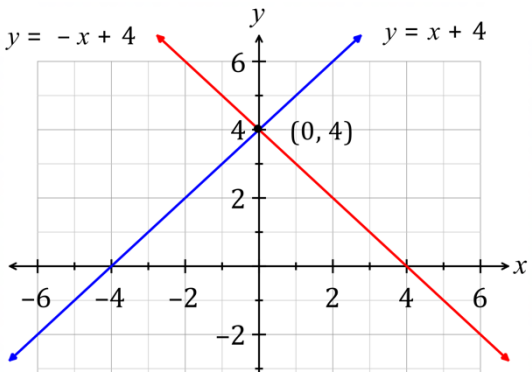
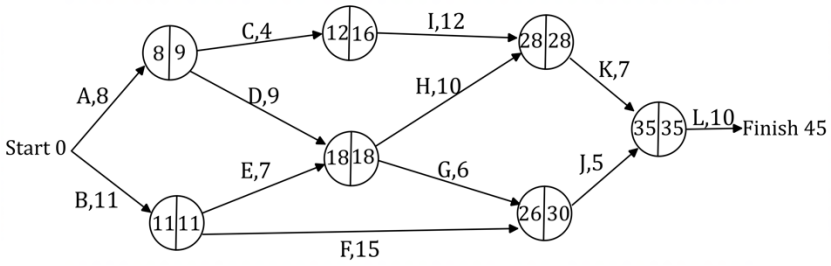
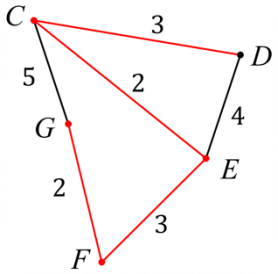
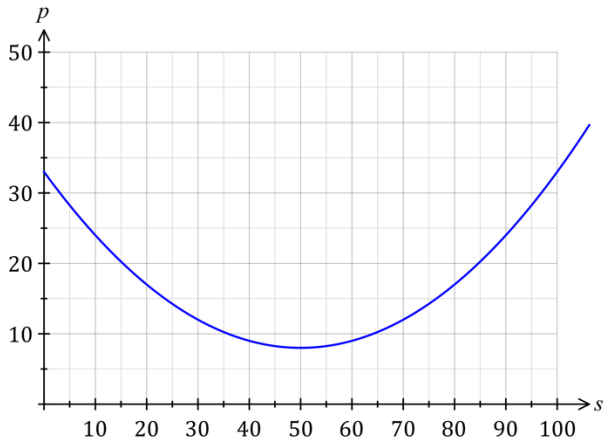
ACE Examination Paper 1
Year 12 Mathematics Standard 2 Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1.	Town A $= 243\,000 \div (18 \times 30)$ Town B $= 340\,000 \div (20 \times 40)$ $= \$450$ per m^2 $= \$425$ per m^2 Difference $= 450 - 425$ $= \$25$	1 Mark: A
2.	Correlation coefficient of –0.5 is a negative linear relationship with medium strength.	1 Mark: A
3.	A path is a walk with no repeated vertices. $\therefore$ S–T–U–V	1 Mark: B
4.	$\cos A = \dfrac{b^2 + c^c - a^2}{2bc}$ $= \dfrac{18^2 + 10^2 - 20^2}{2 \times 18 \times 10}$ $\angle BAC = 86.1774\ldots$ $\approx 86°$	1 Mark: D
5.	180 m $= 1$ mm 30 m $= \dfrac{1}{6}$ mm 240 m $= \dfrac{8}{6}$ mm ≈ 1.33 mm	1 Mark: A
6.	$V_1 = 72\,000 \times 1.004 - 360 = \$71\,928$ $V_2 = 71\,928 \times 1.004 - 360 = \$71\,855.712$ $V_3 = 71\,855.712 \times 1.004 - 360 = \$71\,783.13485$ $\therefore$ Balance of the loan is approximately \$71 783	1 Mark: B
7.	$z = \dfrac{x - \bar{x}}{s}$ $z = \dfrac{x - \bar{x}}{s}$ $= \dfrac{44 - 60}{8}$ $= \dfrac{76 - 60}{8}$ $= -2$ $= 2$ $\therefore$ 95% of scores have a z-score between –2 and 2	1 Mark: C
8.	$FV = PV(1 + r)^n$ $= 3125 \times \left(1 + \dfrac{0.06}{4}\right)^{4\times4}$ $= 3965.5798\ldots$ $\approx \$3966$	1 Mark: C

	Solution	Criteria
9.	 ∴ Point of intersection is (0, 4)	1 Mark: B
10.	$d = \frac{5vt}{18} + \frac{v^2}{170}$ $= \frac{5 \times 70 \times 0.50}{18} + \frac{70^2}{170}$ ≈ 39 m	1 Mark: C
11.	 Float time $= 16 - 12$ $= 4$	1 Mark: A
12.	$\frac{x}{\sin 61°} = \frac{29}{\sin 79°}$ $x = \frac{29 \sin 61°}{\sin 79°}$	1 Mark: C
13.	Total paid $= 2200 + 820 \times 12 \times 4$ $= \$41\,560$	1 Mark: D
