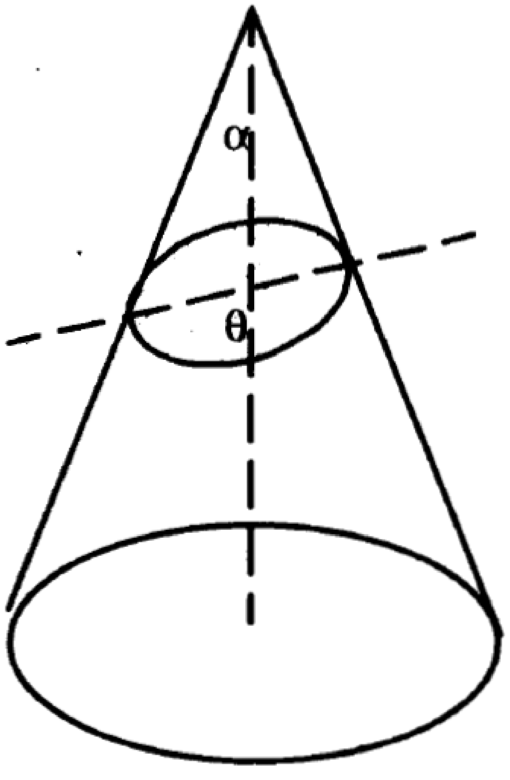
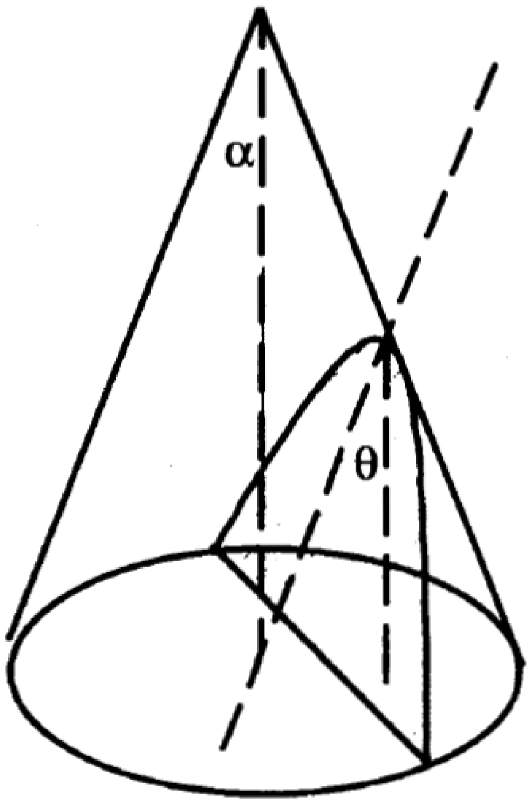
Introduction to Conics (Conic Sections)

