Questions – Senior Division

2015

1. What is the value of 21×2015?

(A) 45 231 (B) 54 321 (C) 42 315 (D) 14 325 (E) 23 514

2. If $K = L + R^2$, $L = 4$, $K = 85$ and R is positive, then R equals

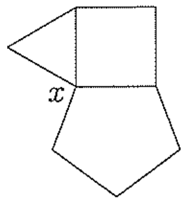
(A) 5 (B) 7 (C) 9 (D) 4 (E) 8

3. On their school holidays, Xenia and Ngoc worked for a farmer picking fruit. Xenia worked for 5 days and Ngoc for 3 days. The farmer paid them \$1000, which they shared in the same ratio as the days they worked. Ngoc's share was

(A) \$325 (B) \$300 (C) \$250 (D) \$375 (E) \$500

4. An equilateral triangle, a square and a regular pentagon are joined as in the diagram. What is the size of angle x?

(A) 108° (B) 105° (C) 90° (D) 120° (E) 102°



5. $3^{-2} - 2^{-3}$ equals

(A) -1 (B) 0 (C) $-\frac{1}{72}$ (D) $\frac{1}{72}$ (E) $\frac{17}{72}$

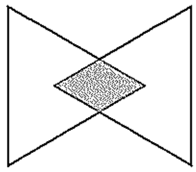
6. Jenna measures three sides of a rectangle and gets a total of 80 cm. Dylan measures three sides of the same rectangle and gets a total of 88 cm. What is the perimeter of the rectangle?

(A) 112 cm (B) 132 cm (C) 96 cm (D) 168 cm (E) 156 cm

7. Two ordinary dice are rolled. The two resulting numbers are multiplied together to create a score. The probability of rolling a score that is a multiple of six is

(A) $\frac{1}{6}$ (B) $\frac{5}{12}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

8. Two congruent equilateral triangles overlap to make a concave hexagon as shown. Each triangle has a vertex on the other's centre. What fraction of the hexagon's area is shaded?

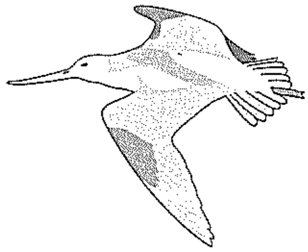


(A) $\frac{2}{9}$ (B) $\frac{1}{8}$ (C) $\frac{1}{5}$ (D) $\frac{1}{7}$ (E) $\frac{1}{6}$

9. The difference between two numbers is 20. When 4 is added to each number the larger is three times the smaller. What is the larger of the two original numbers?

(A) 26 (B) 40 (C) 38 (D) 22 (E) 32

10. A bar-tailed godwit was recorded by satellite tag in 2007 to have flown 11 500 km in eight days.
On average, approximately how many kilometres per hour is that?



(A) 120 (B) 6 (C) 1 (D) 24 (E) 60

11. Let A be the set $\{0, 1, 2\}$. Let B be the set $\{3, 6, x\}$, where x is an integer.
I multiply each number in the first set by each number in the second set.
Let C be the set of all the numbers which are the results of these multiplications.
What could x be such that C has exactly 5 distinct elements?

(A) 12 (B) 4 (C) 24 (D) 0 (E) 6

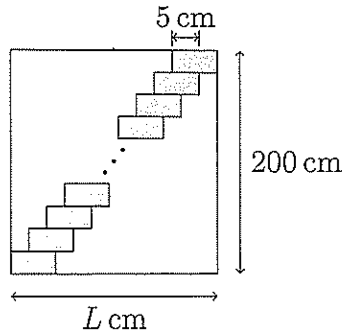
12. What is the value of this expression?

$$\frac{1}{\frac{1}{\frac{1}{2}+\frac{1}{3}}+\frac{1}{\frac{1}{4}+\frac{1}{5}}}$$

(A) $\frac{5}{16}$ (B) $\frac{3}{14}$ (C) $\frac{60}{77}$ (D) $\frac{45}{154}$ (E) $\frac{70}{66}$

13. The diagram indicates a pattern of paving blocks that are laid diagonally across a rectangular floor measuring $L\,\text{cm} \times 200\,\text{cm}$. Each block measures $8\,\text{cm} \times 4\,\text{cm}$, and any two blocks that touch overlap by 5 cm.
What is the value of L?

