Carlingford High School
Mathematics Advanced
Year 11
Preliminary Examination 2020

NESA Number:

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General Instructions

- Working time - 120 minutes
- Reading time - 10 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-30, show relevant mathematical reasoning and/or calculations

Total marks: 80

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 70 marks

- Attempt questions 11-30
- Allow about 1 hour and 45 minutes for this section

Question	Functions	Trigonometric Functions	Calculus	Statistical Analysis	Total
MC	/3	/3	/2	/2	/10
11	/2				/2
12	/2				/2
13			/3		/3
14			/3		/3
15		/3			/3
16		/3			/3
17				/3	/3
18			/5		/5
19	/4				/4
20			/2		/2
21		/3			/3
22	/5				/5
23		/4			/4
24			/8		/8
25			/3		/3
26	/4				/4
27	/4				/4
28	/2				/2
29				/3	/3
30		/4			/4
Total	/26	/20	/26	/8	**/80**

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 8

1. What is the solution to the equation $3x^2 + 7x - 1 = 0$?

   A. $x = \frac{-7 \pm \sqrt{61}}{6}$

   B. $x = \frac{-7 \pm \sqrt{37}}{6}$

   C. $x = \frac{7 \pm \sqrt{37}}{6}$

   D. $x = \frac{7 \pm \sqrt{61}}{6}$

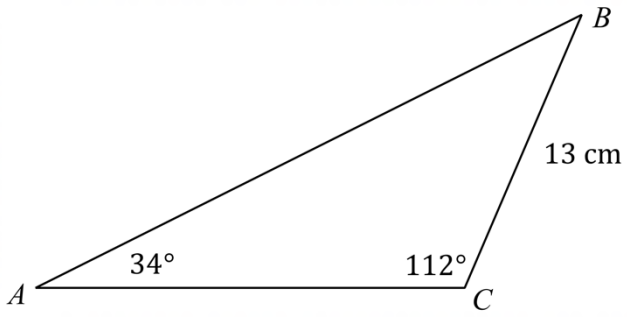
2. Which set of points below is a function?

   A. (–6, 7) , (7 , 6) , (–4, 5) and (–6, –4)

   B. (–6, 7) , (7 , –4) , (–6, 5) and (7, –2)

   C. (–6, 7) , (7 , 5) , (–4, 5) and (7, –6)

   D. (–6, 7) , (7 , –6) , (–4, 5) and (5, –4)

3. 

Not to scale

   What is the length of *AB*, correct to one decimal place?

   A. $13.0cm$

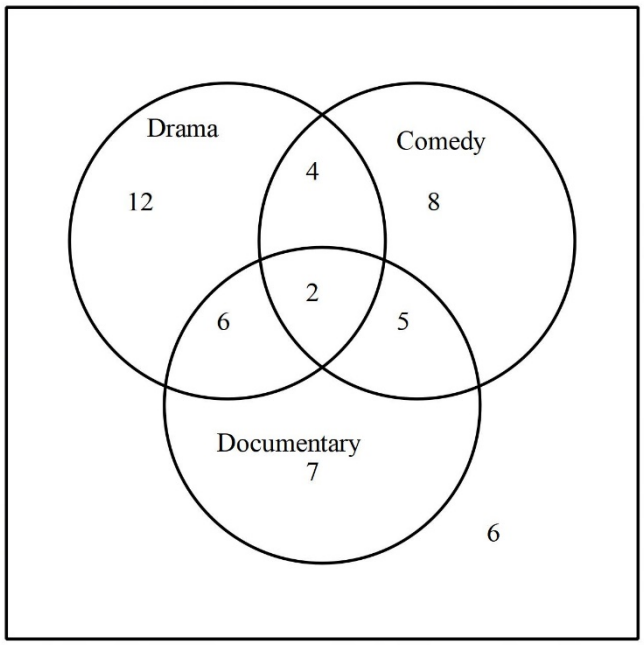
   B. $20.9cm$

   C. $21.6cm$

   D. $40.6cm$

4. A theatrical agent has 50 artists on his books.

The Venn diagram shows the genres in which they perform.



He rings one of his artists at random.

What is the probability that the artist does comedy or documentary or both, but not drama?

