Carlingford High School
Mathematics
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION TERM 3 2019

Student Number: ______________________________

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A Reference & MC Sheet is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 3 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 11

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

	MC	Q11	Q12	Q13	Q14	Q15	Q16	Mark
Basic Arithmetic & Algebra		**/6**	**/3**					**/9**
Plane Geometry			**/1**				**/6**	**/7**
Coordinate Methods in Geometry			**/2**					**/2**
Real Functions	**/2**				**/4**			**/6**
Trigonometric Functions	**/2**	**/3**	**/1**			**/3**		**/9**
Linear Functions	**/1**		**/1**		**/1**			**/3**
Calculus	**/1**	**/2**		**/9**			**/7**	**/19**
Quadratic Polynomial & Parabola			**/3**				**/2**	**/5**
Integration	**/1**				**/3**	**/5**		**/9**
Series & Applications			**/4**			**/7**		**/11**
Logarithms & Exponential Functions	**/2**	**/4**						**/6**
Applications of Calculus to the Physical world	**/1**			**/6**	**/7**			**/14**
Total	**/10**	**/15**	**/15**	**/15**	**/15**	**/15**	**/15**	**/100**

<u>Section</u> I (10 marks)

Attempt Questions 1 – 10. Allow about 15 minutes for this section.

Tear off the multiple-choice answer sheet for Questions 1 – 10 provided at the back.

1. What is the value of $e^{-3\cdot 2}$ correct to two significant figures ?

A 0.04 **B** 0.0 **C** 0.0407 **D** 0.041

2. Find the gradient of the tangent to the function $y = 2x^2 - 3x + 1$ at the point $(-2, 15)$.

A -11 **B** 15 **C** 5 **D** -3

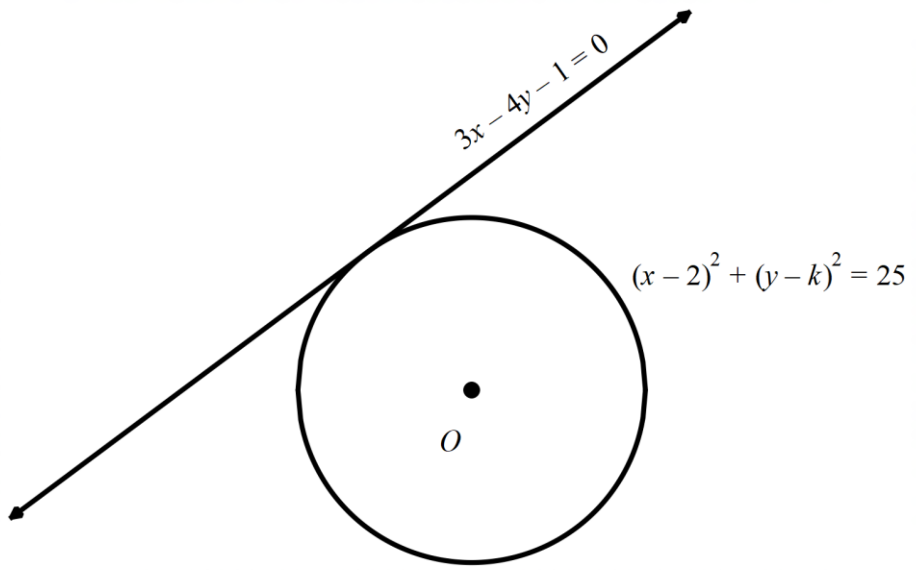
3. What is the derivative of $\ln(\sin x)$?

A $\tan x$ **B** $\operatorname{cosec} x$ **C** $\cot x$ **D** $\ln(\cos x)$

4. For the angle θ, $\tan\theta = \frac{9}{40}$ and $\sin\theta = -\frac{9}{41}$. What is the value of $\cos\theta$?

A $-\frac{9}{40}$ **B** $\frac{40}{41}$ **C** $-\frac{40}{41}$ **D** $\frac{41}{40}$

5. The line $3x - 4y - 1 = 0$ is a tangent to the circle with equation $(x-2)^2 + (y-k)^2 = 25$.
What are the coordinates of O, the centre of the circle ?



