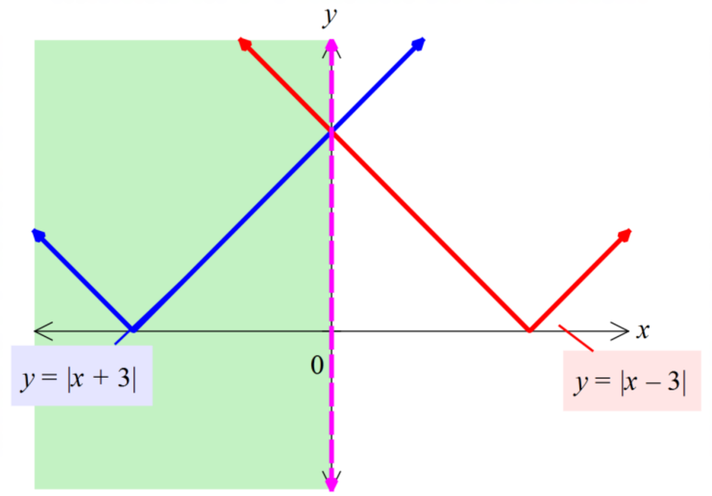
Western Mathematics Exams

2018
TRIAL HSC
EXAMINATION

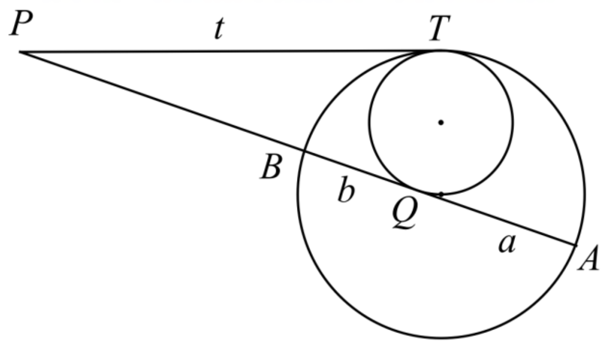
Mathematics
Extension 1

SOLUTIONS

Multiple Choice Worked Solutions

No	Working	Answer
1	$\dfrac{\cos 3\alpha}{\sin\alpha} + \dfrac{\sin 3\alpha}{\cos\alpha} = \dfrac{\cos 3\alpha \ \cos\alpha \ + \ \sin 3\alpha \sin\alpha}{\sin\alpha \cos\alpha}$ $= \dfrac{\cos(3\alpha - \alpha)}{\frac{1}{2}\sin 2\alpha}$ $= \dfrac{2\cos 2\alpha}{\sin 2\alpha}$ $= 2\cot 2\alpha$	**D**
2	Number of arrangements $= \dfrac{11!}{2!2!2!2!}$ $= 2\,494\,800$	**A**
3	$t = 2x \quad y = \dfrac{4 \times (2x)^2}{3}$ $y = \dfrac{16x^2}{3}$ $x^2 = \dfrac{3y}{16}$	**D**
4	$x = 4\sin^2 t - 1$ $= 2(1 - \cos 2t) - 1$ $= 1 - 2\cos 2t$ $\therefore$ Centre of motion is $x = 1$	**C**
5	 $x < 0$	**B**

6	$V = \frac{4}{3} \times \pi \times r^3$ $\frac{dV}{dt} = 4\pi r^2$ $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $2 = 4\pi r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{2}{4\pi r^2}$ $= \frac{1}{2\pi r^2}$	**A**
7	$f(x) = \frac{e^x}{4 + e^x}$ $u = e^x \quad v = 4 + e^x$ $du = e^x \quad dv = e^x$ $f'(x) = \frac{(4 + e^x) \times e^x - e^x \times e^x}{(4 + e^x)^2}$ $= \frac{4e^x + e^{2x} - e^{2x}}{(4 + e^x)^2}$ $= \frac{4e^x}{(4 + e^x)^2}$ $\therefore f''(x) > 0$ for all values of x since $e^x > 0$ $\therefore f''(x)$ has no stat pts	**A**

