CARLINGFORD HIGH SCHOOL ASSESSMENT TASK NOTIFICATION

DEPARTMENT OF MATHEMATICS

YEAR 11

Mathematics Advanced

2019

Assessment Task 2

Investigation Assignment –Functions and Trigonometry

STUDENT NO:______________________________

COURSE COMPONENTS:

Concepts, skills and techniques Use of concepts, skills and techniques to solve mathematical problems in a wide range of theoretical and practical contexts (50%)

Reasoning and communication Application of reasoning and communication in appropriate forms to construct mathematical arguments and proofs and to interpret and use mathematical models (50%)

NOTIFICATION DATE: 3 JUNE 2019

TASK NO: 2 **WEIGHTING:** 30% **DUE DATE:** 17 JUNE 2019

TOPICS/OUTCOMES ASSESSED:

Functions (F1.1-1.4)

- uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
- uses the concepts of functions and relations to model, analyse and solve practical problems MA11-2
- provides reasoning to support conclusions which are appropriate to the context MA11-9

Trigonometry (T1.1 and 1.2)

- uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
- uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes MA11-3
- provides reasoning to support conclusions which are appropriate to the context MA11-9

ADDITIONAL INFORMATION:

Assessment guidelines as per Preliminary Assessment Booklet 2019 and School Policy.

MARKING CRITERIA:

As per Rubric issued with Task Notification

1. (a) This table of values, representing the function $y = 2x + 3$, shows that the difference (d_1) between each of the y values is 2, for $1 \leq x \leq 5$.

x	1	2	3	4	5
y	5	7	9	11	13
d_1		2	2	2	2

(i) Construct a similar table of values for a different linear function, $y = ax + b$, with the $a \neq 2$.

(ii) In your own words, describe the relationship between d_1 and the x-coefficient for a linear function.

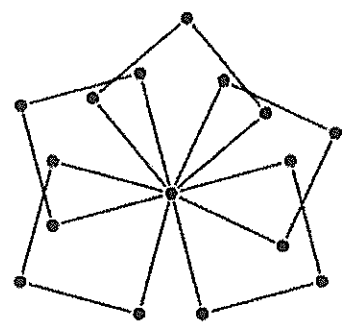
(b) This table of values, representing the function $y = x^2 + x - 1$, shows that the difference (d_2) between each of the d_1 values is 2, for $1 \leq x \leq 6$.

x	1	2	3	4	5	6
y	1	5	11	19	29	41
d_1		4	6	8	10	12
d_2			2	2	2	2

(i) Construct 3 similar tables of values for the quadratic functions $y = 3x^2 - x - 2$, $y = 4x^2 - 9x + 3$ and $y = -6x^2 + 2x + 1$.

(ii) In your own words, describe any relationship you find between d_2 and any particular coefficient(s) of each of these functions.

(c) In this diagram, all 5 squares have a common vertex.



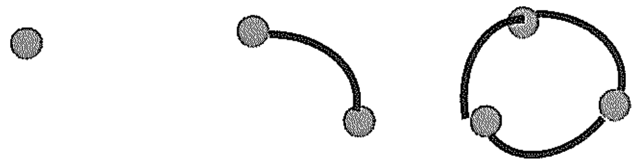
(i) Copy and complete this table, where n represents the number of overlapping squares, and v represents the corresponding number of vertices for each value of n, with one common vertex.

n	1	2	3	4	5
v					

(ii) Write a formula to find the value of v for each value of n. Explain how you found this rule.

(d) A new telephone company wants to develop a formula that find the number of connections required for its increasing number of subscribers.

This set of diagrams shows each connection required for 1, 2, and 3 subscribers.



(i) Using a ruler, draw diagrams for 4, 5 and 6 subscribers.

(ii) Copy and complete this table, where s represents the number of subscribers and c the number of connections.

s	1	2	3	4	5	6
c	0	1	3			

(iii) Write a formula to find the value of c for each value of s. Explain how you found this rule showing all calculations.

2. (a) This table of values, representing the function $y = x^3 - 1$, shows that the difference (d_3) between each of the d_2 values is 6, for $1 \le x \le 6$.

x	1	2	3	4	5	6
y	0	7	26	63	124	215
d_1		7	19	37	61	91
d_2			12	18	24	30
d_3				6	6	6

(i) Construct 2 similar tables of values for the cubic functions $y = -x^3$ and $y = 2x^3 + 4x^2$.

