Trial HSC Examination 2015
Mathematics Course

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

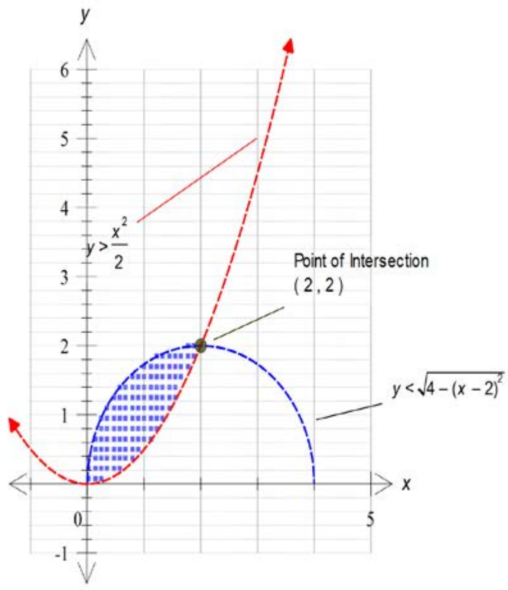
1.	A ○	B ○	C ●	D ○
2.	A ○	B ○	C ●	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ●	C ○	D ○
5.	A ○	B ○	C ●	D ○
6.	A ○	B ●	C ○	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ●
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ●	D ○

Multiple Choice Worked Solutions		
No	**Working**	**Answer**
1	$\left(\frac{2a}{3b}\right)^{-5}$ $\frac{1}{\left(\frac{2a}{3b}\right)^{5}}$ $=\frac{3^5\,b^5}{2^5\,a^5}$ $=\frac{243b^5}{32a^5}$	**C**
2	$2x^2-5x-9=0$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $\alpha+\beta=-\frac{b}{a}=\frac{5}{2}$ $\alpha\beta=\frac{c}{a}=-\frac{9}{2}$ So $\frac{\alpha+\beta}{\alpha\beta}=\frac{-\frac{b}{a}}{\frac{c}{a}}$ $=-\frac{b}{c}$ $=-\frac{5}{9}$	**C**
3	$\lim_{x\to\infty}\frac{3\sqrt{x}}{x-2}$ dividing by highest power of x $=\lim_{x\to\infty}\frac{\frac{3\sqrt{x}}{x}}{\frac{x}{x}-\frac{2}{x}}=\lim_{x\to\infty}\frac{\frac{3}{\sqrt{x}}}{1-\frac{2}{x}}$ $=\frac{0}{1-0}$ (as $x\to\infty,\ \frac{1}{x}\to 0$ and $\frac{1}{\sqrt{x}}\to 0$) $=0$	**D**
4	$y=3\cos 2x$ Amplitude $=3$ Period $=\frac{2\pi}{2}$ $=\pi$	**B**
5	$x^2+2x+y^2+4y-5=0$ $x^2+2x+1+y^2+4y+4=10$ $(x+1)^2+(y+2)^2=10$ Centre $=(-1,-2)$ and Radius $=\sqrt{10}$	**C**

6	$\cos^2\left(\frac{\pi}{2} - \theta\right) \cot\theta$ $= \sin^2\theta \cot\theta$ $= \sin^2\theta \times \frac{\cos\theta}{\sin\theta}$ $= \sin\theta\cos\theta$	**B**
7	$\int_2^7 \frac{5}{x}\,dx$ $= \left[5\ln x\right]_2^7$ $= 5\ln 7 - 5\ln 2$ $= 5\,[\ln 7 - \ln 2]$	**A**
8	$x^2 = 4y$ $y = \frac{x^2}{4}$ $y' = \frac{2x}{4}$ $= \frac{x}{2}$ When $x = 2$ $y' = 1 \therefore m_1 = 1$ So for normal $m_2 = -1$ When $x = 2$, $y = 1$ $y - 1 = -1\,(x - 2)$ $y - 1 = -x + 2$ $y + x - 3 = 0$	**D**
9	$\log_5 200 - 3\log_5 2$ $= \log_5 200 - \log_5 2^3$ $= \log_5\left(\frac{200}{8}\right)$ $= \log_5 25$ $= 2$	**B**
10	$\lvert 5x+4\rvert \le 6$ $-6 \le 5x+4 \le 6$ $-10 \le 5x \le 2$ $-2 \le x \le \frac{2}{5}$	**C**