8	 $PQ = PB + BQ$ $t = PB + b$ $PB = t - b$ $PQ = PT = t$ (tangents from external point are equal) $PA = PQ + QA$ $= t + a$ $PT^2 = PA \cdot PB$ (Square on Tangent = product of secants) $\therefore \quad t^2 = (t + a) \times (t - b)$ $t^2 = t^2 - tb + ta - ab$ $tb - ta = -ab$ $t(b - a) = -ab$ $t = \frac{-ab}{b - a}$ $t = \frac{ab}{a - b}$	**C**
9	$3x^3 - 2x^e + 4$ as it does not have positive whole number powers of x	**D**
10	$\int_0^4 \frac{dx}{x^2 + 16}$ $= \left[\frac{1}{4} \tan^{-1} \frac{x}{4}\right]$ $= \frac{1}{4}(\tan^{-1} 1 - \tan^{-1} 0)$ $= \frac{1}{4}\left[\frac{\pi}{4} - 0\right]$ $= \frac{\pi}{16}$	**B**

Trial HSC Examination 2018
Mathematics Extension 1 Course

Name ______________________ Teacher ________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

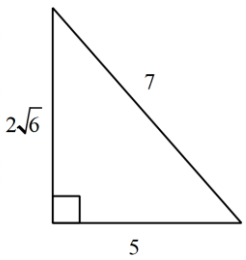
If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

1.	A ○	B ○	C ○	D ●
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ○	C ●	D ○
5.	A ○	B ●	C ○	D ○
6.	A ○	B ○	C ●	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ○	C ○	D ●
10.	A ○	B ●	C ○	D ○

Question 11	**2018**

	Question 11 Solutions	Marks	Allocation of marks
(a)	$y = 2\cos^{-1} x - 1$ $\dfrac{y+1}{2} = \cos^{-1} x$ $\cos\left(\dfrac{y+1}{2}\right) = x$ $\therefore \text{Domain}: -1 \leq x \leq 1$ $\therefore \text{Range}: 0 \leq \dfrac{y+1}{2} \leq \pi$ $0 \leq y+1 \leq 2\pi$ $-1 \leq y \leq 2\pi - 1$	2	2 marks for correct domain and range 1 mark for working which includes correct domain or range
(b)	(i)  $\sin x = \dfrac{2\sqrt{6}}{7}$	1	1 for correct answer
	(ii) $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2 x}$ $= \dfrac{2 \times \dfrac{\sin x}{\cos x}}{1 - \left(\dfrac{\sin x}{\cos x}\right)^2}$ $= \dfrac{-2 \times \dfrac{2\sqrt{6}}{7} \times -\dfrac{7}{5}}{1 - \left(\dfrac{-2\sqrt{6}}{5}\right)^2}$ $= \dfrac{-4\sqrt{6}}{5} \times \dfrac{25}{1}$ $= -20\sqrt{6}$	2	2 marks for correct value 1 mark for working which includes correct substitution in expression for tan 2α with subsequent error or other working of similar merit

	Question 11 Solutions	Marks	Allocation of marks
(c)	$\dfrac{4x-1}{x+2} \geq 1$ $x \neq -2$ $(x+2)(4x-1) \geq (x+2)^2$ $4x^2 - x + 8x - 2 \geq x^2 + 4x + 4$ $3x^2 + 3x - 6 \geq 0$ $3(x^2 + x - 2) \geq 0$ $3(x+2)(x-1) \geq 0$ $\therefore x < -2$ and $x \geq 1$	2	2 marks for correct solution with correct inequality signs 1 mark for working which includes correct solution with error in signs or other working of similar merit
(d)	$y = 3x + 2y - 6$ $\therefore \quad m_1 = -3$ $2y = x + 4$ $\therefore \quad m_2 = \dfrac{1}{2}$ $\tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 m_1} \right\rvert$ $\tan\theta = \left\lvert \dfrac{-3 - \frac{1}{2}}{1 + \left(-3 \times \frac{1}{2}\right)} \right\rvert$ $\tan\theta = \left\lvert \dfrac{-3\frac{1}{2}}{-\frac{1}{2}} \right\rvert$ $\tan\theta = 7$ $\theta = 81° \, 52'$ So obtuse angle $180° - 81° \, 52' = 98° \, 8'$	2	2 marks for correct angle 1 mark for working which includes correct substitution in expression for tan 2θ with subsequent error or other working of similar merit
(e)	Let the ratio be 8 : -3 $x = \dfrac{mx_2 + nx_1}{m+n} \qquad y = \dfrac{my_2 + ny_1}{m+n}$ $P\,(10,13) \quad A\,(x_1, y_1) \quad B\,(7,7)$ $10 = \dfrac{8 \times 7 + -3 \times x}{8 + -3} \qquad 13 = \dfrac{8 \times 7 + -3 \times y}{8 + -3}$ $10 = \dfrac{56 - 3x}{5} \qquad 13 = \dfrac{56 - 3y}{5}$ $50 = 56 - 3x \qquad 13 = 56 - \dfrac{3y}{5}$ $3x = 6 \qquad 3y = -9$ $x = 2 \qquad y = -3$ $\therefore A(2, -3)$	2	2 marks for correct point 1 mark for working which includes correct substitution in equation for ratio with subsequent error or other working of similar merit