14.	$z = \frac{x - \bar{x}}{s}$ $= \frac{190 - 160}{15}$ $= 2$ ∴ 95% of scores have a z-score between −2 and 2. ∴ $5\% \div 2 = 2.5\%$ have a z-score greater than 2.	1 Mark: B
15.	$N = 1000(2.5^t)$ $= 1000 \times 2.5^2$ $= 6250$	1 Mark: D

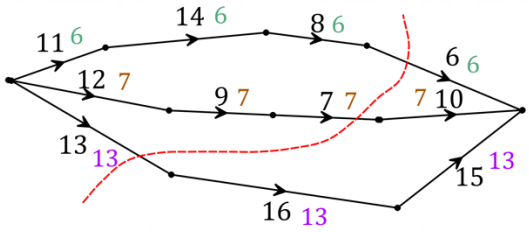
Section II		
	Solution	**Criteria**
16(a)	Cost $= 2000 \times 0.02580 + 3000 \times 0.01620$ $= \$100.20$ $\therefore$ Savannah's gas charge was \$100.20	1 mark: Correct answer.
16(b)	To find the cost of 2000 MJ Cost $= 2000 \times 0.02580$ $= \$51.60$ Gabriel's target of \$80 is greater than \$51.60 $80 - 51.60 = x \times 0.01620$ $x = \dfrac{28.40}{0.01620}$ $= 1732.0864\ldots$ ≈ 1753 MJ $\therefore$ Gabriel's maximum usage is 3753 MJ (2000 MJ + 1753 MJ)	2 Marks: Correct answer. 1 Mark: Shows some understanding.
17	Find the minimum spanning tree.  Length $= 2 + 3 + 2 + 3$ $= 10$ km $\therefore$ Minimum length of pipes is 10 km.	2 Marks: Correct answer. 1 Mark: Shows some understanding
18(a)	Intersection value is 1.89 (4% and 2 years) $PV = 1.89 \times 6000$ $= \$11\,340$	1 mark: Correct answer.
18(b)	Intersection value is 2.58 (8% and 3 years) Let the value of the annuity be x $47\,988 = x \times 2.58$ $x = \dfrac{47\,988}{2.58}$ $= \$18\,600$ $\therefore$ Value of the annuity is \$18 600 per year.	2 marks: Correct answer. 1 mark: Finds the intersection value.
18(c)	Intersection value is 3.47 (6% and 4 years) $PV = 3.47 \times 1000$ $= \$3\,470$	1 mark: Correct answer.
19	Use z-scores to compare results $z = \dfrac{x - \bar{x}}{s}$ $\quad z = \dfrac{x - \bar{x}}{s}$ $= \dfrac{66 - 82}{8}$ $\quad = \dfrac{61 - 71}{10}$ $= -2$ $\quad = -1$ $\therefore$ Jack has improved as his z-score has increased.	2 Marks: Correct answer. 1 Mark: Finds the z-score or shows some understanding.

