Carlingford High School

Mathematics Extension 2

Year 12

HALF YEARLY EXAM

Term 1 2014

Time Allowed: 2 hours

Name:______________________ Teacher: Ms Kellahan

- Use the Multiple Choice Answer Sheet for section One
- Section Two answer each question in a new booklet.
- Do not work in columns.
- Marks may be deducted for careless or badly arranged work.
- Write in blue or black pen. Diagrams and graphs maybe done in pencil.
- Only calculators approved by the Board of Studies may be used.
- There is to be NO LENDING OR BORROWING.
- Write your name on every booklet.
- A Standard Integral sheet is attached.

	MC	Q6	Q7	Q8	Q9	Mark
E3	/2	/12	/5			/19
E4	/2	/3	/3	/17	/8	/33
E6	/1		/8		/9	/18
Total	/5	/15	/16	/17	/17	/70

E3 Uses the relationship between algebraic and geometric representations of complex numbers and of conic sections

E4 Uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials

E6 Combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions

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Section 1

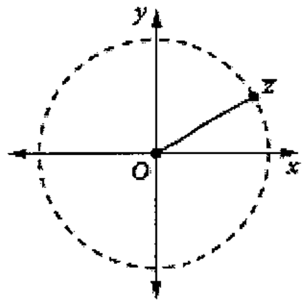
Multiple Choice – Use answer sheet **(5 marks)**

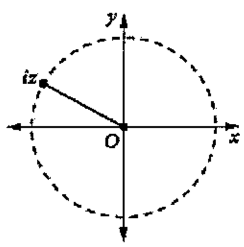
1. The Cartesian equation of the curve whose parametric equations are $x = 2\sec\theta$ and $y = \tan\theta$ is:

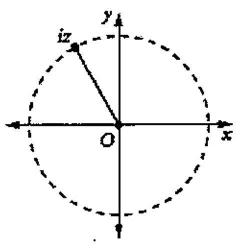
A. $x^2 - \dfrac{y^2}{4} = 1$ B. $\dfrac{x^2}{4} - y^2 = 1$

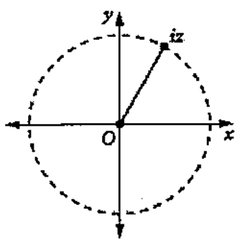
C. $\dfrac{x^2}{4} + y^2 = 1$ D. $x^2 - y^2 = 1$

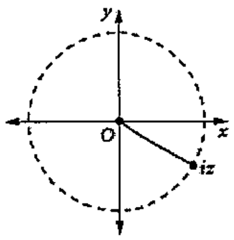
2. The complex number $\overline{z}$ is shown on the Argand diagram. Which of the following best represents iz?



A


B


C


D


3. The equation $x^3 - y^3 + 2xy - 5 = 0$ defines y implicitly as a function of x.

What is the value of $\dfrac{dy}{dx}$ at the point (1, 2)?

A. 14   B. $\dfrac{7}{10}$ C. $\dfrac{11}{4}$ D. $\dfrac{10}{7}$

4. Thirteen students in a Year 10 PDHPE class are to be divided into two teams of six to play touch football, with the remaining person acting as the referee. If two particular students are not to be in the same team, the number of different ways the team can be formed is:

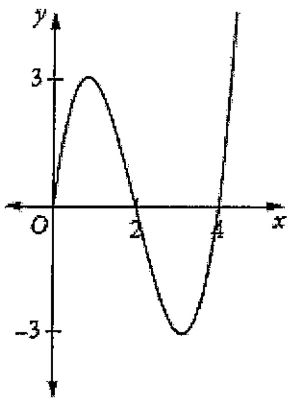
A. ${}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6 \times 2$

