AUSTRALIAN MATHEMATICS COMPETITION
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AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

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CommonwealthBank

NAME ______________________________

YEAR ____________ TEACHER ______________________________

2013
INTERMEDIATE DIVISION
AUSTRALIAN SCHOOL YEARS 9 and 10
TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country or Australian state so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

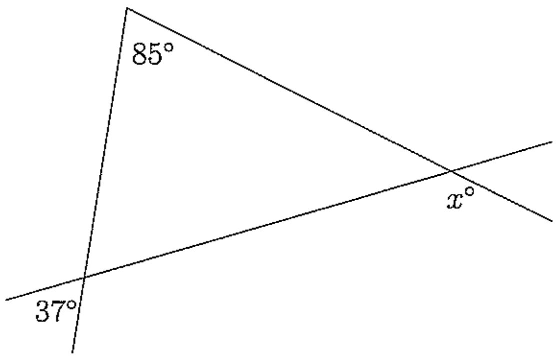
Intermediate Division

Questions 1 to 10, 3 marks each

1. $2013 + 2014 + 2015$ equals

(A) 642 (B) 2016 (C) 6022 (D) 6032 (E) 6042

2. In the diagram below, what is the value, in degrees, of angle x?



(A) 48 (B) 85 (C) 122 (D) 132 (E) 143

3. If every digit of a whole number is either a 3 or a 5, the number will always be

(A) divisible by 3 (B) divisible by 5 (C) prime (D) even (E) odd

4. The average of two numbers is twice the smaller number. The larger number is 12. What is the smaller number?

(A) 2 (B) 3 (C) 4 (D) 6 (E) 8

5. The length of the base of a triangle is 3 times its perpendicular height and the area of the triangle is $24\,\text{cm}^2$. The sum of its base length and its perpendicular height, in centimetres, is

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

6. A regular icosahedron is a solid shape with twenty faces, where each face is directly opposite another face. I label the faces from 1 to 20 so that, for all pairs of opposite faces, the two labels in any pair always add up to the same number. What number is on the face opposite the one labelled 8?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

7. If $p = 4b + 26$ and b is a positive integer, then p could not be divisible by

(A) 2 (B) 4 (C) 5 (D) 6 (E) 7

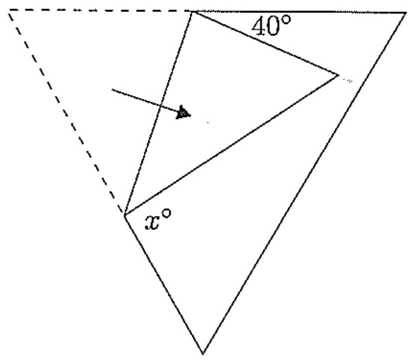
8. My two dogs were running on the beach when I called them back. The faster dog was 100 m away and the slower dog was 70 m away. The faster dog runs twice as fast as the slower dog. How far away was the second dog when the first dog reached me?

(A) 15 m (B) 20 m (C) 30 m (D) 40 m (E) 50 m

9. The value of $x^2 + \frac{1}{x^2}$ when $x = \frac{2}{3}$ is closest to

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10. A piece of paper in the shape of an equilateral triangle has one corner folded over, as shown.



What is the value of x?

(A) 60 (B) 70 (C) 80 (D) 90 (E) 100

Questions 11 to 20, 4 marks each

11. Start with the number 1 and create the sequence

$$1, 2, 4, 8, 16, 22, 24, 28, \ldots$$

where each number is the sum of the previous number and its final digit. How many numbers in the sequence are less than 1000?

