ACE Examination Paper 4
Year 12 Mathematics Standard 2 Yearly Examination
Worked solutions and marking guidelines

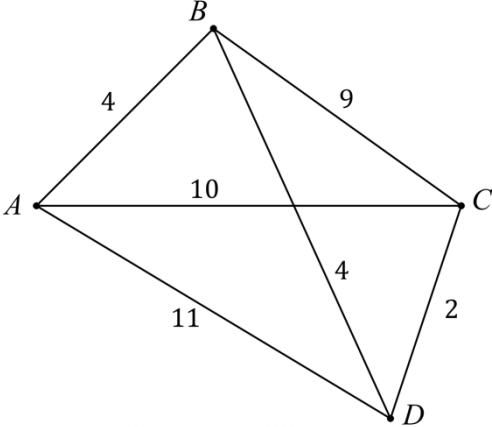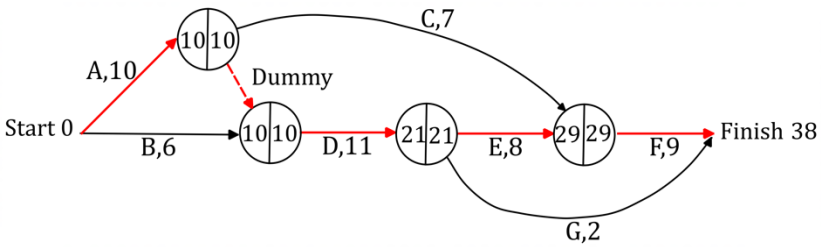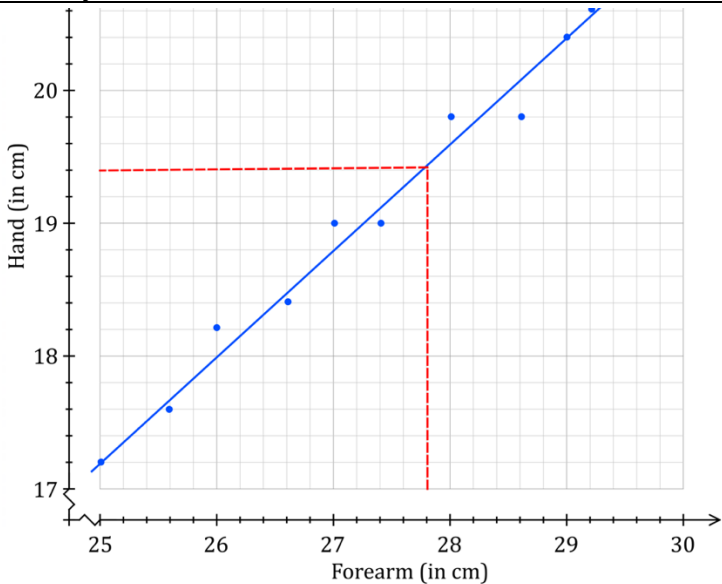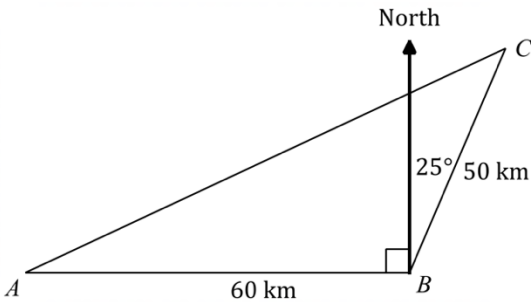| **Section I**		
	Solution	**Criteria**
1.	$z = \dfrac{x - \bar{x}}{s} = \dfrac{75 - 85}{5}$ $= -2$	1 Mark: A
2.	Length $= 11 + 12 + 14$ $= 37$ km	1 Mark: A
3.	$\tan 40° = \dfrac{AB}{15}$ $\quad AB = 15\tan 40°$ $\quad\quad = 12.5864\ldots$ $\quad\quad \approx 13$ m	1 Mark: D
4.	Break-even point the costs equals the income $50n + 75\,000 = 800n$ $\quad\quad 750n = 75\,000$ $\quad\quad\quad n = \dfrac{75\,000}{750} = 100$	1 Mark: B
5.	Data forms a straight line with a positive gradient. $\therefore$ Strong positive correlation	1 Mark: D
6.		1 Mark: D
7.	$m = r\dfrac{s_y}{s_x}$ $\quad = 0.561 \times \dfrac{4.579}{1.987}$ $\quad = 1.29$	1 Mark: B
8.	Let the distance between Callum and Lara be x. Sine rule is needed: $\dfrac{x}{\sin 36} = \dfrac{15}{\sin 59}$	1 Mark: A
9.	Electricity $= 0.120 \times 24 \times 30$ $\quad\quad\quad = 86.4$ kWh $\quad\quad$ Cost $= 86.4 \times 0.248$ $\quad\quad\quad = 21.4272$ $\quad\quad\quad \approx \$21.43$	1 Mark: B

