Student number:_____

2019

YEAR 11
YEARLY
EXAMINATION

Mathematics Advanced

General Instructions

- Working time - 120 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:
80

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 70 marks

- Attempt all questions
- Allow about 1 hour and 45 minutes for this section

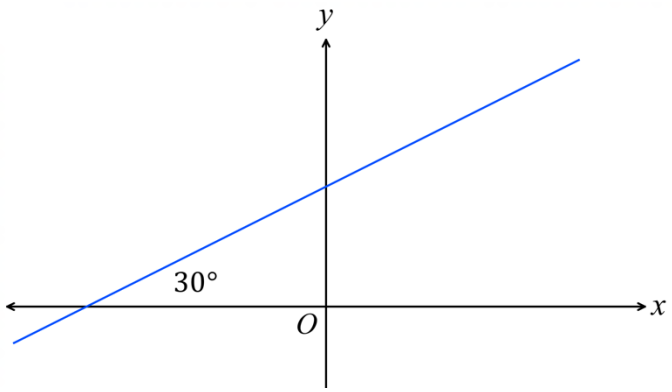
	MC	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Total
Algebra		/6							/6
Trigonometry	/3			/3	/6	/5		/2	/19
Functions	/2		/8		/5	/3		/2	/20
Calculus	/3			/10		/5	/7	/3	/28
Probability	/2	/3			/2				/7
	/10	/9	/8	/13	/13	/13	/7	/7	/80

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. A line makes an angle of 30° with the positive direction of the x-axis as shown.



Not to scale

What is the gradient of the line?

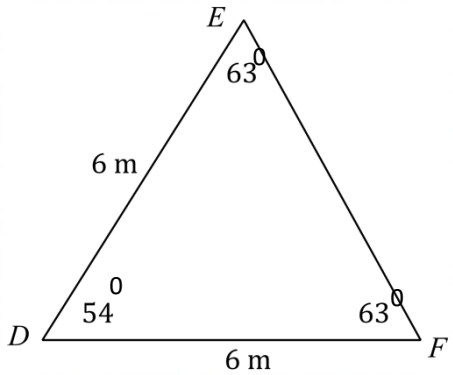
(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

2.



What is the area of the ΔDEF? Answer correct to two decimal places.

(A) 11.71 m²

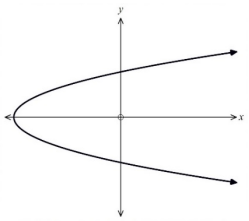
(B) 13.33 m²

(C) 14.56 m²

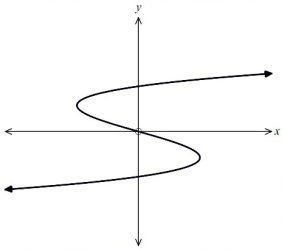
(D) 16.04 m²

3. Which graph represents a function?

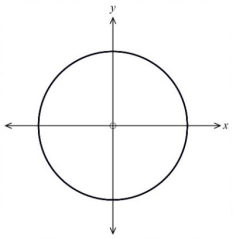
A



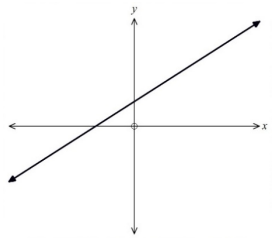
B.



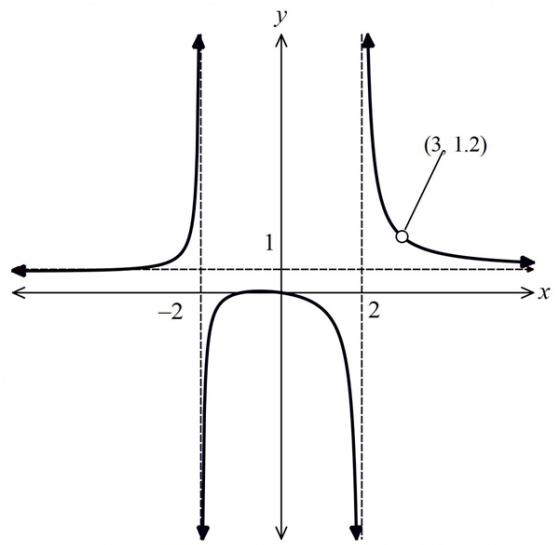
C.



