

# Carlingford High School



Q1-2 = VIR  
Q3-5 = Wilson Young  
Q6-8 = Ben Lobejko  
Q9+10 = GF

## Mathematics

### Year 10 5.3 Term 4 Examination

2019

Time allowed: 1hour 30 minutes

Name: Solitams Class: 10MAT3

Please circle Mrs Virmani Mrs Wilson/ Ms Bennett/ Mr  
your teacher: Mrs Young Mrs Lobejko Fardouly

#### Instructions:

- Use blue or black pen
- Pencil may be used for graphs or diagrams only
- Board approved calculators may be used
- No lending or borrowing
- Show all necessary working out in the space provided
- Marks may be deducted for untidy setting out

Topic	Non-linear relationships	Surface area and volume	Single and Bivariate Data Analysis	Trigonometric Functions	Solving Inequalities	Functions	Logarithms	Polynomials	Probability	Coordinate Methods in Geometry	Total
Mark	/10	/10	/10	/10	/6	/6	/10	/10	/10	/10	/92

## Question 1: Non Linear Relationships

- a) Answer these questions about the following parabola:

$$y = 4x - x^2$$

- i) Find the  $x$ -intercepts

$$y = x(4-x) \text{ when } y=0$$
$$\underline{x=4, 0}$$

[1]

- ii) Find the  $y$ -intercept

$$0$$

[1]

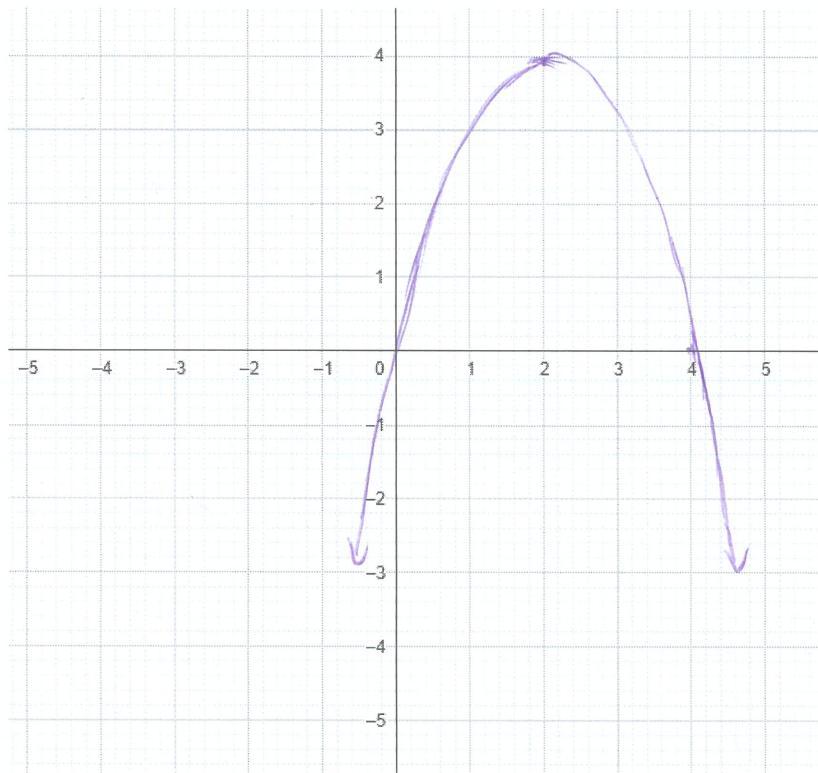
- iii) Find the coordinates of the vertex

when  $x=2$  (using symmetry)

$$y = 4x^2 - x^2 \\ = 4 \quad \therefore \text{vertex } (2, 4)$$

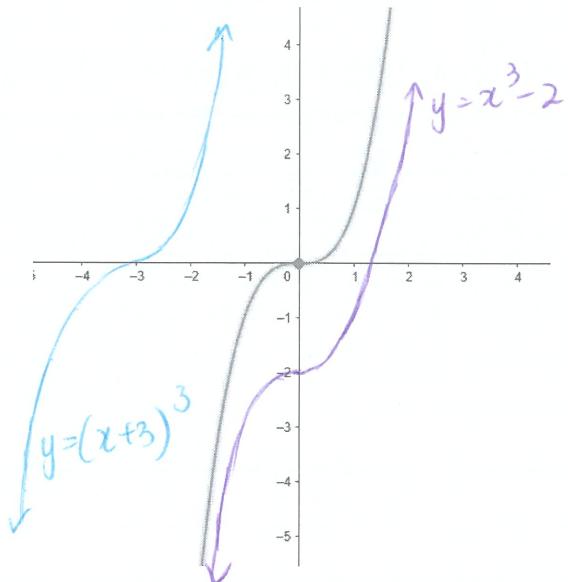
[1]

