Carlingford High School
Mathematics Extension 2
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION
Term 3 2013

Student Name: ______________________

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen.
- Board approved calculators may be used
- A Standard Integrals Sheet is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks 100

Section I Total **10** marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Total **90** marks

- Attempt questions 11 – 16
- Answer on the blank paper provided, unless otherwise instructed
- Start a new sheet for each question
- Allow about 2 hours 45 minutes for this section

	MC	Q11	Q12	Q13	Q14	Q15	Q16	Mark
OC1	/1		/13					/14
OC2	/3			/12	/11			/26
OC3	/1						/15	/16
OC4	/1					/7		/8
OC5	/1			/3	/4			/8
OC6	/1	/15						/16
OC7	/2		/2			/8		/12
Total	/10	/15	/15	/15	/15	/15	/15	/100

Section I 10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section. Use the multiple choice answer sheet for Questions 1 – 10.

1. A square root of $8 + 6i$ is :

(A) $3 - i$ (B) $5 - 3i$ (C) $-3 - i$ (D) $-3 + i$

2. The equation of a curve is given by $x^2 + xy + y^2 = 9$. Which of the following expressions will provide the value of $\frac{dy}{dx}$ at any point on the curve?

(A) $\dfrac{-2x-y}{2y}$ (B) $\dfrac{-2x-y}{x+2y}$ (C) $\dfrac{-2x+y}{2y}$ (D) $\dfrac{-2x+y}{x+2y}$

3. The equation $x^3 + 2x^2 - 4x + 5 = 0$ has roots α, β and γ. Find the value of $a^2 + \beta^2 + \gamma^2$.

(A) -12 (B) -4 (C) 4 (D) 12

4. The area bounded by the curves $y = x^2$ and $x = y^2$ is rotated about the x – axis.
The volume of the solid of revolution formed is:

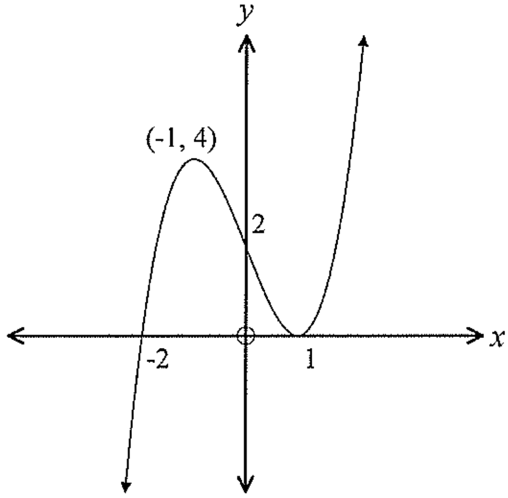
(A) $\frac{9\pi}{70}$ (B) $\frac{3\pi}{10}$ (C) $\frac{7\pi}{10}$ (D) $\frac{3\pi}{2}$

5. The equation of an hyperbola is given by $9x^2 - 4y^2 = 36$. The foci and the directrices of this hyperbola are:

(A) $\left(\pm\sqrt{13}, 0\right)$ and $x = \pm\frac{4\sqrt{13}}{13}$. (B) $\left(0, \pm\sqrt{13}\right)$ and $x = \pm\frac{4\sqrt{13}}{13}$.

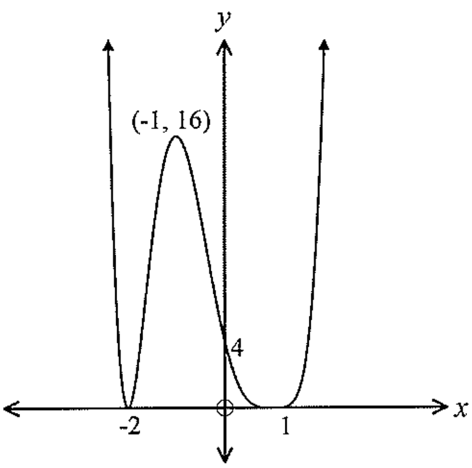
(C) $\left(\pm\sqrt{13}, 0\right)$ and $y = \pm\frac{4\sqrt{13}}{13}$. (D) $\left(0, \pm\sqrt{13}\right)$ and $y = \pm\frac{4\sqrt{13}}{13}$.

6. The graph of the function $y = f(x)$ is drawn below:

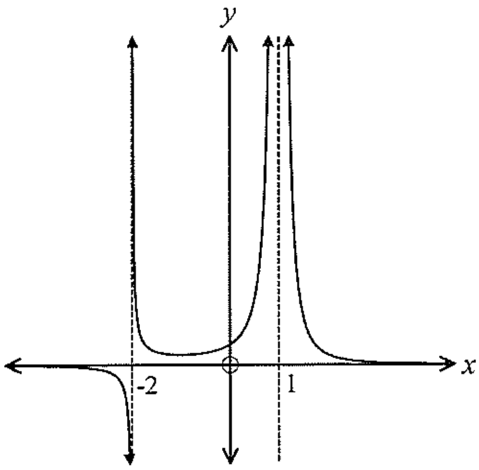


Which of the following graphs best represents the graph $y = \sqrt{f(x)}$?

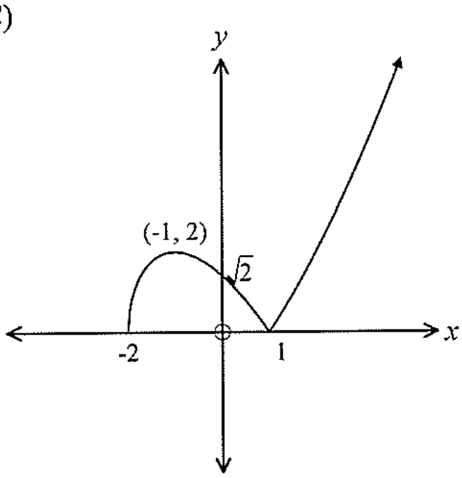
(A)



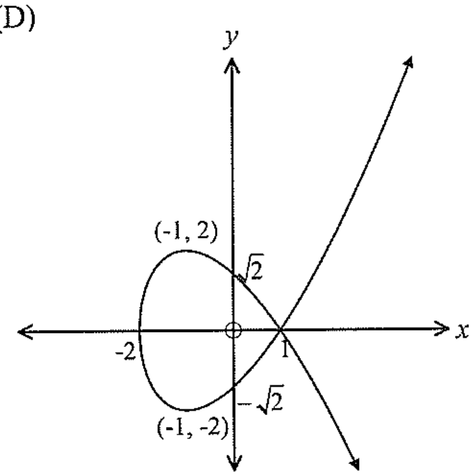
(B)



(C)



(D)



7. $\displaystyle\int \frac{\left(x^3-1\right)dx}{\left(x^4-4x\right)^{\frac{2}{3}}} =$

(A) $\dfrac{3}{4\sqrt[3]{x^4-4x}} + C$

(B) $\dfrac{3}{4\left(x^4-4x\right)} + C$

(C) $\dfrac{3\sqrt[3]{x^4-4x}}{4} + C$

(D) $\dfrac{3\left(x^4-4x\right)}{4} + C$

8. A point is moving in a circular path about a centre O with angular velocity ω.
An expression for the normal acceleration of the point at a time t is:

(A) $\dfrac{dv}{dt}$

(B) $r\dfrac{d\theta}{dt}$

(C) $r\,\omega$

(D) $r\,\omega^2$

9. The general solution of the equation $\sin 4\theta - \sin 2\theta = \cos 3\theta$ is:

(A) $\theta = \dfrac{n\pi}{3} \pm \dfrac{\pi}{6}$ or $\theta = n\pi + \left(-1\right)^n \dfrac{\pi}{3}$.

