CARLINGFORD HIGH SCHOOL

DEPARTMENT OF MATHEMATICS

Year 12 Mathematics Trial HSC 2018

Time allowed : 5 mins reading time 3 hours writing time.

Student Number:________________________________

Instructions:

- All questions should be attempted.
- Answered the multiple choice questions on the MC sheet provide
- Show ALL necessary working.
- Marks may not be awarded for careless or badly arranged work.
- Only board-approved calculators may be used.
- Use your own paper and write on one side only.

	MC	Q11	Q12	Q13	Q14	Q15	Q16	Total
Multiple Choice	/10							/10
App of Calculus		/2	/3		/8	/6	/12	/31
Basic Arith		/5						/5
Trig		/2	/2	/4				/8
Linear Function		/3	/8			/4		/15
Integration		/3		/7	/3			/13
Log& Exp			/2					/2
Geometry				/4			/3	/7
Quad Poly					/4			/4
Seq & series						/5		/5
Total	/10	/15	/15	/15	/15	/15	/15	/100

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. What is the gradient of a line with equation $3x + 2y - 10 = 0$?

   (A) 2

   (B) $-\frac{3}{2}$

   (C) $\frac{2}{3}$

   (D) 5

2. Find $\int \frac{x}{x^2 + 1}\, dx$.

   (A) $\frac{1}{2} \ln (x^2 + 1) + C$

   (B) $\frac{-2x^2 + 2}{(x^2 + 1)^2} + C$

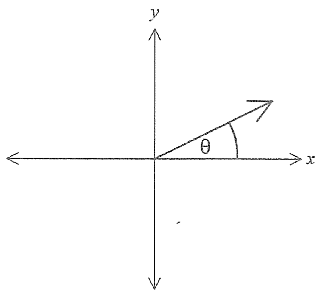
   (C) $\ln 2x + C$

   (D) $2 \ln (x^2 + 1) + C$

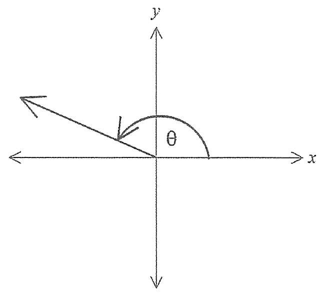
3. For the angle θ, $\sin\theta = -\dfrac{8}{17}$ and $\tan\theta = -\dfrac{8}{15}$.

Which diagram best shows angle θ?

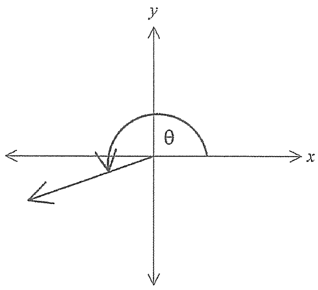
(A)



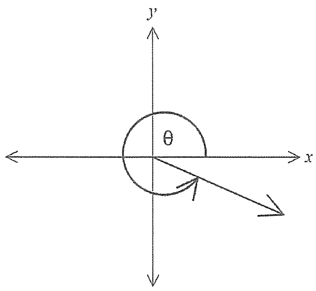
(B)



(C)



(D)



4. Which expression is a factorisation for the expression $27x^3 - 8$?

(A) $(9x - 2)(3x^2 + 18x + 4)$

(B) $(3x - 2)(9x^2 + 6x + 4)$

(C) $(3x + 2)(9x^2 - 18x - 4)$

(D) $(3x - 2)^3$

5. A particle is moving in a straight line. It's distance (x metres) from a fixed point O is given by $x = 2\cos 2t$, where t is the time in seconds.

At which times is the particle at rest?

(A) $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots\ldots$

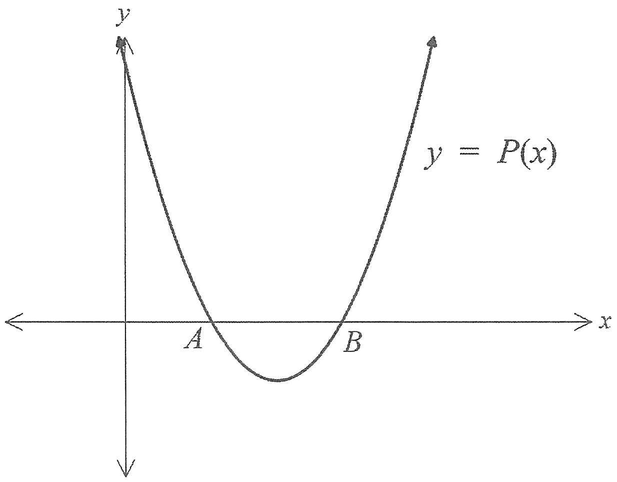
(B) $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots\ldots\ldots$

(C) $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots\ldots\ldots$

(D) $t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots\ldots\ldots$

6. Consider the quadratic polynomial $P(x) = ax^2 + bx + c$ where a, b and c are integers.

The graph of $y = P(x)$ is shown below.



