

PAPER 2

YEAR 12
YEARLY
EXAMINATION

Mathematics Advanced

**General
Instructions**

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-16, show relevant mathematical reasoning and/or calculations

**Total marks:
100**

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

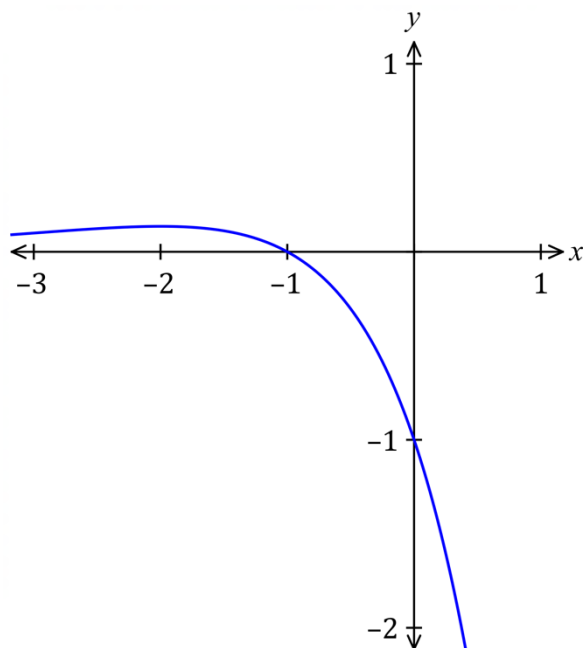
Section I**10 marks****Attempt questions 1 - 10****Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for questions 1-10

1. What is the value of $\int_0^1 (6x^2 - 4)dx$?

- (A) -2
- (B) -1
- (C) 0
- (D) 1

2. The diagram shows the graph of $y = e^x(x - 1)$.



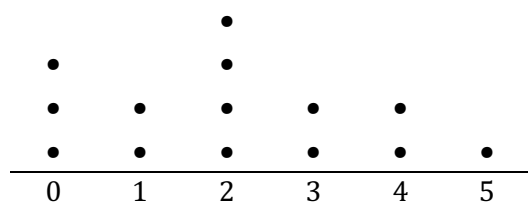
How many solutions are there to the equation $e^x(x - 1) = x^2 - 1$?

- (A) 0
 - (B) 1
 - (C) 2
 - (D) 3
3. An infinite geometric series has a first term of 3 and a limiting sum of 1.8.
What is the common ratio?
- (A) $-0.\dot{3}$
 - (B) $-0.\dot{6}$
 - (C) -1.5
 - (D) -3.75

4. What is the derivative of e^{x^3} ?

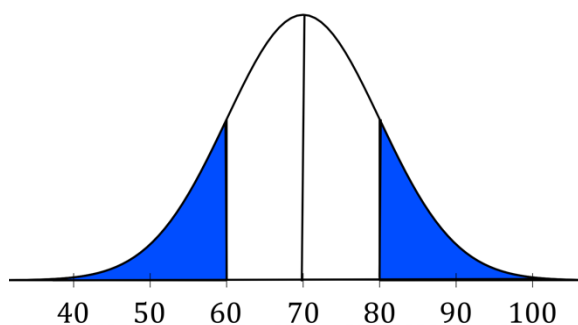
- (A) $3x^2e^{x^3}$
- (B) $3xe^{x^3}$
- (C) $3x^2e^{3x^2}$
- (D) $x^3e^{x^3-1}$

5. A sample of 14 people were asked to indicate the time (in hours) they had spent watching television on the previous night. The results are displayed in the dot plot below.



What is the mean and sample standard deviation of these times? Give your answers correct to one decimal place.

- (A) $\bar{x} = 2.0$ and $s = 1.5$
 - (B) $\bar{x} = 2.1$ and $s = 1.5$
 - (C) $\bar{x} = 2.1$ and $s = 1.6$
 - (D) $\bar{x} = 2.6$ and $s = 1.2$
6. The normal distribution shows the results of a mathematics assessment task. It has a mean of 70 and a standard deviation of 10



What percentage of results lie in the shaded region?