Consider a cone which is cut by a plane…

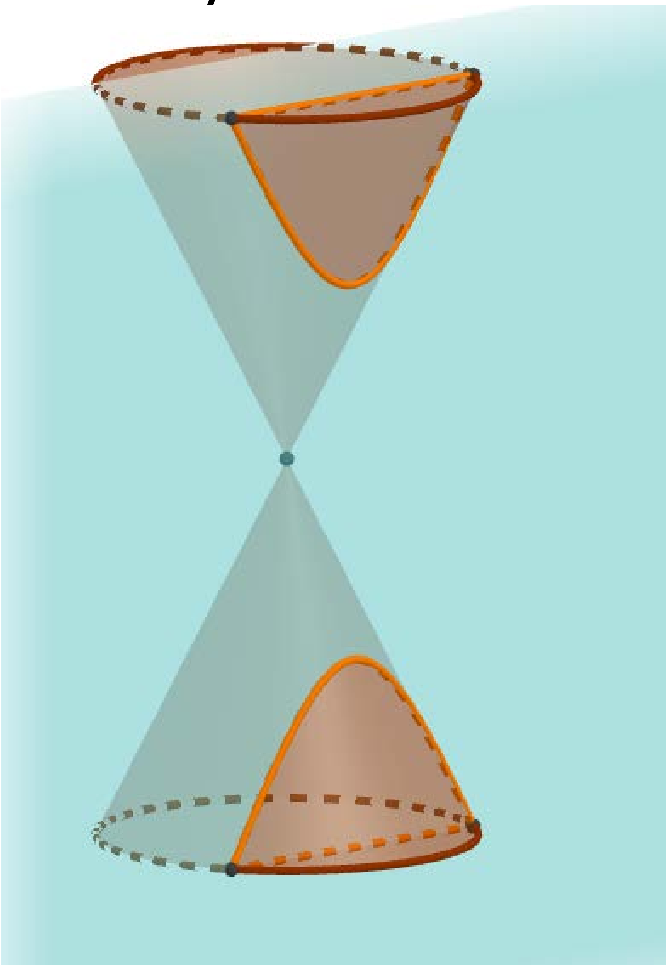


$\theta > \alpha$: Ellipse

($\theta = 90°$: Circle)