The curve crosses the x-axis at A and B.

Given that $\Delta = b^2 - 4ac$ and that A and B are rational, which statement is true?

(A) $a > 0$, Δ is a square number

(B) $a > 0, \Delta < 0$

(C) a is a square number, $\Delta = 0$

(D) $a < 0, \Delta > 0$

7. What is the area under the curve $y = |2x + 1|$ between $x = -2$ and $x = 1$?

(A) 0 square units

(B) 4 square units

(C) 2.25 square units

(D) 4.5 square units

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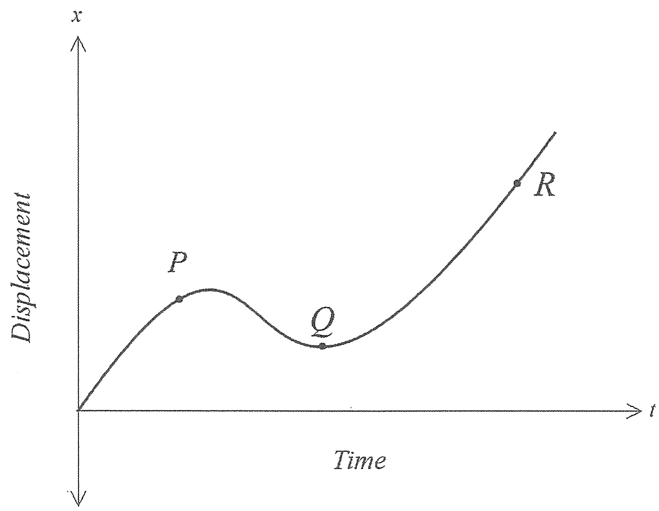
8. The graph shows the displacement x of a particle moving along a straight line as a function of time t.



Which statement correctly describes the motion of the particle?

(A) At point P, its acceleration and velocity are both positive.

(B) At point Q, the particle is stationary and its acceleration is zero.

(C) At point P, its acceleration is negative while its velocity is positive.

(D) At point R, the particle is stationary and its acceleration is zero.

9. Find the limiting sum of the series $5 + \frac{5}{7} + \frac{5}{49} + \ldots$.

(A) ∞

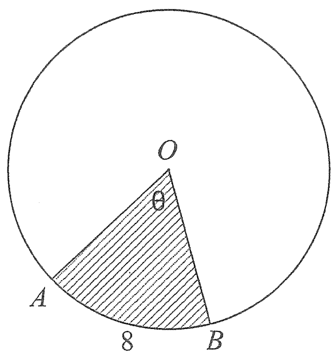
(B) $\frac{5}{42}$

(C) $5\frac{5}{6}$

(D) 6

10. The circle centred at O has radius r.

Arc AB has length 8 units and $\angle AOB = \theta$ radians as shown in the diagram.



Which of the following expressions would give the area of sector OAB?

(A) 32θ square units

(B) $\frac{32}{\theta}$ square units

(C) $64\theta^2$ square units

(D) 8θ square units

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 writing booklet.

(a) Differentiate $\dfrac{3x+2}{2x^3-5x^2}$. **2**

(b) (i) Express $\dfrac{3}{\sqrt{2}-7}$ with a rational denominator. **2**

(ii) Given that $\dfrac{3}{\sqrt{2}-7} = a + b\sqrt{2}$, state the values of a and b. **1**

(c) Factorise the expression $10x^2 - 13x - 3$. **2**

(d) Evaluate $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin 2x \, dx$. **2**

