CHS Extension 1 Mathematics Trial 2020 Solutions

Multiple Choice

1.	$\cos\theta = \dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert} = \dfrac{1\times1+1\times(-1)}{\sqrt{1^2+1^2}\times\sqrt{1^2+(-1)^2}} = 0$ $\theta = \dfrac{\pi}{2}$ Or, can be done graphically	A
2.	$R^2 = (\sqrt{3})^2 + 1^2$ $R = \sqrt{4}$ $= 2$ $\tan\alpha = \sqrt{3}$ $\alpha = \dfrac{\pi}{3}$ $\therefore \sin x + \sqrt{3}\cos x = 2\sin\left(x + \dfrac{\pi}{3}\right)$	C
3.	$f(x) = e^x - 1$ Let $y = e^x - 1$ For $f^{-1}(x)$ we have $x = e^y - 1$ which also gives $e^y = x + 1$ $\dfrac{dx}{dy} = e^y$ $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ $= \dfrac{1}{e^y}$ $= \dfrac{1}{x+1}$	B
4.	The slope in the 1st and 3rd quadrants is always negative. The slope in the 2nd and 4th quadrants is always positive. This rules out options B, C and D.	A
5.	$u = 9 - x^2$ $\dfrac{du}{dx} = -2x$ or $du = -2xdx$	A

	$= -\sqrt{9 - x^2} + C$	
6.	For $x^4 + ax^3 - 3x^2 + bx - 2$ the triple root cannot be 2 since the constant term is -2. A triple root of 2 would lead to a constant of -8. So the triple root is $x = -1$. Substituting -1 into the polynomial gives: $1 - a - 3 - b - 2 = 0$ $-a - b - 4 = 0 \quad (1)$ First derivative of the polynomial is $4x^3 + 3ax^2 - 6x + b$ The second derivative is $12x^2 + 6ax - 6$. Substituting $x = -1$ into second derivative gives: $12 - 6a - 6 = 0$ $-6a = -6$ $a = 1$ Substituting $a = 1$ into (1) gives: $-1 - b - 4 = 0$ $-b = 5$ $b = -5$ OR **substituting x = -1 and 2 into f(x) and solving simultaneously works.**	D

7.	$\frac{dA}{dt} = 0.1 \text{ m}^2/\text{min}$ $A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ $\therefore \quad \frac{dr}{dA} = \frac{1}{2\pi r}$ $\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA}$ $= 0.1 \times \frac{1}{2\pi r}$ $= \frac{0.05}{\pi r}$ $When\ A = 0.3$ $\pi r^2 = 0.3$ $r = \sqrt{\frac{0.3}{\pi}}$ $\frac{dr}{dt} = \frac{0.05}{\pi\sqrt{\frac{0.3}{\pi}}}$ $= 0.05150322694$ $\approx 0.0515 \text{ m/min}$	C
8.	$\text{Proj}_{\underset{\sim}{q}}(\underset{\sim}{p}) = \frac{\underset{\sim}{p}.\underset{\sim}{q}}{\lvert \underset{\sim}{q}\rvert^2} \cdot \underset{\sim}{q}$ $\underset{\sim}{p} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ onto $\underset{\sim}{q} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. $\underset{\sim}{p}.\underset{\sim}{q} = 4 \times -1 + \ -3 \times 2 = -10$ $\lvert \underset{\sim}{q}\rvert^2 = (-1)^2 + 2^2 = 5$ $\text{Proj}_{\underset{\sim}{q}}(\underset{\sim}{p}) = \frac{-10}{5} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $= -2\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$	B
9.	We have ${}^{10}\mathbf{C}_2$ ways of choosing the pair of numbers to use in the code. With the restriction, we have 14 ways of arranging any pair of digits. Eg. If we used 0 and 1 We could have 2 of each digit: 0011 0110 0101 1100 1001 1010 (6 arrangements) We could have 3 x 0 and 1 x 1 0001 0010 0100 1000 (4 arrangements) Or we could have 3 x 1 and 1 x 0 1110 1101 1011 0111 (4 arrangements)	D

