Carlingford High School

Mathematics
Year 10 Yearly Examination
5.3 Course
2015

Name: _____   Class: 105.3 ___

Circle your teacher's name: Mr Wilson   Mr Gong   Mr Cheng   Mrs Lego

Time allowed: 1.5 Hours

- Board approved calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or untidy work.
- Complete the examination in blue or black pen.

Topics	
Co-ordinate Geometry	/9
Non-Linear Relationship and Graphs	/15
Surface Area and Volume	/9
Solving Inequalities and Quadratic	/14
Data Analysis	/12
Trigonometry	/11
Geometric Figures	/8
Probability	/16
Total	/94

KC

PW
plus Trig a-d

AG
Trig e,f
*plus*

Coordinate Geometry (9 marks)

a   The gradient and y-intercept of the linear equation $y = 3x - 7$ when graphed would be:

        (A) y-intercept $= -4$, gradient $= -2\frac{1}{3}$

        (B) y-intercept $= -7$, gradient $= 3$

        (C) y-intercept $= 3$, gradient $= -7$

        (D) y-intercept $= 2\frac{1}{3}$, gradient $= -4$

b   On which line does the point $(2, -3)$ lie?

        (A) $y = x + 5$

        (B) $y = -2x + 1$

        (C) $x + y - 1 = 0$

        (D) $3x + 2y - 12 = 0$

c   In the graph below, A has coordinates of $(6, -3)$. Find the equation of the vertical line which passes through A. (1 mark)

$A\ (6, -3)$

$x = 6$
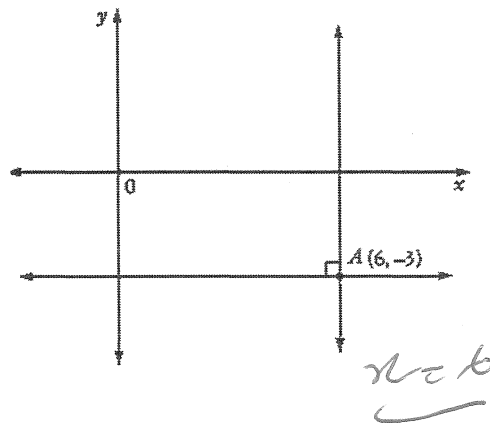
*d   Find the equation of the line that passes through the point $(-1, 3)$ and is parallel to $2x + y + 4 = 0$. Express your answer in general form. (2 marks)
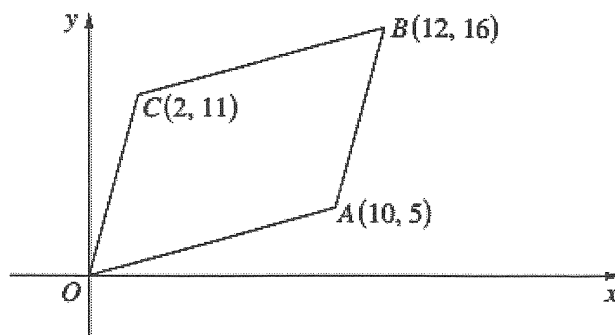
$$2x + y + 4 = 0$$
$$y = -2x - 4$$
$$m_1 = -2 \quad ①$$
$$y - 3 = -2(x + 1)$$
$$y - 3 = -2x - 2$$
$$2x + y - 1 = 0 \quad ①$$

1

e	In the diagram below, A, B and C are the points $(10,5), (12,16)$ and $(2, 11)$ respectively.

i) Find the distance AC. (1 mark)

$$d = \sqrt{(10-2)^2 + (5-11)^2}$$

$$= 10 \text{ units}$$

ii) Find the midpoint of AC. (1 mark)

$$x = \frac{10+2}{2} \qquad y = \frac{5+11}{2}$$

$$= 6 \qquad\qquad = 8$$

$$\therefore (6, 8)$$

iii) Show that OB is perpendicular to AC (2 marks)

$$m \text{ of } OB = \frac{16}{12}$$

$$= \frac{4}{3}$$

$$m \text{ of } AC = \frac{5-11}{10-2}$$

$$= -\frac{6}{8}$$

$$= -\frac{3}{4}$$

$$\text{since } \frac{4}{3} \times -\frac{3}{4} = -1$$

$$\therefore OB \perp AC$$

2

Non-Linear Relationships and Graphs of Physical Phenomena (15 marks)

a   Match each graph to its equation. (4 marks)

_E_

(-2, 25)

_F_

8

_C_

(-2, 2)