(e) (i) Solve $|x-5| > 2$. **2**

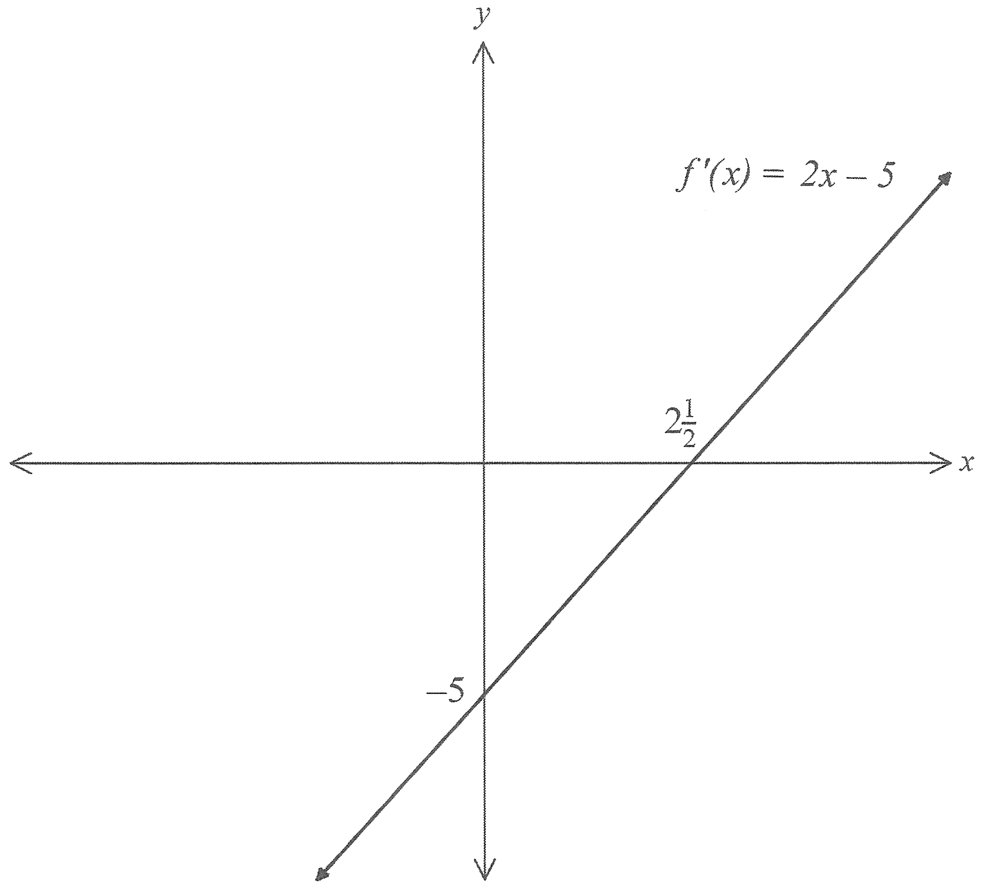
(ii) Sketch your solutions on a number line. **1**

Question 11 continues on page 8

Question 11 continued

(f) The gradient function of a curve $y = f(x)$ is given by $f'(x) = 2x - 5$.

The gradient function is graphed below.



(i) Copy the graph into your answer booklet.

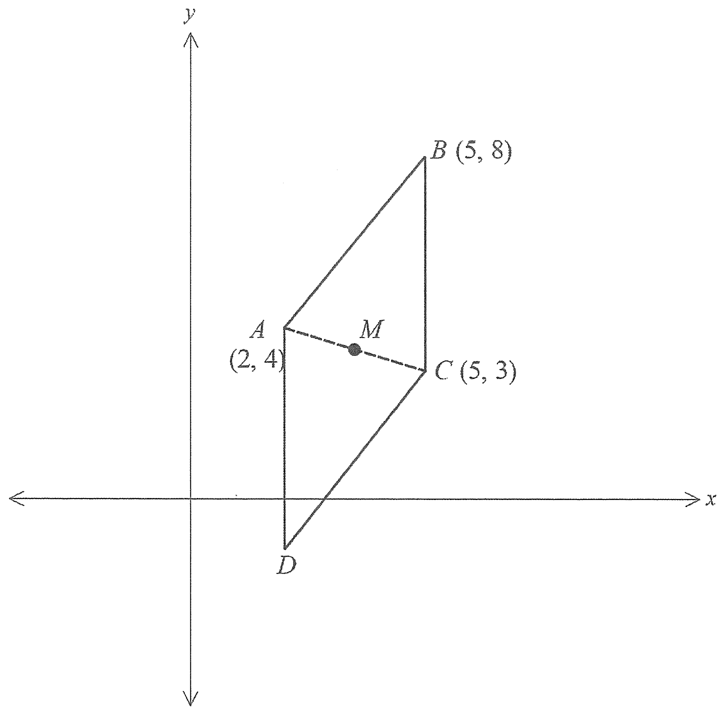
Using the features of the graph of the gradient function, draw a possible sketch of the curve $y = f(x)$ on the same set of axes. **2**

(ii) Given that the curve $y = f(x)$ passes through the point (2, 0), find the equation of the curve. **1**

End of Question 11

Question 12 (15 marks) Use the Question 12 writing booklet.

