PAPER 4

YEAR 12
YEARLY
EXAMINATION

Student name: _____

Mathematics Standard 2

General Instructions
- Working time - 150 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 15 marks
- Attempt Questions 1-15
- Allow about 25 minutes for this section

Section II – 85 marks
- Attempt all questions in Section II
- Allow about 2 hours and 5 minutes for this section

Section I

15 marks
Attempt questions 1 - 15
Allow about 25 minutes for this section

Use the multiple-choice answer sheet for questions 1-15

1. A group of 90 workers took a competency test in literacy and their results were normally distributed. The mean score of the group was 85% with a standard deviation of 5%. Anthony scored 75% on the test. What is this as a z-score?

   (A)   –2

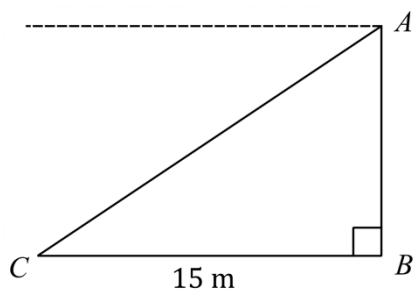   (B)   –1

   (C)   1

   (D)   2

2. The network diagram below shows the distance by road (in km) between four towns.
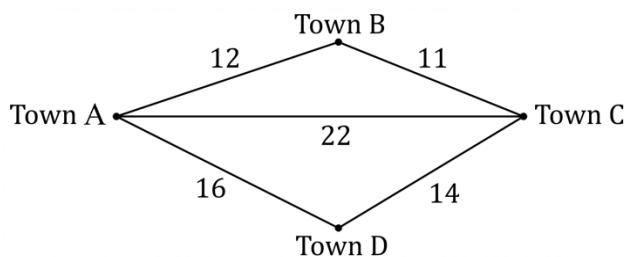
   What is the length of the minimum spanning that connects all the towns?

   (A)   37 km

   (B)   39 km

   (C)   47 km

   (D)   53 km

3. The angle of depression from A to C is 40°. The distance from B to C is 15 metres.

   Not to scale

   How high above B is A, to the nearest metre?

   (A)   10 m

   (B)   11 m

   (C)   12 m

   (D)   13 m

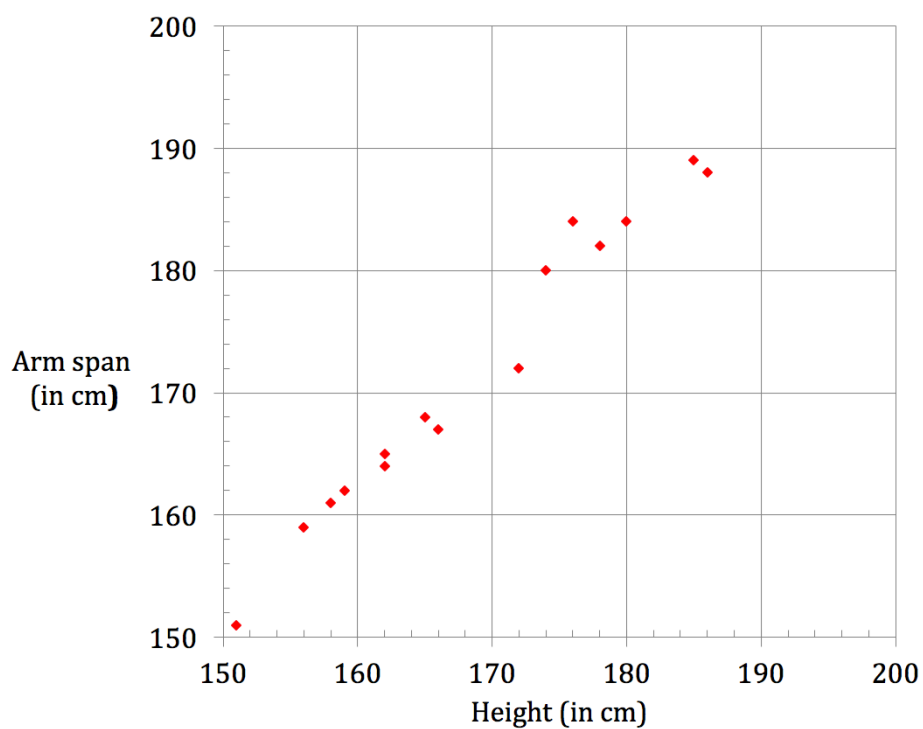4. A company manufactures n bags at a cost (C) according to the equation:

$C = 50n + 75\,000$

The income (I) from the sale of n bags in a particular week was:

$I = 800n$

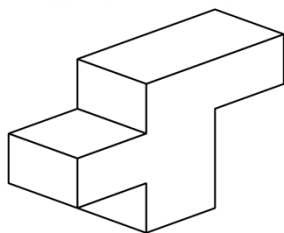If the company "broke even" during the week, how many bags were sold?

