Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 8

1. What is the solution to the equation $3x^2 + 7x - 1 = 0$?

   A. $x = \frac{-7 \pm \sqrt{61}}{6}$

   B. $x = \frac{-7 \pm \sqrt{37}}{6}$

   C. $x = \frac{7 \pm \sqrt{37}}{6}$

   D. $x = \frac{7 \pm \sqrt{61}}{6}$

$\frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times -1}}{2 \times 3}$

$= \frac{-7 \pm \sqrt{61}}{3}$

2. Which set of points below is a function?

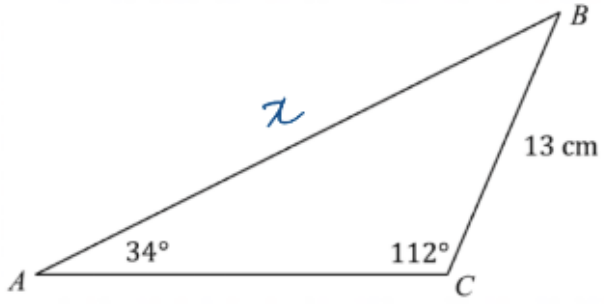
   A. (−6, 7) , (7 , 6) , (−4, 5) and (−6, −4)

   B. (−6, 7) , (7 , 4) , (−6, 5) and (7, −2)

   C. (−6, 7) , (7 , 5) , (−4, 5) and (7, −6)

   D. (−6, 7) , (7 , −6) , (−4, 5) and (5, −4)

3.



Not to scale

What is the length of AB, correct to one decimal place?

A. $13.0cm$

B. $20.9cm$

C. $21.6cm$

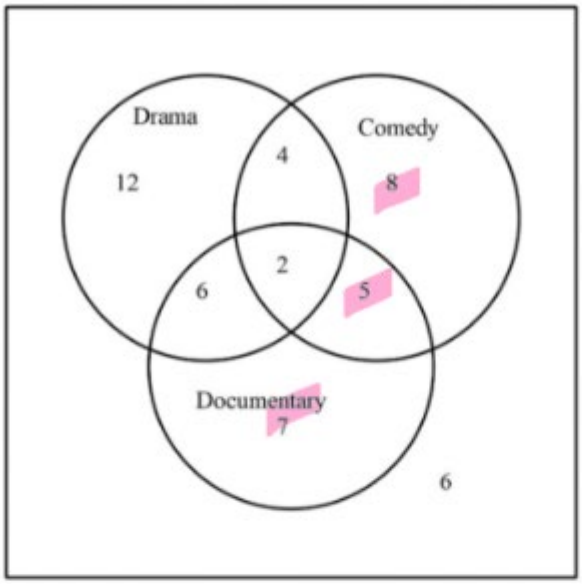
D. $40.6cm$

$\frac{x}{\sin 112} = \frac{13}{\sin 34}$

$x = 21.6$

4. A theatrical agent has 50 artists on his books.

The Venn diagram shows the genres in which they perform.



He rings one of his artists at random.

What is the probability that the artist does comedy or documentary or both, but not drama?

A. $\frac{3}{10}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{16}{25}$

$\frac{20}{50}$

5. Find the exact value of $\tan \frac{5\pi}{3}$.

A. $-\sqrt{3}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

$= -\tan\frac{\pi}{3}$

$= -\sqrt{3}$

6. What is the derivative of $x^{-2} - \frac{1}{x}$?

A. $\frac{-2}{x} - \frac{1}{x^2}$

B. $\frac{-2}{x} + \frac{1}{x^2}$

C. $\frac{-2}{x^3} - \frac{1}{x^2}$

D. $\frac{-2}{x^3} + \frac{1}{x^2}$

$\frac{d}{dx} = -2x^{-3} + \frac{1}{x^2}$

$= -\frac{2}{x^3} + \frac{1}{x^2}$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\frac{S^2}{S^2} + \frac{C^2}{S^2} = \frac{1}{S^2}$

