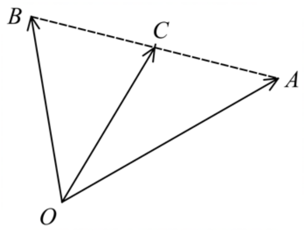
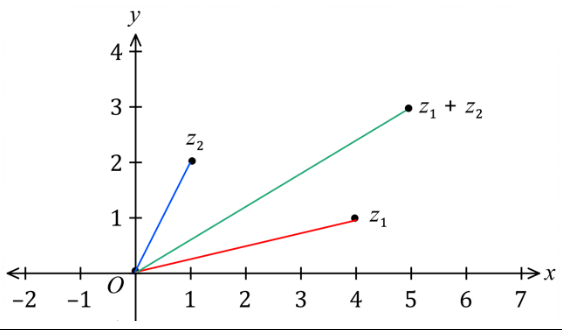
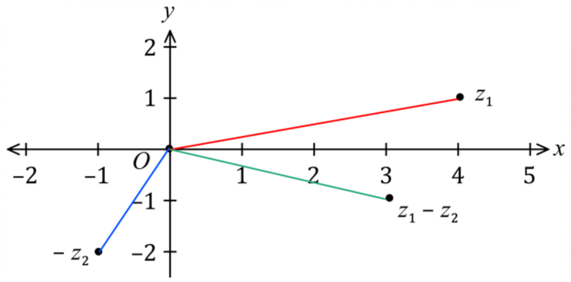
ACE Examination Paper 1
Year 12 Mathematics Extension 2 Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$z=-\sqrt{2}+\sqrt{2}i$ $=2\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)$ $=2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$	1 Mark: D
2	$\int 3\sqrt{x}\ln x\,dx\,3=3\int \ln x\,\frac{d}{dx}\left(\frac{2x^{\frac{3}{2}}}{3}\right)\mathrm{dx}$ $=3\left(\frac{2x^{\frac{3}{2}}}{3}\ln x-\frac{2}{3}\int x^{\frac{3}{2}}\times\frac{1}{x}\,dx\right)$ $=2x^{\frac{3}{2}}\ln x-2\times\frac{2\mathrm{x}^{\frac{3}{2}}}{3}+\mathrm{C}$ $=2x\sqrt{\mathrm{x}}\left(\ln x-\frac{2}{3}\right)+\mathrm{C}$	1 Mark: A
3	$\lvert\underset{\sim}{u}\rvert=\sqrt{5^2+(-1)^2+\sqrt{10}^2}$ $=\sqrt{36}=6$ $\lvert\underset{\sim}{u}\rvert^2=36$	1 Mark: D
4	$\frac{3}{iw}=\frac{3}{-i+i^2}$ $=\frac{3}{-i+i^2}\times\frac{-1+i}{-1+i}=\frac{-3+3i}{1+1}$ $=\frac{-3+3i}{2}$	1 Mark: C
5	$I_6=\int\tan^6 x dx$ $=\frac{\tan^5 x}{5}-I_4$ $=\frac{\tan^5 x}{5}-\left(\frac{\tan^3 x}{3}-I_2\right)$ $=\frac{\tan^5 x}{5}-\frac{\tan^3 x}{3}+\tan x-I_0$ $=\frac{\tan^5 x}{5}-\frac{\tan^3 x}{3}+\tan x-x+C$	1 Mark: A
6	$v^2=36-4x^2$ $=2^2(9-x^2)=n^2(a^2-x^2)$ $a^2=9$ or $a=3, n=2$ and $\alpha=0$ (initially at the origin) $x=a\sin(nt+\alpha)$ $=3\sin(2t)$	1 Mark: B