A $(-2, 4)$ **B** $(2, -5)$ **C** $(-2, -4)$ **D** $(2, -3)$

6. The point P moves such that it remains equidistant from two fixed points.

Which of the following equations might describe the locus of P ?

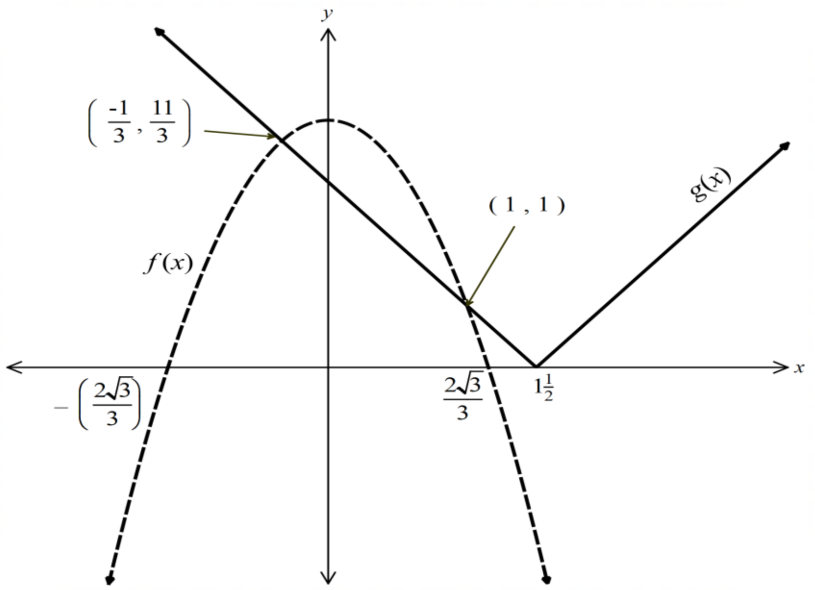
A $3x + 2y - 5 = 0$ **B** $(x-1)^2 + (y+3)^2 = 9$

C $y = \dfrac{3}{x}$ **D** $x^2 = 12y$

7. Convert the angle measurement 1.249 radians into degrees correct to 3 significant figures.

A 7.16º **B** 71.5º **C** 71.6º D 0.398º

8. The graph below shows the functions $f(x) = -3x^2 + 4$ and $g(x) = |2x - 3|$.



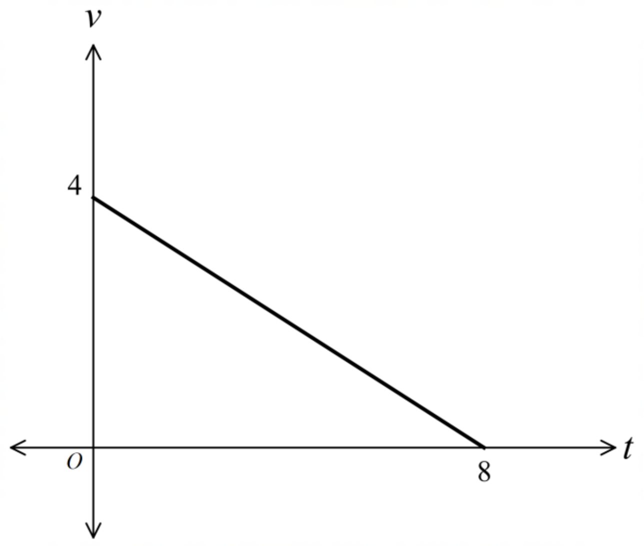
For what values of x is $|2x-3| \geq -3x^2 + 4$?