Question 11		2015	
	Solution	Marks	Allocation of marks
(a)	$x^2 - 2x - 7$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{2 \pm \sqrt{42 - 4 \times 1 \times -7}}{2 \times 1}$ $x = \dfrac{2 \pm \sqrt{4 + 28}}{2}$ $x = \dfrac{2 \pm \sqrt{32}}{2}$ $x = \dfrac{2 \pm \sqrt{16} \times \sqrt{2}}{2}$ $x = \dfrac{2 \pm 4\sqrt{2}}{2}$ $x = 1 \pm 2\sqrt{2}$ ***(arith)***	2	1 for substitution into formula (or use of squares) 1 for simplification of surds.
(b)	$\int \dfrac{3x}{x^2 + 1}\,dx$ $= \dfrac{3}{2}\int \dfrac{2x}{x^2 + 1}\,dx$ $= \dfrac{3}{2}\ln\left(x^2 + 1\right) + C$ ***(calc)***	1	1 for correct answer.
(c)	$\dfrac{2}{\sqrt{7} + 3} - \dfrac{3\sqrt{7}}{\sqrt{7} - 3} = \dfrac{2\sqrt{7} - 6 - 21 - 9\sqrt{7}}{7 - 9}$ $= \dfrac{-7\sqrt{7} - 27}{-2}$ $= \dfrac{7\sqrt{7} + 27}{2}$ ***(arith)***	2	1 (rational denominator) 1 for simplification
(d)	$\dfrac{x}{x + 4.2} = \dfrac{5.6}{8.2}$ $8.2x = 5.6(x + 4.2)$ $8.2x = 5.6x + 23.52$ $2.6x = 23.52$ $x = 9.046$ $x = 9.0$ (nearest mm) ***(geom)***	2	1 for correct ratio 1 for solving equation

Question 11		2015	
	Solution	Marks	Allocation of marks
(e)	$x^2 - 5y + 5 - 0$ $x^2 = 5y - 5$ $x^2 = 5(y - 1)$ $\therefore$ Vertex at$(0\ ,1)$ $4a = 5$ $a = \frac{5}{4}$ $\therefore$ focal length $= \frac{5}{4}$ Focus $\left(0\ ,1 + \frac{5}{4}\right)$ $\therefore$ Focus $= \left(0\ ,\ \frac{9}{4}\right)$ ***(function)***	2	1 for vertex 1 for focus
(f)	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{30} = \frac{30}{2}[2 \times 5 + (29 \times 4)]$ $= 1890$ $S_9 = \frac{9}{2}[2 \times 5 + (8 \times 4)]$ $= 189$ Sum 10th - 30th terms $= S_{30} - S_9$ $= 1890 - 189$ $= 1701$ ***(series)***	2	1 for correct substitution into formula 1 for answer (1 mark only if S_{10} is used)
(g)	$\int_0^{\ln 6} e^x\ dx = \left[e^x\right]_0^{\ln 6}$ $= e^{\ln 6} - e^0$ $= 6 - 1$ $= 5$ ***(calc)***	2	1 for integration 1 for answer

<table>
<tr><td colspan="2">Question 11</td><td colspan="2">2015</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(h)</td><td>
(calc)</td><td>2</td><td>1 for correct functions

1 for correct shading of intersection.

Point of intersection not required for marks.</td></tr>
</table>

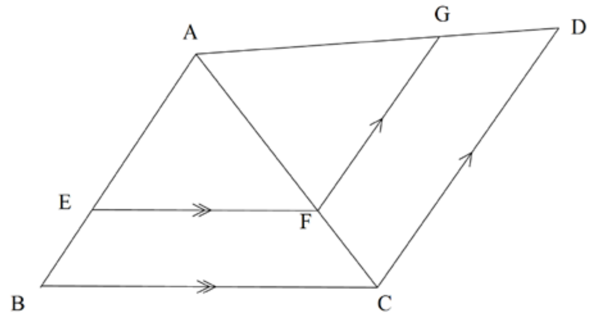
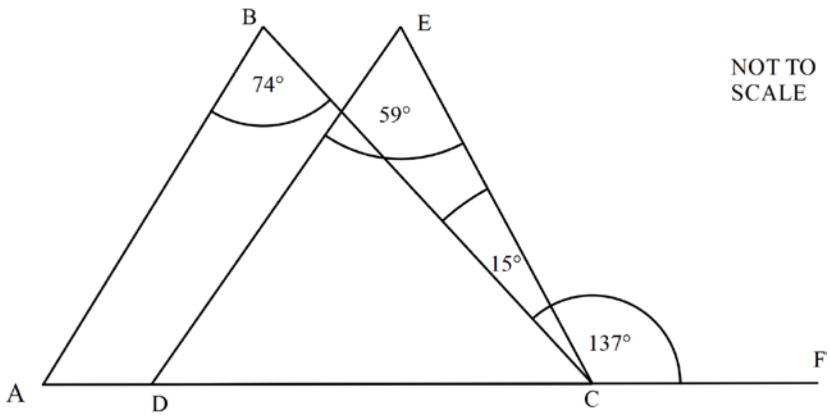
Question 12		**2015**	
	Solution	Marks	Allocation of marks
(a)	(i) $y = \sin^2(4x)$ $y' = 2\sin 4x\ .\cos 4x\ .4$ $= 8\sin 4x \cos 4x$ ***(calc)***	1	1 for correct use of chain rule
	(ii) $y = x^3 e^{3x}$ $y' = x^3\left(3e^{3x}\right) + e^{3x}(3x^2)$ $= 3x^2 e^{3x}(x+1)$ ***(calc)***	1	1 for correct use of product rule
	(iii) $y = \dfrac{e^x}{(x+3)^2}$ Quotient Rule $u = e^x \qquad v = (x+3)^2$ $u' = e^x \qquad v' = 2(x+3)$ $y' = \dfrac{vu' - uv'}{v^2}$ $= \dfrac{(x+3)^2 .(e^x) - (e^x)(2(x+3))}{(x+3)^4}$ ***(calc)*** Further simplification gives: $= \dfrac{(x+3).(e^x) - (e^x)(2)}{(x+3)^3}$ $= \dfrac{e^x(x+3-2)}{(x+3)^3}$ $= \dfrac{e^x(x+1)}{(x+3)^3}$ but is not required.	2	Give 2 marks for a solution which shows a correct use of the quotient rule with correct differentials found including use of the chain rule (or expansion). Full simplification by cancelling is not required. Give 1 mark for as solution which has a minor error in the use of the quotient rule or finding the differentials
(b)	$\sqrt{3}\cos x = \sin x$ $\sqrt{3} = \dfrac{\sin x}{\cos x}$ $\tan x = \sqrt{3}$ $x = \dfrac{\pi}{3}$ tan positive 1st, 3rd quadrant $\therefore \quad x = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$ ***(trig)***	2	1 for determining equation 1 for all solutions

