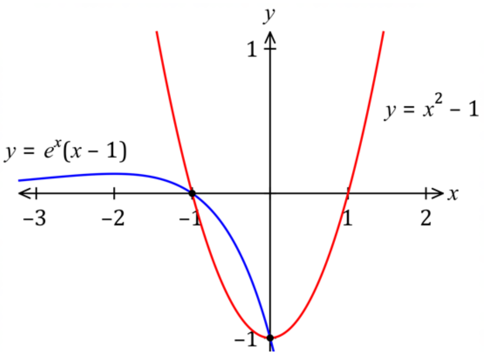
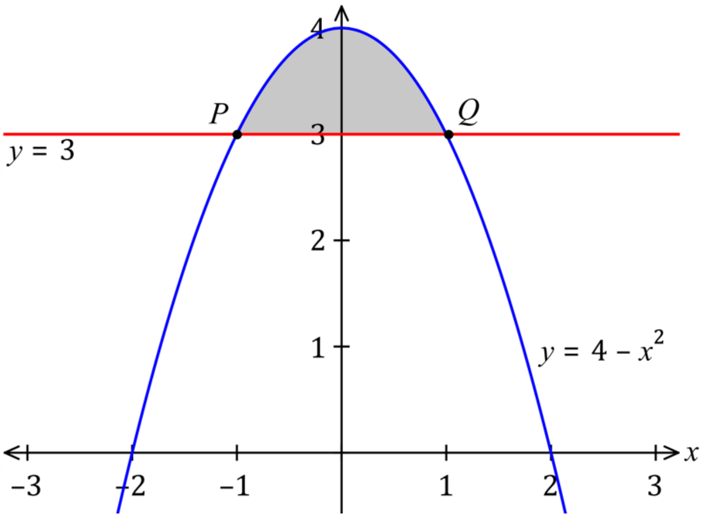
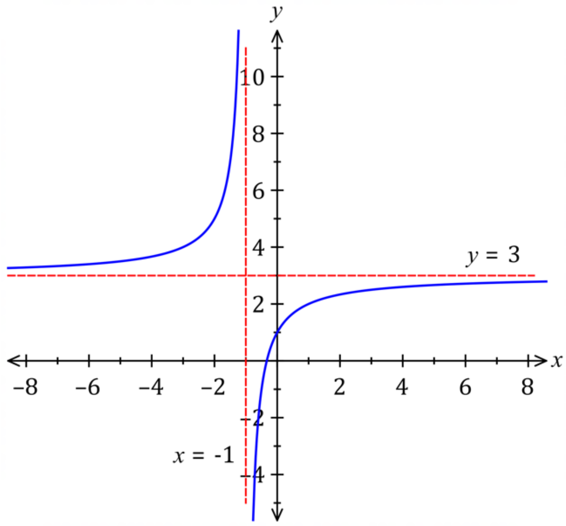
ACE Examination Paper 2
Year 12 Mathematics Advanced Yearly Examination
Worked solutions and marking guidelines

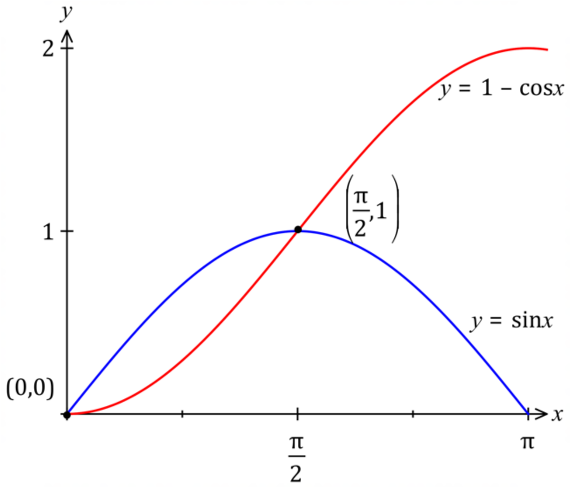
Section I		
	Solution	**Criteria**
1.	$\int_0^1 (6x^2-4)dx = 2\int_0^1 (3x^2-2)dx = 2[x^3-2x]_0^1$ $= 2[(1^3-2\times 1)-(0^3-2\times 0)] = -2$	1 Mark: A
2.	 There are two points of intersection of the two graphs.	1 Mark: C
3.	a = 3 and S = 1.8 $S = \frac{a}{1-r}$ $1.8 = \frac{3}{1-r}$ $1.8 - 1.8r = 3$ $1.8r = -1.2$ $r = -0.\dot{6}$	1 Mark: B
4.	$\frac{d}{dx}e^{x^3} = 3x^2e^{x^3}$	1 Mark: A
5.	Results: {0,0,0,1,1,2,2,2,2,3,3,4,4,5} Calculator: $\bar{x} = 2.1$ and s = 1.6	1 Mark: C
6.	Region is outside one standard deviation 100% – 68% = 32%	1 Mark: B
