Carlingford High School

2017

Mathematics Extension 1

HSC Assessment Task 1

Time allowed 55 min

Name: ....

Teacher: *(Please Circle)*

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Mrs Tomar/Virmarni	Mr Wilson

General Instructions

- Start each question on a new page
- Do not write in columns
- Marks may be deducted for careless or badly arranged work
- Only calculators approved by the Board of Studies may be used
- All answers are to be completed in black pen except graphs and diagrams
- No lending or borrowing

Q1 Polynomials	**Q2** Parametric Equations	**Q3** Series	**Total**
/11	/12	/14	/37

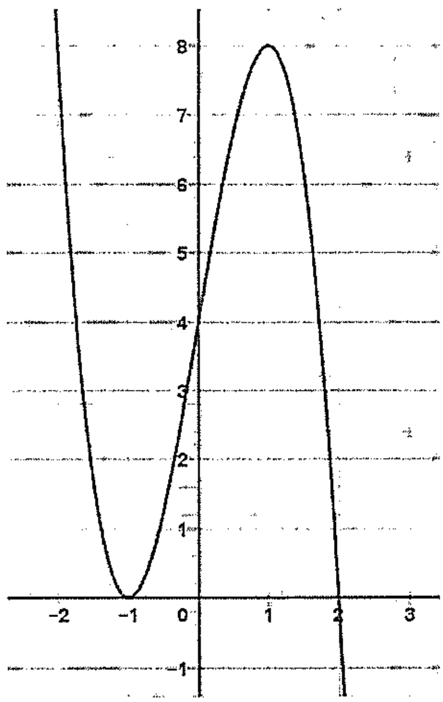
Question 1 (Polynomials) 11 marks

a. Write the equation of this polynomial in factored form. 3

$y = 2x^3 - 3x^2 - 11x + 6$

b. Sketch the graph of $y = 2x^3 - 3x^2 - 11x + 6$, showing all intercepts of the axes. 2

c. Write the equation of this polynomial in factored form. 3



d. If α, β, γ are the roots of $3x^3 - 4x^2 + 7x - 11 = 0$ 3

Find:

i) $\alpha + \beta + \gamma$

ii) $\alpha\beta\gamma$

iii) $(\alpha + 1)(\beta + 1)(\gamma + 1)$

Question 2 (Parametrics) 13 marks

a. Given the parabola $x = 4t^2$ and $y = 8t$, find

i) the Cartesian equation of this parabola 1

ii) the focus 1

iii) the directrix 1

iv) sketch the parabola on the Cartesian plane, showing the directrix, vertex and focus 2

b. The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

The equation of the tangents at P and Q respectively are:
$y = px - ap^2$ and $y = qx - aq^2$.

2

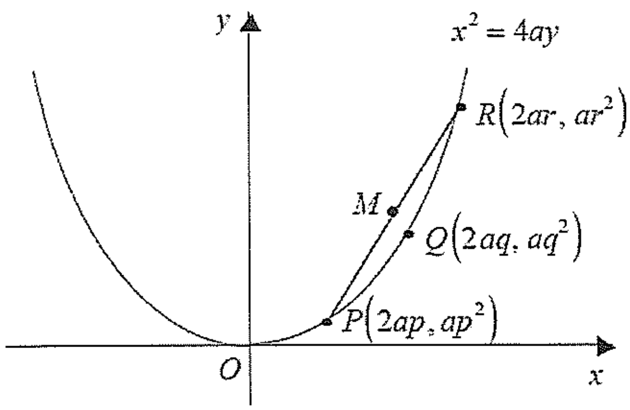
i) The tangents at P and Q meet at the point R. Show that the coordinates of R are $(a(p+q), apq)$.

ii) The equation of the chord PQ is $y = \frac{p+q}{2}x - apq$ (Do NOT show this.) 1

If the chord PQ passes through $(0, a)$, show that $pq = -1$.

iii) Find the equation of the locus of R if the chord PQ passes through $(0, a)$ 2

c.



$Q(2aq, aq^2)$ is a fixed point on the parabola $x^2 = 4ay$ where $a > 0$.
$P(2ap, ap^2)$ and $R(2ar, ar^2)$ are variable points which move on the parabola such that the chord PR is parallel to the tangent to the parabola at Q.

2

Show that $p + r = 2q$.

