EXTENSION 1 Term 2 2019 Solutions

Q.1. $f'(x) = \dfrac{8x^3 - 1}{x^2}$, $x \neq 0$.

(i) At $P(-1,0)$, $f'(x) = -9$

Eq^n of tangent is $y = -9(x+1)$ i.e. $y = -9x - 9$.

(ii) $f(x) = \displaystyle\int \left(\frac{8x^3 - 1}{x^2}\right) dx = \int (8x - x^{-2})\, dx$

$= 4x^2 + x^{-1} + c$

At $x = -1$, $y = 0$ $\therefore$ $0 = 4 - 1 + c$

i.e. $c = -3$

$\therefore f(x) = 4x^2 + \dfrac{1}{x} - 3$.

(iii) At Q, $\dfrac{dy}{dx} = 0$ i.e. $\dfrac{8x^3 - 1}{x^2} = 0$ $(2x-1)(4x^2 + 2x + 1) = 0$

no real sol^{ns}.

$x = \frac{1}{2}$

$\therefore a = \frac{1}{2}$. at Q.

Q2 $\quad y = (4-x)(x+2)(x-1) = (4-x)(x^2+x-2)$

(i) $\frac{dy}{dx} = -1\cdot(x^2+x-2) + (2x+1)(4-x)$

$= -x^2 - x + 2 + 8x - 2x^2 + 4 - x$

$= -3x^2 + 6x + 6$

At $x=2$, $\frac{dy}{dx} = -12 + 12 + 6 = 6$

and $y = 8$

$\therefore$ Eqn of tangent is: $\quad y - 8 = 6(x-2) \quad$ ie. $y = 6x - 4$.

(ii) tangent meets curve when $(4-x)(x^2+x-2) = 6x-4$.

ie. $4x^2 + \cancel{4x} - 8 - x^3 - x^2 + \cancel{2x} = \cancel{6x} - 4$

$-x^3 + 3x^2 + 6x - 8 = 6x - 4$

$-x^3 + 3x^2 - 4 = 0$

Since one soln is known to be $x=2$,

$$\begin{array}{r|l} & -x^2 + x + 2 \\ \hline x-2 & -x^3 + 3x^2 - 4 \\ & -x^3 + 2x^2 \\ \hline & \quad x^2 - 4 \\ & \quad x^2 - 2x \\ \hline & \quad\quad 2x - 4 \\ & \quad\quad 2x - 4 \\ \hline \end{array}$$

the other soln is found from.

$-x^2 + x + 2 = 0$.

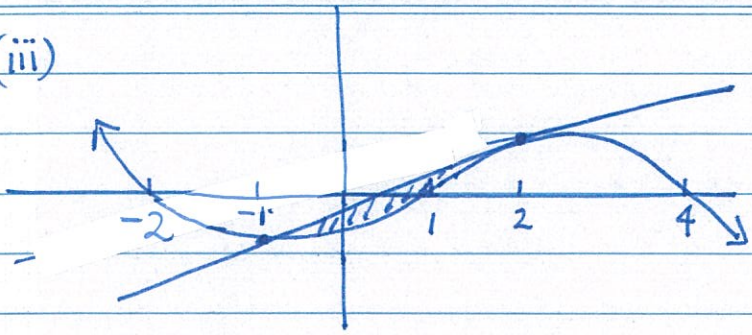
ie. $(-x-1)(x-2) = 0$

ie. $x = -1$

and $y = -10$

$(-1, -10)$

(iii)



$$\text{Area} = \int_{-1}^{2} \left[(\cancel{6x} - 4) \quad - (-x^3 + 3x^2 + \cancel{6x} - 8) \right] dx$$

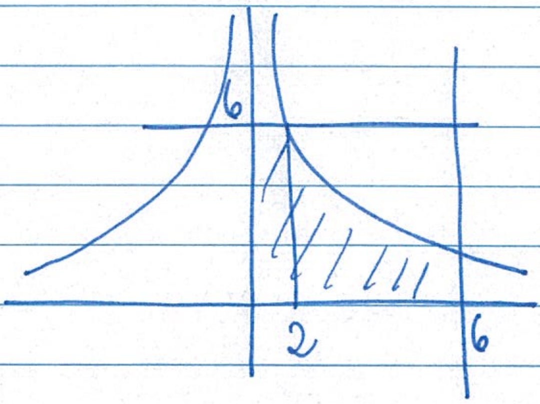
$$= \int_{-1}^{2} (x^3 - 3x^2 + 4)\, dx$$

$$= \left[\frac{x^4}{4} - \frac{\cancel{3}x^3}{\cancel{3}} + 4x \right]_{-1}^{2}$$

$$= (4 - \cancel{8} + \cancel{8}) - \left(\frac{1}{4} + 1 - 4 \right)$$

$$= 6\tfrac{3}{4} \ u^2.$$

Q.3 $y = \dfrac{24}{x^2}$



At $y=6$, $6 = \dfrac{24}{x^2}$

$\Rightarrow x^2 = \dfrac{24}{6} = 4$

$x = 2$ (taking +ve).