(ii) In your own words, describe any relationship you find between d_3 and the coefficients of each of these functions.

(b) In a similar way, investigate the relationship(s) between d_4 and the coefficients of a quartic function, $y = ax^4 + bx^3 + cx^2 + d$.

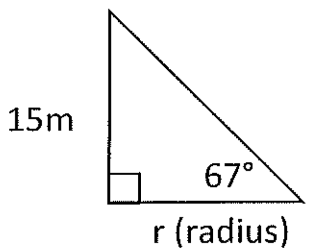
(c) Can you suggest a possible pattern for predicting these relationship(s) across all degrees of polynomials? Provide further working to support this suggestion.

3. (a) (i) Use a pencil, ruler and compass to construct and label two triangles: an acute-angled triangle with side lengths 4cm and 6cm, and an obtuse-angled triangle with side lengths 4cm and 6cm. Measure and label the third side of each triangle.

(ii) Use the cosine rule, showing all working, to find the size of the largest angle of each triangle you have constructed, correct to one decimal place.

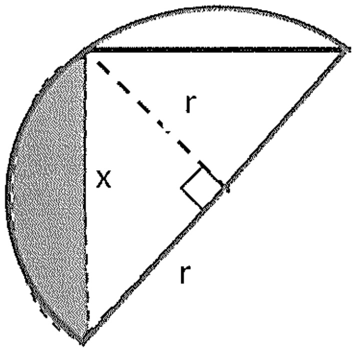
(b) It is a time of drought and the dams that usually provide water for farm crops have dried up.

One farmer considers hiring a plane to drop tanks of water onto a paddock they are preparing to plant. The paddock has dimensions 54 metres by 72 metres. The pilot explains that they usually drop each load from a height of 15m, with the angle of elevation from the circumference of the circle formed by the load of water dropped, equal to 67° (see diagram)



(i) Calculate the radius of each circle, correct to two decimal places.

(ii) The farmer used this diagram showing the circle of water covering one of the paddock corners. Calculate the **shaded** area.

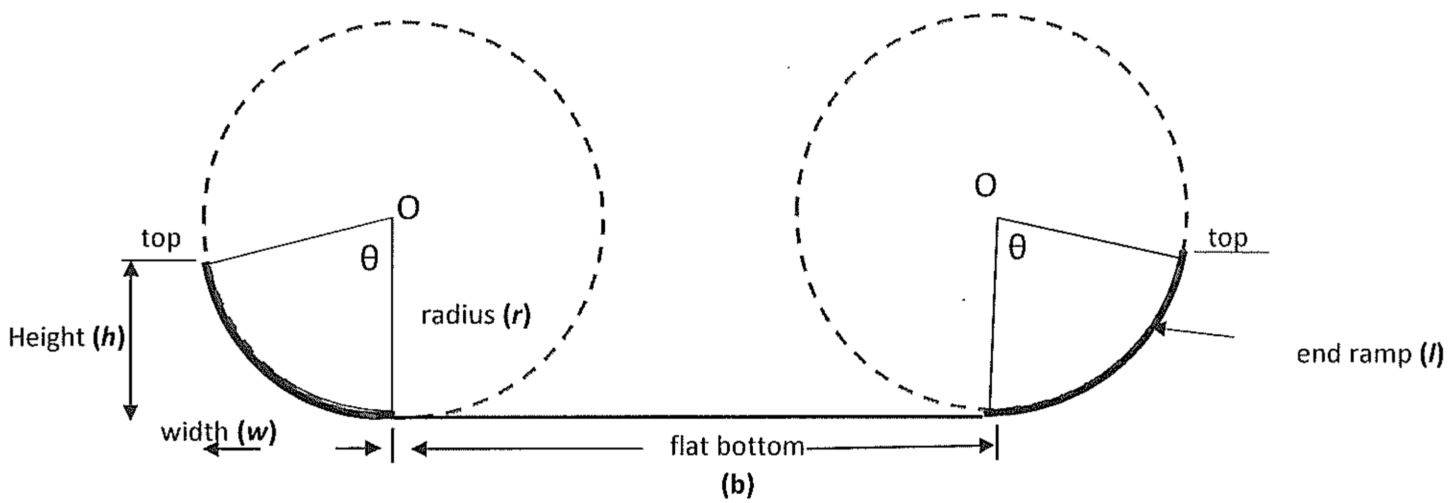


(iii) Showing diagrams and any calculations, find the percentage of water purchased that landed outside the paddock.

