Carlingford High School

Mathematics

Year 10 Term 3 Examination
5.3 Course
2019

Name: SOLUTIONS Class: ________

Circle your teacher's name: Ms Sharma, Ms Wilson/Young, Mrs Lobejko

Time allowed: 50 minutes

- Board approved calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or untidy work.
- Complete the examination in blue or black pen.

COORDINATE METHODS	INEQUATIONS	TRIGONOMETRY
/16	/10	/26
	Total	/52

COORDINATE METHODS
(16 marks)

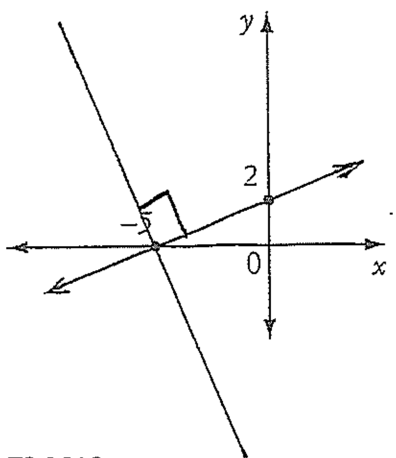
1. Find the equation of the line with gradient 4, passing through (0,-7). Write the equation in general form. 2

$$(y - y_1) = m(x - x_1)$$
$$y - {}^-7 = 4(x - 0)$$
$$y + 7 = 4x$$
$$4x - y - 7 = 0$$

2. (2,3) and (-4,5) lie on the same line. Find the equation of that line. 3

$$m = \frac{5-3}{-4-2} = \frac{2}{-6} = -\frac{1}{3}$$
$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\frac{1}{3}(x - 2)$$
$$3(y - 3) = -(x - 2)$$
$$3y - 9 = -x + 2$$
$$x + 3y - 11 = 0 \quad \text{OR} \quad y = -\frac{1}{3}x + \frac{11}{3}$$

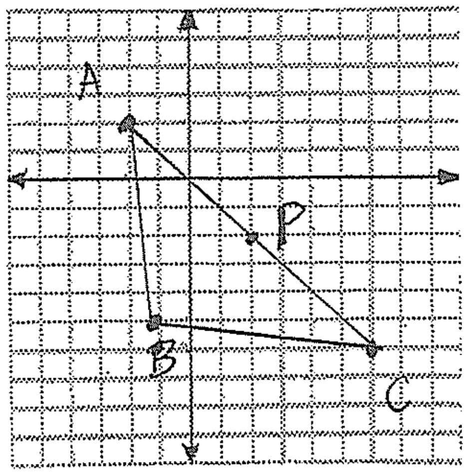
3. With reference to the diagram, find the equation of the line that is perpendicular to the given line and passing through $(-5, 0)$. 3



$$m_1 = \frac{2}{5}$$
$$m_2 = -\frac{5}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{5}{2}(x - {}^-5)$$
$$y = -\frac{5}{2}(x + 5)$$
$$2y = -5(x + 5)$$
$$5x + 2y + 25 = 0$$
$$2y = -5x - 25$$
$$y = -\frac{5}{2}x - \frac{25}{2}$$

4. The points A, B and C have coordinates (-2,2), (-1,-5) and (6,-6) respectively.   A   B   C

(a) On the number plane below, sketch the triangle ABC. 1



(b) Show that the midpoint P of AC, has coordinates (2,-2). 1

$$M = \left(\frac{-2+6}{2}, \frac{2+{}^-6}{2}\right)$$
$$= \left(\frac{4}{2}, -\frac{4}{2}\right)$$
$$= (2, -2)$$

(c) Show that AC and BP are perpendicular. 3

$$m_{AC} = \frac{-6-2}{6-(-2)}$$
$$= -1$$

$$m_{BP} = \frac{-2-(-5)}{2-(-1)}$$
$$= \frac{3}{3} = 1$$

$$m_1 \times m_2 = -1 \times 1$$
$$= -1$$

$\therefore AC \perp BP$

(d) Find the area of the triangle ABC. 3

$$AC = \sqrt{8^2+(-8)^2}$$
$$= \sqrt{128}$$
$$= 8\sqrt{2}$$

$$BP = \sqrt{3^2 + 3^2}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

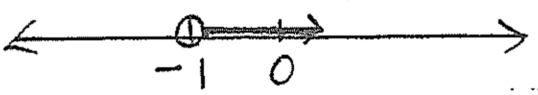
$$A = \frac{1}{2} \times 8\sqrt{2} \times 3\sqrt{2}$$
$$= 24 \text{ units}^2$$

INEQUATIONS (10marks)

