CARLINGFORD HIGH SCHOOL

2016

TRIAL

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Reference Sheets are provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 18

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

	MC	Q11	Q12	Q13	Q14	Q15	Q16	TOTAL
H1	/10	/4						/14
H2			/3					/3
H3				/3				/3
H4			/6		/7	/5		/18
H5		/11		/6		/10	/9	/36
H6			/6		/2			/8
H8					/2		/5	/7
H9				/6	/4		/1	/11
TOTAL	/10	/15	/15	/15	/15	/15	/15	/100

Section I

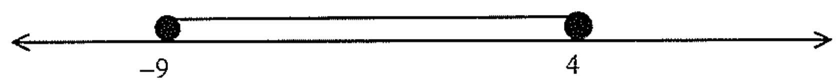
10 marks

Attempt Questions 1 – 10.

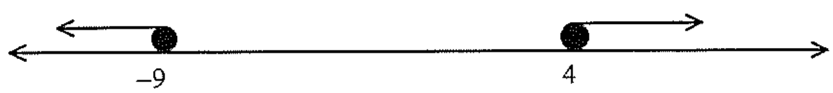
Allow about 15 minutes for this section.

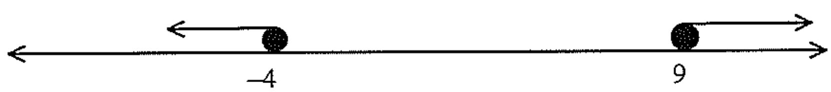
Use the multiple-choice answer sheet for Questions 1 – 10.

1. Which graph shows the solution to $|2x-5| \leq 13$?

(A) 

(B) 

(C) 

(D) 

2. Given that $f(x) = \dfrac{4x^5 - 8x}{x^3}$, what is the value of $f'(2)$?

(A) 2

(B) 8

(C) 12

(D) 18

3. For the parabola $x^2 = 8(y-1)$, which of the following statements is completely correct?

(A) The focal length is 2 and the vertex is (1,0)

(B) The focal length is 2 and the focus is (0,3)

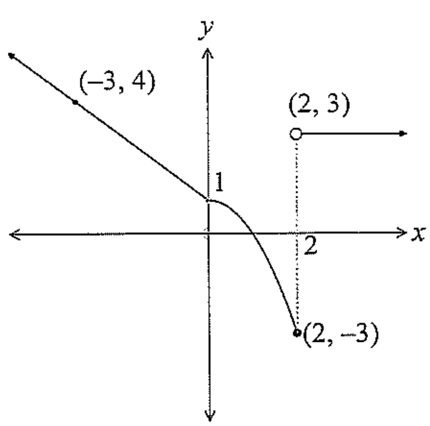
(C) The axis of symmetry is $y = 0$ and the focus is (4,0)

(D) The directrix is $y = -1$ and the focal length is 1.

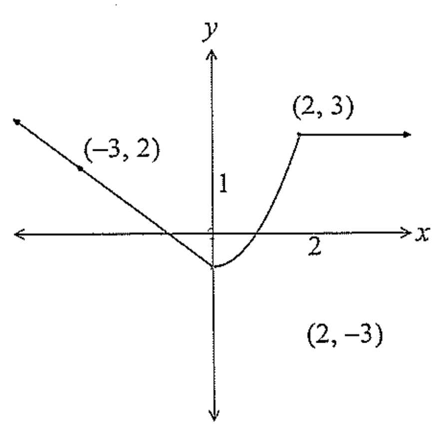
4. Which of the graphs would represent the function below?

$$\begin{cases} y = 1 - x & x < 0 \\ y = 1 - x^2 & 0 \le x \le 2 \\ y = 3 & x > 2 \end{cases}$$

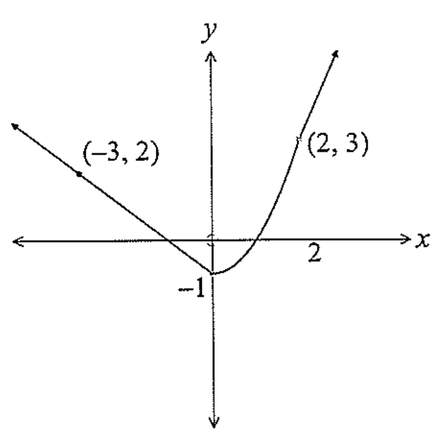
(A)



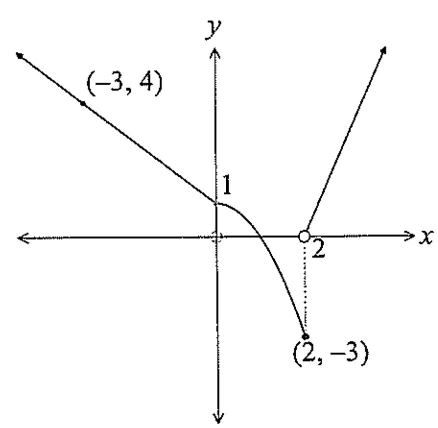
(B)



(C)



(D)



5. Which of the following is the same as cosec $(\pi + \theta)$?

(A) $\dfrac{-1}{\sin\theta}$

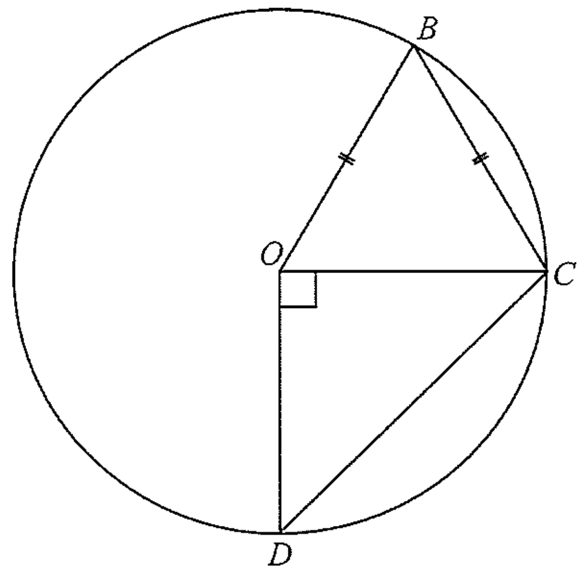
(B) $\dfrac{-1}{\cos\theta}$

(C) $\dfrac{1}{\cos\theta}$

(D) $\dfrac{1}{\sin\theta}$

6. In the diagram below, O is the centre of the circle, and B, C and D are points on the circumference.

$OB = BC$ and $\angle COD$ is a right angle.



What is the size of $\angle BCD$?

(A) 90°

(B) 105°

(C) 125°

(D) 150°

7. Which statement correctly describes the roots of $2x^2 + 4x - 5 = 0$

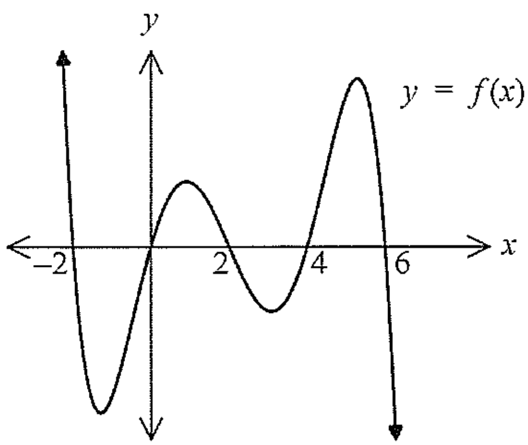
(A) The roots are equal, real and irrational.

(B) The roots are equal, real and rational.

(C) The roots are unequal, real and irrational.

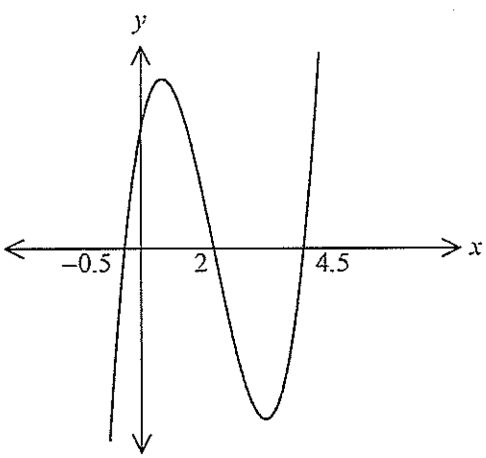
(D) The roots are unequal and unreal.

8. The graph of $y = f(x)$ is shown below.

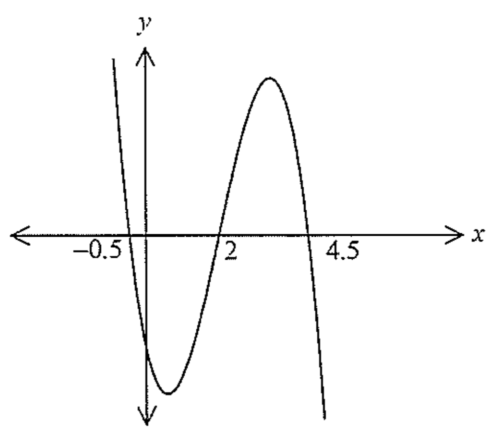


Which of these graphs could represent $y = f'(x)$?

