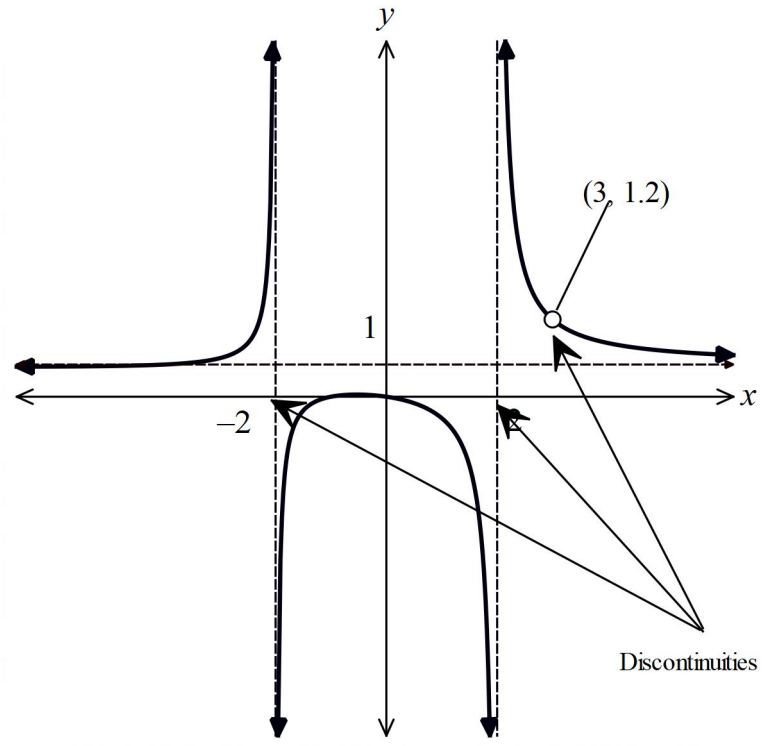
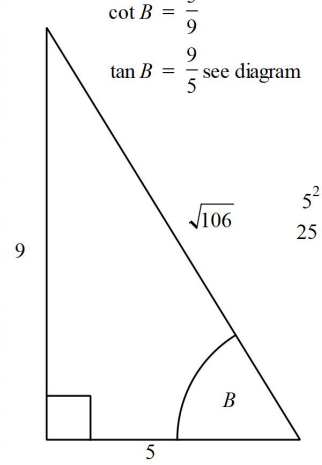
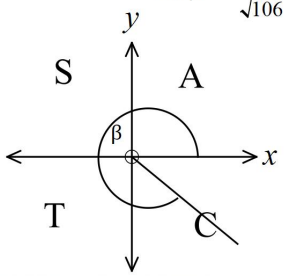
2019