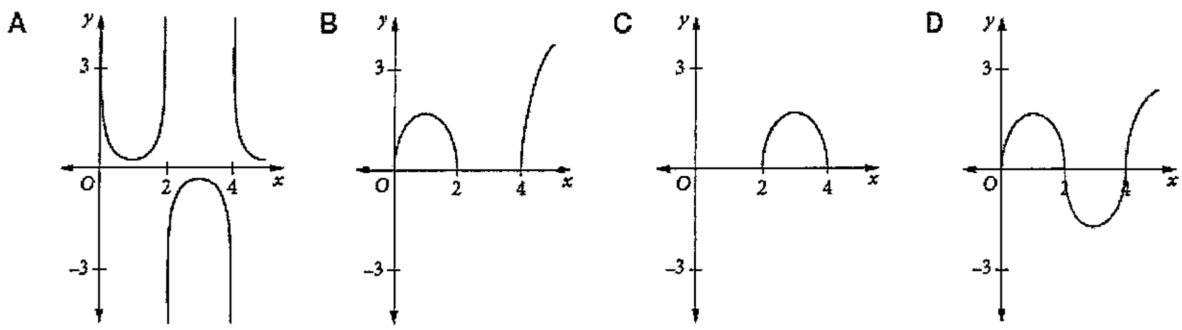
B. ${}^{11}C_5 \times {}^{6}C_5 \times 2 + {}^{12}C_6$

C. ${}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6$

D. ${}^{13}C_6 \times {}^{7}C_6 \div 2$

5. Given the graph of the function $y = f(x)$ shown below, which of the following diagrams could be the graph of $y = \sqrt{f(x)}$ equation of the graph below in relation to $y = f(x)$ is:





END OF SECTION 1

Section 2

Question Six – Start in a new booklet **(15 marks)** **Marks**

a) If $z=2+5i$ and $w=-2-3i$ find:

i) $w+\overline{z}$ **1**

ii) $\frac{w}{\overline{z}}$ **1**

b) Find all the complex numbers $z = a + ib$, a and b real, such that: **3**

$$|z|^2 - iz = 16 - 2i$$

c) In an Argand diagram, a regular hexagon ABCDEF, with the vertices taken in anticlockwise order, has its centre at the origin O and vertex A at $z=2$.

i) Find the set of values $\text{Im}(z)$ for points z on the hexagon. **1**

ii) Find the set of values $|z|$ for points z on the hexagon. **1**

iii) If the hexagon is rotated in a clockwise direction about the origin through an angle of 45°, find the value in modulus / argument form of the complex number which is represented by the new position of the vertex C. **1**

d) i) Express $z = 1 + i$ in mod/arg form. Hence show that $z^9 = 16z$ **2**

ii) Express $(1+i)^9 + (1-i)^9$ in rectangular form where a and b are real. **2**

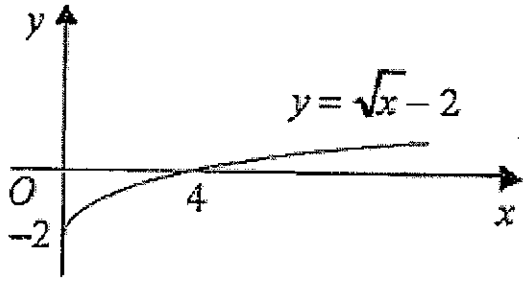
e) The equation $z^2 + (1-2i)z - (7+i) = 0$ has roots α and β.

i) Find the monic equation with numerical coefficients whose roots are $\alpha - i$ and $\beta - i$. **2**

ii) Find the values of α and β. **1**

Question Seven – Start in a new booklet (16 marks) Marks

a) This diagram shows the graph of the function $f(x)=\sqrt{x}-2$.
On separate diagrams sketch the following graphs showing any intercepts on the co-ordinate axes and the equations of any asymptotes.



i) $y=\{f(x)\}^2$ **1**

ii) $y=\log_e f(x)$ **2**

b) Given P(x) = (x + 2)(x – 1)(x – 3)

i) Sketch P(x) showing the intercepts on the coordinate axes. **1**

ii) Without using calculus, draw separate one-third page of the graph of each of the following functions. Indicate clearly any asymptotes and intercepts with axes:

α) $y=|P(x)|$ **1**

β) $y=P(|x|)$ **1**

γ) $y=\dfrac{1}{P(x)}$ **2**

c) i) Use De Moivre's theorem to show that $\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$ **2**

ii) Hence find the exact value of $\cos^4\left(\dfrac{\pi}{12}\right)+\sin^4\left(\dfrac{\pi}{12}\right)$ **3**

d) Consider the polynomial P(x) = $x^3 - x^2 - 21x + 45$ with roots α, β, and γ.

