2021 Practice THSC for Advanced Maths Solutions

Functions

1. B

2. A

3. D

4. C

5. $x > -0.5$

6. $x = 1/3$

Calculus

1. A

2. C

3. B

4. D

5. B

6. $x = 0$

7. $x > 3$

8. 12

Trigonometry

1. D

2. D

3. 4

Financial Mathematics

1. A

2. B

3. 113

Statistics

1. D

2. C

3. B

4. A

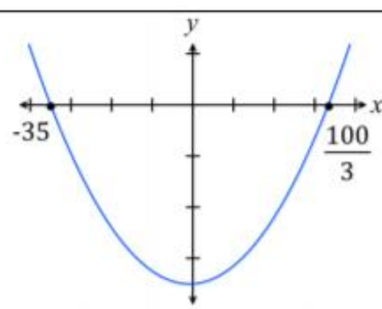
5. 60

Solutions to Extended Response Questions

Question 1 – Financial Mathematics – 9 Marks

Part A (6 marks)

(a)	$\{4, 7, 10, \ldots\}$ Arithmetic series with first term 4 and common difference 3. $T_n = a + (n-1)d$ $= 4 + 3(n-1)$ $= 3n + 1$	2 Marks: Correct answer. 1 Mark: Uses the formula for the nth term of an AP with one correct value.
(b)	To find the sum of the first 10 terms of $\{4, 7, 10, \ldots\}$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{10}{2}[2 \times 4 + (10-1) \times 3]$ $= 175$	1 Mark: Correct answer.
(c)	The number sticks is less than 1750. $S_k < 1750$ $\frac{k}{2}[2 \times 4 + 3(k-1)] < 1750$ $k(8 + 3k - 3) < 3500$ $3k^2 + 5k - 3500 < 0$ $\therefore (3k-100)(k+35) < 0$	2 Marks: Correct answer. 1 Mark: Shows some understanding.

(d)	$(3k-100)(k+35) = 0$ $k = \frac{100}{3}$ or $k = -35$ The value of k is a positive whole number (rows). $\therefore k = 33$	 	1 Mark: Correct answer.

Part B (3 marks)

$n = 25 \times 12 = 300$ $r = 3 \div 12 = 0.25\%$

$r + 1 = 1.0025$

Monthly Balance: $A_1 = P(1.0025) - 3000$

$A_2 = A_1(1.0025) - 3000$

$= P(1.0025)^2 - 3000(1.0025) - 3000$

$= P(1.0025)^2 - 3000(1.0025 + 1)$

$\vdots$

$A_{300} = P(1.0025)^{300} - 3000(1.0025^{299} + \dots + 1.0025 + 1) = 0$

$$P(1.0025)^{300} = \frac{3000(1.0025^{300} - 1)}{1.0025 - 1}$$

$$P = \frac{3000(1.0025^{300} - 1)}{0.0025(1.0025)^{300}}$$

$$= \boxed{\$632629.36}$$

Question 2 – Calculus – 9 Marks

Part A (5 marks)

(a)	$\dot{x} = 8 - 16\sin t$ $\ddot{x} = -16\cos t$ Initially $t = 0$ $\ddot{x} = -16\cos(0)$ $= -16$ $\therefore$ Initially the acceleration is 16 ms² towards the left.	1 Mark: Correct answer.
(b)	Particle at rest when velocity is zero ($\dot{x} = 0$) $\dot{x} = 8 - 16\sin t = 0$ $16\sin t = 8$ $\sin t = \frac{1}{2}$ $t = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$ $\therefore$ First at rest after $\frac{\pi}{6}$ seconds.	1 Mark: Correct answer.
(c)	$\dot{x} = 8 - 16\sin t$ $x = 8t + 16\cos t + C$ When $t = 0$ $x = 0$ (Initially at the origin) $0 = 8 \times 0 + 16\cos(0) + C$ $C = -16$ $\therefore x = 8t + 16\cos t - 16$	1 Mark: Correct answer.