Question 3 (Series) 14 marks

a. x, x^2 and $5x$ are three consecutive terms of an arithmetic series.

i) Show that $2x^2 - 6x = 0$ 1

ii) What is the common difference of this arithmetic series? 2

b. Use the limiting sum formula to rewrite the recurring decimal, $0.\dot{2}\dot{5}$ in simplest fraction form. 2

c. The lengths of the sides of a scalene triangle are in arithmetic progression. It is known that the largest angle is 120^0. Let the sides be a, $a+d$, $a+2d$, and $a, d > 0$

i) Using the cosine rule, show that $2a^2 - ad - 3d^2 = 0$ 2

ii) If the length of the shortest side is 6 cm, find the length of the longest side, leaving your answer in exact form. 2

d. The third and sixth terms of a geometric series are $T_3 = -24$ and $T_6 = 3$ respectively.

i) Find the limiting sum 2

ii) Find the smallest value for n for which $\left|\sum_{k=1}^{\infty} T_k - \sum_{k=1}^{n} T_k\right| < 10^{-3}$ 3

End of Exam

1) a) $y = 2x^3 - 3x^2 - 11x + 6$

Test $x = 2$

$y(2) = 16 - 24 - 22 + 6$

$= -24$ ∴ not a factor

Test $x = 3$

$y(3) = 54 - 27 - 33 + 6 = 0$

∴ $(x-3)$ is a factor.

$$\begin{array}{r|l} & 2x^2 + 3x - 2 \\ x-3 & 2x^3 - 3x^2 - 11x + 6 \\ & \underline{2x^3 - 6x^2} \\ & 3x^2 - 11x \\ & \underline{3x^2 - 9x} \\ & -2x + 6 \\ & \underline{-2x + 6} \\ & 0 \end{array}$$

$(x-3)(2x^2+3x-2)$

$= (x-3)(x+2)(2x-1)$

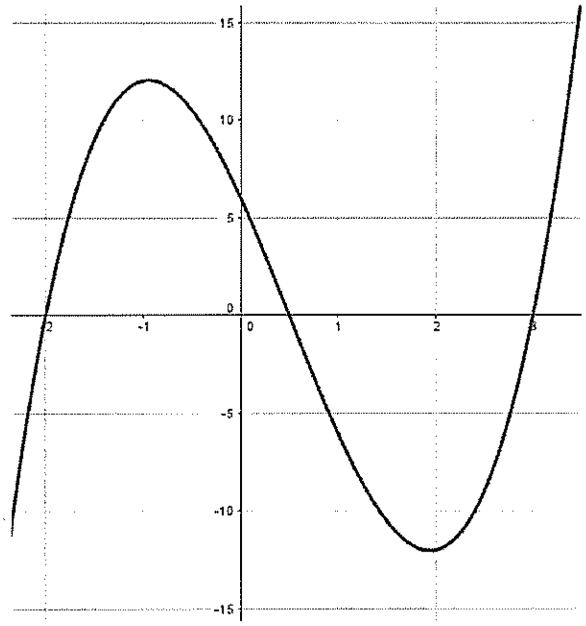
b) x intercepts when $y = 0$

$(x-3)(x+2)(2x-1) = 0$

$x = 3, -2, \frac{1}{2}$

y intercepts when $x = 0$

$y = 6$



c) $y = a(x+1)^2(x-2)$

$x = 0, y = 4$

$4 = a(1)^2(-2)$

$a = -2$

$y = -2(x+1)^2(x-2)$

d) i) $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{4}{3}$

ii) $\alpha\beta\gamma = -\frac{d}{a} = \frac{11}{3}$

iii) $(\alpha+1)(\beta+1)(\gamma+1)$

$= \alpha\beta\gamma + (\alpha\beta + \alpha\gamma + \beta\gamma) + \alpha + \beta + \gamma + 1$

$= \frac{11}{3} + \frac{7}{3} + \frac{4}{3} + \frac{3}{3}$

$= \frac{25}{3}$

2) a) $x = 4t^2,\ y = 8t$

i) $t = \frac{y}{8}$

$\therefore\ x = 4\left(\frac{y}{8}\right)^2$

$x = \frac{4y^2}{64}$

$x = \frac{y^2}{16}$

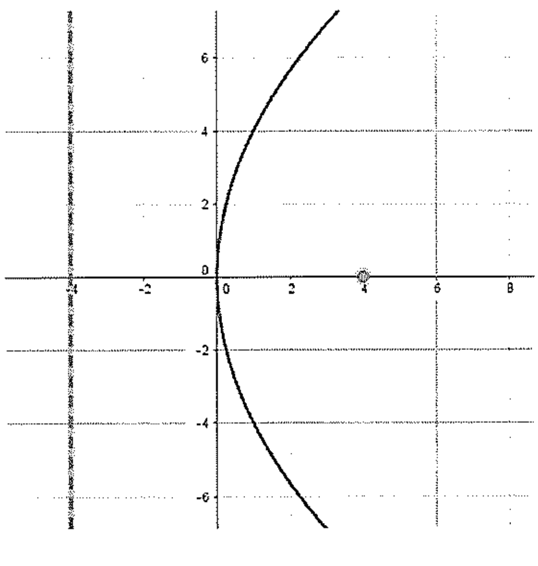
$16x = y^2$

$y^2 = 16x$

ii) focus $(4, 0)$

iii) directrix $x = -4$

iv).



