Carlingford High School
Mathematics Extension 2
Year 11

HSC ASSESSMENT TASK 1
Term 4 2015

Student Name: ______________________ **Teacher: Mr GonG**

- **Time allowed 60 minutes.**
- Start each question on a new page.
- **Do not** work in columns or back to back.
- Marks may be deducted for careless or badly arranged work.
- Only calculators approved by the Board of Studies may be used.
- All answers are to be completed in blue or black pen except graphs and diagrams.
- There is to be NO LENDING OR BORROWING.

	MC	Q6	Q7	Total	
Complex Numbers	/3	/15		/18	
Graphs	/2		/15	/17	
Total	/5	/15	/15	/35	%

Section 1

Multiple Choice – Start a new page (5 marks)

1. i^{-7} equals

A. 1 **B.** $-i$ **C.** i **D.** -1

2. Express $z = 1 + \sqrt{3}\,i$ in mod-arg form:

A. $z = 2cis\left(\frac{\pi}{3}\right)$ **B.** $z = 2cis\left(-\frac{\pi}{3}\right)$ **C.** $z = 2cis\left(-\frac{2\pi}{3}\right)$ **D.** $z = 2cis\left(\frac{2\pi}{3}\right)$

3. Find z^2, if $z = -2 - 3i$

A. $-5 - 12i$ **B.** $4 - 9i^2$ **C.** $7 - 12i$ **D.** $-5 + 12i$

4. If you start with the graph of $y = g(x)$, shift it left 1 unit, then down 2 units and then reflect it in the y-axis, what is the resulting equation?

A. $y = -g(x + 1) + 2$ **B.** $y = g(-x + 1) - 2$

C. $y = -g(x + 1) - 2$ **D.** $y = g(-x - 1) - 2$

5. The gradient of the function $x^3y^2 + x^3 + y = 6$ at the point $(1, 1)$ is

A. 2 **B.** $-\frac{3}{7}$ **C.** $\frac{3}{7}$ **D.** -2

Section 2

Question 6 **– Start a new page –** **(15 marks)** **Marks**

a) Let $z = \dfrac{i}{\sqrt{3} - i}$

i) Sketch z on an Argand diagram. **3**

ii) Find the modulus and argument of z . **2**

b) Solve the equation $z^4 = 2$. **3**

c) Given that $z = \cos\theta + i\sin\theta$, use De Moivre's theorem to express $\sin 5\theta$ in terms of $\cos\theta$ and $\sin\theta$. **3**

d) Draw a single Argand diagram to represent the following region **4**

$|z - 3 - 3i| < 3$ and $\dfrac{\pi}{4} \leq \arg z \leq \dfrac{\pi}{3}$.

Question 7 – Start a new page – (15 marks) **Marks**

a) i) Draw a sketch of $y = x(x-2)^2$. **2**

ii) Hence, or otherwise, sketch the curve whose equation is given by $y^2 = x(x-2)^2$ **2**

b) i) Prove that $\dfrac{(x-1)(x-5)}{x+3} = x - 9 + \dfrac{32}{x+3}$. **2**

ii) Sketch $y = \dfrac{(x-1)(x-5)}{x+3}$, clearly labelling both asymptotes and all intercepts. **2**

iii) Hence sketch the graphs of

α) $y = \left|\dfrac{(x-1)(x-5)}{x+3}\right|$ **2**

β) $y = \sqrt{\dfrac{(x-1)(x-5)}{x+3}}$ **2**

c) i) Sketch $y = \dfrac{9x - x^3}{4x^2 - 3}$ clearly labelling all essential features given that it has three asymptotes, one of which is $y = -\dfrac{x}{4}$. **2**

ii) How many solutions are there to the equation $\dfrac{9x - x^3}{4x^2 - 3} = k$ where k is a constant? (You do not need to actually find the solutions) **1**