1. Solve the following inequations and graph the solution on a number line.

(a) $5 - 3x < 8$ 3

$$-3x < 3$$
$$x > -1$$



(b) $\frac{3-x}{2} \leq -1$ 3

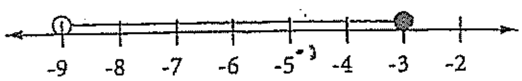
$$3 - x \leq -2$$
$$-x \leq -5$$
$$x \geq 5$$

2. Solve the following equation 3

$$\frac{2x}{3} - \frac{x-1}{2} > 3x$$

$(\times 6)$
$$\frac{2x}{3}\times 6 - \frac{(x-1)}{2}\times 6 > 3x\times 6$$
$$4x - 3(x-1) > 18x$$
$$4x - 3x + 3 > 18x$$
$$x + 3 > 18x$$
$$3 > 17x$$
$$\frac{3}{17} > x$$
$$\therefore x < \frac{3}{17}$$

3. Write the inequality represented on the following number line. 1



$$-9 < x \leq -3$$

TRIGONOMETRY (26 marks)

1. Find the exact value of:

(a) $\tan 30°$ $= \frac{1}{\sqrt{3}}$ OR $\frac{\sqrt{3}}{3}$ 1

(b) $\sin 225°$ $= \sin(180° + 45°)$ 2

S	A
T	C

$= -\sin 45°$

$= -\frac{1}{\sqrt{2}}$

2. Simplify without the use of a calculator.

(a) $\sin 70° + \cos 20°$ 1

$= \sin 70° + \sin 70°$

$= 2\sin 70°$ OR $2\cos 20°$

(b) $\frac{\sin 70°}{\cos 70°}$ $= \tan 70°$ 1

3.
(a) If $\cos\theta = \frac{1}{\sqrt{2}}$ and θ lies in the first quadrant, find θ. 1

$\theta = 45°$

(b) Solve $\sin x = \frac{-\sqrt{3}}{2}$ for $0° \le x \le 360°$ 2

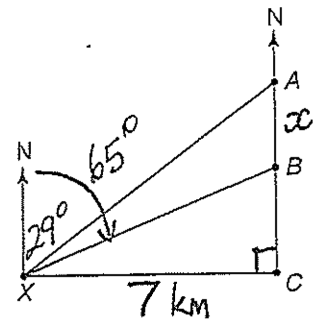
S	A
T ✓	C ✓

$x = 60°$ (1st quad.)

$\therefore x = (180 + 60°), (360° - 60°)$

$= 240°, 300°$

4. A, B and C are three towns where A and B are due north of C. From a position X on a map, A has a bearing of N29°E and B has a bearing of N65°E. Town C is due east of X and 7km from it. Find the distance, correct to one decimal place, between A and B. 3



$\tan 25° = \frac{BC}{7}$

$\therefore BC = 3.264...$

$\tan 61° = \frac{AC}{7}$

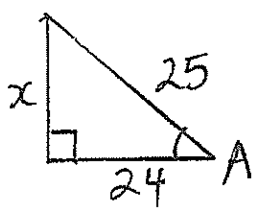
$\therefore AC = 12.628...$

$\therefore x = 12.628... - 3.264...$

$= 9.364...$

$\therefore$ Distance between A and B is 9.4 km.

5. Given that $\cos A = \frac{24}{25}$ find tan A. 2



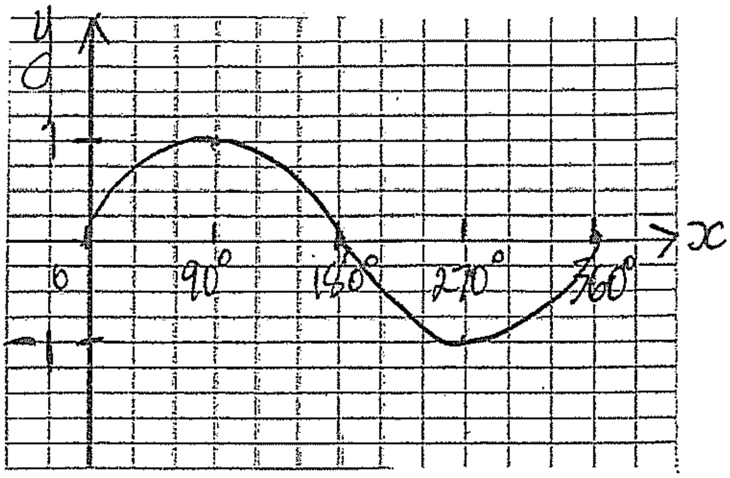
$x^2 = 25^2 - 24^2$

$= 49$

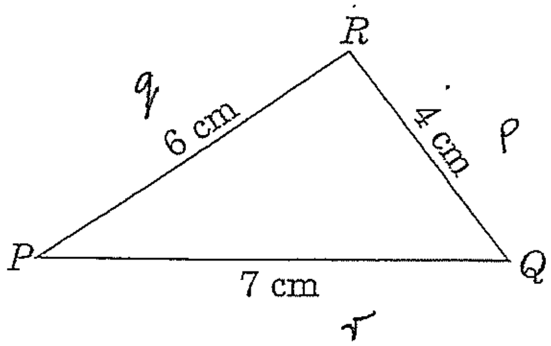
$\therefore x = 7$

$\tan A = \frac{7}{24}$

6. Sketch y = sin x from 0° to 360° 3



7.



