Solutions – Junior Division

1. $1999 + 24 = 2023$,

hence (D).

2. As the angle on a straight line is 180°, $x° = 180° - (10° + 20° + 30°) = 180° - 60° = 120°$,

hence (E).

3. We know that $\frac{1}{2} = 0.5$, so absolute differences are (A) 0.05, (B) 0.1, (C) $\frac{1}{6} > 0.16$, (D) $\frac{1}{8} = 0.125$ and (E) 0.1. The smallest of these differences is 0.05. Thus, the fraction $\frac{1}{2}$ is closest to 0.45,

hence (A).

4. (A) $3 + 2 \times 4 = 11$
(B) $(9 + 5) \times 2 - 4 \times 2 = 20$
(C) $10^2 = 100$
(D) $20 + 20 \div 2 = 30$
(E) $10 \div 2 = 5$

hence (B).

5. From 8:37 am to 9 am is 23 minutes, from 9 am to 10 am is 60 minutes and from 10 am to 10:16 am is 16 minutes: $23 + 60 + 16 = 99$ minutes,

hence (C).

6. A square with an area of 25 cm^2 is 5 cm × 5 cm so the rectangle formed from three of these squares will have sides of 5 cm and 15 cm, giving a perimeter of 40 cm,

hence (C).

7. (Also I3)
If every digit of a whole number is either a 3 or a 5, the number must be odd. Note that 33 is not prime, even or divisible by 5 and 35 is not divisible by 3, showing that odd is the only consistent descriptor,

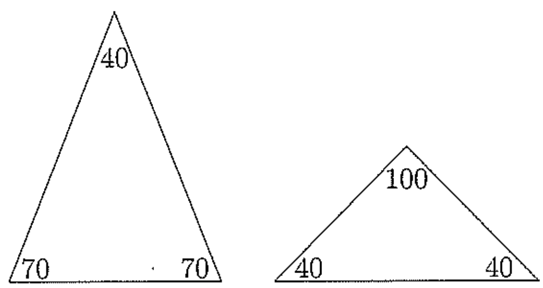
hence (E).

8. The point halfway between P and Q is the average of the two numbers.

$$(0.56 + 1.2) \div 2 = 0.88$$

hence (D).

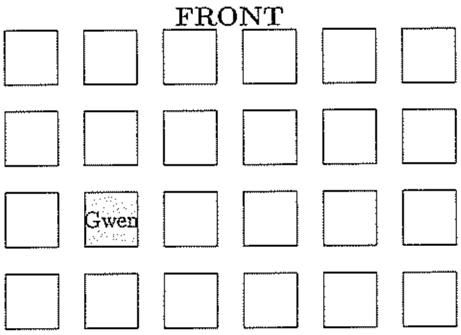
9. If triangle ABC is isosceles with $\angle A = 40°$, there are two possible triangles.



So $\angle B$ could be 40°, 70° or 100°,

hence (E).

10. The diagram shows the position of Gwen's desk, given the information in the question.



So there are 4 rows of 6 desks, giving 24 desks in all,

hence (B).

11. *Alternative 1*
Travelling both ways by bus takes 20 minutes, so the bus journey takes 10 minutes each way. Taking the bus to school and walking home takes 40 minutes, so walking takes $40 - 10 = 30$ minutes. If Will walks both ways it will take 60 minutes,

hence (D).

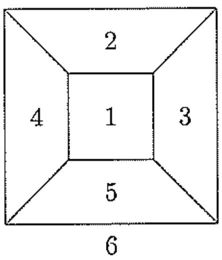
Alternative 2
The following takes 80 minutes: (i) walk to school, (ii) bus home, (iii) bus to school, (iv) walk home. Also, (ii) and (iii) together take 20 minutes. So (i) and (iv) together take 60 minutes,

hence (D).

12. *Alternative 1*
On a standard dice, 1 is opposite 6, 2 is opposite 5 and 3 is opposite 4. When playing *Corners*, by choosing a vertex, we choose the three faces next to that vertex, so we cannot choose two faces which are opposite each other. All other combinations are possible.
(A) $6 = 1 + 2 + 3$, which is possible
(B) $7 = 1 + 2 + 4$, which is possible
(C) $8 = 1 + 2 + 5$ or $1 + 3 + 4$ but both of these contain an opposite pair of faces. There is no other way to break 8 up into 3 different whole numbers
(D) $9 = 1 + 3 + 5$, which is possible
(E) $10 = 1 + 4 + 5$, which is possible,

hence (C).

Alternative 2
Flatten out the dice by puncturing face number 6. Then 6 will occupy the outside region adjacent to regions 2, 3, 4 and 5.



