CARLINGFORD HIGH SCHOOL

DEPARTMENT OF MATHEMATICS

Year 12 Mathematics

Half Yearly Examination

2016

Time allowed : *120 minutes*

Name : ________________________________ **Class : 12M** _____

Teacher : Mr Gong/ Ms Kellahan / Ms Lobejko / Mr Cheng / Ms Strilakos

Instructions

- Start each question on a **new booklet** .
- Board approved calculators may be used
- Show all necessary working by using blue/ black pen except graphs/diagrams
- Marks may be deducted for untidy setting out

Sequence & Series	Calculus	Integration	Total
/37	/38	/18	/93

Question 1 (13 marks) **[BEGIN A NEW BOOKLET]**

(a) The first two terms of an arithmetic series are 8 and 12.

Find:

(i) the third term, T_3 **1**

(ii) the simplest expression for the n^{th} term **2**

(iii) the sum of the first 30 terms, S_{30} **2**

(b) Three numbers a, b and c whose sum is 15 are successive terms of geometric series, while b, a and c are successive terms of an arithmetic series. Find the value of a, b and c. **2**

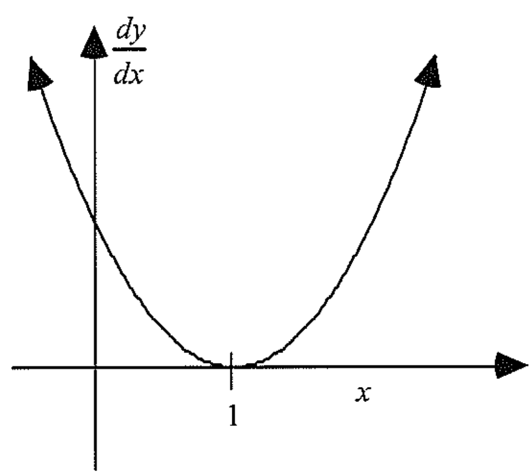
(c) An arithmetic progression has a third term of 7 and the seventh term of −3.

Find the first term and common difference. **2**

(d) For what values of x does the geometric series $1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \ldots\ldots\ldots\ldots$

have a limiting sum? **2**

(e) Consider the graph of the derivative $\frac{dy}{dx}$ given below.



(i) Sketch a possible $\frac{d^2y}{dx^2}$ graph. **1**

(ii) Sketch a possible graph of $y = f(x)$ **1**

Question 2 (13 marks)**[BEGIN A NEW BOOKLET]**

(a) The gradient function of a curve is given by $\frac{dy}{dx} = 6x^2 - 7x + 3$

If the curve passes through the point (2, 5), find the equation of the curve. **3**

(b) A woman invests \$1200 on 1 January each year into a Superannuation fund.

The fund pays interest at the rate of 12% p.a, which is compounded on

30 June and 31 December each year.

(i) Show that the first \$1200 she invests will amount to \$12 342.86 after 20 years, to the nearest cent. **2**

(ii) Find the total value of her investment at the end of the year in which she makes her 20th payment of \$1200. **4**

(c) Use Simpson's rule with five function values to estimate $\int_0^2 \frac{1}{1+x^2}\,dx$ **4**

giving your answer correct to two decimal place.

Question 3 (13 marks) [BEGIN A NEW BOOKLET]

(a) Find the following indefinite integrals

$$\int \frac{dx}{(7x+3)^3}$$ **2**

(b) Evaluate the following definite integral

$$\int_1^2 \frac{3v^2-27}{v-3}\,dv$$ **3**

(c) For the curve $y = x^3 - 3x^2$, find:

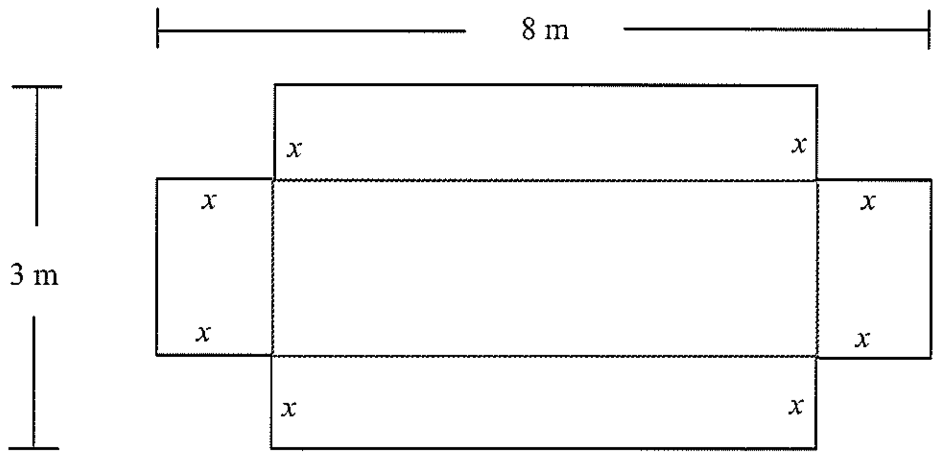
(i) The stationary points and determine their nature. **4**

