Carlingford High School

2021 YEAR 11 ASSESSMENT TASK 2

Mathematics Advanced

STUDENT NUMBER: SOLUTIONS

Teacher: (Please Circle)

11MAA_A (Ms Tang) 11MAA_B (Ms Blakeley) 11MAA_C (Mr Wilson) 11MAA_D (Mr Gong)

11MAA_1 (Ms Strilakos) 11MAA_2 (Ms Bennett) 11MAA_3 (Mr Cheng) 11MAA_4 (Mr Fardouly)

General Instructions

- Working time - 50 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper

TOPIC	MARKS	
Functions Questions: 1 – 7	/22	
Trigonometry Questions: 8 – 14	/20	
TOTAL	/42	%

42 marks
Attempt Questions 1 - 14

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 1 (4 marks)

Solve:

(a) $x^2 + 9x - 36 = 0$ **1**

$(x + 12)(x - 3) = 0$

$x = -12, 3$

(b) $6x^2 = 24x$ **1**

$6x^2 - 24x = 0$

$6x(x - 4) = 0$

$x = 0, 4$

(c) $6x^2 + 13x - 8 = 0$ **2**

$6x^2 + 16x - 3x - 8 = 0$

$2x(3x + 8) - (3x + 8) = 0$

$(3x + 8)(2x - 1) = 0$

$x = -\frac{8}{3}, \frac{1}{2}$

Question 2 (3 marks)

Solve $3x^2 + x = 5$ by completing the square, giving answers correct to 3 significant figures.

$$x^2 + \frac{x}{3} = \frac{5}{3}$$

$$x^2 + \frac{x}{3} + \frac{1}{36} = \frac{5}{3} + \frac{1}{36}$$

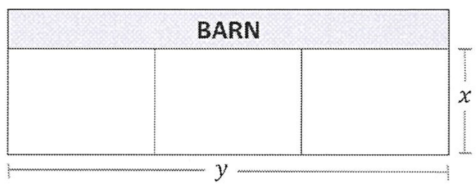
$$\left(x + \frac{1}{6}\right)^2 = \frac{61}{36}$$

$$x + \frac{1}{6} = \frac{\pm\sqrt{61}}{6}$$

$$x = \frac{-1 \pm \sqrt{61}}{6}$$

Question 3 (3 marks)

A farmer bought 240m of fencing to construct three equal rectangular fields. No fencing is required along the side of the barn.



(a) Show that $y = 240 - 4x$ **1**

Perimeter : $240 = y + 4x$

$y = 240 - 4x$

(b) Hence, or otherwise, find the maximum area of the enclosed area. **2**

Area : $A = xy$

$= x(240 - 4x)$

$= -4x^2 + 240x$

$= -4(x^2 - 60x + 900) + 3600$

$= -4(x - 30)^2 + 3600$

$\therefore$ Maximum area $= 3600 \text{ m}^2$

Question 4 (5 marks)

A ball is thrown into the air from a balcony that is 30 metres above the ground. The function that models the height, $h(t)$ in metres above the ground, of the ball over time, t in seconds, is $h(t) = 30 + 12t - 5t^2$.

(a) What is the height of the ball above the ground after 2 seconds? **1**

$h(2) = 30 + 12(2) - 5(2)^2$

$= 34$

$\therefore$ the ball is 34m above the ground after 2 seconds

(b) When does the ball hit the ground? Answer correct to the nearest second. **2**

$30 + 12t - 5t^2 = 0$

$5t^2 - 12t - 30 = 0$

$t = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-30)}}{2(5)}$

$= \dfrac{12 \pm \sqrt{744}}{10}$

$= -1.527\ldots, \ 3.927\ldots$

$\therefore$ the ball hits the ground after 4 seconds (nearest second)

(c) What is the maximum height above the ground reached by the ball? Answer correct to one decimal place. **2**

$t = \dfrac{-(12)}{2(-5)}$

$= 1.2$

$h(1.2) = 30 + 12(1.2) - 5(1.2)^2$

$= 37.2$

$\therefore$ the maximum height above the ground reached by the ball is 37.2m

Question 5 (2 marks)

Prove the quadratic expression $7x^2 + 4x + 1$ is positive definite for all values of x.

$a = 7 \quad \Rightarrow \quad a > 0$

$\Delta = b^2 - 4ac$

$= 4^2 - 4(7)(1)$

$= -12 \quad \Rightarrow \quad \Delta < 0$

$\therefore$ the expression is positive definite for all values of x.

Question 6 (3 marks)

For what values of m does the equation $x^2 - 2mx + 8m - 15 = 0$ have two roots?

