Carlingford High School

2015

Higher School Certificate Trial Examination

Mathematics Extension 1

Name:____________________ Class:12MA1_____

Circle your teacher: Ms Strilakos Mr Cheng Mr Gong/Ms Wilson

- **General Instructions**
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Pencil may be used for graphs and diagrams only
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 12

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

	Q1-10	Q11	Q12	Q13	Q14	Total
Multiple Choice	/10					/10
Algebra		/3				/3
Functions		/2	/6		/8	/16
Calculus		/5	/3	/4		/12
Circle Geometry			/6			/6
Induction				/3		/3
Trigonometry		/5		/3	/3	/11
Motion				/5	/4	/9
	/10	/15	/15	/15	/15	/70

Section I

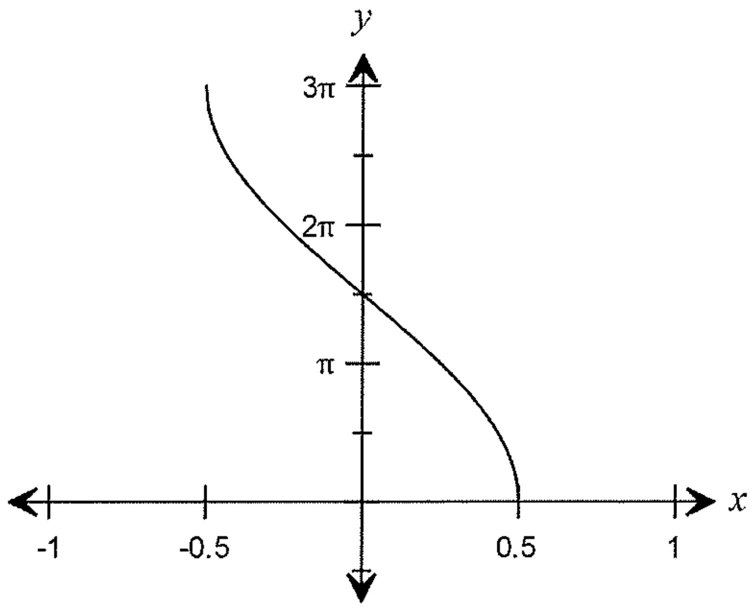
10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1. Which of the following functions is represented in the graph?



(A) $y = 3\sin^{-1}(2x)$

(B) $y = 3\cos^{-1}(2x)$

(C) $y = 2\cos^{-1}(3x)$

(D) $y = 2\left|\sin^{-1}(3x)\right|$

2. Evaluate $\int_0^1 \frac{e^x}{1+e^x}\,dx$.

(A) $\frac{e}{1+e}$

(B) $\frac{e^2}{1+e^2}$

(C) $\ln(1+e)$

(D) $\ln\left(\frac{1+e}{2}\right)$

3. If $(x-3)$ is a factor of the polynomial $P(x) = x^3 - 2x^2 - kx + 6$, what is the value of k?

(A) 3

(B) 5

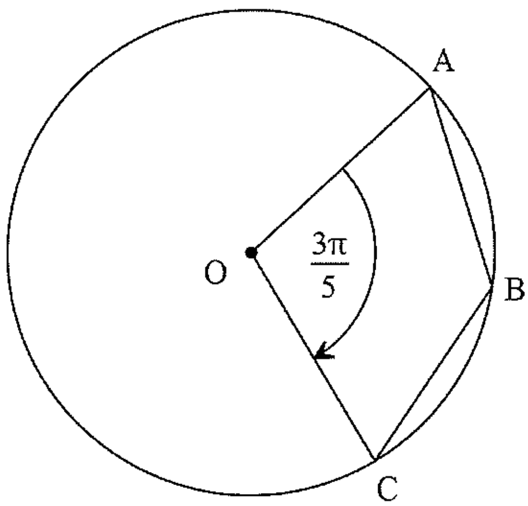
(C) 12

(D) -5

4. The points A, B and C lie on a circle with centre O as shown in the diagram.

The size of $\angle AOC$ is $\frac{3\pi}{5}$.

