Carlingford High School

2015

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Name:_____

Teacher (circle): Cheng, Gong, White, Lobejko, Wilson

Mathematics

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 14

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

	Q1-10	Q11	Q12	Q13	Q14	Q15	Q16	Total
Multiple Choice	/10							**/10**
Arithmetic & Algebra		/4				/3		**/7**
Functions		/2	/2	/6				**/10**
Series		/2		/3			/7	**/12**
Logs & Exponentials					/10			**/10**
Trig			/2		/5	/4		**/11**
Geometry		/2		/4				**/6**
Calculus		/5	/11	/2		/8	/8	**/34**
Total	**/10**	**/15**	**/15**	**/15**	**/15**	**/15**	**/15**	**/100**

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1. $\left(\frac{2a}{3b}\right)^{-5} = ?$

(A) $\frac{2a^5}{3b^5}$

(B) $\frac{3b^5}{2a^5}$

(C) $\frac{243b^5}{32a^5}$

(D) $\frac{1}{243b^5}$

2. Let α and β be the solutions of $2x^2 - 5x - 9 = 0$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(A) $-\frac{9}{2}$

(B) $-\frac{9}{5}$

(C) $-\frac{5}{9}$

(D) $\frac{5}{2}$

3. Find $\lim_{x \to \infty} \frac{3\sqrt{x}}{x-2}$.

(A) $\sqrt{x}$

(B) 3

(C) $\frac{3}{x}$

(D) 0

4. The period and amplitude of $y = 3\cos 2x$ is:

(A) Amplitude = 2, Period = $\frac{2\pi}{3}$

(B) Amplitude = 3, Period = π

(C) Amplitude = π , Period = 3

(D) Amplitude = $\frac{2\pi}{3}$, Period = 2

5. The centre and radius given by the circle $x^2 + 2x + y^2 + 4y - 5 = 0$ is

(A) centre = (-1,-2) and radius = 10

(B) centre = (1,2) and radius = 10

(C) centre = (-1,-2) and radius = $\sqrt{10}$

(D) centre = (1,2) and radius = $\sqrt{10}$

6. When simplified fully $\cos^2\left(\frac{\pi}{2} - \theta\right)\cot\theta$ is:

(A) $\cos^2\theta \cot\theta$

(B) $\sin\theta \cos\theta$

(C) $\frac{\sin^3\theta}{\cos\theta}$

(D) $\sin^2\theta \cot\theta$

7. Find the $\int_2^7 \frac{5}{x}\,dx$.

(A) $5(\ln 7 - \ln 2)$

(B) $\frac{1}{5}(\ln 7 - \ln 2)$

(C) $\frac{5}{49} - \frac{5}{4}$

(D) 0

8. The equation of the normal to the curve $x^2 = 4y$ at the point where $x = 2$ is:

(A) $y = 1$

(B) $x - y - 1 = 0$

(C) $y = -1$

(D) $y + x - 3 = 0$

9. Find the value of $\log_5 200 - 3\log_5 2$.

(A) 1.4

(B) 2.0

(C) 3.2

(D) 2.5

10. Solve $|5x + 4| \le 6$

(A) $\frac{-2}{5} \le x \le 2$

(B) $x \ge \frac{2}{5}$ or $x \le -2$

(C) $-2 \le x \le \frac{2}{5}$

(D) $x \ge 2$ or $x \le \frac{-2}{5}$

End of Section I

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in a *separate* writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a new writing booklet.

