Carlingford High School

Mathematics

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION TERM 3 2017

Student Number: ____________________________

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A Reference & MC Sheet is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 3 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 11

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

	MC	Q11	Q12	Q13	Q14	Q15	Q16	Mark
Algebra	/1	/1						/2
Plane Geometry		/5						/5
Functions & Graphs	/1	/4						/5
Trigonometry	/1				/4		/7	/12
Straight Line Graphs	/1			/8				/9
Calculus	/1	/3	/5	/7		/6	/6	/28
Quadratic Polynomial & Parabola	/1				/4			/5
Integration	/1	/2	/5			/3		/11
Series & Applications	/1				/7			/8
Logarithms & Exponential Functions	/1						/2	/3
Physical Applications of Calculus	/1		/5			/6		/12
Total	/10	/15	/15	/15	/15	/15	/15	/100

Section I **(10 marks)** **Attempt Questions 1 - 10**

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10 from the back of this paper

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1 Which inequality defines the domain of the function $f(x)=\dfrac{1}{\sqrt{x+5}}$?

(A) $x>-5$ (B) $x\geq-5$ (C) $x<-5$ (D) $x\geq-5$

2 What is the solution to the equation $|2x-1|=9$?

(A) $x=-4, x=-5$ (B) $x=-4, x=5$ (C) $x=4, x=-5$ (D) $x=4, x=5$

3 Which expression is a term of the geometric series $4y-8y^2+16y^3-...$?

(A) $-4096y^{10}$ (B) $-4096y^{11}$ (C) $4096y^{10}$ (D) $4096y^{11}$

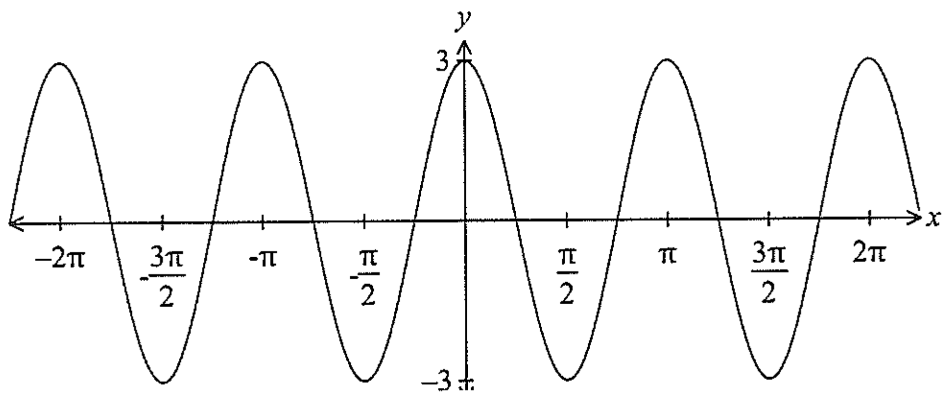
4 What is the value of $\int_{-1}^{1} x^3\,dx$?

(A) -1 (B) 0 (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$

5 Which of the following is the vertex of the quadratic $y=x^2+6x+10$?

(A) $(-3, 1)$ (B) $(3,-1)$ (C) $(-1, 3)$ (D) $(1,-3)$

6 The graph of $y=f(x)$ is show below.



Which of the following functions describes $f(x)$?

(A) $y=3\sin x$ (B) $y=3\sin 2x$ (C) $y=3\cos x$ (D) $y=3\cos 2x$

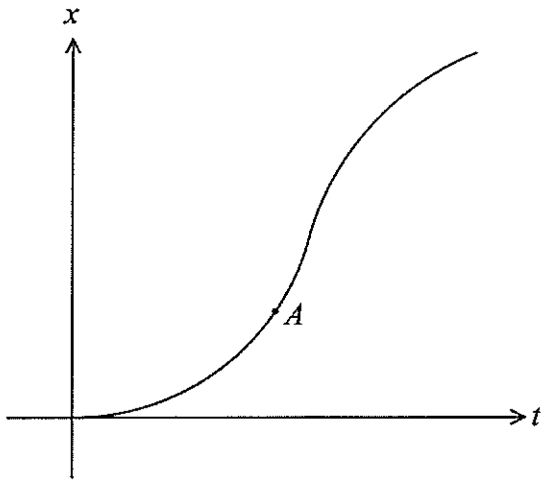
7 The graph $y = f(x)$ passes through the point $(1, 4)$ and $f'(x) = 3x^2 - 2$.
Which of the following expressions is $f(x)$?

(A) $x^3 - 2x$ (B) $2x - 1$ (C) $x^3 - 2x + 3$ (D) $x^3 - 2x + 5$

8 What is the solution of $3^x = 2$?

(A) $x = \dfrac{\log_e 2}{3}$ (B) $x = \dfrac{2}{\log_e 3}$ (C) $x = \dfrac{\log_e 2}{\log_e 3}$ (D) $x = \log_e \left(\dfrac{2}{3}\right)$

9 The graph shows the displacement x of a particle moving along a straight line as a function of time t.



Which statement describes the motion of the particle at the point A?

(A) The velocity is negative and the acceleration is positive.
(B) The velocity is negative and the acceleration is negative.
(C) The velocity is positive and the acceleration is positive.
(D) The velocity is positive and the acceleration is negative.

