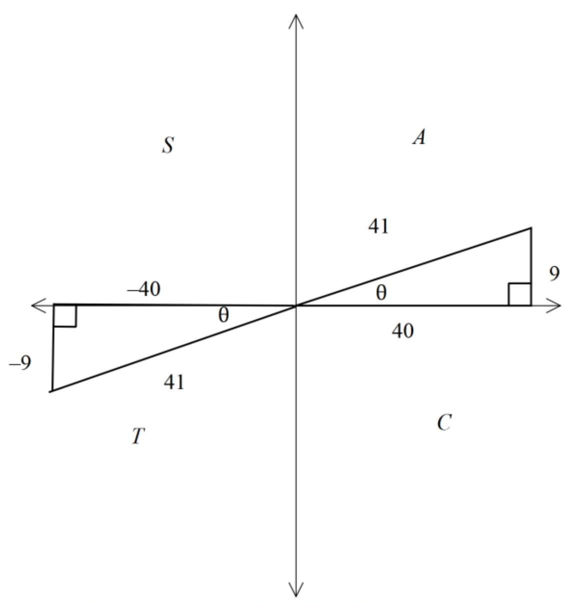
TRIAL HSC 2019

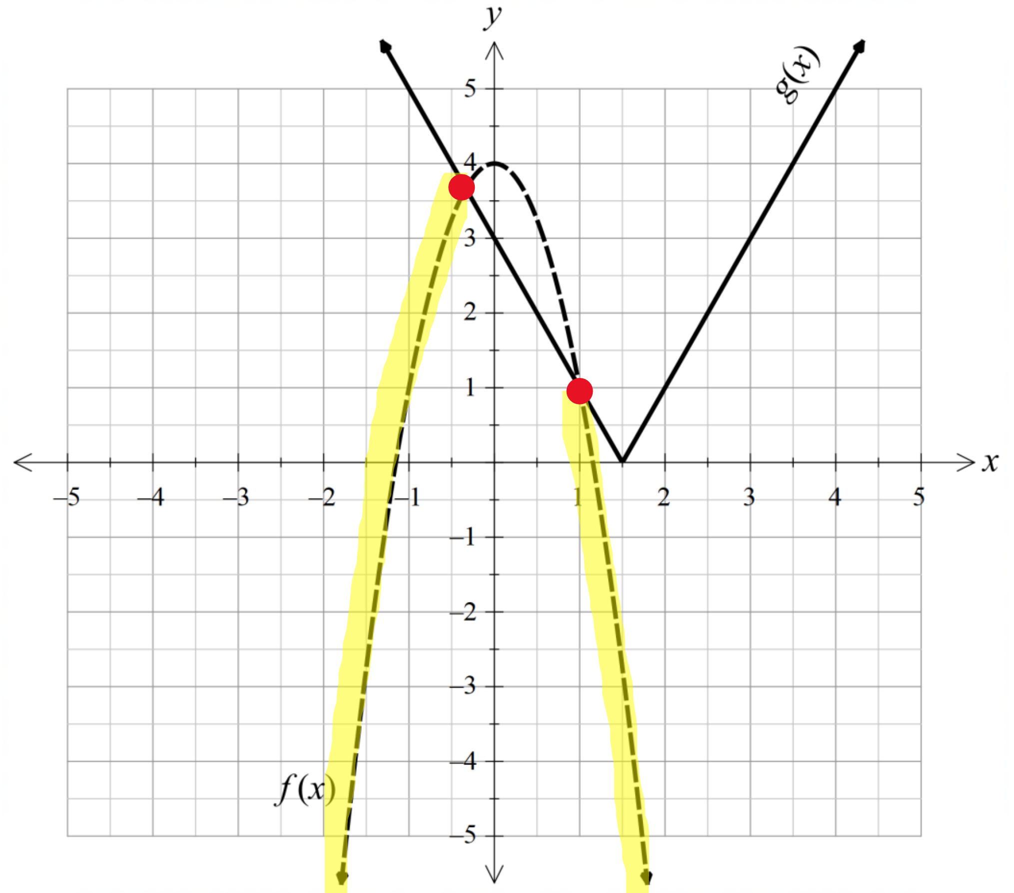
Mathematics

Examination

SOLUTIONS

Multiple Choice Worked Solutions		
No	**Working**	**Answer**
1.	$e^{-3\cdot 2} = 0\cdot 04076220398$ $\approx\ 0\cdot 041$ to two significant figures	**D**
2.	$y = 2x^2 - 3x + 1$ $y' = 4x - 3$ when $x = -2$, $y' = 4(-2) - 3$ $= -11$	**A**
3.	Let $y = \ln(\sin x)$ Let $u = \sin x$ $y = \ln u$ $\dfrac{du}{dx} = \cos x$ $\dfrac{dy}{du} = \dfrac{1}{\sin x}$ $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$ $= \dfrac{1}{\sin x} \times \cos x$ $= \dfrac{\cos x}{\sin x}$ $= \cot x$	**C**
4.	 Since tan is positive and sin is negative we are considering theta in the third quadrant. $\therefore \cos\theta = -\dfrac{40}{41}$	**C**