	Solution	**Criteria**
10.	Number of days from 11 Jun to 19 Jul is 39. $FV = PV(1 + r)^n$ $\quad = 2400 \times \left(1 + \dfrac{18.75\%}{365}\right)^{39}$ $\quad = 2448.5544\ldots$ $\quad \approx \$2448.55$ $\therefore$ Amount due is \$2448.55	1 Mark: C
11.	$r = \dfrac{2\%}{2} = 1\%,$ $n = 2 \times 2 = 4$ Intersection value is 4.0604 $FV = 4.0604 \times 100\,000$ $\quad = \$406\,040$	1 Mark: C
12.	$z = \dfrac{x - \bar{x}}{s}$ $\quad = \dfrac{1000 - 900}{50}$ $\quad = 2$ 95% of scores have a z-score between –2 and 2 $\therefore$ 2.5% have a z-score greater than 2.	1 Mark: B
13.	Quarterly interest rate = 0.12 ÷ 4 = 0.03 First year investment $FV = PV(1 + r)^n$ $\quad = \$2175 \times (1 + 0.03)^4$ $\quad = \$2447.98166$ Second year investment $FV = PV(1 + r)^n$ $\quad = (\$2447.98 + \$1920) \times (1 + 0.03)^4$ $\quad \approx \$4916$	1 Mark: C
14.	 $\therefore$ Minimum completion time is 48.	1 Mark: C
15.	$v = \dfrac{k}{w}$ $\qquad\qquad \therefore v = \dfrac{k}{w}$ $320 = \dfrac{k}{16}$ $\qquad\qquad\quad = \dfrac{16 \times 320}{250}$ $\quad k = 16 \times 320$	1 Mark: D
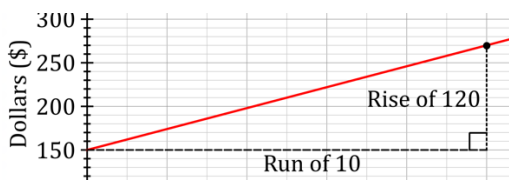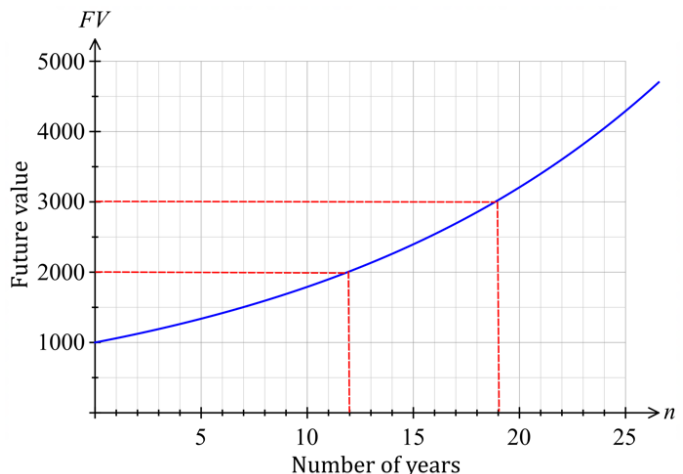
	Section II	
	Solution	**Criteria**
16	Expected saving $= 19.4 \times 0.076 \times 365$ $\qquad\qquad\qquad = \$538.156$ $\qquad\qquad\qquad \approx \$538.16$ $\therefore$ Expected saving is $\$538.16$	2 Marks: Correct answer. 1 Mark: Shows understanding.
17	Weighted edges: $AB = 4, AC = 10, AD = 11, BC = 9, BD = 4, DC = 2$ 	2 marks: Correct answer. 1 mark: Draws the vertices with at least one correct edge.
18(a)	$r = \dfrac{0.084}{12} = 0.0070. \, n = 4 \times 12 = 48$ Intersection value is 40.64856 Let the monthly repayment be x. $\qquad PV = 40.64856 \times x$ $16\,000 = 40.64856 \times x$ $\qquad x = \dfrac{16\,000}{40.64856}$ $\qquad\quad = 393.6178\ldots \approx \$393.62$ $\therefore$ Hayley's monthly repayment is $\$393.62$.	2 marks: Correct answer. 1 mark: Finds the intersection value or shows some understanding.
18(b)	Total repaid $= 393.6178\ldots \times 48$ $\qquad\qquad\quad = 18\,893.6582\ldots \approx \$18\,894$ $\quad$ Interest $= 18\,894 - 16\,000$ $\qquad\qquad = \$2894$ $\therefore$ Hayley's interest on the loan is $\$2894$.	2 marks: Correct answer. 1 mark: Finds the total amount to be repaid.
19	Assessment results increase as height increases. Low positive correlation. Not a strong relationship.	2 marks: Correct answer. 1 mark: Shows understanding
20(a)		3 marks: Correct answer. 2 marks: Finds the EST or LST. 1 mark: Draws a network diagram with some correct edges .
20(b)	Weight $= 38 - 29$ $\qquad\quad = 9$ $\therefore$ Weight of activity F is 9 minutes.	1 mark: Correct answer.