(iv) A month later, the farmer organised another water drop. This time, each load was dropped from a height of 20m. If the angle of elevation from the circumference of the circle formed by the load of water dropped is now 78°, calculate the percentage of this load that landed outside the paddock?

Which, therefore was the better option? Give reasons.

4. You are going to design a skateboard ramp. A cross-section (side view) of a skateboard ramp is shown below.



The end ramps are the shape of an arc of a circle, and there is a flat bottom in between.

You are asked to design a similar cross section for a skateboard ramp, with each end ramp being an arc of a quarter circle (ie $\theta = 90°$) and the platform at the top of each end having height 10 metres.

a) (i) Show all the angles and dimensions used to construct your ramp, including

- the angle subtending the arc, $\theta = 90°$ in this case
- the height of the end ramps, $h = 10\ metres$ in this case
- the width of the end ramps, w, to be calculated by you
- the arc length of the end ramps, l, to be calculated by you
- the radius of the arc's circle, r, to be calculated by you
- the length of the flat bottom, $b = 10\ metres$

(ii) Calculate the full distance the skater will travel from the top of the ramp at one end, to the top of the ramp at the other end.

b) You now wish to change the steepness of the end ramps.

Select a different value for θ, $0 < \theta < 90°$. **You will use this same chosen value of θ for each of the following parts of the question.**

(i) Calculate and describe how the other dimensions, l, r, and b, will change if you maintain **the same height and width of the ramp as was in (a) part (i).**
Also compare and comment on the full distance the skater will now travel.
Show all working using clear, neat diagrams and detailed calculations.

(ii) Again using the same value of θ that you chose for (b) part (i) above, investigate and describe how the other dimensions, l, h, w, and b , will change if you **maintain the same radius of the ramp as was in (a) part (i)**, and assume $h = w$ in this case.
Also compare and comment on the full distance the skater will now travel.
Show all working using clear, neat diagrams and detailed calculations.

(iii) Now consider the impact on variables r, w, and b when you maintain the same l and h values as in **(a) part (i)**, again using your same chosen angle.
Compare and comment on the full distance the skater will now travel.
Show all working using neat, clear diagrams and detailed calculations.

(iv) Describe the impact of reducing the angle θ, on the skater's experience on the ramp. Also comment on how varying the radius length together with reducing the angle, further impacts. Also comment on the effect of maintaining the same ramp curve length and height and reducing the angle.
How would you use these observations to help design skating ramps for skaters with varying abilities?
Which of these three designs do you think would be best to use for a public Skating Park ramp? Give your reasons for making this choice.

Marking Rubric

Investigation Assignment –Functions and Trigonometry

Question	1 mark	2 marks	3 marks	4 marks	Mark
1(a)	Correct table of values or Correct description	Correct table of values and Correct description			
1(b)	One correct table of values and Correct description	Two correct tables of values and No or incorrect description	Two correct tables of values and Correct description	Three correct tables of values and Correct description	
1(c)	Correct table	Correct rule and No or incorrect explanation	Correct rule and Correct explanation		
1(d)	Correct diagrams or Correct table	Correct diagrams and Correct table	Correct diagrams and Correct table and Correct rule	Correct diagrams and Correct table and Correct rule and explanation	
2(a)	One correct table of values and Correct description	Two correct tables of values and No or incorrect description	Two correct tables of values and Correct description		
2(b)	One correct table of values	Two correct tables of values	3 correct tables of values and No or incorrect description	3 correct tables of values and Correct description	
2(c)	Correct table	Correct table and Correct description			
3(a)	Correct triangles and No or incorrect calculations	Correct triangles and One correct calculation	Correct triangles and Two correct calculations		
3(bi,bii,biii)	Correct radius and Correct calculations	Correct area of shaded segment and Correct calculations	Correct percentage and Correct calculations		

