

Carlingford High School



Year 11
YEARLY EXAMINATION 2019

Student Number: _____

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black/blue pen
- Approved calculators may be used
- A reference sheet is provided at the back of this paper
- For all questions in Section II, show relevant mathematical reasoning and/or calculations.
- Extra working space is provided at the back of this paper.

Total marks : 75

Section I – 8 (8 Marks)

- Attempt Questions 1 – 8
- Use the Multiple Choice Answer sheet provided.

Section II – 67 marks

- Attempt Questions 9 - 23

Questions	Further work With functions	Polynomials	Inverse Trigonometric Functions	Further Trigonometric Identities	Rates Of Change	Working with Combinatorics	Total
1						/1	
2					/1		
3					/1		
4					/1		
5	/1						
6			/1				
7				/1			
8		/1					
9						/5	
10		/3					
11		/3					
12	/3						
13						/3	
14			/5				
15					/6		
16				/3			
17	/3						
18				/6			
19				/3			
20					/7		
21					/3		
22		/8					
23						/6	
Total	/7	/15	/6	/13	/19	/15	/75

Section 1

Use the multiple-choice answer sheet for Questions 1 – 8

1. Max has a drawer which contains forty unmatched socks of identical style, with equal numbers of white, blue, grey and brown socks. Max reaches into the drawer in the dark to retrieve a pair of socks. What is the minimum number of socks that Max must take from the drawer, to ensure that his selection includes a pair of matching socks of any colour?

A. 3
B. 4
C. 5
D. 6

2. A particle moves along the x -axis such that its displacement (x metres) from the origin at a time t seconds is given by the equation:

$$x = 5t^2 - \frac{4}{t^5}.$$

What is the acceleration (in m/s^2) of the particle when $t = 1$?

A. -120
B. -110
C. 0
D. 30

3. What is the expression for $\frac{dr}{dt}$ if $V = \frac{4}{3} \pi r^3$ and $\frac{dV}{dt} = 2$?

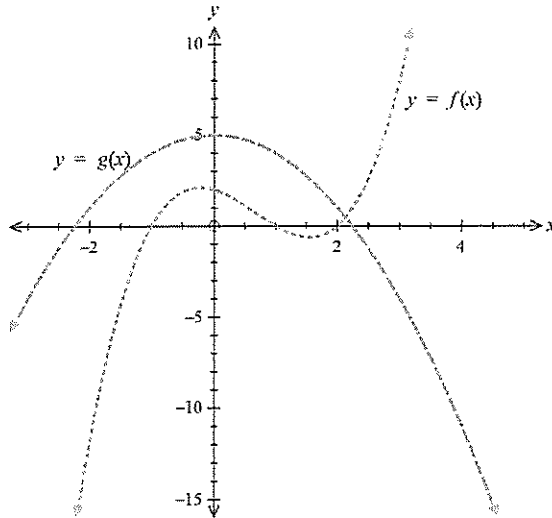
A.	$\frac{1}{2\pi r^2}$	C.	$8\pi r^2$
B.	$\frac{32\pi}{3}$	D.	16π

4. A population grows according to the equation $N(t) = 120e^{0.025t}$, where t is measured in months.

By how many individuals, does the population grow over the first 3 years?