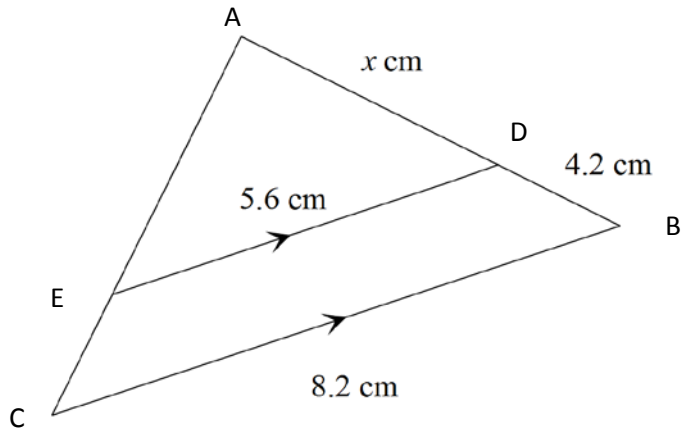
(a) Solve $x^2 - 2x - 7 = 0$, expressing your answer in simplest surd form. **2**

(b) Find $\int \frac{3x}{x^2 + 1}\, dx$. **1**

(c) Simplify fully : **2**

$$\frac{2}{\sqrt{7} + 3} - \frac{3\sqrt{7}}{\sqrt{7} - 3}$$

(d) Find the value of x (correct to the nearest mm), given ΔABC is similar to ΔADE. **2**



(e) Find the coordinates of the vertex and focus of the parabola $x^2 - 5y + 5 = 0$. **2**

Question 11 continues on page 6

Question 11 (continued)

(f) Find the sum of the 10^{th} to the 30^{th} terms of the arithmetic series $5 + 9 + 13 + ...$ **2**

(g) Evaluate $\int_0^{\ln 6} e^x \, dx$. **2**

(h) Shade the following regions bounded by the curves $y < \sqrt{4 - (x-2)^2}$ and $y > \frac{x^2}{2}$. **2**

End of Question 11

Question 12 (15 marks) Use a new writing booklet.

(a) Differentiate:

(i) $y = \sin^2(4x)$. **1**

(ii) $y = x^3 e^{3x}$. **1**

(iii) $y = \dfrac{e^x}{(x+3)^2}$. (Full simplification of your answer is not required.) **2**

(b) Solve $\sqrt{3}\cos x = \sin x$ for $0 \leq \theta \leq 2\pi$. **2**

(c) Use Simpson's Rule with four equal subintervals to find an approximation for **2**

$$\int_0^1 \tan\ x\ dx.$$

(d) Find a primitive of $3 + \dfrac{1}{x}$. **2**

(e) Find the values of A, B and C if $3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$. **2**

(f) A curve has the equation $y = x\cos x$.

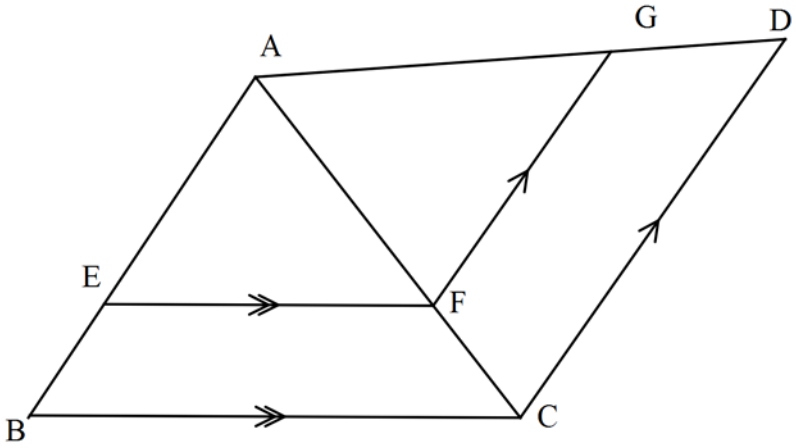
(i) Show that $P\left(\dfrac{\pi}{2}, 0\right)$ is the first point to the right of the origin where the curve crosses the x axis. **1**

