CARLINGFORD HS ADVENTURE IN LEARNING

Carlingford High School

2020 YEAR 12 ASSESSMENT TASK 2

Mathematics Advanced

STUDENT NUMBER:______________________________

Teacher: (Please Circle) 12MAA A (Ms Bennett) 12MAAB (Mr Wilson)

12MAA_1 (Ms Strilakos) 12MAA_2 (Mr Gong) 12MAA_3 (Mr Cheng) 12MAA_4 (Ms Blakeley)

General Instructions

- Working time - 50 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 50

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 10 minutes for this section

Section II – 40 marks (pages 5 – 12)

- Attempt Questions 11 – 21
- Allow about 40 minutes for this section

TOPIC	MARKS	
Graphing Techniques **Questions: 1 – 4, 11 – 14**	**/18**	
Trigonometric Functions and Graphs **Questions: 5 – 8, 15 – 18**	**/17**	
Differential Calculus **Questions: 9 – 10, 19 – 21**	**/15**	
TOTAL	**/50**	%

Section I
10 marks
Attempt Questions 1 - 10.
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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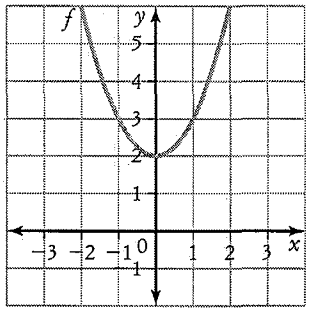
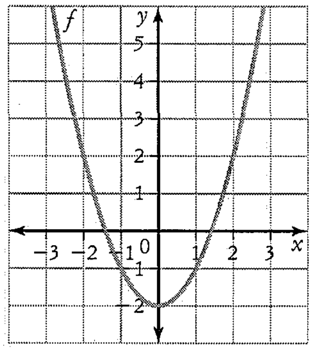
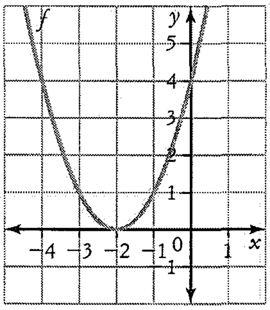
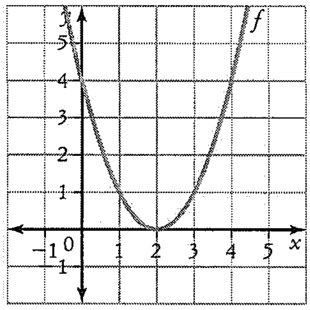
1. Which graph represents $y = (x-2)^2$?

A 

B 

C 

D 

2. The function $y = e^x$ is dilated to $y = \frac{e^x}{7}$, then which of the following is true?

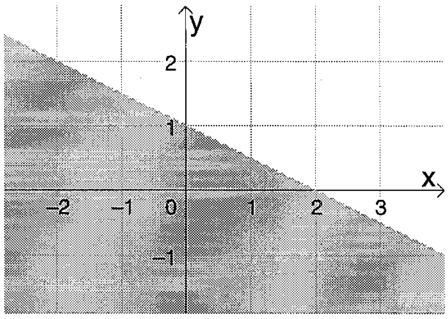
A dilated by a factor of 7 with respect to the x-axis