	So we have ${}^{10}\mathbf{C}_2 \times 14 = 630$ arrangements with exactly 2 digits. Probability of guessing the correct code is $\dfrac{1}{630}$.	
10.	Step 3: To prove true for $n = k + 1$ $2^{k+1} + (-1)^{k+1+1} = 2(2^k) + (-1)^{k+2}$ $= 2[3m - (-1)^{k+1}] + (-1)^{k+2}$ Error Line 4 $= 2 \times 3m - 2 \times (-1)^{k+1} - (-1)^{k+1}$ $= 3[2m - (-1)^{k+1}]$ Which is a multiple of 3 since m and k are integers. Step 4: True by induction	D

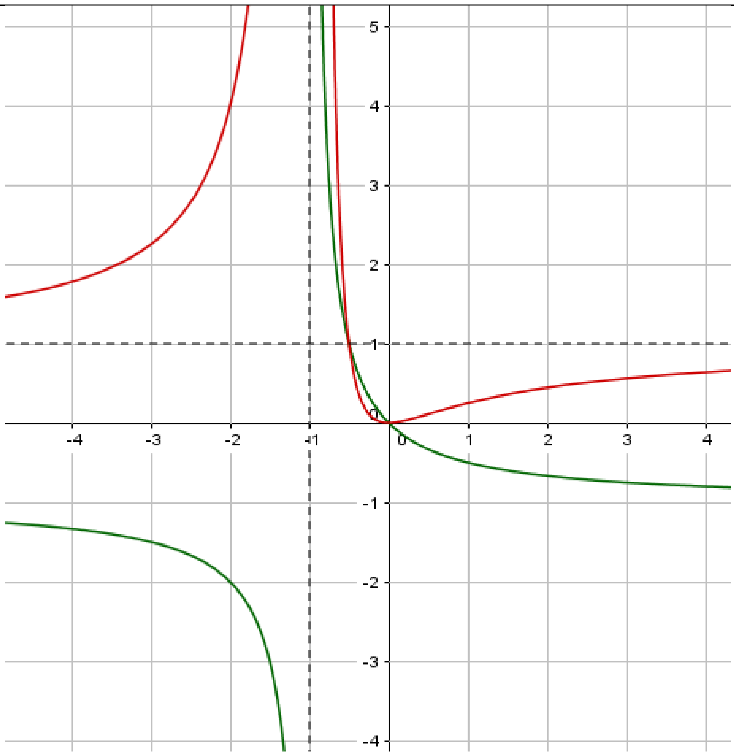
Question 11

11a)	$\dfrac{d}{dx}(e^x \tan^{-1} x) = e^x \tan^{-1} x + \dfrac{e^x}{1+x^2}$	2 Marks: Correct answer. 1 Mark: Uses the product rule
11b)i)	Using $\sin A \sin B = \dfrac{1}{2}[\cos(A-B) - \cos(A+B)]$ $\sin 5x \sin x = \dfrac{1}{2}(\cos(5x-x) - \cos(5x+x))$ $= \dfrac{1}{2}(\cos 4x - \cos 6x)$	1 Mark: Correct answer
11b)ii)	$\displaystyle\int_0^{\frac{\pi}{4}} \sin 5x \ \sin x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 4x - \cos 6x \ dx$ $= \dfrac{1}{2}\left[\dfrac{\sin 4x}{4} - \dfrac{\sin 6x}{6}\right]_0^{\frac{\pi}{4}}$ $= \dfrac{1}{2}\left[\dfrac{\sin(\pi)}{4} - \dfrac{\sin\left(\frac{3\pi}{2}\right)}{6} - 0\right]$ $= \dfrac{1}{2}\left(0 - \left(-\dfrac{1}{6}\right)\right)$ $= \dfrac{1}{12}$	2 marks for the correct answer with sufficient working 1 mark for use of proven identity from part a) to work towards solution with error or omission

