Complex Numbers Revision- Cambridge

1 Find **(a)** $z_1 + z_2$ **(b)** $z_1 - z_2$ **(c)** $z_1 z_2$ **(d)** $\frac{z_1}{z_2}$,
when **(i)** $z_1 = 2 + i$, $z_2 = i$, **(ii)** $z_1 = 4 + i$, $z_2 = 2 + 3i$

2 Find **(a)** $Re\, z$ **(b)** $Im\, z$ **(c)** $\bar{z}$,
when **(i)** $z = 3$ **(ii)** $z = 4i$ **(iii)** $z = 3 + 4i$

3 Find real x and y, such that $(x + iy)^2 = 3 + 4i$

4 Solve **(a)** $x^2 + 2x + 2 = 0$ **(b)** $x^2 + (2 - i)x - 2i = 0$

5 Find $|z|$ and arg z when

(a) $z = 2$ **(b)** $z = 2i$ **(c)** $z = 1 + \sqrt{3}i$ **(d)** $z = -\sqrt{3} - i$

6 Express in modulus/argument form **(a)** $-1 + i$ **(b)** $1 - i$

7 Write z in the form $a + ib$ when

(a) $|z| = 4$ and $\arg z = \frac{2\pi}{3}$ **(b)** $|z| = 2$ and $\arg z = -\frac{\pi}{6}$

8 $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, $z_2 = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$. Find

(a) $|z_1 z_2|$ and $\arg(z_1 z_2)$ **(b)** $\left|\frac{z_1}{z_2}\right|$ and $\arg\left(\frac{z_1}{z_2}\right)$

9 $z = 1 + i$. Find $|z^{10}|$ and $\arg(z^{10})$.

10 $z = 1 + i$. Mark on an Argand diagram the points representing
(a) z **(b)** $\bar{z}$ **(c)** iz **(d)** $z + 1$ **(e)** $z - 2i$

11 Show geometrically how to construct the vectors representing
(a) $z_1 + z_2$ **(b)** $z_1 - z_2$ **(c)** $z_2 - z_1$,
when **(i)** $z_1 = 2$, $z_2 = i$ **(ii)** $z_1 = 4 + 2i$, $z_2 = 1 + 3i$

12 Express $(\cos\theta + i\sin\theta)^4$ in modulus/argument form.

13 Express $\cos 2\theta - i\sin 2\theta$ in the form $(\cos\theta + i\sin\theta)^n$.

14 Use De Moivre's theorem with $n = 2$ to show that
$\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$.
Hence show that $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$.

15 Express $z = 4\sqrt{2}(1 + i)$ in modulus/argument form. Hence find the two square roots of z and mark their representations on an Argand diagram.

16 Indicate on an Argand diagram the locus of the point P representing z when

(a) $Re\, z = -2$ **(b)** $Im\, z = 1$ **(c)** $|z| = 2$
(d) $|z - 2 - i| = 2$ **(e)** $\arg z = -\frac{\pi}{3}$ **(f)** $\arg(z + i) = \frac{3\pi}{4}$

17 Indicate on an Argand diagram the region which contains the point P representing z when

(a) $|z| \leqslant |z - 2|$ and $-\frac{\pi}{4} \leqslant \arg z \leqslant \frac{\pi}{4}$ **(b)** $|z| \leqslant 1$ or $0 \leqslant \arg z \leqslant \frac{\pi}{2}$

Further questions 2

1 Express $(3 + 2i)(5 + 4i)$ and $(3 - 2i)(5 - 4i)$ in the form $a + ib$. Hence find the prime factors of $7^2 + 22^2$.

2 Complex numbers $z_1 = \dfrac{a}{1 + i}$ and $z_2 = \dfrac{b}{1 + 2i}$, where a and b are real, are such that $z_1 + z_2 = 1$. Find a and b.

3 $1 + i$ is a root of the equation $x^2 + (a + 2i)x + (5 + ib) = 0$, where a and b are real. Find the values of a and b.

4 $1 - 2i$ is one root of the equation $x^2 + (1 + i)x + k = 0$. Find the other root and the value of k.

5 a and b are real numbers such that the sum of the squares of the roots of the equation $x^2 + (a + ib)x + 3i = 0$ is 8. Find all possible pairs of values a, b.