D.



4. The graph of $y = f(x)$ is shown below.



Where, in the domain shown, does the curve have discontinuities?

(A) $x = -2$ and $x = 2$ only

(B) $x = -2$, $x = 2$ and $x = 3$

(C) $x = -2$, $x = 1$ and $x = 2$

(D) $x = -2$, $x = 1, x = 1.2$ and $x = 2$

5. The probability distribution of a random variable X is shown below:

x	0	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$2k$	k

What is the value of k?

(A) $\frac{1}{10}$

(B) $\frac{1}{9}$

(C) $\frac{1}{8}$

(D) $\frac{1}{5}$

6. What is the solution to the equation $2\cos x = \sqrt{3}$ for x, where $0 \leq x \leq 2\pi$?

(A) $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

(B) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

(D) $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

7. What is the value of? $\lim_{x \to 4} \frac{x-4}{x^2-16}$

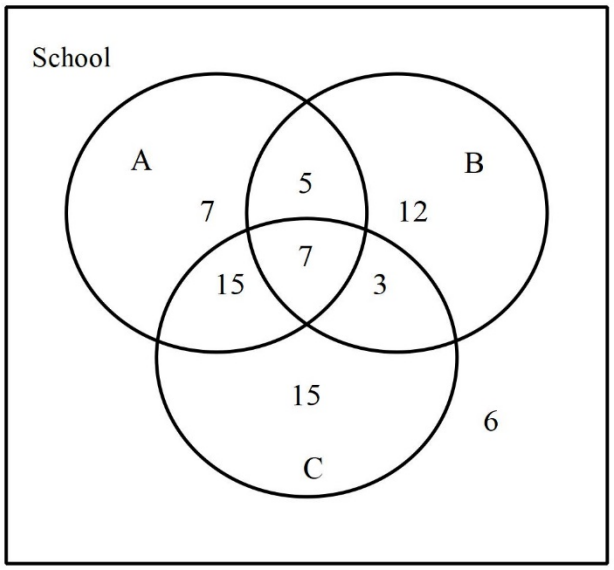
(A) $\frac{x-4}{0}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) No solution

8. The Venn diagram shows the membership of the 70 students in a school, in three groups, the archery club (A), the basketball teams (B) and the choir (C).



A student from the school is chosen at random.

What is the probability that the student is ***not*** a choir member but is in the archery club?

(A) $\frac{6}{35}$

(B) $\frac{11}{35}$

(C) $\frac{24}{35}$

(D) $\frac{26}{35}$

9. Given that $\cot\beta = -\frac{5}{9}$ and that $\sin\beta < 0$, find the value of $\cos\beta$.?

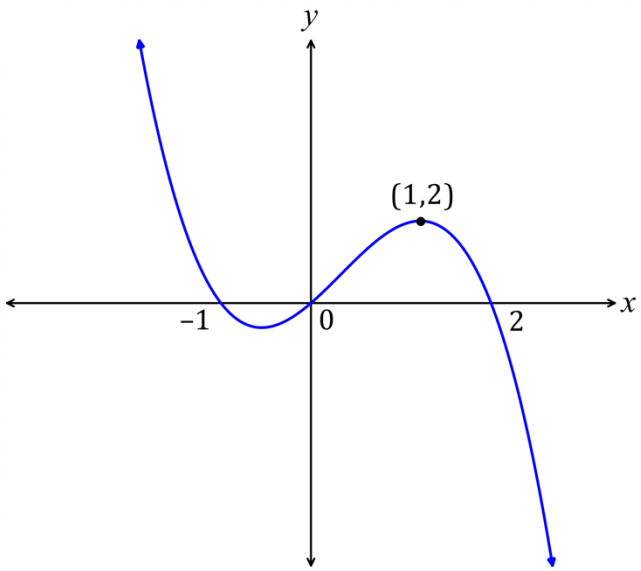
(A) $-\frac{5}{\sqrt{56}}$

(B) $-\frac{5}{\sqrt{106}}$

(C) $\frac{5}{\sqrt{106}}$

(D) $\frac{5}{\sqrt{56}}$

10. The diagram below shows the graph of a cubic function.



Not to scale

What is the equation of this cubic function?