_B_

3

-3   0   3 x

-3
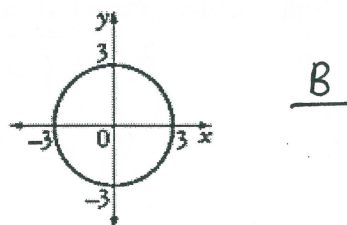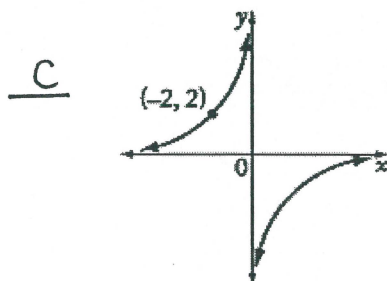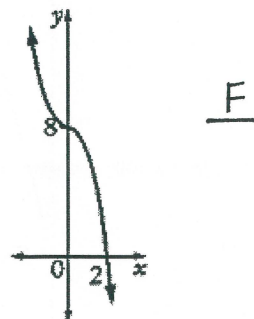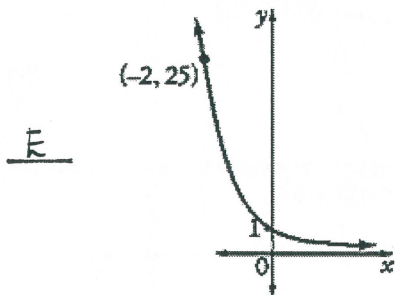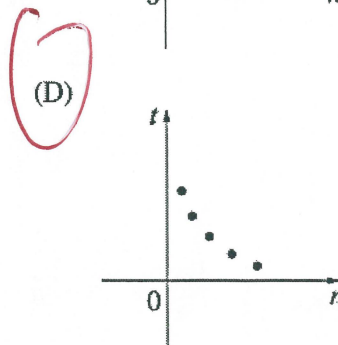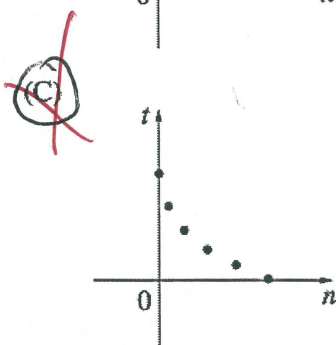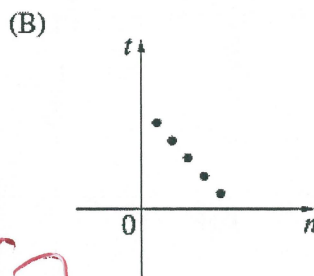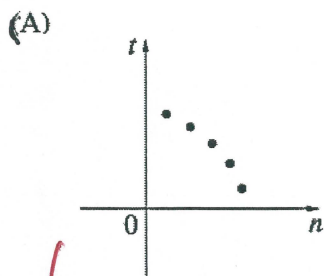
A $y = 2x - 4$

B $x^2 + y^2 = 9$

C $y = -\dfrac{4}{x}$

D $y = 2x^2 - 3$

E $y = 5^{-x}$

F $y = -x^3 + 8$

b   The time (t) taken to clean a house varies inversely with the number (n) of people cleaning the house. Which graph represents this relationship?

(A)

t

0   n

(B)

t

0   n

(C)

t

0   n

(D)

t

0   n

3

c   The height (H) of a particular termite mound is directly proportional to the square root of the number of termites (N). The height of this mound is 35 cm when the number of termites is 4000

    i) Write an equation relating H, N and k, where k is the constant of variation.
      (2 marks)

$$H \propto \sqrt{N}$$
$$H = k\sqrt{N}$$
$$35 = k\sqrt{4000}$$
$$\frac{35}{\sqrt{4000}} = k \qquad\qquad k = 0.6 \;①$$
$$\therefore H = 0.6\sqrt{N} \;①$$

    ii) What is the height of this mound, in centimetres, when there are 10 000 termites?
      (1 mark)

$$H = 0.6\sqrt{N}$$
$$= 0.6\sqrt{10\,000}$$
$$= 60 \text{ cm.}$$

d   The graph below shows the exchange rate to convert Australian dollars to Japanese yen.