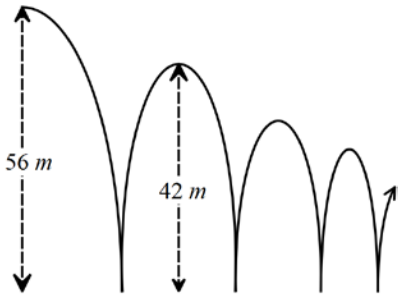
<table>
<tr><td colspan="2">Question 12</td><td colspan="2">2015</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(c)</td><td>$\int_0^1 \tan x \, dx$
Using Simpson's Rule
$h = \frac{1-0}{4} = \frac{1}{4}$
$\int_0^1 \tan x \, dx \approx$
$\frac{h}{3}\left[[\tan 0 + \tan 1] + 4\left[\tan\frac{1}{4} + \tan\frac{3}{4}\right] + 2\left[\tan\frac{1}{2}\right]\right]$
$\approx \frac{1}{12}(7.3977...)$
≈ 0.62 (calc)</td><td>2</td><td>1 for substitution into Simpson’s rule

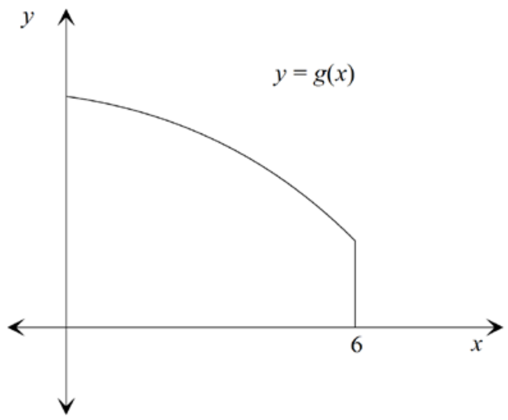
1 for evaluating correct answer
(only 1 mark if radians not used)</td></tr>
<tr><td>(d)</td><td>$3x + \log_e x + C$
(calc)</td><td>2</td><td></td></tr>
<tr><td>(e)</td><td>$3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$
RHS
$= A(x^2 + 2x - x - 2) + Bx + B + C$
$= Ax^2 + Ax - 2A + Bx + B + C$
$= x^2 A + x(A+B) + (-2A + B + C)$
Equating coefficients
$A = 3$
$A + B = 1$ ①
$-2A + B + C = 1$ ②
From ①
$A + B = 1$
$3 + B = 1$
$\therefore \quad B = -2$
From ②
$-2A + B + C = 1$
$-6 - 2 + C = 1$
$C = 9$ (function)</td><td>2</td><td>1 for expansion and determining coefficients

1 for solving to find the values of A, B C.</td></tr>
</table>

Question 12		2015	
	Solution	Marks	Allocation of marks
(f)	(i) $y = x\cos x$ $0 = x\cos x$ $\therefore\ x = 0$ or $\cos x = 0$ $\cos x = 0$ $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ $\therefore$ first after origin is $\frac{\pi}{2}$ $\therefore P\left(\frac{\pi}{2},\ 0\right)$ ***(calc)***	1	1 for any reasonable explanation.
	(ii) $y = x\cos x$ $u = x \quad v = \cos x$ $u' = 1 \quad v' = -\sin x$ $y' = \cos x - x\sin x$ when $x = \frac{\pi}{2}$ $y' = \cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}$ $= -\frac{\pi}{2}$ Equation of tangent $y - 0 = -\frac{\pi}{2}\left(x - \frac{\pi}{2}\right)$ $y = \frac{-\pi x}{2} + \frac{\pi^2}{4}$ ***(calc)***	2	1 for gradient of tangent 1 for equation of tangent.