10 What is the gradient of the line $y = 1 - 2x$?

(A) 1 (B) $2x$ (C) -2 (D) $-2x$

<u>Section II</u> **(90 marks)** **Attempt Questions 11 – 16, show all necessary working.**

Allow about 2 hours and 45 minutes for this section

Question 11 (15 marks) Start a NEW writing booklet **Marks**

(a) Solve $y^2 - 4y = 0$ **1**

(b) Sketch the following graphs, showing important features.

(i) $y = \log_e(x-2)$ **2**

(ii) $y = -|x-2|$ **2**

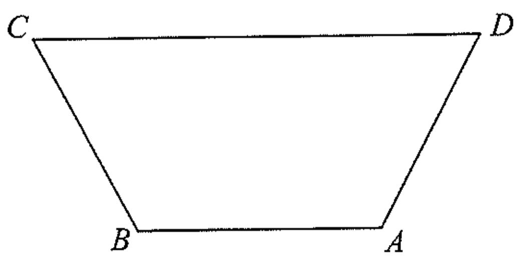
(c) Differentiate with respect to x.

(i) $x\sin 2x$ **1**

(ii) $\log_e\sqrt{4x^2-1}$ **2**

(d) Find the area between the curve $y = x^2$ and the x-axis from $x = 1$ to $x = 3$. **2**

(e) $ABCD$ is quadrilateral with $\angle ABC = \angle BAD$ and $BC = AD$.



Not to scale

(i) Prove that $\Delta ABC \equiv \Delta BAD$. **2**

(ii) Why are $\angle CAB$ and $\angle ABD$ equal? **1**

(iii) Prove that $\angle DBC = \angle CAD$. **2**

Question 12 (15marks) Start a NEW writing booklet **Marks**

(a) Find

(i) $\int e^{6y}dy$ **1**

(ii) $\int_0^{\frac{\pi}{4}}(\sec^2 x - x)dx$ **2**

(b) The line $6x - ky = 4$ passes through the point $(4,2)$. Find the value of k. **2**

(c) The table below shows the values of a function $y = f(x)$ for five values of x. **2**

x	1	1.5	2	2.5	3
$f(x)$	11.2	17.8	9.3	4.1	11.6

Use the Trapezoidal Rule with these five values to estimate $\int_1^3 f(x)\,dx$

(d) A water tank has an initial capacity of 3000 litres and is leaking according to the formula $V = V_0e^{-kt}$, where t is in hours.

(i) Show that $\frac{dV}{dt} = -kV$. **1**

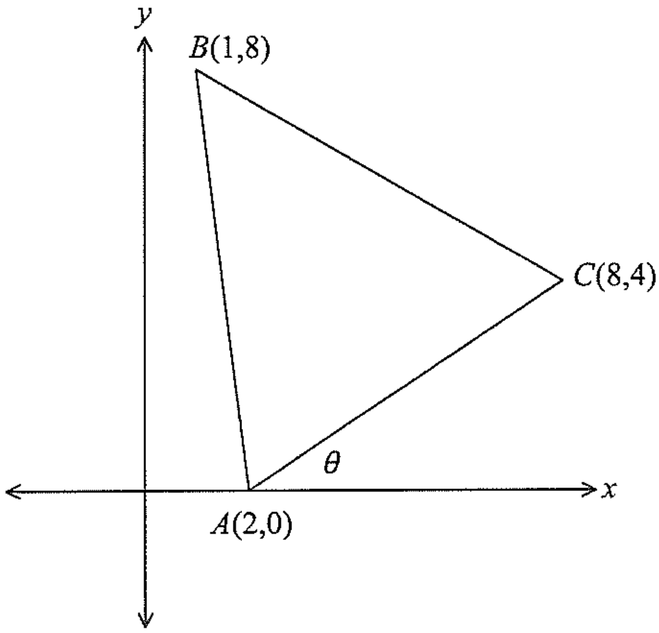
(ii) What is the value of k if after 3 hours the volume is 2000 litres? Answer correct to 3 decimal places. **2**

(iii) How long will it take for the water tank to fall to 250 litres? Answer correct to the nearest minute. **2**

(e) Find the equation of the tangent on $y = \log_e(2x^2 + 1)$ at the point $(2,\ \log_e 9)$. Express your answer in general form. **3**

Question 13 (15 marks) Start a NEW writing booklet **Marks**

(a)



The points A, B and C have coordinates (2,0), (1,8) and (8,4) respectively. The angle between the line AC and the x-axis is θ.

(i) Find the gradient of the line AC. **1**

(ii) Calculate the size of angle θ to the nearest minute. **1**

(iii) Find the equation of the line AC. **2**

(iv) Find the coordinates of D, the midpoint of AC. **2**

(v) Show that AC is perpendicular to BD. **2**

(b) Let $f(x)=x^3-3x^2-9x+18$

(i) Find the turning points and determine their nature. **2**

(ii) Find the coordinates of the point of inflexion. **2**

(iii) Sketch the graph of $y=f(x)$, showing the turning points and point of inflexion. **2**

(iv) What are the values of x for which $f'(x)<0$ and $f''(x)>0$? **1**

Question 14 (15 marks) Start a NEW writing booklet **Marks**

(a) Ellie and Louis worked out they would save \$400 000 in five years by depositing all their combined monthly salary of $\$x$ at the beginning of each month into a savings account and withdrawing \$5000 at the end of each month for living expenses. The savings account paid interest at the rate of 3% p.a. compounding monthly.

(i) Show that at the end of the second month they have **2**

$\$[(1.0025^2 + 1.0025)x - 5000(1.0025 + 1)]$ in their savings account.

