Carlingford High School

Mathematics Ext 1

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION TERM 3 2017

Student Number: ______________________________

- **General Instructions**
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A Reference & MC Sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section

	MC	Q11	Q12	Q13	Q14	Mark
Circle Geo	/1			/4		/5
Inv Trig Function			/2		/3	/5
Division of line	/1					/1
Int of sin^2x & cos^2x	/1		/3			/4
Perm & Comb	/2					/2
Int by Subsituition	/1			/2		/3
SHM	/1				/6	/7
Newton Method	/1					/1
Exp Grow & Decay	/1	/4				/5
Angle btw two lines	/1	/2				/3
Projectile		/4				/4
Parametric			/6			/6
Vel & acc as fn of x			/4			/4
Inductiom				/3		/3
Polynomials				/5		/5
Inverse Functions					/6	/6
Other inequalities		/2				/2
General solution		/3				/3
Harder Application				/1		//1
Total	/10	/15	/15	/15	/15	/70

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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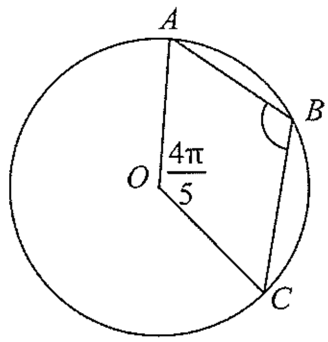
1 The points A, B and C lie on a circle with centre O, as shown in the diagram.

The size of $\angle AOC$ is $\frac{4\pi}{5}$ radians.



Not to scale

What is the size of $\angle ABC$ in radians?

(A) $\frac{3\pi}{10}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{5}$

(D) $\frac{4\pi}{5}$

2 Which of the following is the exact value of $\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4dx}{\sqrt{9-x^2}}$?

(A) $-\pi$

(B) $-\frac{\pi}{4}$

(C) $\frac{\pi}{4}$

(D) π

3 What are the coordinates of the point that divides the interval joining $P(2, 1)$ and $Q(2, 8)$ internally in the ratio 3: 4?

(A) $(1, 7)$

(B) $(2, 4)$

(C) $(2, 7)$

(D) $(4, 2)$

4 What is the exact value of the definite integral $\int_0^{\frac{\pi}{3}} \sin^2 x dx$?

(A) $\dfrac{\pi}{3}-\dfrac{1}{4}$

(B) $\dfrac{\pi}{3}-\dfrac{\sqrt{3}}{4}$

(C) $\dfrac{\pi}{6}-\dfrac{1}{8}$

(D) $\dfrac{\pi}{6}-\dfrac{\sqrt{3}}{8}$

5 How many ways can a football team of eleven be chosen from 15 players?

(A) 15

(B) 165

(C) 1365

(D) 5.44×10^{10}

6 Which integral is obtained when the substitution $u = 1+3x$ is applied to $\int x\sqrt{1+3x}dx$?

(A) $\dfrac{1}{9}\int (u-1)\sqrt{u}du$

(B) $\dfrac{1}{3}\int (u-1)\sqrt{u}du$

(C) $\int (u-1)\sqrt{u}du$

(D) $3\int (u-1)\sqrt{u}du$

7 A particle is moving under SHM in a straight line with an acceleration of $\ddot{x} = 25-5x$, where x is the displacement after t seconds. What is the centre of motion?

(A) $x = 0$

(B) $x = 5$

(C) $x = 10$

(D) $x = 15$

8 The function $f(x) = \sin x - \frac{2x}{3}$ has a real root close to $x = 1.5$.

Let $x = 1.5$ be a first approximation to the root.
What is the second approximation to the root using Newton's method?

(A) 1.495

(B) 1.496

(C) 1.503

(D) 1.504

9 Seven children are seated randomly around a circular table.
How many different arrangement of the seven children if two oldest children sit together?

(A) 6!

(B) 5!

(C) 6!2!

(D) 5!2!

10 A bottle of water has a temperature of 20°C and is placed in a refrigerator whose temperature is 2°C. The cooling rate of the bottle of water is proportional to the difference between the temperature of the refrigerator and the temperature T of the bottle of water. This is expressed by the equation $\frac{dT}{dt} = -k(T-2)$ where k is a constant of proportionality and t is the number of minutes after the bottle of water is placed in the refrigerator. After 20 minutes in the refrigerator the temperature of the bottle of water is 10°C. What is the value of k in the above equation?

