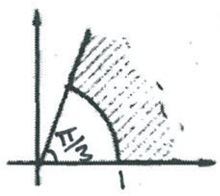
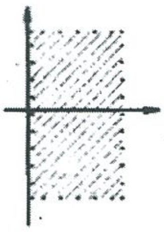
Answers

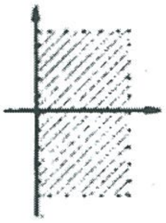
a)



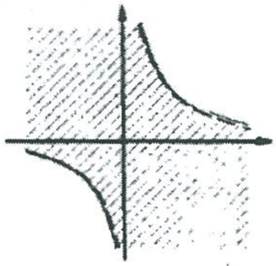
b)



c)



d)



d) excludes hyperbola and axes

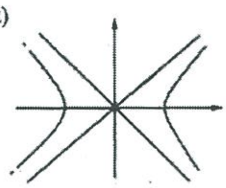
3.

i) $z^2 = x^2 - y^2 + 2xyi$

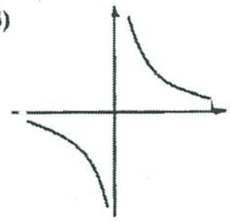
$\mathrm{Re}(z^2) = 3$ gives $x^2 - y^2 = 3$, this is a rectangular hyperbola whose axes are $x = \pm y$ (fig. α)

ii) $\mathrm{Im}(z^2) = 2xy = 4$, this rectangular hyperbola $xy = 2$ has axes coincident with the x-, y-axis (fig. β)

α)



β)



iii) $z^2 = 3 + 4i = 4 + 4i - 1 = 4 + 4i + i^2 = (2 + i)^2$, thus $z = \pm(2 + i)$

iv) Refer to figures (α), and (β) above the locus of z is the intersection of these regions

5.

i) $\dfrac{(2-i)(8+3i)}{3+i} = \dfrac{19-2i}{3+i} = \dfrac{(19-2i)(3-i)}{(3+i)(3-i)}$

$= \dfrac{57-2-19i-6i}{10} = \dfrac{11-5i}{2}$

ii) $\mathrm{Arg}z = \frac{5\pi}{6}$, $|z| = 2$

$\therefore z = 2cis\frac{5\pi}{6}$, so $z^6 = (2cis\frac{5\pi}{6})^6 = 2^6 cis 5\pi = -64$

$\therefore z^6 + 64 = 0$, or $z^7 + 64z = 0$

iii) a) The locus of z is the circle of centre A, radius 5. This circle goes through the origin, hence, $|z|$ is maximum when z lies at the other end of the diameter through the origin. $\therefore$ $|z|_{max} = 10$

b) The locus of z is the ellipse of foci A, B such that the sum of the distance from z to A and B is equal to 12. Notice that this ellipse touches the y-axis. $\therefore$

$\mathrm{Arg}(z)_{max} = \frac{\pi}{2}$