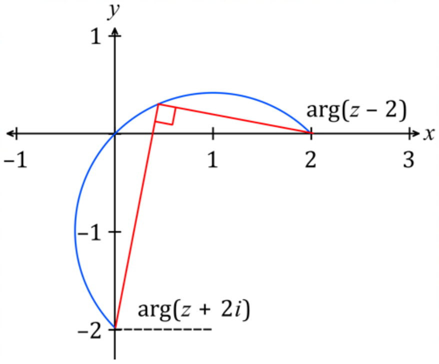
7	$t=\tan\frac{x}{2}$ $dt=\frac{1}{2}\sec^2\frac{x}{2}dx$ or $dx=\frac{2}{1+t^2}dt$ When $x=\frac{\pi}{3}$ then $t=\frac{1}{\sqrt{3}}$ and when $x=\frac{2\pi}{3}$ then $t=\sqrt{3}$ $\sin x+1=\frac{2t+1+t^2}{1+t^2}$ $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{1}{\sin x+1}dx=\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}\frac{1+t^2}{2t+1+t^2}\times\frac{2}{1+t^2}dt$ $=\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}\frac{2}{(1+t)^2}dt=-2\left[\frac{1}{1+t}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$ $=-2\left(\frac{1}{1+\sqrt{3}}-\frac{\sqrt{3}}{\sqrt{3}+1}\right)=-2\left(\frac{(1-\sqrt{3})^2}{1-3}\right)$ $=4-2\sqrt{3}$	1 Mark: B
8	$(\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}).(3\underset{\sim}{i}-x\underset{\sim}{j}+2\underset{\sim}{k})=4$ $3-x-2=4$ $1-x=4$ $\therefore x=-3$	1 Mark: A
9	ab, cd and $ab-cd$ are real ac, bd and $ac-bd$ are imaginary $(ab-cd)^2\geq 0$ $a^2b^2+c^2d^2\geq 2abcd$ $(ac-bd)^2$ is a negative real number $\therefore (ac-bd)^2\leq 0$ $\therefore a^2c^2+b^2d^2\leq 2abcd$	1 Mark: B
10	$a=3x^2$ $v^2=2\int 3x^2dx=2x^3+C$ When $x=1$, $v=-\sqrt{2}$ then $C=0$ $v=-\sqrt{2x^3}\quad (v<0$ when $x=1)$ $\frac{dx}{dt}=-\sqrt{2x^3}$ $\frac{dt}{dx}=-\frac{1}{\sqrt{2}}x^{-\frac{3}{2}}$ $t=\frac{2}{\sqrt{2}}x^{-\frac{1}{2}}+C$ Initially $t=0$ and $x=1$ then $C=-\sqrt{2}$ $t=\sqrt{2}x^{-\frac{1}{2}}-\sqrt{2}$ $x^{-\frac{1}{2}}=\frac{t+\sqrt{2}}{\sqrt{2}}$ $x=\frac{2}{(t+\sqrt{2})^2}$	1 Mark: C

Section II

	Solution	**Criteria**
11(a) (i)	$\lvert\overrightarrow{OA}\rvert = \lvert\overrightarrow{OB}\rvert$ $\sqrt{3^2 + 2^2 + (\sqrt{3})^2} = \sqrt{\alpha^2}$ $\therefore \alpha = 4$	1 Mark: Correct answer.
11(a) (ii)	$\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$ $\overrightarrow{OC} + \overrightarrow{CA} = \overrightarrow{OA}$ Now $\overrightarrow{BC} = \overrightarrow{CA}$ $\therefore \overrightarrow{OC} - \overrightarrow{OB} = \overrightarrow{OA} - \overrightarrow{OC}$ $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$ $= \frac{1}{2}(3\underset{\sim}{i} + 2\underset{\sim}{j} + \sqrt{3}\underset{\sim}{k} + 4\underset{\sim}{i})$ $= \frac{1}{2}(7\underset{\sim}{i} + 2\underset{\sim}{j} + \sqrt{3}\underset{\sim}{k})$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
11(a) (iii)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= 4\underset{\sim}{i} - (3\underset{\sim}{i} + 2\underset{\sim}{j} + \sqrt{3}\underset{\sim}{k})$ $= \underset{\sim}{i} - 2\underset{\sim}{j} - \sqrt{3}\underset{\sim}{k}$ $\overrightarrow{OC} = \frac{1}{2}(7\underset{\sim}{i} + 2\underset{\sim}{j} + \sqrt{3}\underset{\sim}{k})$ $\overrightarrow{OC}.\overrightarrow{AB} = \frac{1}{2}(7 - 4 - 3) = 0$ (Two vectors are perpendicular if and only if $\underset{\sim}{u}.\underset{\sim}{v} = 0$) $\therefore$ $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$	3 Marks: Correct answer. 2 Marks: Applies the statement for perpendicular vectors. 1 Mark: Finds $\overrightarrow{AB}$.
11(b)	Let $z = x + iy$ and given $\lvert z\rvert = 1$ then $x^2 + y^2 = 1$ LHS $= z^{-1}$ $= (x + iy)^{-1}$ $= \frac{1}{x + iy} \times \frac{x - iy}{x - iy}$ $= \frac{x - iy}{x^2 + y^2}$ $= \frac{x - iy}{1}$ $= \bar{z}$ $=$ RHS	2 Marks: Correct answer. 1 Mark: Uses $x^2 + y^2 = 1$ or shows some understanding.
11(c) (i)		1 Mark: Correct answer.

