Carlingford High School
Mathematics Extension 2
Year 12

HSC ASSESSMENT TASK 2
HALF YEARLY
Term 1 2013

Student Name:______________________________ Teacher: Mr GonG

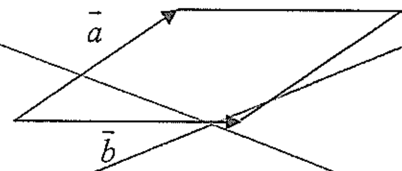
- **Time allowed 120 minutes.**
- Start each question on a new page.
- Write on **ONE SIDE** of the paper only.
- Do not work in columns.
- Marks may be deducted for careless or badly arranged work.
- Only calculators approved by the Board of Studies may be used.
- All answers are to be completed in blue or black pen except graphs and diagrams.
- There is to be NO LENDING OR BORROWING.

	MC	Q6	Q7	Q8	Q9	Total	
E3		/15				/15	
E4	/2		/15	/15		/32	
E6	/2				/15	/17	
Total	/4	**/15**	**/15**	**/15**	**/15**	/64	%

Section 1

Multiple Choice – **Start a new page (5 marks)**

1. The diagram below shows a rhombus, spanned by two vectors $\vec{a}$ and $\vec{b}$.



It follows that

A. $\vec{a} \times \vec{b} = 0$ **B.** $(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = 0$ **C.** $\vec{a} = \vec{b}$ **D.** $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

2. If I start with the graph of $y = h(x)$, shift it right 1 unit, then up 2 units and then reflect it in the y-axis, what is the resulting equation?

A. $y = -h(x+1) + 2$ **B.** $y = -h(x+1) - 2$

C. $y = h(-x-1) + 2$ **D.** $y = h(-x-1) - 2$

3. The hyperbola $\dfrac{(y+3)^2}{9} - \dfrac{(x-6)^2}{4} = 1$ has asymptotes given by

A. $y = \pm\dfrac{3}{2}x$ **B.** $y = \pm\dfrac{3}{2}x - 7$

C. $y = \dfrac{2}{3}x - 7$ & $y = -\dfrac{2}{3}x + 1$ **D.** $y = \dfrac{3}{2}x - 12$ & $y = -\dfrac{3}{2}x + 6$

4. The number of straight line asymptotes of the graph of $y = \dfrac{2x^3 + x^2 - 1}{x^2 - x - 2}$ is

A. 3 **B.** 0 **C.** 1 **D.** 2

5. The polynomial P(z) has real coefficients. Four of the roots of the equation P(z) = 0 are $z = 0$, $z = 1 - 2i$, $z = 1 + 2i$ and $z = 3i$. The minimum number of roots that the equation P(z) = 0 could have is

A. 4 **B.** 5 **C.** 6 **D.** 7

Section 2

Question 6 **– Start a new page –** **(15 marks)** **Marks**

a) Let $z = \dfrac{-i}{1+i\sqrt{3}}$

i) Sketch z on an Argand diagram. **2**

ii) Find the modulus and argument of z. **2**

b) i) Solve the equation $z^4 = 1$. **1**

ii) Hence find all solutions of the equation $z^4 = (z-1)^4$. **3**

c) Use De Moivre's theorem and binomial expansion to express $\cos 4\theta$ in terms of θ. **3**

d) Draw a single Argand diagram to represent the following region **4**

$$1 \le |z+3-2i| \le 3 \text{ and } \frac{\pi}{6} \le \arg(z+3) \le \frac{\pi}{3}.$$

Question 7 **- Start a new page –** **(15 marks)** **Marks**

a) The equation $x^4 + 4x^3 - 3x^2 - 4x - 2 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

Find the equation with roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\gamma}, \dfrac{1}{\delta}$. **3**

b) Solve the polynomial equation $x^5 + 2x^4 - 2x^3 - 8x^2 - 7x - 2 = 0$, given that it has a root of multiplicity 4. **4**

c) Express $\dfrac{3x+1}{(x+1)(x^2+1)}$ in the form $\dfrac{a}{(x+1)} + \dfrac{bx+c}{(x^2+1)}$. **4**

d) Use the substitution $x = 2\sin\theta$ to find $\int_0^2 \sqrt{4-x^2}\,dx$. **4**