(ii) Write down an expression for the balance in their account at the end of the five years? **1**

(iii) What is their combine monthly salary? **2**

(b) Three markers are placed out to sea. Marker B is 4 km north of marker A. However to sail from A to B a boat must first sail from A to C on a bearing 025° and then turn and sail from C to B on a bearing of 335°.

(i) What is the distance from A to C? **2**

Answer correct to one decimal place.

(ii) Calculate the distance from A to B through C. **2**

Answer correct to one decimal place.

(c) The quadratic equation $2x^2 - 3x + 8 = 0$ has roots α and β.

Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ **2**

(d) Find the common ratio of a geometric series with a first term of 2 and a limiting sum of 1.5. **2**

Question 15 (15 marks) Start a NEW writing booklet **Marks**

(a) A particle moves along a straight line about a fixed point O so that its acceleration, a ms^{-2}, at time t seconds is given by $a = 4\cos\left(2t + \frac{\pi}{6}\right)$.

Initially the particle is moving to the right with a velocity of 1 ms^{-1} from a position $\frac{\sqrt{3}}{2}$ metres to the left of O.

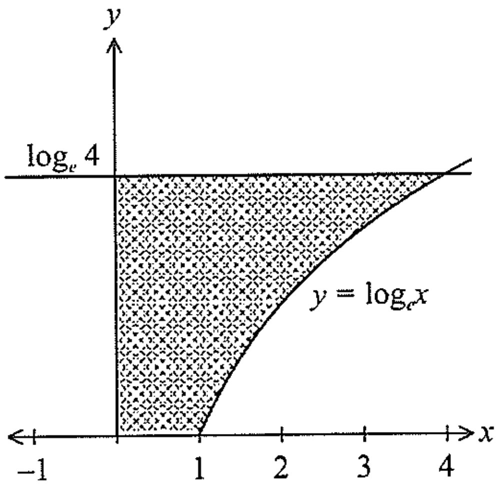
(i) Find an expression for the velocity of the particle after t seconds. **2**

(ii) Find an expression for the position of the particle after t seconds. **2**

(iii) Show that the particle changes directions when $t = \frac{5\pi}{12}$ seconds. **1**

(iv) What time does the particle return to its initial position for the first time? **1**

(b) The shaded region bounded by the curve $y = \log_e x$, the x and y-axes and the line $y = \log_e 4$ is rotated about the y-axis. Find the exact volume of the solid of revolution formed. **3**



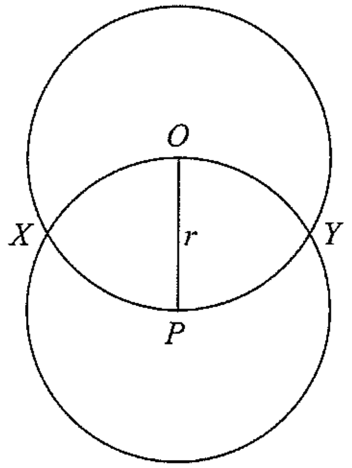
(c) A four-metre piece of wire is cut into three pieces. One piece is bent to form a square and the other two pieces are bent to form two congruent circles.

(i) If the radius of each congruent circle is r metres, show that the area of the square and the two circles is given by $A = 2\pi r^2 + (1 - \pi r)^2$. **2**

(ii) Find the value of r and the length of the side of the square that will minimise the area. **4**

Question 16 (15 marks) Start a NEW writing booklet **Marks**

(a) Two equal circles with centres O and P intersect at X and Y as shown below. The centres of each circle lie on the circumference of the other circle.



(i) Find the exact area of the region $XOYP$. **3**

(ii) What fraction of the circle centre O lies outside the region $XOYP$? **2**

(b) The rate of emission of carbon pollution C, in tonnes per year from a factory from 1st January 2011 is given by:

$$C = 500 - \left(\frac{10}{1+t}\right)^2$$ where t is the time in years.

(i) What was the rate of emission of carbon pollution C on 1st January 2011? **1**

(ii) What was the rate of emission of carbon pollution C on 1st January 2013? **1**

(iii) What value does C approach as time passes? **1**

(iv) Draw a sketch of C as a function of t? **1**

(v) Calculate the total amount of carbon pollution emitted from the factory from 1st January 2011 to 1st January 2017? Answer correct to the nearest tonne. **2**

(c) Find the value of m if $\dfrac{d^2y}{dx^2} = m\sin 3x - 4y$ and $y = 2\sin 3x + 4\cos 2x$. **2**

(d) Solve the equation $\log_2(x-1) - \log_2(x-2) = 2$ **2**

End of paper

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180°$$

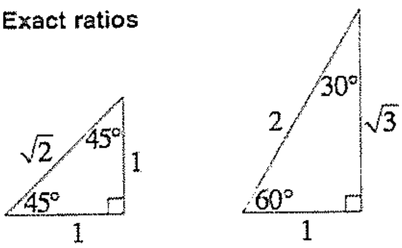
Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \sec\theta = \frac{1}{\cos\theta}$$
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

***n*th term of an arithmetic series**

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

***n*th term of a geometric series**

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Mathematics (continued)

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\dfrac{dy}{dx} = F'(u)\dfrac{du}{dx}$

If $y = e^{f(x)}$, then $\dfrac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\dfrac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\dfrac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

Year 12 Mathematics Section I – Answer Sheet

Student Number: ______________________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ⊗ C ○ D ○