5.	$(x-2)^2 + (y-k)^2 = 25$ is circle with centre $(2, k)$ and radius 5 units. The distance between the line $3x - 4y - 1 = 0$ and the centre of the circle must be 5 units. $d = \dfrac{\lvert ax_1 + by_1 + c\rvert}{\sqrt{a^2 + b^2}}$ $5 = \dfrac{\lvert 3(2) + (-4)(k) - 1\rvert}{\sqrt{3^2 + (-4)^2}}$ $5 = \dfrac{\lvert 6 - 4k - 1\rvert}{\sqrt{25}}$ $25 = 5 - 4k$ $20 = -4k$ $k = -5$ $\therefore$ centre at $(2, -5)$	**B**
6.	Since P remains equidistant from two fixed points as it moves, it must follow a straight line. The only equation that shows a linear function in the options given is $3x + 2y - 5 = 0$.	**A**
7.	Since $\pi = 180^\circ$ then $1^c = \dfrac{180}{\pi}$ So $1.249^c = 1.249 \times \dfrac{180}{\pi}$ $\approx 71.6^\circ$	**C**

8.	 The highlighted section of graph shows where the absolute value function $g(x)$ is above (greater in value than) the parabola $f(x)$. We can see an intersection of the graphs at (1, 1). We can see an intersection at $\left(-\frac{1}{3}, \frac{11}{3}\right)$ So using the x values from these points, the solution is to the left of $x = -\frac{1}{3}$ and to the right of $x = 1$. As the inequality was $\geq$ we also include the two x values. The correct solution : $x \leq -\frac{1}{3}$ and $x \geq 1$.	**B**

9.	From the graph, we see that $v = -\frac{t}{2} + 4$ since the gradient is $-\frac{1}{2}$ and the y-intercept is 4. $\frac{dx}{dt} = -\frac{t}{2} + 4$ $x = \int -\frac{t}{2} + 4 \text{ dt}$ $= -\frac{t^2}{4} + 4t + C$ when $t = 0$, $x = -2$ $\therefore C = -2$ $\therefore x = -\frac{t^2}{4} + 4t - 2$ Maximum displacement occurs when $\frac{dx}{dt} = 0$. This is at $t = 8$ as indicated in the graph. $\therefore$ max displacement $= -\frac{8^2}{4} + 4(8) - 2$ $= 14$ m	**C**
10.	Now $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$ $\therefore \int -\operatorname{cosec}^2 x\,dx = \cot x + C$ $V = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \operatorname{cosec}^2 x \ dx$ $= -\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} -\operatorname{cosec}^2 x \ dx$ $= -\pi\left[\cot x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= -\pi(-\sqrt{3})$ $= \sqrt{3}\,\pi$ cubic units	**A**

Trial HSC Examination 2019
Mathematics Course

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

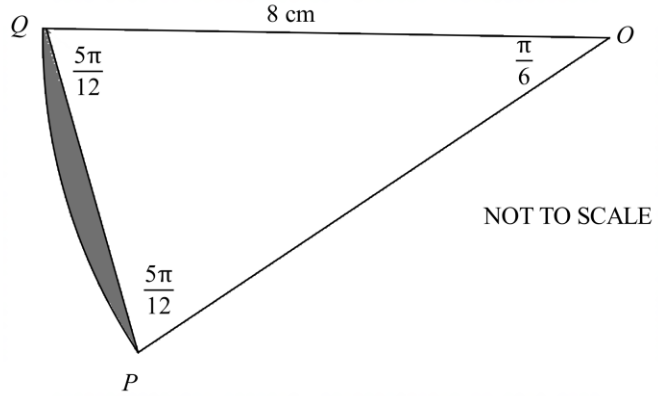
If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

1.	A ○	B ○	C ○	D ●
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ●	D ○
4.	A ○	B ○	C ●	D ○
5.	A ○	B ●	C ○	D ○
6.	A ●	B ○	C ○	D ○
7.	A ○	B ○	C ●	D ○
8.	A ○	B ●	C ○	D ○
9.	A ○	B ○	C ●	D ○
10.	A ●	B ○	C ○	D ○

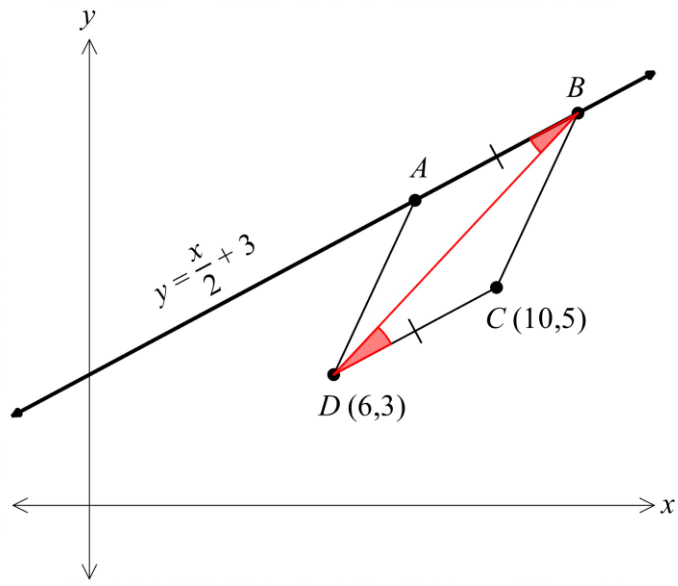
Question 11		2019	
	Solution	Marks	Allocation of marks
a).	$\frac{2}{\sqrt{3}-2} = \frac{2}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$ $= \frac{2\sqrt{3}+4}{3-4}$ $= \frac{2\sqrt{3}+4}{-1}$ $= -2\sqrt{3}-4$	2	2 marks for correct answer 1 mark for solution with correct multiplication by conjugate or similar merit
b).	let $u = 2x^3 \quad v = x-2$ $u' = 6x^2 \quad v' = 1$ $\frac{d}{dx}\left(\frac{2x^3}{x-2}\right) = \frac{vu'-uv'}{v^2}$ $= \frac{6x^2(x-2)-2x^3}{(x-2)^2}$ $= \frac{6x^3-12x^2-2x^3}{(x-2)^2}$ $= \frac{4x^3-12x^2}{(x-2)^2}$ $= \frac{4x^2(x-3)}{(x-2)^2}$	2	2 marks for correct answer 1 mark for solution with correct use of quotient rule or similar merit
c).	$\int \frac{x^2}{3x^3-1}\,dx = \frac{1}{9}\int \frac{9x^2}{3x^3-1}\,dx$ $= \frac{1}{9}\ln(3x^3-1) + C$	2	2 marks for correct answer I mark for solution expressing integral in some form involving $\frac{f'(x)}{f(x)}$,
d).	$-2-5x < 13$ $-5x < 15$ $x > -3$	1	1 mark for correct answer

