3rd

Carlingford High School

Mathematics

Year 10 Yearly Examination

5.3 Course

2017

Name: SOLUTIONS Class: 5.3.___

Circle your teacher's name: Mr Cheng Ms Strilakos Mrs Lego

Time allowed: 1 hour 30 minutes

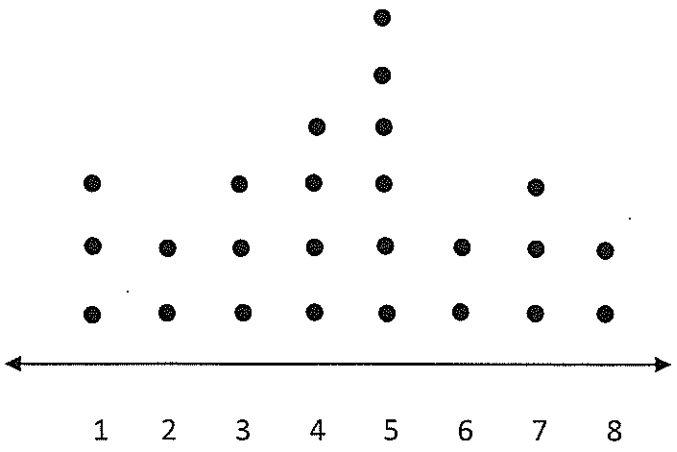
- Board approved calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or untidy work.
- Complete the examination in blue or black pen.

TOPIC	DATA	S.A/ VOLUME	INEQU ATIONS	TRIG	NON-LINEAR RELAT.	SIMULT. EQNS	LOGS/ EXPON.	PROB.	COORD. GEOM.	PROP.OF GEOM.	CIRCLE GEOM.	TOTAL
MARK	/6	/2	/5	/17	/22	/3	/15	/10	/6	/8	/6	/100

SINGLE AND BIVARIATE DATA ANALYSIS

Q.1 Find the median for the following set of data.

(i)



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(ii) Find the interquartile range for the following data. **Show all working.**

Stem	Leaf
4	0 2 4 7
5	1 3 4 6 9
6	8 8 8
7	0 2 4 5 5 6 7
8	1 2 2 4 5

Q_1

Q_3

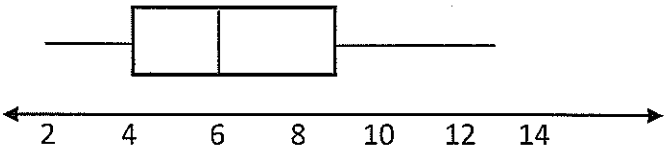
$Q_3 = 6.5$

$Q_1 = 3.5$

∴ Interquartile Range = 3

[1+2=3 marks]

Q.2 The following boxplot represents the number of hours worked by a student each day for a period of 6 weeks during the holidays.



(i) What is the 5-point summary for this data?

lowest – 2
Q_1 – 4
median – 6
Q_3 – 9
highest – 13

(ii) On what percentage of days did this student work more than 9 hours?

25%

[2+1=3 marks]

SURFACE AREA AND VOLUME

Q.1 Two similar rectangular prisms have their surface areas in the ratio 16: 25.

If the volume of the larger prism is $500cm^3$ what is the volume of the smaller prism? **Show all working.**

ratio of lengths 4:5

∴ Ratio of volumes: 64 : 125

$$\frac{V_S}{V_L} = \frac{v}{500} = \frac{64}{125}$$

$v = 4 \times 64 = 256 cm^3$.

[1+1=2 marks]

INEQUATIONS

Q.1 Solve each inequality

(i) $7a + 4 > 32$

$4a > 28, \quad a > 7$

(ii) $\frac{4y+2}{3} \leq 6$

$4y + 2 \leq 18$

$4y \leq 16$

$y \leq 4$

(iii) $2(4x - 7) \geq 6(1 - 3x)$

$8x - 14 \geq 6 - 18x$

$26x \geq 20$

$x \geq \frac{10}{13}$

[1+2+2=5 marks]

TRIGONOMETRY

Q.1 Find the value of each of the following, giving answers in **exact form**:

(i) $sin120° = \frac{\sqrt{3}}{2}$

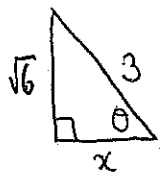
(ii) $cos135° = -\frac{1}{\sqrt{2}}$

(iii) $tan150° = -\frac{1}{\sqrt{3}}$

[3 marks]

Q.2 Given $sin\theta = \frac{\sqrt{6}}{3}$ and $tan\theta = -\sqrt{2}$

find the exact value of $cos\theta$



$\tan\theta = \frac{\sqrt{6}}{x} = -\sqrt{2}$

$x = -\frac{\sqrt{6}}{\sqrt{2}} = -\sqrt{3}$

$\therefore \cos\theta = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

[1+1=2 marks]

Q.3 Solve the equation correct to the nearest minute, if x is between 0° and 180°.

$4tan^2x - 17 = 0$

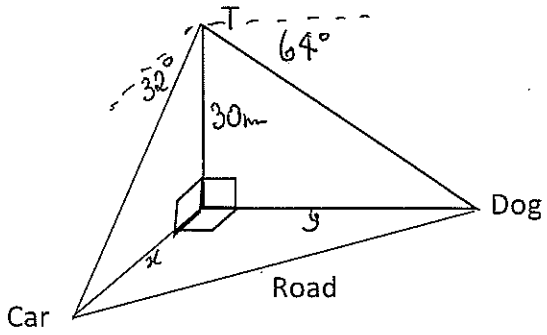
$4tan^2x = \frac{17}{4}$

$tan\,x = \pm\frac{\sqrt{17}}{2}$

$x = 64°7'$ or $180° - 64°7'$

$= 115°53'$ [3 marks]

Q.4 From the top of a building 30 meters high a woman observes a dog on a road due East of the building at an angle of depression of 64°. At the same time a car is observed due South of the building at an angle of depression of 32°. The car is travelling at 40km/hr and is driving on a straight road directly towards the dog.



(i) Put the information into the diagram and find how far apart the dog and the car are at this point in time? (to 1 d.p.)

$x = \frac{30}{\tan 32°} = 48.01m$

$y = \frac{30}{\tan 64°} = 14.63m$

Distance Apart $= \sqrt{(48.01)^2 + (14.63)^2}$

$= 50.2$ metres

(ii) How much time does the dog have to move off the road before it gets hit by the car? (answer in seconds to 1 d.p)

$T = \frac{D}{S}$ $\quad 40km/h = \frac{40000}{60 \times 60}$ m/s

$= 11.11$ m/s.

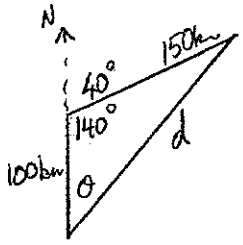
$T = \frac{50.2}{11.1}$

$= 4.5$ seconds

[3+2=5 marks]

Q.5 A ship sails 100km due North and then 150km $N40°E$.

(i) How far from its start point is it? (2 d.p.)



$d^2 = 100^2 + 150^2 - 2 \times 100 \times 150 \cos 140°$

$d = 235.54$ km

(ii) What is its bearing from its start point now? (to nearest degree)

$$\frac{150}{\sin\theta} = \frac{235.54}{\sin 140°}$$

$\theta = 24°$

[2+2=4 marks]

NON LINEAR RELATIONSHIPS

Q.1 Given the parabola with equation $y = x^2 - 4x - 21$ find:

(i) the x -axis intercepts

$(x-7)(x+3)=0 \quad x=7, \quad x=-3$

(ii) the y -axis intercept

$y = -21$

(iii) equation of the axis of symmetry

$$x = \frac{-b}{2a} = 2$$

(iv) coordinates of the vertex

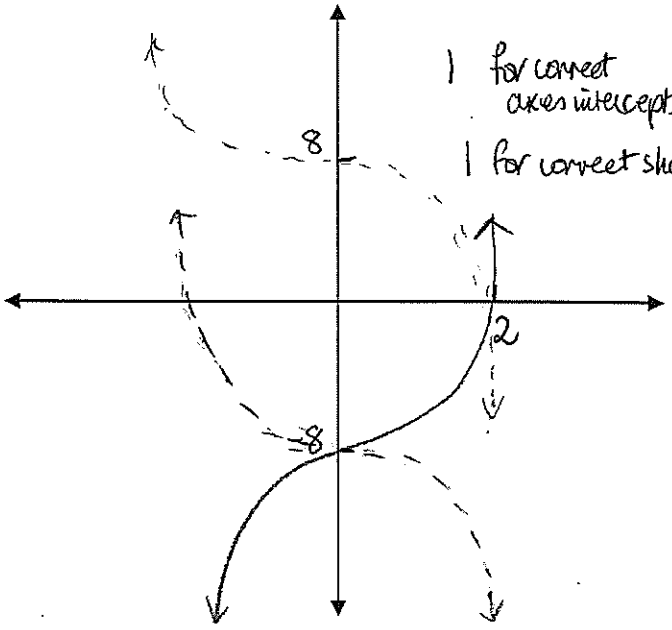
$x=2, \; y=-25$

(v) If this parabola is translated 9 units upwards what will its equation now be?

$y = x^2 - 4x - 21 + 9$

$= x^2 - 4x - 12$ [5X1=5 marks]

Q.2 (i) Sketch the graph of $y = x^3 - 8$ clearly labelling any axes intercepts.



(Draw sketches of the following transformations on your axes above to help you answer the next two parts)

(ii) If this curve is reflected in the y-axis, what will its equation be?

$y = -x^3 - 8$

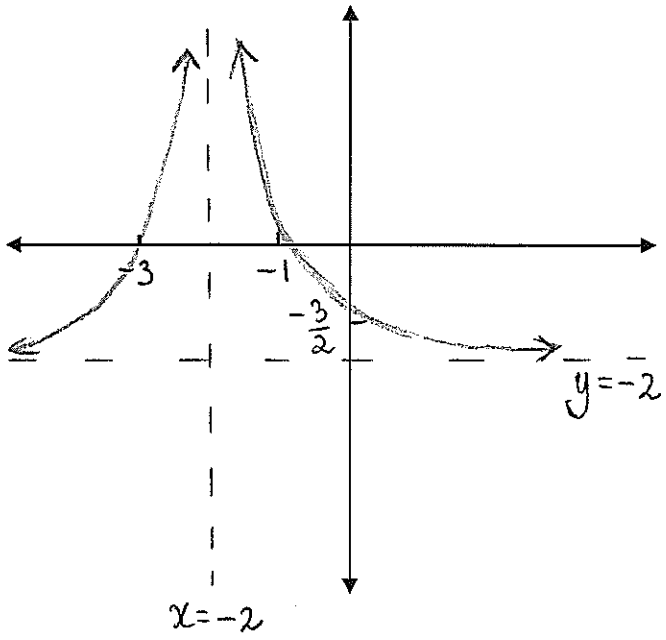
(iii) If the curve in (i) is reflected in the x-axis, what will its equation be?

$y = -x^3 + 8$

[2+1+1=4 marks]

Q.3 Sketch the curve $y = \frac{2}{(x+2)^2} - 2$

clearly labelling asymptotes and any axes intercepts.



When $x=0$, $y = -\frac{3}{2}$

When $y=0$, $\frac{2}{(x+2)^2} = 2$

$\therefore (x+2)^2 = 1$

$x+2 = \pm 1$

$\therefore x = -1, -3.$

[3 marks]

Q.4 Find the centre and radius of the circle given by the equation

$$x^2 - 8x + y^2 + 6y = 11$$

$(x-4)^2 - 16 + (y+3)^2 - 9 = 11$

$(x-4)^2 + (y+3)^2 - 25 = 11$

$(x-4)^2 + (y+3)^2 = 36$

[3 marks]

Centre: $(4, -3)$ Radius: 6

Q.5 Find the coordinates of the points of intersection of the curves with equations:

$x = 2y - 1$ and $3x^2 = x + 2y^2$

$3(2y-1)^2 = 2y - 1 + 2y^2$

$3(4y^2 - 4y + 1) = 2y - 1 + 2y^2$

$12y^2 - 12y + 3 = 2y - 1 + 2y^2$

$10y^2 - 14y + 4 = 0$

$5y^2 - 7y + 2 = 0$

$(5y - 2)(y - 1) = 0$

$y = \frac{2}{5}$ $y = 1$

$x = -\frac{1}{5}$ $x = 1$

Soln: $\left(-\frac{1}{5}, \frac{2}{5}\right)$ $(1, 1)$

[3 marks]

Q.6(i) Find the points of intersection of the graphs of $y = x$ and $y = \frac{2}{x}$.