Question 8 - Start a new page – (15 marks) **Marks**

a) i) Show that the tangent to the ellipse $\frac{x^2}{12}+\frac{y^2}{4}=1$ at the point P(3, 1) has equation $x + y = 4$. **3**

ii) If this tangent cuts the directrix in the fourth quadrant at the point T, and S is the corresponding focus, show that SP and ST are at right angles to each other. **4**

b) $P\left(cp,\ \frac{c}{p}\right)$ and $Q\left(cq,\ \frac{c}{q}\right)$ are two points on the rectangular hyperbola $xy = c^2$ where $c > 0$.

i) Show that the equation of PQ is given by $x + pqy = c(p + q)$. **1**

ii) Hence write down the equations of the tangents at P and Q. **2**

iii) The tangents at P and Q meet at T.
Show that the coordinates of T is $\left(\frac{2cpq}{p+q},\ \frac{2c}{p+q}\right)$. **2**

iv) If PQ passes through the point (1, 0), find the locus of T and describe this locus geometrically. **3**

Question 9 – Start a new page – (15 marks) **Marks**

a) i) Draw a sketch of $y = x^2(2-x)$. **2**

ii) Hence, or otherwise, sketch the curve whose equation is given by $y^2 = x^2(2-x)$ **2**

b) i) Prove that $\dfrac{(x-1)(x-3)}{x+2} = x - 6 + \dfrac{15}{x+2}$. **2**

ii) Sketch $y = \dfrac{(x-1)(x-3)}{x+2}$, clearly labelling both asymptotes and all intercepts. **2**

iii) Hence sketch the graphs of

α) $y = \left|\dfrac{(x-1)(x-3)}{x+2}\right|$ **2**

β) $y = \dfrac{(|x|-1)(|x|-3)}{|x|+2}$ **2**

c) i) Sketch $y = \dfrac{x^3 - 3x}{3x^2 - 1}$ clearly labelling all essential features given that it has three linear asymptotes, one of which is $y = \dfrac{x}{3}$. **2**

ii) How many solutions are there to the equation $\dfrac{x^3 - 3x}{3x^2 - 1} = k$ where k is a constant? (You do not need to actually find the solutions) **1**

END OF EXAM

2013

Term 1 HSC Task 2 (HY) Examination

Ext 2 Mathematics

Solutions

HSC Task 2 (HY) Examination – Ext 2 Mathematics 2013

Section I   Multiple Choice Answer 1 Mark each

1.	A ○	B ●	C ○	D ○	**[E3]**
2.	A ○	B ○	C ●	D ○	**[E6]**
3.	A ○	B ○	C ○	D ●	**[E4]**
4.	A ●	B ○	C ○	D ○	**[E6]**
5.	A ○	B ●	C ○	D ○	**[E4]**

