A C E
EXAM
PAPER

Student name: _____

PAPER 1

YEAR 12
YEARLY
EXAMINATION

Mathematics Advanced

General Instructions

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

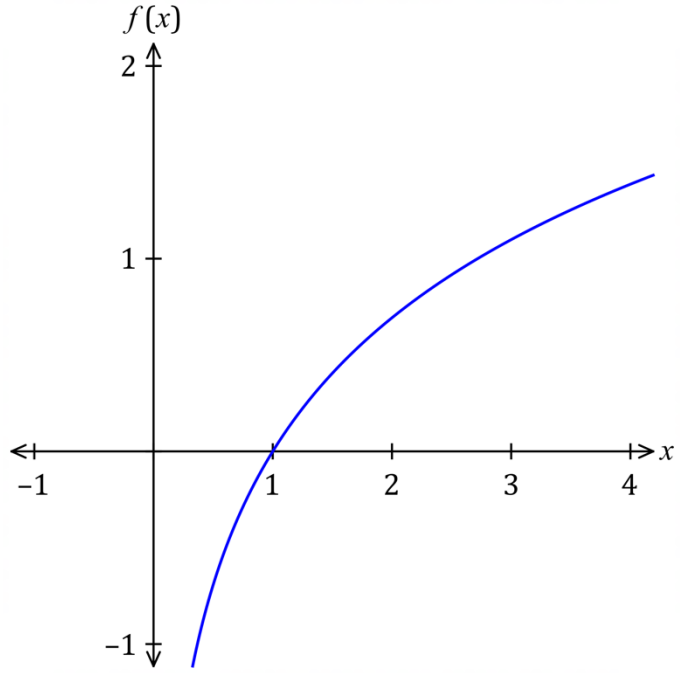
1. What is the solution to the equation $2\cos^2 x - 1 = 0$ in the domain $0 \leq x \leq 2\pi$?

   (A) $x = \frac{\pi}{6}, \frac{11\pi}{6}$

   (B) $x = \frac{\pi}{4}, \frac{7\pi}{4}$

   (C) $x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$

   (D) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

2. 

   Which of the following properties matches the above graph?

   (A) $f'(x) > 0$ and $f''(x) < 0$

   (B) $f'(x) > 0$ and $f''(x) > 0$

   (C) $f'(x) < 0$ and $f''(x) < 0$

   (D) $f'(x) > 0$ and $f''(x) > 0$

3. A factory produces bags of cashews. The weights of the bags are normally distributed, with a mean of 900 g and a standard deviation of 50 g. What is the best approximation for the percentage of bags that weigh more than 1000 g?

(A) 0%

(B) 2.5%

(C) 5%

(D) 16%

4. What is the value of $\int_0^1 (e^{3x} + 1)dx$?

(A) e^3

(B) $\frac{1}{3}e^3$

(C) $\frac{1}{3}(e^3 + 1)$

(D) $\frac{1}{3}(e^3 + 2)$

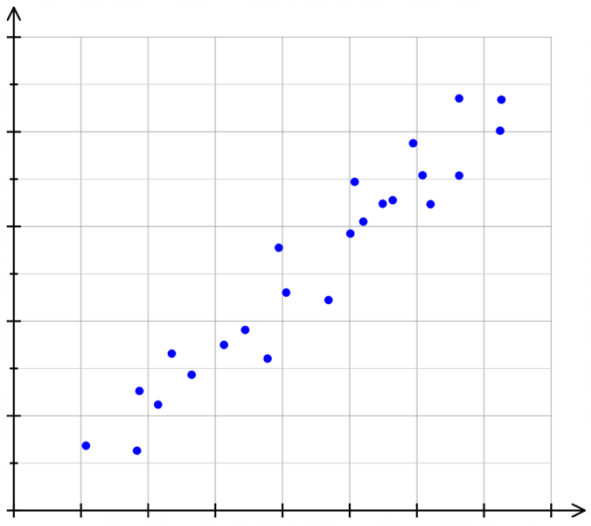
5. What is the gradient to the curve $y = (x - a)(x^2 - 1)$ at the point when $x = -2$?