Year 11 Mathematics Advanced Yearly Examination

Worked solutions and marking guidelines

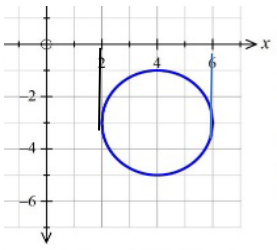
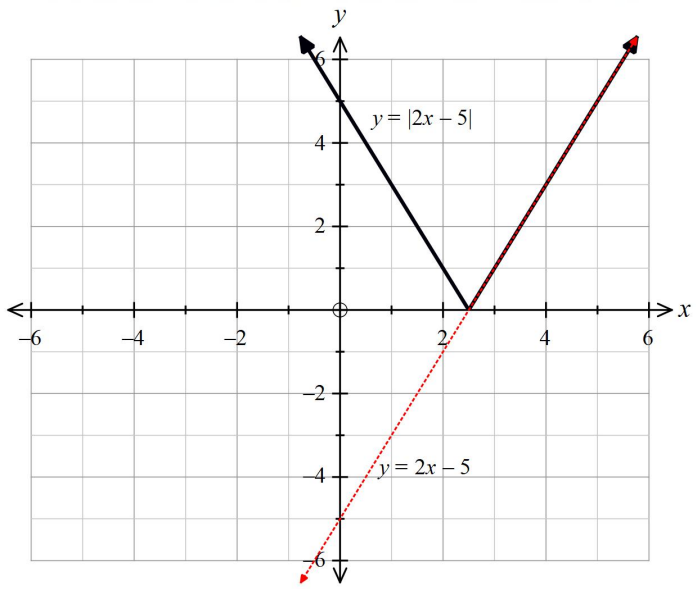
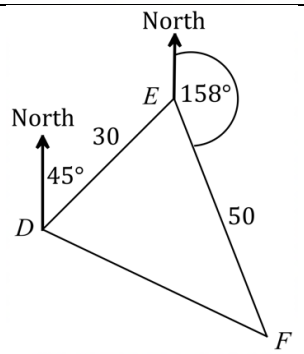
Section I		
	Solution	**Criteria**
1.	$m = \tan\theta$ $= \tan 30°$ $= \frac{1}{\sqrt{3}}$	1 Mark: A ***calculus***
2.	$A = \frac{1}{2}absinC$ $= \frac{1}{2} \times 6 \times 6 \times \sin 54°$ $= 14.5623\ldots$ $\approx 14.56\ \text{m}^2$	1 Mark: C ***Trig***
3.	A vertical line can be drawn to cut all curves except D, the straight line.	1 Mark: D ***functions***
4.	 $x = -2$, $x = 2$ and $x = 3$	1 Mark: B ***calculus***
5.	$k + 2k + 3k + 2k + k = 1$ $9k = 1$ $k = \frac{1}{9}$	1 Mark: B ***prob***

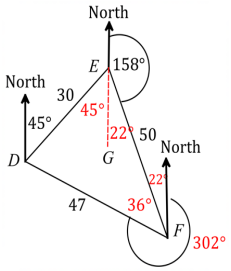
6.	$2\cos x = \sqrt{3}$ $\cos x = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$	1 Mark: D ***Trig***
7.	$\lim_{x\to 4} \frac{x-4}{x^2-16}$ $\lim_{x\to 4} \frac{x-4}{(x-4)(x+4)}$ $\lim_{x\to 4} \frac{1}{(x+4)}$ $= \frac{1}{(4+4)} = \frac{1}{8}$	1 Mark: C ***calculus***
8.	$\frac{7+5}{70} = \frac{12}{70} = \frac{6}{35}$	1 Mark: A ***Prob***
9.	$\cot B = \frac{5}{9}$ $\tan B = \frac{9}{5}$ see diagram  $5^2 + 9^2 = x^2$ $25 + 81 = x^2$ $106 = x^2$ $x = \sqrt{106}$ $\cos B = \frac{5}{\sqrt{106}}$ cot (and tan) are -ve so 2nd or 4th Quad sin is -ve so 3rd or 4th Quad So 4th Quad so cos +ve $\cos \beta = \frac{5}{\sqrt{106}}$  	1 Mark: C ***Trig***
10.	Intercepts are –1, 0 and 2. $y = -x(x+1)(x-2)$ Test point (1, 2) $2 = -1 \times (1+1) \times (1-2)$ True	1 Mark: A or C ***functions***

