Q1

(a)(i) Add 4 next term = 17

(ii) $T_n = a + (n-1)d$

$2013 = 5 + 4(n-1)$

$2013 = 5 + 4n - 4$

$4n = 2012$

$n = 503$

$\therefore$ 2013 is a term in the sequence.

(b) $n=7$ $r = \frac{15}{5} = 3$ $a = 5$

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{5(3^7 - 1)}{3-1}$

$= 5465.$

(c) $a = 90$ $r = \frac{30}{90} = \frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$= \frac{90}{1-\frac{1}{3}}$

$= 135$

(d) $a = x$ $r = \frac{-\frac{x}{4}}{x} = -\frac{1}{4}$

$\therefore \frac{a}{1-r} = S_\infty$

$\frac{x}{1-(-\frac{1}{4})} = \frac{2}{5}$

$\frac{x}{\frac{5}{4}} = \frac{2}{5}$

$x = \frac{2}{5} \times \frac{5}{4}$

$= \frac{1}{2}$

(e) $a = 24$ $r = \frac{12}{24} = \frac{1}{2}.$

$ar^{n-1} = T_n$

$24(\frac{1}{2})^{n-1} = \frac{3}{16}$

$(\frac{1}{2})^{n-1} = \frac{\frac{3}{16}}{24}$

$(\frac{1}{2})^{n-1} = \frac{1}{128}$

$(\frac{1}{2})^{n-1} = (\frac{1}{2})^7$

$\therefore n-1 = 7$

$n = 8$

$\Rightarrow \frac{3}{8}$ is the eighth term.

Q2.

(a)(i) $y = 4 + 3x - x^3$

$\frac{dy}{dx} = 3 - 3x^2$

$\frac{d^2y}{dx^2} = -6x$

(ii) At St pts $\frac{dy}{dx} = 0$

$3 - 3x^2 = 0$

$3(1-x^2) = 0$

$3(1-x)(1+x) = 0$

$\therefore x = 1$ or -1

When $x = 1$ $y = 4 + 3(1) - (1)^3$

$= 6$

$x = -1$ $y = 4 + 3(-1) - (-1)^3$

$= 2$

$\therefore$ St pts $(1,6)$ and $(-1,2)$

At $(1,6)$ $\frac{d^2y}{dx^2} = -6(1)$

$= -6$

$< 0 \Rightarrow (1,6)$ is a max tp.

At $(-1,2)$ $\frac{d^2y}{dx^2} = -6(-1)$

$= 6$

> 0

$\Rightarrow (-1,2)$ is a min tp

(iii) Put $\frac{d^2y}{dx^2} = 0$

$-6x = 0$

$x = 0$

$x = 0$ $y = 4$.

$(0,4)$ may be a POI.

Check change in concavity

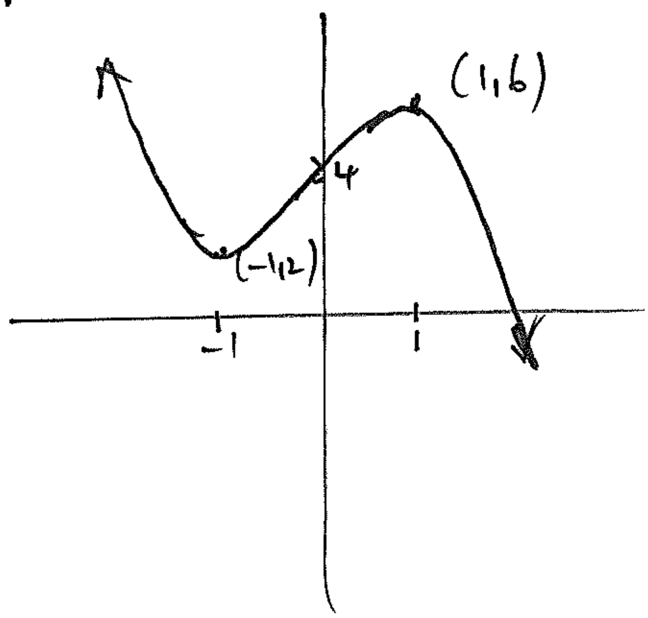
x	0^-	0	0^+
y''	$+$	0	$-$

there is a change in concavity $\Rightarrow (0,4)$ is a POI.