(A) $-3a - 6$

(B) $-5a - 1$

(C) $4a + 11$

(D) $5a + 4$

6. 

What is the correlation between the variables in this scatterplot?

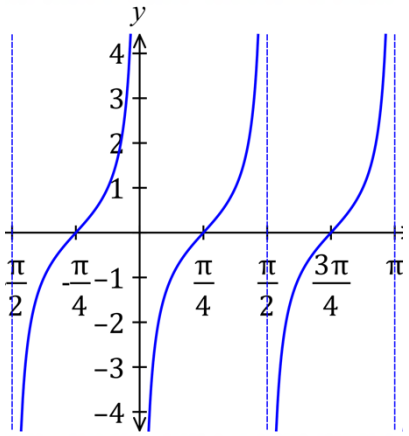
(A) Weak negative

(B) Weak Positive

(C) Moderate negative

(D) Moderate positive

7. A section of the graph $y = f(x)$ is shown below.



Which of the following is the correct function for the above graph?

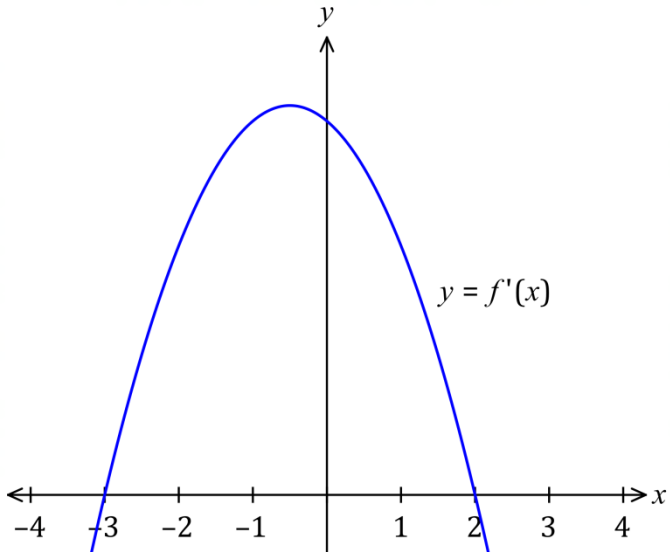
(A) $f(x) = \tan\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right)$

(B) $f(x) = \tan\left(2\left(x - \frac{\pi}{4}\right)\right)$

(C) $f(x) = \tan\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$

(D) $f(x) = \tan\left(2\left(x - \frac{\pi}{2}\right)\right)$

8. The graph of the derivative function is shown below.



Where is the function $y = f(x)$ increasing?

(A) $\{x : x > 0\}$

(B) $\{x : x > 2\}$

(C) $\{x : -3 < x < 2\}$

(D) $\{x : x < -3\}$ or $\{x : x > 2\}$

9. The table below shows the present value of a $1 annuity.

Present value of $1				
End of year	3%	4%	5%	6%
5	4.5797	4.4518	4.3295	4.2124
6	5.4172	5.2421	5.0757	4.9173
7	6.2303	6.0021	5.7864	5.5824
8	7.0197	6.7327	6.4632	6.2098

What is the present value of an annuity where $12,000 is contributed each year for six years into an account earning 3% per annum compound interest?

