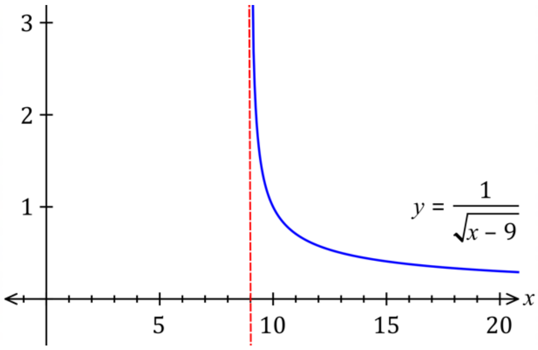
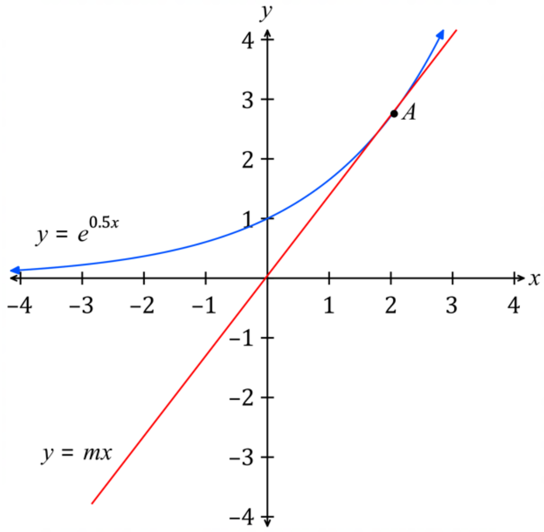
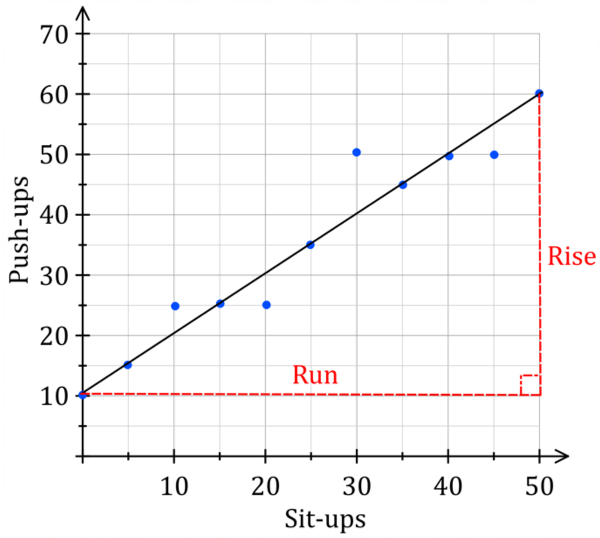
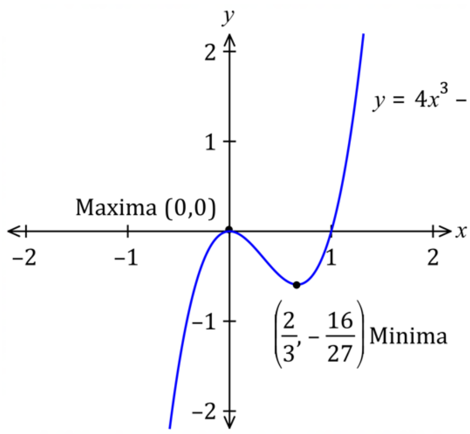
ACE Examination Paper 4
Year 12 Mathematics Advanced Yearly Examination
Worked solutions and marking guidelines

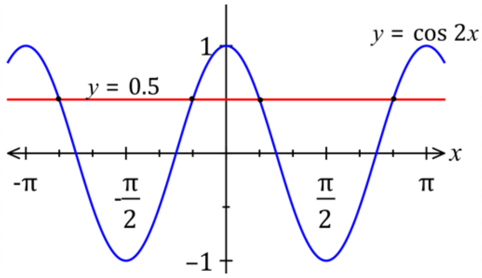
Section I		
	Solution	**Criteria**
1.	$y = 2x\sqrt{x} = 2x \times x^{\frac{1}{2}} = 2x^{\frac{3}{2}}$ $\frac{dy}{dx} = 2 \times \frac{3}{2} \times x^{\frac{1}{2}} = 3\sqrt{x}$	1 Mark: D
2.	$\int_0^5 dx = \int_0^5 1dx = [x]_0^5 = 5 - 0 = 5$	1 Mark: C
3.	Amplitude $= 3$, Period $= \frac{2\pi}{8} = \frac{\pi}{4}$, Vertical shift $= 3$ $\therefore y = 3 + \sin\left(\frac{\pi x}{4}\right)$	1 Mark: A
4.	Concave down when $f''(x) < 0$ $f(x) = x^3 + x^2, f'(x) = 3x^2 + 2x$ $f''(x) = 6x + 2$ $6x + 2 < 0$ $x < -\frac{1}{3}$	1 Mark: D
5.	$z = \frac{x - \bar{x}}{s} = \frac{190 - 160}{15} = 2$ $\therefore$ 95% of scores have a z-score between −2 and 2. $\therefore 5\% \div 2 = 2.5\%$ have a z-score greater than 2.	1 Mark: B
6.	$\sqrt{x-9} \neq 0$ or $x \neq 9$ Also $x - 9 > 0$ or $x > 9$ $\lim_{x\to\infty} \frac{1}{\sqrt{x-9}} = 0$ Domain: $\{x : x > 9\}$ Range: $\{y : y > 0\}$  	1 Mark: B
7.	Correlation between −0.5 and −0.74. Moderate negative.	1 Mark: C