Find the size of $\angle ABC$.



(A) $\frac{\pi}{5}$

(B) $\frac{17\pi}{30}$

(C) $\frac{7\pi}{10}$

(D) $\frac{6\pi}{5}$

5. Find the exact value of $\sin 75^\circ$.

(A) $\frac{\sqrt{2}}{2}$

(B) $\frac{\sqrt{2}+\sqrt{3}}{2}$

(C) $\frac{\sqrt{2}+\sqrt{6}}{4}$

(D) $\frac{\sqrt{2+1}}{2}$

6. A committee of 3 men and 3 women is to be formed from a group of 8 men and 6 women. How many ways can this be done?

(A) 48

(B) 1 120

(C) 40 320

(D) 3003

7. A particle is moving in simple harmonic motion.

The velocity of the particle at a position x is $\dot{x} = 2e^{-\frac{x}{2}}$ metres per second.

Calculate the particle's acceleration when its displacement is – 2 metres.

(A) $-e^{-\frac{x}{2}}$ m/s^2

(B) $-\frac{4}{e^{\frac{2}{2}}}$ m/s^2

(C) $-2e^2$ m/s^2

(D) e^2 m/s^2

8. Find the horizontal asymptote for $y = \dfrac{3x^2 + 2x + 4}{x^2}$

(A) $y = 4$

(B) $y = 2$

(C) $y = 3$

(D) $y = 1$

9. When the polynomial $P(x)$ is divided by $(x+1)(x+2)$ the remainder is $28x + 19$.
What is the remainder when $P(x)$ is divided by $(x+1)$?

(A) -9

(B) 1

(C) -2

(D) 28

10. Which of the following expressions is $\int \cos^2 3x \, dx$?

(A) $2 \cos 3x + C$

(B) $\cos^3 3x \sin^2 3x + C$

(C) $\frac{1}{2}\left(x + \frac{1}{3} \sin 3x\right) + C$

(D) $\frac{1}{2}\left(x + \frac{1}{6} \sin 6x\right) + C$

End of Section I

Section II 60 marks Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a separate booklet. Extra writing booklets are available.

Include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Begin a new writing booklet.

(a) Solve $\dfrac{x}{x-5} \geq 2$. **3**

(b) Monica was trying to divide the polynomial $P(x) = 2x^4 - 5x^3 - x + 12$ by $(x-2)$.
Her working out is shown below.

$$
\begin{array}{rll}
 & 2x^3 - x^2 - 3 & \\
x-2\,\big) & 2x^4 - 5x^3 - x + 12 & \\
 & \underline{2x^4 - 4x^3} & \text{Line 1} \\
 & -x^3 - x & \text{Line 2} \\
 & \underline{-x^3 + 2x^2} & \text{Line 3} \\
 & -3x + 12 & \text{Line 4} \\
 & \underline{-3x + 6} & \text{Line 5} \\
 & 6 & \text{Line 6}
\end{array}
$$

(i) Monika has made an error in her working out. **1**
Explain where the error occurred and what she did wrong.

(ii) Monica's quotient and remainder were incorrect due to her error. **1**
What is the correct remainder for this division?

Question 11 continues on page 7.

Question 11 (continued)

(c) Find $\frac{d}{dx}\tan^{-1}\frac{x}{4}$. **2**

(d) The gradient of the tangent at any point (x, y) on a curve is $\frac{1}{\sqrt{4-x^2}}$ **3**

Find the equation of the curve if it passes through the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$

