2013
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time - 5 minutes
- Working Time - 2 hours
- Write using a blue or black pen. Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-14

	MC	Q11	Q12	Q13	Q14	Total
PE3		/6				/6
HE1	/10					/10
HE2			/4			/4
HE3				/12	/10	/22
HE4					/5	/5
HE5		/9	/11	/3		/23
	/10	/15	/15	/15	/15	/70

Total marks (70)

Section I

Total marks **(10)**

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II

Total marks **(60)**

- Attempt questions 11 – 14
- Answer in the blank answer books provided, unless otherwise instructed
- Start a new book for each question
- All necessary working should be shown for every question
- Allow about 1 hour 45 minutes for this section

Section I – 10 marks **Marks**
Attempt Questions 1–10
All questions are of equal value

Use the multiple-choice answer sheet for Questions 1–10.

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1. What is the value of $\lim_{x \to 0} \left(\frac{\sin \frac{1}{3}x}{2x} \right)$? **1**

(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 6

2. Which of the following is an expression for $\frac{d}{dx}(2^x)$? **1**

(A) $x2^{x-1}$

(B) 2^{x-1}

(C) 2^x

(D) $2^x log_e 2$

3. The equation $2x^3 + x^2 - 13x + 6 = 0$ has roots α, $\frac{1}{\alpha}$ and β. What is the value of β ? **1**

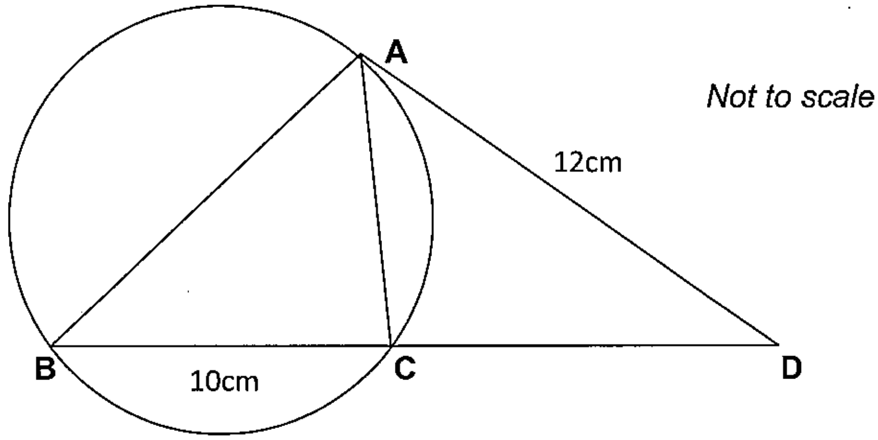
(A) 3

(B) 2

(C) -3

(D) -6

4.



ABC is a triangle inscribed in a circle. The tangent to the circle at *A* meets *BC* produced at *D* where *BC*=10cm and *AD*=12cm. What is the length of *CD*? **1**

(A) 6cm

(B) 7cm

(C) 8cm

(D) 9cm

5. Consider a projectile launched with an initial velocity v at an angle θ to the horizontal. Assume that g = 9.8 m/s2 and that air resistance is negligible.
Which of the following statements is correct? **1**

(A) The acceleration of the projectile decreases during its upward flight.

(B) The acceleration of the projectile is greatest during its upward flight.

(C) The acceleration of the projectile increases during its downward flight.

(D) The acceleration of the projectile remains constant during its entire flight.

