CARLINGFORD HIGH SCHOOL

DEPARTMENT OF MATHEMATICS

Year 12 Mathematics 2U

Term2 Assessment Task 2013

Time allowed: 55 minutes

Name: ________________________________ **Class:** ________**Teacher**________

Gong / Cheng / Strilakos / Nicolaou / Lobejko / Kellahan / White

Instructions:

- All questions should be attempted.
- Show ALL necessary working on your own paper.
- Marks may not be awarded for careless or badly arranged work.
- Only board-approved calculators may be used.
- Start each question on a new page and only write on one side of each sheet of paper.

	Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
H3		/9	/8			/8	/25
H5	/9			/9	/7		/25
TOTAL	/9	/9	/8	/9	/7	/8	/50

Question 1 (9 marks) (Start a new page)

(a) Convert 36° to radians, giving your answer in terms of π. [1]

(b) What is the period of the function $y = \tan\left(x + \frac{\pi}{2}\right)$? [1]

(c) Evaluate correct to three significant figures: [2]

(i) $log_e 8$

(ii) $log_3 7$

(d) What is the exact value of $cot\frac{\pi}{6}$ [1]

(e) Solve $4^{x-3} = 20$ (correct to two decimal places) [2]

(f) An arc length of 5 units subtends an angle θ at the centre of a circle of radius 3 units.

Find the value of θ to the nearest degree. [2]

Question 2 (9 marks) (Start a new page)

(a) Differentiate:

(i) e^{-3x} [1]

(ii) $ln\,(5x-1)$ [1]

(iii) $x^2\, ln\, x$ [2]

(b) Evaluate:

(i) $\int_0^1 6xe^{x^2}\,dx$ [2]

(ii) $\int_1^{e^3} \frac{4}{x}\,dx$ [3]

Question 3 (8 marks) (Start a new page)

(a) Given that $log_a b = 2.75$ and $log_a c = 0.25$ find the value of: [2]

(i) $log_a\left(\frac{b}{c}\right)$

(ii) $log_a(bc)^2$

(b) Find the gradient of the normal to the curve $y = 6lnx$ when $x = e$ [2]

(c) Find $\int \frac{x}{x^2+3}dx$ [2]

(d) Differentiate $log_e \frac{x+1}{3x-4}$ [2]

Question 4 (9 marks) (Start a new page)

(a) Differentiate: [3]

(i) $4x + tanx$

(ii) $sin(3x + 1)$

(iii) $cos(x^2)$

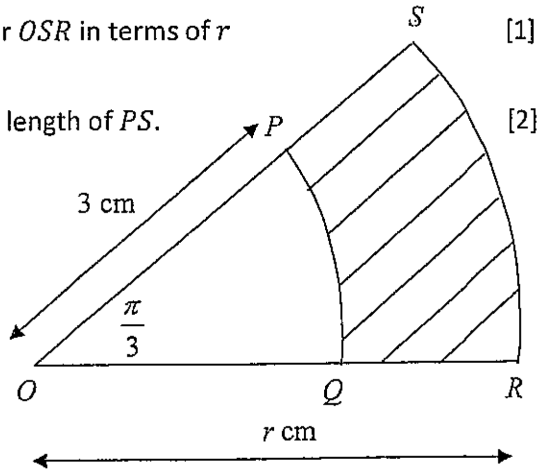
(b) Evaluate $\int_0^{\frac{\pi}{2}} \cos x\, dx$ [2]

(c) In the diagram below PQ and SR are arcs of concentric circles with centre 0.
$< POQ = \frac{\pi}{3} radians$ and $OP = 3cm.$

(i) Find the area of the sector POQ [1]

(ii) If OR is $r\ cm$ find the area of the sector OSR in terms of r [1]

(iii) If the shaded area is $\frac{27\pi}{6}\ cm^2$, find the length of PS. [2]



Question 5 (7 marks) (Start a new page)

In the domain $-2\pi \le x \le 2\pi$ answer the following:

(a) Sketch $y = 5cos\frac{x}{2}$ stating the amplitude and period of the function. [3]

(b) From your graph:

(i) How many solutions are there to the equation $5cos\frac{x}{2} = 1$? [1]

(ii) What are the approximate solutions? [1]

(c) Solve the equation $5cos\frac{x}{2} = 1$ calculating your answer in radians to two decimal places. [2]

Question 6 (8 marks) (Start a new page)

(a) Show that the derivative of $log_e cosx$ is $-tanx$ [2]

(b) Show that $\int \frac{x+5}{x+1} dx = x + 4ln(x+1) + C$ [2]

(c) (i) Sketch the curve $y = lnx$ [1]

(ii) Write $y = lnx$ in index form. [1]

(iii) Hence, find the area between the curve, the y $axis$ and the lines $y = 2$ and $y = 4$ leaving your answer in exact form. [2]

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SOLUTIONS

Yr 12 Mathematics 2U Term 2 2013

Q1 (a) $36^\circ = 36 \times \frac{\pi}{180}$

$= \frac{\pi}{5}$

(b) period $= \pi$

(c) (i) $2 \cdot 08$

(ii) $1 \cdot 77$

(d) $\cot\frac{\pi}{6} = \sqrt{3}$

(e) $4^{x-3} = 20$

$\log_4 20 = x - 3$

$2 \cdot 1609.. = x - 3$

$\therefore x = 5 \cdot 1609\ldots$

$x \doteqdot 5 \cdot 16$ (2dp)

(f) $l = r\theta$

$5 = 3 \times \theta$

$\theta = \frac{5}{3}$ rad.

$= \frac{5}{3} \times \frac{180^\circ}{\pi}$

$= 95 \cdot 49..^\circ$

$\doteqdot 95^\circ$

STRILAKOS

Q2 (a) (i) $\frac{d}{dx}\left(e^{-3x}\right) = -3e^{-3x}$

(ii) $\frac{d}{dx}\ln(5x-1) = \frac{5}{5x-1}$

(iii) $y = x^2 \ln x$

$\frac{dy}{dx} = x^2 \times \frac{1}{x} + \ln x \times 2x$

$= x + 2x\ln x$

$= x(1 + 2\ln x)$

(b) (i) $\int_0^1 6xe^{x^2}dx = 3 \times \int_0^1 2xe^{x^2}dx$

$= 3\left[e^{x^2}\right]_0^1$

$= 3\left(e^1 - e^0\right)$

$= 3(e-1)$

(ii) $\int_1^{e^3} \frac{4}{x}dx = 4\int_1^{e^3}\frac{1}{x}\,dx$

$= 4\left[\ln x\right]_1^{e^3}$

$= 4\left(\ln e^3 - \ln 1\right)$

$= 4(3\ln e - 0)$

$= 4 \times 3$

$= 12$

Q3 (a) (i) $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$

$= 2 \cdot 75 - 0 \cdot 25$

$= 2 \cdot 5$

(ii) $\log_a (bc)^2 = 2\log_a(bc)$

$= 2(\log_a b + \log_a c)$

$= 2(2 \cdot 75 + 0 \cdot 25)$

$= 2 \times 3$

$= 6$

KELLAHAN

(b) $y = 6\ln x$

$\frac{dy}{dx} = \frac{6}{x}$

When $x = e$, $\frac{dy}{dx} = \frac{6}{e}$

$\therefore$ grad. of normal $= -\frac{e}{6}$

(c) $\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int\frac{2x}{x^2+3}\,dx$