7.	$a = 12t + 6$ $v = 6t^2 + 6t + C$ When t = 0 then v = –36 $-36 = 6\times 0^2 + 6\times 0 + C$ or $C = -36$ $v = 6t^2 + 6t - 36 = 6(t+3)(t-2)$ $\therefore$ Particle at rest (v = 0) when t = 2	1 Mark: C
8.	$m = r\frac{s_y}{s_x} = 0.561 \times \frac{4.579}{1.987} = 1.29$	1 Mark: B
9.	Test equations with points $\left(\frac{\pi}{6},0\right)$ and $\left(\frac{2\pi}{3},1\right)$ on the curve. (B) $y = \sin\left(\frac{\pi}{6}-\frac{\pi}{6}\right) = 0$ and $y = \sin\left(\frac{2\pi}{3}-\frac{\pi}{6}\right) = 1$ Correct	1 Mark: B
10.	$y = \frac{2}{3}\sqrt{9-x^2} = \frac{h}{2}[y_0 + y_2 + 2\times y_1]$ $= \frac{1}{2}\left[2 + \frac{2\sqrt{8}}{3} + 2\times\frac{2\sqrt{5}}{3}\right] = 3.6309\ldots \approx 3.63$	1 Mark: D

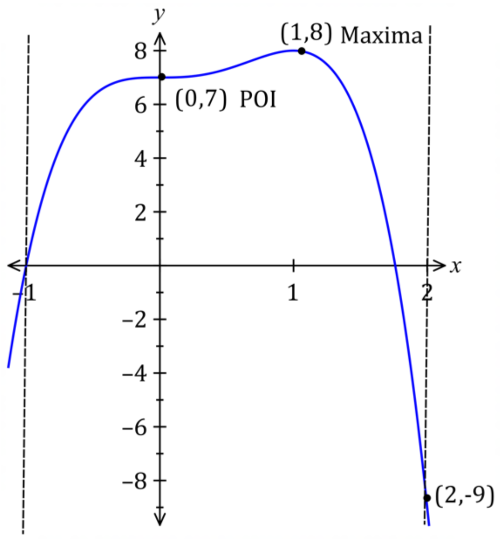
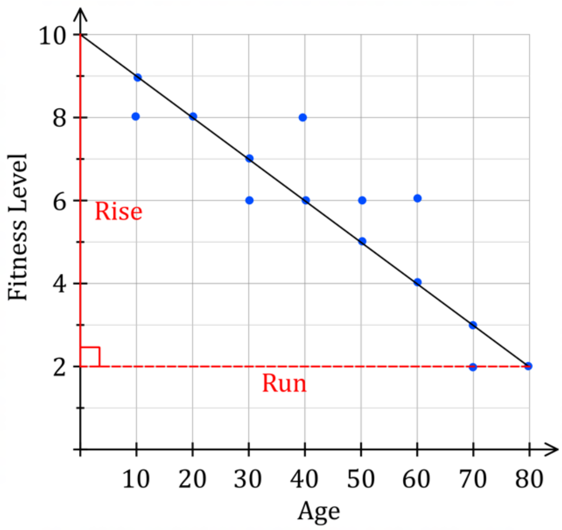
Section II		
11	$\dfrac{y}{y^2-4}-\dfrac{2}{y-2}=\dfrac{y}{(y+2)(y-2)}-\dfrac{2}{y-2}$ $=\dfrac{y-2(y+2)}{(y+2)(y-2)}$ $=\dfrac{-y-4}{y^2-4}$	2 Marks: Correct answer. 1 Mark: Finds a common denominator or shows some understanding.