$\theta = \alpha$: Parabola

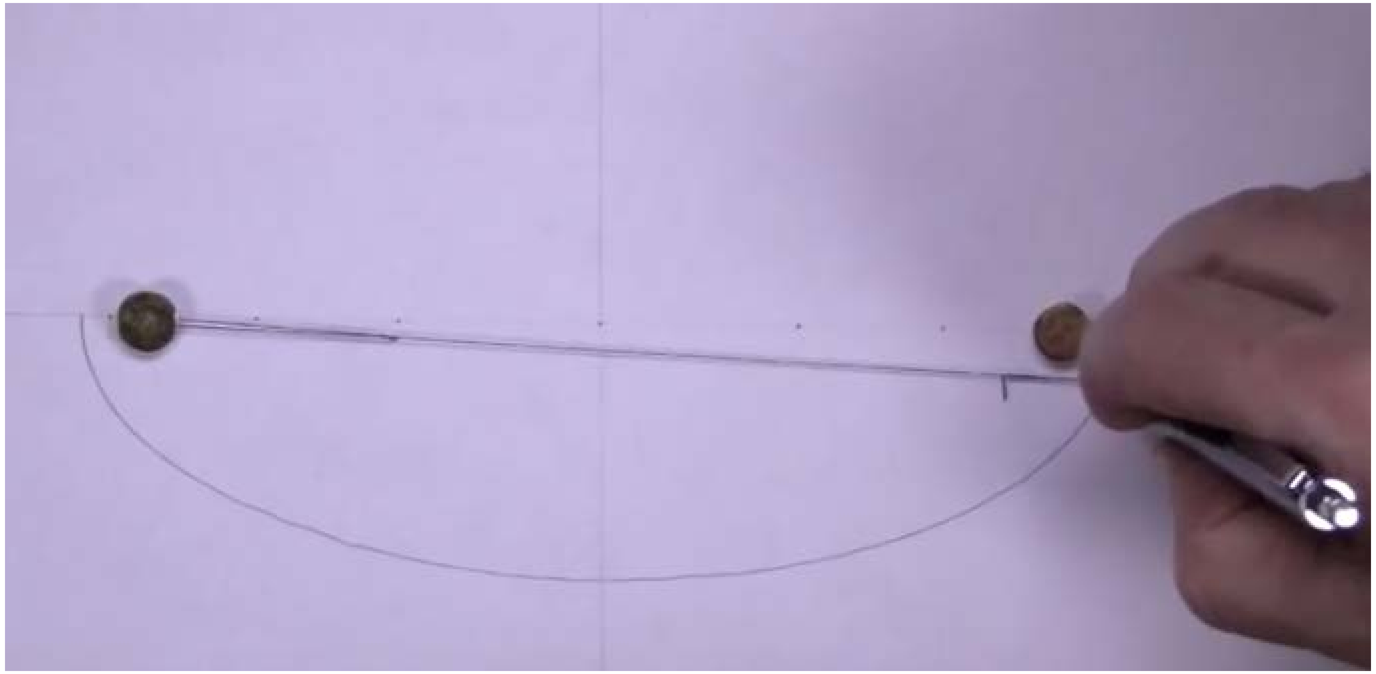


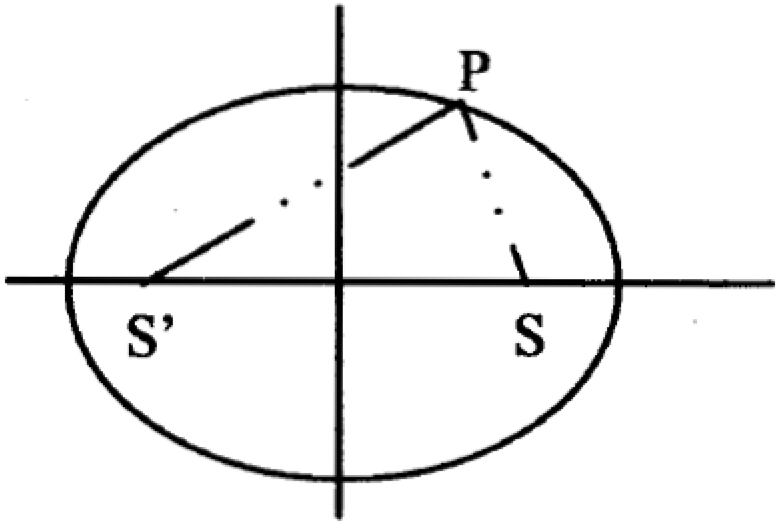
In this diagram, $\theta = 0$

$0 \leq \theta < \alpha$: Hyperbola

An ellipse is the locus of a point P which moves so that the sum of its distances to two fixed points is constant.

