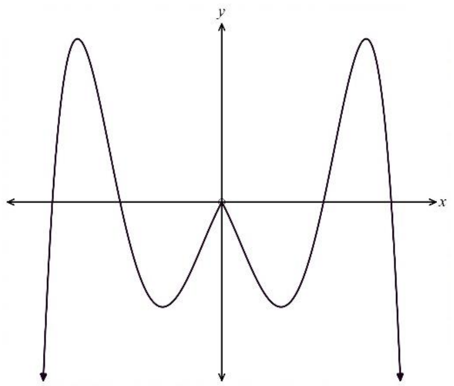
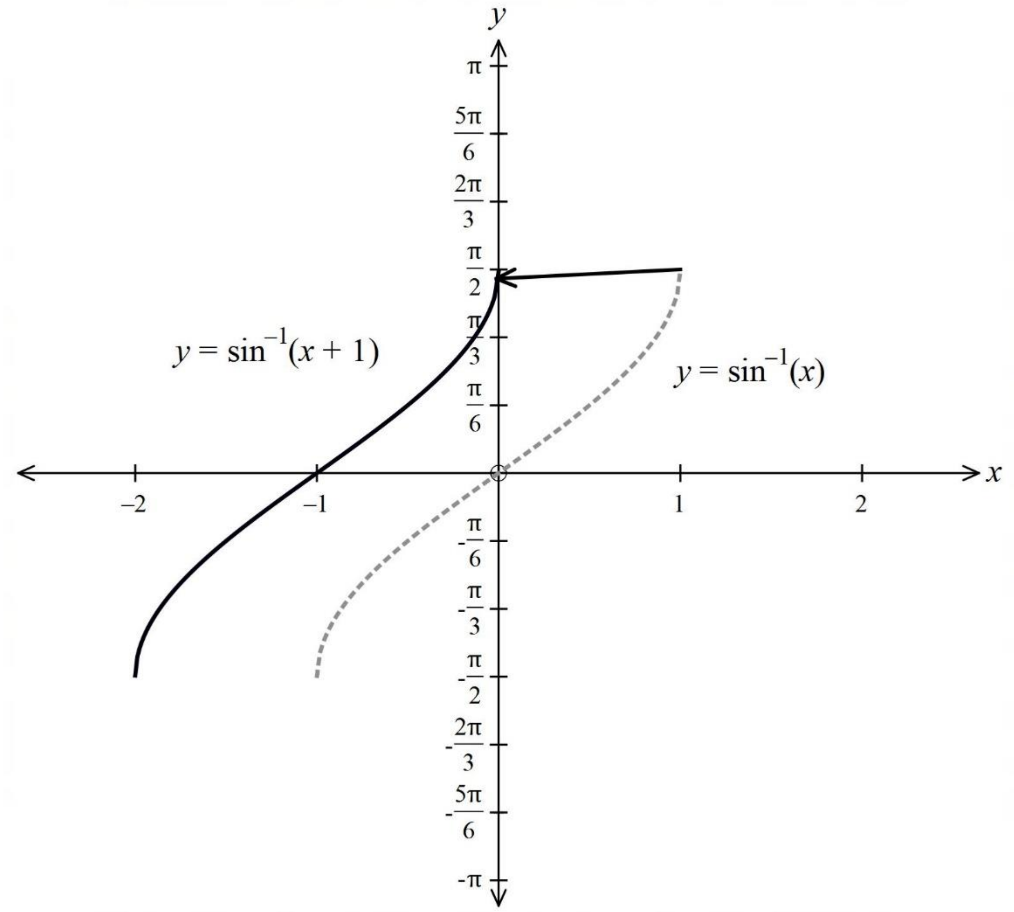
Carlingford High School
YEAR 11 FINAL EXAMINATION 2020
Mathematics Extension 1
SOLUTIONS

	A	B	C	D
1.	●	○	○	○
2.	○	○	●	○
3.	○	●	○	○
4.	○	○	○	●
5.	○	●	○	○

No	Working	Answer
1	There are 11 letters of which there are 5 different vowels and 6 consonants of which there are two T′s . Consider the 5 vowels as one letter, then there are 7 letters $(EUAIO)$, X, L, T, T, N, S 7 Letters can be arranged in 7! ways but as the T′s are repeated this would appear the same so divide by 2! The 5 vowels themselves can be arranged in 5! ways while together Total arrangements $= \dfrac{7!\ 5!}{2!}$	A
2	$y = f(\lvert x\rvert)$ takes all negative values of x and makes them positive before evaluating $f(x)$ So $f(-a) = f(a)$, so for $x < 0$ the graph is a reflection of the positive section of $f(x)$ in the $y -$ axis 	C