7. Which of the following expressions is equal to $4 + 4\cot^2 x$?

A. $4\sec^2 x$

B. $4\operatorname{cosec}^2 x$

C. $\cot^2 x$

D. $8\cot^2 x$

$4(1 + \cot^2 x)$

8. Find the derivative of $y = \sqrt{(x^3 + 4)^3}$.

A. $3(x^3 + 4)^2$

B. $18\sqrt{x^3 + 4}$

C. $\frac{\sqrt{x^3+4}}{18x^2}$

D. $\frac{9x^2\sqrt{x^3+4}}{2}$

$(x^3 + 4)^{3/2}$

$\frac{d}{dx} = \frac{3}{2}(x^3 + 4)^{1/2} \times 3x^2$

$= \frac{9x^2\sqrt{x^3+4}}{2}$

9. Leo owns five blue and seven red ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother. What is the probability that both of the ties, are the same colour?

A. $\frac{1}{2}$

B. $\frac{5}{33}$

C. $\frac{31}{66}$

D. $\frac{31}{72}$

Both blue or both red

$\frac{5}{12} \times \frac{4}{11} + \frac{7}{12} \times \frac{6}{11}$

$= \frac{31}{66}$

10. If $\frac{4}{x-3}+\frac{2}{x}=1, \quad x>0$, then x is equal to:

A. $\frac{9\pm\sqrt{57}}{2}$

B. $\frac{9+\sqrt{57}}{2}$

C. 2

D. 7

$4x+2(x-3)=x(x-3)$

$4x+2x-6=x^2-3x$

$x^2-9x+6=0$

$x=\frac{9\pm\sqrt{9^2-4\times6}}{2}$

$=\frac{9\pm\sqrt{9^2-4\times6}}{2}$

and for both Slns $x>0$

Section II

70 marks
Attempt all questions
Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks)

List all the **points** where $y = x^3 - 4x$ crosses the x-axis. **2**

$y=x(x-2)(x+2)$ — 1m factorise

when $y=0$ — 1m all 3 slns as x-values or coordinates

$x=2,-2,0$

∴ points: $(0,0)(2,0)(-2,0)$

Question 12 (2 marks)

Simplify the expression $\frac{a^2-3a}{2ax} \times \frac{4a^2x}{2ax-6x}$, giving your answer as a simple fraction. **2**

$\frac{x(a-3)}{2ax} \times \frac{4a^2x}{2x(a-3)}$ — 1m factorise

1m simplified

$=\frac{a^2}{x}$

Question 13 (3 marks)

A tangent is drawn to the curve $y = \frac{x^3 - x^2 - 9x}{2}$ at the point where $x = \sqrt{3}$.

Find the angle of inclination of the tangent to the positive x-axis. **3**

$y' = \frac{3x^2}{2} - x - \frac{9}{2}$

when $x = \sqrt{3}$

$y' = \frac{3 \times 3}{2} - \sqrt{3} - \frac{9}{2}$

$= -\sqrt{3}$

$\tan\theta = -\sqrt{3}$

$\theta = 180 - 60^\circ$

$= 120^\circ$

1m differentiate
1m gradient
1m angle

Question 14 (3 marks)

The velocity of a particle moving in a straight line is given by

$v = 2t^2 - 10t - 48$. Find the acceleration, when the velocity is zero. **3**

when $v = 0$

$0 = 2t^2 - 10t - 48$

$0 = t^2 - 5t - 24$

$0 = (t - 8)(t + 3)$

$t = 8, -3$ but $t \geq 0$ so

$t = 8$

$a = 4t - 10$

when $t = 8$

$a = 4 \times 8 - 10$

$= 22$

or $22 cm/s^2$

or any unit.