(A) $k = -\frac{1}{20}\log_e \frac{9}{4}$

(B) $k = -\frac{1}{10}\log_e \frac{4}{9}$

(C) $k = \frac{1}{20}\log_e \frac{9}{4}$

(D) $k = \frac{1}{10}\log_e \frac{4}{9}$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Find the size of the acute angle between the lines $x - y - 4 = 0$ and $3x - y + 4 = 0$. Answer to the nearest degree. **2**

(b) Solve the inequality $\dfrac{1}{|x-1|} < 1$ **2**

(c) Newton's law of cooling states that when an object at temperature T°C is placed in an environment at temperature T_0°C, the rate of the temperature loss is given by the equation:

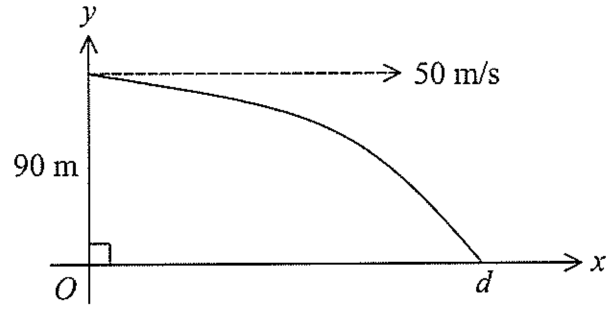
$$\frac{dT}{dt} = -k(T - T_0)$$

where t is the time in minutes and k is a positive constant.

(i) Show that $T = T_0 + Ae^{-kt}$ satisfies the above equation. **1**

(ii) An object whose initial temperature is 60°C is placed in a room in which the internal temperature is maintained at 12°C. **3**
After 25 minutes, the temperature of the object is 30°C.
How long will it take for the object's temperature to reduce to 15°C?
(correct to the nearest minute)

(d) The diagram below shows the trajectory of a ball thrown horizontally, at a speed of 50 ms^{-1}, from the top of a tower 90 metres above ground level.



The ball strikes the ground d metres from the base of the tower.

(i) Show that the equations describing the trajectory of the ball are: **2**

$$x = 50t \text{ and } y = 90 - \frac{1}{2}gt^2$$

where g is the acceleration due to gravity and t is the time in seconds.

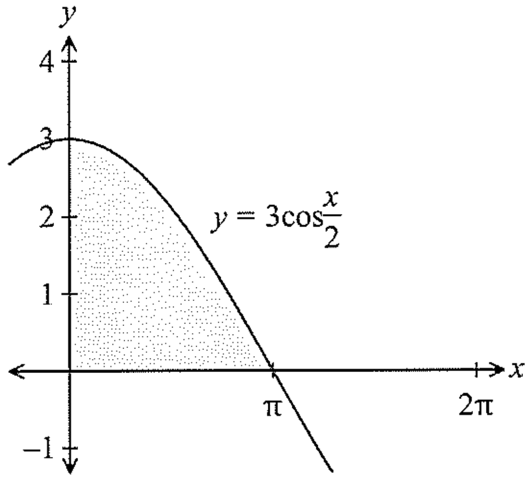
(ii) Prove that the ball strikes the ground at time $t = 6\sqrt{\frac{5}{g}}$ seconds. **1**

(iii) How far from the base of the tower does the ball strike the ground? **1**

(e) Find the general solution for $2\sin x = -1$ **3**

Question 12 (15 marks) **Marks**

(a) The region bounded by the graph $y = 3\cos\frac{x}{2}$ and the x-axis between $x = 0$ and $x = \pi$ is rotated about the x-axis to form a solid. **3**



Find the exact volume of the solid.

(b) $P(2at, at^2)$ is any point on the parabola $x^2 = 4ay$. The line d is parallel to the tangent at P and passes through the focus S of the parabola.

(i) Find the equation of the line d. **3**

(ii) The line d intersects the x-axis at the point R. **2**
Find the coordinates of the midpoint, M, of the interval RS.

(iii) Find the equation of the locus of M. **1**

(c) Find $\displaystyle\int \frac{1}{x^2 + 2x + 2}\,dx$ **2**

(d) A particle moves in a straight line so that its acceleration is given by $a = x + 1.5$ ms^{-2}. Initially, the particle is 5 metres to the right of O and moving towards O with a velocity of 6 ms^{-1}.