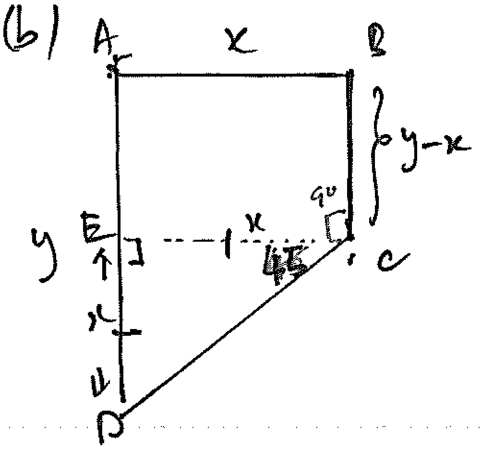
(iv) Put $x = 0$

$y = 4$. $(0,4)$ y intercept

✓



Q 2 (b)



$\angle ECD = 135 - 90 = 45°$

$\Rightarrow \triangle CED$ is an isos right angled triangle.

$\Rightarrow ED = x$

$\Rightarrow BC = y - x$

$\therefore$ Area $= \frac{1}{2} \times x \times [y + y - x]$

$= \frac{1}{2}x[2y - x]$

$= \frac{1}{2} \times 2xy - \frac{1}{2}x^2$

$A = xy - \frac{1}{2}x^2$

$y = 117 - x$

$A = x(117 - x) - \frac{1}{2}x^2$

$= 117x - x^2 - \frac{1}{2}x^2$

$A = 117 - \frac{3}{2}x^2$

(ii) $A = 117x - \frac{3}{2}x^2$

$\frac{dA}{dx} = 117 - 3x$

At St pts $\frac{dA}{dx} = 0$

$117 - 3x = 0$

$3x = 117$

$x = 39$

$\frac{d^2A}{dx^2} = -3$

$\Rightarrow x = 39$ will give max value.

$\therefore y = 117 - 39$

$= 78$

$= 2(39)$

$\Rightarrow y = 2x$

Q3 (a) $\frac{dv}{dt} = 2-t$

$$V = 2t - \frac{t^2}{2} + C$$

When $t=6$ $V=10$

$$\therefore 10 = 12 - 18 + C$$

$$C = 16$$

$$\therefore V = 2t - \frac{t^2}{2} + 16$$

b(i) $\int \left(\sqrt{x} - x^{-2}\right) dx$

$$= \int \left(x^{\frac{1}{2}} - x^{-2}\right) dx$$

$$= \frac{2x^{\frac{3}{2}}}{3} - \frac{x^{-1}}{-1} + C$$

$$= \frac{2x^{\frac{3}{2}}}{3} + x^{-1} + C$$

$$= \frac{2x\sqrt{x}}{3} + x^{-1} + C$$

(ii) $\int (3x+5)^7 dx$

$$= \frac{(3x+5)^8}{3(8)} + C$$

$$= \frac{1}{24}(3x+5)^8 + C$$

c(i) $\int_{-1}^{3} (3t^2 - t)\, dt$

$$= \left[t^3 - \frac{t^2}{2}\right]_{-1}^{3}$$

$$= \left(3^3 - \frac{3^2}{2}\right) - \left((-1)^3 - \frac{(-1)^2}{2}\right)$$

$$= \left(27 - \frac{9}{2}\right) - \left(-1 - \frac{1}{2}\right)$$

$$= 24$$

(ii) $\int_0^1 (4-x)^{-\frac{1}{2}} dx$

$$= \frac{(4-x)^{\frac{1}{2}}}{\frac{1}{2}(-1)}$$

$$= \left[-2(4-x)^{\frac{1}{2}}\right]_0^1$$

$$= \left[-2(3)^{\frac{1}{2}}\right] - \left(-2(4)^{\frac{1}{2}}\right)$$

$$= -2\sqrt{3} + 4$$

3(d) $a + ar = 18$ ... (1)