6. The area enclosed between the curve $y = x^3 - 1$, the y-axis and the lines $y = 1$ and $y = 2$ is given by: **1**

(A) $\int_1^2 (x^3 - 1)\, dy$

(B) $\int_1^2 (\sqrt[3]{y+1})dy$

(C) $\int_1^2 (\sqrt[3]{y} + 1)\, dy$

(D) $\int_1^2 (y + 1)\, dy$

7. Which equation shows a particle **not** moving in simple harmonic motion? **1**

(A) $x = a \sin{(nt + \alpha)}$

(B) $x = a \tan{(nt + \alpha)}$

(C) $x = a \cos\ (nt + \alpha) - a \sin(nt + \alpha)$

(D) $x = a \cos\ (nt + \alpha)$

8. Find $\int \frac{dx}{\sqrt{4-x^2}}$ **1**

(A) $\frac{1}{4}\sin^{-1}\left(\frac{x}{4}\right) + C$

(B) $\sin^{-1}\left(\frac{x}{4}\right) + C$

(C) $\frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right) + C$

(D) $\sin^{-1}\left(\frac{x}{2}\right) + C$

9. Which of the following is an expression for $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)$? **1**

(A) $\frac{-x^2}{1+x^2}$

(B) $\frac{-1}{1+x^2}$

(C) $\frac{1}{1+x^2}$

(D) $\frac{x^2}{1+x^2}$

10. Which of the following lines is a horizontal asymptote of the curve $y = \frac{e^x-2}{e^x+2}$?

(A) $y = -2$

(B) $y = -1$

(C) $y = 0$

(D) $y = 2$

Section II – 60 marks
Attempt Questions 11–14
All questions are of equal value
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) A particle moves in a straight line so that its acceleration is given by $\frac{dv}{dt} = x - 1$,

where v is its velocity and x is its displacement from the origin.

Initially the particle is at the origin and has $v = 1$.

(i) Show that $v^2 = (x-1)^2$. **2**

(ii) Find x as a function of t, that is, $x(t)$. **4**

(b) Find the coordinates of the point, P that divides the interval MN with $M\,(1,4)$ and $N\,(5,2)$ in the ratio $-1:3$. **2**

(c) Using the substitution $u = 1 + 2x$, find $\int \frac{6}{\sqrt{(1+2x)^3}}\,dx$. **3**

(d) The polynomial $P(x) = x^3 + px^2 + qx + 5$, where p and q are constants, leaves remainders of 7 and 17 when divided by $x - 2$ and $x + 3$ respectively.

(i) Find the value of p and q. **3**

(ii) Find the remainder when $P(x)$ is divided by $x - 4$. **1**

Marks

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Seven people are sitting around a table.

(i) How many seating arrangements are possible? **1**

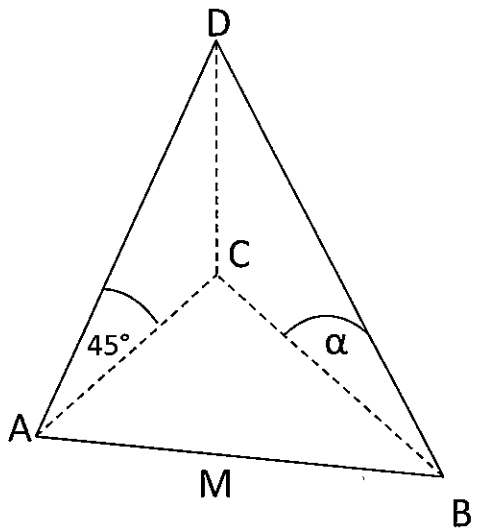
(ii) Two people, Kevin and Julia, do not sit next to each other. **2**

How many seating arrangements are now possible?

(b) Use Mathematical Induction to show that $n! > e^n$ for all **3**

positive integers $n \geq 6$.

(c)



CD is a vertical pole of height 1 metre that stands with its base *C* on horizontal ground. *A* is a point due South of *C* such that the angle of elevation of *D* from *A* is 45°. *B* is a point due East of *C* such that the angle of elevation of *D* from *B* is α. *M* is the midpoint of *AB*.