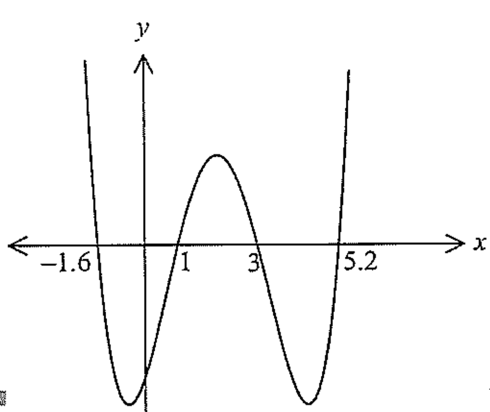
(A)



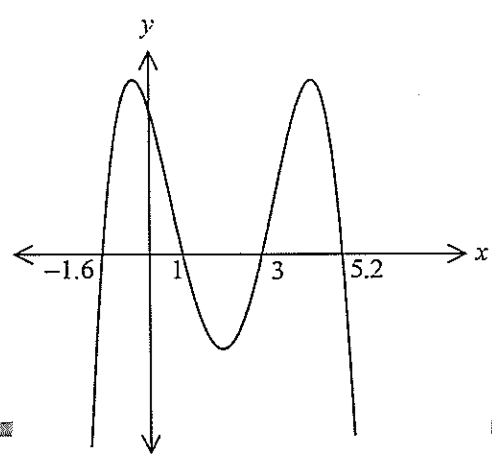
(B)



(C)



(D)



9. What is the value of $\int_{-2}^{2} |x| \, dx$?

(A) 0

(B) 2

(C) 4

(D) 8

10. The function $f(x) = x(x^2 - 9)$ is an

(A) odd function

(B) odd function with an axis of symmetry $x = 0$

(C) even function

(D) even function with an axis of symmetry $x = 0$

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 writing booklet.

(a) Expand and simplify $(2\sqrt{2} - \sqrt{3})(\sqrt{2} - \sqrt{3})$. **2**

(b) Simplify $\dfrac{a^4 - ab^3}{a^4 - a^2b^2}$. **2**

(c) Find the equation of the tangent to the curve $y = (x^2 - 8)^4$ at the point where $x = 3$. **2**

(d) Find $\displaystyle\int_2^4 \frac{6x^4 - 3x^3 - 1}{x^2}\,dx$. **3**

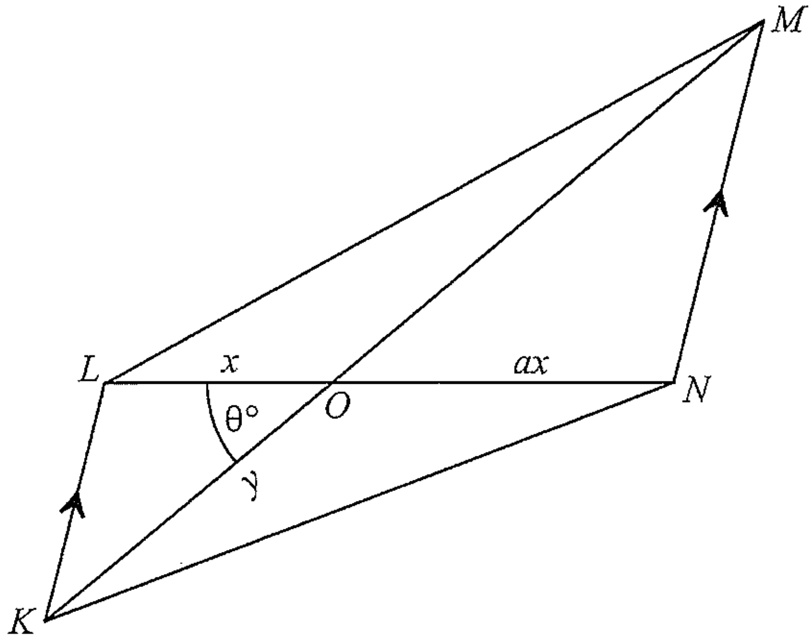
Question 11 continues on next page

Question 11 continued.

(e) $LMNK$ is a trapezium with $KL \parallel MN$.

$LO = x$ cm, $KO = y$ cm and $ON = ax$ cm.

$\angle LOK = \theta°$.



(i) Prove that $\Delta KOL \parallel\!\mid \Delta MON$. **2**

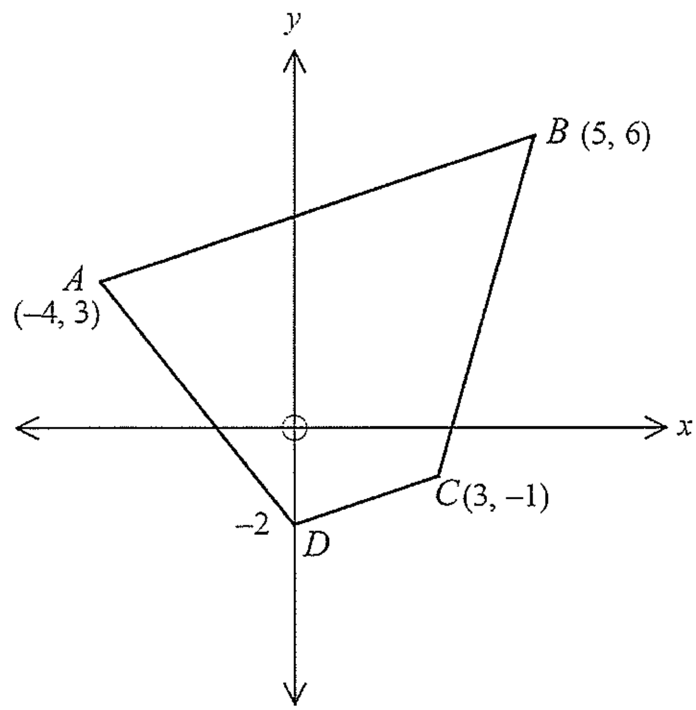
(ii) Hence or otherwise prove that Area ΔMOL = Area ΔNOK. **2**

(f) Find $\dfrac{d}{dx}(\sqrt{x}\, e^{x})$. **2**

End of Question 11

Question 12 (15 marks)

(a) A quadrilateral is formed by the points $A(-4, 3)$, $B(5, 6)$, $C(3, -1)$ and $D(0, -2)$ as shown in the diagram.



(i) Show that the quadrilateral is a trapezium, with $AB \parallel DC$. **2**

(ii) Show that the equation of AB is $x - 3y + 13 = 0$. **1**

(iii) Find the perpendicular distance from D to AB. **1**

(iv) Find the area of the trapezium $ABCD$. **2**

Question 12 continues on next page

Question 12 continued.

(b) For what values of k does the equation $(k + 6)x^2 - 2(2 + k)x + k + 2 = 0$ have two real unequal roots. **3**

(c) (i) Show that the curve $y = x^3 - 18x^2 + 60x$ passes through the origin and also has x intercepts at $x = 9 + \sqrt{21}$ and $x = 9 - \sqrt{21}$. **1**

(ii) Find the coordinates of all the stationary points and inflexion points on the curve $y = x^3 - 18x^2 + 60x$. **3**

(iii) Draw a neat half page sketch of the curve $y = x^3 - 18x^2 + 60$ showing all the features determined in parts (i) and (ii). **2**

End of Question 12

Question 13 (15 marks)

(a) Show that $\frac{d}{dx}(\tan^2 x \cos x) = \frac{\sin x(\cos^2 x + 1)}{\cos^2 x}$. **2**

(b) (i) For what value of k is $log_{10}(3x^2 - 2x) = \frac{log_e(3x^2-2x)}{k}$? **1**

Give your answer as an exact value.

(ii) For $f(x) = \log_{10}(3x^2 - 2x)$, find $f'(2)$. **2**

(c) Use Simpson's Rule with 5 function values to approximate $\int_1^9 (log_e x)^2\, dx$,

giving your answer correct to 2 significant figures. **4**

(d) On a number plane diagram show the region defined by the intersection of the inequalities: **2**

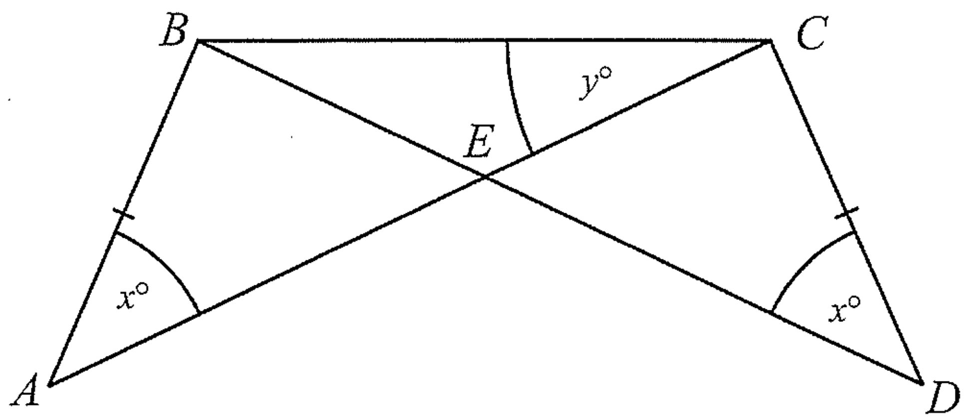
$$x^2 + y^2 < 16$$
$$y > 4 - x^2$$
$$x - y > 0$$

Label axes clearly, but it is not necessary to label all points of intersection between curves.

Question 13 continues on next page

Question 13 continued

(e) In the diagram below, $AB = CD$ and $\angle BAC = \angle CDB = x°$.
Also $\angle BCA = y°$.



(i) Prove that $\Delta ABE \equiv \Delta DCE$. **2**

(ii) Show that $\angle ABE = 180° - (x + 2y)°$. **2**

End of Question 13

Question 14 (15 marks)

(a) A particular curve passes through the point (2, 7). **2**

For this curve $\dfrac{dy}{dx} = 6e^{3x-6}$.

