

1. $30, 35, 40, 45, \dots$ $a=30$
 $d=5$

(i) $T_{21} = 30 + (21-1) \times 5$ ✓
 $= 130 \text{ min}$
 $2 \text{ hrs } 10 \text{ min}$ } ✓

(ii) $S_{21} = \frac{21}{2} [2(30) + 20 \times 5]$ ✓
 $= \frac{21}{2} \times 160$
 $= 1680 \text{ min}$
 28 hrs } ✓

(4)

2. (i) $A = 800(1+0.1)^{30}$
 $= \$13959.52$
 $\$13960$ ✓

(ii) $A_1 = 800(1.1)^{30}$
 $A_2 = 800(1.1)^{29}$
 $A_3 = 800(1.1)^{28}$
 \vdots

$A_{30} = 800(1.1)^1$ ✓

Total $= 800(1.1 + 1.1^2 + 1.1^3 + \dots + 1.1^{30})$

$= 800 \times \frac{1.1(1.1^{30}-1)}{1.1-1}$ ✓

$= 144754.74$ } ✓

$= \$144755$

(4)

$$3. \quad 6\% \text{ pa} = 0.06 \text{ pa} \\ = 0.005 \text{ per month}$$

$$(i) \quad A_1 = 100000 \times 1.005 - M \\ = \$ (100500 - M)$$

$$A_2 = A_1 \times 1.005 - M \\ = (100000 \times 1.005 - M) 1.005 - M \\ = 100000 (1.005)^2 - 1.005M - M \quad \checkmark$$

$$A_3 = A_2 \times 1.005 - M \\ = (100000 (1.005)^2 - 1.005M - M) 1.005 - M \\ = 100000 (1.005)^3 - 1.005^2 M - 1.005M - M \\ = 100000 (1.005)^3 - M (1 + 1.005 + 1.005^2)$$

⋮

$$A_n = 100000 (1.005)^n - M (1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1}) \quad \checkmark$$

OR

$$(ii) \quad A_n = 100000 (1.005)^n - M \frac{(1.005^n - 1)}{0.005} \quad \checkmark$$

After 144 payments $A_n = 0$

$$0 = 100000 \times 1.005^{144} - M \frac{(1.005^{144} - 1)}{0.005} \quad \checkmark$$

$$M = \frac{100000 \times 1.005^{144} \times 0.005}{1.005^{144} - 1}$$

$$= \$ 975.85 \quad \checkmark$$

4. (i) For stat pts $\frac{dy}{dx} = 0$

$$(x-1)^2(x-2) = 0$$

$$x = 1, 2$$

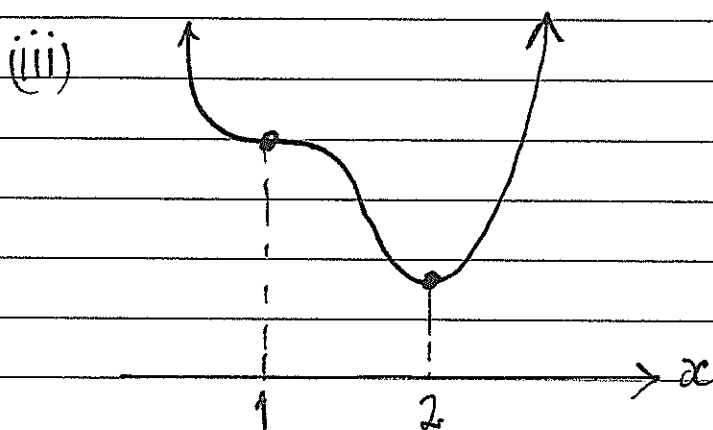
✓

(ii)

| | | | | | |
|-----------------|---------------|---|----------------|---|----------------|
| x | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ |
| $\frac{dy}{dx}$ | - | 0 | - | 0 | + |

At $x=1$ horizontal point of inflexion ✓

At $x=2$ minimum TP ✓



✓

(4)

5. (i) Point L when $y=0$

$$6x^2 - x^3 = 0$$

✓

$$L = (6, 0)$$

✓

$$x^2(6-x) = 0$$

$$x = 0, 6$$

(ii) $\frac{dy}{dx} = 12x - 3x^2$

For local max^m $\frac{dy}{dx} = 0$

$$12x - 3x^2 = 0$$

✓

$$3x(4-x) = 0$$

$$x = 0, 4$$

When $x=4$,

$$M = (4, 32)$$

✓

$$y = 6(4)^2 - (4)^3$$
$$= 32$$

(7)

6. $y = x^3 - 6x^2 + 9x - 4$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\frac{d^2y}{dx^2} = 6x - 12$$