(B) $\theta = \dfrac{n\pi}{2} \pm \dfrac{\pi}{4}$ or $\theta = n\pi + \left(-1\right)^n \dfrac{\pi}{6}$.

(C) $\theta = \dfrac{2n\pi}{3} \pm \dfrac{\pi}{6}$ or $\theta = n\pi + \left(-1\right)^n \dfrac{\pi}{3}$.

(D) $\theta = \dfrac{2n\pi}{3} \pm \dfrac{\pi}{6}$ or $\theta = n\pi + \left(-1\right)^n \dfrac{\pi}{6}$.

10. Thirteen students in a Year 10 PDHPE class are to be divided into two teams of six to play touch football, with the remaining person acting as the referee. If two particular students are not to be in the same team, the number of different ways the teams can be formed is:

(A) ${}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6 \times 2$

(B) ${}^{11}C_5 \times {}^{6}C_5 \times 2 + {}^{12}C_6$

(C) ${}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6$

(D) ${}^{13}C_6 \times {}^{7}C_6 \div 2$

End of Section I

Section II 90 marks

Attempt Questions 11-16. Allow about 2 hours 45 minutes for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.

Question 11 **(15 marks)** Use a separate sheet of paper **Marks**

a) i) If $\dfrac{x}{(x-2)^2(x-1)} = \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x-1}$, find the values of A, B & C. **2**

ii) Hence evaluate $\displaystyle\int \frac{x}{(x-2)^2(x-1)}\,dx$. **2**

b) i) Derive the reduction formula **1**

$$\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx.$$

ii) Hence or otherwise, find $\displaystyle\int_0^2 x^3 e^x\,dx$. **2**

c) Find

i) $\displaystyle\int \frac{5x-3}{x^2+6x-14}\,dx$. **3**

ii) $\displaystyle\int \frac{dx}{\left(25+x^2\right)^{\frac{3}{2}}}$. using the trigonometric substitution $x = 5\tan\theta$. **3**

iii) $\displaystyle\int \frac{dx}{\sqrt{4+2x-x^2}}$. **2**

End of Question 11

Question 12 **(15 marks)** Use a separate sheet of paper **Marks**

a) If $z = \dfrac{1 + \sqrt{3}\, i}{\sqrt{3} + i}$, find :

i) $|z|$ **1**

ii) $Arg\,(z)$ **1**

iii) z^4 **1**

iv) the five fifth roots of z. **2**

b) Sketch the region in the Argand plane consisting of those points z for which:

$$|\arg(z+1)| < \frac{\pi}{6},\ z + \bar{z} \leq 6 \text{ and } |z+1| > 2.$$ **3**

c) i) Use De Moivre's theorem to express $\cos 4\theta$ in terms of $\cos\theta$. **2**

ii) Hence express $\cot 4\theta$ as a rational function of x where $x = \cot\theta$. **1**

iii) By considering the roots of $\cot 4\theta = 0$, prove that

$$\cot\frac{\pi}{8}\,.\,\cot\frac{3\pi}{8}\,.\,\cot\frac{5\pi}{8}\,.\,\cot\frac{7\pi}{8} = 1.$$ **2**

d) Prove that $\dbinom{n-1}{k-1} = \dfrac{k}{n}\dbinom{n}{k}$. **2**

End of Question 12

Question 13 **(15 marks)** Use a separate sheet of paper **Marks**

a) i) Show that if $x = a$ is a double root of the polynomial $P(x) = 0$, then $P'(a) = P(a) = 0$. **2**

ii) Find the roots of the equation $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$, given that it has a double root. **3**

iii) Given that one root of the equation $x^4 - 5x^3 + 5x^2 + 25x - 26 = 0$ is $3 + 2i$, solve the equation. **3**

b) The equation $2x^3 - x^2 + 3x - 1 = 0$ has roots α, β, and γ. Find the equation which has roots:

i) $2\alpha, 2\beta$ and 2γ. **2**

ii) α^2, β^2 and γ^2. **2**

c) The region bounded by $y = \sin x$ and the $x-$axis between $x = 0$ and $x = \pi$ is rotated about the $y-$axis.

Find the volume of the resulting solid, using the method of **Cylindrical Shells.** **3**

End of Question 13

Question 14 (15 marks) Use a separate sheet of paper **Marks**

a) An ellipse has equation $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

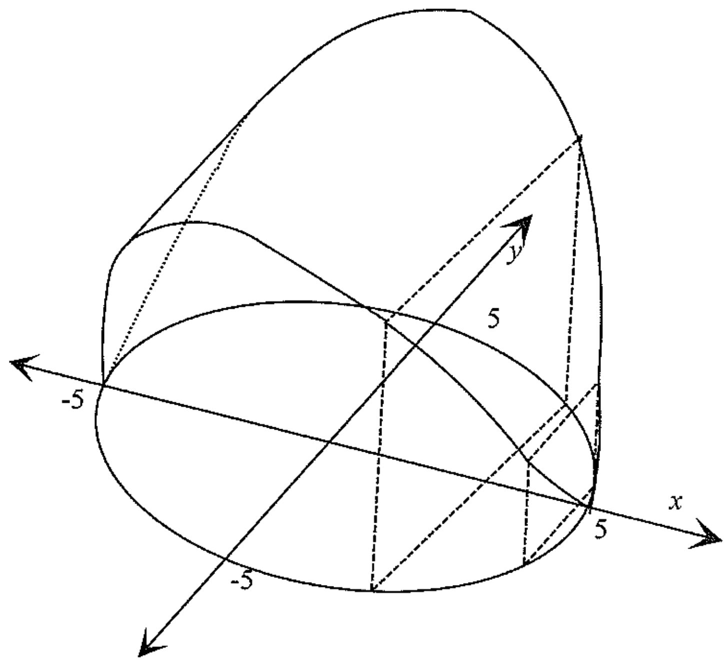
i) Show that this is the equation of the locus of a point $P(x, y)$ that moves such that the sum of its distances from $A(0, 3)$ and $B(0, -3)$ is 10 units. **4**

ii) Find the equations of the tangents to the ellipse when $y = 4$. **4**

b) The hyperbola H has equation $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

P is an arbitrary point $(2\sec\theta, 3\tan\theta)$. Show that P lies on H and show that the equation of the tangent at P is $\frac{x\sec\theta}{2} - \frac{y\tan\theta}{3} = 1$ **3**

c) Let S be the solid having for its base the region bounded by the circle $x^2 + y^2 = 25$.



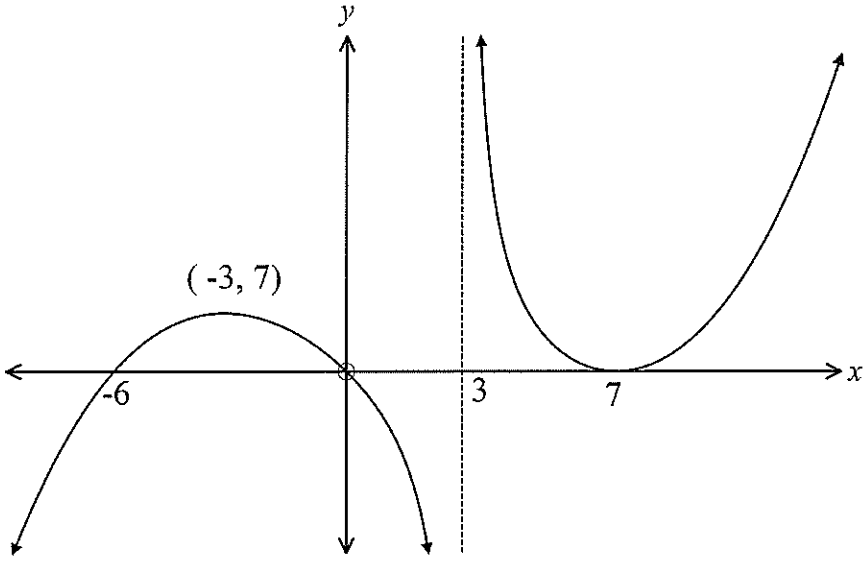
Every vertical plane section of the solid perpendicular to the x- axis is a square.