END OF EXAM

2015

Term 4 HSC Task 1 Examination

Ext 2 Mathematics

Solutions

HSC Task 1 Examination – Ext 2 Mathematics 2015

Section I Multiple Choice Answer 1 Mark each

1. A ○ B ○ C ● D ○
2. A ● B ○ C ○ D ○
3. A ○ B ○ C ○ D ●
4. A ○ B ● C ○ D ○
5. A ○ B ○ C ○ D ●

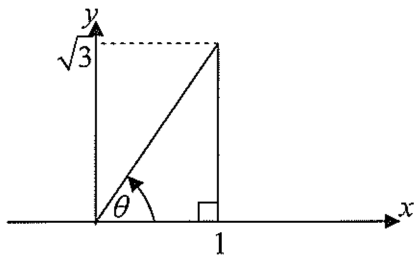
Working Out

1

$$i^{-7}=\frac{1}{i^7}=\frac{1}{i^2\times i^2\times i^2\times i}=\frac{1}{-1\times i}=-\frac{1}{i}$$
$$=-\frac{1}{i}\times\frac{i}{i}=-\frac{i}{-1}=i\rightarrow C$$

2

$z=1+\sqrt{3}i$



$$|z|=\sqrt{1^2+\left(\sqrt{3}\right)^2}$$
$$\therefore |z|=2$$
$$\tan\theta=\sqrt{3}$$
$$\therefore \quad \theta=\frac{\pi}{3}$$

Hence $z=2cis\left(\frac{\pi}{3}\right)\ \rightarrow A$

3

$$z^2=(-2-3i)(-2-3i)$$
$$=4+12i+9i^2$$
$$=4+12i-9$$
$$=-5+12i\ \rightarrow D$$

4

Since $y=g(x)$ then shift 1 unit to the left get $y=g(x+1)$ then 2 unit down gives $y=g(x+1)-2$, now reflect in the y-axis gives the resulting equation $y=g(-x+1)-2\quad\rightarrow B$

5

$$x^3y^2+x^3+y=6$$
$$3x^2y^2+2x^3y\frac{dy}{dx}+3x^2+\frac{dy}{dx}=0$$
$$\frac{dy}{dx}(2x^3y+1)=-3x^2y^2-3x^2$$
$$\frac{dy}{dx}=\frac{-3x^2y^2-3x^2}{2x^3y+1}$$

At (1, 1)

$$\frac{dy}{dx}=\frac{-3-3}{2+1}=\frac{-6}{3}=-2$$

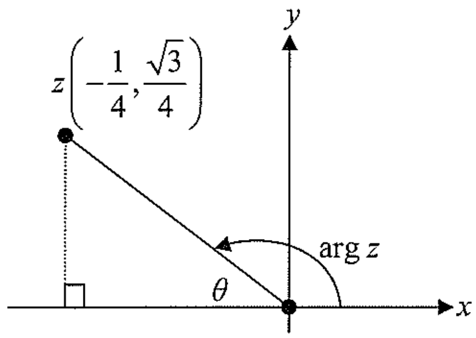
$\rightarrow D$

Section II Solutions

Question 6 [E3]

a) i)

$$\begin{aligned} z &= \frac{i}{\sqrt{3}-i}\times\frac{\sqrt{3}+i}{\sqrt{3}+i} \\ &= \frac{\sqrt{3}i+i^2}{3-i^2} \\ &= \frac{-1+\sqrt{3}i}{4} \\ &= -\frac{1}{4}+\frac{\sqrt{3}}{4}i \end{aligned}$$



2 mark for rationalise z correctly.

1 mark for correct diagram

ii)

$$\begin{aligned} |z| &= \sqrt{\frac{1}{16}+\frac{3}{16}} \\ &= \frac{1}{2} \end{aligned}$$

$$\tan\theta = \frac{\sqrt{3}}{4}\div\frac{1}{4} = \sqrt{3}$$

$$\therefore\quad \theta = \frac{\pi}{3}$$

$$\begin{aligned} \text{Thus }\arg z &= \pi-\frac{\pi}{3} \\ &= \frac{2\pi}{3} \end{aligned}$$

1 mark for correct $|z|$.

1 mark for correct arg z

b)

$$z^4-2=0$$

$$\left(z^2\right)^2-\left(\sqrt{2}\right)^2=0$$

$$\left[z^2+\sqrt{2}\right]\left[z^2-\sqrt{2}\right]=0$$

1 mark for correct working.

$$\left[z^2-i^2\sqrt{2}\right]\left[z^2-\sqrt{2}\right]=0$$

$$\left[z^2-\left(i\sqrt[4]{2}\right)^2\right]\left[z^2-\left(\sqrt[4]{2}\right)^2\right]=0$$

1 mark for correct working.

$$\left(z+i\sqrt[4]{2}\right)\left(z-i\sqrt[4]{2}\right)\left(z+\sqrt[4]{2}\right)\left(z-\sqrt[4]{2}\right)=0$$

$$\therefore\ z = \pm i\sqrt[4]{2}\quad\text{or }\pm\sqrt[4]{2}$$

1 mark for correct working & answer.

c)

$\cos 5\theta + i\sin 5\theta = \left(\cos\theta + i\sin\theta\right)^5$ by De Moivre's Theorem

Now let $c=\cos\theta$ & $s=\sin\theta$ then

$$\left(c+is\right)^5 = c^5+5c^4is+10c^3i^2s^2+10c^2i^3s^3+5ci^4s^4+i^5s^5$$

1 mark for correct working.