	Question 11 Solutions	Marks	Allocation of marks
(f)	When $u = 2x^3 - 4x^2 + 5$ $\frac{du}{dx} = 6x^2 - 8x$ $du = 2(3x^2 - 4)\,dx$ $\int (3x^2 - 4)(2x^3 - 4x^2 + 5)\,dx = \frac{1}{2}\int 2(2x^3 - 4x^2 + 5)^4\ (3x^2 - 4)dx$ $= \frac{1}{2}\int u^4\ du$ $= \frac{1}{2}\left[\frac{u^5}{5}\right]$ $= \frac{(2x^3 - 4x^2 + 5)^5}{10} + C$	2	2 marks for correct value 1 mark for working which includes correct substitution in expression with subsequent error or other working of similar merit
(g)	$2x - 1$ is a factor if $P\left(\frac{1}{2}\right) = 0$ $2\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) - 1 = 2 \times \frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} - 1$ $= \frac{1}{8} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} - 1$ $= 0$ $\therefore 2x - 1$ is a factor of $P(x)$ Test for other factors $P(-1) = 2 \times 1 - 1 + 1 - 1 - 1$ $= 0$ So $(2x - 1)(x + 1) = 2x^2 + x - 1$ is a factor $x^2 \quad + \quad 1$ $2x^2 + x - 1 \,\big)\, 2x^4 + x^3 + x^2 + x - 1$ $2x^4 + x^3 - x^2$ $2x^2 + x - 1$ $2x^2 + x - 1$ 0 $x^2 + 1$ cannot be factorised further over the reals $(\Delta = 0^2 - 4 \times 1 \times 1 = -4)$ So factorisation is $2x^4 + x^3 + x^2 + x - 1 = (2x - 1)(x + 1)(x^2 + 1)$	2	2 marks for correct factors in any form 1 mark for working which includes showing $2x - 1$ is a factor or which finds at least one other factor or working of equal merit.