Working Out

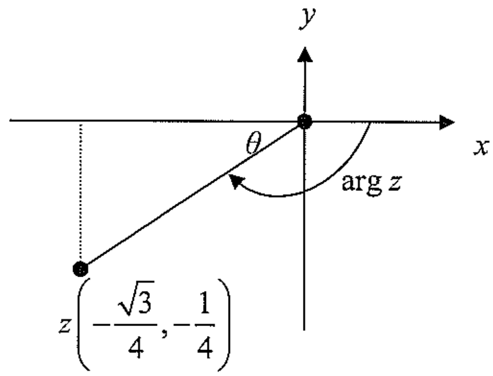
1	Definition: The dot product (also called the inner product or scalar product) of 2 vectors is defined as A.B =|A|.|B|.cosθ where |A| & |B| represents the magnitudes of vectors A &B & θ is the angle betwwen vectors A & B. Thus the dot product of the diagonals of rhombus =0. i.e. $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=0 \Rightarrow$ B	**3**	$\frac{(y+3)^2}{9}-\frac{(x-6)^2}{4}=1$ $4(y+3)^2-9(x-6)^2=36$ $(y+3)^2=\frac{9(x-6)^2+36}{4}$ $y+3=\pm\sqrt{\frac{9(x-6)^2+36}{4}}$ $y=-3\pm\sqrt{\frac{9(x-6)^2+36}{4}}$ $y=-3\pm\sqrt{\frac{9\left[(x-6)^2+4\right]}{4}}$	$y=-3\pm\frac{3}{2}\sqrt{(x-6)^2+4}$ $y=-3\pm\frac{3}{2}\sqrt{(x-6)^2\left(1+\frac{4}{(x-6)^2}\right)}$ $y=-3\pm\frac{3}{2}(x-6)\sqrt{1+\frac{4}{(x-6)^2}}$ As $x\to\infty$ then $\frac{4}{(x-6)^2}\to 0$ $\therefore\ y=-3\pm\frac{3}{2}(x-6)$ i.e. $y=\frac{3}{2}x-12$ or $y=-\frac{3}{2}x+6\ \Rightarrow$ D
2	$\because\ y=h(x)$ then shift 1 unit to right get $y=h(x-1)$ then 2 unit up gives $y=h(x-1)+2$, now reflect in the $y-$axis gives the resulting equation $y=h(-x-1)+2\Rightarrow$ C	**4**	$\because y=\frac{2x^3+x^2-1}{x^2-x-2}$ then by division get $y=2x+3+\frac{7x+5}{(x-2)(x+1)}$ Thus $y=2x+3$ is an oblique asymptote, $x=2$ and $x=-1$ are vertical asymptotes. i.e. there are 3 straight line asymptotes, $\Rightarrow$ A	
		5	The polynomial have minimum of 5 roots namely z=0, z=1-2i, z=1+2i these are conjugate pairs, z=3i and z= -3i are possible conjugate pairs. =>B	

Section II Solutions

Question 6 [E3]

a) i)

$$z=\frac{-i}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$
$$=\frac{-i+i^2\sqrt{3}}{1-i^2\times 3}$$
$$=\frac{-i-\sqrt{3}}{4}$$
$$=-\frac{\sqrt{3}}{4}-\frac{1}{4}i$$



1 mark for rationalise z correctly.

1 mark for correct diagram

ii)

$$|z|=\sqrt{\frac{3}{16}+\frac{1}{16}}$$
$$=\frac{1}{2}$$

$$\tan\theta=\frac{1/4}{\sqrt{3}/4}=\frac{1}{\sqrt{3}}\quad\therefore\quad\theta=30^\circ\text{ or }\frac{\pi}{6}$$

Thus $\arg z=-180^\circ+30^\circ$ or $-\pi+\dfrac{\pi}{6}$

$$=-150^\circ\text{ or }-\frac{5\pi}{6}$$

1 mark for correct $|z|$.

1 mark for correct arg z

b) i)

$$z^4-1=0\Rightarrow\left(z^2+1\right)\left(z^2+1\right)=0$$
$$(z+i)(z-i)(z+1)(z-1)=0$$
$$\therefore\ z=\pm1,\ \pm i$$

1 mark for correct working & answer.

ii)

$$z^4=(z-1)^4$$
$$\left(\frac{z}{z-1}\right)^4=1$$
$$\frac{z}{z-1}=\sqrt[4]{1}$$

i.e. $\dfrac{z}{z-1}=\pm1,\ \pm i$

Now the 4 solutions are:

[Case 1]: $\dfrac{z}{z-1}=1$
$$z=z-1$$
$$0=-1$$
$\therefore$ No solution.