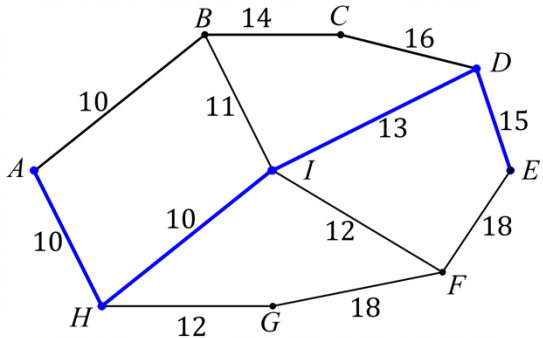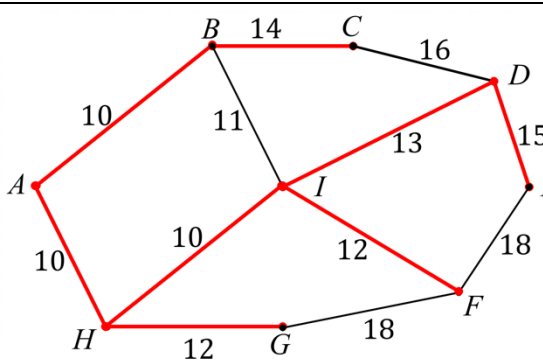| 20(c)	Float time = LST − EST $\qquad$ = 36 − 21 $\qquad$ = 15 min ∴ Float time for activity G is 15 minutes.	1 mark: Correct answer.
21(a)	A z-score of 2 is two standard deviations above the mean. That is, Charlie scored 89% in the class test.	1 mark: Correct answer.
21(b)	$z = \dfrac{x - \bar{x}}{s}$ $-2 = \dfrac{x - 64}{12.5}$ $x = (-2 \times 12.5) + 64$ $\quad = 39\%$ ∴ Hannah's mark is 39%	1 mark: Correct answer.
21(c)	$z = \dfrac{x - \bar{x}}{s}$ $\quad = \dfrac{51.5 - 64}{12.5}$ $\quad = -1$ ∴ Jacob's z-score is −1	1 mark: Correct answer.
21(d)	$z = \dfrac{x - \bar{x}}{s}$ $3 = \dfrac{x - 64}{12.5}$ $x = (3 \times 12.5) + 64$ $\quad = 101.5\%$ ∴ Lucy needs to score 101.5% in the test (impossible).	1 mark: Correct answer.
22	$FV = \$50\,000, r = 4.8\% \div 12 = 0.4\%, n = 3 \times 12 = 36$ $PV = \dfrac{FV}{(1 + r)^n}$ $\quad = \dfrac{50\,000}{(1 + 0.004)^{36}}$ $\quad = 43\,306.82482$ $\quad \approx \$43\,307$ ∴ Present value is \$43 307.	2 marks: Correct answer. 1 mark: Uses the present value formula with one correct value.
23(a)	Leaking at 0.2 litres per minute implies $m = -0.2$ $y = mx + c$ $V = -0.2t + 4$	1 mark: Correct answer.
23(b)	Time is measure in minutes. Therefore $t = 150$ s = 2.5 min $V = -0.2t + 4$ $\quad = -0.2 \times 2.5 + 4$ $\quad = 3.5$ L ∴ There is 3.5 L remaining after 150 seconds.	1 mark: Correct answer.