i) Find the monic polynomial with roots $\alpha - 3$, $\beta - 3$, and $\gamma - 3$ **2**

ii) Hence solve P(x) = 0 **1**

Question Eight- Start in a new booklet (17marks) Marks

a) Find the constants A, B, C and D such that: **2**

$$\frac{x^3+2x^2+4x+2}{(x^2+1)(x^2+4)} \equiv \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)}$$

b) $P(x)=x^4+3x^3-6x^2-28x+c$ has a zero of multiplicity 3. **3**
Find the value of c.

c) An ellipse is defined by the parametric equations:

$$x=2\cos\theta$$
$$y=3\sin\theta$$

for $0\le\theta\le2\pi$.

i) Find the Cartesian equation of the ellipse. **1**

ii) Find the eccentricity of the ellipse **1**

iii) Sketch the ellipse showing the intercepts, foci and directrices. **3**

d) The hyperbola, H has the Cartesian Equation $5x^2-4y^2=20$ has asymptotes at $y=\pm\frac{\sqrt{5}}{2}x$. P is an arbitrary point $(2\sec\theta, \sqrt{5}\tan\theta)$ that lies on H.

i) Show that the tangent to H at P is: **2**

$$\frac{x\sec\theta}{2}-\frac{y\tan\theta}{\sqrt{5}}=1$$

ii) The tangent at P cuts the asymptotes at L and M. **3**
Prove $LP = PM$.

iii) O is the origin.
Show that the area of ΔOLM is independent of the position P on H. **2**

Question Nine- Start in a new booklet (18 marks) Marks

a) For the rectangular hyperbola $H: xy = c^2$.

$P(cp, \frac{c}{p})$ and $Q(cq, \frac{c}{q})$ are two points on the hyperbola.

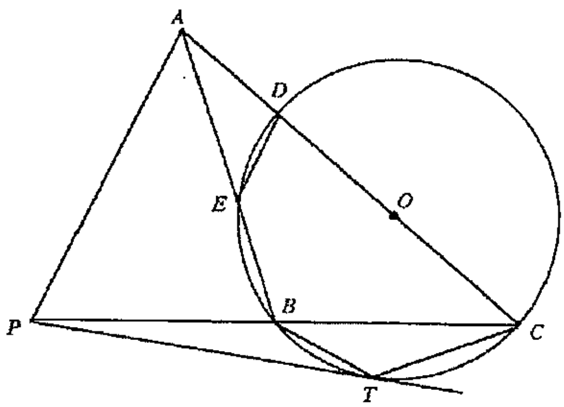
i) Find the equation of the chord PQ. **2**

ii) Hence determine the equation of the focal chord. **2**

iii) The tangent at P is given is $x + p^2y = 2cp$. (Do not prove this). **2**
This tangent cuts the x and y axes respectively at the points A and B while O is the origin.
Show that the area of the triangle AOB is constant.

iv) If R is a vertex of a rectangle OARB, find the equation of the locus of R. **3**
Describe the locus geometrically.

b)



A is a point outside a circle with centre O.
P is a second point outside the circle such that PT = PA where PT is a tangent to the circle at T.
AO cuts the circle at D and C.
PC cuts the circle at B.
AB cuts the circle at E.

i) Copy or trace the diagram on to your paper **1**

ii) Show that ΔPBT ||| Δ PTC **2**

ii) Show that ΔAPB ||| Δ CPA **3**

iii) Hence prove DE is parallel to AP **3**

END OF EXAM

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Half Yearly HSC – Mathematics Extension 2 2014

Section I – Multiple Choice Answer Sheet

Name_______________________________.

