

# Mathematics Extension 1

Year 12 Half Yearly Assessment Task

2014

Time allowed: 2 hours

Name: ..... Class: 12MA1....

12MA11 (Ms Kellahan) 12MA12 (Mr White) 12MA13 (Mrs Lobjko) 12MA14 (Mr Fardouly)

## Instructions

- Start each question in a new booklet
- Write on one side of the paper only, and not in columns
- Marks may be deducted for careless or badly arranged work
- Only calculators approved by the Board of Studies may be used
- All answers are to be completed in blue or black pen except graphs and diagrams
- No lending or borrowing

## Outcomes and Marks

Section A	Section B				Mark
	Question 11	Question 12	Question 13	Question 14	
HE7	/10				/10
HE4		/15		/7	/22
H8		/15			/15
H3			/15		/15
HE2				/8	/8
Total	/10	/15	/15	/15	/70
%					



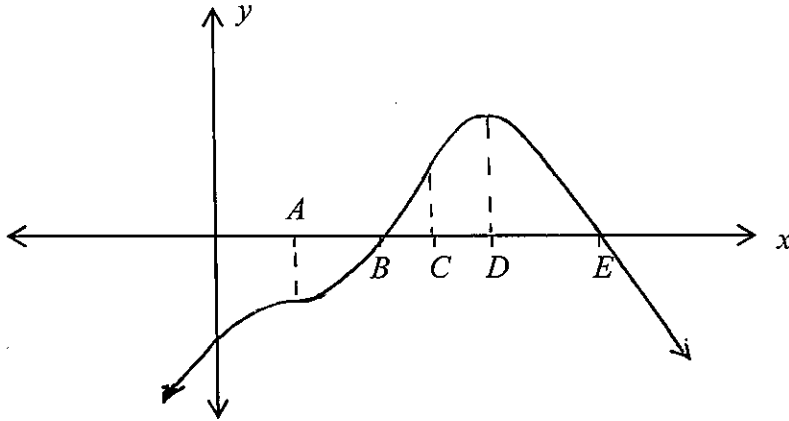
**Section A** (Please use the attached multiple choice answer sheet)

1.  $|9 - 5x| > 1$   
A.  $x < \frac{8}{5}$       B.  $\frac{8}{5} < x < 2$       C.  $x < \frac{8}{5}$  or  $x > 2$       D.  $x > 2$
2. If  $\tan \frac{2A}{3} = \sqrt{3}$ , for  $0^\circ \leq A \leq 360^\circ$ ,  $A =$   
A.  $45^\circ$       B.  $90^\circ$  or  $360^\circ$       C.  $45^\circ$  or  $315^\circ$       D.  $90^\circ$
3.  $\lim_{h \rightarrow \infty} \frac{3(x+h)^2 - 3x^2}{h}$   
A. 0      B.  $3x^2$       C.  $3(x+h)^2$       D.  $6x$
4. The Cartesian equation represented by the parametric equations  $x = 3t$  and  $y = t^2 - 2$  is  
A.  $x^2 = 9y + 18$       B.  $y = t^2 + 3t - 2$       C.  $y = \frac{x^2 - 2}{9}$       D.  $x^2 y = 9 - 2x^2$
5. If  $3^{2x} - 10(3^x) + 9 = 0$ ,  $x =$   
A. 0 or 1      B. 1 or 2      C. 0 or 2      D. 1 or 3
6. The shortest distance between the lines  $x + y + 1 = 0$  and  $(2, -1)$  is  
A.  $\frac{\sqrt{2}}{2}$       B.  $\frac{2}{\sqrt{2}}$       C. 2      D.  $\sqrt{2}$
7. If  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of  $2x^3 + 4x^2 - x + 6$ , the value of  $\alpha^2 + \beta^2 + \gamma^2 =$   
A. 3      B. 4      C. 5      D. 6
8. How many different ways can the letters of the word *EXAMINATION* be arranged?  
A.  $11!$       B.  $\frac{11!}{2!}$       C.  $\frac{11!}{2!2!}$       D.  $\frac{11!}{2!2!2!}$
9.  $\lim_{x \rightarrow \infty} \frac{x-1}{x+1} =$   
A. 0      B. 1      C. -1      D.  $\infty$
10. How many different ways can a committee of 4 people be selected from a group of 7 people?  
A.  $7C_4$       B.  $7P_4$       C.  $7!4!$       D.  $\frac{7!}{4!}$

