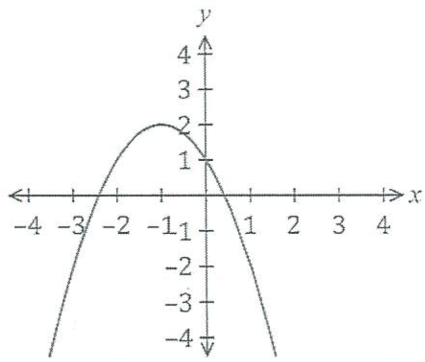
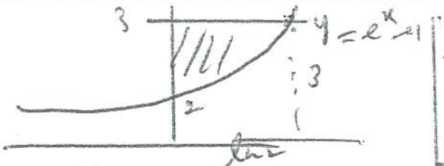
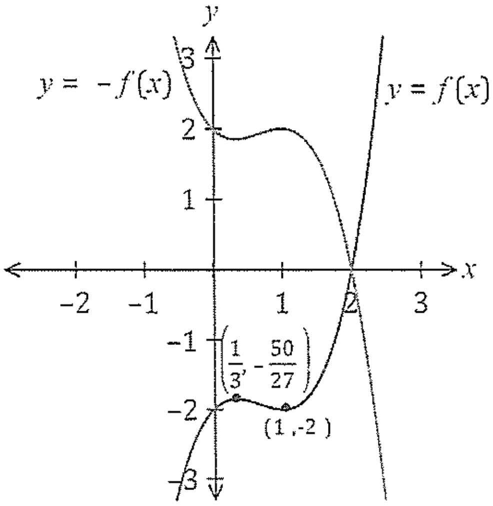
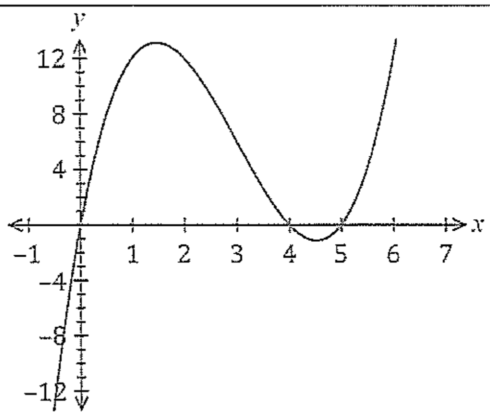
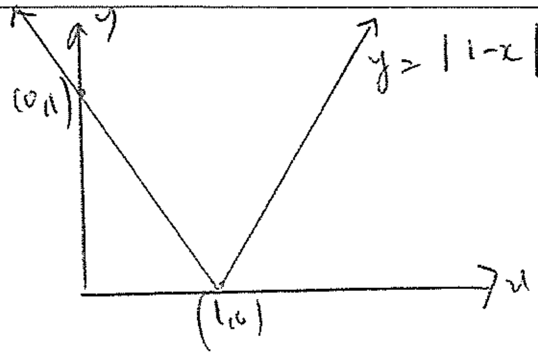
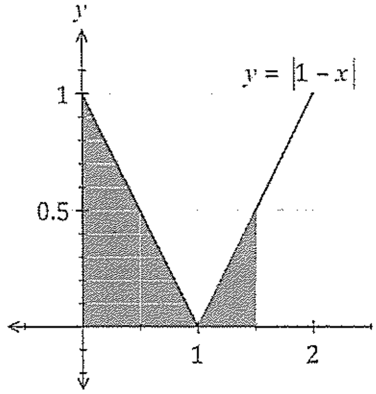
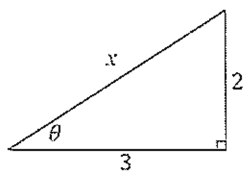
ACE Examination 2020
Year 12 Mathematics Advanced Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1.	$f(x) = 3x^4(4-x)^3$ $f'(x) = 3x^4 \times 3(4-x)^2 \times -1 + (4-x)^3 \times 12x^3$ $= 3x^3(4-x)^2[-3x + 4(4-x)]$ $= 3x^3(4-x)^2(16-7x)$	1 Mark: D Calculus
2.	A moderate negative correlation.	1 Mark: A Statistics
3.	$f(x) = 2x^3 + x^2$ $f'(x) = 6x^2 + 2x$ $f''(x) = 12x + 2$ Concave down $f''(x) < 0$ $f''(x) < 0$ $12x + 2 < 0$ $x < -\frac{1}{6}$	1 Mark: A Calculus
4.	Amplitude = 1 Period $= \frac{2\pi}{\pi} = 2$	1 Mark: A Trig functions
5.		1 Mark: D Functions ~~Graphs~~
6.	$z = \frac{(680-750)}{35} = -2$ Below Z= -2 is 2.5 %	1 Mark: B Statistics
7.	Intersection value is 4.25 (4% and 4 years). Let x be the amount to be invested. $FV = 4.25 \times x$ $28\,475 = 4.25 \times x$ $x = \frac{28\,475}{4.25}$ $= \$6700$ $\therefore$ \$6700 is invested every quarter.	1 Mark: C Financial
8.	$z = \frac{x-\mu}{\sigma} = \frac{16-11}{2} = 2.5$ $P(X > 16) = P(Z > 2.5)$ $= P(Z < -2.5)$	1 Mark: B Statistics

	Solution	Criteria
9.	$2\sin x + \sqrt{3} = 0$ $2\sin x = -\sqrt{3}$ $\sin x = -\frac{\sqrt{3}}{2}$ $x = \frac{4\pi}{3}, \frac{5\pi}{3}$	1 Mark: C