8.	$\frac{d}{dx}(e^{-4x}\cos 2x)dx = (e^{-4x} \times -2\sin 2x) + (\cos 2x \times -4e^{-4x})$ $= -2e^{-4x}(\sin 2x + 2\cos 2x)$	1 Mark: A
9.	30, 35, 40,...,105. AP with $a = 30$, $d = 5$ and $T_n = 105$ $T_n = a + (n-1)d$ $105 = 30 + (n-1) \times 5$ $105 = 30 + 5n - 5$ $5n = 80$ $n = 16$	1 Mark: B
10.	$\int_1^{1.3} \frac{3}{4}(x^2 - 1)dx = \frac{3}{4}\left[\frac{x^3}{3} - x\right]_1^{1.3}$ $= \frac{3}{4}\left[\left(\frac{1.3^3}{3} - 1.3\right) - \left(\frac{1^3}{3} - 1\right)\right]$ $= 0.07425 \approx 0.07$	1 Mark: A

Section II		
11(a)	Sample answer 	1 mark: Correct answer.
11(b)	At A the y-values of the line and the curve coincide: $y = mx = e^{0.5x}$ (1) At A the slopes of the line and the tangent to the curve coincide: $m = \frac{d}{dx}e^{0.5x} = 0.5e^{0.5x}$ (2) Substitute (1) into (2): $m = 0.5mx$ $x = 2$ $y = e^{0.5\times 2} = e$ $\therefore$ Coordinates of A are $(2, e)$	2 Marks: Correct answer. 1 Mark: Finds one of the coordinates or shows some understanding.
11(c)	At A $mx = e^{0.5x}$ $m \times 2 = e^{0.5\times 2}$ $m = 0.5e$	1 Mark: Correct answer.
12	$(\cos x + 2)(2\cos x + 1) = 0$ $\cos x = -\frac{1}{2}$ or $\cos x = -2$ $x = \frac{2\pi}{3}$ or (No soln) In the domain $0 \le x \le 2\pi$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$	2 Marks: Correct answer. 1 Mark: Finds one solution or shows some understanding.