6 Solve $x^2 - 4x + (1 - 4i) = 0$.

7 Find the modulus and argument of each of the complex numbers $z_1 = 2i$ and $z_2 = 1 + \sqrt{3}i$. Mark on an Argand diagram the points P, Q, R and S representing z_1, z_2, $z_1 + z_2$ and $z_1 - z_2$ respectively. Deduce the exact values of $\arg(z_1 + z_2)$ and $\arg(z_1 - z_2)$.

8 On an Argand diagram, the points A, B, C and D represent z_1, z_2, z_3 and z_4 respectively. Show that if $z_1 - z_2 + z_3 - z_4 = 0$, then ABCD is a parallelogram, and if also $z_1 + iz_2 - z_3 - iz_4 = 0$, then ABCD is a square.

9 If $|z| = r$ and $\arg z = \theta$, show that $\dfrac{z}{z^2 + r^2}$ is real and give its value.

10 1, ω and ω^2 are the cube roots of unity. State the values of ω^3 and $1 + \omega + \omega^2$. Hence show that $(1 + \omega^2)^{12} = 1$ and $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^5)(1 - \omega^7)(1 - \omega^8) = 27$.

11 1, ω and ω^2 are the three cube roots of unity. Show that if the equations $z^3 - 1 = 0$ and $pz^5 + qz + r = 0$ have a common root, then $(p + q + r)(p\omega^5 + q\omega + r)(p\omega^{10} + q\omega^2 + \mathrm{r}) = 0$.

12 Show that the roots of $z^6 + z^3 + 1 = 0$ are among the roots of $z^9 - 1 = 0$. Hence find the roots of $z^6 + z^3 + 1 = 0$ in modulus/argument form.

13 Indicate on an Argand diagram the region defined by the pair of inequalities $|z| \leqslant 6$ and $|z - 5| \leqslant 5$. Write down the range of values of arg z for such z. Find the values of z for which both $|z| = 6$ and $|z - 5| = 5$.

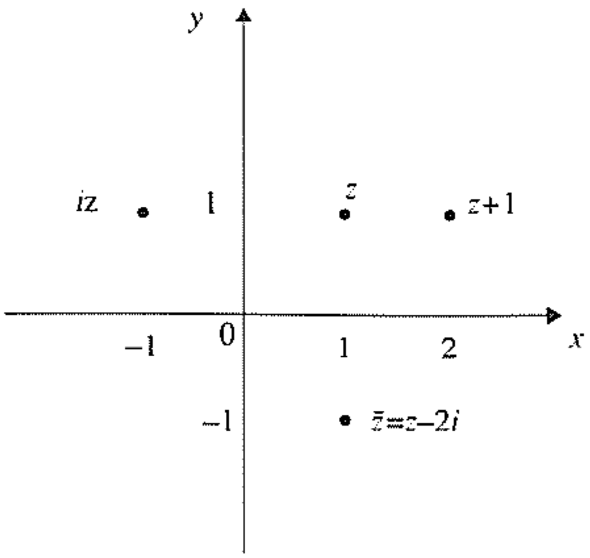
14 The point P represents the complex number z on an Argand diagram. Describe the locus of P in each of the following cases

(a) $|z| = |z - 2|$ (b) $\arg(z - 2) = \arg(z + 2) + \dfrac{\pi}{2}$

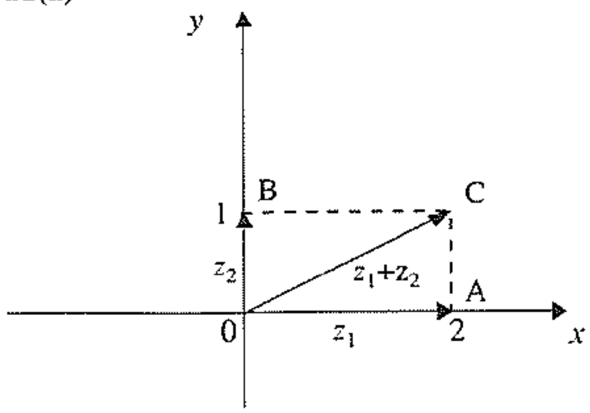
Find the complex number z which satisfies both of these equations.