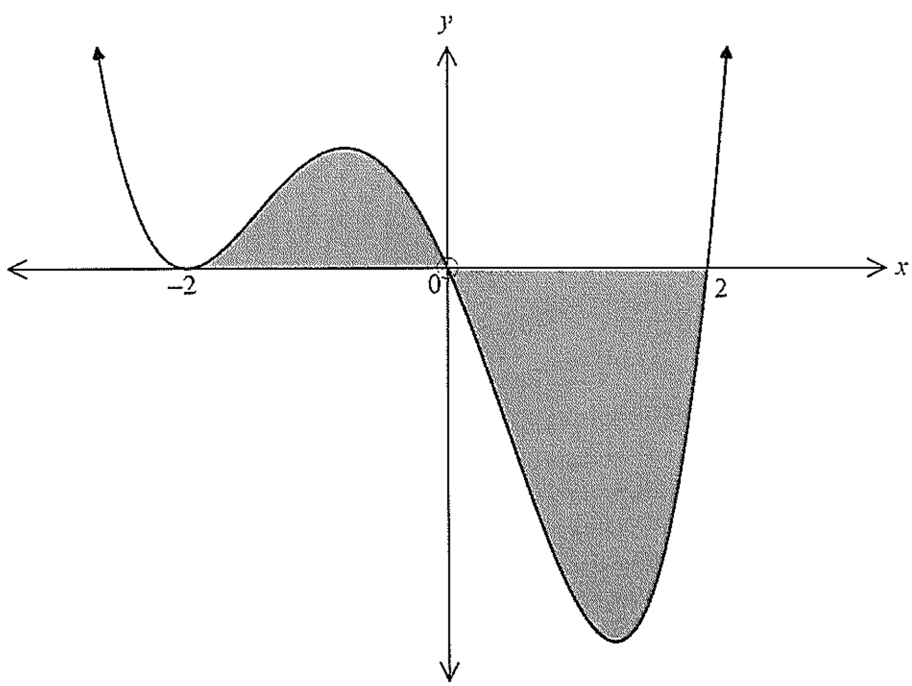
Write down the equation of the curve.

(b) (i) Find the exact values of u for which $2u^2 + \sqrt{3}\,u - 3 = 0$. **2**

(ii) Hence or otherwise solve $2\cos^2 x + \sqrt{3}\cos x - 3 = 0$ for $0 \le x \le 2\pi$. **2**

(c) The diagram below shows the curve $y = x^4 + 2x^3 - 4x^2 - 8x$. **2**

Calculate the shaded area.



Question 14 continues on next page

Question 14 continued

(d) The sum of the first two terms of a geometric series is 18 and the sum of the third and fourth terms of the series is 72. **3**

Show that there are two possible series which meet the criteria above and write down the first four terms of each series.

(e)

A man borrows \$10 000 at 15% p.a. reducible interest, and pays it off in 12 equal annual instalments at the end of each year for twelve years.

(i) What should his instalments be (to the nearest cent)? **3**

(ii) Determine the rate % p.a. simple interest charged on the loan, **1**

correct to 2 d.p.s

End of Question 14

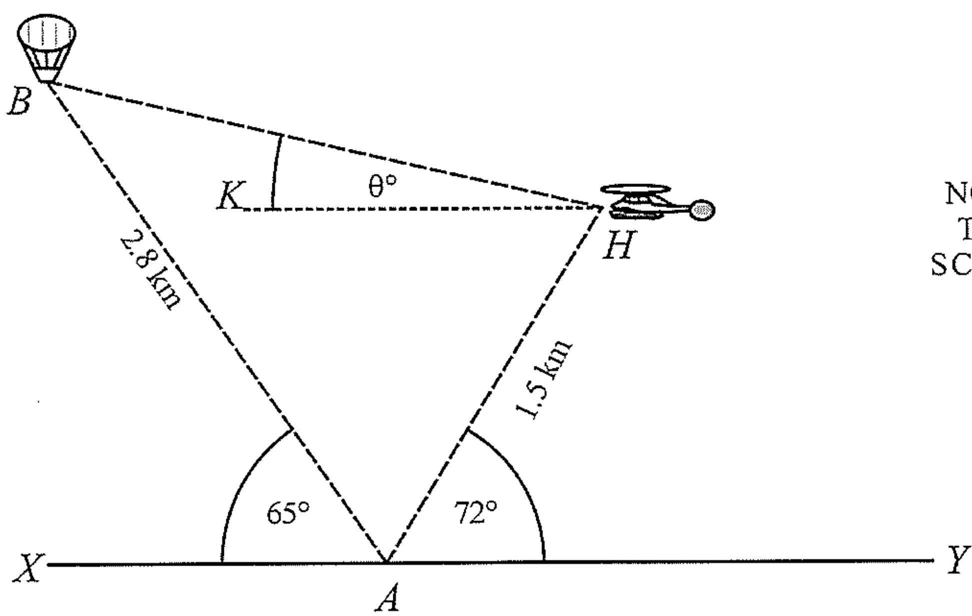
Question 15 (15 marks)

(a) Consider the series $S_n = -18 + (-15) + (-12) + \dots \dots + u_n$ **2**

For what value of n is $S_n = 0$?

(b) From a point A on level ground an observer sees a balloon B and a helicopter H which are both stationary at the time.

The balloon is positioned due west of point A, at a distance of 2.8 km on an angle of elevation of 65° and the helicopter is positioned due east of point A, at a distance of 1.5 km on an angle of elevation of 72°, as shown in the diagram.



NOT TO SCALE

(i) Show that the distance between the helicopter and the balloon is approximately 2.0 km. **1**

(ii) Calculate the angle of elevation of the balloon as seen from the helicopter (θ). Answer correct to the nearest degree. **2**

Question 15 continues on next page

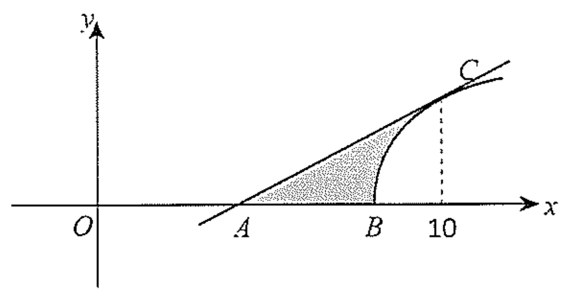
(c) Determine the absolute maximum and minimum values of y for the curve: 3

$y = e^2 + e^x - x\,e^2$ in the domain $0 \le x \le 5$.

(Give your answers as exact values.)

(d) A curve has equation $y = (2x - 16)^2$.

The diagram shows part of the curve and the tangent to it at the point C, where $x = 10$.



NOT TO SCALE

(i) Show that the equation of the tangent to this curve at the point C, where $x = 10$, is $x - 2y - 6 = 0$. **2**

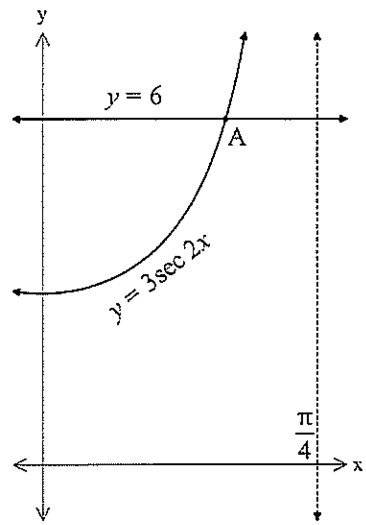
(ii) Find the coordinates of the points A and B. **2**

(iii) Calculate the shaded area ABC as shown on the diagram. **3**

Question 16 (15 marks)

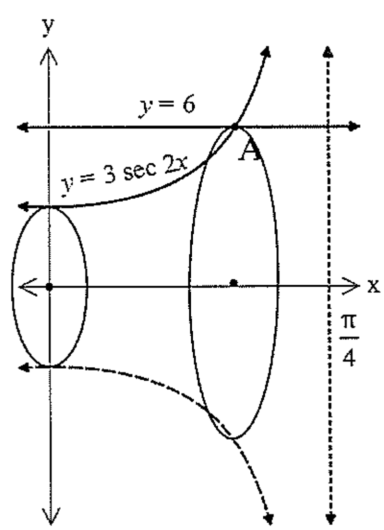
(a) Find the equation of the parabola with focus at (-2, 1) and having directrix with equation $x = 2$. **1**

(b) The graph below shows the line $y = 6$ and the curve $y = 3\sec 2x$ for $0 \leq x \leq \dfrac{\pi}{4}$.



(i) By solving the equation $3\sec 2x = 6$, show that the point A where the line and curve intersect has coordinates $\left(\dfrac{\pi}{6}, 6\right)$. **2**

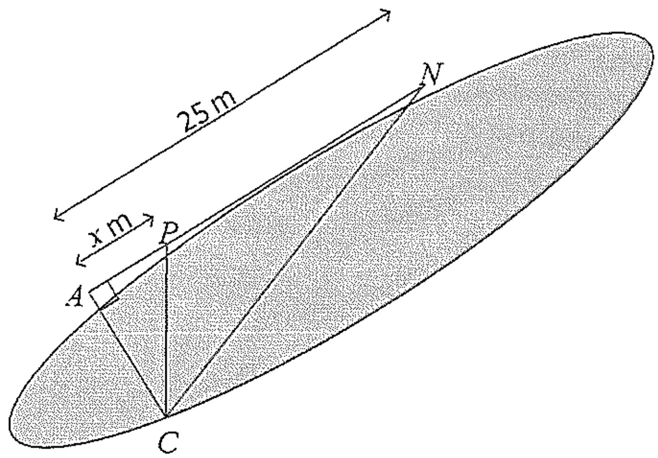
(ii) The region enclosed between the curve $y = 3\sec 2x$ and the x axis between $x = 0$ and $x = \dfrac{\pi}{6}$ is rotated about the x axis. **3**



Find the volume of the solid formed.

