Carlingford High School

Mathematics Extension 2

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION Term 3 2016

Student Name: ____________________________

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A Reference Sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 13

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

	MC	Q11	Q12	Q13	Q14	Q15	Q16	Mark
Complex	/1	/9		/4				/14
Graphs	/2	/3		/8				/13
Conics	/1				/6		/4	/11
Polynomials	/1	/3	/6		/3	/4		/17
Integration	/1		/9				/4	/14
Volumes	/1			/3	/4			/8
Mechanics	/2					/8	/4	/14
Harder 3U	/1				/2	/3	/3	/9
Total	/10	/15	/15	/15	/15	/15	/15	/100

Section I (10 marks)

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. The hyperbola $16x^2 - 9y^2 = 144$ has foci $S(5, 0)$ and $S'(-5, 0)$.

What are the equations of its directrices?

(A) $y = \frac{9}{5}$ and $y = -\frac{9}{5}$

(B) $x = \frac{9}{5}$ and $x = -\frac{9}{5}$

(C) $y = \frac{12}{5}$ and $y = -\frac{12}{5}$

(D) $x = \frac{12}{5}$ and $x = -\frac{12}{5}$

2. Evaluate $\int \frac{dx}{x^2-4x+13}$.

(A) $\frac{1}{9}tan^{-1}\frac{x-2}{9} + c$

(B) $\frac{1}{9}tan^{-1}\frac{x-2}{3} + c$

(C) $\frac{1}{3}tan^{-1}\frac{x-2}{9} + c$

(D) $\frac{1}{3}tan^{-1}\frac{x-2}{3} + c$

3. Which expression gives the gradient of the normal to the curve $x^3 + xy + y^2 = 7$ at any point on the curve?

(A) $\frac{-3x^2 - y}{x + 2y}$

(B) $\frac{x + 2y}{3x^2 + y}$

(C) $\frac{3x^2+y}{x + 2y}$

(D) $\frac{-x - 2y}{3x^2 + y}$

4. A five-digit number is formed from the numerals 5, 6, 7, 8 and 9.
What is the probability that the number will be less than 89 765?

(A) $\frac{5 \times 4!}{5!}$

(B) $\frac{5! - 4!}{5!}$

(C) $\frac{5! - 4! - 1}{5!}$

(D) $\frac{4! \times 3! \times 2!}{5!}$

5. Given that $z = 3\left(cos\frac{\pi}{6} + isin\,\frac{\pi}{6}\right)$, what is the value of $(\bar{z})^3$?

(A) $9\left(cos\frac{\pi}{2} - isin\,\frac{\pi}{2}\right)$

(B) $9\left(cos\frac{\pi}{2} + isin\,\frac{\pi}{2}\right)$

(C) $27\left(cos\frac{\pi}{2} - isin\,\frac{\pi}{2}\right)$

(D) $27\left(cos\frac{\pi}{2} + isin\,\frac{\pi}{2}\right)$

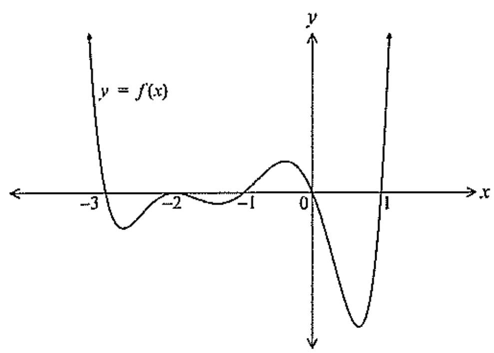
6. A particle of mass 4 kg moves in a circular motion on a smooth frictionless table at a speed of 3 m/s. It is attached to a fixed point in the middle of the table by a light, inelastic string of length 2 metres. What is the tension in the string?

(A) 6N
(B) 12N
(C) 18N
(D) 36N

7. The area enclosed by the curve $y = 3x^2 - x^3$, the x-axis and the lines $x = 0$ and $x = 3$ is rotated about the y-axis. What is the volume of the solid generated?

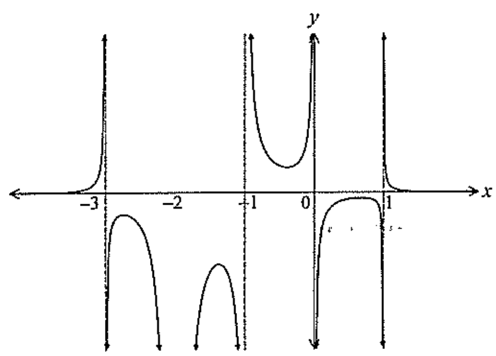
(A) $\frac{27\pi}{4}$
(B) 12π
(C) $\frac{116\pi}{5}$
(D) $\frac{243\pi}{10}$

8. The graph of $y = f(x)$ is shown below:

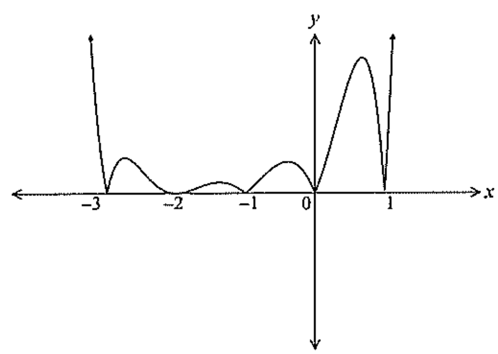


Which of the graphs below could represent the graph of $y = \frac{1}{f(x)}$?

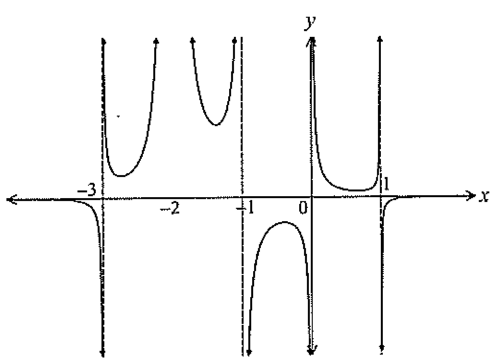
(A)



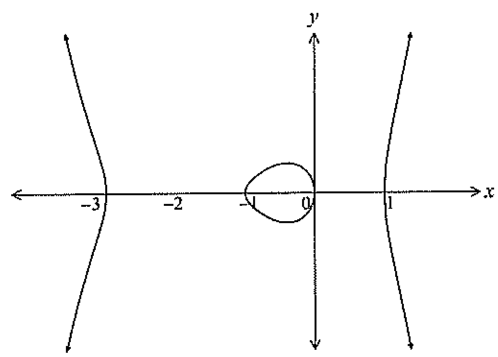
(B)



(C)



(D)



9. The polynomial $x^3 + 3x^2 + 2x - 1 = 0$ has roots α, β and γ.

Which polynomial has roots $\frac{2}{\alpha}, \frac{2}{\beta}$ and $\frac{2}{\gamma}$?

(A) $x^3 + 4x^2 - 12x + 8 = 0$

(B) $x^3 - 4x^2 - 12x - 8 = 0$

(C) $8x^3 - 12x^2 - 4x + 1 = 0$

(D) $8x^3 + 12x^2 + 4x - 1 = 0$

10. A particle of mass 0·8 kilograms is moving in uniform circular motion. The particle is rotating at 5 radians per second and there is a force of 40N acting on it towards the centre.

What is the radius of the circle?