$\Delta > 0$

$b^2 - 4ac > 0$

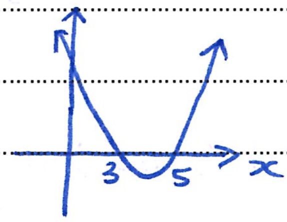
$(-2m)^2 - 4(1)(8m-15) > 0$

$4m^2 - 32m + 60 > 0$

$m^2 - 8m + 15 > 0$

$(m-5)(m-3) > 0$

$\therefore \quad m < 3 \text{ or } m > 5$



Question 7 (2 marks)

Prove the line $y = 6x + 1$ is a tangent to the curve with equation $y = x^2 + 4x + 2$.

$y = 6x + 1 \quad \ldots (1)$

$y = x^2 + 4x + 2 \quad \ldots (2)$

sub (1) into (2) : $\quad 6x + 1 = x^2 + 4x + 2$

$x^2 - 2x + 1 = 0$

for tangent : $\quad \Delta = 0$

$(-2)^2 - 4(1)(1) = 0$

$\therefore$ the line $y = 6x + 1$ is a tangent to parabola $y = x^2 + 4x + 2$.

Question 8 (4 marks)

Find the exact value of:

(a) $\tan 30°$ **1**

$\tan 30° = \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

(b) $\sin 300°$ **1**

$\sin 300° = \sin(360° - 60°)$

$= -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

(c) $\cot(-30°)$ **1**

$\cot(-30°) = -\cot 30°$

$= -\sqrt{3}$

(d) $\operatorname{cosec} 150°$ **1**

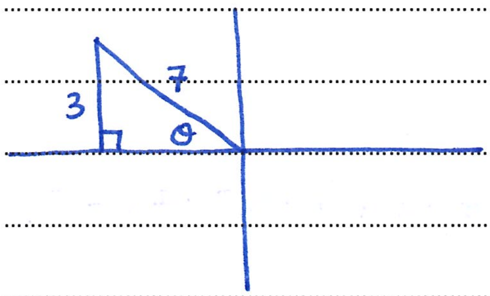
$\operatorname{cosec} 150° = \operatorname{cosec}(180° - 30°)$

$= \operatorname{cosec} 30°$

$= 2$

Question 9 (2 marks)

Given $\sin\theta = \frac{3}{7}$ and $\cos\theta < 0$, find the exact value of $\tan\theta$.



$x^2 = 7^2 - 3^2$

$x = \sqrt{40}$

$\tan\theta = -\frac{3}{\sqrt{40}}$

$= \frac{-3}{2\sqrt{10}}$

$= \frac{-3\sqrt{10}}{20}$

Question 10 (2 marks)

Show that $\tan(90° + \theta) = -\cot\theta$

$\tan(90° + \theta) = \tan(180° - (90° - \theta))$

$= -\tan(90° - \theta)$

$= -\cot\theta$

Question 11 (2 marks)

Find all values of $x, 0° \leq x \leq 360°$ for which $2\cos^2 x - 1 = 0$.

$2\cos^2 x - 1 = 0$

$\cos^2 x = \dfrac{1}{2}$

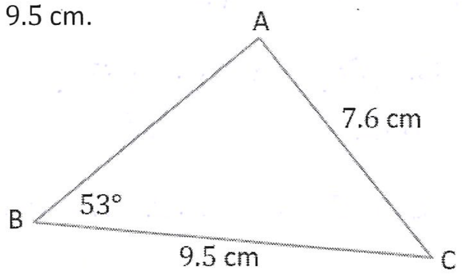
$\cos x = \dfrac{\pm 1}{\sqrt{2}}$

$x = 45°, 180° - 45°, 180° + 45°, 360° - 45°$

$= 45°, 135°, 225°, 315°$

Question 12 (2 marks)

In triangle ABC, $\angle B = 53°$, $AC = 7.6$ cm and $BC = 9.5$ cm.
Find $\angle A$ to the nearest degree.