A. $\frac{3}{10}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{16}{25}$

5. Find the exact value of $\tan \frac{5\pi}{3}$.

A. $-\sqrt{3}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

6. What is the derivative of $x^{-2} - \frac{1}{x}$?

A. $\frac{-2}{x} - \frac{1}{x^2}$

B. $\frac{-2}{x} + \frac{1}{x^2}$

C. $\frac{-2}{x^3} - \frac{1}{x^2}$

D. $\frac{-2}{x^3} + \frac{1}{x^2}$

7. Which of the following expressions is equal to $4 + 4\cot^2 x$?

A. $4\sec^2 x$

B. $4\operatorname{cosec}^2 x$

C. $\cot^2 x$

D. $8\cot^2 x$

8. Find the derivative of $y = \sqrt{(x^3 + 4)^3}$.

A. $3(x^3 + 4)^2$

B. $18\sqrt{x^3 + 4}$

C. $\frac{\sqrt{x^3+4}}{18x^2}$

D. $\frac{9x^2\sqrt{x^3+4}}{2}$

9. Leo owns five blue and seven red ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother. What is the probability that both of the ties, are the same colour?

A. $\frac{1}{2}$

B. $\frac{5}{33}$

C. $\frac{31}{66}$

D. $\frac{31}{72}$

10. If $\frac{4}{x-3}+\frac{2}{x}=1, \quad x>0$, then x is equal to:

A. $\frac{9\pm\sqrt{57}}{2}$

B. $\frac{9+\sqrt{57}}{2}$

C. 2

D. 7

Section II

70 marks
Attempt all questions
Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks)

List all the **points** where $y=x^3-4x$ crosses the x-axis. **2**

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Question 12 (2 marks)

Simplify the expression $\frac{a^2-3a}{2ax} \times \frac{4a^2x}{2ax-6x}$, giving your answer as a simple fraction. **2**

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Question 13 (3 marks)

A tangent is drawn to the curve $y = \frac{x^3 - x^2 - 9x}{2}$ at the point where $x = \sqrt{3}$.

Find the angle of inclination of the tangent to the positive *x*-axis. **3**

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Question 14 (3 marks)

The velocity of a particle moving in a straight line is given by

$v = 2t^2 - 10t - 48$. Find the acceleration, when the velocity is zero. **3**

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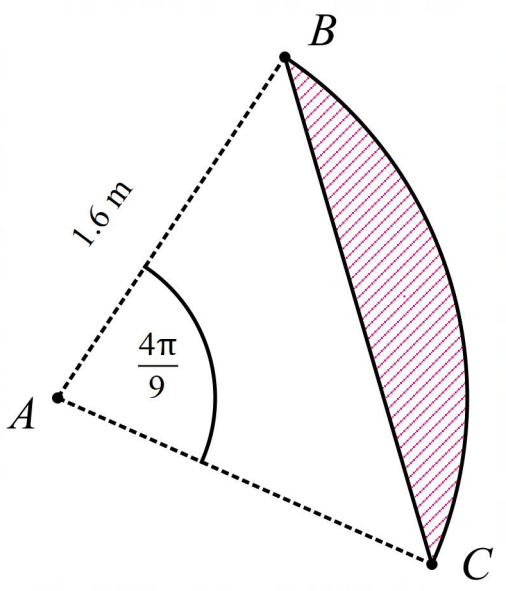
Question 15 (3 marks)

The shaded segment shown is formed by an arc *BC* of a circle radius 1.6m, which subtends an angle of $\frac{4\pi}{9}$ radians at its centre *A*.



Find the area of the shaded segment, correct to 3 significant figures.

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Question 16 (3 marks)

Solve the equation $\sin\theta = 5\cos\theta$ in the interval $0 \leq \theta \leq 360°$.
Answer correct to the nearest minute.

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Question 17 (3 marks)

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

(a) What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?

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(b) What is the probability that the sum of the numbers on the two balls is 5?

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(c) Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

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Question 18 (5 marks)

Differentiate, with respect to x.

(a) $4x^{-3}$ **1**

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(b) $(5+x^2)^{\frac{1}{2}}$ **2**

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(c) $\frac{3x^2-5}{2x+1}$ **2**

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Question 19 (4 marks)

(a) Find the equation of the axis of symmetry of the parabola $y = 5 + 4x - x^2$ and the coordinate of its vertex **2**

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(b) Hence, write down the domain and range of $y = 5 + 4x - x^2$, using interval notation **2**

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Question 20 (2 marks)

Use the result $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ to find $f'(x)$ if $f(x) = 2x^2$

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Question 21 (3 marks)

Show that $\sin \alpha \cot \alpha - \cos\alpha \sin^2\alpha = \cos^3 \alpha$.