11(c) (ii)		1 Mark: Correct answer.
11(d)	$\int \frac{1}{\sqrt{12+4x-x^2}}dx = \int \frac{1}{\sqrt{12-(x^2-4x)}}dx$ $= \int \frac{1}{\sqrt{16-(x^2-4x+4)}}dx$ $= \int \frac{1}{\sqrt{16-(x-2)^2}}dx$ $= \sin^{-1}\left(\frac{x-2}{4}\right) + C$	2 Marks: Correct answer. 1 Mark: Completes the square.
11(e)	$\int \frac{x^2}{x^2+1}dx = \int \frac{x^2+1-1}{x^2+1}dx$ $= \int \left(1 - \frac{1}{x^2+1}\right)dx$ $= x - \tan^{-1}x + C$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
12(a)	De Moivre's theorem. $\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$ $(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5\cos^4\theta\sin\theta i + 10\cos^3\theta(\sin\theta i)^2$ $+10\cos^2\theta(\sin\theta i)^3 + 5\cos\theta(\sin\theta i)^4 + (\sin\theta i)^5$ $= \cos^5\theta - 10\cos^3\theta\sin^3\theta + 5\cos\theta\sin^4\theta$ $+i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$ Equating the real components $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta$ $+5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$ $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	3 Marks: Correct answer. 2 Marks: Expands the binomial. 1 Mark: Applies De Moivre's theorem.
12(b) (i)	$\frac{8-2x}{(1+x)(4+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{4+x^2}$ Using partial fractions to find A, B and C $A(4+x^2) + (Bx+C)(1+x) = 8-2x$ $(A+B)x^2 + (B+C)x + (4A+C) = 8-2x$ $A + B = 0$ (1) $B + C = -2$ (2) $4A + C = 8$ (3) Equation (1) − (2) $A - C = 2$ (4) Equation (3) + (4) $5A = 10$ $A = 2$ $\therefore A = 2, B = -2$ and $C = 0$	2 Marks: Correct answer. 1 Mark: Makes progress in finding A, B or C.