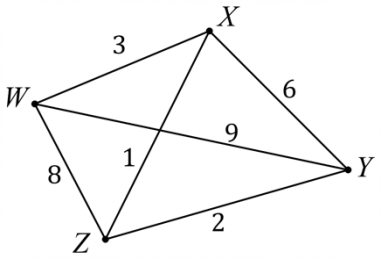
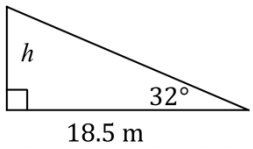
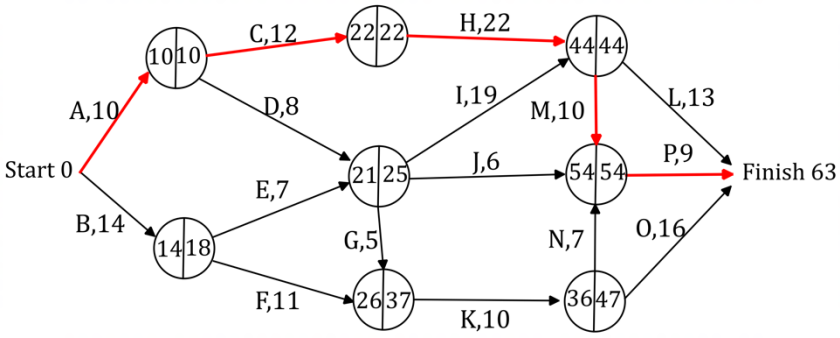
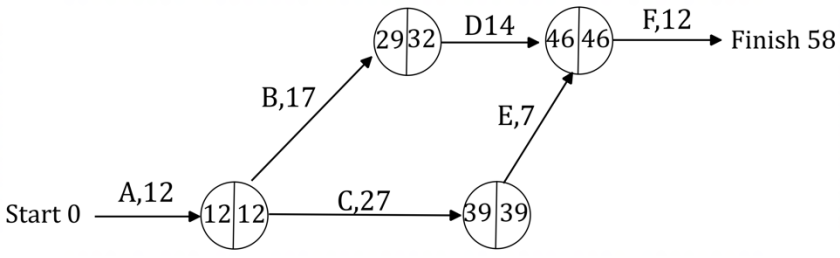
<table>
<tr><td>20(a)</td><td>
<table>
<tr><td>s</td><td>0</td><td>20</td><td>40</td><td>50</td><td>60</td><td>80</td><td>100</td></tr>
<tr><td>p</td><td>33</td><td>17</td><td>9</td><td>8</td><td>9</td><td>17</td><td>33</td></tr>
</table>
</td><td>1 mark: Correct answer.</td></tr>
<tr><td>20(b)</td><td> 
</td><td>1 mark: Correct answer.</td></tr>
<tr><td>20(c)</td><td>When $s = 30$
$p = 0.01s^2 - s + 33$
$= 0.01 \times 30^2 - 30 + 33$
$= 12$ litres per 100 km
Litres of petrol $= 12$L for 100 km
$= 1.2$L for 10 km
$= 4.8$L for 40 km
$\therefore$ The car used 4.8 L of petrol.</td><td>1 mark: Correct answer.</td></tr>
<tr><td>20(d)</td><td>When $s = 0$
$p = 0.01s^2 - s + 33$
$= 0.01 \times 0^2 - 0 + 33$
$= 33$ litres per 100 km
However the car is not moving so no petrol is being used.</td><td>1 mark: Correct answer.</td></tr>
<tr><td>21(a)</td><td>
<table>
<tr><td>Vertex</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr>
<tr><td>Degree</td><td>1</td><td>4</td><td>1</td><td>1</td><td>3</td><td>2</td></tr>
</table>
</td><td>1 mark: Correct answer.</td></tr>
<tr><td>21(b)</td><td>No. Path (Eulerian trail) only exists if the graph has exactly two vertices with an odd degree. There are 4 vertices with odd degree.</td><td>1 mark: Correct answer.</td></tr>
<tr><td>22(a)</td><td>$S = V_0(1 - r)^n$
$= 16000 \times (1 - 0.20)^2$
$= \$10\,240$
$\therefore$ Salvage value of the car is \$10 240</td><td>1 mark: Correct answer.</td></tr>
<tr><td>22(b)</td><td>$S = V_0(1 - r)^n$
$4000 = 16\,000 \times (1 - 0.20)^n$
$\frac{4000}{16\,000} = 0.80^n$
$0.80^n = 0.25$
$n = \frac{\log 0.25}{\log 0.80} = 6.21126 \approx 7$
$\therefore$ Number of years is 7 to be less than \$4 000.
