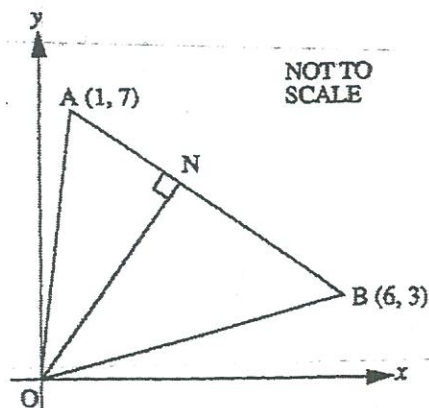


### QUESTION 1 (30marks)

(a) [5marks]



(i) Find the gradient of AB.

$$m = \frac{7-3}{1-6} \\ = -\frac{4}{5}$$

(ii) Calculate the length of AB in exact form.

$$AB = \sqrt{(6-1)^2 + (3-7)^2} \\ = \sqrt{5^2 + (-4)^2} \\ = \sqrt{41}$$

(iii) Find the equation of the line ON.

$$y - y_1 = m(x - x_1) \\ y - 0 = \frac{5}{4}(x - 0) \\ y = \frac{5}{4}x$$

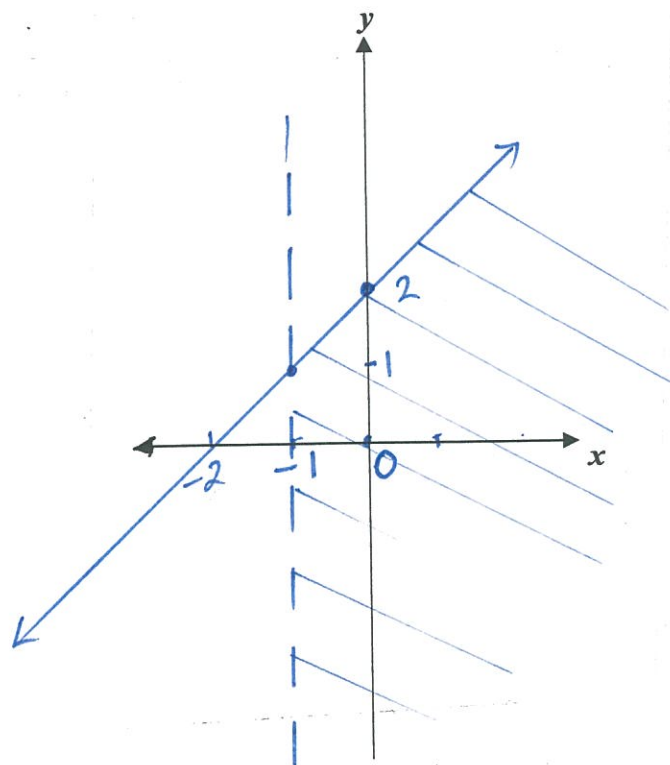
OR  $5x - 4y = 0$

(iv) Find the midpoint of OA.

$$M = \left( \frac{0+1}{2}, \frac{0+7}{2} \right) = \left( \frac{1}{2}, \frac{7}{2} \right)$$

(b) Graph the region defined by

$$y \leq x + 2 \text{ and } x > -1 \quad [3\text{marks}]$$



Test (0,0)

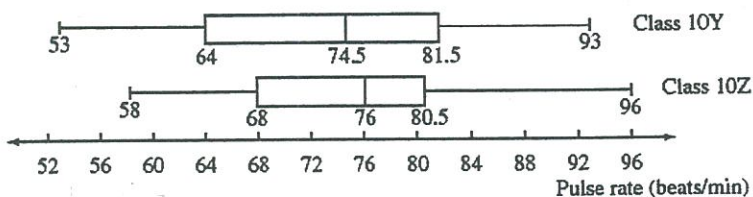
(c) For this question circle A, B, C or D. [1mark]

Six kitchen hands at a restaurant earn the following weekly wages: \$520, \$610, \$610, \$630, \$660, \$710. How will another wage of \$400 affect the mean and the standard deviation (SD) of this set of data?

- A. The mean and SD will both increase.
- B. The mean and SD will both decrease
- C. The mean will increase and the SD will decrease.
- ☒ D. The mean will decrease and the SD will increase.

- (d) Two PE classes of students have their pulse rates (in beats per minute) measured. The results are shown in the box-and-whisker plots.