11c)	Let $u = 1 - x$ $du = -\ dx$ $x = 1 - u$ when $x = 0$, $u = 1$ when $x = -8$, $u = 9$ $\int_{-8}^{0} \frac{x}{1-x}\,dx = \int_{9}^{1} \frac{1-u}{\sqrt{u}}\ -\ du$ $= \int_{1}^{9} \frac{1}{\sqrt{u}} - \frac{u}{\sqrt{u}}\,du$ $= \int_{1}^{9} u^{-\frac{1}{2}} - u^{\frac{1}{2}}\,du$ $= \left[2u^{\frac{1}{2}} - \frac{2}{3}u^{\frac{3}{2}} \right]_{1}^{9}$ $= \left[2\sqrt{9} - \frac{2}{3} \times 9\sqrt{9} - \left(2 - \frac{2}{3}\right) \right]$ $= 6 - 18 - 1\frac{1}{3}$ $= -\frac{40}{3}$ or $-13\frac{1}{3}$	3 marks for correct answer obtained from correct use of given substitution 2 marks for obtaining correct integral in terms of u, including upper and lower bounds or equivalent merit 1 mark for some relevant work towards using the given substitution
11d)i)	The parametric equations are: $x = \cos^2 t \qquad (1)$ $y = 4\sin^2 t \qquad (2)$ $\frac{(2)}{4}$ gives $\frac{y}{4} = \sin^2 t. \qquad (3)$ $(1) + (3)$ and using $\cos^2 t + \sin^2 t = 1$ gives $x + \frac{y}{4} = 1 \Rightarrow 4x + y = 4$.	1 Mark: Correct answer in any form
11d)ii)	$0 \le \cos^2 t \le 1$ and so $0 \le x \le 1$. Therefore, $y = 4 - 4x$ for $0 \le x \le 1$.	1 Mark: Correct answer
11 e)	$\int 2\cos^2 x dx = 2\int \frac{1}{2}(1 + \cos 2x)dx$ $= x + \frac{1}{2}\sin 2x + C$	2 Marks: Correct answer. 1 Mark: Uses double angle formula to simplify the integral.

11 f)	Step 1: To prove true for n = 1 $$4^1 + 14 = 18$$ Result is true for n = 1 Step 2: Assume true for $n = k$ $$4^k + 14 = 6\text{m}$$ where m is an integer Step 3: To prove true for $n = k + 1$ $$4^{k+1} + 14 = 6\text{p}$$ where p is an integer $$\begin{aligned}\text{LHS} &= 4^{k+1} + 14 \\ &= 4\left(4^k\right) + 14 \\ &= 4\left(4^k\right) + 4 \times 14 - 3 \times 14 \\ &= 4\left(4^k + 14\right) - 3 \times 14 \\ &= 4(6\text{m}) - 42 \\ &= 6(4\text{m} - 7) \\ &= 6p \\ &= \text{RHS}\end{aligned}$$ Step 4: True by induction	3 marks: Correct answer. 2 marks: Proves the result true for n = 1 and attempts to use the result of $n = k$ to prove the result for $n = k$ + 1 1 mark: Proves the result true for n = 1.
11 g) i)	$$\begin{aligned}\overrightarrow{AO} &= \overrightarrow{AB} - \overrightarrow{OB} \\ &= \underset{\sim}{v} - \underset{\sim}{u}\end{aligned}$$ $\Delta ABO = \Delta OCD$ Congruent triangles $\therefore \overrightarrow{AO} = \overrightarrow{OD}$ $$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{AO} + \overrightarrow{OD} \\ &= (\underset{\sim}{v} - \underset{\sim}{u}) + (\underset{\sim}{v} - \underset{\sim}{u}) \\ &= 2\underset{\sim}{v} - 2\underset{\sim}{u}\end{aligned}$$	1 Mark: Correct answer
11 g) ii)	$$\begin{aligned}\overrightarrow{BD} &= \overrightarrow{AD} - \overrightarrow{AB} \\ &= (2\underset{\sim}{v} - 2\underset{\sim}{u}) - \underset{\sim}{v} \\ &= \underset{\sim}{v} - 2\underset{\sim}{u}\end{aligned}$$	1 Mark: Correct answer
12 a) i)	Substituting $\underset{\sim}{F} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\underset{\sim}{s} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ into $W = \underset{\sim}{F} \cdot \underset{\sim}{s}$ gives: $$\begin{aligned}W &= \underset{\sim}{F} \cdot \underset{\sim}{s} \\ &= \begin{pmatrix} 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix} \\ &= 20\end{aligned}$$	2 Marks: Correct final answer 1 Mark: Obtains correct value for vector: **s**