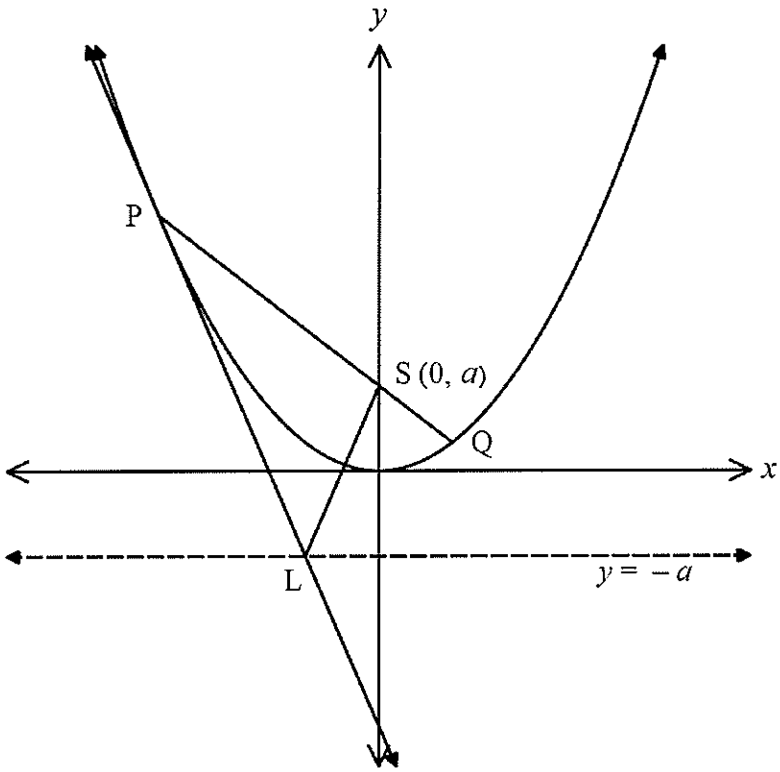
(e) Evaluate $\int_0^1 x^3\left(\sqrt{x^4+1}\right)\,dx$ using the substitution $u = x^4 + 1$. **3**

(f) The radius r of a circle is increasing at a constant rate of 0.1 cm/s. **2**
What is the rate at which the area of the circle is increasing when $r = 10$ cm?

End of Question 11

Question 12 (15 marks) Begin a new writing booklet.

(a) $x^2 = 4ay$ is a parabola with focus $S(0, a)$.



$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are the endpoints of a focal chord to the parabola.

L is the point where the tangent at P meets the directrix of the parabola.

You may assume without proof that $pq = -1$ and that the tangent at P has equation $y = px - ap^2$.

(i) Show that $SP = a(p^2 + 1)$. **1**

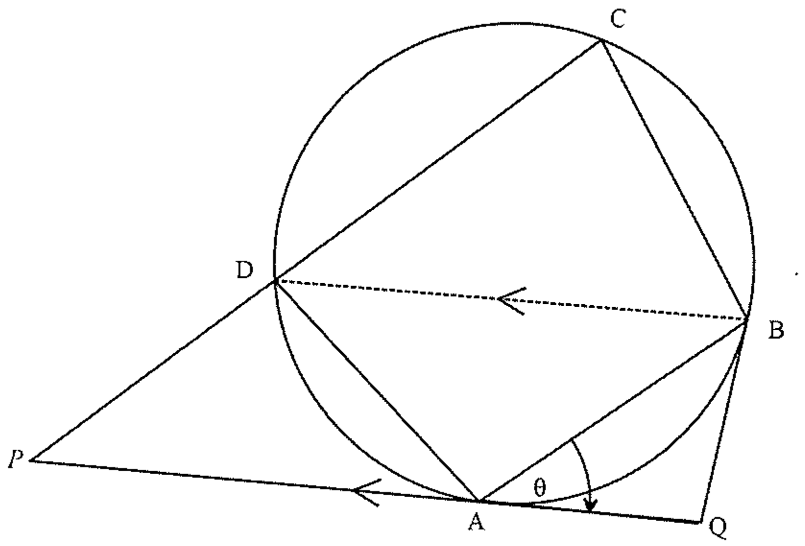
(ii) Show that L has coordinates $\left(ap - \frac{a}{p}, -a\right)$. **2**

(iii) Show that $SP \times SQ = SL^2$. **3**

Question 12 continues on page 9.

Question 12 (continued)

(b)



In the diagram above, $ABCD$ is a cyclic quadrilateral.

The tangents from Q touch the circle at A and B.

The diagonal DB is parallel to the tangent AQ.