(A) 10 (B) 100 (C) 101 (D) 200 (E) 201

12. A six-sided dice has the numbers 1, 2, 2, 3, 3 and 3 on its faces. Two such dice are rolled and a score is made by adding the numbers on the uppermost faces. The probability of rolling an odd score is

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{1}{3}$ (D) $\frac{4}{9}$ (E) $\frac{5}{9}$

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13. If $x^2 = x + 3$, then x^3 equals

(A) $x + 6$ (B) $2x + 6$ (C) $3x + 9$ (D) $4x + 3$ (E) $27x + 9$

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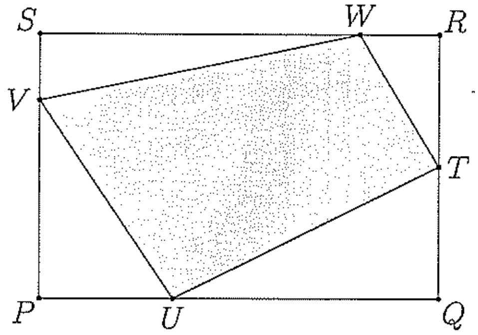
14. The point T divides the side QR of the rectangle $PQRS$ into two equal segments. The point U divides PQ such that $PU : UQ = 1 : 2$. Point V divides SP such that $SV : VP = 1 : 3$ and finally, point W divides RS such that $RW : WS = 1 : 4$. Find the area of the quadrilateral $TUVW$ if the area of $PQRS$ equals 120.



(A) 67 (B) 70 (C) 72 (D) 75 (E) 77

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15. Three line segments of lengths 1, a and $2a$ are the sides of a triangle. Which of the following defines all possible values of a?

(A) $\frac{1}{3} < a < 1$ (B) $0 < a < \frac{1}{3}$ (C) $a < 1$ (D) for all $a > 0$ (E) for no a

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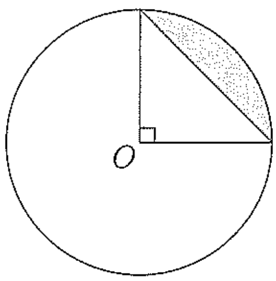
16. The shaded segment in the circle below, centre O, has an area of $1\,\text{cm}^2$. The radius of the circle, in centimetres, is



(A) $\sqrt{\frac{4}{\pi}}$ (B) $\frac{8}{\pi}$ (C) $\sqrt{\frac{4}{\pi - 2}}$ (D) $\frac{4}{\pi}$ (E) $2\sqrt{\pi}$

17. Dan and Jane each have a measuring tape of length 1 m. Dan's tape got stuck in a door and was extended by 4 cm. Jane left her tape in a pocket and it shrank by 5 cm after washing. However, the centimetre marks on both tapes remained evenly distributed.

Measuring the schoolyard, Dan noted the length as 23.75 m. What length will Jane get measuring the same schoolyard with her tape?

(A) 23 m (B) 24 m (C) 25 m (D) 26 m (E) 27 m

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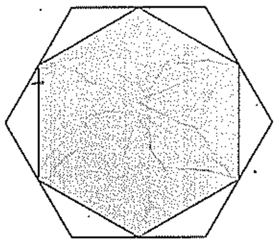
18. In the regular hexagon pictured, the midpoints of the sides are joined to form the shaded regular hexagon. What fraction of the larger hexagon is shaded?



(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{1}{2}$ (E) $\frac{7}{8}$

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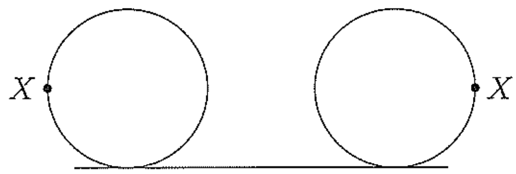
19. A circular wheel of radius r rolls, without slipping, through half a revolution. The point X is on the horizontal diameter at the start.



The distance between the starting and finishing position of the point X is

(A) $2\pi r$ (B) $(\pi + 2)r$ (C) $(\pi - 2)r$ (D) $2(\pi + 1)r$ (E) $2(\pi - 1)r$

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20. The sport of bingbong involves two players. Each match consists of a number of rounds and each round consists of a number of points. The first player to win four points in a round wins the round. The first player to win six rounds in a match wins the match.