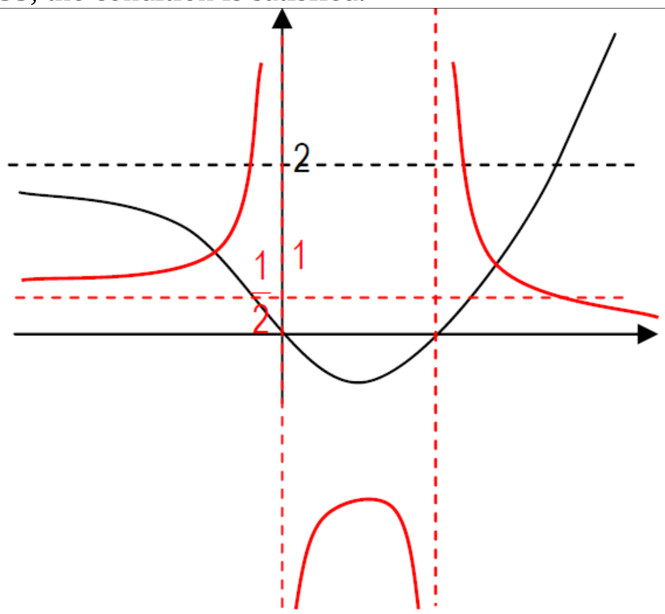
3		**B**
4	$\cos(15°) = \cos(45° - 30°)$ $= \cos 45° \cos 30° + \sin 45° \sin 30°$ $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$ $= \frac{\sqrt{3} + 1}{2\sqrt{2}}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$	**D**
5	If $\lvert 3x - 4 \rvert \leq 16$ Then $-16 \leq 3x - 4 \leq 16$ $-12 \leq 3x \leq 20$ $-4 \leq x \leq 6\frac{2}{3}$	**B**