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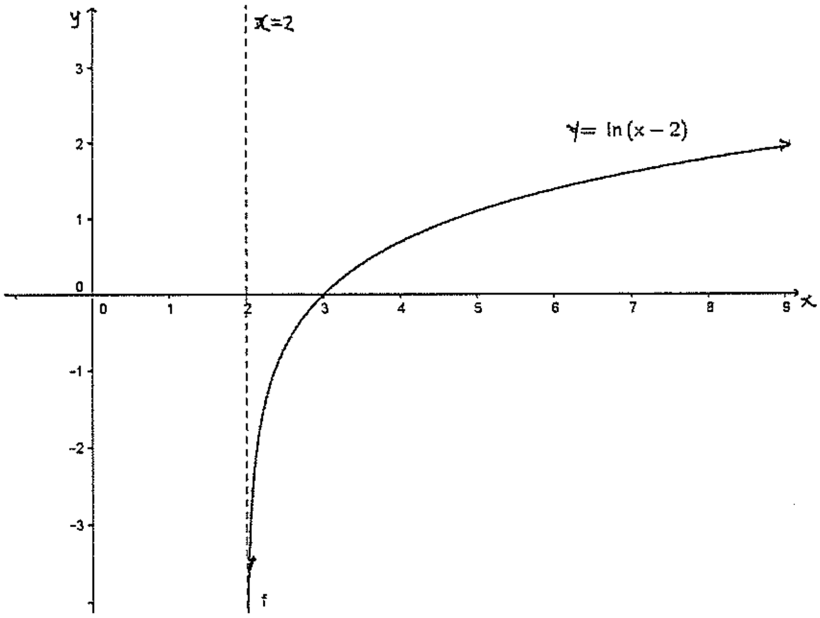
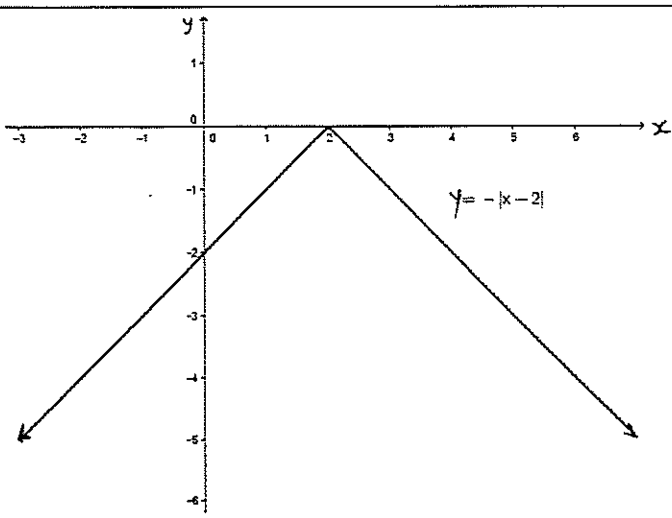
1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

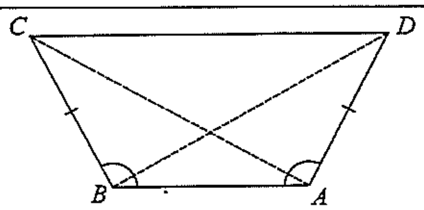
CARLINGFORD HIGH SCHOOL

Mathematics Trial HSC Examination 2017

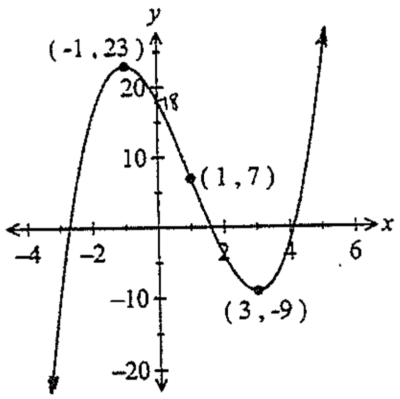
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$\sqrt{x+5}>0$ $x+5>0$ $x>-5$ Note – the denominator cannot be zero	1 Mark: A
2	$2x-1=9$ or $2x-1=-9$ $2x=10$ $2x=-8$ $x=5$ $x=-4$	1 Mark: B
3	$a=4y,\ r=-2y$ $T_n=4y(-2y)^{n-1}$ $T_{11}=4y(-2y)^{10}$ $=4096y^{11}$	1 Mark: D
4	$\int_{-1}^{1}x^3\,dx=\left[\frac{x^4}{4}\right]_{-1}^{1}=\frac{1^4}{4}-\frac{(-1)^4}{4}=0$	1 Mark: B
5	$x^2+6x=y-10$ $x^2+6x+9=y-1$ $(x+3)^2=(y-1)$ Vertex is (–3, 1)	1 Mark: A
6	Amplitude of the $y=f(x)$ is 3. Period $=\frac{2\pi}{n}=\pi$ or $n=2$. $y=3\cos 2x$	1 Mark: D
7	$f'(x)=3x^2-2$ $f(x)=x^3-2x+C$ Point $(1,4)$ satisfies the function. $4=1^3-2\times1+C$ or $C=5$ $\therefore f(x)=x^3-2x+5$	1 Mark: D