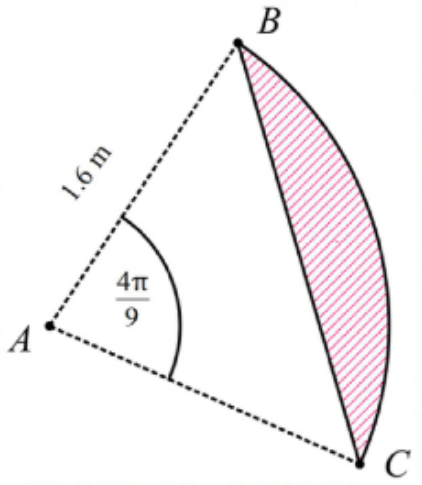
1m find t

1m differentiate for a

1m final answer.

Question 15 (3 marks)

The shaded segment shown is formed by an arc BC of a circle radius 1.6m, which subtends an angle of $\frac{4\pi}{9}$ radians at its centre A.



Find the area of the shaded segment, correct to 3 significant figures.

$A = \frac{1}{2}(1.6)^2\left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$

$= 0.527$ (3 s.f.)

3

1m uses correct formula
1m obtains correct answer
1m correct s.f.

Question 16 (3 marks)

Solve the equation $\sin\theta = 5\cos\theta$ in the interval $0 \leq \theta \leq 360°$.
Answer correct to the nearest minute.

$\tan\theta = 5$ ✓ ✓

$\theta = 78°41', 258°41'$

3

1m for tan
1m one correct angle
1m two correct angles

Question 17 (3 marks)

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

(a) What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

1

1m correct

(b) What is the probability that the sum of the numbers on the two balls is 5?

1, 4 or 4, 1 or 2, 3 or 3, 2

$\therefore \frac{4}{12} = \frac{1}{3}$

1

1m correct simplified

(c) Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

first ball had to be a 4

so $\frac{1}{3}$

1

1m correct

Question 18 (5 marks)

Differentiate, with respect to x.

* deduct 1 mark if uses = instead of $\frac{d}{dx}$ =

(a) $4x^{-3}$ 1

$\frac{d}{dx} = -12x^{-4}$ or $\frac{-12}{x^4}$

1m correct

(b) $(5+x^2)^{\frac{1}{2}}$ 2

$\frac{d}{dx} = \frac{1}{2}(5+x^2)^{-1/2} \times 2x$ 1m progress

$= x(5+x^2)^{-1/2}$ or $\frac{x}{\sqrt{5+x^2}}$ 1m correct

(c) $\frac{3x^2-5}{2x+1}$ 2

$\frac{d}{dx} = \frac{6x(2x+1)-2(3x^2-5)}{(2x+1)^2}$ $u = 3x^2-5$ $v = 2x+1$

$u' = 6x$ $v' = 2$

$= \frac{12x^2+6x-6x^2+10}{(2x+1)^2}$ 1m uses quotient rule

$= \frac{6x^2+6x+10}{(2x+1)^2}$ 1m correct simplified

Question 19 (4 marks)

(a) Find the equation of the axis of symmetry of the parabola $y = 5 + 4x - x^2$ and the coordinate of its vertex 2

$x = \frac{-b}{2a} = \frac{-4}{2\times -1} = 2$ 1m AOS

$\therefore x = 2$ ← axis of symmetry 1m vertex

$\therefore$ vertex: $(2, 9)$

(b) Hence, write down the domain and range of $y = 5 + 4x - x^2$, using interval notation 2

D: $(-\infty, \infty)$ 1m correct Domain and range

R: $(-\infty, 2]$ 1m interval notation

Question 20 (2 marks)

Use the result $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ to find $f'(x)$ if $f(x) = 2x^2$

2

$f(x) = \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$

$= \lim_{h \to 0} 4x + 2h$

$= 4x$

1m correct substitution and notation
1m obtains correct answer via Diff by F.P.

Question 21 (3 marks)

Show that $\sin\alpha \cot\alpha - \cos\alpha \sin^2\alpha = \cos^3\alpha$.