23(c)	$V = -0.2t + 4$ $\quad 0 = -0.2 \times t + 4$ $0.2t = 4$ $\quad t = 20 \text{ min}$ $\therefore$ There is no water remaining in the can after 20 minutes.	1 mark: Correct answer.
24(a)	Distance $= 2 \times 500\,000$ $\qquad\quad = 1\,000\,000 \text{ cm} = 10 \text{ km}$	1 mark: Correct answer.
24(b)	Distance $= \dfrac{7\,500\,000}{500\,000} = 15 \text{ cm}$	1 Mark: Correct answer.
25(a)	Daily interest rate $= \dfrac{21.45}{365}$ $\qquad\qquad\qquad\quad = 0.058767\ldots \approx 0.059\%$	1 mark: Correct answer.
25(b)	Interest $= 0.0587\ldots\% \times \$8200 \times 30$ $\qquad\quad = \$144.5671\ldots$ $\qquad\quad \approx \$145$ $\therefore$ Lilly is charged \$145 in interest.	1 mark: Correct answer.
26(a)	Interest $= 100.00 + 94.00 + 87.96$ $\qquad\quad = \$281.96$ $\therefore$ Moly paid \$281.96 interest in the first 3 months.	1 mark: Correct answer.
26(b)	4th month: $P = \$12\,281.96$ $I = Prn = 12\,281.96 \times \dfrac{0.08}{12} = 81.8797\ldots \approx \$81.88$ $P + I - R = 12\,281.96 + 81.88 - 1000 = \$11\,363.84$ Repaid $= 15\,000 - 11\,363.84$ $\qquad\quad = \$3636.16$ $\therefore$ Molly has repaid \$3636 16	2 marks: Correct answer. 1 mark: Calculates the interest.
26(c)	Interest without gift $\qquad\qquad$ Interest with gift $I = Prn \qquad\qquad\qquad\qquad I = Prn$ $\quad = 11\,363.84 \times \dfrac{0.08}{12} \qquad\quad = 4\,363.84 \times \dfrac{0.08}{12}$ $\quad = 75.7589\ldots \approx \$75.76 \qquad = 29.0922\ldots \approx \$29.09$ Interest saving $= 75.76 - 29.09 = \$46.67$ $\therefore$ Molly saved \$46.67	2 marks: Correct answer. 1 mark: Makes some progress towards the solution.
27(a)		1 mark: Correct answer.