Exchange rate, Australian $ to Japanese yen
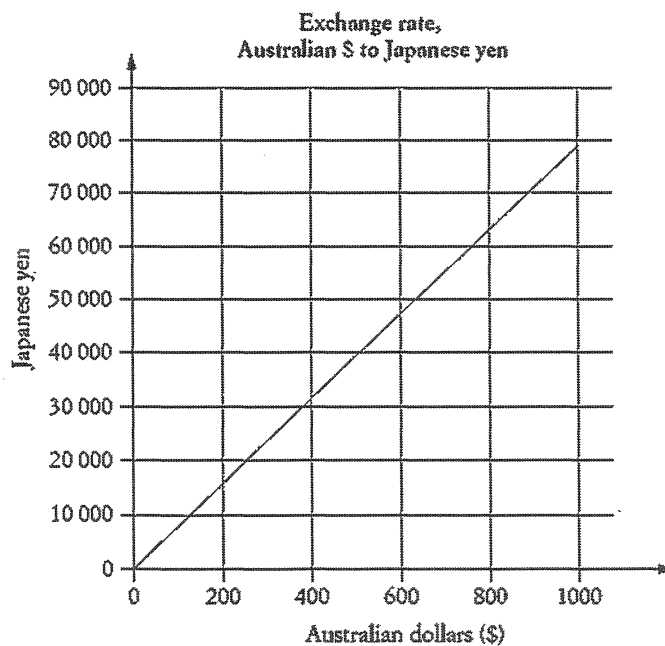
Convert $A900 to Japanese yen. (1 mark)

70000 yen

4

e   Ben leaves Solartown and cycles to Eagletown, a distance of 90 km. At the same time,
Luke leaves Eagletown and cycles to Solartown and later returns to Eagletown. The
graph shows the first part of Luke's journey. Ben's journey is shown until 1 pm.

Bicycle Journeys
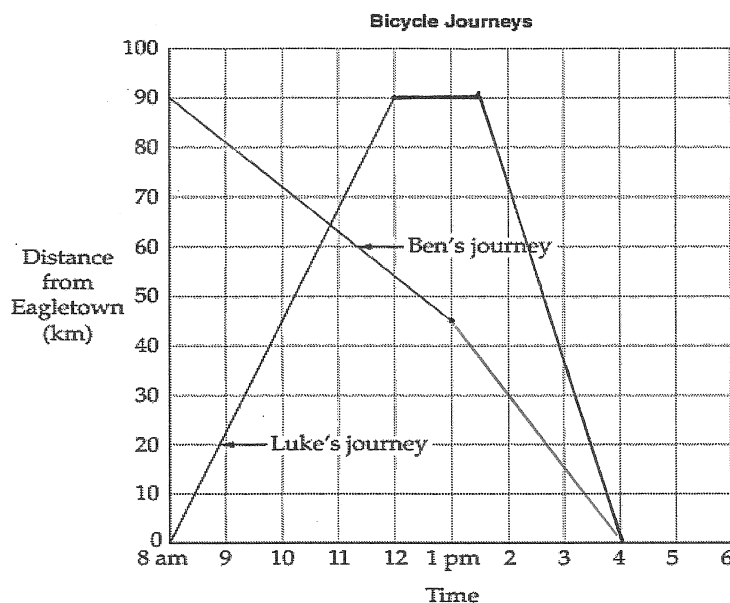
i)   After arriving at Solartown, Luke rests for $1\frac{1}{2}$ hours. He then cycles back to
Eagletown at a constant spped, arriving at 4 pm. Complete the graph of Luke's
journey between 12 and 4 pm. (1 mark)

ii) What is Luke's average speed for the first 4 hours of his journey? (1 mark)
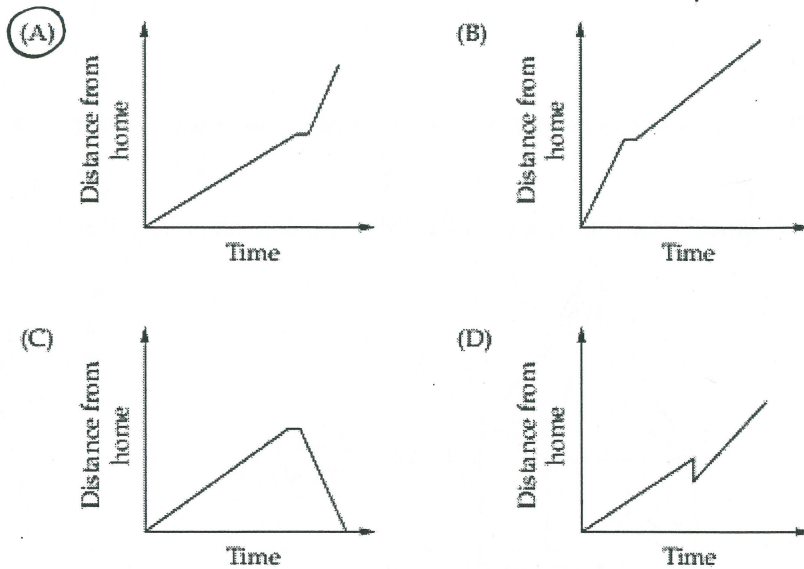
$$Speed = \frac{90}{4}$$