(i) Show that $AB = cosec\ \alpha$. **2**

(ii) Show that $CM = \frac{1}{2} cosec\ \alpha$. **2**

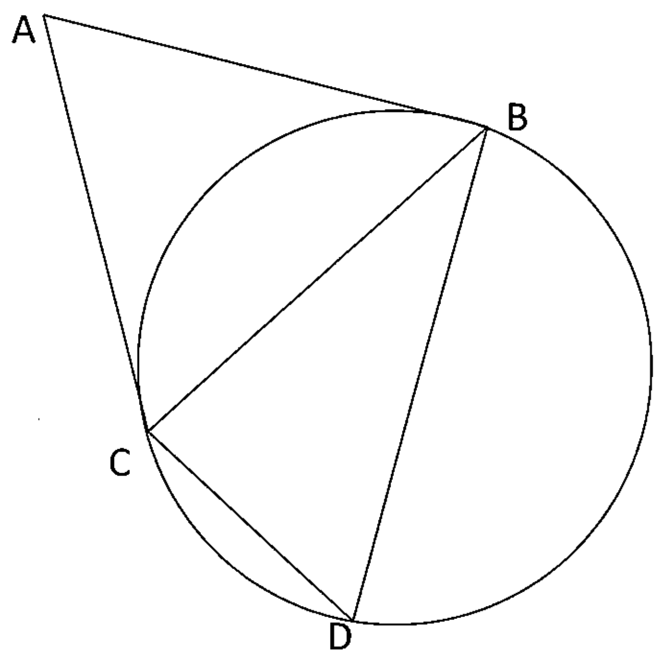
(Question 12 continued on next page)

(d) AB and AC are tangents to a circle. D is a point on the circle such that

$\angle BDC = \angle BAC$ and $2 \times \angle DBC = \angle BAC$.

(i) Show that $BC = AB$. **4**

(ii) Show that DB is a diameter. Show that $BC = AB$. **1**



Marks

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) $P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$ with focus $F(0, a)$.

(i) Use differentiation to show that the tangent to the parabola at P has gradient t and equation $tx - y - at^2 = 0$. **2**

(ii) Show that the shortest distance between the focus and this tangent is $a\sqrt{1+t^2}$. **1**

(b) Consider the function $f(x) = sin^{-1}(x-1)$.

(i) Find the domain of the function. **1**

(ii) Sketch the graph of the curve $y = f(x)$ showing the endpoints and the $x-$axis intercept. **2**

(iii) The region in the first quadrant bounded by the curve $y = f(x)$ and the $y-$axis between the lines $y = 0$ and $y = \frac{\pi}{2}$ is rotated through one complete revolution about the $y-$axis. Find in simplest exact form the volume of the solid of revolution. **3**

(c) A particle is performing Simple Harmonic Motion in a straight line.
At time t seconds its displacement from a fixed point O on the line, is x metres, given by $x = 4\sqrt{2}\cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$, its velocity is $v\ \mathrm{ms}^{-1}$ and its acceleration is $\ddot{x}\ ms^{-2}$.

(i) Find the amplitude and period of the motion. **2**

(ii) Find the initial position of the particle and determine if it is initially moving towards or away from O. **2**

(iii) Find the distance travelled by the particle in the first 3 seconds of its motion. **2**

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Express $5sinx + 12cosx$ in the form $Asin(x + \alpha)$ where $0 \leq \alpha \leq \frac{\pi}{2}$. **2**

(Give the value of α in radians, correct to two decimal places.)

(ii) Hence or otherwise, solve $5sinx + 12cosx = 8$ for $0 \leq \alpha \leq \pi$. **2**

(Give the value, or values, of x in radians, correct to two decimal places.)

(b) A vertical building of height 60 metres stands on horizontal ground. A particle is projected from a point O at the top of the building with speed $V = 20\sqrt{2}\, ms^{-1}$ at an angle α above the horizontal. It moves in a vertical plane under gravity where the acceleration due to gravity is $= 10ms^{-2}$, and hits the ground at a distance of 120 metres from the foot of the building. At time t seconds its horizontal and vertical displacements from O are x metres and y metres respectively, given by

$x = 20\sqrt{2}\, t\, cos\alpha$ and $y = 20\sqrt{2}\, t\, sin\alpha - 5t^2$. (Do NOT prove these results.)