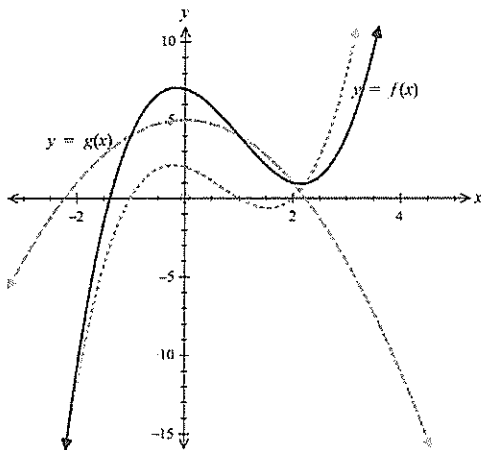
A. 120
B. 125
C. 175
D. 295

5. The graphs of $y = f(x)$ and $y = g(x)$ are shown on the grid below.

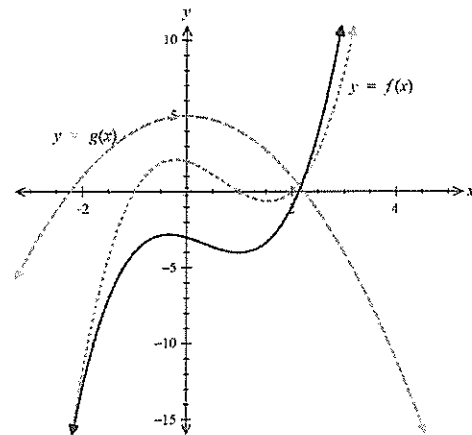


In which diagram has the graph of $y = f(x) + g(x)$ been drawn on the same grid?

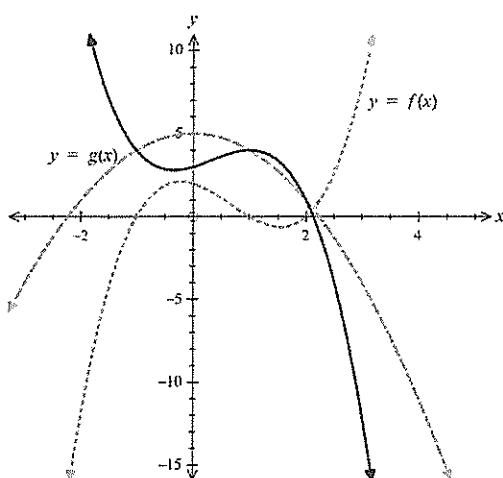
A.



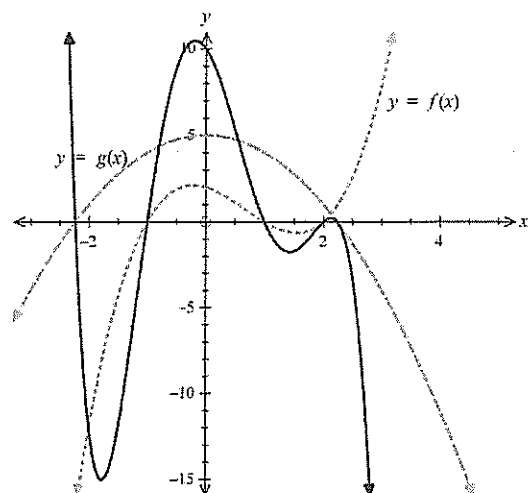
B.



C.



D.



6. Given that $\cos^{-1}(\sqrt{1-t^2}) = \alpha$, which expression represents $\sin \alpha$.

- A. $\sqrt{1-t^2}$
- B. $\sqrt{1+t^2}$
- C. 1
- D. t

7. The expansion of $\cos(2x + 60^\circ)$ equals:

- A. $\frac{\cos 2x}{2} + \frac{\sqrt{3}\sin 2x}{2}$
- B. $\frac{\cos 2x}{2} - \frac{\sqrt{3}\sin 2x}{2}$
- C. $\frac{\sqrt{3}\cos 2x}{2} + \frac{\sin 2x}{2}$
- D. $\frac{\sqrt{3}\cos 2x}{2} - \frac{\sin 2x}{2}$

8. What is the multiplicity of the root $x = 1$ if $P(x) = 3x^5 - 5x^4 + 5x - 3$?

- A. 1
- B. 2
- C. 3
- D. 4

Section 2

Question 9 (5 marks)

There are 8 girls and 7 boys in a class.

- (a) The class elects a captain and vice-captain. In how many ways is this possible? **1**

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- (b) The class elects four representatives for the student council. How many different groups of representatives are possible? **2**

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- (c) How many groups of representatives for the student council are possible, if the class decide that they want two girls and two boys in the group? **2**

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Question 10 (3 marks)

Marks

The equation $x^3 + 2x^2 + 3x + 6 = 0$ has roots α , β and γ . Find the value of

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$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

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Question 11 (3 marks)

Consider the polynomial $P(x) = 2x^3 - x^2 + ax + 6$ where a is a real number.