B dilated by a factor of $\frac{1}{7}$ with respect to the x-axis

C dilated by a factor of 7 with respect to the y-axis

D dilated by a factor of $\frac{1}{7}$ with respect to the y-axis

3. Which inequality defines the shaded region?



A $2y + x - 2 > 0$

B $2y + x - 2 < 0$

C $2y - x + 2 > 0$

D $2y - x + 2 < 0$

4. Find the coordinates of the image of (x, y) when the function $y = f(x)$ is transformed to $y = -2f(x+1) + 4$.

A $(x+1, -2y-4)$

B $(x+1, -2y+4)$

C $(x-1, -2y+4)$

D $(-x+1, 2y+4)$

5. The function $y = 2\sin(3x) + 5$ has:

A amplitude 2, period 3 and centre 5

B amplitude 5, period $\frac{1}{3}$ and centre 2

C amplitude 2, period $\frac{2\pi}{3}$ and centre 5

D amplitude 2, period $\frac{2\pi}{3}$ and centre -5

6. The equation of a function with phase π units to the left is:

A $y = \tan(x + \pi)$

B $y = \tan(\pi x)$

C $y = \tan x + \pi$

D $y = \tan(x - \pi)$

7. The solution of $\cos 2x = 1$ in the domain $[0, 2\pi]$ is:

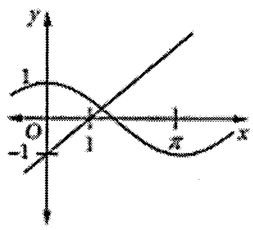
A $x = 0, 2\pi$

B $x = 0, \pi, 2\pi$

C $x = \frac{\pi}{2}, \frac{3\pi}{2}$

D $x = \pi$

8. Which graph could be used to solve the equation $\cos x = x - 1$?

A


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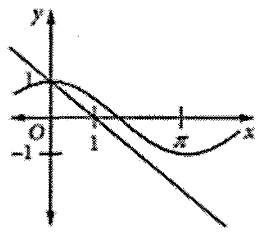

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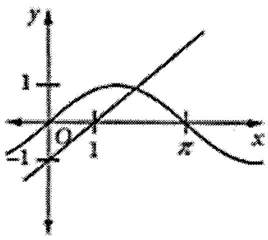

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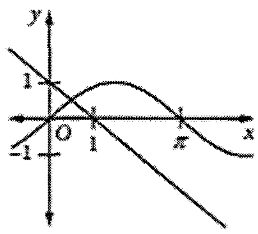


9. The derivative of $\cos^2(5t)$ is:

A $-10 \sin 5t \cos 5t$

B $-10 \cos 5t$

C $-5 \sin 5t \cos 5t$

D $-2 \sin 5t \cos 5t$

10. The derivative of $y = \sqrt[3]{\tan x}$ is:

A $\frac{dy}{dx} = \frac{2}{3\sqrt[3]{\tan^2 x}}$

B $\frac{dy}{dx} = \frac{2\sqrt[3]{\tan^2 x}}{3\sec^2 x}$

C $\frac{dy}{dx} = 2 \sec x \sqrt[3]{\tan^2 x}$

D $\frac{dy}{dx} = \frac{\sec^2 x}{3\sqrt[3]{\tan^2 x}}$

Section II
40 marks
Attempt Questions 11 – 21.
Allow about 40 minutes for this section.

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks)

Find the equation for each transformed function.

(a) $y = 2^x$ translated 2 units to the right **1**

....

(b) $f(x) = |2x| - 2$ translated 2 units to the left **1**

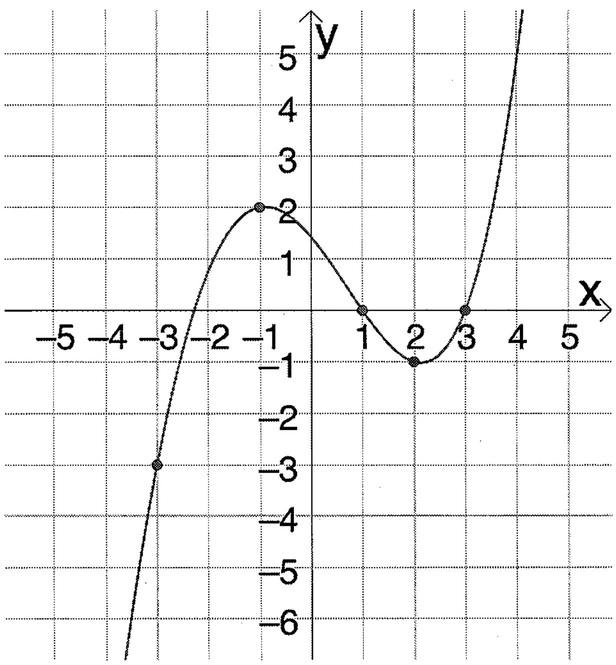
....

