Complex Numbers Formula Test

1 For a complex number, if the real part $\text{Re}(z)$ is x and the imaginary part $\text{Im}(z)$ is y, then $z = \square$.

2 For complex numbers $a+ib$ and $c+id$, the following occur:

(a) equality: $a+ib = c+id \Rightarrow \square = \square$ and $\square = \square$

(b) addition: $(a+ib)+(c+id) = \square + i\square$

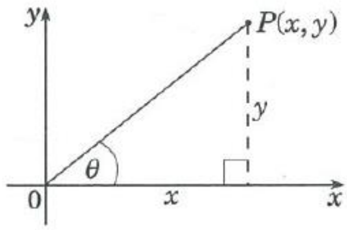
(c) subtraction: $(a+ib)-(c+id) = \square + i\square$

(d) multiplication: $(a+ib)(c+id) = \square + i\square$

(e) division: $\dfrac{a+ib}{c+id} = \dfrac{a+ib}{c+id} \times \dfrac{\square}{\square} = \square$

3 The powers of i may be used to simplify complex number arithmetic using:

$i^2 = \square$; $\quad i^3 = \square$; $\quad i^4 = \square$



4 The complex conjugate $\bar{z}$ of $z = x+iy$ is $\bar{z} = \square$.

5 $z = \square$ in mod-arg format where the modulus is given by

$r = \square = \square$ and the argument θ is given by $\tan\theta = \square$

for $\square \leq \theta \leq \square$.

6 For the complex numbers $z_1 = r_1(\cos\theta + i\sin\theta)$ and $z_2 = r_2(\cos\phi + i\sin\phi)$ then:

(a) $z_1 z_2 = \square$; $\quad |z_1 z_2| = \square$ **(b)** $\dfrac{z_1}{z_2} = \square$; $\quad \left|\dfrac{z_1}{z_2}\right| = \square$

(c) $\arg(z_1 z_2) = \square$ **(d)** $\arg\left(\dfrac{z_1}{z_2}\right) = \square$

7 If $z = r(\cos\theta + i\sin\theta)$, then $z^n = \square$

8 If $z = \cos\theta + i\sin\theta$, then $z^n = \square$. This is known as $\square$.

9 By expanding $(\cos\theta + i\sin\theta)^n$ as a binomial expansion and equating real and imaginary parts,

(a) $\cos n\theta = \square$ **(b)** $\sin n\theta = \square$

10 If $z^n = r(\cos\theta + i\sin\theta)$, then $z = \square$. This gives the n roots of z^n which lie on a circle of radius $\square$, and a sector angle of $\square$ between successive roots.

11 If the square roots of a complex number are given by $\pm\sqrt{a+ib} = z = x+iy$, then $a+ib = \square$, and $a = \square$, $b = \square$.

12 If $z^n = 1$ is solved to give the n roots of unity, $z = \square$ in mod-arg format.

13 The n roots of unity can be drawn in the Argand plane and are the vertices of a regular polygon of $\square$ sides inscribed in a circle $|z| = \square$, whose Cartesian equation is $\square$ with one vertex at $z = \square$.

14 Complex numbers have the following properties:

(a) $|z| = |\square| = \square$ **(b)** $\arg \bar{z} = \square$

(c) $z + \bar{z} = \square$ **(d)** $z - \bar{z} = \square$

(e) $z\bar{z} = |\square|^2 = |\square|^2 = \square$ **(f)** $\overline{z_1 \pm z_2} = \square$

(g) $\overline{z_1 z_2} = \square$ **(h)** $\overline{\dfrac{z_1}{z_2}} = \square$

(i) $\dfrac{1}{z} = \square = \dfrac{\square}{\square}$

15 $z^n - 1$ can be factorised over the complex field as

(a) $z^n + 1 = \square$ if n is odd **(b)** $z^n - 1 = \square$ if n is odd

(c) $z^n - 1 = \square$ if n is even

16 From the results in 15(a), (b), then:

(a) $\dfrac{z^n + 1}{z + 1} = \square$ if n is odd, $z \neq -1$ **(b)** $\dfrac{z^n - 1}{z - 1} = \square$ if $z \neq 1$

17 If the kth root of $z^n - 1$ is given by $z_k = \cos\dfrac{2\pi k}{n} + i\sin\dfrac{2\pi k}{n}$, then $z_k + \bar{z}_k = \square$ and $z_k \bar{z}_k = \square$.

