Carlingford High School

2021 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Instructions - please read carefully.

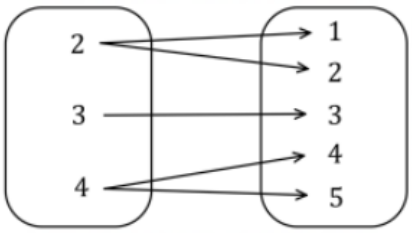
1. Time allowed: 2 hours exam time + 10 mins reading time
2. Equipment required: black pen, calculator, at least 5 pieces of paper (for Extended Response Questions)
3. Sections 2 to 6 : Google Quiz - A mix of multiple choice questions and short answer questions (25 marks) - allow 40 mins - type your answers for the short answer questions.
4. Section 7 : Five extended response questions (42 marks) - allow 1 h 20 mins.
5. For the extended response questions: Start a new piece of paper for each question. You must upload your hand-written solutions to each question to SEPARATE assignment sections in Google Classroom. Write your FULL NAME at the top of the first sheet of each submission.
6. You are given 20 mins at the completion of the exam to upload your 5 solutions, to 5 different sections in GC. When taking the photos, position your camera directly above, and in a light-filled place. Make sure it is in portrait view and the correct way up. Using Notes, CamScanner, Adobe scan or a similar app - is recommended. Ensure that your submission includes your full name in this format (Last Name_First Name_Class).

Google Quiz Section (MC and Short answer questions)

Functions (6 marks)

1

What type of relation is shown?



(A) Many-to many

(B) One-to-many

(C) One-to-one

(D) Many-to-one

2 1 point

Which of the following statements is true for the function $f(x) = e^{|x|} - 1$?

(A) The function is not differentiable at $x = 0$.

(B) The function is not continuous at $x = 0$.

(C) The function has a stationary point at $x = 0$.

(D) The function has an asymptote at $y = -1$.

3 1 point

What is the domain and range of a circle with the equation $x^2 + 4x + y^2 = 5$?

A. domain $= \left[-2-\sqrt{5},\ -2+\sqrt{5}\right]$; range $= \left[-\sqrt{5},\ \sqrt{5}\right]$

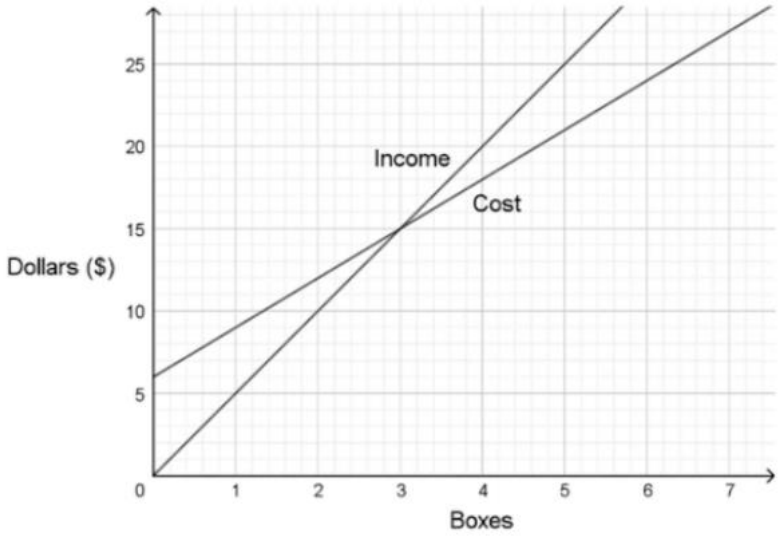
B. domain $= \left[2-\sqrt{5},\ 2+\sqrt{5}\right]$; range $= \left[-\sqrt{5},\ \sqrt{5}\right]$

C. domain $= [-1,\ 5]$; range $= [-3,\ 3]$

D. domain $= [-5,\ 1]$; range $= [-3,\ 3]$

4 1 point

The graph below shows the cost of producing boxes of chocolates and the income received from their sale.



Use the graph to determine the number of boxes that need to be sold to break even.

A. 5

B. 4

C. 3

D. 2

5 1 point

What is the natural domain of the function $y = \dfrac{1}{\sqrt{2x+1}}$?

6 1 point

Solve the equation $5^{1-x} = 25^{x}$.

Calculus (8 marks)

1

What is the derivative of $\cos^2 3x$ with respect to x?