13	$\frac{5}{x-2} - \frac{2}{x-3} = \frac{5(x-3)}{(x-2)(x-3)} - \frac{2(x-2)}{(x-2)(x-3)}$ $= \frac{5x - 15 - 2x + 4}{(x-2)(x-3)}$ $= \frac{3x - 11}{(x-2)(x-3)}$	2 Marks: Correct answer. 1 Mark: Finds a common denominator or shows some understanding.
14	$y = 2\sin 3x - 3\tan x$ $y' = 2 \times 3\cos 3x - 3 \times \sec^2 x$ $= 6\cos 3x - 3\sec^2 x$ At $x = 0$ $y' = 6 - 3 = 3$	2 Marks: Correct answer. 1 Mark: Finds the derivative.
15	Assessment results increase as head circumference increases. Low positive correlation. Not a strong relationship.	2 Marks: Correct answer. 1 Mark: Shows understanding

16(a)	Point A is an x-intercept $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $\therefore x = 4$ or $x = -2$ The x value of A is positive (diagram) $\therefore$ Coordinates of A is $(4, 0)$	1 Mark: Correct answer.
16(b)	$\left\lvert \int_0^4 (x^2 - 2x - 8)dx \right\rvert + \int_4^6 (x^2 - 2x - 8)dx$ $= \left\lvert \left[\frac{x^3}{3} - x^2 - 8x\right]_0^4 \right\rvert + \left[\frac{x^3}{3} - x^2 - 8x\right]_4^6$ $= \left\lvert \left[\frac{4^3}{3} - 4^2 - 8\times 4\right] \right\rvert + \left(\left[\frac{6^3}{3} - 6^2 - 8\times 6\right] - \left[\frac{4^3}{3} - 4^2 - 8\times 4\right]\right)$ $= 41\frac{1}{3}$ square units	2 Marks: Correct answer. 1 Mark: Calculates the primitive function or shows some understanding of the problem.
17	$T_n = a + (n-1)d$ $T_2 = a + d = 37$ (1) $T_6 = a + 5d = 17$ (2) Equation (2) − (1) $4d = -20$ $d = -5$ Substitute $d = -5$ into equation (1) $a - 5 = 37$ $a = 42$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{10}{2}[2\times 42 + (10-1)\times(-5)]$ $= 195$	3 Marks: Correct answer. 2 Marks: Finds the first term and the common difference. 1 Mark: Finds two equations using the nth term of a AP or shows some understanding.
18(a)	 $m = \frac{\text{Rise}}{\text{Run}} = \frac{50}{50} = 1$ $\therefore$ Gradient is 1.	2 marks: Correct answer. 1 mark: Finds the line of best fit or shows some understanding.

18(b)	When $s = 36$ then $p = 46$ (from the scatterplot) Alyssa should score 46 on the push-up test.	1 mark: Correct answer.
18(c)	Data: (0,10)(5,15)(10,25)(15,25)(20,25)(25,35) (30,50)(35,45)(40,50)(45,50)(50,60) $r = 0.968450\ldots$ ≈ 0.97	2 marks: Correct answer. 1 mark: Finds a value of r close to 0.9.
19	$5y = \sin\left(2x - \frac{\pi}{3}\right)$ $y = \frac{1}{5}\sin\left(2x - \frac{\pi}{3}\right)$ Amplitude $= \frac{1}{5}$ Period $= \frac{2\pi}{2} = \pi$	2 Marks: Correct answer. 1 Mark: Finds either amplitude or the period.
20(a)	$f(x) = 4x^3 - 4x^2$ Stationary points $f'(x) = 0$ $f'(x) = 12x^2 - 8x$ $4x(3x - 2) = 0$ $x = 0, x = \frac{2}{3}$ $\therefore$ Stationary points are $(0, 0)$ and $(\frac{2}{3}, -\frac{16}{27})$ $f''(x) = 24x - 8$ At $(0, 0), f''(0) = -8 < 0$ Maxima At $(\frac{2}{3}, -\frac{16}{27})$ $f''\left(\frac{2}{3}\right) = 8 > 0$ Minima	3 Marks: Correct answer. 2 Marks: Finds both of the stationary points. 1 Mark: Finds one of the stationary points or recognises $12x^2 - 8x = 0$.