(c) $f(x) = \ln x$ dilated vertically with factor 5 and reflected in the $y-$ axis **1**

....

Question 12 (4 marks)

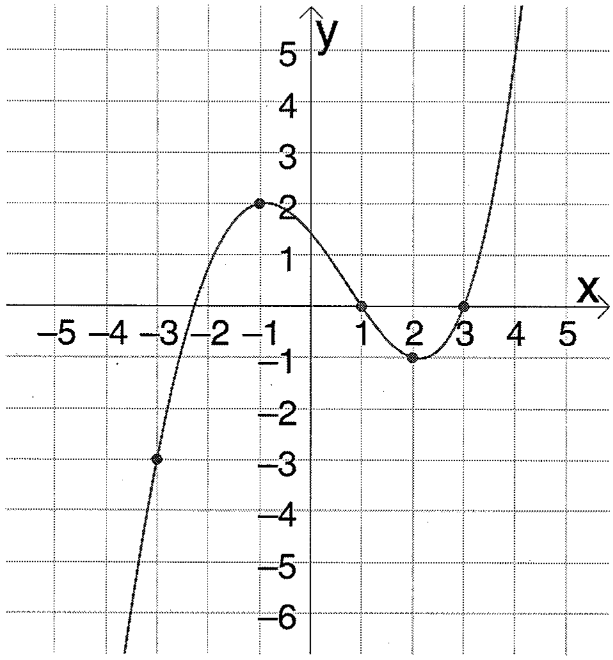
(a) From the graph of $y = f(x)$ shown, draw the graph of $y = 2f(x+1)$. **2**



Question 12 continues on page 6

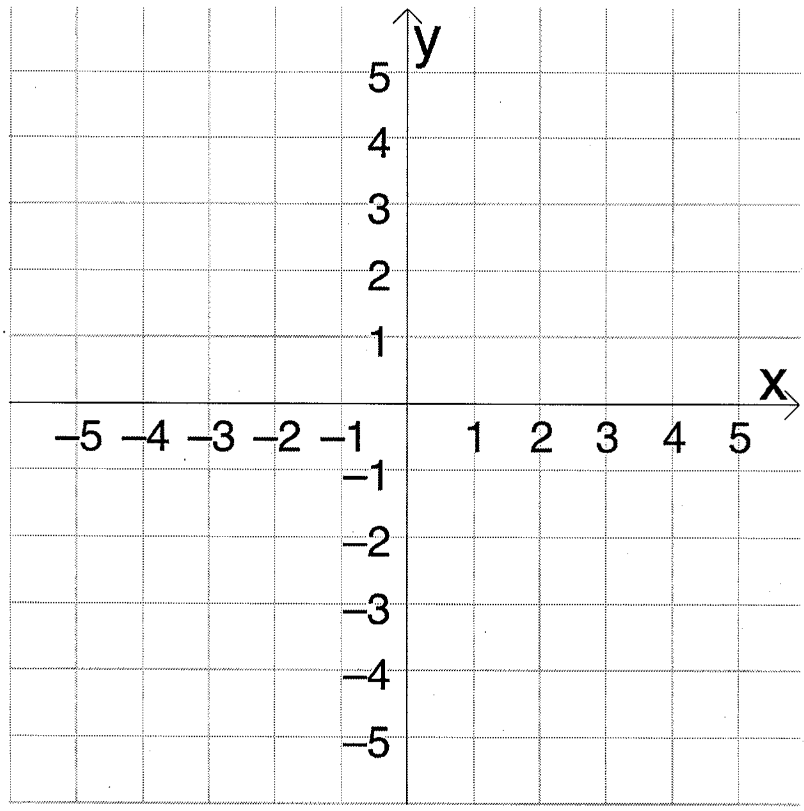
Question 12 (continued)