3(biv)	Correct area of shaded segment and Correct calculations	Incorrect percentage and Some correct calculations	Correct percentage and Correct calculations		
4(a)(i)	Produces a design	Produces a design with some dimensions correctly calculated	Produces a design with all dimensions correctly calculated.		
4(a)(ii)	Some correct calculations	Correct calculations			
4(b)(i)	Produces clear diagrams indicating relative changes	Produces clear diagrams with some correct calculations	Produces clear diagrams with correct calculations	Produces clear diagrams with correct calculations and description of changes	
4(b)(ii)	Produces clear diagrams indicating relative changes	Produces clear diagrams with some correct calculations	Produces clear diagrams with correct calculations	Produces clear diagrams with correct calculations and description of changes	
4(b)(iii)	Produces clear diagrams indicating relative changes	Produces clear diagrams with some correct calculations	Produces clear diagrams with correct calculations	Produces clear diagrams with correct calculations and description of changes	
4(b)(iv)	Has limited understanding of the impact of changing the variables.	Understands the impact of changing the variables, but only some understanding of the effect of the interaction.	Understands the impact of changing the variables, and also of the interaction between them all, but little idea of how this would affect Skate Park ramp design.	Understands the impact of changing the variables, and also of the interaction between them all, and can use this to predict impact on Skate Park ramp design.	
				TOTAL MARKS	**/52**

YEAR 11 — INVESTIGATION ASSIGNMENT

FUNCTIONS & TRIGONOMETRY

SOLUTIONS

TOTAL

52 marks

1(a)(i) $y = 3x + 4$

x	1	2	3	4	5
y	7	10	13	16	19
d_1		3	3	3	3

(iii) d_1 defines the gradient of the line.

(b) (i) $y = 3x^2 - x - 2$

x	1	2	3	4	5	6
y	0	8	22	42	68	100
d_1		8	14	20	26	32
d_2			6	6	6	6

$y = 4x^2 - 9x + 3$

x	1	2	3	4	5	6
y	−2	+1	12	31	58	93
d_1		3	11	19	27	35
d_2			8	8	8	8

$y = -6x^2 + 2x + 1$

x	1	2	3	4	5	6
y	−3	−19	−47	−87	−139	−203
d_1		−16	−28	−40	−52	−64
d_2			−12	−12	−12	−12

(iii) d_2 is twice the coeff of x^2.

(c) (i)

n	1	2	3	4	5	
v	4	7	10	13	16	

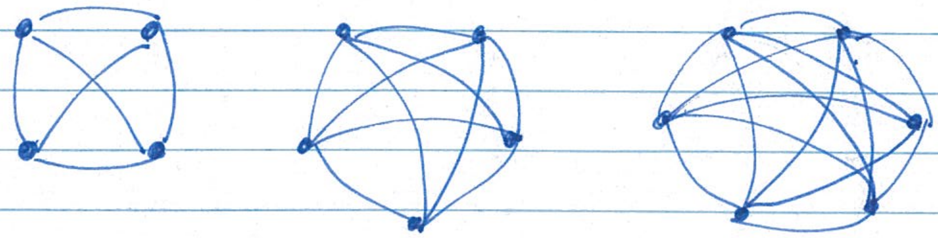
3

(ii) $v = 3n + 1$

Using $d_1 = 3$ (constant) ∴ like gradient in Q.1.

or by inspection - each square shares 1 common vertex and has 3 others of its own.

(d) (i)



S	1	2	3	4	5	6
C	0	1	3	6	10	15

1 2 3 4 5

1 1 1 1

(iii) 2nd diff = 1

∴ Quadratic $\quad c = \frac{1}{2}s^2 + bs + c$

$c = 0,\ s = 1 \qquad 0 = \frac{1}{2} + b + c \qquad —①$

$c = 1,\ s = 2 \qquad 1 = 2 + 2b + c \qquad —②$

$② - ① \qquad 1 = \frac{3}{2} + b \qquad b = -\frac{1}{2}$

② becomes $c = 0$. ∴ eqⁿ $c = \frac{1}{2}s^2 - \frac{1}{2}s$

Q.2 (a) (i) $y = -x^3$

x	1	2	3	4	5	6
y	-1	-8	-27	-64	-125	-216
d_1		-7	-19	-37	-61	-91
d_2			-12	-18	-24	-30
d_3				-6	-6	-6

$y = 2x^3 + 4x^2$

x	1	2	3	4	5	6
y	6	32	90	192	350	576
d_1		26	58	102	158	226
d_2			32	44	56	68
d_3				12	12	12

(ii) Coeff of x^3 is $\frac{1}{6} \times d_3$.