(ii) Verify that there is a point of inflexion at $(1,-2)$. **2**

(iii) Sketch the curve, showing all critical points. **2**

Question 4 (12 marks) **[BEGIN A NEW BOOKLET]**

(a)



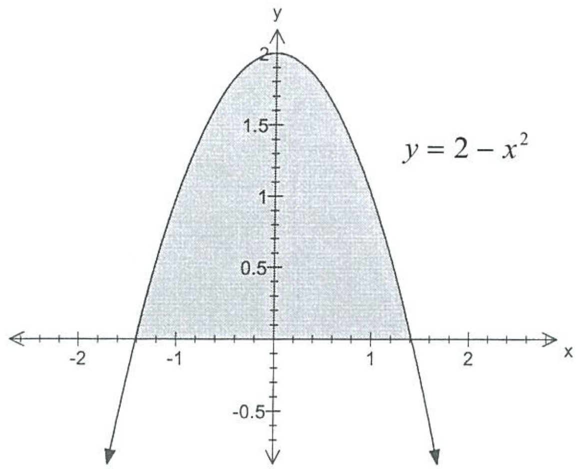
A rectangular sheet of metal is 8 m by 3 m. Equal squares of side x m are cut from each corner. The flaps are folded to form an open rectangular box.

(i) Show that the volume of the box is given by $V = 4x^3 - 22x^2 + 24x$ **2**

(ii) Find the dimensions of the box so that the volume is a maximum. **4**

(b) Find the area of the region bounded by the curve $y = (x-2)(x-3)$, the x-axis and the lines $x = 2$ and $x = 4$ **3**

(c) Find the volume of the solid of revolution obtained when the region in the diagram is rotated about the y-axis. **3**



Question 5 (12 marks) **[START A NEW BOOKLET]**

(a) Robert has decided to establish a superannuation fund. His financial advisor told him he needs \$600 000 in the fund when he retires in 30 years time. He decided that he will make equal contributions of \$$C$ at the beginning of each year. The fund pays 6% p.a. interest compounded annually.

(i) Show that his account balance after three years (before making the fourth contribution) is given by:

$A_3 = C\{(1.06)^3 + (1.06)^2 + (1.06)\}$

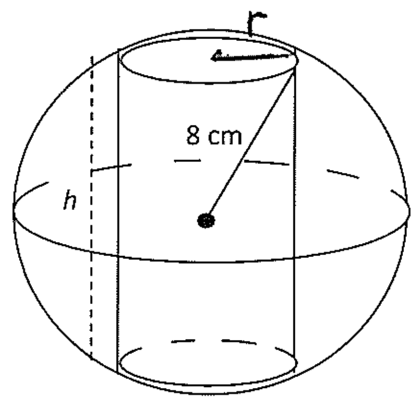
~~$A_2 = \{(1.06)^3 + (1.06)^2 + (1.06)\}$~~ **2**

(where A_n *is* the accumulated value of the fund after n contributions)

(ii) Present an **expression** for the total amount in the fund when he retires after 30 years. **1**

(iii) Calculate the value of C, if Robert wishes to reach his goal of \$600 000 upon retirement. **3**

(b)



A cylinder with height h cm and base radius r cm is inscribed in a sphere of radius 8 cm.

(i) Show that $r^2 = 64 - \dfrac{h^2}{4}$ **2**

(ii) Find an expression for the volume of the cylinder in terms of π and h **1**

(Given $V = \pi r^2 h$)