12(b) (ii)	$\int_0^4 \frac{8-2x}{(1+x)(4+x^2)}dx = \int_0^4 \frac{2}{1+x} + \frac{-2x}{4+x^2}dx$ $= [2\ln(1+x) - \ln(4+x^2)]_0^4$ $= (2\ln 5 - \ln 20) - (2\ln 1 - \ln 4)$ $= 2\ln 5 - (\ln 4 + \ln 5) - (2\ln 1 - \ln 4)$ $= \ln 5$	2 Marks: Correct answer. 1 Mark: Correctly finds one of the integrals.
12(c) (i)	Simple harmonic motion occurs when $\ddot{x} = -n^2(x-b)$ $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left(\frac{1}{2} \times (x^2+2x+8)\right)$ $= \frac{1}{2} \times (-2x+2)$ $= -x+1$ $= -1(x-1)$ $\therefore$ SHM about the position $x = -1$ ($n = 1$ and $b = 1$)	2 Marks: Correct answer. 1 Mark:R the formula for SHM.
12(c) (ii)	Centre of motion $x = 1$ Period $= \frac{2\pi}{n} = \frac{2\pi}{1} = 2\pi$ To find the amplitude $v^2 = -x^2 + 2x + 8$ $= 1^2(8 + 2x - x^2)$ $= 1^2(9 - (x-1)^2)$ $= n^2(a^2 - x^2)$ $\therefore$ Amplitude is 3 metres Alternatively Amplitude occurs at the extremes when $v = 0$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ Displacement occurs between $x = 4$ and $x = -2$ (distance of 6) Hence the amplitude is 3 metres $\therefore$ Centre of motion $x = 1$, period 2π and amplitude is 3 metres.	3 Marks: Correct answer. 2 Marks: Finds two correct answers. 1 Mark: Finds one correct answer.
12(c) (iii)	Now $a = 3$, $n = 1$ and $b = 1$ (centre of motion) $x = a\cos(nt + \alpha) + 1$ $= 3\cos(t + \alpha) + 1$ Initially $t = 0$ and $x = 2.5$ $2.5 = 3\cos(0 + \alpha) + 1$ $\cos\alpha = \frac{1.5}{3}$ $\alpha = \frac{\pi}{3}$ $x = 3\cos\left(t + \frac{\pi}{3}\right) + 1$ $\therefore b = 1$ and $\alpha = \frac{\pi}{3}$	1 Mark: Correct answer.

12(d)	$\arg\left(\frac{z-2}{z+2i}\right) = \arg(z-2) - \arg(z+2i) = \frac{\pi}{2}$ Angle in a semicircle. 	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.
13(a)	$P(x) = x^4 + x^2 + 6x + 4 = 0$ $P'(x) = 4x^3 + 2x + 6$ $\quad = 2(2x^3 + x + 3)$ To determine the roots of $2x^3 + x + 3$ $P'(-1) = 2(2 \times (-1)^3 + (-1) + 3)$ $\quad = 0$ Therefore –1 is a zero of multiplicity 2 of $P(x)$ $P(x) = x^4 + x^2 + 6x + 4$ $\quad = (x+1)^2(x^2 + bx + c)$ $\quad = (x^2 + 2x + 1)(x^2 + bx + c)$ $\quad = x^4 + bx^3 + cx^2 + 2x^3 + 2bx^2 + 2cx + x^2 + bx + c$ $\quad = x^4 + (b+2)x^3 + (c + 2b + 1)x^2 + (b + 2c)x + 4$ Hence $c = 4, b = -2$ $P(x) = x^4 + x^2 + 6x + 4$ $\quad = (x+1)^2(x^2 - 2x + 4)$ $\quad = (x+1)^2((x-1)^1 - 1 + 4)$ $\quad = (x+1)^2((x-1)^2 + 3)$ $\quad = (x+1)^2(x - 1 + \sqrt{3}i)(x - 1 - \sqrt{3}i)$ $\therefore$ Zeros of $P(x)$ are $-1, 1 + \sqrt{3}i$ and $1 - \sqrt{3}i$	4 Marks: Correct answer. 3 Marks: Factorises the polynomial. 2 Marks: Recognises $(x+1)^2$ as a factor of the polynomial. 1 Mark: Calculates the derivative and finds its zeros.
13(b)	$\frac{dx}{dt} = x$ $\frac{dt}{dx} = \frac{1}{x}$ $t = \ln x + C$ Given $x = 1$ when $t = 3$ $3 = \ln 1 + C$ $C = 3$ $t = \ln x + 3$ $\ln x = t - 3$ $x = e^{t-3}$	2 Marks: Correct answer. 1 Mark: Finds t in terms of x.