- (A) 16%
 - (B) 32%
 - (C) 34%
 - (D) 68%
7. The acceleration of a particle moving in a straight line is given by the formula $a = 12t + 6$. Initially the particle is at $x = 5$ metres and the initial velocity of the particle is -36 m/s. When is the particle at rest?
- (A) $t = 0$
 - (B) $t = 1$
 - (C) $t = 2$
 - (D) $t = 3$

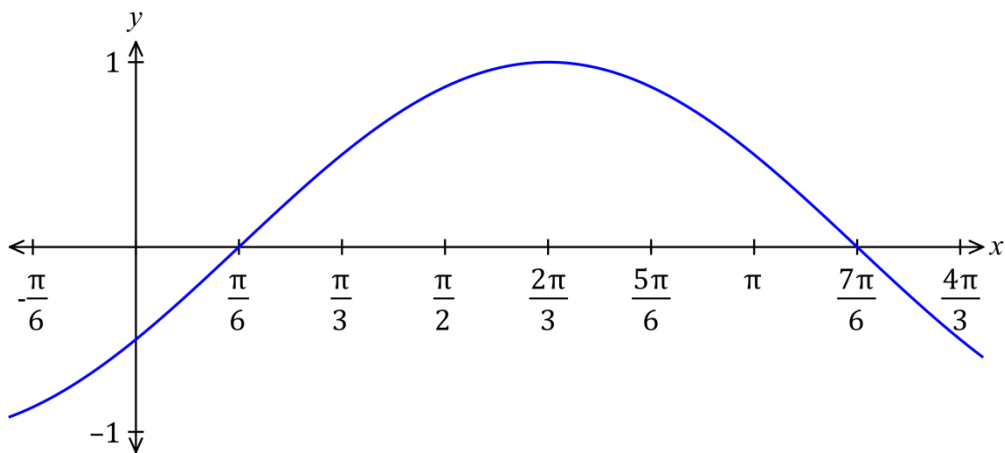
8. The equation of least-squares line of best fit is given by $y = mx + c$ where

$$m = r \frac{s_y}{s_x} \text{ and } c = \bar{y} - m\bar{x}$$

What is the gradient of the least-squares line of best fit given $r = 0.561$, $s_x = 1.987$ and $s_y = 4.579$?

- (A) 0.24
- (B) 1.29
- (C) 7.13
- (D) 16.21

9.



A possible equation for the graph shown above is :

- (A) $y = \cos\left(x + \frac{\pi}{6}\right)$
 - (B) $y = \sin\left(x - \frac{\pi}{6}\right)$
 - (C) $y = \sin\left(x + \frac{\pi}{6}\right)$
 - (D) $y = -\sin\left(x - \frac{\pi}{6}\right)$
10. An area is bounded by the curve $y = \frac{2}{3}\sqrt{9 - x^2}$ the coordinate axes and line $x = 2$.

What is an approximation for this area using the trapezoidal rule and three function values?

- (A) 1.82
- (B) 2.69
- (C) 3.26
- (D) 3.63

Section II**90 marks****Attempt questions 11 - 16****Allow about 2 hours and 45 minutes for this section**

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)**Marks**

Simplify $\frac{y}{y^2 - 4} - \frac{2}{y - 2}$

2**Question 12 (2 marks)**

(a) Express $\sin\theta\cos\theta + \frac{\cos^3\theta}{\sin\theta}$ as a single trigonometric ratio.

1

(b) Hence solve $\sin\theta\cos\theta + \frac{\cos^3\theta}{\sin\theta} = 1$ for $0 \leq \theta \leq 2\pi$.

1

Question 13 (3 marks)**Marks**

Differentiate

(a) $\tan 5x$

1

(b) $\frac{\ln x}{x}$

1

(c) $x \cos x$

1

Question 14 (3 marks)

The second term of an arithmetic series is 39 and the sixth term is 19.
What is the sum of the first ten terms?

3

Question 15 (2 marks)Find the anti-derivative of $4 - x^{-3}$.**2**

Question 16 (5 marks)**Marks**

- (a) Sketch the graphs of $y = 4 - x^2$ and $y = 3$ on the same number plane.

2

- (b) The graph of $y = 3$ cuts the parabola at P and Q .
What are the coordinates of P and Q ?

1

- (c) Calculate the area bounded by the graphs of $y = 4 - x^2$ and $y = 3$.

2**Question 17** (2 marks)

Riley's class achieved a 72% mean and 8% standard deviation for their project work. What was Riley's mark if he achieved a z-score of -2.5 ?

2

Question 18 (4 marks)**Marks**

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

<i>Present value of \$1</i>				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

Jessica borrows \$16 000 for a car. She arranges to repay the loan with monthly repayments over 4 years. She is charged 8.4% per annum interest.

- (a) Find Jessica's monthly repayment. Answer to the nearest cent.

2

- (b) Calculate the amount of interest Jessica will pay over the term of the loan. Answer to the nearest dollar.

2

Question 19 (2 marks)

Evaluate $\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x) dx$. Write your answer correct to three decimal places.

2

Question 20 (3 marks)**Marks**

Sketch the graph of $f(x) = 3 - \frac{2}{x+1}$.

3

Label all axis intercepts. Label each asymptote with its equation.