(A) $15 183.83

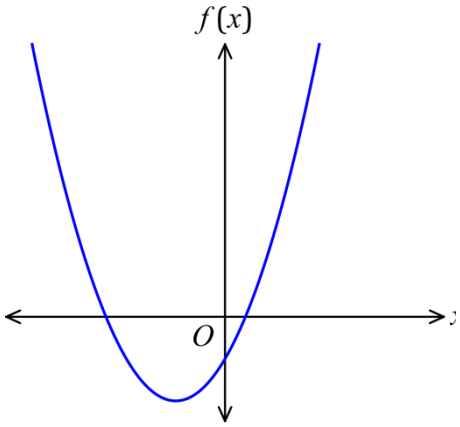
(B) $54 956.40

(C) $65 006.40

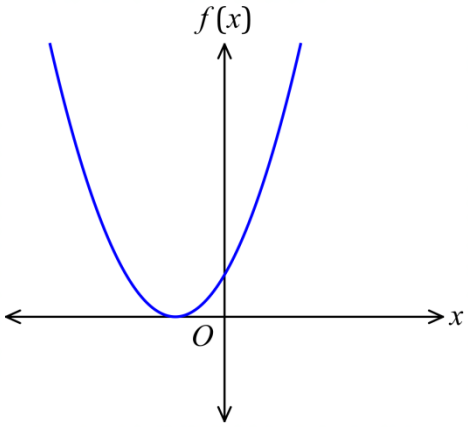
(D) $72 000.00

10 Which of the following graphs could *not* represent a probability density function $f(x)$?

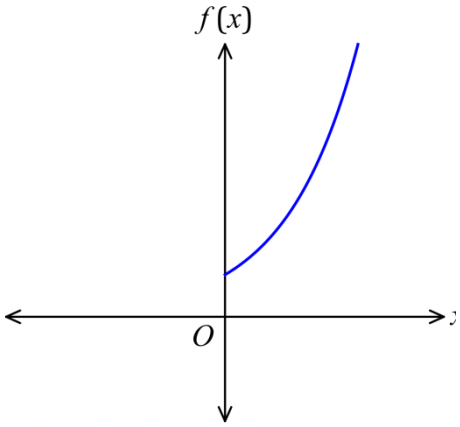
(A)



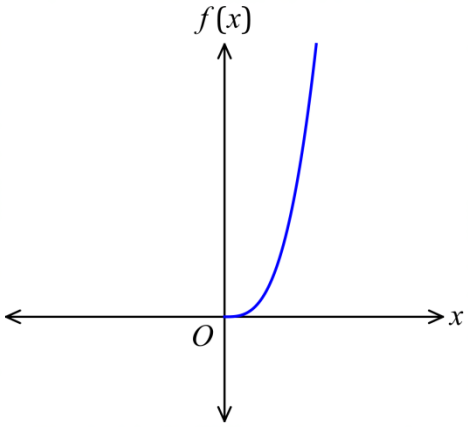
(B)



(C)



(D)



Section II

90 marks
Attempt questions 11 - 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

Differentiate the following functions with respect to x.

(a) $f(x) = \sin x + x^2$ **1**

(b) $f(x) = \ln(x^2 + 1)$? **1**

Question 12 (3 marks)

For the arithmetic sequence 4, 9, 14, 19, ...

(a) Write the rule to describe the nth term. **1**

(b) What is the 25^{th} term? **1**

(c) Find the sum of the first 100 terms. **1**

Question 13 (4 marks) **Marks**

A continuous random variable X has a function f given by

$$f(x) = \begin{cases} |3 - x| & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(2 \leq X \leq 3.5)$ **2**

(b) Find $P(2 \leq X \leq 2.5)$ **2**

Question 14 (4 marks)

Differentiate

(a) $2e^x \cos x$ **2**

(b) $\dfrac{\tan x}{x}$ **2**

Question 15 (1 mark)

Find $\int (2x + 3)^{10}\, dx$ **1**

Question 16 (2 marks) **Marks**

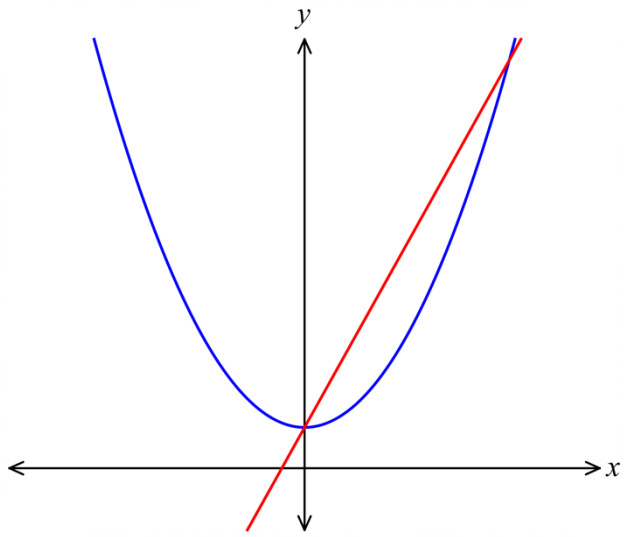
Tran's industrial unit produces aluminium rods. In the past week the industrial unit has produced aluminium rods with a mean weight of 12.5 kilograms and a standard deviation of 0.5 kilograms.