(i) Show that $\alpha = \frac{\pi}{4}$ or $\alpha = tan^{-1}\frac{1}{3}$. **2**

(ii) If $\alpha = tan^{-1}\frac{1}{3}$, find the exact time taken for the particle to hit the ground. **2**

(iii) If $\alpha = \frac{\pi}{4}$, find the exact speed of the particle after 6 seconds. **2**

(c) Consider the function $f(x) = x + e^{-x},\ x \geq 0$.

(i) Show that for all values of $x > 0$, the function is increasing and the curve $y = f(x)$ is concave up. **2**

(ii) Sketch the graph of $y = f(x)$ showing clearly the coordinates of the endpoint and the equation of the asymptote. **2**

(iii) On the same diagram, sketch the graph of the inverse function $y = f^{-1}(x)$. **1**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a>0, \quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$

STUDENT NUMBER/NAME: ....

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

ALEX Q11 + 14(b)

YR 12
Ext^n 1 2013
Trial Solutions

Section I

Questions 1-10 (1 mark each)

Question	Answer	Solution	Outcomes
1.	**A**	$\lim\limits_{x\to 0}\left(\dfrac{\sin\frac{1}{3}x}{2x}\right)=\frac{1}{6}\lim\limits_{x\to 0}\left(\dfrac{\sin\frac{1}{3}x}{\frac{1}{3}x}\right)=\frac{1}{6}\times 1=\frac{1}{6}$	H5
2.	**D**	$\dfrac{d}{dx}\left(2^x\right)=\dfrac{d}{dx}\left(e^{\log_e 2^x}\right)=\dfrac{d}{dx}\left(e^{x\log_e 2}\right)=e^{x\log_e 2}\log_e 2=2^x\log_e 2$	H5
3.	**C**	$\alpha\times\frac{1}{\alpha}\times\beta=-\frac{6}{2}\qquad\therefore\beta=-3$	PE3
4.	**C**	Let $\cancel{BD}$ CD $=x$ cm. Then $(10+x)x=12^2$ and $x>0$ $\therefore (x+18)(x-8)=0\qquad\therefore x=8\qquad CD=8$ cm	PE3
5.	**D**	The fact that gravity, g is constant indicates that the acceleration remains constant throughout flight.	HE3 Band 2-3
6.	**B**	**Question 6** B As it is an area to the y-axis we need $\int_1^2 f(y).dy$, so given $y=x^3-1$ we get $f(y)=\sqrt[3]{y+1}$. The area is $\int_1^2\sqrt[3]{y+1}.dy$.	HE7 Band 4-5
7.	**B**	**Question 7** B The functions $a\sin(nt+\alpha)$, $a\cos(nt+\alpha)$, $a\sin(nt+\alpha)-a\cos(nt+\alpha)$ are all functions that represent SHM. Therefore, $a\tan(nt+\alpha)$ does not represent SHM.	HE3 Band 4-5
8.	**D**	$\int\dfrac{dx}{\sqrt{4-x^2}}=\int\dfrac{dx}{\sqrt{2^2-x^2}}$. Hence using standard integrals $\int\dfrac{dx}{\sqrt{2^2-x^2}}=\sin^{-1}\left(\dfrac{x}{2}\right)+C$	HE4 Band 3-4
9.	**B**	$\dfrac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)=\dfrac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right)=\dfrac{-1}{1+x^2}$	HE4
10.	**B**	$\lim\limits_{x\to-\infty}\left(\dfrac{e^x-2}{e^x+2}\right)=\dfrac{0-2}{0+2}=-1$. Hence $y=-1$ is an asymptote as $x\to-\infty$.	H5

4(a) (i)

$a=(x-1)$

$\dfrac{d\left(\frac{1}{2}v^2\right)}{dx}=(x-1)$

$\frac{1}{2}v^2=\frac{1}{2}(x-1)^2+C_1$ ✓

When $x=0, v=1$

$C_1=0$ and

$\frac{1}{2}v^2=\frac{1}{2}(x-1)^2$ ✓

$v^2=(x-1)^2$

HE5

HE5 Band 5-6

- Correctly finds $C=0$ and shows the final result 2
- Uses $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$ 1