(ii) Find the equation of the tangent at point P. **2**

End of Question 12

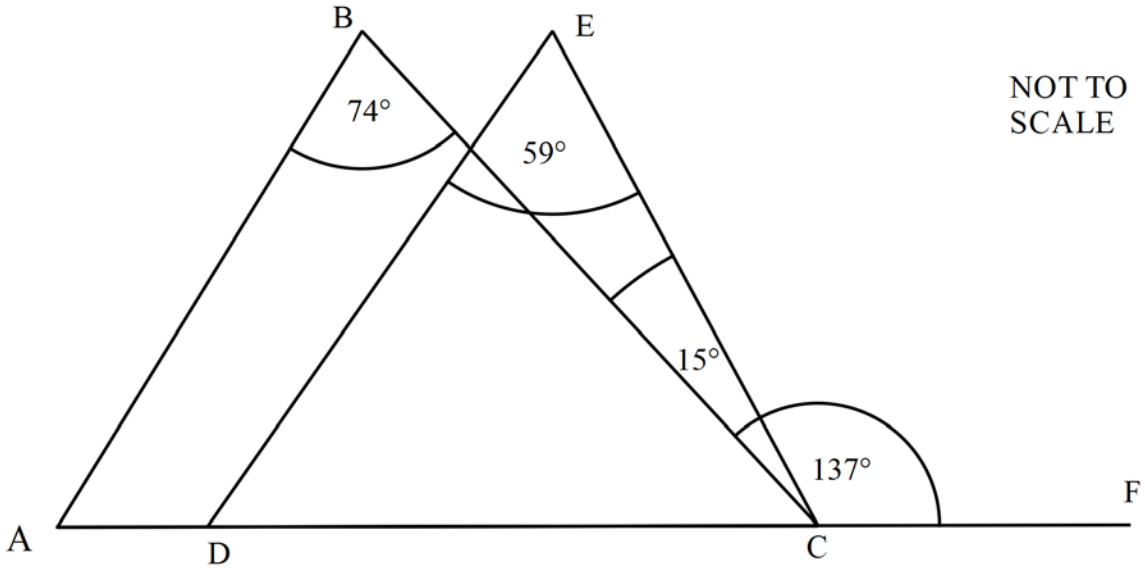
Question 13 (15 marks) Use a new writing booklet.

(a) In the figure below, EF || BC and CD || FG.



Prove that $\frac{AE}{AB} = \frac{AG}{AD}$ **2**

(b) In the diagram below *AF* is a straight line, $\angle B = 74°$, $\angle E = 59°$, $\angle BCF = 137°$ and $\angle BCE = 15°$.



Prove that AB||DE **2**

Question 13 continues on page 9

Question 13 (continued)

(c) Jack drops a super bouncy ball from the top of a 56 m building on to a concrete surface below. Its first rebound is 42 m, and each subsequent rebound is three quarters the height of the previous one.

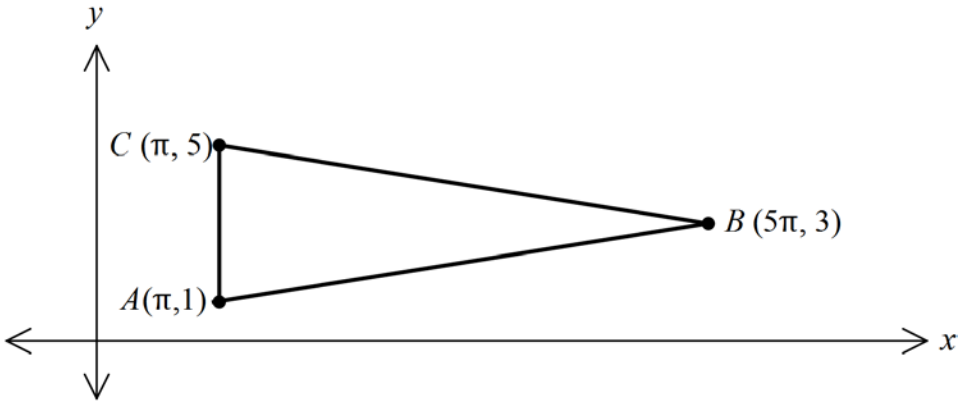
(i) How high will it rise on the fifth rebound? **2**

(ii) How far will it travel in total? **1**

(d) For the domain $0 \leq x \leq 6$, a function $y = f(x)$ satisfies $f'(x) < 0$ and $f''(x) < 0$. **2**

Sketch a possible graph of $y = f(x)$ in this domain.