12a) ii)	A unit vector in the direction of $\overrightarrow{PQ}$ is $\hat{\underset{\sim}{s}} = \frac{1}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. Substituting $\underset{\sim}{F} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, $\hat{\underset{\sim}{s}} = \frac{1}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\lvert\underset{\sim}{s}\rvert = 5$ into $W = (\underset{\sim}{F} \cdot \hat{\underset{\sim}{s}})\lvert\underset{\sim}{s}\rvert$ gives: $W = \left(\begin{pmatrix} 4 \\ -2 \end{pmatrix} \cdot \frac{1}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}\right)5$ $= 20$	2 Marks: Correct working to demonstrate proof 1 Mark: Correctly finds unit vector **s**.
iii)	The component of $\underset{\sim}{F}$ in the direction of l is given by $\left(\frac{\underset{\sim}{F} \cdot \underset{\sim}{s}}{\underset{\sim}{s} \cdot \underset{\sim}{s}}\right)\underset{\sim}{s}$. Substituting $\underset{\sim}{F} \cdot \underset{\sim}{s} = 20$, $\underset{\sim}{s} \cdot \underset{\sim}{s} = 25$ and $\underset{\sim}{s} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ into $\left(\frac{\underset{\sim}{F} \cdot \underset{\sim}{s}}{\underset{\sim}{s} \cdot \underset{\sim}{s}}\right)\underset{\sim}{s}$ gives: $\left(\frac{\underset{\sim}{F} \cdot \underset{\sim}{s}}{\underset{\sim}{s} \cdot \underset{\sim}{s}}\right)\underset{\sim}{s} = \frac{20}{25}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $= \frac{4}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $= \begin{pmatrix} 2.4 \\ -3.2 \end{pmatrix}$ Alternatively, the component of $\underset{\sim}{F}$ in the direction of l is $(\underset{\sim}{F} \cdot \hat{\underset{\sim}{s}})\hat{\underset{\sim}{s}}$.	1 mark: Correct solution

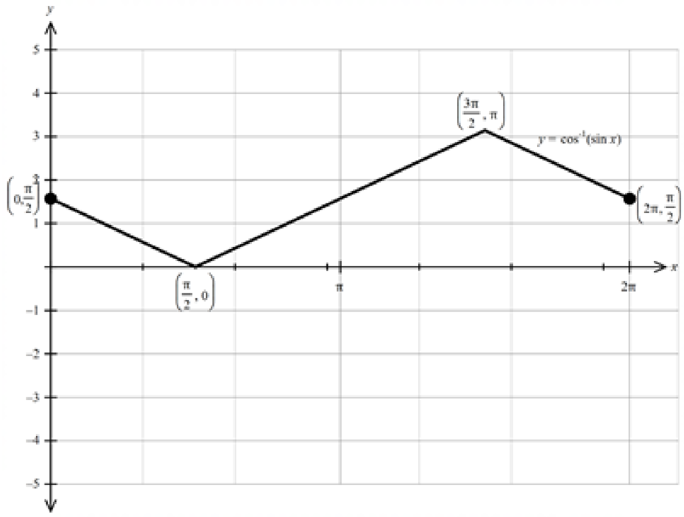
12b) i)		2 marks: also, correct intercept with f(x) at (-1/2,1) and intercept (0,0) and correct placement of f(x)^2 in relation to f(x) 1 mark: Correct shape and asymptotes
12 b) ii)	$(f(x))^2 = f(x) \Rightarrow f(x)(f(x)-1)=0$ So $f(x)=1$ or $f(x)=0$. $-\frac{x}{x+1}=1 \Rightarrow x=-\frac{1}{2}$ Hence $x=-\frac{1}{2}$ or $x=0$. **OR** The graphs of $y=f(x)$ and $y=(f(x))^2$ intersect at O, where $x=0$. The graphs of $y=f(x)$ and $y=(f(x))^2$ intersect on the line $y=1$, where $x=-\frac{1}{2}$.	2 Marks: Two correct solutions 1 Mark: one Correct solution
12 c) i)	$y=\cos^{-1}(\sin x)$ $\frac{dy}{dx} = -\frac{\cos x}{\sqrt{1-\sin^2 x}}$ $= -\frac{\cos x}{\sqrt{\cos^2 x}}$ $= \mp 1$	1 Mark: correct working to demonstrate result
ii)	Since $\frac{dy}{dx} \neq 0$ this function has no stationary points / no points where the graph is horizontal.	1 Mark: Correct reasoning