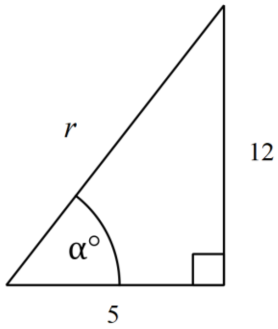
Question 12	**2018**

	Question 12 Solutions	Marks	Allocation of marks
(a)	i) $x^2 = 4ay$ $y = \frac{x^2}{4a}$ $\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$ At $P(2ap, ap^2)$ $\frac{dy}{dx} = \frac{2ap}{2a} = p$ Equation of the tangent at $P(2ap, ap^2)$ $y - ap^2 = p(x - 2ap)$ $y - ap^2 = px - 2ap^2$ $\therefore px - y - ap^2 = 0$	1	1 mark: Finds the equation of the tangent at $P(2ap, ap^2)$.
	ii) Similarly, at Q the equation of the tangent is $qx - y - aq^2 = 0$ Solving simultaneously to find the co-ordinates of T $px - y - ap^2 = 0$ (1) $qx - y - aq^2 = 0$ (2) Equation (1) – equation (2) $px - qx - ap^2 + aq^2 = 0$ $(p - q)x = a(p + q)(p - q)$ $x = a(p + q)$ Substituting $a(p + q)$ for x into equation (1) $pa(p + q) - y - ap^2 = 0$ $y = ap^2 + apq - ap^2$ $= apq$ $\therefore$ Coordinates of T are $(a(p + q), apq)$	2	2 marks: Correct answer. 1 mark: Finds one of the coordinates or shows some understanding.
	iii) $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ $\tan 45° = \frac{p - q}{1 + pq}$ $1 = \frac{p - q}{1 + pq}$ $\therefore p - q = 1 + pq$	1	1 mark: Correct answer.
	iv) Evaluating the expression $x^2 = 4ay$ at T $x^2 - 4ay = [a(p + q)]^2 - 4a(apq)$ $= a^2(p + q)^2 - 4a^2pq$ $= a^2(p + q)^2 - 4pq$ $= a^2(p^2 - 2pq + q^2)$ $= a^2(p - q)^2$ Now using the result in part (iii) $x^2 - 4ay = a^2(1 + pq)^2$ $= a^2(1 + 2pq + p^2q^2)$ $= a^2 + 2a(apq) + (apq)^2$ $= a^2 + 2ay + y^2$ $x^2 - y^2 = a^2 + 6ay$ $\therefore$ Locus of T is $x^2 - y^2 = a^2 + 6ay$.	2	2 marks: Correct answer. 1 mark: Makes some progress towards the solution.

	Question 12 Solutions	Marks	Allocation of marks
(b)	$\frac{d}{dx}\, x \sin^{-1} x = x \,.\, \frac{d}{dx}(\sin^{-1} x) + \sin^{-1}$ $= x \,.\, \frac{1}{\sqrt{1-x^2}} + \sin^{-1} x$ $= \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x$ $\frac{d}{dx}\sqrt{1-x^2} = \frac{d}{dx}\left((1-x^2)^{\frac{1}{2}} \right)$ $= \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times (-2x)$ $= -\frac{x}{\sqrt{1-x^2}}$ $\frac{d}{dx}\, x \sin^{-1} x + \sqrt{1-x^2} = \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x$ $-\frac{x}{\sqrt{1-x^2}}$ $= \sin^{-1} x$	2	2 marks for correct solution 1 mark for working which includes correct derivative for one of the terms or working with a with a minor error of equal merit.
	(ii) $\int_0^1 \sin^{-1} x \; dx$ $= \left[x \sin^{-1} x + \sqrt{1-x^2} \right]_0^1$ $= \left(1 \sin^{-1} 1 + \sqrt{1-1^2}\right) - \left(0 \sin^{-1} 0 + \sqrt{1-0^2}\right)$ $= \left(\frac{\pi}{2} - 1\right)$	1	1 for correct answer

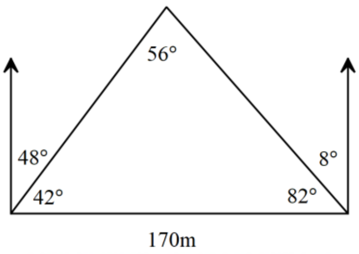
<table>
<tr><td>(c)</td><td>(i)
$$2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots\ldots + 2\left(\frac{1}{3}\right)^{n-1} = -3\left[\left(\frac{1}{3}\right)^{n} - 1\right]$$
Show true for $n = 1$
$$\text{LHS} = 2\left(\frac{1}{3}\right)^{0} = 2 \times 1 = 2$$
$$\begin{aligned}\text{RHS} &= -3\left[\left(\frac{1}{3}\right)^{1} - 1\right]\\ &= -3\left(\frac{1}{3} - 1\right)\\ &= -3\left(-\frac{2}{3}\right)\\ &= 2\\ &= \text{LHS}\end{aligned}$$
Assume true for $n = k$
$$2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots\ldots + 2\left(\frac{1}{3}\right)^{k-1} = -3\left[\left(\frac{1}{3}\right)^{k} - 1\right]$$
Prove true for $n = k + 1$
$$2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots + 2\left(\frac{1}{3}\right)^{k-1} + 2\left(\frac{1}{3}\right)^{(k+1)-1} = -3\left[\left(\frac{1}{3}\right)^{k+1} - 1\right]$$
LHS
$$\begin{aligned}&= 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots + 2\left(\frac{1}{3}\right)^{k-1} + 2\left(\frac{1}{3}\right)^{(k+1)-1}\\ &= -3\left[\left(\frac{1}{3}\right)^{k} - 1\right] + 2\left[\left(\frac{1}{3}\right)^{k}\right]\\ &= -3\left(\frac{1}{3}\right)^{k} + 3 + 2\left(\frac{1}{3}\right)^{k}\\ &= \left(\frac{1}{3}\right)^{k}[-3 + 2] + 3\\ &= -1\left(\frac{1}{3}\right)^{k} + 3\\ &= -3\left[\frac{1}{3} \times \left(\frac{1}{3}\right)^{k} - 1\right]\\ &= -3\left[\left(\frac{1}{3}\right)^{1} \times \left(\frac{1}{3}\right)^{k} - 1\right]\\ &= -3\left[\left(\frac{1}{3}\right)^{k+1} - 1\right]\\ &= \text{RHS}\end{aligned}$$
$\therefore$ since true for $n = 1$ it is also true for all $n \geq 1$</td><td>3</td><td>3 marks for a valid and complete proof