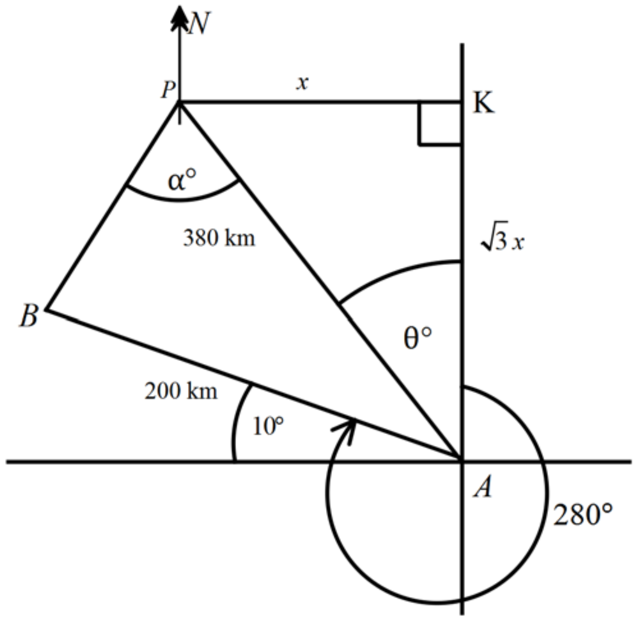
Question 13		2015	
	Solution	Marks	Allocation of marks
(a)	 AE/AB = AF/AC (ratios of intercepts) AF/AC = AG/AD (ratios of intercepts) ∴ AE/AB = AG/AD (equating ratios) ***(geom)***	2	1 for use of ratios of intercepts 1 for conclusion
(b)	 Prove AB‖DE ∠ BAD + 74° = 137° (exterior∠ Δ) ∴ ∠ BAD = 63° ∠ EDC + 59° = 137° – 15° (exterior∠ Δ) ∠ EDC = 63° ∠ BAD = ∠ EDC (both = 63°) ∴ AB || DE (equal corresponding ∠'s) ***(geom)***	2	1 for showing angles are 63^{o} 1 for stating lines parallel with reason

Question 13		**2015**	
	Solution	Marks	Allocation of marks
(c)	(i) ***(series)*** Taking 42 as the first term as it is the first completed rebound.  $T_n = ar^{n-1}$ For the rise on the 5th bounce $a = 42 \quad r = \frac{3}{4} \quad n = 5$ $T_5 = 42\left(\frac{3}{4}\right)^{5-1}$ $T_5 = 42\left(\frac{3}{4}\right)^{4}$ $\approx 13.29m$ Note if take 56 as first term, need to find the 6th term.	2	1 for determining the series 1 for finding correct term (only 1 mark if found T_5 after using a = 56.)
	(ii) ***(series)*** $\lvert r \rvert < 1 \qquad r = \frac{3}{4}$ Consider one bounce up and down as a term, so $t_1 = 84$ $\therefore s_\infty = \frac{a}{1-r}$ $= \frac{84}{1-\frac{3}{4}}$ $= \frac{84}{\frac{1}{4}}$ $= 336m$ Total distance travelled will be $336 + 56 = 392m$ Alternately take 42 as first term and double result from $S\infty$	1	1 for correct answer

Question 13		**2015**	
	Solution	Marks	Allocation of marks
(d)	$f'(x) < 0 \therefore$ negative gradient (decreasing) $f''(x) < 0 \therefore$ concave down  ***(calc)***	2	Graph needs to have negative gradient (1 mark) and concave down (1 mark).
(e)	(i) $A\,(\pi\,,1)$ $\quad B\,(5\pi\,,\,3)$ $\quad C\,(\pi\,,\,5)$ Midpoint $M = \left[\dfrac{5\pi+\pi}{2}, \dfrac{1+3}{2}\right]$ $= (3\pi\,,\,2)$ *(function)*	1	1 for correct answer
	(ii) $m_{AB} = \dfrac{3-1}{5\pi - \pi}$ $= \dfrac{2}{4\pi} = \dfrac{1}{2\pi}$ For Perpedicular line $m_1 \times m_2 = -1$ $\dfrac{1}{2\pi} \times m_2 = -1$ $m_2 = -2\pi$ Equation of line $y - 5 = -2\pi\,(x - \pi)$ $y - 5 = -2\pi x + 2\pi^2$ $y + 2\pi x - 5 - 2\pi^2 = 0$ ***(function)***	2	1 for gradient of the perpendicular 1 for finding the equation of the line
	(iii) $\overline{AB} = \sqrt{(5\pi-\pi)^2 + (3-1)^2}$ $= \sqrt{(4\pi)^2 + (2)^2}$ $= \sqrt{16\pi^2 + 4}$ ***(function)***	1	1 for correct answer