(e) The points $A(\pi, 1)$, $B(5\pi, 3)$ and $C(\pi, 5)$ form an isosceles triangle, with $AB = BC$.



(i) Find the midpoint of AB. **1**

(ii) Show that the equation of the line which is perpendicular to AB and which passes through point C is: **2**

$$y + 2\pi x - 5 - 2\pi^2 = 0$$

(iii) Calculate the distance AB. **1**

(iv) Using the distances AB, BC and AC, or otherwise, find $\angle CAB$ to the nearest degree. **2**

End of Question 13

Question 14 (15 marks) Use a new writing booklet.

(a) Connor buys a new car, which begins to depreciate immediately. The value ($\$V$) of the car after t years is given by $V = A e^{-kt}$

Where A - is the initial value

k – constant of depreciation

t - time in years

If the car is worth \$30 000 after 5 years and \$18 000 after 10 years, find the following:

(i) The depreciation constant k **2**

(ii) The initial value of the car **1**

(iii) How many whole years will it take before the car's value falls below \$1 000? **2**

(b) A plane leaves an airport (A) and travels due north $\sqrt{3}\,x$ kilometres to a point K and then turns due west and travels a further x kilometres until it reaches a point P which is 380 kilometres from A. Due to storms the plane is then diverted to a new airport (B) which is 200 kilometres on a bearing of 280^{o} from A.

(i) Draw a diagram and label it to show the above information. **1**

(ii) Find the exact distance AK. **1**

(iii) Show that the plane needs to travel 294 kilometres from P to the new airport (B). **2**

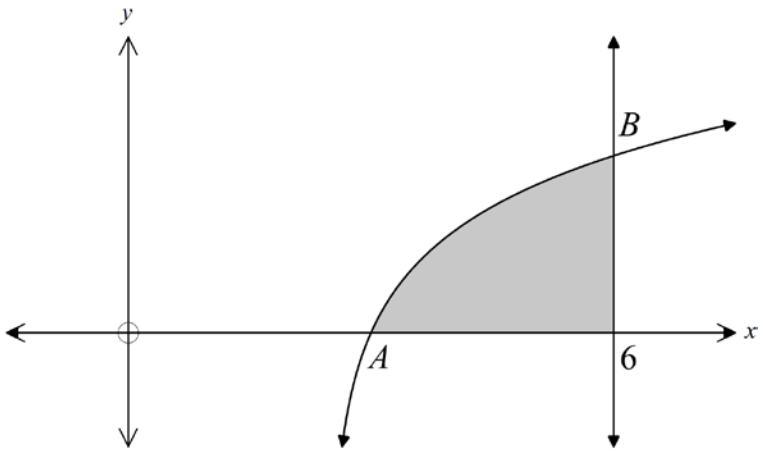
(iv) Hence or otherwise find the bearing (to the nearest degree) on which the plane flies from P to B. **1**

Question 14 continues on page 11

Question 14 (continued)

(c) The diagram shows a shaded region which is bounded by the curve $y = \ln(2x - 5)$, the x axis and the line $x = 6$.

The curve $y = \ln(2x - 5)$ intersect the x axis at A and the line $x = 6$ at B.



(i) Show that the coordinates of points A and B are (3, 0) and (6, ln7) respectively. **1**

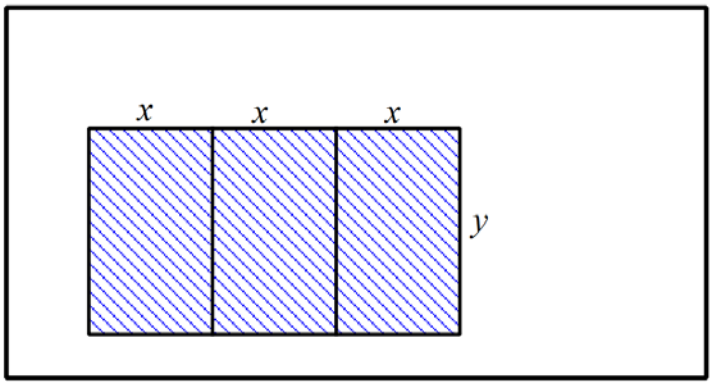
(ii) Show that if $y = \ln(2x - 5)$, then $x = \dfrac{e^y + 5}{2}$. **1**

(iii) Hence find the exact area of the shaded region. **3**

End of Question 14

Question 15 (15 marks) Use a new writing booklet.