12(a)	$\sin\theta\cos\theta+\dfrac{\cos^3\theta}{\sin\theta}=\dfrac{\cos\theta}{\sin\theta}(\sin^2\theta+\cos^2\theta)$ $=\cot\theta$	1 Mark: Correct answer.
12(b)	$\cot\theta=1$ $\theta=\dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$	1 Mark: Correct answer.
13(a)	$\dfrac{d}{dx}(\tan5x)=\sec^2 5x\times\dfrac{d}{dx}(5x)$ $=5\sec^2 5x$	1 Mark: Correct answer.
13(b)	$\dfrac{d}{dx}\left(\dfrac{\ln x}{x}\right)=\dfrac{x\times\frac{1}{x}-\ln x\times1}{x^2}$ $=\dfrac{1-\ln x}{x^2}$	1 Mark: Correct answer.
13(c)	$\dfrac{d}{dx}(x\cos x)=-x\sin x+\cos x$	1 Mark: Correct answer.
14	$T_n=a+(n-1)d$ $T_2=a+d=39$ (1) $T_6=a+5d=19$ (2) Equation (2) − (1) $4d=-20$ $d=-5$ Substitute $d=-5$ into equation (1) $a-5=39$ $a=44$ $S_n=\dfrac{n}{2}[2a+(n-1)d]$ $=\dfrac{10}{2}[2\times44+(10-1)\times(-5)]$ $=215$	3 Marks: Correct answer. 2 Marks: Finds the first term and the common difference. 1 Mark: Finds two equations using the *n*th term of a AP or shows some understanding.
15	$\displaystyle\int 4-x^{-3}\,dx=4x+\frac{1}{2}x^{-2}+C$ $=4x+\dfrac{1}{2x^2}+C$	2 Marks: Correct answer. 1 Mark: Finds one of the terms.

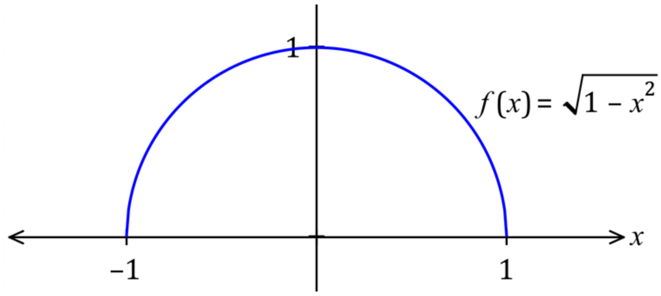
16(a)		2 Marks: Correct answer. 1 Mark: Draws one of the graphs correctly.
16(b)	$y = 4 - x^2$ ① $y = 3$ ② Substitute 3 for y into equation ① $3 = 4 - x^2$ $x^2 = 1$ $x = \pm 1$ $\therefore$ Coordinates are $P(-1, 3)$ and $Q(1, 3)$	1 Mark: Correct answer.
16(c)	$A = 2\int_0^1 [(4 - x^2) - 3]dx$ $= 2\int_0^1 -x^2 + 1\,dx$ $= 2\left[-\frac{x^3}{3} + x\right]_0^1$ $= 2\left[\left(\frac{-1^3}{3}\right) + 1\right]$ $= \frac{4}{3}$ square units	2 Marks: Correct answer. 1 Mark: Correctly sets up the integral.
17	$z = \frac{x - \bar{x}}{s}$ $-2.5 = \frac{x - 72}{8}$ $x = (-2.5 \times 8) + 72$ $= 52\%$ $\therefore$ Riley's mark was 52%.	2 Marks: Correct answer. 1 Mark: Uses the z-score formula with at least one correct vale.
18(a)	$r = \frac{0.084}{12} = 0.0070, n = 4 \times 12 = 48$ Intersection value is 40.64856 Let the monthly repayment be x. $PV = 40.64856 \times x$ $16\ 000 = 40.64856 \times x$ $x = \frac{16\ 000}{40.64856}$ $= 393.6178 \ldots \approx \$393.62$ $\therefore$ Jessica's monthly repayment is \$393.62.	2 Marks: Correct answer. 1 Mark: Finds the intersection value or shows some understanding.