## Section B

### Question 11 (Please start a new booklet)

- (a) Given this graph for the function  $y = f(x)$  [2]



Draw the graph of  $y = f'(x)$ , showing any of the points A, B, C, D or E as required

- (b) If  $y = \frac{2}{x-1}$

(i) Show that  $y \times \frac{dy}{dx} + \frac{d^2y}{dx^2} = 0$  [2]

- (ii) For what values of  $x$  is the function

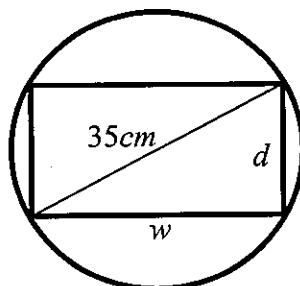
( $\alpha$ ) decreasing? [1]

( $\beta$ ) concave upwards? [1]

- (c) Sketch the graph of  $y = \frac{2x}{x-1}$  showing any asymptotes and labelling at least two points on your graph. [2]

- (d) A curve with equation  $y = ax^3 - 4x + c$  has a turning point at  $(1, 2)$ . Find the values of  $a$  and  $c$  [2]

- (e) The strength of a rectangular beam varies as the product of the width  $w$  and the square of its depth  $d$ , that is,  $s = kwd^2$ , for some constant  $k$



- (i) If a cylindrical log has diameter  $35\text{cm}$ , show that  $s = kw(1225 - w^2)$  [1]
- (ii) Find  $\frac{ds}{dw}$  [1]
- (iii) Find the dimensions of the strongest rectangular beam that can be cut from the log given in (i). (Leave your answers correct to the nearest  $\text{mm}$ ) [3]

**Question 12** (Please start a new booklet)

- (a) (i) Evaluate  $\int_{-1}^2 \frac{x^2 - 2x + 1}{5} dx$  [2]
- (ii) Find  $\int \frac{5}{x^2 - 2x + 1} dx$  [2]
- (b) (i) Sketch the graph of  $y = \sqrt{x^2 + 1}$  [2]
- (ii) Use two applications (4 sub-intervals) of the trapezoidal rule to find an approximation for the area enclosed by the graph of  $y = \sqrt{x^2 + 1}$  and the  $x$ -axis, between  $x = -1$  and  $x = 3$  [2]  
(Write your answer correct to 2 decimal places)
- (iii) Find the volume of the solid formed by revolving the enclosed region in (ii) around the  $x$ -axis. (Leave your answer in terms of  $\pi$ ) [2]
- (c) (i) The graphs of  $y = (2 - x)^2$  and  $y = (2 - x)^3$  intersect at  $(1, 4)$  and one other point. Find the coordinates of the second point [1]
- (ii) Expand and simplify  $(2 - x)^3$  [1]
- (iii) Calculate the area enclosed by the graphs of  $y = (2 - x)^2$  and  $y = (2 - x)^3$  [3]

**Question 13** (Please start a new booklet)

- (a) Evaluate, correct to 2 decimal places
  - (i)  $2e$  [1]
  - (ii)  $\log_2 10$  [2]
- (b) If  $\log_a x = 10 \cdot 218$  and  $\log_a y = 2 \cdot 109$ , use the log laws to
  - (i) Evaluate  $\log_a \left(\frac{x}{y^2}\right)$  [2]

- (ii) If  $y = \frac{\sqrt{x}}{2}$ , find the value of  $a$ , correct to 2 decimal places [2]
- (c) Differentiate, with respect to  $x$
- (i)  $3e^{4-x}$  [1]
- (ii)  $\frac{1}{\log_e x}$  [2]
- (d) If  $\frac{dy}{dx} = xe^{-x^2}$ , and  $y = e$  when  $x = 1$ . Write an expression for  $y$  in terms of  $x$  [2]
- (e) (i) What value does  $e^{-x^2}$  approach as  $x$  approaches  $\infty$ ? [1]
- (iii) What value does  $e^{-x^2}$  approach as  $x$  approaches  $-\infty$ ? [1]
- (iv) What value does  $xe^{-x^2}$  approach as  $x$  approaches  $\pm\infty$ ? [1]