<table>
<tr><td>iii)</td><td>Range: $\text{Using } -1 \le \sin x \le 1$

$\text{when } \sin x = 1$
$y = \cos^{-1}(1)$
$= 0$
$\text{when } \sin x = -1$
$y = \cos^{-1}(-1)$
$= \pi$

$\therefore$ range is $[0,\pi]$

Naturally, the domain for $y = cos^{-1}x$ would be $-1 \le x \le 1$. But since $-1 \le sinx \le 1$ for all real x, this will satisfy the natural domain for $y = cos^{-1}x$. Hence, the domain is $(-\infty, \infty)$</td><td>2 Marks: Correct range and correct justification of the domain

1 Mark: one of these correct</td></tr>
<tr><td>iv)</td><td>
</td><td>2 Marks: correct graph with all important points correct

1 Mark: Graph that matches given domain and range</td></tr>
<tr><td>13a) i)</td><td>Surface area of a cube with a side length of x is $6x^2$
$S = 6x^2$
$\frac{dS}{dx} = 12x$
$\text{iven } \frac{dS}{dt} = 8$
$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \frac{8}{12x}$
$= \frac{\frac{2}{3}}{x} \text{ with } k = \frac{2}{3}$</td><td>1 Mark: Correct answer</td></tr>
<tr><td>13a) ii)</td><td>$V = x^3$ (volume of a cube with side length x)
$\frac{dV}{dx} = 3x^2$
$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$
2</td><td>2 Marks: Correct answer

1 Mark: Makes some progress</td></tr>
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	$x = \frac{1}{2}\frac{dV}{dt}$ $x = V^{\frac{1}{3}}$ $\frac{1}{2}\frac{dV}{dt} = V^{\frac{1}{3}}$ $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	
iii)	$\frac{dV}{dt} = 2V^{\frac{1}{3}}$ $V^{-\frac{1}{3}}dV = 2\mathrm{dt}$ $\int V^{-\frac{1}{3}}\,dV = \int 2\,\mathrm{dt}$ $\frac{3}{2}V^{\frac{2}{3}} = 2t + C$ Given that V = 8 when t =0 $\frac{3}{2}8^{\frac{2}{3}} = 2 \times 0 + C \text{ or } C = 6$ $\frac{3}{2}V^{\frac{2}{3}} = 2t + 6$ Find t when V =16√2 $\frac{3}{2}\left(16\sqrt{2}\right)^{\frac{2}{3}} = 2t + 6$ $12 = 2t + 6$ $t = 3$	3 Marks: Correct answer. 2 Marks: Finds the expression for V and t. 1 Mark: Separates the variables and attempts to integrate both sides.
13b) i)	$\mathrm{LHS} = \mathrm{cosec}x + \mathrm{cot}x$ $= \frac{1+t^2}{2t} + \frac{1-t^2}{2t}$ $= \frac{1+t^2+1-t^2}{2t}$ $= \frac{1}{t}$ $= \cot\frac{x}{2}$ $= \mathrm{RHS}$	2 Marks: Correct answer. 1 Mark: Writes $\mathrm{cosec}x$ and $\mathrm{cot}x$ in terms of t.
13 b) ii)	$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\mathrm{cosec}x + \mathrm{cot}x)dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\frac{x}{2}dx$ $= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{0.5\cos\frac{x}{2}}{\sin\frac{x}{2}}dx$ $= 2\left[\ln\left(\sin\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Finds the primitive function.