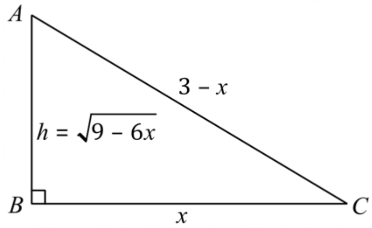
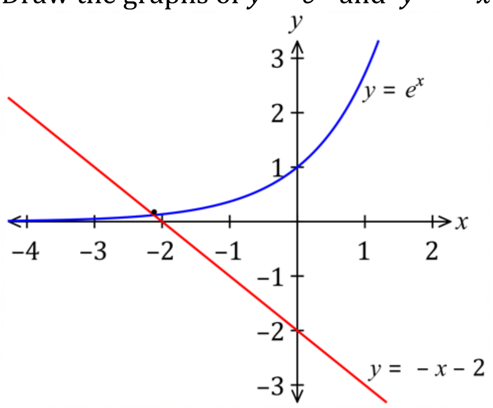
18(b)	Total repaid $= 393.6178\ldots \times 48$ $= 18\,893.6582\ldots \approx \$18\,894$ Interest $= 18\,894 - 16\,000$ $= \$2894$ $\therefore$ Jessica's interest on the loan is \$2894.	2 Marks: Correct answer. 1 Mark: Finds the total amount to be repaid.
19	$\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x)\,dx = \left[\frac{x^3}{3} - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$ $= \left[\frac{\left(\frac{\pi}{6}\right)^3}{3} - \frac{1}{2}\cos\left(2 \times \frac{\pi}{6}\right)\right] - \left[\frac{0^3}{3} - \frac{1}{2}\cos 2 \times 0\right]$ $= \left[\frac{\pi^3}{648} - \frac{1}{4}\right] + \frac{1}{2}$ $= 0.2978\ldots$ ≈ 0.298	2 Marks: Correct answer. 1 Mark: Finds the primitive function or shows some understanding.
20	Intercept is (0, 1). Asymptotes are $x = -1$ and $y = 3$.  	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Draws the general shape of the function.
21	Strong positive correlation indicates that when one variable increases the other variable increases. $\therefore$ Increased spending of advertising are associated with increased profits.	2 Marks: Correct answer. 1 Mark: Shows some understanding.
22(a)	A z-score of 2 is two standard deviations above the mean. That is, Marcus scored 89% in the class test.	1 Mark: Correct answer.
22(b)	$z = \frac{x - \bar{x}}{s}$ $= \frac{51.5 - 64}{12.5}$ $= -1$ $\therefore$ Fletcher's z-score is -1	1 Mark: Correct answer.

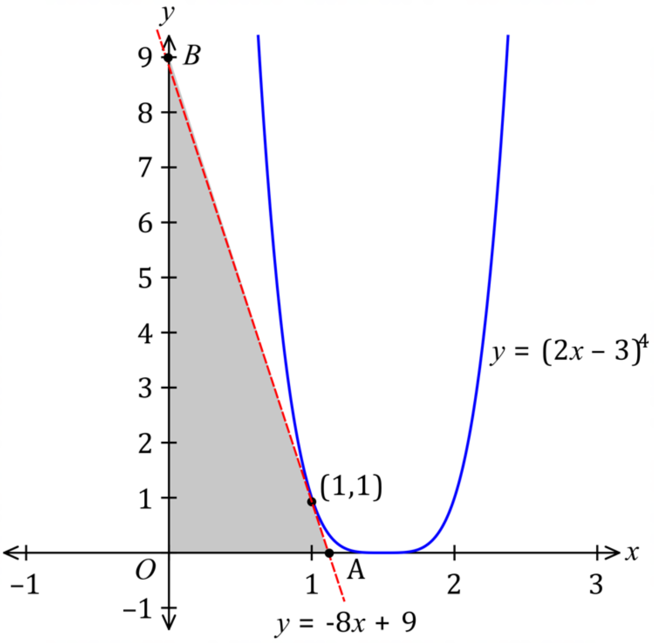
22(c)	$z = \dfrac{x - \bar{x}}{s}$ $3 = \dfrac{x - 64}{12.5}$ $x = (3 \times 12.5) + 64$ $\quad = 101.5\%$ $\therefore$ Ayla needs to score 101.5% in the test (impossible).	1 Mark: Correct answer.