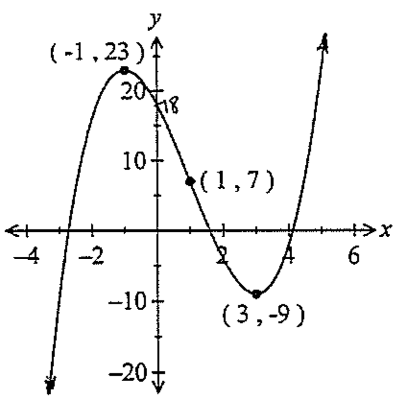
8	$3^x = 2$ $\log_e 3^x = \log_e 2$ $x \times \log_e 3 = \log_e 2$ $x = \dfrac{\log_e 2}{\log_e 3}$	1 Mark: C
9	Gradient of the tangent at A represents the velocity. It is positive. Concavity at A indicates the acceleration. It is concave up and positive.	1 Mark: C
10	Given $y = 1 - 2x$. It's gradient is -2.	1 Mark: C
Section II		
11(a)	$y^2 - 4y = 0$ $y(y-4) = 0$ Therefore $y = 0$ or $y = 4$.	1 – for correct answer
11(b) (i)		1 – for asymptote & x-intercept 1 – for the shape of curve
11(b) (ii)		1 – for y-intercept & cusps 1 – for correct shape

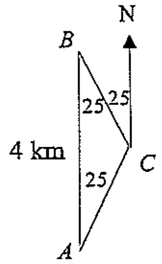
11(c) (i)	$\frac{d}{dx}x\sin 2x = x\times 2\cos 2x + \sin 2x \times 1$ $= 2x\cos 2x + \sin 2x$	1 – for correct answer
11(c) (ii)	$\frac{d}{dx}\log_e\sqrt{4x^2-1} = \frac{d}{dx}\log_e(4x^2-1)^{\frac{1}{2}}$ $= \frac{d}{dx}\left[\frac{1}{2}\log_e(4x^2-1)\right]$ $= \frac{1}{2}\times\frac{8x}{4x^2-1}$ $= \frac{4x}{4x^2-1}$	2 – for correct answer 1 – for applies the log laws or shows some understanding
11(d)	$A = \int_1^3 x^2\,dx$ $= \left[\frac{x^3}{3}\right]_1^3$ $= \left[\frac{3^3}{3}\right] - \left[\frac{1^3}{3}\right]$ $= \frac{26}{3} = 8\frac{2}{3}$ square units	2 – for correct answer 1 – for correctly sets up the integral
11(e) (i)	 Consider ΔABC and ΔBAD $BC = AD$ (given) $\angle ABC = \angle BAD$ (given) $AB = AB$ (common side) $\Delta ABC \equiv \Delta BAD$ (SAS)	2 – for correct answer 1 – for one correct statement
11(e) (ii)	$\angle CAB = \angle ABD$ (matching angles in congruent triangles)	1 – for correct answer
11(e) (iii)	$\angle ABC = \angle BAD$ $\angle DBC + \angle ABD = \angle CAD + \angle CAB$ (adjacent angles) Since $\angle CAB = \angle ABD$ (from part (ii)) Therefore $\angle DBC = \angle CAD$	2 – for correct answer 1 – for show some understanding

12(a) (i)	$\int e^{6y}dy = \frac{1}{6}e^{6y} + C$	1 – for correct answer
12(a) (ii)	$\int_0^{\frac{\pi}{4}} (\sec^2 x - x)dx = \left[\tan x - \frac{x^2}{2}\right]_0^{\frac{\pi}{4}}$ $= \left(\tan\frac{\pi}{4} - \frac{\left(\frac{\pi}{4}\right)^2}{2}\right) - \left(\tan 0 - \frac{0^2}{2}\right) = 1 - \frac{\pi^2}{32}$	2 – for correct answer 1 – for integrates correctly
12(b)	$(4,2)$ satisfies the equation $6x - ky = 4$ $6\times 4 - k\times 2 = 4$ $24 - 2k = 4$ $-2k = -20$ $k = 10$	2 – for correct answer 1 – for substitutes $(4,2)$ into the equation
12(c)	$A = \frac{h}{2}\left[y_0 + y_4 + 2\times(y_1 + y_2 + y_3)\right]$ $= \frac{0.5}{2}\left[11.2 + 11.6 + 2\times(17.8 + 9.3 + 4.1)\right]$ $= 21.3$	2 – for correct answer 1 – for use trapezoidal rule
12(d) (i)	$V = 3000e^{-kt}$ $\frac{dV}{dt} = -k\times 3000e^{-kt} = -kV$	1 – for correct answer
12(d) (ii)	When $t = 3$ then $V = 2000$ $2000 = 3000e^{-3k}$ $e^{-3k} = \frac{2}{3}$ $\log_e e^{-3k} = \log_e 0.\dot{6}$ $-3k = \log_e 0.\dot{6}$ $k = \frac{\log_e 0.\dot{6}}{-3} = 0.135155036... \approx 0.135$	2 – for correct answer 1 – for substitutes $t = 3$ and $V = 2000$ into the formula
12(d) (iii)	We need to find t when $V = 250$. $250 = 3000e^{-k\times t}$ $e^{-kt} = 0.08\dot{3}$ $-kt = \log_e 0.08\dot{3}$ $t = -\frac{1}{k}\log_e 0.08\dot{3} = -\frac{\log_e 0.08\dot{3}}{0.135155..}$ $= 18.385601.. \approx 18$ h 23 min	2 – for correct answer 1 – for determnes $e^{-kt} = 0.8\dot{3}$ or shows some understanding of the problem