(i) Is the particle speeding up or slowing down? Give a reason. **1**

(ii) Show that $v^2 = x^2 + 3x - 4$. **2**

(iii) Where does the particle first change direction? **1**

Question 13 (15 marks) **Marks**

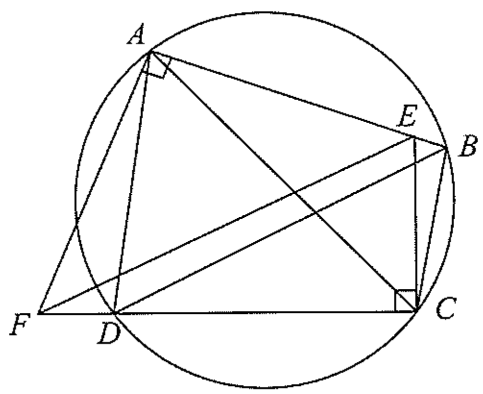
(a) (i) Prove that $\dfrac{\sec^2 x}{\tan x} = \operatorname{cosec} x \sec x$ **1**

(ii) Use the substitution $u = \tan x$ to find the exact value of this integral: **2**

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{cosec} x \sec x \, dx$$

(b) Prove by mathematical induction that $5^n + 12n - 1$ is divisible by 16 for all positive integers n $(n \geq 1)$. **3**

(c) $ABCD$ is a cyclic quadrilateral with $\angle FAE = \angle ECD = 90^\circ$.



(i) Why is $AECF$ a cyclic quadrilateral? **1**

(ii) Hence show that EF is parallel to BD. **3**

(d) It is given that $P(x) = (x-a)^3 + (x-b)^2$ and the remainder when $P(x)$ is divided by $(x-b)$ is -8.

(i) What is the remainder when $P(x)$ is divided by $(x-a)$? **2**

(ii) Prove that $x = \dfrac{a+b}{2}$ is a zero of $P(x)$. **1**

(iii) Prove that $P(x)$ has no stationary points. **2**

Question 14 (15 marks) **Marks**

(a) Find the gradient of the tangent to the curve $y = \ln(\tan^{-1} 3x)$ at the point where $x = \dfrac{1}{\sqrt{3}}$. **3**

Give your answer as an exact value.

(b) Consider the function $f(x) = \dfrac{x}{x+4}$.

(i) Show that $f'(x) > 0$ for all x in the domain. **1**

(ii) State the equation of the horizontal asymptote of $y = f(x)$. **1**

(iii) Without using any further calculus, sketch the graph of $y = f(x)$. **2**

(iv) Explain why $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(v) Find an expression for the inverse function $f^{-1}(x)$. **1**

(c) A particle is moving in a straight line under SHM. At any time (t seconds) its displacement (x metres) from a fixed point O is given by:

$$x = A\cos\left(\frac{\pi}{4}t + \alpha\right) \text{ where } A > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

After 1 second the particle is 2 metres to the right of O and after 3 seconds the particle is 4 metres to the left of O.

(i) Show that $A\sin\alpha - A\cos\alpha = -2\sqrt{2}$ and $A\sin\alpha + A\cos\alpha = 4\sqrt{2}$ **2**

(ii) Show that $A = 2\sqrt{5}$ and $\alpha = \tan^{-1}\dfrac{1}{3}$ **2**

(ii) When does the particle first pass through O. **2**

End of paper

Year 12 Mathematics Extension 1 Section I – Answer Sheet

Student Name/Number ______________________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ⊗ C ○ D ○

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180^\circ$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

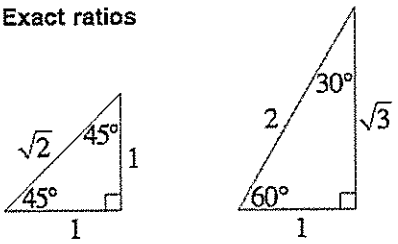
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

***n*th term of an arithmetic series**

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a+l)$$

***n*th term of a geometric series**

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r-1} \quad \text{or} \quad S_n = \frac{a(1-r^n)}{1-r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Mathematics (continued)

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\dfrac{dy}{dx} = F'(u)\dfrac{du}{dx}$

If $y = e^{f(x)}$, then $\dfrac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\dfrac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\dfrac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\dfrac{b}{a}$ $\qquad$ $\alpha\beta = \dfrac{c}{a}$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$\text{Area} = \dfrac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$$

$$\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$$

$$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$$

***t* formulae**

If $t=\tan\frac{\theta}{2}$, then

$$\sin\theta=\frac{2t}{1+t^2}$$

$$\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\tan\theta=\frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta=a,\quad \theta=n\pi+(-1)^n\sin^{-1}a$