Question 21 (2 marks)

The variables profit made and amount spent on advertising are strongly correlated with a correlation coefficient $r = 0.9$. What conclusions can you draw from this information?

2

Question 22 (3 marks)

Marks

The mean for a class test is 64% and the standard deviation is 12.5.

- (a) In a class test Marcus has a z-score of 2. What does that mean?

1

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840.

- (b) Fletcher has a mark of 51.5%. What is his z-score?

1

[L'Espresso](#)

- (c) Ayla said she had a z-score of 3 but Hannah is unconvinced. Why?

1

[illegible]

Question 23 (3 marks)

The displacement of an object at time (t) seconds is given by:

3

$$x = 3e^{-2t} + 10e^{-t} + 4t$$

Find the time the object comes to rest.

1. The first step in the process of creating a new product is to identify a market need. This involves conducting market research to understand the preferences and behaviors of potential customers.

2. Once a market need is identified, the next step is to develop a concept. This involves brainstorming ideas and creating a rough sketch of the product.

3. The third step is to create a prototype. This involves building a small-scale model of the product to test its functionality and appearance.

4. After the prototype is created, the next step is to conduct a feasibility study. This involves evaluating the technical, financial, and market viability of the product.

5. If the feasibility study is positive, the next step is to develop a business plan. This involves outlining the marketing, sales, and financial strategies for the product.

6. The final step is to launch the product. This involves manufacturing the product, distributing it to retailers, and promoting it to the target market.

Question 24 (5 marks)**Marks**

Lara borrows \$50,000 to purchase furniture for her small business. The interest is calculated monthly at a rate of 2% per month. She intends to repay the loan with interest in two annual instalments of \$ M at the end of the first and second years.

- (a) Write an expression involving M for the total amount owed by Lara after 12 months, just after the first instalment of \$ M has been paid. **1**

- (b) Show that $M = \frac{\$50\,000 \times 1.02^{24}}{1.02^{12} + 1}$ **2**

- (c) What will be the total amount of interest paid on this loan? **2**

Question 25 (2 marks)

The probability density function for the continuous random variable X is:

2

$$f(x) = \begin{cases} x^3 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of $E(X)$?

Question 26 (5 marks)**Marks**

- (a) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \leq \theta \leq \pi$. **2**

- (b) Find the values of x for which $\sin x = 1 - \cos x$ in the domain $0 \leq \theta \leq \pi$. **1**

- (c) Find the area between $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \leq \theta \leq \pi$. **2**

Question 27 (2 marks)

What is the equation of the normal to the curve $y = x^2 - 4x$ at the point $(1, -3)$? **2**

Question 28 (7 marks)**Marks**

A function $f(x)$ is defined by $f(x) = 7 + 4x^3 - 3x^4$.

- (a) Find the coordinates of the stationary points for the curve $y = f(x)$. **2**

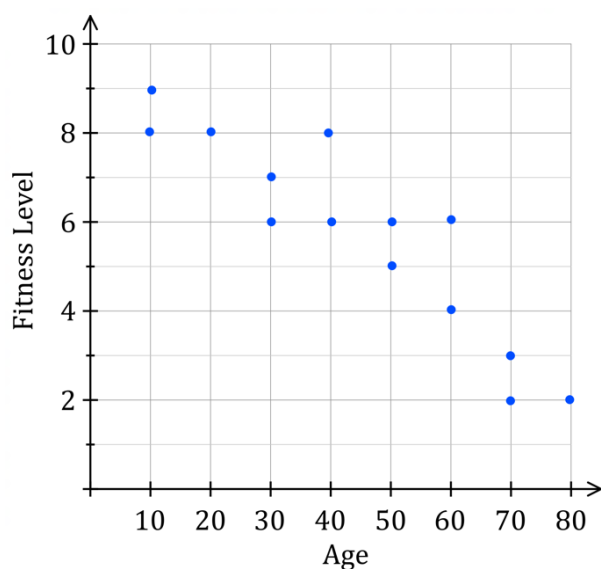
- (b) Find all values of x for which $f''(x) = 0$. **1**

- (c) Determine the nature of the stationary points. **2**

- (d) Sketch the graph of $y = f(x)$ for the domain $-1 \leq x \leq 2$. **2**

Question 29 (5 marks)**Marks**

The scatterplot below shows the relationship between age and fitness level.



- (a) Draw a line of best fit on the scatterplot. Find the gradient of this line.

2

- (b) Lachlan is 30 years old. What is his expected fitness level?

1

- (c) Calculate the value of the Pearson's correlation coefficient. Answer correct to two decimal places.

2

Question 30 (2 marks)

State the domain and range of $f(x) = \sqrt{1 - x^2}$.