(A)   89

(B)   100

(C)   750

(D)   1500

5. The data in the scatterplot compares the height (in cm) to arm span (in cm).

What relationship is suggested by the data between the height and arm span?

(A)   No correlation

(B)   Perfect positive correlation

(C)   Weak negative correlation

(D)   Strong positive correlation

6.

What is the plan view of the above object?

(A)
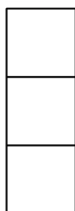
(B)

(C)
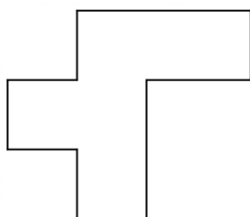
(D)
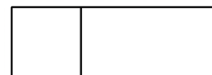
7. The equation of least-squares line of best fit is given by $y = mx + c$ where

$$m = r\frac{S_y}{S_x} \text{ and } c = \bar{y} - m\bar{x}$$

What is the gradient of the least-squares line of best fit given $r = 0.561$, $S_x = 1.987$ and $S_y = 4.579$?

(A) 0.24

(B) 1.29

(C) 7.13

(D) 16.21

8.

Lara

15 m

Not to scale

36°

59°

Keira   Callum
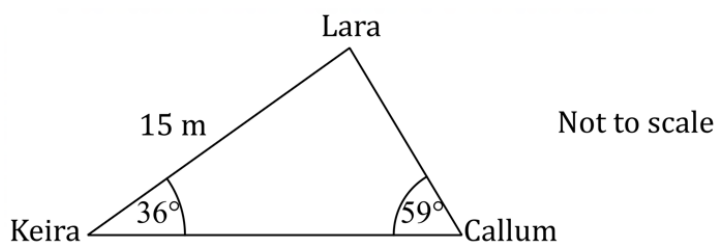
Which formula should be used to calculate the distance between Callum and Lara?

(A) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

(B) $c^2 = a^2 + b^2$

(C) $A = \dfrac{1}{2} ab\sin C$

(D) $c^2 = a^2 + b^2 - 2ab \sin C$

9. A 120 watt ceiling fan is run for 24 hours each day. If electricity is charged at 24.8 c/kWh, what is the cost of running the ceiling fan for 30 days, to the nearest cent?

    (A) $15.68

    (B) $21.43

    (C) $86.40

    (D) $2142.73

10. A computer was purchased for $2400 on 11 June 2018 using a credit card. Compound interest is charged at a rate of 18.75% per annum for purchases using a credit card. No other purchases were made and there was no interest-free period. The period for which interest was charged included the date of purchase and the date of payment.
    What amount was required to pay the account in full on 19 July 2018?

    (A) $2447.20

    (B) $2448.08

    (C) $2448.55

    (D) $2449.32

11. A table of future value interest factors is shown below.

Period	Interest rate per period				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101

A certain annuity involves making equal contributions of $100 000 into an account every 6 months for 2 years at an interest rate of 2% per annum. Based on the information provided, what is the future value of this annuity?

    (A) $201 000

    (B) $202 000

    (C) $406 040

    (D) $412 160

12. A factory produces bags of cashews. The weights of the bags are normally distributed, with a mean of 900 g and a standard deviation of 50 g. What is the best approximation for the percentage of bags that weigh more than 1000 g?

    (A)   0%

    (B)   2.5%

    (C)   5%

    (D)   16%

13. On the 1st July 2017 Violet invested $2175 into an account which pays 12% p.a. interest, compounded quarterly. On the 1st July 2018 she added a further $1920 to the account. How much has she in the account on the 1st July 2019?