Question 16 continued

(c) A female crocodile, C, notices a man taking her eggs from her nest, N, on the opposite side of a billabong 25 m further along the bank, as shown in the diagram.



The time taken to reach the nest will be minimised if the crocodile swims to point P, x metres further along the billabong, and then covers the rest of the distance by land. The time taken, T, measured in tenths of a second, is given by

$$T = 13\sqrt{49 + x^2} + 12(25 - x)$$

(i) Calculate the time taken, in seconds, if the crocodile does the entire distance by water. Give your answer to the nearest tenth of a second. **2**

(ii) Calculate the time taken, in seconds, if the crocodile does the minimum distance by water. Give your answer to the nearest tenth of a second. **2**

(iii) Prove that the value of x which minimises the time taken is 16.8 m, and hence, calculate the minimum time possible, in seconds. Give your answer to the nearest tenth of a second. **5**

End of paper

Trial HSC Examination 2016

Mathematics Course

Name ____________________ Teacher ________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180^\circ$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

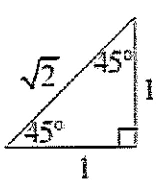
$$\sec\theta = \frac{1}{\cos\theta}$$

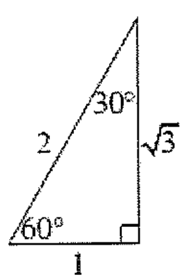
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios





Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

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*n*th term of an arithmetic series

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

*n*th term of a geometric series

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\frac{dy}{dx} = F'(u)\frac{du}{dx}$

If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\frac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\frac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\frac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

Teresa Q11 & Q12a,b.
Ken Q12(c) & Q13
Angela Q14 & Q15a,b,c,
Sandra Q15(d) & Q16

Trial HSC Examination 2016
Mathematics Course

Name ______________________ Teacher ________________

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A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

	A	B	C	D
1.	○	●	○	○
2.	○	○	○	●
3.	○	●	○	○
4.	●	○	○	○
5.	●	○	○	○
6.	○	●	○	○
7.	○	○	●	○
8.	○	○	○	●
9.	○	○	●	○
10.	●	○	○	○

Mark own M.C.

Q.11 & Q12(a)(b) 24 marks

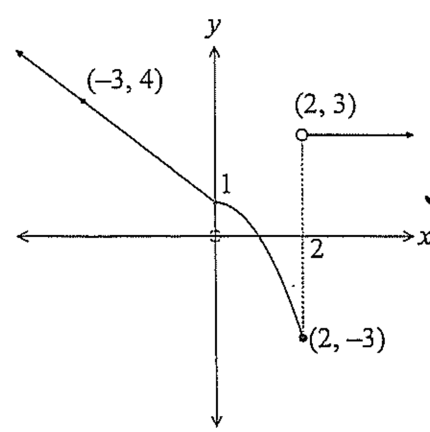
Q12(c) & Q13. 21 marks.

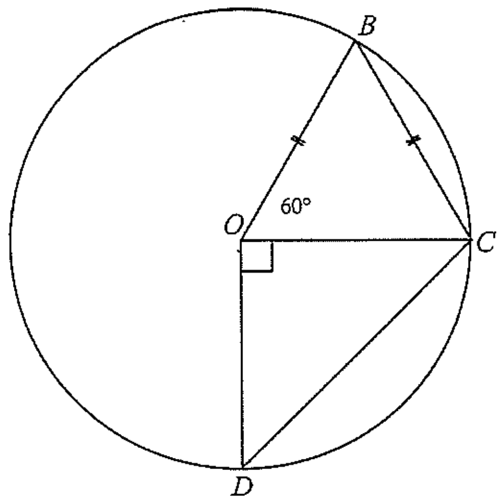
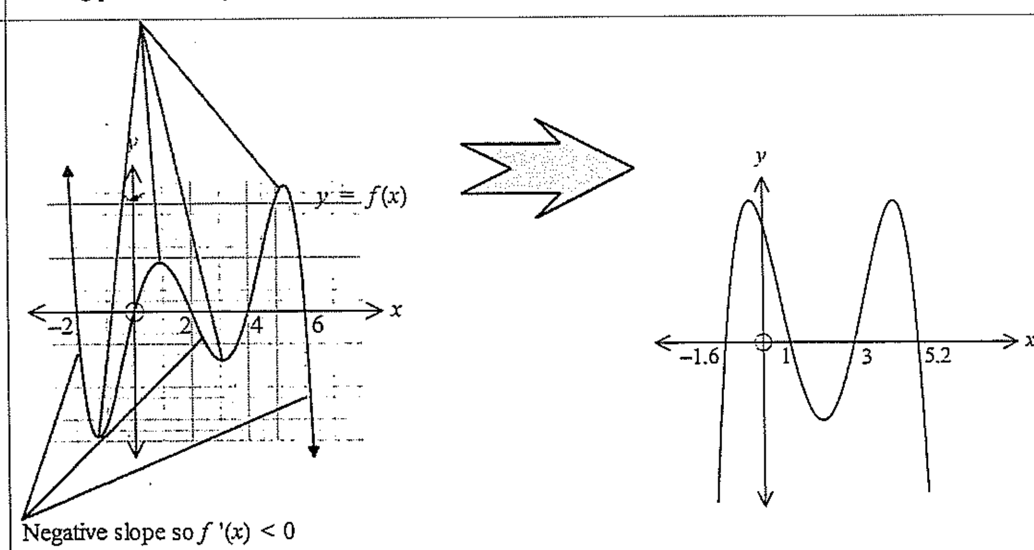
Q.14 & Q15 a,b,c. 23 marks

Q15(d) & Q16 22 marks.

BDBAA BCDCA

Multiple Choice Worked Solutions

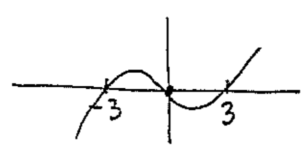
No	Working	Answer
1	$\lvert 2x-5\rvert \le 13$ $-13 \le 2x-5 \le 13$ $-8 \le 2x \le 18$ $-4 \le x \le 9$ 	B
2	$f(x) = \dfrac{4x^5-8x}{x^3}$ $= 4x^2 - 8x^{-2}$ $f'(x) = 8x + 16x^{-3}$ $f'(2) = 8(2) + 16(2)^{-3}$ $= 16 + \dfrac{16}{8}$ $= 18$	D
3	$x^2 = 8(y-1)$ $= 4(2)(y-1)$ focal length = 2, vertex = (0, 1), focus = (0, 3) H5, Band 3	B
4	Graph for $x < 0$ is a straight line with a negative gradient and intercept of 1 on y axis, It does not include upper domain endpoint but it is common with next section. Graph for $0 \le x \le 2$ is a is a parabola which is concave down and has an intercept of 1 on y axis, includes both endpoints. Graph for $x > 2$ is a horizontal straight line through 3 on y axis,. does not include lower domain endpoint. 	A
5	$\operatorname{cosec}(\pi+\theta) = \dfrac{1}{\sin(\pi+\theta)}$ $= -\dfrac{1}{\sin\theta}$	A

6	$OB = OC$ since both are radii, $\therefore$ $OB = OC = OB$ so $\Delta\ OBC$ is equilateral. $\therefore$ $\angle BOC = 60°$ ($\angle$ in equilateral Δ) $OC = OD$ (equal radii) $\therefore$ $\angle ODC = \angle OCD$ ($\angle$ in isosceles Δ) $\therefore$ $2 \times \angle OCD + 90° = 180°$ ($\angle$ sum $\Delta\ OCD$) $2 \times \angle OCD = 180° - 90° = 90°$ $\angle OCD = \frac{90°}{2} = 45°$ $\angle BCD = 60° + 45° = 105°$ (adjacent $\angle$)  	B
7	$2x^2 + 4x - 5 = 0$ $\Delta = b^2 - 4ac$ $= 4^2 - 4(2)(-5)$ $= 16 + 40$ $= 56$ (which is positive and not a perfect square.) $\therefore$ roots are unequal, real and irrational.	C
8	Turning points where $f\,'(x) = 0$  	D

<table>
<tr>
<td>9</td>
<td>$$\int_{-2}^{2} |x|\, dx = 2\int_{0}^{2} x\, dx = 2\left[\frac{x^2}{2}\right]_0^2 = 4$$</td>
<td>C</td>
</tr>
<tr>
<td>10</td>
<td>
$ln(2e) = ln(2) + \ln e = ln(2) + 1$
$ln(3e) = ln(3) + \ln e = ln(3) + 1$
<table>
<tr><td>x</td><td>e</td><td>$2e$</td><td>$3e$</td></tr>
<tr><td>$\ln(x)$</td><td>1</td><td>$\ln(2) + 1$</td><td>$\ln(3) + 1$</td></tr>
</table>
$$\begin{aligned}\int_{e}^{3e} ln x\, dx &\approx \frac{e}{3}(1 + 4(ln(2) + 1) + ln(3) + 1) \\ &\approx \frac{e}{3}(1 + 4ln(2) + 4 + ln(3) + 1) \\ &\approx \frac{e}{3}(4ln(2) + ln(3) + 6) \\ &\approx \frac{e}{3}(ln(2^4) + ln(3) + 6) \\ &\approx \frac{e}{3}(ln(16) + ln(3) + 6) \\ &\approx \frac{e}{3}(ln(16 \times 3) + 6) \\ &\approx \frac{e\,(ln(48) + 6)}{3}\end{aligned}$$
</td>
<td>D</td>
</tr>
</table>