2

Question 31 (2 marks)

Marks

The probability density function for the continuous random variable X is:

2

$$f(x) = \begin{cases} \frac{1}{9}(4x - x^2) & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find value of $P(X \leq 2)$.

| | | | | |
|--|--|--|--|--|
| <p> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 </p> | <p> 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 </p> | <p> 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 </p> | <p> 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 </p> | <p> 401 402 403 404 405 406 407 408 409 410 411 </p> |
|--|--|--|--|--|

Question 32 (3 marks)

Determine the equation of a curve given by $\frac{d^2y}{dx^2} = 12x + 6$ and $(1, -2)$ as a stationary point on the curve.

3

1. The first step in the process of creating a new product is to identify a market need. This involves conducting market research to understand the preferences and behaviors of potential customers.

2. Once a market need is identified, the next step is to develop a concept for the product. This involves brainstorming ideas and creating a rough sketch of the product.

3. The third step is to create a prototype. This is a physical model of the product that allows the designer to test and refine the design.

4. After the prototype is created, the next step is to conduct a feasibility study. This involves evaluating the technical, financial, and market viability of the product.

5. If the feasibility study is positive, the next step is to develop a business plan. This document outlines the marketing, financial, and operational strategies for the product.

6. The final step in the process is to launch the product. This involves manufacturing the product, distributing it to retailers, and promoting it to the target market.

Question 33 (2 marks)

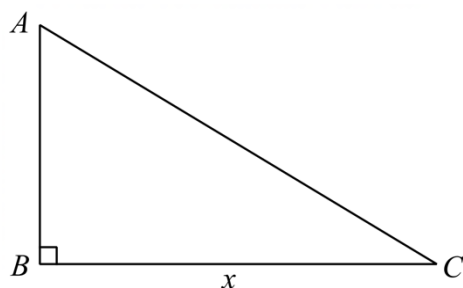
Simplify $\lim_{x \rightarrow 0} \frac{\sin 6x}{x}$

2

1. The first part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...
 2. The second part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...
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 9. The ninth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...
 10. The tenth part of the document is a list of the names of the members of the committee, which is headed by the Chairman, Mr. J. H. ...

Question 34 (7 marks)**Marks**

A cable of length 3 metres is to be bent to form the hypotenuse and base of a right-angled triangle ABC . Let the length of the base BC is x metres.



- (a) What is the length of the hypotenuse AC in terms of x ? **1**

.....

- (b) Show that the area of the triangle ABC is $0.5x\sqrt{9-6x}$. **2**

.....

.....

.....

.....

- (c) What value of x gives the maximum possible area of the triangle? **3**

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- (d) Find the maximum possible area of the triangle. **1**

.....

.....

.....

.....

.....

Question 35 (3 marks)**Marks**

How many solutions exist for x of the equation $e^x + x + 2 = 0$?

3

Hint: Draw graphs.

Question 36 (3 marks)

The number of students absent from year 12 for the past nine days was as follows:

15, 18, 14, 17, 18, 13, 12, 29, 20

(a) What is the mean? Answer correct to one decimal place.

1

(b) Find the interquartile range?

1

(c) Is 29 an outlier for this set of data? Justify your answer with calculations.

1

Question 37 (6 marks)**Marks**Consider the function $f(x) = (2x - 3)^4$.(a) Find the value of $f'(1)$.**2**(b) Find equation of the tangent at the point $(1, 1)$ to the curve $y = (2x - 3)^4$.**1**(b) The equation of the tangent cuts the x -axis at A and the y -axis at B .
Find the area of $\triangle OAB$, where O is the origin.**3****End of paper**



NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Mathematics Extension 1

Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

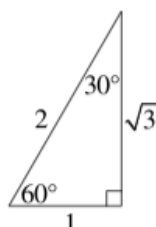
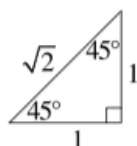
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

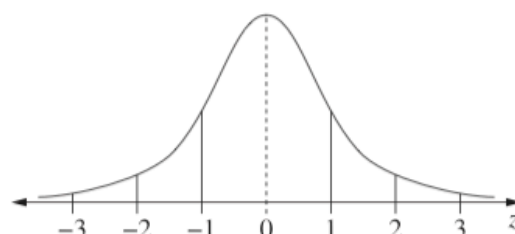
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus**Function****Derivative**

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \{f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})]\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1 x_2 + y_1 y_2,$$

$$\text{where } \underline{u} = x_1 \underline{i} + y_1 \underline{j}$$

$$\text{and } \underline{v} = x_2 \underline{i} + y_2 \underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z &= a + ib = r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n (\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2 x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$