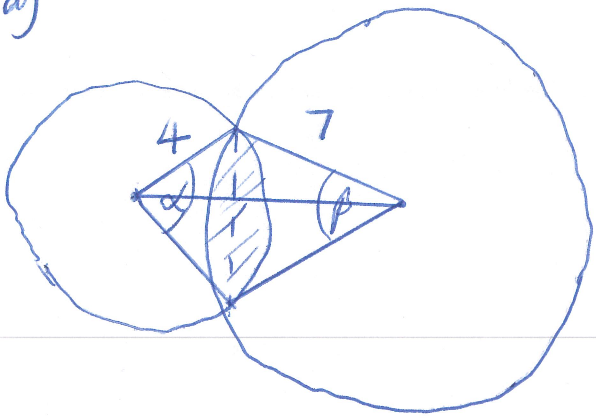
1) a)



$$\alpha = 2\times\cos^{-1}\left[\frac{4^2+10^2-7^2}{2\times4\times10}\right]$$

$$= 2\cos^{-1}\left(\frac{67}{80}\right)$$

$$= 1.156^c \ (3\text{ dp})$$

$$A_{seg_{r=4}} = \frac{1}{2}(4)^2(1.156 - \sin 1.156)$$

$$= 1.93\text{ cm}^2 \ (2\text{dp})$$

$$\beta = 2\cos^{-1}\left[\frac{7^2+10^2-4^2}{2\times7\times10}\right]$$

$$= 2\cos^{-1}\left(\frac{133}{140}\right)$$

$$\beta = 0.635^c$$

$$A_{seg_{r=7}} = \frac{1}{2}(7)^2\left[0.635 - \sin 0.635\right]$$

$$= 1.02 \ (2\text{dp})$$

Area of common $= 1.93 + 1.02$

$$= 2.95\text{cm}^2$$

b) $\cot\theta - 2\cot 2\theta = \tan\theta$

LHS: $= \cot\theta - \dfrac{2}{\tan 2\theta}$

$$= \frac{\cos\theta}{\sin\theta} - 2 \div \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \frac{\cos\theta}{\sin\theta} - \frac{2-2\tan^2\theta}{2\tan\theta}$$

$$= \frac{\cos\theta}{\sin\theta} - \frac{2 - 2\frac{\sin^2\theta}{\cos^2\theta}}{2\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta}{\sin\theta} - \frac{2\cos^2\theta - 2\sin^2\theta}{\cos^2\theta} \times \frac{\cos\theta}{2\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta} - \frac{2(\cos^2\theta - \sin^2\theta)}{\cos\theta} \times \frac{1}{2\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta} - \frac{(\cos^2\theta - \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta - \cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cancel{\sin\theta}\,\sin\theta}{\cancel{\sin\theta}\,\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

$\therefore$ LHS = RHS

1) c)

$$\cos\theta - \sin\theta \equiv A\cos(\theta+\alpha)$$
$$\equiv A(\cos\theta\sin\alpha - \sin\theta\sin\alpha)$$
$$\equiv A\cos\theta\cos\alpha - A\sin\theta\sin\alpha$$

$A\cos\alpha = 1$ ...... (1)

$A\sin\alpha = 1$ ...... (2)

$1^2 + 2^2$

$A^2\sin^2\alpha + A^2\cos^2\alpha = 2$

$A^2(\sin^2\alpha + \cos^2\alpha) = 2$

$A^2 = 2$

$A = \sqrt{2}$ ...... (3)

Subst (3) into (1) or (2)

$\sqrt{2}\cos\alpha = 1$ $\quad$ $\sqrt{2}\sin\alpha = 1$

$\cos\alpha = \frac{1}{\sqrt{2}}$ $\quad$ $\sin\alpha = \frac{1}{\sqrt{2}}$

$\alpha = \frac{\pi}{4}$

$\therefore \cos\theta - \sin\theta \equiv \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$

ii) $\sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right) = 1$

$\cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{7\pi}{4}$ or $\frac{9\pi}{4}$

$\theta = 0, \frac{3\pi}{2}$ or 2π

$\therefore \theta = 0°, 270°, 360°$

1) d) $3\left(\frac{2t}{1+t^2}\right) - 2\left(\frac{1-t^2}{1+t^2}\right) = 3$

$$\frac{6t}{1+t^2} - \frac{2-2t^2}{1+t^2} = 3$$

$$\frac{6t-2+2t^2}{1+t^2} = 3$$

$$2t^2+6t-2 = 3t^2+3$$

$$t^2-6t+5=0$$

$$(t-5)(t-1)=0$$

$$t=5 \text{ or } t=1$$

ie: $\tan\frac{\theta}{2} = 5$ or $\tan\frac{\theta}{2} = 1$

$$\frac{\theta}{2} = 78^\circ 41' \text{ or } 258^\circ 41', 45^\circ \text{ or } 225^\circ$$

$$\theta = 157^\circ 22', 517^\circ 22', 90, 450^\circ$$

$$\therefore \theta = 90^\circ \text{ or } 157^\circ 22'$$

2) a)

$$-1 \leq \frac{x}{2} \leq 1$$

$$-2 \leq x \leq 2$$

b) i) $f(x) = \frac{x}{x+3}$ Domain $x \in R, x \neq -3$

$$f'(x) = \frac{(x+3)(1) - x(1)}{(x+3)^2}$$

$$= \frac{x+3-x}{(x+3)^2}$$

$$= \frac{3}{(x+3)^2}$$

$\therefore f'(x) > 0$ for all x in the domain as $(x+3)^2 > 0$ for all x.