$\cos\theta=a,\quad \theta=2n\pi\pm\cos^{-1}a$

$\tan\theta=a,\quad \theta=n\pi+\tan^{-1}a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2=4ay$,

$$x=2at,\quad y=at^2$$

At $(2at,\ at^2)$,

tangent: $y=tx-at^2$

normal: $x+ty=at^3+2at$

At $(x_1,\ y_1)$,

tangent: $xx_1=2a(y+y_1)$

normal: $y-y_1=-\frac{2a}{x_1}(x-x_1)$

Chord of contact from $(x_0,\ y_0)$: $xx_0=2a(y+y_0)$

Acceleration

$$\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$$x=b+a\cos(nt+\alpha)$$

$$\ddot{x}=-n^2(x-b)$$

Further integrals

$$\int\frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\frac{x}{a}+C$$

$$\int\frac{1}{a^2+x^2}\,dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

Sum and product of roots of a cubic equation

$$\alpha+\beta+\gamma=-\frac{b}{a}$$

$$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}$$

$$\alpha\beta\gamma=-\frac{d}{a}$$

Estimation of roots of a polynomial equation

Newton's method

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k}=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

ACE Examination 2017
HSC Mathematics Extension 1 Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	Reflex $\angle AOC = 2\pi - \frac{4\pi}{5} = \frac{6\pi}{5}$ $\angle ABC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times \frac{6\pi}{5} = \frac{3\pi}{5}$ Angle at the centre is twice the angle at the circumference standing on the same arc.	1 Mark: C
2	$\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4}{\sqrt{9-x^2}}dx = 4 \times \int_{\frac{3}{\sqrt{2}}}^{3} \frac{1}{\sqrt{9-x^2}}dx = 4\left[\sin^{-1}\frac{x}{3}\right]_{\frac{3}{\sqrt{2}}}^{3}$ $= 4\left[\left(\sin^{-1}\frac{3}{3}\right) - \left(\sin^{-1}\frac{1}{\sqrt{2}}\right)\right]$ $= 4\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \pi$	1 Mark: D
3	$P(2, 1)$ and $Q(2, 8)$. Internally 3:4. $x = \frac{mx_2 + nx_1}{m+n} = \frac{3\times 2 + 4\times 2}{3+4} = 2$ $y = \frac{my_2 + ny_1}{m+n} = \frac{3\times 8 + 4\times 1}{3+4} = 4$ The coordinates are (2, 4).	1 Mark: B
4	$\int_0^{\frac{\pi}{3}} \sin^2 x dx = \int_0^{\frac{\pi}{3}} \frac{1}{2}(1-\cos 2x)dx$ $= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{3}}$ $= \frac{1}{2}\left[\left(\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right) - \left(0 - \frac{1}{2}\sin 0\right)\right]$ $= \frac{1}{2}\left[\frac{\pi}{3} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right] = \frac{\pi}{6} - \frac{\sqrt{3}}{8}$	1 Mark: D
5	Unordered selection ${}^{15}C_{11} = 1365$	1 Mark: C

6	$u = 1 + 3x$ or $x = \frac{1}{3}(u-1)$ $\frac{du}{dx} = 3$ or $dx = \frac{1}{3}du$ $\int x\sqrt{1+3x}\,dx = \int \frac{1}{3}(u-1)\sqrt{u}\,\frac{1}{3}du = \frac{1}{9}\int (u-1)\sqrt{u}\,du$	1 Mark: A
7	$\frac{d^2x}{dt^2} = 25 - 5x = -5(x-5)$ Centre of motion at $x = 5$ (SHM $\frac{d^2x}{dt^2} = -n^2(x-b)$ with centre of motion at $x = b$)	1 Mark: B
8	$f(x) = \sin x - \frac{2x}{3}$ $f'(x) = \cos x - \frac{2}{3}$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= 1.5 - \frac{\sin 1.5 - \frac{2}{3}\times 1.5}{\cos 1.5 - \frac{2}{3}} = 1.49579... \approx 1.496$	1 Mark: B
9	No restrictions $= (7-1)! = 6!$ Arrangements $= 2\times(6-1)! = 2!5!$ $P(E) = \frac{5!\cancel{2!}}{\cancel{6!}}$	1 Mark: A
10	$T = 2 + Ae^{-kt}$ satisfies the equation $\frac{dT}{dt} = -k(T-2)$ Initially $t = 0$ and $T = 20$ $T = 2 + Ae^{-kt}$ $20 = 2 + Ae^{-k\times 0}$ $A = 18$ Also $t = 20$ and $T = 10$ $T = 2 + 18e^{-kt}$ $10 = 2 + 18e^{-k\times 20}$ $e^{-k\times 20} = \frac{8}{18}$ $-20k = \log_e \frac{4}{9}$ $k = -\frac{1}{20}\log_e \frac{4}{9} = \frac{1}{20}\log_e \frac{9}{4}$	1 Mark: C