- iv) Hence, sketch the parabola



[2]

- b) The following shows the graph of  $y = x^3$  [2]



On the same graph, sketch and clearly label the graph of:

- i)  $y = x^3 - 2$
- ii)  $y = (x + 3)^3$

- c) Find the centre and the radius of the following circle: [3]

$$x^2 - 10x + y^2 + 8y + 32 = 0$$

$$x^2 - 10x + 25 + y^2 + 8y + 16 = -32 + 25 + 16$$

$$(x-5)^2 + (y+4)^2 = 9$$

$$\text{Centre} = (5, -4)$$

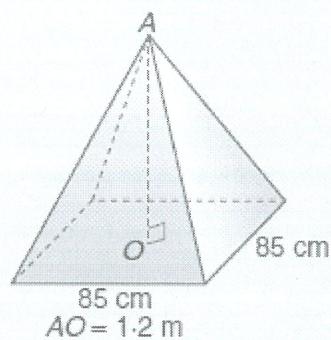
$$\text{radius} = 3$$

## Question 2: Surface Area and Volume

- a) Calculate the volume of a sphere with  $r = 5\text{cm}$  (1d.p.) [2]

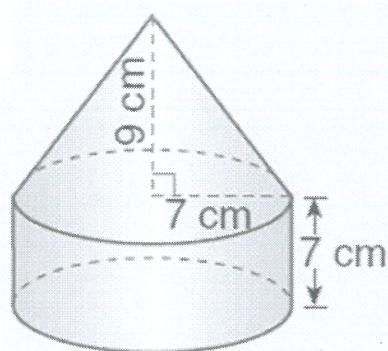
$$\begin{aligned}
 V &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \pi \times 5^3 \\
 &= 523.6 \text{ cm}^3 \quad (1\text{d.p})
 \end{aligned}$$

- b) Find the volume of this pyramid: [3]



$$\begin{aligned}
 V &= \frac{1}{3} b h \\
 &= \frac{1}{3} \times 1.2 \times 0.85^2 \\
 &= 0.289 \text{ m}^3 \\
 \text{or } 289000 \text{ cm}^3 \\
 1 \text{ m}^3 &= 1000000 \text{ cm}^3
 \end{aligned}$$

- c) Find the surface area of the following solid: (4 s.f) [4]



$$\begin{aligned}
 SA &= \pi r l + 2\pi r h + \pi r^2 \\
 &= \pi \times 7 \times \sqrt{130} + 2 \times \pi \times 7 \times 7 \\
 &\quad + \pi \times 7^2 \\
 &= 712.6 \text{ cm}^2 \text{ (4 sf)}
 \end{aligned}$$

$$\begin{aligned}
 l &= \sqrt{9^2 + 7^2} \\
 &= \sqrt{130}
 \end{aligned}$$

### Question 3: Single and Bivariate Data Analysis

- a) Match up the following types of data with their category: [2]

How many times do you order using Uber Eats each month?

Categorical

Which company is the carrier of your mobile phone?

Numerical (continuous)

How long does it take you to drive to school?

Numerical (discrete)

- b) The following data represents the number of cars that were counted on each day for 16 days, on a quiet suburban street:

~~10, 9, 15, 14, 10, 17, 15, 0, 12, 14, 8, 15, 15, 11, 13, 11~~

0 8 9 10 | 10 11 11 12 | 13 14 14 15 | 15 15 15 17

[2]

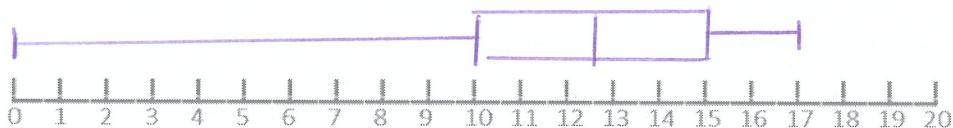
- i) Write down the 5 number summary, and the interquartile range for this data.

