Carlingford High School

Mathematics

Year 10 5.3 Term Three Examination

2016

Time allowed: 55 minutes

Name:_____ Class: 10MAT3__

Please circle your teacher:	**Ms Kellahan**	**Mrs Wilson / Mrs Young**	**Mrs Lego**	**Mr Wilson**

Instructions:

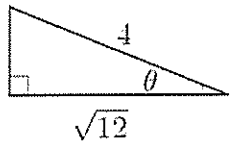
- Use blue or black pen
- Board approved calculators may be used
- Show all necessary working out in the space provided
- Marks may be deducted for untidy setting out
- Extension level questions are marked with an asterisk (*)

Topic	Trigonometry	Coordinate Geometry	Probability	Inequations & Logarithms	Properties of Geometric Figures	Total
Mark	/12	/6	/7	/9	/10	/44
Extension *	/4	/3	/2	/2	/3	/14
Total	/16	/9	/9	/11	/13	/58

Trigonometry (16 marks)

1 a) Find the value of A if $\cos 32°46' = \sin A$. **1**

b) Find the exact value of θ in this triangle. **2**



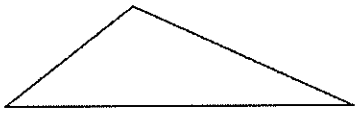
c) If θ is obtuse, find the value of θ if $\tan 55° = -\tan \theta$. **1**

d) Solve the equation $\tan x = 0.4$ correct to the nearest degree if x is between $0°$ and $180°$. **1**

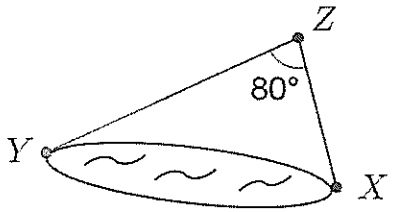
2 a) In ΔKLM, $\angle M = 27°51'$, $KL = 8.7$ cm and $LM = 9.8$ cm.

i) Complete the diagram below, showing all given information. **1**

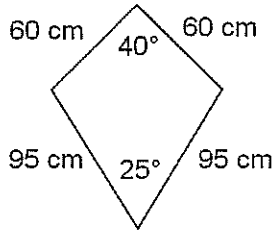
ii) Find $\angle K$ correct to the nearest minute. **2**



b) To find the length of a lake, a surveyor walks 390 metres from point X to point Z, then turns $80°$ and walks 450 metres to point Y. Find the length of the lake correct to the nearest metre. **2**



3 a) Calculate the area of the kite below, correct to the nearest square centimetre. **2**



* b) Find the exact value of $\sin\theta$ if $\cos\theta = \frac{\sqrt{5}}{4}$ and θ is acute. **2**

* c) Solve the equation $1 - 3\sin x = 0$ correct to the nearest degree if x is between $0°$ and $180°$. **2**

Coordinate Geometry (9 marks)

1 a) Find the gradient and y-intercept for the linear equation $y = \frac{12-3x}{4}$ **2**

b) Find, in general form the equation of the line which passes through the points $(-1, 2)$ and $(3, -4)$. **2**

c) Find the equation of the line which is perpendicular to $y = 3x - 1$ and passes through the point $(6,3)$. **2**

* d) Use calculations to show that the quadrilateral with vertices $A(3,1)$, $B(8,4)$, $C(5,9)$ and $D(0,6)$ is a square. Write which test you have used. **3**

Probability (9 marks)

1 At Xford High School, students who use mobile phones were surveyed. Some of the results are shown in the table.

	Pre-paid	Plan	Total
Female students	_____	147	319
Male students	158	103	261
Total	330	250	_____

a) Complete the table. **1**

b) Of the female students surveyed, one is chosen at random. What is the probability that she is on a plan? **1**

_____

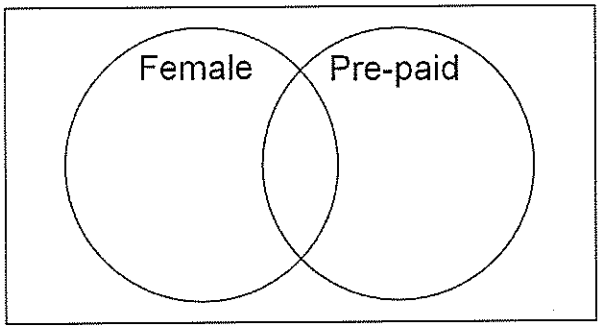
c) What is the probability that a student who completed the survey is male, given that they use a pre-paid phone? **1**