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Question 22 (5 marks)

(a) Find the centre and radius of the circle represented by the equation:

$x^2 - 4x + y^2 + 6y + 4 = 0.$ **3**

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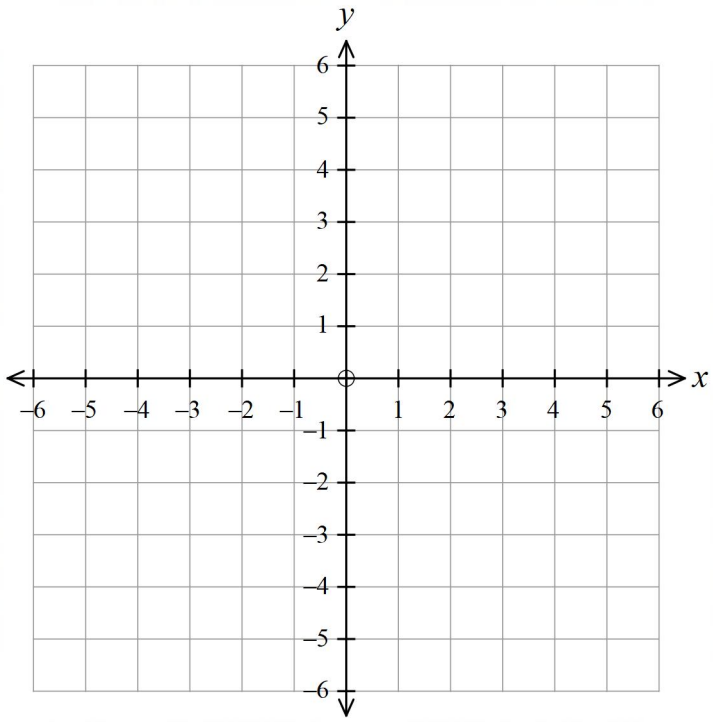
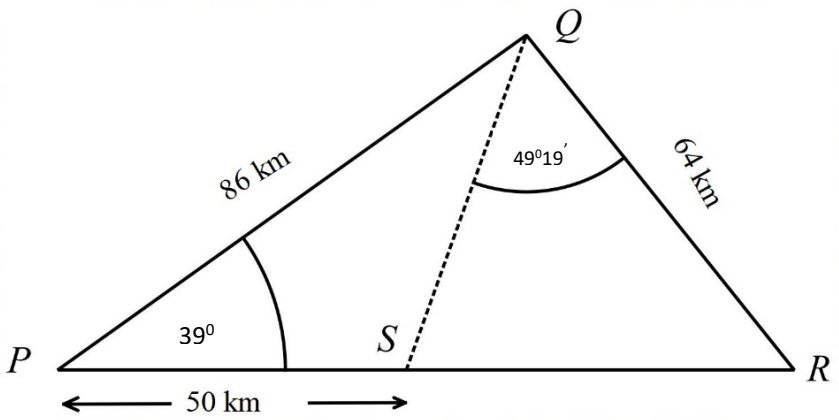
(b) Sketch the circle on the axes below, showing important points. (Note: you do not need to find the y-intercept) **2**



Question 23 (4 marks)

Triangles PQS and QRS have a common side QS, as shown.



(a) Calculate the length of QS, correct to 3 significant figures

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(b) Find the area of $\Delta\, PQR$, correct to the nearest square kilometre.

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Question 24 (8 marks)

$P\,(1, 4)$ lies on the curve $y = x^3 - 4x^2 + x + 6$.

(a) Find the equation of the tangent to the curve at P. **3**

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(b) Find the equation of the normal to the curve at P, writing your answer in general form. **2**

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(c) The tangent and normal at P intersect the x-axis at T and N respectively.
What are the coordinates of T and N? **2**

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(d) Find the area of triangle PTN. **1**

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Question 25 (3 marks)

Given that $y = 3x^4(2-x)^5$, find $\frac{dy}{dx}$, writing your answer in simplified form **3**

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Question 26 (4 marks)

(a) For the function $f(x) = \frac{1}{x^2-4}$ determine whether $f(x)$ is odd, even or neither, justifying your answer algebraically. **2**

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(b) Describe what the result from [a] means geometrically **1**

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(c) Write down any x values on $f(x)$ that are discontinuities. **1**

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Question 27 (4 marks)

(a) On the axes below, draw the graphs of $y = |x - 3|$ and $y = |3x + 3|$.