A $-\dfrac{1}{3} \leq x \leq 1$ **B** $x \leq -\dfrac{1}{3}$ and $x \geq 1$

C $-\dfrac{2\sqrt{3}}{3} \leq x \leq 1$ **D** $x \leq -\dfrac{2\sqrt{3}}{3}$ and $x \geq 1\frac{1}{2}$

9. The graph below shows the velocity of a particle as it moves along the x-axis.

The velocity is measured in metres per second and time is measured in seconds.

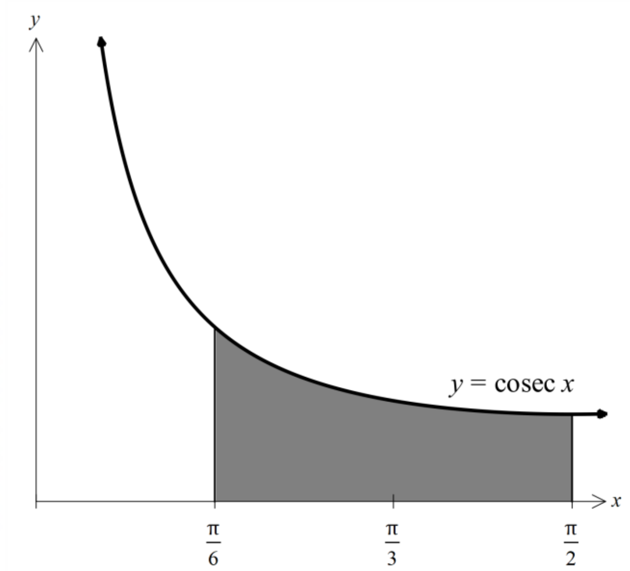


When $t = 0$ the particle's displacement, x, is -2 metres.

What is the maximum value of the particle's displacement?

A 4 m **B** 12 m **C** 14 m D 32 m

10. Given that $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$, find the volume of the solid formed when the shaded region below is rotated about the x-axis.



A $\sqrt{3}\,\pi$ cubic units **B** 2π cubic units **C** π cubic units **D** $\frac{\sqrt{3}\,\pi}{2}$ cubic units

Section II (90 marks)

Attempt Questions 11 – 16. Allow about 2 hours and 45 minutes for this section.

Answer each question in a **SEPARATE** writing **Booklet**. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and / or calculations.

Question 11 (**15 marks**) Use a new writing booklet.

a. Rationalise the denominator of $\dfrac{2}{\sqrt{3}-2}$. **2**

b. Differentiate $\dfrac{2x^3}{x-2}$ with respect to x. **2**

c. Find $\displaystyle\int \frac{x^2}{3x^3-1}\,dx$. **2**

d. Solve $-2-5x<13$. **1**

e. Find the points of intersection of $y=4-x^2+2x$ and $x+y=0$. **3**

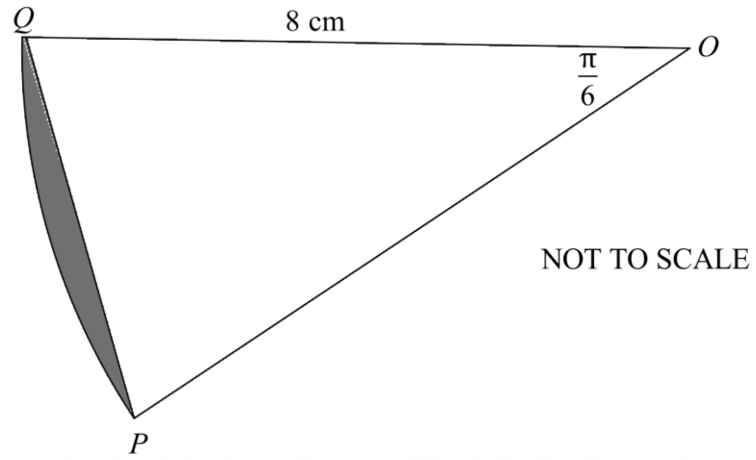
f. Differentiate $e^{3x}\tan x$. **2**

Question 11 continues on page 7

Question 11 continued

g. A sector OPQ of a circle, is shown in the diagram.