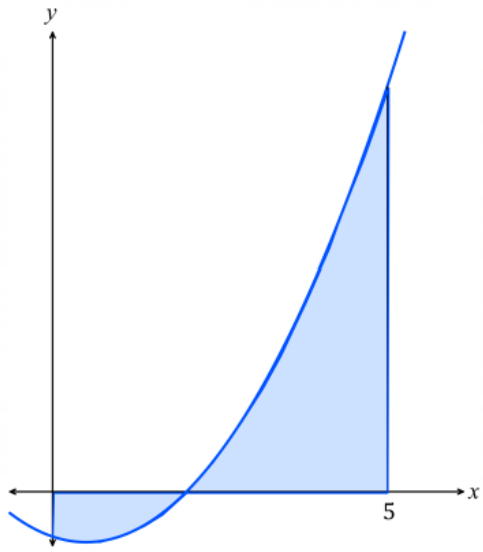
(A) $-6\sin 3x\cos 3x$

(B) $-2\sin 3x\cos 3x$

(C) $2\sin 3x\cos 3x$

(D) $6\sin 3x\cos 3x$

2 1 point



Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x-axis and the lines $x = 0$ and $x = 5$?

(A) $A = \left|\int_0^1 (x^2 - x - 2)dx\right| + \int_1^5 (x^2 - x - 2)dx$

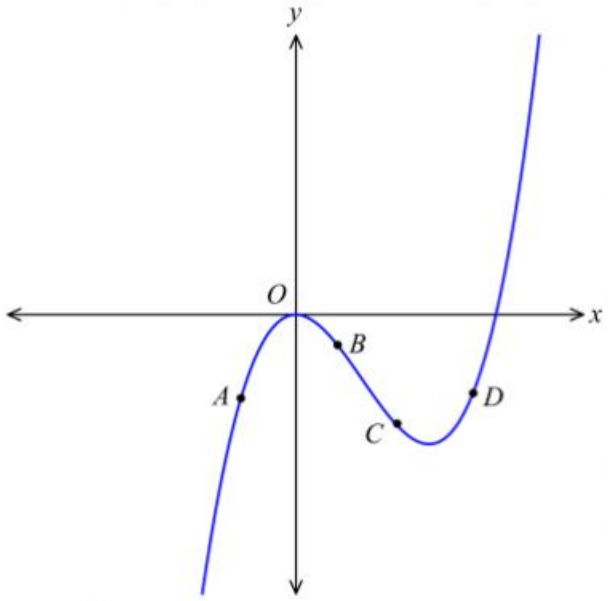
(B) $A = \int_0^1 (x^2 - x - 2)dx + \left|\int_1^5 (x^2 - x - 2)dx\right|$

(C) $A = \left|\int_0^2 (x^2 - x - 2)dx\right| + \int_2^5 (x^2 - x - 2)dx$

(D) $A = \int_0^2 (x^2 - x - 2)dx + \left|\int_2^5 (x^2 - x - 2)dx\right|$

3 1 point

At which point on this curve are the first and second derivatives both negative?



(A) A
(B) B
(C) C
(D) D

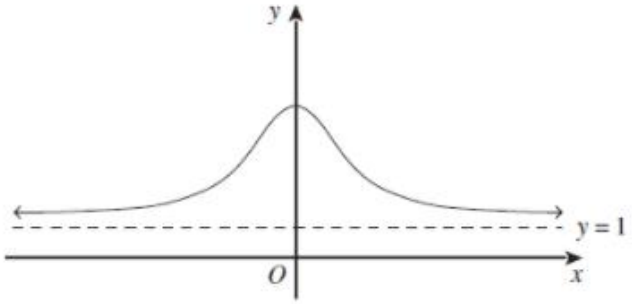
4 1 point

The curve $y = 2x^2 + ax + 9$ has a stationary point at $x = -2$. What is the value of a?

(A) −8
(B) −4
(C) 4
(D) 8

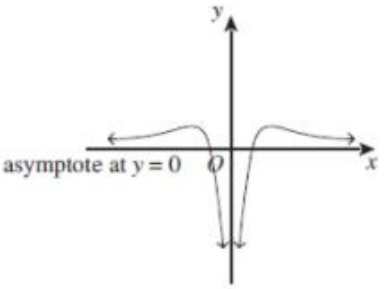
5 1 point

The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = 1$.

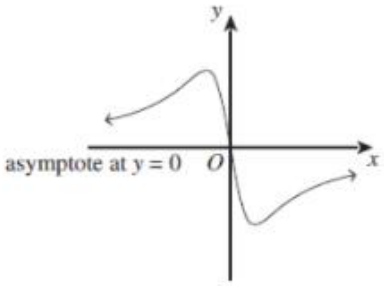


Which of the following could be the graph of $y = f'(x)$?

