Introduction to Polynomials (Review)

A polynomial $P(x)$ is an algebraic expression of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

- n is a positive integer, called the degree of the polynomial
- The a_i s are real or complex numbers, the coefficients
- The leading term is $a_n x^n$

When the leading coefficient a_{n} is equal to 1, the polynomial is said to be monic.

Complex numbers r which satisfy the polynomial equation $P(r) = 0$ are called the roots of the equation or the zeroes of the polynomial function $P(x)$

A polynomial equation of degree n with real coefficients will have at most n real roots.

The Remainder Theorem

When a polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

The Factor Theorem

For a polynomial $P(x)$ if $P(a) = 0$ then $x - a$ is a factor of $P(x)$.

The Fundamental Theorem of Algebra

Every polynomial of degree n has at least one complex root.

Corollary: Every polynomial of degree n has exactly n complex roots.

This is proved by repeated applications of the Fundamental Theorem of Algebra.

Example 1. Factorise the polynomial $x^4 + x^2 - 12$ over

a) $\mathbb{Q}$ (the set of rational numbers)

b) $\mathbb{R}$

c) $\mathbb{C}$

Example 2.

Given that $x + 3$ is a factor of $x^3 + 4a^2x^2 - 7x - 6a$, find the values of a.