13(c)	Step 1: To prove true for $n = 1$ $\text{LHS} = 1 + x$ $\text{RHS} = \dfrac{x^{1+1} - 1}{x - 1} = \dfrac{(x+1)(x-1)}{(x-1)} = 1 + x$ Result is true for $n = 1$ Step 2: Assume true for $n = k$ $S_k = \dfrac{x^{k+1} - 1}{x - 1}$ Step 3: To prove true for $n = k + 1$ $S_{k+1} = \dfrac{x^{k+2} - 1}{x - 1}$ $S_k + T_{k+1} = S_{k+1}$ $\text{LHS} = \dfrac{x^{k+1} - 1}{x - 1} + x^{k+1}$ $= \dfrac{x^{k+1} - 1}{x - 1} + \dfrac{x^{k+1}(x-1)}{(x-1)}$ $= \dfrac{x^{k+1} - 1 + x^{k+2} - x^{k+1}}{x - 1}$ $= \dfrac{x^{k+2} - 1}{x - 1}$ $= \text{RHS}$ Step 4: True by induction	3 marks: Correct answer. 2 marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$ 1 mark: Proves the result true for $n = 1$.
13(d)(i)	$\underset{\sim}{k}$ component $2 + 4\lambda = -2$ $\lambda = -1$ $\underset{\sim}{i}$ component $3 - 1 = \mu$ $\mu = 2$ Substitute –1 for λ into the equation of l_1 $3\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k} + (-1)(\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}) = 2\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ $\therefore A\text{: } 2\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$	2 Marks: Correct answer. 1 Mark: Finds either λ or μ.
13(d)(ii)	Line l_1 : $(\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k})$ $\lvert\underset{\sim}{u}\rvert = \sqrt{1^2 + (-1)^2 + 4^2} = \sqrt{18}$ Line l_2 : $(\underset{\sim}{v} = \underset{\sim}{i} - \underset{\sim}{j})$ $\lvert\underset{\sim}{v}\rvert = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ $\underset{\sim}{u} \cdot \underset{\sim}{v} = (1 \times 1) + (-1 \times -1) + (4 \times 0) = 2$ $\cos\theta = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert} = \dfrac{2}{\sqrt{18} \times \sqrt{2}}$ $= \dfrac{1}{3}$	3 Marks: Correct answer. 2 Marks: Uses the angle between two vectors. 1 Mark: Shows some understanding.

<table>
<tr>
<td>14(a)</td>
<td>Inequality is only defined for $x(1-x) \geq 0$
(cannot find the square root of a negative number)
$\therefore 0 \leq x \leq 1$ ①
Using the result $|x| = \sqrt{x^2}$ or $|4x-1| = \sqrt{(4x-1)^2}$
$\sqrt{(4x-1)^2} > 2\sqrt{x(1-x)}$
$(4x-1)^2 > 4x(1-x)$
$16x^2 - 8x + 1 > 4x - 4x^2$
$20x^2 - 12x + 1 > 0$
$(10x-1)(2x-1) > 0$
$\therefore x < \frac{1}{10}$ or $x > \frac{1}{2}$ ②
Combining results ① and ②
$\therefore 0 \leq x < \frac{1}{10}$ or $\frac{1}{2} < x \leq 1$</td>
<td>3 Marks: Correct answer.

2 Marks: Finds one correct region or makes significant progress.

1 Mark: Finds $0 \leq x \leq 1$ or uses $|x| = \sqrt{x^2}$ or shows some understanding.</td>
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<tr>
<td>14(b)</td>
<td>Let $z = x + iy$ and $\bar{z} = x - iy$
$z^2 = i\bar{z}$
$x^2 - y^2 + 2xyi = i(x - iy)$
$x^2 - y^2 + 2xyi = y + ix$
Equating the real and imaginary parts.
$x^2 - y^2 = y$ ①
$2xy = x$ ②
Rearranging equation ②
$x(2y-1) = 0$
$\therefore x = 0$ and $y = \frac{1}{2}$
Substitute $x = 0$ into equation ①
$-y^2 = y$
$y(y+1) = 0$
$\therefore y = 0$ and $y = -1$
Substitute $y = \frac{1}{2}$ into equation ①
$x^2 - \frac{1}{4} = \frac{1}{2}$
$x^2 = \frac{3}{4}$
$x = \pm\frac{\sqrt{3}}{2}$
$\therefore$ Solution is $(0,0), (0,-1), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$</td>
<td>3 Marks: Correct answer.