Note: answer can also be obtained by trial and error.</td><td>2 marks: Correct answer.

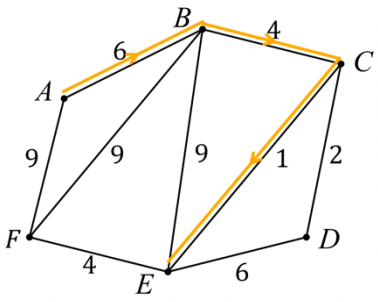
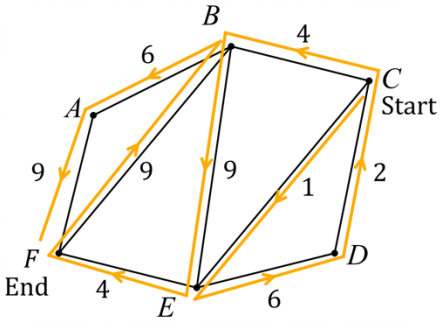
1 mark: Substitutes at least two correct values into depreciation formula.</td></tr>
</table>

23	$c = \frac{k}{n}$ $\quad c = \frac{280}{n}$ $14 = \frac{k}{20}$ $\quad = \frac{280}{15}$ $k = 280$ $\quad = 18.6666\ldots \approx \$18.67$ $\therefore$ Cost per passenger is \$18.67	2 Marks: Correct answer. 1 Mark: Finds the value of k or shows some understanding
24(a)	A z-score of 2.5 is 2.5 standard deviations above the mean.	1 mark: Correct answer.
24(b)	$z = \frac{x - \bar{x}}{s}$ $2.5 = \frac{x - 56}{9.5}$ $23.75 = x - 56$ $x = 79.75$ $\therefore$ Claire scored 79.75 in the class test.	1 mark: Correct answer.
25(a)	Total paid $= 1910 \times 26 \times 7$ $= \$347\ 620$ $\therefore$ Total paid is \$347 620	1 mark: Correct answer.
25(b)	Interest $= 347\ 620 - 220\ 000$ $= \$127\ 620$ $\therefore$ Interest on the loan is \$127 620	1 mark: Correct answer.
25(c)	$I = Prn$ $127\ 620 = 220\ 00 \times r \times 7$ $r = \frac{127\ 620}{220\ 000 \times 7}$ $= 0.08287\ldots$ $\approx 8.3\%$ $\therefore$ Equivalent flat interest rate is 8.3%	1 mark: Correct answer.
26(a)	True bearing is 045˚ (A is NE of O)	1 mark: Correct answer.
26(b)	To find $\angle BOA$ $\angle BOA + 115 + 45 + 45 = 360$ $\angle BOA = 155°$ $\therefore$ True bearing is 200˚ (155˚ + 45˚)	1 mark: Correct answer.
26(c)	$BC^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos 115°$ $BC^2 = 18.0714\ldots$ $= 4.2510\ldots$ $BC \approx 4.25$ m $\therefore$ Distance from B to C is approximately 4.25 metres	2 Marks: Correct answer. 1 Mark: Recognises the use of the cosine rule and makes progress.