23	The object comes to rest when $\dot{x} = 0$ $x = 3e^{-2t} + 10e^{-t} + 4t$ $\dot{x} = -6e^{-2t} - 10e^{-t} + 4$ $\quad = -2(3e^{-2t} + 5e^{-t} - 2)$ Let $m = e^{-t}$ $-2(3m^2 + 5m - 2) = 0$ $-2(3m - 1)(m + 2) = 0$ Hence $3m - 1 = 0$ or $m + 2 = 0$ (No solution: $e^{-t} \neq -2$) $m = \dfrac{1}{3}$ $e^{-t} = \dfrac{1}{3}$ $t = -\ln\left(\dfrac{1}{3}\right)$ $\quad = \ln 3$ $\therefore$ Object comes to rest after ln3 seconds.	3 Marks: Correct answer. 2 Marks: Finds and factorises the quadratic equation. 1 Mark: Correctly differentiates x.
24(a)	P = \$50 000, r = 0.02 per month, n = 12 months $FV = PV(1 + r)^n$ $A_{12} = 50\,000 \times (1 + 0.02)^{12} - M$ $\quad = 50\,000 \times 1.02^{12} - M$	1 Mark: Correct answer.
24(b)	Amount owed at the end of the second year. $A_{24} = (50\,000 \times 1.02^{12} - M) \times 1.02^{12} - M$ $\quad = 50\,000 \times 1.02^{24} - M \times 1.02^{12} - M$ $\quad = 50\,000 \times 1.02^{24} - M(1.02^{12} + 1)$ Now $A_{24} = 0$ when the loan is paid off. $0 = 50\,000 \times 1.02^{24} - M(1.02^{12} + 1)$ $M(1.02^{12} + 1) = 50\,000 \times 1.02^{24}$ $M = \dfrac{50\,000 \times 1.02^{24}}{1.02^{12} + 1}$	2 Marks: Correct answer. 1 Mark: Finds the correct expression for A_{24} or shows some understanding of the problem.
24(c)	$M = \dfrac{50\,000 \times 1.02^{24}}{1.02^{12} + 1}$ $\quad = \$35\,455.5950\ldots$ Total paid $= \$35\,455.5950\ldots \times 2$ $\quad = \$70\,911.1900\ldots$ Interest $= \$70\,911.1900\ldots - \$50\,000$ $\quad = \$20\,911.1900\ldots$ $\quad \approx \$20\,911.19$ $\therefore$ Total amount of intertest paid was \$20 911.19	2 Marks: Correct answer. 1 Mark: Makes significant progress.

25	To find the expected value or mean $\int_0^2 x(x^3)dx = \int_0^2 x(x^3)\,dx$ $= \int_0^2 x^4 dx$ $= \left[\frac{x^5}{5}\right]_0^2$ $= 6.4$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
26(a)		2 Marks: Correct answer. 1 Mark: Draws one of the curves.
26(b)	$x = 0$ or $x = \frac{\pi}{2}$ (from the graph)	1 Mark: Correct answer.
26(c)	$A = \int_0^{\frac{\pi}{2}}[\sin x - (1 - \cos x)]dx + \int_{\frac{\pi}{2}}^{\pi}[(1 - \cos x) - \sin x]dx$ $= [-\cos x - x + \sin x]_0^{\frac{\pi}{2}} + [x - \sin x + \cos x]_{\frac{\pi}{2}}^{\pi}$ $= \left(0 - \frac{\pi}{2} + 1 - (-1 - 0 + 0)\right) + \left(\pi - 0 - 1 - \left(\frac{\pi}{2} - 1 + 0\right)\right)$ $= 2 - \frac{\pi}{2} + \frac{\pi}{2}$ $= 2$ square units	2 Marks: Correct answer. 1 Mark: Shows some understanding.