QA produced intersects with CD produced at P.

Let $\angle QAB = \theta$.

Copy or trace the diagram into your writing booklet.

(i) Given that $PD = 5$ cm and DC is 7 cm in length, calculate the exact length of AP. **1**

(ii) Show that $\angle BCD = 2\theta$, giving reasons. **3**

(iii) Show that $PQBC$ is a cyclic quadrilateral, giving reasons. **2**

(c) An oven which had been heated to $180^\circ C$ was switched off when the cook was finished baking at 11:30 am. The oven was in a kitchen which was kept at a constant temperature of 22°. **3**

After t minutes, the temperature, $(T^\circ C)$, of the oven is given by:

$$T = A + Be^{-kt}.$$

A, B and k are positive constants.

After ten minutes, the oven's temperature has dropped to $115^\circ C$.

At what time will the oven's temperature drop to 23° ?

End of Question 12

Question 13 (15 marks) Begin a new writing booklet.

(a) Given that x is a positive integer, prove by the method of mathematical induction that $(1+x)^n - 1$ is divisible by x for all positive integers $n \geq 1$. **3**

(b) A particle is moving in a straight line according to the equation

$$x = 4\cos 3t + 6\sin 3t - 5.$$

Displacement is measured in metres and time in hours.

(i) Find an equation to represent the acceleration of this particle and prove that it is moving in simple harmonic motion. **2**

(ii) Given that the particle is one metre below the origin at noon, between what times will the particle be greater than one metre above the origin for the first time? **3**

(Let the time at $t = 0$ be noon. Give your times correct to the nearest minute.)

(c) The area bounded by the function $y = 2 + e^{-x}$ and the lines $x = 0$, $x = 1$ and $y = 1$ is rotated about the x-axis.

(i) Show that the volume generated is given by the integral: **2**

$$V = \pi\int_0^1 (\ 4 + 4e^{-x} + e^{-2x}\ )\, dx \ - \ \pi \ \text{ cubic units}$$

(ii) Show that the exact volume is $\pi\left(\dfrac{15e^2 - 8e - 1}{2e^2}\right)$ cubic units. **2**

(d) (i) Given that the limiting sum exists, show that

$$\tan x + \tan^2 x + \tan^3 x + \cdots = \frac{1}{2}\tan 2x.$$ **2**

(ii) Hence find the exact value of $\tan\frac{\pi}{8} + \tan^2\frac{\pi}{8} + \tan^3\frac{\pi}{8} + \cdots$. **1**

End of Question 13

Question 14 (15 marks) Begin a new writing booklet.

(a) Solve the equation $\sin^{-1}x = 3\cos^{-1}x$, giving the solution correct to 2 decimal places. **3**

(b) (i) Find the domain and range of the function $f(x) = \cos^{-1}(2x-1) - \frac{\pi}{2}$. **2**

(ii) Find the inverse function $f^{-1}(x)$. **3**

(iii) State the domain of this inverse function. **1**

(iv) Draw a neat sketch of $f^{-1}(x)$. **2**

(c) The acceleration of a particle P along the x axis is given by $a = -e^{-x}(1+e^{-x})$, where x is the displacement of the particle from the origin in metres. Initially, the particle is at the origin and its velocity is 2 m/s. **4**

Find the velocity v in terms of x.

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Trial HSC Examination 2015

Mathematics Extension 1 Course

Name ____________________ Teacher ____________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Carlingford High School Mathematics Extension 1 Trial Examination 2015 Answers