	Solution	Marks	Allocation of marks
e).	$y = -x^2 + 2x + 4$ ① $y = -x$ ② substituting ② into ① gives: $-x^2 + 2x + 4 = -x$ $-x^2 + 3x + 4 = 0$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $\therefore$ $x = 4$ or $x = -1$ when $x = 4$, $y = -4$ when $x = -1$, $y = 1$ $\therefore$ points of intersection are (4, -4) and (-1,1)	3	3 marks for correct answer 2 marks for valid use of any method of solving simultaneous equations with a minor error in calculation or logic or equivalent merit 1 marks for valid working towards finding the value of one pronumeral only (*x* or *y* values) or equivalent merit
f).	Let $u = e^{3x}$ $v = \tan x$ $u' = 3e^{3x}$ $v' = \sec^2 x$ $\frac{d}{dx}(e^{3x} \tan x) = vu' + uv'$ $= 3e^{3x} \tan x + e^{3x} \sec^2 x$ $= e^{3x}(3\tan x + \sec^2 x)$	2	2 marks for correct solution. 1 mark for working towards solution using product rule.

	Solution	Marks	Allocation of marks
g).	**i).**  OP = OQ (equal radii) $\therefore \Delta OPQ$ is isosceles $\therefore \angle OPQ = \angle OQP = \dfrac{\pi - \frac{\pi}{6}}{2}$ $= \dfrac{5\pi}{12}$ By sine rule: $\dfrac{PQ}{\sin\frac{\pi}{6}} = \dfrac{8}{\sin\frac{5\pi}{12}}$ $PQ = \dfrac{8}{\sin\frac{5\pi}{12}} \times \sin\frac{\pi}{6}$ $= \dfrac{8 \times \frac{1}{2}}{\sin\frac{5\pi}{12}}$ $= \dfrac{4}{\sin\frac{5\pi}{12}}$	2	2 marks for correct solution 1 mark for correctly finding the missing angle/s in the triangle or using the sine rule or equivalent merit

	Solution	Marks	Allocation of marks
	ii). Perimeter of segment = chord PQ + arc PQ $Arc\ \text{PQ} = l$ $= r\theta$ $= \dfrac{8\pi}{6}$ $= \dfrac{4\pi}{3}$ $\text{chord PQ} = \dfrac{4}{\sin\frac{5\pi}{12}}$ $\therefore \text{perimeter} = \dfrac{4\pi}{3} + \dfrac{4}{\sin\frac{5\pi}{12}}$ $= 8{\cdot}329894926$ $\approx 8{\cdot}3\ \text{cm}$	1	1 mark for correct solution.

Question 12		2019	
	Solution	Marks	Allocation of marks
a).	$\dfrac{6x^3}{8x^3 - 27y^3} \times \dfrac{4x^2 - 9y^2}{8x^2 + 12xy}$ $= \dfrac{6x^3}{(2x - 3y)(4x^2 + 6xy + 9y^2)} \times \dfrac{(2x - 3y)(2x + 3y)}{4x(2x + 3y)}$ $= \dfrac{(3x^2)(\cancel{6x^3})}{\cancel{(2x - 3y)}(4x^2 + 6xy + 9y^2)} \times \dfrac{\cancel{(2x - 3y)}\cancel{(2x + 3y)}}{2(\cancel{4x})\cancel{(2x + 3y)}}$ $= \dfrac{3x^2}{2(4x^2 + 6xy + 9y^2)}$	3	3 marks for correct solution 2 marks for solution showing all correct factorisation without full simplification or equivalent merit 1 mark for some correct factorisation and simplification
b).	**i).** To have two equal roots, the discriminant is equal to zero. $b^2 - 4ac = 0$ $64 - 4(2)k = 0$ $64 - 8k = 0$ $-8k = -64$ $k = 8$	2	2 marks for correct solution. 1 mark for setting $\Delta = 0$ and attempting to solve equation, or similar merit
	ii). To have two distinct roots $\Delta > 0$ and to have rational roots Δ is a square number The smallest number for which $\Delta > 0$ and Δ is a square number is $\Delta = 1$. $\therefore\ 64 - 8k = 1$ $-8k = -63$ $k = \dfrac{63}{8}$	1	1 mark for correct answer
c).	**i).** AB has equation $y = \dfrac{x}{2} + 3$ $\therefore m_{AB} = \dfrac{1}{2}$ $m_{DC} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{5 - 3}{10 - 6}$ $= \dfrac{2}{4}$ $= \dfrac{1}{2}$ $= m_{AB}$ $\therefore AB \parallel CD$	1	1 mark for demonstrating equal gradients.

