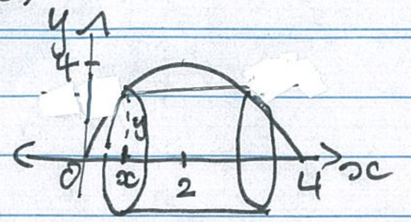
The region bounded by   <u>Cylindrical Shells</u>

1b) the curve $y = 4x - x^2$ and the x axis is rotated around the x-axis ...



$$\delta V = \pi\left[(y+\delta y)^2 - y^2\right]h$$
$$h = 2(2-x)$$
$$= 4-2x$$
$$\delta V = 2\pi y(4-2x)\delta y \text{ ignoring terms in } (\delta y)^2$$

$$V = \lim_{\delta y \to 0} \sum_0^4 2\pi y(4-2x)\,\delta y$$
$$= 2\pi\int_0^4 y(4-2x)\,dy$$

Method 1: Integrating with respect to x $\qquad y = 4x - x^2$
$\qquad dy = (4-2x)\,dx$

Now when $y = 0$ $\quad x = 0$
$\qquad\qquad y = 4 \quad x = 2$

$$V = 2\pi\int_0^2 (4x - x^2)(4-2x)^2\,dx$$

$$= 8\pi\int_0^2 (4x-x^2)(4-4x+x^2)\,dx$$
$$= 8\pi\int_0^2 16x - 16x^2 + 4x^3 - 4x^2 + 4x^3 - x^4\,dx$$
$$= 8\pi\int_0^2 16x - 20x^2 + 8x^3 - x^4\,dx$$
$$= 8\pi\left[8x^2 - \frac{20}{3}x^3 + 2x^2 - \frac{x^5}{5}\right]_0^2$$
$$= \frac{512\pi}{15}\,u^3$$

Method 2: Express $4-2x$ in terms of y

$$y = 4x - x^2$$
$$= -(x-2)^2 + 4$$
$$4 - y = (x-2)^2$$

choose $x \le 2$ $\rightarrow$ $\sqrt{4-y} = 2 - x$

$$\therefore 4 - 2x = 2\sqrt{4-y}$$

$$V = 2\pi\int_0^4 2y\sqrt{4-y}\,dy \qquad \begin{matrix} g = y & f = \sqrt{4-y} \\ g' = 1 & F = -\tfrac{2}{3}\sqrt{(4-y)^3} \end{matrix}$$

$$= 4\pi\left(\underbrace{\left[-\tfrac{2}{3}y\sqrt{(4-y)^3}\right]_0^4}_{=0} + \frac{2}{3}\int_0^4 \sqrt{(4-y)^3}\,dy\right)$$

$$= 4\pi \cdot \frac{4}{15}\left[(4-y)^{5/2}\right]_0^4 = \frac{512\pi}{15}\,u^3$$