$x = \frac{2}{x}$

$2 = x^2$

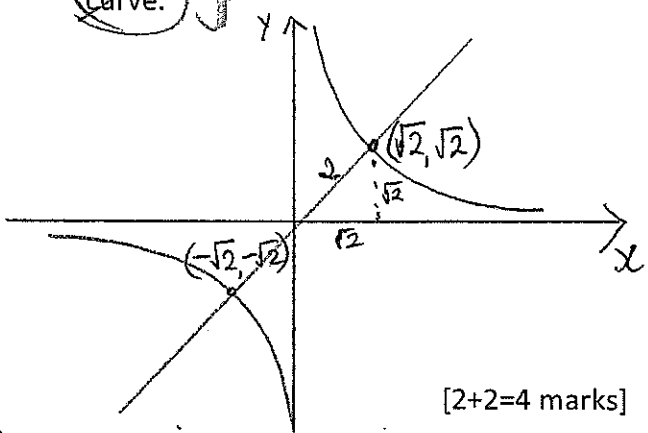
$x = \sqrt{2}$, $y = \sqrt{2}$

and

$x = -\sqrt{2}$, $y = -\sqrt{2}$

(ii) Sketch the two curves on the same set of axes and hence find the shortest distance between the two branches of the curve.

hyperbola



[2+2=4 marks]

Shortest Distance $= 2 \times 2$

$= 4$ units

SIMULTANEOUS EQUATIONS

Q.1 Solve the following pair of simultaneous equations:

$2x + 3y = -1$ —① and

$3x - 4y = 24$ —②

①×3 $6x + 9y = -3$

②×2 $6x - 8y = 48$

①'-②' $17y = -51$ $y = -3$

Subst. $y = -3$ in ① $\therefore 2x = 8$

$x = 4$

Solution: $(4, -3)$

[3 marks]

LOGARITHMS

Q.1 Solve the following equations:

(i) $\log_3(7x + 3) = \log_3(5x + 9)$

$7x + 3 = 5x + 9$

$2x = 6$

$x = 3$

(ii) $\log_2(5x + 7) = 5$

$2^5 = 5x + 7$

$32 = 5x + 7$

$5x = 25$

$x = 5$

[1+2=3 marks]

Q. 2 Solve each of the following exponential equations:

(i) $3^{2x} = 81$

$3^{2x} = 3^4$

$x = 2$

(ii) $2^{-3-x} = 2\sqrt{2}$

$2^{-3-x} = 2^{3/2}$

$-3 - x = 3/2$

$x = -9/2$

(iii) $4^{m-3} = \frac{1}{16\sqrt{2}}$

$4^{m-3} = \frac{1}{2^{9/2}} = 2^{-9/2}$

$2^{2m-6} = 2^{-9/2}$

$2m - 6 = -9/2$

$2m = 3/2$, $m = 3/4$

[1+2+3=6 marks]

Q.2 Evaluate:

$$4\log_8 2 + \frac{2}{3}\log_8 8$$

$= \log_8 2^4 + \log_8 8^{2/3}$ 1

$= \log_8 (2^4 \times 4)$

$= \log_8 64$ 1 $= \log_8 8^2 = 2$ 1

[3 marks]

Q.3 Solve:

$2\log_{10} x = \log_{10} 2 + \log_{10}(3x - 4)$

$\log_{10} x^2 = \log_{10} 2(3x - 4)$ 1

$\log_{10} x^2 = \log_{10}(6x - 8)$

$x^2 = 6x - 8$ 1

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$ 1

$x = 2, 4.$

[3 marks]

PROBABILITY

Q.1 A standard die is tossed and the uppermost number is noted.