26(d)	$A = \frac{1}{2}\ ab\sin C$ $= \frac{1}{2} \times 2 \times 3 \times \sin 115°$ $= 2.7189\ldots \approx 2.72\ \text{m}^2$	1 mark: Correct answer.

27(a)		1 mark: Correct answer.
27(b)	Length $\approx 3.1 \times 100$ ≈ 310 cm ≈ 3.1 m Breadth $\approx 1.5 \times 100$ ≈ 150 cm ≈ 1.5 m 	1 mark: Correct answer.
27(c)	Length $\approx 5.4 \times 100$ ≈ 540 cm ≈ 5.4 m Breadth $\approx 4.6 \times 100$ ≈ 460 cm ≈ 4.6 m $A = lb$ $= 5.4 \times 4.6$ $= 24.84 \approx 25$ m^2 $\therefore$ Area of the extension is 25 m^2	1 mark: Correct answer.
28(a)	Daily interest rate $= \frac{16.3\%}{365}$ $= 0.044657\ldots$ $\approx 0.04466\%$	1 mark: Correct answer.
28(b)	12 days (27,28,29,30,31,1,2,3,4,5,6,7) Interest $= 1029 \times 0.04466\% \times 12$ $= 5.5146\ldots$ $\approx \$5.51$ Total paid $= 1029 + 5.51$ $= \$1034.51$ $\therefore$ Total amount paid for the TV is \$1034.51	1 mark: Correct answer.
29(a)	$m = \frac{\text{Rise}}{\text{Run}}$ $= -\frac{70}{100}$ $= -0.7$ $\therefore$ Gradient is –0.7 	1 mark: Correct answer.
29(b)	y-intercept is 100 $y = mx + b$ $h = -0.7 + 100$	1 mark: Correct answer.
29(c)	Correlation coefficient is about –0.8	1 mark: Correct answer.

30	Maximum flow $= 13 + 7 + 6$ $= 26$ 	3 Marks: Correct answer. 2 Marks: Finds the minimum cut. 1 Mark: Shows understanding.
31(a)	When $t = 0$ then $N = 15$ $\therefore$ Initial number of bacteria is 15 000	1 mark: Correct answer.
31(b)	Using the graph when $N = 45$ then $t \approx 4.9$ (Acceptable range 4.8 to 5.0)	1 mark: Correct answer.
31(c)	Using the graph when $N = 30$ then $t \approx 3.1$ (Acceptable range 3.0 to 3.2)	1 mark: Correct answer.
32	Assessment results increase as head circumference increases. Low positive correlation. Not a strong relationship.	2 Marks: Correct answer. 1 Mark: Shows understanding
33(a)	Almost certainly – 99.7% of the scores. 3 standard deviations above and below the mean. $4.50 - 3 \times 0.03 = 4.41$ cm $4.50 + 3 \times 0.03 = 4.59$ cm $\therefore$ Interval range is from 4.41 cm to 4.59 cm	1 mark: Correct answer.
33(b)	The manager is concerned because 4.62 cm is 4 standard deviations above the mean. This is extremely unlikely to occur and indicates the machine is not working correctly.	1 mark: Correct answer.
34(a)	Intersection value is 3.2464 (8% per year for 3 years) $FV = 3.2464 \times \$16\,000$ $= \$51\,942.40$ $\approx \$51\,942$	1 mark: Correct answer.
34(b)	Intersection value is 5.1010 (1% per month for 5 months) $FV = 5.1010 \times 2100$ $= \$10\,712.10$ $\approx \$10\,712$	1 mark: Correct answer.
35(a)	To find the value of k substitute a value from the table. $L = km + 32$ $41.2 = 2k + 32$ $2k = 9.2$ $k = 4.6$	1 mark: Correct answer.
35(b)	When no item is attached then $m = 0$ $L = 4.6m + 32$ $= 4.6 \times 0 + 32$ $= 32$ $\therefore$ The length of the spring is 32 cm.	1 mark: Correct answer.