[6marks]



- (i) What is the lowest pulse rate across both classes? **53 beats/min**

- (ii) By how much do the medians differ?

$$76 - 74.5 = 1.5$$

- (iii) Find the interquartile range for class 10Z

$$80.5 - 68 = 12.5$$

- (iv) If there are 24 students in class 10Y how many had a pulse rate between 64 and 93?

$$75\% \times 24 = 18$$

- (v) Which class had the more consistent results? Give a reason for your answer.

$$10Y \quad \text{Range} = 40 \\ \text{IQR} = 17.5$$

$$10Z \quad \text{Range} = 38 \\ \text{IQR} = 12.5$$

**Class 10Z had more consistent results because pulse rates are less spread out**

- (e) [2marks]

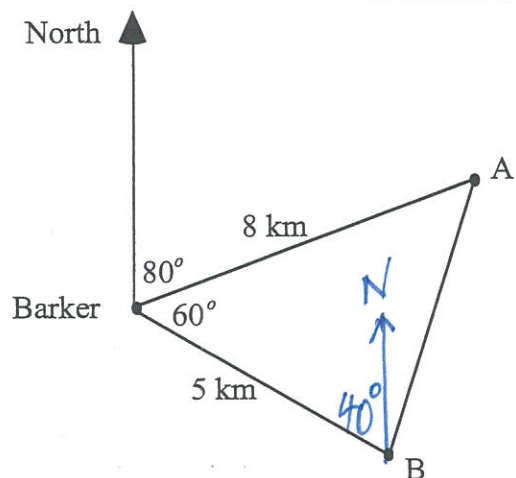
	Mark	$\bar{x}$	$\sigma_n$
Maths	63	70	7
Science	55	60	10

Julie compared her test results in Science and Maths. In which subject did Julie perform better? Give a reason for your answer.

**Julie performed better in Science, because her result in science is less than one standard deviation below the mean.**

**In Maths it is exactly one SD below mean.**

- (f) Molly leaves point A and walks 8km to Barker. Tom leaves point B and walks 5km to Barker. [3marks]



- (i) Use the cosine rule to show the distance from A to B is 7km.

$$AB^2 = 8^2 + 5^2 - 2(8)(5) \cos 60^\circ$$

$$= 49$$

$$AB = \sqrt{49}$$

$$= 7 \text{ km}$$

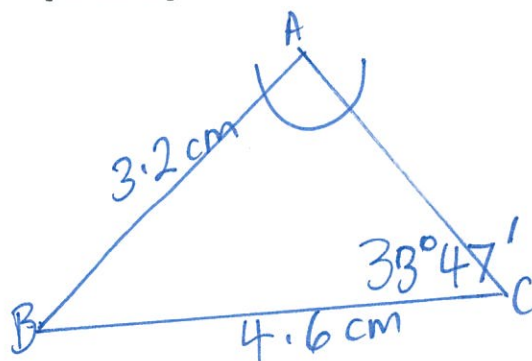
- (ii) What is the bearing of Barker from B.

$$360^\circ - 40^\circ$$

$$320^\circ$$

$$\text{OR } N40^\circ W$$

- (g) In a triangle ABC,  $AB = 3.2 \text{ cm}$ ,  $BC = 4.6 \text{ cm}$  and angle C is  $33^\circ 47'$ . Find angle A to the nearest minute. [3marks]



$$\frac{\sin A}{4.6} = \frac{\sin 33^\circ 47'}{3.2}$$

$$\sin A = 0.79932 \dots$$

$$\therefore \angle A = 53^\circ 4'$$

$$\text{or } (180^\circ - 53^\circ 4')$$

$$126^\circ 56' + 33^\circ 47' < 180^\circ$$

$\therefore$  accept both answers

$$\angle A = 53^\circ 4' \text{ and } 126^\circ 56'$$

- (h) For what value of  $\theta$  is  $\tan \theta = -3.75$ , correct to the nearest minute and given  $0^\circ \leq \theta \leq 180^\circ$  [2marks]

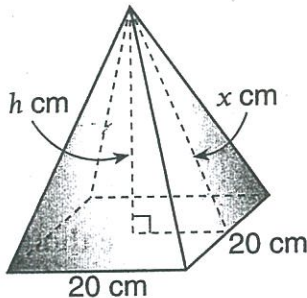


$$\theta = (180^\circ - 75^\circ 4')$$

$$= 104^\circ 56'$$



- (j) A square pyramid has a total surface area of  $2000 \text{ cm}^2$ . If the base edge is  $20 \text{ cm}$ , calculate the:  
[5marks]