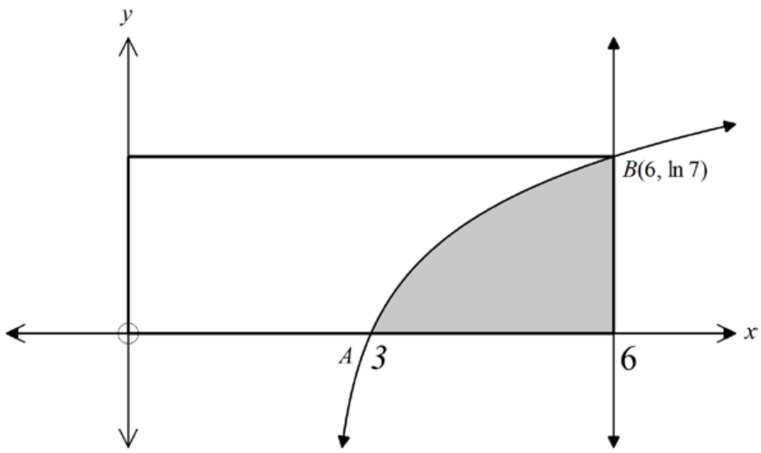
Question 13		2015	
	Solution	Marks	Allocation of marks
(e)	(iv) $\overline{AB} = \sqrt{16\pi^2 + 4}$ $\overline{BC} = \overline{AB} = \sqrt{16\pi^2 + 4}$ (given) $\overline{AC} = 4$ (vertical line) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \dfrac{16\pi^2 + 4 + 16 - (16\pi^2 + 4)}{2 \times 4 \times \left(\sqrt{16\pi^2 + 4}\right)}$ $\cos A = \dfrac{16}{8\sqrt{16\pi^2 + 4}}$ $A = 81°$ (nearest degree) ***(function)*** **Alternate Solution** **Can also be done using the inclination of a line** $m_{AB} = \dfrac{1}{2\pi}$ $\tan\theta = m_{AB}$ (where θ is the inclination to the positive x axis) $\tan\theta = \dfrac{1}{2\pi}$ $\theta = \tan^{-1}\left(\dfrac{1}{2\pi}\right) = 9°$ Now AC is vertical so inclination $= 90°$ $\angle BAC = 90° - 9° = 81°$	2	1 for use of cosine rule 1 for answer 1 for angle with *x* axis 1 for answer

Question 14		**2015**	
	Solution	Marks	Allocation of marks
(a)	(i) $V = Ae^{-kt}$ $30000 = Ae^{-5t}$.....(1) $18000 = Ae^{-10t}$......(2) 2 $\div$ 1 $\dfrac{18000}{30000} = \dfrac{Ae^{-10k}}{Ae^{-5k}}$ $e^{-5k} = \dfrac{3}{5}$ $-5k = \ln\left(\dfrac{3}{5}\right)$ $k = -\dfrac{\ln\left(\frac{3}{5}\right)}{5}$ $k = 0.102165124$ ***(log)***	2	1 for eliminating A 1 for value of k
	(ii) ***(log)*** $V = Ae^{-0.102... \times t}$ when $t = 5$ $V = \$30000$ $30000 = Ae^{-0.102 \times 5}$ $A = \$50000$	1	1 for value of A
	(iii) ***(log)*** $V = 50000e^{-0.102t}$ $50000e^{-0.102t} < 1000$ $e^{-0.102t} < \dfrac{1}{50}$ $-0.102t < \ln\left(\dfrac{1}{50}\right)$ $t > \dfrac{\ln\left(\frac{1}{50}\right)}{-0.102}$ $t > 38.29$ $\therefore$ It will take 39 years to fall below \$1000	2	1 for correct inequality in t 1 for value of t

<table>
<tr><td colspan="2">Question 14</td><td colspan="2">2015</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(b)</td><td>(i)


(trig)</td><td>1</td><td>1 for diagram</td></tr>
<tr><td></td><td>(ii) (trig)
$$\begin{aligned} x^2 + (\sqrt{3}\,x)^2 &= 380^2 \\ x^2 + 3x^2 &= 380^2 \\ 4x^2 &= 380^2 \\ x^2 &= \frac{380^2}{4} = \frac{380}{2} \\ x &= 190 \end{aligned}$$
The distance AK = $\sqrt{3}\,x = \sqrt{3} \times 190 = 190\sqrt{3}$</td><td>1</td><td>1 for answer</td></tr>
<tr><td></td><td>(iii)
$$\begin{aligned} \tan\theta &= \frac{x}{\sqrt{3}\,x} \\ \tan\theta &= \frac{1}{\sqrt{3}} \\ \theta &= 30^\circ \\ \angle PAB &= 90 - 30 - 10 \\ &= 50^\circ \\ (PB)^2 &= 200^2 + 380^2 - (2 \times 200 \times 380 \times \cos 50^\circ) \\ &= 86696.28 \\ PB &= 294.44 \\ &= 294 \text{ km} \end{aligned}$$
(trig)</td><td>2</td><td>1 for angle θ.

