A C E
EXAM
PAPER

Student name: _____

PAPER 3

YEAR 12
YEARLY
EXAMINATION

Mathematics Advanced

General Instructions

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

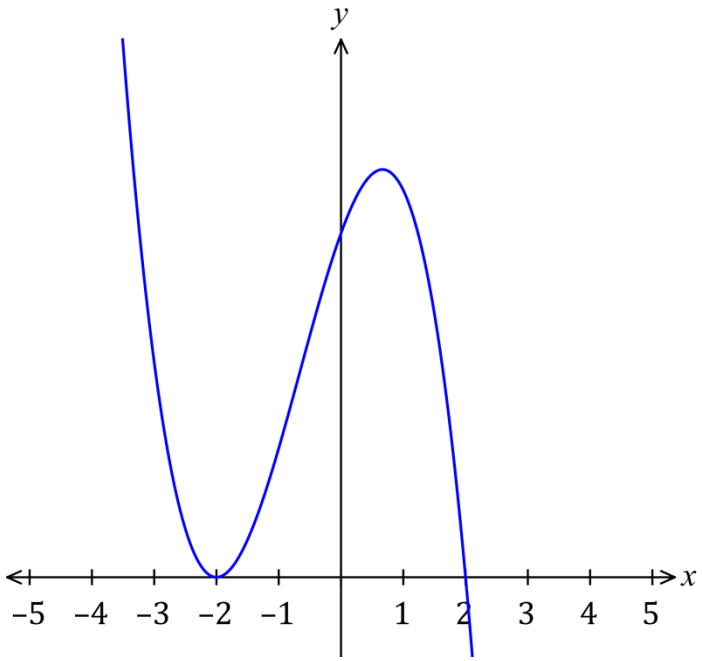
- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. Which of the following equations describes the graph below?



(A) $y = x(x-2)(x+2)$

(B) $y = -x(x+2)(x-2)$

(C) $y = -(x+2)^2(x-2)$

(D) $y = -(x-2)^2(x+2)$

2. $\int (4 + 2x + 3x^2)dx$ is equal to:

(A) $4 + x^2 + x^3 + C$

(B) $4x + x^2 + x^3 + C$

(C) $4 + x^2 + \frac{x^3}{3} + C$

(D) $4x + x^2 + \frac{x^3}{3} + C$

3. What is the value of $\sum_{n=1}^{5} (4n - 2)$?

(A) 18

(B) 39

(C) 50

(D) 90

4. What is the gradient of the tangent to the curve $y = (2 - x)^3 + 1$ at the point $(1, 2)$?

   (A) -3
   (B) -2
   (C) 2
   (D) 3

5. The probability density function for the continuous random variable X is:

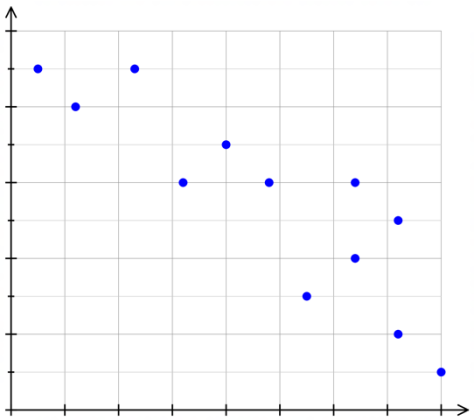
$$f(x) = \begin{cases} \frac{1}{2}\sin x & 0 < x < k \\ 0 & x \geq k \text{ or } x \leq 0 \end{cases}$$

   What is the value of k?

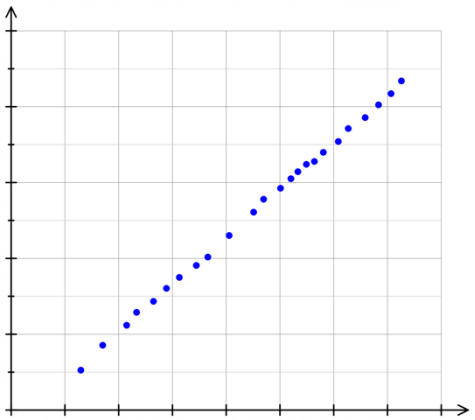
   (A) 1
   (B) 2
   (C) π
   (D) 2π

6. The correlation coefficient for two quantities was -0.5. Which scatterplot could represent this result?

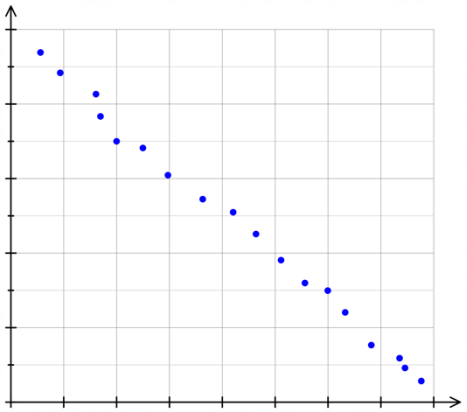
(A)