Section II		
11(a)	$x-y-4=0 \qquad 3x-y+4=0$ $y=x-4 \qquad y=3x+4$ $m_1=1 \qquad m_2=3$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|=\left\|\frac{1-3}{1+1\times3}\right\|=\frac{1}{2}$ $\theta=26.56505118...$ $\approx 27^\circ$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the lines or shows some understanding.
11(b)	$\frac{1}{\|x-1\|}<1 \qquad x\neq1$ $\|x-1\|>1$ $x-1>1$ or $x-1<-1$ $x>2 \qquad x<0$	2 Marks: Correct answer. 1 Mark: Finds one correct region or makes significant progress.
11(c) (i)	$T=T_0+Ae^{-kt}$ or $Ae^{-kt}=T-T_0$ $\frac{dT}{dt}=-kAe^{-kt}$ $=-k(T-T_0)$	1 Mark: Correct answer.
11(c) (ii)	Initially $t=0$ and $T=60$, $T_0=12$ $T=T_0+Ae^{-kt}$ $60=12+Ae^{-k\times0}$ or $A=48$ Also $t=25$ and $T=30$ $30=12+48e^{-k\times25}$ $e^{-25k}=\frac{18}{48}=\frac{3}{8}$ $-25k=\log_e\frac{3}{8}$ $k=-\frac{1}{25}\log_e\frac{3}{8}=\frac{1}{25}\log_e\frac{8}{3}$ We need to find t when $T=15$ $15=12+48e^{-kt}$ $e^{-kt}=\frac{3}{48}=\frac{1}{16}$ $-kt=\log_e\frac{1}{16}$ $t=\frac{1}{k}\log_e16=25\frac{\log_e16}{\log_e\frac{8}{3}}=70.66950..\approx 71$ minutes	3 Marks: Correct answer. 2 Marks: Finds the value of A and an expression for k. 1 Mark: Finds the value of A.

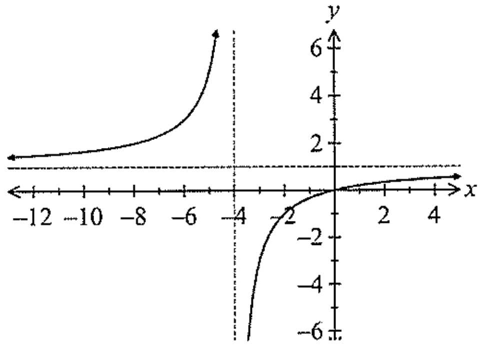
11(d) (i)	Horizontal $\ddot{x}=0$ $\dot{x}=50\cos 0^\circ = 50$ $x=50t+c$ When $t=0$, $x=0$ implies $c=0$ $x=50t$ Vertical $\ddot{y}=-g$ $\dot{y}=-gt+50\sin 0^\circ=-gt$ $y=-\frac{1}{2}gt^2+c$ When $t=0$, $y=90$ implies $c=90$ $y=90-\frac{1}{2}gt^2$	2 Marks: Correct answer. 1 Mark: Finds horizontal or vertical parametric equations or shows some understanding of the problem.
11(d) (ii)	Ball strikes the ground $y=0$ $90-\frac{1}{2}gt^2=0$ $\frac{1}{2}gt^2=90$ $t^2=\frac{180}{g}$ $t=\sqrt{\frac{180}{g}}=6\sqrt{\frac{5}{g}}$ as $t>0$	1 Mark: Correct answer.
11(d) (iii)	Ball strikes the ground when $t=6\sqrt{\frac{5}{g}}$ seconds. Now $x=50t$ $d=50\times 6\sqrt{\frac{5}{g}}=300\sqrt{\frac{5}{g}}$ metres	1 Mark: Correct answer.
11(e)	$T_{k+1}={}^{11}C_k(3x^8)^{11-k}\left(-\frac{2}{x^3}\right)^k$ $={}^{11}C_k\times 3^{11-k}\times x^{88-8k}\times(-2)^k\times x^{-3k}$ $={}^{11}C_k(-2)^k\times 3^{11-k}\times x^{88-11k}$ The term independent of x: $88-11k=0$ $k=8$ Required term is ${}^{11}C_8(-2)^8\times 3^{11-8}=1{,}140{,}480$ Refer to last page	3 Marks: Correct answer. 2 Marks: Finds the value of k or makes significant progress. 1 Mark: Uses the expression for the general term of a binomial expansion.
12(a)	$V=\pi\int_a^b y^2\,dx=\pi\int_0^\pi 9\cos^2\frac{x}{2}dx$ $=\frac{9\pi}{2}\int_0^\pi(1+\cos x)\,dx$ $=\frac{9\pi}{2}[x+\sin x]_0^\pi=\frac{9\pi^2}{2}$ cubic units	3 Marks: Correct answer. 2 Marks: Applies the double angle trig identity. 1 Mark: Sets up the integral for volume