0, 10, 12.5, 15, 17

$$IQR = 15 - 10 = 5$$

- ii) Draw a box-and-whisker plot from the data.

[2]



- iii) Identify the outlier and suggest a possible reason for it.

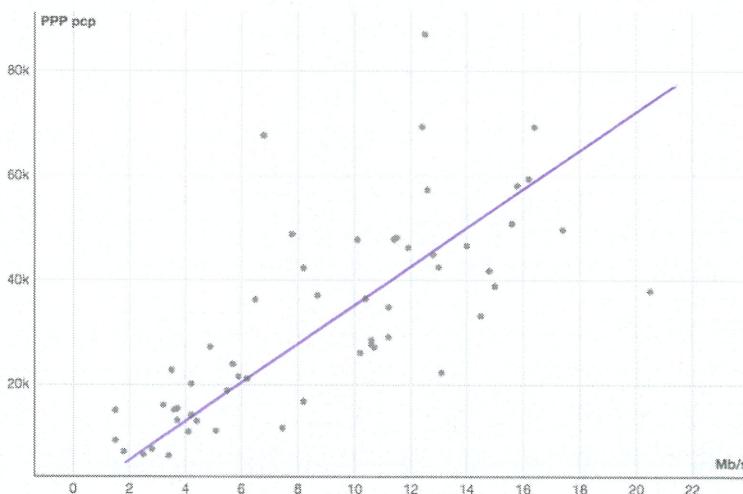
[1]

0 is the outlier  
possibly road closed for repairs  
(various)

- c) i) For the following data shown on a scatter plot, draw a line of best fit, by eye.

[1]

Comparing GDP per capita with internet speeds



## Internet speeds

ii) Describe the correlation between GDP and Life Expectancy.

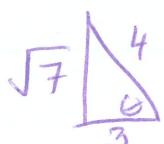
weak positive correlation [2]

between GDP + life expectancy.

i.e. The higher the GDP, the higher the Internet speeds

### Question 4: Trigonometric Functions

- a) If  $0 \leq \theta \leq 180^\circ$  and  $\cos\theta = -\frac{3}{4}$ , find the value of  $\sin\theta$

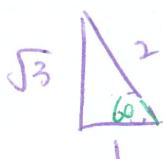


$$\sin\theta = \frac{\sqrt{7}}{4}$$

✓

[3]

- b) Solve for  $\theta$ , where  $0 \leq \theta \leq 360^\circ$  and  $\sin\theta = -\frac{\sqrt{3}}{2}$



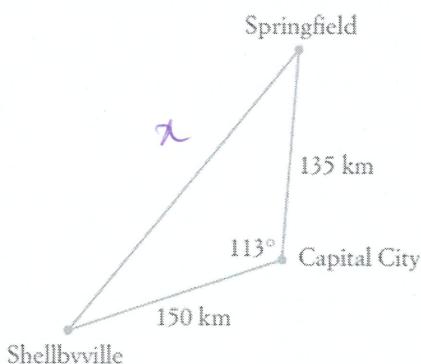
$$\begin{aligned} \theta &= 180^\circ + 60^\circ, \quad 360^\circ - 60^\circ \\ &= 240^\circ, \quad 300^\circ \end{aligned}$$

✓✓

[2]

- c) Three towns are joined by straight roads. What distance (correct to the nearest km) is saved by going directly from Shellbyville to Springfield, rather than going via Capital City?

[3]



$$x^2 = 135^2 + 150^2 - 2 \times 135 \times 150 \times \cos 113^\circ$$

$$x^2 = 56549.6107$$

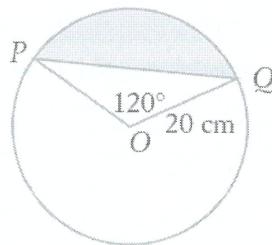
$$x = 237.8 \text{ km (1dp)}$$

Distance saved

$$= 135 + 150 - 237.8$$

$$= 47 \text{ km (Nearest km)}$$

- d) Calculate the area of the shaded segment in the following circle. Give [2]  
your answer to the nearest cm<sup>2</sup>.