https://www.youtube.com/watch?v=Et3OdzEGX_w

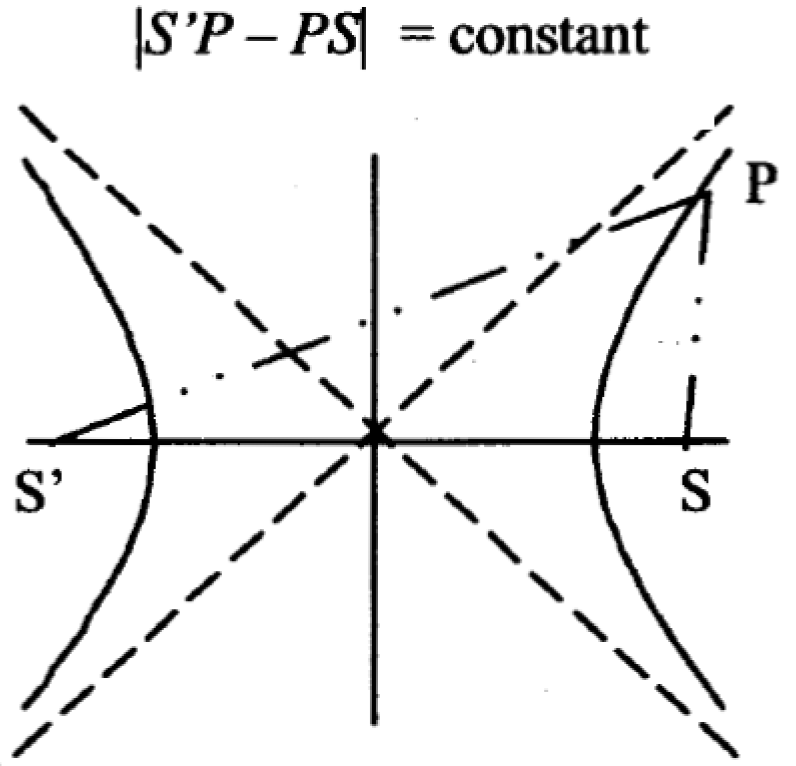




$$PS + PS' = \text{constant}$$

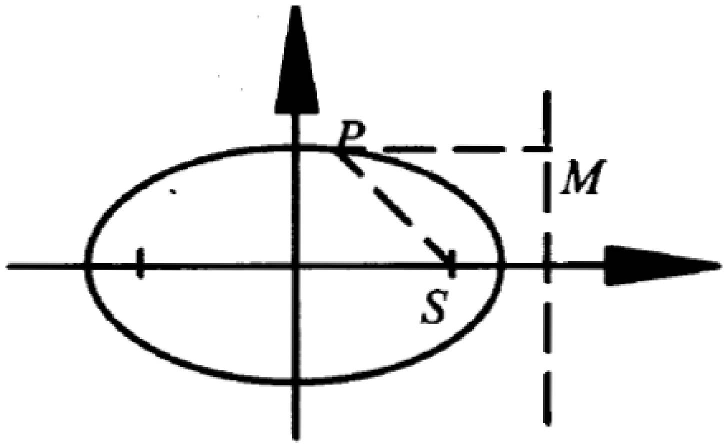
A hyperbola is the locus of a point P which moves so that the difference between its distances to two fixed points is constant.

https://www.youtube.com/watch?v=bAAppgMqeJ8



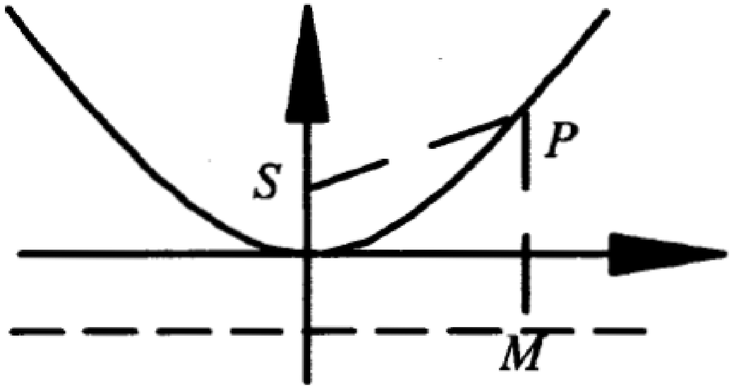
Conics as Loci

A conic section may be defined as the locus of all points P in a plane such that the ratio of distances from P to a fixed point S and to a fixed straight line m is a constant e.



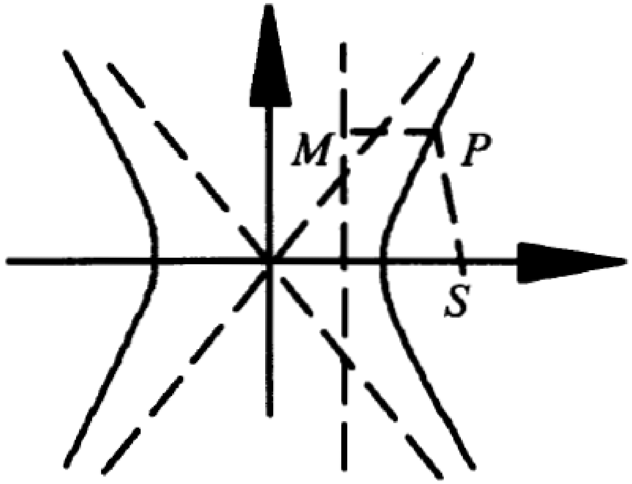
$$e = \frac{PS}{PM} < 1$$

Ellipse $\quad$ ($e = 0$ Circle)



$$e = \frac{PS}{PM} = 1$$

Parabola



$$e = \frac{PS}{PM} > 1$$

Hyperbola

The fixed point is called the focus, the fixed straight line is called the directrix and e is called the eccentricity.

Ellipses and hyperbolas have two directrices, two foci and two axes of symmetry.

To obtain the standard form of the equations for the ellipse and the hyperbola, we take their axes of symmetry to be the x- and y-axes, and the foci to be points on the x-axis.