

# Carlingford High School



## Year 11 Mathematics

### Term 2 Assessment Task 2020

Time allowed: 50 minutes

Student Number: Solutions ..... Class: 11MAA.....

Teacher: (Please Circle)      Mr Cheng      Ms Strilakos      Ms Tang  
Mr Fardouly/ Mrs Wilson      Ms Bennett      Mr Gong      Mr Wilson

#### Instructions

- Answer each question in the space provided
- Marks may be deducted for careless or badly arranged work
- All answers are to be completed in blue or black pen except graphs and diagrams
- No lending or borrowing
- Board approved calculators may be used

Functions	Trigonometry	Total	
/16	/25	/41	%

F 1-4 Ken

F 5-6 + T 1-3 Peter

T 4-8 Grace

T 9-11 Alex

## Functions (16 marks)

1. Which equation represents the line perpendicular to  $2x - 3y = 1$ , passing through the point  $(0, 2)$ ?

(A)  $3x + 2y = 4$   
C  $3x - 2y = -4$

B  $3x + 2y = 6$   
D  $3x - 2y = 6$

1

$$m_1 = \frac{2}{3}$$

$$m_2 = -\frac{3}{2}$$

2. Find the equation of the straight line with angle of inclination  $30^\circ$  passing through the point  $(\sqrt{3}, 2)$ . 2

$$m = \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}} \quad \checkmark$$

$$y - 2 = \frac{1}{\sqrt{3}}(x - \sqrt{3})$$

$$y = \frac{1}{\sqrt{3}}x + 1 \quad \text{or} \quad x - \sqrt{3}y + \sqrt{3} = 0 \quad \checkmark$$

3. The lines  $x + 2y + 5 = 0$  and  $3x - y + 1 = 0$  intersect at a point  $P$ .

- a) Find the coordinates of  $P$

2

$$y = 3x + 1$$

$$x + 2(3x + 1) + 5 = 0$$

$$7x + 7 = 0$$

$$x = -1 \quad \checkmark$$

$$y = -3 + 1$$

$$y = -2$$

$$P = (-1, -2) \quad \checkmark$$

- b) Find the equation of the line passing through  $P$  which is parallel to the line  $4x - y + 1 = 0$ . Give your answer in general form.

2

$$y = 4x + 1$$

$$\therefore m = 4 \quad \checkmark$$

$$y + 2 = 4(x + 1)$$

$$4x - y + 4 - 2 = 0$$

$$4x - y + 2 = 0 \quad \checkmark$$

4. The SRC is selling cards as a fundraiser. They spend \$30 on equipment, and estimate that it costs \$2 in supplies to make each pack of cards. They plan to sell the cards for \$3.50 per pack.

a) Set up the cost,  $C$ , and revenue,  $R$ , functions for the sale of  $x$  items. 2

$$C = 30 + 2x \checkmark$$

$$R = 3.5x \checkmark$$

b) Determine the point at which cost is equal to revenue. 1

$$30 + 2x = 3.5x$$

$$30 = 1.5x$$

$$x = 20 \checkmark$$

c) How many packs of cards would need to be sold to raise \$450? 1

$$450 = 3.5x - (30 + 2x)$$

$$450 = 1.5x - 30$$

$$480 = 1.5x$$

$$x = 320$$

320 packs.  $\checkmark$

5. Solve the simultaneous equations:  $y = x^2 - 1$  2

$$y = x + 1$$

$$x^2 - 1 = x + 1$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2, -1 \checkmark$$

$$\text{if } x = 2, y = 3$$

$$\text{if } x = -1, y = 0.$$

The solutions are

$$(2, 3) \text{ and } (-1, 0) \checkmark$$

6. For what values of  $m$  do the line  $y = mx - 4$  and the parabola  $y = x^2 - 2x - 2$  intersect twice? 3

Intersection:  $x^2 - 2x - 2 = mx - 4$

$$x^2 - (2 + m)x + 2 = 0$$

$\Delta > 0$  for 2 points of intersection

$$\Delta = (2 + m)^2 - 4 \times 2$$

$$\Delta = m^2 + 4m - 4 \checkmark$$

When  $\Delta = 0$ ,  $m = \frac{-4 \pm \sqrt{16 + 16}}{2}$

$$= -2 \pm 2\sqrt{2}$$

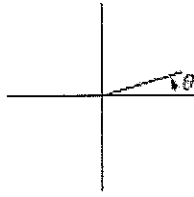


$$m < -2 - 2\sqrt{2} \text{ or } m > -2 + 2\sqrt{2}$$

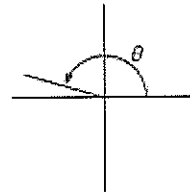
# Trigonometry (25 marks)

1. For the angle  $\theta$ ,  $\sin \theta = -\frac{7}{25}$  and  $\cos \theta = \frac{24}{25}$ . Which diagram best shows the angle  $\theta$ ?