20(b)		2 Marks: Correct answer. 1 Mark: Makes some progress towards sketching the curve.
20(c)	x-intercepts ($y = 0$) $4x^3 - 4x^2 = 0$ $4x^2(x - 1)$ $x = 0, x = 1$ $\therefore$ The curve cuts the x-axis at $x = 0$ and $x = 1$.	1 Mark: Correct answer.

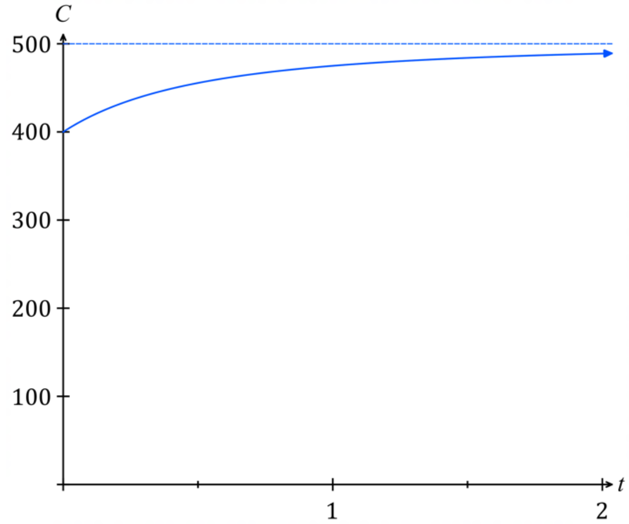
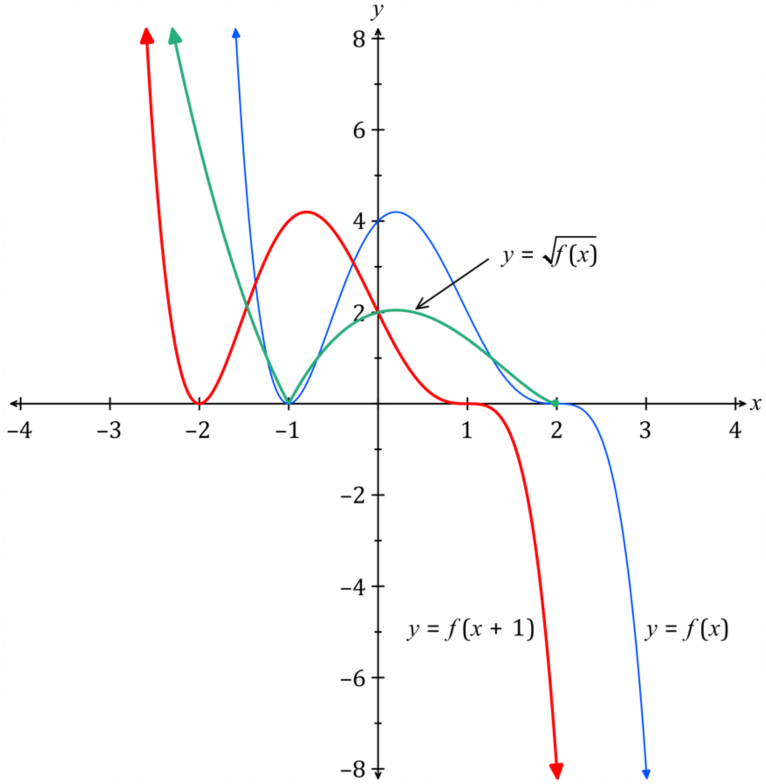
20(d)	$f(x) > 0$ when $x > 1$ (see the graph)	1 Mark: Correct answer.