- (i) perpendicular height,  $x \text{ cm}$  of one of the triangular faces

$$\text{SA of 4 } \Delta\text{'s} = 2000 - (20 \times 20) \\ = 1600$$

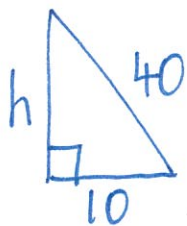
$$\text{SA of } \Delta = 400$$

$$A = \frac{1}{2} b h$$

$$400 = \frac{1}{2} \times 20 \times x$$

$$\therefore x = 40 \text{ cm}$$

- (ii) perpendicular height,  $h \text{ cm}$  of the pyramid in surd form.



$$h^2 = 40^2 - 10^2 \\ = 1500$$

$$\therefore h = \sqrt{1500} = 10\sqrt{15}$$

- (iii) volume of the pyramid. (1dp)

$$V = \frac{1}{3} A h \\ = \frac{1}{3} \times 400 \times 10\sqrt{15} \\ \doteq 5164.0 \text{ cm}^3$$

## QUESTION 2 (30 marks)

- (a) For  $y = -x^2 - 2x + 8$ , find the:  
[7marks]

- (i)  $y$  intercept

$$\text{When } x=0, y=8$$

- (ii)  $x$  intercepts

$$x^2 + 2x - 8 = 0 \\ (x+4)(x-2) = 0 \\ x = -4, 2$$

- (iii) equation of the axis of symmetry

$$x = \frac{-b}{2a} \\ = \frac{-(-2)}{2(-1)} \\ = \frac{2}{-2} \quad x = -1$$

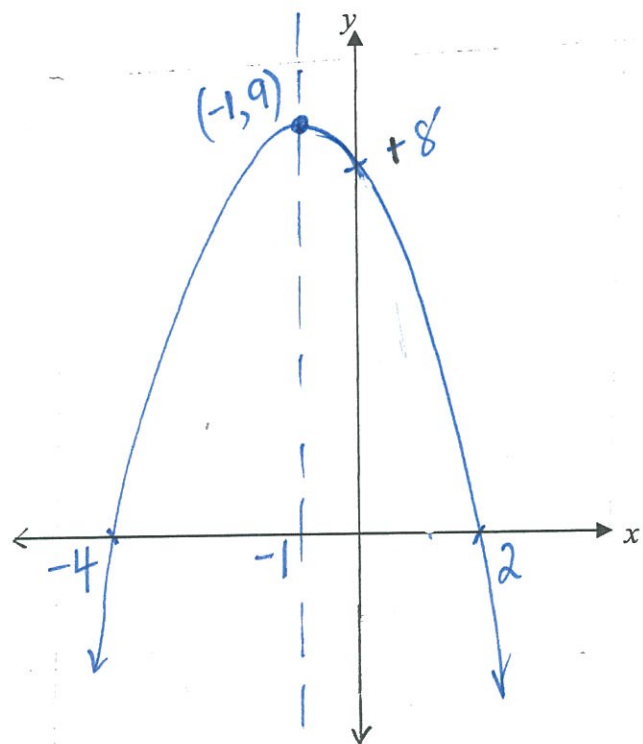
- (iv) coordinates of the vertex

$$\text{When } x = -1$$

$$y = -(-1)^2 - 2(-1) + 8 \\ = -1 + 2 + 8 \\ = 9$$

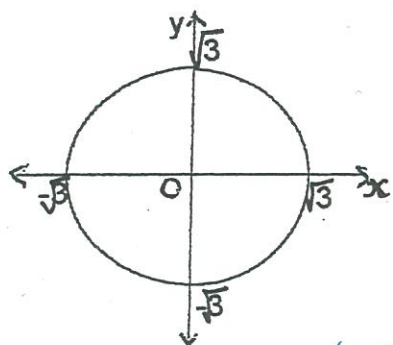
$$(-1, 9)$$

(v) hence, sketch the graph showing all these features.