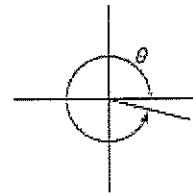
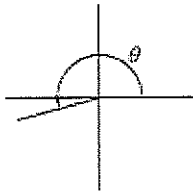
A



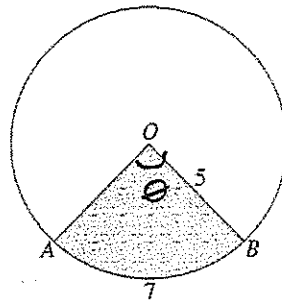
B



C



2. The circle centred at  $O$  has radius 5 and arc length  $AB$  is 7 as shown in the diagram.



$$\theta = \frac{7}{5}$$

$$A = \frac{\theta}{2\pi} \times \pi r^2$$

$$= \frac{7}{2} \times 5$$

What is the area of the shaded sector  $OAB$ ?

A  $\frac{35\pi}{2}$

C  $\frac{125\pi}{14}$

B

D

$\frac{35}{2}$   
 $\frac{125}{14}$

3. Find the exact value of

a)  $\sin 300^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

1

b)  $\cos 765^\circ = \cos 405^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$

1

c)  $\operatorname{cosec} \frac{4\pi}{3} = \frac{1}{\sin(\pi + \frac{\pi}{3})} = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

1

4. Find all values of  $\theta$ ,  $0 \leq \theta \leq 2\pi$  for which  $\cos \theta + \sqrt{3} \sin \theta = 0$

2

$$\sqrt{3} \sin \theta = -\cos \theta$$

$$\sqrt{3} \tan \theta = -1$$

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

$$\theta = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6}, \text{ or } \frac{11\pi}{6} \quad \checkmark \checkmark$$

150°, 330°

1 mark for 1 correct solution  
OR 1st quadrant value  
in radians

5. If  $\sin A = x$ , express  $\cos(90^\circ - A) \sin(180^\circ - A)$  in terms of  $x$ .

2

$$\cos(90^\circ - A) = \sin A$$

$$\sin(180^\circ - A) = \sin A$$

[1 mark for either identity]

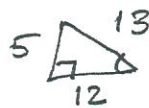
$$\therefore \cos(90^\circ - A) \sin(180^\circ - A) = x^2$$

6. If  $\operatorname{cosec} a = -\frac{13}{5}$  and  $\cot a > 0$ , find the exact value of  $\cos a$ .

2

$$\sin a = -\frac{5}{13}$$

3rd quadrant  $\therefore \cos -ve$



$$\sqrt{13^2 - 5^2} = 12 \quad \checkmark$$

$$\cos a = -\frac{12}{13} \quad \checkmark$$

7. Prove the identity  $(1 + \tan x)^2 + (1 - \tan x)^2 = 2 \sec^2 x$

2

$$\begin{aligned} \text{LHS} &= 1 + 2\tan^2 x + \tan^2 x + 1 - 2\tan^2 x + \tan^2 x \\ &= 2 + 2\tan^2 x \quad \checkmark \end{aligned}$$

$$= 2(1 + \tan^2 x) = 2 \sec^2 x = \text{RHS}$$

$$\boxed{\sec^2 x = 1 + \tan^2 x}$$

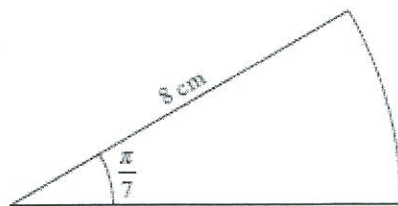
$$= 2 \left( \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \right)$$

$$= 2 \left( \frac{1}{\cos^2 x} \right)$$

$$= 2 \sec^2 x$$

$$= \text{RHS} \quad \checkmark$$

8. The angle of a sector in a circle of radius 8 cm is  $\frac{\pi}{7}$  radians, as shown in the diagram.



NOT TO SCALE

Find the exact value of the perimeter of the sector.