**Question 14** (Please start a new booklet)

- (a) Given  $2^k \geq k$  for any positive integer  $k$ , show that  $2^{k+1} \geq k + 1$  [2]
- (b) Use mathematical induction to prove that
- (i)  $1 + x + x^2 + \dots + x^{n-1} = \frac{1-x^n}{1-x}$ , for all positive integers  $n$  [3]
- (ii)  $9^{n+2} - 4^n = 5m$ , for any integer  $m$ , for all integers  $n$ ,  $n \geq 1$  [3]
- (c) For the graph of  $y = (2x + 5)^{\frac{7}{3}}$
- (i) Find all axes intercepts [2]
- (ii) Find any stationary points, and state their nature [3]
- (iii) Sketch its graph [2]

**Section A (Multiple Choice) Answer Sheet**  
**(Please detach)**

<b>Question</b>				
<b>1</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>2</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>3</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>4</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>5</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>6</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>7</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>8</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>9</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>10</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>

## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$

**Section A (Multiple Choice) Answer Sheet**  
(Please detach)

Question				
1	A	B	<input checked="" type="radio"/> C	D
2	A	<input checked="" type="radio"/> B	C	D
3	A	B	C	<input checked="" type="radio"/> D
4	<input checked="" type="radio"/> A	<input checked="" type="radio"/> B	C	D
5	A	B	<input checked="" type="radio"/> C	D
6	A	B	C	<input checked="" type="radio"/> D
7	A	B	<input checked="" type="radio"/> C	D
8	A	B	C	<input checked="" type="radio"/> D
9	A	<input checked="" type="radio"/> B	C	D
10	<input checked="" type="radio"/> A	B	C	D

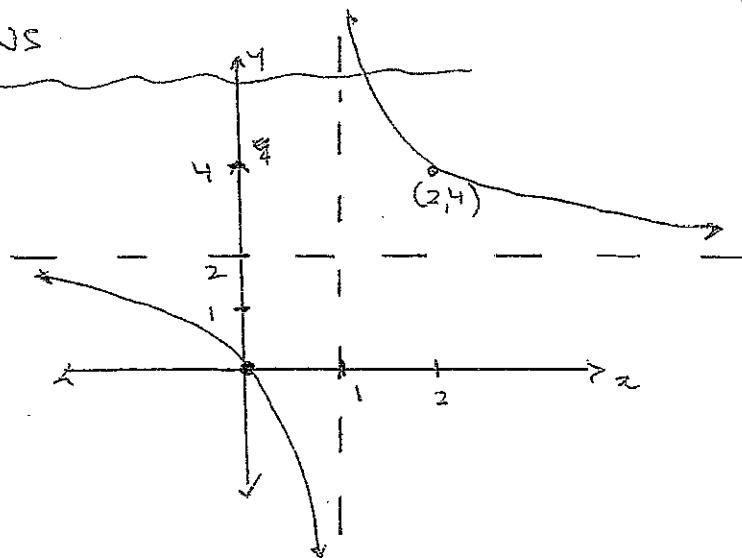
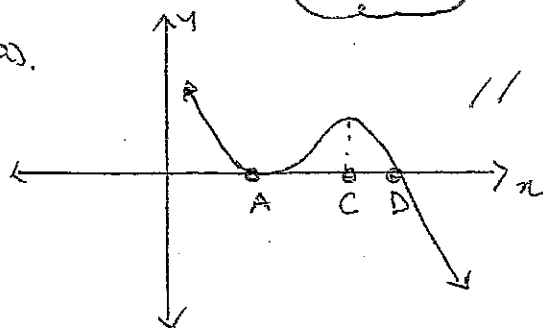


EXAM SOLUTIONS

QUESTION 11

WHITE

(a)



$$(b) (i) \left. \begin{aligned} \frac{dy}{dx} &= -\frac{2}{(x-1)^2} \\ \frac{d^2y}{dx^2} &= \frac{4}{(x-1)^3} \end{aligned} \right\} \checkmark$$

✓ for asymptotes  
✓ for graph, including 2 pts.

$$\begin{aligned} y \times \frac{dy}{dx} + \frac{d^2y}{dx^2} &= \frac{2}{x-1} \times \frac{-2}{(x-1)^2} + \frac{4}{(x-1)^3} \\ &= \frac{-4}{(x-1)^3} + \frac{4}{(x-1)^3} \\ &= 0 \end{aligned} \checkmark$$