$ar^2 + ar^3 = 72$ ... (2)

$\Rightarrow$ $a(1+r) = 18$ ... (3)

$ar^2(1+r) = 72$ ... (4)

$\frac{(4)}{(3)}$ $r^2 = 4$

$r = \pm 2$

$r = 2$ $a = \frac{18}{3} = 6$

$\therefore$ 6, 12, 24, 48

$r = -2$ $a = \frac{18}{-1} = -18$

$-18, 36, -72, 144$

Q4

a (i) 16

(ii) $20-16=4$

(iii) $T_n = a+(n-1)d$

$T_n = 16+4(n-1)$

$= 16+4n-4$

$T_n = 4n+12$

(iv) $T_{12} = 4(12)+12$

$= 60$

(v) $S_n = \frac{n}{2}[a+l]$

$= \frac{12}{2}[16+60]$

$= 6(76)$

$= \$456$

b(i) $A_1 = 20000\left(1+\frac{15}{100}\right) - M$

$= 20000(1+0.005) - M$

$A_1 = 20000(1.015) - M$

(ii) $A_2 = \cancel{20000}\ A_1(1.015) - M$

$= \cancel{20000[2}$

$= \left(20000(1.015)-M\right)1.015 - M$

$= 20000(1.015)^2 - M(1+1.015)$

$A_{36} = 20000(1.015)^{36} - M(1+1.015+\dots 1.015^{35})$

(iii) $A_{36}=0$

$M = \dfrac{20000(1.015)^{36}}{1+1.015+\dots 1.015^{35}}$

$= \dfrac{20000(1.015)^{36}}{\dfrac{1.015^{36}-1}{1.015-1}}$

$= \$723.05$

c) $6\% \div 12 = 0.5\%$ per month.

$A_2 = P\left(1+\frac{0.5}{100}\right)^2$ after 2 mths

$= P(1.005)^2$

$A_1 = P(1.005)$ after 1 mth.

Total $= P(1.005) + P(1.005)^2$

$= P(1.005+1.005^2)$

(ii) After 1 year

$= P(1.005+1.005^2+\dots 1.005^{12})$

(c) After 20 years.

$$f = P(1.005 + 1.005^2 + \ldots 1.005^{240})$$

$$\therefore P = \frac{450000}{1.005 + \ldots 1.005^{240}}$$

$$= \frac{450000}{\frac{1.005\left[1.005^{240} - 1\right]}{1.005 - 1}}$$

$$= \frac{450000}{464.35}$$

$$= \$969.09 \text{ or } \$969.10$$

Q5 

$h + 2r = 18 \quad \cdots (1)$

$h = 18 - 2r$

(i) $V = \pi r^2 h$

$V = \pi r^2 (18 - 2r)$

$= 18\pi r^2 - 2\pi r^3$

(ii) $\frac{dV}{dr} = 36\pi r - 6\pi r^2$

At St pt $\frac{dV}{dr} = 0$

$36\pi r - 6\pi r^2 = 0$

$6\pi r \left[6 - r\right] = 0$

$\therefore r = 0$ (NA) $\quad r = 6$

$\frac{d^2V}{dr^2} = 36\pi - 12\pi r$

At $r = 6$ $\quad \frac{d^2V}{dr^2} = -36\pi$

< 0

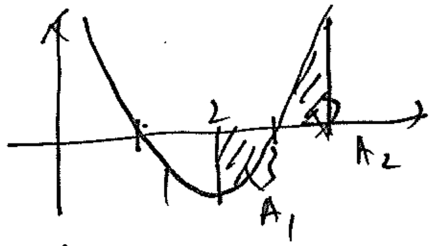
$\Rightarrow r = 6$ will give max V

(iii) $V_{max} = 18\pi(6)^2 - 2\pi(6)^3$

$= 648\pi - 432\pi$

$= 216\pi$

$= 679 \text{ cm}^3$ #

(b) 