[Case 2]: $\dfrac{z}{z-1}=-1$
$$z=-z+1$$
$$2z=1$$
$$\therefore\ z=\frac{1}{}$$

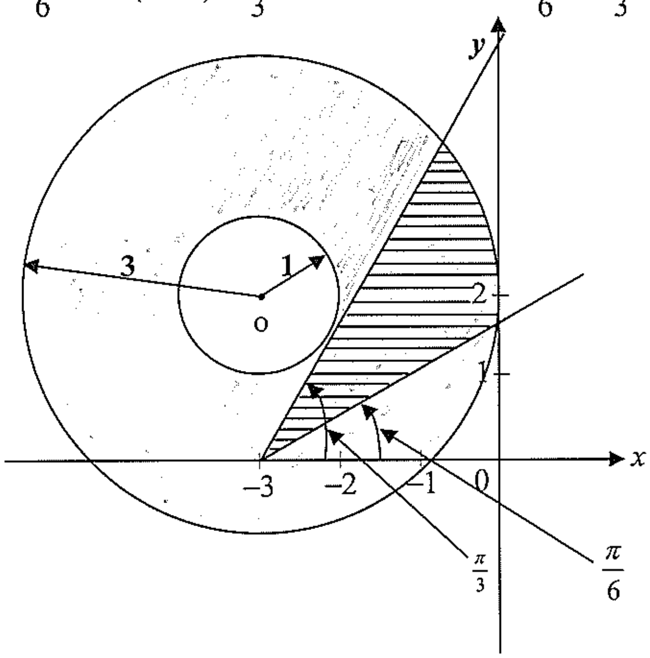
[Case 3]: $\dfrac{z}{z-1}=i$
$$z=(z-1)i$$
$$z=zi-i$$
$$z-zi=-i$$
$$\therefore\ z=\frac{-i}{1-i}$$

[Case 4]: $\dfrac{z}{z-1}=-i$
$$z=(z-1)\times -i$$
$$z=-zi+i$$
$$z+zi=i$$
$$\therefore\ z=\frac{i}{1+i}$$

1 mark for working up to & include case 2.

1 mark for case 3.

1 mark for case 4.

c)	$\cos 4\theta + i\sin 4\theta = (\cos\theta + i\sin\theta)^4$ by De Moivre's Theorem Now let $c = \cos\theta$ & $s = \sin\theta$ then $(c+is)^4 = c^4 + 4c^3is + 6c^2i^2s^2 + 4ci^3s^3 + i^4s^4$ $= c^4 + 4c^3is - 6c^2s^2 - 4cs^3i + s^4$ $= c^4 - 6c^2s^2 + s^4 + (4c^3s - 4cs^3)i$ By equating real parts we get $\cos 4\theta = c^4 - 6c^2s^2 + s^4$ $= c^4 - 6c^2(1-c^2) + (1-c^2)^2$ $= c^4 - 6c^2 + 6c^4 + 1 - 2c^2 + c^4$ $= 8c^4 - 8c^2 + 1$ $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	**1 mark for correct working to line 4.** **1 mark for correct working for $\cos\theta$ =.** **1 mark for correct answer, last line.**
d)	For $1 \le \lvert z+3-2i \rvert \le 3$, solutions between the circles centre at $(-3,\ 2i)$. For $\frac{\pi}{6} \le \arg(z+3) \le \frac{\pi}{3}$, solutions between $\frac{\pi}{6}$ & $\frac{\pi}{3}$ centre at $(-3,\ 0)$.  	**1 mark for correct drawing & shading the region between the circles** **1 mark for the two correct centres.** **1 mark for drawing the correct angles of the 2 lines.** **1 mark for correctly shading the final region between the circles & the lines.**