3

LHS $= \frac{\sin\alpha \cos\alpha}{\sin\alpha} - \cos\alpha \sin^2\alpha$

$= \cos\alpha(1 - \sin^2\alpha)$

$= \cos\alpha \cdot \cos^2\alpha$

$= \cos^3\alpha$

$=$ RHS

1m uses LHS $=$ $=$ $=$ RHS
1m some progress
1m correct proof

Question 22 (5 marks)

(a) Find the centre and radius of the circle represented by the equation:

$x^2 - 4x + y^2 + 6y + 4 = 0.$

3

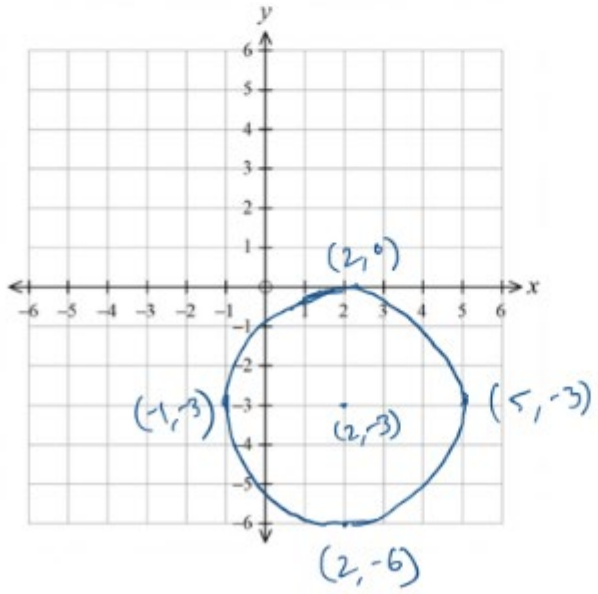
$(x-4x+4) + (y^2+6y+9) = -4+9+4$ 1m one complete square

$(x-2)^2 + (y+3)^2 = 9$ 1m centre

∴ centre $(2,-3)$ 1m radius

$r = 3$

(b) Sketch the circle on the axes below, showing important points. (Note: you do not need to find the y-intercept)



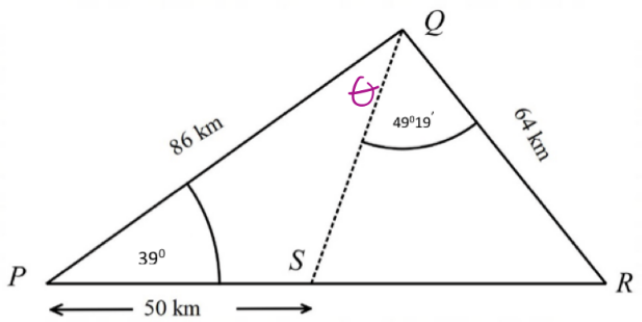
2

1m centre + shape

1m endpoints shown or obvious.

Question 23 (4 marks)

Triangles PQS and QRS have a common side QS, as shown.



(a) Calculate the length of QS, correct to 3 significant figures

2

$QS = \sqrt{86^2 + 50^2 - 2 \times 86 \times 50 \times \cos 39}$

$= 56.7$ km (3 s.f.)

1m cos rule used

1m correct answer.

(b) Find the area of ΔPQR, correct to the nearest square kilometre.

2

Let $\angle PQS = \theta$

$\frac{\sin\theta}{50} = \frac{\sin 39}{56.7}$

$\theta = 33^\circ 42'$ (Nearest min)

$\angle PQR = 83^\circ 1'$

So Area $= \frac{1}{2} \times 86 \times 64 \times \sin(83^\circ 1')$

$= 2732$ km^2 (Nearest km^2)

or/ $A = \frac{1}{2} \times 50 \times 86 \times \sin 39 + \frac{1}{2} \times 64 \times 56.7 \times \sin 49^\circ 19'$

$= 2729$ km^2

Question 24 (8 marks)

$P(1, 4)$ lies on the curve $y = x^3 - 4x^2 + x + 6$.

