

CARLINGFORD HIGH SCHOOL
DEPARTMENT OF MATHEMATICS

Year 12

Mathematics

Term 2

ASSESSMENT TASK 3

2018



Time allowed: 50 minutes

Student Number: _____

Instructions:

- All questions should be attempted on your own paper.
- Show ALL necessary working.
- Marks may not be awarded for careless or badly arranged work.
- Only board-approved calculators may be used.
- Please write on one side of each sheet of paper only, and do not use multiple columns on the page.

TOPIC	Trigonometric Functions	Logarithmic & Exponential Functions	TOTAL
MARKS	/16	/18	/34

Question 1

Find $\frac{d}{dx}(7 \cos x^2)$

[2 marks]

Question 2

Evaluate the following definite integral, leaving your answer in exact form.

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{1 + \sin x} dx$$

[2 marks]

Question 3

- (i) State the period and the amplitude of the curve with equation $y = 3 \sin 2x$
- (ii) Sketch the graph for this curve for $0 \leq x \leq 2\pi$ clearly labelling axes intercepts.

[1+2=3 marks]

Question 4

Solve the following trigonometric equations across the domain stated:

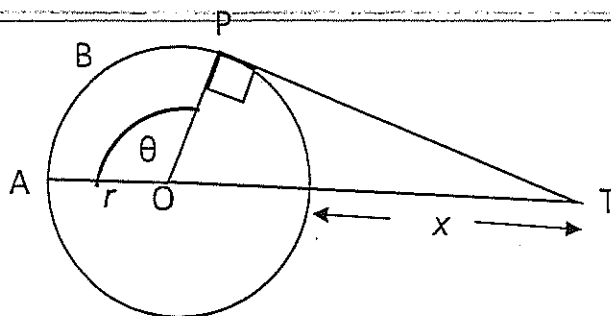
- (i) $3\sin 3x + \sqrt{3} \cos 3x = 0, \quad 0 \leq x < \pi$
- (ii) $4\sin^2 \theta - \cos^2 \theta = 7\sin \theta + 5, \quad 0^\circ \leq \theta < 360^\circ$

[3+3=6 marks]

Question 5

A string attached at A passes around arc ABP and is fastened at T .

Find the length of the string in terms of x , r and θ .



[3 marks]

Question 6

Solve each of the following equations for x :

(i) $\log_{10} x = \log_{10} 4 + \log_{10} 8$

(ii) $2 \log_e 54 - x \log_e 3 = \log_e 12$

[1+2=3 marks]

Question 7

Evaluate $\int_0^1 \frac{2e^x}{1+e^x} dx$

[2 marks]

Question 8

Solve for w , leaving your answer in exact form:

$$1 - 25e^{-4w} = \frac{24}{25}$$

[3 marks]

Question 9

Find $f'(x)$ if $f(x) = \log_e \sqrt{(x^3 + 1)}$

[2 marks]