11 a b, 12 K Cheng
11 c d, 13 L Wilson
11 e f, 14 S. Strilakos

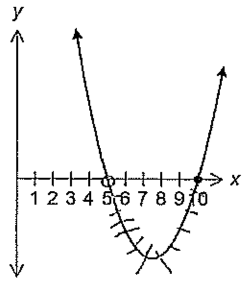
Section I - Multiple Choice

Note change to mark allocation in Q13

1	B
2	D
3	B
4	C
5	C
6	B
7	C
8	C
9	A
10	D

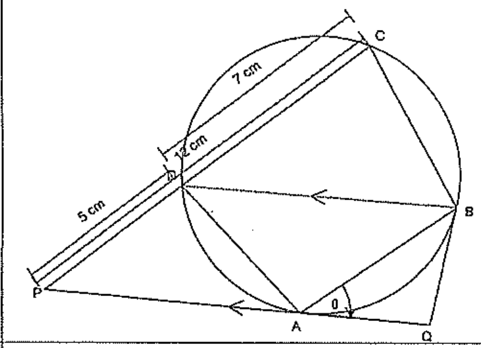
	Q1-10	Q11	Q12	Q13	Q14	Total
Multiple Choice	/10					/10
Algebra		/3				/3
Functions		/2	/6		/8	/16
Calculus		/5	/3	/4		/12
Circle Geometry			/6			/6
Induction				/3		/3
Trigonometry		/5		/2	/3	/10
Motion				/6	/4	/10
	/10	/15	/15	/15	/15	/70

Section II

	Solution	Outcome/Marks
	Question 11	
a)	$\frac{x}{x-5} \geq 2$ $x(x-5) \geq 2(x-5)^2$ $0 \geq 2(x-5)^2 - x(x-5)$ $0 \geq (x-5)(2(x-5)-x)$ $0 \geq (x-5)(2x-10-x)$ $0 \geq (x-5)(x-10)$  Since the inequality is true in the shaded region, the solution is $5 < x \leq 10$. Note - $x \neq 5$.	Algebra/3
b)	(i) The response should indicate that Monica hasn't allowed for the fact that there is a term of $0x^2$ in the polynomial P(x). She has brought down the term in x in line 2 and has subtracted the term in x^2 from a term in x in lines 3 and 4. (ii) The remainder upon dividing by (x-2) will be P(2). $P(2) = 2(2)^4 - 5(2)^3 - 2 + 12$ $= 2$	Functions/2 1 1

	Solution	Outcome/Marks
c)	From the table of standard integrals: $\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$ $\therefore \frac{d}{dx}\left(\frac{1}{a}\tan^{-1}\frac{x}{a}\right) = \frac{1}{a^2+x^2}$ $\frac{d}{dx}\frac{1}{4}\tan^{-1}\frac{x}{4} = \frac{1}{16+x^2}$ $\therefore \frac{d}{dx}\tan^{-1}\frac{x}{4} = \frac{4}{16+x^2}$ 1 mark for $\frac{1}{16+x^2}$ or $\frac{4}{4+x^2}$ or $\frac{16}{16+x^2}$	~~Calculus~~/2 Trigonometry
d)	$y = \int \frac{1}{\sqrt{4-x^2}}dx = \sin^{-1}\frac{x}{2} + c$ $\frac{\pi}{4} = \sin^{-1}\frac{\pi}{4} + c$ $\frac{\pi}{4} = \sin^{-1}\frac{\pi}{4} + c$ $c = \frac{\pi}{4} - \sin^{-1}\frac{\pi}{4}$ $y = \sin^{-1}\frac{x}{2} + \left(\frac{\pi}{4} - \sin^{-1}\frac{\pi}{4}\right)$	Trig /3
e)	$\int_0^1 x^3\sqrt{x^4+1}$ let u $= x^4 + 1$, then $du = 4x^3\ dx$ when $x = 0$, $u = 1$ when $x = 1$, $u = 2$ $\int_1^2 \frac{1}{4}u^{\frac{1}{2}}\,du$ $= \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^2$ $= \frac{1}{6}\left(2^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$ $= \frac{1}{6}(\sqrt{8} - 1)$ $= \frac{2\sqrt{2}-1}{6}$	Calculus / 3
f)	$A = \pi r^2, \frac{dr}{dt} = 0.1$ $\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$ $= 2\pi r \times 0.1$ When $r = 10, \frac{dA}{dt} = 2\pi$ cm/s	Calculus / 2