2 Marks: Finds one possible solution to the equation.

1 Mark: Correctly expresses the equation in terms of x and y.</td>
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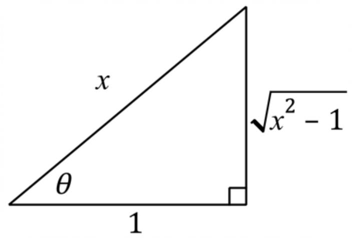
14(c) (i)	$\text{LHS} = \dfrac{n!}{r!\,(n-r)!} + \dfrac{n!}{(r-1)!\,(n-r+1)!}$ $= \dfrac{n!\,(n-r+1) + n!\,r}{r!\,(n-r+1)!}$ $= \dfrac{n!\,(n-r+1+r)}{r!\,(n-r+1)!}$ $= \dfrac{(n+1)n!}{r!\,(n-r+1)!}$ $= \dfrac{(n+1)!}{r!\,(n-r+1)!}$ $= \text{RHS}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Makes some progress.
14(c) (ii)	$\text{RHS} = {}^nC_r$ $= \dfrac{n!}{[n-(n-k)]!\,(n-k)!}$ $= \dfrac{n!}{(k!\,(n-k)!)}$ $= {}^nC_{n-k}$ $= \text{LHS}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Makes some progress.
14(d) (i)	$I_n = \displaystyle\int_1^e (\ln x)^n dx$ $= [x(\ln x)^n]_1^e - \displaystyle\int_1^e x \times n(\ln x)^{n-1} \times \frac{1}{x} dx$ $= e(\ln e)^n - 1(\ln 1)^n - n\displaystyle\int_1^e (\ln x)^{n-1} dx$ $I_n = e - nI_{n-1}$ for $n = 1, 2, 3, \ldots$	2 Marks: Correct answer. 1 Mark: Sets up the integration by parts.
14(d) (ii)	Now $I_0 = \displaystyle\int_1^e (\ln x)^0 dx = [x]_1^e = e - 1$ $I_1 = e - 1 \times I_0 = e - (e-1) = 1$ $I_2 = e - 2 \times I_1 = e - 2 \times 1 = e - 2$ $\therefore I_3 = e - 3 \times I_2 = e - 3(e-2)$ $= -2e + 6$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
15(a) (i)	All the coefficients of $P(z)$ are real. Then any complex roots occur in conjugate pairs. Since $(2+i)$ is a root then $(2-i)$ is a root.	1 Mark: Correct answer.
15(a) (ii)	Roots are $(2+i)$, $(2-i)$ and α $(2+i)(2-i)\alpha = -\dfrac{d}{a} = -\dfrac{20}{1}$ $(4-i^2)\alpha = -20$ $5\alpha = -20$ $\alpha = -4$ $P(z) = \big(z-(-4)\big)[z-(2+i)][z-(2-i)]$ $= (z+4)(z^2-4z+5)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