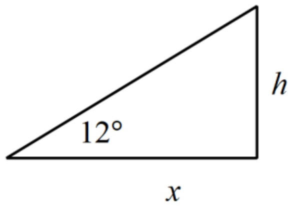
1 mark for
prove true for $n = 1$
1 mark for assumption true for $n = k$.</td></tr>
</table>

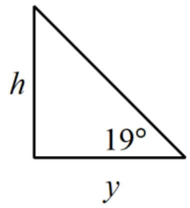
	Question 12 Solutions	Marks	Allocation of marks
(d)	$r = \sqrt{5^2 + 12^2} = 13$ $\alpha = tan^{-1}\left[\frac{12}{5}\right] = 1.18\ (2d.p.)$ $13\cos(\theta + 1.18) = -3$ $\theta = 0.62, 3.30$  	2	3 marks for correct solution 1 mark for working which includes correct value of r or/and or α or other working of equal merit.

Question 13	2018

	Question 13 Solutions	Marks	Allocation of marks
(a)	(i) $v^2 = 6 + 4x - 2x^2$ Extreme at $v = 0$ $0 = 6 + 4x - 2x^2$ $0 = -2(-3 - 2x + x^2)$ $0 = -2(x - 3)(x + 1)$ $\therefore$ particle oscillates between -1 and 3	1	1 mark for correct answer
	(ii) Centre of motion $x = \dfrac{-1 + 3}{2}$ $x = 1$ Max speed at centre of motion $v^2 = 6 + 4x - 2x^2$ $= 6 + 4 - 2$ $= 8$ $v = \pm\sqrt{8}$ $= \pm 2\sqrt{2}\ m\ s^{-1}$	1	1 mark for correct answer
	(iii) $v^2 = 6 + 4x - 2x^2$ $\frac{1}{2}v^2 = \frac{1}{2}(6 + 4x - 2x^2)$ $\frac{1}{2}v^2 = 3 + 2x - x^2$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}(3 + 2x - x^2)$ $\therefore \dfrac{d^2x}{dt^2} = 2 - 2x$ $\therefore a = 2(1 - x)$	2	2 marks for correct value 1 mark for working which includes correct substitution in expression with subsequent error or other working of similar merit

<table>
<tr><td></td><td>Question 13 Solutions</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(b)</td><td>From above