Suppose that after a match of bingbong, the winner has won W points while the loser has won L points. What is the largest possible value of $L - W$?

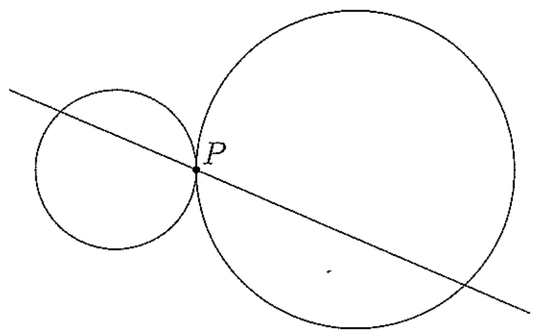
(A) -6 (B) -4 (C) 0 (D) 12 (E) 14

Questions 21 to 25, 5 marks each

21. In how many ways can the numbers 1, 2, 3, 4, 5, 6 be arranged in a row so that the product of any two adjacent numbers is even?

(A) 64 (B) 72 (C) 120 (D) 144 (E) 720

22. Two circles, one of radius 1 and the other of radius 2, touch externally at P. A straight line through P cuts the area formed by these two circles in the ratio $1:2$. In what ratio does this line cut the area of the smaller circle?



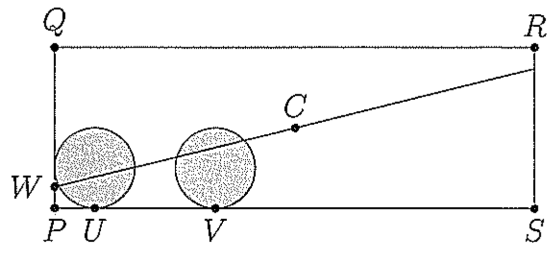
(A) $1:2$ (B) $2:5$ (C) $1:3$ (D) $2:7$ (E) $1:4$

23. How many positive integers n are there such that $2n+1$ is a divisor of $8n+46$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

24. The rectangle $PQRS$ shown has $PQ = 4$, $PS = 12$ and centre C. The two shaded circles have radius 1 and touch PS at U and V where $PU = 1$ and $PV = 4$.

The line CW divides the unshaded area in half. The length of PW is



(A) $\frac{2}{7}$ (B) $\frac{2}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

25. In 3013, King Warren of Australia is finally deposed. The five remaining earls argue about which one of them will be king, and which one of the others will be treasurer.

Akaroa will be satisfied only if Darlinghurst or Erina is treasurer.

Bairnsdale will be satisfied only if Claremont is treasurer.

Claremont will be satisfied only if Darlinghurst is either king or treasurer.

Darlinghurst will be satisfied only if Akaroa is either king or treasurer.

Erina will be satisfied only if Akaroa is not king.

It is not possible for all five to be satisfied, so in the end they appoint king and treasurer so that the other three earls are satisfied. Who becomes king?

(A) Akaroa (B) Bairnsdale (C) Claremont (D) Darlinghurst (E) Erina

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For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

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26. The 4-digit number $pqrs$ has the property that $pqrs \times 4 = srqp$. If $p = 2$, what is the value of the 3-digit number qrs?

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27. Three different non-zero digits are used to form six different 3-digit numbers. The sum of five of them is 3231. What is the sixth number?

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28. A hockey game between two teams is 'relatively close' if the number of goals scored by the two teams never differ by more than two. In how many ways can the first 12 goals of a game be scored if the game is 'relatively close'?

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29. How many pairs (a, b) of positive integers are there such that a and b are factors of 6^6 and a is a factor of b?

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30. All the digits of the positive integer N are either 0 or 1. The remainder after dividing N by 37 is 18. What is the smallest number of times that the digit 1 can appear in N?