(iii) Find the value of h for the cylinder with greatest volume. **3**

Question 6 (10 marks) **[START A NEW BOOKLET]**

(a) Consider the function defined by $f(x) = \dfrac{x^4}{4} + x^3$

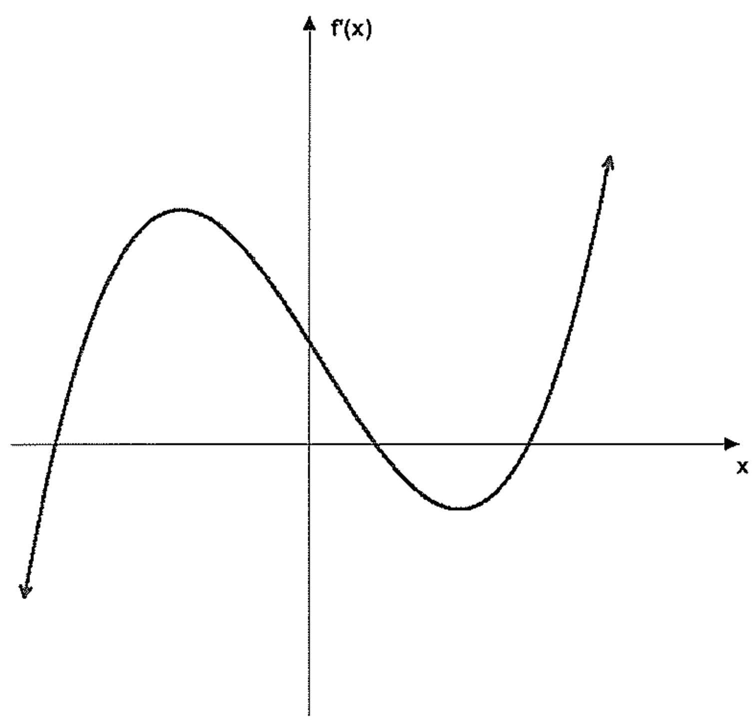
(i) Find $f'(x)$ **1**

(ii) Find the coordinates of any stationary points and determine their nature. **4**

(iii) Find the coordinates of any points of inflexion. **2**

(iv) Draw a neat sketch of the curve, identifying **all** key features. **1**

(b)



Copy the sketch of $y = f'(x)$ onto your answer sheet.

On a separate set of axes, sketch a possible diagram for $y = f(x)$ **2**

Question 7 (5 Marks)**(BEGIN ON A NEW BOOKLET)**

Rebecca borrows $100 000 at 12% p.a. reducible interest over 20 years, repaid in monthly Instalments , $M.

(i) Show that the amount owing, A_2, at the end of the second month, after the second repayment, is given by:

$$A_2 = 100\,000 \times 1\cdot01^2 - M(1 + 1\cdot01)$$ [2]

(ii) Show that the amount owing, A_n, at the end of the *n*th month, after the *n*th repayment, is given by:

$$A_n = 100\,000 \times 1\cdot01^n - M\left(\frac{1\cdot01^n - 1}{0\cdot01}\right)$$ [3]

(iii) Hence find the monthly repayment after 20 years.

QUESTION 8 (6 Marks) (2 MARKS EACH)(BEGIN ON A NEW BOOKLET)

(a) Place 5 terms between the numbers 15 and 483 so that they form an arithmetic series.

(b) If $x + 1$, $x - 1$, $2x - 5$ are consecutive terms of a geometric sequence, find all possible values of x.

(c) Evaluate :

$$\sum_{k=4}^{162} 2 - 3k$$

QUESTION 9 [9 Marks] (BEGIN ON A NEW BOOKLET)

(a) If $0.\dot{6}4\dot{5}$ is expressed as a geometric series, find the first term and the common ratio. **2**

(b) A certain geometric sequence has $T_2 = 8$ and $T_6 = 128$.

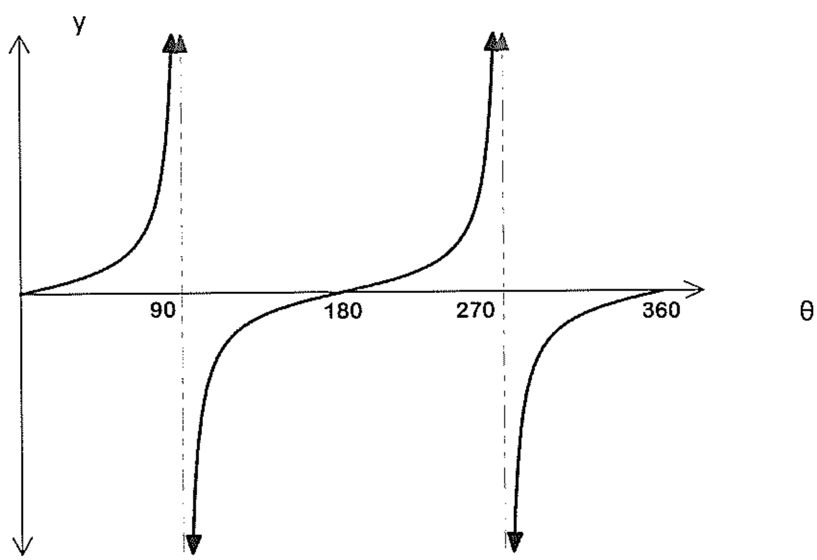
Find the common ratio. **3**

(c) For the geometric series:

$$1 + \tan^2\theta + \tan^4\theta + \tan^6\theta + \ldots$$

(i) What is the common ratio? **1**

(ii) The following graph shows $y = \tan\theta$ for $0^\circ \le \theta \le 360^\circ$.