(a) Find the equation of the tangent to the curve at P. **3**

$y' = 3x^2 - 8x + 1$ — Im y'

$x = 1$ $y' = 3 \times 1^2 - 8 + 1$ — Im find m

$y' = -4$ — Im equn

$y - 4 = -4(x - 1)$

$y - 4 = -4x + 4$ T: $y = -4x + 8$

(b) Find the equation of the normal to the curve at P, writing your answer in general form. **2**

N: $m = \frac{1}{4}$ $x - 4y + 15 = 0$

$y - 4 = \frac{1}{4}(x - 1)$ — Im equn

$4y - 16 = x - 1$ — Im gen. form

(c) The tangent and normal at P intersect the x-axis at T and N respectively. What are the coordinates of T and N? **2**

when $y = 0$ N: $x = -15$ — Im both x correct

T: $0 = -4x + 8$ $(-15, 0)$

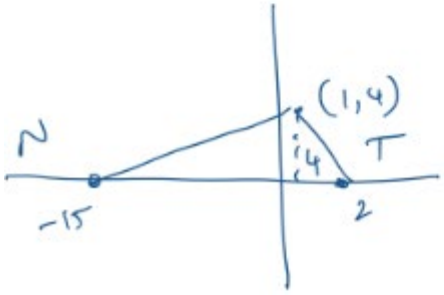
$x = 2$ — Im writes coordinates

$(2, 0)$

(d) Find the area of triangle PTN. **1**

$A = \frac{17 \times 4 \times 1}{2}$ — Im correct answer.

$= 34 u^2$



Question 25 (3 marks)

Given that $y = 3x^4(2-x)^5$, find $\frac{dy}{dx}$, writing your answer in simplified form

3

$u = 3x^4 \quad v = (2-x)^5$

$u' = 12x^3 \quad v' = -5(2-x)^4$

$\frac{dy}{dx} = 12x^3(2-x)^5 - 15x^4(2-x)^4$

$= 3x^3(2-x)^4[4(2-x) - 5x]$ 1m product rule

$= 3x^3(2-x)^4(8-4x-5x)$ 1m correct $\frac{dy}{dx}$

$= 3x^3(2-x)^4(8-9x)$ 1m simplified

Question 26 (4 marks)

(a) For the function $f(x) = \frac{1}{x^2-4}$ determine whether $f(x)$ is odd, even or neither, justifying your answer algebraically. 2

$f(-x) = \frac{1}{(-x)^2-4}$

$= \frac{1}{x^2-4}$

$= f(x) \quad \therefore$ even

(b) Describe what the result from [a] means geometrically 1

reflective symmetry about y-axis or $x=0$

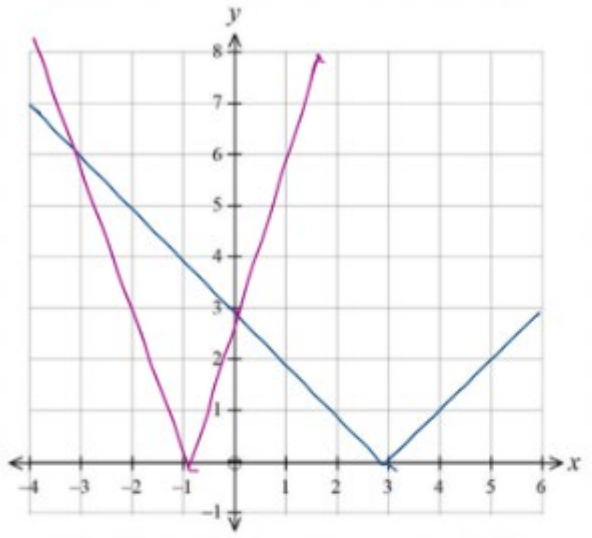
x values

(c) Write down the coordinates of any points on $f(x)$ that are discontinuities. 1

$x = 2, \quad x = -2$

Question 27 (4 marks)

(a) On the axes below, draw the graphs of $y = |x - 3|$ and $y = |3x + 3|$.



2

1m each correct

(b) Hence, or otherwise, find solutions to $|x - 3| = |3x + 3|$.

2

$x = 0$ and

$x = -3$ 1m each

Question 28 (2 marks)

For what values of m will the equation $y = x^2 - 2mx + 4m$, have exactly one solution?