(A) 1·28 metres

(B) 2 metres

(C) 2·5 metres

(D) 5 metres

End of Section I

Section II (90 marks)

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question on a **new writing booklet**. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a new writing booklet.

a) Consider the complex numbers $\omega = -1 + \sqrt{3}\,i$ and $Z = \sqrt{3} + 2i$.

i) Evaluate $\omega\bar{z}$. **1**

ii) Evaluate $|\omega|$. **1**

iii) Find the value of $arg\ (\omega)$. **1**

iv) Find the value of ω^5. **1**

v) Evaluate $\frac{\omega}{Z}$. **2**

b) Find the values of A, B and C such that: **3**

$$\frac{6x^2 + 17x + 15}{x(x+2)(x-3)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-3}$$

c) Sketch the region in the Argand diagram where $-\frac{\pi}{6} \leq arg\ z \leq \frac{\pi}{3}$ and $z\bar{z} \leq 4$. **3**

d) Use logarithms, implicit differentiation and the product rule to find the derivative of $y = x^x$. **3**

End of Question 11

Question 12 (15 marks) Use a new writing booklet.

a) **i)** Evaluate $\int_0^4 x\sqrt{x^2+9}\,dx$. **3**

ii) Find $\int \frac{\sqrt{x^2-25}}{x}\,dx$, using the trigonometric substitution $x = 5\sec\theta$. **3**

iii) Find $\int \frac{dx}{9x^2+6x+5}$. **3**

b) The cubic equation $x^3 - 5x^2 + 3x - 2 = 0$ has roots α, β and γ.

Find the value of:

i) $\alpha + \beta + \gamma$ **1**

ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ **1**

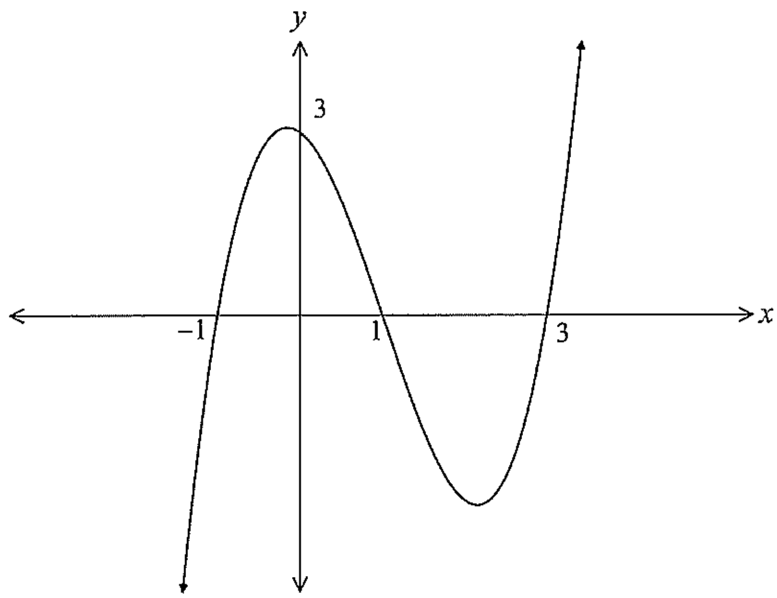
iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

iv) $\alpha^3 + \beta^3 + \gamma^3$ **2**

End of Question 12

Question 13 (15 marks) Use a new writing booklet.

a) A sketch of the function $f(x)$ is shown below.



Draw a separate half-page graph for each of the following functions, showing all asymptotes and intercepts.

i) $y = |f(x)|$ **2**

ii) $y^2 = f(x)$ **2**

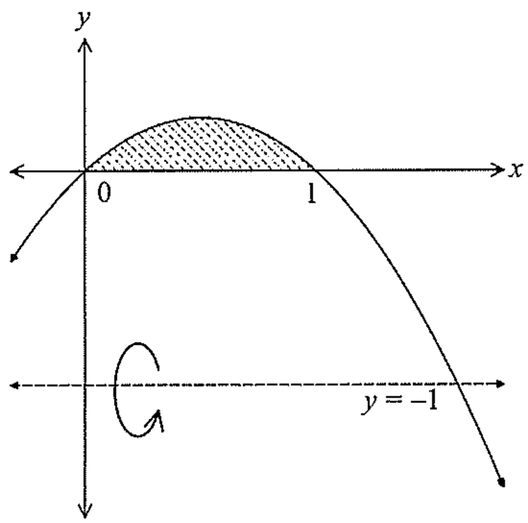
iii) $y = (f(x))^2$ **2**

iv) $y = e^{f(x)}$ **2**

Question 13 continues on page 8.

Question 13 continued

b) The area enclosed by the curve $y = x(1-x)$ and the x-axis is rotated about the line $y = -1$.



Find the volume of the solid of revolution formed. **3**

c) Solve the equation $x^4 - 5x^3 + 5x^2 + 25x - 26 = 0$, given that one of the roots is $3 + 2i$ **4**

End of Question 13

Question 14 (15 marks) Use a new writing booklet.

a) **i)** Show that for all values of θ the point $P(3\cos\theta, 4\sin\theta)$ lies on the ellipse **1**

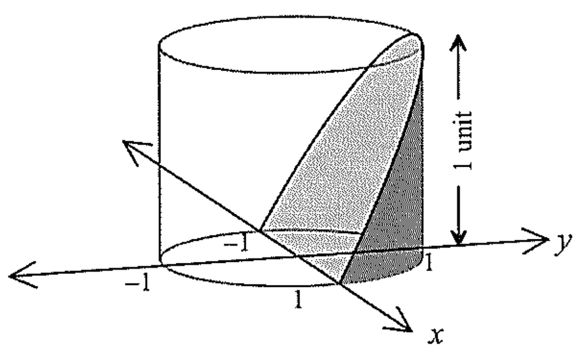
$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

ii) Find the equation of the tangent to the ellipse at the point P. **2**

iii) Show that the point $Q(-3\sin\theta, 4\cos\theta)$ also lies on the ellipse. **1**

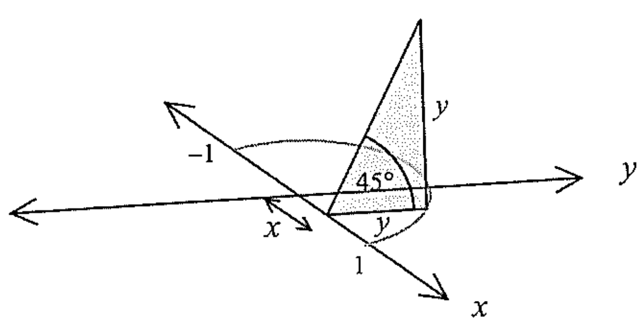
iv) Find the equation of the normal to the ellipse at the point Q. **2**

b) A cylinder has the circle $x^2 + y^2 = 1$ as its base and is 1 unit in height.
The shaded wedge is formed by a plane which passes along the x-axis and is angled at $45°$ to the base of the cylinder.

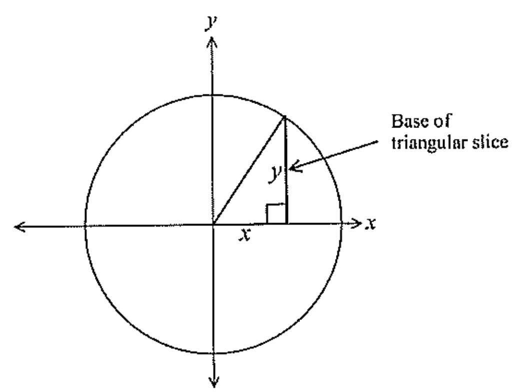


Slices are taken through this wedge at right angles to the x-axis, and perpendicular to the base of the cylinder, through a point (x, y) on the circle.

Triangular slice through the wedge



Base of the cylinder



Find the volume of the wedge. **4**

Question 14 continues on page 10.

Question 14 continued

c) Solve the polynomial equation $x^4 - 6x^3 + 9x^2 + 4x - 12 = 0$, given that the equation has a double root. **3**

d) i) If x, y, z are real and unequal, show that $x^2 + y^2 > 2xy$ and hence deduce that $x^2 + y^2 + z^2 > xy + yz + xz$. **1**

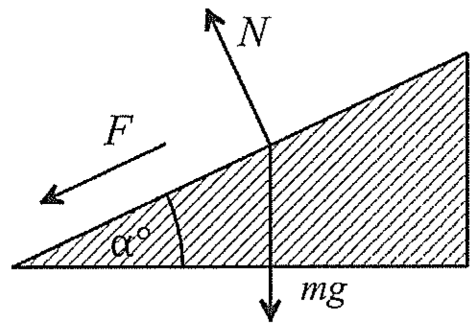
ii) If $x + y + z = 3$, show that $xy + yz + xz < 3$. **1**

End of Question 14

Question 15 (15 marks) Use a new writing booklet.

a) A car of mass m travels with a constant speed v around a circular track of radius r.