1 for distance</td></tr>
<tr><td></td><td>(iv)
Find $\angle APB$ (trig)
$$\begin{aligned} \frac{\sin\alpha}{200} &= \frac{\sin 50^\circ}{294.44} \\ \alpha &= \sin^{-1}\left[\frac{200\sin 50^\circ}{294.44}\right] \\ &= 31^\circ 21' \\ &= 31^\circ \\ \theta &= 30 \text{ so } \angle APK = 60^\circ \\ \text{Bearing} &= 90^\circ + 60^\circ + 31^\circ \\ &= 181^\circ T \end{aligned}$$</td><td>1</td><td>1 for bearing</td></tr>
</table>

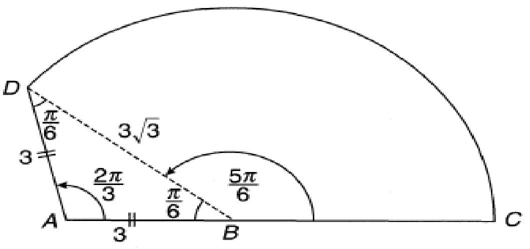
Question 14		**2015**	
	Solution	Marks	Allocation of marks
(c)	(i) A is on the x axis so $y = 0$ $\ln(2x-5) = 0$ $2x - 5 = e^0 = 1$ $2x = 6$ $x = 3$ ***(log)*** A is the point $(3, 0)$ For B, $x = 6$ so $y = \ln(2 \times 6 - 5)$ $y = \ln 7$ B is the point $(6, \ln 7)$  	1	1 for use of logs to show both values
	(ii) Given $\ln(2x-5)$ change subject to x. $2x - 5 = e^y$ $2x = e^y + 5$ $x = \dfrac{e^y + 5}{2}$ ***(log)***	1	1 for changing the subject.

Question 14		2015	
	Solution	Marks	Allocation of marks
(c)	(iii) Can't integrate $\ln(2x-5)$ so use the area between the curve and the *y* axis and subtract from the rectangle shown. Area to y axis $= \int_0^{\ln 7} \frac{e^y + 5}{2}\,dy$ $= \left[\frac{e^y + 5y}{2}\right]_0^{\ln 7}$ $= \frac{\left(e^{\ln 7} + 5(\ln 7)\right)}{2} - \frac{e^0 + 5 \times 0}{2}$ $= \frac{(7 + 5(\ln 7) - 1)}{2}$ $= \frac{(6 + 5\ln 7)}{2}$ Area Rectangle $= 6 \times \ln 7 = 6\ln 7$ Shaded area $= 6\ln 7 - \frac{(6 + 5\ln 7)}{2}$ $= \frac{(12\ln 7 - (6 + 5\ln 7))}{2}$ $= \frac{7\ln 7 - 6}{2}$ square units ***(log)***	3	1 for correct integral 1 for finding area to *y* axis 1 for shaded area

Question 15		2015	
	Solution	Marks	Allocation of marks
(a)	(i) $3x + 3x + 4y = 300$ $4y = 300 - 6x$ $y = 75 - \frac{3x}{2}$ ***(calc)***	1	1 for correct expression
	(ii) $A = 3x \times y$ $A = 3x\left[75 - \frac{3x}{2}\right]$ $A = 225x - \frac{9x^2}{2}$ Maximum Area find A' $A' = 225 - \frac{18x}{2}$ $= 225 - 9x$ $A' = 0$ $0 = 225 - 9x$ $9x = 225$ $x = 25m$ *When* $x = 25m$ $y = 37.5m$ Test maximum point $A'' = -9$ < 0 $\therefore$ Maximum Area $\therefore$ $x = 25m$ will produce the maximum area ***(calc)***	3	1 for A' 1 for x 1 for test that it is maximum
	(iii) $A = 25 \times 37.5$ $= 937.5m^2$ ***(calc)***	1	1 for area
	(iv) $3 \times 937.5 = 2812.5m^2$ $1Ha = 10000m^2$ $10000 - 2812.5 = 7187.5m^2$ So Greg and his wife will have 7187.5 m^2 left. ***(calc)***	1	1 for answer

<table>
<tr><td colspan="2">Question 15</td><td colspan="2">2015</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(b)</td><td>

(i)



In $\triangle ABD$,

$\angle ABD = \pi - \angle CBD$ (ABC is a straight angle, π)

$= \pi - \frac{5\pi}{6}$

$= \frac{\pi}{6}$

Also $\angle ABD = \angle ADB$ (angles opp. equal sides; $AD = AB$)

$= \frac{\pi}{6}$

Now $\angle DAB + \angle ABD + \angle ADB = \pi$ (angle sum of $\triangle ABD$ is π)

$\therefore \angle DAB + \frac{\pi}{6} + \frac{\pi}{6} = \pi$

$\therefore \angle DAB = \pi - \frac{2\pi}{6}$

$= \frac{2\pi}{3}$.