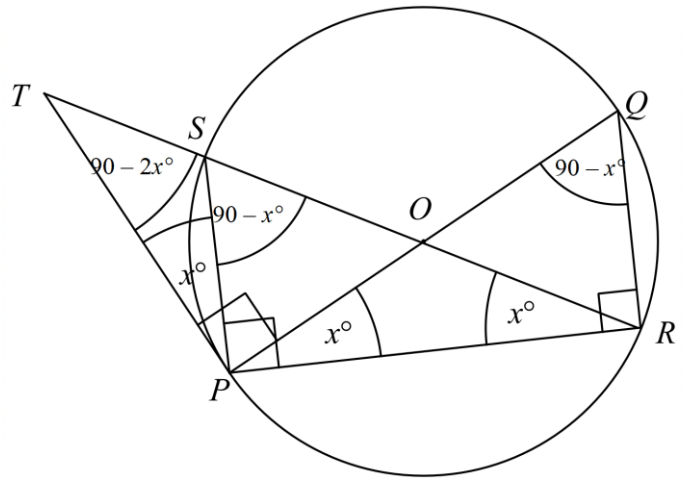
$x = \dfrac{h}{\tan 12^0}$ $\quad y = \dfrac{h}{\tan 19^0}$
$$170^2 = \left[\frac{h}{\tan 12°}\right]^2 + \left[\frac{h}{\tan 19°}\right]^2 - 2 \times \left[\frac{h}{\tan 19°}\right] \times \left[\frac{h}{\tan 12°}\right] \times \cos 56°$$
$$= h^2\left(\frac{1}{\tan^2 12°} + \frac{1}{\tan^2 19°} - \frac{2\cos 56°}{\tan 12° \tan 19°}\right)$$
$= h^2(15.28717)$
$h^2 = \dfrac{170^2}{15.28717}$
$h = 43.479$
$h = 43 \text{ m } (\text{nearest m})$
$\therefore$ the height of the tower is 43 metres</td><td>3</td><td>3 marks for correct solution

2 marks for a solution which links x and y with h and uses cosine rule with a minor error or working with similar merit

1 mark for solution which links x and y with h or uses cosine rule but is incomplete or has multiple errors</td></tr>
</table>

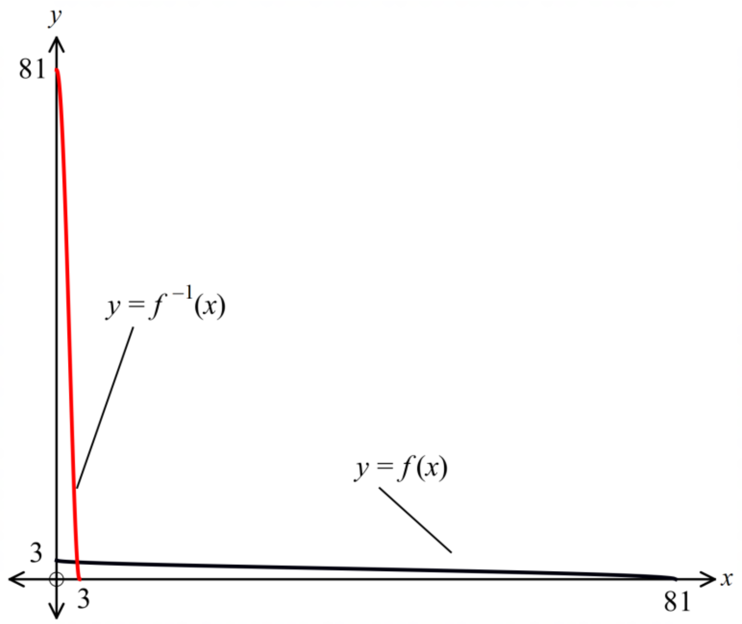
	Question 13 Solutions	Marks	Allocation of marks
(c)	(i) At $t = 0$ $P = 1$ $P = \dfrac{1200}{1 + ce^{-600t}}$ $1 = \dfrac{1200}{1 + ce^{0}}$ $1 + c = 1200$ $\therefore \quad c = 1199$ $\therefore \quad P = \dfrac{1200}{1 + 1199e^{-600t}}$ When $P =$ 600 students $600 = \dfrac{1200}{1 + 1199e^{-600t}}$ $600 + 600 \times 1199e^{-600t} = 1200$ $600 \times 1199e^{-600t} = 600$ $1199e^{-600t} = 1$ $e^{-600t} = \dfrac{1}{1199}$ $\ln[e^{-600t}] = \ln\left[\dfrac{1}{1199}\right]$ $-600t = \ln\left[\dfrac{1}{1199}\right]$ $t = \dfrac{\ln\left[\left[\dfrac{1}{1199}\right]\right]}{-600}$ $t = 0.011815405$ years $t = 4.3$ days	2	2 marks for solution with correct number of days 1 mark for solution which correctly finds c then has error following this or if an error is made in finding c and subsequent working is correct