(A) $4609

(B) $4625

(C) $4916

(D) $5187

14.
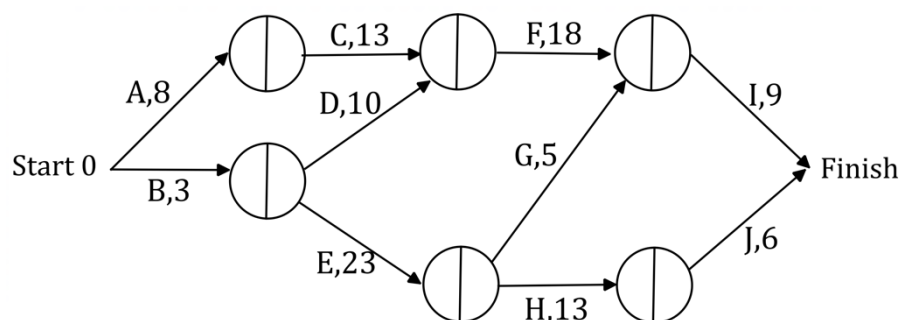
What is the minimum completion time for the above network?

(A)   40

(B)   44

(C)   48

(D)   52

15. The speed (v), in km/h, of a ski lift is inversely proportional to the weight (w kg) it carries. A ski lift carrying a weight of 320 kg can travel at 16 km/h. What is the speed of the ski lift if weight decreases to 250 kg?

(A) $\dfrac{16 \times 250}{320}$

(B) $\dfrac{320}{16 \times 250}$

(C) $\dfrac{250 \times 320}{16}$

(D) $\dfrac{16 \times 320}{250}$

Section II

85 marks
Attempt all questions
Allow about 2 hours and 5 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 16 (2 marks)   **Marks**

A solar PV system exports 19.4 kWh per day of energy to the grid. An energy   **2**
retailer pays $0.076 per kWh for energy. What is the expected saving from the solar
PV system for the year? Answer to the nearest cent.

Question 17 (2 marks)

2

	A	*B*	*C*	*D*
A	–	4	10	11
B	4	–	9	4
C	10	9	–	2
D	11	4	2	–

Represent the table shown above as a weighted network.

Question 18 (4 marks)   **Marks**

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

Present value of $1				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

Hayley borrows $16 000 for a car. She arranges to repay the loan with monthly repayments over 4 years. She is charged 8.4% per annum interest.

(a)   Find Hayley's monthly repayment. Answer to the nearest cent.   **2**

(b)   Calculate the amount of interest Hayley will pay over the term of the loan.   **2**
      Answer to the nearest dollar.

Question 19 (2 marks)

The Pearson's correlation coefficient between students assessment result and their   **2**
height was 0.12. What is the meaning of this correlation?

Question 20 (5 marks)   **Marks**

Activity	Duration (min)	Immediate predecessors
A	10	–
B	6	–
C	7	A
D	11	A, B
E	8	D
F	?	C, E
G	2	D

(a) Construct a network diagram using the activity chart. Show the earliest   **3**
    starting times (EST) and latest starting times (LST).

(b) The critical path is *ADEF*. It has a critical time of 38 minutes.   **1**
     What is weight of activity *F*?

(c) What is the float time of activity *G*?   **1**

Question 21 (4 marks)

Marks

The mean for a class test is 64% and the standard deviation is 12.5.

(a) In a class test Charlie has a z-score of 2. What does that mean? **1**

(b) Hannah obtains a z-score of –2. What is her mark? **1**

(c) Jacob has a mark of 51.5%. What is his z-score? **1**

(d) Lucy said she had a z-score of 3 but Hannah is unconvinced. Why? **1**

Question 22 (2 marks)

What single amount of money would need to be invested today at 4.8% p.a., compounded monthly, to allow $50 000 to be in the account after 3 years? Assume no other amounts are deposited into the account over the 3 years. Give your answer to the nearest dollar. **2**

Question 23 (3 marks)   **Marks**

Lucas's watering can is initially filled with 4 litres of water. However the watering can has a small hole in the base and is leaking at a rate of 0.2 litres per minute.

(a)   Write a linear equation in the form $y = mx + c$ to describe this situation. Use the variables V for volume and t for time.   **1**

(b)   What volume of water remains after 150 seconds?   **1**

(c)   How long would it take for all the water to leak out?   **1**

Question 24 (2 marks)

A map has a scale of 1:500 000.