_____

d) Ten new male students are surveyed, and are all on a plan. What percentage of male students are now on a plan? Give your answer to the nearest percent. **1**

_____

e) Use the information in the table to complete the Venn Diagram below. **2**



2 Three raffles are drawn, each with 100 tickets and 1 prize. Jane buys 2 tickets in one raffle and Mary buys one ticket in each of the other two raffles.

a) Fill in the correct name to make this statement true. **1**

_____ winning with her first ticket and winning with her second ticket are examples of **independent** events.

* b) i) Calculate the probability of Jane winning at least one prize, correct to 4 decimal places. **1**

_____

_____

ii) Does Mary have a better chance of winning at least one prize than Jane? Show working to support your answer. **1**

_____

_____

Inequations & Logarithms (11 marks)

1 a) Solve each inequality:

i) $2t + 1 \geq 5$ **1**

_____

_____

ii) $\frac{3-5x}{4} \leq -9$ **2**

_____

_____

_____

2 a) Simplify $\log_x 4 + \log_x 5$ **1**

_____

_____

b) Evaluate $\log_2 36 - 2\log_2 3$ **2**

c) Solve each equation, writing your answer to 3 decimal places where necessary

i) $\log_7 x = 3$ **1**

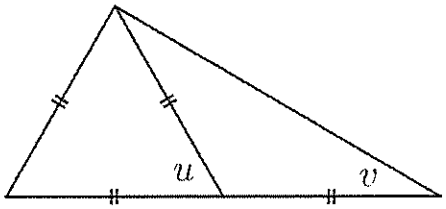
ii) $5^{x-1} = 81$ **2**

* iii) $\log_2 \sqrt{x} - 3\log_2\left(\frac{1}{x}\right) = 21$ **2**

Properties of Geometric Figures (13 marks)

1 a) A parallelogram has one angle equal to 55°. Give the values of the other three angles. **1**

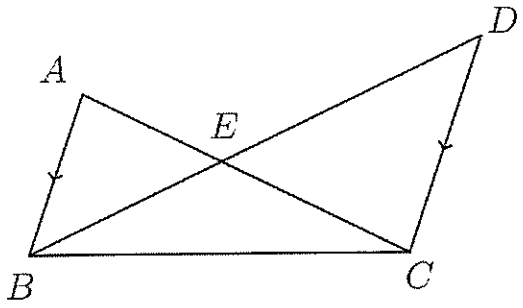
b) Find the values of u and v in the triangle below, giving reasons. **3**



2 a) In the diagram below, $AB = 19$ cm, $AE = 10$ cm, $BE = 23$ cm, $EC = 15$ cm and $AB \parallel DC$.

i) Prove that $\triangle AEB$ is similar to $\triangle CED$. **2**

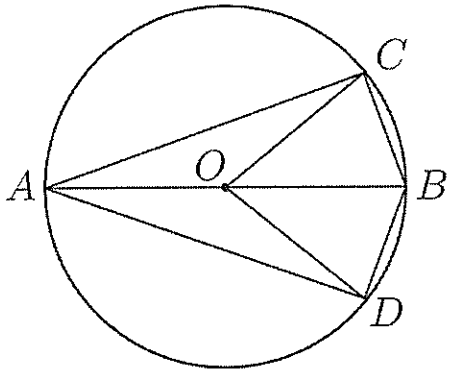
ii) Find the value of CD. **2**



b) In the diagram below, O is the centre of a circle of radius 5 cm. $BC = BD = 3$ cm.

i) Prove that $\triangle OCB$ is congruent to $\triangle ODB$. **2**

* ii) Hence, prove that $AC = AD$. **3**



End of Exam – Please check your work

Carlingford High School

AK Geometry
JY Coord Geom + Prob Q1
PW Prob Q2 + Ineq + Logs
VL Trig

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VL

Trigonometry (16 marks)

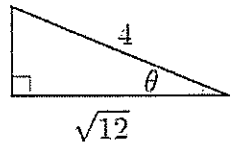
1 a) Find the value of A if $\cos 32°46' = \sin A$. **1**

$\cos(90° - A) = \sin A$

$A = 90° - 32°46'$

$= 57°14'$

b) Find the exact value of θ in this triangle. **2**



$\cos\theta = \frac{\sqrt{12}}{4} = \frac{2\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