Find the volume of the solid S. **4**

End of Question 14

Question 15 (15 marks) Use a separate sheet of paper **Marks**

a) The graph of $y = f(x)$ is shown below:



Draw separate half page sketches of the following (indicate important features).

i) $y = \dfrac{1}{f(x)}$. **2**

ii) $y = |f(x)|$. **2**

iii) $y = e^{f(x)}$. **3**

b) i) Use the principle of Mathematical induction to prove that:

$$\sum_{i=1}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}.$$ **3**

ii) Hence evaluate $\lim_{n \to \infty} \dfrac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{n^3}$ **2**

c) To create the daily passcode for entry to an animal enclosure at the zoo, four letters from the word FERRETS are chosen in a random order.

e.g. the code might be RRES or TERF.

How many distinct four letter passcodes are possible? **3**

End of Question 15

Question 16 **(15 marks)** Use a separate sheet of paper **Marks**

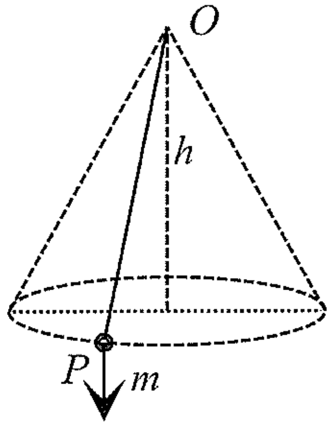
a) A body of mass 50 kg falls from a height at which gravitational acceleration is g. Assuming that air resistance is proportional to the speed v with a constant of proportion being $\frac{1}{10}$, find :

i) The velocity after time t. **3**

ii) The terminal velocity. **1**

iii) The distance the object has fallen after time t. **2**

b) A particle P, of mass m kg, is suspended by a light inextensible string from a point O. It describes a circle with constant speed in a horizontal plane whose vertical distance below O is h metres.



i) By resolving forces find an equation for the period of revolution. **2**

ii) Find the period if the distance below the point of suspension is 50 cm and $g = 10$ m/s. **1**

Question 16 continues on page 13

Question 16 continued. **Marks**

c) A circular track has a radius of 100 metres.
Using $g = 10ms^{-2}$, calculate the angle to the nearest minute at which the track should be banked in order to allow cars to travel at 80 kilometres per hour with no sideways force on the tyres? **3**

d) An object of mass 5kg is attached to a fixed point on a smooth table by a string of length 1.5 metres. If the object moves with a velocity of 3m/s:

i) Find the tension in the string. **1**

ii) If the maximum tension that the string can withstand is 150N, find the greatest number of revolutions per second (in exact value) that the particle can make without breaking the string. **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Trial HSC Examination – Mathematics - Extension 2 2013

Section I – Multiple Choice Answer Sheet

Student Name ______________________________.

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ correct C ○ D ○

Start Here ➔

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

CARLINGFORD
ADVENTURE IN LEARNING

Carlingford High School

Mathematics Extension 2

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Term 3 2013

SOLUTIONS

Trial HSC Examination 2013 – Mathematics Extension 2

Section I – Multiple Choice Answer Sheet

Name ________________________________.

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

Start Here →

1.	A ○	B ○	C ●	D ○	OC1
2.	A ○	B ●	C ○	D ○	OC1
3.	A ○	B ○	C ○	D ●	OC2
4.	A ○	B ●	C ○	D ○	OC5
5.	A ●	B ○	C ○	D ○	OC2
6.	A ○	B ○	C ●	D ○	OC4
7.	A ○	B ○	C ●	D ○	OC6
8.	A ○	B ○	C ○	D ●	OC3
9.	A ○	B ○	C ○	D ●	OC7
10.	A ○	B ○	C ●	D ○	OC7

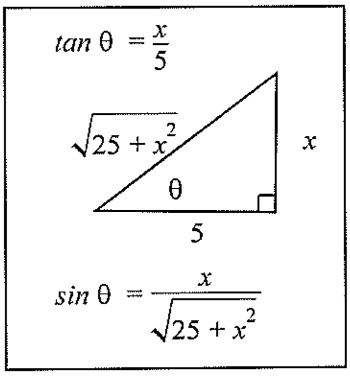
Multiple Choice Worked Solutions.	
1. Let $(x+iy)^2 = 8+6i$ then $x^2+2xyi - y^2 = 8+6i$ $\therefore x^2 - y^2 = 8 \ldots\ldots\ldots [1]$ and $2xy = 6$ $\because (x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$ $= 8^2 + (6)^2$ $= 100$ $\therefore x^2+y^2 = 10 \ldots\ldots\ldots [2]$ From [1] + [2] gives $2x^2 = 18$ $x^2 = 9 \quad \therefore x = \pm 3$ From [2] – [1] gives $2y^2 = 2$ $y^2 = 1 \quad \therefore y = \pm 1$ Since $2xy = 6$ then the square root is $\pm(\mathbf{3}+\boldsymbol{i})$	**C**
2. $\because x^2+xy+y^2 = 9$ then $2x+y+x\frac{dy}{dx}+2y\frac{dy}{dx} = 0$ $\frac{dy}{dx}(x+2y) = -2x-y$ $\therefore \frac{dy}{dx} = \frac{-2x-y}{x+2y}$	**B**
3. $\because x^3+2x^2-4x+5 = 0$ then $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $= \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$ $= \left(\frac{-2}{1}\right)^2 - 2\left(\frac{-4}{1}\right)$ $= 4+8$ $\mathbf{= 12}$	**D**
4. $\because Volume = \pi\int_0^1\left((\sqrt{x})^2-(x^2)^2\right)dx = \pi\int_0^1(x-x^4)\,dx$ $= \pi\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$ $= \pi\left[\left(\frac{1}{2}-\frac{1}{5}\right)-0\right]$ $= \frac{\mathbf{3\pi}}{\mathbf{10}}$	**B**
5. $\because 9x^2-4y^2 = 36$ then $\frac{x^2}{4} - \frac{y^2}{9} = 1$ where $a = 2,\ b = 3$ So $a^2e^2 = 2^2+3^2 = 13$ $ae = \sqrt{13}$ $\therefore \quad e = \frac{\sqrt{13}}{2}$ **Hence Focus** $= (\pm\sqrt{\mathbf{13}}, \mathbf{0})$ **and** **Directrix** $x = \pm\frac{a}{e} = \pm\frac{4}{\sqrt{13}} = \pm\frac{4\sqrt{13}}{13}$	**A**
6. $\because$ Square root of $f(x)$ cannot be negative & square root of 4 is 2. Thus the graph is	**C**