(d) when  $x=1, y=2$

$$2 = a - 4 + c$$

$$\therefore a + c = 6$$

$$\frac{dy}{dx} = 3ax^2 - 4 \quad \checkmark$$

$$\frac{dy}{dx} = 0 \text{ when } x=1$$

$$3a - 4 = 0$$

$$a = \frac{4}{3}$$

$$\therefore c = 6 - \frac{4}{3} = 4\frac{2}{3} \quad \checkmark$$

(ii) (a) Decreasing for  $\frac{dy}{dx} \leq 0$

$$\frac{-2}{(x-1)^2} < 0 \text{ for all } x, x \neq 1 \quad \checkmark$$

(b) Concave upwards when  $\frac{d^2y}{dx^2} > 0$  (c) (i)  $d^2 = 3s^2 - w^2$  (Pythagoras)

$$\therefore s = kw(1225 - w^2)$$

$$\frac{4}{(x-1)^3} > 0 \text{ for } x > 1 \quad \checkmark \quad (ii) \frac{ds}{dw} = 1225k - 3kw^2 \quad \checkmark$$

(c) Asymptotes -

vertical when  $x=1$

horizontal when

$$y = \lim_{x \rightarrow 0} \frac{2}{1 - \frac{1}{x}}$$

$$(iii) \text{ Let } \frac{ds}{dw} = 0$$

$$1225k - 3kw^2 = 0$$

$$k(1225 - 3w^2) = 0$$

$$k \neq 0 \therefore 3w^2 = 1225$$

$$w^2 = \frac{1225}{3}$$

$$\frac{d^2s}{dw^2} = -6kw$$

$$-6 \times k \times 20.2 < 0 \quad \checkmark$$

$\therefore$  Maximum strength when  $w = 20.2 \text{ cm}$

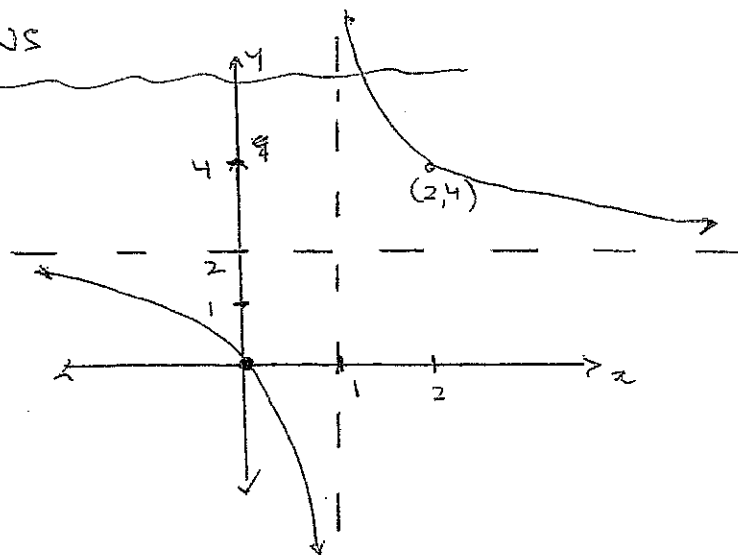
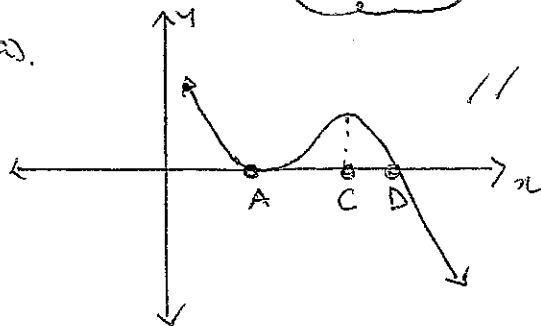
$$\text{and } d = \sqrt{1225 - 1225}$$

EXAM SOLUTIONS

QUESTION 11

WHITE

(a)



✓ for asymptotes  
✓ for graph, including 2 pts.

$$(b) (i) \left. \begin{aligned} \frac{dy}{dx} &= -\frac{2}{(x-1)^2} \\ \frac{d^2y}{dx^2} &= \frac{4}{(x-1)^3} \end{aligned} \right\} \quad \checkmark$$

$$\begin{aligned} y \times \frac{dy}{dx} + \frac{d^2y}{dx^2} &= \frac{2}{x-1} \times \frac{-2}{(x-1)^2} + \frac{4}{(x-1)^3} \\ &= \frac{-4}{(x-1)^3} + \frac{4}{(x-1)^3} \\ &= 0 \quad \checkmark \end{aligned}$$