	$= 2\left[\ln\left(\sin\frac{\pi}{4}\right) - \ln\left(\sin\frac{\pi}{6}\right)\right]$ $= 2\left(\ln\frac{1}{\sqrt{2}} - \ln\frac{1}{2}\right)$ $= 2\ln\left(2^{-\frac{1}{2}} \div 2^{-1}\right)$ $= 2\ln 2^{\frac{1}{2}}$ $= \ln 2$	
c)	Rearranging $y = \frac{1}{x^2+1}$ to express x^2 in terms of y gives $x^2 = \frac{1}{y} - 1$. $V = \pi\int_{\frac{1}{2}}^{1}\left(\frac{1}{y} - 1\right)dy$ $= \pi\left[\ln\lvert y\rvert - y\right]_{\frac{1}{2}}^{1}$ $= \pi\left(\ln 1 - 1 - \left(\ln\frac{1}{2} - \frac{1}{2}\right)\right)$ $= \pi\left(\ln 2 - \frac{1}{2}\right)$ Rearranging $y = 1 - \frac{x}{2}$ to express x in terms of y gives $x = 2(1-y)$. $V = \pi\int_{\frac{1}{2}}^{1}(4(1-y)^2)dy$ $= -\frac{4\pi}{3}\left[(1-y)^3\right]_{\frac{1}{2}}^{1}$ $= -\frac{4\pi}{3}\left(0 - \frac{1}{8}\right)$ $= \frac{\pi}{6}$ Hence, total volume = $\pi\left(ln2 - \frac{1}{2}\right) - \frac{\pi}{6}$	4 Marks: Correct final answer 3 Marks: Correct primitive 2 marks: correctly applies volume formulae 1 Mark: rearranges to make x the subject

	$=\pi ln2-\frac{2\pi}{3}$	
14 a) i)	Start with the RHS and show that it equals the LHS $\text{RHS}=\frac{1}{50}\left(\frac{(50-A)+A}{A(50-A)}\right)$ $=\frac{1}{A(50-A)}$ $=\text{LHS}$	1 Mark: Correct solution
ii)	This is a differential equation of the form $\frac{dA}{dt}=g(A)$. Attempt to separate variables and integrate both sides. $\int 1\,dt=\int\frac{25}{A(50-A)}dA$ $t=\frac{1}{2}\int\left(\frac{1}{A}+\frac{1}{50-A}\right)dA$ (using the part (i) result) $=\frac{1}{2}(\ln\lvert A\rvert-\ln\lvert 50-A\rvert)+c$ $=\frac{1}{2}\ln\left\lvert\frac{A}{50-A}\right\rvert+c$ Rearranging gives $A_0e^{2t}=\frac{A}{50-A}$ where $A_0=e^{-2c}$ and hence $A_0>0$. When $t=0$, $A=\frac{1}{2}$ and so $A_0=\frac{1}{99}$. *Note: There are various possible ways to find the value of the constant.* $e^{2t}=\frac{99A}{50-A}$ $99Ae^{-2t}=50-A$ $A(1+99e^{-2t})=50$ So $A=\frac{50}{1+99e^{-2t}}$.	3 Marks: Correct solution 2 Marks: Correctly applies initial condition 1 Marks: Uses a) i) result and separation of variables to find t in terms of A
iii)	As $t\to\infty$, $1+99e^{-2t}\to 1$ and so $A\to\frac{50}{1}=50$. The limiting area of the bacteria colony is 50 cm^2.	1 Mark: Correct Solution