(a) The diagram shows the rhombus $ABCD$.



(i) Show that the diagonal from point $A(2, 4)$ to point $C\,(5, 3)$ lies on the line $x + 3y - 14 = 0$. **2**

(ii) State the gradient of the line on which the diagonal from point B to point D lies. Justify your answer. **1**

(iii) The point M is the midpoint of diagonal AC. **2**

Show that the distance from point $B\,(5, 8)$ to point M is exactly $\dfrac{3\sqrt{10}}{2}$ units.

(iv) Calculate the area of rhombus $ABCD$. **3**

(b) Show that $2 - \log_3 a$ can be expressed as $\log_3\left(\dfrac{9}{a}\right)$. **2**

Question 12 continues on page 10

Question 12 continued

(c) For the function $y = 3 \sin 2x$, state the period and the range. **2**

(d) A curve has the equation $y = x \cos x$.

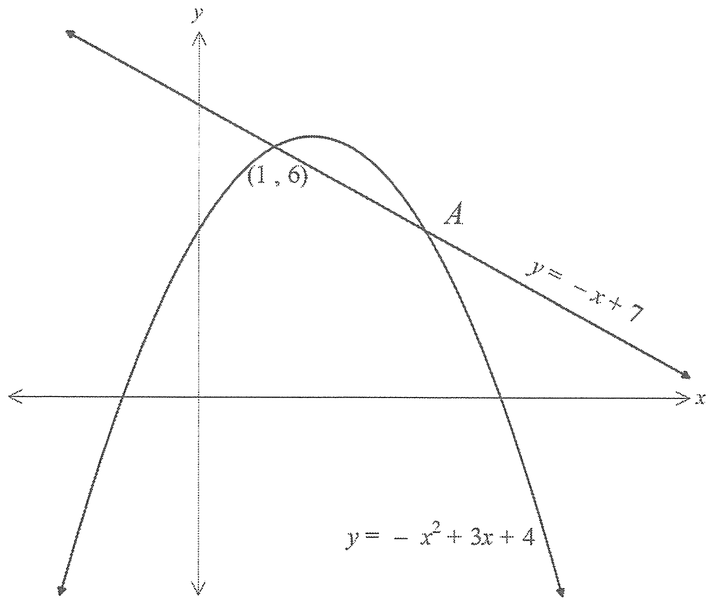
(i) Show that $P\left(\frac{\pi}{2}, 0\right)$ is the first point to the right of the origin where the curve crosses the x axis.. **2**

(ii) Find the equation of the tangent at point P. **1**

End of Question 12

Question 13 (15 marks) Use the Question 13 writing booklet.

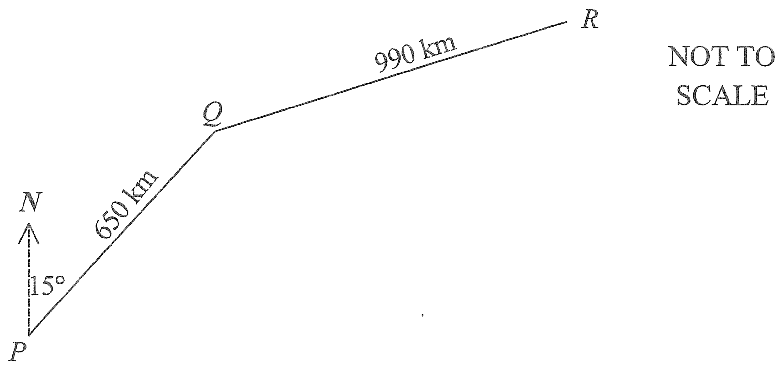
(a) The parabola $y = -x^2 + 3x + 4$ and the line $y = -x + 7$ intersect at the point (1 ,6) and at point A.