12(b) (i)	To find the gradient of the tangent $y=\frac{1}{4a}x^2$ and $\frac{dy}{dx}=\frac{1}{2a}x$ At $P(2at,at^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2at=t$ Line d has a gradient of t and passes through $S(0,a)$ $y-y_1=m(x-x_1)$ $y-a=t(x-0)$ $tx-y+a=0$	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Finds or states the gradient of the tangent at P.
12(b) (ii)	To find the coordinates of R Substitute $y=0$ into $tx-y+a=0$ then $x=-\frac{a}{t}$ $R(-\frac{a}{t},0)$ To find the coordinates of M $x=\frac{x_1+x_2}{2}$ $=\frac{-\frac{a}{t}+0}{2}$ $=-\frac{a}{2t}$ $y=\frac{y_1+y_2}{2}$ $=\frac{0+a}{2}$ $=\frac{a}{2}$ $M(-\frac{a}{2t},\frac{a}{2})$	2 Marks: Correct answer. 1 Mark: Finds the coordinates of R.
12(b) (iii)	To find the equation of the locus eliminate t. However y is independent of t. $y=\frac{a}{2}$	1 Mark: Correct answer.
12(c)	$\int\frac{1}{x^2+2x+2}dx=\int\frac{1}{(x+1)^2+1}dx$ $=\tan^{-1}(x+1)+c$	2 Marks: Correct answer. 1 Mark: Completes the square.
12(d) (i)	Initially $x=5$ and $v=-6$ Acceleration $a=x+1.5=5+1.5=6.5$ Therefore $a>0$ and $v<0$ (different signs) The particle is slowing down.	1 Mark: Correct answer.
12(d) (ii)	$\frac{d}{dx}(\frac{1}{2}v^2)=x+1.5$ $\frac{1}{2}v^2=\frac{1}{2}x^2+1.5x+c_1$ $v^2=x^2+3x+c_2$ When $x=5$, $v=-6$ then $(-6)^2=5^2+3\times 5+c_2$ or $c_2=-4$ Therefore $v^2=x^2+3x-4$	2 Marks: Correct answer. 1 Mark: Determines $\frac{1}{2}v^2=\frac{1}{2}x^2+1.5x+c_1$ or makes similar progress.

12(d) (iii)	Particle changes direction when $v = 0$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ Particle starts at $x = 5$ and is moving to the left ($v = -6$). At $x = 1$ the particle is at rest $v = 0$ and $a = 2.5 > 0$ It then changes direction and moves to the right ($v > 0$) $\therefore\ x = 1$ metres	1 Mark: Correct answer.
13(a) (i)	$\text{LHS} = \dfrac{\sec^2 x}{\tan x}$ $= \dfrac{1}{\cos^2 x} \div \tan x$ $= \dfrac{1}{\cos^2 x} \times \dfrac{\cos x}{\sin x}$ $= \dfrac{1}{\cos x} \times \dfrac{1}{\sin x}$ $= \operatorname{cosec} x \sec x$ $= \text{RHS}$	1 Mark: Correct answer.
13(a) (ii)	$u = \tan x \qquad u = \tan\dfrac{\pi}{3} = \sqrt{3} \qquad u = \tan\dfrac{\pi}{4} = 1$ $du = \sec^2 x\,dx$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{cosec} x \sec x\ dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \dfrac{\sec^2 x}{\tan x} dx$ $= \int_1^{\sqrt{3}} \dfrac{1}{u} du$ $= \left[\log_e u\right]_1^{\sqrt{3}}$ $= \log_e \sqrt{3} - \log_e 1$ $= \log_e \sqrt{3}$	2 Marks: Correct answer. 1 Mark: Recognises the use of part (i) or makes progress in the substitution.
13(b)	Step 1: To prove the statement true for $n = 1$ $5^1 + 12 \times 1 - 1 = 16$ (Divisible by 16) Result is true for $n = 1$ Step 2: Assume the result true for $n = k$ $5^k + 12k - 1 = 16P$ where P is an integer (1)	3 Marks: Correct answer. 2 Marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$.