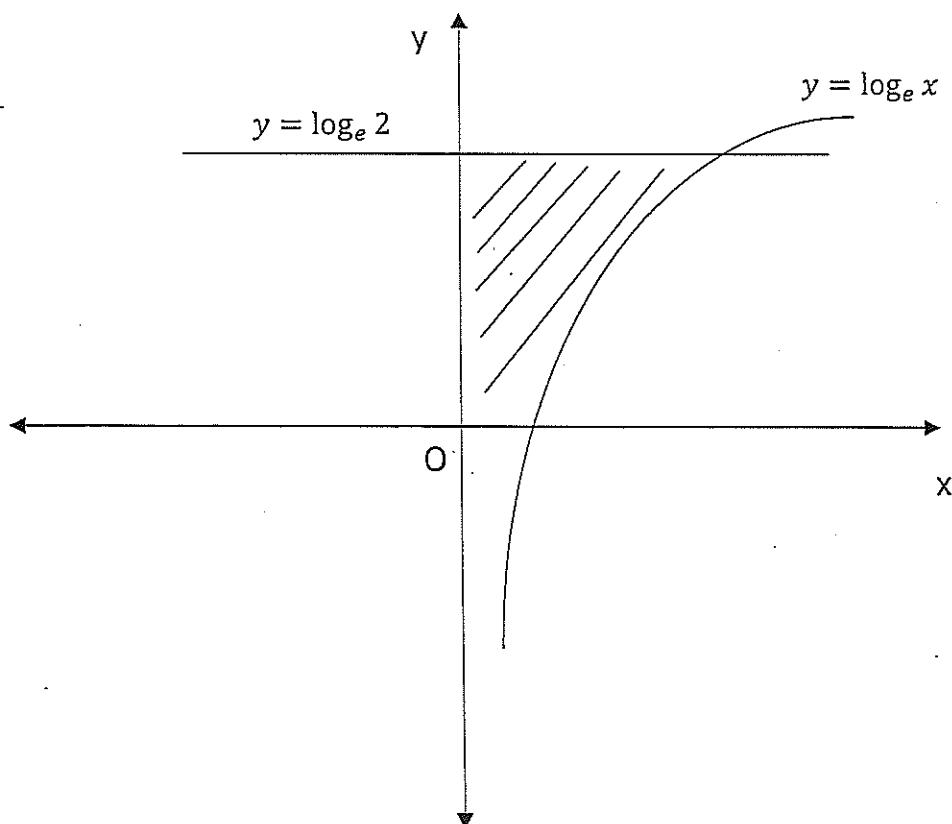
Question 10

- (i) Differentiate $x \log_e x$
- (ii) Hence find an expression for $\int \log_e x$
- (iii) Use your result in (ii) to find the area bounded by the curve $\log_e x$, the x -axis and the line $x = 3$.

[1+1+2=4]

Question 11

Find the volume of the solid generated when the area bounded by the x and y -axes, the curve with equation $y = \log_e x$ and the line $y = \log_e 2$ is rotated about the y -axis.



[4 marks]

END OF TEST

YEAR 12 MATHEMATICS TERM 2 ASSESSMENT TASK 3 2018
SOLUTIONS

QUESTION 1

$$\frac{d}{dx}(7\cos x^2) = -2x \times 7\sin x^2$$

$$= -14x \sin x^2$$

(2)

QUESTION 2

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{1+\sin x} dx$$

$$= \left[\log_e (1+\sin x) \right]_0^{\frac{\pi}{6}}$$

$$= \log_e \left(\frac{3}{2} \right)$$

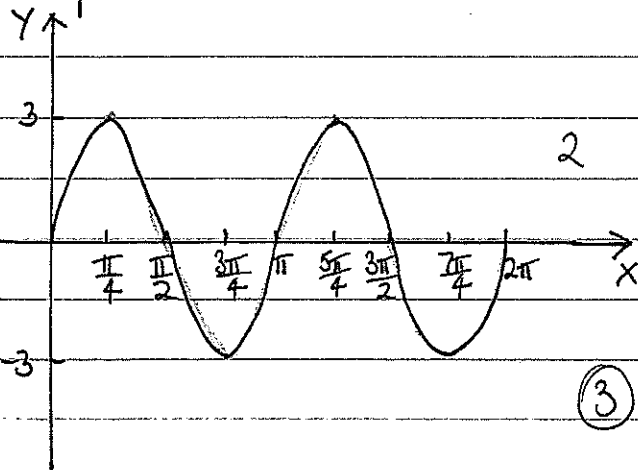
(2)

QUESTION 3

(i) $y = 3\sin 2x$

$$T = \frac{2\pi}{n} = \pi$$

amplitude 3



(3)

QUESTION 4

(i) $3\sin 3x + \sqrt{3}\cos 3x = 0$ $0 \leq x < 180^\circ$

$$3\sin 3x = -\sqrt{3}\cos 3x$$

$$\frac{3\tan 3x}{\sqrt{3}} = -1$$

$$\tan 3x = -\frac{1}{\sqrt{3}}$$

$$3x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}$$

$$\Rightarrow x = \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}$$

Indegrees, $x = 50^\circ, 110^\circ, 170^\circ$

(3)

(ii) $4\sin^2 \theta - \cos^2 \theta = 7\sin \theta + 5$

$$0^\circ \leq \theta < 360^\circ$$

$$4\sin^2 \theta - (1 - \sin^2 \theta) - 7\sin \theta - 5 = 0$$

$$5\sin^2 \theta - 7\sin \theta - 6 = 0$$

$$(5\sin \theta + 3)(\sin \theta - 2) = 0$$

$$\sin \theta = 2$$

\therefore no solutions here

$$\sin \theta = -\frac{3}{5}$$

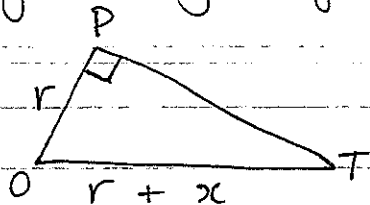
$$\theta = 180^\circ + 36^\circ 52' = 216^\circ 52'$$

$$\text{or } \theta = 360^\circ - 36^\circ 52' = 323^\circ 08'$$

(3)