2

$\Delta = 0$

$2m^2 - 4 \times 4m = 0$ 1m $\Delta = 0$

$4m^2 - 16m = 0$ 1m solutions

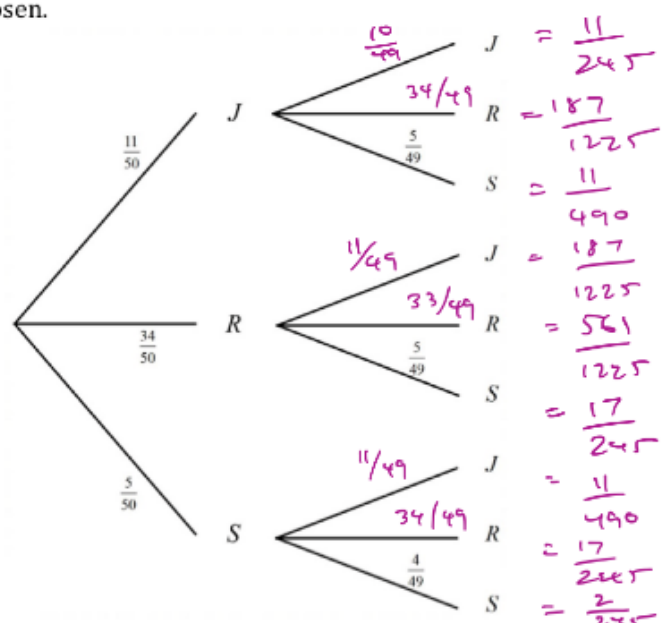
$m(m - 4) = 0$

$m = 0$ or $m = 4$

Question 29 (3 marks)

Employees in a company are classified as Junior if aged less than 18, Senior if aged over 60 or Regular for all others. There are 50 employees in the company and 2 are to be chosen to attend a conference.

The probability tree diagram has been started to show how the two employees could be chosen. **3**



= 11/245
= 187/1225
= 11/490
= 187/1225
= 561/1225
= 17/245
= 11/490
= 17/245
= 2/245

1m correct prob. on arms

1m multiply across

Complete the missing probabilities on the probability tree and hence find the probability that the two employees chosen will come from different categories.

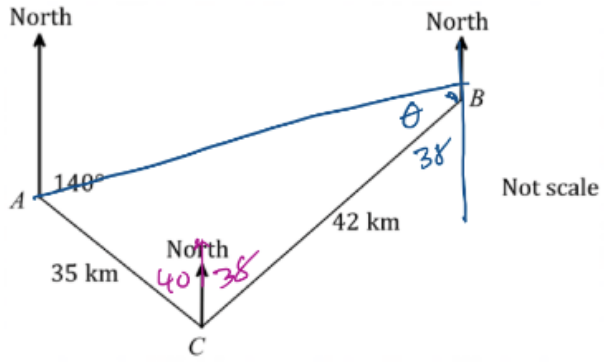
1m correct

P(2 from diff categories) = $1 - \left(\frac{11}{245} + \frac{561}{1225} + \frac{2}{245}\right)$

= $\frac{599}{1225}$

or 0.489 (3dp).

Question 30 (4 marks)



Tyler drives 35km from town *A* on a true bearing of 140°T to town *C*.
He then drives 42km on a true bearing of 38°T to town of *B*.

(a) ~~Show that~~ Find $\angle ACB = 78°$.

$40 + 38 = 78°$ 1

1m

(b) Find the bearing of *A* from B, to the nearest minute.

$AB = \sqrt{35^2 + 42^2 - 2 \times 35 \times 42 \times \cos 78}$ 3

$= 48.76$ km (2dp) 1m AB

$\frac{\sin\theta}{35} = \frac{\sin 78}{48.76}$ 1m $\angle CBA$

$\theta = 44°36'$

$\therefore$ Bearing $= 44°36' + 38 + 180$ 1m correct answer.

$= 262°36'$