13(b) (i)	$f(x) = x^3 - 3x^2 - 9x + 18$ Stationary points $f'(x) = 3x^2 - 6x - 9$ $= 3(x^2 - 2x - 3)$ $f''(x) = 6x - 6$ $f'(x) = 0$ $3(x^2 - 2x - 3) = 0$ $3(x-3)(x+1) = 0$ $x = -1,\ x = 3$ When $x = -1, y = 23$ then $f''(x) = -12 < 0$ Maxima. When $x = 3, y = -9$ then $f''(x) = 12 > 0$ Minima. Maximum turning point at $(-1, 23)$. Minimum turning point at $(3, -9)$.	2 – for correct answer 1 – for finding the stationary points
13(b) (ii)	Possible points of inflexion $f''(x) = 0$ $\quad 6x - 6 = 0$ $6(x-1) = 0$ $x = 1$ When $x = 1, y = 7$ Check for change in concavity When $x = 0.9$ then $f''(x) = 6 \times 0.9 - 6 < 0$ When $x = 1.1$ then $f''(x) = 6 \times 1.1 - 6 > 0$ Hence (1,7) is a point of inflexion.	2 – for correct answer 1 – for finding the point of inflexion without checking concavity
13(b) (iii)		2 – for correct answer 1 – for correct shape or shows some understanding
13(b) (iv)	Function has a negative tangent and is concave up when $1 < x < 3$ (from the graph) Alternatively: $f'(x) > 3(x-3)(x+1) < 0$ when $-1 < x \lessgtr 3$ $f''(x) > 6x - 6 > 0$ when $x > 1$	1 – for correct answer
14(a) (i)	3% p.a $= \dfrac{0.03}{12} = 0.0025$ per month After 1 month $A_1 = \$\left[x \times 1.0025 - 5000\right]$ After 2 months $A_2 = \$\left[\left(x \times 1.0025 - 5000 + x\right) \times 1.0025 - 5000\right]$ $= \$[(1.0025^2 + 1.0025)x - 5000(1.0025 + 1)]$	2 – for correct answer 1 – for sets up the first month

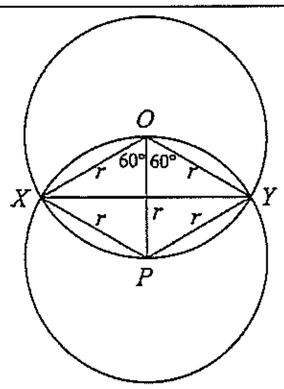
12(e)	$y=\log_e(2x^2+1)$ $\frac{dy}{dx}=\frac{4x}{2x^2+1}$ At $x=2$ then $m=\frac{4\times 2}{2\times 2^2+1}=\frac{8}{9}$ $y-\log_e 9=\frac{8}{9}(x-2)$ $8x-9y+9\log_e 9-16=0$	3 – for correct answer 2 – for finding the gradient of the tangent 1 – for finding the derivative
13(a) (i)	Gradient of AC $M=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{4-0}{8-2}$ $=\frac{2}{3}$	1 – for correct answer
13(a) (ii)	Gradient of AC $\tan\theta=\frac{2}{3}$ $\theta=33°41'$	1 – for correct answer
13(a) (iii)	Point slope formula $y-y_1=m(x-x_1)$ $y-0=\frac{2}{3}(x-2)$ $3y=2(x-2)$ $2x-3y-4=0$	2 – for correct answer 1 – for substitutes into point-slope formula
13(a) (iv)	Mid-point formula $x=\frac{x_1+x_2}{2}=\frac{2+8}{2}=5$ $\quad y=\frac{y_1+y_2}{2}=\frac{0+4}{2}=2$ Midpoint is $D(5, 2)$	2 – for correct answer 1 – for finding one solution
13(a) (v)	AC is perpendicular to BD if $m_1m_2=-1$. Gradient of AC is $^2/_3$ Gradient of BD $m=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{8-2}{1-5}=\frac{6}{-4}=-\frac{3}{2}$ Now $m_1m_2=-1$ $\frac{2}{3}\times-\frac{3}{2}=-1$ (True)	2 – for correct answer 1 – for make some progress towards the solution

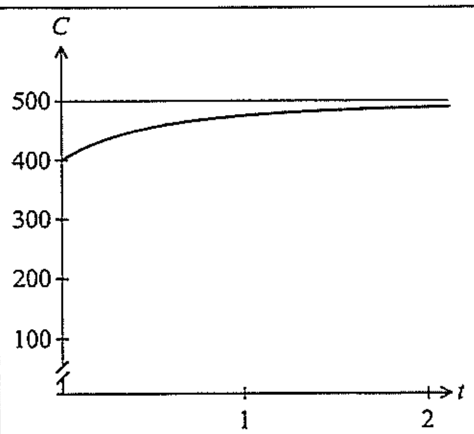
13(b) (i)	$f(x)=x^3-3x^2-9x+18$ Stationary points $f'(x)=3x^2-6x-9$ $=3(x^2-2x-3)$ $f''(x)=6x-6$ $f'(x)=0$ $3(x^2-2x-3)=0$ $3(x-3)(x+1)=0$ $x=-1, x=3$ When $x=-1, y=23$ then $f''(x)=-12<0$ Maxima. When $x=3, y=-9$ then $f''(x)=12>0$ Minima. Maximum turning point at $(-1,23)$. Minimum turning point at $(3,-9)$.	2 – for correct answer 1 – for finding the stationary points
13(b) (ii)	Possible points of inflexion $f''(x)=0$ $6x-6=0$ $6(x-1)=0$ $x=1$ When $x=1, y=7$ Check for change in concavity When $x=0.9$ then $f''(x)=6\times0.9-6<0$ When $x=1.1$ then $f''(x)=6\times1.1-6>0$ Hence (1,7) is a point of inflexion.	2 – for correct answer 1 – for finding the point of inflexion without checking concavity
13(b) (iii)		2 – for correct answer 1 – for correct shape or shows some understanding
13(b) (iv)	Function has a negative tangent and is concave up when $1<x<3$ (from the graph) Alternatively: $f'(x)>3(x-3)(x+1)<0$ when $-1<x>3$ $f''(x)>6x-6>0$ when $x>1$	1 – for correct answer
14(a) (i)	3% p.a $=\dfrac{0.03}{12}=0.0025$ per month After 1 month $A_1=\$\left[x\times1.0025-5000\right]$ After 2 months $A_2=\$\left[\left(x\times1.0025-5000+x\right)\times1.0025-5000\right]$ $=\$[(1.0025^2+1.0025)x-5000(1.0025+1)]$	2 – for correct answer 1 – for sets up the first month