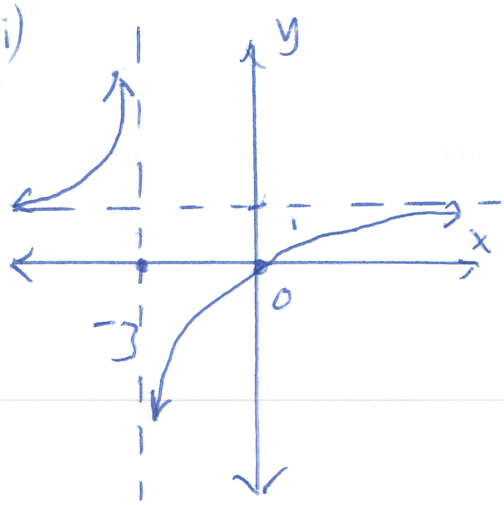
ii) $f(x) = \frac{x+3-3}{x+3}$

$$= 1 - \frac{3}{x+3}$$

As $x \to \pm\infty$ $\frac{3}{x+3} \to 0$

$\therefore$ horizontal asymptote is $y = 1$

iii)



iv) $y = f(x)$ is a one to one increasing function. (passes the horizontal line test).

v) $x = \frac{y}{y+3}$

$$xy + 3x = y$$

$$(1-x)y = 3x$$

$$y = \frac{3x}{1-x}$$

$$f^{-1}(x) = \frac{3x}{1-x}$$

vi)

Domain of $f^{-1}(x)$

All real $x, x \neq 1$

2) c) $\displaystyle\int_{\sqrt{2}}^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$

$= \displaystyle\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{2^2-x^2}}\,dx$

$= \left[\sin^{-1}\frac{x}{2}\right]_{\sqrt{2}}^{\sqrt{3}}$

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{\sqrt{2}}{2}$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{4\pi - 3\pi}{12}$

$= \frac{\pi}{12}$

3) a) $\displaystyle\int_0^{\pi} \cos^2 3x\,dx$

$= \displaystyle\int_0^{\pi} \frac{1}{2}(1+\cos 6x)\,dx$

$= \frac{1}{2}\displaystyle\int_0^{\pi}(1+\cos 6x)\,dx$

$= \frac{1}{2}\left[x + \frac{1}{6}\sin 6x\right]_0^{\pi}$

$= \frac{1}{2}\left[\pi + \frac{1}{6}\sin 6\pi - (0 + \frac{1}{6}\sin 6\times 0)\right]$

$= \frac{1}{2}(\pi + 0)$

$= \frac{\pi}{2}$

b) $\displaystyle\int_0^1 x^2\sqrt{3x^3+1}\,dx$

let $u = 3x^3+1$

$\frac{du}{dx} = 9x^2$

$x^2dx = \frac{1}{9}du$

when $x=0$, $u=1$

when $x=1$, $u=4$

$\displaystyle\int_0^1 \sqrt{3x^3+1}\;x^2dx$

$= \frac{1}{9}\displaystyle\int_1^4 u^{\frac{1}{2}}\,du$

$= \frac{1}{9}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^4$

$= \frac{2}{27}\left[\sqrt{u^3}\right]_1^4$

$= \frac{2}{27}\left[\sqrt{4^3} - 1\right]$

$= \frac{2}{27}(8-1)$

$= \frac{14}{27}$

c) $\displaystyle\int_0^{\frac{\pi}{4}} \sin x\cos^3 x\,dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$\sin x\,dx = -du$

when $x=0$, $u=1$

when $x=\frac{\pi}{4}$, $u=\frac{1}{\sqrt{2}}$

$= -\displaystyle\int_1^{\frac{1}{\sqrt{2}}} u^2\,du$

$= -\frac{1}{3}\left[u^3\right]_1^{\frac{1}{\sqrt{2}}}$

$= -\frac{1}{3}\left[\left(\frac{1}{\sqrt{2}}\right)^3 - 1\right]$

$= -\frac{1}{3}\left[\frac{1}{2\sqrt{2}} - 1\right]$

$= -\frac{1}{3}\left[\frac{1-2\sqrt{2}}{2\sqrt{2}}\right]$

$= \frac{2\sqrt{2}-1}{6\sqrt{2}}$

or $\frac{2\sqrt{2}-1}{6\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4-\sqrt{2}}{12}$

Q3d) i)

$\operatorname{cosec} x + \cot x = \cot \frac{x}{2}$

LHS

$\frac{1}{\sin x} + \frac{1}{\tan x}$

$= \frac{1+t^2}{2t} + \frac{1-t^2}{2t}$

$= \frac{1+t^2+1-t^2}{2t}$

$= \frac{2}{2t}$

$= \frac{1}{t}$

$= \frac{1}{\tan \frac{x}{2}}$

$= \cot\left(\frac{x}{2}\right)$

$\therefore$ LHS = RHS

Q3 d ii)

$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\operatorname{cosec} x + \cot x)\, dx$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2}\, dx$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}}\, dx$

$= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\frac{1}{2}\cos \frac{x}{2}}{\sin \frac{x}{2}}\, dx$

$= 2\left[\ln\left(\sin \frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= 2\left[\ln\left(\sin \frac{\pi}{4}\right) - \ln\left(\sin \frac{\pi}{6}\right)\right]$

$= 2\left[\ln \frac{1}{\sqrt{2}} - \ln \frac{1}{2}\right]$

$= 2\ln\left(\frac{1}{\sqrt{2}} \div \frac{1}{2}\right)$

$= 2\ln \sqrt{2}$

$= \ln(\sqrt{2})^2$

$= \ln 2$