Find the probability that the number is:

(i) less than or equal to four and a six 0

(ii) less than or equal to three or a six $\frac{2}{3}$

(iii) even and less than or equal to four $\frac{1}{3}$

[3X1=3 marks]

Q.2 A dice is tossed. What is the probability that an outcome greater than 4 is obtained **given:**

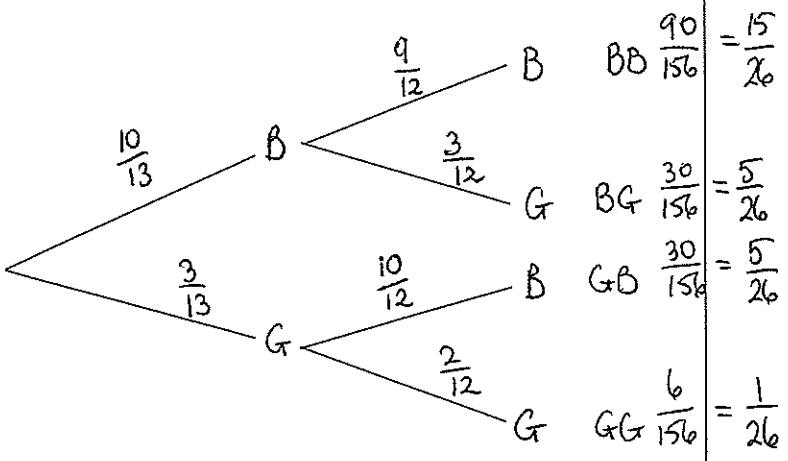
(i) an even number is obtained $\frac{1}{3}$

(ii) a number greater than 2 is obtained. $\frac{1}{2}$

[2 marks]

Q.3 A bag contains 10 blue balls and 3 green balls. A ball is taken out and its colour noted. **It is not replaced**. A second ball is taken out and its colour noted.

(i) Complete the tree diagram and indicate the outcomes and probabilities for each stage of the branches on the tree.



(ii) Find the probability of obtaining

(a) a green ball followed by a blue ball

$\frac{5}{26}$

(b) a green ball and a blue ball

$\frac{10}{26}$ or $\frac{5}{13}$

(c) two green balls.

$\frac{1}{26}$

[2+1+1+1=5 marks]

COORDINATE METHODS IN GEOMETRY

Q.1 A line with a gradient of $\frac{4}{5}$ passes through the midpoint of $(-3, 7)$ and $(7, 1)$.

Find its equation and give it in General Form. **Show all working.**

midpoint $(2, 4)$

$y - 4 = \frac{4}{5}(x - 2)$

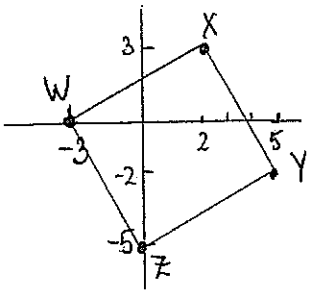
$5y - 20 = 4x - 8$

$4x - 5y + 12 = 0$

[3 marks]

Q.2 $W(-3,0), X(2,3), Y(5,-2)$ and $Z(0,-5)$ are the vertices of a quadrilateral.

(i) Find the lengths of the diagonals



$d_{WY} = \sqrt{(-3-5)^2 + 2^2}$

$= \sqrt{68}$

$d_{XZ} = \sqrt{2^2 + 8^2}$

$= \sqrt{68}$

(ii) Find the gradients of the diagonals.

$m_{WY} = \frac{\Delta y}{\Delta x} = -\frac{2}{8}$

$m_{XZ} = \frac{8}{2}$

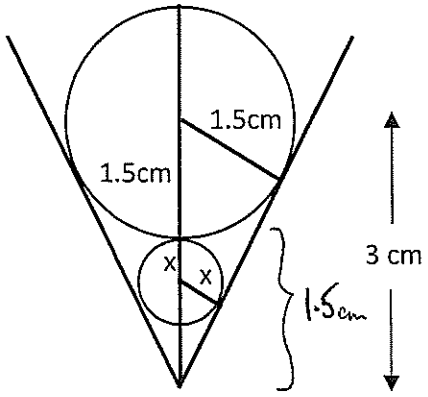
(iii) Hence state what type of quadrilateral this is.

Equal length Diagonals with ⊥ gradients

∴ Square [3 marks]

PROPERTIES OF GEOMETRICAL FIGURES

Q.1 A spherical icecream of radius 1.5cm will just fit in a cone so that the centre of the icecream is 3cm from the vertex. Find the radius of the largest spherical icecream which will fit in the cone underneath the first icecream.