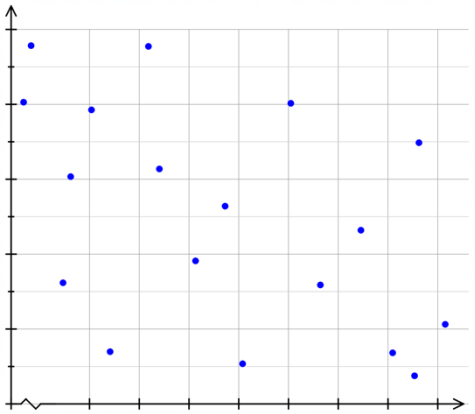
(B)



(C)



(D)



7. The gradient function of a curve is $\frac{dy}{dx} = 3 - \frac{2}{x^2}$

   What is the equation of the curve if it passes through the point (1, –2)?

   (A) $y = \frac{4}{x^3}$ (B) $y = \frac{2}{x} - 4$

   (C) $y = 3x - \frac{2}{x} - 3$ (D) $y = 3x + \frac{2}{x} - 7$

8. The curve $y = 2x^2 + ax + 5$ has a stationary point at $x = -3$. What is the value of a?

   (A) –12

   (B) –6

   (C) 6

   (D) 12

9. The function $f(x) = \cos 3x - 1$ is defined in the interval $0 \leq x \leq \frac{2\pi}{9}$.

   What is the range of the function?

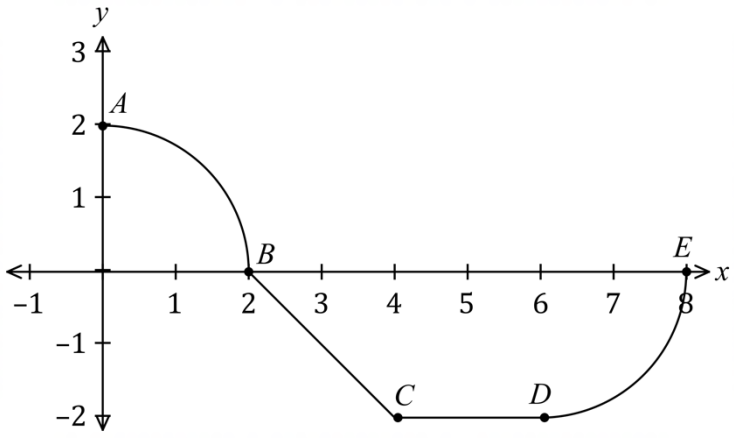
   (A) $-1 \leq y \leq 0$

   (B) $-\frac{3}{2} \leq y \leq 0$

   (C) $-2 \leq y \leq 0$

   (D) $-3 \leq y \leq 0$

10. The graph of the function $y = f(x)$ consists of quarter of a circle AB, straight-line segment BC, a horizontal straight-line segment CD, and a quarter circle DE.



   What values of x is the function increasing?

   (A) $0 < x < 2$

   (B) $2 < x < 4$

   (C) $4 < x < 6$

   (D) $6 < x < 8$

Section II

90 marks
Attempt questions 11 - 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

Find $\int (3x+5)^2 dx$ **2**

Question 12 (2 marks)

The table below shows the future value of an annuity with a contribution of $1.

Future value of $1				
Period	1%	4%	8%	12%
1	1.0000	1.0000	1.0000	1.0000
3	3.0301	3.1216	3.2464	3.3744
5	5.1010	5.4163	5.8666	6.3528

(a) Find the future value of $16 000 invested at the end of each year for 3 years at 8% p.a. compounding annually? Answer correct to the nearest whole number. **1**

(b) Find the future value of $2100 invested at the end of each month for 5 months at 12% p.a. compounding monthly? Answer correct to the nearest whole number. **1**

Question 13 (4 marks) **Marks**

Boxes are stacked in layers, where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether.