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✖ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ✖ B ✖ correct C ○ D ○

Start Here → **1.** A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

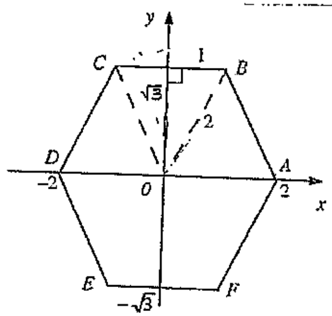
3. A ○ B ○ C ○ D ○

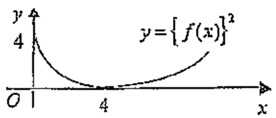
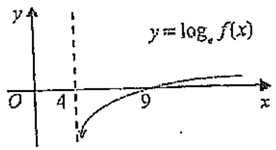
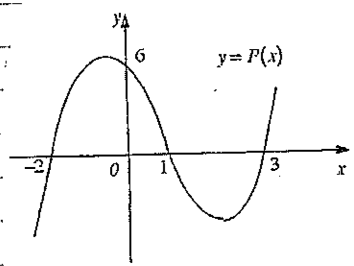
4. A ○ B ○ C ○ D ○

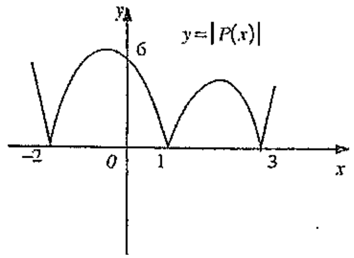
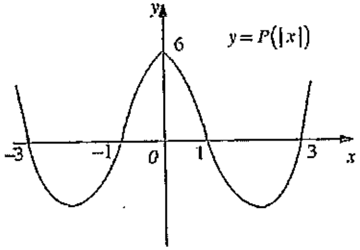
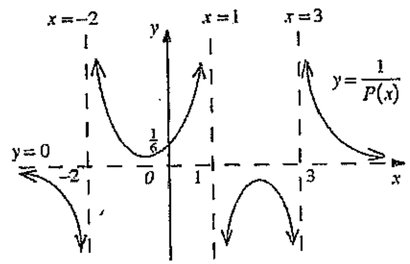
5. A ○ B ○ C ○ D ○

Extension 2 Mathematics
Half-yearly Solutions

Q	Solution: Multiple Choice.	M	O
1	B	1	E3
2.		1	E3
3.	B	1	E4
4.	C total ways $= {}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6 \div 2 \times 2$	1	E4
	$= {}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6$		
5.	B	1	E6.
	Question Six.		
a)	i) $w + \bar{z} = -2 - 3i + (2 - 5i)$		
	$= -8i$	1	E3
	ii) $\frac{w}{\bar{z}} = \frac{-2-3i}{2-5i} \times \frac{2+5i}{2+5i}$		
	$= \frac{-4-10i-6i+15}{4+25}$		
	$= \frac{11-16i}{29}.$	1	E3.
b)	$z = a + ib$ a, b real		E3
	$\lvert z\rvert^2 - iz = a^2 + b^2 - ia + b$		
	$\therefore 16 - 2i = (a^2 + b^2 + b) - ia$		
	Equating imaginary: $a = 2$.	1	
	Equating real: $a^2 + b^2 + b = 16$		
	$4 + b^2 + b = 16$		
	$b^2 + b - 12 = 0$	1	
	$(b+4)(b-3) = 0$		
	$\therefore a = 2$ & $b = -4$ or $a = 2$ & $b = 3$		
	$\therefore z = 2 - 4i$ or $z = 2 + 3i$.	1	E3

c)			E3
	i) $-\sqrt{3} \le \text{Im}(z) \le \sqrt{3}$	1	
	ii) $\sqrt{3} \le \lvert z\rvert \le 2$	1	
	iii) All triangles are equilateral with sides 2 units		
	$\angle AOC = 2 \times \frac{\pi}{3}$		
	$= \frac{2\pi}{3}$		
	$2\,\text{cis}\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$ After rotation thr' $\frac{\pi}{4}$, OC makes an angle of $\frac{5\pi}{12}$ with the positive x-axis.		
	$\therefore C = 2\,\text{cis}\frac{5\pi}{2}.$	1.	
d)	i) $z = 1 + i$		E3
	$z = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$		
	$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$	1	
	$z^9 = (\sqrt{2})^9\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)$		
	$= 16\sqrt{2}\left[\cos\left(2\pi + \frac{\pi}{4}\right) + i\sin\left(2\pi + \frac{\pi}{4}\right)\right]$		
	$= 16\left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]$	1	
	$= 16z.$		
	ii) $(1+i)^9 + (1-i)^9 = z^9 + \bar{z}^9$		E3
	$= z^9 + \overline{z^9}$	1	
	$= 16(z + \bar{z})$		
	$= 16 \times 2\,\text{Re}\,z$		
	$= 32.$	1	