(trig)

(ii) By the sine rule:

$$\frac{BD}{\sin\frac{2\pi}{3}} = \frac{3}{\sin\frac{\pi}{6}}$$

$$\therefore BD = \frac{3 \times \sin\frac{2\pi}{3}}{\sin\frac{\pi}{6}}$$

$$= \frac{3 \times \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{6 \times \sqrt{3}}{2}$$

$$= 3\sqrt{3} \text{ m.}$$

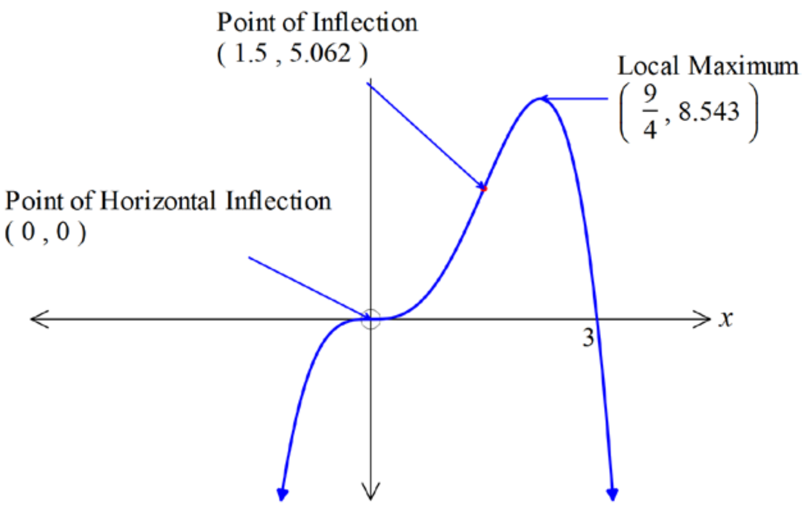
(trig)

</td><td>2

2</td><td></td></tr>
</table>

Question 15		2015	
	Solution	Marks	Allocation of marks
(c)	Let V be the volume of the water in the pond at any time. $\frac{dV}{dt} = -(5+2t)$ *(the volume is decreasing)* $= -5-2t$ $V = \int(-5-2t)dt$ $= -5t-t^2+c$ At $t = 0$, $V = 50$ *(initially there are 50 lt. of water)* $\therefore \quad c = 50$ $\therefore \quad V = -5t-t^2+50$ ***(calc)***	2	
(d)	$\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$ ***(arith)*** LHS $= \frac{(10^3)^n \times (5^2)^{n+2}}{(2^3)^n}$ $= \frac{(1000)^n \times (5^2)^{n+2}}{(2^3)^n}$ $= \frac{(2^3 \times 5^3)^n \times 5^{2n+4}}{(2^{3n})}$ $= \frac{(2^{3n} \times 5^{3n}) \times (5^2)^{n+2}}{(2^{3n})}$ $= 5^{3n} \times 5^{2n+4}$ $\therefore 5^{5n+4} = 1$ $5^0 = 1$ $\therefore 5n + 4 = 1$ $n = -\frac{4}{5}$	3	1 for expanding the terms 1 for collecting powers of 2 and of 5 1 for solving for n

Question 16		2015	
	Solution	Marks	Allocation of marks
(a)	$x^2-4x+4+y^2=9$ is a circle $x^2-4x+4+y^2=5+4$ $(x-2)^2+y^2=9$ $\therefore$ Centre (2,0) Radius = 3 So $V=\frac{4}{3}\pi r^3$ $=\frac{4}{3}\times\pi\times 3^3$ $=36\pi\ units^3$ OR $x^2-4x+y^2=5$ $y^2=5-x^2+4x$ Intercepts $y=0$ so $x^2-4x-5=0$ $(x+1)(x-5)=0$ Intercepts are $x=-1$ or $x=5$ $V=\int_a^b y^2\,dx$ $=\int_{-1}^{5} 5-x^2+4x\ \ dx$ $=\pi\left[5x-\frac{x^3}{3}+2x\right]_{-1}^{5}$ $=\pi\left(25-\frac{125}{3}+50\right)-\left(-5+\frac{1}{3}+2\right)$ ***(calc)*** $=36\pi$ unit3	2	1 for circle and finding end points 1 for volume either method
(b)	(i) $y=x^3(3-x)=3x^3-x^4$ $y'=9x^2-4x^3$ Stationary points where $y'=0$ $9x^2-4x^3=0$ $x^2(9-4x)=0$ $x=0$ or $x=\frac{9}{4}$ $y=0$ or $y=8.543$ $y''=18x-12x^2$ $x=0, y''=0$ so possible inflexion test $x=-1,\ y''=,-30\ x=1\ y''=6$ so change of concavity **so (0,0) is horizontal inflexion** $x=\frac{9}{4},\ y''=-20\frac{1}{4}$ $\therefore$ concave down **so $\left(\frac{9}{4}, 8.543\right)$ is a local maximum.** ***(calc)***	3	1 for the two x values of stationary pts 1 for second derivative used to determine possible nature. 1 for checking inflexion and naming the two points and their nature.