(i) Find the x–coordinate of point A. **2**

(ii) Calculate the area enclosed by the parabola and the line. **3**

(b) An aeroplane flies directly from town P to town Q. The distance from P to Q is 650 kilometres. The bearing of Q from P is $015°$.
At town Q, the aeroplane turns onto a bearing of $040°$ and heads to town R which is 990 kilometres from town Q.



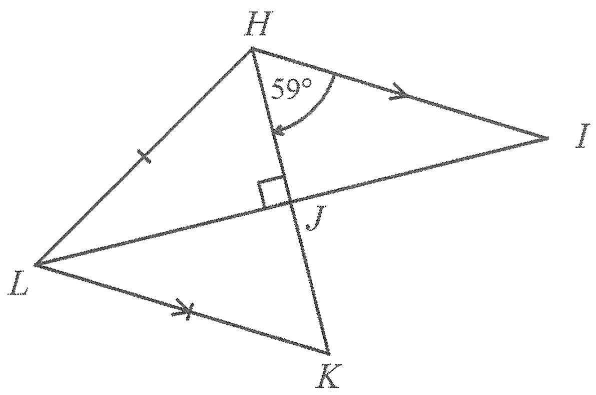
(i) Show that $\angle PQR = 155°$. **2**

(ii) Calculate the distance from town P to town R, correct to the nearest kilometre. **2**

Question 13 continues on page 12

Question 13 continued

(c) In the diagram, $HI || LK$, $LH = LK$, $\angle KHI = 59°$ and $\angle HJL = 90°$.



NOT TO SCALE

(i) Calculate the size of $\angle HLK$, giving reasons. **2**

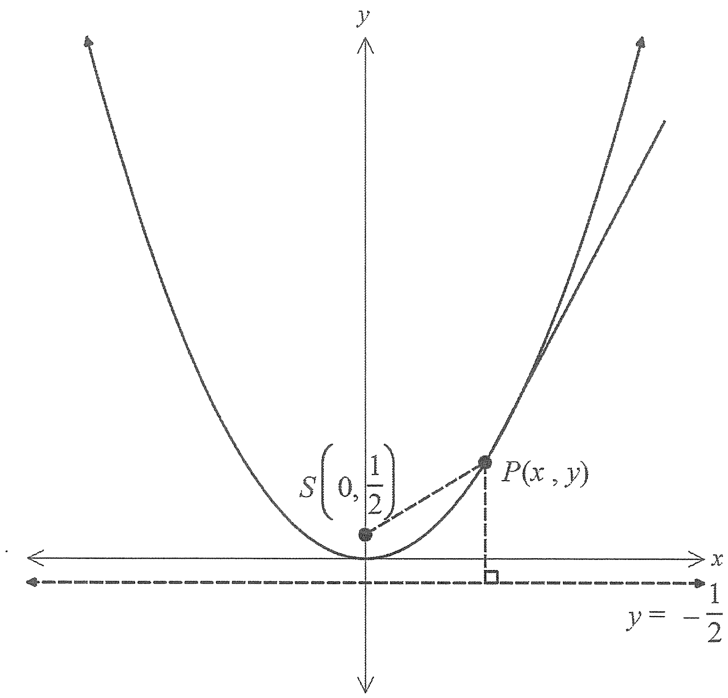
(ii) Prove that $LH = IH$. **2**

(d) Use the trapezoidal rule with 4 subintervals to approximate the value of $\int_0^2 xe^x \, dx$, correct to 2 decimal places. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 writing booklet.

(a) The parabola below shows the locus of the point P which remains equidistant from the fixed point $S\left(0, \frac{1}{2}\right)$ and the fixed line $y = -\frac{1}{2}$.



(i) Show that the equation of the parabola is $x^2 = 2y$. **2**

(ii) When P has coordinates (2, 2), show that the tangent at P has the equation $2x - y - 2 = 0$. **2**

(b) Consider the curve $y = 3x^2 - x^3$.