b) i) $y = px - ap^2 \ldots ①$

$y = qx - aq^2 \ldots ②$

Find R. $\therefore$ ① = ②

$px - ap^2 = qx - aq^2$

$(p-q)x = ap^2 - aq^2$

$(p-q)x = a(p^2 - q^2)$

$x = a(p+q)$

Sub into ①

$y = p(a(p+q)) - ap^2$

$= ap^2 + apq - ap^2$

$= apq$

$R = (a(p+q),\ apq)$

ii) PQ: $y = \frac{p+q}{2}x - apq$

Passes $(0, a)$ $a = -apq$

$1 = -pq$

$pq = -1$

2 b) iii) locus of R when PQ focal chord $(pq=-1)$

from i) coordinates of R are $(a(p+q),\ apq)$

and $pq = -1 \therefore R = (a(p+q), -a)$

$\therefore$ R lies on directrix $y = -a$

2) c) PR || tangent at Q

$$M_{PR} = \frac{ar^2 - ap^2}{2a(r-p)} = \frac{r+p}{2}$$

Tangent at Q:

$y = qx - aq^2 \quad m_T = q$

parallel $\therefore M_T = M_{PQ}$

$$\therefore q = \frac{r+p}{2}$$

$$p + r = 2q.$$

3) a) i) $x, x^2, 5x$ AP

$\therefore x^2 - x = 5x - x^2$

$2x^2 - 6x = 0$

ii) $2x^2 - 6x = 0$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\therefore \underline{x = 0}$ and $x = 3$

no solution

$\therefore x = 3$

common difference $= x^2 - x$

$= 3^2 - 3$

$= 6$

b) $0.\dot{2}\dot{5} = 0.25 + 0.0025 + 0.000025 + \ldots$

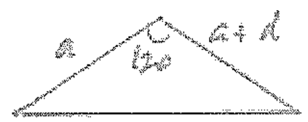
GP: $a = 0.25 \quad r = \frac{1}{100}$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{0.25}{0.99}$$

$$= \frac{25}{99}$$

c) i)



Cosine rule

$(a+2d)^2 = a^2 + (a+d)^2 - 2a(a+d)\cos 120°$

$(a+2d)^2 = a^2 + (a+d)^2 + a(a+d)$

$a^2 + 4ad + 4d^2 = a^2 + a^2 + 2ad + d^2 + a^2 + ad$

$ad + 3d^2 = 2a^2$

$2a^2 - ad - 3d^2 = 0$

ii) $a = 6$

$2(6^2) - 6d - 3d^2 = 0$

$72 - 6d - 3d^2 = 0$

$d^2 + 2d - 24 = 0$

$$d = \frac{-2 \pm \sqrt{4 + 96}}{2}$$

$$= \frac{-2 \pm 10}{2}$$

$\therefore 4$ or -6

$\therefore d = 4$

3 d i)

$T_6 = T_3 \times r^3$ $\qquad T_3 = ar^2$

$r^3 = \frac{3}{-24}$ $\qquad -24 = \frac{1}{4}a$

$r^3 = -\frac{1}{8}$ $\qquad a = -96$

$r = -\frac{1}{2}$

$S_\infty = \frac{a}{1-r}$

$= \frac{-96}{1.5}$

$= -64$

b) $\sum_{k=1}^{n} T_k = S_n = \frac{-96[(-\frac{1}{2})^n - 1]}{-1.5}$

$= 64[(-\frac{1}{2})^n - 1]$

$|S_\infty - S_n| = |-64 - 64(-\frac{1}{2})^n + 64|$

$= |-64(-\frac{1}{2})^n|$

need $64 \times (0.5)^n < 10^{-3}$

$2^{-n} < \frac{1}{64000}$

$-n \log 2 < \log(\frac{1}{64000})$

$n > \frac{\log(64000)}{\log 2}$

$n > 15.965$

The smallest value of n is 16.