(b) $y = 2x^4 + 3x^3 + x^2 + 4$

x	1	2	3	4	5	6	7
y	10	64	256	724	1654	3280	
d_1		54	192	468	930	1628	
d_2			138	276	462	696	
d_3				138	186	234	
d_4					48	48	48.

d_4 contains $24 \times$ coeff of x^4

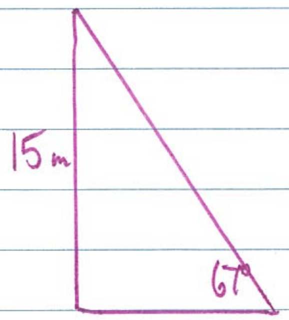
(c)

degree	1	2	3	4	5
multip factor	1	2	6	24	120.

check

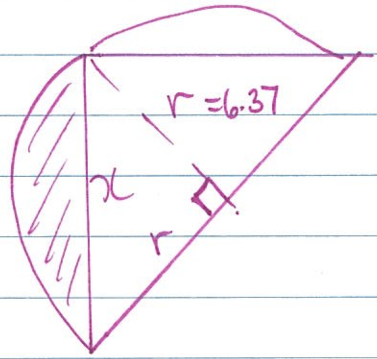
3(a) Students own choice

Q.3(b)(i)



$$r = \frac{15}{\tan 67^\circ} = 6.37\,m$$

(ii)



$$x = \sqrt{2 \times 6.37^2}$$

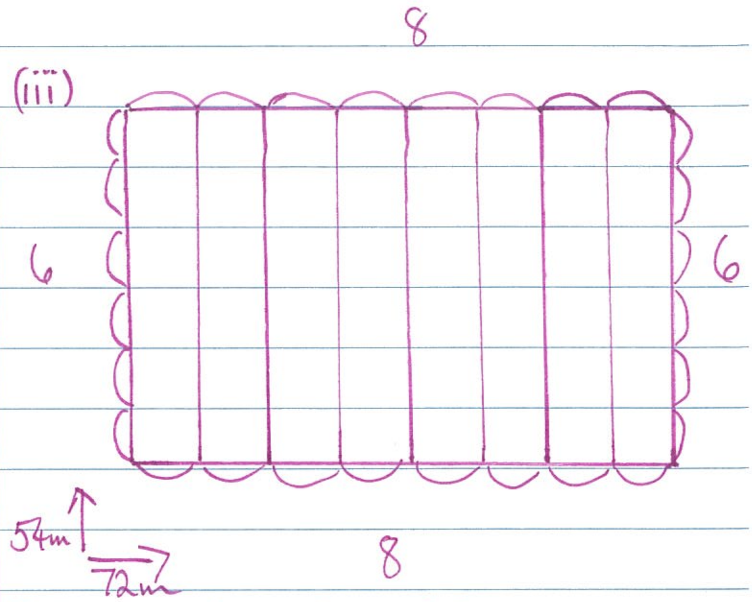
$$x = 9\ cm$$

$\therefore$ Shaded area $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$$= \frac{1}{2} \times 6.37^2 \times \frac{\pi}{2} - \frac{1}{2} \times 6.37^2$$

$$= \frac{6.37^2}{2}\left(\frac{\pi}{2} - 1\right) = 11.58\ m^2$$

(iii)



Wastage $= 2 \times 8 + 2 \times 6 = 28 \times$ segments.

Water wasted $= 28 \times 11.58\ m^2$ area

$= 324.24\ m^2$

Total no. of drops required

$= 6 \times 8$

$\Rightarrow$ Area of water dropped $= 48 \times \pi \times 9^2$

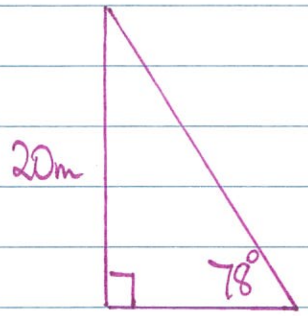
$\therefore$ Have dropped a total area of $12214.51\ m^2$

$\therefore$ % wastage

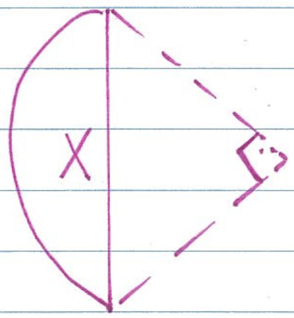
$$= \frac{324.24}{12214.51} \times 100$$

$$= 2.654\%$$

Q.3(b)
(iv)

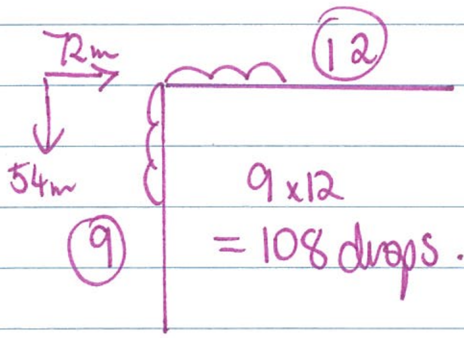


$R = \dfrac{20}{\tan 78°} = 4.25$ m.