(i) For stat. pts $\frac{dy}{dx} = 0$

$$3x^2 - 12x + 9 = 0$$

$$3(x^2 - 4x + 3) = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3, 1$$

When $x=3$, $\frac{d^2y}{dx^2} > 0$ \cup ✓

\therefore Min^m TP at $(3, -4)$

When $x=1$, $\frac{d^2y}{dx^2} < 0$ \cap ✓

\therefore Max^m TP at $(1, 0)$

(ii) For inflexion pt. $\frac{d^2y}{dx^2} = 0$

$$6x - 12 = 0$$

$$6x = 12$$

$$x = 2$$

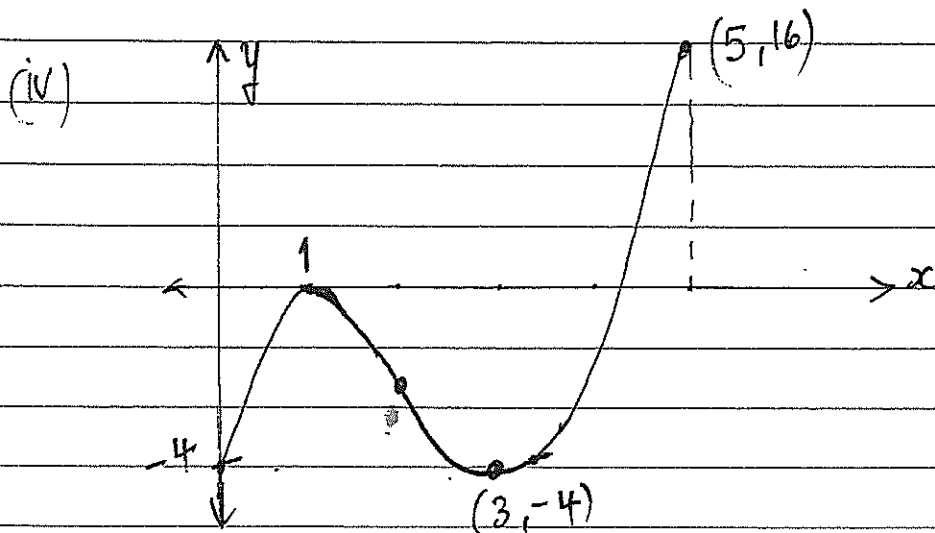
| | | | |
|---------------------|-------|-----|-------|
| x | 2^- | 2 | 2^+ |
| $\frac{d^2y}{dx^2}$ | $-$ | 0 | $+$ |

\therefore change of concavity ✓

\therefore point of inflexion at $(2, -2)$ ✓

(iii) When $x=5$, $y = 5^3 - 6(5)^2 + 9(5) - 4$
 $= 16$

\therefore absolute max^m value is 16 ✓



Let y be length of rectangle

7. (i) $P = 2x + y - 2$

$$100 = 2x + y - 2 \quad \checkmark$$

$$102 = 2x + y$$

$$y = 102 - 2x$$

$$A = x \times y$$

$$= x \times (102 - 2x) \quad \checkmark$$

$$= 102x - 2x^2$$

(ii) $\frac{dA}{dx} = 102 - 4x$

$$\frac{d^2A}{dx^2} = -4$$

$$102 - 4x = 0 \text{ for stat. pts.}$$

$$4x = 102$$

$$x = 25.5 \quad \checkmark$$

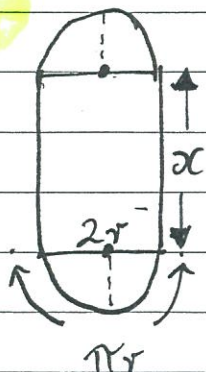
$$\text{At } x = 25.5 \quad \frac{d^2A}{dx^2} < 0 \quad \wedge$$

$\therefore \text{Max}^m \text{TP}$

(iii) $A = 102(25.5) - 2(25.5)^2$
 $= 1300.5 \text{ m}^2 \quad \checkmark$

(5)

8.



$$A = \pi r^2 + 2r \times x$$

$$1 = \pi r^2 + 2rx$$

$$1 - \pi r^2 = 2rx$$

$$\therefore x = \frac{1 - \pi r^2}{2r}$$

Note: window is a circle.

(3)

$$P = 2\pi r + 2x$$

$$P = 2\pi r + 2 \left(\frac{1 - \pi r^2}{2r} \right)$$

$$P = 2\pi r + \frac{1}{r} - \pi r$$

For stat. pts $\pi - r^{-2} = 0$

$$r^{-2} = \pi$$

$$\frac{1}{r^2} = \pi$$

$$r^2 = \frac{1}{\pi}$$

$$\therefore r = 0.56418..$$

$$P = 2\pi r + r^{-1} - \pi r = \pi r + \frac{1}{r}$$

When $r = 0.56..$ $\frac{d^2P}{dr^2} > 0 \quad \vee$

$\therefore \text{Min}^m \text{TP}$

and $x = \frac{1 - \pi(0.564..)^2}{2(0.564..)}$
 $= 0.0$

$$\frac{dP}{dr} = 2\pi - r^{-2} - \pi = \pi - r^{-2}$$

$$\frac{d^2P}{dr^2} = 2r^{-3}$$

$\therefore \text{height of window} = 0.56.. + 0.56.. = 1.128... \text{ m}$