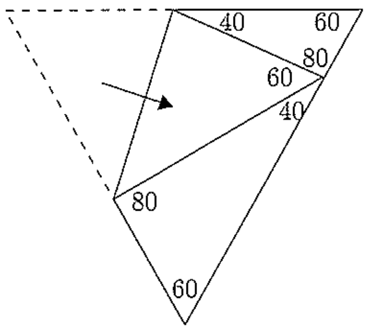
There are 8 possible totals and 8 is missing:

$$\begin{array}{ll} 1+2+3=6 & 6+2+3=11 \\ 1+2+4=7 & 6+2+4=12 \\ 1+3+5=9 & 6+3+5=14 \\ 1+4+5=10 & 6+4+5=15 \end{array}$$

hence (C).

13. The following diagram shows, using the facts that each corner of an equilateral triangle is 60° and the angles of any triangle add up to 180°, that x is equal to 80,

hence (C).



14. The lengths are $1, 1, 2, 2, 3, 3, 4, 4, \ldots$, with length n occurring in the $(2n-1)$th and $2n$th segments. Since $97 = 2 \times 49 - 1$, the 97th segment has length 49 centimetres,

hence (D).

15. Filling in around the discs with four touching greens, we get:

O	*O*	*G*	*O*
O	*G*	*O*	*G*
G	*O*	*G*	*O*
O	*G*	*O*	*O*

Then, surrounding the discs with no touching greens by reds, we obtain:

O	*O*	*G*	*R*
R	*G*	*R*	*G*
G	*R*	*G*	*R*
R	*G*	*R*	*O*

The remaining disc colours can then be identified. The correct pattern is below:

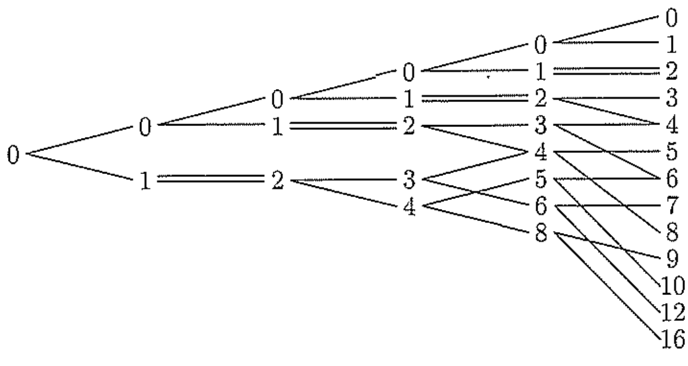
R	*G*	*G*	*R*
R	*G*	*R*	*G*
G	*R*	*G*	*R*
R	*G*	*R*	*R*

hence (E).

16. The second purchase has three fewer loaves of bread and three more bottles of milk, for a saving of $3 \times \$1.40$, so each loaf of bread costs \$1.40 more than a bottle of milk,

hence (A).

17. (Also S10)
The following diagram shows all possibilities after each step:



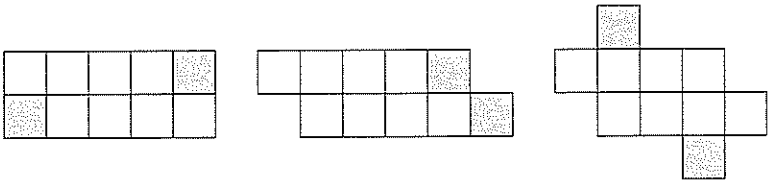
After five steps, the smallest missing number is 11,

hence (A).

18. By symmetry, $TM||DC$ and $AM||DN$, so the angle between TM and AM is the same as the angle between DC and DN. Many other solutions are also possible. $\angle AMT = 60°$,

hence (D).

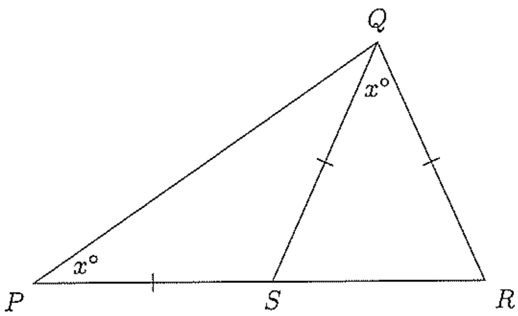
19. (Also UP23)
The perimeter of the original shape is 14.



As demonstrated in the examples shown, the perimeter of the new shapes could be 14, 16 or 18. The perimeter cannot be longer than 18 as adding a single square can add at most 2 to the perimeter. The sum is $14 + 18 = 32$,

hence (C).

20.



In the isosceles triangle ΔPQS, $\angle PQS = \angle QPS = x°$. So the exterior angle at S is $\angle QSR = x° + x° = 2x°$.
Since ΔQRS is isosceles, $\angle QRS = \angle QSR = 2x°$.
Consequently in ΔPQR, $180° = \angle QPR + \angle PQR + \angle PRQ = x° + 2x° + 2x° = 5x°$ and $x = 36$,

hence (B).