10.	 $e^x + 1 = 3$ $e^x = 2$ $x = \ln 2$ pt of intersection $(\ln 2, 3)$ Area of shaded region $= 3\ln 2 - \int_0^{\ln 2} (e^x + 1)\,dx$ $= 3\ln 2 - [e^x + x]_0^{\ln 2}$ $= 3\ln 2 - [e^{\ln 2} + \ln 2] - (1)$ $= 3\ln 2 - \{[2 + \ln 2] - 1\}$ $= 3\ln 2 - \{\ln 2 + 1\}$ $= 3\ln 2 - \ln 2 - 1$ $= 2\ln 2 - 1$	1 Mark: D
Section II		
11	$\int \frac{1}{1-2x}\,dx = -\frac{1}{2}\ln\|1-2x\| + C$	2 Marks: Correct answer. 1 Mark: Finds the integral as a log function.
12(a)	$f(x) = \frac{(x+3)(2x+1)}{\sqrt{x}}$ $= \frac{2x^2+7x+3}{\sqrt{x}}$ $= 2x^{\frac{3}{2}} + 7x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$ $\therefore A = 2, B = 7$ and $C = 3$	2 Marks: Correct answer. 1 Mark: Finds A or B or C.
12(b)	$f(x) = 2x^{\frac{3}{2}} + 7x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$ $f'(x) = 3x^{\frac{1}{2}} + \frac{7}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$	1 Mark: Correct answer.
13(a)	$P(1 < X \leq 3) = 0.4 + 0.2$ $= 0.6$	1 Mark: Correct answer.
13(b)	$\mu = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1$ $= 2$ $Var(X) = E(X^2) - \mu^2$ $= 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1 - 2^2$ $= 1.2$ $\therefore$ Variance is 1.2	2 Marks: Correct answer. 1 Mark: Shows some understanding

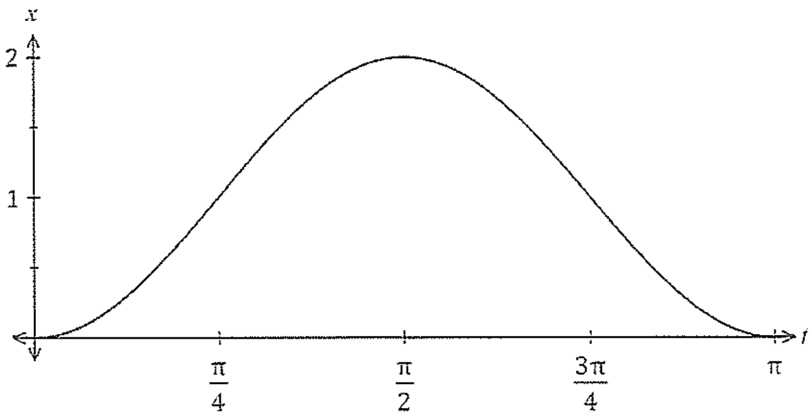
14	$\int 6x^2 + 2 + x^{-\frac{1}{2}}\,dx = \frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$ $= 2x^3 + 2x + 2x^{\frac{1}{2}} + C$	2 Marks: Correct answer. 1 Mark: Integrates one term correctly.