(a) Find the number of boxes in the bottom layer. **2**

(b) Show that there are $\frac{1}{2}n(n+1)$ boxes. **2**

Question 14 (2 marks)

Lex scored 66% in the first assessment task for which the mean was 82% and the standard deviation was 8. In the second assessment task the mean was 71% and standard deviation was 10. Lex scored 61%. Did Lex improve? Justify your answer. **2**

Question 15 (3 marks)

A particle moves along the x-axis with acceleration $3t - 2$. Initially it is 4 units to the right of the origin, with a velocity of 2 units per second. What is the position of the particle after 5 seconds? **3**

Question 16 (4 marks) **Marks**

(a) Find $\int \frac{x}{x^2+3}dx$ **2**

(b) Evaluate $\int_0^{\frac{\pi}{3}} \cos 2x dx$ **2**

Question 17 (3 marks)

How many solutions are there to the equation $4\cos x = 2 - x$ in the domain $-2\pi \leq x \leq 2\pi$? Hint: Solve by sketching the graphs. **3**

Question 18 (6 marks) **Marks**

A continuous random variable X has a function f given by

$$f(x) = \begin{cases} 6x(1-x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Check that $f(x)$ is a probability density function. **2**

(b) Sketch the function. **2**

(c) Find $P(0 \leq X \leq 1)$. **1**

(d) Find $P(0.5 \leq X \leq 1)$. **1**

Question 19 (3 marks) **Marks**

Differentiate

(a) $(e^x - 3)^4$ **1**

(b) $x\tan x$ **1**

(c) $\ln(\cos x)$ **1**

Question 20 (4 marks)

The third term of a geometric series is 0.75 and the seventh term is 12.

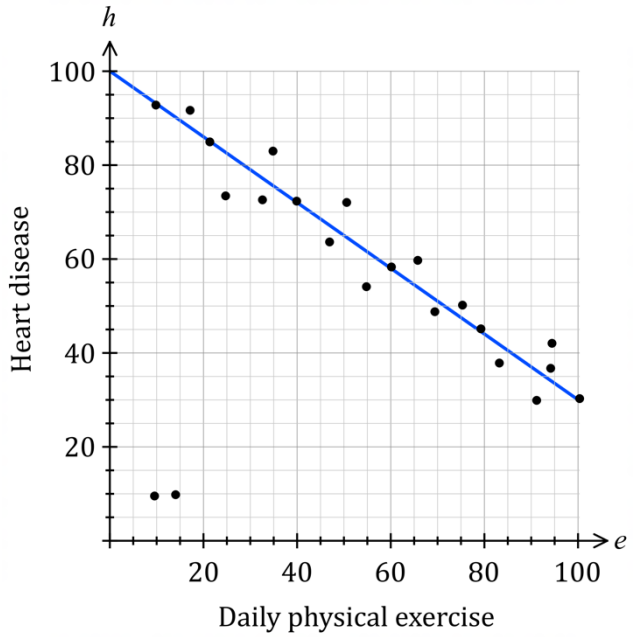
(a) Find the common ratio. **2**

(b) Find the first term. **1**

(c) What is the tenth term? **1**

Question 21 (3 marks) **Marks**

The scatterplot shows daily physical exercise (e) versus heart disease (h).



(a) Calculate the gradient of the line. **1**

(b) What is the equation of the line of best fit drawn? **1**

(c) Estimate the value of the correlation coefficient. **1**

Question 22 (2 marks)

Elijah starts on a salary of \$55 000 with an annual increase of \$1650. **2**
What is the total amount Elijah would earn in twelve years of employment?

Question 23 (9 marks) **Marks**

A function $f(x)$ is defined by $f(x) = 2x^3 - 3x^2$.

(a) Find all the solutions for $f(x) = 0$. **1**

(b) Find the turning points for the curve $y = f(x)$ and determine their nature. **3**

(c) Find the coordinates of the point of inflexion. **2**

(d) Sketch the graph of $y = f(x)$ showing the essential features. **2**

(e) Find the values of x for which $f(x) < 0$. **1**

Question 24 (6 marks)

Marks

(a) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = \cos x$ over the domain $0 \leq x \leq 2\pi$. **2**

(b) The graphs intersect at points A and B. What are the coordinates of A and B? **2**