(a) Quality control requires any aluminium rod with a z-score less than –1 to be rejected. What is the minimum weight that will be accepted? **1**

(b) Aluminium rods with a z-score greater than 2 are also rejected. What is the maximum weight that will be accepted? **1**

Question 17 (2 marks)

What is the area enclosed between the curves $y = x^2 + 1$ and $y = 3x + 1$? **2**

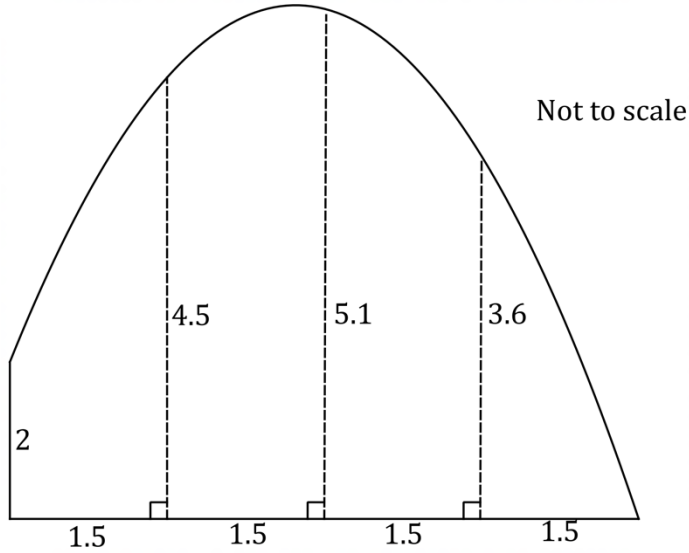


Question 18 (3 marks) **Marks**

Find the value of k if $y = e^{kx}\sin x$ and $\frac{dy}{dx} - 3y = e^{kx}\cos x$. **3**

Question 19 (3 marks)

The diagram below shows a native garden. All measurements are in metres.



(a) Use the Trapezoidal Rule with 4 intervals to find an approximate value for the area of the native garden. **2**

(b) If 25 millimetres of rain fell overnight, how many litres of rain fell on the native garden? Assume 1 $m^3 = 1000$ L. **1**

Question 20 (3 marks) **Marks**

Consider the functions $y = x^2$ and $y = x^2 - 3x + 2$.

(a) Sketch the two functions on the same axes. **2**

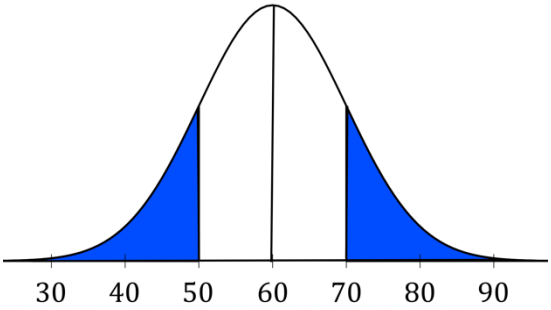
(b) Hence or otherwise find the values of x such that $x^2 > (x-1)(x-2)$. **1**

Question 21 (2 marks)

State the amplitude and period of the function $f(x) = 4 + 3\cos\left(\frac{\pi x}{2}\right)$ **2**

Question 22 (2 marks) **Marks**

The normal distribution shows the results of a mathematics assessment task. It has a mean of 60 and a standard deviation of 10.