QUESTION 5

Length of string = length of arc ABP + PT



$$PT = \sqrt{(r+x)^2 - r^2}$$

$$= \sqrt{r^2 + 2rx + x^2 - r^2}$$

$$= \sqrt{2rx + x^2}$$

$$\text{length of arc ABP} = r\theta$$

$$\therefore \text{Total length} = r\theta + \sqrt{2rx + x^2}$$

(3)

QUESTION 6

$$(i) \log_{10} x = \log_{10} 4 + \log_{10} 8$$

$$= \log_{10} 32$$

$$\therefore x = 32$$

(1)

$$(ii) 2 \ln 54 - x \ln 3 = \ln 12$$

$$\ln 54^2 - \ln 12 = x \ln 3$$

$$\ln \left(\frac{54 \times 54}{12} \right) = \ln 3^x$$

$$\ln \frac{9 \times 6 \times 9 \times 6}{6 \times 2} = \ln 3^x, \ln 3^5 = \ln 3^x, \therefore x = 5$$

(2)

QUESTION 7

$$\int_0^1 \frac{2e^x}{1+e^x} dx$$

$$= \left[2 \ln(1+e^x) \right]_0^1$$

$$= 2 \ln(1+e) - 2 \ln 2$$

$$= 2 \ln \frac{(1+e)}{2}$$

(2)

QUESTION 8

$$1 - 25e^{-4w} = \frac{24}{25}$$

$$\frac{-25}{e^{4w}} = \frac{-1}{25}$$

$$e^{4w} = 625 = 5^4$$

$$\log_e 5^4 = 4w$$

$$w = \frac{1}{4} \log_e 5^4$$

$$= \log_e 5$$

(3)

QUESTION 9.

$$f(x) = \log_e \sqrt{(x^3+1)} = \log_e (x^3+1)^{\frac{1}{2}}$$

$$f'(x) = \frac{\frac{3x^2}{2\sqrt{x^3+1}}}{(x^3+1)^{\frac{1}{2}}}$$

$$= \frac{3x^2}{2(x^3+1)}, \quad x \neq -1$$

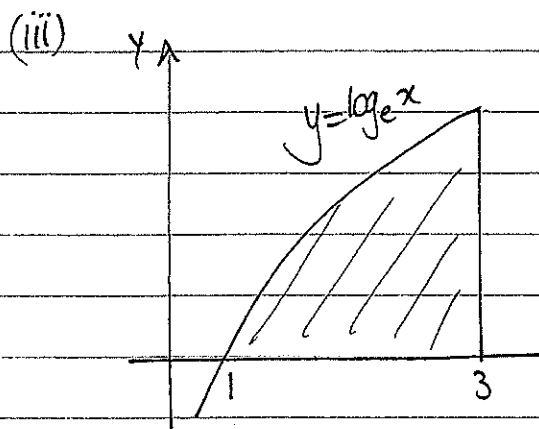
(2)

QUESTION 10

$$(i) \frac{d}{dx} (x \log_e x) = \log_e x + 1$$

$$(ii) \int \log_e x = x \log_e x - \int 1 \, dx$$

$$= x \log_e x - x$$



$$\text{Area} = \int_1^3 \log_e x \, dx$$

$$= \left[x \log_e x - x \right]_1^3 = 3 \ln 3 - 3 - (-1)$$

QUESTION 11

$$V = \pi \int_0^{\log_e 2} x^2 \, dy$$

Since $y = \log_e x \Rightarrow x = e^y$

$$\therefore x^2 = e^{2y}$$

$$\therefore V = \pi \int_0^{\log_e 2} e^{2y} \, dy$$

$$= \pi \left[\frac{1}{2} e^{2y} \right]_0^{\log_e 2}$$

$$= \frac{\pi}{2} \left[e^{2 \log_e 2} - e^0 \right]$$

$$= \frac{\pi}{2} (e^{\log_e 4} - 1)$$

$$= \frac{\pi}{2} (4 - 1) = \frac{3\pi}{2} \text{ units}^3$$

(4)

$$= 3 \ln 3 - 2$$