If $0^\circ \le \theta \le 360^\circ$, for what values of θ does the series have a limiting sum?
(HINT: Use the graph and remember $\tan 45^\circ = 1$). **3**

END OF EXAM

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180°$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

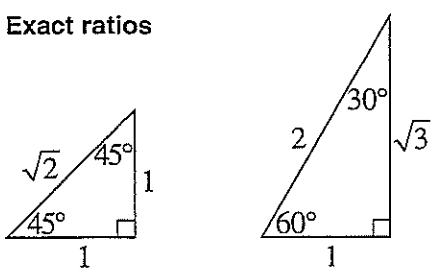
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

*n*th term of an arithmetic series

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

*n*th term of a geometric series

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Mathematics (continued)

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\dfrac{dy}{dx} = F'(u)\dfrac{du}{dx}$

If $y = e^{f(x)}$, then $\dfrac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\dfrac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\dfrac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

Q1 to Q4

Q1

(a)(i) 8, 12, 16

$a=8 \quad d=4$

(b)(ii) $T_n = a + (n-1)d$
$= 8 + 4(n-1)$
$= 8 + 4n - 4$
$= 4n + 4$

(iii) $S_{30} = \frac{30}{2}\left[2(8) + (29)4\right]$
$= 15\left[16 + 116\right]$
$=$

(b) $a + b + c = 15 \quad \dots (1)$

a, b, c are GP

$\Rightarrow b^2 = ac \quad \dots (2)$

b, a, c are AP

$\Rightarrow a = \frac{b+c}{2} \quad \dots (3)$

$b + c = 2a \quad \dots (4)$

Sub (4) into (1)

$a + 2a = 15$
$3a = 15$
$a = 5$

$\Rightarrow b + c = 10 \quad (5)$

Sub (5) into (2)

$b^2 = 5(10 - b)$
$b^2 = 50 - 5b$
$b^2 + 5b - 50 = 0$
$(b+10)(b-5) = 0$
$b = -10$ or $b = 5$

Not applicable

$\therefore a=5 \; b=5 \; c=5$ Not AP

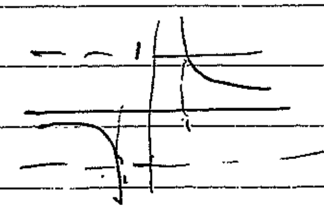
$c = 10 - (-10) = 20$

$\therefore a = 5 \quad b = -10 \quad c = 20$

(d) $-1 < r < 1$ for infinity sum

$\therefore -1 < -\frac{1}{x} < 1$

$-1 < \frac{1}{x} < 1$



$x < -1$ or $x > 1$

(c) $a + 2d = 7 \quad \dots (1)$

$a + 6d = -3 \quad (2)$

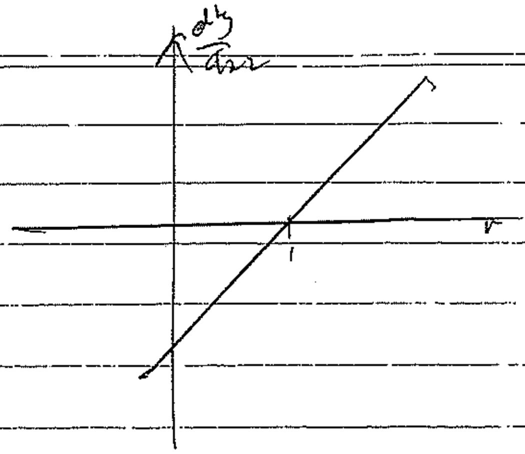
(2) − (1) $\quad 4d = -10$

$\boxed{d = -\frac{5}{2}}$

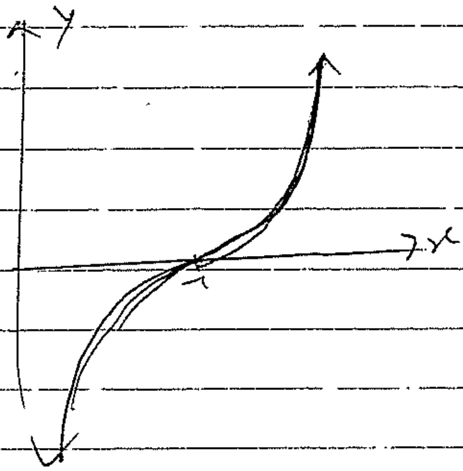
$a - 5 = 7$

$\boxed{a = 12}$

(i)