(a) What is the mathematics assessment result with a z-score of –2? **1**

(b) What is the z-score of a mathematics assessment result of 65? **1**

Question 23 (2 marks)

Find $\int_0^{\frac{\pi}{8}} (\sec^2 2x)dx$ **2**

Question 24 (2 marks)

How many solutions does the equation $|\cos(2x)| = 1$ have for $0 \leq x \leq 2\pi$? **2**

Question 25 (5 marks) **Marks**

A function $f(x)$ is defined by $f(x) = x^2(3 - x)$.

(a) Find the stationary points for the curve $y = f(x)$ and determine their nature. **2**

(b) Sketch the graph of $y = f(x)$ showing the stationary points and x-intercepts. **2**

(c) Find the equation of the tangent to the curve at the point $P(1,2)$. **1**

Question 26 (2 marks)

Construct a recurrence relation in the form $V_{n+1} = V_n \times (1 + r) - D$ to model the balance of a loan of \$58 000 borrowed at 6% per annum, compounding monthly, with payments of \$810 per month. **2**

Question 27 (4 marks) **Marks**

Ten kilograms of chlorine is placed in water and begins to dissolve. After t hours the amount A kg of undissolved chlorine is given by $A = 10e^{-kt}$

(a) Calculate the value of k given that $A = 3.6$ when $t = 5$. Answer correct to three decimal places. **2**

(b) After how many hours does one kilogram of chlorine remain undissolved? Answer correct to one decimal place. **2**

Question 28 (2 marks)

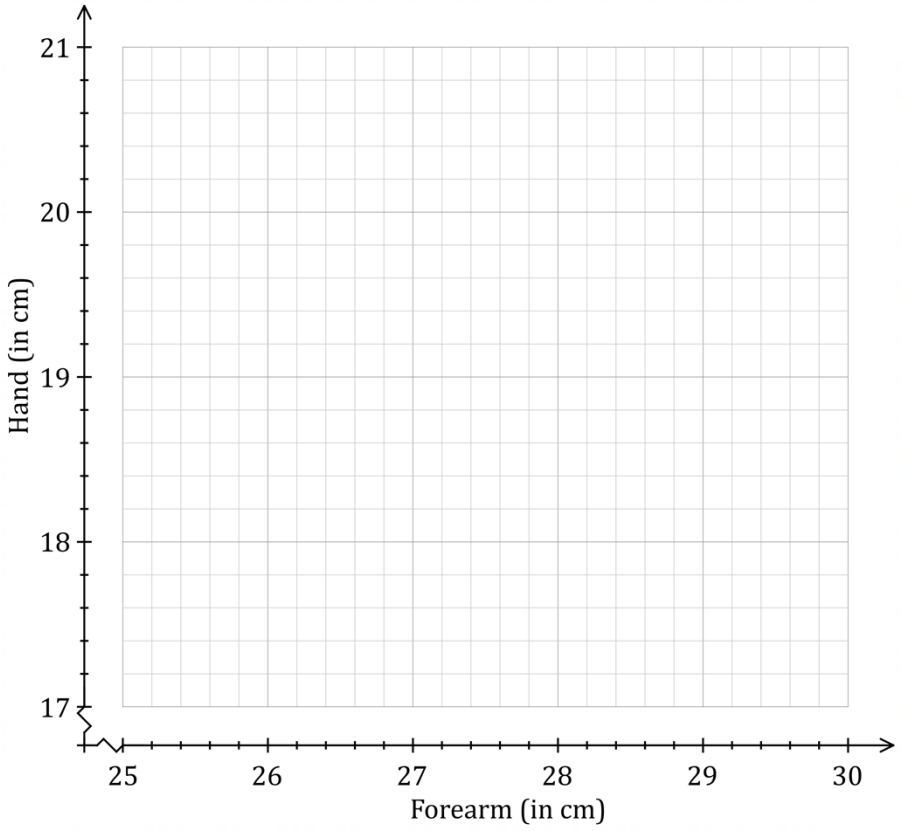
The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term? **2**

Question 29 (5 marks) **Marks**

The table below shows forearm length and hand length.