	Step 3: To prove the result is true for $n = k+1$ $5^{k+1} + 12(k+1) - 1 = 16Q$ where Q is an integer. $\text{LHS} = 5^{k+1} + 12(k+1) - 1$ $= 5^{k+1} + 12k + 11$ $= 5(5^k + 12k - 1) - 48k + 16$ $= 5(5^k + 12k - 1) + 16(1 - 3k)$ $= 5(16P) + 16(1 - 3k)$ from (1) $= 16(5P + 1 - 3k)$ $= 16Q$ $= \text{RHS}$ Q is an integer as P and k are integers. Result is true for $n = k+1$ if true for $n = k$ Step 4: Result true by principle of mathematical induction.	1 Mark: Proves the result true for $n = 1$.
13(c) (i)	$\angle FAE = \angle ECF = 90^\circ$ (given) $\therefore AECF$ is a cyclic quadrilateral (if two opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic)	1 Mark: Correct answer.
13(c) (ii)	$\angle BDC = \angle BAC$ (Angles in the same segment of a circle are equal) $\angle EAC = \angle EFC$ (Angles in the same segment of a circle are equal) $\angle BAC = \angle EAC$ (same angle) $\therefore \angle BDC = \angle EFC$ (corresponding angles are equal if and only if $EF \parallel BD$) Therefore EF is parallel to BD	3 Marks: Correct answer. 2 Marks: Makes some progress towards the solution. 1 Mark: States one relevant statement and circle theorem.
13(d) (i)	Given the remainder of -8 when $P(x)$ is divided by $(x-b)$ $P(b) = (b-a)^3 + (b-b)^2$ $= (b-a)^3$ $(b-a)^3 = -8$ $b - a = -2$ $a = b + 2$ To find the remainder when $P(x)$ is divided by $(x-a)$ $P(a) = (a-a)^3 + (a-b)^2$ $= (a-b)^2$ $= (b+2-b)^2$ $= 4$ Therefore the remainder is 4.	2 Marks: Correct answer. 1 Mark: Applies the remainder theorem.

13(d) (ii)	If $x=\frac{a+b}{2}$ is a zero of $P(x)$ then the remainder is 0. $P(\frac{a+b}{2})=(\frac{a+b}{2}-a)^3+(\frac{a+b}{2}-b)^2$ $=(\frac{a+b-2a}{2})^3+(\frac{a+b-2b}{2})^2$ $=(\frac{b-a}{2})^3+(\frac{a-b}{2})^2$ $=-1^3(\frac{a-b}{2})^3+(\frac{a-b}{2})^2$ $=-(\frac{a-b}{2})^2\left(\frac{a-b}{2}-1\right)$ $=-(\frac{b+2-b}{2})^2\left(\frac{b+2-b}{2}-1\right)$ $=-(1)(0)=0$	1 Mark: Correct answer.
13(d) (iii)	$P(x)=(x-a)^3+(x-b)^2$ $P'(x)=3(x-a)^2+2(x-b)$ Stationary points occur when $P'(x)=0$ $3(x-a)^2+2(x-b)=0$ $3x^2-6ax+3a^2+2x-2b=0$ $3x^2+(2-6a)x+(3a^2-2b)=0$ $\Delta=b^2-4ac$ $=(2-6a)^2-4\times3\times(3a^2-2b)$ $=4-24a+36a^2-36a^2+24b$ $=4-24a+24b$ $=4-24\times(b+2)+24b$ $=4-24b-48+24b=-44<0$ $P(x)$ has no stationary points.	2 Marks: Correct answer. 1 Mark: Finds the derivative and uses the discriminate.
14(a) (i)	Let p be the probability of hitting the target ($p=0.87$) Let q be the probability of not hitting the target ($q=0.13$) $P(k \text{ successes})={}^nC_k(0.87)^k(0.13)^{n-k}$ $P(40 \text{ targets})={}^{50}C_{40}(0.87)^{40}(0.13)^{10}$ ≈ 0.0539 refer to last page	1 Mark: Correct answer.
14(a) (ii)	Misses at most 2 targets then $k=48$, 49 and 50 $P(\text{At most 2 misses})$ $={}^{50}C_{48}0.87^{48}0.13^2+{}^{50}C_{49}0.87^{49}0.13^1+{}^{50}C_{50}0.87^{50}$ $\approx 0.0.339$	2 Marks: Correct answer. 1 Mark: Makes some progress.