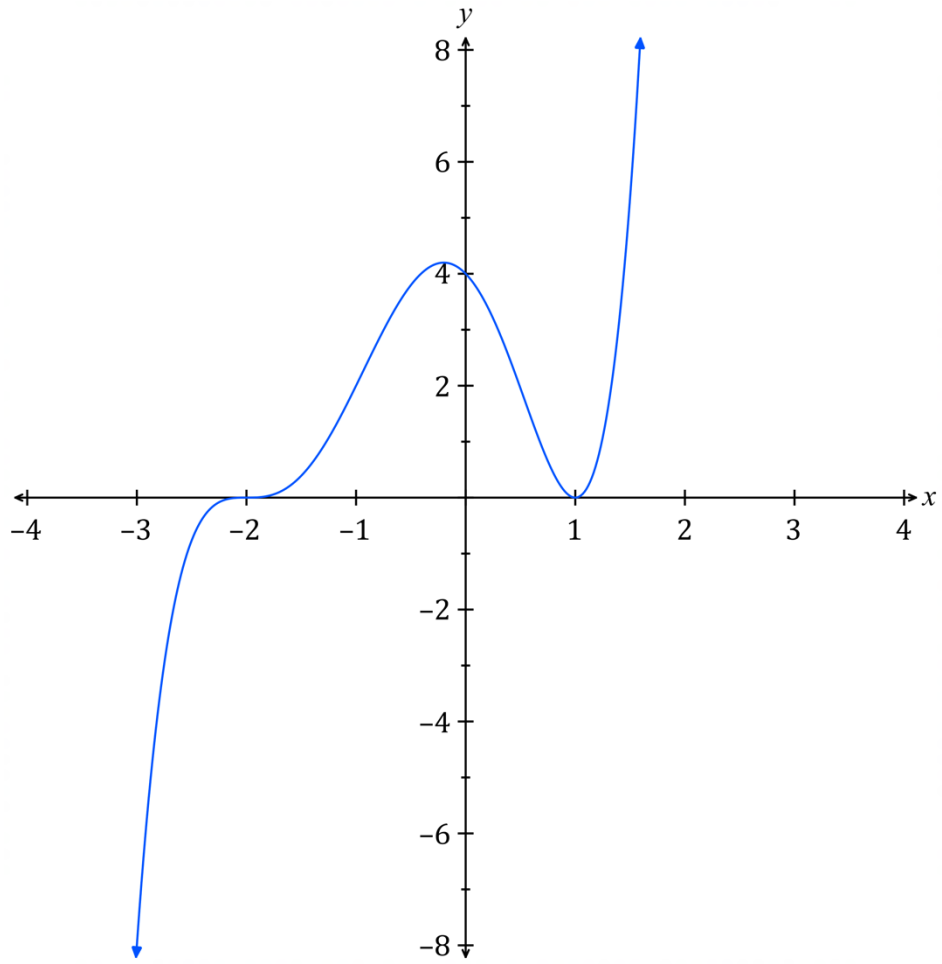
(c) Find the area between $y = \sin x$ and $y = \cos x$ over the domain $0 \leq x \leq \pi$. **2**

Question 25 (2 marks)

Differentiate $\ln_2 x^2$. **2**

Question 26 (4 marks) **Marks**

The graph of $y = f(x)$ is shown below.



Draw sketches of the following functions on the above number plane.
Clearly label each sketch. Indicate any asymptotes and intercepts with the axes.

(a) $y = f(x - 1)$ **2**

(b) $y = \dfrac{1}{f(x)}$ **2**

Question 27 (2 marks)

An infinite geometric series has a first term of 10 and a limiting sum of 15. **2**
Calculate the common ratio.

Question 28 (3 marks) **Marks**

The cost of preparing meals in a school canteen is linearly related to the number of meals prepared. To help the caterers predict the costs, data was collected on the cost of preparing meals for different levels of demands. The data is shown below.

Number (meals)	30	35	40	45	50	55	60	65	70	75	80
Cost (dollars)	138	154	159	182	198	198	214	208	238	234	244

(a) Find the equation of the least-squares line of best fit. Answer correct to one decimal place. **1**

(b) What is the predicted cost of producing 48 meals? **1**

(c) What is the predicted number of meals if the cost is \$249.50? **1**

Question 29 (2 marks)

In a normally distributed set of scores, the mean is 74 and the standard deviation is 6. Approximately what percentage of the scores will lie between 62 and 86? **2**

Question 30 (2 marks)

For $f(x) = \dfrac{\cos x}{2x + 2}$ find $f'(x)$. **2**

Question 31 (2 marks) **Marks**

A machine creates metal disks with mean diameter of 4.50 cm and a standard deviation of 0.03 cm. The diameters of these metal disks are normally distributed.