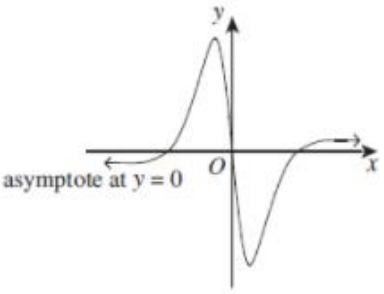
A.



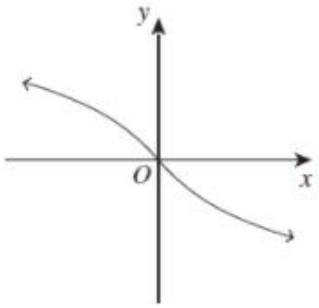
B.



C.

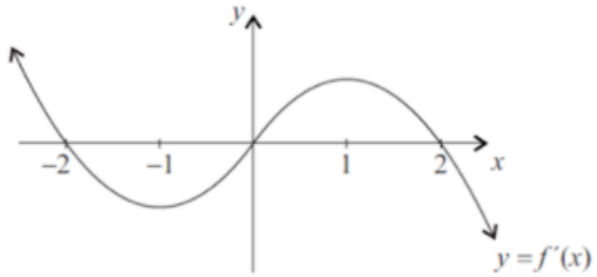


D.



6 1 point

The diagram shows the graph of the derivative function $y = f'(x)$.



At which value of x does a minimum turning point occur on the graph of $y = f(x)$?

7

If $f'(x) = 2x - 6$, for what values of x is $f(x)$ increasing?

8

What is value of $\int_1^3 (3x^2 - 4x + 1)\ dx$

Trigonometry (3 marks)

1 1 point

What is the amplitude and period for the function $f(x) = 4\sin\left(\frac{x+\pi}{3}\right)$?

(A) Amplitude 3 and period $\frac{\pi}{2}$

(B) Amplitude 3 and period 6π

(C) Amplitude 4 and period $\frac{\pi}{2}$

(D) Amplitude 4 and period 6π

2 1 point

What is the maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ and the value of x occurring in the domain $[0, 2\pi]$?

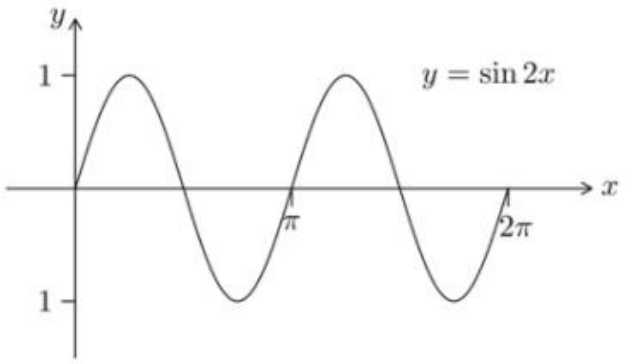
A. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 2 occurring at $x = \frac{\pi}{2}$.

B. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 3 occurring at $x = \frac{\pi}{2}$.

C. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 2 occurring at $x = \frac{3\pi}{2}$.

D. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 3 occurring at $x = \frac{3\pi}{2}$.

3 1 point



How many solutions does the equation $5\sin 2x = x$ for $0 \leq x \leq 2\pi$ have?

Financial Mathematics (3 marks)

1 1 point

An infinite geometric series has a first term of 12 and a limiting sum of 15. What is the common ratio?

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

2 1 point

The table shows the future value of a \$1 annuity for varying interest rates over different time periods.

Time period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

What is the present value of an annuity, correct to the nearest dollar, that would provide a future value of \$34 800 after 4 years at 2% per annum, compounded half-yearly?