The track is banked at a constant angle of α to the horizontal, as shown in the diagram below.



i) Write equations for the forces which are acting on the car in the horizontal and the vertical direction. **2**

ii) Find expressions for $F\sin\alpha$ and $F\cos\alpha$. **2**

iii) Prove that $F = \dfrac{m(v^2 - g\,r\tan\alpha)}{r}\cos\alpha$. **2**

iv) If $r = 150$ metres and the road is banked so that a car travelling at 90 km/h has no sideways frictional force acting upon it, find the value of α, correct to the nearest minute. (use $g = 10$ m/s^2) **2**

b)

i) If $\dfrac{x}{x^2 - x - 6} \equiv \dfrac{A}{x-3} + \dfrac{B}{x+2}$, find the values of A and B. **2**

ii) Hence find $\displaystyle\int \frac{\sin\theta\cos\theta}{\sin^2\theta - \sin\theta - 6}\,d\theta$ **2**

c) Ten teams compete in a car rally lasting three days. Each team consists of 2 cars and if a car does not complete a day then the team is eliminated.

The probability that a car completes a day is 0.8.

i) Find the probability that a team completes all three days of the rally.

Leave your answer in index notation. **1**

ii) Write down a calculation which would give the probability that at least three teams successfully complete all three days of the rally. **2**

You are not required to calculate the value of this probability.

End of Question 15

Question 16 (15 marks) Use a new writing booklet.

a) **i)** Derive the reduction formula **2**

$$\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$$

ii) Hence evaluate $\int_1^e (\ln x)^3 dx$ **2**

b) Show that the normals at the points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ to the rectangular hyperbola **4**

$xy = c^2$ meet at the point $\left(\dfrac{cpq(p^2+q^2+pq)+c}{pq(p+q)}, \dfrac{cp^3q^3+c(p^2+q^2+pq)}{pq(p+q)}\right)$.

c) A particle of mass 40kg experiences a force numerically equivalent to $\frac{1}{10}$ of the square of its velocity in metres per second when moving through the air.

The particle is projected vertically upwards with a velocity of u metres per second.

Assuming the value of g is 10m/s^2,

i) Find the time taken for the particle to reach its maximum height. **2**

ii) Find the maximum height reached by the particle. **2**

d) Use Mathematical induction to prove that for any real θ,

$$\cos 4\theta + i\sin 4\theta = (\cos\theta + i\sin\theta)^4$$ **3**

End of Examination

Trial HSC Examination 2016
Mathematics Extension 2 Course

Name ____________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1.	**A** ○	**B** ○	**C** ○	**D** ○
2.	**A** ○	**B** ○	**C** ○	**D** ○
3.	**A** ○	**B** ○	**C** ○	**D** ○
4.	**A** ○	**B** ○	**C** ○	**D** ○
5.	**A** ○	**B** ○	**C** ○	**D** ○
6.	**A** ○	**B** ○	**C** ○	**D** ○
7.	**A** ○	**B** ○	**C** ○	**D** ○
8.	**A** ○	**B** ○	**C** ○	**D** ○
9.	**A** ○	**B** ○	**C** ○	**D** ○
10.	**A** ○	**B** ○	**C** ○	**D** ○

Carlingford High School

Mathematics Extension 2

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION Term 3 2016

SOLUTIONS

Trial HSC Examination 2016
Mathematics Ext 2 Course

Name ______________________ Teacher __________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

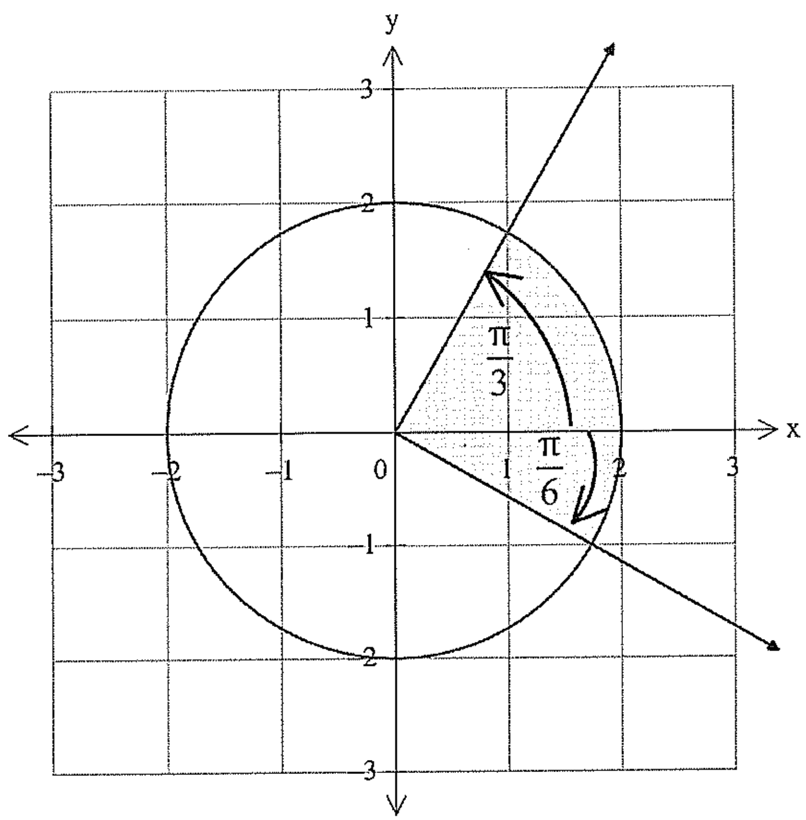
1. A ○ B ● C ○ D ○
2. A ○ B ○ C ○ D ●
3. A ○ B ● C ○ D ○
4. A ○ B ○ C ● D ○
5. A ○ B ○ C ● D ○
6. A ○ B ○ C ● D ○
7. A ○ B ○ C ○ D ●
8. A ● B ○ C ○ D ○
9. A ○ B ● C ○ D ○
10. A ○ B ● C ○ D ○

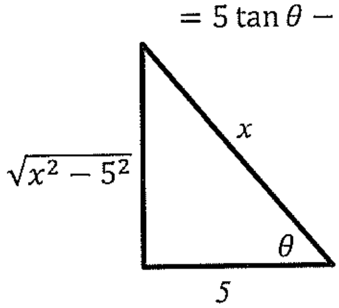
Multiple Choice Worked Solutions

No	Working	Answer
1	$16x^2 - 9y^2 = 144 \rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1 \qquad \therefore a = 3, b = 4$ Foci are $(ae, 0)$ and $(-ae, 0) = (5, 0)$ and $(-5, 0)$ $ae = 5$ $3e = 5$ $\therefore e = \frac{5}{3}$ Equation of directrices is: $x = \pm\frac{a}{e} = \pm\frac{3}{\frac{5}{3}} = \pm\frac{9}{5}$	B
2	$\int \frac{dx}{x^2 - 4x + 13} = \int \frac{dx}{x^2 - 4x + 4 + 9}$ $= \int \frac{dx}{(x-2)^3 + 9}$ $= \frac{1}{3}\tan^{-1}\frac{x-2}{3} + c$	D
3	$x^3 + xy + y^2 = 7$ By implicit differentiation $3x^2 + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ $\frac{dy}{dx}(x + 2y) = -3x^2 - y$ Gradient of Tangent is $\frac{dy}{dx} = \frac{-3x^2 - y}{x + 2y}$ Gradient of normal is $\frac{x+2y}{3x^2+y}$	B
4	Number of possible digits without repetition = 5! Numbers greater than 89 765 must begin with 9. Therefore 4! possible numbers greater than 89 765. Rule out the number 89 765 as it has to be less than this. Therefore number less than 89 765 = 5! – 4! – 1 Probability of number less than 89 765 is $\frac{5! - 4! - 1}{5!}$.	C
5	If $z = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ then $\bar{z} = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$ $(\bar{z})^3 = 3^3\left(\cos\frac{3\pi}{6} - i\sin\frac{3\pi}{6}\right)$ $(\bar{z})^3 = 27\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$	C
6	$F = \frac{mv^2}{r}$ $= \frac{4(3)^2}{2}$ $= 18N$	C