$$= 22.5 \ km/h.$$

*iii) At 1 pm, Ben realises that in order to arrive at Eagletown by 4 pm he must
increase his average speed. By how much must Ben increase his average speed?
All calculations must be shown. (2 marks)

$$initial \ speed = \frac{45}{5}$$

$$= 9 \ km/h$$

$$(1pm-4pm) \ spead = \frac{45}{3}$$

$$= 15 \ km/h$$

$$\therefore increase = 6 \ km/h$$

5

f Mark left home and drove to work. Along the way he had to stop at a school crossing. He then travelled at an increased speed, until he arrived at work. Which graph below best shows this journey?

(A)

Distance from home | Time

(B)

Distance from home | Time

(C)

Distance from home | Time

(D)

Distance from home | Time

g John drove from O to S as shown by the graph below, in 3.5 hours. His speed varied due to traffic and road conditions.
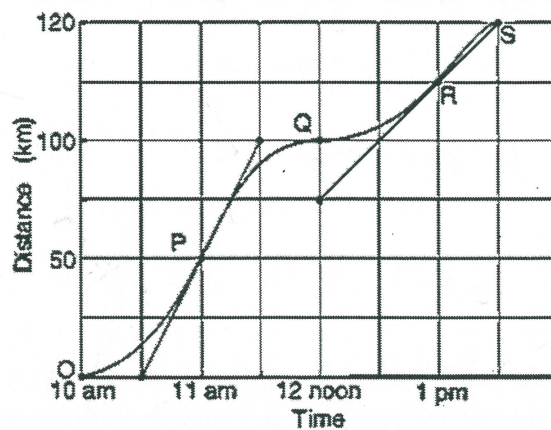
Using the graph, determine whether John was driving faster at point Q or point R?
(1 mark)

Faster at R

Surface Area and Volume (9 marks)

a   A sphere has a volume of 360 cm^3. What is its radius, correct to one decimal place?
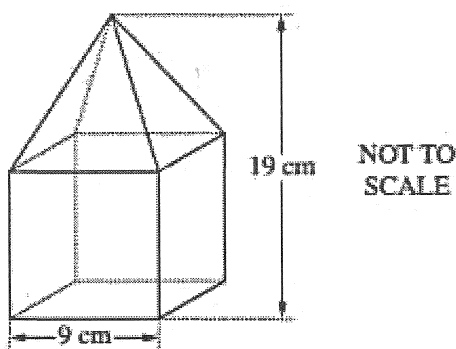
    (A) 1.7 cm

    (B) 4.4 cm

    (C) 8.1 cm

    (D) 9.3 cm

b   A square pyramid fits exactly on top of a cube to form a solid, as shown below.

19 cm   NOT TO SCALE

9 cm

Calculate the volume of the solid. (2 marks)

$$V \text{ of cube} = 9^3$$
$$= 729 \text{ cm}^3$$

$$V \text{ of pyramid} = \tfrac{1}{3} \times 9^2 \times 10$$
$$= 270 \text{ cm}^3 \quad ①$$
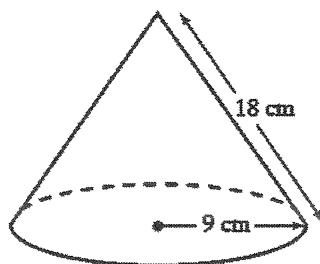
$$\therefore \text{ total } V = 999 \text{ cm}^3$$

c   The game of Beach Quidditch is played with a large hollow spherical ball made from gold vinyl. The diameter of the ball is 1.2 metres. If the vinyl costs $32 per square metre, calculate the cost of the vinyl for one ball. (2 marks)

$$SA = 4\pi \times 0.6^2$$
$$= 4.5238\ldots \quad ①$$

$$\text{Cost} = 4.5238\ldots \times \$32$$
$$= \$144.76 \quad ①$$

d   For the cone, find correct to one decimal place:

i) the curved surface area.(1 mark)

$$curved \ S.A. = \pi r l$$
$$= \pi \times 9 \times 18$$
$$= 508.9 \ cm^2$$

ii) the total surface area. (1 mark)

$$total \ SA = 508.9 + \pi \times 9^2$$
$$= 763.4070 \cdots$$
$$= 763.4 \ cm^2$$

e   Two similar cylinders have their surface areas in the ratio 25 : 36. If the volume of the larger cylinder is 432 cm^3, find the volume of the smaller solid. (2 marks).