(d)	Distance between $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ $t = \frac{\pi}{6}$ $x = 8 \times \frac{\pi}{6} + 16\cos\frac{\pi}{6} - 16 = \frac{4\pi}{3} + 8\sqrt{3} - 16$ $t = \frac{5\pi}{6}$ $x = 8 \times \frac{5\pi}{6} + 16\cos\frac{5\pi}{6} - 16 = \frac{20\pi}{3} - 8\sqrt{3} - 16$ Distance travelled $= \left(\frac{20\pi}{3} - 8\sqrt{3} - 16\right) - \left(\frac{4\pi}{3} + 8\sqrt{3} - 16\right)$ $= \frac{16\pi}{3} - 16\sqrt{3}$	2 Marks: Correct answer. 1 Mark: Makes some progress.

Part B (4 marks)

For the point A:

$$4 - 3x^2 = -x$$

$$0 = 3x^2 - x - 4$$

$$0 = (3x - 4)(x + 1)$$

$$x = \frac{4}{3} \text{ or } -1$$

Therefore, $x = -1$ according to the diagram.

When $x = -1$:

$$y = 4 - 3(-1)^2$$

$$= 1$$

Therefore, $A(-1, 1)$.

Due to the symmetry of $y = 4 - 3x^2$, $C(1, 1)$.

$$A_{ABC} = \int_{-1}^{1} 4 - 3x^2 dx - A_{\text{rectangle}}$$

$$= \left[4x - x^3\right]_{-1}^{1} - 2$$

$$= (4 - 1) - (-4 + 1) - 2$$

$$= 4$$

$$A_{\text{logo}} = 4 \times A_{ABC} + 2 \times A_{\text{rectangle}}$$

$$= 20 \text{ units}^2$$

MA–C4 Integral Calculus

MA12–7 Bands 5–6

- Gives the correct solution 4

- Correctly uses the points $A(-1, 1)$ and $C(1, 1)$ to find the area of ABC OR equivalent merit. 3

- Correctly uses the points $A(-1, 1)$ and $C(1, 1)$ to develop an integral that represents the area of ABC OR equivalent merit. 2

- Develops an equation to show either point $A(-1, 1)$ OR point $C(1, 1)$ 1

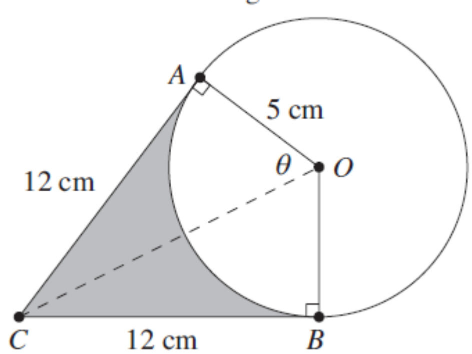
Question 3 – Trigonometric Functions – 10 Marks

Part A (3 marks)

$\int_0^a 5\sin 3x\,dx = \frac{10}{3}$ $\left[-\frac{5}{3}\cos 3x\right]_0^a = \frac{10}{3}$ $-\frac{5}{3}(\cos 3a - \cos 0) = \frac{10}{3}$ $(\cos 3a - \cos 0) = -2$ $\cos 3a = -1$ $3a = \pi, 3\pi, \ldots$ $a = \frac{\pi}{3} \quad (0 \le a < \pi)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Integrates correctly.