<table>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td></td><td>ii).

In ΔABD and ΔCDB :
BD is common side
$AB = CD$ (given)
$\angle ABD = \angle CDB$ (alternate angles on parallel lines)
$\therefore \Delta ABD \equiv \Delta CDB$ (SAS)</td><td>2</td><td>2 marks for congruence proof with all necessary reasoning included.

1 mark for working towards congruence proof with missing or incorrect statements or reasons</td></tr>
<tr><td></td><td>iii). Since $AB \| CD$ and $AB = CD$, quadrilateral ABCD must be a parallelogram.</td><td>1</td><td>1 mark for statement that includes the two minimum conditions.</td></tr>
<tr><td>d).</td><td>i). $T_5 = 22 \Rightarrow a + 4d = 22$ (1)
$S_5 = 50 \Rightarrow \frac{5}{2}(2a + 4d) = 50$ (2)

Simplifying (2) gives $5a + 10d = 50$ (3)
(1) × 5 gives $5a + 20d = 110$ (4)
(4) – (3) gives $10d = 60$
$\therefore d = 6$

substituting $d = 6$ into (1) gives $a + 24 = 22$
$\therefore a = -2$</td><td>2</td><td>2 marks for correct solutions for a and d.

1 mark for solution which includes setting up of appropriate equations to solve simultaneously</td></tr>
</table>

	Solution	Marks	Allocation of marks
	ii). $S_n > 1000$ $\frac{n}{2}(2a + (n-1)d) > 1000$ $\frac{n}{2}(-4 + 6(n-1)) > 1000$ $n(-4 + 6(n-1)) > 2000$ $-4n + 6n^2 - 6n - 2000 > 0$ $6n^2 - 10n - 2000 > 0$ By quadratic formula: Let $n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{10 \pm \sqrt{100 - 4(6)(-2000)}}{12}$ $= 19{\cdot}10976017$ or $-17{\cdot}4430935$ Since n must be a positive integer, $n > 19{\cdot}1$ So we need 20 terms to reach a sum greater than 1000.	2	2 marks for correct solution 1 mark for setting up quadratic inequation from S_n formula and attempting to solve.
e).	$Area = \frac{1}{2}\text{absinC}$ $= \frac{1}{2} \times 71 \times 125 \times \sin 48°$ $= 3297{\cdot}705163$ $\approx 3298 \text{ m}^2$	1	1 mark for correct answer

<table>
<tr><td colspan="2">Question 13</td><td colspan="2">2019</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>a).</td><td>i). $y = 2x^3 - 9x^2 + 12x - 5$
Stationary points at $y' = 0$
$y' = 6x^2 - 18x + 12$
Let $y' = 0$
$6x^2 - 18x + 12 = 0$
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$\therefore$ $x = 1$ or $x = 2$

When $x = 1$:
$y = 2(1)^3 - 9(1)^2 + 12(1) - 5$
$= 0$
When $x = 2$:
$y = 2(2)^3 - 9(2)^2 + 12(2) - 5$
$= -1$
$\therefore$ stationary points at $(1,0)$ and $(2,-1)$

To determine the nature of the stationary points consider y''.
$y'' = 12x - 18$
When $x = 1$:
$y'' = 12 - 18$
< 0
$\therefore$ maximum turning point at $(1,0)$
When $x = 2$:
$y'' = 24 - 18$
> 0
$\therefore$ minimum turning point at $(2,-1)$</td><td>4</td><td>4 marks for finding the two correct turning points and correctly stating their nature, using either 1st or 2nd derivative to determine max/min.