Let $(x - 1)$ be a factor of $P(x)$.

(a) What is the value of a ?

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(b) Find the remainder when $P(x)$ is divided by $(2x + 1)$.

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Question 12 (3 marks)

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Solve: $\frac{2x-4}{3-x} \geq 2$.

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Question 13 (3 marks)

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Find the number of ways in which 3 males and 3 females can be arranged in a line so that the two end positions are occupied by males and no two males are next to each other.

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Question 14 (5 marks)

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For the function $f(x) = 4\sin^{-1}\left(\frac{x-5}{2}\right)$.

a) Give the domain and range of the function.

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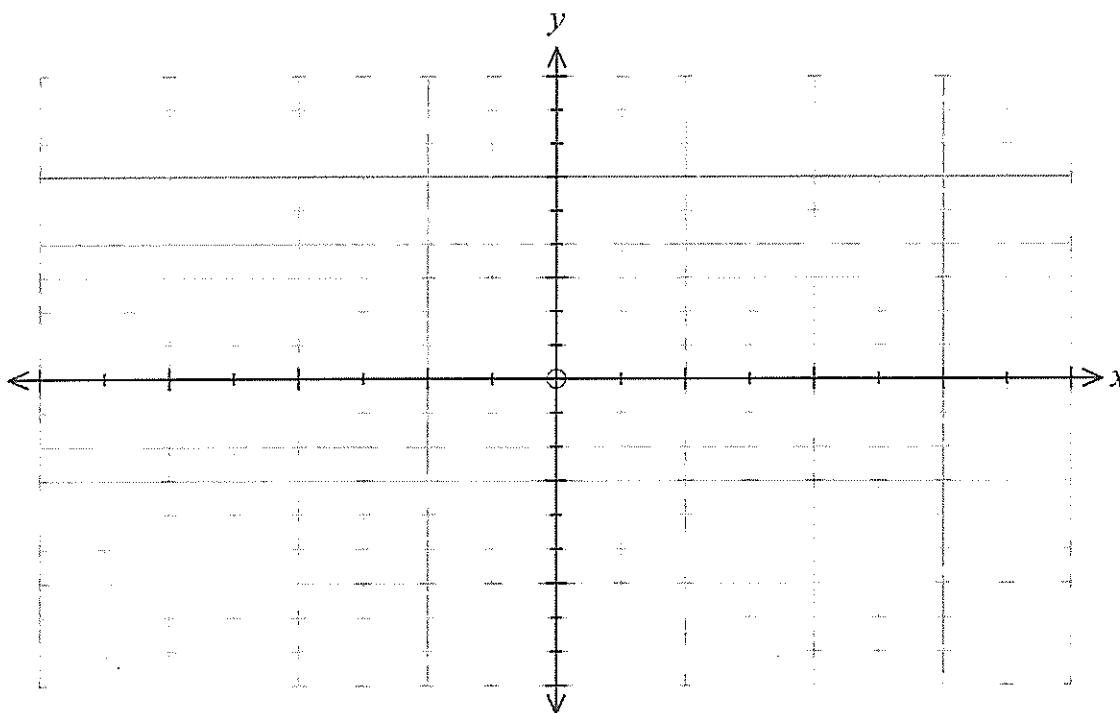
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b) Draw a sketch of the function.

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Question 15 (6 marks)

The number of bacteria N a person has after being infected with a virus after t hours is given by:

$$N = 10\,000e^{0.05t}$$

- (a) Find the number of bacteria after 10 hours. 1

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- (b) Find the time required for the number of bacteria to reach 100 000.
Answer to the nearest hour. 3

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- (c) What is the rate at which bacteria is increasing after one day? 2

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Question 16 (3 marks)

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Show that $\frac{\cos 3\beta}{\sin \beta} + \frac{\sin 3\beta}{\cos \beta} = 2\cot 2\beta$

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Question 17 (3 marks)

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Find the Cartesian equation that is represented by the pair of parametric equations below:

$$\begin{array}{ll} x = 2p + 3 & \dots \textcircled{1} \\ y = p^2 - 6p + 9 & \dots \textcircled{2} \end{array}$$

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Question 18 (6 marks)

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- (a) Use the substitution $t = \tan \frac{\theta}{2}$ to show that the equation $2\sin\theta - \cos\theta = 1$ is equivalent to the equation $t^2 + 2t - 2 = 0$.