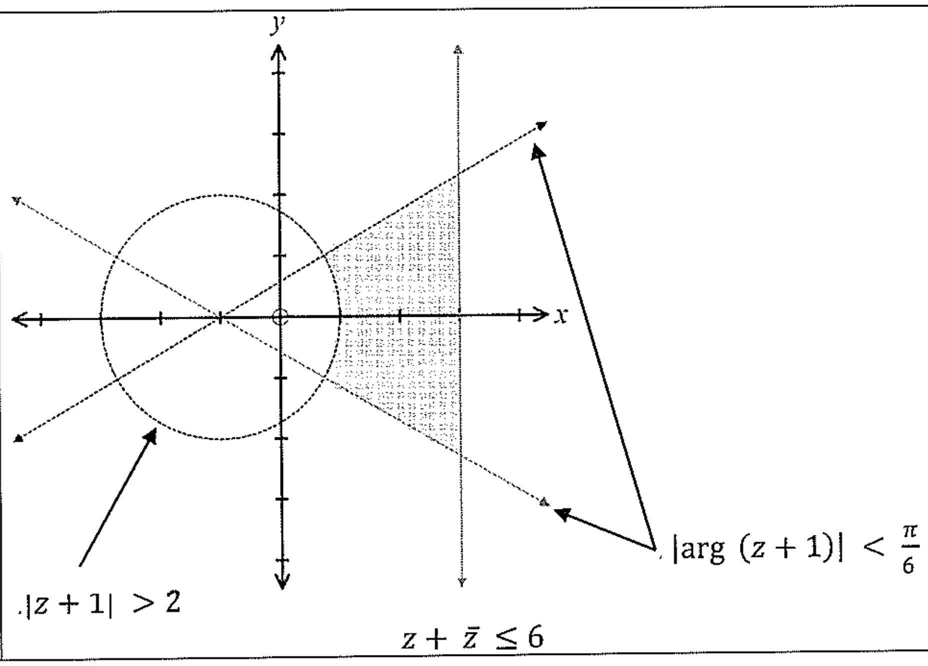
7. $\because \int \frac{x^3-1}{(x^4-4x)^{\frac{2}{3}}}\,dx = \frac{1}{4}\int \frac{4x^3-4}{(x^4-4x)^{\frac{2}{3}}}\,dx$ Now Let $u = x^4 - 4x$ then $du = 4x^3 - 4\,dx$ $= \frac{1}{4}\int u^{\frac{-2}{3}}\,du$ $= \frac{1}{4}\frac{u^{\frac{1}{3}}}{\frac{1}{3}} + c$ $= \frac{3}{4}\sqrt[3]{\boldsymbol{x^4 - 4x}} + \mathbf{c}$	C
8. Normal component of acceleration $= -\ddot{x}\cos\theta - \ddot{y}\sin\theta$ $= i\cos\theta\,(-r\omega^2\cos\theta - r\dot{\omega}\sin\theta) - \sin\theta\,(-r\omega^2\sin\theta - r\dot{\omega}\cos\theta)$ $= r\omega^2(\cos^2\theta + \sin^2\theta)$ $= \boldsymbol{r\omega^2}$	D
9. $\because Sin\,4\theta - \sin 2\theta = \cos 3\theta$ then $2\cos\left(\frac{4\theta+2\theta}{2}\right)\sin\left(\frac{4\theta-2\theta}{2}\right) = \cos 3\theta$ $2\cos 3\theta\sin\theta - \cos 3\theta = 0$ $\cos 3\theta\,(2\sin\theta - 1) = 0$ $\therefore \cos 3\theta = 0$ *or* $\sin\theta = \frac{1}{2}$ Thus $3\theta = 2n\pi \pm \cos^{-1}0$ *or* $\theta = n\pi + (-1)^n\sin^{-1}\frac{1}{2}$ **Hence** $\boldsymbol{\theta = \frac{2n\pi}{3} \pm \frac{\pi}{6}}$ ***or*** $\boldsymbol{n\pi + (-1)^n\frac{\pi}{6}}$	D
10. Call the students who cannot be on the same team A and B. **<u>Case 1</u>** students A and B are in separate teams. This leaves ${}^{11}C_5 \times {}^{6}C_5$ ways the remaining students can be placed with the last student being the referee. **<u>Case 2</u>** Student A is the referee which leaves ${}^{12}C_6 \times {}^{6}C_6 = {}^{12}C_6$ ways the team 1 and team 2 can be formed. However those chosen for team 1 and team 2 could be interchanged, and it is still the same arrangement, so the number of ways = ${}^{12}C_6 \div 2$ ways. Similarly if student B is the referee this again leaves ${}^{12}C_6 \div 2$ ways the teams can be formed. **Total Ways** = ${}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6 \div 2 \times 2$ $= {}^{11}C_5 \times {}^{6}C_5 + {}^{12}C_6$	C

Question 11	Trial HSC Examination 2013		
Part	Solution	Marks	Comment
a) i)	Now $\frac{x}{(x-2)^2(x-1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-1}$ $\therefore x = A(x-2)(x-1) + B(x-1) + C(x-2)^2$		**(OC6)**
	Let $x = 1$ $\therefore C = 1$ and let $x = 2$ $\therefore B = 2$	1	For both B and C
	Thus $x = A(x-2)(x-1) + 2(x-1) + (x-2)^2$ Now let $x = 0$ then $0 = 2A - 2 + 4$ $2A + 2 = 0$ $2A = -2$ $A = -1$	1	For the value of A
	Therefore $A = -1$, $B = 2$ and $C = 1$. **Hence** $\frac{x}{(x-2)^2(x-1)} = -\frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x-1}$		
ii)	$\therefore \int \frac{x}{(x-2)^2(x-1)}\, dx = \int (-\frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x-1})dx$ $= \int -\frac{1}{x-2}\, dx + 2\int (x-2)^{-2}\, dx + \int \frac{1}{x-1}\, dx$	1	For the integral
	$= -ln(x-2) - 2(x-2)^{-1} + ln(x-1) + C$ $= \boldsymbol{ln\left(\frac{x-1}{x-2}\right) - \frac{2}{x-2} + C}$	1	Correct answer
b) i)	Given $\int x^n e^x\, dx$ then let $u = x^n$ & $v' = e^x$ $\therefore u' = nx^{n-1}$ $v = e^x$ Thus $\int x^n e^x\, dx = uv - \int vu'$ $= x^n e^x - \int e^x nx^{n-1}\, dx$ $\therefore$ $\boldsymbol{\int x^n e^x\, dx = x^n e^x - n\int x^{n-1} e^x\, dx}$	1	Reduction Formula
ii)	Now $\int_0^2 x^3 e^x\, dx = [x^3 e^x]_0^2 - 3\int x^2 e^x\, dx$ $= [x^3 e^x]_0^2 - 3\{[x^2 e^x]_0^2 - 2\int xe^x\, dx\}$ $= [x^3 e^x - 3x^2 e^x]_0^2 + 6\left\{[xe^x]_0^2 - \int_0^2 e^x dx\right\}$	1	Substitution
	$= [x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x]_0^2$ $= [8e^2 - 12e^2 + 12e^2 - 6e^2] - [0 + 0 + 0 - 6]$ $= \boldsymbol{2e^2 + 6}$	1	Correct answer