(a) Greg has a one hectare (Ha) block of land. He is going to fence off three identical rectangular plots within his block for his three children. Each plot will measure x m by y m as shown in the diagram below. He will retain the remainder of the block for himself and his wife. Greg can only afford 300 m of fencing to go around the children's plots.



(i) Show $y = 75 - \frac{3x}{2}$. **1**

(ii) Find the value of x for which the area will be a maximum. **3**

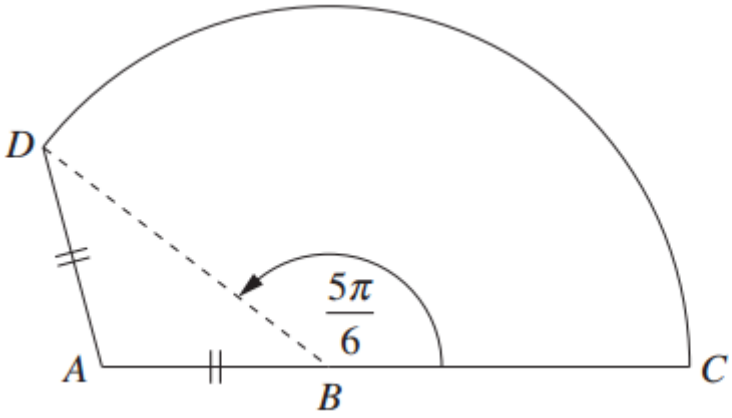
(iii) Find the maximum area of one of the children's blocks. **1**

(iv) How much of Greg's 1 Ha block is left for him and his wife? **1**

Question 15 continues on page 13

Question 15 (continued)

(b)



NOT TO SCALE

In the diagram, $ABCD$ represents a garden. The sector BCD has centre B and $\angle DBC = \frac{5\pi}{6}$.

The points A, B and C lie on a straight line and $AB=AD$ = 3 metres

(i) Show that $\angle DAB = \frac{2\pi}{3}$. **2**

(ii) Find the length of BD. **2**

(c) A pond has 50 litres of water. Water is taken out of the pond in the rate **2**

$\frac{dv}{dt} = 5 + 2t$ *L/min*

Find a formula to give the volume of the water in the pond after t minutes.

(d) Find the value of n such that: **3**

$$\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$$

End of Question 15

Question 16 (15 marks) Use the Question 14 writing booklet.

(a) The relation $x^2 - 4x + y^2 = 5$ is rotated about the x-axis to form a solid. Find the exact volume of this solid of revolution. **2**

(b) For the curve $y = x^3(3 - x)$

(i) Find any stationary points and determine their nature. **3**

(ii) Draw a sketch of the curve showing the stationary points, inflexion points and intercepts on the axes. **3**

(c) Georgina borrows \$650 000 to purchase her first home. She takes out a loan over 30 years, to be repaid in equal monthly instalments. The interest rate is 5.4% per annum reducible, calculated monthly.

(i) Show that the amount, $\$A_n$, owing after the nth repayment is given by the formula: **2**

$$A_n = 650\,000(1.0045)^n - M\left(1 + 1.0045 + 1.0045^2 + ... + 1.0045^{n-1}\right)$$

(ii) Find the monthly repayment required to repay the loan in 30 years. **2**

(iii) Georgina wants pay the loan off in less than 30 years. If she can afford to pay \$5 000 per month, how many months will it take her to pay off the home loan? **2**

(iv) How much will Georgina save in interest if she pays \$5 000 per month? **1**

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Trial HSC Examination 2015

Mathematics Course

Name _____ Teacher _____

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○