$\dfrac{\sin A}{9.5} = \dfrac{\sin 53°}{7.6}$

$\sin A = \dfrac{9.5 \sin 53°}{7.6}$

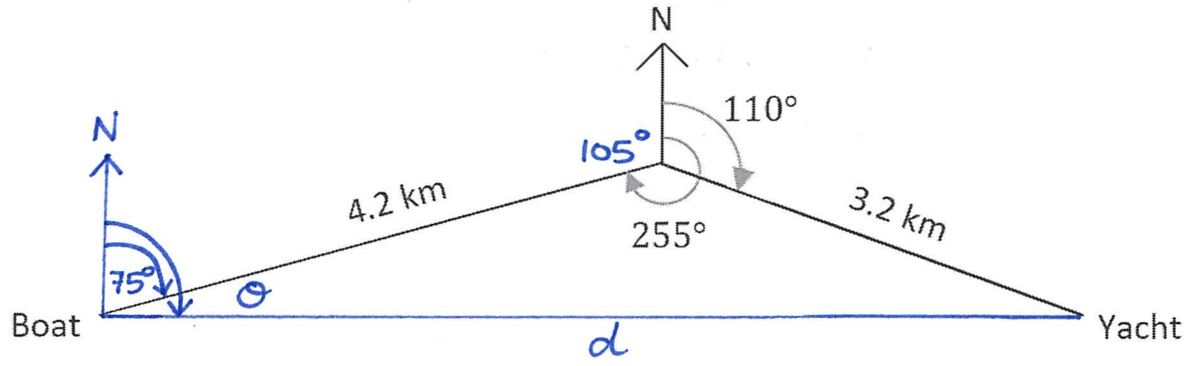
$A = 87°$

Check $A = 180° - 87° = 93°$ $(93° + 53° < 180°)$

$\therefore A = 87°$ or $93°$

Question 13 (4 marks)

The bearings of a yacht and a boat from a lighthouse are 110° and 255° respectively. The yacht is 3.2km and the boat 4.2 km from the lighthouse.



(a) Find the distance between the yacht and the boat. Answer correct to one decimal place. **2**

$255° - 110° = 145°$

$d^2 = 3.2^2 + 4.2^2 - 2 \times 3.2 \times 4.2 \times \cos 145°$

$d = \sqrt{49.898\ldots}$

$= 7.063\ldots$

∴ the distance between the yacht and boat is 7.1 km (1dp)

(b) Find the true bearing of the yacht from the boat. Answer correct to the nearest degree. **2**

$360° - 255° = 105°$

$180° - 105° = 75°$

$\frac{\sin \theta}{3.2} = \frac{\sin 145°}{d}$

$\sin \theta = \frac{3.2 \sin 145°}{d}$

$\theta = \sin^{-1}(0.259\ldots)$

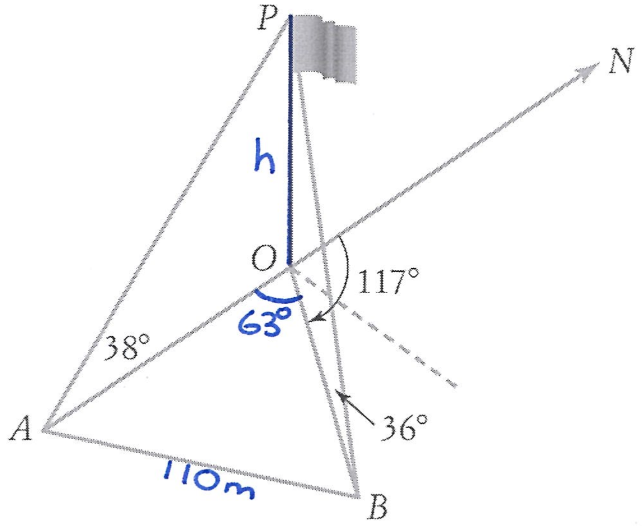
$= 15°$

∴ Bearing $= 75° + 15°$

$= 090°$

Question 14 (4 marks)

From a point A due south of a flagpole, the angle of elevation of the top of the pole P, is 38°. From another point B, on a bearing of 117° from the pole, the angle of elevation of P is 36°. The distance AB is 110 metres. Let h be the height of the flagpole in metres.



(a) $OA = \dfrac{h}{\tan 38°}$. Show that $OB = \dfrac{h}{\tan 36°}$. **1**

In $\triangle PBO$: $\tan 36° = \dfrac{OP}{OB}$

$OB = \dfrac{h}{\tan 36°}$ $(OP = h)$

(b) Hence find, correct to one decimal place, the height of the flagpole. **3**

$\angle AOB = 180° - 117° = 63°$

In $\triangle AOB$: $110^2 = \dfrac{h^2}{\tan^2 38°} + \dfrac{h^2}{\tan^2 36°} - 2 \times \dfrac{h}{\tan 38} \times \dfrac{h}{\tan 36°} \times \cos 63°$

$110^2 = h^2 \left(\dfrac{1}{\tan^2 38°} + \dfrac{1}{\tan^2 36°} - \dfrac{2\cos 63°}{\tan 38° \tan 36°} \right)$

$h = \sqrt{\dfrac{110^2}{1.933\ldots}}$

$= 79.116\ldots$

∴ the flagpole is 79.1 m in height (1 dp)

END OF EXAM

Extra writing space

If you use this space, clearly indicate which question you are answering.

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

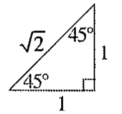
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

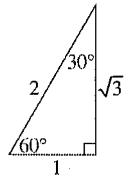
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A\sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

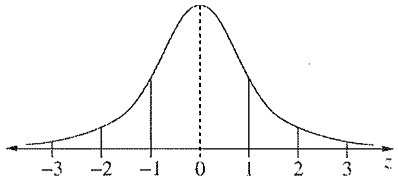
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x-\mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = |x\underset{\sim}{i} + y\underset{\sim}{j}| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$