(A) $y = -x(x+1)(x-2)$

(B) $y = -x(x-1)(x+2)$

(C) $y = -x(x+1)(x-2)$

(D) $y = -x(x-1)(x+2)$

Section II

70 marks
Attempt all questions
Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (9 marks) **Marks**

Simplify the expression:

(a) $\dfrac{6x^3 \times 5x^{-1}}{2x^3 \times 5x^{-2}}$. **2**

Simplify the following algebraic fractions

(b) $\dfrac{1}{3+\sqrt{3}}$ **2**

(c) $\dfrac{x+2y}{3} - \dfrac{2x-y}{4}$ **2**

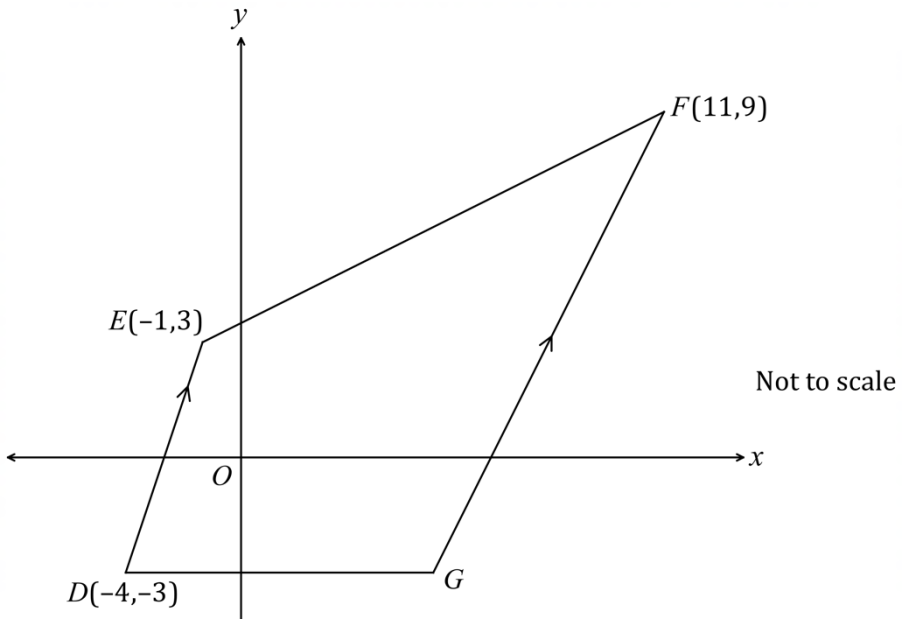
(d)
The number of people who will ring a hotline between 9 am and 10 am is described by the probability distribution below.

X	0	1	2	3	4	5	6
P(X)	0.08	0.12	0.2	0.25	0.18	0.13	0.04

i What is the expected value of *X*? **1**

ii What is the standard deviation of *X*, correct to 3 significant figures? **2**

Question 12 (8 marks) **Marks**



On a number plane the points D, E and F have coordinates (–4, –3), (–1, 3) and (11, 9) respectively. DE is parallel to GF and DG is parallel to the x-axis.

(a) What is the gradient of DE? **1**

(b) What is the midpoint of DE? **1**

(c) Find the equation of the line FG. **2**

(d) Show that the coordinates of the point G are (5, –3). **2**

(e) Find the distance EF. **1**

(f) Find the equation of the circle centred at F with radius EF. **1**

Question 13 (13 marks) **Marks**

(a) Find the derivative **by first principles**. $f(x) = 3x^2 + x$ **3**

Use the definition: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$,

Differentiate the following:

(b) $3\sqrt{x^3}$. **1**

(c) $\frac{-2}{x^3}$. **1**

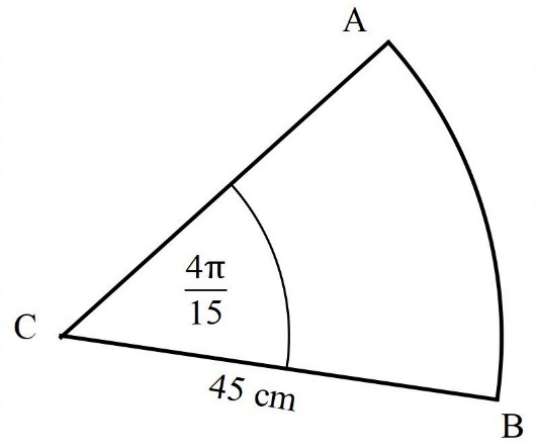
(d) $\frac{x^2+1}{x+1}$. **1**

(e) $(x^3+2)^5$. **2**

(f) $x^7(x^4+5x)^6$. **2**

(g)