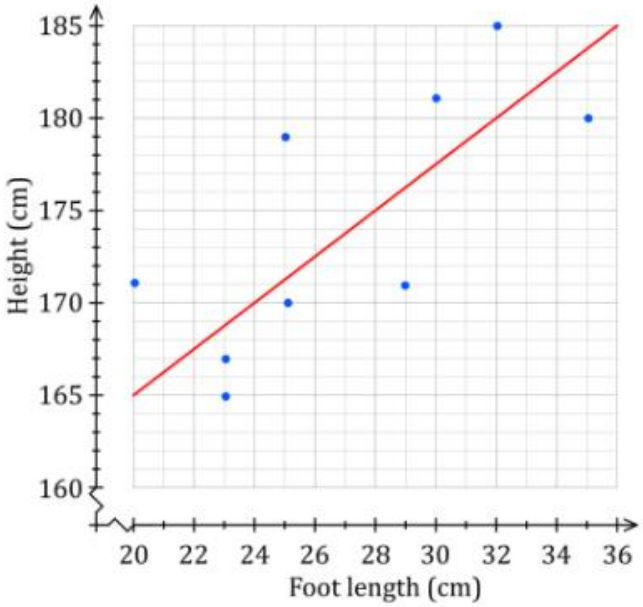
A. \$4055
B. \$4200
C. \$8195
D. \$8443

3. In an arithmetic series, the first term is 18 and the sum of the first 20 terms is 1310. Find the 20th term.

Statistics (5 marks)

1 1 point

The height (in cm) and foot length (in cm) for nine students are displayed in the scatterplot below. A least-squares line of best fit has been fitted to the data as shown.



What is the equation of the least-squares line of best fit?

(A) $height = 0.80 \times foot\ length + 165$

(B) $height = 0.80 \times foot\ length + 140$

(C) $height = 1.25 \times foot\ length + 165$

(D) $height = 1.25 \times foot\ length + 140$

2

The probability distribution of random variable X is shown below.

x	−3	−2	−1	0	1	2	3
P(X= x)	0.05	0.05	a	0.20	0.15	a	0.05

Find the value of a.

(A) 0.15

(B) 0.20

(C) 0.25

(D) 0.30

3 1 point

The table shows the accidents recorded on a motorway.

	Cause		
	Speeding	*Alcohol consumption*	*Driver fatigue*
Fatal	35	41	12
Non-fatal	75	149	55

What is the probability that an accident is fatal, given that alcohol consumption was NOT the cause?

A. $\frac{47}{367}$

B. $\frac{47}{177}$

C. $\frac{47}{88}$

D. $\frac{177}{367}$

4 1 point

John works in a cake shop, and based on sales over two weeks, he conducted a survey of the five most popular cakes. What type of data is this?

A. Categorical nominal

B. Categorical ordinal

C. Quantitative continuous

D. Quantitative discrete

5 1 point

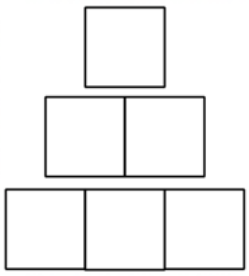
A bag contains 20 yellow marbles and n red marbles. If a marble is drawn at random from the bag, the probability of drawing a red marble is 0.75. What is the value of n?

Extended Response Questions

Q1 (9 marks) – Financial Mathematics

Part A (6 marks)

James has some sticks that are all of the same length. He arranges them in squares and has made the following 3 rows of patterns:



Row 1

Row 2

Row 3

He notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row he needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row. **2**

(b) James continues to make squares following the same pattern. He makes 4 squares in the 4th row and so on until he has completed 10 rows. Find the total number of sticks James uses in making these 10 rows. **1**

(c) James started with 1750 sticks. Given that James continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)$th row, show that k satisfies $(3k-100)(k+35)<0$. **2**

(d) What is the value of k? **1**

Part B (3 marks)

Ms Tang has just retired from work. She has received a large superannuation payment and has worked out that she will require \$3000 per month for the next 25 years to pay for living expenses. She intends to invest a set amount ($\boldsymbol{P}$) at 3% p.a. interest compounding monthly and withdraw the \$3000 from this account at the end of each month.

Find the amount $\boldsymbol{P}$ that will need to be invested if she is to withdraw \$3000 each month for the next 25 years.

Q2 (9 marks) - Calculus

Part A (5 marks)

A particle moves such that its velocity at a given time is given by:

$\dot{x} = 8 - 16\sin t$

(a) What is the initial acceleration of the particle? **1**

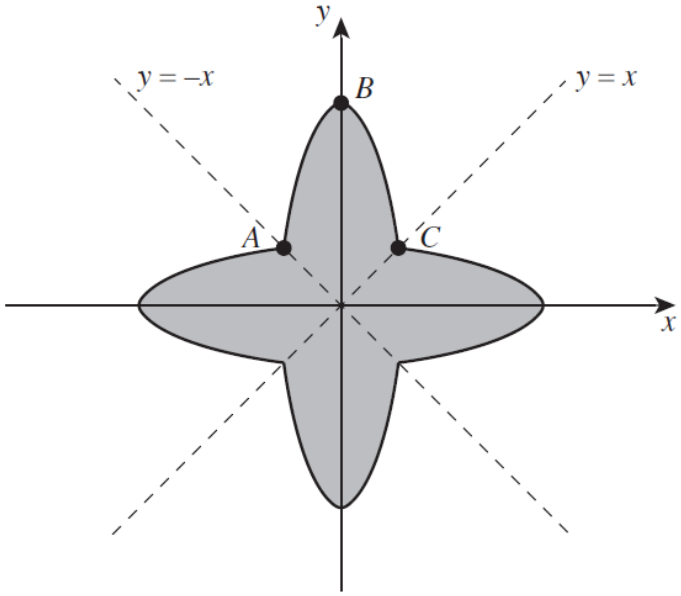
(b) When is the particle first at rest? **1**

(c) Given that the particle is initially at the origin, find an equation for the displacement (x), at a time t. **1**

(d) Find the distance travelled between the first two times when the particle is at rest. **2**

Part B (4 marks)

A company's logo is designed using the curve ABC, as shown by the shaded region in the diagram. The curve ABC shows part of the function $y = 4 - 3x^2$. This curve is reflected along the x-axis and the lines $y = x$ and $y = -x$ to form the rest of the logo.



Calculate the area of the company's logo.

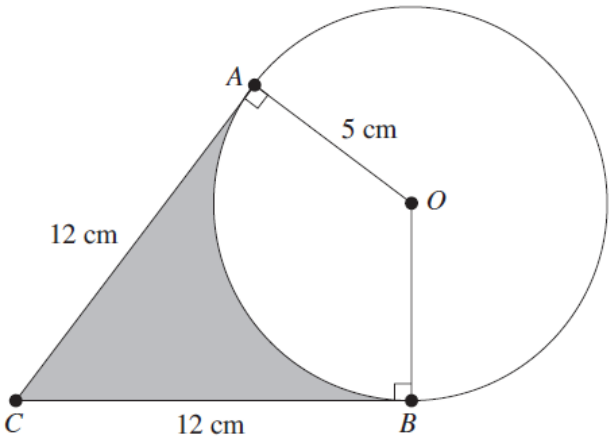
Q3 (10 marks) - Trigonometry

Part A (3 marks)

Given that $\int_0^a 5\sin 3x dx = \frac{10}{3}, 0 \leq a < \pi$, calculate the value of a.

Part B (4 marks)

The diagram shows a circle with centre O and radius 5 cm. Tangents are drawn from the points A and B, which meet at point C.



NOT TO SCALE

(a) Show that $\angle AOB = 2.4$ radians correct to two significant figures. **2**

(b) Hence, find the area of the shaded region. **2**

Part C (3 marks)

Show that $\sin x + 1 + \cos x \cot x - \operatorname{cosec} x = 1$

Q4 (8 marks) - Statistics

Part A (3 marks)

A discrete random variable X has the probability distribution table shown.

$X = x$	20	21	22	23
$P(x)$	0.24	0.2	m	0.4

By finding the value of m, calculate the expected value and the variance of X.

Part B (5 marks)

A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.

The results indicate that:

- 8 students chose to study neither Physics nor Chemistry
- 8 students chose to study Chemistry
- 12 students chose to study Physics.

(a) Draw a Venn diagram showing this information. **1**

(b) A student choosing to study Chemistry is event 1. A student choosing to study Physics is event 2. **2**

If a student is chosen at random, determine whether these events are independent of each other.

Must provide mathematical reasoning.

(c) The entire year group is given the same survey. The results indicate that: **2**

- the probability that a student chose to study Chemistry is $\frac{1}{3}$
- the probability that a student chose to study Physics is $\frac{2}{5}$
- given that a student chose to study Chemistry, the probability that they chose to study Physics is $\frac{3}{7}$.

Calculate the probability that a student picked at random has chosen to study Chemistry or Physics.

Q5 (6 marks) – Exponential & Logarithms

The mass of two substances, A and B, are decaying exponentially according to

$$M_A = 200e^{-0.05t}$$
$$M_B = 400 \times 3^{-0.12t},$$

where M is the mass in grams and t is the time in minutes.

(a) Find the time taken for substance A to decrease to half of its original value. Give your answer correct to the nearest minute. **2**

(b) At what instance do both particles decay at the same rate? Give your answer correct to the nearest second. **4**