Question 11 Trial HSC Examination 2013

Part	Solution	Marks	Comment
c) i).	$\int \frac{5x-3}{x^2+6x-14}\,dx = \frac{5}{2}\int \frac{2x+6}{x^2+6x-14}\,dx - \int \frac{18}{x^2+6x-14}\,dx$	**1**	Splitting integral
	$= \frac{5}{2}\int \frac{2x+6}{x^2+6x-14}\,dx - 18\int \frac{dx}{x^2+6x+9-23}\,dx$		
	$= \frac{5}{2}ln(x^2+6x-14) - 18\int \frac{dx}{(x+3)^2-23}$	**1**	First integral
	$= \frac{5}{2}ln(x^2+6x-14) - \frac{18}{2\sqrt{23}}\,ln\,\frac{x+3-\sqrt{23}}{x+3+\sqrt{23}} + C$		
	$= \boldsymbol{\frac{5}{2}ln(x^2+6x-14) - \frac{9}{\sqrt{23}}\,ln\,\frac{x+3-\sqrt{23}}{x+3+\sqrt{23}} + C}$	**1**	Other integral
ii).	$I = \int \frac{dx}{(25+x^2)^{\frac{3}{2}}}$ Let $x = 5\tan\theta$ then $dx = 5\sec^5\theta\,d\theta$		
	$I = \int \frac{1}{(25+25\tan^2\theta)^{3/2}}\,5\sec^2\theta\,d\theta$	**1**	Initial Substitution
	Now $(25+25\tan^2\theta)^{3/2} = 125\,(1+\tan^2\theta)^{3/2}$ $= 125\,(\sec^2\theta)^{3/2}$ $= 125\sec^3\theta$		
	$I = \int \frac{5\sec^2\theta\,d\theta}{125\sec^3\theta}$	**1**	Manipulation
	$= \frac{1}{25}\int \frac{1}{\sec\theta}\,d\theta$		
	$= \frac{1}{25}\int \cos\theta\,d\theta$		
	$= \frac{1}{25}\sin\theta + C$		
	$= \frac{1}{25}\,\frac{x}{\sqrt{25+x^2}} + c = \boldsymbol{\frac{x}{25\sqrt{25+x^2}} + C}$	**1**	Correct answer
iii).	$\int \frac{dx}{\sqrt{4+2x-x^2}} = \int \frac{dx}{\sqrt{4-(x^2-2x)}}$		
	$= \int \frac{dx}{\sqrt{5-(x^2-2x+1)}}$		
	$= \int \frac{dx}{\sqrt{5-(x-1)^2}}$	**1**	Splitting integral
	Let $u = x-1$ and $du = dx$ $= \int \frac{du}{\sqrt{5-u^2}}$ $= sin^{-1}\frac{u}{\sqrt{5}} + C$		
	$= \boldsymbol{sin^{-1}\left[\frac{x-1}{\sqrt{5}}\right] + C}$	**1**	Correct answer



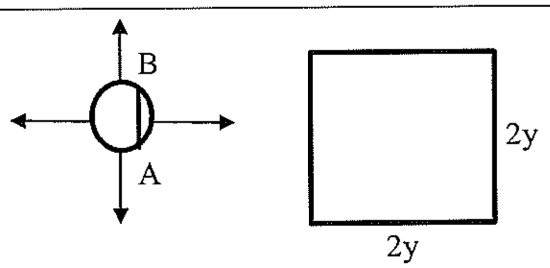
Question 12	Trial HSC Examination	2013	
Part	Solution	Marks	Comment
a) i).	$z = \frac{1+\sqrt{3}i}{\sqrt{3}+i}$ $\lvert z\rvert = \frac{\lvert 1+\sqrt{3}i\rvert}{\lvert\sqrt{3}+i\rvert}$ $= \frac{\sqrt{1^2+(\sqrt{3})^2}}{\sqrt{(\sqrt{3})^2+1^2}}$ $= \frac{2}{2} = \mathbf{1}$ or $z = \frac{1+\sqrt{3}i}{\sqrt{3}+i}$ $z = \frac{1+\sqrt{3}i}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i}$ $z = \frac{\sqrt{3}-i+3i-\sqrt{3}i^2}{3-i^2}$ $z = \frac{2\sqrt{3}+2i}{4} = \frac{\sqrt{3}+i}{2}$ $\lvert z\rvert = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{1} = 1$	1	**(OC1)** Mod z
ii).	$Arg\ z = \frac{Arg\,(1+\sqrt{3}i)}{Arg\,(\sqrt{3}+i)}$ $= Arg\,(1+\sqrt{3}i) - Arg\,(\sqrt{3}+i)$ $= \frac{\pi}{3} - \frac{\pi}{6}$ $= \frac{\pi}{6}$ or $Arg\left(\frac{\sqrt{3}+i}{2}\right) = \frac{\pi}{6}$	1	Arg z
iii).	$z = cis\,\frac{\pi}{6}$ $z^4 = \left(cis\,\frac{\pi}{6}\right)^4$ $z^4 = \boldsymbol{cis}\,\frac{2\pi}{3}$	1	z^4
iv).	$z^5 = cis\,\frac{\pi}{6}$ $z = \frac{cis\,\frac{\pi}{6}+2k\pi}{5}$, $k = 0, 1, 2, 3, 4$ $z = cis\left(\frac{\pi}{30} + \frac{2k\pi}{5}\right)$, $k = 0, 1, 2, 3, 4$ $Z_0 = cis\,\frac{\pi}{30}$ $Z_1 = cis\left(\frac{\pi}{30} + \frac{2\pi}{5}\right) = cis\left(\frac{13\pi}{30}\right)$ $Z_2 = cis\left(\frac{\pi}{30} + \frac{4\pi}{5}\right) = cis\left(\frac{25\pi}{30}\right) = cis\left(\frac{5\pi}{6}\right)$ $Z_3 = cis\left(\frac{\pi}{30} + \frac{6\pi}{5}\right) = cis\left(\frac{37\pi}{30}\right)$ $Z_4 = cis\left(\frac{\pi}{30} + \frac{8\pi}{5}\right) = cis\left(\frac{49\pi}{30}\right)$	1 1	Working Roots
b)		3	One for each graph section

Question 12	Trial HSC Examination 2013		
Part	Solution	Marks	Comment
c) i).	$cis\,4\theta = (cis\,\theta)^4 = cos^4\theta + 4cos^3\theta sin\theta i + 6cos^2\theta sin^2\theta i^2 + 4cos\theta sin^3 i^3 + sin^4\theta i^4$ $cis\,4\theta = (cis\,\theta)^4 = cos^4\theta + 4cos^3\theta sin\theta i - 6cos^2\theta sin^2\theta - 4icos\theta sin^3\theta + sin^4\theta$ $cos\,4\theta = cos^4\theta - 6cos^2\theta sin^2\theta + sin^4\theta$ $= cos^4\theta - 6cos^2\theta(1 - cos^2\theta) + (1 - cos^2\theta)^2$ $= cos^4\theta - 6cos^2\theta + 6cos^4\theta + 1 - 2cos^2\theta + cos^4\theta$	1	expansion
	$\therefore\ \boldsymbol{cos\,4\theta = 8cos^4\theta - 8cos^2\theta + 1}$	1	$cos\,4\theta$
ii).	From above $sin\,4\theta = 4cos^3\theta\,sin\,\theta - 4\,cos\,\theta sin^3\theta$ $cos\,4\theta = cos^4\theta - 6cos^2\theta sin^2\theta + sin^4\theta$ $\therefore\ cot\,4\theta = \dfrac{cos\,4\theta}{sin\,4\theta} = \dfrac{cos^4\theta - 6cos^2\theta sin^2\theta + sin^4\theta}{4cos^3\theta\,sin\,\theta - 4\,cos\,\theta sin^3\theta}$ Divide by $sin^4\theta$, $cot\,4\theta = \dfrac{\frac{cos^4\theta}{sin^4\theta} - \frac{6cos^2\theta sin^2\theta}{sin^4\theta} + \frac{sin^4\theta}{sin^4\theta}}{\frac{4\sin\theta\,cos^3\theta}{sin^4\theta} - \frac{4\,cos\,\theta sin^3\theta}{sin^4\theta}}$		
	Let $x = \cot\theta$ then $\boldsymbol{\cot 4\theta = \dfrac{x^4 - 6x^2 + 1}{4x^3 - 4x}}$	1	$\cot 4\theta$
iii).	If $cot\,4\theta = 0$ then $4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $\therefore\ \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$ Also $\frac{x^4-6x^2+1}{4x^3-4x} = 0$ then $x^4 - 6x^2 + 1 = 0$		
	Since $x = \cot\theta$ So the roots are $cot\frac{\pi}{8}$, $cot\frac{3\pi}{8}$, $cot\frac{5\pi}{8}$, $cot\,\frac{7\pi}{8}$	1	Roots in terms of θ
	From $x^4 - 6x^2 + 1 = 0$ product of the roots is 1 $\therefore\ \boldsymbol{cot\frac{\pi}{8}\ cot\frac{3\pi}{8}\ cot\frac{5\pi}{8}\ cot\,\frac{7\pi}{8} = 1}$	1	Product of roots to obtain result.
d)	LHS $= \binom{n-1}{k-1} = \dfrac{(n-1)!}{(k-1)!\,(n-1-k+1)!}$		**(OC7)**
	$= \dfrac{(n-1)!}{(k-1)!\,(n-k)!}$	1	LHS Working
	RHS $= \dfrac{k}{n}\,\dfrac{n!}{k!\,(n-k)!}$		
	$= \dfrac{(n-1)!}{(k-1)!\,(n-k)!}$	1	RHS Working
	$\therefore$ LHS = RHS		