10. $f(x) = x(x^2-9)$

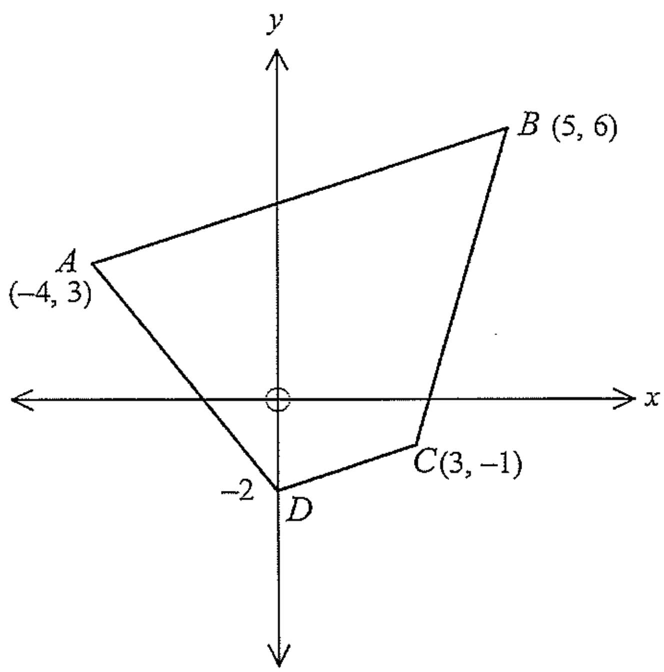
$f(-x) = -x(x^2-9) = -f(x)$ ∴ odd.



A

Question 11		**2016**	
	Solution	Marks	Allocation of marks
(a)	$(2\sqrt{2}-\sqrt{3})(\sqrt{2}-\sqrt{3}) = 2\sqrt{4}-2\sqrt{6}-\sqrt{6}+\sqrt{9}$ $= 4-3\sqrt{6}+3$ $= 7-3\sqrt{6}$	2	2 marks for correct answer 1 mark if either the expansion or simplification are correct with an error elsewhere.
(b)	$\dfrac{a^4-ab^3}{a^4-a^2b^2} = \dfrac{a(a^3-b^3)}{a^2(a^2-b^2)}$ $= \dfrac{a(a-b)(a^2+ab+b^2)}{a^2(a+b)(a-b)}$ $= \dfrac{a^2+ab+b^2}{a^2+ab}$	2	2 marks for correct answer 1 mark if at least one of the factorisations is correct and/or simplification are correct with an error elsewhere.
(c)	$y=(x^2-8)^4$ when $x = 3, y = ((3)^2-8)^4 = 1$ $y' = 4(x^2-8)^3.(2x)$ $= (8x)(x^2-8)^3$ When $x = 3, y' = 8 \times 3(9-8)^3 = 24$ Equation : $y-1 = 24(x-3)$ $y-1 = 24x-72$ $24x-y-71 = 0$	2	2 marks for correct answer 1 mark if either the differentiation or the method for finding the equation of the line are done correctly, with error elsewhere.
(d)	$\displaystyle\int_2^4 \frac{6x^4-3x^3-1}{x^2}\,dx = \int_2^4 \left(\frac{6x^4}{x^2}-\frac{3x^3}{x^2}-\frac{1}{x^2}\right)dx$ $\displaystyle= \int_2^4 6x^2-3x-x^{-2}\,dx$ $\displaystyle= \left[\frac{6x^3}{3}-\frac{3x^2}{2}-\frac{x^{-1}}{-1}\right]_2^4$ $\displaystyle= \left(128-24+\frac{1}{4}\right) - \left(16-6+\frac{1}{2}\right)$ $\displaystyle= 104\frac{1}{4}-10\frac{1}{2}$ $\displaystyle= 93\frac{3}{4}$	3	3 marks for correct value 2 marks for a solution which includes 2 of these Correct simplification prior to integration Finding the indefinite integral Substitution into the indefinite integral to obtain definite integral 1 mark for solution which includes at least one part of the above.

Question 11		2016	
	Solution	Marks	Allocation of marks
(e) i)	In ΔKOL and ΔMON $\angle KLO = \angle MNO$ (alt ang on || lines) $\angle LKO = \angle NMO$ (alt ang on || lines) $\angle KOL = \angle MON = \theta$ (Vert Opp ang) $\therefore \Delta KOL \parallel\!\mid \Delta MON$ (Corresponding ang equal)	2	2 marks for correct proof 1 mark for an incorrect or incomplete proof with some correct and relevant statements
e) ii)	Since $\Delta KOL \parallel\!\mid \Delta MON$ Corresponding sides are in the same ratio. $\frac{ON}{LO} = \frac{ax}{x} = a$ $\therefore \frac{MO}{KO} = \frac{MO}{y} = a$ $\therefore MO = ay$ Area $\Delta MOL = \frac{1}{2}LO \,.\, MO.\sin MON$ $= \frac{1}{2}.\, x \,.ay.\sin \theta$ $= \frac{axy\sin\theta}{2}$ Area $\Delta NOK = \frac{1}{2}ON \,.\, OK.\sin KON$ $= \frac{1}{2}.\, ax.y.\sin\theta$ $= \frac{axy\sin\theta}{2}$ $=$ Area ΔMOL	2	2 marks for correct proof 1 mark for an incorrect or incomplete proof with some correct and relevant statements
(f)	$\frac{d}{dx}(\sqrt{x}\,.e^x) = \frac{d}{dx}\left(x^{\frac{1}{2}}.\,e^x\right)$ $= \left(x^{\frac{1}{2}}\right)(e^x) + (e^x)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$ $= \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)(e^x)$ $= \frac{e^x(1+2x)}{2\sqrt{x}}$ $= \frac{e^x + 2xe^x}{2\sqrt{x}}$	2	2 marks for any of the last three lines of working or other simplified equivalent expression. 1 mark for a solution which shows correct use of product rule and differentiation applied to the individual components, with a minor error in one component, or correct differentiation of components, with an error in use of product rule, or similar merit.

Question 12		2016	
	Solution	Marks	Allocation of marks
(a)	First term $a = -18$ Common difference $d = 3$ Number of terms $n = ?$ $S_n = 0$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $0 = \frac{n}{2}(2 \times -18 + (n-1) \times 3)$ $0 = n \times (-36 + 3n - 3)$ $0 = n(3n - 39)$ $\therefore\ n = 0$ or $3n = 39$ $n = 0$ is not a solution $3n = 39$ $n = 13$ So 13 terms are needed to give a sum of 0.	2	2 marks for correct value of n 1 mark for worked solution which finds an incorrect value for a, or d but then substitutes and solves correctly to find a value for n Or 1 mark for worked solution which finds correct values for a, and d and has an error in substitution and solution to find a value for n
(a) i)	 $m_{AB} = \frac{6-3}{5--4}$ $= \frac{3}{9} = \frac{1}{3}$ $m_{DC} = \frac{-1--2}{3-0}$ $= \frac{1}{3}$ $\therefore AB \parallel DC$ $\therefore ABCD$ is a trapezium.	2	2 marks for correct answer 1 mark if only one of the gradients is calculated correctly or if a similar error is made in the solution.

Question 12		2016	
	Solution	Marks	Allocation of marks
(a) ii)	Equation AB using $m_{AB} = \frac{1}{3}$ and point $(-4, 3)$ $y - 3 = \frac{1}{3}(x + 4)$ $3y - 9 = x + 4$ $x - 3y + 13 = 0$	1	1 mark for correct answer {can also use the point (6, 5)}
(a) iii)	$D = (x_1, y_1) = (0, -2)$ $d = \left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\rvert$ $= \left\lvert \frac{1 \times 0 - 3 \times (-2) + 13}{\sqrt{1^2 + 3^2}} \right\rvert$ $= \left\lvert \frac{19}{\sqrt{10}} \right\rvert$ $= \frac{19}{\sqrt{10}}$	1	1 mark for correct answer
(a) iv)	$AB = \sqrt{(6-3)^2 + (5+4)^2}$ $= \sqrt{(3)^2 + (9)^2}$ $= \sqrt{9 + 81}$ $= \sqrt{90}$ $= 3\sqrt{10}$ $DC = \sqrt{(3-0)^2 + (-1+2)^2}$ $= \sqrt{(3)^2 + (1)^2}$ $= \sqrt{9 + 1}$ $= \sqrt{10}$ Area $= \frac{h}{2}(a + b)$ $= \frac{1}{2} \times \frac{19}{\sqrt{10}}(3\sqrt{10} + \sqrt{10})$ $= \frac{19}{2\sqrt{10}}(4\sqrt{10})$ $= 38$ sq units	2	2 marks for correct answer 1 mark if only one of the distances is calculated correctly or if an error is made in the calculation of the area.