(a)   Two towns are 2 cm apart on the map. What is the actual distance between the towns, in kilometres?   **1**

(b)   The distance between two cities is 75 km. How far apart are the two cities on the map, in centimetres?   **1**

Question 25 (2 marks)

Lilly has a credit card with a simple interest rate of 21.45% per annum.

(a)   What is the daily percentage interest rate, correct to 3 decimal places?   **1**

(b)   Lilly has an outstanding balance of $8200 for a period of thirty days. How much interest, to the nearest dollar, will she be charged?   **1**

Question 26 (5 marks)   **Marks**

Molly borrowed $15 000 from the bank to buy a used car. The first three months of loan repayments are shown below.

Amount borrowed	$15 000	This table assumes the same number of days in each month. $I = Prn$ or $I = P \times \frac{r}{12}$		
Annual interest rate (r)	8%			
Monthly repayment (R)	$1000			
Month n	Principal P	Interest I	$P + I$	$P + I - R$
1	$15 000.00	$100.00	$15 100.00	$14 100.00
2	$14 100.00	$94.00	$14 0194.00	$13 194.00
3	$13 194.00	$87.96	$13 281.96	$12 281.96

(a) How much interest did Molly pay in the first three months?   1

(b) How much of the principal would be repaid after the 4th payment?   2

(c) Molly received a gift of $7000 and used all of it to make the 4th repayment.   2
How much interest did she save in the 5th month?

Question 27 (5 marks) **Marks**

The table below shows forearm length and hand length.

Forearm (in cm)	25.0	25.6	26.0	26.6	27.0	27.4	28.0	28.6	29.0	29.2
Hand (in cm)	17.2	17.6	18.2	18.4	19.0	19.0	19.8	19.8	20.4	20.6

(a) Draw a scatterplot using the above table. **1**
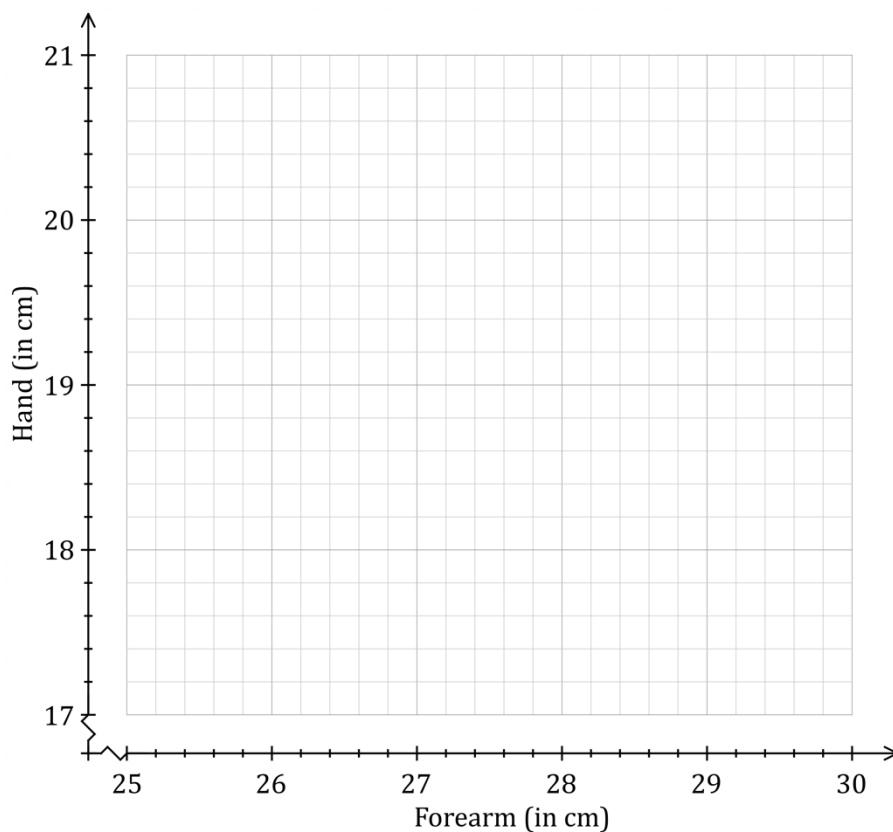
(b) Draw a line of best fit on the scatterplot. **1**

(c) Lara has a forearm whose length is 27.8 cm. What is her expected hand length? **1**

(d) Calculate the value of the Pearson's correlation coefficient. Answer correct to four decimal places. **2**

13

Question 28 (2 marks) **Marks**

Asha travelled 696 km in his car, using 39 L of petrol. At this consumption rate, how many litres does he use per 100 km? (Answer correct to 3 decimal places). **2**

Question 29 (6 marks)

The diagram shows three towns. Town A is due west of town B and the bearing of town C from town B is 025°.