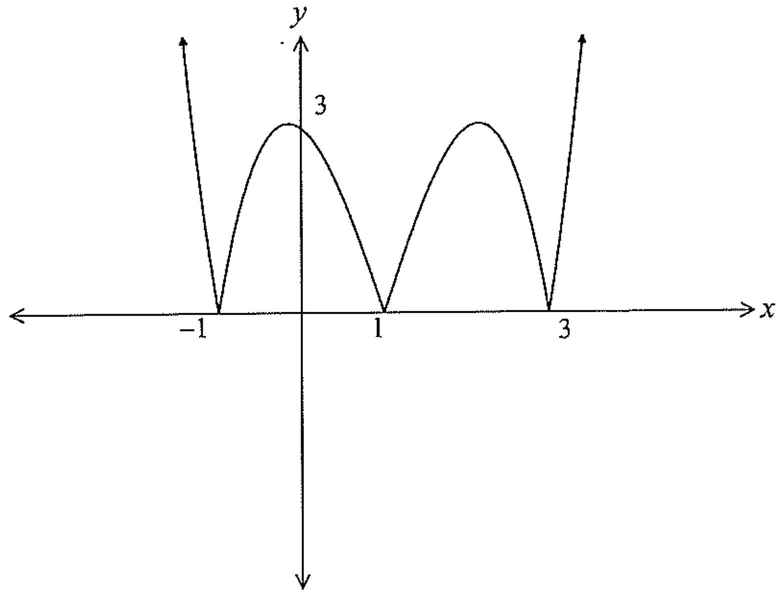
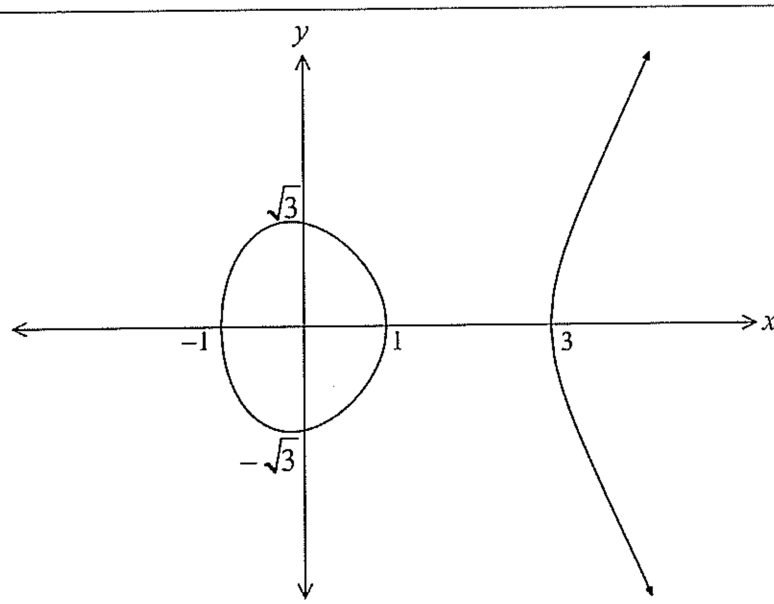
7	Use the method of cylindrical shells $V = \int_a^b 2\pi xy\, dx$ $V = \int_0^3 2\pi x(3x^2 - x^3)\, dx$ $V = 2\pi \int_0^3 (3x^3 - x^4) dx$ $V = 2\pi \left[\frac{3}{4}x^4 - \frac{1}{5}x^5\right]_0^3$ $V = 2\pi \left\{\left[\frac{3}{4}(3)^4 - \frac{1}{5}(3)^5\right] - \left[\frac{3}{4}(0)^4 - \frac{1}{5}(0)^5\right]\right\}$ $V = \frac{243\pi}{10}$	D
8	Graph A as zero's become discontinuities and the sign of the function values remains unchanged, and $\frac{1}{\text{small } y \text{ value}} = \text{large } y \text{ value and}$ $\frac{1}{\text{large } y \text{ value}} = \text{small } y \text{ value}$	A
9	$x^3 + 3x^2 + 2x - 1 = 0$ Roots $\frac{2}{\alpha}, \frac{2}{\beta}$ *and* $\frac{2}{\gamma}$ Let $y = \frac{2}{x}, \therefore x = \frac{2}{y}$ *i.e. sub* $\frac{2}{x}$ $\left(\frac{2}{x}\right)^3 + 3\left(\frac{2}{x}\right)^2 + 2\left(\frac{2}{x}\right) - 1 = 0$ $\frac{8}{x^3} + \frac{12}{x^2} + \frac{4}{x} - 1 = 0$ Multiply through by x^3 $8 + 12x + 4x^2 - x^3 = 0$ i.e. $x^3 - 4x^2 - 12x - 8 = 0$	B
10	$F = mr\omega^2$ $40 = (0 \cdot 8)r(5)^2$ $40 = 20r$ $r = 2\, metres$	B

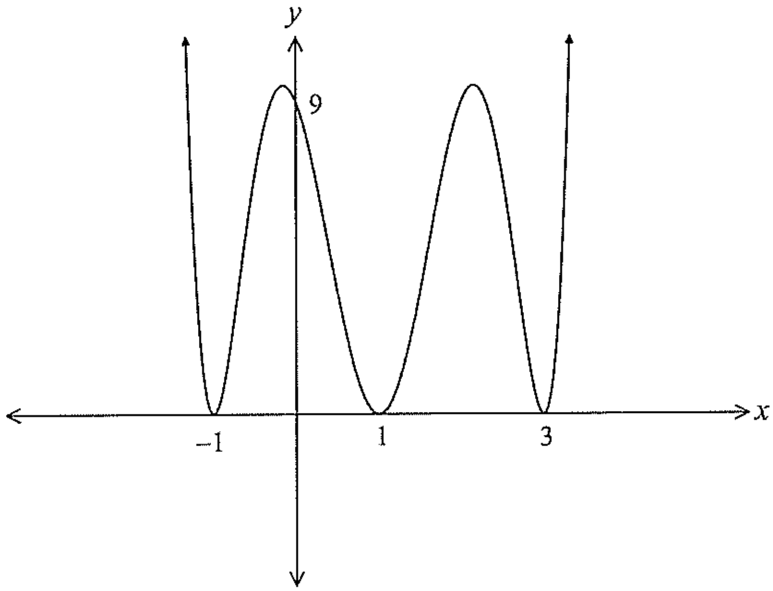
Question 11		2016	
	Solution	Marks	Allocation of marks
a) **i)**	$\omega = -1 + \sqrt{3}\, i \; and \; Z = \sqrt{3} + 2i$ $\omega\bar{z} = (-1 + \sqrt{3}i)(\sqrt{3} - 2i)$ $= -\sqrt{3} + 2i + 3i + 2\sqrt{3}$ $= \sqrt{3} + 5i$	1	1 mark for correct answer.
ii)	$\lvert\omega\rvert = \sqrt{(-1)^2 + (\sqrt{3})^2}$ $= \sqrt{4}$ $= 2$	1	1 mark for correct answer.
iii)	Arg ω $\quad tan\,\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$ $tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ $\omega = (-1 + \sqrt{3}i)$ is in the second quadrant, so Arg $\omega = \frac{2\pi}{3}$	1	1 mark for correct answer.
iv)	$\omega^5 = \left[2\left(cos\frac{2\pi}{3} + isin\frac{2\pi}{3}\right)\right]^5$ $= 32\left(cos\frac{10\pi}{3} + isin\frac{10\pi}{3}\right)$ $= 32\left(cos\frac{-2\pi}{3} + isin\frac{-2\pi}{3}\right)$	1	1 mark for correct answer.
v)	$\frac{\omega}{Z} = \frac{-1+\sqrt{3}\,i}{\sqrt{3}+2i} \times \frac{\sqrt{3}-2i}{\sqrt{3}-2i} = \frac{-\sqrt{3} + 2i + 3i - 2\sqrt{3}i^2}{3 - 4i^2}$ $= \frac{\sqrt{3}+5i}{7}$	2	2 marks for correct answer. 1 mark for significant progress toward correct answer
b)	$\frac{6x^2 + 17x + 15}{x(x+2)(x-3)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-3}$ $\therefore \quad 6x^2 + 17x + 15 \equiv A(x+2)(x-3) + Bx(x-3) + Cx(x+2)$ Try $x = 0, 15 = -6A \qquad \therefore A = -2\frac{1}{2}$ $x = -2, 5 = 10B \qquad \therefore B = \frac{1}{2}$ $x = 3, 120 = 15C \qquad \therefore C = 8$ $\frac{6x^2 + 17x + 15}{x(x+2)(x-3)} = -\frac{5}{2x} + \frac{1}{2x+4} + \frac{8}{x-3}$	3	3 marks for correct answer. 2 marks for significant progress toward correct answer with at least 2 of the values of A, B or C 1 mark for some progress toward correct answer