(a) Use the cosine rule to calculate angle RPQ to the nearest degree. 2

$$p^2 = q^2 + r^2 - 2qr \cos P$$

$$4^2 = 6^2 + 7^2 - 2(6)(7) \cos P$$

$$16 = 85 - 84 \cos P$$

$$84 \cos P = 69$$

$$\therefore \cos P = 0.82142\ldots$$

$$\angle P = 34.77\ldots$$

$$= 35^\circ$$

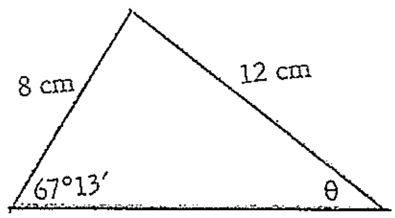
(b) Calculate the area of the triangle PQR, giving your answer to two significant figures. 2

$$A = \frac{1}{2} \times 6 \times 7 \times \sin 35^\circ$$

$$= 12.045\ldots$$

$$= 12 \text{ cm}^2$$

8. Calculate angle θ in triangle ABC, correct to the nearest minute. 3



$$\frac{\sin\theta}{8} = \frac{\sin 67^\circ 13'}{12}$$

$$\sin\theta = \frac{\sin 67^\circ 13'}{12} \times 8$$

$$= 0.6146\ldots$$

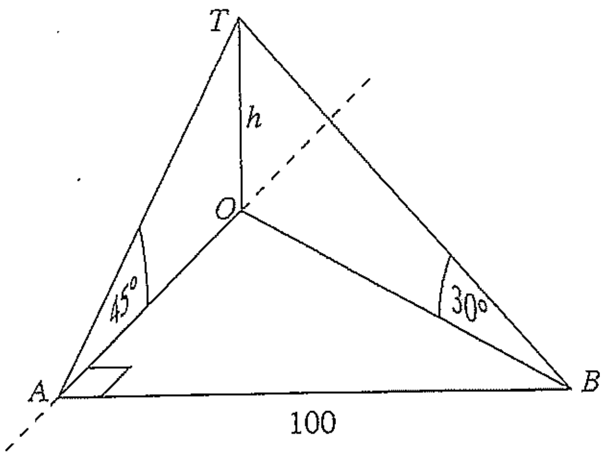
$$\therefore \theta = 37^\circ 56', (180^\circ - 37^\circ 56')$$

$$= 37^\circ 56', 142^\circ 4'$$

Since $67^\circ 13' + 142^\circ 4' > 180^\circ$

accept only $37^\circ 56'$

9. A surveyor stands at a point A, which is due south of a tower OT of height h m. The angle of elevation of the top of a tower from A is 45°. The surveyor then walks 100 m due east to point B, from where she measures the angle of elevation of the top of the tower to be 30°.



(a) Express the length of OB in terms of h. 1

$$\begin{aligned}
&\text{Now } \angle TOB = 90^\circ \quad \text{(given)}\\
&\therefore \angle OTB = 90^\circ - 30^\circ\\
&\qquad = 60^\circ\\
&\tan \angle OTB = \frac{OB}{h}\\
&\text{i.e. } \tan 60^\circ = \frac{OB}{h}\\
&\therefore \sqrt{3} = \frac{OB}{h}\\
&\therefore OB = \sqrt{3}\,h.
\end{aligned}$$

(b) Show that $h = 50\sqrt{2}$. 2

$$\begin{aligned}
&\text{Now } \angle TOA = 90^\circ \quad \text{(given)}\\
&\therefore \angle OTA = 90^\circ - 45^\circ\\
&\qquad = 45^\circ\\
&\tan \angle OTA = \frac{OA}{OT}\\
&\text{i.e. } \tan 45^\circ = \frac{OA}{h}\\
&\therefore 1 = \frac{OA}{h}\\
&\therefore OA = h\\
&\text{Now } OB^2 = OA^2 + 100^2 \quad \text{(Pythagoras' theorem)}\\
&\text{i.e. } (\sqrt{3}\,h)^2 = h^2 + 100^2\\
&\therefore 3h^2 = h^2 + 100^2\\
&\therefore 2h^2 = 100^2\\
&\therefore \sqrt{2}\,h = 100\\
&h = \frac{100}{\sqrt{2}}\\
&\quad = \frac{100\sqrt{2}}{2}\\
&\quad = 50\sqrt{2}.
\end{aligned}$$