Using Similar Δ's.:

$\frac{1.5}{x} = \frac{3}{1.5-x}$ 1

$\Rightarrow 1.5^2 - 1.5x = 3x$

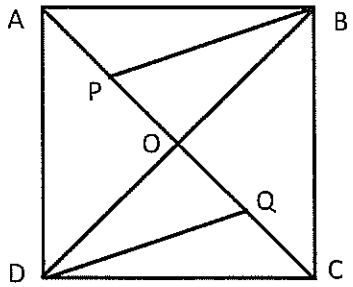
$1.5^2 = 4.5x$

$2.25 = 4.5x$

$x = 0.5$ cm

[4 marks]

Q.2



$ABCD$ is a square with diagonals intersecting at O. P and Q are the midpoints of $\overline{AO}$ and $\overline{OC}$ respectively.

(i) Prove that $\Delta POB \equiv \Delta QOD$.

Proof: In ΔPOB and ΔQOD

$\angle POB = \angle QOD = 90°$ (diagonals of square bisect at 90°).

$OB = OD$ (diagonals of square bisect)

$OP = OQ$ (midpoints of equally bisected diagonal of square)

∴ $\Delta POB \equiv \Delta QOD$ SAS

(ii) Hence show that $d\overline{PB} = d\overline{DQ}$.

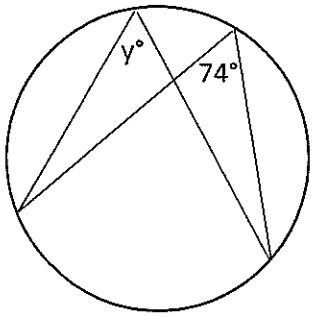
$d\overline{PB} = dDQ$

Equal corresponding sides of congruent Δ's.

[3+1=4 marks]

CIRCLE GEOMETRY

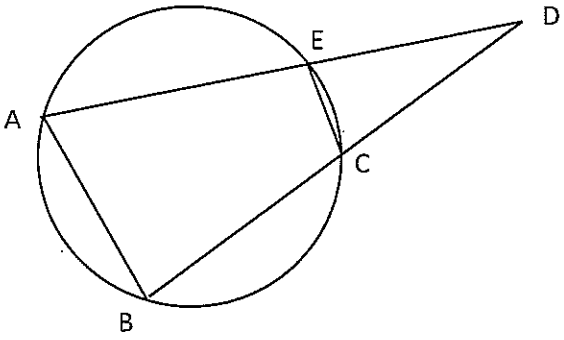
Q.1 What is the value of y?



$y = 74°$.

[1 mark]

Q.2 Prove that ΔADB is similar to ΔCED.



Proof: In ΔADB and ΔCED

$\angle ADB = \angle CDE$ (shared angle) !

$\angle ABC = \angle CED$

(exterior angle of cyclic quadrilateral equal to opposite interior angle) !

$\therefore \angle BAD = \angle ECD$

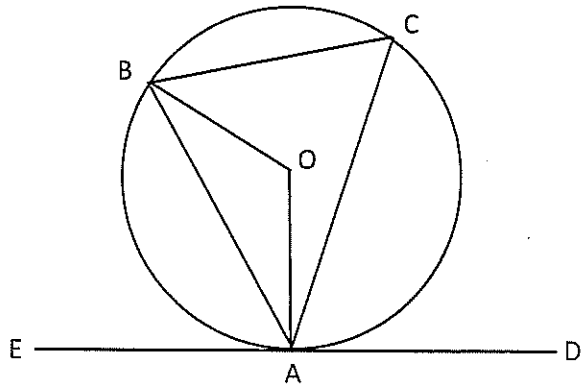
(angle sum of Δ). !

$\therefore \Delta ADB \,|||\, \Delta CED$

(three pairs of equal corresponding angles)

[3 marks]

Q.3 Prove that $\angle BOA = 2\angle EAB$



Proof: $\angle EAB = \angle BCA$

(angle in the alternate segment)

$\angle BOA = 2 \times \angle BCA$

(angle subtended by chord AB at centre is twice angle subtended at circumference)

$\therefore \angle BOA = 2 \times \angle EAB$

as required.

[2 marks]

END OF EXAM