(d) when  $x=1, y=2$

$$2 = a - 4 + c$$

$$\therefore a + c = 6$$

$$\frac{dy}{dx} = 3ax^2 - 4 \quad \checkmark$$

$$\frac{dy}{dx} = 0 \text{ when } x=1$$

$$3a - 4 = 0$$

$$a = \frac{4}{3}$$

$$\therefore c = 6 - \frac{4}{3} = 4\frac{2}{3} \quad \checkmark$$

(ii) (a) Decreasing for  $\frac{dy}{dx} \leq 0$

$$\frac{-2}{(x-1)^2} < 0 \text{ for all } x, x \neq 1 \quad \checkmark$$

(b) Concave upwards when  $\frac{d^2y}{dx^2} > 0$  (c) (i)  $d^2 = 35^2 - w^2$  (Pythagoras' ✓)  
 $\therefore s = kw(1225 - w^2)$

$$\frac{4}{(x-1)^3} > 0 \text{ for } x > 1 \quad \checkmark \quad (ii) \frac{ds}{dw} = 1225k - 3kw^2 \quad \checkmark$$

(c) Asymptotes -  
~~horizontal~~ vertical when  $x=1$   
horizontal when

$$y = \lim_{x \rightarrow 0} \frac{2}{1-1}$$

$$(iii) \text{ Let } \frac{ds}{dw} = 0$$

$$1225k - 3kw^2 = 0$$

$$k(1225 - 3w^2) = 0$$

$$k \neq 0 \therefore 3w^2 = 1225$$

$$w^2 = \frac{1225}{3}$$

$$\frac{d^2s}{dw^2} = -6kw$$

$$-6 \times k \times 20.2 < 0 \quad \checkmark$$

$\therefore$  Maximum strength  
when  $w = 20.2 \text{ cm}$

$$\text{and } d = \sqrt{1225 - 1225}$$

# QUESTION 12 (KILLAHAN)

$$(a)(i). \frac{1}{5} \left[ \frac{x^3}{3} - x^2 + x \right]_{-1}^2 \checkmark$$

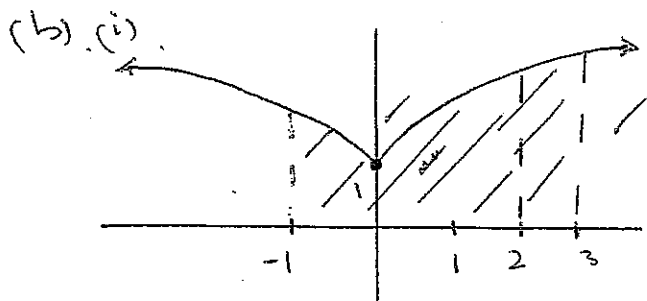
$$= \frac{1}{5} \left[ \left( \frac{8}{3} - 4 + 2 \right) - \left( -\frac{1}{3} - 1 - 1 \right) \right]$$

$$= \frac{3}{5} \checkmark$$

$$(ii). \int \frac{5}{(x-1)^2} dx = \int 5(x-1)^{-2} dx \checkmark$$

$$= \frac{5(x-1)^{-1}}{-1} + C$$

$$= \frac{-5}{x-1} + C \checkmark$$



$$(ii). A \doteq \frac{1}{2} [f(-1) + 2f(0) + f(1) + f(2) + f(3)]$$

$$\doteq \frac{1}{2} [\sqrt{2} + 2(1 + \sqrt{2} + \sqrt{5}) + \sqrt{10}] \checkmark$$