27(b)	See line of best fit on the above scatterplot.	1 mark: Correct answer.
27(c)	When forearm length = 27.8 then hand length = 19.4 cm (from the scatterplot) $\therefore$ Lara's hand length should be 19.4 cm.	1 mark: Correct answer.
27(d)	Use the calculator to find Pearson's correlation coefficient. $r = 0.990691 \ldots$ ≈ 0.9907	2 marks: Correct answer. 1 mark: Finds a value of r close to 0.99.
28	$696 \text{ km} = 39 \text{ L}$ $1 \text{ km} = \dfrac{39}{696} \text{ L}$ $100 \text{ km} = \dfrac{39}{696} \times 100 \text{ L}$ $= 5.603 \text{ L}$ $\therefore$ Asha used 5.603 L	2 marks: Correct answer. 1 mark: Shows some understanding.
29(a)	$\angle ABC = 90 + 25$ $\qquad = 115°$	1 mark: Correct answer.
29(b)	$a^2 = b^2 + c^2 - 2bc\cos A$ $AC^2 = 50^2 + 60^2 - 2 \times 50 \times 60 \times \cos 115°$ $AC = 92.9285\ldots$ $\qquad \approx 93 \text{ km}$ $\therefore$ The distance from town A to town C is 93 km.	2 marks: Correct answer. 1 mark: Uses the cosine rule with at least one correct value.
29(c)	Use the sine rule to find $\angle BAC$ $\dfrac{\sin \angle BAC}{50} = \dfrac{\sin 115}{92.9285\ldots}$ $\sin \angle BAC = \dfrac{50 \times \sin 115}{92.9285\ldots}$ $\angle BAC = 29.1853\ldots \approx 29°$ Bearing $= 90 - 29$ $\qquad = 061$ $\therefore$ Bearing of town C from town A is 061°.	3 marks: Correct answer. 2 marks: Finds the size of $\angle BAC$ 1 mark: Uses the sine rule with at least one correct value.
30	A spanning tree is a tree that connects all vertices of a graph. A tree with n vertices has $n – 1$ edges. $\therefore$ The network has 7 vertices.	1 mark: Correct answer.

31(a)	When $h = 40$ m then $d \approx 25$ m (Read from the graph) ∴ The distance to the horizon is about 25 kilometres.	1 mark: Correct answer.
31(b)	When $d = 20$ km then $h \approx 25$ m (Read from the graph) When $d = 24$ km then $h \approx 35$ m (Read from the graph) Difference $= 35 - 25 = 10$ m ∴ Difference is about 10 metres.	1 mark: Correct answer.
31(c)	$$d = 8\sqrt{\frac{h}{4}}$$ $$= 8\sqrt{\frac{40}{4}}$$ $$= 25.298221\ldots$$ $$\approx 25 \text{ km}$$ ∴ Distance to the horizon is 25 km	1 mark: Correct answer.
32(a)	The length of the rectangular sheet forms the circumference. $C = 2\pi r$ $1.25 = 2\pi r$ $r = \dfrac{1.25}{2\pi}$ $= 0.1989\ldots$ ≈ 0.2 m	1 mark: Correct answer.
32(b)	$V = \pi r^2 h$ $= \pi \times (0.1989\ldots)^2 \times 0.9$ $= 0.1119\ldots$ $= 0.11 \text{ m}^3$	1 mark: Correct answer.
32(c)	$1 \text{ m}^3 = 1000$ L $V = 0.1119\ldots \times 1000$ ≈ 112 L	1 mark: Correct answer.
33		2 marks: Correct answer. 1 mark: Shows some understanding.
34(a)	Students with a z-score of −2 is two standard deviations below the mean $(60 - (2 \times 10) = 40$. ∴ A score of 40 has a z-score of −2 .	1 mark: Correct answer.
34(b)	z-score for 80 $z = \dfrac{x - \bar{x}}{s}$ $= \dfrac{65 - 60}{10}$ $= 0.5$ ∴ z-score is 0.5	1 mark: Correct answer.