Question 6	**2020**

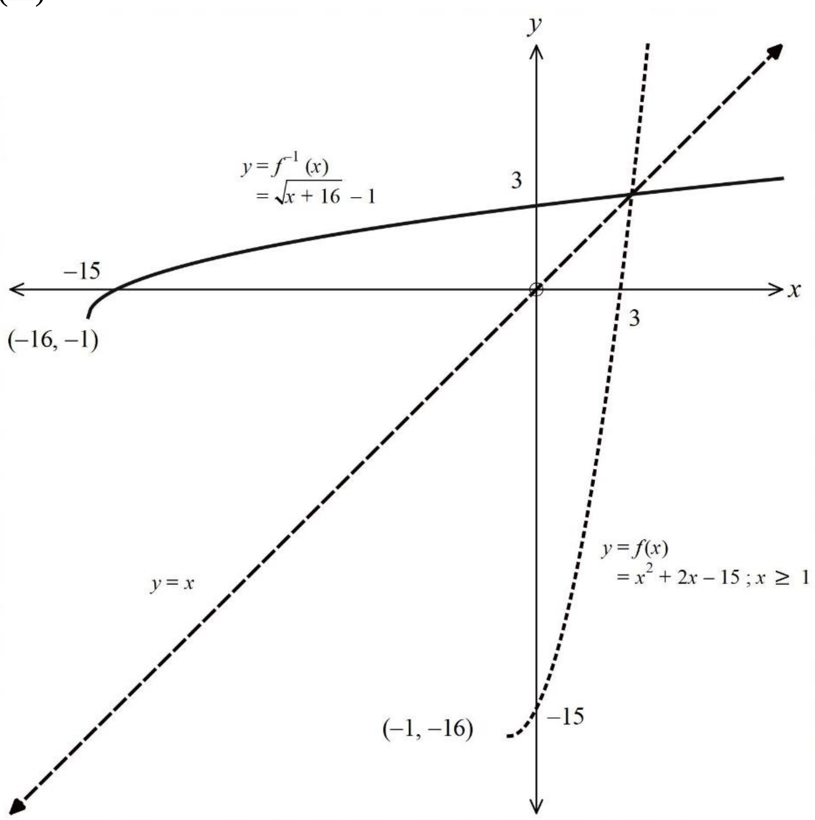
	Solution	Marks	Allocation of marks
(a)	$\begin{array}{r} 2x^3+2x+1 \\ 2x^2-3\,\overline{)\,4x^5-2x^3+2x^2\quad-7} \\ \underline{4x^5\ -6x^3\qquad\qquad} \\ 4x^3+2x^2\quad-7 \\ \underline{4x^3\qquad-6x\quad} \\ 2x^2+6x-7 \\ \underline{2x^2\qquad-3} \\ 6x\ -4 \end{array}$ The remainder is $R(x) = (6x-4)$	2	2 marks for correct quotient and remainder 1 mark for correct method of division with a minor error leading to an incorrect quotient and/or remainder
(b)	$P(x) = x^3 + kx + m,$ $P(-3) = 0 \therefore -27 - 3k + m = 0$ (1) $P(-1) = 30 \therefore -1 - k + m = 30$ (2) $m = 31 + k$ (3) sub (3) into (1): $-27 - 3k + 31 + k = 0$ $4 - 2k = 0$ k=2, $m = 33$.	2	1 mark for k or m or correct method with minor error
(c)	$\binom{n}{n-1} = \frac{n!}{(n-1)!\,(n-(n-1))!}$ $= \frac{n!}{(n-1)!\,1!}$ $= \frac{n!}{(n-1)!}$ $= \frac{n(n-1)(n-2)(n-3)\ldots\times 3\times 2\times 1}{(n-1)(n-2)(n-3)\ldots\times 3\times 2\times 1}$ $= n$	2	2 marks for any correct derivation 1 mark for correct use of definition and attempt to simplify
(d)	From 4 Blue and 3 White cards, four cards can be chosen as shown in these combinations 4 Blue and 0 White can be arranged in $\frac{4!}{4!} = 1$ way 3 Blue and 1 White can be arranged in $\frac{4!}{3!\,1!} = 4$ ways 2 Blue and 2 White can be arranged in $\frac{4!}{2!\,2!} = 6$ ways 1 Blue and 3 White can be arranged in $\frac{4!}{1!\,3!} = 4$ ways 4 White is not possible as there are only 3 Total possible arrangements $= 1 + 4 + 6 + 4 = 15$ distinct arrangements	2	2 marks for the correct answer 1 mark for a valid attempt to list arrangements with some missing or with an error in calculation

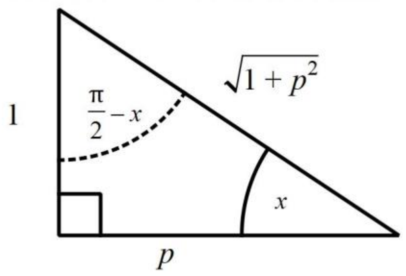
	Solution	Marks	Allocation of marks
(e)	$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ $\sin \theta \sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{2}\left[\cos\left(\theta - \left(\frac{\pi}{2} - \theta\right)\right)\right.$ $\left. - \cos\left(\theta + \left(\frac{\pi}{2} - \theta\right)\right)\right]$ $= \frac{1}{2}\left[\cos\left(2\theta - \frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right)\right)$ $= \frac{1}{2}\left[\cos\left(2\theta - \frac{\pi}{2}\right) - 0\right]$ $\sin \theta \sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{2}\cos\left(2\theta - \frac{\pi}{2}\right)$ OR $\sin \theta \sin\left(\frac{\pi}{2} - \theta\right) = \sin\theta \cos\theta$ $= \frac{1}{2}(2 \sin\theta \cos\theta)$ $= \frac{1}{2}\sin 2\theta$ $= \frac{1}{2}\cos\left(2\theta - \frac{\pi}{2}\right)$	2	2 marks for use of formula to show result 1 mark for correct choice of formula with a minor error leading to an incorrect or incomplete result