A sector of a circle is shown below, with the internal angle measured in radians.



i. Calculate the exact length of the arc *AB* **2**

ii. Find the exact area of the sector *ABC*. **1**

Question 14 (13 marks)

Paul buys three tickets in a raffle which has three equal prizes.

There are 100 tickets sold in the raffle.

(a) What is the probability that he wins all three prizes? **1**

(b) If he does not win the first prize which is drawn, what is the probability that he wins the next two prizes? **1**

(c)

A circle on the number plane has equation $x^2 + y^2 - 8x + 6y + 21 = 0$.

i Find the centre and radius of the circle. **2**

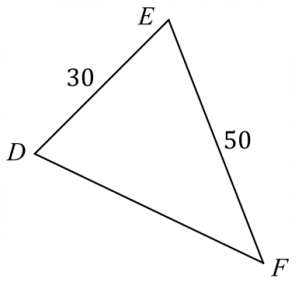
ii. Give the domain of the circle. **1**

(d)

Draw a neat sketch of $y = |2x - 5|$ in your booklet showing all axis intercepts. **2**

(e)

Three girls are standing in the park. *D* is 30 metres from *E* and *E* is 50 metres from *F*. The bearing of *E* from *D* is 045° and the bearing of *F* from *E* is 158°.



i. Copy the diagram above in your booklet showing all information given. **1**

ii. Show that $\angle DEF = 67°$. **1**

iii. Show that the distance of *F* from *D* is approximately 47 metres. **1**

iv. Hence, or otherwise, find the bearing of *D* from *F*.
Answer to the nearest degree. **3**

Question 15 (13 marks)

The curve *C* has the equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point *P* has coordinates (3, 0)

(a) Show that *P* lies on *C*. **1**

(b) Find the equation of the tangent to *C* at *P*. **2**

(c) Another point *Q* also lies on *C*. The tangent to *C* at *Q* is parallel to the tangent to *C* at *P*. What are the coordinates of *Q*? **2**

(d)

Given that $f(x) = x^2 - x$ and $h(x) = 2x + 1$, find:

i. $f(-2)$ **1**

ii. $f\big(h(x)\big)$ **1**

iii. x where $h(x) = 0$ **1**

(e)

i. Show that $\dfrac{(1 + \tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$ **3**

ii. Find the exact value of $\sin(315^0)$ **1**

iii. Solve $6^{x-2} = \frac{1}{36}$ **1**

Question 16 (7 marks)

A water tank is being filled at a variable rate. The height of the water, h cm, at any time, t minutes can be described by

$h(x) = 2t^2 - 4t + 50$

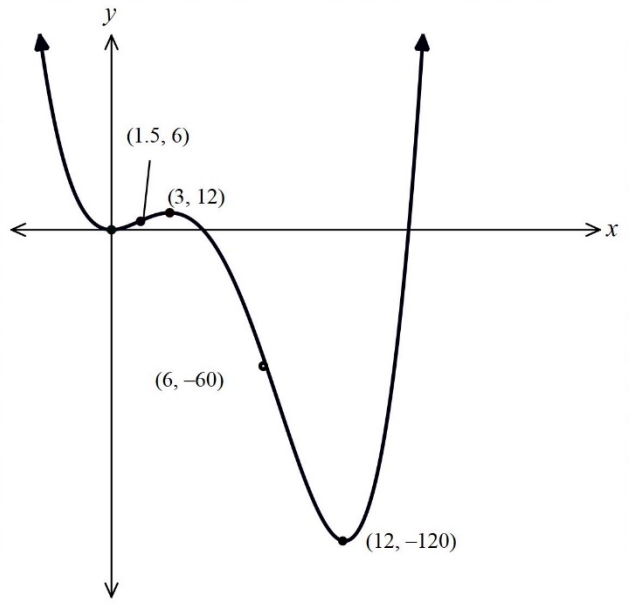
Find

(a) the initial height of water in the tank **1**

(b) the instantaneous rate at which the height is changing at 4 minutes **2**

(c) the average rate at which the height has changed over the first 4 minutes **2**