Question 7 [E4]		
a)	$\because$ the given eqt is $x^4+4x^3-3x^2-4x-2=0$ has roots $\alpha, \beta, \gamma, \delta$, then the new eqt with roots $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta}$ is $\left(\frac{1}{x}\right)^4+4\left(\frac{1}{x}\right)^3-3\left(\frac{1}{x}\right)^2-4\left(\frac{1}{x}\right)-2=0$ $1+4x-3x^2-4x^3-2x^4=0$ i.e. $2x^4+4x^3+3x^2-4x-1=0$	**1 mark for correct substitution.** **1 mark for multiply throughout by x^4 correctly.** **1 mark for correct final arrangement.**
b)	Let $\mathrm{P}(x)=x^5+2x^4-2x^3-8x^2-7x-2$ then $\mathrm{P}'(x)=5x^4+8x^3-6x^2-16x-7$ $\mathrm{P}''(x)=20x^3+24x^2-12x-16$ and $\mathrm{P}'''(x)=60x^2+48x-12$ If $\mathrm{P}(x)$ has a root of multiplicity 4, then $\mathrm{P}'''(x)$ has single root i.e. $60x^2+48x-12=0$ or $12(5x^2+4x-1)=0$ $(5x-1)(x+1)=0$ $\therefore\ x=\frac{1}{5}$ or -1 Now $\mathrm{P}\left(\frac{1}{5}\right)\neq 0$ & $\mathrm{P}(-1)=0$, thus $x=-1$ is a root of multiplicity 4. Hence $\mathrm{P}(x)=(x+1)^4(x+k)$, where $k=-2$ by inspection $=(x+1)^4(x-2)$ $\therefore\ x=-1$ or 2	**1 mark for correct differentiations.** **1 mark for solve correctly.** **1 mark for correct testing.** **1 mark for correct final answer.**
c)	$\because\ \frac{3x+1}{(x+1)(x^2+1)}\equiv\frac{a}{x+1}+\frac{bx+c}{x^2+1}$ then $3x+1=a(x^2+1)+(bx+c)(x+1)$ when $x=-1$ then $-2=2a \Rightarrow a=-1$ when $x=0$ then $1=a+c$ i.e. $1=-1+c \Rightarrow c=2$ Now equate coefficients of x^2 get $ax^2+bx^2=0$ i.e. $a+b=0$ $-1+b=0 \Rightarrow b=1$ Hence $\frac{3x+1}{(x+1)(x^2+1)}=\frac{-1}{x+1}+\frac{x+2}{x^2+1}$	**1 mark for express in the correct form.** **1 mark for the correct value *a*.** **1 mark for the correct value *c*.** **1 mark for the correct value *b*.**

d)	$\because\ x = 2\sin\theta$ then $\dfrac{dx}{d\theta} = 2\cos\theta$ $\therefore\ dx = 2\cos\theta\ d\theta$ also when $x = 0$ then $\theta = 0$ and when $x = 2$ then $\theta = \dfrac{\pi}{2}$	**1 mark for correct manipulation.**
	Now $\int_0^2 \sqrt{4 - x^2}\ dx = \int_0^{\frac{\pi}{2}} \sqrt{4 - 4\sin^2\theta}\ \times 2\cos\theta\ d\theta$ $= \int_0^{\frac{\pi}{2}} \sqrt{4(1 - \sin^2\theta)}\ \times 2\cos\theta\ d\theta$ $= \int_0^{\frac{\pi}{2}} \sqrt{4\cos^2\theta}\ \times 2\cos\theta\ d\theta$	**1 mark for correctly shown this result.**
	$= \int_0^{\frac{\pi}{2}} 2\cos\theta\ \times 2\cos\theta\ d\theta$ $= \int_0^{\frac{\pi}{2}} 4\cos^2\theta\ d\theta$ $= 2\int_0^{\frac{\pi}{2}} 2\cos^2\theta\ d\theta$ $\because\ \cos 2\theta = 2\cos^2\theta - 1$ then $2\cos^2\theta = \cos 2\theta + 1$ $= 2\int_0^{\frac{\pi}{2}} \left(\cos 2\theta + 1\right) d\theta$ $= 2\left[\dfrac{1}{2}\sin 2\theta + \theta\right]_0^{\frac{\pi}{2}}$	**1 mark for correct integration.**
	$= \left[\sin 2\theta + 2\theta\right]_0^{\frac{\pi}{2}}$ $= \sin\pi + \pi - \left(\sin 0 + 0\right)$ $= \pi$	**1 mark for correct answer.**

Question 8 [E4]

a) i)

$\because \dfrac{x^2}{12}+\dfrac{y^2}{4}=1$ then by implicit differentiation get $\dfrac{2x}{12}+\dfrac{2y}{4}\dfrac{dy}{dx}=0$

i.e. $\dfrac{x}{6}+\dfrac{y}{2}\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=-\dfrac{x}{3y}$

1 mark for correct derivative.

At the point $(3,\ 1)$ we get $\dfrac{dy}{dx}=m=-\dfrac{3}{3}=-1$

1 mark for correct gradient.