18 Using the results of question 17, $z^n \pm 1$, $n \geq 2$ has a quadratic factor

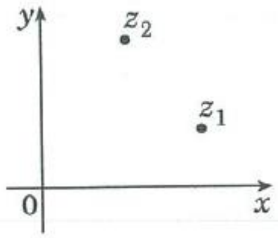
$$z^2 - \square z + \square = z^2 - \square z + \square$$

19 For the Argand diagram drawn below showing the complex numbers z_1 and z_2, draw the position of:

(a) $z_1 + z_2$

(b) $z_1 - z_2$

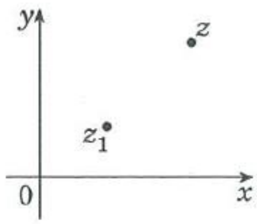
(c) $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$



20 The complex number iz is equivalent to a rotation of the complex number z through an angle of $\square$.

21 Show on an Argand diagram the position of: **(a)** iz **(b)** $z + iz$

22 Show on an Argand diagram the representation of $z - z_1 = r(\cos\theta + i\sin\theta)$, given the positions of z, z_1 as shown below:



***By means of a sketch, show the locus of the following*:**

23 **(a)** $|z| = r$ **(b)** the locus is $\boxed{}$

24 **(a)** $|z - z_0| = r$ **(b)** the locus is $\boxed{}$

25 **(a)** $|z - z_1| = |z - z_2|$ **(b)** the locus is $\boxed{}$

26 **(a)** $\arg(z - z_1) = \alpha$, a constant **(b)** the locus is $\boxed{}$

27 **(a)** $\arg(z - z_1) - \arg(z - z_2) = \alpha$, **(b)** the locus is $\boxed{}$

alternatively $\arg\left(\dfrac{z - z_1}{z - z_2}\right) = \alpha$,

where $0 < \alpha < \pi$ and z_1, z_2 represent two fixed points A, B.

28 **(a)** $\arg z_1 = \pm\dfrac{\pi}{2}$ **(b)** the locus is $\boxed{}$

29 **(a)** $\arg z_1 = 0$ or π **(b)** the locus is $\boxed{}$

Answers to formula test

1 $z = x + iy$

2 **(a)** $a = c$ and $b = d$

(b) $(a+c)+i(b+d)$

(c) $(a-c)+i(b-d)$

(d) $(ac-bd)+i(ad+bc)$

(e) $\dfrac{a+ib}{c+id} \times \dfrac{c-id}{c-id} = \dfrac{(ac+bd)+i(bc-ad)}{c^2+d^2}$

3 $i^2 = -1;\ \ i^3 = -i;\ \ i^4 = 1$

4 $\bar{z} = x - iy$

5 $z = r(\cos\theta + i\sin\theta);\ \ r = |z| = \sqrt{x^2+y^2};\ \ \tan\theta = \dfrac{y}{x}$ for $-\pi \le \theta \le \pi$

6 **(a)** $z_1 z_2 = r_1 r_2[\cos(\theta+\phi)+i\sin(\theta+\phi)];\ \ |z_1 z_2| = |z_1||z_2|$

(b) $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta-\phi)+i\sin(\theta-\phi)];\ \ \left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

(c) $\arg(z_1 z_2) = \arg z_1 + \arg z_2 \pm (2\pi)$

(d) $\arg\left(\dfrac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 \pm (2\pi)$

7 $z^n = r^n(\cos n\theta + i\sin n\theta)$

8 $z^n = \cos n\theta + i\sin n\theta$; De Moivre's theorem

9 **(a)** $\cos n\theta = c^n - {}^nC_2\, c^{n-2} s^2 + \cdots$

(b) $\sin n\theta = {}^nC_1\, c^{n-1} s - {}^nC_3\, c^{n-3} s^3 + \cdots$ where $c = \cos\theta,\ s = \sin\theta$

10 $z = r^{\frac{1}{n}}\left[\cos\left(\dfrac{\theta+2k\pi}{n}\right)+i\sin\left(\dfrac{\theta+2k\pi}{n}\right)\right]$ for $k = 0, 1, \ldots, (n-1)$; radius $= r^{\frac{1}{n}}$; sector angle $= \dfrac{2\pi}{n}$

11 $a + ib = z^2;\ \ a = x^2 - y^2;\ \ b = 2xy$

12 $z = \cos\dfrac{2k\pi}{n} + i\sin\dfrac{2k\pi}{n};\ \ k = 0, 1, 2, \ldots, n-1$