$$25 : 36$$
$$5 : 6$$
$$5^3 : 6^3$$
$$125 : 216$$
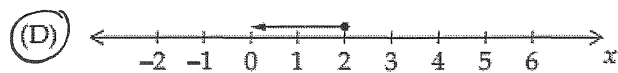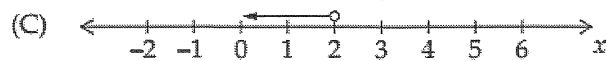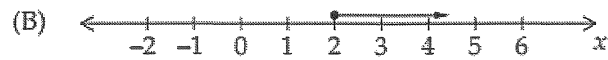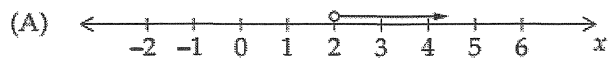$$250 : 432$$

$$\therefore \ v \ of \ smaller \ solid = 250 \ cm^3$$

Solving Inequalities and Quadratic (14 marks)

a   Which graph represents the inequality $x \leq 2$?

(A)
$$\begin{array}{ccccccccc} -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \quad x$$

(B)
$$\begin{array}{ccccccccc} -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \quad x$$

(C)
$$\begin{array}{ccccccccc} -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \quad x$$

(D)
$$\begin{array}{ccccccccc} -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \quad x$$

b   Solve the following quadratic equations: (3 marks)

i) $(m + 4)(m - 2) = 0$

$m = -4 \text{ or } m = 2$ ①

ii) $2y^2 + 7y + 6 = 0$

$(2y + 3)(y + 2) = 0$

$2y + 3 = 0 \qquad \text{or} \quad y = -2$

$y = -\frac{3}{2}$

$y = -1\frac{1}{2}$

c   Solve the following equation by **completing the square**. Leave your answer in exact form. (2 marks)

$$h^2 + 2h - 5 = 0$$

$h^2 + 2h + 1^2 = 5 + 1^2$

$(h + 1)^2 = 6$

$h + 1 = \pm\sqrt{6}$

$h = \pm\sqrt{6} - 1$

9

d   Solve the following equation using the **quadratic formula.** Answer correct to one decimal place.(2 marks)

$$3k^2 - 8k + 2 = 0$$

$$k = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= 2.4 \quad \text{or} \quad 0.3$$

e   Solve the following, correct to two decimal places. (2 marks)

$$m^6 + 6m^3 + 5 = 0$$

let $x = m^3$

$$x^2 + 6x + 5 = 0$$
$$(x + 1)(x + 5) = 0$$

$x + 1 = 0$ or $x + 5 = 0$

$m^3 + 1 = 0$ $m^3 + 5 = 0$

$m^3 = -1$ $m^3 = -5$

$m = \sqrt[3]{-1}$ $m = \sqrt[3]{-5}$

$m = -1$ $m = -1.71$

f   For the parabola $y = x^2 - 6x - 40$, calculate the: (4 marks)

i) the x-intercepts
ii) the y-intercept
iii) the equation of the axis of symmetry
iv) the coordinates of the vertex

$\underline{x\text{-int}}$

$0 = x^2 - 6x - 40$

$0 = (x - 10)(x + 4)$

$x = 10, \ -4$ ①

$\underline{y\text{-int when } x = 0}$

$y = -40$ ①

$\underline{\text{axis of symmetry}}$

$x = \dfrac{--6}{2 \times 1}$

$= 3$ ①

vertex $= (3, -49)$ ①

Data Analysis (12 marks)

a   The box and whisker plot shows the results of a test out of fifty of 80 students.
(4 marks)
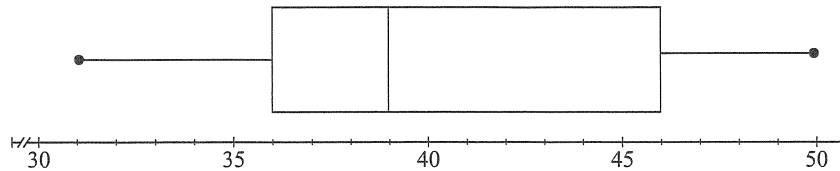
i) Find the range of the test results.

19

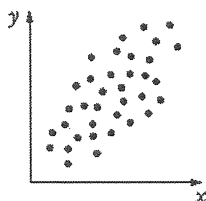ii) Find the median test score.

39

iii) What is the interquartile range?