			M	O
e)	i) $P(z) = z^2 + (1-2i)z - (7+i)$		½	E4
	$P(z) = 0$ has roots α and β.			
	$\therefore P(z+i) = 0$ has root $\alpha - i$, $\beta - i$.			
	$P(z+i) = (z+i)^2 + (1-2i)(z+i) - (7+i)$		1	
	$= z^2 + 2iz - 1 + z + i - 2iz + 2 - 7 - i$			
	$= z^2 + z - 6$		1	
	ii) $\therefore P(z+i) = 0$ has roots $-3, 2$.			
	$\therefore \alpha$ and β are $-3+i$, $2+i$		1	E4
	Question Seven.			
a)	i) 	correct shape and asymptotes	1	E6
	ii) 	1 correct shape and position & behaviour near asymptote. 1 shows x-intercept.	2	E6
b)	i) 	shows correct shape & intercepts.	1	E6

7b)	ii) α) 	correct shape & intercepts.	1	E6
	β) 	correct shape & intercepts	1	E6
	γ) 	1 asymptotes & intercepts. 1 correct shape	2	E6
7c)	i) Using De Moivre's $\cos 4\theta = \text{Re}(\cos\theta + i\sin\theta)^4$			E3
	$\therefore \text{Re}(\cos\theta + i\sin\theta)^4 = \cos^4\theta + {}^4C_2\cos^2\theta(i\sin\theta)^2 + (i\sin\theta)^4$		1	
	$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$		1	
	ii) $\cos^4\theta + \sin^4\theta = \cos 4\theta + 6\cos^2\theta\sin^2\theta$			E3
	$= \cos 4\theta + \frac{3}{2}\sin^2 2\theta$		1	
	$\cos^4\frac{\pi}{12} + \sin^4\frac{\pi}{12} = \cos\frac{4\pi}{12} + \frac{3}{2}\sin^2\frac{2\pi}{12}$		1	
	$= \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$		1	

7		M	O
7d	i) $P(x) = x^3 - x^2 - 21x + 45$ $\quad \alpha - 3 = x$		E4
	$\therefore \alpha = x + 3$		
	$\therefore$ monic polynomial :		
	$(x+3)^3 - (x+3)^2 - 21(x+3) + 45$	1	
	$= x^3 + {}^3C_1 3x^2 + {}^3C_2 3^2 x + 27 - x^2 - 6x - 9 - 21x - 63 + 45$		
	$= x^3 + 9x^2 + 27x + 27 - x^2 - 6x - 9 - 21x - 63 + 45$		
	$= x^3 + 8x^2$	1	
	ii) $x^3 + 8x^2 = x^2(x+8)$ has roots $\alpha - 3 = 0$		E4
	$\beta - 3 = 0$		
	$\gamma - 3 = -8$		
	$\therefore \alpha = 3,\ \beta = 3 \quad \gamma = -5$		
	$\therefore x = 3, -5$ are the solutions to $P(x) = 0$	1	