(b) Determine the equation of each of the curves below: [6marks]

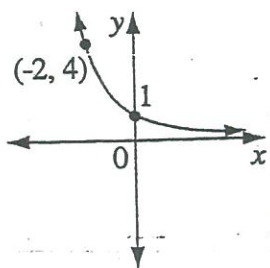
(i)



$$x^2 + y^2 = (\sqrt{3})^2$$

$$x^2 + y^2 = 3$$

(ii)

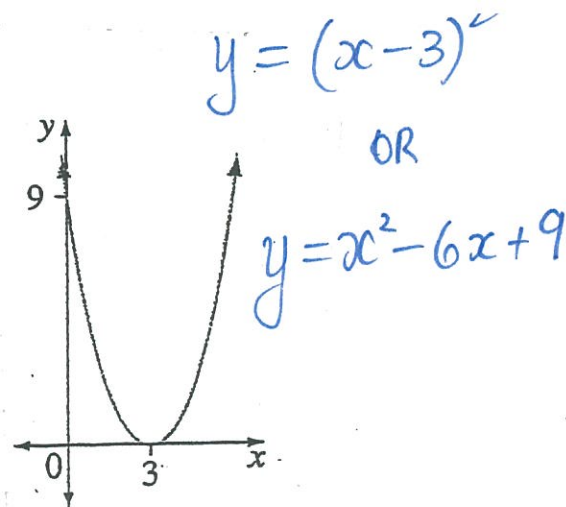


$$y = 2^{-x}$$

OR

$$y = \left(\frac{1}{2}\right)^x$$

(iii)

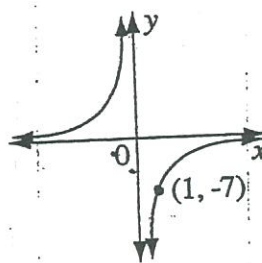


$$y = (x-3)^2$$

OR

$$y = x^2 - 6x + 9$$

(iv)

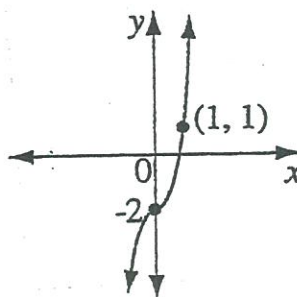


$$xy = -7$$

OR

$$y = -\frac{7}{x}$$

(v)



$$y = ax^3 + d$$

$$y = ax^3 - 2$$

$$1 = a(1)^3 - 2$$

$$\therefore a = 3$$

$$y = 3x^3 - 2$$

- (c) Express the equation of the circle  $x^2 + y^2 - 10y + 9 = 0$  in the form that shows the radius and centre, and hence find the radius and centre of the circle. [3marks]

$$x^2 + y^2 - 10y = -9$$

$$x^2 + y^2 - 10y + 25 = -9 + 25$$

$$x^2 + (y-5)^2 = 16$$

$$\text{centre} = (0, 5)$$

$$\text{radius} = 4$$

- (d) Solve the following equations simultaneously [3marks]
- $$4x^2 + y^2 = 25$$
- $$x + y = 5$$

$$\therefore y = (5-x)$$

$$4x^2 + (5-x)^2 = 25$$

$$4x^2 + 25 - 10x + x^2 = 25$$

$$5x^2 - 10x = 0$$

$$5x(x-2) = 0$$

$$\therefore x = 0, 2$$

$$\text{When } x=0, y=5$$

$$\text{When } x=2, y=3$$

- (e) Given the function  $f(x) = x^2 + 5$  [6marks]

(i) Find  $f(-1)$   $= (-1)^2 + 5$   
 $= 6$

- (ii) What is the domain of the function?

$$D: \text{All real } x$$

- (iii) What is the range of the function?

$$R: y \geq 5$$

- (iv) Find the equation of the inverse.

$$y = x^2 + 5$$

$$x = y^2 + 5$$

$$y^2 = x - 5$$

$$\therefore y = \pm \sqrt{x-5}$$

- (v) What restriction needs to be placed on  $f(x) = x^2 + 5$  so that it does have an inverse function?

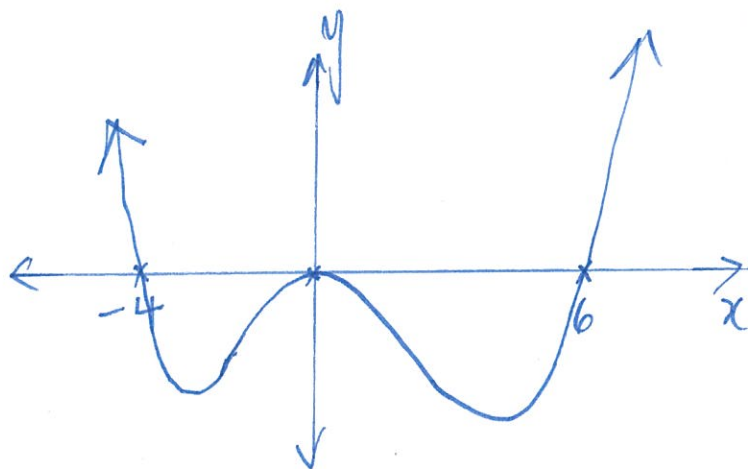
$$x \leq 0 \quad \text{OR} \quad x \geq 0$$



(f) Sketch the curve

$$y = x^2(x - 6)(x + 4)$$

[2marks]



(g) Given that  $px - 2y - 5 = 0$  meets the line through  $(3,2)$  and  $(-3,4)$  at right angles, find the value of  $p$ .