(A) 253 (B) 155 (C) 400 (D) 250 (E) 158

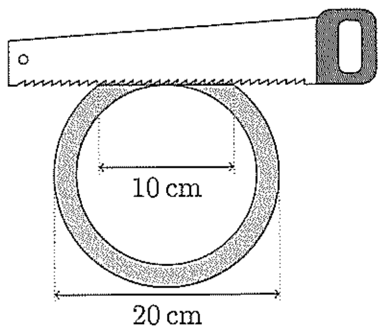


14. In a class of 25 students, 11 students are fifteen years old and the rest are sixteen years old. There are 15 boys. There are twice as many sixteen-year-old boys as fifteen-year-old boys. What fraction of the class are sixteen-year-old girls?

(A) $\frac{7}{25}$ (B) $\frac{2}{5}$ (C) $\frac{4}{25}$ (D) $\frac{1}{5}$ (E) $\frac{6}{25}$

15. Peter was cutting a pipe with an outside diameter of 20 cm. When the cut was just through the wall of the pipe, it was 10 cm in length. How thick was the wall of the pipe in centimetres?

(A) 5 (B) $5\sqrt{3} - 5$ (C) $10 - 5\sqrt{2}$ (D) $4 - \sqrt{10}$ (E) $5(2 - \sqrt{3})$



16. Three different dinosaur fossils have been found in the kingdom of Mathemania. Euleraptor is twice as old as Gaussasaurus, though when Gaussasaurus was alive Euleraptor was twice as old as Fermatops. The sum of the ages of the three fossils is 360 million years. How many million years after Fermatops did Gaussasaurus live?

(A) 120 (B) 40 (C) 80 (D) 60 (E) 20

17. At Gunaroo High School, a two-weekly (10-day) timetable is used, with 5 periods in each day.

		Mon	Tue	Wed	Thu	Fri
	1	Engl	PE			
	2	Maths	Arts			
Period	3	Sci	Engl			
	4	Geog	⋮			
	5	Hist				

Mon	Tue	Wed	Thu	Fri

Students take seven subjects, English, Maths, Science, Geography, History, PE and Arts, in a rotating sequence in the order given, starting with English on Day 1, period 1. This takes up 49 of the 50 available periods. The one remaining period is an assembly which is held on the first period of one of the days of the second week. Which lesson can never be held during period 1 of one of the other days of the second week?

(A) English (B) Maths (C) Geography (D) PE (E) Arts

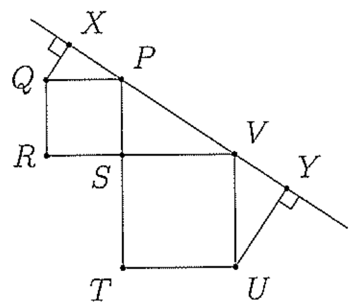
18. Positive integers x and y satisfy $(2^x + 1)(2^y - 1) = 2015$. What is the value of xy?

(A) 36 (B) 40 (C) 12 (D) 30 (E) 24

19. Given an integer N greater than 1, the sum of N and the second largest factor of N can be found. For example, with $N = 55$, the sum is $55 + 11 = 66$. For how many integers is this sum equal to 42?

(A) 3 (B) 4 (C) 1 (D) 0 (E) 2

20. Unequal squares $PQRS$ and $STUV$ are aligned with the straight line PST. XQ and YU are perpendicular to XY. The length $XQ + YU$ is the same as

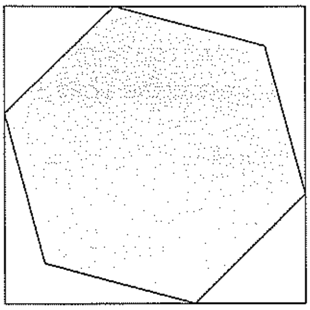


(A) SU (B) RV (C) UQ

(D) PR (E) PV

21. The diagram shows a regular hexagon with sides of length 1 inside a square. Four vertices of the hexagon lie on sides of the square; the other two vertices lie on a diagonal of the square.

What is the side length of the square?