(ii)



Q2

$$\frac{dy}{dx} = 6x^2 - 7x + 3$$

$$y = \frac{6x^3}{3} - \frac{7x^2}{2} + 3x + c$$

$$= 2x^3 - \frac{7x^2}{2} + 3x + c$$

At $(2,5)$

$$5 = 2(2)^3 - \frac{7(2)^2}{2} + 3(2) + c$$

$$\cancel{5 = 16 - 14 + 6 + c}$$

$$\therefore \quad c = -3$$

$$\therefore \text{Eq } y = 2x^3 - \frac{7x^2}{2} + 3x - 3$$

Q2(b) (i) $r = 12\%$ pa $= 6\%$ per six months

$$A_{40} = 1200\left(1 + \frac{6}{100}\right)^{40}$$

$$= 1200(1.06)^{40}$$

$$= \$12342.86$$

$$A_{38} = 1200(1.06)^{38}$$

$$\therefore T = 1200\left(1.06^2 + 1.06^4 + \dots 1.06^{4}\right)$$

$$= 1200\left[\frac{1.06^2(1.06^2 - 1)}{1.06^2 - 1}\right]$$

$$=$$

(c)

x	0	0.5	1	1.5	2
$f(x)$	$\frac{1}{1}$	$\frac{1}{1+0.5^2}$	$\frac{1}{2}$	$\frac{1}{1+1.5^2}$	$\frac{1}{5}$
	y_1	y_2	y_3	y_4	y_5

$$\int_0^2 \frac{1}{1+x^2}\,dx$$

$$= \frac{0.5}{3}\left[1 + \frac{1}{5} + 4\left[\frac{1}{1+0.5^2} + \frac{1}{1+1.5^2}\right] + 2\left(\frac{1}{2}\right)\right]$$

$$=$$

Q7(a) $\int (7x+3)^{-3}\,dx$

$= \dfrac{(7x+3)^{-2}}{7(-2)} + C$

$= \dfrac{-1}{14(7x+3)^{2}} + C$

(b) $\int_1^2 \dfrac{3(v^2-9)}{v-3}\,dv$

$= 3\int_1^2 \dfrac{\cancel{(v-3)}(v+3)}{\cancel{v-3}}\,dv$

$= 3\int_1^2 (v+3)\,dv$

$= 3\left[\dfrac{v^2}{2} + 3v\right]_1^2$

$= 3\left[\left(\dfrac{4}{2}+6\right) - \left(\dfrac{1}{2}+3\right)\right]$

$= 3\left[8 - 3\tfrac{1}{2}\right]$

$= 3\left[4\tfrac{1}{2}\right]$

$= 13\tfrac{1}{2}$

(c) $y = x^3 - 3x^2$
$= x^2(x-3)$

cuts x axis $y=0$

$x=0$ or $x=3$

cuts y-axis $x=0$
$\Rightarrow (0,0)$

$\dfrac{dy}{dx} = 3x^2 - 6x$

At st pt $\dfrac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x = 0$
$3x(x-2) = 0$
$x=0$ or $x=2$