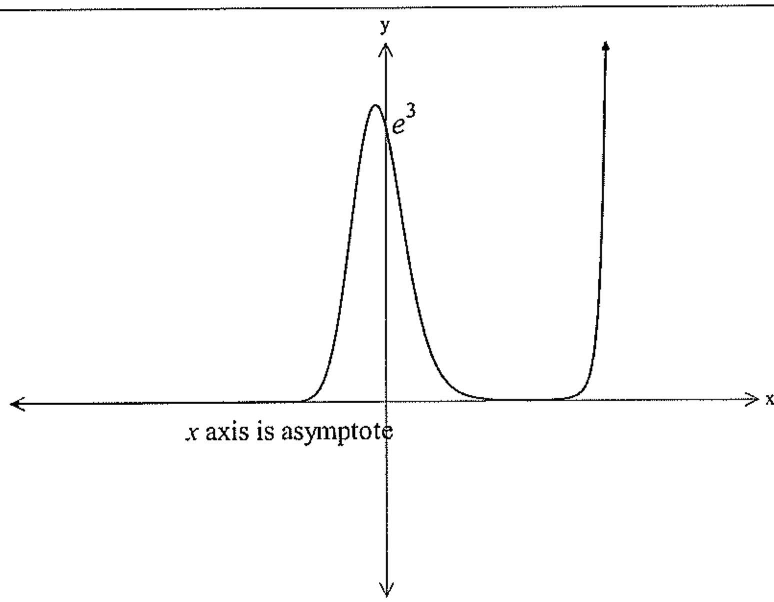
Question 11		2016	
	Solution	Marks	Allocation of marks
c)	$Z.\overline{Z} = 4$ $(x+iy)(x-iy) = 4$ $x^2+y^2 = 4$ $Z.\overline{Z} \leq 4$ is the interior of this circle. 	3	3 marks for correct region 2 marks for region which is mainly correct, with a minor error. 1 mark for a region which includes at least one of the lines or curves shown, as a boundary.
d)	$y = x^x$ Take logarithms of both sides $\ln y = \ln x^x$ $\ln y = x\ln x$ Implicitly Differentiate $\frac{1}{y}\frac{dy}{dx} = \ln x + \frac{x}{x}$ $\frac{1}{y}\frac{dy}{dx} = \ln x + 1$ $\frac{dy}{dx} = y(\ln x + 1)$ $\frac{dy}{dx} = x^x(\ln x + 1)$	3	3 marks for correct answer. 2 marks for significant progress toward correct answer with correct use of logs and/or correct differentiation 1 mark for some progress toward correct answer

Question 12		2016	
	Solution	Marks	Allocation of marks
a) **i)**	$\int_0^4 x\sqrt{x^2+9}\,dx$ Let $u = x^2+9$ $\therefore du = 2x\,dx$ When $x=0$, $u=9$ When $x=4$, $u=25$ $\int_0^4 x\sqrt{x^2+9}\,dx = \frac{1}{2}\int_0^4 2x\sqrt{x^2+9}\,dx$ $= \frac{1}{2}\int_9^{25} u^{\frac{1}{2}}\,du$ $= \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_9^{25}$ $= \frac{1}{3}\left[25^{\frac{3}{2}} - 9^{\frac{3}{2}}\right]$ $= \frac{1}{3}[125-27]$ $= \frac{98}{3}$	3	3 marks for a correct solution giving all the roots. 2 marks for a solution which has a minor error in one of: *substitution including limits*, or *integration* or *evaluating numerical value* 1 mark for a solution that shows some progress in at least one of the above.
ii)	$\int \frac{\sqrt{x^2-25}}{x}\,dx$ *Let* $x = 5\sec\theta$. $\therefore dx = 5\sec\theta\tan\theta\,d\theta$ $\int \frac{\sqrt{x^2-25}}{x}\,dx = \int \frac{\sqrt{(5\sec\theta)^2-25}}{5\sec\theta}\,5\sec\theta\tan\theta\,d\theta$ $= \int \frac{\sqrt{25\sec^2\theta-25}}{5\sec\theta}\,5\sec\theta\tan\theta\,d\theta$ $= \int \frac{\sqrt{25\,(\sec^2\theta-1)}}{5\sec\theta}\,5\sec\theta\tan\theta\,d\theta$ $= \int \frac{\sqrt{25(\sec^2\theta-1)}}{5\sec\theta}\,5\sec\theta\tan\theta\,d\theta$ $= \int \frac{\sqrt{25\tan^2\theta}}{5\sec\theta}\,5\sec\theta\tan\theta\,d\theta$ $= \int \frac{5\tan\theta}{5\sec\theta}\,5\sec\theta\tan\theta\,d\theta$ $= \int 5\tan^2\theta\,d\theta$ $= 5\int(\sec^2\theta-1)\,d\theta$ $= 5\tan\theta - 5\theta + c$  $= 5\left(\frac{\sqrt{x^2-25}}{5}\right) - 5\sec^{-1}\left(\frac{x}{5}\right) + c$ $= \sqrt{x^2-25} - 5\sec^{-1}\left(\frac{x}{5}\right) + c$	3	3 marks for a correct solution giving all the roots. 2 marks for a solution which has a minor error in one of: *substitution*, or *integration* or *writing in terms of x* 1 mark for a solution that shows some progress in at least one of the above.

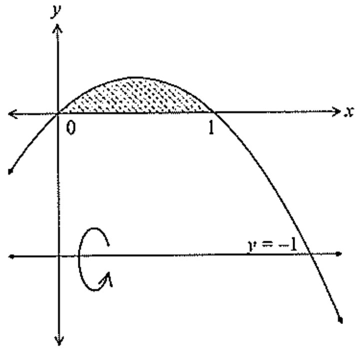
Question 12		2016	
	Solution	Marks	Allocation of marks
iii)	$\displaystyle\int \frac{dx}{9x^2 + 6x + 5}$ Use complete the square $9x^2 + 6x + 5 = 9\left(x^2 + \frac{2}{3}x\right) + 5$ $= 9\left(x^2 + \frac{2}{3}x + \frac{1}{9}\right) - 1 + 5$ $= 9\left(x + \frac{1}{3}\right)^2 + 4$ $= (3x+1)^2 + 2^2$ Now, $\displaystyle\int \frac{dx}{9x^2+6x+5} = \int \frac{dx}{(3x+1)^2 + 2^2}$ Let $u = 3x + 1$ $du = 3dx$ $dx = \frac{1}{3}\,du$ $= \frac{1}{3}\displaystyle\int \frac{du}{u^2+4}$ $= \frac{1}{6}\tan^{-1}\left(\frac{u}{2}\right) + c$ $= \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + c$	3	3 marks for a correct solution giving all the roots. 2 marks for a solution which has a minor error in one of: *completing the square* or *substitution*, or *integration* 1 mark for a solution that shows some progress in at least one of the above.
b) i)	$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{-5}{1} = 5$	1	1 mark for correct answer
ii)	$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{3}{1} = 3$	1	1 mark for correct answer
iii)	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$ $= 5^2 - 2(3)$ $= 25 - 6$ $= 19$	2	2 marks for correct answer. 1 mark for significant progress toward correct answer
iv)	$x^3 - 5x^2 + 3x - 2 = 0$ $\therefore x^3 = 5x^2 - 3x + 2$ $\therefore \alpha^3 = 5\alpha^2 - 3\alpha + 2$ $\therefore \beta^3 = 5\beta^2 - 3\beta + 2$ $\therefore \gamma^3 = 5\gamma^2 - 3\gamma + 2$ $\therefore \alpha^3 + \beta^3 + \gamma^3 = 5(\alpha^2 + \beta^2 + \gamma^2) - 3(\alpha + \beta + \gamma) + 6$ $= 5(19) - 3(5) + 6$ $= 86$	2	2 marks for correct answer. 1 mark for significant progress toward correct answer