34(c)	$z = \dfrac{x - \bar{x}}{s}$ $\qquad\qquad\qquad$ $z = \dfrac{x - \bar{x}}{s}$ $= \dfrac{50 - 60}{10} \qquad\qquad\qquad = \dfrac{70 - 60}{10}$ $= -1 \qquad\qquad\qquad\qquad = 1$ 68% of scores have a z-score between -1 and 1 $\therefore$ Percentage above 70 and below 50 is 100% – 68% = 32%	2 marks: Correct answer. 1 mark: Finds the z-score of 50 and 70.
35(a)	10 boxes of paper are required to break-even. (intersection of income and costs)	1 mark: Correct answer.
35(b)	Loss = \$135 − \$210 $\qquad$ = \$75 (Reading values from the graph. Accept answers from \$70 to \$80)	1 mark: Correct answer.
35(c)	Intial costs are \$150	1 mark: Correct answer.
35(d)	Gradient is 12 and y-intercept is 150. $y = mx + b$ $C = 12n + 150$	1 mark: Correct answer.
36(a)	$\therefore$ It takes about 12 years to double in value (from the graph)	2 marks: Correct answer. 1 mark: Uses a future value of \$2000.
36(b)	The time to triple is about 19 years, compared to 12 years to double, so it is increasing at an increasing rate, or it is increasing exponentially.	2 marks: Correct answer. 1 Mark: Finds the 19 years to triple.
37	$z = \dfrac{x - \bar{x}}{s}$ $= \dfrac{179 - 167}{12}$ $= 1$ 68% of scores have a z-score between −1 and 1. $\therefore 32\% \div 2 = 16\%$ have a z-score greater than 1.	2 marks: Correct answer. 1 mark: Finds the z-score.

38(a)	Time taken $= 10 + 14 + 16 + 13$ $\qquad = 53$ mim	1 mark: Correct answer.
38(b)	Vertices with an even degree are A, C, E, G and I.	1 mark: Correct answer.
38(c)	There are 4 vertices that are odd (B, D, F and H) Eulerian trail exist if the graph has 2 vertices with an odd degree. $\therefore$ This network does not contain a walk that visits every edge exactly once.	1 mark: Correct answer.
38(d)	 Shortest time $= 10 + 10 + 13 + 15$ $\qquad = 48$ min $\therefore$ Shortest time from A to E is 48 minutes.	2 marks: Correct answer. 1 mark: Finds the shortest path or shows some understanding.
38(e)		1 mark: Correct answer.
39(a)	Total paid $=$ Loan repayment $\times$ Number of repayments $\qquad = \$1580.75 \times 20 \times 12$ $\qquad = \$379\,380$	1 mark: Correct answer.
39(b)	Interest $=$ Total paid $-$ Principal $\qquad = \$379\,380 - \$200\,000$ $\qquad = \$179\,380$	1 mark: Correct answer.
39(c)	30 years Total paid $=$ Loan repayment $\times$ Number of repayments $\qquad = \$1364.35 \times 30 \times 12$ $\qquad = \$491\,166$ Extra paid $= \$491\,166 - \$379\,380$ $\qquad = \$111\,786$	2 marks: Correct answer. 1 mark: Finds the total paid for 30 years.

40		2 marks: Correct answer.
	Maximum flow $= 5 + 3 + 8$ $\qquad\qquad\quad = 16$	1 mark: Finds the minimum cut.

41(a)		1 mark: Correct answer.

Matches (m)	1	2	3	4	5	6
Players (p)	0	1	3	6	10	15

41(b)		1 mark: Correct answer.

41(c)	$m = \dfrac{1}{2}\,(p^2 - p)$ $ = \dfrac{1}{2} \times (7^2 - 7)$ $ = 21$ $\therefore$ The model predicts 21 matches for 7 players.	1 mark: Correct answer.

41(d)	P represents the number of players. When there is one player there are no matches. Clearly p must be a positive whole number and negative values have no meaning.	1 mark: Correct answer.