15(b) (i)	When $x = 1$ $\ddot{x} = \frac{5}{1^3} - \frac{2}{1^2} = 3$ Particle starts from rest at $x = 1$ with a positive acceleration. Hence it is moving in the positive x direction.	1 Mark: Correct answer.
15(b) (ii)	$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{5}{x^3} - \frac{2}{x^2}$ $\frac{1}{2}v^2 = \int 5x^{-3} - 2x^{-2}dx$ $= -\frac{5}{2}x^{-2} + 2x^{-1} + C$ When $x = 1$, $v = 0$ $0 = -\frac{5}{2} \times 1^{-2} + 2 \times 1^{-1} + C$ $C = \frac{1}{2}$ $\frac{1}{2}v^2 = -\frac{5}{2}x^{-2} + 2x^{-1} + \frac{1}{2}$ $v^2 = \frac{-5}{x^2} + \frac{4}{x} + 1$ $= \frac{x^2 + 4x - 5}{x^2}$ Now $x^2 + 4x - 5 = (x + 5)(x - 1)$ $\therefore v = \frac{\sqrt{x^2 + 4x - 5}}{x}$ for $x \geq 1$	3 Marks: Correct answer. 2 Marks: Finds the correct expression for v^2 1 Mark: Integrates to find $\frac{1}{2}v^2$.
15(b) (iii)	For $x = 3$ $\ddot{x} = \frac{5}{3^3} - \frac{2}{3^2} = -\frac{1}{27}$ The acceleration is negative and the particle is slowing. For $x > 2.5$ $v = \lim\limits_{x \to \infty} \frac{\sqrt{x^2 + 4x - 5}}{x}$ $= \lim\limits_{x \to \infty} \sqrt{1 + \frac{4}{x} - \frac{5}{x^2}}$ $\to 1$ $\therefore$ The velocity decreases and approaches 1.	2 Marks: Correct answer. 1 Mark: Determines that the particle is slowing or approaches 1.
15(c) (i)	$\underset{\sim}{c} = \underset{\sim}{a} + \underset{\sim}{b}$ $= (2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}) + (\underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k})$ $= 3\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{k}$	1 Mark: Correct answer.

15(c) (ii)	*OACB* is a rectangle if opposite sides are equal (parallelogram) and one angle is a right angle. $\overrightarrow{OA} = 2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ $\lvert OA \rvert = \sqrt{2^2 + 2^2 + 1^2} = 3$ $\overrightarrow{OB} = \underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k}$ $\lvert\overrightarrow{OB}\rvert = \sqrt{1^2 + 1^2 + (-4)^2} = \sqrt{18} = 3\sqrt{2}$ $\overrightarrow{BC} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ $\lvert\overrightarrow{BC}\rvert = \sqrt{2^2 + 2^2 + (-1)^2} = 3$ $\overrightarrow{AC} = \underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k}$ $\lvert\overrightarrow{AC}\rvert = \sqrt{1^2 + 1^2 + (-4)^2} = \sqrt{18} = 3\sqrt{2}$ $\therefore$ Opposite sides of *OACB* are equal. $\overrightarrow{OA} \cdot \overrightarrow{OB} = (2 \times 1) + (2 \times 1) + \big(1 \times (-4)\big)$ $\quad = 0$ $\therefore$ $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular. $\therefore$ *OACB* is a rectangle.	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Shows that the opposite sides of *OACB* are equal. 1 Mark: Finds the distance of one side of *OACB*.
15(c) (iii)	$A = lb$ $\quad = 3\sqrt{2} \times 3$ $\quad = 9\sqrt{2}$ square units	1 Mark: Correct answer.
16(a) (i)	Resolving forces (m =40 kg, g = 10 ms^{-2}) $ma = -mg - 0.1v^2$ $40a = -400 - 0.1v^2$ $400a = -4000 - v^2$ $a = -\frac{1}{400}(v^2 + 4000)$ $\frac{dv}{dt} = -\frac{1}{400}(v^2 + 4000)$ $\frac{dt}{dv} = \frac{-400}{v^2 + 4000}$ $t = \int \frac{-400}{4000 + v^2}\,dv$ The particle has an initial speed of u ms^{-1} and reaches maximum height when $v = 0$. $t = \int_u^0 \frac{-400}{4000 + v^2}\,dv$ $= 400\int_0^u \frac{1}{4000 + v^2}\,dv$ $= 400\left[\frac{1}{20\sqrt{10}}\tan^{-1}\frac{v}{20\sqrt{10}}\right]_0^u$ $= 2\sqrt{10}\left[\tan^{-1}\frac{v}{20\sqrt{10}}\right]_0^u$ $= 2\sqrt{10}\left[\tan^{-1}\frac{u}{20\sqrt{10}} - \tan^{-1}\frac{0}{20\sqrt{10}}\right]$ $= 2\sqrt{10}\tan^{-1}\frac{u}{20\sqrt{10}}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Resolves the forces.