Question 13		2016	
	Solution	Marks	Allocation of marks
a) **i)**		2	2 marks for a correct graph with intercepts shown. 1 mark for graph with wrong intercepts or wrong orientation, or other minor error.
ii)		2	2 marks for a correct graph with intercepts shown. 1 mark for graph with wrong intercepts or wrong orientation, or other minor error.

<table>
<tr><th colspan="2">Question 13</th><th colspan="2">2016</th></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>iii)</td><td>
</td><td>2</td><td>2 marks for a correct graph with intercepts shown.

1 mark for graph with wrong intercepts or wrong orientation, or other minor error.</td></tr>
<tr><td>iv)</td><td>
</td><td>2</td><td>2 marks for a correct graph with intercepts shown.

1 mark for graph with wrong intercepts or wrong orientation, or other minor error.</td></tr>
</table>

Question 13		2016	
	Solution	Marks	Allocation of marks
b)	$y = x(1-x) = x - x^2$  Use Washer Method: $V = \pi \int_a^b (R^2 - r^2)\ dx$ Here $R = y + 1$ $r = 1$ $V = \pi \int_0^1 [(y+1)^2 - 1^2] dx$ $= \pi \int_0^1 [(x - x^2 + 1)^2 - 1]\ dx$ $= \pi \int_0^1 (1 + 2x - x^2 - 2x^3 + x^4 - 1) dx$ $= \pi \left[x^2 - \frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1$ $= \pi \left[(1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{5}) - (0)\right]$ $= \frac{11\pi}{30}$ cubic units	3	3 marks for correct volume. 2 marks for method of washers with a minor error, or an attempt to use another method which has proceeded to close to a correct answer. 1 mark for working which shows some knowledge of using methods of finding volumes, and includes some correct working toward an answer.
c)	$x^4 - 5x^3 + 5x^2 + 25x - 26 = 0,$ If $3 + 2i$ is one root then $3 - 2i$ is another root. $\alpha + \beta = (3 + 2i) + (3 - 2i) = 6$ $\alpha\beta = (3 + 2i)(3 - 2i) = 9 - 4i^2 = 13$ Therefore divisible by $x^2 - 6x + 13$ By division, $x^4 - 5x^3 + 5x^2 + 25x - 26 = (x^2 - 6x + 13)(x^2 + x - 2) = 0$ $= (x^2 - 6x + 13)(x + 2)(x - 1) = 0$ $\therefore$ Roots are $3 + 2i$, $3 - 2i$, -2 and 1	4	4 marks for a correct solution giving all the roots. 3 marks for a solution which has a minor error in one of: giving the *conjugate root,* or *division* or *factorisation* or *giving final roots.* 2 marks for a solution which has more than one error in one or more of the above 1 mark for a solution that shows some progress in at least one of the above.

Question 14		2016	
	Solution	Marks	Allocation of marks
a) **i)**	$\frac{x^2}{9}+\frac{y^2}{16}=1$ At $P(3\cos\theta, 4\sin\theta)$ we have $\frac{(3\cos\theta)^2}{9}+\frac{(4\sin\theta)^2}{16}=1$ $\frac{9cos^2\theta}{9}+\frac{16sin^2\theta}{16}=1$ $cos^2\theta+ sin^2\theta=1$ $1=1$ $\therefore$ P lies on the curve.	1	1 mark for correct demonstration.
ii)	When $x=3\cos\theta$ and $y=4\sin\theta$ then $\frac{dx}{d\theta}=-3\sin\theta \quad \frac{dy}{d\theta}=4\cos\theta$ So $\frac{dy}{dx}=\frac{4\cos\theta}{-3\sin\theta}$ $y-y_1=m(x-x_1)$ $y-4\sin\theta=\frac{4\cos\theta}{-3\sin\theta}(x-3\cos\theta)$ $3y\sin\theta-12sin^2\theta=-4x\cos\theta+12cos^2\theta$ $4x\cos\theta+3y\sin\theta=12cos^2\theta+12sin^2\theta$ $4x\cos\theta+3y\sin\theta=12(cos^2\theta+sin^2\theta)$ $\frac{x\cos\theta}{3}+\frac{y\sin\theta}{4}=1$	2	2 marks for correct equation. 1 mark for significant correct progress toward equation
iii)	Given $\frac{x^2}{9}+\frac{y^2}{16}=1$ At $Q(-3\sin\theta, 4\cos\theta)$ we have $\frac{(-3\sin\theta)^2}{9}+\frac{(4\cos\theta)^2}{16}=1$ $\frac{9sin^2\theta}{9}+\frac{16cos^2\theta}{16}=1$ $sin^2\theta+cos^2\theta=1$ $1=1$ $\therefore$ Q lies on the curve.	1	1 mark for correct demonstration.

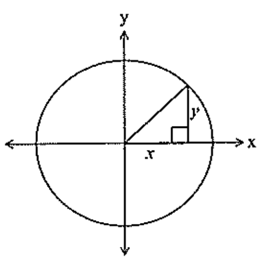
<table>
<tr><td colspan="2">Question 14</td><td colspan="2">2016</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>iv)</td><td>Tangent at Q
Given $x = -3\sin\theta$ and $y = 4\cos\theta$
$\frac{dx}{d\theta} = -3\cos\theta \qquad \frac{dy}{d\theta} = -4\sin\theta$

Thus $\frac{dy}{dx} = \frac{-4\sin\theta}{-3\cos\theta} = \frac{4\sin\theta}{3\cos\theta}$

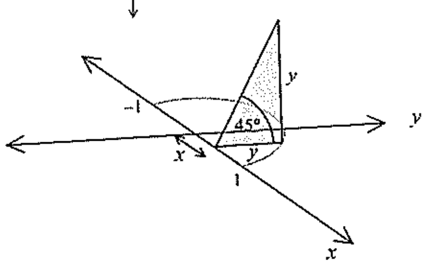
$\therefore$ Normal at Q has gradient $m_N = -\frac{3\cos\theta}{4\sin\theta}$
$y - y_1 = m(x - x_1)$

$y - 4\cos\theta = \frac{-3\cos\theta}{4\sin\theta}(x - -3\sin\theta)$
$4y\sin\theta - 16\sin\theta\cos\theta = -3x\cos\theta - 9\sin\theta\cos\theta$
$3x\cos\theta + 4y\sin\theta = 7\sin\theta\cos\theta$

Divide by $\sin\theta\cos\theta$, the equation of Normal becomes
$\frac{3x}{\sin\theta} - \frac{4y}{\cos\theta} = 7$ or
$3x\,\mathrm{cosec}\,\theta - 4y\sec\theta = 7$</td><td>2</td><td>2 marks for correct equation in either format.