$\theta = 30°$

c) If θ is obtuse, find the value of θ if $\tan 55° = -\tan\theta$. **1**

$\theta = 180° - 55°$

$= 125°$

d) Solve the equation $\tan x = 0.4$ correct to the nearest degree if x is between $0°$ and $180°$. **1**

$\tan x = 0.4$

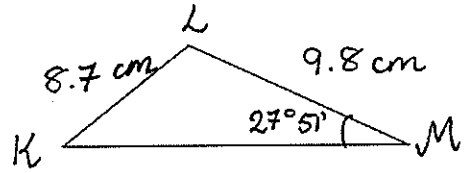
$x = \tan^{-1}(0.4)$

$x \approx 22°$

2 a) In ΔKLM, $\angle M = 27°51'$, $KL = 8.7$ cm and $LM = 9.8$ cm.

i) Complete the diagram below, showing all given information. **1**

ii) Find $\angle K$ correct to the nearest minute. **2**



$\frac{\sin K}{9.8} = \frac{\sin 27°51'}{8.7}$

$\sin K = \frac{9.8 \sin 27°51'}{8.7}$

$\angle K = 31°45'$

OR $\angle K = 180° - 31°45'$

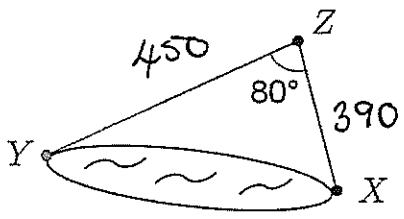
$= 148°15'$

full marks awarded for either acute only OR acute + obtuse angles

check: $180° - 148°15' - 27°51' = 3°54' > 0$

$\angle K = 148°15'$ or $31°45'$

b) To find the length of a lake, a surveyor walks 390 metres from point X to point Z, then turns 80° and walks 450 metres to point Y. Find the length of the lake correct to the nearest metre. 2



$x^2 = 390^2 + 450^2 - 2 \times 390 \times 450 \cos 80°$

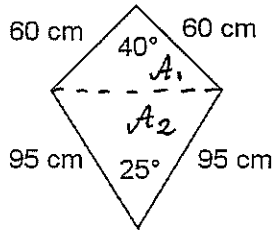
$= 293649.4896...$

$x = 541.894...$

$x \approx 542$ m (nearest metre)

The lake is 542 m long.

3 a) Calculate the area of the kite below, correct to the nearest square centimetre. 2



$A_1 = \frac{1}{2} \times 60^2 \times \sin 40°$

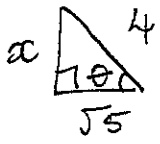
$= 1157.018...$

$A_2 = \frac{1}{2} \times 95^2 \times \sin 25°$

$= 1907.064...$

Total Area $= A_1 + A_2 \approx 3064 \text{ cm}^2$

* b) Find the exact value of $\sin\theta$ if $\cos\theta = \frac{\sqrt{5}}{4}$ and θ is acute. 2



$x^2 + 5 = 16$

$x^2 = 9$

$x = 3$

$\sin\theta = \frac{3}{4}$

* c) Solve the equation $1 - 3\sin x = 0$ correct to the nearest degree if x is between 0° and 180°. 2

$3\sin x = 1$

$\sin x = 1/3$

$\sin^{-1}\left(\frac{1}{3}\right) = 19.4712...$

sin ve in 1st and 2nd quadrant

$\theta = 19°$ or $180° - 19°$

$\theta = 19°$ or $161°$

Coordinate Geometry (9 marks)

1 a) Find the gradient and y-intercept for the linear equation $y = \frac{12-3x}{4}$ **2**

$y = \frac{12}{4} - \frac{3}{4}x$ $\quad m = -\frac{3}{4}$

$y\text{-int} = 3$

b) Find, in general form the equation of the line which passes through the points $(-1, 2)$ and $(3, -4)$. **2**

$m = \frac{-4-2}{3--1}$ $\quad y - 2 = -\frac{3}{2}(x+1)$

$= -\frac{3}{2}$ $\quad 2y - 4 = -3x - 3$

$3x + 2y - 1 = 0$

c) Find the equation of the line which is perpendicular to $y = 3x - 1$ and passes through the point (6,3). **2**