50 km

Not to scale

A 60 km B
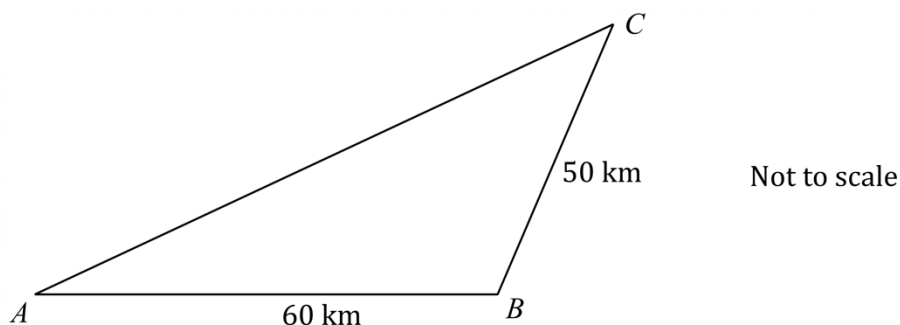
(a) What is the size of $\angle ABC$? **1**

(b) Find the distance (to nearest kilometre) from town A to town C. **2**

(c) What is the bearing of town C from town A? **3**

Question 30 (1 mark)

A spanning tree contains six edges. How many vertices are in the network? **1**

Question 31 (3 marks)

The graph below shows the distance (d) in kilometres to the horizon that can be seen from different heights (h) above sea level

(a)   What is the distance to the horizon, if the height above sea-level is 40 m?   **1**
Answer to the nearest kilometre.

(b)   Steve records a distance of 20 km to the horizon from his position at a   **1**
lookout. Steve climbs to a second lookout at a higher position above sea level where he can see a further 4 kilometres. What is the difference in the height of the two vintage points? Answer to the nearest metre.

(c)   The formula $d = 8\sqrt{\dfrac{h}{4}}$ can be used to calculate the distance (d) to the horizon   **1**
from a height (h) above sea level. Using this formula, calculate the distance to the horizon when the height above sea-level is 40 metres. Answer to the nearest kilometre.

Question 32 (3 marks)   **Marks**

A rectangular sheet of aluminium is rolled into an open cylindrical drum.

0.9 m

1.25 m
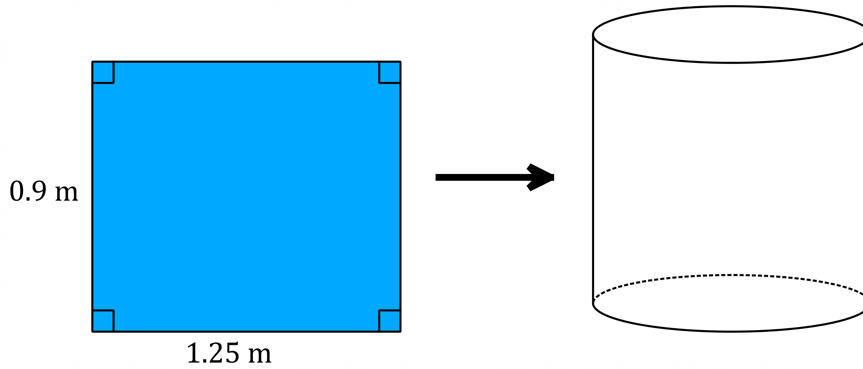
(a)   Find the radius of the drum. Answer correct to one decimal place.   **1**

(b)   What is the volume of the drum in cubic metres? Answer correct to two   **1**
significant figures.