1 $2+2i$, 2, $-1+2i$, $1-2i$; $6+4i$, $2-2i$, $5+14i$, $\frac{11}{13}-\frac{10}{13}i$ **2** $3, 0, 3; 0, 4, -4i; 3, 4, 3-4i$
3 $x=2, y=1$ or $x=-2, y=-1$ **4** $x=-1\pm i$; $x=-2, i$ **5** $2, 0; 2, \frac{\pi}{2}; 2, \frac{\pi}{3}; 2, \frac{-5\pi}{6}$
6 $\sqrt{2}\text{ cis }\frac{3\pi}{4}, \sqrt{2}\text{ cis}\left(\frac{-\pi}{4}\right)$ **7** $-2+2\sqrt{3}i, \sqrt{3}-i$ **8** $2\sqrt{2}, \frac{\pi}{12}; \sqrt{2}, \frac{7\pi}{12}$ **9** $32, \frac{\pi}{2}$

10



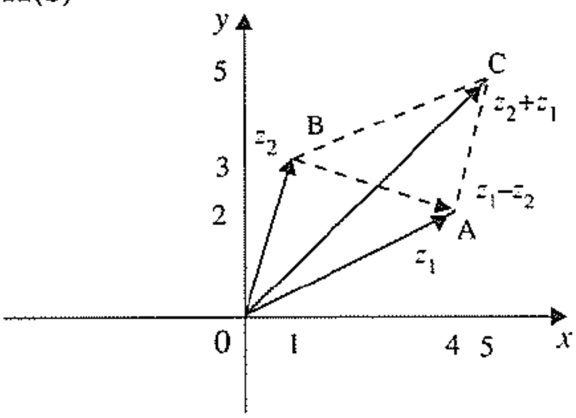
11(a)



$\overrightarrow{BA}=z_1-z_2$

$\overrightarrow{AB}=z_2-z_1$

11(b)



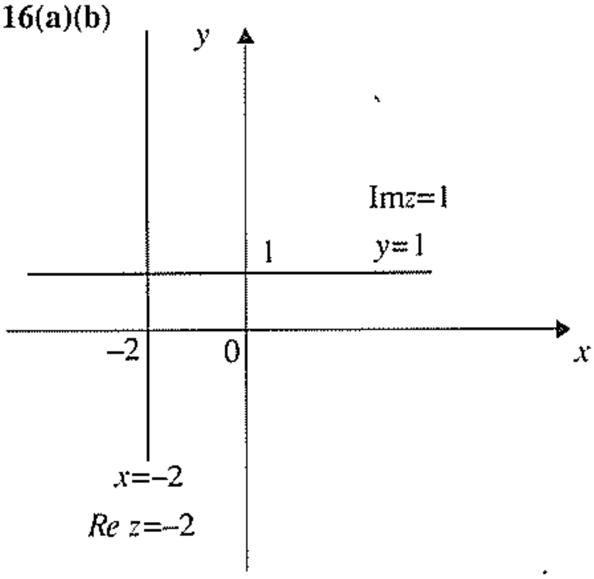
$\overrightarrow{BA}=z_1-z_2$

$\overrightarrow{AB}=z_2-z_1$

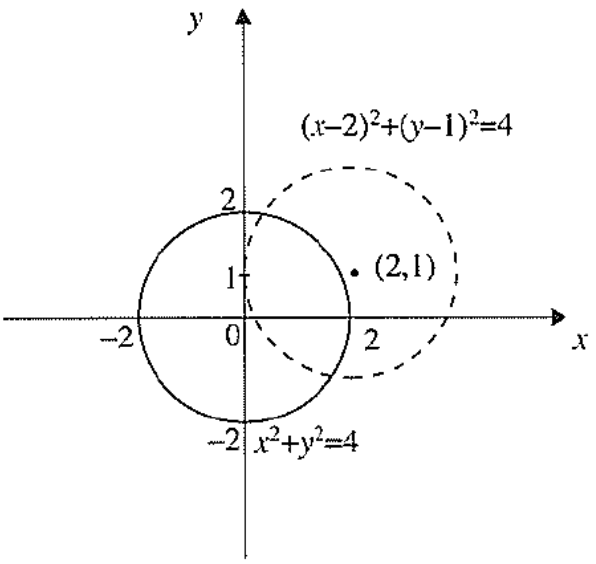
12 cis 4θ **13** $(\text{cis }\theta)^{-2}$