Hence, area $= 2 \times 6 + \displaystyle\int_2^6 \frac{24}{x^2}\,dx$

$= 12 + \left[-\dfrac{24}{x}\right]_2^6$

$= 12 + \left[-4 + 12\right] = 20$ units^2.

Q4. $f(x) = \dfrac{x^2}{(x+2)(x-3)}$

(i) $x = -2$, $x = 3$.

(ii) $f(x) = \dfrac{x^2}{x^2 - x - 6} = \dfrac{1}{1 - \frac{1}{x} - \frac{6}{x^2}} \rightarrow 1$ as $x \rightarrow \pm\infty$.

(iii) $f'(x) = \dfrac{2x(x^2 - x - 6) - (2x-1)x^2}{((x+2)(x-3))^2}$

$= \dfrac{\cancel{2x^3} - 2x^2 - 12x - \cancel{2x^3} + x^2}{((x+2)(x-3))^2}$

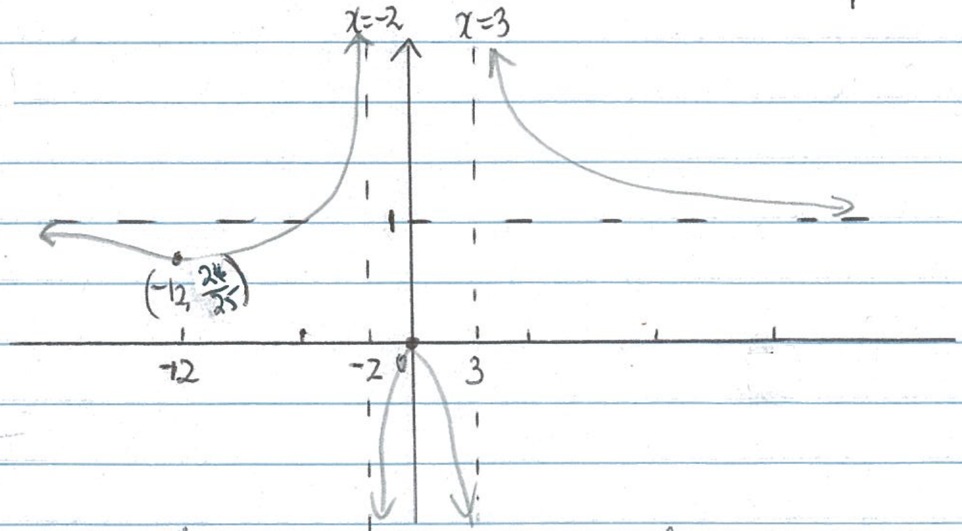
$= \dfrac{-x^2 - 12x}{(\quad)^2} = \dfrac{-x(x+12)}{(\quad)^2} = 0$ when $x = 0, -12$.

x	-13	-12	-10	0	1
$f'(x)$	\	—	/	—	\

$\therefore$ min^m t.p at $\left(-12, \frac{144}{150}\right)$ $\frac{24}{25}$

max^m t.p at $(0, 0)$

(v)



(iv) As $x \rightarrow +\infty$, $f(x) \rightarrow 1^+$, As $x \rightarrow 3^+$, $f(x) \rightarrow +\infty$

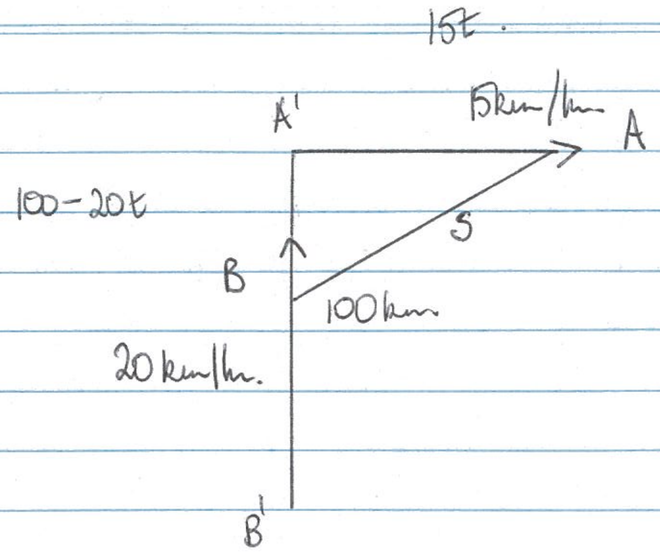
As $x \rightarrow 3^-$, $f(x) \rightarrow -\infty$, As $x \rightarrow -2^+$ $f(x) \rightarrow -\infty$