(d)
The function below has turning points at the origin and at (3, 12) and (12, –120).
It has points of inflection at (1.5, 6) and (6, –60).



i. For what x-values is the function increasing? **1**

ii. For what x-values is the curve decreasing at an increasing rate? **1**

Question 17 (7 marks)

A particle moves along the *x*-axis such that its displacement from the origin (in cm) at a time *t* (seconds) is given by the equation:

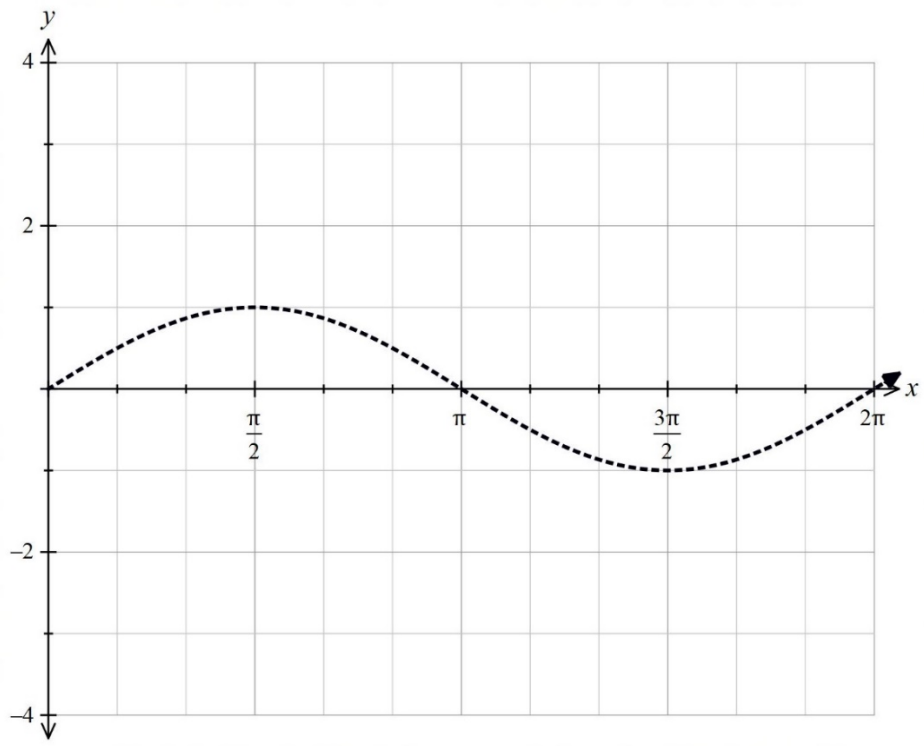
$x = 2t^3 + \sqrt{t}$.

(a) Find the velocity and acceleration of the particle after 9 seconds. **3**

(b)

The graph of $y = \sin x$ for $0 \leq x \leq 2\pi$ is drawn on the axes below.

Copy the graph below into your booklet and on the same axes, draw and label neat sketches of $y = \cos x$ and $y = \text{cosec } x$ for $0 \leq x \leq 2\pi$ **2**



(c)

Show that the function below is odd, even or neither. **2**

$f(x) = 2x^4 - 3x^2 - 1$

End of paper

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

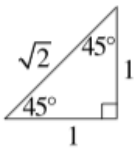
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

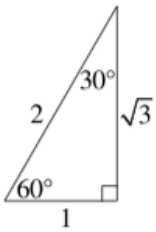
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

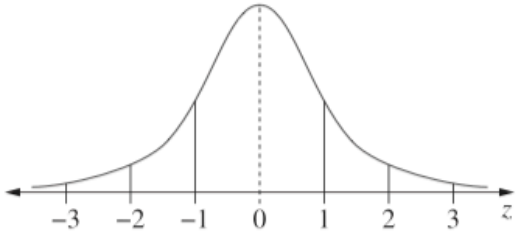
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$