The circle has its centre at O and a radius of 8 cm. $\angle QOP = \dfrac{\pi}{6}$.



i). Show that the length of the interval PQ is given by $\dfrac{4}{\sin\left(\dfrac{5\pi}{12}\right)}$. **2**

ii). Calculate the perimeter of the shaded segment, correct to 1 decimal place. **1**

End of Question 11

Question 12 (15 marks) Use a new writing booklet.

a. Simplify $\dfrac{6x^3}{8x^3 - 27y^3} \times \dfrac{4x^2 - 9y^2}{8x^2 + 12xy}$ fully. **3**

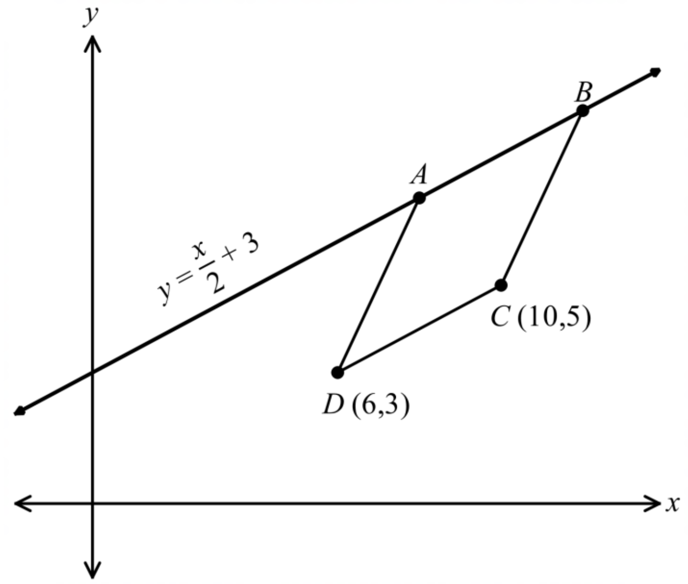
b. Consider the quadratic function $f(x) = 2x^2 - 8x + k$.

i). For what value(s) of k does $f(x) = 0$ have two equal real roots? **2**

ii). Given that the discriminant has an integer value, find the smallest value of k for which $f(x) = 0$ has two rational roots. **1**

c. A quadrilateral *ABCD* is pictured on the Cartesian plane below.

The equation of *AB* is $y = \dfrac{x}{2} + 3$. Point C is (10, 5) and D is (6, 3). $AB = CD$.



i). Show that $DC \parallel AB$. **1**

ii). Show that the line *DB* divides *ABCD* into congruent triangles: $\Delta\, ABD$ and $\Delta\, CDB$. **2**