1 mark for significant correct progress toward equation</td></tr>
<tr><td>b)</td><td>

From diagram, $x^2 + y^2 = 1$
$\therefore y = \sqrt{1 - x^2}$



$A(x) = \frac{1}{2}bh = \frac{1}{2}y^2$
$= \frac{1}{2}(1 - x^2)$

$V = \int_a^b A(x)dx$
$= \int_{-1}^{1}\frac{1}{2}(1 - x^2)\,dx$
$= 2\int_0^1\frac{1}{2}(1 - x^2)dx$
$= \int_0^1(1 - x^2)dx$
$= \left[x - \frac{x^3}{3}\right]_0^1$
$= \left(1 - \frac{1}{3}\right) - 0$
$= \frac{2}{3}$ cubic units</td><td>4</td><td>4 marks for a correct solution giving the correct volume.

3 marks for a solution which has a minor error in one of: finding the expression for y, or expression for area or integral or finding the volume.

2 marks for a solution which has more than one error in one or more of the above

1 mark for a solution that shows some progress in at least one of the above.</td></tr>
</table>

Question 14		2016	
	Solution	Marks	Allocation of marks
c)	$x^4 - 6x^3 + 9x^2 + 4x - 12 = 0$ Let $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$ $f'(x) = 4x^3 - 18x^2 + 18x + 4$ Now $f'(2) = f(2) = 0 \;\; \therefore (x-2) \textit{is a double root.}$ Dividing $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$ by $x^2 - 4x + 4$ Gives: $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12 = (x^2 - 4x + 4)(x^2 - 2x - 3)$ $= (x-2)(x-2)(x-3)(x+1)$ Solution is: $x = -1, 2, 2,$ and 3.	3	3 marks for a correct solution giving all the roots. 2 marks for a solution which has a minor error in one of: giving the *double root,* or *division* or *giving final roots.* 1 mark for a solution that shows some progress in at least one of the above.
d) i)	$(x-y)^2 > 0$ $\therefore \; x^2 - 2xy + y^2 > 0$ i.e. $\quad x^2 + y^2 > 2xy$ ----- (1) Similarily, $x^2 + z^2 > 2xz$ ----- (2) And $y^2 + z^2 > 2yz$ ----- (3) From (1) + (2) + (3) gives: $x^2 + y^2 + x^2 + z^2 + y^2 + z^2 > 2xy + 2xz + 2yz$ $2(x^2 + y^2 + z^2) > 2(xy + xz + yz)$ i. e. $\quad x^2 + y^2 + z^2 > xy + xz + yz$	1	1 mark for any correct demonstration.
ii)	$x^2 + y^2 + z^2 = (x+y+z)^2 - 2(xy + xz + yz)$ But $x^2 + y^2 + z^2 > xy + xz + yz$ (Proven above) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + xz + yz)$ Therefore $(x+y+z)^2 > xy + xz + yz + 2(xy + xz + yz)$ i.e. $\quad (x+y+z)^2 > 3(xy + xz + yz)$ Since $\quad x + y + z = 3$ $(3)^2 > 3(xy + xz + yz)$ $3 > (xy + xz + yz)$ i. e. $\; xy + yz + xz < 3.$	1	1 mark for any correct demonstration.

<table>
<tr><td colspan="2">Question 15</td><td colspan="2">2016</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>a)
i)</td><td>Horizontally: $\frac{mv^2}{r} = N\sin\alpha + F\cos\alpha$

Vertically: $N\cos\alpha = mg + F\sin\alpha$</td><td>2</td><td>2 marks for two correct equations.

1 mark for only one correct equation</td></tr>
<tr><td>ii)</td><td>$F\cos\alpha = \frac{mv^2}{r} - N\sin\alpha$

$F\sin\alpha = N\cos\alpha - mg$</td><td>2</td><td>2 marks for two correct equations.

1 mark for only one correct equation</td></tr>
<tr><td>iii)</td><td>$F\cos\alpha = \frac{mv^2}{r} - N\sin\alpha \ \ldots\ldots\ldots..(1)$
$F\sin\alpha = N\cos\alpha - mg \ \ldots\ldots\ldots..(2)$

From (1) $\times \cos\alpha$: $F\cos^2\alpha = \frac{mv^2}{r}\cos\alpha - N\sin\alpha\cos\alpha$
From (2) $\times \sin\alpha$: $F\sin^2\alpha = N\sin\alpha\cos\alpha - mg\sin\alpha$

Now by adding, we get
$F = \frac{mv^2}{r}\cos\alpha - mg\sin\alpha$ (since $\sin^2\alpha + \cos^2\alpha = 1$)

$= \frac{mv^2\cos\alpha - mgr\sin\alpha}{r}$

$= \frac{m(v^2\cos\alpha - gr\sin\alpha)}{r}$

$= \frac{m(v^2 - gr\tan\alpha)}{r} . \cos\alpha$</td><td>2</td><td>2 marks for correct expression for F.

1 mark for significant correct progress toward expression</td></tr>
<tr><td>iv)</td><td>If $F = 0, \frac{m(v^2 - gr\tan\alpha)}{r} . \cos\alpha = 0$

i.e. $v^2 - gr\tan\alpha = 0$

$\tan\alpha = \frac{v^2}{gr}$

Since $r = 150, v = 90\frac{km}{h} = \frac{\frac{90\times1000}{60^2}m}{s}$ & $g = 10$ then

$\tan\alpha = \frac{v^2}{gr} = \left(\frac{90\times1000}{60^2}\right)^2 \div (10\times150)$

$= 0{\cdot}416\ldots\ldots$

$\therefore \quad \alpha = 22^\circ 37'$</td><td>2</td><td>2 marks for correct value of α.

1 mark for significant correct progress toward answer</td></tr>
</table>

Question 15		2016	
	Solution	Marks	Allocation of marks
b) i)	Now $\frac{x\,dx}{x^2-x-6} \equiv \frac{A}{x-3} + \frac{B}{x+2}$ $\therefore x = A(x+2) + B(x-3)$ Let $x=-2$, then $-2 = A(0) + -5B$ $\boldsymbol{B = \frac{2}{5}}$ Let $x = 3$, then $3 = 5A + B(0)$ $\boldsymbol{A = \frac{3}{5}}$ $\therefore \frac{x}{x^2 - x - 6} \equiv \frac{\frac{3}{5}}{x-3} + \frac{\frac{2}{5}}{x+2}$ $\equiv \frac{3}{5(x-3)} + \frac{2}{5(x+2)}$	2	2 marks for correct values of both A and B. 1 mark for either A or B correct or significant correct progress toward this.
ii)	Given $\int \frac{\sin\theta\cos\theta\,d\theta}{\sin^2\theta - \sin\theta - 6}$ Now let $u = \sin\theta$ then $du = \cos\theta\,d\theta$ So $\int \frac{\sin\theta\cos\theta\,d\theta}{\sin^2\theta - \sin\theta - 6} = \int \frac{u\,du}{u^2-u-6}$ $= \int \left(\frac{3}{5(u-3)} + \frac{2}{5(u+2)}\right) du$ From part **i)** above $= \frac{3}{5}\int \frac{1}{u-3}\,du + \frac{2}{5}\int \frac{1}{u+2}\,du$ $= \frac{3}{5}\ln(u-3) + \frac{2}{5}\ln(u+2) + c$ $= \frac{3}{5}\ln(\sin\theta - 3) + \frac{2}{5}\ln(\sin\theta + 2) + c$	2	2 marks for correct expression for integral. 1 mark for significant correct progress toward equation
c) i)	P(Car Successful each day) = 0·8 P(Team successful each day) = $(0\cdot 8)^2 = 0\cdot 64$ P(Team completes rally) = $(0\cdot 64)^3$	1	1 mark for correct answer
ii)	This is a binomial probability where probabilities can be found using the expansion of $[(0\cdot 64)^3 + (1 - 0\cdot 64^3)]^{10}$ P(≥ 3 teams) = $1 - {}^{10}\mathbf{C}_2\,(0\cdot 64^3)^2(1 - 0\cdot 64^3)^8$ $- {}^{10}\mathbf{C}_1\,(0\cdot 64^3)^1(1 - 0\cdot 64^3)^9$ $- {}^{10}\mathbf{C}_0\,(1 - 0\cdot 64^3)^{10}$	2	2 marks for correct expression for probability. 1 mark for significant correct progress toward probability