	Solution	Outcome/Marks
Question 12		
a)	(i) $SP=\sqrt{(2ap)^2+(ap^2-a)^2}$ $=\sqrt{4a^2p^2+a^2p^4-2a^2p^2+a^2}$ $=\sqrt{2a^2p^2+a^2p^4+a^2}$ $=\sqrt{a^2(p^4+2p^2+1)}$ $=\sqrt{a^2(p^2+1)^2}$ $=a(p^2+1)$ OR $SP = PM$ $= ap^2-(-a)$ $= ap^2+a$ $= a(p^2+1)$	Functions / 1
	(ii) Find the coordinates where the tangent intersects with the directrix by solving simultaneous equations: $y=px-ap^2$ ① $y=-a$ ② $-a=px-ap^2$ (*substitute* ① *into* ②) $px=ap^2-a$ $x=\frac{ap^2-a}{p}$ $=ap-\frac{a}{p}$ Therefore the coordinates are $\left(ap-\frac{a}{p},\ -a\right)$.	Functions / 2
iii)	$SQ=a(q^2+1)$ $SP\times SQ=a(p^2+1)a(q^2+1)$ $=a^2(p^2+1)(q^2+1)$ *But* $pq=-1\therefore q=-\frac{1}{p}$ $=a^2(p^2+1)\left(\left(-\frac{1}{p}\right)^2+1\right)$ $=a^2(p^2+1)\left(\frac{1}{p^2}+1\right)$ $=a^2\left(1+p^2+\frac{1}{p^2}+1\right)$ $=a^2(p^2+p^{-2}+2)$	

	Solution	Outcome/Marks
iii ctd	$SL^2=\left(ap-\frac{a}{p}\right)^2+(-a-a)^2$ $=a^2p^2-2ap.\frac{a}{p}+\frac{a^2}{p^2}+4a^2$ $=a^2p^2-2a^2+\frac{a^2}{p^2}+4a^2$ $=a^2p^2+2a^2+\frac{a^2}{p^2}$ $=a^2(p^2+p^{-2}+2)$ $=SP\times SQ$	~~Circle geometry~~ /
b)		Circle geom
i)	$AP^2=PC\times PD$ $=5\times12$ $=60$ $AP=\sqrt{60}$ $=2\sqrt{15}\ cm$	CG / 1

	Solution	Outcome/Marks
ii)	$\angle ADB = \angle QAB = \theta$ (angle between tangent and chord is equal to angle in alternate segment) $\angle ABD = \angle QAB = \theta$ (alternate angles on parallel lines) OR AQ = BQ (tangent from an external point are equal) $\therefore \angle QAB = \angle QBD = \theta$ (isosceles Δ) $\therefore \angle BAD = 180 - 2\theta$ (angle sum of triangle) $\therefore \angle BCD = 180 - (180 - 2\theta) = 2\theta$ (opposite angles in cyclic quadrilateral are supplementary)	CG / 3
	(iii) $\angle ABQ = \angle ADB = \theta$ (angle between tangent and chord is equal to angle in alternate segment) $\therefore \angle AQB = 180 - 2\theta$ (angle sum of a triangle) $\angle PCB + \angle PQB = 2\theta + 180 - 2\theta$ $= 180°$ Therefore PQBC is a cyclic quadrilateral (opposite angles supplementary)	CG / 2
c)	$T = A + Be^{-kt}$ $= 22 + Be^{-kt}$ when $t = 0$, $T = 180°C$ $180° = 22 + B$ $B = 158$ $\therefore \quad T = 22 + 158e^{-kt}$ when $t = 10, T = 115°C$ $115 = 22 + 158e^{-10k}$ $93 = 158e^{-10k}$ $e^{-10k} = \frac{93}{115}$ $-10k = \ln\left(\frac{93}{115}\right)$ $k = \frac{\ln\left(\frac{93}{115}\right)}{-10}$ ≈ 0.05299955399 $\therefore \quad T = 22 + 158e^{-0.05299955399t}$	Calculus /3
	To find when T = 23: $23 = 22 + 158e^{-0.05299955399t}$ $1 = 158e^{-0.05299955399t}$ $e^{-0.05299955399t} = \frac{1}{158}$ $t = \frac{\ln\left(\frac{1}{158}\right)}{-0.05299955399}$ $= 95.52146485$ min $\approx 1h\ 36$ min $\therefore$ The oven will reach 23° at 1:06 pm	