Forearm (in cm)	25.0	25.6	26.0	26.6	27.0	27.4	28.0	28.6	29.0	29.2
Hand (in cm)	17.2	17.6	18.2	18.4	19.0	19.0	19.8	19.8	20.4	20.6

(a) Draw a scatterplot using the above table. **1**



(b) Draw a line of best fit on the scatterplot. **1**

(c) Charlotte has a forearm whose length is 27.8 cm. What is her expected hand length? **1**

(d) Calculate the value of the Pearson's correlation coefficient. Answer correct to four decimal places. **2**

Question 30 (3 marks) **Marks**

Florence left \$1000 in her will for World Vision. Her instructions were that this money be invested at 5% interest, compounded annually.

(a) How much money would be given to World Vision after 100 years? Give your answer to the nearest dollar. **1**

(b) Florence has requested her family invest a further \$1000 at the beginning of each subsequent year at the same interest rate. How much money would be given to World Vision after 100 years if her family followed Florence's instructions? Give your answer to the nearest dollar. **2**

Question 31 (3 marks)

Evaluate the following definite integrals.

(a) $\int_{-1}^{2} x^2 + 1 dx$ **1**

(b) $\int_{-1}^{4} \sqrt{3x + 4} dx$ **2**

Question 32 (2 marks) **Marks**

The table below shows the future value of a $1 annuity.

Future value of $1				
End of year	4%	6%	8%	10%
1	1.00	1.00	1.00	1.00
2	2.04	2.06	2.08	2.10
3	3.12	3.18	3.25	3.31
4	4.25	4.37	4.51	4.64

(a) What would be the future value of a $32 000 per year annuity at 8% per annum for 4 years, with interest compounding annually? **1**

(b) An annuity of $6300 is invested every six months at 8% per annum, compounded biannually for 2 years. What is the future value of the annuity? **1**

Question 33 (4 marks)

Consider the function $f(x) = \dfrac{1}{1+x^2}$

(a) Find the value of $f'(x)$. **2**

(b) Find the coordinates of the point on the curve $y = f(x)$ at which the tangent is parallel to the x-axis. **2**

Question 34 (7 marks) **Marks**

An object is moving in a straight line and its velocity is given by;

$v = 1 - 2\sin 2t$ for $t \geq 0$

where v is measured in metres per second and t in seconds.
Initially the object is at the origin.

(a) Find the displacement x, as a function of t. **2**

(b) What is the position of the object when $t = \frac{\pi}{3}$? **1**

(c) Find the acceleration a, as a function of t. **1**

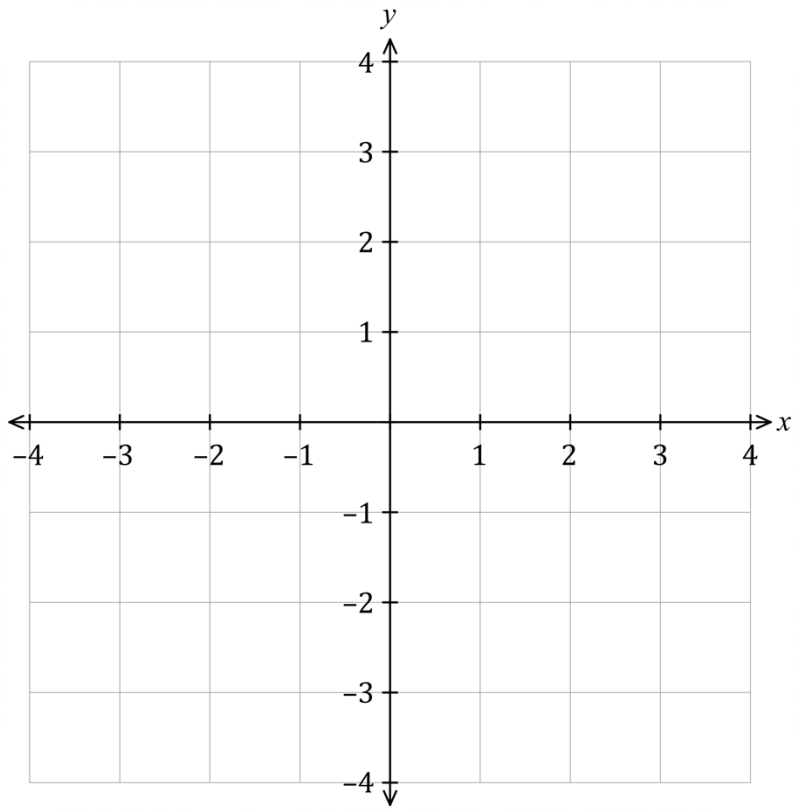
(d) Sketch the graph of a, as a function of t, for $0 \leq t \leq \pi$. **2**