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- (b) Hence, solve the equation $2\sin\theta - 3\cos\theta = 1$ for $0 \leq \theta \leq 360^\circ$.
Answer correct to the nearest degree.

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Question 19 (3 marks)

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Show that: $2\sin(A + B)\cos(A - B) = \sin 2A + \sin 2B$.

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Question 20 (7 marks)

A particle moves along the x -axis such that its displacement (x metres) from the origin at a time t seconds is given by the equation:

$$x = 2t^3 - 15t^2 + 24t .$$

- (a) The motion of the particle begins at the origin when $t = 0$. After how many seconds does the particle first return to the origin? (Answer to the nearest tenth of a second). 2

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- (b) Find the times at which the particle is at rest. 3

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- (c) Find the initial acceleration of the particle. 2

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Question 21 (3 marks)

The displacement of a particle at time t (in seconds) is given by:

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$$x = 3e^{-2t} + 4e^{-t} + 2t$$

Find the exact time at which the particle comes to rest.

[illegible]

Question 22 (8 marks)

Consider the function $f(x) = (x + 2)^2 - 5$.

- (a) What is the largest domain of $f(x)$, including $x = 0$ that has an inverse? 1

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- (b) Find the equation of the inverse function $f^{-1}(x)$. 2

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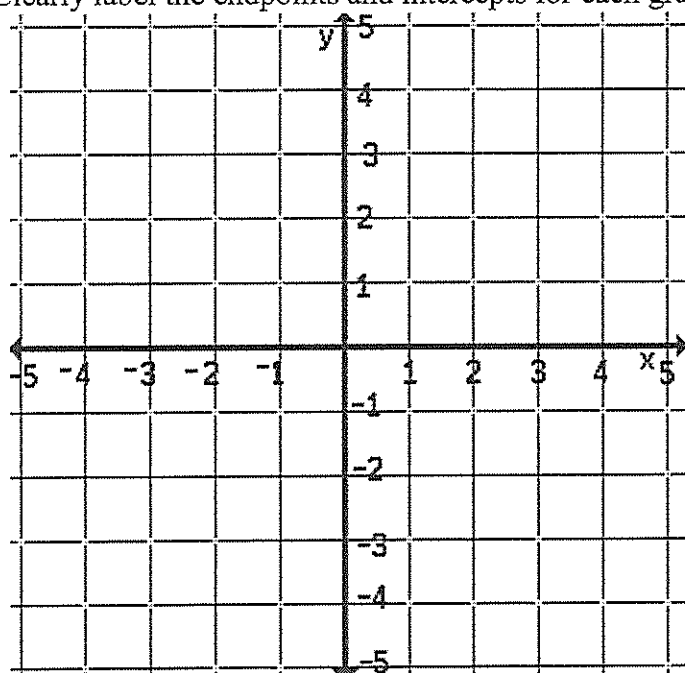
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- (c) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same number plane. 3

Clearly label the endpoints and intercepts for each graph.



- (d) What is the x -coordinate of the point of intersection of the curves $y = f(x)$ and $y = f^{-1}(x)$? Answer in simplest exact form. 2

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Question 23 (6 marks)

- (a) Show that $(1 - x)^5 \left(1 + \frac{1}{x}\right)^5 = \left(\frac{1}{x} - x\right)^5$. 2

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- (b) Write the expansion of $\left(\frac{1}{x} - x\right)^5$, leaving coefficients in the form $\binom{n}{r}$. 2

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- (c) Determine the coefficient of x^3 in the expansion of $(1 - x)^5 \left(1 + \frac{1}{x}\right)^5$ and hence show that $\binom{5}{4}\binom{5}{1} - \binom{5}{5}\binom{5}{2} - \binom{5}{3}\binom{5}{0} = \binom{5}{4}$.

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End of Paper

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.