(i) Find the stationary points and determine their nature. **3**

(ii) Show that there is a point of inflexion at coordinates (1, 2). **2**

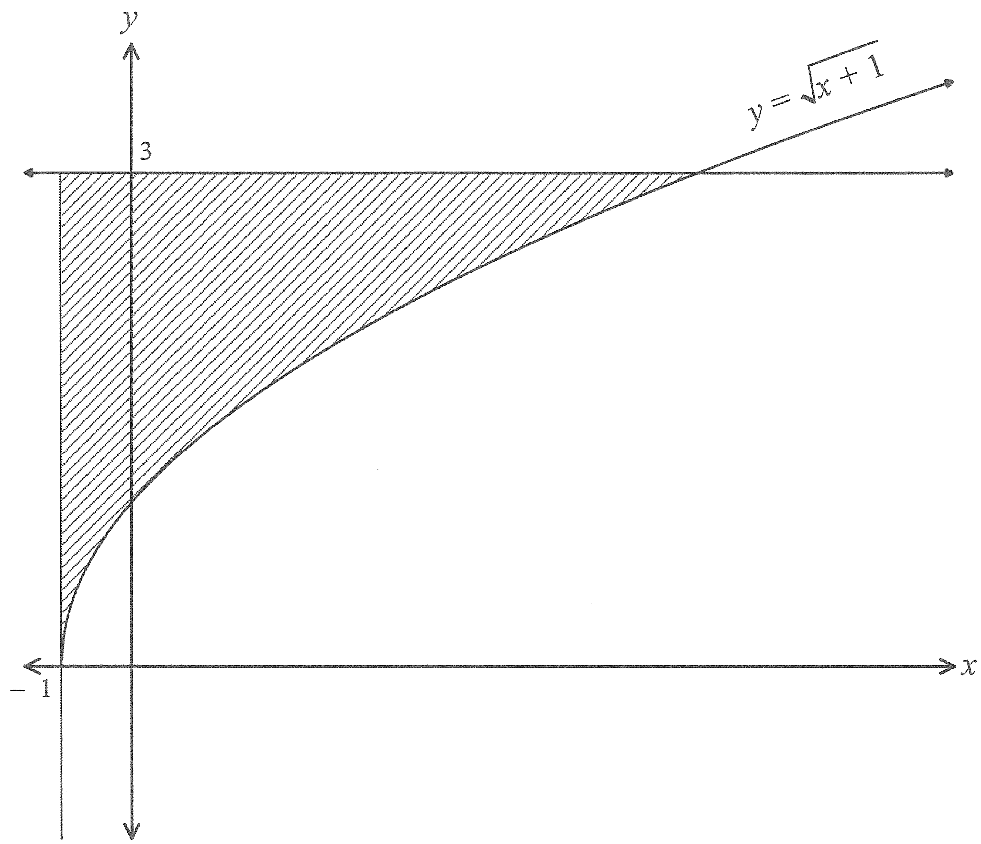
(iii) Explain the difference between the inflexion point on this curve and a horizontal inflexion. **1**

(iv) Sketch the curve, labelling the stationary points, the inflexion and the intercepts. **2**

Question 14 continues on page 14

Question 14 continued

(c) Find the exact volume of the solid formed when the region bounded by $y = \sqrt{x+1}$, $y = 3$ and $x = -1$ is rotated about the x-axis. **3**



End of Question 14

Question 15 (15 marks) Use the Question 15 writing booklet.

(a) When James stops drinking, the amount of alcohol in his blood, C, decreases according to the equation:

$$\frac{dC}{dt} = -0.18C,$$

where C is the amount of alcohol in milligrams and t is the time in hours.

(i) Show that $C = Ae^{-0.18t}$ is a solution to $\frac{dC}{dt} = -0.18C$, where A is a constant. **1**

(ii) When $t = 0$, there are 24 milligrams of alcohol in James' blood. **1**

Find the value of A.

(iii) Calculate the amount of alcohol in James' blood after 3 hours. **1**

(Give your answer correct to 2 significant figures.)

(iv) What is the time taken for the amount of alcohol in James' blood to be below 5 mg? **3**
(Give your answer correct to the nearest minute.)

(b) (i) Given that $\cos 2x = \cos^2 x - \sin^2 x$, show that: **1**

$$\cos 2x + \cos x = 2\cos^2 x + \cos x - 1 .$$

(ii) Hence, or otherwise, solve the equation: **3**

$$\cos 2x + \cos x = 0 \text{ , for } 0 \leq x \leq 2\pi$$

Question 15 continues on page 16

Question 15 continued

(c) At the start of the month, Katrina opens a bank account and deposits \$300 into the account.

At the start of each subsequent month, Katrina makes a deposit which is 1.5% more than the previous deposit.

At the end of every month, the bank pays Katrina interest at a rate of 3% per annum on the balance of the account.