$A_1 = \left| \int_2^3 (x^2 - 4x + 3)\,dx \right|$

$= \left[\frac{x^3}{3} - 2x^2 + 3x \right]_2^3$

$= \left| \left(\frac{27}{3} - 18 + 9\right) - \left(\frac{8}{3} - 8 + 6\right) \right|$

$= \left| -\frac{2}{3} \right|$

$= \frac{2}{3}$

$A_2 = \int_3^4 (x^2 - 4x + 3)\,dx$

$= \left[\frac{x^3}{3} - 2x^2 + 3x \right]_3^4$

$= \left(\frac{64}{3} - 32 + 12\right) - \left(\frac{27}{3} - 18 + 9\right)$

$= \frac{4}{3}$

$\therefore$ Total area $= \frac{2}{3} + \frac{4}{3}$

$= \frac{6}{3}$

$= 2 \text{ units}^2$

c(i) $x=2$

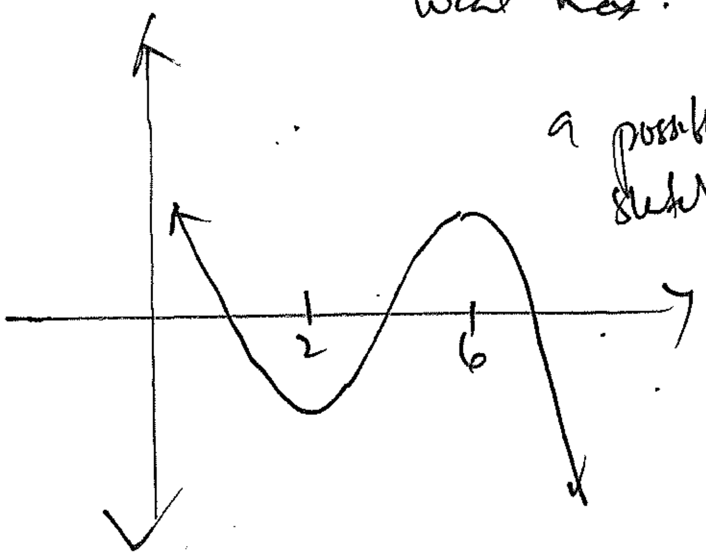
x	2^-	2	2^+
y'	$-$	0	$+$
	\	—	/

$\Rightarrow$ $x=2$ is a local min.

(ii) $x=6$

x	6^-	6	6^+
y'	$+$	0	$-$
	/	—	\

$\Rightarrow$ $x=6$ gives local max.



a possible sketch

Q6(a)

$= (3^2+3) + (4^2+3) + (5^2+3)$

$= 12 + 19 + 28$

$= 59$

(b) $x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3-1) = 0$

$x=0$ or $x=1$

$y=0$ $\quad$ $y=1$

$(0,0)$ $(1,1)$ pt. of intersection

$$A = \int_0^1 (\sqrt{x} - x^2)\,dx$$

$$= \int_0^1 (x^{\frac{1}{2}} - x^2)\,dx$$

$$= \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) - 0$$

$$= \frac{1}{3} \text{ unit}^3$$

(c)

x	0	10	20	30	40
y	0	15.2	x	15.3	13.8
	y_1	y_2	y_3	y_4	y_5

$$A = \frac{10}{3}\left[0 + 13.8 + 4(15.2+15.3) + 2(x)\right]$$

$$\therefore \frac{10}{3}\left[135.8 + x\right] = 546$$

$$10(135.8 + x) = 1638$$

$$135.8 + x = 163.8$$

$$x = 28$$

$$V = \pi \int_1^2 \left(\frac{1}{x}\right)^2 dx$$

$$= \pi \int_1^2 x^{-2}\,dx$$

$$= \pi \left[\frac{x^{-1}}{-1}\right]_1^2$$

$$= \pi \left[-\frac{1}{x}\right]_1^2$$

$$= \pi \left[\left(-\frac{1}{2}\right) - \left(-\frac{1}{1}\right)\right]$$

$$= \pi \left[-\frac{1}{2} + 1\right]$$

$$= \frac{\pi}{2} \text{ units}^3$$