	Question 8	M	
a)	i) $\dfrac{x^3 + 2x^2 + 4x + 2}{(x^2+1)(x^2+4)} \equiv \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{x^2+4}$		E
	$x^3 + 2x^2 + 4x + 2 \equiv (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$		
	equating x^3 coeff : $1 = A + C$ ①		
	equating x coeff : $4 = 4A + C$ ②		
	② − ① $3 = 3A$		
	$\therefore A = 1$.		
	$C = 0$	1	
	equating x^2 coeff : $2 = B + D$ ③		
	sub $x = 0$: $2 = 4B + D$ ④		
	④ − ③ $0 = 3B$		
	$\therefore B = 0$		
	$D = 2$.	1	
b)	$P(x) = x^4 + 3x^3 - 6x^2 - 28x + c$		E
	$P'(x) = 4x^3 + 9x^2 - 12x - 28$		
	$P''(x) = 12x^2 + 18x - 12$		
	$= 6(x+2)(2x-1)$	1	
	$\therefore P''(x) = 0$ when $x = -2, \frac{1}{2}$.		
	Now $P'(\frac{1}{2}) \neq 0$		
	$P'(-2) = 0$		
	$\therefore x = -2$ is the zero of multiplicity 3.	1	
	$P(-2) = 0$		
	$\therefore -c = 16 + -24 - 24 + 56$		
	$\therefore c = -24$.	1	

8c) i) $x = 2\cos\theta \qquad y = 3\sin\theta$

$\cos\theta = \frac{x}{2} \qquad \sin\theta = \frac{y}{3}$

$\cos^2\theta + \sin^2\theta = 1$

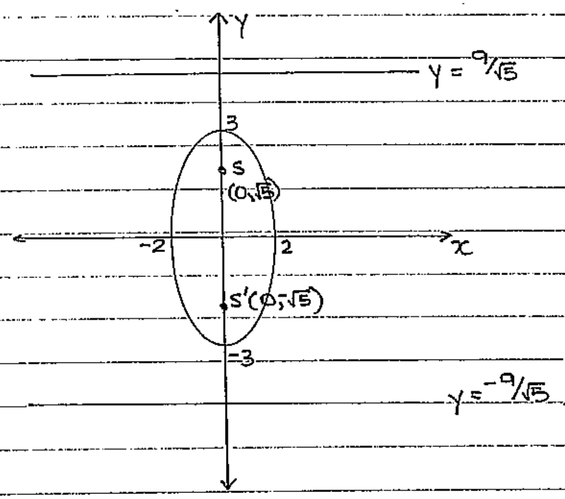
$\therefore \frac{x^2}{4} + \frac{y^2}{9} = 1$ — 1 — E4

ii) $4 = 9(1-e^2) \qquad b > a.$ — E4

$e = \frac{\sqrt{5}}{3}$ — 1.

iii) $S(0, \pm be) = \left(0, \pm 3 \times \frac{\sqrt{5}}{3}\right)$ — 3 — E4

$= (0, \pm\sqrt{5})$



-1 for missing item

-1 2nd missing item.

8d) i) $5x^2 - 4y^2 = 20$ — E4

$10x - 8y\frac{dy}{dx} = 0$ by implicit differentiation.

$\therefore \frac{dy}{dx} = \frac{5x}{4y}$

At $P(2\sec\theta, \sqrt{5}\tan\theta)$ $\frac{dy}{dx} = \frac{10\sec\theta}{4\sqrt{5}\tan\theta}$

$= \frac{\sqrt{5}\sec\theta}{2\tan\theta}$ — 1.

Eqn of tangent : $y - \sqrt{5}\tan\theta = \frac{\sqrt{5}\sec\theta}{2\tan\theta}(x - 2\sec\theta)$

$2y\tan\theta - 2\sqrt{5}\tan^2\theta = \sqrt{5}x\sec\theta - 2\sqrt{5}\sec^2\theta$

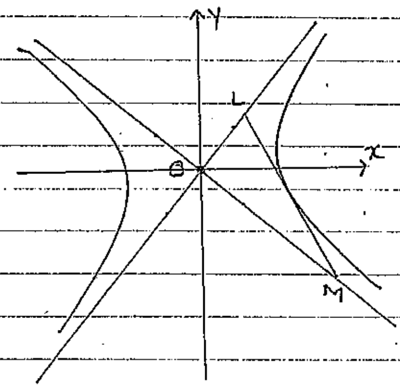
$-2y\tan\theta + \sqrt{5}x\sec\theta = 2\sqrt{5}\sec^2\theta - 2\sqrt{5}\tan^2\theta$