16(a) (ii)	$v\dfrac{dv}{dx} = -\dfrac{1}{400}(v^2+4000)$ $\dfrac{dv}{dx} = -\dfrac{v^2+4000}{400v}$ $\dfrac{dx}{dv} = \dfrac{-400v}{v^2+4000}$ $x = \displaystyle\int \dfrac{-400v}{v^2+4000}\,dv$ Maximum height reached when travelling from $v = u$ to $v = 0$. Max height $= \displaystyle\int_u^0 \dfrac{-400v}{v^2+4000}\,dv$ $= \displaystyle\int_0^u \dfrac{400v}{v^2+4000}\,dv$ $= 200\displaystyle\int_0^u \dfrac{2v}{v^2+4000}\,dv$ $= 200[\ln(v^2+4000)]_0^u$ $= 200[\ln(u^2+4000) - \ln(4000]$ $= 200\ln\left(\dfrac{u^2+4000}{400}\right)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Uses $a = v\dfrac{dv}{dx}$
16(b) (i)	$(1-3i)^2 = 1-6i+9i^2$ $= 1-6i-9$ $= -8-6i$	1 Mark: Correct answer.
16(b) (ii)	Quadratic formula $z = \dfrac{8 \pm \sqrt{64 - 4\times 2\times(12+3i)}}{4}$ $= \dfrac{8 \pm \sqrt{64-96-24i}}{4}$ $= \dfrac{8 \pm \sqrt{-32-24i}}{4}$ $= \dfrac{8 \pm \sqrt{4(-8-6i)}}{4}$ $= \dfrac{4 \pm (1-3i)}{2}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

16(c)	Let $f(x) = 1 + x + \frac{x^2e^x}{2} > e^x$ $f'(x) = 1 + \frac{1}{2}(x^2e^x + e^x2x) - e^x$ $= 1 + xe^x + \frac{x^2e^x}{2} - e^x$ $f'(0) = 0$ $f''(x) = xe^x + e^x + \frac{1}{2}(x^2e^x + e^x2x) - e^x$ $= 2xe^x + \frac{x^2e^x}{2} > 0$ for $x > 0$ Therefore $f'(x) > 0$ (increasing) for $x > 0$ and $f(0) = 0$ $\therefore 1 + x + \frac{x^2e^x}{2} - e^x > 0$ $\therefore 1 + x + \frac{x^2e^x}{2} > e^x$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Sets up $f(x)$ and uses calculus.
16(d)	$x = \sec\theta$ $\frac{dx}{d\theta} = tan\theta sec\theta$ Also $\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1}$ $= \sqrt{\tan^2\theta} = \tan\theta$ $\int \frac{\sqrt{x^2-1}}{x^2}dx = \int \frac{\tan\theta \tan\theta \sec\theta \, d\theta}{\sec^2\theta}$ $= \int \frac{\tan^2\theta \, d\theta}{\sec\theta}$ $= \int \left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos\theta}{1}\right) d\theta = \int \frac{\sin^2\theta}{\cos\theta} d\theta$ $= \int \frac{1 - \cos^2\theta}{\cos\theta} d\theta$ $= \int (\sec\theta - \cos\theta) d\theta$ $= \ln(\sec\theta + \tan\theta) - \sin\theta + C$  $\therefore \int \frac{\sqrt{x^2-1}}{x^2}dx = \ln\left(x + \sqrt{x^2-1}\right) - \frac{\sqrt{x^2-1}}{x} + C$	4 Marks: Correct answer. 3 Marks: Finds the integral in terms of θ. 2 Marks: Simplifies the integral using the substitution. 1 Mark: Sets up an appropriate substitution.