$$\begin{aligned}
 A &= \frac{120}{360} \times \pi r^2 - \frac{1}{2} ab \sin C \\
 &= \frac{1}{3} \pi \times 20^2 - \frac{1}{2} \times 20^2 \sin 120 \\
 &= 246 \text{ cm}^2 \text{ (Nearest cm}^2\text{)}
 \end{aligned}$$

### Question 5: Solving Inequations

- a) Solve the following inequations and graph the solution on a number line.

i)  $\frac{x}{9} + 6 \geq 4$

[2]

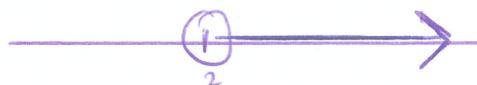
$$\begin{aligned}
 \frac{x}{9} &\geq -2 \\
 x &\geq -18
 \end{aligned}$$



ii)  $4 - 3x < -2$

[2]

$$\begin{aligned}
 -3x &< -6 \\
 x &> 2
 \end{aligned}$$



iii)  $7(1 - x) \geq 3(2 + 3x)$

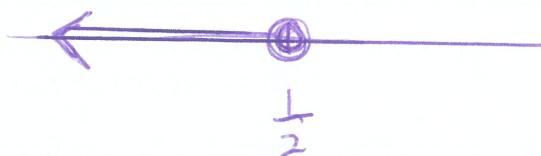
[2]

$$7 - 7x \geq 6 + 9x$$

$$7 \geq 6 + 16x$$

$$1 \geq 16x$$

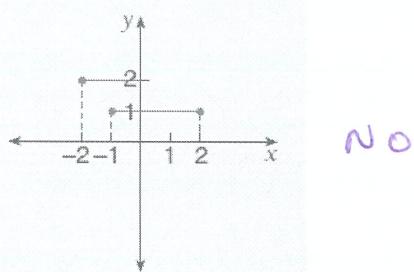
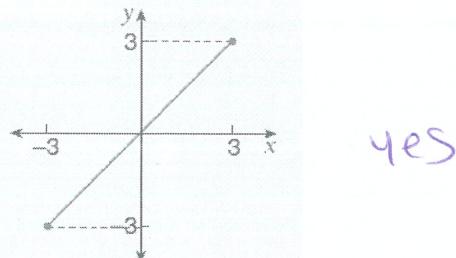
$$x \leq \frac{1}{16}$$



## Question 6: Functions

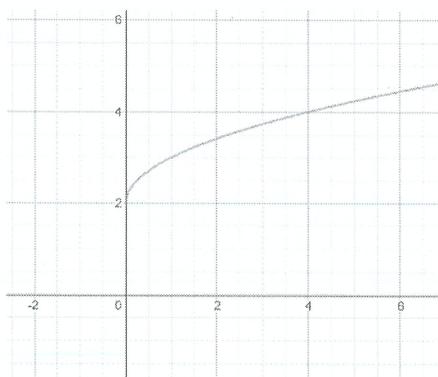
a) Which of the following graphs are functions?

[2]



b) Write down the domain and range for the following function:

[2]



Domain:  $x \geq 0$

Range:  $y \geq 2$

c) If  $F(x) = 5^x + 1$ , find:

[2]

$$\text{i) } F(2) = 5^2 + 1 \\ = 26$$

$$\text{ii) } F(-3) = 5^{-3} + 1 \\ = \frac{1}{125} + 1 \\ = 1 \frac{1}{125}$$

## Question 7: Logarithms

a) Evaluate:

[2]

$$\text{i)} \log_3 27 = 3$$

$$\text{ii)} \log_7 \frac{1}{49} = -2$$

b) Solve:

[2]

$$\text{i)} \log_x 144 = 2 \quad x = 12$$

$$\text{ii)} \log_4 x = -3 \quad x = \frac{1}{4^3} = \frac{1}{64}$$

c) Evaluate:

[2]

$$\begin{aligned} & \log_3 24 - 3 \log_3 2 \\ &= \log_3 24 - \log_3 2^3 \\ &= \log_3 \frac{24}{8} = \log_3 3 = 1 \end{aligned}$$

d) If  $\log_a 2 = 0.506$  and  $\log_a 3 = 0.701$ , evaluate:

[2]

a)  $\log_a 6$

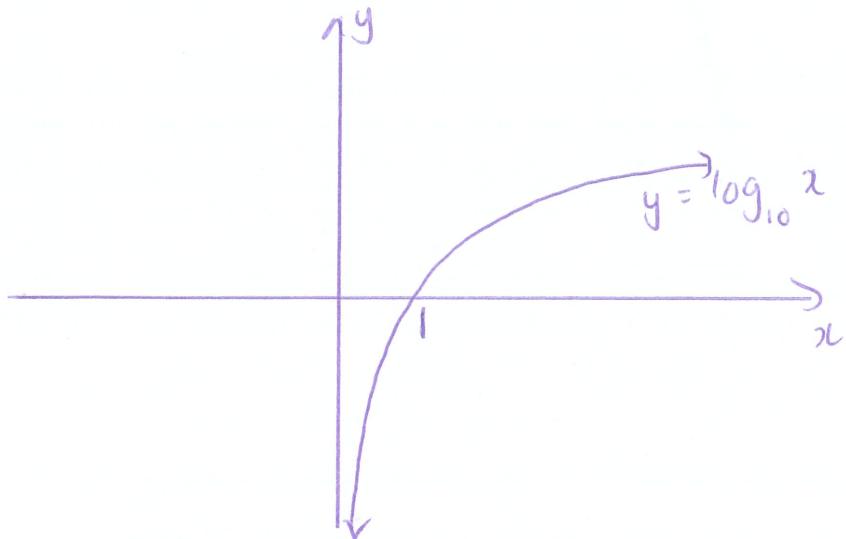
$$\begin{aligned} \log_a (2 \times 3) &= \log_a 2 + \log_a 3 \\ &= 0.506 + 0.701 \\ &= 1.207 \end{aligned}$$

b)  $\log_a 18$

$$\begin{aligned} & \log_a (2 \times 3^2) \\ &= \log_a 2 + 2 \log_a 3 \\ &= 0.506 + 2 \times 0.701 \\ &= 1.908 \end{aligned}$$

e) Sketch  $y = \log_{10}x$

[2]



### Question 8: Polynomials

a) Answer the following questions about this polynomial:

[4]

$$6 + 2x^2 - x^3$$

i) Is the polynomial monic or non-monic?

Non-monic

ii) What is the degree?

3

iii) What is the value of the leading coefficient?

-1

iii) What is the value of the constant?

6

b) If  $P(x) = 3x^3 + 5x^2 - x + 7$  and  $Q(x) = 3x^2 - 4x + 9$

[2]

Find  $P(x) - Q(x)$

$$\begin{aligned} & 3x^3 + 5x^2 - x + 7 - (3x^2 - 4x + 9) \\ &= 3x^3 + 5x^2 - x + 7 - 3x^2 + 4x - 9 \\ &= 3x^3 + 2x^2 + 3x - 2 \end{aligned}$$

c) Complete the following division, writing your answer as: [2]

$$\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

$$(2x^3 - x^2 + x - 1) \div (x - 2)$$

$$\begin{array}{r} 2x^2 + 3x + 7 \\ \hline x-2 ) 2x^3 - x^2 + x - 1 \\ 2x^3 - 4x^2 \\ \hline 3x^2 + x \\ 3x^2 - 6x \\ \hline 7x - 1 \\ 7x - 14 \\ \hline 13 \end{array}$$

$$2x^3 - x^2 + x - 1 = (x - 2)(2x^2 + 3x + 7) + 13$$

d) Show  $(x - 2)$  is a factor of  $P(x) = x^3 - 5x^2 + 8x - 4$  [2]

$$\begin{aligned} P(2) &= 2^3 - 5(2)^2 + 8(2) - 4 \\ &= 0 \\ \therefore (x-2) &\text{ is a factor.} \end{aligned}$$

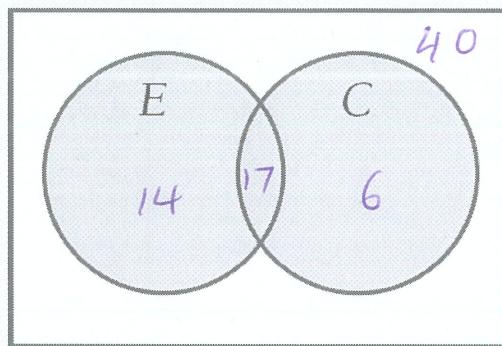
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## Question 9: Probability

- a) An engineering firm employs 87 people. Of these, 31 have special skills in engineering (E) and 23 have special skills in computing (C). Of those, 17 have special skills in both engineering and computing.

i) Complete the following Venn diagram for this information:

[2]



What is the probability that a randomly selected employee will have:

ii) special skills in engineering but not computing?