21. The pictures below show that he can use as few as 6 black mice by putting a black mouse in each 'x' cage.

.	.	x	x
x	.	.	.
x	.	.	.
.	.	x	x

.	x	.	.
.	x	.	x
.	.	.	x
x	x	.	.

Four black mice in two adjacent pairs will have at most eight neighbouring squares, which is not enough to guarantee a black mouse neighbour for all remaining squares. Five mice will have to be in a block of two and a block of three (as no black square can be isolated from a black neighbour and a block of five will obviously not provide a black neighbour for all remaining squares). If a straight 3-block is placed along an edge as in the picture below, square 1 requires a black neighbour, so square 2 cannot be reached, however the remaining 2-block is placed.

.	.	.	2
.	.	.	.
1	.	.	.
.	x	x	x

If a straight 3-block is placed away from an edge as in the picture below, square 1 is isolated.

.	.	.	.
.	.	.	.
.	x	x	x
1	.	.	.

Finally, if an L-shaped 3-block is used, then if it does not sit on any edge, all 4 corners are isolated. If it sits on an edge, however placed, the remaining un-neighboured squares are too far apart to be reached by any placement of a 2-block. Hence at least 6 black mice are needed,

hence (C).

22. Considering all combinations and their sums, we get

$$(1 + c) + (1 + d) + (b + c) + (b + d) = 8 + 9 + 10 + 11 = 38$$
$$2 + 2(b + c + d) = 38$$
$$b + c + d = 18$$

hence (B).

23. *Alternative 1*
The digits need to sum to an odd multiple of 3: 3, 9, 15, 21 or 27. Possibilities in each case:

- 111
- 117, 135, 153, 171, 315, 333, 351, 513, 531, 711
- 159, 177, 195, 339, 357, 375, 393, 519, 537, 555, 573, 591, 717, 735, 753, 771, 915, 933, 951
- 399, 579, 597, 759, 777, 795, 939, 957, 975, 993
- 999

The number of possibilities in each case is $1, 10, 19, 10, 1$, a total of 41 oddie numbers,

hence (D).

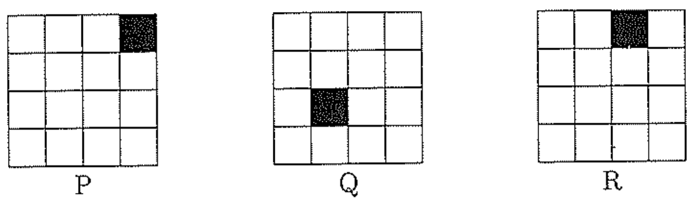
Alternative 2
Either all three digits are the same, two are the same with one different or all three are different. Possibilities in each case:

- 111, 333, 555, 777, 999
- 117, 171, 711, 177, 717, 771, 339, 393, 933, 399, 939, 993
- 135, 153, 315, 351, 513, 531, 357, 375, 537, 573, 735, 753, 579, 597, 759, 795, 957, 975, 159, 195, 519, 591, 915, 951

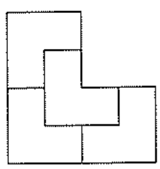
The number of possibilities is $5 + 12 + 24 = 41$,

hence (D).

24.



It is possible to tile the following shape in some orientation to fit each case,



and then the remaining shape is one more L-shaped tile,

hence (E).

25. (Also UP25)
There are only three 9s, and since the last has only 2 digits after it, it is not possible to have $N = 999xxx$. The next largest possibility is $N = 9989xx$, which is possible, but only with $N = 998930$. The sum of the digits is then $9 + 9 + 8 + 9 + 3 + 0 = 38$,

hence (B).

26. As we tally, the number of As and Bs may differ by no more than one, so after any even number of letters, the tallies must be the same. The next two letters must then be either AB or BA to leave the scores the same once again. Hence there are two possibilities for every pair of tallied letters. For 18 letters this gives 2^9, which is 512,

hence (512).

27. A date $d/m/2013$ will give the fraction $d \div (\frac{m}{2013}) = \frac{2013d}{m}$, which will be a whole number whenever $2013d$ is a multiple of m. Now $2013 = 3 \times 11 \times 61$, so all days in months 1, 3, 11 give integers, and in months $6, 9, 12$, the number d needs to be a multiple of $2, 3, 4$ respectively. In all other months, d needs to be a multiple of m. In summary:

Month	length	integer days	number of integer days
1	31	all	31
2	28	2,4,...,28	14
3	31	all	31
4	30	4,8,...,28	7
5	31	5,10,...,30	6
6	30	2,4,6,...,30	15
7	31	7,14,21,28	4
8	31	8,16,24	3
9	30	3,6,9,...,30	10
10	31	10,20,30	3
11	30	all	30
12	31	4,8,...,28	7
			161

hence (161).