Thus the equation of the tangent at $(3,\ 1)$ & $m=-1$ is

$$\left.\begin{aligned} y-y_1 &= m(x-x_1) \\ \text{i.e. } y-1 &= -1(x-3) \\ y-1 &= -x+3 \\ \therefore\ x+y &= 4 \end{aligned}\right\}$$

1 mark for correct equation.

ii)

$\because$ the ellipse is $\dfrac{x^2}{12}+\dfrac{y^2}{4}=1$ then

$a=\sqrt{12}=2\sqrt{3}$ & $b=2$.

Now $e=\sqrt{1-\frac{4}{12}}=\dfrac{\sqrt{2}}{\sqrt{3}}$;

focus $(ae,\ 0)=\left(2\sqrt{3}\times\dfrac{\sqrt{2}}{\sqrt{3}},\ 0\right)$

$=\left(2\sqrt{2},\ 0\right)$;

1 mark for correct focus.

directrix $x=\dfrac{a}{e}=\dfrac{2\sqrt{3}}{\sqrt{2}/\sqrt{3}}=3\sqrt{2}$.

1 mark for correct directrix.

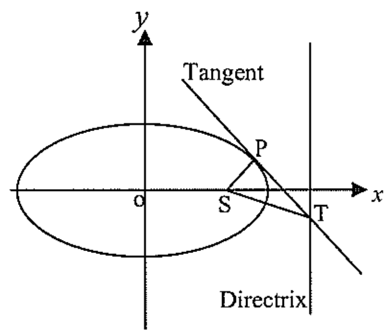
$\because$ T is on the tangent & the directrix then

sub $x=3\sqrt{2}$ in $x+y=4$ get $y=4-3\sqrt{2}$

Thus we have the points

$P(3,\ 1)$, $S\left(2\sqrt{2},\ 0\right)$, $T\left(3\sqrt{2},\ 4-3\sqrt{2}\right)$.

1 mark for correct T.



If SP $\perp$ ST then

Grad SP $\times$ Grad ST $=-1$

i.e. LHS= $\dfrac{1}{3-2\sqrt{2}}\times\dfrac{4-3\sqrt{2}}{3\sqrt{2}-2\sqrt{2}}$

$=\dfrac{4-3\sqrt{2}}{(3-2\sqrt{2})\left(\sqrt{2}\right)}$

$=\dfrac{4-3\sqrt{2}}{3\sqrt{2}-4}$

$=-1$

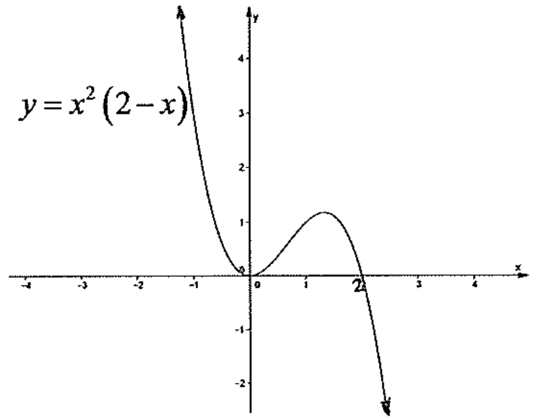
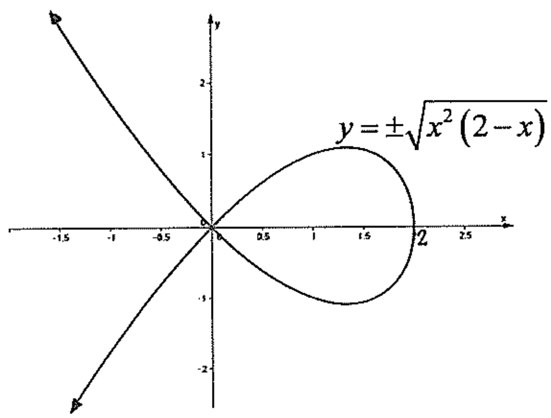
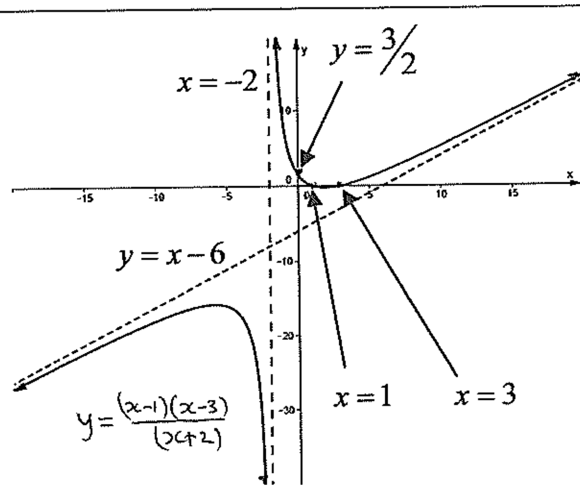
$=$ RHS