[3marks]

$$\begin{aligned} px - 2y - 5 &= 0 \\ 2y &= px - 5 \\ y &= \frac{p}{2}x - \frac{5}{2} \\ \therefore m_1 &= \frac{p}{2} \end{aligned}$$

$$\begin{aligned} m_2 &= \frac{4-2}{-3-3} \\ &= \frac{2}{-6} \\ &= -\frac{1}{3} \end{aligned}$$

$$m_1 \times m_2 = -1$$

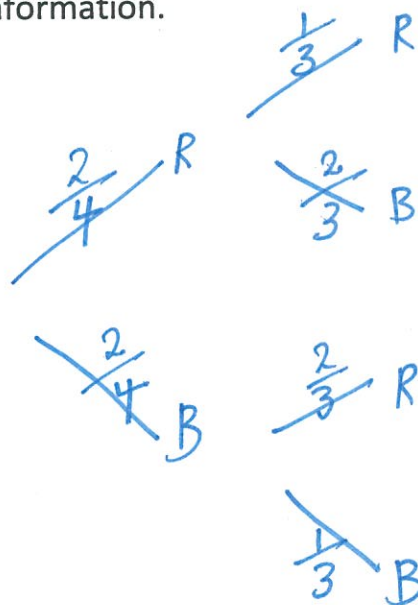
$$\begin{aligned} \frac{p}{2} \times -\frac{1}{3} &= -1 \\ \therefore -\frac{p}{6} &= -1 \\ \therefore p &= 6 \end{aligned}$$

### QUESTION 3 (30 marks)

(a) A bag contains two red and two black marbles. One marble is selected at random, the colour noted and NOT replaced. A second marble is then selected in the same way.

[4marks]

(i) Draw a probability tree to show this information.



(ii) Find the probability of selecting a red marble followed by a black marble.

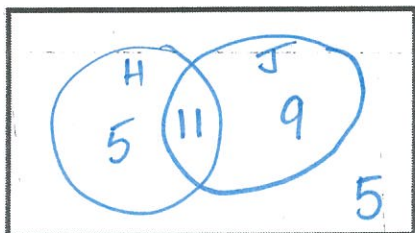
$$\begin{aligned} P(RB) &= \frac{2}{4} \times \frac{2}{3} \\ &= \frac{4}{12} \\ &= \frac{1}{3} \end{aligned}$$

(iii) Find the probability of selecting two marbles of the same colour.

$$2 \times \left( \frac{2}{4} \times \frac{1}{3} \right) = \frac{1}{3}$$

- (b) There are 30 students in a class of which 16 study History, 20 study Japanese and 5 study neither of these subjects. [2marks]

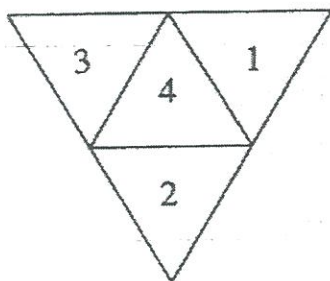
(i) Complete the Venn diagram.



(ii) What is the probability that a student chosen at random studies only History?

$$\frac{5}{30} = \frac{1}{6}$$

(c) A die is made in the form of a tetrahedron. The net is shown below:

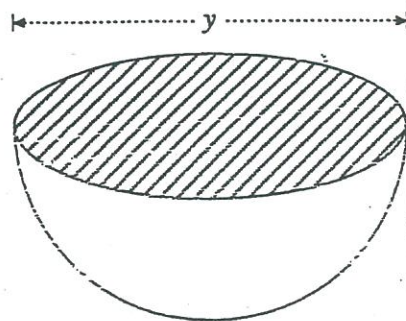


If the die is rolled twice and the number on the face is recorded each time, find the probability that the sum of the two recorded numbers is 4. [2marks]

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$$\frac{3}{16}$$

- (d) The outer surface of a closed hemisphere with a diameter of  $y$  units, is to be painted. [4marks]