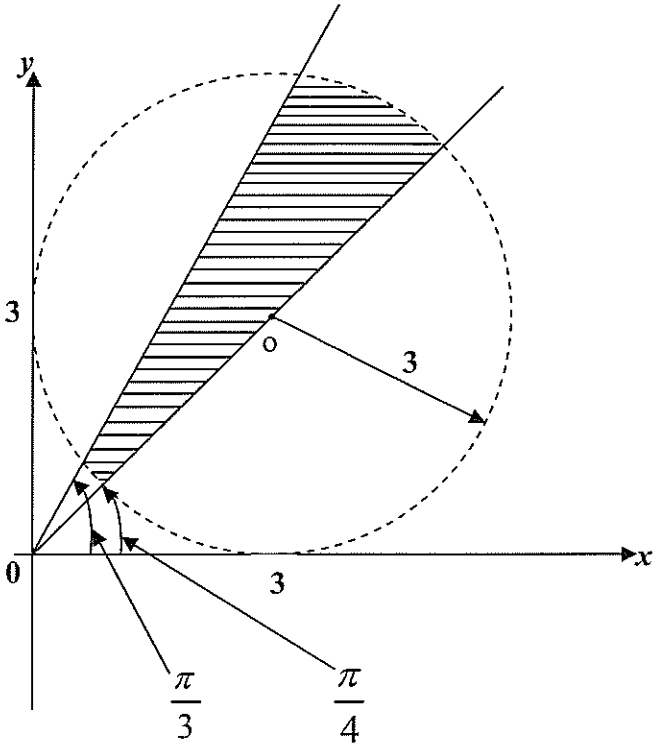
$$\begin{aligned} &= c^5+5ic^4s-10c^3s^2-10ic^2s^3+5cs^4+is^5 \\ &= \left(c^5-10c^3s^2+5cs^4\right)+i\left(5c^4s-10c^2s^3+s^5\right) \end{aligned}$$

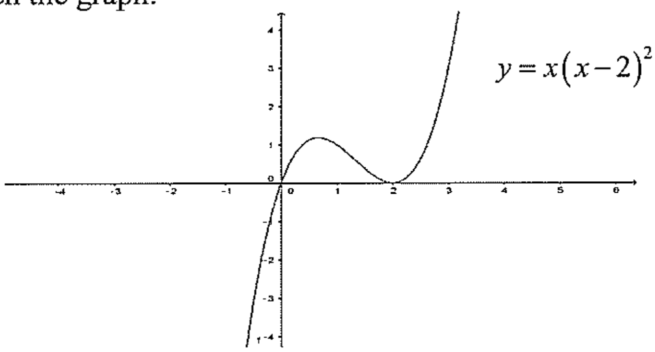
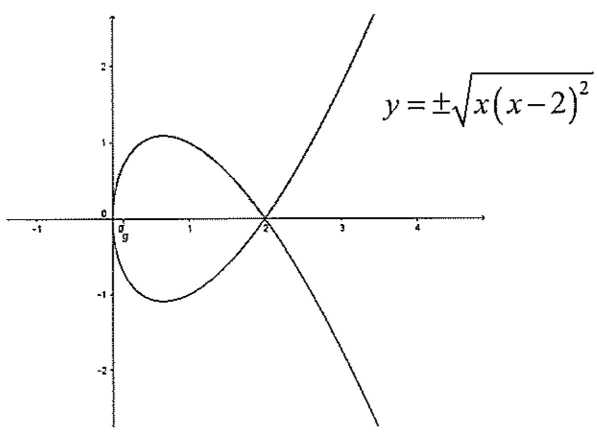
1 mark for correct working.