Question 7	2020

	Solution	Marks	Allocation of marks
(a)	$G(x) = 2x^3 - 3x^2 + 7x - 5$ $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{3}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{7}{2}$ $\alpha\beta\gamma = -\frac{d}{a} = \frac{5}{2}$ $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \alpha\beta\gamma(\alpha + \beta + \gamma)$ $= \frac{5}{2} \times \frac{3}{2} = \frac{15}{4}$	2	2 marks for correct answer 1 mark for correct substitution into equations for sums and differences leading to incorrect result or equivalent merit
(b)	(i) Combinations of 6 chosen from 15 $= {}^{15}\mathbf{C}_6$ $= 5005$	1	1 mark for correct answer
	(ii) If at least four vans, then either 4 or 5 vans. 4 vans in ${}^{5}\mathbf{C}_4 \times {}^{10}\mathbf{C}_2 = 5 \times 45 = 225$ ways 5 vans in ${}^{5}\mathbf{C}_5 \times {}^{10}\mathbf{C}_1 = 10$ ways P (At least 4 vans) $= \frac{225 + 10}{5005}$ $= \frac{235}{5005}$ $= \frac{47}{1001} = 0.0470$ (3 sig fig)	2	2 marks for correct answer 1 mark for obtaining one of the correct amounts for 4 or 5 vans and finding probability from this
(c)	(i) $12 \times 4 + 1 = 49$ students	1	
	(ii) Grouping the months in pairs we have 6 pairs of months. By the Pigeonhole principle we will have 10 in at least one of these pairs if there are at least $6 \times 9 + 1 = 55$ students present. Since $56 \geq 55$, the condition is satisfied.	2	1 mark for evidence of correct logic with incomplete detail
(d)		3	1 mark vertical asymptotes 1 mark horizontal asymptotes 1 mark shape

Question 8	**2020**

	Solution	Marks	Allocation of marks
(a)	$(1+2y)^6 = {}^6\mathbf{C}_0\, 1^6(2y)^0 + {}^6\mathbf{C}_1\, 1^5(2y)^1 + {}^6\mathbf{C}_2\, 1^4(2y)^2$ $+ {}^6\mathbf{C}_3\, 1^3(2y)^3 + {}^6\mathbf{C}_4\, 1^2(2y)^4 + {}^6\mathbf{C}_5\, 1^1(2y)^5$ $+ {}^6\mathbf{C}_6\, 1^0(2y)^6$ $= 1.1.1 + 6.1(2y) + 15.1(4y^2) + 20.1(8y^3) + 15.1(16y^4)$ $+ 6.1(32y^5) + 1.1(64y^6)$ $= 1 + 12y + 60y^2 + 160y^3 + 240y^4 + 192y^5 + 64y^6$	2	2 marks for correct expansion 1 mark for correct use of combinations with minor error
(b)	(i) From graph axis is at $x = -1$ Or algebraically $f(x) = x^2 + 2x - 15$ $y = (x-3)(x+5)$ $x -$ intercepts at $x = 3$ and $x = -5$ axis at $x = -1$ So for curve to have an inverse each y must have only one x Restriction is $x \geq -1$	1	1 mark for correct answer Also accept $x \leq -1$
	(ii) Inverse $f^{-1}(x)$ found by $x = y^2 + 2y - 15$ $y^2 + 2y = x + 15$ $y^2 + 2y + 1 = x + 16$ $(y+1)^2 = x + 16$ $y + 1 = \sqrt{x+16}$ $y = \sqrt{x+16} - 1$ $f^{-1}(x) = \sqrt{x+16} - 1$	2	2 marks for correct equation 1 mark for correct substitution to obtain inverse, with a minor algebraic error or incomplete answer Also accept $f^{-1}(x) = -\sqrt{x+16} - 1$
	(iii) 	1	1 mark for correct shaped inverse curve, don't penalise if all intercepts are not shown. $y = f(x)$ does not need to be shown