(a) State the interval where the mean diameter of the metal disks will almost certainly lie. **1**

(b) A metal disk is produced at random with a diameter of 4.62 cm. Why is the manager concerned? **1**

Question 32 (3 marks)

William invests $25,000 into an account at the beginning of each year for ten years. If the account earns interest at 6% p.a. compounded yearly, find the amount of money in the account at the end of ten years. Answer to the nearest dollar. **3**

Question 33 (6 marks) **Marks**

A can is the shape of a closed cylinder with a height h cm and a radius r cm.
The volume of the can is of 200 cm^3.

(a) Find an expression for h in terms of r. **1**

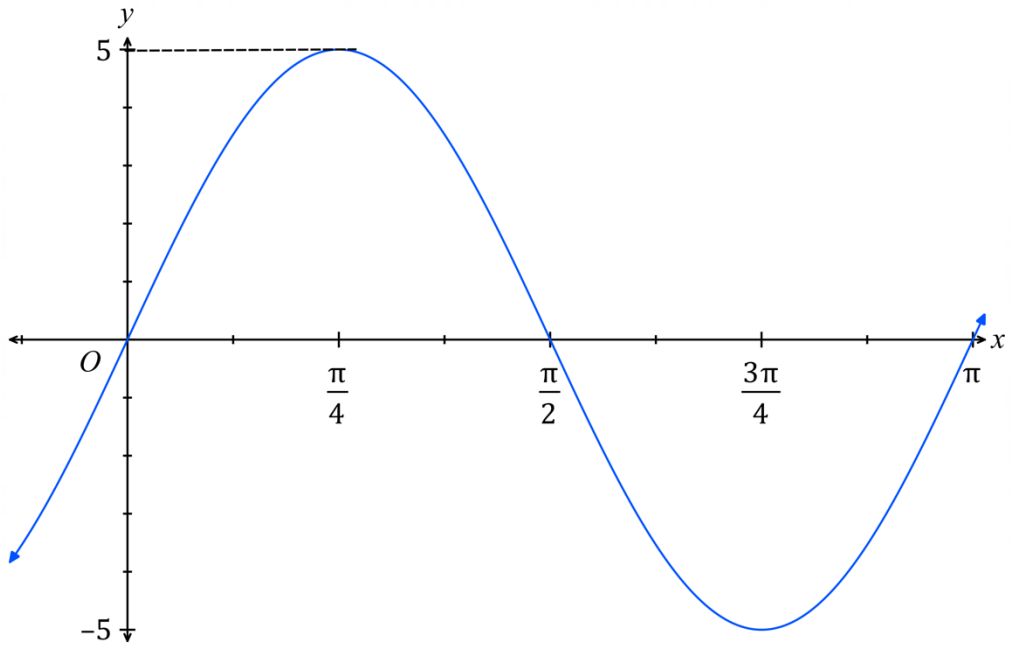
(b) Show that the surface area of the can is given by: **2**

$$A = 2\pi r^2 + \frac{400}{r}$$

(c) If the area of the metal used to make the can is to be minimized, find the radius of the can. Answer correct to two decimal places. **3**

Question 34 (4 marks) **Marks**

The graph shown is $y = a\sin bx$



(a) Find the value of a. **1**

(b) Find the value of b. **1**

(c) Draw on the above diagram, the graph of $y = 4\sin x + 1$, for $0 \leq x \leq \pi$. **2**

Question 35 (2 marks)

Find the gradient of the curve $y = \tan x$ at the point where $x = \frac{\pi}{16}$. **2**
Give your answer correct to 3 significant figures.

Question 36 (5 marks) **Marks**

A factory produces mobile phones. The annual production, *M* phones at time *t* years, is given by:

$$M = M_0 e^{kt}$$

Initially the production at the factory was 2000 phones per annum.
Five years later it had increased to 3200 phones per annum.

(a) Find the values of M_0 and *k* (Answer correct to three decimal places). **2**

(b) What is the predicted production after 10 years? **1**

(c) How many years will it take for the production to double its original output? Answer correct to one decimal place. **2**

End of paper

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $$\alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

$$S = \frac{a}{1 - r}, \quad |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

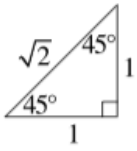
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

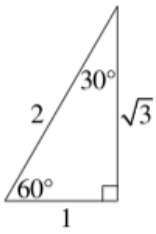
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

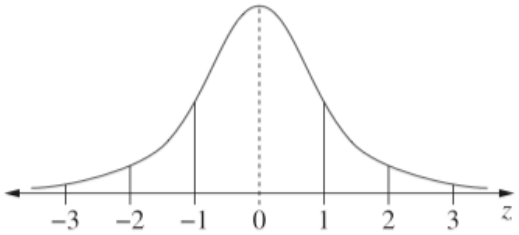
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$