Question 12		2016	
	Solution	Marks	Allocation of marks
(c) (i)	$y = x^3 - 18x^2 + 60x$ x intercepts where $y = 0$ $x^3 - 18x^2 + 60x = 0$ $x(x^2 - 18x + 60) = 0$ x intercept when $x = 0$ and $y = 0^3 - 18 \times 0^2 + 60 \times 0 = 0$ hence there is an intercept at the origin. Other x intercepts when $x^2 - 18x + 60 = 0$ $x = \dfrac{18 \pm \sqrt{324 - 240}}{2 \times 1}$ $= \dfrac{18 \pm 2\sqrt{21}}{2}$ $= 9 \pm \sqrt{21}$ $x \approx 4.1$ and 13.6	1	1 mark for solving correctly the equation $x^3 - 18x^2 + 60x = 0$

(b) $(k+6)x^2 - 2(2+k)x + k + 2 = 0$

$\Delta > 0$ for 2 unequal roots

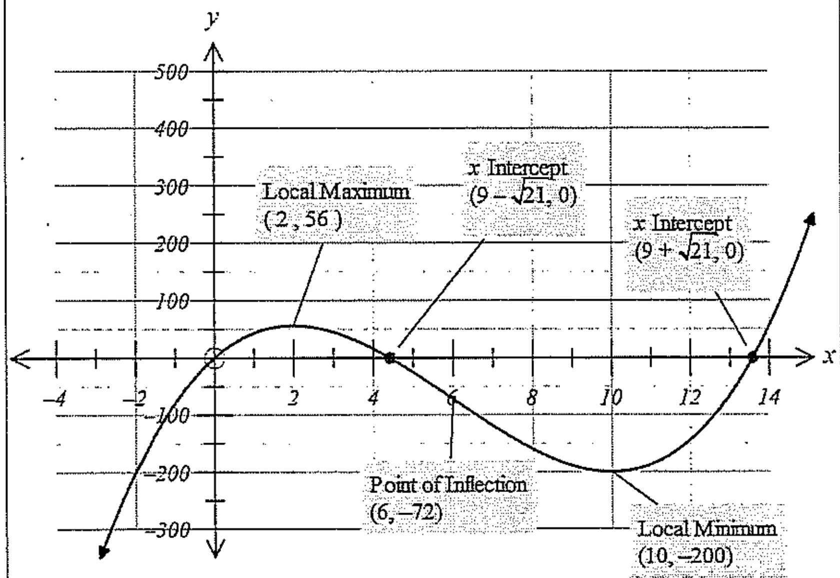
i.e. $\Delta = 4(2+k)^2 - 4(k+6)(k+2)$

$= 4(k+2)\left[(k+2) - (k+6)\right]$

$= 4(k+2)(-4)$

$= -16(k+2) > 0$ when $k+2 < 0$ i.e. $k < -2$

Question 12		2016	
	Solution	Marks	Allocation of marks
(c) (ii)	$y = x^3 - 18x^2 + 60x$ $\frac{dy}{dx} = 3x^2 - 36x + 60$ $= 3(x^2 - 12x + 20)$ $= 3(x-10)(x-2)$ $\frac{d^2y}{dx^2} = 6x - 36$ $= 6(x-6)$ Turning points when $\frac{dy}{dx} = 0$ $3(x-10)(x-2) = 0$ $x = 2$ and 10 $x = 2, y = 2^3 - 18 \times 2^2 + 60 \times 2 = 56$ $x = 10, y = 10^3 - 18 \times 10^2 + 60 \times 10 = -200$ Stationary points are $(2, 56)$, $(10, -200)$ Determine nature $x = 2, \frac{d^2y}{dx^2} = 6 \times 2 - 36 = -24 \therefore$ concave down $x = 10, \frac{d^2y}{dx^2} = 6 \times 10 - 36 = 24 \therefore$ concave up Local maximum at $(2, 56)$ and local minimum at $(10, -200)$ Inflexion where $\frac{d^2y}{dx^2} = 0$ $6x - 36 = 0$ $6x = 36$ $x = 6, y = -72.$ Since there is a change in concavity between $x = 2$ and $x = 10$, there is an inflexion at $(6, -72)$	3	3 marks for finding both the local maxima and minima and their nature and the inflexion. 2 marks for a solution that is missing one of the above, or has only a minor error in calculation of the above. 1 mark for solution that finds the first and second derivatives at least

Question 12		2016	
	Solution	Marks	Allocation of marks
(c) (iii)		2	2 mark for accurate sketch showing all features 1 mark for an untidy sketch which does not clearly indicate the features or is missing some features, or is incorrect in concavity.

Question 13		**2016**	
	Solution	Marks	Allocation of marks
(a)	**Method 1** $y = \tan^2 x.\cos x$ $\frac{dy}{dx} = \tan^2 x.(-\sin x) + \cos x(2\tan x .\sec^2 x)$ $= \frac{\sin^2 x}{\cos^2 x}(-\sin x) + \cos x\left(\frac{2\sin x}{\cos x}\right)\left(\frac{1}{\cos^2 x}\right)$ $= \frac{-\sin^3 x}{\cos^2 x} + \frac{2\sin x}{\cos^2 x}$ $= \frac{\sin x(2-\sin^2 x)}{\cos^2 x}$ $= \frac{\sin x(\cos^2 x + 1)}{\cos^2 x}$ **Method 2** $y = \tan^2 x.\cos x$ $y = \frac{\sin^2 x}{\cos^2 x}\cos x$ $= \frac{\sin^2 x}{\cos x}$ $\frac{dy}{dx} = \, = \frac{\cos x.\,2\sin x.\cos x - \sin^2 x.(-\sin x)}{\cos^2 x}$ $= \frac{\sin x(2\cos^2 x + \sin^2 x.)}{\cos^2 x}$ $= \frac{\sin x(\cos^2 x + \cos^2 x + \sin^2 x.)}{\cos^2 x}$ $= \frac{\sin x(\cos^2 x + 1)}{\cos^2 x}$ $= \frac{\sin x(1+\cos^2 x)}{\cos^2 x}$	2	2 marks for deriving the correct result 1 mark if derivative done correctly, but unable to obtain result required, 1 mark if a minor error in differentiation makes it impossible to obtain result, but a reasonable attempt is made to do so.
(b) (i)	$\log_{10}(3x^2-2x) = \frac{\log_e(3x^2-2x)}{\log_e(10)} = \frac{\ln(3x^2-2x)}{k}$ where $k = \ln(10)$.	1	1 mark for correct value of k

Question 13		2016	
	Solution	Marks	Allocation of marks
(b) (ii)	$f(x) = log_{10}(3x^2 - 2x)$, $= \dfrac{ln(3x^2 - 2x)}{k}$ $f'(x) = \dfrac{1}{k}\left(\dfrac{6x-2}{3x^2-2x}\right)$ $= \dfrac{1}{ln(10)}\left(\dfrac{6x-2}{3x^2-2x}\right)$ $f'(2) = \dfrac{1}{ln(10)}\left(\dfrac{10}{8}\right)$ $= \dfrac{10}{8ln(10)}$	2	2 marks for correct answer. 1 mark for a solution which shows some correct use of differentiation of log function.

(c) **Outcomes assessed: H8**

Marking Guidelines

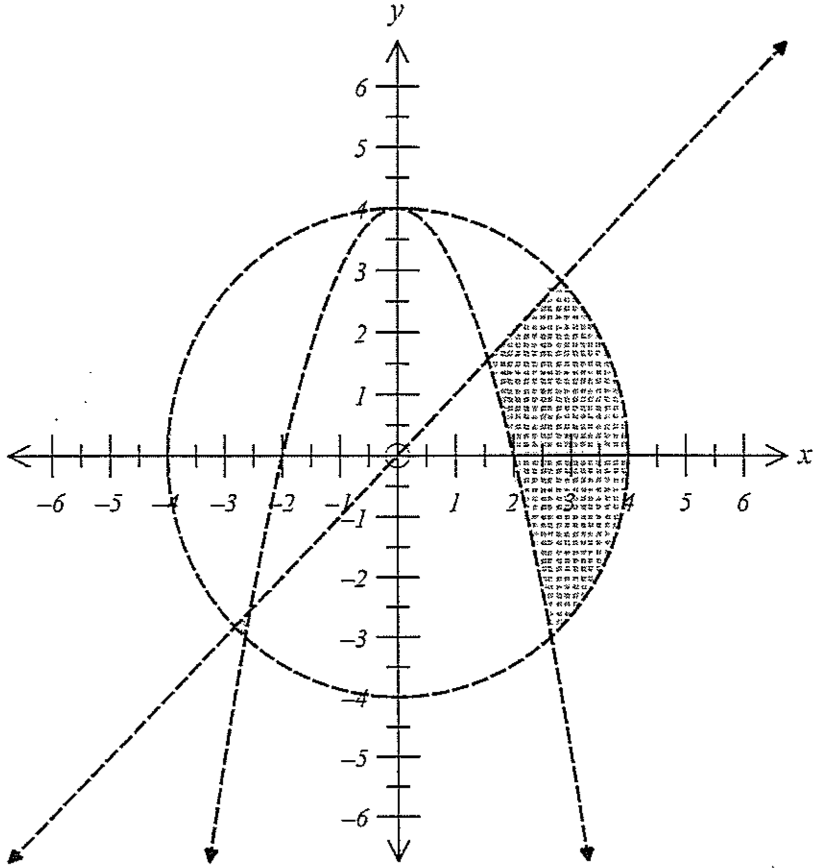
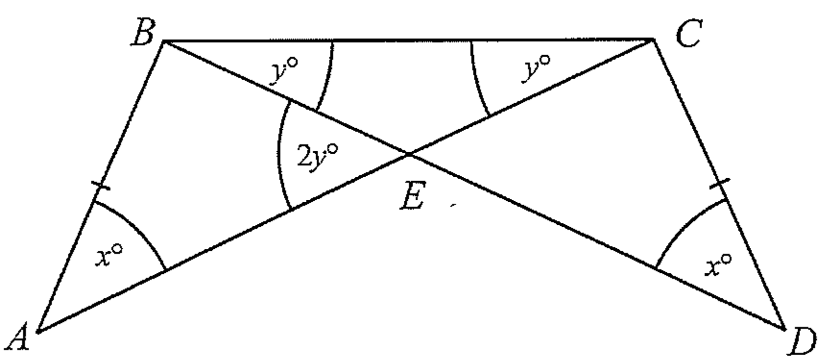
Criteria	Marks
• uses correct x values and value of h	1
• substitutes correctly into formula	1
• calculates correctly	1
• makes no intermediate rounding errors and expresses approximation to 2 significant figures	1