(e) What is the maximum acceleration of the object? **1**

Question 35 (3 marks) **Marks**

Sketch the follow graphs on the same number plane. **3**

$y = \sqrt{x}, \quad y = \sqrt{x-1}, \quad y = \sqrt{x} - 1$

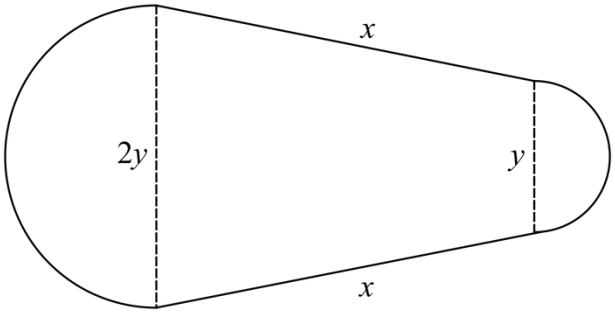


Question 36 (2 marks)

Class A has 24 students and achieved a mean on an assessment task of 75.5%. Class B has 28 students and achieved a mean on the same assessment task of 80.5%. What was the mean mark for both classes. Answer correct to one decimal places. **2**

Question 37 (5 marks) **Marks**

A V8 supercars racetrack consists of two semi-circular curves and two straights. The dimensions of the racetrack are shown below. The total length of the racetrack is 4.8 km.



(a) Let x km represent the length of the straight and y km represent the diameter of the smaller semicircle. Show that: **2**

$$y = \frac{9.6 - 4x}{3\pi}$$

(b) The average speed of a V8 supercar on this racetrack is dependent on the length of the straight. It is given by: **3**

$$S = 200 - \left(\frac{x^3}{27} + \frac{\pi}{6}y\right)$$

What is the length of the straight that maximizes the speed?

Question 38 (4 marks)

Marks

(a) Sketch the graph $y = |2x - 4|$. **2**

(b) Using the graph from part (a), or otherwise, find all values of m for which the equation $|2x - 4| = mx + 1$ has exactly one solution. **2**

Question 39 (2 marks)

The Pearson's correlation coefficient between students assessment result and their height was 0.12. What is the meaning of this correlation? **2**

Question 40 (2 marks)

The heights of a group of friends are normally distributed with a mean of 167 cm and a standard deviation of 12 cm. What percentage of the group are more than 179 cm tall? **2**

End of paper

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $$\alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

$$S = \frac{a}{1 - r}, \quad |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

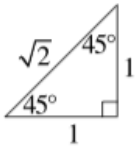
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

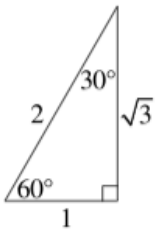
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

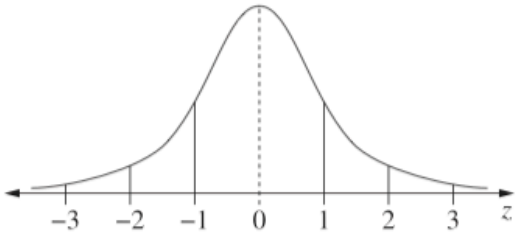
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$