15	$S_n = \frac{a}{1-r}$ $1.8 = \frac{3}{1-r}$ $1.8 - 1.8r = 3$ $1.8r = -1.2$ $r = -\frac{1.2}{1.8} = -\frac{2}{3}$ $\therefore$ The common ratio is $-\frac{2}{3}$	2 Marks: Correct answer. 1 Mark: Uses the limiting sum with at least one correct value.
16(a)	x-intercept then $y = 0$ $0 = (x-2)(x^2+1)$ $x = 2$ y-intercept then $x = 0$ $y = (0-2)(0^2+1)$ $= -2$ $\therefore$ Intercepts are (2, 0) and (0,–2)	2 Marks: Correct answer. 1 Mark: Finds either the x or y intercept.
16(b)	$f(x) = (x-2)(x^2+1)$ $f'(x) = (x-2)2x + (x^2+1) \times 1$ $= 3x^2 - 4x + 1$ $f''(x) = 6x - 4$ Stationary points occur when first derivative is equal to zero. $3x^2 - 4x + 1 = 0$ $(3x-1)(x-1) = 0$ $x = \frac{1}{3}$ or $x = 1$ When $x = \frac{1}{3}$ then $y = -\frac{50}{27}$ and when $x = 1$ then $y = -2$ $\therefore$ Stationary points $(\frac{1}{3}, -\frac{50}{27})$ $(1, -2)$ At $(\frac{1}{3}, -\frac{50}{27})$ $f''(x) = 6 \times \frac{1}{3} - 4 = -2 < 0$ Max At $(1, -2)$ $f''(x) = 6 \times 1 - 4 = 2 > 0$ Min	3 Marks: Correct answer. 2 Marks: Finds the stationary points. 1 Mark: Finds the derivative.
16(c)		3 Marks: Correct answer. 2 Marks: Finds $y = f(x)$ or $y = -f(x)$. 1 Mark: Finds the general shape of $y = f(x)$ or shows some understanding.

17	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right)$ $= 1 - \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{2}-1}{\sqrt{2}}$ $= \frac{2-\sqrt{2}}{2}$	2 Marks: Correct answer. 1 Mark: Correctly integrates.
18(a)	$f(x) = (x^2 - 6x)(x-3) + 2x$ $= x[(x-6)(x-3)+2]$ $= x(x^2 - 9x + 20)$	2 Marks: Correct answer. 1 Mark: Makes some progress.
18(b)	$f(x) = x(x^2 - 9x + 20)$ $= x(x-4)(x-5)$	1 Mark: Correct answer.
18(c)		2 Marks: Correct answer. 1 Mark: Shows the general shape of the curve or finds the intercepts.
19(a)	$\frac{d}{dx}\ln(x^2+2) = \frac{2x}{x^2+2}$	1 Mark: Correct answer.
19(b)	$\frac{d}{dx}\left(\frac{\sin x}{x^2}\right) = \frac{x^2\cos x - \sin x \times 2x}{(x^2)^2}$ $= \frac{x\cos x - 2\sin x}{x^3}$	2 Marks: Correct answer. 1 Mark: Uses the quotient rule correctly.
20	$A = \int_0^2 x^3 - 5x^2 + 2x + 8dx$ $= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + x^2 + 8x\right]_0^2$ $= \left(\frac{1}{4}(2)^4 - \frac{5}{3}(2)^3 + 2^2 + 8 \times 2\right) - 0$ $= \frac{32}{3} = 10\frac{2}{3}$	2 Marks: Correct answer. 1 Mark: Integrates one term correctly.