(c)   What is the capacity of the drum to the nearest litre?   **1**

Question 33 (2 marks)   **Marks**
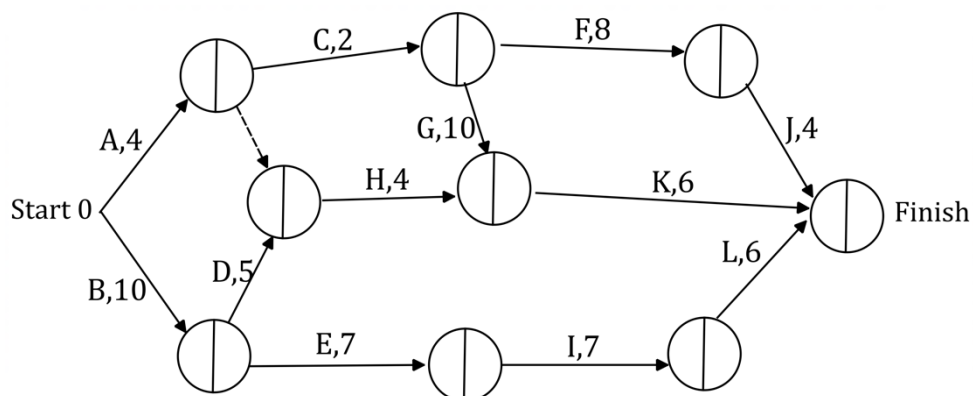
Write the earliest starting times (EST) and latest starting times (LST) on the above   **2**
network diagram.

Question 34 (4 marks)

The normal distribution shows the results of a mathematics assessment task. It has
a mean of 60 and a standard deviation of 10.
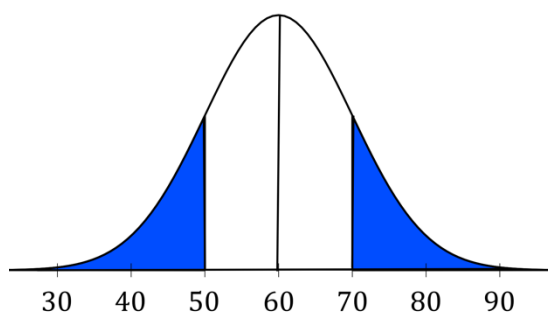
(a)   What is the mathematics assessment task with a *z*-score of –2?   **1**

(b)   What is the z-score of a mathematics assessment task of 65?   **1**

(c)   What percentage of results lie in the shaded region?   **2**

Question 35 (4 marks)   **Marks**

The graph below shows the cost of producing boxes of paper and the income received from their sale.
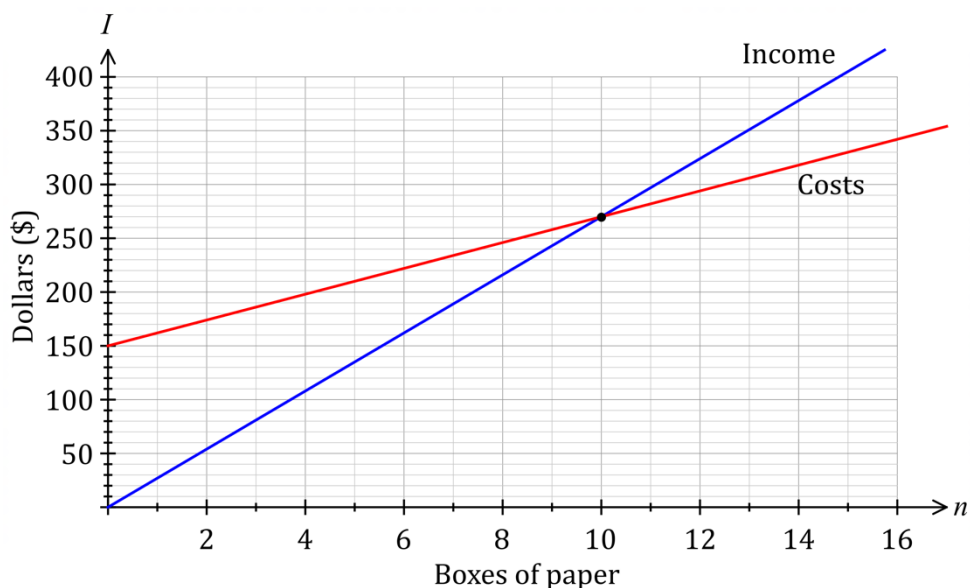
(a) Use the graph to determine the number of boxes which need to be sold to   **1**
     break even.

(b) How much profit or loss is made when 5 boxes are sold?   **1**

(c) What are the initial costs?   **1**

(d) Write an equation to describe the relationship between the costs and the   **1**
     number of boxes.