21(a)	$v = \int 4\cos\left(2t + \frac{\pi}{6}\right) dt$ $= 2\sin\left(2t + \frac{\pi}{6}\right) + C$ Initially $t = 0$ and $v = 1$ $1 = 2\sin\left(2 \times 0 + \frac{\pi}{6}\right) + C$ $C = 0$ $\therefore v = 2\sin\left(2t + \frac{\pi}{6}\right)$	2 Marks: Correct answer. 1 Mark: Integrates to find velocity function.
21(b)	$x = \int 2\sin\left(2t + \frac{\pi}{6}\right) dt$ $= -\cos\left(2t + \frac{\pi}{6}\right) + C$ Initially $t = 0$ and $x = -0.5\sqrt{3}$ $-0.5\sqrt{3} = -\cos\left(2 \times 0 + \frac{\pi}{6}\right) + C$ $-\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2} + C$ $C = 0$ $\therefore x = -\cos\left(2t + \frac{\pi}{6}\right)$	2 Marks: Correct answer. 1 Mark: Integrates to find position function.
21(c)	Particle changes direction when $v = 0$ $0 = 2\sin\left(2t + \frac{\pi}{6}\right)$ $2t + \frac{\pi}{6} = 0, \pi, 2\pi, \ldots$ $2t = -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}, \ldots$ $t = \frac{5\pi}{12}, \frac{11\pi}{12}, \ldots$ $\therefore$ Particle changes direction at $t = \frac{5\pi}{12}$	2 Marks: Correct answer. 1 Mark: Uses $v = 0$.
22(a)	Intersection value is 1.89 (4% and 2 years) $PV = 1.89 \times 6000$ $= \$11\,340$	1 mark: Correct answer.
22(b)	Intersection value is 2.58 (8% and 3 years) Let the value of the annuity be x $47\,988 = x \times 2.58$ $x = \frac{47\,988}{2.58}$ $= \$18\,600$ $\therefore$ Value of the annuity is \$18 600 per year.	2 marks: Correct answer. 1 mark: Finds the intersection value.
22(c)	Intersection value is 3.47 (6% and 4 years) $PV = 3.47 \times 1000$ $= \$3\,470$	1 mark: Correct answer.

23	$f'(x) = 3x^2 - 2$ $f(x) = x^3 - 2x + C$ Point (1, 4) satisfies the function. $4 = 1^3 - 2 \times 1 + C$ $C = 5$ $\therefore f(x) = x^3 - 2x + 5$	2 Marks: Correct answer. 1 Mark: Correctly integrates the first derivative.
24(a)	$FV = PV(1+r)^n$ $= 20\,000(1+0.07)^{10}$ $= 39\,343.02715\ldots$ $\approx \$39\,343.03$ $\therefore$ Account balance is \$39 343.03	1 Mark: Correct answer.
24(b)	$A_{10} = 2000(1.07)^9 + 2000(1.07)^8 + \cdots + 2000(1.07)^0$ $= 2000(1.07^9 + 1.07^8 + \cdots + 1.07^1 + 1)$ GP with $a = 1$, $r = 1.07$ and $n = 10$ $A_{10} = 2000 \times \dfrac{1[1.07^{10} - 1]}{1.07 - 1}$ $= 27\,632.8959\ldots$ $\approx \$27\,632.90$ Account balance $= \$27\,632.90 + \$39\,343.03$ $= \$66\,975.93$ $\therefore$ Account balance is \$27 632.90	3 Marks: Correct answer. 2 Mark: Finds the amount of the annuity or makes significant progress. 1 Mark: Identifies a G.P. with 10 terms.
24(c)	$FV = PV(1+r)^n$ $49\,565 = 20\,000 \times (1+r)^{10}$ $(1+r)^{10} = 2.47825$ $1 + r = \sqrt[10]{2.47825}$ $r = \sqrt[10]{2.47825} - 1$ $= 0.095000989\ldots$ $\approx 9.5\%$ $\therefore$ Annual rate of compound interest is 9.5%.	2 Marks: Correct answer. 1 Mark: Uses the future value interest formula with one correct value.