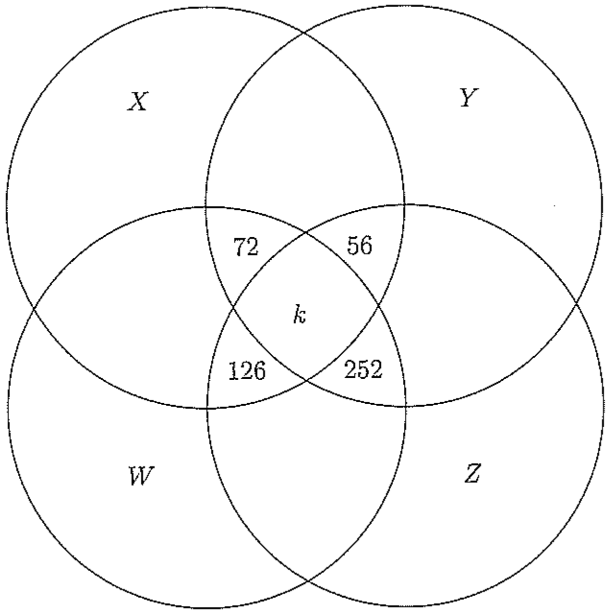
28. *Alternative 1*
Let N be the required integer. If a, b and c are the first numbers of each set of consecutive integers which sum to N, then N must be $9a + 36$, $10b + 45$ and $11c + 55$. So N is a multiple of 9 and 11, and its last digit is 5. The smallest multiple of 99 with last digit 5 is $5 \times 99 = 495$. 495 is of the form $9a + 36, 10b + 45$ and $11c + 55$,

hence (495).

Alternative 2
The sum of nine consecutive integers must be divisible by 9 (because the middle number will be the average of the nine numbers). Conversely, every multiple of 9 is expressible as the sum of nine consecutive integers because $9n = (n - 4) + (n - 3) + (n - 2) + (n - 1) + n + (n + 1) + (n + 2) + (n + 3) + (n + 4)$. This idea can be extended to any sequence of $2n + 1$ consecutive integers, which must be divisible by $2n + 1$. In particular a number which is the sum of 11 consecutive numbers will be divisible by 11. The sum of 10 consecutive integers must be an odd multiple of 5, because the sum will be 5 times the sum of the middle two numbers. Conversely every odd multiple of 5 is expressible as the sum of 10 consecutive integers because $5(2n + 1) = (n-4)+(n-3)+(n-2)+(n-1)+n+(n+1)+(n+2)+(n+3)+(n+4)+(n+5)$. In general, the sum of $2n$ consecutive numbers will be divisible by n. In this case, the required number must be divisible by 9, 5 and 11. Given that 9, 5 and 11 are co-prime, the smallest such positive number is $5 \times 9 \times 11 = 495$,

hence (495).

29.



Let the numbers assigned to the circles be X, Y, Z, W as shown above. Then since the region labelled 56 is part of the circles labelled X, Y and Z we have

$$
\begin{aligned}
XYZ &= 56 && (1)\\
YZW &= 252 && (2)\\
XWZ &= 126 && (3)\\
WXY &= 72 && (4)
\end{aligned}
$$

Also $k = WXYZ$. Using the symmetry we multiply (1), (2), (3) and (4) and obtain

$$k^3 = X^3Y^3Z^3W^3 = 56 \times 252 \times 126 \times 72 = 2^9.3^6.7^3$$

Thus $k = 2^3.3^2.7 = 504$. Also we can divide $k = 504$ by (1), (2), (3) and (4) to obtain $W = 9, X = 2, Y = 4$ and $Z = 7$. Thus $X + k = 506$,

hence (506).

30. (Also I27)

Each digit appears twice in the hundreds, the tens and the units column amongst the six numbers which can be formed. If the digits are a, b, c and the missing number is x, then $3231 + x = 222(a + b + c)$. Then $x = 222m - 3231$ must have three distinct digits that add to m. So $m > 3231/222 = 15\frac{1}{222}$. Try $m = 16, 17, \ldots$:

$$
\begin{aligned}
&222 \times 16 - 3231 = 3552 - 3231 = 321, \quad 3 + 2 + 1 = 6 \quad \neq 16\\
&222 \times 17 - 3231 = 3774 - 3231 = 543, \quad 5 + 4 + 3 = 12 \quad \neq 17\\
&222 \times 18 - 3231 = 3996 - 3231 = 765, \quad 7 + 6 + 5 = 18
\end{aligned}
$$

So the sixth number is 765,

hence (765).