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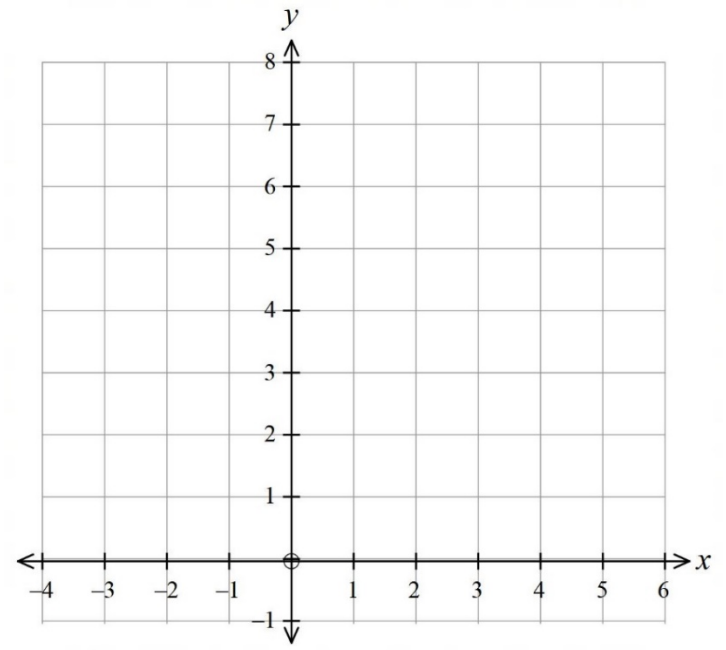

(b) Hence, or otherwise, find solutions to $|x - 3| = |3x + 3|$.

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Question 28 (2 marks)

For what values of m will the equation $y = x^2 - 2mx + 4m$, have exactly one solution?

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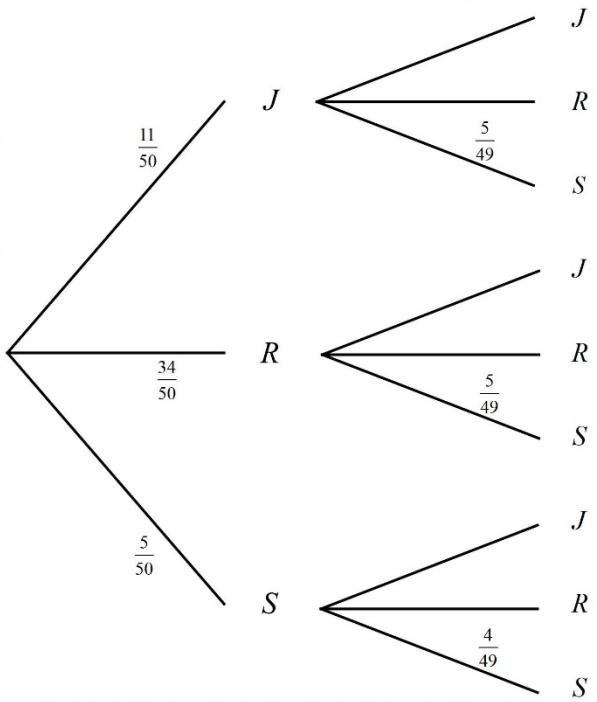
Question 29 (3 marks)

Employees in a company are classified as Junior if aged less than 18, Senior if aged over 60 or Regular for all others. There are 50 employees in the company and 2 are to be chosen to attend a conference.

The probability tree diagram has been started to show how the two employees could be chosen. **3**



Complete the missing probabilities on the probability tree and hence find the probability that the two employees chosen will come from different categories.

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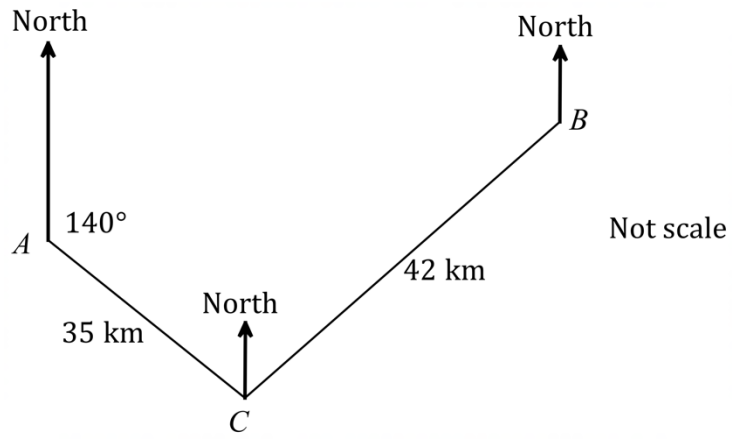
Question 30 (4 marks)



Tyler drives 35km from town *A* on a true bearing of 140°T to town *C*.
He then drives 42km on a true bearing of 38°T to town of *B*.

(a) Find $\angle ACB$

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(b) Find the bearing of *A* from *B*, to the nearest minute.

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End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Preliminary HSC Examination 2020
Mathematics Advanced Course

NESA Number _____

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

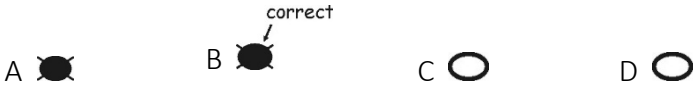
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○