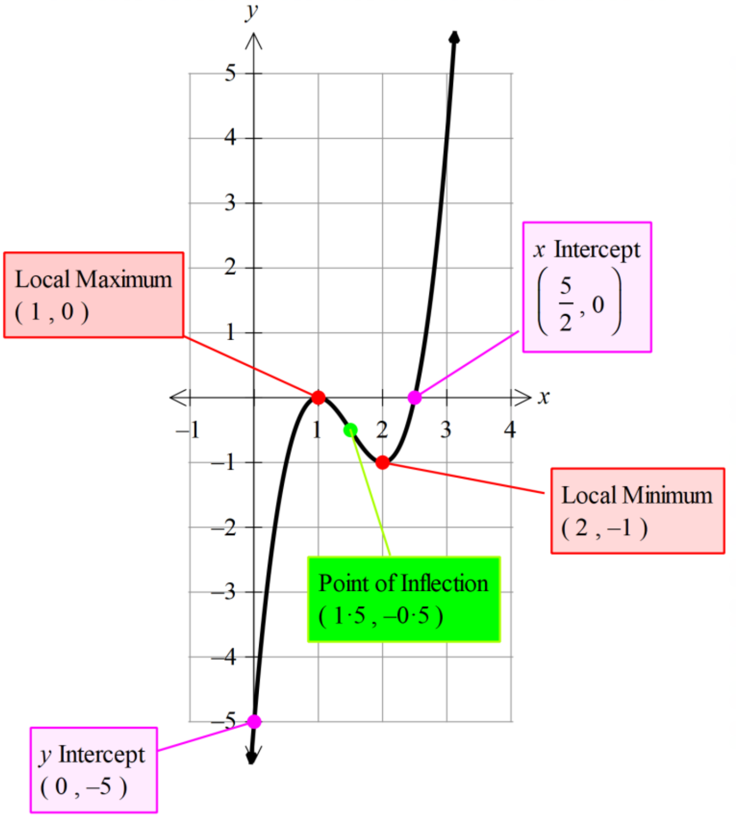
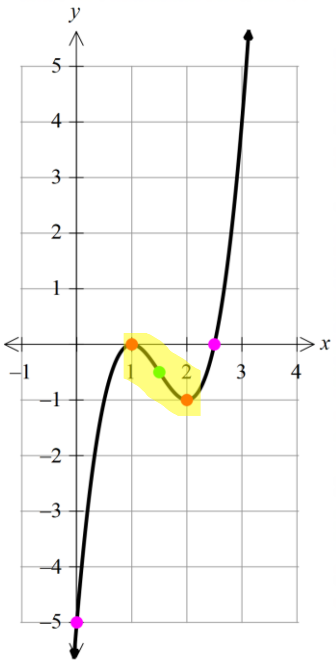
3 marks for correctly finding the stationary points and attempting to determine their nature or similar merit

2 marks for finding derivative and identifying that y’=0 at stationary points or similar merit

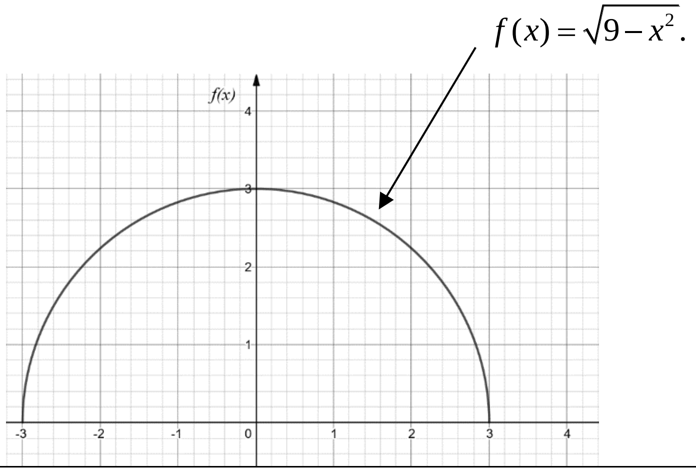
1 mark for finding the derivative or similar merit

Valid method used for identifying nature of turning points. (Could test y' either side of the points instead.)</td></tr>
<tr><td></td><td>ii). Inflexions have $y'' = 0$
and concavity changes either side
$y'' = 12x - 18$
When $x = 1{\cdot}5$,
$y'' = 12(1{\cdot}5) - 18$
$= 0$
From (ii) we know:
when $x = 1$, $y'' < 0$, concave down
when $x = 2$, $y'' > 0$, concave up
$\therefore$ inflexion point occurs when $x = 1{\cdot}5$</td><td>2</td><td>2 marks for correct inflexion point found and tested

1 mark for y'' and setting $y'' = 0$ or similar merit</td></tr>
</table>

	Solution	Marks	Allocation of marks
	iii). 	2	2 marks for graph which has the correct shape and all the important features clearly shown 1 mark if graph has correct shape but not all features included or similar merit
	iv).  From inspection of the graph, we can see that the function is decreasing $\left(\frac{dy}{dx} < 0\right)$ between $x = 1$ and $x = 2$.	1	1 mark for correct x-values from graph or working.

	Solution	Marks	Allocation of marks
b).	**i).** $P = Ae^{kt}$ $\dfrac{dP}{dt} = kAe^{kt}$ $= kP$	1	1 mark for correct expression
	ii). In 2000 at $t = 0$, $P = 19\ 000\ 000$ $P = Ae^{kt}$ $19\ 000\ 000 = Ae^{0}$ $\therefore\ A = 19\ 000\ 000$	1	1 mark for correct answer
	iii). In 2000, $t = 0$ At $t = 0$, $P = 19\ 000\ 000$ At $t = 19$, $P = 25\ 000\ 000$ $P = Ae^{kt}$ $25\ 000\ 000 = 19\ 000\ 000e^{19k}$ $\dfrac{25}{19} = e^{19k}$ $19k = \ln\left(\dfrac{25}{19}\right)$ $k = \dfrac{\ln\left(\frac{25}{19}\right)}{19}$ $= 0{\cdot}014444$ $\approx\ 0{\cdot}0144$	2	2 marks for valid working which leads to the required calculator solution 1 mark for setting up correct exponential equation or similar merit
	iv). In 2050, $t = 50$ $P = 19\ 000\ 000e^{50k}$ $= 19\ 000\ 000e^{50 \times 0{\cdot}0144}$ $= 39\ 120\ 288$ (or $39\ 034\ 231$ using rounded value) $< 50\ 000\ 000$ The article's claim is incorrect as the population will not have reached 50 million by 2050.	2	2 marks for correct deduction based on valid calculations 1 mark for appropriate calculations with incorrect conclusion or similar merit