$x=0$ $y=0$
$x=2$ $y = 8 - 3(4)$
$= -4$

$\therefore$ st pts are
$(0,0)$ $(2,-4)$

$\dfrac{d^2y}{dx^2} = 6x - 6$

At $(0,0)$ $\dfrac{d^2y}{dx^2} = -6 < 0$
$\Rightarrow (0,0)$ is a
Max pt

At $(2,-4)$ $\dfrac{d^2y}{dx^2} = 6 > 0$
$\Rightarrow (2,-4)$ is

For pt of inflexion $\frac{d^2y}{dx^2}=0$

$\Rightarrow \quad 6x-6=0$

$6x=6$

$x=1$

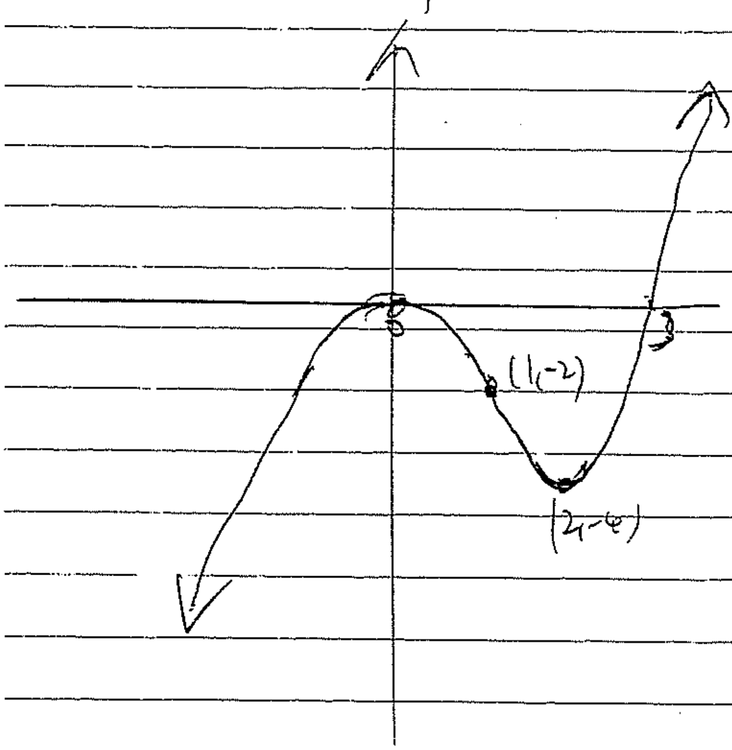
At $x=1$ $y=-2$

Check

x	1^-	1	1^+
$\frac{d^2y}{dx^2}$	$-$	0	$+$

$\Rightarrow$ change in concavity

$\Rightarrow$ $(1,-2)$ is a pt of inflexion.



Q4

(a) (i) $V = (8-2x)(3-2x)x$

$= (24 - 16x - 6x + 4x^2)\,x$

$= (24 - 22x + 4x^2)\,x$

$V = 4x^3 - 22x^2 + 24x$.

For max vol differentiate,

$\frac{dV}{dx} = 12x^2 - \cancel{44}x + 24$

At st pt, $\frac{dV}{dx} = 0$

$\cancel{12(x^2 - \frac{22x}{} + 2)}$

$12x^2 - 44x + 24 = 0$

$4(3x^2 - 11x + 6) = 0$

$4\frac{(3x-9)(3x-2)}{3} = 0$

$4(3x-2)(x-3) = 0$

At st pt $\frac{dV}{dx} = 0$

$x = \frac{2}{3}$ or $x = 3$.

$\frac{d^2V}{dx^2} = 24x - 44$

At $x = \frac{2}{3}$

$\frac{d^2V}{dx^2} = 24\left(\frac{2}{3}\right) - 44$

< 0

$\Rightarrow x = \frac{2}{3}$ will give max vol.

$\therefore$ Dimension =

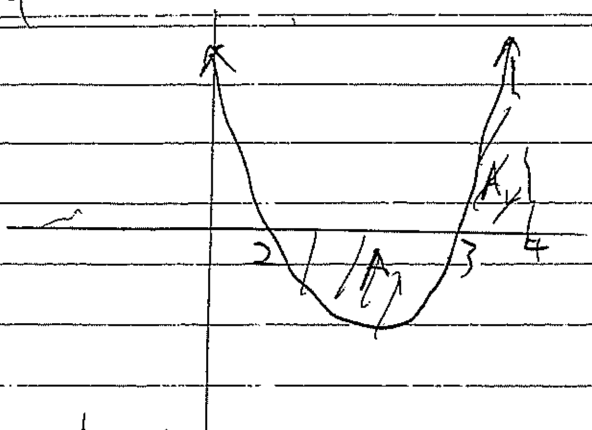
$\left(8 - 2\times\frac{2}{3}\right) \times \left(3 - 2\times\frac{2}{3}\right) \times \frac{2}{3}$

$= \left(8 - \frac{4}{3}\right) \times \left(3 - \frac{4}{3}\right) \times \frac{2}{3}$

$= \frac{20}{3} \times \frac{5}{3} \times \frac{2}{3}$

Q4

(b)



$A = \left| \int_2^3 (x^2 - 5x + 6) \right| + \int_3^4 (x^2 - 5x + 6)\,dx$

Q4(5)

$V = \pi \int_0^{2} \left(\cancel{} \right)$ $2-y$

$= \pi \int_0^{2} \left(\cancel{} \right)$ $(2-y)$

$= \pi \left[\cancel{}\ 2y - \frac{y^2}{2} \right]_0^{2}$

$= \pi \left[\left(\cancel{8 - \frac{16}{2}} \right) (4 - 2) - 0 \right]$

$= 2\pi$ units^3.