(b) From the graph of $y = f(x)$ shown, draw the graph of $y = -f(x) + 2$. **2**



Question 13 (2 marks)

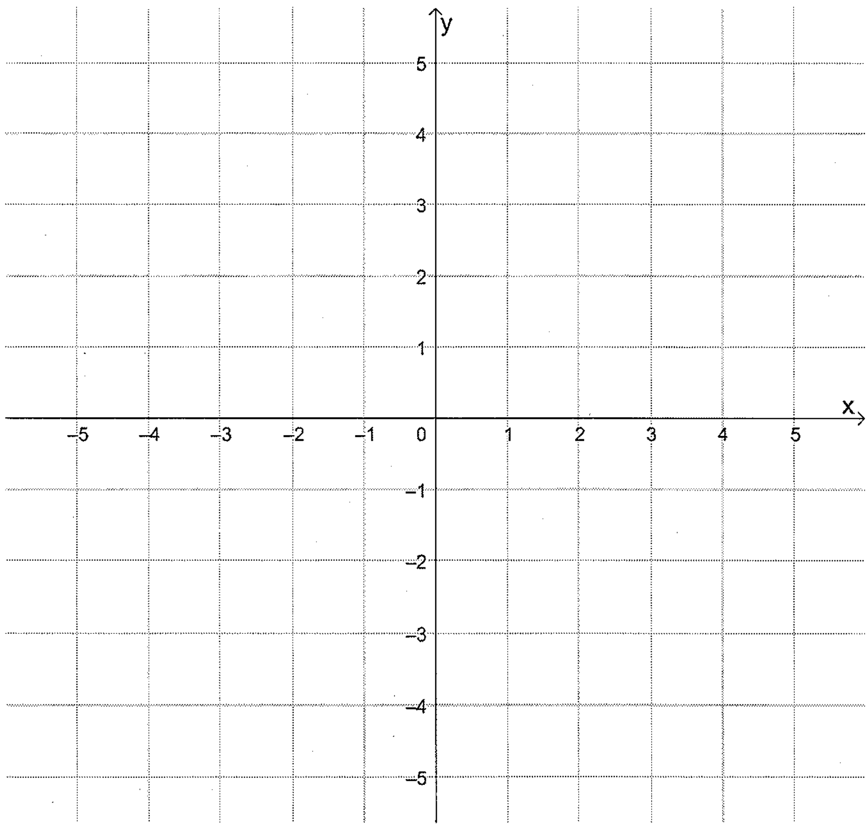
The parabola $y = x^2$ meets the line $y = -x + 2$ at the points $(-2, 4)$ and $(1, 1)$. **DO NOT** prove this.
By first sketching the graphs of $y = x^2$ and $y = -x + 2$, shade the region which simultaneously satisfies the inequalities $y \geq x^2$ and $y \geq -x + 2$.



Question 14 (5 marks)

(a) Show that $\dfrac{2x-5}{x-3} = \dfrac{1}{x-3} + 2$ **2**

(b) Sketch the graph of $y = \dfrac{2x-5}{x-3}$ **2**



(c) Hence solve $\dfrac{2x-5}{x-3} > 2$ **1**

Question 15 (6 marks)

Find all the solutions for the following equations where $0 \leq x \leq 2\pi$.

(a) $\sqrt{2}\sin x = 1$ **1**

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(b) $2\cos\left(x - \frac{\pi}{3}\right) = \sqrt{3}$ **2**

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(c) $2\sin^2 x + \cos x - 2 = 0$ **3**