10

iv) How many students had a test mark between 31 and 36?
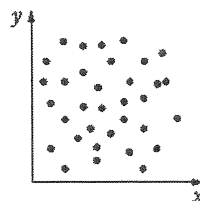
25% x 80 = 20 students.

b   Describe the strength and direction of the relationship shown in each of the scatter plot below. (3 marks)
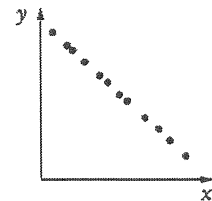
i)

weak positive relationship

ii)

no relationship

iii)

perfect negative relationship

11

c   Data was collected from 30 students on the number of text messages they had sent in the previous 24 hours. The set of data collected is displayed below.

Male		Female
9 9 8 7 6 5 5 4 2 1	0	8 9
1 1 0 0	1	1 1 2 5 6 8 8 8
0	2	0 1 7
	3	4
	4	
	5	
	6	
1	7	

i) What is the outlier for this set of data? (1 mark)

71

iii) Describe the shape of the distribution for the male students (1 mark).

positively skewed

d   William and Kate's times for running 100 metres time trials are given below.

William :   13.0   14.2   13.7   13.2   14.7   13.5   14.3
Kate   :   13.2   15.2   13.9   13.8   14.2   15.5   12.9

i)   Calculate the mean and standard deviation for each runner, correct to one decimal place.(2 marks)

william

$\bar{x}$ = 13.8

SD = 0.6

kate

$\bar{x}$ = 14.1

SD = 0.9

ii) Which runner is more consistent? Give reasons. (1 mark)

William is more consistent as his standard deviation is lower.

Trigonometry (11 marks)
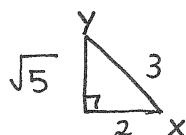
a   If θ is acute, find θ if $\tan 140° = -\tan \theta°$ (1 mark)

$$\theta = 180° - 140°$$
$$= 40°$$

$$\therefore \tan 140° = -\tan 40°.$$

b   If $X + Y = 90°$ and $\cos X = \frac{2}{3}$, find the exact value of $\sin X$. (1 mark)
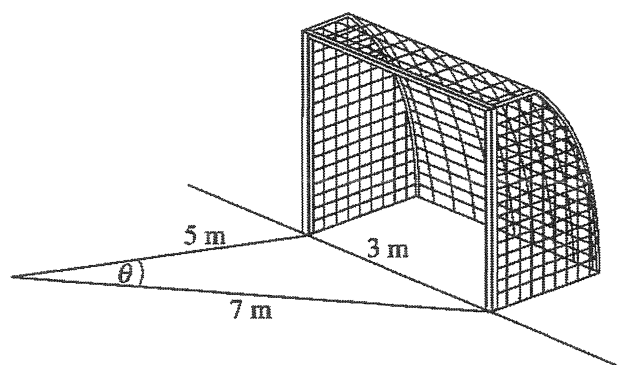

$$\sin X = \frac{\sqrt{5}}{3}$$

c   Solve the trigonometric equation $\sin x = 0.64$, if x is obtuse. Answer correct to the nearest minute. (2 marks)

$$\sin x = 0.64$$
$$x = 39.7918..$$

but x is obtuse, $\quad \therefore \quad x = 180 - 39.7918$
$$= 140°12'$$

d

NOT TO SCALE

In a game of Goal-ball the goal is 3 m wide. Scott has the ball in a position 5 metres from the closest post and 7 m from the other post. Calculate the size of the angle θ, to the nearest degree. (2 marks)

$$\cos \theta = \frac{5^2 + 7^2 - 3^2}{2 \times 5 \times 7}$$

$$= 22°.$$

e   During a trek in Nepal, Anne recorded the angle of elevation from various points to the top of a mountain. In the diagram below, AM represents the height of the mountain and ACB is a straight line.
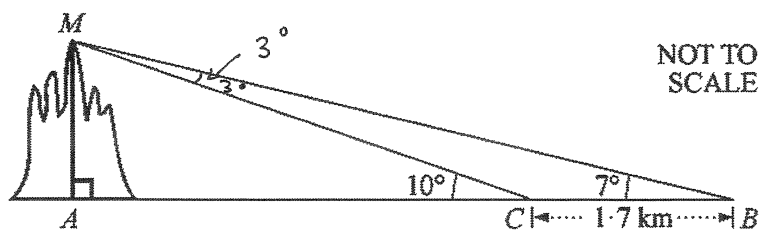
i) Find the size of $< BMC$. (1 mark)
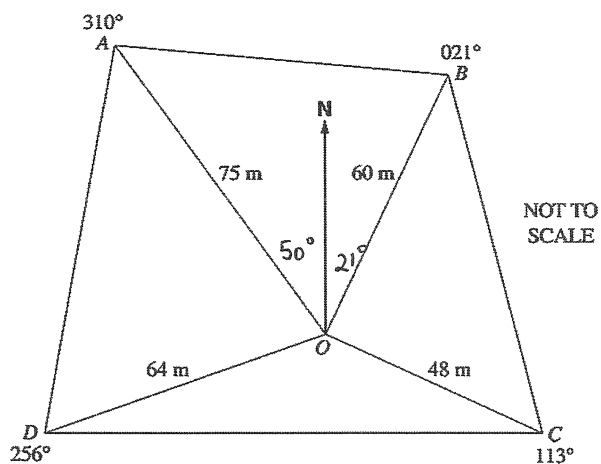
$$3^{\circ}$$