Section II		
	Solution	**Criteria**
11(a) **KC**	$\frac{6x^3 \times 5x^{-1}}{2x^3 \times 5x^{-2}} = \frac{30x^2}{10x^1} = 3x^1 = 3x$	2 mark for correct answer 1 mark: Simplifies the fractions ***Algebra***
11(b) **KC**	$\frac{1}{3+\sqrt{3}} = \frac{1}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}$ $= \frac{3-\sqrt{3}}{6}$	2 marks: Correct answer. 1 mark: Simplifies the fractions by multiplying by the conjugate. ***Algebra***
11(c) **KC**	$\frac{x+2y}{3} - \frac{2x-y}{4} = \frac{4(x+2y)-3(2x-y)}{12}$ $= \frac{4x+8y-6x+3y}{12}$ $= \frac{11y-2x}{12}$	2 marks: Correct answer. 1 mark: Finds common denominator and converts numerators. ***Algebra***
11(di) **KC**	Mean (Exp Value) $= 0 \times 0.08 + 1 \times 0.12 + 2 \times 0.2 + 3 \times 0.25 + 4 \times 0.18 + 5 \times 0.13 + 6 \times 0.04$ $= 2.88$	1 marks for correct answer ***prob***
11(dii) **KC**	Variance $= (-2.88)^2 \times 0.08 + (-1.88)^2 \times 0.12 + (-0.88)^2 \times 0.2 + (0.12)^2 \times 0.25 + (1.12)^2 \times 0.18 + (2.12)^2 \times 0.13 + (3.12)^2 \times 0.04$ $= 0.663552 + 0.424128 + 0.15488 + 0.0036 + 0.225792 + 0.584272 + 0.389376$ $= 2.4456$ Standard deviation $= \sqrt{2.4456}$ $= 1.563841424185969$ $= 1.56\ (3sf)$	2 marks for correct answer 1 mark for relevant working involving finding deviations from the mean and squaring them ***prob***
12(a) **KC**	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-3)}{-1-(-4)} = \frac{6}{3} = 2$	1 marks: Correct answer. ***functions***

12(b) **KC**	$x = \frac{x_1 + x_2}{2} = \frac{-4 + (-1)}{2} = -\frac{5}{2}$ $y = \frac{y_1 + y_2}{2} = \frac{-3 + 3}{2} = 0$ $\therefore$ Midpoint is (–2.5, 0)	1 mark: Correct answer. ***Functions***
12(c) **AG**	Line *FG* has the same gradient as *DE* (parallel lines) $y - y_1 = m(x - x_1)$ $y - 9 = 2(x - 11)$ $y - 9 = 2x - 22$ $2x - y - 13 = 0$	2 mark: Correct answer. (any form is fine) ***Functions***
12(d) **AG**	Point *G* is on the line *DG* or $y = -3$ To find x when $y = -3$ substitute into the equation of *FG* $2x - (-3) - 13 = 0$ $2x = 10$ $x = 5$ $\therefore$ Coordinates of *G* are (5, –3)	2 marks: Correct answer. 1 mark: Used the point slope formula with correct slope or point. ***Functions***
12(e) **AG**	Use Pythagoras theorem $EF^2 = 12^2 + 6^2$ $EF = \sqrt{12^2 + 6^2}$ $= \sqrt{180}$ $= 3\sqrt{20}$	1 marks: Correct answer. ***Functions***
12(f) **AG**	$(x - a)^2 + (y - b)^2 = r^2$ $(x - 11)^2 + (y - 9)^2 = \left(\sqrt{180}\right)^2$ $(x - 11)^2 + (y - 9)^2 = 180$	1 mark: Correct answer. ***Functions***
13(a) **AG**	$f(x) = 3x^2 + x \qquad f(x+h) = 3(x+h)^2 + (x+h)$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{3(x+h)^2 + (x+h) - 3x^2 - x}{h}$ $= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + (x+h) - 3x^2 - x}{h}$ $= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + x + h - 3x^2 - x}{h}$ $= \lim_{h \to 0} \frac{6xh + 3h^2 + h}{h}$ $= \lim_{h \to 0} 6x + 3h + 1$ $= 6x + 1$	3 marks for complete and correct derivation 2 marks for correct substitution and manipulation with minor errors or not completed fully eg: mistake with limit notation 1 mark for some correct and valid working ***calculus***
13(b) **AG**	$\frac{d}{dx}\left(3\sqrt{x^3}\right) = \frac{d}{dx}\left(3x^{\frac{3}{2}}\right) = 3 \times \frac{3}{2}x^{\frac{1}{2}}$ $= \frac{9}{2}x^{\frac{1}{2}}$ $= \frac{9\sqrt{x}}{2}$	1 mark for correct answer in either form of last 2 lines ***calculus***