iv)	The graph of $\frac{dA}{dt}$ versus A (inverted parabola) has a maximum at $A = 25$. It requires us to find the value of t such that $25 = \frac{50}{1+99e^{-2t}}$. $25(1+99e^{-2t}) = 50$ $1+99e^{-2t} = 2$ $e^{-2t} = \frac{1}{99}$ $e^{2t} = 99$ $t = \frac{1}{2}\ln 99$ (days) The rate of change of the area is at its maximum at $t = \frac{1}{2}\ln 99$ (days). *Note: There are other valid but more time-consuming methods of determining this solution.* *Method 1:* *Finding* $\frac{d^2A}{dt^2} = \frac{1}{25^2}A(50-A)(50-2A)$, *determining that* $\frac{dA}{dt}$ *is a maximum when* $A = 25$ *and then solving for t as above.* *Method 2:* *Determining the value of t when the (non-stationary) point of inflection occurs by finding* $\frac{d^2A}{dt^2}$ *in terms of t and then finding the value of t such that* $\frac{d^2A}{dt^2} = 0$.	2 Marks: Correct solution 1 Mark: recognises that the graph of $\frac{dA}{dt}$ versus A has a maximum at A=25 OR equivalent merit.

b) i)	Max range when $y = 0$ $110t\sin\theta - 4\cdot9t^2 = 0$ $t(110\sin\theta - 4\cdot9t) = 0$ $t = 0$ or $110\sin\theta = 4\cdot9t$ $t = \dfrac{110\sin\theta}{4\cdot9}$ When $t = \dfrac{110\sin\theta}{4\cdot9}$ $x = 110\left(\dfrac{110\sin\theta}{4\cdot9}\right)\cos\theta$ $= \dfrac{110^2\sin\theta\cos\theta}{4\cdot9}$ $= \dfrac{12100 \times 2\sin\theta\cos\theta}{9\cdot8}$ $= \dfrac{12100\sin 2\theta}{9\cdot8}$	2 marks for correct proof showing solving quadratic equation and several steps of the proof, including $2\sin\theta\cos\theta$ 1 mark for working towards proof either using solution of quadratic equation or double angle result
ii)	We want max range to be between 400 and 450 metres. $400 < \dfrac{12100\sin 2\theta}{9\cdot8} < 450$ $\dfrac{400 \times 9\cdot8}{12100} < \sin 2\theta < \dfrac{450 \times 9\cdot8}{12100}$ $18°54'10\cdot8'' < 2\theta < 21°22'28\cdot3''$ $9°27' < \theta < 10°41'$ <u>Or</u> $158^0 38' < 2\theta < 161^0 6'$ $79^0 19' < 2\theta < 80^0 33'$	2 Marks: Correct range of angles- both sets 1 Mark: one angle correct
iii)	When t=3.4 sec, y=8 m To find the angle the ball was hit at: $y = 110t\sin\theta - 4\cdot9t^2$ $8 = 110(3\cdot4)\sin\theta - 4\cdot9(3\cdot4)^2$ $\sin\theta = \dfrac{8 + 4\cdot9(3\cdot4)^2}{110(3\cdot4)}$ $\theta = 9°57'11\cdot77''$ Since this falls within the range of angles acceptable in b) ii), we can conclude that it would have landed on the green. OR, they may have gone further to calculate the exact distance: To find the distance travelled using this angle, using time of flight from (a):	2 Marks: Correct conclusion with correct working 1 Mark: Progress towards solution

	$t = \dfrac{110\sin\theta}{4{\cdot}9}$ $= \dfrac{110\sin 9°57''}{4{\cdot}9}$ $= 3{\cdot}880192077$ sec $x = 110 \times 3{\cdot}880192077\cos 9°57'11{\cdot}77''$ $= 420{\cdot}3970664$ m Therefore, if the ball hadn't hit the drone, it would have made the green.	