$= \frac{1}{2}\ln(x^2+3) + C$

(d) $y = \log_e \frac{x+1}{3x-4}$

$= \log_e(x+1) - \log_e(3x-4)$

$\frac{dy}{dx} = \frac{1}{x+1} - \frac{3}{3x-4}$

$= \frac{1(3x-4)-3(x+1)}{(x+1)(3x-4)}$

$= \frac{3x-4-3x-3}{(x+1)(3x-4)}$

$= \frac{-7}{(x+1)(3x-4)}$

Q4 (a) (i) $\frac{d}{dx}(4x+\tan x)$

$= 4 + \sec^2 x$

(ii) $\frac{d}{dx}\sin(3x+1)$

$= 3\cos(3x+1)$

(iii) $\frac{d}{dx}\cos(x^2)$

$= -2x\sin(x^2)$

(b) $\int_0^{\frac{\pi}{2}} \cos x \, dx$

$= [\sin x]_0^{\frac{\pi}{2}}$

$= \sin\frac{\pi}{2} - \sin 0$

$= 1$

(c) (i) Area of sector POQ

$= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(3)^2\left(\frac{\pi}{3}\right)$

$= \frac{3\pi}{2} \text{ cm}^2$

(ii) Area of sector OSR

$= \frac{1}{2}(r)^2\left(\frac{\pi}{3}\right)$

$= \frac{\pi r^2}{6}$

(iii) $\frac{\pi r^2}{6} - \frac{3\pi}{2} = \frac{27\pi}{6}$

$\frac{\pi r^2}{6} - \frac{9\pi}{6} = \frac{27\pi}{6}$

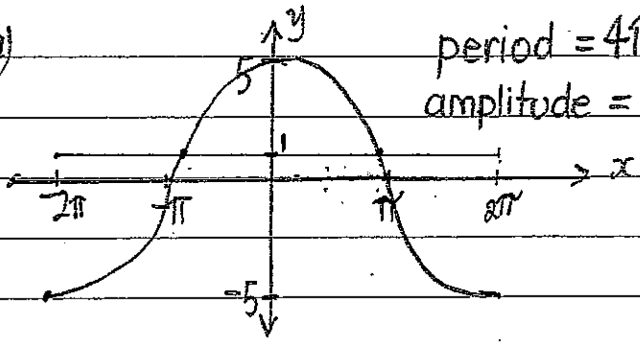
$r^2 - 9 = 27$

$r^2 = 36$

$r = 6$

$\therefore PS = 3 \text{ cm}$

Q5 (a)



period $= 4\pi$

amplitude $= 5$

(b) (i) 2 solutions (must graph $y=1$)

(ii) -3 and 3 to -2.5 and 2.5

(c) $5\cos\frac{x}{2} = 1$ $\qquad -2\pi \le x \le 2\pi$

$\cos\frac{x}{2} = \frac{1}{5}$ $\qquad -\pi \le \frac{x}{2} \le \pi$

$\therefore \frac{x}{2} = 1.3694\ldots$ or $-1.3694\ldots$

$\therefore x = 2.7388\ldots$ or $-2.7388\ldots$

$x \doteqdot 2.74$ or -2.74

Q6 (a) $y = \log_e \cos x$

$\frac{dy}{dx} = \frac{1}{\cos x} \times -\sin x$

$= \frac{-\sin x}{\cos x}$

$= -\tan x$

(b) $\int \frac{x+5}{x+1} dx$

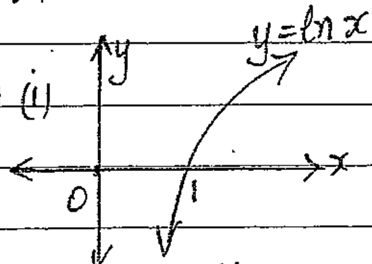
$= \int \frac{x+1+4}{x+1} dx$

$= \int \frac{x+1}{x+1} dx + \int \frac{4}{x+1} dx$

$= \int 1 \, dx + \int \frac{4}{x+1} dx$

$= x + 4\ln(x+1) + C$

(c) (i)



(ii) $x = e^y$

(iii) $A = \int_2^4 e^y \, dy$

$= \left[e^y\right]_2^4$

$= e^4 - e^2$

$= e^2(e^2-1)$ unit

NICOLAOU

LOBEJKO