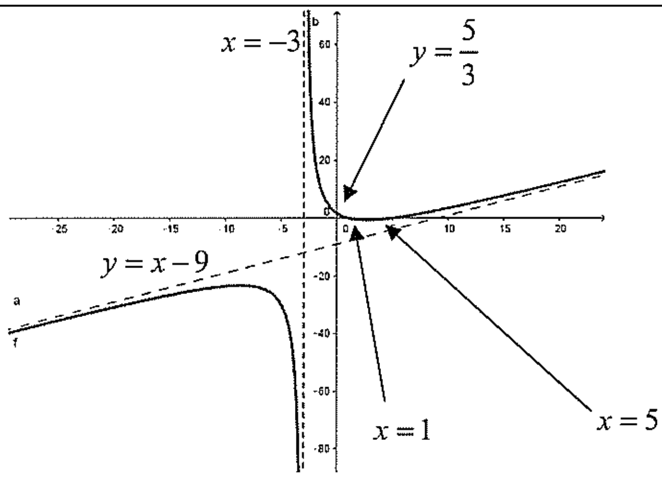
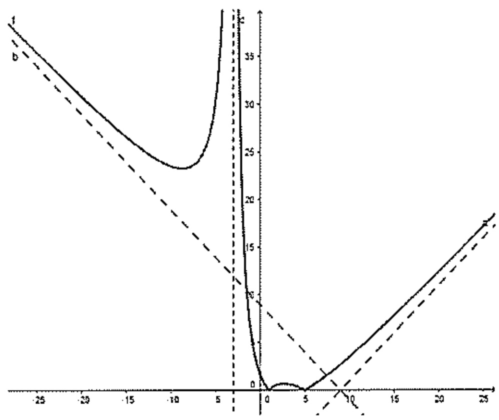
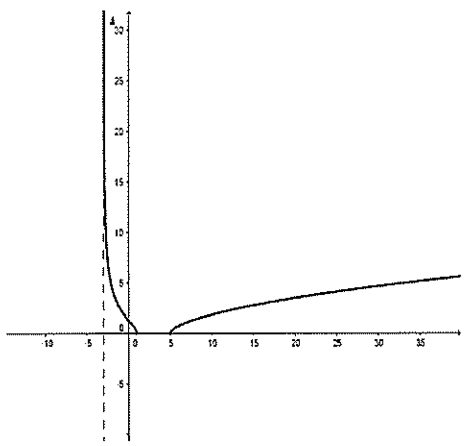
By equating imaginary parts we get

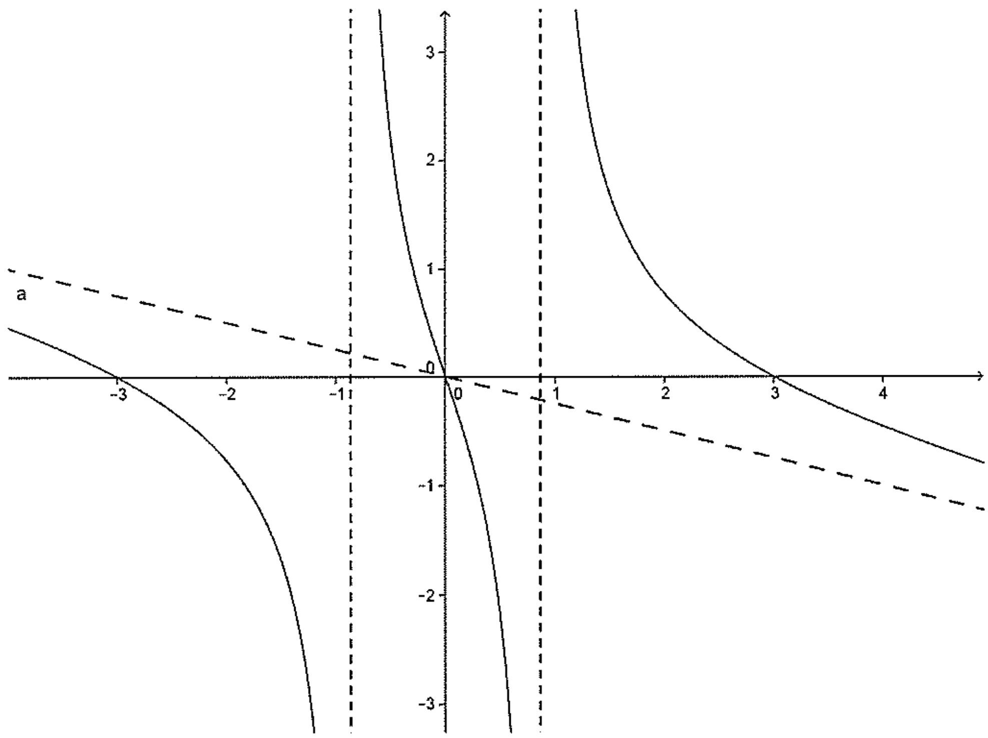
$$\begin{aligned} \sin 5\theta &= 5c^4s-10c^2s^3+s^5 \\ &= 5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta \end{aligned}$$

