Graphs Revision Questions

1 Sketch the graphs of the following curves.

    (i) $y = \frac{1}{x} - 1$

    (ii) $y = \left| -\frac{1}{x} \right| + 1$

    (iii) $|y| = 2|\cos x|$ for $-2\pi \leq x \leq 2\pi$

    (iv) $y = \sqrt{1-x} + \sqrt{x}$

2 (a) If $f(x) = (x-2)(x+1)$, sketch graphs of the following on separate diagrams.

    (i) $y = f(x)$

    (ii) $y = \frac{1}{|f(x)|}$

    (iii) $y = \log[f(x)]$

    (iv) Sketch the graph of $y^2 = g(x)f(x)$ if $g(x) = -x^2$ and $y = f(x)$ is the curve in (i). By using calculus, describe the nature of the curve at $x = -1$, $x = 0$ and $x = 2$.

3 (a) Sketch the graph of $y = 1 + x^2$.

    (b) On separate diagrams sketch the graphs of

    (i) $y = \frac{1}{1+x^2}$

    (ii) $y = \frac{x}{1+x^2}$

    (iii) $y = \left| \frac{x}{1+x^2} \right|$

    (iv) $y^2 = \frac{x}{1+x^2}$

    Indicate on your diagrams all important features.

4 Let $f(x) = 1 - x^2$. On separate diagrams and without using calculus, sketch the following graphs. Indicate clearly any asymptotes and intercepts with the axes.

    (i) $y = f(x)$

    (ii) $y = \frac{1}{f(x)}$

    (iii) $|y| = |f(x)|$

    (iv) $y = \log f(x)$

    (v) $y = e^{f(x)}$

5 Let $f(x) = x^2 - 3x$. On separate diagrams sketch the graphs of the following functions. For each graph label any asymptote and the coordinates of any turning points.

    (i) $y = |f(x)|$

    (ii) $y = \frac{1}{f(x)}$

    (iii) $y = \sqrt{f(x)}$

    (iv) $y^2 = f(x)$

    (v) $y = [f(x)]^2$

6 (a) (i) On a single diagram, draw the graphs of $y = |x-2|$ and $y = |x+2|$ in the domain $-6 \leq x \leq 6$.

       (ii) On your diagram from (i), draw the graph of $y = |x-2| - |x+2|$ in the domain $-4 \leq x \leq 4$. Identify each graph clearly.

    (b)
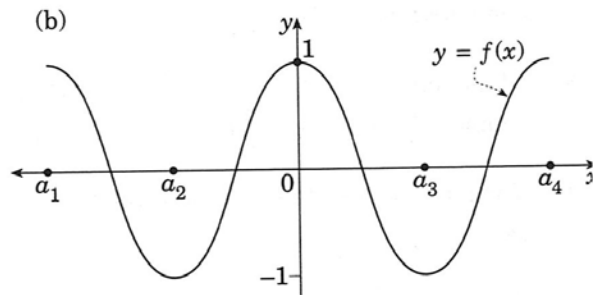
The diagram shows the graph of the continuous function $y = f(x)$. Turning points occur at $x = a_1$, a_2, 0, a_3 and a_4. On separate diagrams sketch each of the following graphs in the domain $a_1 \leq x \leq a_4$

    (i) $y = |f(x)|$

    (ii) $y = [f(x)]^2$

    (iii) $y = \sqrt{f(x)}$

    (iv) $y = xf(x)$

7 (a) The function $y = f(x)$ is defined as

$$f(x) = \begin{cases} \frac{1}{2}(1-x)(2+x) & \text{for } x < 0 \\ \frac{1}{2}(1+x)(2-x) & \text{for } x \geq 0 \end{cases}$$

Answers

1 Graphs

1 (i)

(ii) $(-1, 2)$, $(1, 2)$, $y = 1$

(iii)

(iv) $y = \sqrt{1-x} + \sqrt{x}$, $y = \sqrt{1-x}$, $\left(\frac{1}{2}, \sqrt{2}\right)$, $y = \sqrt{x}$

2 (i) $\left(\frac{1}{2}, -2\frac{1}{4}\right)$

(ii) $\frac{1}{2}$

(iii)

(iv) $x = -1$, $x = 2$
$\frac{dy}{dx}$ does not exist; vertical tangents at $x = -1, 2$

3 (a) $y = 1 + x^2$

(b) (i) $y = \dfrac{1}{1+x^2}$

(ii) $\left(1, \frac{1}{2}\right)$, $\left(-1, -\frac{1}{2}\right)$

(iii) $\left(1, -\frac{1}{2}\right)$, $\left(1, \frac{1}{2}\right)$

(iv) 0.5, $\left(1, \dfrac{1}{\sqrt{2}}\right)$

4 (i) $y = f(x)$

(ii)

(iii)

(iv)

(v) e

5

(i) $\left(\frac{3}{2}, \frac{9}{4}\right)$

(ii) $y = 0$, $\left(\frac{3}{2}, -\frac{9}{4}\right)$, $x = 0$, $x = 3$

(iii)

(iii) $\left(\frac{3}{2}, -\frac{9}{4}\right)$

(iv) -1

(v) $\left(\frac{3}{2}, \frac{81}{16}\right)$

6 (a) (i) $y = |x+2|$, $y = |x-2|$, $y = |x+2|$, $y = |x-2|$

(ii) $y = |x+2|$, $y = |x-2|$, $y = |x+2|$, $y = |x-2|$
$y = |x-2| - |x+2|$

(b) (i) a_1 a_2 a_3 a_4

(ii) a_1 a_2 a_3 a_4

(iii) a_1 a_2 a_3 a_4

(iv) $y = xf(x)$, $y = -x$, $y = x$, a_1 a_2 a_3 a_4, $y = xf(x)$