14(a) (ii)	After 5 years or 60 months $A_{60} = \$[(1.0025^{60} + ... + 1.0025)x - 5000(1.0025^{59} + ... + 1)]$	1 – for correct answer
14(a) (iii)	After 60 months the amount required is \$400 000 $400{,}000 = \$[(1.0025^{60} + ... + 1.0025)x - 5000(1.0025^{59} + ... + 1)]$ Use the formula for the sum of a GP. $400{,}000 = \$[\left(1.0025 \times \frac{1.0025^{60} - 1}{1.0025 - 1}\right)x - 5000 \times \frac{1.0025^{60} - 1}{1.0025 - 1}$ $x = \left(400{,}000 + 5000 \times \frac{1.0025^{60} - 1}{1.0025 - 1}\right) \div \left(1.0025 \times \frac{1.0025^{60} - 1}{1.0025 - 1}\right)$ $= \$11{,}159.58$	2 – for correct answer 1 – for make significant progress towards the solution
14(b) (i)	$\frac{AC}{\sin 25} = \frac{4}{\sin 130}$ $AC = \frac{4 \sin 25}{\sin 130}$ $= 2.206755838...$ ≈ 2.2 km 	2 – for correct answer 1 – for make some progress towards the solution
14(b) (ii)	ΔABC is an isosceles triangle $\angle CAB = \angle CBA$ Total distance travelled $= 2 \times AC$ $= 4.413511676...$ ≈ 4.4 km	2 – for correct answer 1 – for use an isosceles triangle or calculates BC
14(c) (i)	$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{2} = \frac{3}{2}$	1 – for correct answer
14(c) (ii)	$\alpha\beta = \frac{c}{a} = \frac{8}{2} = 4$	1 – for correct answer
14(c) (iii)	$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$ $= \frac{\left(\frac{3}{2}\right)^2 - 2 \times (4)}{(4)^2} = -\frac{23}{64}$	2 – for correct answer 1 – for show some understanding of the problem
14(d)	$a = 2$ and $S = 1.5 = \frac{3}{2}$ $S = \frac{a}{1-r}$ or $\frac{3}{2} = \frac{2}{1-r}$ $3 - 3r = 4$ or $3r = -1$ or $r = -\frac{1}{3}$	2 – for correct answer 1 – for use the limiting sum of a GP formula with one correct value

15(a) (i)	$v=\int 4\cos\left(2t+\frac{\pi}{6}\right)dt$ $=2\sin\left(2t+\frac{\pi}{6}\right)+C$ When $t=0, v=1$ $1=2\sin\left(2\times 0+\frac{\pi}{6}\right)+C$ $C=1-2\sin\left(2\times 0+\frac{\pi}{6}\right)=0$ $\therefore\ v=2\sin\left(2t+\frac{\pi}{6}\right)$	2 – for correct answer 1 – for correctly integrates to find the velocity
15(a) (ii)	$x=\int 2\sin\left(2t+\frac{\pi}{6}\right)dt$ $=-\cos\left(2t+\frac{\pi}{6}\right)+C$ When $t=0, x=-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2}=-\cos\left(2\times 0+\frac{\pi}{6}\right)+C$ $C=-\frac{\sqrt{3}}{2}+\cos\left(2\times 0+\frac{\pi}{6}\right)=0$ $\therefore\ x=-\cos\left(2t+\frac{\pi}{6}\right)$	2 – for correct answer 1 – for correctly integrates to find the position
15(a) (iii)	Particle changes direction when $v=0$. $2\sin\left(2t+\frac{\pi}{6}\right)=0$ $\sin\left(2t+\frac{\pi}{6}\right)=0$ $2t+\frac{\pi}{6}=0,\pi,2\pi,\ldots$ $2t=-\frac{\pi}{6},\frac{5\pi}{6},\frac{11\pi}{6},\ldots$ $t=\frac{5\pi}{12},\frac{11\pi}{12},\ldots$ Particle changes direction at $t=\frac{5\pi}{12}$ seconds.	1 – for correct answer