18

Question 36 (4 marks)

A sum of \$1 000 is invested at 6% pa compounding annually. The graph of the equation $FV = 1000(1.06)^n$ illustrates how the investment grows.

(a) Estimate the number of years it would take this investment to double in value. **2**

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(b) Compare the time it takes the investment to triple in value with the time that it takes to double in value. Explain the difference. **2**

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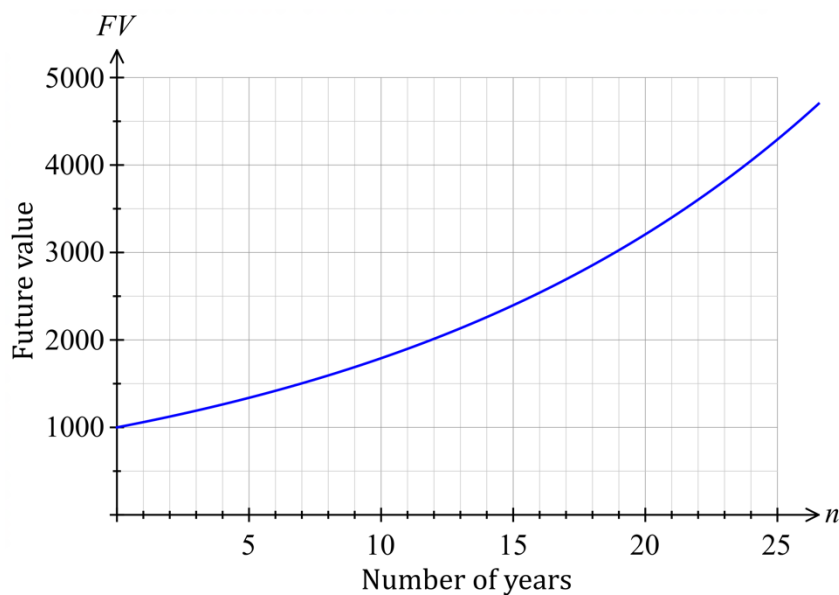
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Question 37 (2 marks)

The heights of a group of friends are normally distributed with a mean of 167 cm and a standard deviation of 12 cm. What percentage of the group are more than 179 cm tall? **2**

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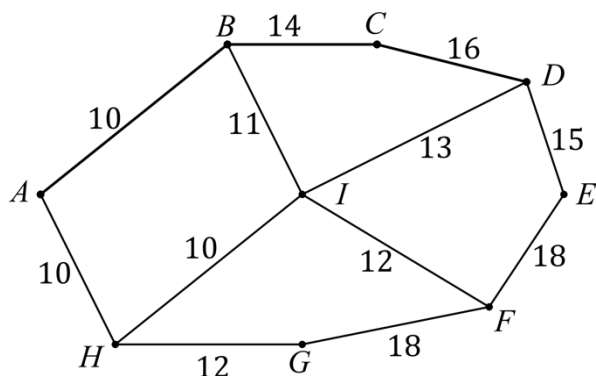**Question 38** (6 marks)   **Marks**

The network below shows the time (in minutes) taken to travel between towns.

(a)   What is the time taken to travel *ABCDI*?   **1**

(b)   List the vertices with an even degree?   **1**

(c)   Does this network contain a walk that visits every edge exactly once? Give a   **1**
      reason for your answer.

(d)   Find the shortest time it would take to travel from *A* to *E*?   **2**

(e)   Find a minimum spanning tree for this network.   **1**

Question 39 (4 marks)   **Marks**

A table for borrowing $200 000 at 7.25% p.a. interest is shown below.

Loan period in years	15	20	25	30
Monthly repayments	$1825.73	$1580.75	$1445.61	$1364.35

(a) Find the total amount that must be repaid if the loan is taken over 20 years.   **1**

(b) Calculate the amount of interest to paid if the loan is taken over 30 years.   **1**