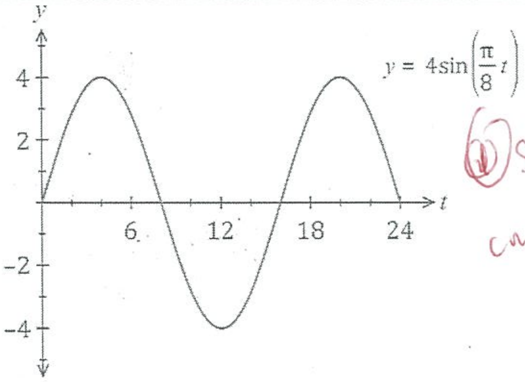
21(a)		2 Marks: Correct ~~answer~~, graph label + the intercepts 1 Mark: Sketches the function or shows some understanding.
21(b)	$P(X \leq 1.5)$ $= \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 0.5 \times 0.5$ $= 0.625$ 	2 Marks: Correct answer. 1 Mark: Sketches the function or shows some understanding.
22(a)	AP: {119, 117, 115, ...} with a = 119, d = −2 and n =25 $T_n = a + (n-1)d$ $= 119 + (25-1) \times (-2)$ $= 71$ ∴ Sonny's repayment is \$71 in the 25th month.	2 marks: Correct answer. 1 mark: Uses the formula for the nth term of an AP with one correct value.
22(b)	S_n = 3200, a = 119 and d = −2 $S_n = \frac{n}{2}[2a + (n-1)d]$ $3200 = \frac{n}{2} \times [2 \times 119 + (n-1) \times (-2)]$ $6400 = n \times (238 - 2n + 2)$ $= 240n - 2n^2$ $2n^2 - 240n + 6400 = 0$ $n^2 - 120n + 3200 = 0$	2 marks: Correct answer. 1 mark: Uses the formula for the sum of an AP with one correct value.
22(c)	$n^2 - 120n + 3200 = 0$ $(n-80)(n-40) = 0$ $n = 40$ or $n = 80$ When n = 40 then $T_{40} = 119 + (40-1) \times (-2) = \$41$ When n = 80 then $T_{80} = 119 + (80-1) \times (-2) = -\$39$ ∴ n = 80 is not a sensible answer as the repayments are −\$39.	2 marks: Correct answer. 1 mark: Solves the quadratic equation.
23	$x^2 = 2^2 + 3^2$ $x = \sqrt{13}$ $\sin\theta = \frac{2}{\sqrt{13}}$ 	2 Marks: Correct answer. 1 Mark: Finds the value of x or makes some progress.

24(a)	$\frac{dy}{dx} = x^3 + 2x - 7$ $\frac{d^2y}{dx^2} = 3x^2 + 2$	1 Mark: Correct answer.
24(b)	Since $x^2 \geq 0$ (for all values of x) then $3x^2 \geq 0$ $\therefore \frac{d^2y}{dx^2} \geq 2$	1 Mark: Correct answer.
24(c)	Finding the anti-derivative. $y = \frac{x^4}{4} + x^2 - 7x + C$ $P(2, 4)$ is on the curve and satisfies the equation. $4 = \frac{2^4}{4} + 2^2 - 14 + C$ $C = 10$ $\therefore y = \frac{x^4}{4} + x^2 - 7x + 10$	2 Marks: Correct answer. 1 Mark: Finds the anti-derivative.
24(d)	Gradient of the tangent at $P(2, 4)$ $m = \frac{dy}{dx} = 2^3 + 2 \times 2 - 7 = 5$ Gradient of the normal at $P(2, 4)$ $m_1 m_2 = -1$ $m = -\frac{1}{5}$ Equation of the normal at $P(2, 4)$ $y - y_1 = m(x - x_1)$ $y - 4 = -\frac{1}{5}(x - 2)$ $\therefore x + 5y - 22 = 0$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent at $P(2, 4)$
25(a)	95% of the data lie within two standard deviations of the mean. 42 hours is a z-score of –2 and 54 hours has a z-score of 2. The mean is midway between 42 and 54 hours. $\therefore$ Mean is 48 hours.	1 Mark: Correct answer.