(ii)

$\left(\dfrac{dx}{dt}\right)^2=(x-1)^2$

$\dfrac{dx}{dt}=\pm(x-1)$ ✓

Since, $v=1$ when $x=0$

$\dfrac{dx}{dt}=-(x-1)$ ✓

$\displaystyle\int\frac{dx}{(1-x)}=\int dt$

$-\ln(1-x)=t+C_2$ ✓

Since, $t=0,\ x=0$

$C_2=0,$

$1-x=e^{-t}$

$x=1-e^{-t}$ ✓

HE5

HE5 Band 4-5

- Correctly writes $x(t)$ 4
- Correctly integrates. 3
- Correctly shows $\dfrac{dx}{dt}=-(x-1)$ 2
- Uses $v=\dfrac{dx}{dt}$ 1

(b)

$P\left(\dfrac{mx_2+nx_1}{m+n},\dfrac{my_2+ny_1}{m+n}\right)$

$P\left(\dfrac{-1(5)+3(1)}{2},\dfrac{-1(2)+3(4)}{2}\right)$ ✓

$P(-1,5)$ ✓

$\boxed{\text{PE3.}}$

PE3 Band 3-4

- Gives the correct answer. 2
- Uses correct formula. 1

(c)

Putting $u=1+2x\quad\therefore du=2dx$

$\displaystyle\int\frac{6}{\sqrt{(1+2x)^3}}dx$

$\displaystyle=3\int\frac{2}{\sqrt{(1+2x)^3}}dx$

$\displaystyle=3\int\frac{du}{u^{\frac{3}{2}}}$ ✓

$\displaystyle=3\int u^{-\frac{3}{2}}du$

$=3\left[-2u^{-\frac{1}{2}}\right]+C$ ✓

$=-\dfrac{6}{\sqrt{(1+2x)}}+C$ ✓

HE5

HE7 Band 4-5

- Gives the correct answer. 3
- Correctly integrates. 2
- Correctly changes variable 1

11(d) (i) $P(x)=x^3+px^2+qx+5$ $P(2)=7, P(-3)=17$ $\therefore 8+4p+2q+5=7$ and $\therefore -27+9p-3q+5=17$ $4p+2q=-6$ and $9p-3q=39$ $2p+q=-3$ and $3p-q=13$ Adding these equations; $5p=10$ $p=2, q=-7$ PE3	PE3 Band 3-4 • Correctly solves equations for p and q. . . . . 3 • Determines simultaneous equations . . . 2 • Uses remainder theorem 1
(ii) $\therefore P(x)=x^3+2x^2-7x+5$ $P(4)=64+32-28+5$ $P(4)=73$	PE3 Band 3-4 • Gives the correct answer. 1
Question 12	
(a) (i) $(7-1)!=720$	PE3 Band 3-4 • Gives the correct answer. 1
(ii) Kevin and Julia sit together in $2\times 5!=240$ ways Hence, Kevin and Julia do not sit together in $720-240=480$ ways.	PE3 Band 4-5 • Gives the correct answer. . . . . 2 • Finds ways of sitting together. 1

Q12 (cont)

(b) Outcomes assessed : HE2

Marking Guidelines

Criteria	Marks
• defines an appropriate sequence of statements and verifies that the first is true	1
• shows that if $S(k)$ is true, then $(k+1)! > (k+1)e^k$	1
• deduces that since $k \geq 6$, $S(k)$ true implies $S(k+1)$ true, and completes the induction process	1

Answer

Let $S(n)$, $n=6,7,8,\ldots$ be the sequence of statements defined by $S(n):\ n! > e^n$

Consider $S(6)$: $\quad 6! = 720 > e^6 \approx 403\cdot 4 \quad$ Hence $S(6)$ is true.