$m = -\frac{1}{3}$ $\quad y - 3 = -\frac{1}{3}(x - 6)$

$y = -\frac{1}{3}x + 5$

$x + 3y - 15 = 0$

* d) Use calculations to show that the quadrilateral with vertices $A(3,1)$, $B(8,4)$, $C(5,9)$ and $D(0,6)$ is a square. Write which test you have used. **3**

$m_{AB} = 3/5$ $\quad m_{AB} = m_{CD}$

$m_{BC} = -5/3$ $\quad \therefore AB \parallel CD$

$m_{CD} = 3/5$ $\quad m_{BC} = m_{AD}$

$m_{AD} = -5/3$ $\quad \therefore BC \parallel AD$

$m_{AB} \times m_{BC} = \frac{3}{5} \times -\frac{5}{3} = -1$

$\therefore AB \perp BC$

$\therefore ABCD$ is a rectangle (2 pairs of opp sides $\parallel$ and 1 angle 90°)

$AB^2 = 5^2 + 3^2 = 34$

$BC^2 = (5-8)^2 + (9-4)^2 = 34$

$\therefore AB = BC$

$\therefore ABCD$ is a square (rectangle with 1 pair of adjacent sides equal)

JY

Probability (9 marks)

1 At Xford High School, students who use mobile phones were surveyed. Some of the results are shown in the table.

	Pre-paid	Plan	Total
Female students	172	147	319
Male students	158	103	261
Total	330	250	580

a) Complete the table. **1**

b) Of the female students surveyed, one is chosen at random. What is the probability that she is on a plan? **1**

$\frac{147}{319}$

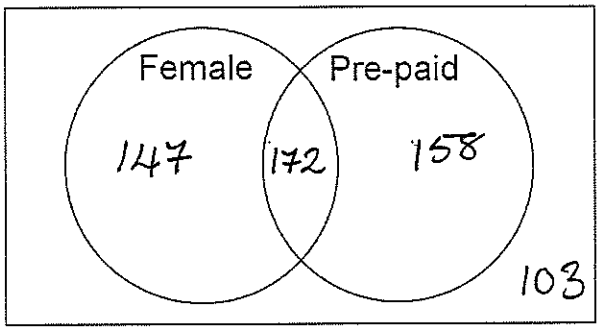
c) What is the probability that a student who completed the survey is male, given that they use a pre-paid phone? **1**

$\frac{158}{330} = \frac{79}{165}$

d) Ten new male students are surveyed, and are all on a plan. What percentage of male students are now on a plan? Give your answer to the nearest percent. **1**

$\frac{113}{271} \approx 42\%$

e) Use the information in the table to complete the Venn Diagram below. **2**



2 Three raffles are drawn, each with 100 tickets and 1 prize. Jane buys 2 tickets in one raffle and Mary buys one ticket in each of the other two raffles.

a) Fill in the correct name to make this statement true. **1**

Mary winning with her first ticket and winning with her second ticket are examples of **independent** events.

* b) i) Calculate the probability of Jane winning at least one prize, correct to 4 decimal places. **1**

P(Jane loses with both tickets) $= \frac{99}{100} \times \frac{98}{99} = \frac{98}{100}$

P(Jane wins) $= 1 - \frac{98}{100} = 0.0200$

ii) Does Mary have a better chance of winning at least one prize than Jane? Show working to support your answer. **1**

P(Mary LL) $= \frac{99}{100} \times \frac{99}{100}$

P(Mary wins ≥ 1) $= 1 - \frac{99}{100} \times \frac{99}{100} = 0.0199 < 0.02$

Jane has a better chance.

Inequations & Logarithms (11 marks)

1 a) Solve each inequality:

i) $2t + 1 \geq 5$ **1**

$2t \geq 4$

$t \geq 2$

ii) $\frac{3-5x}{4} \leq -9$ **2**

$3 - 5x \leq -9 \times 4$

$5x - 3 \geq 36$

$5x \geq 39 \qquad x \geq 7.8$

2 a) Simplify $\log_x 4 + \log_x 5$ **1**

$\log_x (20)$

PW

b) Evaluate $\log_2 36 - 2\log_2 3$ **2**

$\log_2 36 - 2\log_2 3$

$= \log_2 36 - \log_2 9$

$= \log_2 \left(\frac{36}{9}\right)$

$= \log_2 4$

$= 2$

c) Solve each equation, writing your answer to 3 decimal places where necessary

i) $\log_7 x = 3$ **1**

$x = 7^3$

$= 343$

ii) $5^{x-1} = 81$ **2**

$x - 1 = \frac{\log_{10} 81}{\log_{10} 5}$

$x = 1 + \frac{\log_{10} 81}{\log_{10} 5}$

≈ 3.730

* iii) $\log_2 \sqrt{x} - 3\log_2 \left(\frac{1}{x}\right) = 21$ **2**

$\frac{1}{2}\log_2 x + 3\log_2 x = 21$

$3\frac{1}{2}\log_2 x = 21$

$\log_2 x = 6$

$x = 2^6 = 64$

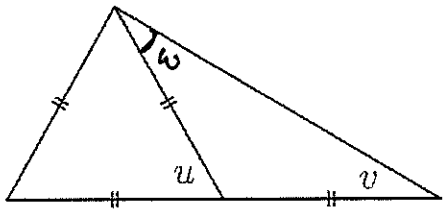
AK

Properties of Geometric Figures (13 marks)

1 a) A parallelogram has one angle equal to 55°. Give the values of the other three angles. **1**

55°, 125°, 125°

b) Find the values of u and v in the triangle below, giving reasons. **3**



$u = 60^\circ$ (angle in an equilateral triangle)

$w = v$ (base angle in isosceles triangle)

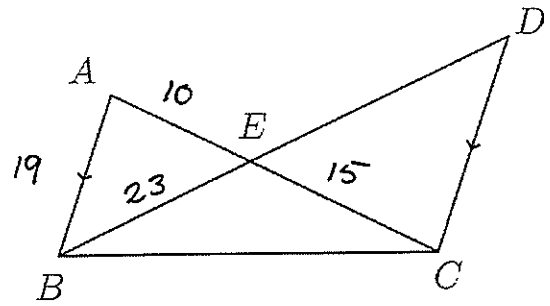
$u = w + v = 2v$ (exterior angle of triangle)

$v = 30^\circ$

2 a) In the diagram below, $AB = 19$ cm, $AE = 10$ cm, $BE = 23$ cm, $EC = 15$ cm and $AB \parallel DC$.

i) Prove that $\triangle AEB$ is similar to $\triangle CED$. **2**

ii) Find the value of CD. **2**



i) In $\triangle AEB$ and $\triangle CED$

$\angle AEB = \angle CED$ (vertically opposite angles)

$\angle BAE = \angle DCE$ (alternate angles, $AB \parallel DC$)

$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel DC$)

$\therefore \triangle AEB \parallel\parallel \triangle CED$ (3 pairs of corresponding angles equal)

ii) $\frac{CD}{19} = \frac{15}{10}$

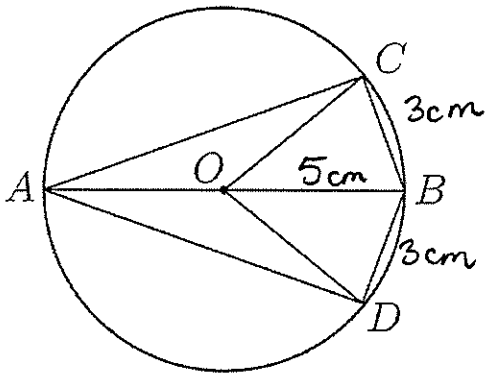
$CD = 28.5$ cm.

AK

b) In the diagram below, O is the centre of a circle of radius 5 cm. $BC = BD = 3$ cm.

i) Prove that ΔOCB is congruent to ΔODB. **2**

* ii) Hence, prove that $AC = AD$. **3**



i) In ΔOCB and ΔODB

$OB = OB$ (common side)

$OC = OD$ (equal radii)

$BC = BD$ (given)

$\therefore \Delta OCB \equiv \Delta ODB$ (SSS)

ii) $\angle COB = \angle DOB$ (corresponding angles $\Delta OCB \equiv \Delta ODB$)

$\angle AOC = 180° - \angle COB$ (adjacent supplementary angles

$\angle AOD = 180° - \angle DOB$ (similarly)

$\therefore \angle AOC = \angle AOD$

In ΔAOC and ΔAOD

$OA = OA$ (common side)

$\angle AOC = \angle AOD$ (proved above)

$OC = OD$ (equal radii)

$\therefore \Delta AOC \equiv \Delta AOD$ (SAS)

End of Exam – Please check your work

$\therefore AC = AD$ (corresponding sides, $\Delta AOC \equiv \Delta AOD$