Year 11 Advanced Task 1 2020

Question 1

a) $2x + 2w = P$

$w = \frac{1}{2}(P - 2x)$

b) length $= x - 4$ width $= w - 4$ height ~~length~~ $= 2$

$V(x) = 2(x-4)(w-4)$

$= 2(x-4)\left[\frac{1}{2}(P-2x) - 4\right]$

$= (x-4)(P-2x-8)$

$= xP - 2x^2 - 8x - 4P + 8x + 32$

$= -2x^2 + Px + 32 - 4P$

c) $P = 120 \therefore V(x) = -2x^2 + 120x + 32 - 480$

$= -2(x^2 + 60x) - 448$

$= -2[(x+30)^2 - 30^2] - 448$

$= -2(x-30)^2 + 1352$

d) We must have $x - 4 > 0$ and $w - 4 > 0$

$\therefore \; P/2 - x - 4 > 0$

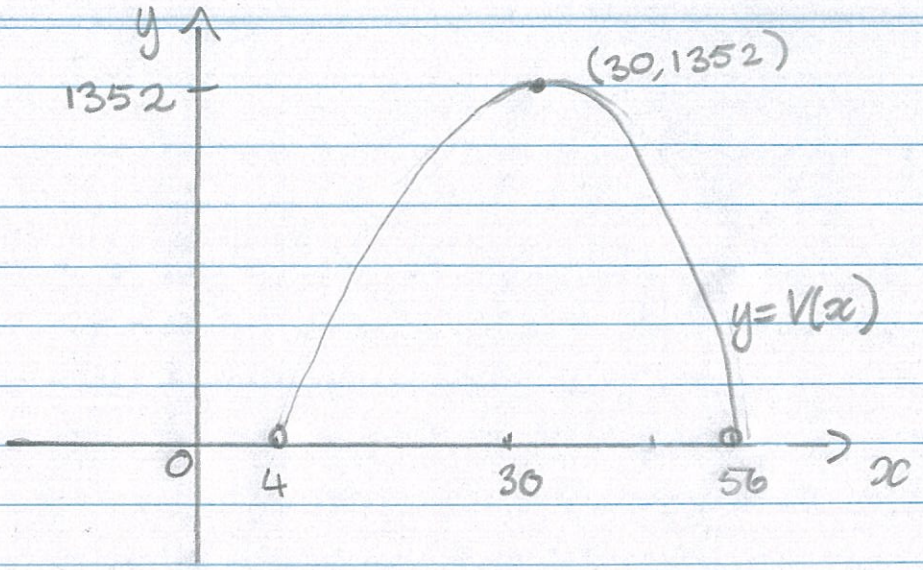
$60 - x - 4 > 0$

$56 > x$

Domain $x \in (4, 56)$

f) The turning point of the graph represents the maximum possible volume of the box and the value of x for which it occurs.
At this point $x = w = 30$ and the box is a square prism with sides of 22 cm.
(Allow 'has a square base'.)

1 e)



g) length $= x - 2$ width $= \frac{p}{2} - x - 2 = 58 - x$
height $= 1$

$\therefore V(x) = (x-2)(58-x)$
$= -x^2 + 60x - 116$

Turning point at $x = \frac{-b}{2a} = \frac{-60}{2 \times -1}$
$= 30$

when $x = 30$ $V = (30-2)(58-30)$
$= 784 \text{ cm}^3$

Turning point: $(30, 784)$

The maximum volume is smaller than when $s = 2$ but it occurs at the same value of x, when the base of the box is square.

h) Lengths of $2s$ are cut out of each side to form the box. Therefore $4 \times 2s < 120$

$s < \frac{120}{8}$

$s < 15$.

i) length $= x - 2s$
width $= w - 2s = 60 - x - 2s$
height $= s$

j) The maximum volume of the box initially increases with s, then decreases ~~[illegible]~~.
The maximum volume is attained when $s = 5$, $x = 30$ and the length and width of the box are both 20 cm.

Note. Third mark should not be given if $x = 30$ is used without checking other values.
~~Investigations should~~

k) Investigations should look at values of s with $0 < s < P/8$ and show that the maximum volume occurs when $s = P/24$ for a box with square base of sidelength $\frac{P}{4} - 2s$

$P = 66$ $\quad 0 < s < 8.25$ $\quad$ max value $s = 2.75$
length = width = 11 $\quad$ Volume $= 332.75$ $\quad x = 16.5$

$P = 78$ $\quad 0 < s < 9.75$ $\quad$ max value when $s = 3.25$
length = width = 13 $\quad$ Vol $= 549.25$ $\quad x = 19.5$

$P = 84$ $\quad 0 < s < 10.5$ $\quad$ max value when $s = 3.5$
length = width = 14 $\quad V(x) = 686$ when $x = 21$

$P = 90$ $\quad 0 < s < 11.25$ $\quad$ max value when $s = 3.75$
length = width = 15 $\quad V(x) = 787.5$ when $x = 22.5$

Not all of these values need to be specified in the answer, and an approximation to s to 1 decimal place eg 'around 3.8' may still get full marks.

Question 2

a) No it is not a function it would fail the vertical line test

b) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55
After the first two numbers which equal 1 the next Fibonacci number is found by adding the last two together

c) 1st: centre (0,0) radius 1 $y = -\sqrt{1-x^2}$
2nd: centre (-1,0) radius 2 $y = \sqrt{4-(x+1)^2}$
3rd: centre (0,0) radius 3 $y = -\sqrt{9-x^2}$
4th: centre (-2,0) radius 5 $y = \sqrt{25-(x+2)^2}$

d) Radius of 6^{th} semicircle $= 7^{th}$ Fibonacci number
$= 13$
From Figure 4 it has an x intercept at 9 and its centre is on the x axis
$9 - 13 = -4$ $\therefore$ centre is at $(-4, 0)$

e) $y = -1 + |x|$ for $-1 \leq x \leq 1$

$$y = \begin{cases} -1 + x & 0 \leq x \leq 1 \\ -1 - x & -1 \leq x < 0 \end{cases}$$

$\therefore$ the equation describes portions of the the straight lines with y-intercept -1 and gradients 1 and -1 between their x and y intercepts which touch the semicircle $y = -\sqrt{1-x^2}$ at its x and y intercepts.

f) All line segments shown have gradient $m = \pm 1$
The second pair touch at $(-1, 2)$ and are defined for $x \in [-3, 1]$ hence they satisfy

$$y = 2 - |x + 1|, \ -3 \leq x \leq 1$$

The third pair touch at $(0, -3)$ and have x intercepts 3 and -3 so they satisfy

$$y = -3 + |x|, \ -3 \leq x \leq 3$$

g) Radius 13 corresponds to the 6^{th} semicircle.
From (d) the maximum value of the equation is at $(-4, 13)$ and the x intercepts are $-4-13$ and $-4+13$ so the equation is $y = 13 - |x + 4|, \ -17 \leq x \leq 9$
and my friend was wrong.