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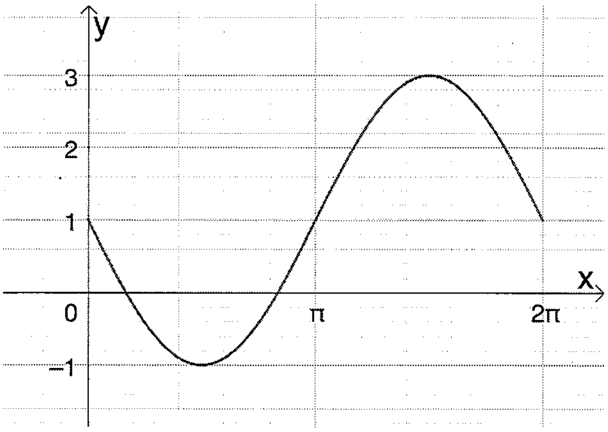
Question 16 (2 marks)

The sketch below shows part of a trigonometric function for the given domain $0 \leq x \leq 2\pi$. Determine the equation of the function.



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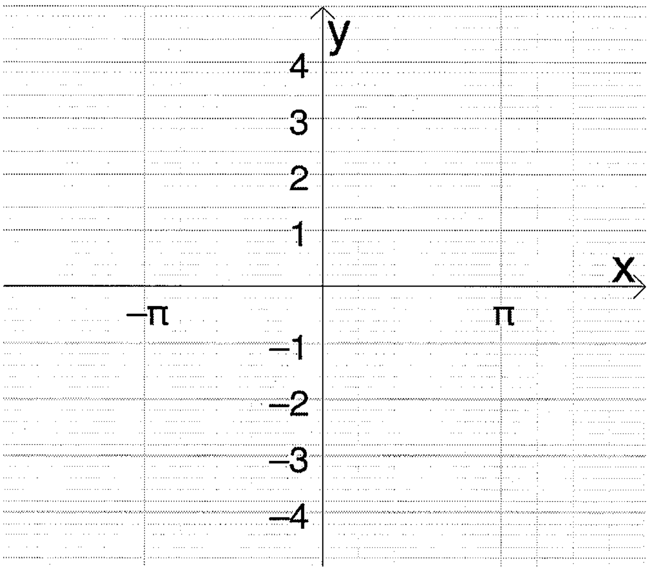
Question 17 (2 marks)

By drawing appropriate graphs, determine the **number of solutions** to the equation $3\cos 2x = e^x$ in the domain $-\pi \leq x \leq \pi$.



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Question 18 (4 marks)

A vertical spring is pulled down and then let go. It bounces back up and down again according to the equation $h = 12\cos t + 15$ where h is the height of the spring in cm and t is the time in seconds.

(a) Describe the significance of the 15 in the equation **1**

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(b) What are the maximum and minimum heights of the spring? **1**

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(c) What is the height of the spring after π seconds? **1**

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(d) At what time will the spring first be at its minimum height? **1**

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Question 19 (9 marks)

Differentiate the following with respect to x:

(a) $\tan(5x - \pi)$ **1**

....

(b) $\cot^2(2x)$ **2**

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(c) $e^{x \sin x}$ **2**

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(d) $\ln \sqrt{x}$ **2**

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(e) $\ln\left(\dfrac{x^3 + 1}{x}\right)$ **2**

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Question 20 (2 marks)

If $f(x) = \cos x$ find the exact value of x if $f'(x) = \frac{\sqrt{3}}{2}$ for the domain $0 \leq x \leq 2\pi$.

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Question 21 (2 marks)

Find the equation of the tangent to the curve $y = 2 + e^{3x}$ at the point $x = 0$.

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END OF PAPER

NSW GOVERNMENT

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

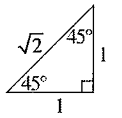
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

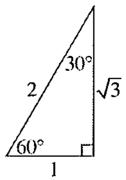
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A\sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\sin A\cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A\sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

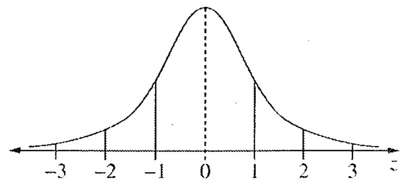
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

STUDENT NUMBER: ______________________________

Carlingford High School

2020 YEAR 12 ASSESSMENT TASK 2

Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○