Q8

~~Q6~~ 1st contribution $= C(1.06)^3$

~~2nd contribution $= C(1.06)^2$~~

3rd contribution $= C(1.06)$

$A_1 = C\left(1+\frac{6}{100}\right)^1$

$A_2 = C\left(1+\frac{6}{100}\right) + C(1.06)^2$

$A_3 = C\left[1.06 + 1.06^2 + 1.06^3\right]$

$\therefore$ Total $= C\left(1.06^3 + 1.06^2 + 1.06\right)$

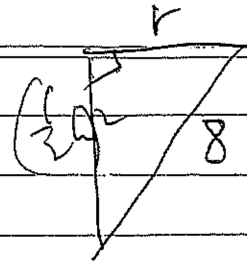
~~$A_3 = 1.06^3$~~

$A_{30} = C\left(1.06 + 1.06^2 + \ldots\ 1.06^{30}\right)$

$A_{30} = 600000$

$$C = \frac{60000}{\left[\frac{1.06\left[1.06^{30}-1\right]}{0.06}\right]}$$

$=$



$r^2 + \left(\frac{1}{2}h\right)^2 = 8^2$

$r^2 + \frac{h^2}{4} = 64$

$\therefore r^2 = 64 - \frac{h^2}{4}$.

$V = \pi r^2 h$

$V = \pi\left(64 - \frac{h^2}{4}\right)h$.

$V = 64\pi h - \frac{1}{4}\pi h^3$.

$\frac{dV}{dh} = 64\pi - \frac{3}{4}\pi h^2$

At st pt $\frac{dV}{dh} = 0$

$\therefore \frac{3}{4}\pi h^2 = 64\pi$

$h^2 = \frac{64 \times 4}{3}$.

$h = \frac{16}{\sqrt{3}}$

$\frac{d^2V}{dh^2} = -\frac{3}{2}\pi h < 0 \ \forall h$.