	Solution	Outcome/Marks
	Question 13	
a)	RTP: $(1+x)^n - 1$ is divisible by x for all positive integers $n \geq 1$ <u>Prove true for n=1</u> $(1+x)^1 - 1 = 1 + x - 1$ $= x$ which is divisible by x, so true for n=1. <u>Assume true for n=k.</u> $(1+x)^k - 1 = c_1 x$ where c_1 is a polynomial $\therefore (1+x)^k = c_1 x + 1$ <u>Prove true for n=k+1</u> $(1+x)^{k+1} - 1 = c_2 x$ where c_2 is a polynomial $LHS = (1+x)(1+x)^k - 1$ $= (1+x)(c_1 x + 1) - 1$ (by assumption) $= c_1 x + 1 + x^2 c_1 + x - 1$ $= c_1 x + x c_1 x + x$ $= x(c_1 + x c_1 + 1)$ which is divisible by x. So true for $n=k+1$ when true for $n=k$. <u>Conclusion</u> Since true for $n=k+1$ when true for $n=k$ and already proven for $n=1$, must be true for all $n \geq 1$.	Induction/3
b)	i) $x = 4\cos 3t + 6\sin 3t - 5$ $\dot{x} = -12\sin 3t + 18\cos 3t$ $\ddot{x} = -36\cos 3t - 54\sin 3t$ $= -9(4\cos 3t + 6\sin 3t)$ $= -9\left(x + \frac{45}{9}\right)$ $= -9(x+5)$ Since this is in the form $\ddot{x} = -n^2(x-c)$, the particle is moving in simple harmonic motion. (ii) We want to find when $x \geq 1$. Let $x = 1$ to find the first time when this occurs. $4\cos 3t + 6\sin 3t - 5 = 1$ $4\cos 3t + 6\sin 3t = 6$ $2\cos 3t + 3\sin 3t = 3$ $\sqrt{13}\sin(3t+\alpha) = 3$ (since $a\sin\theta + b\cos\theta = r\sin(\theta+\alpha)$) $\sin(3t+\alpha) = \frac{3}{\sqrt{13}}$ $\quad \alpha = \tan^{-1}\left(\frac{2}{3}\right) \approx 0.5880026035$	Motion /2 Motion /4

Solution	Outcome/Marks
Look at 1st and 2nd quadrants for first two times where $x = 1$ $3t+\alpha = \sin^{-1}\left(\frac{3}{\sqrt{13}}\right)$ $3t+\alpha = 0.9827937232, 2.15879893$ $3t = 0.3947911197, 1.570796327$ $t = 0.1315970399, 0.5235987757$ $\approx$ 8 min and 31 min Therefore the particle is more than 1 metre above the origin between 12:08 pm and 12:31 pm.	
Volume for function revolved about x-axis is given by $V = \pi\int_a^b y^2\,dx$ $V = \pi\int_a^b \left(2+e^{-x}\right)^2 dx$ $= \pi\int_a^b \left(4+4e^{-x}+e^{-2x}\right) dx$ Since we want only the volume between the curve and the line y=1, we must subtract the volume of the cylinder created by the revolving area between the x-axis, x=1, y-axis and y =1. This cylinder has height 1 unit and radius 1 unit, so its volume is $V = \pi r^2 h$ $= \pi \times 1^2 \times 1$ $= \pi$ cubic units So for our solid: $V = \pi\int_0^1 \left(4+4e^{-x}+e^{-2x}\right) dx - \pi$	Calculus/2
(ii) $V = \pi\int_0^1 \left(4+4e^{-x}+e^{-2x}\right) dx - \pi$ $= \pi\left[4x - 4e^{-x} - \frac{1}{2}e^{-2x}\right]_0^1 - \pi$ $= \pi\left[\left(4-4e^{-1}-\frac{1}{2}e^{-2}\right)-\left(0-4e^{0}-\frac{1}{2}e^{0}\right)\right]-\pi$ $= \pi\left(4-\frac{4}{e}-\frac{1}{2e^2}+4+\frac{1}{2}\right)-\pi$ $= \pi\left(\frac{8e^2-8e-1}{2e^2}+\frac{9}{2}-1\right)$ $= \pi\left(\frac{8e^2-8e-1+7e^2}{2e^2}\right)$ $= \pi\left(\frac{15e^2-8e-1}{2e^2}\right)$ cubic units	Calculus /2