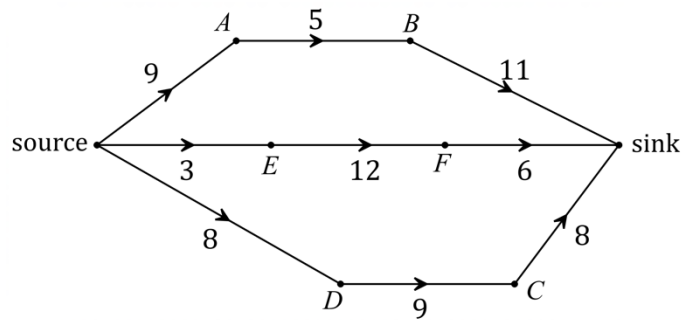(c) How much extra is repaid if the loan is taken over 30 years rather 20 years?   **2**

Question 40 (2 marks)

What is the maximum flow for this network?   **2**

Question 41 (4 marks)   **Marks**

A tennis competition involves every player having a match against every other player. The number of matches is calculated using the formula:

$$m = \frac{1}{2}(p^2 - p)$$

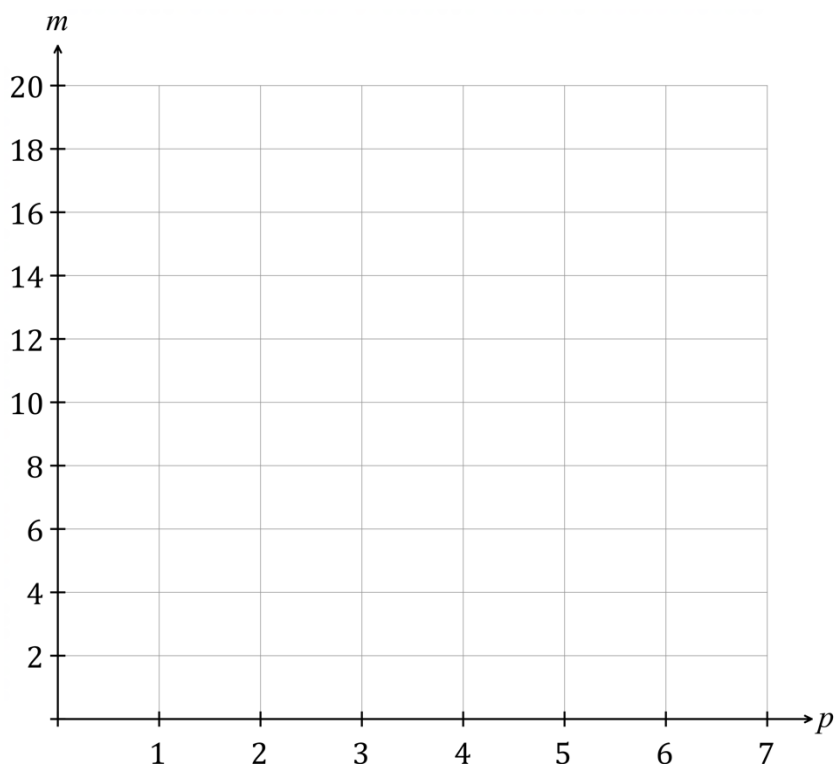where m is the number of matches and p the number of players.

(a)   Complete the following table of values for the above formula.   **1**

Matches (m)	1	2	3	4	5	6
Players (p)						

....

....

(b)   Plot the points and join them to make a parabola.   **1**

(c)   Use the model to predict the number of matches for 7 players.   **1**

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....

(d)   Explain why you would not use the model when p is less than one.   **1**

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End of paper

NSW Education Standards Authority

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Standard 1
Mathematics Standard 2

REFERENCE SHEET

Measurement

Precision

Absolute error $= \dfrac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement − absolute error

Length, area, surface area and volume

$$l = \frac{\theta}{360} \times 2\pi r$$

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(x + y)$$

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$A = \frac{1}{2}ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV(1 + r)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

Statistical Analysis

$$z = \frac{x - \overline{x}}{s}$$

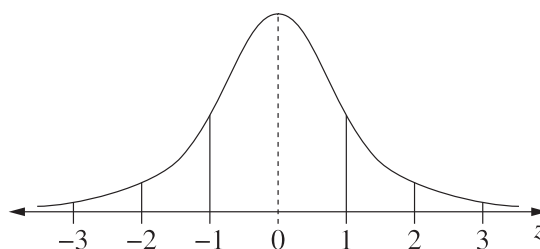An outlier is a score

    less than $Q_1 - 1.5 \times IQR$
    or
    more than $Q_3 + 1.5 \times IQR$

Normal distribution

- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3