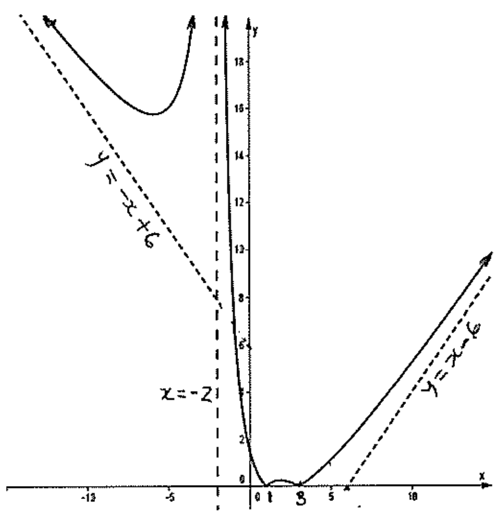
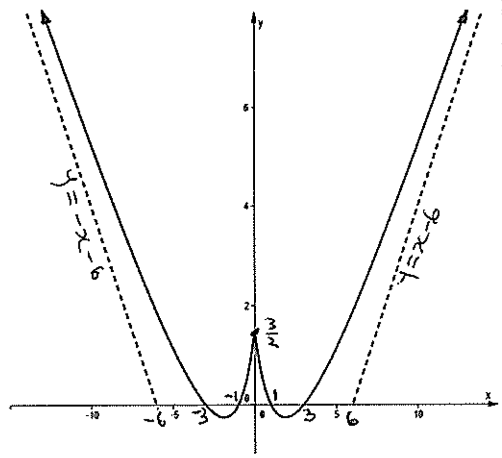
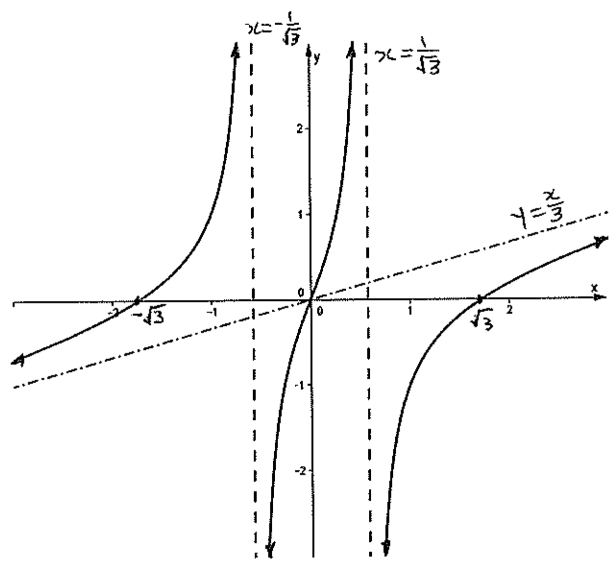
$\Rightarrow$ SP $\perp$ ST

1 mark for justify SP perpendicular to ST.

