32 \times 60}{70}$

$= 27.42\ldots = 27\text{mL}$ (must round) ✓

(b) Kane's brother, Abel, is given a dosage, according Clark's rule, of 12mL. [2]

How much does Abel weigh?

$12 = \frac{w \times 60}{70}$ ✓

$840 = w \times 60$

$w = \frac{840}{60} = 14\text{ kg}$ ✓

Question 26 (3 marks)

Eve invests $2000 over 4 years at 6.25%.

(a) Calculate the simple interest for this investment. [1]

$I = Prn$

$= 2000 \times 6.25\% \times 4$

$= \$500$

(b) Eve decides to re-invest the $2500 at the same interest rate until it is worth more than $10 000. How many years will it take for this to happen? [2]

$I = Prn$

Interest $= 10000 - 2500$
$= 7500$

(1) $7500 = 2500 \times 6.25\% \times n$

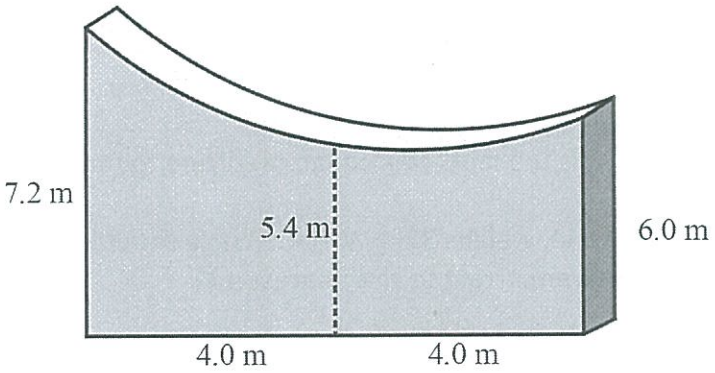
$7500 = 156.25 \times n$

$\frac{7500}{156.25} = n$

$n = 48$ years (1)

Question 27 (4 marks)

A decorative wall in a garden has a curved top with the dimensions shown.



(a) Use one application of the Trapezoidal rule to estimate the area of the front face of the wall. [1]

$A = \frac{h}{2}(a+b)$

$= 4(7.2 + 6.0)$

$= 52.8\ m^2$

(b) Use two applications of the Trapezoidal rule to get another estimate for the area of the front face of the wall. [2]

$A = \frac{4}{2}(7.2 + 5.4) + \frac{4}{2}(5.4 + 6.0)$

$= 25.2 + 22.8$

$= 48\ m^2$

-1 one mistake
-1 using one application

(c) Explain why the second estimate should be more accurate than the first. [1]

The top edge of each trapezium is closer to the curved side than the top edge of one large trapezium.

mention of curved edge

no marks for "more accurate as using more applications"

Question 28 (4 marks)

The percentages of people of different blood types in Australia are shown below.

Blood Type	O+	A+	B+	AB+	O-	A-	B-	AB-
Percentage	40.0%	31.0%	8.0%	2.0%	9.0%	7.0%	2.0%	1.0%

(a) There are 208 students in Year 11 at an Australian school. [1]

How many of these students would you expect to have O+ blood type?

40% x 208 = 83.2

(or 83 students)

(b) If a person is selected at random from a list of Australians, what is the probability they will have blood type A+ or B+? [1]

31% + 8% = 39% (or $\frac{39}{100}$ or 0.39)

(0 mark for $\frac{81}{208}$)

(c) Mr Trump wants to use the 21 students in his class as a sample to see if the school results are similar to the Australian results.

Mrs Merkel explains to him that this is not a **random** sample.

(i) Explain why she is right. [1]

Not all Y11 students have a chance of being selected.

(must reference whole school population)

(ii) What would be a better way for Mr Trump to select his 21 student sample? [1]

(must mention a method: random systematic stratified)

Allocate each student in Y11 (or whole school) a number from the alphabetical roll and randomly selecting 21 students, using a random number generator or systematic sampling technique.

Question 29 (4 marks)

The cost of fuel used in running a stationary motor is directly proportional to the time that the motor is running.

When the motor has been running for 2.5 hours, the cost of fuel is $21.00.

The equation of variation can be written as $C = kT$.

Where C is the cost of fuel (in dollars) and T is the time that the motor is running (in hours), and k is the constant of variation.

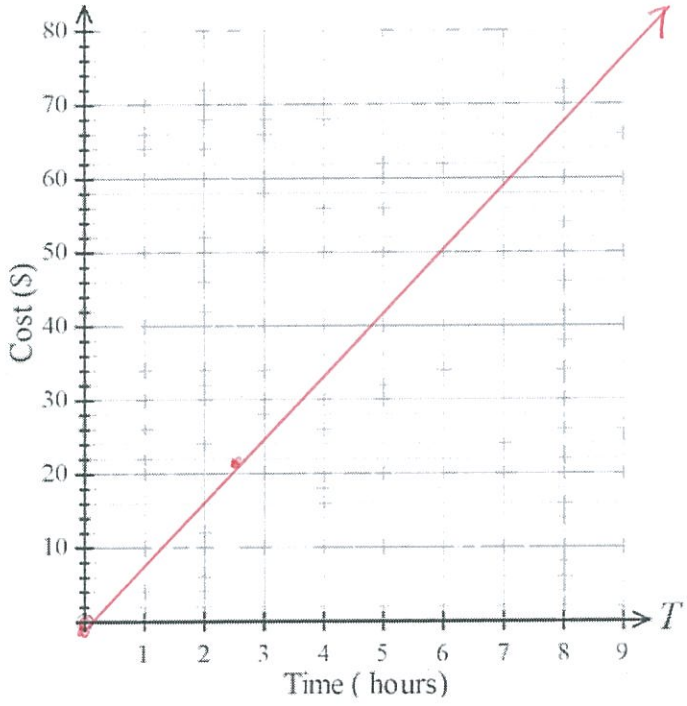
(a) Find the constant of variation, k. [1]

$C = kT$

$21 = k \times 2.5$

$8.4 = k$

(b) Draw a sketch of the equation of variation on the axes below. [1]



(c) **Use your graph** to find the cost of running the motor for 7 hours. [1]

$58 ✓

(or answer from graph with working line) ✓

(no mark for using equation or $58.80)

(d) How long could the motor run on $50 worth of fuel? [1]

$50 = 8.4T$

$T = \frac{50}{8.4}$

$= 5.95...$ } either.

$= 6$ hours.

(or 5 hrs 57 mins)

Question 30 (4 marks)

Lexie is a carpenter who is paid $45.40 per hour plus an additional 5% allowance when she works outdoors or in dusty conditions.

Her normal working hours are 36 hours per week. All overtime is paid at time and a half.

How much does Lexie earn for a week where she worked outdoors for the 40 hours. [3]

Normal time = 45.40 × 36
= $1634.40

allowance = 5% × 1634.40
= $81.72

} total = 1716.12

Overtime = 45.40 × 4 × 1.5
= $272.40

allowance = 5% × 272.40
= $13.62

} $286.02

∴ total earnings = $1716.12 + $286.02
= $2002.14

1 mark — one correct calculation

2 marks — some error in calculations

3 marks — correct answer.

Section 2 Extra writing space

If you use this space, clearly indicate which question you are answering.