If $S(k)$ is true: $\quad k! > e^k \quad **$

Consider $S(k+1)$:

$$\begin{aligned}(k+1)! &= (k+1)\,k! \\ &> (k+1)e^k \quad \textit{if } S(k) \textit{ is true using } ** \\ &> e\,.\,e^k \quad \textit{for } k\geq 6 \\ &= e^{k+1}\end{aligned}$$

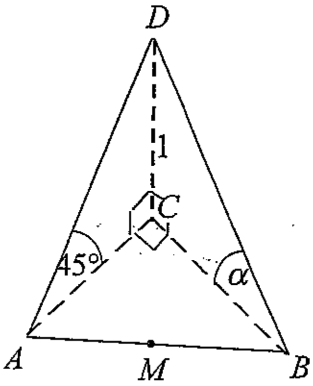
Hence if $S(k)$ is true, then $S(k+1)$ is true. But $S(6)$ is true, hence $S(7)$ is true and then $S(8)$ is true and so on. Therefore by Mathematical Induction, $n! > e^n$ for all integers $n \geq 6$.

6. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
i • writes down the length of AC and writes an expression for BC in terms of α	1
• uses Pythagoras' theorem in ΔABC to find AB in terms of α	1
ii • deduces that A, B, C lie on a circle with centre M	1
• uses equal radii to deduce result	1

Answer



i. In ΔABC : $AC = 1$, $BC = \cot\alpha$ and $\angle ACB = 90°$

$\therefore AB^2 = 1 + \cot^2\alpha = \operatorname{cosec}^2\alpha$

$AB = \operatorname{cosec}\alpha$

ii. A unique circle can be drawn through A, B and C. Since $\angle ACB = 90°$, AB is a diameter of this circle and hence M is its centre and CM, BM are radii.

$\therefore CM = BM = \frac{1}{2}\operatorname{cosec}\alpha$

Sample answer	Syllabus outcomes and marking guide
Question 12	
(a) (i) Let $\angle DBC = x$ $AB = AC$, (tangents drawn from an external point are equal) $\angle BAC = 2x$, (given) $\angle BAC = \angle BDC = 2x$, (given) $\angle BDC = \angle ABC = 2x$, (angle between the tangent to a circle and a chord through the point of contact is equal to the angle in the alternate segment) $\angle ACB = \angle ABC = 2x$, (base angles of isosceles triangle are equal) $6x = 180$, (angle sum of a triangle is 180) $x = 30$ hence, $\angle BCD = 90$, (angle sum of a triangle is 180) So, DB is diameter of circle as angle in semi-circle is 90.	HE2 Band 5-6 • Correctly shows angle in semicircle is 90 degrees . . . . 4 • Using base angles of isosceles triangle . . . . 3 • Uses angle in alternate segment . . . . . . 2 • States $AB = AC$ 1
(ii) ΔABC is an equilateral triangle (all angles are equal to 60°) $BC = AB$, (opposite sides of an equilateral triangle are equal)	HE2 Band 4-5 • Gives the correct answer. 1

Q13(a) Outcomes assessed : PE4, H5

Marking Guidelines

Criteria	Marks
i • uses differentiation to show tangent has gradient t	1
• finds equation of tangent	1
ii • writes expression for perpendicular distance from F to the tangent and simplifies	1

Answer

i.

$$y = at^2 \Rightarrow \frac{dy}{dt} = 2at$$

$$x = 2at \Rightarrow \frac{dx}{dt} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = t$$

Hence tangent at $P(2at, at^2)$ has gradient t

and equation $y - at^2 = t(x - 2at)$

$$tx - y - at^2 = 0.$$

ii. $\perp$ distance from $F(0, a)$ to line $tx - y - at^2 = 0$ is $d = \dfrac{|0 - a - at^2|}{\sqrt{t^2 + (-1)^2}} = \dfrac{a(1+t^2)}{\sqrt{1+t^2}} = a\sqrt{1+t^2}$

b. Outcomes assessed : HE4, H8

Marking Guidelines

Criteria	Marks
i • states domain	1
ii • sketches curve with correct shape and x-intercept	1
• shows endpoints of curve	1
iii • writes definite integral for V	1
• expands and finds primitive function	1
• evaluates by substitution of limits	1