Question 16		2016	
	Solution	Marks	Allocation of marks
a) i)	Given $\int (ln\,x)^n\,dx$ Now let $u = (ln\,x)^n$ then $u' = n\left(\frac{1}{x}\right)(ln\,x)^{n-1}$ and $v' = 1$ then $v = x$ Since $uv - \int vu'dx = x\,(ln\,x)^n - \int x\left(\frac{n}{x}\right)(ln\,x)^{n-1}\,dx$ $= x\,(ln\,x)^n - \int n\,(ln\,x)^{n-1}\,dx$	2	2 marks for correct expression for integral. 1 mark for significant correct progress toward expression
ii)	Thus $\int_1^e (ln\,x)^3 dx = [x(ln\,x)^3]_1^e - 3\int_1^e (ln\,x)^2 dx$ $= [e-0] - 3\left\{[x(ln\,x)^2]_1^e - 2\int_1^e (ln\,x)^1 dx\right\}$ $= e - 3(e-0) + 6\left\{[x(ln\,x)^1]_1^e - 1\int_1^e (ln\,x)^0 dx\right\}$ $= e - 3e + 6e - 6[x]_1^e$ $= e - 3e + 6e - [6e - 6]$ $= e - 3e + 6e - 6e + 6$ $= 6 - 2e$	2	2 marks for correct value. 1 mark for significant correct progress toward answer

Question 16		2016	
	Solution	Marks	Allocation of marks
b)	$xy = c^2$ $y + x\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{y}{x}$ --- Gradient of Tangent $\therefore$ Gradient of Normal $= \frac{x}{y}$ Normal at $P\left(cp, \frac{c}{p}\right)$ $\quad m = \frac{x}{y} = \frac{cp}{\frac{c}{p}} = p^2$ $y - y_1 = m(x - x_1)$ $y - \frac{c}{p} = p^2(x - cp)$ $y - \frac{c}{p} = p^2x - cp^3$ $p^2x - y = cp^3 - \frac{c}{p}$ $p^2x - y = \frac{c}{p}(p^4 - 1)$ --- (1) $q^2x - y = \frac{c}{q}(q^4 - 1)$ --- (2) From (1) – (2) $(p^2 - q^2)x = \frac{c}{p}(p^4 - 1) - \frac{c}{q}(q^4 - 1)$ $(p+q)(p-q)x = \frac{cq(p^4-1) - (q^4-1)cp}{pq}$ $x = \frac{cp^4q - cq - cpq^4 + cp}{pq\,(p-q)(p+q)}$ $x = \frac{cp - cq + cpq(p^3 - q^3)}{pq\,(p-q)(p+q)}$ $x = \frac{c(p-q) + cpq(p-q)(p^2 + pq + q^2)}{pq\,(p-q)(p+q)}$ $x = \frac{cpq(p^2 + pq + q^2) + c}{pq\,(p+q)}$ Sub into (1) $p^2\left(\frac{cpq(p^2 + pq + q^2) + c}{pq\,(p+q)}\right) - y = \frac{c}{p}(p^4 - 1)$ $y = p^2\left(\frac{cpq(p^2 + pq + q^2) + c}{pq\,(p+q)}\right) - \frac{c(p^4-1)q(p+q)}{pq\,(p+q)}$ $y = \frac{cp^5q + cp^4q^2 + cp^3q^3 + cp^2 - cp^5q - cp^4q^2 + cpq + cq^2}{pq\,(p+q)}$ $y = \frac{cp^3q^3 + cp^2 + cpq + cq^2}{pq\,(p+q)}$ $y = \frac{cp^3q^3 + c(p^2 + pq + q^2)}{pq\,(p+q)}$	4	4 marks for a correct derivation including equations of the normals and solution to show they meet at the required point. 3 marks for a solution which has a minor error in one of: finding the *equations of normals,* or *solving simultaneously* or *finding x value* of point or *finding the y value.* 2 marks for a solution which has more than one error in one or more of the above 1 mark for a solution that shows some progress in at least one of the above.

Question 16		2016	
	Solution	Marks	Allocation of marks
c) i)	Since $ma = mg - \frac{1}{10}v^2$ $40a = -400 - \frac{1}{10}v^2$ $400a = -4000 - v^2$ $a = -10 - \frac{v^2}{400}$ $= -\frac{1}{400}(v^2 + 4000)$ $\therefore \frac{dv}{dt} = \frac{-4000 \;\; - v^2}{400}$ $\frac{dt}{dv} = \frac{-400}{4000 + v^2}$ $t = \int \frac{-400}{4000 + v^2}\, dv$ The time *t* seconds, for the particle to travel from its initial position O (where $v = u$) the initial speed, to its highest point A, where $v = 0$ i.e. $t = \int_u^0 \frac{-400}{4000 + v^2}\, dv$ $= 400 \int_0^u \frac{1}{4000 + v^2}\, dv$ $= 400 \left[\frac{1}{20\sqrt{10}}\, tan^{-1} \frac{v}{20\sqrt{10}}\right]_0^u$ $= 2\sqrt{10}\left[\, tan^{-1} \frac{v}{20\sqrt{10}}\right]_0^u$ $= 2\sqrt{10}\left\{tan^{-1} \frac{u}{20\sqrt{10}} - \; tan^{-1} \frac{0}{20\sqrt{10}}\right\}$ $= 2\sqrt{10}\, tan^{-1}\left(\frac{u}{20\sqrt{10}}\right)$ Time to reach greatest height = $2\sqrt{10}\, tan^{-1}\left(\frac{u}{20\sqrt{10}}\right)$	2	2 marks for correct value. 1 mark for significant correct progress toward answer

Question 16		2016	
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ii)	Using $a = v\frac{dv}{dx}$ then $v\frac{dv}{dx} = -\frac{1}{400}(v^2 + 4000)$ $\frac{dv}{dx} = -\frac{(v^2 + 4000)}{400v}$ $\frac{dx}{dv} = \frac{-400v}{v^2 + 4000}$ i.e. $x = \int \frac{-400v}{v^2 + 4000}\,dv$ The height OA (the distance reached by the particle in travelling from O where $v = u$ to A where $v = 0$) is given by $OA = \int_u^0 \frac{-400v}{v^2 + 4000}\,dv$ $= \int_0^u \frac{400v}{v^2 + 4000}\,dv$ This becomes $= 200\int_0^u \frac{2v}{v^2 + 4000}\,dv$ $= 200[ln(v^2 + 4000)]_0^u$ $= 200[ln(u^2 + 4000) - ln\,4000]$ $= 200\,ln\left(\frac{u^2 + 4000}{4000}\right)$ Greatest height $= 200\,ln\left(\frac{u^2 + 4000}{4000}\right)$	2	2 marks for correct value. 1 mark for significant correct progress toward answer

Question 16		2016	
	Solution	Marks	Allocation of marks
d)	We prove first that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ where n is a positive integer. <u>Test $n = 0$</u> $(\cos\theta + i\sin\theta)^0 = 1 = \cos(0)\theta + i\sin(0)\theta$ <u>Test $n = 1$</u> $(\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta = \cos(1)\theta + i\sin(1)\theta$ $\therefore$ True for $n = 0, 1$. <u>Let k be a value for which result is true</u> i.e. $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ <u>Test $n = k + 1$</u> $(\cos\theta + i\sin\theta)^{k+1}$ $= (\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)^1$ $= (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$ using above $= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ $= \cos(k\theta + \theta) + i\sin(k\theta + \theta)$ $= \cos(k+1)\theta + i\sin(k+1)\theta$ $\therefore$ True for $n = k + 1$. $\therefore$ Since shown true for $n = 1$, is also true for $n = 2, 3 \ldots$ true for all integer n i.e. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for all integer n. Thus result is true for $n = 4$ i.e. $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$	3	3 marks for a correct proof giving all steps. 2 marks for a solution which has a minor error in one of: the steps 1 mark for a solution that shows some progress in at least the $k + 1$ step, or which completes all the other steps correctly, but is way off course in the $k + 1$ step.