As $x \rightarrow -2^-$, $f(x) \rightarrow +\infty$, As $x \rightarrow -\infty$, $f(x) \rightarrow 1^-$

(vi) A horizontal line $y=k$ will cut twice for $k > 1$, $\frac{24}{25} < k < 1$

and $k < 0$.

Q5.



(i) $s^2 = (100 - 20t)^2 + (15t)^2$ 1

$s = \sqrt{10000 - 4000t + 400t^2 + 225t^2}$

$= \sqrt{10000 - 4000t + 625t^2}$

$= \sqrt{25(400 - 160t + 25t^2)}$

$= 5\sqrt{400 - 160t + 25t^2}$ 1

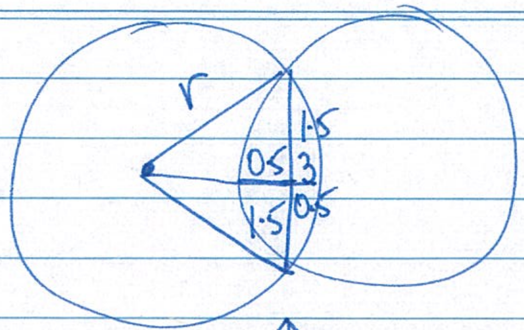
(ii) $\dfrac{ds}{dt} = \dfrac{5(-160 + 50t)}{2\sqrt{400 - 160t + 25t^2}}$ 1

(iii) $\dfrac{ds}{dt} = 0$ for max/min when $50t = 160$ i.e. $t = 16/5$ 1

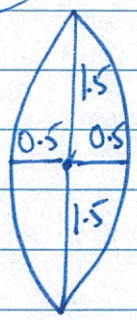
If $t < 16/5$, say 3 $\dfrac{ds}{dt} < 0$ if $t > \dfrac{16}{5}$, say 4, $\dfrac{ds}{dt} > 0$

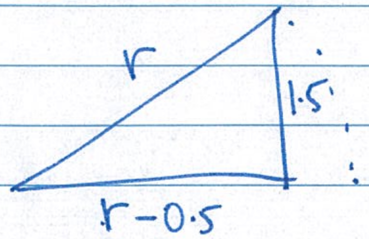
$\therefore$ s is a min^{m} when $t = \dfrac{16}{5}$ hrs.

Q.6.



(i)



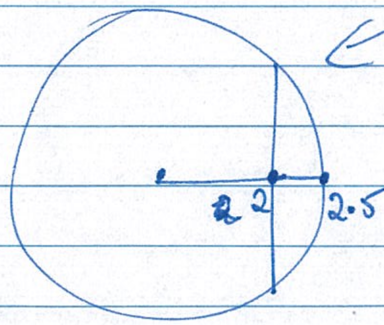


$$r^2 = (r-0.5)^2 + \left(\frac{3}{2}\right)^2$$

$$\cancel{r^2} = \cancel{r^2} - r + 0.25 + \frac{9}{4}$$

$$r = 5/2.$$

(ii)



$$x^2 + y^2 = \frac{25}{4}$$

$$V_{\frac{1}{2}\text{ lens}} = \pi \int_2^{5/2} \left(\frac{25}{4} - x^2\right) dx$$

$$= \pi \left[\frac{25}{4}x - \frac{x^3}{3}\right]_2^{5/2}$$

$$= \pi \left[\left(\frac{25}{4} \times \frac{5}{2} - \frac{125}{24}\right) - \left(\frac{50}{4} - \frac{8}{3}\right)\right]$$

$$= \pi \left[\left(\frac{125}{8} - \frac{125}{24}\right) - \left(\frac{150-32}{12}\right)\right]$$

$$= \pi \left[\frac{250}{24} - \frac{236}{24}\right] = \pi \times \frac{14}{24}$$

∴ Volume of full lens is $\pi \times \frac{7}{6} \approx 3.67$ cm