Answer

x	1	3	5	7	9
$f(x)$	0	$(\ln 3)^2$	$(\ln 5)^2$	$(\ln 7)^2$	$(\ln 9)^2$
$\times$	1	4	2	4	1

$h = 2$

$$\int_1^9 (\log_e x)^2\,dx \approx \tfrac{2}{3}\left\{0 + 4\times(\ln 3)^2 + 2\times(\ln 5)^2 + 4\times(\ln 7)^2 + (\ln 9)^2\right\}$$
$$\approx 19\cdot 988$$
$$\approx 20 \quad (to\ 2\ sig.\ fig.)$$

Question 13		**2016**	
	Solution	Marks	Allocation of marks
(d)		2	2 marks for correct graph with shading shown correctly. 1 mark for graph which has one of the graphs drawn incorrectly, or has correct graphs with incorrect region shaded.
(e)	i)  In $\Delta\,ABE$ and $\Delta\,DCE$ $\angle A = \angle D = x^\circ$ (given) $AB = CD$ (given) $\angle BEA = \angle CED$ (vertically opposite $\angle$) $\therefore \Delta\,ABE \equiv \Delta\,DCE$ (AAS)	2	2 marks for correct proof 1 mark for an incorrect or incomplete proof with some correct and relevant statements
	ii) $BE = CE$ (corresponding sides of congruent Δs) $\angle\,EBC = \angle\,ECB = y^\circ$ ($\Delta\,EBC$ is isosceles from above) $\angle\,BEA = 2y^\circ$ (exterior angle of $\Delta\,EBC$) $\angle\,ABE = 180^\circ - x - 2y$ ($\angle$ sum $\Delta\,ABE$) $\angle\,ABE = 180^\circ - (x + 2y)^\circ$	2	2 marks for correct proof 1 mark for an incorrect or incomplete proof with some correct and relevant statements

Question 14		2016	
	Solution	Marks	Allocation of marks
(a)	$\frac{dy}{dx} = 6e^{3x-6}$ $y = 6\int e^{3x-6}\,dx$ $y = 6 \times \frac{1}{3} e^{3x-6} + C$ $y = 2e^{3x-6} + C$ When $x = 2, y = 7$ $7 = 2e^{3\times 2-6} + C$ $7 = 2e^{0} + C$ $7 = 2 + C$ $C = 5$ $y = 2e^{3x-6} + 5$	2	2 marks for correct equation for y. 1 mark if valid attempt at solution which has a minor error in calculations, differentiation or algebra, or which is correct to a point but incomplete.
(b) (i)	$2u^2 + \sqrt{3}\,u - 3 = 0$ $u = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $= \frac{-(\sqrt{3}) \pm \sqrt{3-4(2)(-3)}}{2(2)}$ $= \frac{-(\sqrt{3}) \pm \sqrt{27}}{4}$ $= -\frac{\sqrt{3}}{4} \pm \frac{3\sqrt{3}}{4}$ $u = \frac{\sqrt{3}}{2}, -\sqrt{3}$	2	2 marks for 2 correct exact solutions for u. 1 mark if valid attempt at solution with an error in calculation or algebra including giving extra incorrect answers.
(ii)	$2\cos^2 x + \sqrt{3}\cos x - 3 = 0$ Let $u = \cos x$ So, from part i) $\cos x = \frac{\sqrt{3}}{2},$ $x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ or $\cos x = -\sqrt{3}$ No solution Solutions are $x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$	2	2 marks for exactly 2 correct solutions for x. 1 mark if valid attempt at solution with an error in calculation or algebra, or for answers in the wrong quadrants, including giving extra incorrect answers.

Question 14		2016	
	Solution	Marks	Allocation of marks
(c)	$\text{Area} = \left\lvert \int_{-2}^{0} x^4 + 2x^3 - 4x^2 - 8x \; dx \right\rvert + \left\lvert \int_{0}^{2} x^4 + 2x^3 - 4x^2 - 8x \; dx \right\rvert$ $= \left\lvert \left[\frac{x^5}{5} + \frac{x^4}{2} - \frac{4x^3}{3} - 4x^2 \right]_{-2}^{0} \right\rvert + \left\lvert \left[\frac{x^5}{5} + \frac{x^4}{2} - \frac{4x^3}{3} - 4x^2 \right]_{0}^{2} \right\rvert$ $= \left\lvert \left[(0) - \left(-\frac{32}{5} + 8 + \frac{32}{3} - 16 \right) \right] \right\rvert + \left\lvert \left[\left(\frac{32}{5} + 8 - \frac{32}{3} - 16 \right) - (0) \right] \right\rvert$ $= \left\lvert 3\frac{11}{15} \right\rvert + \left\lvert -12\frac{4}{15} \right\rvert$ $= 16$ sq units	2	2 marks for correct answer 1 mark for worked solution which has a minor error such as failing to use absolute values.
(d)	**Method 1** $u_1 + u_2 = 18$ $a + ar = 18$ $a(1 + r) = 18 \quad \ldots (1)$ $u_3 + u_4 = 72$ $ar^2 + ar^3 = 72$ $ar^2(1 + r) = 72 \quad \ldots. (2)$ $\frac{ar^2(1+r)}{a(1+r)} = \frac{72}{18} \quad (2) \div (1)$ $r^2 = 4$ $r = \pm 2$ Case A $r = 2$ $a(1 + 2) = 18$ sub in (1) $3a = 18$ $a = 6$ So when $r = 2$, $a = 6$ 1*st* four terms of Series Aare 6, 12, 24, 48 Case *B* $r = -2$ $a(1 - 2) = 18$ $-a = 18$ $a = -18$ So when $r = -2$, $a = -18$ 1*st* four terms of Series *B* are –18, 36, –72, 144 (Alternate method on next page)	3	3 marks for four terms of two series which meet the criteria 2 marks for only one series which is correct, or two incorrect series which result from only a single minor error. 1 mark for an attempt at solution which includes some basic correct working such as substituting into the correct formulas for geometric series.

Question 14		2016	
	Solution	Marks	Allocation of marks
(d)	**Method 2** $S_2 = 18$ $\dfrac{a(r^2-1)}{r-1} = 18 \quad \ldots (1)$ $u_3 + u_4 = 72$ $S_4 - S_2 = 72$ $\dfrac{a(r^4-1)}{r-1} - 18 = 72$ $\dfrac{a(r^4-1)}{r-1} = 90 \ldots\ (2)$ $\dfrac{r^4-1}{r^2-1} = \dfrac{90}{18} \quad (2) \div (1)$ $\dfrac{(r^2+1)(r^2-1)}{r^2-1} = 5$ $r^2+1=5$ $r^2=4$ $r=\pm 2$ Case A $r = 2$ $\dfrac{a(2^2-1)}{2-1} = 18$ sub $r = 2$ in (1) $3a = 18$ $a = 6$ So when $r = 2$, $a = 6$ 1*st* four terms of Series A are 6, 12, 24, 48 Case *B* $r = -2$ $\dfrac{a((-2)^2-1)}{(-2)-1} = 18$ sub $r = -2$ in (1) $a\left(\dfrac{3}{-3}\right) = 18$ $-a = 18$ $a = -18$ So when $r = -2$, $a = -18$ 1*st* four terms of Series *B* are -18, 36, -72, 144		

Q14(e)

(i) Let A be the amount owing after n years, and R the amount of each repayment.

$$A_1 = 10000(1.15) - R$$

$$A_2 = (10000(1.15) - R) \times 1.15 - R$$
$$= 10000(1.15)^2 - R(1.15 + 1)$$

$$A_3 = \left(10000(1.15)^2 - R(1 + 1.15)\right) \times 1.15 - R$$
$$= 10000(1.15)^3 - R(1 + 1.15 + 1.15^2)$$

$\vdots$

$$A_{12} = 10000(1.15)^{12} - R(1 + 1.15 + 1.15^2 + \dots + 1.15^{11})$$

$= 0$ after 12 years

$\therefore 10000(1.15)^{12} = R\left(\frac{a(r^n - 1)}{r-1}\right)$ where $a = 1$ $r = 1.15$ $n = 12$

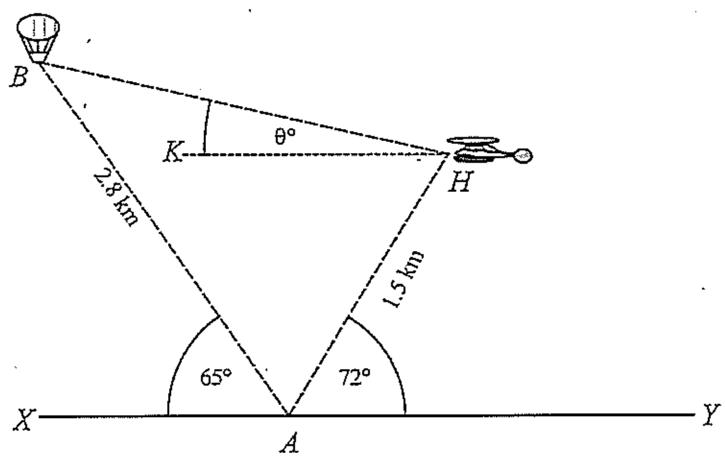
$$= R\left(\frac{1.15^{12} - 1}{0.15}\right)$$

$\therefore R = \dfrac{10000(1.15)^{12} \times 0.15}{1.15^{12} - 1} = \$1844.81^{¢}$

(ii) For simple interest rate:

Total Interest Paid $= 1844.81 \times 12 - 10000 = 12137.72$

$\therefore R = \dfrac{100 \times 12137.72}{10000 \times 12} = 10.11\%$ p.a.