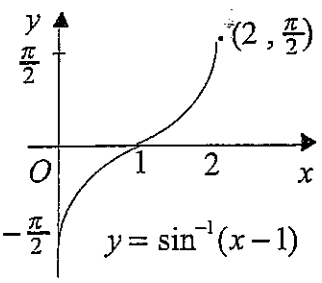
Answer

$f(x) = \sin^{-1}(x-1)$

i. Domain : $-1 \le x - 1 \le 1$

$\{x : 0 \le x \le 2\}$

ii.



iii.

$$V = \pi\int_0^{\frac{\pi}{2}} (1 + \sin y)^2 \, dy$$

$$= \pi\int_0^{\frac{\pi}{2}} \left\{1 + 2\sin y + \tfrac{1}{2}(1 - \cos 2y)\right\} dy$$

$$= \pi\left[\tfrac{3}{2}y - 2\cos y - \tfrac{1}{4}\sin 2y\right]_0^{\frac{\pi}{2}}$$

$$= \pi(\tfrac{3\pi}{4} + 2)$$

Volume is $\pi(\frac{3\pi}{4} + 2)$ cubic units.

Q13 (cont)
c. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • writes down the amplitude	1
• finds the period	1
ii • finds the initial position	1
• determines the initial direction of travel	1
iii • finds the position of the particle when $t=3$	1
• determines the distance travelled in the first three seconds.	1

Answer

$x=4\sqrt{2}\cos\left(\frac{\pi}{4}t-\frac{\pi}{4}\right)$, $\dot{x}=-\pi\sqrt{2}\sin\left(\frac{\pi}{4}t-\frac{\pi}{4}\right)$

i. Amplitude $4\sqrt{2}$ m, period $2\pi\div\frac{\pi}{4}=8$ s

ii. $t=0\Rightarrow x=4$, $v=\pi>0$. Particle is initially 4m to the right of O and moving away from O.

iii. Particle first reaches its positive extreme at $x=4\sqrt{2}$ when $t=1$. In the next 2 seconds ($\frac{1}{4}$ period) the particle travels from this extreme back to O. Hence the distance travelled in the first three seconds is $\left(4\sqrt{2}-4\right)+4\sqrt{2}=8\sqrt{2}-4$ metres.

Q14(a)

(i) $A\sin(x+\alpha)=A\sin x\cos\alpha+A\cos x\sin\alpha$
$=5\sin x+12\cos x$ (2)

2 marks for correct Answer

$\therefore$ $A\cos\alpha=5$ and $A\sin\alpha=12$

1 mark for α correct only

$\dfrac{A\sin\alpha}{A\cos\alpha}=\dfrac{12}{5}=\tan\alpha$

1 mark for A correct only

$\therefore$ $\alpha=\tan^{-1}\frac{12}{5}$, $\alpha=1.18^c$ (to 2d.p.)

$A=\sqrt{5^2+12^2}$
$=13$

$\therefore$ $5\sin x+12\cos x=13\sin(x+1.18^c)$

(ii) $5\sin x+12\cos x=8$ (2)

2 marks for correct answer in radians

ie. $13\sin(x+1.18^c)=8$

2 marks for correct answer calculated from wrong answer in

$\sin(x+1.18^c)=\frac{8}{13}$

1 mark for an error in an otherwise correct solution.