1 mark for correct answer.

d)	For $\lvert z-3-3i\rvert<3$, solution is inside the circle centre at $(3,\ 3i)$ radius 3. For $\frac{\pi}{4}\le\arg z\le\frac{\pi}{3}$, solutions between $\frac{\pi}{4}$ & $\frac{\pi}{3}$ centre at $(0,\ 0)$. 	**1 mark for correct drawing the dotted circle.** **1 mark for the two correct centres.** **1 mark for drawing the correct angles of the 2 lines.** **1 mark for correctly shading the final region between the circle & the lines.**

Question 7 [E6]			
a)	i)	For x-intercepts, put $y=0$, i.e. $x(x-2)^2=0$ => $x=0$ or $x=2$ Also this is a positive cubic. Now sketch the graph: 	**1 mark for correct x- intercepts.** **1 mark for correct direction of graph.**
	ii)	$\because\ y^2=x(x-2)^2$ then $y=\pm\sqrt{x(x-2)^2}$ 	**1 mark for correct x-intercepts.** **1 mark for correct graph.**

b)	i)	$\because \dfrac{(x-1)(x-5)}{(x+3)} = x-9+\dfrac{32}{x+3}$ then RHS $= \dfrac{(x-9)(x+3)+32}{x+3}$ $= \dfrac{x^2-6x-27+32}{x+3}$ $= \dfrac{x^2-6x+5}{x+3}$ $= \dfrac{(x-1)(x-5)}{x+3}$ $\therefore$ RHS = LHS	**1 mark for express RHS as a single fraction.** **1 mark for simplify & factorise.**
	ii)	The asymptotes are: $x=-3$; As $x \to \pm\infty$, $y \to x-9$ $\therefore\ y = x-9$ The x & y intercepts: When $x=0$, $y=\dfrac{5}{3}$ When $y=0$, $x=1$ or 5 	**1 mark for correct asymptotes.** **1 mark for correct x-y intercepts & graph.**
	iii)	α) $y=\lvert f(x)\rvert$ i.e. anything below x-axis is reflected in x-axis. Asymptotoes $x=-3$, $y=x-9$ & $y=-x+9$. y-intercept $=\dfrac{5}{3}$, x-intercept $=1$ & 5.  β) $y=\sqrt{f(x)}$ i.e. anything left of $x=-3$ & $1<x<5$ is deleted. $x=-3$ is a vertical asymptote. y-intercept is $\sqrt{\dfrac{5}{3}}$, x-intercept is 1 & 5. 	**α) 1 mark for correct asymptotes.** **1 mark for correct x-y intercepts & graph.** **β) 1 mark for correct asymptotes & x-y intercepts.** **1 mark for correct graph.**

c)	i)	Now $f(x)=\dfrac{9x-x^3}{4x^2-3}$ and $f(-x)=\dfrac{9(-x)-(-x)^3}{4(-x)^2-3}=\dfrac{-9x+x^3}{4x^2-3}=-\left(\dfrac{9x-x^3}{4x^2-3}\right)$ $\therefore f(x)=-f(x)$ Thus this is an odd function. The other 2 asymptotes are: $4x^2-3=0 \Rightarrow x=\pm\dfrac{\sqrt{3}}{2}$ When $x=0$ then $y=0$ When $y=0$ then $9x-x^3=0$ $x\left(9-x^2\right)=0$ $\therefore\ x=0,\ \pm 3$ Check $f\left(\dfrac{1}{2}\right)=-\dfrac{35}{16}$ for shape of the middle curve.  	**1 mark for all the correct asymptotes.** **1 mark for *x-y* intercepts & graph.**
	ii)	Looking for maximum number of times a horizontal line intersect the graph. $\therefore$ there are **3** solutions in this case.	**1 mark for correct answer.**