Question 14		**2019**	
	Solution	Marks	Allocation of marks
a.	**i).** The domain D: $\{ -3 \le x \le 3 \}$	1	1 mark for correct answer
	ii). 	2	2 marks for correct sketch with all important features
	iii). The range R: $\{ 0 \le f(x) \le 3 \}$	1	1 mark for correct answer
b.	**i).** $y = 1 + 2\sqrt{x}$ At P, $x = 4$ $\therefore y = 1 + 2\sqrt{4}$ $= 5$ $\therefore B$ is at $(0, 5)$	1	1 mark for correct answer

	Solution	Marks	Allocation of marks
	ii). $y = 1 + 2\sqrt{x}$ $y - 1 = 2\sqrt{x}$ $\frac{y-1}{2} = \sqrt{x}$ $x = \left(\frac{y-1}{2}\right)^2$ $V = \pi\int_1^5 \left(\frac{y-1}{2}\right)^4 dy$ $= \frac{\pi}{2^4}\int_1^5 (y-1)^4 \; dy$ $= \frac{\pi}{16}\left[\frac{(y-1)^5}{5}\right]_1^5$ $= \frac{\pi}{80}[(5-1)^5 - (1-1)^5]$ $= \frac{\pi}{80} \times 4^5$ $= \frac{1024\pi}{80}$ $= \frac{64\pi}{5}$ cubic units	3	3 marks for correct answer correct answer 2 marks for answer which includes rearranging to make *x* the subject and correct set up of volume integral followed by incorrect result or working of equal merit 1 mark for answer which includes attempt at rearranging to make *x* the subject and/or attempt set up of volume integral or equal merit
c.	**i).** $v = -\frac{7}{t+1}$ when $t = 0$ $v = -\frac{7}{0+1}$ $= -7 \; ms^{-1}$	1	1 mark for correct answer

	Solution	Marks	Allocation of marks
	ii). $x = \int v \ \mathrm{dt}$ $= \int -\dfrac{7}{t+1} \ \mathrm{dt}$ $= -7\int \dfrac{1}{(t+1)} \mathrm{dt}$ $= -7 \ln(t+1) + C$ When $t = 0$, $x = 8$ $8 = -7 \ln(1) + C$ $8 = 0 + C$ $C = 8$ $\therefore \ x = -7 \ln(t+1) + 8$ When $t = 3$, $x = -7 \ln(4) + 8$ $= -1{\cdot}704060528$ $\approx -1{\cdot}7$ m	3	3 marks for finding correct integral to obtain x and evaluating it correctly for $t = 3$ 2 marks for correct integration but not evaluated correctly or equivalent merit 1 mark for some relevant work to find expression for x or equivalent merit
	iii). $a = \dfrac{d^2x}{dt^2}$ $= \dfrac{d}{dt} v$ $= \dfrac{d}{dt}\left(-\dfrac{7}{t+1}\right)$ $= \dfrac{d}{dt} - 7(t+1)^{-1}$ $= 7(t+1)^{-2}$ $= \dfrac{7}{(t+1)^2}$ For any value of t, the denominator will be positive. The numerator is positive. Therefore, acceleration will always take a positive value.	2	2 marks for correct expression for acceleration followed by an appropriate explanation 1 marks for correct expression for acceleration with no appropriate explanation or equivalent merit
	iv). For the particle to be at rest, its velocity must be zero. On inspection of the expression for velocity, we see that $v \neq 0$ for any values of t. Therefore the particle will never be at rest. $-\dfrac{7}{t+1} = 0$ $-7 = 0$ (multiplying both sides by $(t+1)$) This is impossible. $\therefore \ -\dfrac{7}{t+1} \neq 0$	1	1 mark for an appropriate explanation

<table>
<tr><td colspan="2">Question 15</td><td colspan="2">2019</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>a.</td><td>
$$\begin{aligned} 2\cos^2 x + \sin x &= 2 \\ 2\cos^2 x + \sin x - 2 &= 0 \\ 2(1 - \sin^2 x) + \sin x - 2 &= 0 \\ 2 - 2\sin^2 x + \sin x - 2 &= 0 \\ -2\sin^2 x + \sin x &= 0 \\ -\sin x(2\sin x - 1) &= 0 \\ \therefore \quad \sin x &= 0 \\ x &= 0, \pi \text{ or } 2\pi \end{aligned}$$
$$\begin{aligned} \text{or } 2\sin x &= 1 \\ \sin x &= \frac{1}{2} \\ x &= \frac{\pi}{6} \text{ in 1st quadrant or } \frac{5\pi}{6} \text{ in 2nd quad} \end{aligned}$$
$$\therefore \quad x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi \text{ and } 2\pi$$
</td><td>3</td><td>3 marks for all four solutions