15 $8\text{ cis }\frac{\pi}{4}; \pm 2\sqrt{2}\text{ cis }\frac{\pi}{8}$

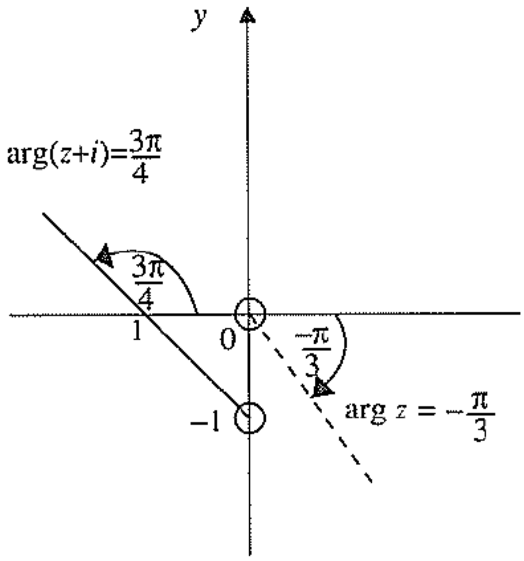
16(a)(b)



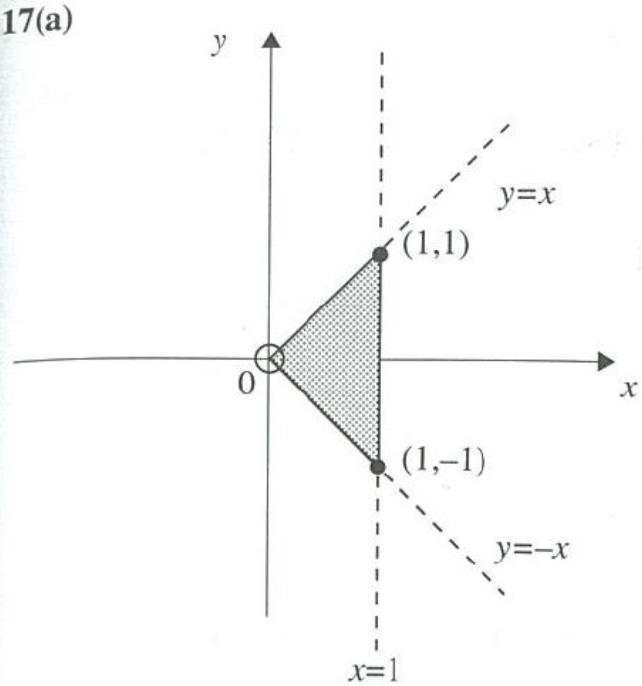
16(c)(d)



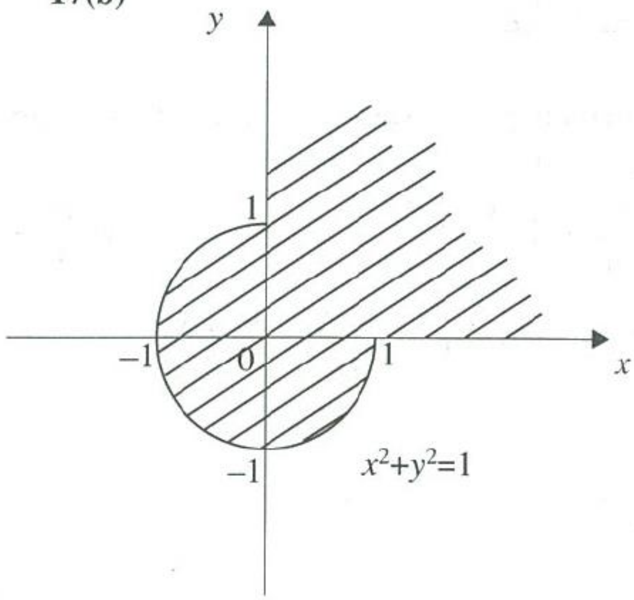
16(e)(f)



17(a)



17(b)



Further questions 2

1 $7 + 22i, 7 - 22i; 7^2 + 22^2 = (3^2 + 2^2)(5^2 + 4^2)$ **2** $a = 4, b = -5$ **3** $a = -3, b = -1$

4 $-2 + i; k = 5i$ **5** $a = 3, b = 1; a = -3, b = -1$ **6** $x = 4 + i, x = -i$

7 $2, \frac{\pi}{2}; 2, \frac{\pi}{3}; \frac{5\pi}{12}, \frac{11\pi}{12}$ **9** $\frac{1}{2r\cos\theta}$ **12** $\operatorname{cis}\left(\pm\frac{2\pi}{9}\right), \operatorname{cis}\left(\pm\frac{4\pi}{9}\right), \operatorname{cis}\left(\pm\frac{8\pi}{9}\right)$

13 $\frac{-\pi}{2} < \arg z < \frac{\pi}{2}; \frac{18}{5} \pm \frac{24}{5}i$ **14** $x = 1$ and $y = \sqrt{(4 - x^2)}; 1 + \sqrt{3}i$