2

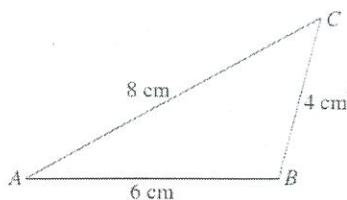
$$l = r\theta$$

$$= \frac{8\pi}{7} \quad \checkmark \quad 3.59$$

$$P = \boxed{\frac{8\pi}{7} + 16} \text{ cm} \quad \text{or} \quad \boxed{\frac{112 + 8\pi}{7}} \text{ cm} \quad \checkmark$$

19.59

9. The diagram shows  $\triangle ABC$  with  $AB = 6$  cm,  $AC = 8$  cm and  $CB = 4$  cm.



NOT TO SCALE

- a) Show that  $\cos A = \frac{7}{8}$

1

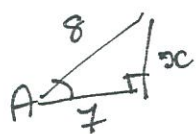
$$4^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \cos A$$

$$\cos A = \frac{8^2 + 6^2 - 4^2}{2 \times 8 \times 6}$$

$$= \frac{84}{96} = \frac{12 \times 7}{12 \times 8} = \frac{7}{8}$$

- b) By finding the exact value of  $\sin A$ , find the exact area of  $\triangle ABC$ .

2



$$x^2 = 8^2 - 7^2$$

$$= 15$$

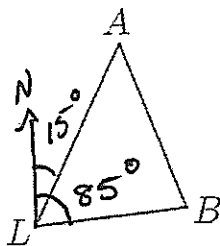
$$\sin A = \frac{\sqrt{15}}{8} \quad \checkmark$$

$$\text{Area} = \frac{1}{2} \times 8 \times 6 \sin A$$

$$= \frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{15}}{8}$$

$$= 3\sqrt{15} \text{ cm}^2 \quad \checkmark$$

10. Boat A is 30 km from lighthouse L on a bearing of  $015^\circ$  and boat B is 20 km from L on a bearing of  $085^\circ$
- a) How far apart are the boats? Give your answer correct to 1 decimal place. 2



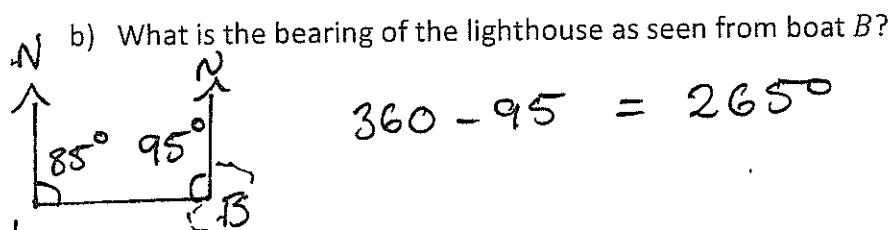
$$\angle ALB = 85 - 15 = 70^\circ$$

$$AB^2 = 30^2 + 20^2 - 2 \times 30 \times 20 \cos 70^\circ$$

$$AB = \sqrt{889.5758 \dots}$$

$$\approx 29.8257 \dots$$

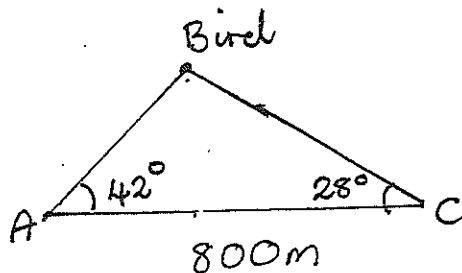
$$\approx 29.8 \text{ km} \checkmark$$



$$360 - 95 = 265^\circ$$

11. From the ends of a straight horizontal section of road 800 m long, a bird hovering directly above the road is observed to have angles of elevation of  $42^\circ$  and  $28^\circ$  respectively.

- a) Using a ruler, draw a diagram representing this information. 1



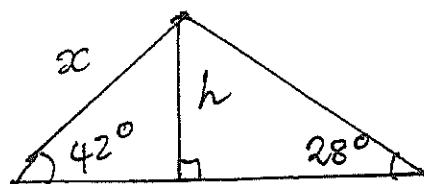
- b) Find the height of the bird above the road, correct to the nearest metre. 3

$$\angle ABC = 180 - 42 - 28 = 110^\circ \checkmark$$

$$\frac{BC}{\sin 28^\circ} = \frac{800}{\sin 110^\circ}$$

$$BC = \frac{800 \sin 28^\circ}{\sin 110^\circ} \checkmark$$

(or equivalent using  $\sin 42^\circ$ )



$$h = x \sin 42^\circ$$

$$= \frac{800 \sin 28^\circ \sin 42^\circ}{\sin 110^\circ}$$

$$= 267.4387 \dots$$

$$\approx 267 \text{ m} \checkmark$$

Can also evaluate

$$x \approx 399.68 \text{ m}$$

$$h = 399.68 \sin 42^\circ \dots$$

End of Exam. Please check your work.

