CARLINGFORD HIGH SCHOOL

DEPARTMENT OF MATHEMATICS

Year 12 Mathematics 2U

Term2 Assessment Task 2015

Time allowed: 55 minutes

Name: _____ **Class:** _____**Teacher**_____

Cheng/ Gong, Ms Wilson / White / Lobejko / Mr Wilson

Instructions:

- All questions should be attempted.
- Show ALL necessary working on your own paper.
- Marks may not be awarded for careless or badly arranged work.
- Only board-approved calculators may be used.
- Start each question on a new page and only write on one side of each sheet of paper.

	Q1	Q2	Q3	Q4	Q5	TOTAL
H3	/8	/8	/8			/24
H5				/8	/8	/16
TOTAL	/8	/8	/8	/8	/8	/40

QUESTION 1 (8 Marks) **Marks**

a) Evaluate $e^2 - log_e 20$ correct to 3 significant figures. 1

b) Differentiate $y = 2xe^{x^2}$ 2

c) Find the exact value of $\int_1^3 e^2 - e^{2x}\, dx$ 3

d) Find the coordinates of the stationary point of $y = 4 + e^{3x^2}$ 2

QUESTION 2 (8 Marks)

a) Simplify $log_e x^3 + 2log_e x^2 - log_e 4x^2$ 2

b) Differentiate $y = \frac{log_e x}{x^2}$ 2

c) Find $\int \frac{3x}{3x^2-4} dx$ 1

d) The area under the curve $y = \frac{5}{\sqrt{x}}$ for $1 \leq x \leq e^3$ is rotated about the *x* axis. Find the exact value of the solid of revolution formed. 3

QUESTION 3 (8 Marks)

a) Use Simpson's rule with 5 function values to approximate $\int_1^3 e^x log_e x\, dx$ correct to 3 decimal places. 2

b) (i) Find the *x* intercept of $y = 1 + log_e 3x$. 1
(ii) Hence sketch $y = 1 + log_e 3x$. 1

c) An area between $y = \frac{4}{x+1}$ and the *x* axis from $x = 0$ to $x = 8$ is cut into two parts of equal areas by a vertical line $x = k$. Find the value of k. 4

QUESTION 4 (8 Marks)

a) Find the exact value of $os\frac{7\pi}{6}$. 1

b) Change 35^o to radians in terms of π. 1

c) Convert $\frac{5\pi}{12}$ radians to degrees. 1

d) Differentiate

(i) $y = \tan(\frac{\pi}{2} - x)$ 1

(ii) $y = sin^2(3x + 1)$ 2

e) Solve $2sin^2 x = 1 \ for \ 0 \leq x \leq 2\pi$ 2

QUESTION 5 (8 Marks) **Marks**

a) Find the exact area of the minor segment formed by a chord that subtends an angle of 60^o at the centre of a circle of radius 6mm. 2

b) (i) Sketch $y = 5cos4x \; for \; 0 \leq x \leq \pi$. 2

(ii) State the period of the curve. 1

(iii) Find the area between the curve and the x axis for the given domain. 2

(iv) Without calculation evaluate $\int_0^{\pi} 5cos4x \; dx$. 1

END OF EXAM

Question 1

a) 4.39

b) $y' = e^{x^2}(2) + 2x(2xe^{x^2})$

$= 2e^{x^2}(1 + 2x^2)$

c) $e^2x - \frac{1}{2}e^{2x}\Big]_1^3$

$= 3e^2 - \frac{1}{2}e^6 - (e^2 - \frac{1}{2}e^2)$

$= \frac{5e^2}{2} - \frac{1}{2}e^6$

d) $\therefore y' = 6xe^{3x^2}$ (1)

Want $y' = 0$

$\therefore 6xe^{3x^2} = 0$

$\therefore x = 0$

When $x = 0$ $y = 5$

$\therefore$ S.P at $(0, 5)$ (1)

Question 2

a) $3\log_e x + 4\log_e x - (\log_e 4 + 2\log_e x)$ (1)