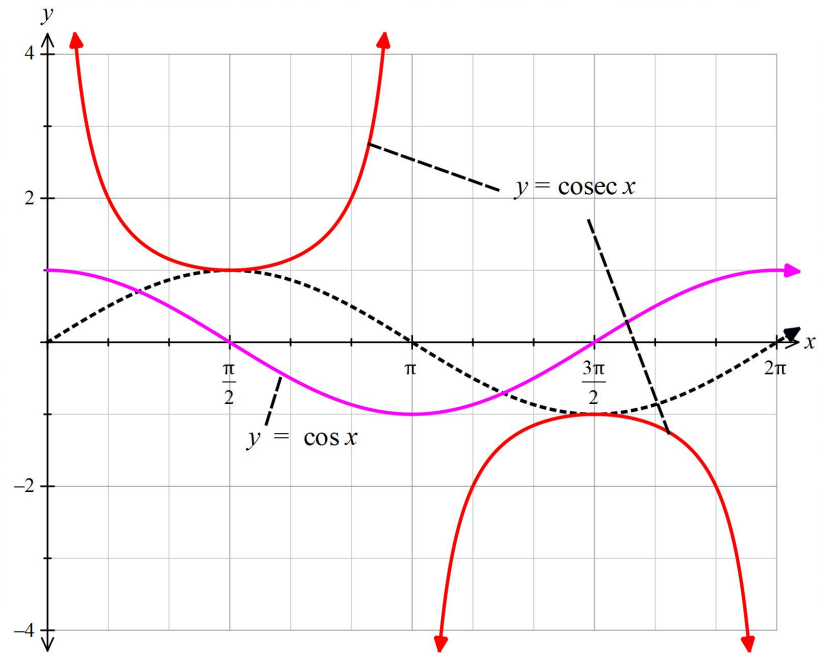
13(c) **AG**	$\frac{d}{dx}\left(\frac{-2}{x^3}\right) = \frac{d}{dx}(-2x^{-3})$ $= -2 \times -3x^{-4}$ $= 6x^{-4}$ $= \frac{6}{x^4}$	1 mark for correct answer in either form of last 2 lines ***calculus***
13(d) **AG**	$\frac{d}{dx}\left(\frac{x^2+1}{x+1}\right) = \frac{(x+1)(2x)-(x^2+1)(1)}{(x+1)^2}$ $= \frac{2x^2+2x-x^2-1}{(x+1)^2}$ $= \frac{x^2+2x-1}{(x+1)^2}$	1 mark for correct equation ***calculus***
13(e) **MV**	$\frac{d}{dx}((x^3+2)^5) = 5(x^3+2)^4(3x^2)$ $= 15x^2(x^3+2)^4$	2 mark for correct equation 1 mark for working which includes some correct differentiation ***calculus***
13(f) **MV**	$\frac{d}{dx}(x^7(x^4+5x)^6) = x^7(6(x^4+5x)^5(4x^3+5)) + 7x^6(x^4+5x)^6$ $= (24x^{10}+30x^7)(x^4+5x)^5 + 7x^6(x^4+5x)^6$ Can be further simplified but acquires no extra marks. $= (x^4+5x)^5(24x^{10}+30x^7+7x^{10}+35x^7)$ $= (x^4+5x)^5(31x^{10}+65x^7)$	2 marks for correct expression in any reasonable form 1 mark for working which includes some correct differentiation using product and/or chain rule ***calculus***
13(gi) **MV**	$l = r\theta$ $= 45 \times \frac{4\pi}{15}$ $= 12\pi$ cm	2 marks for correct answer 1 mark for ***non-exact*** answer (37.699) ***Trig***
13(gii) **MV**	Area $= \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 45^2 \times \frac{4\pi}{15}$ $= 270\pi$ cm^2	1 mark for correct answer ***Trig***
14(a) **MV**	$P(WWW) = \frac{3}{100} \times \frac{2}{99} \times \frac{1}{98}$ $= \frac{1}{161\,700}$	1 mark for correct answer ***Prob***
14(b) **MV**	If doesn't win first, he still has 3 tickets from 99, since one drawn P(Wins next 2 given doesn't win first) $= P(WW$ after $L)$ $= \frac{3}{99} \times \frac{2}{98}$ $= \frac{1}{1\,617}$	1 mark for correct answer ***Prob***