(A) $\sqrt{3}$ (B) $6\left(\sqrt{3}-\sqrt{2}\right)$ (C) $\frac{1}{2}\left(\sqrt{3}+\sqrt{2}\right)$ (D) $\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)$ (E) 2

22. This list

$$3 \quad 5 \quad 15 \quad \boxed{\tfrac{1}{3}} \quad 5 \quad \boxed{\tfrac{1}{15}} \quad \cdots$$

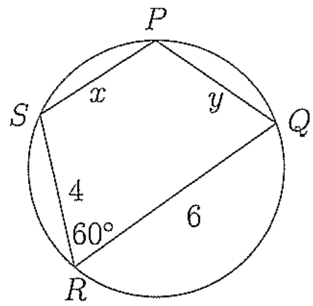
was made by Norm and Zoltan, starting with 3 and 5 and then taking turns to add one number to the list. Norm went first.

Whenever it is his turn, Norm's new number (shown circled) is the product of the last two numbers in the list. Whenever it is his turn, Zoltan's new number (shown boxed) is found by dividing the second-last number by the last number.

What is the 2015th number in this list?

(A) $\frac{1}{15}$ (B) 3 (C) 5 (D) $\frac{1}{3}$ (E) $\frac{1}{5}$

23. Points P, Q, R and S lie on a circle and $\angle SRQ = 60°$. If $RS = 4$, $RQ = 6$, $SP = x$ and $PQ = y$, then a possible solution for x and y is

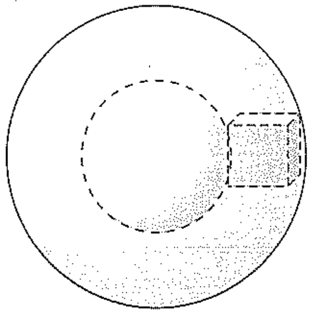


(A) $x = 4$ and $y = 2$ (B) $x = y = 3$ (C) $x = y = 4$ (D) $x = 4$ and $y = 3$ (E) $x = 5$ and $y = 2$

24. How many sequences of seven positive integers $a_1 < a_2 < a_3 < a_4 < a_5 < a_6 < a_7$ are there such that each is less than 100 and each number apart from the last is a factor of the next number in the sequence?

(A) 3 (B) 7 (C) 4 (D) 6 (E) 1

25. Two spheres, one of radius 2 and the other of radius 4, have the same centre. What is the edge size of the largest cube that fits between them?

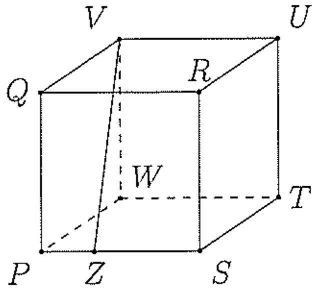


(A) $\frac{6}{5}$ (B) $\frac{1}{3}(\sqrt{19}+1)$ (C) $\frac{\sqrt{21}-2}{3}$ (D) $\frac{2}{3}(\sqrt{22}-2)$ (E) $\frac{12}{5}$

The answers to questions 26–30 are three digit numbers (000 to 999).

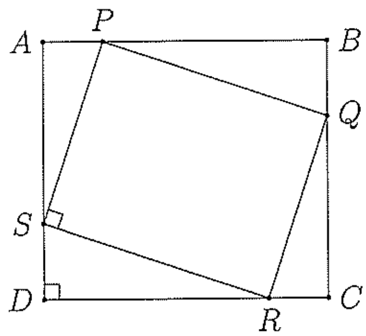
26. How many positive integers n less than 2015 have the property that $\frac{1}{3}+\frac{1}{n}$ can be simplified to a fraction with denominator less than n?

27. A $100 \times 100 \times 100$ cube $PQRSTUVW$ is made of $1 \times 1 \times 1$ non-overlapping cubes. Z is a point on PS such that $PZ = 33$. Through how many of these $1 \times 1 \times 1$ cubes does VZ pass?



28. At Berracan station, northbound trains arrive every three minutes starting at noon and finishing at midnight, while southbound trains arrive every five minutes starting at noon and finishing at midnight. Each day, I walk to Berracan station at a random time in the afternoon and wait for the first train in either direction. On average, how many seconds should I expect to wait?

29. In a 38×32 rectangle $ABCD$, points P, Q, R and S are chosen, one on each side of $ABCD$ as pictured. The lengths AP, PB, BQ, QC, CR, RD, DS and SA are all positive integers and $PQRS$ is a rectangle. What is the largest possible area that $PQRS$ could have?



30. The set S consists of distinct integers such that the smallest is 0 and the largest is 2015. What is the minimum possible average value of the numbers in S?