25(b)	There are 4 standard deviations between 42 and 54 hours. Standard deviation $= \frac{54 - 42}{4}$ $= 3$ hours	1 Mark: Correct answer.
25(c)	To find the z-score of 51 and 57 $z = \frac{x - \bar{x}}{s} = \frac{51 - 48}{3} = 1$ $z = \frac{x - \bar{x}}{s} = \frac{57 - 48}{3} = 3$ Percentage $= \frac{99.7\%}{2} - \frac{68\%}{2}$ $= 15.85\%$ $\therefore$ 15.85% have sleep between 51 and 57 hours of sleep per week	2 Marks: Correct answer. 1 Mark: Uses the z-score formula or shows some understanding.

26	Width of the strip $h = \frac{b-a}{n} = \frac{420-0}{6} = 70$ Top half of the lake. $A \approx \frac{h}{2}[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)$ $\approx \frac{70}{2}[30 + 30 + 2(40 + 60 + 80 + 60 + 40)]$ $\approx 21\,700$ Area of the entire lake $A \approx 2 \times 21\,700 \approx 43\,400$ m $\therefore$ Area of the lake is about 43 400 metres.	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Uses the trapezoidal rule with at least one correct value.
27(a)	Investment $= 400 \times 12 \times 45$ $= \$216\,000$ $\therefore$ John contributed \$216 00 over the 45 years.	1 Mark: Correct answer.
27(b)	$r = \frac{0.06}{12} = 0.005\%$ per month After 1 month $A_1 = (400 \times 1.005^1)$ After 2 months $A_2 = (400 \times 1.005^2) + (400 \times 1.005^1)$ After 45 years (540 months) $A_{540} = 400 \times 1.005^{540} + (400 \times 1.005^{539}) \ldots + (400 \times 1.005^1)$ Use the formula for the sum of a GP $A_{540} = 400 \times \frac{1.005(1.005^{540} - 1)}{1.005 - 1}$ $= \$1\,107\,909.03$ $\therefore$ John's investment is worth \$1 107 909.03 after 45 years.	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
27(c)	$FV = PV(1+r)^n$ $300\,000 = PV(1+0.08)^{10}$ $PV = 138\,958.0464\ldots$ $\approx \$138\,958$ $\therefore$ John needs to reinvest \$138 958.	2 Marks: Correct answer. 1 Mark: Uses the *FV* formula with one correct value.
28	$\frac{dP}{dt} = 1200e^{0 \cdot 3t}$ $P = \frac{1200}{0.3}e^{0 \cdot 3t} + C$ $P = 4000e^{0 \cdot 3t} + C$ Given $P = 5000$ when $t = 1$ $5000 = 4000e^{0 \cdot 3t} + C$ $C = -399.4352 \ldots \approx -399.4$ To find t when $P = 100\,000$ $P = 4000e^{0 \cdot 3t} - 399.4352 \ldots$ $100\,000 = 4000e^{0 \cdot 3t} - 399.435 \ldots$ $e^{0 \cdot 3t} = 25.099$ $\ln 25.099 \ldots = 0.3t$ $t = 10.742 \ldots$ ≈ 11 $\therefore$ The nest reaches a viable stage after 11 months.	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Finds the anti-derivative of the differential equation.

29(a)	Amplitude = 1 Period $= \dfrac{2\pi}{1} = 2\pi$ 	2 Marks: Correct answer. 1 Mark: Finds the general shape of the curve or makes some progress.
29(b)	The particle is a rest when $v = 0$ or $\frac{dx}{dt} = 0$ $\therefore t = 0, \frac{\pi}{2}, \pi$ $\therefore$ Position of the particle at these times: $x = 0, 2, 0$	2 Marks: Correct answer. 1 Mark: Finds the times or positions.
29(c)	$x = 1 - \cos 2t$ $v = \dfrac{dx}{dt} = 2\sin 2t$ At $t = \frac{\pi}{4}$. $v = 2\sin\left(2 \times \frac{\pi}{4}\right)$ $\quad = 2$ m/s $\therefore$Velocity of the particle is 2 m/s.	1 Mark: Correct answer.