27	$y = x^2 - 4x$ $y' = 2x - 4$ At the point (1, –3)) $y' = 2 \times 1 - 4 = -2$ Gradient of the normal $m_1 m_2 = -1$ $m \times -2 = -1$ $m = \frac{1}{2} = 0.5$ Equation of the normal $y - y_1 = m(x - x_1)$ $y - (-3) = 0.5(x - 1)$ $2y + 6 = x - 1$ $x - 2y - 7 = 0$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the normal.

28(a)	$f(x) = 7 + 4x^3 - 3x^4$ Stationary points $f'(x) = 0$ $f'(x) = 12x^2 - 12x^3$ $12x^2(1 - x) = 0$ $x = 0, x = 1$ When $x = 0, y = 7 + 4 \times 0^3 - 3 \times 0^4 = 7$ When $x = 1, y = 7 + 4 \times 1^3 - 3 \times 1^4 = 8$ $\therefore$ Stationary points are $(0, 7)$ and $(1, 8)$	2 Marks: Correct answer. 1 Mark: Finds the first derivative and equates it to zero
28(b)	$f''(x) = 0$ $24x - 36x^2 = 0$ $12x(2 - 3x) = 0$ $x = 0$ or $x = \frac{2}{3}$	1 Mark: Correct answer.
28(c)	$f''(x) = 12x(2 - 3x)$ At $(0, 7)$, $f''(0) = 0$ Possible of inflexion At $(1, 8)$, $f''(1) = -12 < 0$ Maxima At $(0, 7)$ check for change in concavity $x = -0.1, f''(x) = 12 \times -0.1(2 - 3 \times -0.1) = -2.76 < 0$ $x = 0.1, f''(x) = 12 \times 0.1(2 - 3 \times 0.1) = 2.04 > 0$ $\therefore (0, 7)$ is a point of inflexion and $(1, 8)$ is a maxima.	2 Marks: Correct answer. 1 Mark: Finds the nature of one of the points.
28(d)	 	2 Marks: Correct answer. 1 Mark: Obtains the correct general shape of the curve or shows some understanding.
29(a)	$m = \frac{\text{Rise}}{\text{Run}}$ $= -\frac{8}{80}$ $= -0.1$ $\therefore$ Gradient is -0.1  	2 Marks: Correct answer. 1 Mark: Finds the line of best fit or shows some understanding.

29(b)	When age = 30 then fitness level = 7 (from the scatterplot) $\therefore$ Lachlan's fitness level should be 7.	1 Mark: Correct answer.
29(c)	Data: (10,8)(10,9)(20,8)(30,6)(30,7)(40,6)(40,8)(50,5)(50,6)(60,4)(60,6)(70,2)(70,3)(80,2) $r = -0.9115\ldots$ ≈ -0.91	2 Marks: Correct answer. 1 Mark: Finds a value of r close to −0.9.
30	 Domain: $-1 \leq x \leq 1$ Range: $0 \leq y \leq 1$	2 Marks: Correct answer. 1 Mark: Domain or range
31	$\int_0^2 \frac{1}{9}(4x - x^2)dx = \frac{1}{9}\left[2x^2 - \frac{x^3}{3}\right]_0^2$ $= \frac{1}{9}\left[\left(2 \times 2^2 - \frac{2^3}{3}\right) - \left(2 \times 0^2 - \frac{0^3}{3}\right)\right]$ $= \frac{16}{27}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
32	$\frac{d^2y}{dx^2} = 12x + 6$ $\frac{dy}{dx} = 6x^2 + 6x + C_1$ At $(1, -2)$ $\frac{dy}{dx} = 0$ $6 \times 1^2 + 6 \times 1 + C_1 = 0$ $C_1 = -12$ $\frac{dy}{dx} = 6x^2 + 6x - 12$ $y = 2x^3 + 3x^2 - 12x + C_2$ $(1, -2)$ satisfies the equation of the curve $2 \times 1^3 + 3 \times 1^2 - 12 \times 1 + C_2 = -2$ $C_2 = 5$ $\therefore y = 2x^3 + 3x^2 - 12x + 5$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds the first derivative.
33	$\lim_{x \to 0} \frac{\sin 6x}{x} = 6 \lim_{5x \to 0} \frac{\sin 6x}{6x}$ $= 6$	2 Marks: Correct answer. 1 Mark: Shows understanding.