$x+1.18^c=0.66^c, 2.48^c, 6.94^c$

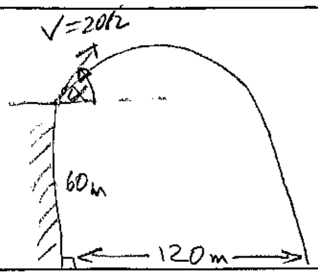
$\therefore$ $x=-0.52, 1.30, 5.76$

$\therefore$ $x=1.30$ for $0\leq x\leq\pi$ $\{\pi=3.14^c\}$

Q14 (cont)
b. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • writes equation for $\tan\alpha$	1
• solves for $\tan\alpha$ to deduce required values of angle of projection	1
ii • finds exact value of $\cos\alpha$ given $\tan\alpha$	1
• uses this value to find exact time to hit the ground	1
iii • finds expressions for horizontal and vertical components of velocity	1
• uses Pythagoras' theorem to find the speed	1



Answer

HE3

i. $x=20\sqrt{2}\,t\cos\alpha$, $y=20\sqrt{2}\,t\sin\alpha-5t^2$
Ground is 60m below O.
When $x=120$, $y=-60$.

$$20\sqrt{2}\sin\alpha\left(\frac{120}{20\sqrt{2}\cos\alpha}\right)-5\left(\frac{120}{20\sqrt{2}\cos\alpha}\right)^2=-60$$

$$120\tan\alpha-5\times6\times3\sec^2\alpha=-60$$

$$4\tan\alpha-3(1+\tan^2\alpha)=-2 \ ✓$$

$$3\tan^2\alpha-4\tan\alpha+1=0$$

$$(\tan\alpha-1)(3\tan\alpha-1)=0 \ ✓$$

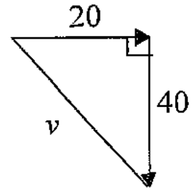
$$\therefore\ \tan\alpha=1 \quad\text{or}\quad \tan\alpha=\tfrac{1}{3}$$

$$\alpha=\tfrac{\pi}{4} \qquad\qquad \alpha=\tan^{-1}\tfrac{1}{3}$$

ii. $\tan\alpha=\frac{1}{3}\Rightarrow\sec^2\alpha=\frac{10}{9}$, $\cos\alpha=\frac{3}{\sqrt{10}}$ ✓
Particle hits ground when

$$t=\frac{120}{20\sqrt{2}\cos\alpha}=\frac{6}{\sqrt{2}}\cdot\frac{\sqrt{10}}{3}=2\sqrt{5}\ \text{s} \ ✓$$

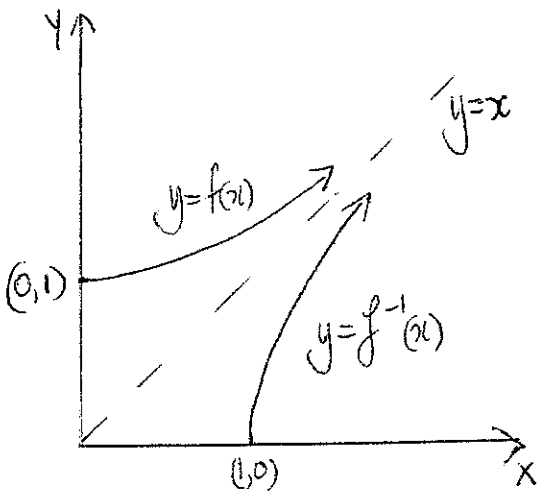
iii. $\alpha=\frac{\pi}{4}\Rightarrow\dot{x}=20$ and $\dot{y}=20-10t$
Then $t=6\Rightarrow\dot{x}=20$ and $\dot{y}=-40$ ✓



$$v=\sqrt{20^2+40^2}=20\sqrt{5}\ \text{ms}^{-1} \ ✓$$

(c) (i) $f(x)=x+e^{-x}$, $x\geq 0$
$f'(x)=1-e^{-x}>0$ for $x>0$ (1)
$\therefore$ Function is increasing for $x>0$
$f''(x)=e^{-x}>0$ for $x>0$ (1)
Hence curve is concave up for $x>0$.

(ii) & (iii)



(1) for correct graph of $f(x)$ showing endpoint $(0,1)$
(1) for showing oblique asymptote $y=x$
(1) for correctly sketching $f^{-1}(x)$.