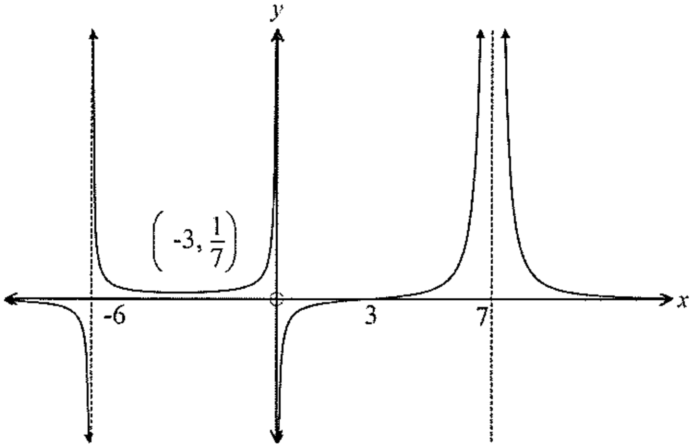
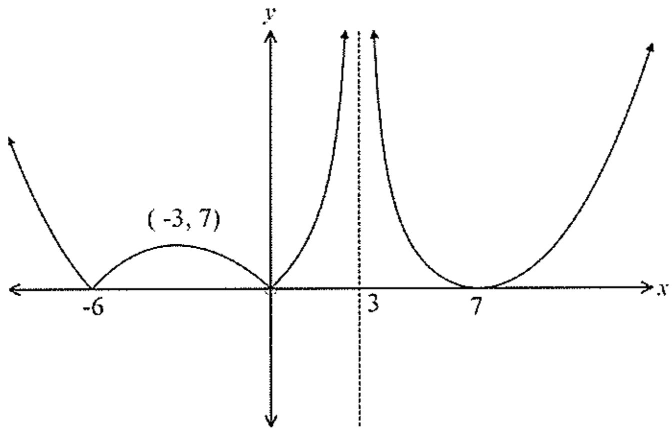
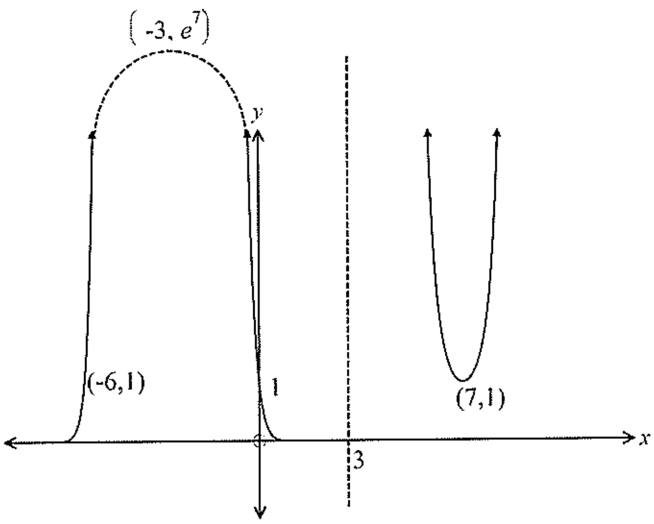
Question 13	Trial HSC Examination 2013		
Part	Solution	Marks	Comment
a)			**(OC2)**
i).	$\because x = a$ is a double root of the polynomial $P(x) = 0$		
	$\therefore P(x) = (x-a)^2.Q(x)$ where $Q(x)$ is a polynomial in x.	1	Expression
	So $P'(x) = 2(x-a)Q(x) + (x-a)^2Q'(x)$		
	$\therefore P'(x) = (x-a)[2Q(x) + (x-a)Q'(x)]$		
	At $x = a$ then		
	$P'(a) = (a-a)[2Q(a) + (a-a)Q'(a)] = 0$	1	Showing $P'(a) = 0$
	$\therefore$ if $x = a$ is a double root of the polynomial $P(x) = 0$, then		
	$\boldsymbol{P'(a) = P(a) = 0}$.		
ii).	$\because f(x) = x^4 - 2x^3 + x^2 + 12x + 8$ then		
	$f'(x) = 4x^3 - 6x^2 + 2x + 12$		
	Try $x = 1$, then $f'(1) = 4 - 6 + 2 + 12 = 12 \neq 0$		
	$x = -1$, then $f'(-1) = -4 - 6 - 2 + 12 = 0$		
	So $f(-1) = 1 + 2 + 1 - 12 + 8 = 0$		
	Thus $(x+1)$ is a double root.	1	Finding double root
	$\therefore (x+1)^2 = x^2 + 2x + 1$ is a root of $f(x)$		
	After dividing $f(x)$ by $x^2 + 2x + 1$ gives		
	$f(x) = (x+1)^2(x^2 - 4x + 8)$	1	Division
	$= (x+1)^2(x - 2 + 2i)(x - 2 - 2i)$		
	$\therefore$ **Roots are** $\boldsymbol{-1, \quad -1, \quad 2 \pm 2i}$	1	Roots
iii).	Given $x^4 - 5x^3 + 5x^2 + 25x - 26 = 0$		
	If one root is $3 + 2i$ then $3 - 2i$ is also a root.		
	$\therefore$ P(x) is divisible by $x^2 - 6x + 13$.	1	Quadratic
	By division P(x) = $(x^2 - 6x + 13)(x^2 + x - 2)$	1	Division
	i.e. P(x) = $(x^2 - 6x + 13)(x + 2)(x - 1)$		
	$\boldsymbol{\therefore x = 3 + 2i, \ 3 - 2i, \ -2, \ 1}$	1	Roots
b)			
i).	Given $2x^3 - x^2 + 3x - 1 = 0$		
	Let $x = 2X$, $\therefore X = \frac{x}{2}$		
	Thus $2\left(\frac{x}{2}\right)^3 - \left(\frac{x}{2}\right)^2 + 3\left(\frac{x}{2}\right) - 1 = 0$	1	Substituting new variable
	$2\left(\frac{x^3}{8}\right) - \left(\frac{x^2}{4}\right) + \frac{3x}{2} - 1 = 0$		
	$2x^3 - 2x^2 + 12x - 8 = 0$		
	$\therefore \boldsymbol{x^3 - x^2 + 6x - 4 = 0}$	1	Correct Equation
ii).	Let $x = X^2$ $\therefore X = \sqrt{x}$		
	Thus $2(\sqrt{x})^3 - (\sqrt{x})^2 + 3(\sqrt{x}) - 1 = 0$	1	Substituting new variable
	$2x\sqrt{x} - x + 3\sqrt{x} - 1 = 0$		
	$\sqrt{x}\,(2x + 3) = x + 1$		
	$x(2x+3)^2 = (x+1)^2$		
	$x(4x^2 + 12x + 9) = x^2 + 2x + 1$		
	$4x^3 + 12x^2 + 9x - x^2 - 2x - 1 = 0$		
	$\boldsymbol{\therefore 4x^3 + 11x^2 + 7x - 1 = 0}$	1	Correct Equation