$\sqrt{5}x\sec\theta - 2y\tan\theta = 2\sqrt{5}(\sec^2\theta - \tan^2\theta)$

$\sqrt{5}x\sec\theta - 2y\tan\theta = 2\sqrt{5}$

$\frac{1}{2}x\sec\theta - \frac{1}{\sqrt{5}}y\tan\theta = 1.$ — 1

ii) — E4



$\frac{1}{2}x\sec\theta - \frac{1}{\sqrt{5}}y\tan\theta = 1$

$y = \frac{1}{2}\sqrt{5}x$

$\therefore \frac{1}{2}x\sec\theta - \frac{1}{\sqrt{5}} \times \frac{1}{2}\sqrt{5}x\tan\theta = 1$

$\frac{1}{2}x\sec\theta - \frac{1}{2}x\tan\theta = 1$

$x\left(\frac{\sec\theta}{2} - \frac{\tan\theta}{2}\right) = 1$

$\therefore x = \frac{2}{\sec\theta - \tan\theta}$

$\therefore y = \frac{1}{2}\sqrt{5}x$

$= \frac{1}{2}\sqrt{5} \times \frac{2}{\sec\theta - \tan\theta}$

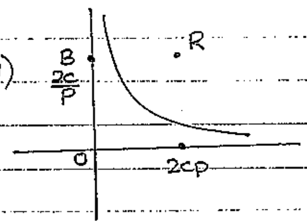
$= \frac{\sqrt{5}}{\sec\theta - \tan\theta}$

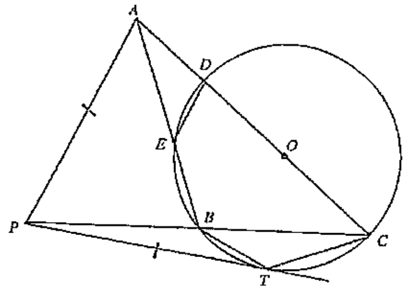
$\therefore L\left(\frac{2}{\sec\theta - \tan\theta}, \frac{\sqrt{5}}{\sec\theta - \tan\theta}\right)$ — 1

		M.	O
8d	ii) cont		
	Similarly solve $\frac{1}{2}x\sec\theta - \frac{1}{\sqrt{5}}y\tan\theta = 1$ and $y = -\frac{1}{2}\sqrt{5}x$		E4
	$\therefore \frac{1}{2}x\sec\theta - \frac{1}{\sqrt{5}}\times -\frac{1}{2}\sqrt{5}x\tan\theta = 1$		
	$\frac{1}{2}x\sec\theta + \frac{1}{2}x\tan\theta = 1$		
	$x\left(\frac{\sec\theta + \tan\theta}{2}\right) = 1$		
	$\therefore x = \frac{2}{\sec\theta + \tan\theta}$		
	$\therefore y = -\frac{1}{2}\sqrt{5}x$		
	$= -\frac{1}{2}\sqrt{5} \times \frac{2}{\sec\theta + \tan\theta}$		
	$= \frac{-\sqrt{5}}{\sec\theta + \tan\theta}$		
	$\therefore M\left(\frac{2}{\sec\theta + \tan\theta}, \frac{-\sqrt{5}}{\sec\theta + \tan\theta}\right)$	1.	
	Now if $LP = PM$, then P is the midpt.		
	$\therefore$ Midpt x $= \frac{1}{2}\left(\frac{2}{\sec\theta + \tan\theta} + \frac{2}{\sec\theta - \tan\theta}\right)$		
	$= \frac{1}{2} \times \frac{2\sec\theta - 2\tan\theta + 2\sec\theta + 2\tan\theta}{\sec^2\theta - \tan^2\theta}$		
	$= \frac{1}{2} \times \frac{4\sec\theta}{1}$		
	$= 2\sec\theta$		
	Midpt y $= \frac{1}{2}\left(\frac{-\sqrt{5}}{\sec\theta + \tan\theta} + \frac{\sqrt{5}}{\sec\theta - \tan\theta}\right)$		
	$= \frac{1}{2}\left(\frac{-\sqrt{5}\sec\theta + \sqrt{5}\tan\theta + \sqrt{5}\sec\theta + \sqrt{5}\tan\theta}{\sec^2\theta - \tan^2\theta}\right)$		
	$= \frac{1}{2} \times \frac{2\sqrt{5}\tan\theta}{1}$		
	$= \sqrt{5}\tan\theta$		