25	Draw graphs of $y = \cos 2x$ and $y = 0.5$  Solutions are the points of intersection of the graphs $\therefore x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, -\dfrac{\pi}{6}, -\dfrac{5\pi}{6}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Draws one of the graphs or finds one of the solutions.

26(a)	A z-score of 2.5 is 2.5 standard deviations above the mean.	1 mark: Correct answer.
26(b)	$z = \frac{x - \bar{x}}{s}$ $2.5 = \frac{x - 56}{9.5}$ $23.75 = x - 56$ $x = 79.75$ $\therefore$ Claire scored 79.75 in the class test.	1 mark: Correct answer.
27	$c = \bar{y} - m\bar{x}$ $= 65 - 0.6 \times 50$ $= 35$ $\therefore y$-intercept is 35.	1 mark: Correct answer.
28(a)	$\frac{d}{dx}(e^x - 2)^4 = 4(e^x - 2)^3 e^x$ $= 4e^x(e^x - 2)^3$	1 Mark: Correct answer.
28(b)	$\frac{d}{dx}\left(\frac{3x}{\sin 2x}\right) = \frac{\sin 2x \times 3 - 3x \times 2\cos 2x}{(\sin 2x)^2}$ $= \frac{3\sin 2x - 6x\cos 2x}{(\sin 2x)^2}$	2 Marks: Correct answer. 1 Mark: Applies the quotient rule.
29	$z = \frac{x - \bar{x}}{s}$ $= \frac{70 - 56}{14}$ $= 1$ Percentage $= 50\% + \frac{68\%}{2}$ $= 84\%$ $\therefore 84\%$ of scores have a z-score less than1.	2 Marks: Correct answer. 1 mark: Calculates the z-score.
30	$y = x\ln x$ $y' = x \times \frac{1}{x} + \ln x \times 1 = 1 + \ln x$ When $x = 1$ $y' = 1 + \ln 1 = 1$ (gradient of the tangent) Gradient of the normal $m_1 m_2 = -1$ $m \times 1 = -1$ $m = -1$ When $x = 1$ then $y = 1 \times \ln 1 = 0$ $(1, 0)$ Equation of the normal $y - y_1 = m(x - x_1)$ $y - 0 = -1(x - 1)$ $x + y - 1 = 0$	3 Marks: Correct answer. 2 Marks: Finds the gradient of the normal. 1 Mark: Finds the derivative of the function.

31	X is a random variable $1 \le x \le 6$, $\therefore \int_1^6 (Ax + B)dx = 1$ $\left[\frac{Ax^2}{2} + Bx\right]_1^6 = 1$ $\frac{35A}{2} + 5B = 1$ $35A + 10B = 2$ ① Also $\int_1^3 (Ax + B)dx = \frac{1}{2}$ (Median 3) $\left[\frac{Ax^2}{2} + Bx\right]_1^3 = \frac{1}{2}$ $8A + 2B = \frac{1}{2}$ ② Multiply equation (2) by 5 $40A + 10B = \frac{5}{2}$ ③ Equation ①–Equation ③ $-5\text{A} = -\frac{1}{2}$ or $A = \frac{1}{10}$ Substitute $A = \frac{1}{10}$ in equation (1) $35 \times \frac{1}{10} + 10B = 2$ or $B = -\frac{3}{20}$ $\therefore A = \frac{1}{10}$ and $B = -\frac{3}{20}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds one equation relating A and B.
32(a)	Students with a z-score of –2 is two standard deviations below the mean $(70 - (2 \times 10) = 50$. $\therefore$ Weight of the student is 50 kg.	1 mark: Correct answer.
32(b)	68% of scores have a z-score between -1 and 1 (or from 60 to 80) Region A $= \frac{68\%}{2} = 34\%$	1 mark: Correct answer.