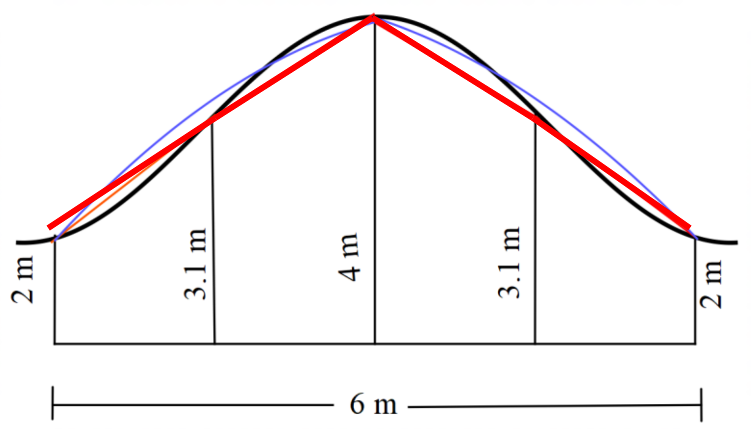
2 mark for rearranging to quadratic in terms of sin x and attempting to solve or similar merit

1 mark for some valid and relevant working toward solution</td></tr>
<tr><td>b.</td><td>i).
$$\begin{aligned} \$A_1 &= 600\,000 + 0\cdot003 \times 600\,000 - M \\ &= 600\,000(1\cdot003) - M \\ \$A_2 &= \$A_1 \times 1\cdot003 - M \\ &= 600\,000(1\cdot003)^2 - M(1\cdot003) - M \\ &= 600\,000(1\cdot003)^2 - M(1\cdot003 + 1) \end{aligned}$$
</td><td>1</td><td>1 mark for correct development to formula</td></tr>
<tr><td></td><td>ii).
$$\begin{aligned} \$A_3 &= \$A_2 \times 1\cdot003 - M \\ &= 600\,000(1\cdot003)^3 - M(1\cdot003 + 1)(1\cdot003) - M \\ &= 600\,000(1\cdot003)^3 - M(1\cdot003^2 + 1\cdot003 + 1) \end{aligned}$$
$$\therefore \$A_n = 600\,000(1\cdot003)^n - M(1\cdot003^n + 1\cdot003^{n-1} + \ldots + 1\cdot003 + 1)$$
Since $1 + 1\cdot003 + 1\cdot003^2 + \ldots + 1\cdot003^{n-1} + 1\cdot003^n$ is a geometric series with $a = 1$ and $r = 1\cdot003$ we can find the sum of n terms:
$$\begin{aligned} S_n &= \frac{a(r^n - 1)}{r - 1} \\ &= \frac{1\cdot003^n - 1}{0\cdot003} \end{aligned}$$
$$\therefore \$A_n = 600\,000(1\cdot003)^n - M\left(\frac{1\cdot003^n - 1}{0\cdot003}\right)$$
</td><td>1

1</td><td>2 marks for setting up $\$A_n$ using repeated steps and getting to series form and using sum of geometric series formula to simplify expression to the one required.

1 mark for some relevant working using series to develop the required expression</td></tr>
</table>

	Solution	Marks	Allocation of marks
	iii). $\$A_n = 600\ 000(1\cdot003)^n - 2800\left(\frac{1\cdot003^n - 1}{0\cdot003}\right)$ Let $\$A_n = 0$ to find when balance owing is zero. $600\ 000(1\cdot003)^n - 2800\left(\frac{1\cdot003^n - 1}{0\cdot003}\right) = 0$ $\frac{0\cdot003 \times 600\ 000(1\cdot003)^n - 2800(1\cdot003^n - 1)}{0\cdot003} = 0$ $0\cdot003 \times 600\ 000(1\cdot003)^n - 2800(1\cdot003)^n + 2800 = 0$ $1\cdot003^n(0\cdot003 \times 600\ 000 - 2800) = -2800$ $1\cdot003^n = -\frac{2800}{0\cdot003 \times 600\ 000 - 2800}$ $1\cdot003^n = 2\cdot8$ $n = \log_{1\cdot003} 2\cdot8$ $= \frac{\ln 2\cdot8}{\ln 1\cdot003}$ $= 343\cdot7210251$ ≈ 344 months Repayments total over 360 months $360 \times 2728 = \$982\ 080$ Repayments total over 344 months $344 \times 2\ 800 = \$963\ 200$ Savings = $982\ 080 - 963\ 200$ $= \$18\ 880$	4	4 marks for working which includes finding the value of *n* and appropriate calculations to find the savings amount 3 marks for working that includes most correct steps required to find *n*, the repayments and the savings, but which includes an error or is incomplete 2 marks for substantial correct working towards an expression for *n* and working towards the total repayments 1 mark for some relevant working towards an expression with *n* as the subject
c.	**i).** Because of symmetry, we can calculate application of Simpsons Rule and then double it. $Area = 2 \times \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$ $= 2 \times \frac{3}{6}(2 + 4(3\cdot1) + 4)$ $= 18\cdot4\ m^2$	1	1 mark for correct answer
	ii). As in **i)**, we can use symmetry to reduce calculations. We find two trapezia to the left and double. Area of each strip $= \frac{b-a}{2}[f(a) + f(b)]$ Area $= 2 \times \left(\frac{1\cdot5}{2}[(2 + 3\cdot1) + (3\cdot1 + 4)]\right)$ $= 18\cdot3\ m^2$	1	1 mark for correct answer

	Solution	Marks	Allocation of marks
	iii). Area $= 2 \times \int_0^3 (3 + \cos x)\ dx$ $= 2 \times \left[3x + \sin x\right]_0^3$ $= 2[(9 + \sin 3) - (0)]$ $= 18{\cdot}28224002$ $\approx 18{\cdot}28\, m^2$	2	2 marks for correct answer 1 mark for relevant working including integral toward the area
	iv).  Trapezoidal rule finds the area between a straight line and the *x*-axis by finding the area of trapezia while Simpsons rule finds the area between a parabolic arc and the *x*-axis.	1	1 mark for any correct explanation that mentions the different way that the Trapezoidal Rule and Simpson's Rule approximate areas