		M	E
8d	ii) cont		
	$\therefore$ Midpt $= (2\sec\theta, \sqrt{5}\tan\theta)$ which is P.	1	E
	$\therefore LP = PM$		
	iii) Area $\triangle OLM = \frac{1}{2}OL \times OM \times \sin(\angle LOM)$	1	E
	but $\frac{1}{2}OL \times OM$ = constant & is independent of θ		
	$\therefore$ independent of P.		
	Now $\angle LOM$ is a combination of angles that the asymptotes make with the x-axis.		
	i.e. $\tan^{-1}\frac{\sqrt{5}}{2}$ and $\pi - \tan^{-1}\left(-\frac{\sqrt{5}}{2}\right)$		
	$\therefore \sin\angle LOM$ is a contant and independent of θ so is independent of P.	1	

9.

9.		M	D
i)	$P\left(cp, \frac{c}{p}\right)$ & $Q\left(cq, \frac{c}{q}\right)$		E4
	$mPQ = \dfrac{\frac{c}{p} - \frac{c}{q}}{cp - cq}$		
	$= \dfrac{\frac{cq - cp}{pq}}{cp - cq}$		
	$= \dfrac{cq - cp}{-pq(cq - cp)}$		
	$= -\dfrac{1}{pq}$	1	
	Equation of chord:		
	$y - \frac{c}{p} = -\frac{1}{pq}(x - cp)$		
	$-pqy - \frac{c}{p} \times -pq = x - cp$		
	$\therefore x + pqy = cp + cq$	1	
ii)	$S(c\sqrt{2}, c\sqrt{2})$		E4
	$y - c\sqrt{2} = \frac{-1}{pq}(x - c\sqrt{2})$		
	$-pqy + pqc\sqrt{2} = x - c\sqrt{2}$		
	$x + pqy = c\sqrt{2} + pqc\sqrt{2}$	1	
iii)	$\Delta AOB = \frac{1}{2} OA \times OB$ $\quad$ $x + p^2y - 2cp = 0$		E4
	$= \frac{1}{2} \times 2cp \times \frac{2c}{p}$ $\quad$ $A: y = 0,\ x = 2cp$		
	$= 2c^2$ $\quad$ $B: x = 0,\ y = \frac{2c}{p}$	2	
	which is a constant.		
iv)	 $R\left(2cp, \frac{2c}{p}\right)$	1	E4
	$x = 2pc$		
	$\therefore p = \frac{x}{2c}$		
	$y = 2c \times \frac{2c}{x}$	1	
	$\therefore xy = 4c^2$ is a		

9b			E
i)	 Copied neatly & labelled	1.	E
ii)	In ΔPBT and ΔPTC:		
	$\angle BPT = \angle CPT$ common.		
	$\angle BTP = \angle TCP$ $\angle$ in alt. segment	1	
	$\therefore \Delta PBT \parallel\!\mid \Delta PTC$ (2 pairs corres. are equal)	1.	
iii)	In ΔAPB and ΔCPA		
	$\frac{PB}{PT} = \frac{PT}{PC}$ (corres sides in same ratio, $\Delta PBT \parallel\!\mid \Delta PTC$)	1	
	$\therefore \frac{PB}{PA} = \frac{PA}{PC}$ $(PT = PA)$	1	
	$\angle APB = \angle CPA$ (common angle)		
	$\Delta APB \parallel\!\mid \Delta CPA$ (2 pairs of sides in same ratio & included $\angle$ equal)	1	
iv)	$\angle PAE = \angle BCD$ (corres $\angle$ of $\parallel\!\mid$ ΔAPB, ΔCPA are =)	1	
	$\angle BCD = \angle DEA$ (ext. $\angle$ cyclic $\Delta BCDE$ is equal to interior opp. $\angle$)	1	
	$\therefore \angle PAE = \angle DEA$		
	$\therefore DE \parallel AP$ (equal alt. $\angle$'s on transversal AE)	1	