Part B (4 marks)

(a) Construct the line segment OC and let $\angle AOC = \theta$.



$$\tan\theta = \frac{12}{5}$$

$$\theta = \tan^{-1}\left(\frac{12}{5}\right)$$

$$\therefore \angle AOB = 2\times\tan^{-1}\left(\frac{12}{5}\right)$$

$$= 2.35$$

$$\approx 2.4 \quad \text{(to 2 significant figures)}$$

MA–T1 Trigonometry and Measure of Angles
MA11–3 Bands 3–4

- Gives the correct solution 2

- Shows progress towards the correct solution 1

(b) $A_{\text{sector } AOB} = \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 5^2 \times 2.4$ $= 30$ $A_{AOBC} = 2 \times \left(\frac{1}{2} \times 5 \times 12\right)$ $= 60$ $A_{\text{shaded region}} = 60 - 30$ $= 30 \text{ cm}^2$	MA–T1 Trigonometry and Measure of Angles MA11–3 Band 4 • Gives the correct solution .... 2 • Correctly calculates the area of sector *AOB* OR the area of *AOBC* 1

Part C (3marks)

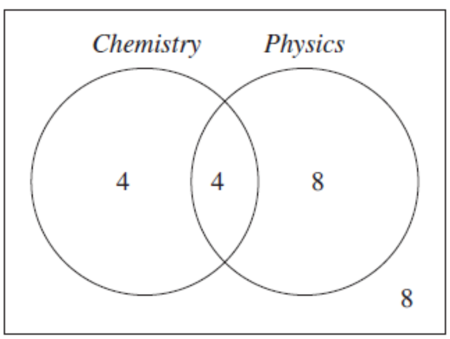
$\text{LHS} = \sin x + 1 + \cos x \cot x - \operatorname{cosec} x$ $= \sin x + 1 + \cos x \times \frac{\cos x}{\sin x} - \frac{1}{\sin x}$ $= \frac{\sin^2 x + \sin x + \cos^2 x - 1}{\sin x}$ $= \frac{\sin^2 x + \cos^2 x + \sin x - 1}{\sin x}$ $= \frac{1 + \sin x - 1}{\sin x} = \frac{\sin x}{\sin x}$ $= 1$ $=$ RHS	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Correctly uses one trig identity.

Question 4 – Statistics – 8 Marks

Part A (3 Marks)

$0.24 + 0.2 + m + 0.4 = 1$ $m = 0.16$ $E(X) = 20 \times 0.24 + 21 \times 0.2 + 22 \times 0.16 + 23 \times 0.4$ $E(X) = 21.72$ $Var(X) = E(X^2) - [E(X)]^2$ $Var(X) = 20^2 \times 0.24 + 21^2 \times 0.2 + 22^2 \times 0.16 + 23^2 \times 0.4 - 21.72^2$ $Var(X) = 1.4816$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Finds the correct value of m

Part B (5 marks)

(a) $\lvert \text{CHE} \cup \text{PHY} \rvert = \lvert \text{CHE} \rvert + \lvert \text{PHY} \rvert - \lvert \text{CHE} \cap \text{PHY} \rvert$ $16 = 8 + 12 - \lvert \text{CHE} \cap \text{PHY} \rvert$ $\lvert \text{CHE} \cap \text{PHY} \rvert = 20 - 16$ $= 4$ 	MA–S1 Probability and Discrete Probability Distributions MA11–8 Band 3 • Draws the correct diagram. 1

(b) $P(\text{PHY}) = \frac{12}{24}$

$= \frac{1}{2}$

$P(\text{CHE}) = \frac{8}{24}$

$= \frac{1}{3}$

$P(\text{CHE} \cap \text{PHY}) = \frac{4}{24}$

$= \frac{1}{6}$

$P(\text{PHY}) \times P(\text{CHE}) = \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$

$= P(\text{CHE} \cap \text{PHY})$

As $P(\text{CHE}) \times P(\text{PHY}) = P(\text{CHE} \cap \text{PHY})$, the two events are independent.