<table>
<tr><td colspan="2">Question 16</td><td colspan="2">2019</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>a.</td><td>$20y = x^2 - 4x + 24$
$20y = (x-2)^2 + 20$
$(x-2)^2 = 20y - 20$
$(x-2)^2 = 20(y-1)$
$\therefore$ vertex at $(2, 1)$ focal length = 5
parabola is in the form $(x-h)^2 = 4a(y-k)$
$\therefore$ concave up
Focus is at (2, 6)</td><td>2</td><td>2 marks for correct focus

1 mark for working which attempts to rearrange the form of the parabola into standard form and to find focal length and focus</td></tr>
<tr><td>b.</td><td>i). The lengths of all the pieces of frame, including the semicircular arc must total 24 metres.
$\therefore 2(w+h) + \frac{\pi w}{2} = 24$
$2w + 2h + \frac{\pi w}{2} = 24$
$2h = 24 - 2w - \frac{\pi w}{2}$
$h = 12 - w - \frac{\pi w}{4}$
$= 12 - w\left(1 + \frac{\pi}{4}\right)$ metres</td><td>2</td><td>2 marks for setting up correct perimeter statement and manipulating it to the required factorised expression

1 mark for obtaining an expression for perimeter and some working towards the required formula</td></tr>
<tr><td></td><td>ii). $P = 60 \times \text{area of rectangle} + 10 \times \text{area of semicircle}$
$= 60wh + \frac{10\pi w^2}{8}$
$= 60w\left[12 - w\left(1 + \frac{\pi}{4}\right)\right] + \frac{10\pi w^2}{8}$
$= 720w - 60w^2 - \frac{60w^2\pi}{4} + \frac{5\pi w^2}{4}$
$= 720w - 60w^2 - \frac{55w^2\pi}{4}$
$= 720w - 60w^2 - \frac{110w^2\pi}{8}$
$= 720w - 10w^2\left(6 + \frac{11\pi}{8}\right)$ dollars</td><td>2</td><td>2 marks for setting up correct profit statement and manipulating it to the required factorised expression

1 mark for obtaining an expression for profit and some working towards the required formula</td></tr>
</table>

	Solution	Marks	Allocation of marks
	iii). $P' = 720 - 20w\left(6 + \frac{11\pi}{8}\right)$ Let $P' = 0$ to find stationary points $720 - 20w\left(6 + \frac{11\pi}{8}\right) = 0$ $20w\left(6 + \frac{11\pi}{8}\right) = 720$ $w\left(6 + \frac{11\pi}{8}\right) = 36$ $w = \dfrac{36}{6 + \frac{11\pi}{8}}$ $= 3{\cdot}4884769..$ $\approx 3{\cdot}5$ metres Check that this is a maximum turning point: $P'' = -20\left(6 + \frac{11\pi}{8}\right)$ < 0 for all w $\therefore$ concave down and maximum turning point Substitute found value of w into equation for h $h = 12 - w\left(1 + \frac{\pi}{4}\right)$ $= 12 - 3{\cdot}488\left(1 + \frac{\pi}{4}\right)$ $= 5{\cdot}7716797...$ $\approx 5{\cdot}8$ metres Solution $w \approx 3.5$ metres and $h \approx 5.8$ metres	3	3 marks for correct differentiation and finding values of w and h 2 marks for a solution with minor error(s) but which includes differentiation, checking that the turning point is a maximum and finding values for w and h 1 mark for some relevant working involving differentiation and solving a resulting equation
c.	**i).** In $\Delta\, ACY$ and $\Delta\, AZW$: $\angle A$ is common $\angle AZW = \angle ACY$ (corresponding angles on parallel lines ZX and CB) $\therefore \Delta\, ACY \parallel\!\mid \Delta\, AZW$ (2 matching angles are equal)	2	2 marks for complete and correct similarity proof 1 mark for some working towards a similarity proof
	ii). $\frac{AZ}{AC} = \frac{AW}{AY} = \frac{AX}{AB}$ (ratio of intercepts on parallel lines) $\frac{AX}{AB} = \frac{p}{p+q}$ $\therefore \frac{AZ}{AC} = \frac{p}{p+q}$	2	2 marks for setting up correct ratios and obtaining the required expression by representing AB as $p + q$ 1 mark for setting up correct ratios and some working toward required solution

	Solution	Marks	Allocation of marks
	iii). The ratio of side lengths in ΔAXZ and ΔACB is $\frac{p}{p+q.}$ $\therefore$ the ratio of the areas is $\left(\frac{p}{p+q}\right)^2$. $\therefore \frac{A_2}{A_1} = \left(\frac{p+q}{p}\right)^2$ $A_2 = A_1\left(\frac{p+q}{p}\right)^2$ Area of trapezium $= A_2 - A_1$ $= \left(\frac{p+q}{p}\right)^2 A_1 - A_1$ $= A_1\left(\left(\frac{p+q}{p}\right)^2 - 1\right)$	2	2 marks for correct reasoning to obtain the required result 1 mark for relevant working which includes the ratio of areas being the square of the ratio of sides