[1]

$$\frac{14}{87}$$

iii) special skills in neither engineering nor computing?

[1]

$$\frac{40}{87}$$

- b) A raffle has sold 120 tickets, of which Eric has bought 5. There is a first prize and a second prize available.

i) What is the probability Eric wins first prize?

[1]

$$\frac{5}{120} = \frac{1}{24}$$

ii) What is the probability that Eric wins both first and second prize?

[2]

$$\frac{5}{120} \times \frac{4}{119} = \frac{1}{714}$$

iii) What is the probability that Eric wins exactly one of the prizes?

$$\frac{5}{120} \times \frac{115}{119} + \frac{115}{120} \times \frac{5}{119} = \frac{115}{1428} \approx 0.08 \text{ (2dp)}$$

iv) What is the probability that, given Eric did not win the first prize, that he then wins the second prize?

[1]

$$\frac{5}{119}$$

### Question 10: Coordinate Methods in Geometry

- a) Find the equation of the line that is perpendicular to  $y = -4x - 1$  [3] and passes through  $(-1, 3)$ . Write your answer in general form.

$$m_1 = -4 \quad m_2 = \frac{1}{4}$$

$$y - 3 = \frac{1}{4}(x + 1)$$

$$4y - 12 = x + 1$$

$$x - 4y + 13 = 0$$

- b) Determine whether the three points are collinear, giving reasons for [3] your answer.

$$A(0, -7), B(-3, 5), C(2, -15)$$

$$m(AB) = \frac{5 + 7}{-3 - 0}$$

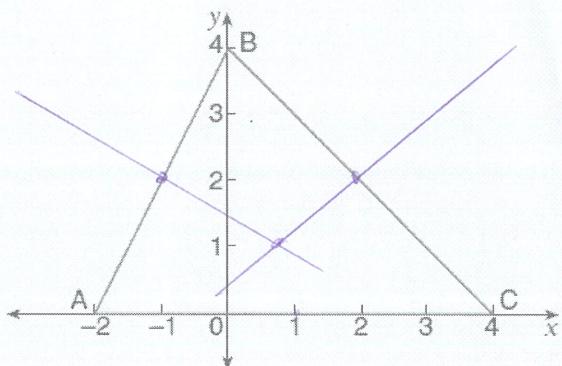
$$= \frac{12}{-3} = -4$$

$$m(BC) = \frac{-15 - 5}{2 + 3}$$

$$= \frac{-20}{5} = -4$$

$\therefore$  the points A, B and C  
are collinear.

c) The points  $A(-2, 0)$ ,  $B(0, 4)$  and  $C(4, 0)$  form a triangle.



i) Find the equation of the perpendicular bisectors of the sides AB, BC and AC. [2]

$$m(AB) = \frac{4-0}{0-(-2)} = 2$$

$$\perp m = -\frac{1}{2}$$

$$\text{midpoint} = (-1, 2)$$

$$y-2 = -\frac{1}{2}(x+1)$$

$$2y-4 = -x-1$$

$$\textcircled{2} \quad \underline{x+2y-3=0}$$

$$m(BC) = -1$$

$$\perp m = 1$$

$$\text{midpoint} = (2, 2)$$

$$y-2 = 1(x-2)$$

$$\underline{x-y=0} \quad \textcircled{1}$$

$$m_{AC} = 0$$

$$\underline{x=1} \quad \textcircled{3}$$

ii) Find the point of intersection of the perpendicular bisectors of the sides AB and AC.

Sub  $\textcircled{3}$  into  $\textcircled{2}$

[1]

$$1+2y-3=0$$

$$2y=2$$

$$y=1$$

$$\therefore (1, 1)$$

iii) Show that the perpendicular bisector of the side BC passes through the point of intersection found in (ii)

Sub  $(1, 1)$  into  $BC$

[1]

$$1-1=0$$

$$0=0$$

which is true.

$\therefore$  the perpendicular bisector of  $BC$  passes through  $(1, 1)$