13 n sides; $|z| = 1$ with $x^2 + y^2 = 1$; vertex at $z = 1$

14 **(a)** $|z| = |\bar{z}| = \sqrt{x^2+y^2}$

(b) $\arg\bar{z} = -\arg z$

(c) $z + \bar{z} = 2x$

(d) $z - \bar{z} = 2yi$

(e) $z\bar{z} = |z|^2 = |\bar{z}|^2 = x^2 + y^2$

(f) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

(g) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

(h) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}$

(i) $\dfrac{1}{z} = z^{-1} = \dfrac{\bar{z}}{|z|^2}$

15 **(a)** $z^n+1 = (z+1)\left(z^{n-1}-z^{n-2}+\cdots-z+1\right)$ **(b)** $z^n-1 = (z-1)\left(z^{n-1}+z^{n-2}+\cdots+z+1\right)$

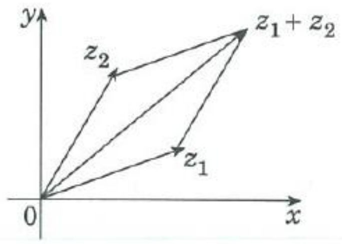
(c) $z^n-1 = (z-1)(z+1)\left(z^{n-2}+z^{n-4}+\cdots+z^2+1\right)$

16 **(a)** $\dfrac{z^n+1}{z+1} = z^{n-1}-z^{n-2}+z^{n-3}-\cdots-z+1$ **(b)** $\dfrac{z^n-1}{z-1} = z^{n-1}+z^{n-2}+\cdots+z+1$

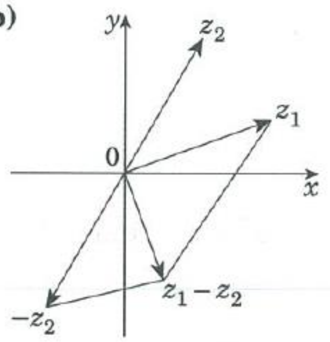
17 $z_k+\bar{z}_k = 2\cos\dfrac{2k\pi}{n}$; $z_k\bar{z}_k = 1$

18 $z^2-\left(z_k+\bar{z}_k\right)z+z_k\bar{z}_k = z^2-\left(2\cos\dfrac{2k\pi}{n}\right)z+1$

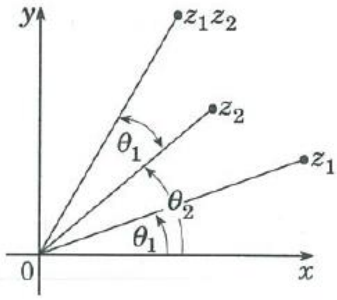
19 **(a)**



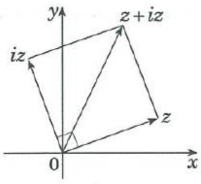
(b)



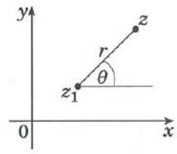
(c)



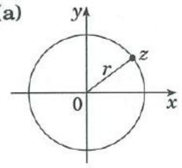
21



22

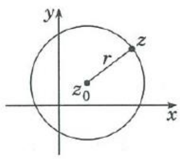


23 **(a)**



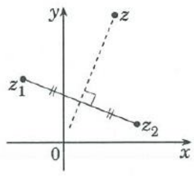
(b) a circle $x^2+y^2 = r^2$

24 **(a)**



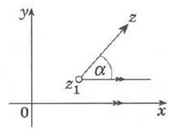
(b) A circle $(x-a)^2+(y-b)^2 = r^2$ with centre (a, b), given $z_0 = a+ib$

25 **(a)**



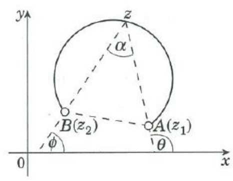
(b) The perpendicular bisector of the line joining z_1, z_2

26 **(a)**



(b) The half ray commencing at z_1, making an angle α parallel to the x axis, the point z_1 not included.

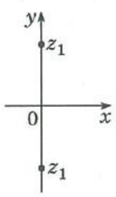
27 **(a)**



(b) The arc of a circle on the chord AB which subtends angle α at the circumference.

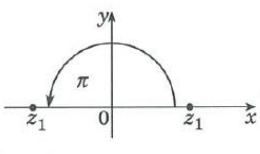
$\arg(z-z_1) = \theta$, $\arg(z-z_2) = \phi$, and $\alpha = \theta-\phi$, with A, B excluded.

28 **(a)**



(b) The purely imaginary number z_1

29 **(a)**



(b) The purely real number z_1