$\Rightarrow h = \frac{16}{\sqrt{3}}$ will give max

6b $f(x) = \frac{x^4}{4} + x^3$

(i) $f'(x) = x^3 + 3x^2$

(ii) At st pts $f'(x) = 0$

$x^3 + 3x^2 = 0$

$x^2(x+3) = 0$

$\therefore x = 0$ or $x = -3$

$f(0) = 0 \quad f(-3) = \frac{81}{4} - 27$

$\therefore$ St pts

$(0,0)$, $(-3, -7.25)$

$f''(x) = 3x^2 + 6x$

$f''(0) = 0 \Rightarrow$ ~~[illegible]~~ cannot determine

x	0^-	0	0^+
$f'(x)$	+	0	+

$\Rightarrow$ horizontal pt of inflexion at $(0,0)$

$f''(-3) = 3(-3)^2 + 6(-3)$
$= 27 - 18$
$= 9 > 0$

(iii) $f''(x) = 0$

$3x^2 + 6x = 0$

$3x(x+6) = 0$

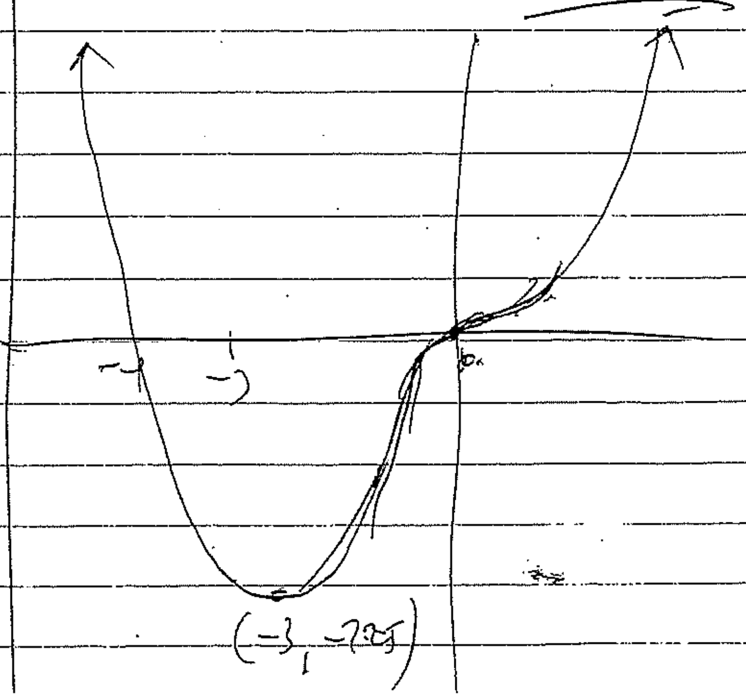
$x = 0$ or $x = -6$

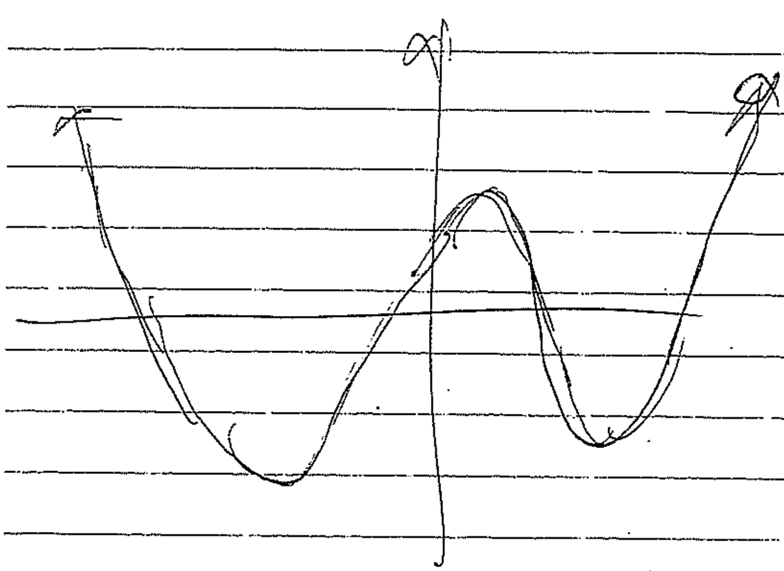
	$x = 0^-$	$x = 0$	$x = 0^+$
$f''(x)$	$-$	0	+

$(0,0)$ is a pt of inflexion $\Rightarrow$ horizontal

	$x = -7$	$x = -6$	$x = -5$
$f''(x)$	+	0	+

$\Rightarrow$ no change in concavity $\Rightarrow (-6, \;)$ is not a pt of inflexion





Q7   12% = 1% per month.

$A_1 = 100000\left(1+\frac{1}{100}\right) - M$

$= 100000(1.01) - M$

$A_2 = \cancel{100000(1.01) - M}$

$= A_1(1.01) - M$

$= \left[100000(1.01) - M\right] - M$

$= \left(100000(1.01) - M\right)1.01 - M$

$= 100000(1.01)^2 - M(1.01) - M$

$= 100000(1.01)^2 - M(1 + 1.01)$

(iii)

$\therefore A_n = 100000(1.01)^n - M\left(1 + 1.01 + 1.01^2 + \dots 1.01^{n-1}\right)$

$\therefore A_n = 100000(1.01)^n - M\left(\frac{1\left[(1.01)^n - 1\right]}{1.01 - 1}\right)$

$= 100000(1.01)^n - M\left(\frac{1.01^n - 1}{0.01}\right)$.

After 20 years $A_{20} = 0$

$\Rightarrow M = \frac{100000(1.01)^{20}}{\frac{1.01^{20} - 1}{0.01}}$

$=$

Q8

(a) 15, _, _, _, _, _, 483

$a = 15 \quad T_7 = 483$

$a + 6d = 483$

$15 + 6d = 483$

$6d = 468$

$d = 93$

15, 108, 201, 294, 387, 480

$a + 6d = 483$

$15 + 6d = 483$

$6d = \cancel{483}\ 468$

$d = 78$

15, 93, 171, 249, 327, 405, 483

\# of terms = 162 − 4 + 1
= 159

$a = 2 - 3(4) \quad L = 2 - 3(162)$
$= -10 \quad = -484$

$\therefore S_{159} = \frac{159}{2}\left[\cancel{-10 + (-484)}\right]$

=

8(b)

$(x-1)^2 = (x+1)(2x-5)$

$x^2 - 2x + 1 = 2x^2 - 3x - 5$

$0 = x^2 - x - 6$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

Ld

$x = 3, \quad 4, 2, 1$

$x = -2 \quad -1, -3, -9 \quad$ de

Q9

(a) Let $x = 0.645645\ldots$

$$x = \frac{645}{1000} + \frac{645}{1000000} + \ldots$$

$$a = \frac{645}{1000} \quad r = \frac{645}{1000000} \times \frac{1000}{645}$$

$$r = \frac{1}{1000}$$

(b) $ar = 8 \quad \ldots(1)$

$ar^5 = 128 \quad \ldots(2)$

$$\frac{(2)}{(1)} \quad r^4 = \frac{128}{8}$$

$$r^4 = 16$$

$$r = \pm 2$$

(c) $r = \tan^2\theta$

(ii) $|r| < 1$

$$-1 < r < 1$$

$$-1 < \tan^2\theta < 1$$

$$-1 < \tan\theta < 1$$

From graph

$0 < \theta < 45$

$135 < \theta < 225$

$315 < \theta < 360$