14(ci) **MV**	Recognise the curve as a circle, so complete the square: $x^2 + y^2 - 8x + 6y + 21 = 0$ $x^2 - 8x + y^2 + 6y = -21$ $x^2 - 8x + 4^2 + y^2 + 6y + 3^2 = 16 + 9 - 21$ $x^2 - 8x + 16 + y^2 + 6y + 9 = 4$ $(x-4)^2 + (y+3)^2 = 2^2$ Curve is a circle, centre $(4, -3)$ and radius 2	2 marks for correct description 1 mark for attempt at completing the square or equivalent merit ***Functions***
14(cii) **MV**	Domain minimum = $4 - 2 = 2$ Domain maximum = $4 + 2 = 6$ Domain $2 \leq x \leq 6$ OR From Sketch $2 \leq x \leq 6$  	1 mark for correct domain ***Functions***
14(d) **PW**	The line $y = 2x - 5$ has a gradient of 2 and a y intercept of -5 so has the graph shown with dotted line Absolute value graph is the same when above x axis and the reflection when below the axis  	2 marks for correct graph 1 mark for graph of $y = 2x - 5$ or working of similar merit ***Functions***
14(ei) **PW**	 	1 mark: Correct answer. ***Trig***

14(eii) **PW**	$\angle DEG = 45°$ (Alternate angle to the bearing of E from D) $\angle FEG = 180 - 158$ $= 22°$ (Straight angle of 180°) $\angle DEF = \angle DEG + \angle FEG$ $= 45 + 22$ $= 67°$  	1 mark: Correct answer. ***Trig***
14(eiii) **PW**	$a^2 = b^2 + c^2 - 2bc\cos A$ $DF^2 = 30^2 + 50^2 - 2 \times 30 \times 50 \times \cos 67$ $= 2227.8066$ $DF = 47.1996\ldots$ $= 47$ m $\therefore$ Distance of F from D is approximately 47 m	1 mark: Correct answer. ***Trig***
14(eiv) **PW**	To find $\sin \angle DFE$ $\frac{\sin \angle DFE}{30} = \frac{\sin 67}{47.1996\ldots}$ $\sin \angle DFE = \frac{\sin 67 \times 30}{47.1996\ldots}$ $\angle DFE = 35.8080\ldots$ $\approx 36°$ Bearing $= 360 - (36 + 22)$ $= 302°$ (See above diagram) $\therefore$ Bearing of D from F is 302°	3 marks: Correct answer. 2 marks: Finds $\angle DFE$ 1 mark: Used the sine rule (or alternatively the cosine rule) with at least one correct. ***Trig***
15(a) **PW**	Substitute P into the equation of C $y = \frac{1}{3} \times 3^3 - 4 \times 3^2 + 8 \times 3 + 3$ $= 9 - 36 + 27$ $= 0$ $\therefore$ P lies on C	1 mark: Correct answer. ***calculus***
15(b) **PW**	$\frac{dy}{dx} = x^2 - 8x + 8$ When $x = 3$ $m = 3^2 - 8 \times 3 + 8$ $= -7$ Equation of the tangent $y - y_1 = m(x - x_1)$ $y - 0 = -7(x - 3)$ $y = -7x + 21$ $7x + y - 21 = 0$	2 marks: Correct answer. 1 mark: Finds the gradient of the tangent at P. ***calculus***