	Question 13 Solutions	Marks	Allocation of marks
	(ii) $P = \dfrac{1200}{1 + ce^{-600t}}$ $P = 1200\,(1 + ce^{-600t})^{-1}$ $\dfrac{dP}{dt} = -1200(1 + ce^{-600t})^{-2}\,(-600ce^{-600t})$ $= \dfrac{1200 \times 600(ce^{-600t})}{(1 + ce^{-600t})^{2}}$ $= \dfrac{600ce^{-600t}}{1 + ce^{-600t}} \times \dfrac{1200}{1 + ce^{-600t}}$ $= \dfrac{1200}{1 + ce^{-600t}}\left[\dfrac{600 + 600ce^{-600t} - 600}{1 + ce^{-600t}}\right]$ $= P\left[\dfrac{600(1 + ce^{-600t})}{1 + ce^{-600t}} - \dfrac{600}{1 + ce^{-600t}}\right]$ $= P\left[600 - \dfrac{1}{2} \times \dfrac{1200}{1 + ce^{-600t}}\right]$ $= P\left[600 - \dfrac{1}{2}P\right]$ $= P\left[600 - \dfrac{P}{2}\right]$	2	2 marks for correctly showing the required result 1 mark for working which includes correct derivative with other minor errors or which is incomplete or for a solution with a minor error in finding the derivative followed by correct working which may be incomplete
(d)	$^{13}C_6 5! = \dfrac{13!}{7!\,6!}5!$ $= \dfrac{13!}{7!\,6}$	2	2 marks correct answer 1 mark for $^{13}C_6$ or 5!

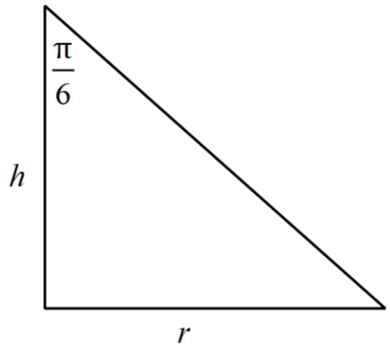
	Question 13 Solutions	Marks	Allocation of marks
(e)	 $\angle TPS = x$ (ang in alt segment) $OP = OR$ (equal radii) $\angle OPR = x$ (ang in isos triangle) $\angle PRQ = \angle SPR = 90°$ (ang in semi circle) $\angle TPO = 90°$ (ang between tangent and radius) $\angle PSR = 90° - x$ (ang sum right triangle) $\angle TPR = \angle SPR + \angle TPS = 90° + x$ (*adjacent* angles) $\angle PTR = 180 - (90 + x) - x$ $= 180 - 90 - x - x$ $= 90 - 2x$ $\angle PSR - \angle SRP = 90 - x - x$ $= 90 - 2x$ $\therefore \angle PTR = \angle PSR - \angle SRP$	2	2 marks for correct answer 1 mark for a well set out proof which involves some relevant and correct comparisons between angles but which is incomplete or has minor errors or missing reasons

Question 14	2018

	Question 14 Solution	Marks	Allocation of marks
(a)	(i) $y = \sqrt{9 - \sqrt{x}}$ $\therefore \; x = \sqrt{9 - \sqrt{y}}$ $x^2 = 9 - \sqrt{y}$ $\sqrt{y} = 9 - x^2$ $y = (9 - x^2)^2$	2	2 marks for correct equation 1 mark for solution that involves interchanging *x* and *y* but does not reach required equation

	Question 14 Solution	Marks	Allocation of marks
	(ii) On $f(x)$ when $x = 0$, $y = 3$ and when $x = 81$, $y = 0$. So by symmetry on $f^{-1}(x)$ when $x = 0$, $y = 81$ and when $x = 3$, $y = 0$.  $\int_0^{81} \sqrt{9 - \sqrt{x}}\ dx = \int_0^{3} (9 - x^2)^2\ dx$ $= \int_0^{3} 81 - 18x^2 + x^4\ dx$ $= \left[81x - 6x^3 + \frac{x^5}{5} \right]_0^3$ $= \left(81 \times 3 - 6 \times 3^3 \times \frac{3^5}{5} \right)$ $= 129\frac{3}{5}$	2	2 marks for correct result by any method. 1 mark for a solution with some merit in calculation of relevant integrals