(i) Show that the balance of the account at the end of the second month is **2**

$$\$300(1.0025)^2 + \$300(1.015)(1.0025)$$

(ii) Show that the balance of the account at the end of the nth month is given by:

$$\$300(1.0025)^n\left(\frac{\left(\frac{1.015}{1.0025}\right)^n - 1}{\left(\frac{1.015}{1.0025}\right) - 1}\right)$$

(iii) Calculate the balance of the account at the end of the 60th month, correct to the nearest dollar. **1**

End of Question 15

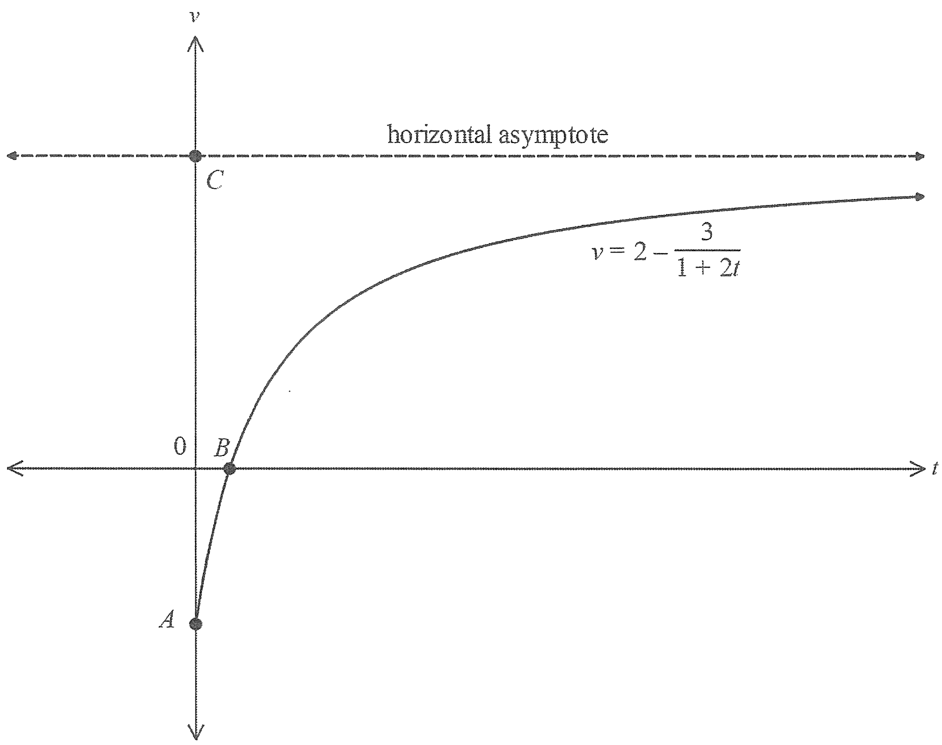
Question 16 (15 marks) Use the Question 16 writing booklet.

(a) The velocity of a particle, moving in a straight line, is given by the equation:

$$v = 2 - \frac{3}{1 + 2t}$$

where t is the time in seconds and v is measured in metres per second.

The velocity-time graph is shown below.



(i) Show that the coordinates of point A are $(0, -1)$. **1**

(ii) Show that the coordinates of point B are $\left(\frac{1}{4}, 0\right)$. **1**

(iii) Find the acceleration of the particle when it is stationary. **2**

(iv) By considering the behaviour of v for large values of t, find the coordinates of point C. **1**

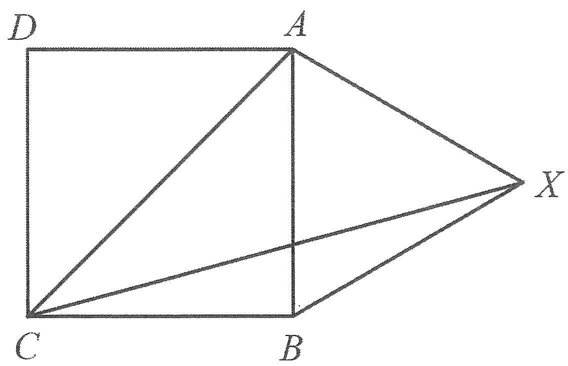
(v) Find the distance travelled by the particle in the first second, correct to 3 decimal places. **3**