$X = \sqrt{2 \times R^2}$

$= 6$ metres



9×12
$= 108$ drops.

Wastage $= 2 \times 12 + 2 \times 9$
$= 24 + 18$
$= 42$ segments.

Area of each segment

$= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2} \times 4.25^2 \times \frac{\pi}{2} - \frac{1}{2} \times 4.25^2$

$= 5.155$

$\therefore$ Total wastage $= 42 \times 5.155$ m^2

$= 216.51$ m^2

Total no. of drops required

$= 108 \times \pi \times 6^2$

$= 12214.51$

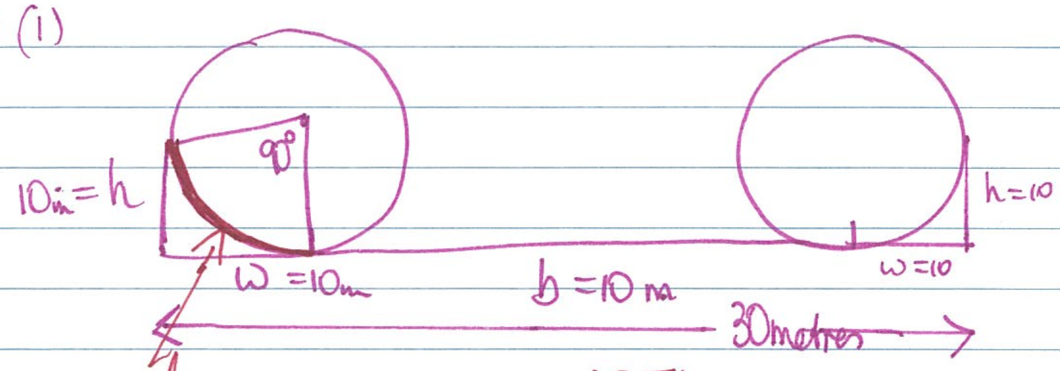
$\therefore$ % wastage $= \dfrac{216.51}{12214.51} \times \dfrac{100}{1}$

$= 1.77$ %.

The better option is dropping from 20m as it reduces the radius of each circle and therefore reduces wastage outside paddock ie. More goes onto the actual paddock

Q.4.

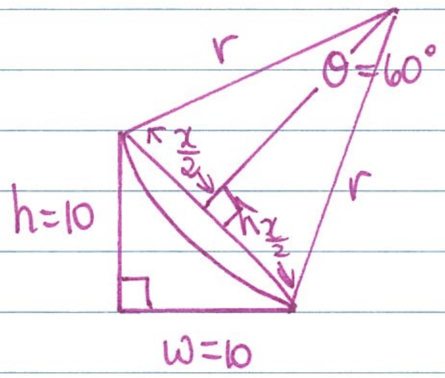
(a) (i)



$l = r\theta = 10 \times \frac{\pi}{2} = 15.71$ metres.

(ii) $d_{TOTAL} = 2 \times 5\pi + 10 = 41.42$ metres.

(b) Let $\theta = 60°$



$x^2 = 10^2 + 10^2 = 200$

$\therefore x = 10\sqrt{2}$ $\quad \frac{x}{2} = 5\sqrt{2}$.

$\rightarrow \therefore r = \frac{5\sqrt{2}}{\sin 30°} = 10\sqrt{2}$.

$\rightarrow$ b does not change

$\rightarrow l = r\theta \quad \therefore l = 10\sqrt{2} \times \frac{\pi}{3} = \frac{\pi 10\sqrt{2}}{3}$ metres

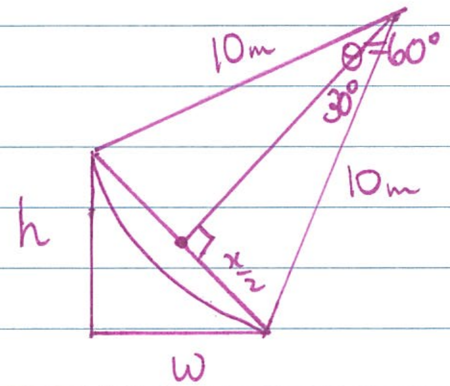
≈ 14.81 metres.