34(a)	$AC = (3 - x)$ metres	1 Mark: Correct answer.
34(b)	$(3 - x)^2 = h^2 + x^2$ $h^2 = 9 - 6x + x^2 - x^2$ $h = \sqrt{9 - 6x}$ ($h > 0$ as h is a height) $A = \frac{1}{2}bh$ $= 0.5x\sqrt{9 - 6x}$ m^2 	2 Marks: Correct answer. 1 Mark: Finds the height of the triangle or shows some understanding.
34(c)	Maximum occurs when $\frac{dA}{dx} = 0$ $A = 0.5x\sqrt{9 - 6x}$ $\frac{dA}{dx} = 0.5\left[x \times \frac{1}{2}(9 - 6x)^{-\frac{1}{2}} \times (-6) + (9 - 6x)^{\frac{1}{2}} \times 1\right]$ $= \frac{1}{2}(9 - 6x)^{-\frac{1}{2}}[-3x + (9 - 6x)^1]$ $= \frac{-9x + 9}{2\sqrt{9 - 6x}} = \frac{9(1 - x)}{2\sqrt{9 - 6x}}$ Now $\frac{dA}{dx} = \frac{9(1 - x)}{2\sqrt{9 - 6x}} = 0$ $\therefore x = 1$ $x = 0.9, \quad \frac{dA}{dx} = \frac{9(1 - 0.9)}{2\sqrt{9 - 6 \times 0.9}} > 0$ $x = 1.1, \quad \frac{dA}{dx} = \frac{9(1 - 1.1)}{2\sqrt{9 - 6 \times 1.1}} < 0$ $\therefore$ Maximum occurs when $x = 1$.	3 Marks: Correct answer. 2 Marks: Finds $x = 1$ 1 Mark: Calculates the first derivative or has some understanding of the problem.
34(d)	$A = 0.5x\sqrt{9 - 6x}$ $= 0.5 \times 1 \times \sqrt{9 - 6 \times 1}$ $= 0.5\sqrt{3}$ m^2 $\therefore$ Maximum possible area is $0.5\sqrt{3}$ m^2	1 Mark: Correct answer.
35	Draw the graphs of $y = e^x$ and $y = -x - 2$  $\therefore$ There is 1 solution for $e^x + x + 2 = 0$ (point of intersection)	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Draws one graph correctly.
36(a)	Using the statistic mode on the calculator. $\bar{x} = 17.3333\ldots \approx 17.3$	1 Mark: Correct answer.

36(b)	Using the statistic mode on the calculator. $Q_1 = 13.5$ and $Q_3 = 19$ $\text{IQR} = Q_3 - Q_1$ $= 19 - 13.5 = 5.5$	1 Mark: Correct answer.
36(c)	Outlier Upper limit $= Q_3 + 1.5 \times \text{IQR}$ $= 19 + 1.5 \times 5.5$ $= 27.25$ $\therefore$ 29 is an outlier as it is above the upper limit of 27.25	1 Mark: Correct answer.
37(a)	$f(x) = (2x-3)^4$ $f'(x) = 4(2x-3)^3 \times 2$ $= 8(2x-3)^3$ $f'(1) = 8(2 \times 1 - 3)^3$ $= -8$	2 Marks: Correct answer. 1 Mark: Finds $f'(x)$.
37(b)	Equation of the tangent at (1, 1) with gradient –8. $y - y_1 = m(x - x_1)$ $y - 1 = -8(x-1)$ $y = -8x + 9$ or $8x + y - 9 = 0$	1 Mark: Correct answer.
37(c)	 To find the x-intercept ($y = 0$) $y = -8x + 9$ $= -8 \times 0 + 9 = 9$ $\therefore B(0, 9)$ To find the y-intercept ($x = 0$) $y = -8x + 9$ $0 = -8x + 9$ $x = \frac{9}{8}$ $\therefore A(\frac{9}{8}, 0)$ $A = \frac{1}{2}bh = \frac{1}{2} \times \frac{9}{8} \times 9$ $= \frac{81}{16}$ square units $\therefore$ Area of ΔOAB is $\frac{81}{16}$ square units.	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds point A or point B.