Question 16 continues on page 18

Question 16 continued

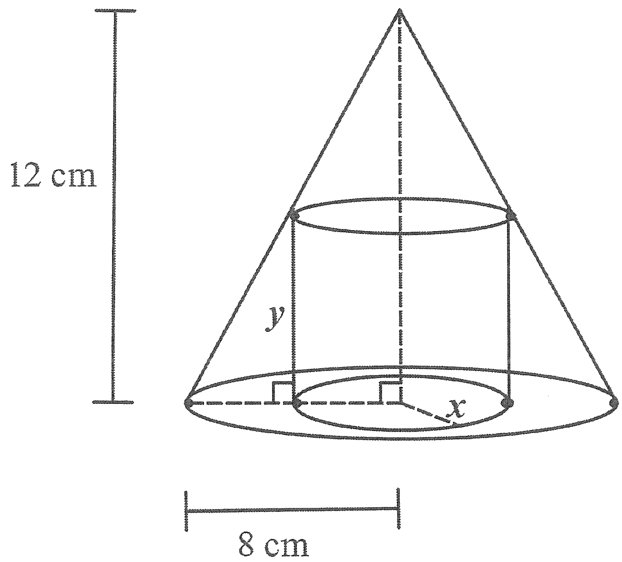
(b) In the diagram, $ABCD$ is a square and ΔABX is equilateral. **3**

$\angle ACX$ is adjacent to $\angle XCB$.



Show that $\angle AXC + \angle ACX = 5 \times \angle BXC$.

(c) The diagram below shows a cylinder of radius x cm and height y cm, inscribed in a cone. The cone has a radius of 8 cm and its height is 12 cm.



(i) Show that $y = \frac{3}{2}(8 - x)$. **1**

(ii) Show that the volume of the cylinder is given by $V = \frac{3}{2}\pi x^2(8 - x)$ cm^3. **1**

(iii) Find the value of x for which the volume is a maximum. **2**

End of Paper

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180^\circ$$

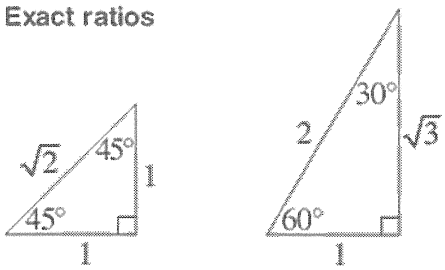
Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \csc\theta = \frac{1}{\sin\theta}$$
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \sec\theta = \frac{1}{\cos\theta}$$
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

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nth term of an arithmetic series

$$T_n = a + (n-1)d$$

Sum to n terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

nth term of a geometric series

$$T_n = ar^{n-1}$$

Sum to n terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\frac{dy}{dx} = F'(u)\frac{du}{dx}$

If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\frac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\frac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\frac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

Equation of a parabola

$(x-h)^2 = \pm 4a(y-k)$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

Area $= \frac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$$

$$\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$$

$$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$$

***t* formulae**

If $t=\tan\frac{\theta}{2}$, then

$$\sin\theta=\frac{2t}{1+t^2}$$

$$\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\tan\theta=\frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta=a,\qquad \theta=n\pi+(-1)^n\sin^{-1}a$

$\cos\theta=a,\qquad \theta=2n\pi\pm\cos^{-1}a$

$\tan\theta=a,\qquad \theta=n\pi+\tan^{-1}a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2=4ay$,

$$x=2at,\quad y=at^2$$

At $(2at,\ at^2)$,

tangent: $y=tx-at^2$

normal: $x+ty=at^3+2at$

At $(x_1,\ y_1)$,

tangent: $xx_1=2a(y+y_1)$

normal: $y-y_1=-\frac{2a}{x_1}(x-x_1)$

Chord of contact from $(x_0,\ y_0)$: $xx_0=2a(y+y_0)$

Acceleration

$$\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$$x=b+a\cos(nt+\alpha)$$

$$\ddot{x}=-n^2(x-b)$$

Further integrals

$$\int\frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\frac{x}{a}+C$$

$$\int\frac{1}{a^2+x^2}\,dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

Sum and product of roots of a cubic equation

$$\alpha+\beta+\gamma=-\frac{b}{a}$$

$$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}$$

$$\alpha\beta\gamma=-\frac{d}{a}$$

Estimation of roots of a polynomial equation

Newton's method

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k}=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

Trial HSC Examination 2018

Mathematics Course

Student Number ______________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○