iii). Show that *ABCD* is a parallelogram. **1**

Question 12 continues on page 9

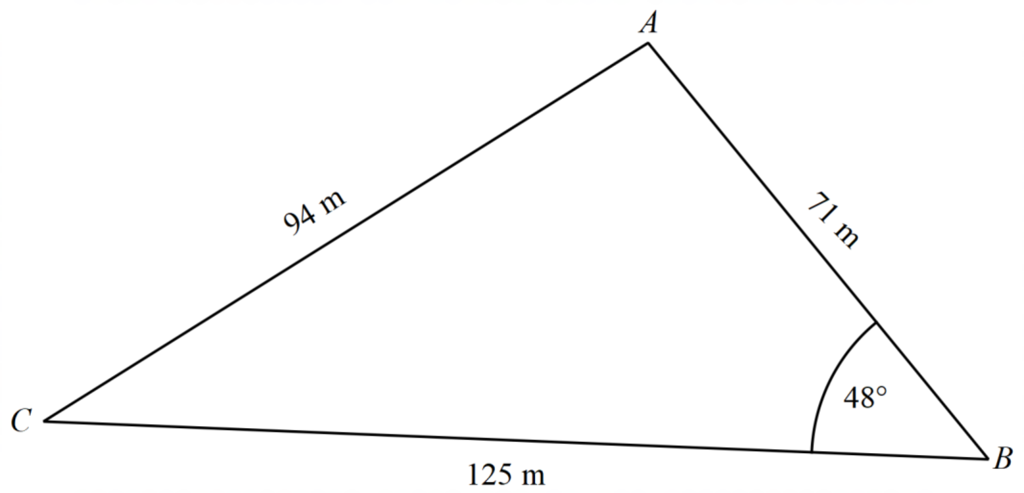
Question 12 continued

d). The fifth term of an arithmetic series is 22. The sum of the first five terms is 50.

i). Find the values of a, the first term and d, the common difference for this series. **2**

ii). How many terms of this series are required to reach a sum greater than 1000? **2**

e). Calculate the area of ΔABC, giving your answer correct to the nearest square metre. **1**



End of Question 12

Question 13 (15 marks) Use a new writing booklet.

a. Consider the curve $y = 2x^3 - 9x^2 + 12x - 5$.

i). Find the stationary points of the curve and determine their nature. **4**

ii). Given that the point $(1 \cdot 5, -0 \cdot 5)$ lies on the curve, show that it is a point of inflexion. **2**

iii). Sketch the curve, labelling its important features. **2**

iv). Hence, or otherwise, find the values of x for which $\dfrac{dy}{dx} < 0$. **1**

b. Australia's population was 19 million at the start of the year 2000.
By the beginning of 2019, it had grown to 25 million.
Assuming that the population of Australia is increasing exponentially,
it can be represented by an equation in the form $P = Ae^{kt}$,
where A and k are constants, and t is measured in years from the beginning of 2000.

i). Show that $P = Ae^{kt}$ satisfies the equation $\dfrac{dP}{dt} = kP$. **1**

ii). What is the value of A? **1**

iii). Show that $k \approx 0 \cdot 0144$. **2**

iv). A newspaper article, published in 1980, forecast that the Australian population would reach 50 million by the year 2050. Using calculations to justify your response, state whether you agree or disagree with this article. **2**

End of Question 13

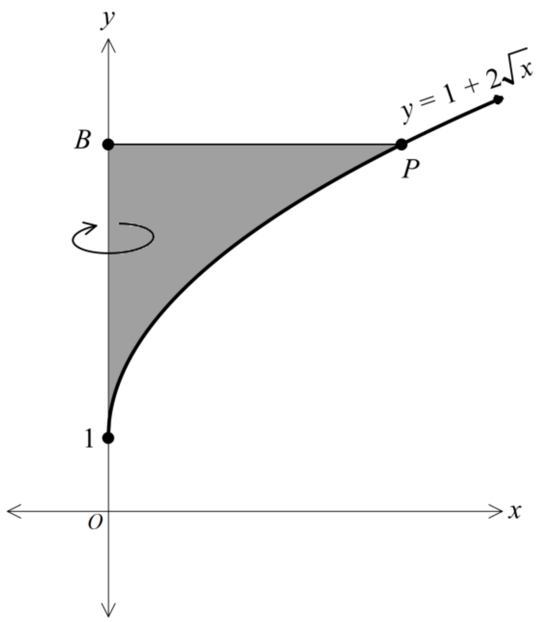
Question 14 (15 marks) Use a new writing booklet.

a. **i).** State the domain of the function $f(x)=\sqrt{9-x^2}$. **1**

ii). Sketch the function $y = f(x)$, showing all important features. **2**

iii). Write down the range of the function. **1**

b. The shaded region in the diagram is bounded by the curve $y = 1 + 2\sqrt{x}$, the y-axis and the horizontal interval BP. The x-coordinate of point P is 4.



i). Find the coordinates of the point B. **1**

ii). Calculate the volume of the solid of revolution formed when the shaded region is rotated about the y-axis. **3**