ii) Calculate the length of CM, correct to two decimal places. (2 marks)

$$\frac{CM}{\sin 7^{\circ}} = \frac{1.7}{\sin 3^{\circ}}$$

$$CM = \frac{1.7}{\sin 3^{\circ}} \times \sin 7^{\circ}$$

$$= 3.96 \, km$$

f   A compass radial survey of the field ABCD has been conducted from O.

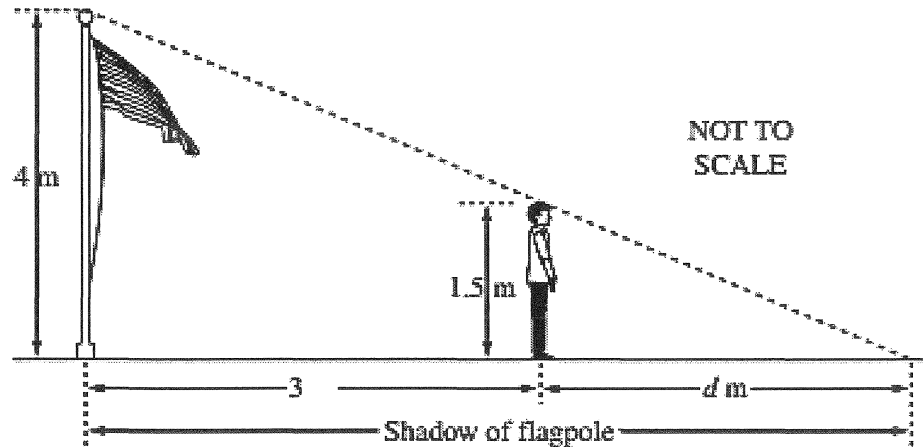

Find the area of the section ABO, to the nearest square metre. (2 marks)

$$A = \frac{1}{2} \times 75 \times 60 \times \sin 71^{\circ}$$
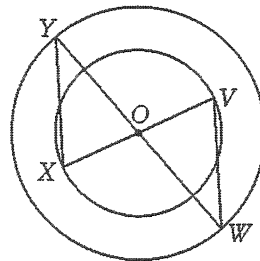
$$= 2127 \, m^2$$

Geometric Figures (8 marks)

a   Jacques and a flagpole both cast shadows on the ground, as shown below. The difference between the lengths of their shadow is 3 metres.

What is the value of d, the length of Jacques' shadow? (2 marks)

$$\frac{d}{3+d} = \frac{1.5}{4}$$

$$4d = 1.5(3+d)$$
$$4d = 4.5 + 1.5d$$
$$2.5d = 4.5$$
$$d = 1.8\,m$$

b O is the centre of both circles as shown below. Prove that $\triangle XOY \equiv \triangle VOW$. (2 marks)

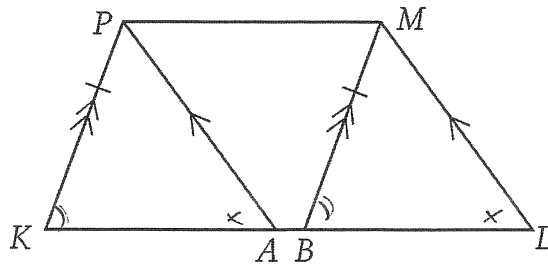$$XO = VO \quad (\text{radii of circle})$$
$$YO = WO \quad (\text{radii of circle})$$
$$\angle YOX = \angle WOV \quad (\text{vertically opposite angles are equal})$$

$$\therefore \triangle XOY \equiv \triangle VOW \quad (SAS)$$

c   In the diagram below, $KP \parallel BM$, $AP \parallel LM$ and $KP = BM$.