$\therefore$ Full distance skater will travel is :

$2 \times 14.81 + 10 = 39.62$ metres.

Q.4 (b) (ii)

$\theta = 60°$ $h = w$ $r = 10$.



$l = r\theta = 10 \times \frac{\pi}{3} = \frac{10\pi}{3} = 10.47m$

$\sin 30° = \frac{x}{20}$

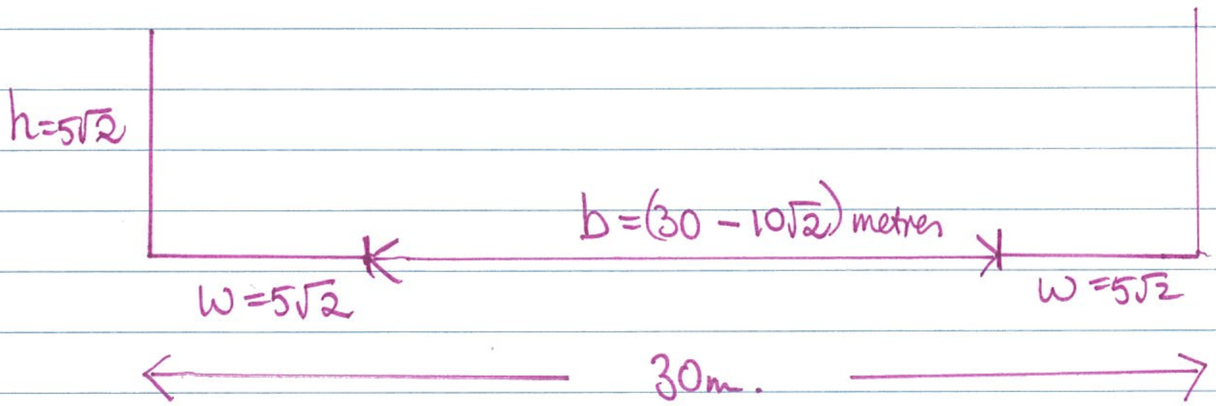
$x = 20 \sin 30°$

$\Rightarrow x = 10$ metres

$h^2 + w^2 = x^2$

$\therefore 100 = 2h^2$ since $h = w$.

$50 = h^2$ $\therefore h = w = 5\sqrt{2}$



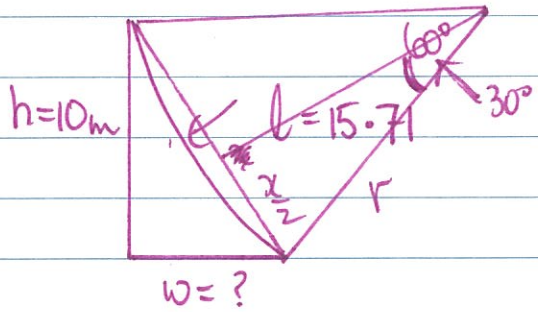
$h = w = 5\sqrt{2}$

$l = 10.47$ metres

$b = (30 - 10\sqrt{2})$ metres $= 15.86$ metres

Full distance skater now travels is $2l + b = 2 \times 10.47 + 15.86$
$= 36.80$ metres

Q4(b) (iii) $h = 10$ metres.
$l = 15.71$ metres



$l = 15.71 = r\theta$

ie, $15.71 = r \times \frac{\pi}{3}$

$\Rightarrow r = \frac{15.71}{\pi} \times 3$

$r = 15$ metres

$\therefore \sin 30° = \frac{\frac{x}{2}}{r} = \frac{x}{30}$

$\Rightarrow x = 30 \sin 30° = 15$ metres.

$\therefore w^2 = 15^2 - 10^2 = 225 - 100 = 125$

$\therefore w = 5\sqrt{5}$ metres.

$\therefore b = 30m - 2 \times 5\sqrt{5}$

$= (30 - 10\sqrt{5})\,m$

$b = 7.64$ metres

$\therefore$ Full distance skater travels $= 2l + b$

$= 2 \times 15.71 + 7.64$

$= 39.06$ metres.