Question 13 Trial HSC Examination 2013			
Part	Solution	Marks	Comment
c)	Now $V = 2\pi \int_0^{\pi} xy\,dx$ $= 2\pi \int_0^{\pi} x \sin x\,dx$ Let $u = x$ $v' = \sin x$ $uv - \int vu'$ Then $u' = 1$ $v = -\cos x$ $-x \cos x + \cos x$ $= 2\pi\{ [-x \cos x]_0^{\pi} + \int_0^{\pi} \cos x\,dx\}$ $= 2\pi[-x \cos x + \sin x]_0^{\pi}$ $= 2\pi[\pi - 0]$ $= \mathbf{2\pi^2}$	 **1** **1** **1**	 Correct Integral Integrating by parts Answer

Question 14	Trial HSC Examination 2013		
Part	Solution	Marks	Comment
a) i).	Given $P(x,\ y),\ A(0,\ 3),\ B(0,\ -3)$ $\because\ PA + PB = 10$ then $\sqrt{x^2 + (y-3)^2} + \sqrt{x^2 + (y+3)^2} = 10$	1	(OC2) Substitution
	$\sqrt{x^2 + (y-3)^2} = 10 - \sqrt{x^2 + (y+3)^2}$		
	$x^2 + y^2 - 6y + 9 = 100 - 20\sqrt{x^2 + (y+3)^2} + x^2 + y^2 + 6y + 9$	1	Working
	$20\sqrt{x^2 + (y+3)^2} = 100 + 12y$		
	$400(x^2 + y^2 + 6y + 9) = 10000 + 2400y + 144y^2$	1	Working
	$400x^2 + 400y^2 + 2400y + 3600 = 10000 + 2400y + 144y^2$		
	$400x^2 + 256y^2 = 6400$		
	Now dividing by 6400 gives $\frac{x^2}{16} + \frac{y^2}{25} = 1$	1	Answer
ii).	$\because\ \frac{x^2}{16} + \frac{y^2}{25} = 1$ then $25x^2 + 16y^2 = 400$ When $y = 4$ then $25x^2 + 16(4)^2 = 400$ $25x^2 + 256 = 400$ $25x^2 = 144$ $\therefore x = \pm\frac{12}{5}$	1	x values
	Now $50x + 32y\frac{dy}{dx} = 0$		
	then $\frac{dy}{dx} = \frac{-50x}{32y} = \frac{-25x}{16y}$		
	At $x = \frac{12}{5}$, then $m = \frac{-60}{64} = \frac{-15}{16}$	1	Gradients of tangents
	Thus the equation of the tangent is $y - 4 = \frac{-15}{16}\left(x - \frac{12}{5}\right)$		
	$16y - 64 = -15x + 36$ $\therefore\ \mathbf{15x + 16y - 100 = 0}$	1	Equation
	At $x = -\frac{12}{5}$, then $m = \frac{60}{64} = \frac{15}{16}$		
	thus the equation of the tangent is $y - 4 = \frac{15}{16}\left(x + \frac{12}{5}\right)$ $16y - 64 = 15x + 36$ $\therefore\ \mathbf{15x - 16y + 100 = 0}$	1	Equation

Question 14 Trial HSC Examination 2013			
Part	Solution	Marks	Comment
b)	Given $\frac{x^2}{4} - \frac{y^2}{9} = 1$ at $P(2\sec\theta, 3\tan\theta)$ $LHS = \frac{(2\sec\theta)^2}{4} - \frac{(3\tan\theta)^2}{9}$ $= \frac{4\sec^2\theta}{4} - \frac{9\tan^2\theta}{9} = 1$ $= \sec^2\theta - \tan^2\theta$ $= 1 + \tan^2\theta - \tan^2\theta$ $LHS = RHS$ $\therefore$ P lies on the Hyperbola H	1	**(OC2)** Showing that P satisfies equation of hyperbola
	$\because\ x = 2\sec\theta$ and $y = 3\tan\theta$ then $\frac{dx}{d\theta} = 2\sec\theta\tan\theta$ and $\frac{dy}{d\theta} = 3\sec^2\theta$ Thus $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ $\therefore\ \frac{dy}{dx} = \frac{3\sec^2\theta}{2\sec\theta\tan\theta} = \frac{3\sec\theta}{2\tan\theta}$	1	Derivative (Implicit derive also possible)
	Eqt of tangent at P is $y - 3\tan\theta = \frac{3\sec\theta}{2\tan\theta}(x - 2\sec\theta)$ $2y\tan\theta - 6\tan^2\theta = 3x\sec\theta - 6\sec^2\theta$ $3x\sec\theta - 2y\tan\theta = 6\sec^2\theta - 6\tan^2\theta$ $3x\sec\theta - 2y\tan\theta = 6[\tan^2\theta + 1 - \tan^2\theta]$ $3x\sec\theta - 2y\tan\theta = 6$ Now divide by 6 gives $\boldsymbol{\frac{x\sec\theta}{2} - \frac{y\tan\theta}{3} = 1}$	1	Required result
c)	 		**(OC5)**
	Now $AB = 2y = 2\sqrt{25 - x^2}$ So Area of Cross-section is $A(x) = 4(25 - x^2)$	1 1	Length AB Area
	$\therefore$ Volume $= \int_{-5}^{5} 4(25 - x^2)\,dx$ $= 4\left[25x - \frac{x^3}{3}\right]_{-5}^{5}$ $= 4\left\{\left[125 - \frac{125}{3}\right] - \left[-125 - \frac{-125}{3}\right]\right\}$	1	Integral
	$= \frac{\mathbf{2000}}{\mathbf{3}}$ **cubic units** NB. (May also use $2\int_0^5 etc$ also)	1	Answer

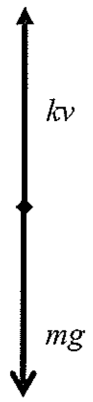
Question 15 Trial HSC Examination 2013

Part	Solution	Marks	Comment
a) i).		**2**	**(OC4)** Lose 1 mark for each mistake.
ii).		**2**	Lose 1 mark for each mistake.
iii).		**3**	Lose 1 mark for each mistake.