Prove that :

i) $\triangle KAP \equiv \triangle BLM$ (2 marks)

$\angle PKA = \angle MBL$ (alt. $\angle$s on $\parallel$ lines are equal)

$\angle PAK = \angle MLB$ (alt $\angle$s on $\parallel$ lines are equal)

$KP = BM$ (given)

$\therefore \triangle KAP \equiv \triangle BLM$ (AAS).

ii) $ALMP$ is a parallelogram. (2 marks)

since $\triangle KAP \equiv \triangle BLM$

$AP = LM$ (matching sides of congruent triangles)

$AP \parallel LM$ (given)

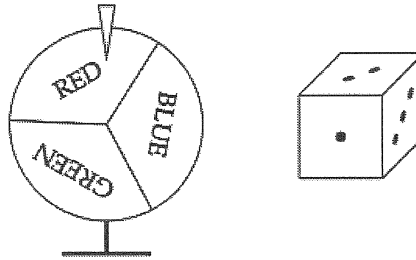$\therefore ALMP$ is a parallelogram   (one pair of opposite sides are parallel and equal).

Probability (16 marks)

a   A set of traffic lights shows red for 45 seconds, green for 30 seconds and amber for 5 seconds.

At any instant, what is the probability that the lights show green?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{5}$ (D) $\frac{3}{8}$

b   The diagram shows a wheel and a fair die.

The wheel is spun and the die is rolled. Which of the following events has the greatest chance of occurring?

(A)   Rolling a 6 $\frac{1}{6}$

(B)   Landing on RED $\frac{1}{3}$

(C)   Landing on RED or on GREEN $\frac{2}{3}$

(D)   Rolling a number greater than 3 $\frac{2}{6}$

c   A die was repeatedly rolled and the results are shown in the table below. (3 marks)

Outcome	Frequency
1	90
2	110
3	113
4	96
5	91
6	100

i) How many times was the die rolled?

600

ii) Find the experimental probability of rolling a number less than 3.

$$\frac{200}{600} = \frac{2}{3}$$

iii) Find the theoretical probability of rolling a number less than 4 or odd.

$$\frac{4}{6} = \frac{2}{3}$$

d   At Riverside College, Year 10 students are asked what language they are studying. 64 students take French, 47 students take German, 15 students take both French and German, and 27 do not study a language.

i) Show the above information on a Venn diagram. (2 marks)
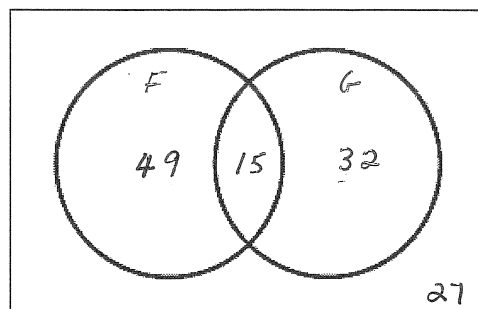
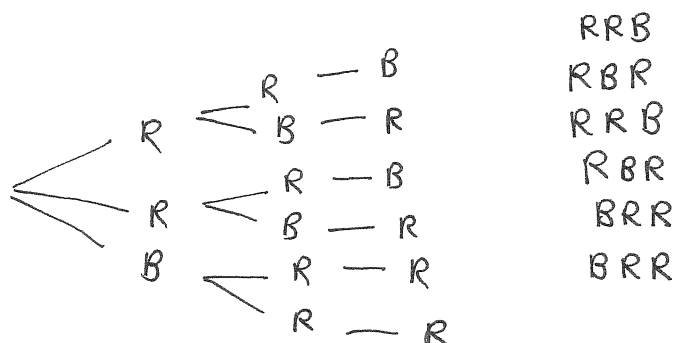ii) How many students studied only one language? (1 mark)

81

ii) Find the probability of selecting a Year 10 student at random who studies French but not German. (1 mark)

$$\frac{49}{123}$$

e   A bag contains 2 red marbles and a blue marble. Three marbles are drawn at random without replacement.

i) Use a tree diagram to display all possible outcomes. (2 marks)

RRB
RBR
RRB
RBR
BRR
BRR

ii) Find the probability of choosing two red marbles. (1 mark)

1

iii) Find the probability of choosing 2 red marbles followed by a blue. (1 mark)

$$\frac{2}{6} = \frac{1}{3}$$

f   A coin is tossed 3 times and the result is tails each time.

i) Are each of the three coin tosses dependent or independent? (1 mark)

independent

ii) Find the probability of tossing 2 heads and one tail in any order. (1 mark)

O

g   Students studying vocational education courses were surveyed about their living arrangements.

	Females	Males	Totals
Living with parent(s)	46	155	201
Not living with parent(s)	182	122	304
Totals	228	277	505

One of these students is selected at random. What is the probability that this student is male and living with his parents (1 mark).

$$\frac{155}{505} = \frac{31}{101}$$

THE END