$$\doteq \frac{1}{2} [3\sqrt{2} + 2 + 2\sqrt{5} + \sqrt{10}]$$

$$\doteq 6.94 \text{ units}^2 \checkmark$$

$$(iii). V = \pi \int_{-1}^3 (x^2 + 1) dx \checkmark$$

$$= \pi \left[ \frac{x^3}{3} + x \right]_{-1}^3$$

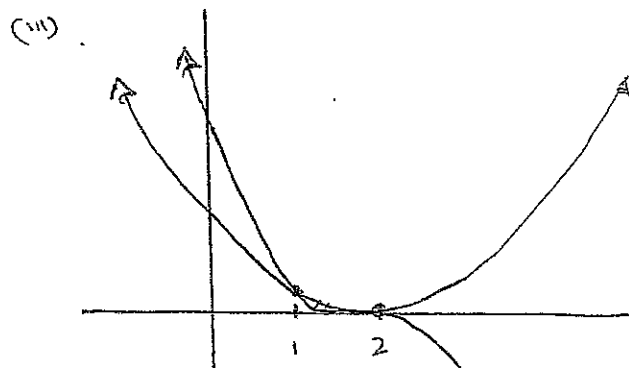
$$= \pi \left[ (9 + 3) - \left( -\frac{1}{3} - 1 \right) \right]$$

$$= \frac{40\pi}{3} \text{ units}^3 \checkmark$$

(c)(i), when  $x=2$ ,  
 $LHS = RHS = 0$   
 $\therefore (2, 0) \checkmark$

$$(ii). 2^3 - 3(2^2)x + 3(2)x^2 - x^3$$

$$= 8 - 12x + 6x^2 - x^3$$



(No marks for diagram)

$$A = \int_1^2 (2-x)^2 - (2-x)^3 dx$$

$$= \int_1^2 (4 - 4x + x^2) - (8 - 12x + 6x^2 - x^3) dx$$

$$= \int_1^2 (-4 + 8x - 5x^2 + x^3) dx$$

$$= \left[ -4x + 4x^2 - \frac{5x^3}{3} + \frac{x^4}{4} \right]_1^2 \checkmark$$

$$= \left[ \left( -8 + 16 - \frac{40}{3} + 4 \right) - \left( -4 + 4 - \frac{5}{3} + \frac{1}{4} \right) \right]$$

$$= \frac{1}{12} \text{ units}^2 \checkmark$$

# QUESTION 13 (LOBOSKO)

(a) (i) 5.44 ✓

(ii)  $\frac{\log 10}{\log 2} = 3.32$  ✓

(b) (i)  $\log x - \log y^2$   
 $= \log x - 2 \log y$  ✓  
 $= 10.218 - 2(2.109)$   
 $= 6$  ✓

(ii)  $x = a^{10.218}$ ,  $y = a^{2.109}$

$y = \frac{\sqrt{x}}{2}$

$a^{2.109} = \frac{(a^{10.218})^{1/2}}{2}$  ✓

$2 = \frac{a^{5.109}}{a^{2.109}}$

$2 = a^3$

$\therefore a = 1.26$  ✓

(c) (i)  $-3e^{4-x}$  ✓

(ii)  $\frac{d}{dx} (\log x)^{-1}$   
 $= -(\log x)^{-2} \times \frac{1}{x}$  ✓

$= \frac{-1}{(\log_e x)^2}$  ✓

(d) (i)  $y = -\frac{1}{2} \int -2x e^{-x^2} dx$

$y = -\frac{1}{2} e^{-x^2} + c$  ✓

when  $x=1$ ,  $y=e$

$e = -\frac{1}{2} e^{-1} + c$

$c = e + \frac{1}{2e}$

$\therefore y = -\frac{1}{2} e^{-x^2} + e + \frac{1}{2e}$  ✓

(e) (i) 0 ✓

(ii) 0 ✓

(iii) 0 ✓

Question 12. (KELLAHAN)

$$(a)(i). \frac{1}{5} \left[ \frac{x^3}{3} - x^2 + x \right]_{-1}^2 /$$

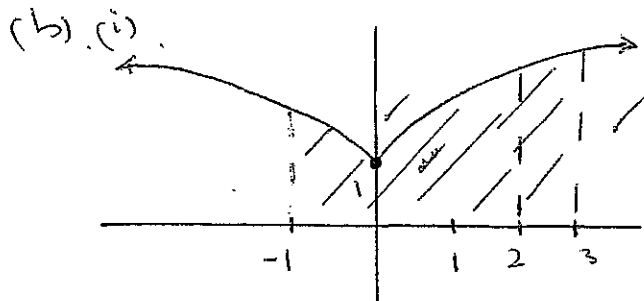
$$= \frac{1}{5} \left[ \left( \frac{8}{3} - 4 + 2 \right) - \left( -\frac{1}{3} - 1 - 1 \right) \right]$$