b)	**i)**	By using the two point formula we get $\dfrac{y-\frac{c}{p}}{x-cp}=\dfrac{\frac{c}{p}-\frac{c}{q}}{cp-cq}$ $=\dfrac{\frac{q-p}{pq}}{p-q}$ $\dfrac{py-c}{px-cp^2}=-\dfrac{1}{pq}$ $pq(py-c)=-(px-cp^2)$ $\therefore$ the eqt of PQ is $x+pqy=c(p+q)$	**1 mark for derive the eqt of PQ.**
	ii)	Thus the equations of the tangents at P & Q are: $x+p^2y=2cp \quad \& \quad x+q^2y=2cq$	**1 mark for P.** **1 mark for Q.**
	iii)	Let the equations of the tangents at P & Q be: $x+p^2y=2cp$ .... [1] $x+q^2y=2cq$ .... [2] From [1] −[2] get $(p^2-q^2)y=2c(p-q)$ $\therefore \quad y=\dfrac{2c}{p+q}$ Now sub y in [1] get $x=-p^2\left(\dfrac{2c}{p+q}\right)+2cp$ $=\dfrac{-2cp^2+2cp(p+q)}{p+q}$ $\therefore x=\dfrac{2cpq}{p+q}$ Hence the coordinates of T is $\left(\dfrac{2cpq}{p+q},\ \dfrac{2c}{p+q}\right)$.	**1 mark for finding y.** **1 mark for finding x.**
	iv)	Now sub $(1,\ 0)$ in the line PQ get $1=c(p+q)$. For T: $y=\dfrac{2c}{p+q}=\dfrac{2c}{1/c}=2c^2$ $\therefore$ the locus of T is $y=2c^2$. This is a horizontal straight line passing through the point $(0,\ 2c^2)$.	**1 mark for the expression after substitution.** **1 mark for finding the locus.** **1 mark for describe the locus.**

Question 9 [E6]

a)	i)	For x-intercepts, put $y = 0$, i.e. $x^2(2-x) = 0 \Rightarrow x = 0$ or $x = 2$ Also this is a negative cubic. Now sketch the graph: 	**1 mark for correct x- intercepts.** **1 mark for correct direction of graph.**
	ii)	$\because \quad y^2 = x^2(2-x)$ then $y = \pm\sqrt{x^2(2-x)}$ 	**1 mark for correct x-intercepts.** **1 mark for correct graph.**
b)	i)	$\because \dfrac{(x-1)(x-3)}{(x+2)} = x - 6 + \dfrac{15}{x+2}$ then RHS $= \dfrac{(x-6)(x+2)+15}{x+2}$ $= \dfrac{x^2-4x-12+15}{x+2}$ $= \dfrac{x^2-4x+3}{x+2}$ $= \dfrac{(x-1)(x-3)}{x+2}$ $\therefore$ RHS = LHS	**1 mark for express RHS as a single fraction.** **1 mark for simplify & factorise.**
	ii)	The asymptotes are: $x = -2$; As $x \to \pm\infty,\ y \to x - 6$ $\therefore\ y = x - 6$ The x & y intercepts: When $x = 0,\ y = \dfrac{3}{2}$ When $y = 0,\ x = 1$ or 3 	**1 mark for correct asymptotes.** **1 mark for correct x-y intercepts & graph.**

	iii)	$\alpha)\ y=\left\|f(x)\right\|$ if $f(x)=\dfrac{(x-1)(x-3)}{x+2}$ i.e. anything below x-axis is reflected in x-axis.  	$\beta)\ y=f(\|x\|)$ i.e. anything left of y-axis is deleted & replaced with reflection of curve on the right of y-axis.  	α) **1 mark for correct asymptotes.** **1 mark for correct x-y intercepts & graph.** β) **1 mark for correct asymptotes & x-y intercepts.** **1 mark for correct graph.**
c)	**i)**	Now $f(a)=\dfrac{a^3-3a}{3a^2-1}$ and $f(-a)=\dfrac{(-a)^3-3(-a)}{3(-a)^2-1}$ $=\dfrac{-a^3+3a}{3a^2-1}$ $\therefore f(a)=-f(a)$ Thus this is an odd fn. The other 2 asymptotes are: $3x^2-1=0 \Rightarrow x=\pm\dfrac{1}{\sqrt{3}}$ When $x=0$ then $y=0$ When $y=0$ then $x^3-3x=0$ $x(x^2-3)=0$ $\therefore x=0,\ \pm\sqrt{3}$ Check $f\left(\dfrac{1}{2}\right)=5\dfrac{1}{2}$ for shape of the middle curve.	 	**1 mark for all the correct asymptotes.** **1 mark for x-y intercepts & graph.**
	ii)	How many times will a horizontal line intersect the graph? $\therefore$ there are **3** solutions in this case.		**1 mark for correct answer.**