Question 14 continues on page 12

Question 14 continued

c. The velocity of a particle moving in a straight line has its velocity, in metres per second, given by

$$v = -\frac{7}{t+1}$$

Initially, the particle's displacement is 8 metres to the right of the origin.

i). What is the particle's initial velocity? **1**

ii). Calculate the displacement of the particle at $t = 3$ seconds, correct to 1 decimal place. **3**

iii). Show that the acceleration of the particle is always positive. **2**

iv). Is the particle ever at rest? Give reasons for your answer. **1**

End of Question 14

Question 15 (15 marks) Use a new writing booklet.

a. Solve the equation $2\cos^2 x + \sin x = 2$, where $0 \leq x \leq 2\pi$. **3**

b. Kelsey borrowed \$600 000 for the purchase of a home.

The interest rate on the loan is 3.6% per annum and the loan term is 30 years.

Let $\$A_n$ be the amount owing at the end of *n* months and $\$M$ be the monthly repayment amount.

i). Show that $A_2 = 600\,000(1{\cdot}003)^2 - M(1{\cdot}003 + 1)$. **1**

ii). Show that $A_n = 600\,000(1{\cdot}003)^n - M\left(\dfrac{1{\cdot}003^n - 1}{0{\cdot}003}\right)$. **2**

iii). The monthly repayments are set at \$2 728 in order for Kelsey to repay the loan by the end of 30 years. **4**

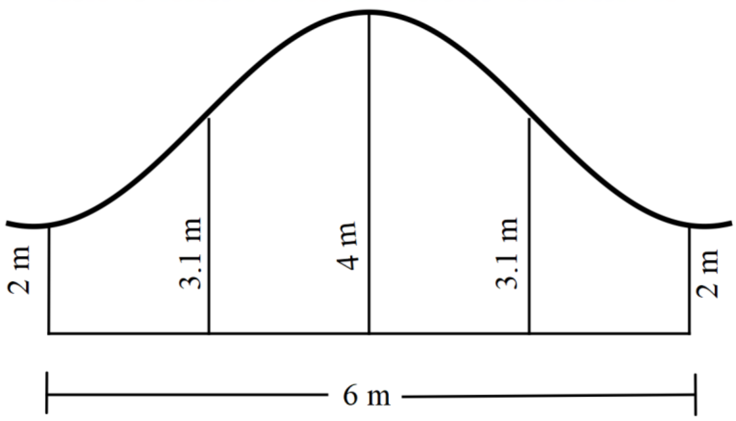
Calculate Kelsey's total saving on the home loan if she decides to pay \$2 800 per month so that the loan is paid out sooner.

Assume that the interest rate remains the same.

Question 15 continues on page 14

Question 15 continued

c. The diagram shows the front view of a farm shed. The area of the front of the shed has been split into 4 sub-intervals of equal width and their heights are shown.



i). Use two applications of Simpson's Rule to approximate the area of the front of the shed. **1**

ii). Use four applications of the trapezoidal rule to approximate the area of the front of the shed. **1**

iii). The front of the shed has an area that is equivalent to the area between the curve $y = 3 + \cos x$, the x-axis and the lines $x = -3$ and $x = 3$. Calculate this area, correct to 2 decimal places. **2**

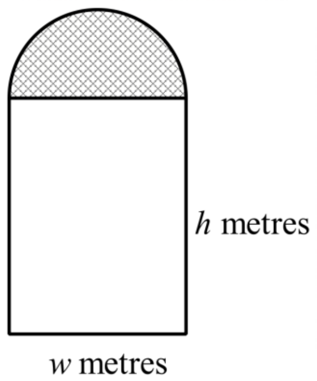
iv). Explain the difference in the way that the trapezoidal rule and Simpson's Rule approximate the area under a curve. **1**

End of Question 15

Question 16 (15 marks) Use a new writing booklet.

a. Find the coordinates of the focus of the parabola $20y = x^2 - 4x + 24$. **2**

b. A design for a new mirror is shown below. The semi-circular section at the top will be made of decorative pressed tin. The bottom section is rectangular and will be made from mirrored glass.
The frame of the window, including the horizontal piece that separates the two sections, will be made from thin metal which is 24 metres in length.
The width of the mirror is *w* metres. The height of the rectangular section is *h* metres.