Question 15		**2016**	
	Solution	Marks	Allocation of marks
(a)	First term $a = -18$ Common difference $d = 3$ Number of terms $n = ?$ $S_n = 0$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $0 = \frac{n}{2}(2 \times -18 + (n-1) \times 3)$ $0 = n \times (-36 + 3n - 3)$ $0 = n(3n - 39)$ $\therefore \; n = 0$ or $3n = 39$ $n = 0$ is not a solution $3n = 39$ $n = 13$ So 13 terms are needed to give a sum of 0.	2	2 marks for correct value of n 1 mark for worked solution which finds an incorrect value for a, or d but then substitutes and solves correctly to find a value for n Or 1 mark for worked solution which finds correct values for a, and d and has an error in substitution and solution to find a value for n
(b) (i)	 $\angle BAH = 180 - 72 - 65 = 43°$ (supplementary ∠) $HB^2 = BA^2 + AH^2 - 2 \times BA \times AH \times \cos \angle BAH$ (Cos Rule) $HB^2 = 2.8^2 + 1.5^2 - 2 \times 2.8 \times 1.5 \times \cos 43°$ $= 3.9466....$ $HP = \sqrt{3.9466..}$ $= 1.9866....$ $= 2.0$ km (2 s.f)	1	1 mark for correct working to achieve the answer required.

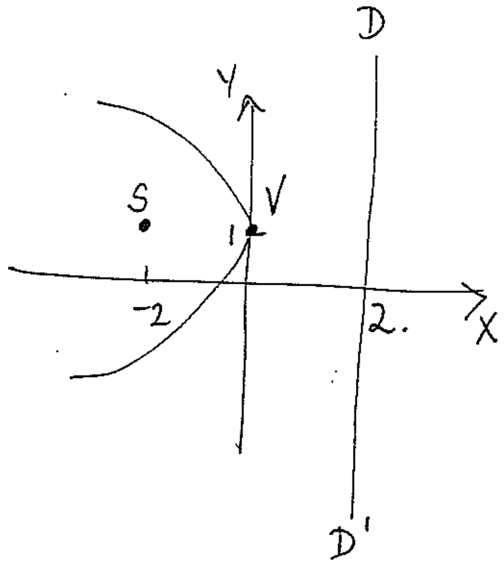
Question 15		2016	
	Solution	Marks	Allocation of marks
(b) (ii)	In ΔBAH $cosB = \dfrac{AB^2 + BH^2 - AH^2}{2 \times AB \times BH}$ $= \dfrac{2.8^2 + 2.0^2 - 1.5^2}{2 \times 2.8 \times 2.0}$ $= 0.85722..$ $B = cos^{-1}(0.85722...)$ $= 30.993826138853172244588323111713$ $= 31°$ (nearest degree) $\angle BHA = 180 - 43 - 31 = 106°$ $\angle KHA = 72°$ (alt angles) $\angle KHB = \theta = 106 - 72 = 34°$ Angle of elevation $\theta = 34°$	2	2 marks for correct answer. 1 mark for a valid attempt at a solution which has a minor error or is not quite complete.

Question 15		2016	
	Solution	Marks	Allocation of marks
(c)	$y = e^2 + e^x - e^2 x$ $\dfrac{dy}{dx} = e^x - e^2$ $\dfrac{dy}{dx} = 0$ when $e^x - e^2 = 0$ $e^x = e^2$ $x = 2, y = e^2 + e^2 - 2e^2 = 0$ $\dfrac{d^2y}{dx^2} = e^x$ and since $e^x > 0$ for all x, the curve is always concave up. Hence the point (2,0) is a minimum turning point, and hence $y = 0$ is the absolute minimum. Absolute maximum, compare the end points of the domain. $x = 0, y = e^2 + e^0 - 0.e^2 = e^2 + 1 - 0 = e^2 + 1 \approx 8.4$ $x = 5, y = e^2 + e^5 - 5e^2$ $= e^5 - 4e^2 \approx 118.8$ (This is Absolute maximum) Absolute minimum is $y = 0$ when $x = 2$ and absolute maximum $y = e^5 - 4e^2$, when $x = 5$.	3	3 marks for a solution which has the correct absolute maximum and minimum and which has verified that they are the absolute values. 2 marks for a solution that is missing one of the above, or has only a minor error in calculation of the above, or which has at least found the first derivative an attempted to find maximum and minimum. 1 mark for solution that finds the first derivative or finds the values at the bounds.

(d) (i) $y = (2x-16)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{(2x-16)^{\frac{1}{2}}}$ $x = 10, \frac{dy}{dx} = m = \frac{1}{2}, y = 2$ $y - 2 = \frac{1}{2}(x-10)$ $x - 2y - 6 = 0$	H5, Band 5 • Gives the correct proof . . . . 2 • Correct differentiation 1
(ii) Point A: $x - 2y - 1 = 0$ If $y = 0$ and $x = 1$: $A(6, 0)$ Point B: $y = (2x-16)^{\frac{1}{2}}$ If $y = 0$, $x = 8$: $B(8, 0)$ Take 1 mark off if pts not given in correct pt. form.	P5, Band 3 • Gives both correct solutions . . . . 2 • Gives one correct solution 1
(iii) area $= \frac{1}{2} \times 4 \times 2 - \int_8^{10} (2x-16)^{\frac{1}{2}} dx$ $= 4 - \left(\frac{(2x-16)^{\frac{3}{2}}}{2 \times \frac{3}{2}} \right)_8^{10}$ $= 4 - \left(\frac{8}{3} - 0 \right)$ $= 1\frac{1}{3}$ units^2	H8, Band 5 • Gives the correct solution . . . . 3 • Correct integration . . . . 2 • Correct first step, area of triangle and integrand 1

Question 16	2016

Q16(a)



vertex is at $(0,1)$. $a=2$

$(y-1)^2 = -8x$

(b) i)	$y = 6$ $y = 3\sec 2x$ $3\sec 2x = 6$ $\sec 2x = 2$ $\cos 2x = \frac{1}{2}$ $2x = \frac{\pi}{3}, \frac{5\pi}{3} \quad \{0 \le 2x \le 2\pi\}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \quad \{0 \le x \le \pi\}$ Point of intersection is $\left(\frac{\pi}{6}, 6\right) \quad \left\{0 \le x \le \frac{\pi}{4}\right\}$	2	2 marks for correct worked solution of the equation which results in the point required. 1 mark for solution that has some relevant and correct algebraic and or trigonometric reasoning

Question 16		2016	
	Solution	Marks	Allocation of marks
(b) ii)	Find the volume enclosed between $y = 3\sec 2x$ and x axis from $x = 0$ to $x = \frac{\pi}{6}$. Volume $= \pi \int y^2 \; dx$ $= \pi \int_0^{\frac{\pi}{6}} (3\sec \; 2x)^2 \; dx$ $= \pi \int_0^{\frac{\pi}{6}} 9 \; \sec^2 2x \; dx$ $= \pi \left[\frac{9}{2} \tan 2x \right]_0^{\frac{\pi}{6}}$ $= \pi \left[\left(\frac{9}{2} \tan \frac{\pi}{3} \right) - \left(\frac{9}{2} \tan 0 \right) \right]$ $= \pi \left[\frac{9}{2} \times \sqrt{3} - \frac{9}{2} \times 0 \right]$ $= \frac{9\sqrt{3}\,\pi}{2}$	3	3 marks for correct worked integration to give the correct solution. 2 marks for a solution that has only a minor error in calculation or logic 1 mark for solution that has some correct integration or other relevant calculations or reasoning.

Q.16. (c) (i) $T = 13\sqrt{49 + x^2} + 12(25 - x),\ x = 25$ $\therefore T = 13\sqrt{49 + 625} + 0$ $= 337.4996$ $\cong 33.7$ secs	H1, Band 4 • Gives the correct solution .... 2 • Correct substitution 1
(ii) $T = 13\sqrt{49 + x^2} + 12(25 - x),\ x = 0$ $\therefore T = 13\sqrt{49 + 0} + 12 \times 25$ $= 391$ $= 39.1$ secs	H1, Band 4 • Gives the correct solution .... 2 • Correct substitution 1
(iii) $T = 13\sqrt{49 + x^2} + 12(25 - x)$ $\frac{dT}{dx} = \frac{13}{2}(49 + x^2)^{-\frac{1}{2}} \times 2x - 12$ $\frac{13x}{(49 + x^2)^{\frac{1}{2}}} - 12 = 0$ $13x = 12(49 + x^2)^{\frac{1}{2}}$ $169x^2 = 144(49 + x^2)$ $49 \times 144 = 169x^2 - 144x^2$ $x^2 = \frac{49 \times 144}{25}$ $x = \frac{84}{5}$ $= 16.8$ Test neighbourhood for minimum: (see table below) $\therefore$ minimum time at $x = 16.8$ m $T = 13\sqrt{49 + 16.8^2} + 12(25 - 16.8)$ $= 335$ $\therefore$ minimum time $T = 33.5$ sec	H5, Band 6 • Gives the correct solution............. 5 • Achieves minimum $x = 16.8$........... 4 • Achieves $x = 16.8$.................. 3 • Achieves $x^2 = \frac{84}{5}$ .... 2 • Correct differentiation 1

x	16	16.8	17
$\frac{dT}{dx}$	−ve	0	+ve