	Question 14 Solution	Marks	Allocation of marks
(b)	(i) $4x^3 + 0x^2 - 6x + 10 = 0$ $\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{0}{4} = 0$ $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -\frac{6}{4} = -\frac{3}{2}$ $\alpha\beta\gamma = -\frac{d}{a} = -\frac{10}{4} = -\frac{5}{2}$ $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$ $= 0^2 - 2 \times \left(-\frac{3}{2}\right)$ $= 0 + 3$ $= 3$	1	1 mark for correct value
	(ii) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\alpha^2\gamma^2} + \frac{1}{\beta^2\gamma^2} = \frac{\alpha^2 + \beta^2 + \gamma^2}{\alpha^2\beta^2\gamma^2}$ $= \frac{\alpha^2 + \beta^2 + \gamma^2}{(\alpha\beta\gamma)^2}$ $= \frac{3}{\left(-\frac{5}{2}\right)^2}$ $= 3 \times \frac{4}{25}$ $= \frac{12}{25}$	2	2 marks for correct answer 1 mark for solution which expresses the expression in term of sums and products but is incomplete or has a minor error

	Question 14 Solution	Marks	Allocation of marks
(c)	(i) Angle = $\frac{\pi}{6}$ because $\frac{1}{2} \times \frac{\pi}{3}$  $\therefore \tan\frac{\pi}{6} = \frac{r}{h}$ $r = h \tan\frac{\pi}{6}$ $\therefore \quad r = h \times \frac{1}{\sqrt{3}}$ $r = \frac{h}{\sqrt{3}}$	1	1 for showing $r = \frac{h}{\sqrt{3}}$
	(ii) $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3} \times \pi \times \left[\frac{h}{\sqrt{3}}\right]^2 \times h$ $= \frac{1}{3} \times \pi \times \frac{h^2}{3} \times h$ $V = \frac{\pi h^3}{9}$	1	1 for correct substitution

	Question 14 Solution	Marks	Allocation of marks
	(iii) $$\frac{dV}{dt} = \frac{d}{dh}\left[\frac{\pi h^3}{9}\right] \times \frac{dh}{dt}$$ $$= \frac{3\pi h^2}{9} \times \frac{dh}{dt}$$ $$= \frac{\pi h^2}{3} \times \frac{dh}{dt}$$ But $\frac{dV}{dt} = 1.5$ $\therefore \quad 1.5 = \frac{\pi h^2}{3} \times \frac{dh}{dt}$ dh/dt = $1.5 \div \frac{\pi h^2}{3}$ $= 1.5 \times \frac{3}{\pi h^2}$ $= \frac{4.5}{\pi h^2}$ $\therefore$ When $h = 6m$ $\frac{dh}{dt} = \frac{4.5}{\pi \times 6^2}$ $= \frac{1}{8\pi}$ m/s	2	2 marks for correct answer 1 mark for relevant working including related rates with minor errors or incomplete

	Question 14 Solution	Marks	Allocation of marks
(d)	(i) Given $\ddot{x} = 0, \ \ddot{y} = -10$ At $t = 0, \ y = 405, \ \dot{y} = 0$ Vertical Motion: $\ddot{y} = -10$ $\therefore \dot{y} = \int -10 \, dt$ $= -10t + C_2$ At $t = 0 \ \dot{y} = 0 \ \therefore C_2 = 0$ $\therefore \dot{y} = -10t$ $y = \int -10t \, dt$ $= -10\dfrac{t^2}{2} + C_3$ At $t = 0 \ y = 405 \ \therefore C_3 = 405$ $\therefore y = 405 - 5t^2$	2	2 marks for deriving the vertical motion 1 mark if there is a minor error in working of the derivation
	(ii) The life raft hits the water when $y = 0$ $y = 405 - 5t^2$ $5t^2 = 405$ $t^2 = 81$ $t = \sqrt{81}$ since $t > 0$ $t = 9$ seconds	1	1 mark for correct answer
	(iii) AO is the horizontal displacement of the payload when $y = 0$. $y = 0$ when $t = 9$ $AO = x = 60t = 60 \times 9 = 540 \ m$ $\tan \varphi = \dfrac{405}{540}$ $\varphi = \tan^{-1}\left(\dfrac{405}{540}\right)$ $= \tan^{-1}(0.75)$ $= 36.8698976$ $= 36° \ 52'$	1	1 mark for correct answer