15(c) **PW**	Tangent at Q is parallel and has the same gradient (m =–7) $\frac{dy}{dx} = x^2 - 8x + 8 = -7$ $x^2 - 8x + 15 = 0$ $(x-3)(x-5) = 0$ $x = 3$ or $x = 5$ The x-coordinate of Q is 5 $y = \frac{1}{3} \times 5^3 - 4 \times 5^2 + 8 \times 5 + 3 = -15\frac{1}{3}$ $\therefore$ Point Q is $\left(5, -15\frac{1}{3}\right)$	2 marks: Correct answer. 1 mark: Finds the x coordinate of Q or makes some progress. ***calculus***
15(di) **MB**	$f(-2) = (-2)^2 - (-2)$ $= 6$	1 mark: Correct answer. ***Functions***
15(dii) **MB**	$f\big(h(x)\big) = f(2x+1)$ $= (2x+1)^2 - (2x+1)$ $= 4x^2 + 2x$ or $2x(2x+1)$	1 mark: Correct answer. ***Functions***
15(diii) **MB**	$h(x) = 0$ $2x + 1 = 0$ $x = -\frac{1}{2}$	1 mark: Correct answer. ***Functions***
15(ei) **MB**	LHS $= \frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta}$ $= \frac{\sec^2\theta\cot\theta}{\text{cosec}^2\theta}$ $= \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin^2\theta}$ $= \frac{1}{\cos\theta\sin\theta} \times \sin^2\theta$ $= \frac{\sin\theta}{\cos\theta}$ $= \tan\theta$ = RHS	3 marks: Correct answer. 2 marks: Makes significant progress. 1 mark: Used one correct trigonometric identity. ***Trig***
15(eii) **MB**	$\sin(315°) = \frac{-1}{\sqrt{2}}$	1 mark: Correct answer. ***Trig***
15(eiii) **MB**	$6^{x-2} = \frac{1}{36}$ $6^{x-2} = 6^{-2}$ $x - 2 = -2$ $x = 0$	1 mark: Correct answer. ***Trig***
16(a) **MB**	Initially t = 0 $h(0) = 2 \times 0^2 - 4 \times 0 + 50$ $= 50$ cm $\therefore$ Initial height is 50 cm	1 mark: Correct answer. ***Calc***
16(b) **MB**	Use the derivatives to find the instantaneous rate of change $h'(t) = 4t - 4$ $h'(4) = 4 \times 4 - 4$ $= 12$ $\therefore$ Instantaneous rate is 12 cm/min	2 marks: Correct answer. 1 mark: Finds the first derivative. ***Calc***

16(c) **MB**	To find the height when $t = 4$ $h(4) = 2 \times 4^2 - 4 \times 4 + 50$ $= 66$ cm Average rate $= \dfrac{\text{Total change in depth}}{\text{Total time taken}}$ $= \dfrac{66 - 50}{4}$ $= 4$ cm/min $\therefore$ Average rate is 4 cm/min	2 marks: Correct answer. 1 mark: Finds the height when $t = 4$ or shows some understanding. ***Calc***
16(di) **SS**	Curve is increasing for $0 < x < 3$ and $x > 12$.	1 mark for both correct inequalities ***Calc***
16(dii) **SS**	Curve is decreasing at an increasing rate for $3 < x < 6$.	1 mark for correct inequality ***Calc***

17(a) **SS**	$x = 2t^3 + t^{\frac{1}{2}}$ $\dot{x} = 6t^2 + \frac{1}{2}t^{-\frac{1}{2}}$ $\ddot{x} = 12t - \frac{1}{4}t^{-\frac{3}{2}}$ When $t = 9$ $\dot{x} = 6(9)^2 + \frac{1}{2}(9)^{-\frac{1}{2}}$ $\dot{x} = 6(9)^2 + \frac{1}{2} \times \frac{1}{\sqrt{9}}$ Velocity $= 486\frac{1}{6}$ When $t = 9$ $\ddot{x} = 12(9) - \frac{1}{4}(9)^{-\frac{3}{2}}$ $= 108 - \frac{1}{4} \times \frac{1}{\sqrt{9^3}}$ $= 108 - \frac{1}{108}$ Acceleration $= 107\frac{107}{108}$	3 marks for velocity and acceleration 2 mark for either velocity or acceleration or working of equivalent merit. 1 mark for velocity ***and*** acceleration functions. ***Calc***
17(b) **SS**		1 mark ***for each*** graph (2) ***Trig***
17(c) **SS**	$f(x) = 2x^4 - 3x^2 - 1$ $f(-x) = 2x^4 - 3x^2 - 1$ $\therefore f(x) = f(-x) \therefore$ *even function*	2 marks for correct working and answer 1 mark for finding f(-x) ***Functions***