MA–S1 Probability and Discrete Probability Distributions
MA11–8 Bands 4–5

- Gives the correct solution 2

- Correctly calculates either $P(\text{CHE} \cap \text{PHY})$ OR $P(\text{CHE}) \times P(\text{PHY})$ OR equivalent condition for independence 1

(c) $P(\text{CHE}) = \frac{1}{3}$, $P(\text{PHY}) = \frac{2}{5}$ and $P(\text{PHY} | \text{CHE}) = \frac{3}{7}$.

$$P(\text{CHE} \cup \text{PHY}) = P(\text{CHE}) + P(\text{PHY}) - P(\text{CHE} \cap \text{PHY})$$

$$= \frac{1}{3} + \frac{2}{5} - \left(P(\text{CHE}) \times P(\text{CHE} | \text{PHY})\right)$$

$$= \frac{1}{3} + \frac{2}{5} - \left(\frac{1}{3} \times \frac{3}{7}\right)$$

$$= \frac{62}{105}$$

MA–S1 Probability and Discrete Probability Distributions
MA11–8 Bands 5–6

- Gives the correct solution 2

- Correctly calculates $P(\text{CHE} \cap \text{PHY})$. 1

Question 5 – Exponential & Logarithms – 6 Marks

(a) As the initial amount of substance A is 200 grams, the time taken to decrease to half its original value is calculated as follows. Let $M_A = 100$. $100 = 200e^{-0.05t}$ $\frac{1}{2} = e^{-0.05t}$ $\ln\left(\frac{1}{2}\right) = -0.05t$ $\ln 1 - \ln 2 = -0.05t$ $\ln 2 = 0.05t$ $t = \frac{\ln 2}{0.05}$ $= 13.86\ldots$ ≈ 14 minutes Therefore, it will decrease to half its original value in 14 minutes.	MA–E1 Exponential and Logarithmic Functions MA11–8 Bands 4–5 • Gives the correct solution . . . . 2 • Correctly arrives at the expression $100 = 200e^{-0.05t}$ and attempts to solve for t. 1

(b) The rate of change of both substances:	MA–E1 Exponential and Logarithmic Functions MA11–8 Bands 5–6 • Gives the correct solution . . . . 4 • Writes an expression using the same base . . . . 3 • Correctly finds the rates of decay for both substances AND attempts to solve the equation $-10e^{-0.05t} = -48\ln 3 \times 3^{-0.12t}$ . . . . 2 • Find the rate of change for substance *A* OR substance *B* 1

(b) The rate of change of both substances:

$$\frac{dM_A}{dt} = -0.05 \times 200e^{-0.05t}$$
$$= -10e^{-0.05t}$$

$$\frac{dM_B}{dt} = 400 \times \ln 3 \times -0.12 \times 3^{-0.12t}$$
$$= -48\ln 3 \times 3^{-0.12t}$$

Equate the two rates:

$$-10e^{-0.05t} = -48\ln 3 \times 3^{-0.12t}$$
$$\frac{-10}{-48\ln 3} = \frac{3^{-0.12t}}{e^{-0.05t}}$$
$$= \frac{e^{\ln\left(3^{-0.12t}\right)}}{e^{-0.05t}}$$
$$= \frac{e^{(-0.12\ln 3)t}}{e^{-0.05t}}$$
$$= e^{(-0.12\ln 3 + 0.05)t}$$
$$0.1895\ldots = e^{-0.0818t}$$
$$\ln(0.1895)\ldots = -0.0818t$$
$$t = \frac{\ln 0.1895\ldots}{-0.0818\ldots}$$
$$= 20.317\ldots \text{ minutes}$$
$$\approx 20 \text{ minutes } 19 \text{ seconds}$$

Therefore, both substances decay at the same rate at 20 minutes and 19 seconds.

MA–E1 Exponential and Logarithmic Functions

MA11–8 Bands 5–6

- Gives the correct solution 4
- Writes an expression using the same base 3
- Correctly finds the rates of decay for both substances AND attempts to solve the equation $-10e^{-0.05t} = -48\ln 3 \times 3^{-0.12t}$ 2
- Find the rate of change for substance *A* OR substance *B* 1