(i) Find the area to be painted in terms of  $\pi$ .

$$\begin{aligned} SA &= 2\pi r^2 + \pi r^2 \\ &= 3\pi r^2 \\ &= 3\pi \times \left(\frac{y}{2}\right)^2 \\ &= \frac{3\pi}{4} y^2 \text{ units}^2 \end{aligned}$$

(ii) Given another hemisphere with half the diameter, what would be the ratio of their volumes?

$$\begin{aligned} \text{Radius} & \quad y : \frac{y}{2} \\ \text{Volume} & \quad y^3 : \left(\frac{y}{2}\right)^3 \\ & \quad y^3 : \frac{y^3}{8} \\ & \quad 8 : 1 \end{aligned}$$



(e) [4marks]

(i) Make  $n$  the subject of the formula

$$\frac{1}{f} = \frac{1}{m} + \frac{1}{n}$$

$$\frac{1}{n} = \frac{1}{f} - \frac{1}{m}$$
$$\frac{1}{n} = \frac{m - f}{fm}$$

$$\therefore n = \frac{fm}{m - f}$$

(ii) Determine any restrictions on the values of the variables in the equation given in part (i).

$$f \neq 0$$

$$m \neq 0$$

$$n \neq 0$$

(iii) What is the restriction on the formula with subject  $n$ ?

$$m - f \neq 0$$

$$\therefore m \neq f$$

(f) Given the polynomial with equation

$$2x^3 - 3x^2 - 8x - 3 = 0 \quad [6\text{marks}]$$

(i) Show that  $x = -1$  is a solution.

$$2(-1)^3 - 3(-1)^2 - 8(-1) - 3$$
$$= -2 - 3 + 8 - 3$$
$$= 0$$

(ii) If  $Q(x) = (2x^3 - 3x^2 - 8x - 3) \div (x + 1)$ , find  $Q(x)$ .

$$\begin{array}{r} 2x^2 - 5x - 3 \\ x+1 \overline{) 2x^3 - 3x^2 - 8x - 3} \\ \underline{2x^3 + 2x^2} \phantom{- 3} \\ -5x^2 - 8x \phantom{- 3} \\ \underline{-5x^2 - 5x} \phantom{- 3} \\ -3x - 3 \\ \underline{-3x - 3} \\ 0 \end{array}$$

$$\therefore Q(x) = 2x^2 - 5x - 3$$

(iii) Write the equation of the polynomial as a product of its three linear factors.

$$(x+1)(2x^2 - 5x - 3) = 0$$

$$(x+1)(2x+1)(x-3) = 0$$

(iv) Hence, find the roots of the equation.

$$x = -1, 3, -\frac{1}{2}$$

(g) If  $\log_x 5 = 0.56$ , find: [4marks]

$$\begin{aligned}\log_x 25 &= \log_x 5^2 \\ &= 2 \times \log_x 5 \\ &= 2 \times 0.56 \\ &= 1.12\end{aligned}$$

$$\begin{aligned}\log_x 0.2 &= \log_x \frac{1}{5} \\ &= \log_x 5^{-1} \\ &= -\log_x 5 \\ &= -0.56\end{aligned}$$

(h) Solve  $4^x = 29$  correct to two decimal places.

[2marks]

$$\begin{aligned}4^x &= 29 \\ \log 4^x &= \log 29 \\ x \log 4 &= \log 29 \\ \therefore x &= \frac{\log 29}{\log 4} \\ &= 2.4289 \dots \\ &\approx 2.43\end{aligned}$$

(i) Solve the equation for  $m$  [2marks]

$$4^{m+1} = \frac{1}{4\sqrt{2}}$$

$$\log 4^{m+1} = \log \frac{1}{4\sqrt{2}}$$

$$(m+1) \log 4 = -\log 4\sqrt{2}$$

$$m+1 = \frac{-\log 4\sqrt{2}}{\log 4}$$

$$m+1 = -1.25$$

$$\therefore m = -2.25$$

OR

$$2^{2(m+1)} = \frac{1}{2^2 \times 2^{\frac{1}{2}}}$$

$$2^{2m+2} = \frac{1}{2^{\frac{5}{2}}}$$

$$2^{2m+2} = 2^{-\frac{5}{2}}$$

$$\therefore 2m+2 = -\frac{5}{2}$$

$$2m = -\frac{9}{2}$$

$$\therefore m = -\frac{9}{4}$$

$$= -2.25$$

END OF EXAM !!