The company's profit on pressed tin is \$10 per square metre.
They make \$60 profit per square metre on their mirrored glass.
Let the total profit per mirror be represented by $\$P$.

i). Show that $h = 12 - w\left(1 + \dfrac{\pi}{4}\right)$ metres. **2**

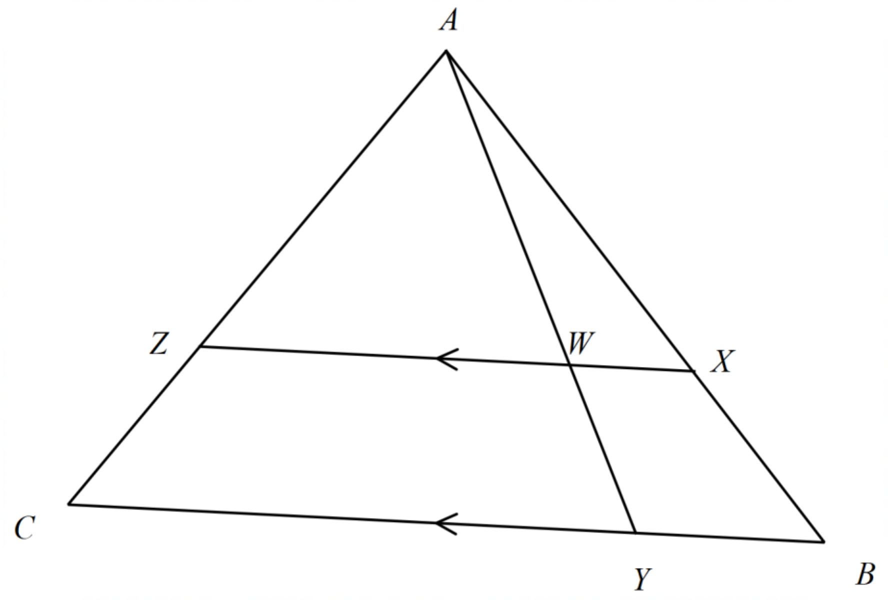
ii). Show that $P = 720w - 10w^2\left(6 + \dfrac{11\pi}{8}\right)$ dollars. **2**

iii). Find the values of *w* and *h* that will maximise the profit made per mirror, correct to 1 decimal place. **3**

Question 16 continues on page 16

Question 16 continued

c. In ΔABC, AB has been divided by point X and AC has been divided by point Z such that $ZX \parallel CB$. CB has been divided by point Y. AY meets ZX at W
$AX = p$ and $XB = q$.



i). Show that $\Delta ACY \parallel\!| \Delta AZW$. **2**

ii). Show that $\dfrac{\text{AZ}}{\text{AC}} = \dfrac{p}{p+q}$. **2**

iii). If A_1 is the area of ΔAZW and A_2 is the area of ΔACY,

show that the area of the trapezium *ZWYC* is given by $A_1\left(\left(\dfrac{p+q}{p}\right)^2 - 1\right)$. **2**

<u>End of Examination</u>

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180^\circ$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

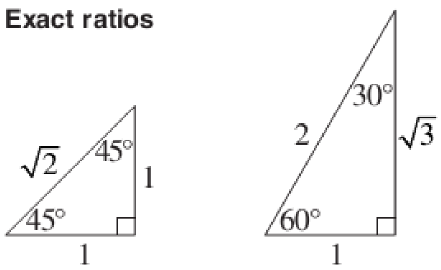
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

***n*th term of an arithmetic series**

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

***n*th term of a geometric series**

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\frac{dy}{dx} = F'(u)\frac{du}{dx}$

If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\frac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\frac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\frac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

Trial HSC Examination 2019
Mathematics Course

Student Number ________________________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○