35(c)	To find m when $L = 78$ $L = 4.6m + 32$ $78 = 4.6m + 32$ $4.6m = 46$ $m = 10$ kg $\therefore$ Mass of 10 kg makes the spring 78 cm long.	1 mark: Correct answer.

36	Weighted edge: $WX = 3$, $WY = 9$, $WZ = 8$, $XY = 6$, $XZ = 1$, $YZ = 2$ 	2 marks: Correct answer. 1 mark: Draws the vertices with at least one correct edge.
37	$\tan 32° = \frac{h}{18.5}$ $h = 18.5 \times \tan 32°$ $= 11.5600\ldots$ ≈ 11.6 m $\therefore$ Difference in height is 11.6 m 	2 marks: Correct answer. 1 mark: Shows some understanding.
38(a)		3 Marks: Correct answer. 2 Marks: Finds the EST or LST. 1 Mark: Shows some understanding.
38(b)	Critical path is A–C–H–M–P	1 mark: Correct answer.
39(a)	Students with a z-score of –2 is two standard deviations below the mean $(70 - (2 \times 10) = 50$. $\therefore$ Weight of the student is 50 kg.	1 mark: Correct answer.
39(b)	68% of scores have a z-score between -1 and 1 (or from 60 to 80) Region A $= \frac{68\%}{2} = 34\%$	1 mark: Correct answer.
39(c)	$z = \frac{x - \bar{x}}{s} = \frac{100 - 70}{10} = 3$ Percentage of scores less than a z-score of 3 is 99.85% Number of students $= 99.85\% \times 400$ $= 399.4 = 399$ $\therefore$ There are 399 students with a mass less than 105 kg.	2 Marks: Correct answer. 1 Mark: Finds the z-score or shows some understanding.
40		3 Marks: Correct answer. 2 Marks: Completes the EST or LST 1 Mark: Draws a network diagram with some correct edges .

41(a)	Intersection of the two linear graphs (15, 400) ∴ Businesses charge the same amount for 15 people.	1 mark: Correct answer.
41(b)	Business A: when $n = 10$ then $c \approx 300$ Business B: when $n = 10$ then $c \approx 340$ ∴ Recommend business A as it is \$40 cheaper.	1 mark: Correct answer.
41(c)	Business A: when $n = 25$ then $c \approx 600$ Cost per person $= 600 \div 25 = \$24.00$ Business B: when $n = 25$ then $c \approx 520$ Cost per person $= 520 \div 25 = \$20.80$ Difference $= 24 - 20.8$ $= \$3.20$ ∴ There is a difference of \$3.20 per person.	2 Marks: Correct answer. 1 Mark: Finds the cost per person for one of the businesses.
42	$P = 0.05n + 4.5$ $= 0.05 \times 100\,000\,000 + 4.5$ $= \$5\,000\,450$ This is not an accurate prediction. It is extremely unlikely that a worker can produce 100 million units to make a profit of \$5 000 045.	2 Marks: Correct answer. 1 Mark: Finds the profit.
43	Total parts $= 7 + 6 + 12 = 25$ 25 parts $= 100$ 1 part $= 4$ 7 parts $= 28$ g/m^2 6 parts $= 24$ g/m^2 12 parts $= 48$ g/m^2 ∴ Nitrogen is 28 g, phosphorus is 24 g and potassium is 48 g	2 Marks: Correct answer. 1 Mark: Makes some progress.
44(a)	Identifying the alternative routes from A to E. Shortest path is A–B–C–E Length $= 6 + 4 + 1$ $= 11$		2 Marks: Correct answer. 1 Mark: Finds the shortest path or shows some understanding.
44(b)	There are two vertices with an odd degree. deg(C) = 3 and deg(F) = 3. Walk C–E–D–C–B–E–F–B–A–F Note: other answers are possible.		2 Marks: Correct answer. 1 Mark: Shows some understanding.