15(a) (iv)	Initial position is $x=-\frac{\sqrt{3}}{2}$. $-\cos\left(2t+\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}$ $\cos\left(2t+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $2t+\frac{\pi}{6}=\frac{\pi}{6},\frac{11\pi}{6},...$ $2t=0,\frac{10\pi}{6},...$ $t=0,\frac{5\pi}{6},...$ Particle returns when $t=\frac{5\pi}{6}$ seconds.	1 – for correct answer
15(b)	If $y=\log_e x$ then $x=e^y$ $V=\pi\int_a^b x^2 dy$ $=\pi\int_0^{\log_e 4}(e^y)^2 dy$ $=\pi\left[\frac{e^{2y}}{2}\right]_0^{\log_e 4}$ $=\frac{\pi}{2}\left[e^{2\log_e 4}-e^0\right]$ $=\frac{15\pi}{2}$ cubic units	3 – for correct answer 2 – for make significant progress 1 – for correctly set up the integral
15(c) (i)	Let the side length of the square be x Perimeter of a circle is given by $C=2\pi r$ $4x+2\pi r+2\pi r=4$ $4x+4\pi r=4$ $x+\pi r=1$ $x=1-\pi r$ Area of a circle is given by $A=\pi r^2$ $A=\pi r^2+\pi r^2+(1-\pi r)^2$ $=2\pi r^2+(1-\pi r)^2$	2 – for correct answer 1 – for find x in terms of r

15(c) (ii)	$A=2\pi r^2+(1-\pi r)^2$ $\dfrac{dA}{dr}=4\pi r+2(1-\pi r)\times -\pi=4\pi r-2\pi(1-\pi r)$ Minimal A occurs when $\dfrac{dA}{dr}=0$ $4\pi r-2\pi(1-\pi r)=0$ $2\pi(2r-1+\pi r)=0$ $2r-1+\pi r=0$ $(2+\pi)r=1$ $r=\dfrac{1}{2+\pi}$ Check $\dfrac{d^2A}{dr^2}=4\pi+2\pi^2>0$ for all values of r Therefore $r=\dfrac{1}{2+\pi}$ is a minima To find the side length of the square. $x=1-\pi r=1-\pi\times\left(\dfrac{1}{2+\pi}\right)$ $=\dfrac{2+\pi-\pi}{2+\pi}$ $=\dfrac{2}{2+\pi}$	4 – for correct answer 3 – for check the value of r is a minima by using the first or second derivative 2 – for find the value of r 1 – for find the derivative
16(a) (i)	Area of segment XYP $A=\dfrac{1}{2}r^2(\theta-\sin\theta)$ $=\dfrac{1}{2}r^2(\dfrac{2\pi}{3}-\sin\dfrac{2\pi}{3})$ $=\dfrac{1}{2}r^2\left(\dfrac{2\pi}{3}-\dfrac{\sqrt{3}}{2}\right)$  Area of the region $XOYP$ is twice the area of segment XYP $A=2\times\dfrac{1}{2}r^2\left(\dfrac{2\pi}{3}-\dfrac{\sqrt{3}}{2}\right)$ $=r^2\left(\dfrac{2\pi}{3}-\dfrac{\sqrt{3}}{2}\right)$	3 – for correct answer 2 – for make significant progress towards the solution 1 – for recognise an equilateral triangle or similar understanding

16(a) (ii)	Area outside the region $XOYP$ $A=\pi r^2-r^2\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ $=r^2\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$ Fraction required is $=\dfrac{r^2\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)}{\pi r^2}=\frac{1}{3}+\frac{\sqrt{3}}{2\pi}$	2 – for correct answer 1 – for make progress towards the solution
16(b) (i)	Initial calculation occurs on 1st January 2011 or $t=0$ $C=500-\left(\frac{10}{1+0}\right)^2$ $=400$ tonnes per year	1 – for correct answer
16(b) (ii)	1st January 2013 requires $t=2$ $C=500-\left(\frac{10}{1+2}\right)^2$ $=488.\dot{8}$ tonnes per year	1 – for correct answer
16(b) (iii)	$C=\lim\limits_{t\to\infty}500-\left(\frac{10}{1+t}\right)^2 \quad \left(\lim\limits_{t\to\infty}\frac{10}{1+t}\approx 0\right)$ ≈ 500 tonnes per year	1 – for correct answer
16(b) (iv)		1 – for correct answer
16(b) (v)	Area under the curve represents the amount of carbon pollution. $\int_0^6 500-\left(\frac{10}{1+t}\right)^2 dt=\int_0^6 500-100(1+t)^{-2}dt$ $=\left[500t+100(1+t)^{-1}\right]_0^6$ $=\left[\left(500\times 6+100(1+6)^{-1}\right)-\left(100(1+0)^{-1}\right)\right]$ $=2914.285714...$ ≈ 2914 tonnes	2 – for correct answer 1 – for set up the area under the curve

16(c)	$y = 2\sin 3x + 4\cos 2x$ $\frac{dy}{dx} = 6\cos 3x - 8\sin 2x$ and $\frac{d^2y}{dx^2} = -18\sin 3x - 16\cos 2x$ $\frac{d^2y}{dx^2} = m\sin 3x - 4y$ $-18\sin 3x - 16\cos 2x = m\sin 3x - 4(2\sin 3x + 4\cos 2x)$ $-18\sin 3x - 16\cos 2x = (m-8)\sin 3x - 16\cos 2x$ $m - 8 = -18$ $m = -10$	2 – for correct answer 1 – for find the first & second derivative
16(d)	$\log_2(x-1) - \log_2(x-2) = 2$ $\log_2\left(\frac{x-1}{x-2}\right) = 2$ $\frac{x-1}{x-2} = 2^2$ $x - 1 = 4x - 8$ $3x = 7$ $x = \frac{7}{3}$	2 – for correct answer 1 – for remove the logarithms or shows some understanding