Question 16		2015	
	Solution	Marks	Allocation of marks
(b)	(ii) Use second derivative to check for other turning points. $y'' = 18x - 12x^2$ $y'' = 0$ when $18x - 12x^2 = 0$ $6x(3-2x) = 0$ $x = 0$ or $x = \frac{3}{2}$ $x = 0$ is horizontal inflexion found in part i) $x = \frac{3}{2},\ y = 5\frac{1}{16}$ $x = 2,\quad y'' = -12$ $x = 1,\quad y'' = 6$ $\therefore$ change of concavity so inflexion at $\left(\frac{3}{2}, 5\frac{1}{16}\right)$ Intercepts on x axis $x^3(3-x) = 0$ $x = 0$ or $x = 3$  ***(calc)***	3	1 for determining other inflexion 1 for general shape of sketch 1 for showing all features
(c)	(i) ***(series)*** $P = \$650000 \quad r = 5.4 \div 100 \div 12 = 0.0045$ $A = P(1+r)^n - M$ $A_1 = 650000(1.0045)^1 - M$ $A_2 = A_1(1.0045)^1 - M$ $A_2 = [650000(1.0045)^1 - M](1.0045) - M$ $A_2 = 650000(1.0045)^2 - M(1.0045) - M$ $A_2 = 650000(1.0045)^2 - M[1 + 1.0045]$ $A_3 = (650000(1.0045)^2 - M[1 + 1.0045])(1.0045) - M$ $A_3 = 650000(1.0045)^3 - M[1 + 1.0045 + 1.0045^2]$ . . . $A_n = 650000(1.0045)^n - M\left[1 + 1.0045 + \ldots + 1.0045^{n-1}\right]$	2	1 for setting up initial terms as examples 1 for following pattern to establish required formula

Question 16		**2015**	
	Solution	Marks	Allocation of marks
(c)	(ii) ***(series)*** Months = 30 × 12 = 360 repayments A_{360} = 0 (loan repaid) $A_n = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$ $0 = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$ $M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right] = 650000(1.0045)^n$ $M = \dfrac{650000(1.0045)^n}{1 + 1.0045 + ... + 1.0045^{n-1}}$ The denominator is a geometric series with $a = 1$, $r = 1.0045$ and $n = 360$ $S_n = \dfrac{a(r^n - 1)}{r}$ $S_{360} = \dfrac{1((1.0045)^{360} - 1)}{0.0045}$ $S_{360} = \dfrac{(1.0045)^{360} - 1}{0.0045}$ $\therefore M = \dfrac{(650000(1.0045)^{360}) \times 0.0045}{(1.0045)^{360} - 1}$ $M = \$3649.95$	2	1 for expression for *M* 1 for substituting into sum of series and finding *M* (can use rounded answer for S_n)
	(iii) ***(series)*** $A_n = 650000(1.0045)^n - 5000S_n$ A_n = \$0 paid off $5000S_n = 650000(1.0045)^n$ $5000\left[\dfrac{(1.0045)^n - 1}{0.0045}\right] = 650000(1.0045)^n$ $5000(1.0045)^n - 5000 = 2925(1.0045)^n$ $5000(1.0045)^n - 2925(1.0045)^n = 5000$ $(1.0045)^n[5000 - 2925] = 5000$ $(1.0045)^n = \dfrac{5000}{2075}$ $\ln\left(1.0045^n\right) = \ln\left[\dfrac{5000}{2075}\right]$ $n \ln(1.0045) = \ln\left[\dfrac{5000}{2075}\right]$ $n = \dfrac{\ln\left[\dfrac{5000}{2075}\right]}{\ln 1.0045}$ $n = 195.88$ $= 196$ months	2	1 for using sum to establish equation 1 for solving to find *n*

Question 16		2015	
	Solution	Marks	Allocation of marks
(c)	(iv) ***(series)*** Total of loan over 30 years $360 \times \$3\ 649.95 = \$1\ 313\ 982$ Total of loan by paying \$5000/month $196 \times \$5\ 000 = \$980\ 000$ Interest Saving" \$1 313 982 - \$980 000 = \$333 982	1	1 for answer