14(b) (i)	$f(x)=\dfrac{x}{x+4}$ is defined for all $x\neq -4$ $f'(x)=\dfrac{(x+4)\times 1-x\times 1}{(x+4)^2}=\dfrac{4}{(x+4)^2}>0$ for all $x\neq -4$	1 Mark: Correct answer.
14(b) (ii)	$f(x)=\dfrac{x+4-4}{x+4}$ $=1-\dfrac{4}{x+4}$ As $x\to\pm\infty$ $\dfrac{4}{x+4}\to 0$ Horizontal asymptote is $y=1$	1 Mark: Correct answer.
14(b) (iii)		2 Marks: Correct answer. 1 Mark: Shows asymptotes or basic shape of the curve.
14(b) (iv)	The graph of $y=f(x)$ indicates a one-to-one increasing function (it satisfies the horizontal line test)	1 Mark: Correct answer.
14(b) (v)	The inverse function is $x=\dfrac{y}{y+4}$ $xy+4x=y$ $(1-x)y=4x$ $y=\dfrac{4x}{1-x}$ $f^{-1}(x)=\dfrac{4x}{1-x}$	1 Mark: Correct answer.
14(c) (i)	When $t=1$ then $x=2$ $2=A\cos\left(\dfrac{\pi}{4}\times 1+\alpha\right)$ $=A\left(\cos\dfrac{\pi}{4}\cos\alpha-\sin\dfrac{\pi}{4}\sin\alpha\right)$ $=A\left(\dfrac{1}{\sqrt{2}}\cos\alpha-\dfrac{1}{\sqrt{2}}\sin\alpha\right)$ $2\sqrt{2}=A\cos\alpha-A\sin\alpha$ $A\sin\alpha-A\cos\alpha=-2\sqrt{2}$	2 Marks: Correct answer. 1 Mark: Finds one of the equations or uses the compound angle formula with the given information.

	When $t = 3$ then $x = -4$ $-4 = A\cos\left(\frac{\pi}{4}\times 3+\alpha\right)$ $= A\left(\cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha\right)$ $= A\left(-\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{\sqrt{2}}\sin\alpha\right)$ $-4\sqrt{2} = -A\cos\alpha - A\sin\alpha$ $A\sin\alpha + A\cos\alpha = 4\sqrt{2}$	
14(c) (ii)	$A\sin\alpha - A\cos\alpha = -2\sqrt{2}$ (1) $A\sin\alpha + A\cos\alpha = 4\sqrt{2}$ (2) Adding equations (1) and (2) then $2A\sin\alpha = 2\sqrt{2}$ Subtracting equation (1) from (2) then $2A\cos\alpha = 6\sqrt{2}$ $(2A\sin\alpha)^2 + (2A\cos\alpha)^2 = (2\sqrt{2})^2 + (6\sqrt{2})^2$ $4A^2(\sin^2\alpha + \cos^2\alpha) = 8 + 72$ $A^2 = 20$ or $A = 2\sqrt{5}$ $\frac{2A\sin\alpha}{2A\cos\alpha} = \frac{2\sqrt{2}}{6\sqrt{2}}$ $\tan\alpha = \frac{1}{3}$ or $\alpha = \tan^{-1}\frac{1}{3}$	2 Marks: Correct answer. 1 Mark: Finds A or α. Alternatively shows some understanding of the problem.
14(c) (iii)	Particle passes through O when $x = 0$ $A\cos\left(\frac{\pi}{4}t+\alpha\right) = 0$ $\frac{\pi}{4}t + \alpha = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ First passes through O $\frac{\pi}{4}t + \alpha = \frac{\pi}{2}$ $\frac{\pi}{4}t + \tan^{-1}\frac{1}{3} = \frac{\pi}{2}$ $\frac{\pi}{4}t = \frac{\pi}{2} - \tan^{-1}\frac{1}{3}$ $\frac{\pi}{4}t = \tan^{-1}3$ $t = \frac{4}{\pi}\tan^{-1}3$ Accept $t = 2 - \frac{4}{\pi}\tan^{-1}\frac{1}{3}$ or 1.59 seconds	2 Marks: Correct answer. 1 Mark: Finds $\frac{\pi}{4}t + \alpha = \frac{\pi}{2}$ or shows some understanding.

11(e) $2\sin x = -1$

$\sin x = -\frac{1}{2}$ (1)

$x = \sin^{-1}(-\frac{1}{2})$ (1)

$= -\frac{\pi}{6}$

GS

$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$ (1)

14a $y = \ln(\tan^{-1} 3x)$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$= \frac{3}{1+(3x)^2} \times \frac{1}{\tan^{-1}(3x)}$$

$$= \frac{3}{(1+9x^2)\tan^{-1}(3x)} \quad 1$$

When $x = \frac{1}{\sqrt{3}}$

$$\frac{dy}{dx} = \frac{3}{\left(1+9\left(\frac{1}{3}\right)\right)\tan^{-1}\left(\frac{3}{\sqrt{3}}\right)} \quad 1$$

$$= \frac{3}{4 \times \frac{\pi}{3}}$$

$$\frac{dy}{dx} = \frac{9}{4\pi} \quad 1$$