$= 5\log_e x - \log_e 4$

$= \log_e \frac{x^5}{4}$ (1)

b) $y' = \frac{x^2(\frac{1}{x}) - 2x\log_e x}{x^4}$ (1)

$= \frac{x - 2x\log_e x}{x^4}$

$= \frac{1 - 2\log_e x}{x^3}$ (1)

c) $= \frac{1}{2}\int \frac{6x}{3x^2 - 4}\,dx$

$= \frac{1}{2}\log_e(3x^2 - 4) + C$

d) $V = \pi \int_1^{e^3} \frac{25}{x}\,dx$ (1)

$= \pi\left[25\log_e x\right]_1^{e^3}$ (1)

$= \pi(25\log_e e^3 - 25\log_e 1)$

$= 75\pi$ units^3 (1)

Question 3

a) $\frac{0.5}{3}\left[e\log 1 + e^3\log_e 3 + 4(e^{1.5}\log_e 1.5) + 2(e^2\log_e 2) + 4(e^{2.5}\log_e 2.5)\right]$

$= 14.038$

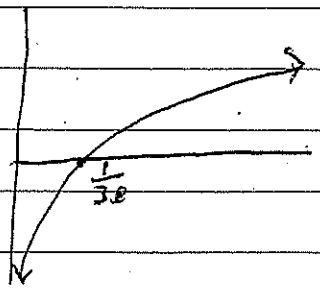
b) (i) $0 = 1 + \log_e 3x$

$\log_e 3x = -1$

$3x = e^{-1}$

$x = \frac{1}{3e}$

(ii)



c) $\int_0^k \frac{4}{x+1}\,dx = \int_k^8 \frac{4}{x+1}\,dx$ (1)

$\therefore \; 4\log(x+1)\Big]_0^k = 4\log(x+1)\Big]_k^8$ (1)

$4\log(k+1) - 4\log 1 = 4\log 9 - 4\log(k+1)$

$8\log(k+1) = 4\log 9$

$\log(k+1) = \frac{1}{2}\log 9$

$= \log 3$ (1)

$\therefore k+1 = 3$

$k = 2$ (1)

Question 4

(a) $\cos\left(\pi + \frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

(b) $35 \times \frac{\pi}{180} = \frac{7\pi}{36}$

(c) $\frac{5\pi}{12} \times \frac{180}{\pi} = 75^\circ$

(d) (i) $-\sec^2\left(\frac{\pi}{2} - x\right)$

(ii) $2\sin(3x+1)(3\cos(3x+1))$

$= 6\sin(3x+1)(\cos(3x+1))$

(iii) $\frac{\pi}{2}$

(iv) C

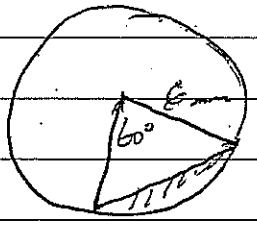
(e) $\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Question 5

a)

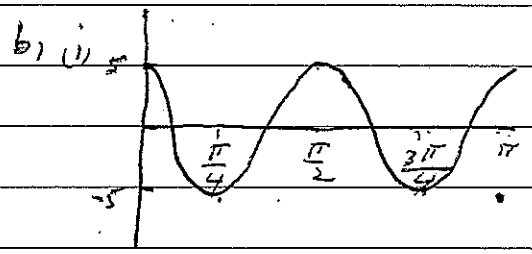


$A = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} - \frac{1}{2} \times 6^2 \times \sin\frac{\pi}{3}$ (1)

$= 6\pi - \frac{18\sqrt{3}}{2}$

$= 6\pi - 9\sqrt{3}$ mm^2 (1)

b) (i)



(ii) $\frac{\pi}{2}$

(iii) $A = 4\left|\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 5\cos 4x\,dx\right|$ (1)

$= 4\left|\left[\frac{5}{4}\sin 4x\right]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\right|$

$= 4\left|\left(\frac{5}{4}\sin\frac{3\pi}{2} - \frac{5}{4}\sin\frac{\pi}{2}\right)\right|$

$= 4\left|\left(\frac{-5}{4} - \frac{5}{4}\right)\right|$

$= 10$ units^2 (1)