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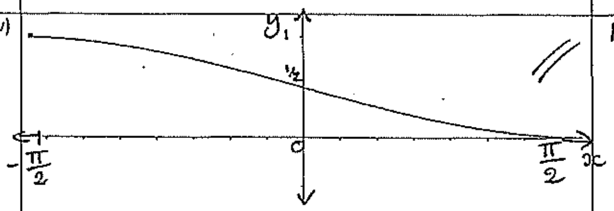
	Solution	Outcome/Marks
d)	There was a mistake in this question: (The series should have been $\tan x + \tan^3 x + \tan^5 x + \cdots$) i) 1 mark awarded for correct expression for LHS: This is a geometric series so $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{\tan x}{1-\tan x}$ 1 mark awarded for correct expression for $\tan 2x$: $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	Trigonometry /2
	ii) No marks awarded for this question : 1 additional mark awarded for 13bii	0
a)	$\sin^{-1}x = 3\cos^{-1}x$ $\frac{\pi}{2} - \cos^{-1}x = 3\cos^{-1}x$ $\frac{\pi}{2} = 4\cos^{-1}x$ $\cos^{-1}x = \frac{\pi}{8}$ $x = \cos\frac{\pi}{8}$ $x = 0.92$ (to 2 decimal places)	Trigonometry/3
b) i)	Domain: $-1 \le 2x-1 \le 1$ $0 \le 2x \le 2$ $0 \le x \le 1$ Range: $0 \le \cos^{-1}(2x-1) \le \pi$ $-\frac{\pi}{2} \le \cos^{-1}(2x-1) \le \frac{\pi}{2}$ $-\frac{\pi}{2} \le f(x) \le \frac{\pi}{2}$	Functions/~~3~~ 2
ii)	$f: y = \cos^{-1}(2x-1) - \frac{\pi}{2}$ $f^{-1}: x = \cos^{-1}(2y-1) - \frac{\pi}{2}$ $x + \frac{\pi}{2} = \cos^{-1}(2y-1)$ $\cos(x+\frac{\pi}{2}) = 2y-1$ $2y = \cos\ (x+\frac{\pi}{2}) + 1$ $y = \frac{1}{2}\left(\cos\ (x+\frac{\pi}{2}) + 1\right)$	Functions /3
iii)	Domain of $y = f^{-1}(x)$ is the same as the range of $y = f(x)$. $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	Functions /1
iv)		Functions /2

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	Solution
c)	$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = -e^{-x} - e^{-2x}$ $\frac{1}{2}v^2 = \int\left(-e^{-x}-e^{-2x}\right)dx$ $\frac{1}{2}v^2 = e^{-x} + \frac{1}{2}e^{-2x} + c$ when $x = 0, v = 2$ $\therefore 2 = 1 + \frac{1}{2} + c \therefore c = \frac{1}{2}$ $\therefore \frac{1}{2}v^2 = e^{-x} + \frac{1}{2}e^{-2x} + \frac{1}{2}$ $\therefore v^2 = 2e^{-x} + e^{-2x} + 1$ $= \frac{2}{e^x} + \frac{1}{e^{2x}} + 1$ $= \frac{2e^x + 1 + e^{2x}}{e^{2x}}$ $= \frac{e^{2x} + 2e^x + 1}{e^{2x}}$ $= \frac{(e^x+1)^2}{e^{2x}}$ $\therefore v = \frac{e^x+1}{e^x}$ since $v = 2$ when $x = 0$

Motion/4