$$= \frac{3}{5} \quad \checkmark$$

$$(ii). \int \frac{5}{(x-1)^2} dx = \int 5(x-1)^{-2} dx \quad \checkmark$$

$$= \frac{5(x-1)^{-1}}{-1} + C$$

$$= \frac{-5}{x-1} + C \quad \checkmark$$



$$(ii). A \doteq \frac{1}{2} [f(-1) + 2(f(0) + f(1) + f(2)) + f(3)]$$

$$\doteq \frac{1}{2} [\sqrt{2} + 2(1 + \sqrt{2} + \sqrt{5}) + \sqrt{10}] \quad \checkmark$$

$$\doteq \frac{1}{2} [3\sqrt{2} + 2 + 2\sqrt{5} + \sqrt{10}]$$

$$\doteq 6.94 \text{ units}^2 \quad \checkmark$$

$$(ii). V = \pi \int_{-1}^3 (x^2 + 1) dx \quad \checkmark$$

$$= \pi \left[ \frac{x^3}{3} + x \right]_{-1}^3$$

$$= \pi \left[ (9 + 3) - \left( -\frac{1}{3} - 1 \right) \right]$$

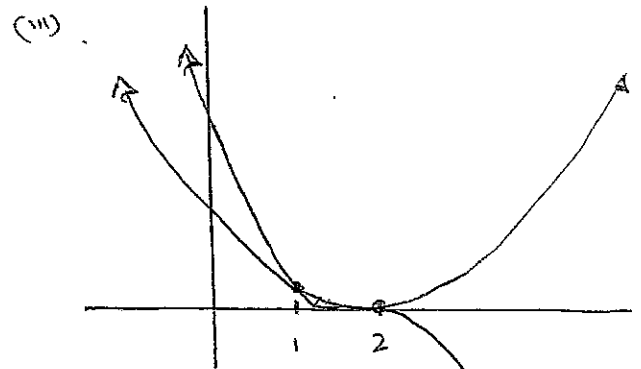
$$= \frac{40\pi}{3} \text{ units}^3 \quad \checkmark$$

(c)(i). when  $x=2$ ,  
LHS = RHS = 0

$$\therefore (2, 0) \quad \checkmark$$

$$(ii). 2^3 - 3(2^2)x + 3(2)x^2 - x^3$$

$$= 8 - 12x + 6x^2 - x^3$$



(No marks for diagram)

$$A = \int_1^2 (2-x)^2 - (2-x)^3 dx$$

$$= \int_1^2 (4 - 4x + x^2) - (8 - 12x + 6x^2 - x^3) dx$$

$$= \int_1^2 (-4 + 8x - 5x^2 + x^3) dx$$

$$= \left[ -4x + 4x^2 - \frac{5x^3}{3} + \frac{x^4}{4} \right]_1^2 \quad \checkmark$$

$$= \left[ \left( -8 + 16 - \frac{40}{3} + 4 \right) - \left( -4 + 4 - \frac{5}{3} + \frac{1}{4} \right) \right]$$

$$= \frac{1}{12} \text{ units}^2 \quad \checkmark$$

# QUESTION 14

FARDOOLY

(a).  $2^k \geq k$

$\therefore 2 \times 2^k \geq 2k$

$2^{k+1} \geq 2k$  ✓

and  $2k \geq k+1$  for  $k \geq 1$  ✓

(b). (i). Prove true for  $n=1$

LHS = 1

RHS =  $\frac{1-x}{1-x} = 1$

$\therefore$  true for  $n=1$  ✓

Assume true for  $n=k$ , that is

$1 + x + x^2 + \dots + x^{k-1} = \frac{1-x^k}{1-x}$

Hence prove true for  $n=k+1$ , that is

$1 + x + x^2 + \dots + x^{k-1} + x^k = \frac{1-x^{k+1}}{1-x}$

or  $\frac{1-x^k}{1-x} + x^k = \frac{1-x^{k+1}}{1-x}$  ✓

LHS =  $\frac{1-x^k + (1-x)x^k}{1-x}$

$= \frac{1-x^k + x^k - x^{k+1}}{1-x}$

$= \frac{1-x^{k+1}}{1-x}$

$=$  RHS. ✓

$\therefore$  if true for  $n=k$ , then true for  $n=k+1$ .