32(c)	$z = \frac{x - \bar{x}}{s} = \frac{100 - 70}{10} = 3$ Percentage of scores less than a z-score of 3 is 99.85% Number of students $= 99.85\% \times 400 = 399.4 = 399$ $\therefore$ There are 399 students with a mass less than 105 kg.	2 Marks: Correct answer. 1 Mark: Finds the z-score or shows some understanding.
33(a)	Area of a sector: $A = \frac{\theta}{360} \times \pi r^2$ (need to eliminate θ) Perimeter $= r + r + \frac{\theta}{360} \times 2\pi r$ $40 = 2r + \frac{2\pi r\theta}{360}$ $\frac{2\pi r\theta}{360} = 40 - 2r$ $\theta = \frac{360}{2\pi r}(40 - 2r)$ $A = \frac{\theta}{360} \times \pi r^2 = \frac{\pi r^2}{360} \times \theta = \frac{\pi r^2}{360} \times \frac{360}{2\pi r}(40 - 2r)$ $= \frac{r}{2}(40 - 2r)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Uses the perimeter to eliminate θ or shows some understanding.

33(b)	$A = \frac{r}{2}(40 - 2r) = 20r - r^2$ $\frac{dA}{dr} = 20 - 2r$ $\frac{d^2A}{dr^2} = -2$ Maximum A occurs when $\frac{dA}{dr} = 0$ $20 - 2r = 0$ $2r = 20$ $r = 10$ Check if $r = 10$ is a maximum $\frac{d^2A}{dr^2} = -2 < 0$ $\therefore r = 10$ gives the maximum value of A	2 Marks: Correct answer. 1 Mark: Differentiates the formula for A with respect to r.
33(c)	Maximum area when $r = 10$ $A = \frac{r}{2}(40 - 2r)$ $= \frac{10}{2}(40 - 2 \times 10)$ $= 100 \text{ m}^2$	1 Mark: Correct answer.
34	Trapezoidal rule with 2 intervals. $A = \frac{h}{2}[y_0 + y_2 + 2y_1]$ $612 = \frac{18}{2}[24 + 24 + 2 \times x]$ $68 = 48 + 2x$ $2x = 20$ $x = 10 \text{ cm}$	2 Marks: Correct answer. 1 Mark: Uses the trapezoidal rule with at least one correct value.
35	Initially $t = 163$ and $M = 5$ $M(t) = 10e^{-kt}$ $5 = 10e^{-k \times 163}$ $\frac{1}{2} = e^{-k \times 163}$ $\ln 0.5 = -163k$ $k = \ln 0.5 \div -163$ $= 0.004252436 \ldots$ ≈ 0.0043	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
36(a)	Initial calculation occurs on 1st January 2014 or $t = 0$ $C = 500 - \left(\frac{10}{1 + 0}\right)^2$ $= 400$ tonnes per year $\therefore$ Rate of emission is 400 tonnes per year.	1 Mark: Correct answer.

36(b)	$C = \lim_{t\to\infty} 500 - \left(\frac{10}{1+t}\right)^2 \quad \left(\lim_{t\to\infty} \frac{10}{1+t} = 0\right)$ ≈ 500 tonnes per year $\therefore C$ approaches 500 as time passes.	1 Mark: Correct answer.
36(c)		1 Mark: Correct answer.
36(d)	Area under the curve represents the amount of carbon pollution. $\int_0^6 500 - \left(\frac{10}{1+t}\right)^2 dt = \int_0^6 500 - 100(1+t)^{-2} dt$ $= [500t + 100(1+t)^{-1}]_0^6$ $= [500 \times 6 + 100(1+6)^{-1} - (100(1+0)^{-1})]$ $= 2914.2857\ldots$ ≈ 2914 tonnes $\therefore$ There was 2914 tonnes of carbon emitted from the factory.	2 Marks: Correct answer. 1 Mark: Sets up the area under the curve.
37(a) 37(b)		2 Marks: Correct answer. 1 Mark: Draws the general shape of the curve or shows some understanding.