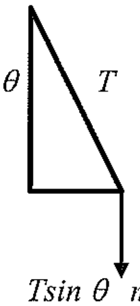
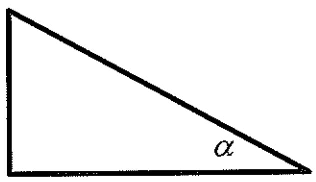
Question 15	Trial HSC Examination 2013		
Part	Solution	Marks	Comment
b) i).	Let $\sum_{i=1}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$ Test $n=1$ $\quad 1 = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1 \quad \therefore$ True for $n = 1$ Now assume it is true for $n = k$ i.e. $\sum_{i=1}^{k} i^2 = \frac{k^3}{3} + \frac{k^2}{2} + \frac{k}{6}$ Prove for $n = k + 1$ is true i.e. $\sum_{i=1}^{k+1} i^2 = \frac{k^3}{3} + \frac{k^2}{2} + \frac{k}{6} + (k+1)^2$ $\text{LHS} = \frac{2k^3 + 3k^2 + k + 6(k+1)^2}{6}$ $= \frac{2k^3 + 3k^2 + k + 6k^2 + 12k + 6}{6}$ $= \frac{2k^3 + 9k^2 + 13k + 6}{6}$ By division, taking out a factor of $(k + 1)$ $\therefore \ \mathbf{LHS} = \frac{(\boldsymbol{k}+\mathbf{1})(\mathbf{2}\boldsymbol{k}^2 + \mathbf{7}\boldsymbol{k} + \mathbf{6})}{\mathbf{6}}$ Now $\text{RHS} = \frac{(k+1)^3}{3} + \frac{(k+1)^2}{2} + \frac{k+1}{6}$ $= \frac{2(k+1)^3 + 3(k+1)^2 + (k+1)}{6}$ $= \frac{(k+1)[2(k+1)^2 + 3(k+1) + 1]}{6}$ $= \frac{(k+1)(2k^2 + 4k + 2 + 3k + 3 + 1}{6}$ $\therefore \ \mathbf{RHS} = \frac{(\boldsymbol{k}+\mathbf{1})(\mathbf{2}\boldsymbol{k}^2 + \mathbf{7}\boldsymbol{k} + \mathbf{6})}{\mathbf{6}}$ $\therefore$ True for $n = k + 1$ if true for $n = k$, but true for $n = 1$ $\therefore$ true for $n = 1 + 1 = 2$ etc Hence by Mathematical induction, true for all $n \geq 1$	1 1 1	**(OC7)** Tests $n = 1$ Assume $n = k$ Case for LHS of $n = k+1$ Case for RHS of $n = k+1$
ii).	Now $\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 \ldots\ldots\ldots + n^2}{n^3} = \lim_{n \to \infty} \frac{1}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$ $= \lim_{n \to \infty} \left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right)$ $= \mathbf{\frac{1}{3}}$	1 1	Correct limit value

Question 15	Trial HSC Examination 2013		
Part	Solution	Marks	Comment
c)	Possible passcodes could contain All different letters chosen from F,E,R,T,S in ${}^5\mathbf{C}_4 \times 4! = 120\ \mathit{ways}$. 2 R's and no E's (ie R,R and two of F, T and S) in ${}^3\mathbf{C}_2 \times \frac{4!}{2!} = 36\ \mathit{ways}$ 2 E's and no R's (ie E,E and two of F, T and S) in ${}^3\mathbf{C}_2 \times \frac{4!}{2!} = 36\ \mathit{ways}$ 2 R's and one E (ie R,R,E and one of F, T and S) in ${}^3\mathbf{C}_1 \times \frac{4!}{2!} = 36\ \mathit{ways}$ 2 E's and one R (ie R,E,E and one of F, T and S) in ${}^3\mathbf{C}_1 \times \frac{4!}{2!} = 36\ \mathit{ways}$ 2 E's and 2 R's (ie R,R,E, E) in $\frac{4!}{2!.2!} = 6\ \mathit{ways}$	2	**(OC7)** For identifying the different cases.
	Total possible passcodes = $120 + 36 \times 4 + 6 = 270\ \mathit{ways}$	1	For calculating the correct result

Question 16 Trial HSC Examination 2013

Part	Solution	Marks	Comment
a) i).			**(OC3)**
	Now $R \propto v$ then $R = kv$ $\therefore R = \frac{1}{10}v$		
	$\because F = ma$ then $ma = mg - kv$ $a = g - \frac{kv}{m}$ $a = g - \frac{v}{500}$	**1**	Expression for a
	$\frac{dv}{dt} = g - \frac{v}{500}$ $\frac{dv}{dt} = \frac{500g - v}{500}$ $\frac{dt}{dv} = \frac{500}{500g - v}$		
	$\therefore t = -500\,ln(500g - v) + c$	**1**	Equation for t
	When $v = 0, t = 0$ then $0 = -500\,ln(500g) + c$ $\therefore c = 500\,ln(500g)$ $\therefore t = -500\,ln(500g - v) + 500\,ln(500g)$ $= 500[ln\,500g - ln(500g - v)]$ $t = 500\ln\frac{500g}{500g - v}$ $e^{\frac{t}{500}} = \frac{500g}{500g - v}$ $500g - v = 500g\,e^{\frac{-t}{500}}$		
	$\therefore \boldsymbol{v = 500g\left(1 - e^{\frac{-t}{500}}\right)}$	**1**	Finding v
ii).	Terminal velocity when $t \to \infty$ $As\ t \to \infty,\ e^{\frac{-t}{500}} \to 0 \quad \therefore \boldsymbol{v \to 500g}$	**1**	Terminal Velocity
iii).	$\because v = \frac{dx}{dt} = 500g\left(1 - e^{\frac{-t}{500}}\right)$ then $x = \int(500g - 500g\,e^{\frac{-t}{500}})\,dt$ $= 500gt + 250000g e^{\frac{-t}{500}} + c$	**1**	Equation for x
	When $t = 0,\ x = 0$ then $0 = 0 + 250000g + c$ $\therefore c = -250000g$		
	$\therefore \boldsymbol{x = 500gt + 250000g\left(e^{\frac{-t}{500}} - 1\right)}$	**1**	Result

Question 16 Trial HSC Examination 2013

Part	Solution	Marks	Comment
b) i).	 Vertical: $Tcos\,\theta = mg$ ------[1] Horizontal: $Tsin\,\theta = mr\omega^2$ Now $r = htan\,\theta$ So $Tsin\,\theta = m.htan\,\theta\omega^2$ --[2] Eliminating T we obtain: $\frac{[2]}{[1]} = \frac{Tsin\,\theta}{Tcos\,\theta} = \frac{m.htan\,\theta\omega^2}{mg}$ Thus $Tan\,\theta = \frac{htan\,\theta\omega^2}{g}$ $\therefore\ \omega^2 = \frac{g}{h}$ i.e. $\omega = \sqrt{\frac{g}{h}}\ rad/s$ $\therefore$ Time for 1 revolution $= 2\pi \Big/ \sqrt{\frac{g}{h}}$		
	$\therefore\ \boldsymbol{Period} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}}$	1 1	Resolving forces to obtain value of Tan θ Period
ii).	When h = 50 cm (i.e. 0·5m) and g = 10 m/s^2 then $\boldsymbol{T = 2\pi\sqrt{\frac{0\cdot5}{10}} = 1\cdot4\ \text{sec}}$	1	Correct value
c)	 Now $v = \frac{80\times1000}{60^2} = 22\frac{2}{9}$ m/s $\because\ \tan\alpha = \frac{v^2}{gr}$ then $= \frac{\left(22\frac{2}{9}\right)^2}{10\times100} = 0.49\ldots$ $\therefore\ \boldsymbol{\alpha = 26°17'}$	1 1 1	Velocity Sub into equation Angle
d) i).	$T = \frac{mv^2}{r} = \frac{5(3)^2}{1\cdot5} = \mathbf{30\ N}$	1	Tension
ii).	$\because\ T = \frac{mv^2}{r}$ then $150 = \frac{5v^2}{1\cdot5} \rightarrow 45 = v^2$ $\therefore\ v = 3\sqrt{5}\ m/s$ $\because\ \omega = \frac{v}{r}$ then $\omega = \frac{3\sqrt{5}}{1\cdot5}$ $= \frac{3\sqrt{5}}{1\cdot5\times2\pi}$ $\boldsymbol{\therefore\ \omega = \frac{3\sqrt{5}}{3\pi}\ rad/s}$	1 1	Velocity Number of Revolutions