$\therefore$  true for

(ii). Prove true for  $n=1$

LHS =  $9^3 - 4$

$= 725$

$= 5 \times 145$  ✓

$\therefore$  true for  $n=1$

Assume true for  $n=k$ , that is

$9^{k+2} - 4^k = 5m$  ①

Hence prove true for  $n=k+1$ ,

that is,  $9^{k+3} - 4^{k+1} = 5p$  ②

From ①  $9^{k+2} = 5m + 4^k$

From ② LHS =  $9 \times 9^{k+2} - 4 \times 4^k$

$= 9(5m + 4^k) - 4 \times 4^k$  ✓

$= 45m + 9 \times 4^k - 4 \times 4^k$

$= 45m + 5 \times 4^k$

$= 5(9m + 4^k)$  ✓

$\therefore$  if true for  $n=k$ , then true for  $n=k+1$

if true ...

(c). (i),  $x=0$ ,  $y = 5^{7/3} = 42.7$  ✓

$y=0$ ,  $x = -\frac{5}{2}$  ✓

(ii).  $\frac{dy}{dx} = \frac{7}{3}(2x+5)^{4/3} \times 2$   
 $= \frac{14}{3}(2x+5)^{4/3}$

$\frac{dy}{dx} = 0$  when  $x = -\frac{5}{2}$  ✓

$\therefore$  stationary point when  $x = -\frac{5}{2}$  ✓

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{56}{9} (2x+5)^{\frac{1}{3}} \times 2 \\ &= \frac{112}{9} (2x+5)^{\frac{1}{3}}\end{aligned}$$

$$\frac{d^2y}{dx^2} = 0 \text{ when } x = -\frac{5}{2}$$

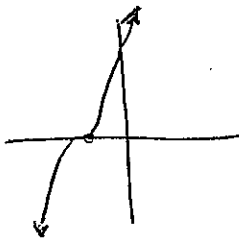
∴ possible stationary  
point of inflexion when  $x = -\frac{5}{2}$  ✓

$x$	-3	$-\frac{5}{2}$	-2
$\frac{d^2y}{dx^2}$	$-\frac{112}{9}$	0	$\frac{112}{9}$

change of concavity i.

stationary point of inflexion  
at  $(-\frac{5}{2}, 0)$  ✓

(iii).



✓ for intercepts

✓ for shape.

if true for  $n=1$  then true for  $n=2$ , and so on,  $\therefore$  the statement is true for  $n \geq 1$

$\therefore$  if true for  $n=k$ , then true for  $n=k+1$ .

$$\begin{aligned} \text{LHS} &= 1 - x^k + (1-x)x^k \\ &= 1 - x^k + x^k - x^{k+1} \\ &= 1 - x^{k+1} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 1 - x^{k+1} \\ &= 1 - x^k + x^k - x^{k+1} \\ &= 1 - x^{k+1} \end{aligned}$$

Assume true for  $n=k$ , that is  $1 + x + x^2 + \dots + x^{k-1} = 1 - x^k$

$$\begin{aligned} \text{LHS} &= 1 \\ \text{RHS} &= 1 - x^{k+1} \end{aligned}$$

$\therefore$  true for  $n=1$

and  $2k \geq k+1$  for  $k \geq 1$

$$2k \geq k+1 \geq 2k$$

$$(a) \quad 2^k \geq k$$

$\therefore$  stationary point when  $x = -\frac{2}{5}$

$$\begin{aligned} \frac{dy}{dx} &= 0 \text{ when } x = -\frac{2}{5} \\ \frac{dy}{dx} &= \frac{3}{7}(2x+5)^{\frac{2}{3}} \times 2 = \frac{3}{14}(2x+5)^{\frac{2}{3}} \end{aligned}$$

$$(c) \quad (i) \quad x=0, y=\frac{5}{2} = 2.5$$

if true for  $n=k$ , then true for  $n=k+1$

$$\begin{aligned} \text{From (1)} \quad 9^{k+2} &= 5m + 4k \\ \text{From (2)} \quad \text{LHS} &= 9 \times 9^{k+2} - 4 \times 4^k \\ &= 9(5m + 4k) - 4 \times 4^k \\ &= 45m + 36k - 4 \times 4^k \\ &= 45m + 36k - 4 \times 4^k \end{aligned}$$

$$\begin{aligned} \text{Assume true for } n=k, \text{ that is } 9^{k+2} - 4^k &= 5m \\ \therefore \text{ true for } n=1 \end{aligned}$$