

# Carlingford High School



## Mathematics Extension 1

### HSC Assessment Task 1

Term 4 2016

Time allowed: 55 minutes

Name: \_\_\_\_\_ Class: 11MA1\_\_

Please circle:            Ms Strilakos            Mr Fardouly            Mr Cheng

#### Instructions:

- Start a **new** page for every question
- Use black pen. Pencil may be used for graphs and diagrams.
- Board approved calculators may be used.
- Show all necessary working on your paper.
- Marks may be deducted for illegible or badly set out work.

Topic	Q1	Q2	Q3	Total
Polynomials	/12			
Parametric Equations		/12		
Series			/13	
Total				/37

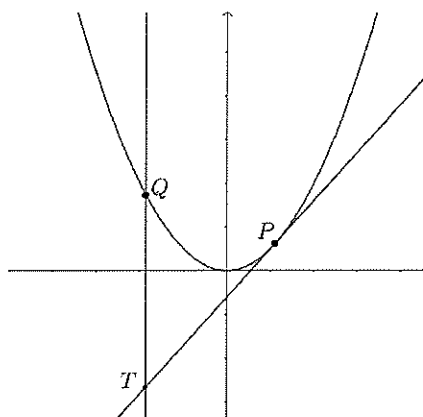
### Question 1

- a) Find the remainder when the polynomial  $P(x) = x^5 - 10x^3 + 5x$  is divided by  $x - 3$ . 1
- b) The polynomial  $f(x) = 8x^3 + 12x^2 - 18x - 20$  has a zero at  $x = -2$ .
- i) Find all roots of the polynomial equation  $f(x) = 0$ .  
(Leave your answer in **exact** form.) 2
  - ii) Sketch the graph of the curve, showing all axis intercepts. 1
  - iii) For what values of  $x$  is  $f(x) > 0$ ? 2
- c) Find the value of  $k$  if  $x - 3$  is a factor of  $P(x) = x^3 - 3kx + 6$ . 1
- d) The roots of the equation  $x^3 - 8x^2 + kx + 2 = 0$  are given by  $\alpha$ ,  $\beta$  and  $\frac{1}{\alpha}$
- i) Find the value of  $\beta$  1
  - ii) Find the two possible values of  $\alpha$ . 2
  - iii) Hence find the value of  $k$ . 2

### Question 2 (Please start a new page)

- a) Find the Cartesian equation of the curve with parametric equations 3
- $$x = 3 + 2\cos t, y = 2\sin t$$
- and describe its locus geometrically.
- b) Consider the parabola  $x^2 = -12y$ .
- i) Write down a set of parametric equations which describe this curve. 1
  - ii)  $P(2, -\frac{1}{3})$  is a point on the parabola. Find the coordinates of the point  $Q$  so that  $PQ$  is a focal chord. 2

- c) Two points  $P(2ap, ap^2)$  and  $Q(2aq, aq^2)$  lie on the parabola  $x^2 = 4ay$ . The point  $T$  is the intersection between the tangent at  $P$  and the line through  $Q$  parallel to the  $y$ -axis.



- i) Show that the equation of the tangent to the parabola at  $P$  is  $y = px - ap^2$ . 2
- ii) Find the coordinates of the point  $T$ . 2
- iii) Find the coordinates of  $M$ , the midpoint of  $PT$ . 1
- iv) Determine the locus of  $M$  when  $pq = -1$ . 1

### Question 3 (Please start a new page)

- a) The first three terms of a geometric sequence are 2,  $b$ , 50. Write down two possible values for  $b$ . 1
  
- b) Consider the arithmetic sequence 12, 8.5, 5, ...
  - i) Write an expression for the general term  $T_n$ . 1
  - ii) Find the value of  $m$  if  $m$  is the smallest integer such that 2

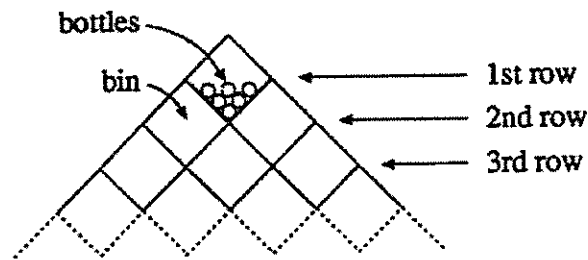
$$\sum_{i=1}^m T_i < 0$$
  
- c) i) For which values of  $t$  does the geometric series 2

$$3t + \frac{3}{4}t^3 + \frac{3}{16}t^5 + \dots$$

have a limiting sum?

  
- ii) Hence find an expression for the limiting sum. 1

- d) A bottle storage rack consists of  $n$  rows of bins stacked in such a way that the number of bins in the  $r$ th row is  $r$ , counting from the top.



- i) Show that the total number of bins in the storage rack is  $\frac{1}{2}n(n+1)$ . 1
- ii) Each bin in the  $r$ th row contains  $r+5$  bottles, so a bin in the top row contains 6 bottles, a bin in the second row contains 7 bottles and so on. Write an expression for the total number of bottles in the storage rack, using sigma notation. 1
- iii) Show that the total number of bottles in the storage rack is  $\frac{1}{3}n(n+1)(n+8)$ . 2  
 (You may assume that  $1^2 + 2^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ .)
- iv) The average number of bottles per bin in the storage rack is 10. Calculate the number of rows in the storage rack. 2

**End of Exam (Please check your work)**