29(d)	$2\sin 2t = 1$ $\sin 2t = \dfrac{1}{2}$ $2t = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ $t = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ Using the graph $\therefore \dfrac{\pi}{12} < t < \dfrac{5\pi}{12}$	1 Mark: Correct answer.

30	Minimum value occurs when $\frac{dy}{dx} = 0$ $\frac{dy}{dx} = 4x - \frac{0.5}{\frac{x}{2}}$ $0 = 4x - \frac{1}{x}$ $4x^2 = 1$ $x^2 = \frac{1}{4}$ $x = \pm\frac{1}{2}$ Since x cannot take a negative value, $x = 0.5$. 1 $y = 2(0.5)^2 - \ln\left(\frac{0.5}{2}\right) - 4 = \ln 4 - 3\frac{1}{2}$ 4 $\therefore \left(\frac{1}{2}, \ln 4 - 3\frac{1}{2}\right)$ Check if a minima $\frac{d^2y}{dx^2} = 4 + \frac{1}{x^2}$ When $x = 0.5$ $\frac{d^2y}{dx^2} = 4 + \frac{1}{0.5^2} = 8 > 0$ Minima	3 Marks: Correct answer. 2 Marks: Shows the minimum value at $\left(\frac{1}{2}, \ln 4 - 3\frac{1}{2}\right)$ without testing for a minima 1 Mark: Finds the derivative.
31	To find the expected value or mean $\int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x(x^3 - x + 4)\,dx$ $= \int_0^1 x^4 - x^2 + 4x\,dx$ $= \left[\frac{x^5}{5} - \frac{x^3}{3} + 2x^2\right]_0^1$ $= \frac{28}{15} = 1\frac{13}{15}$	2 Marks: Correct answer. 1 Mark: shows some understanding.
32(a)	Intersection value is 38.78231 (0.0075 and 46 months) $PV = 38.78231 \times 3200$ $= 124\,103.392$ $\approx \$124\,103.39$	1 Mark: Correct answer.
32(b)	$r = \frac{0.078}{12} = 0.0065 \quad n = 4 \times 12 = 48$ Intersection value is 41.11986 Let the monthly repayment be x. $PV = 41.11986 \times x$ $27\,000 = 41.11986 \times x$ $x = \frac{27\,000}{41.11986}$ $= 656.6170 \ldots \approx \$656.62$ $\therefore$ Annabelle's monthly repayment is \$656.62.	2 marks: Correct answer. 1 mark: Finds the intersection value or shows some understanding.

33(a)	$r = 0.995193611\ldots$ ≈ 0.9952	1 mark: Correct answer.
33(b)	$y = mx + c$ $= BX + A$ $\text{H} = 1.19t - 0.85$	1 mark: Correct answer.
33(c)	When $t = 5$ years $H = 1.19t - 0.85$ $= 1.19 \times 5 - 0.85$ ≈ 5.1 m $\therefore$ Height of the tree after 5 years is 5.1 metres.	1 mark: Correct answer.
33(d)	When $H = 20$ m $H = 1.19t - 0.85$ $20 = 1.19 \times t - 0.85$ $1.19t = 20.85$ $t \approx 17.5$ years $\therefore$ It takes 17.5 years for the tree to reach a height of 20 metres.	1 mark: Correct answer.
33(e)	Strong positive linear association Question (C) involves interpolation. Very reliable. Question (D) involves extrapolation. Less reliable.	1 mark: Correct answer.
34(a)	 (1) Shape correct (1)	2 Marks: Correct answer. 1 Mark: Draws the general shape of the curve.
34(b)	High tide occurs when $h(t) = 4$ Using the graph $\therefore t = 4$ or $t = 20$	1 Mark: Correct answer.
34(c)	The high tide has height 4 metres above the mean height.	1 Mark: Correct answer.
34(d)	$h(t) = 4\sin\left(\frac{\pi}{8}t\right)$ $h(8) = 4\sin\left(\frac{\pi \times 10}{8}\right) = 4\sin\left(\frac{5\pi}{4}\right)$ $= -2.8284\ldots$ ≈ -2.8 $\therefore$ Water is about 2.8 metres below mean height.	2 Marks: Correct answer. 1 Mark: Substitutes 10 into the equation.