	Solution	Marks	Allocation of marks
(c)	(i) $\sin^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad = \frac{\pi}{3}$	1	1 for correct answer
	(ii)  $\cos x = \frac{p}{\sqrt{1+p^2}}$ $\sin^{-1}(\cos x) = \sin^{-1}\left(\frac{p}{\sqrt{1+p^2}}\right)$ $= \frac{\pi}{2} - x$	2	2 marks for correct answer 1 mark for any correct and relevant diagrams equations leading to an incorrect answer
(d)	(i) $\sin X = \frac{2t}{1+t^2}$ $\cos X = \frac{1-t^2}{1+t^2}$ $\sin X - 2\cos X = \frac{2t}{1+t^2} - 2\frac{1-t^2}{1+t^2}$ $= \frac{2t}{1+t^2} - \frac{2-2t^2}{1+t^2}$ $= \frac{2t^2+2t-2}{1+t^2}$	1	1 mark for correct algebraic manipulation to get required result.
	(ii) $\frac{2t^2+2t-2}{1+t^2} = 0$ $t^2 + t - 1 = 0$ $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $= \frac{-1 \pm \sqrt{1-4\times 1\times -1}}{2\times 1}$ $= \frac{-1\pm\sqrt{5}}{2}$ $t = 0.618$ or $t = -1.618$ $\tan\frac{X}{2} = 0.618$ or $\tan\frac{X}{2} = t = -1.618$ $\frac{X}{2} = 31.716°, 211.716°, 121.718°, 301.718°$ $X = 63.432, 423.432°, 243.436°, 603.436°$ $X = 63, , 243°$ for $0 \le X \le 360^o$.	2	2 marks for correct solutions 1 mark for working with minor errors such as extra or missing solutions or algebraic error leading to incorrect result

Question 9	2020

	Solution	Marks	Allocation of marks
(a)	$\sin^{-1}(-x) + \cos^{-1}(-x) + \tan^{-1}(\tan x)$ $= -\sin^{-1}x + \pi - \cos^{-1}x + x$ (Since $\tan^{-1}(\tan x) = x$) $= -(\sin^{-1}x + \cos^{-1}x) + \pi + x$ $= -\frac{\pi}{2} + \pi + x$ (Since $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$) $= x + \frac{\pi}{2}$	3	3 marks for correct derivation of result 2 marks for derivation with minor error in logic or algebra 1 mark for statement of some correct and relevant results
(b)	$\frac{2x}{(x+3)(x-2)} \leq 1\ x \neq -3, x \neq 2$ Case 1: $(x+3)(x-2) > 0, x < -3\ or\ x > 2$ $2x \leq (x+3)(x-2)$ $2x \leq x^2 + x - 6$ $0 \leq x^2 - x - 6$ $0 \leq (x-3)(x+2)$ $x \geq 3\ or\ x \leq -2$ Comparing with restrictions, we get $x < -3, x \geq 3$. Case 2: $(x+3)(x-2) < 0, -3 < x < 2$. $0 \geq (x-3)(x+2)$, $-2 \leq x \leq 3$ Comparing with restrictions we get $-2 \leq x < 2$ So the inequality is satisfied for $x < -3, x \geq 3, -2 \leq x < 2$	3	1 mark for correct working up to $x \geq 3\ or\ x \leq -2$ 2 marks for $x < -3, x \geq 3$.
(c)	The general term is $^{12}C_k \left(\frac{x}{2}\right)^{12-k} \left(\frac{2}{x^2}\right)^k$ We want the power of x to be 0, ie $12 - k - 2k = 0$ $k = 4$ The constant term is $^{12}C_4 \times 2^{-4} = \frac{495}{16}$.	2	1 mark for correct working with minor error
(d)	(i) $x = 4\sin t + 3$ $x - 3 = 4\sin t$ $(x-3)^2 = 16\sin^2 t$ $y = 4\cos t - 1$ $y + 1 = 4\cos t$ $(y+1)^2 = 16\cos^2 t$ $\sin^2 t + \cos^2 t = 1$ $16\sin^2 t + 16\cos^2 t = 16$ $(x-3)^2 + (y+1)^2 = 16$	2	2 marks for manipulation reaching required answer 1 mark for correct manipulations of equations and use of Pythagorean result but not reaching required result or equivalent merit

	Solution	Marks	Allocation of marks
	(ii) $y = x^2 - 6x - 8$ and $(x-3)^2 + (y+1)^2 = 16$ Completing the square on $y = x^2 - 6x - 8$ $y = (x-3)^2 - 17$ $y + 17 = (x-3)^2$ Subbing into the circle equation $y + 17 + (y+1)^2 = 16$ $y^2 + 3y + 18 = 16$ $y^2 + 3y + 2 = 0$ $(y+1)(y+2) = 0$ When y=-1, x=-1 or x=7 The points (-1,-1) and (7, -1) lie on a diameter, since they lie on the line y=-1 which passes through the centre of the circle.	2	