Student Number: ________________

2019 | YEAR 12
HSC TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Working time - 120 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt questions 11-14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. Which of the following is an expression for $\int \frac{x}{(2-x^2)^3}dx$?

   Use the substitution $u = 2 - x^2$.

   (A) $\frac{1}{2(2-x^2)^2} + c$

   (B) $\frac{1}{4(2-x^2)^2} + c$

   (C) $\frac{1}{4(2-x^2)^4} + c$

   (D) $\frac{1}{8(2-x^2)^4} + c$

2. Which expression is equal to $\int \sin^2 2x dx$?

   (A) $\frac{1}{8}(4x + \sin 4x) + c$

   (B) $\frac{1}{8}(4x - \sin 4x) + c$

   (C) $\frac{-\cos^3 2x}{6} + c$

   (D) $\frac{\sin^3 2x}{6} + c$

3. What is the remainder when $x^3 - 10x$ is divided by $x + 5$?

   (A) -75

   (B) 75

   (C) $x^2 - 2x$

   (D) $x^2 - 5x + 15$

4. Find the Cartesian equation of the curve defined by the parametric equations:

$x = \sin\theta$

$y = \cos^2\theta - 3.$

(A) $y = -2 - x^2$

(B) $y = \sin^2 x - 3$

(C) $y = -3 + 3x^2$

(D) $y = \sin 2x + 3\cos^2 x$

5. The Cartesian equation of the tangent to the parabola $x = t - 3, y = t^2 + 2$ at $t = -3$ is:

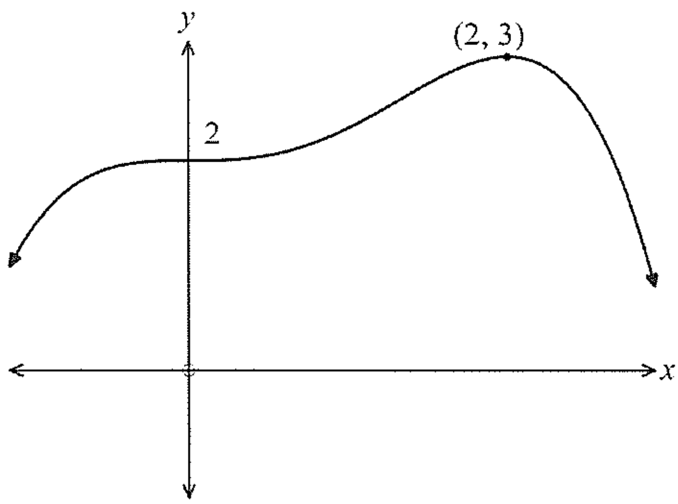
(A) $6x - y - 25 = 0$

(B) $6x - y - 36 = 0$

(C) $6x + y + 25 = 0$

(D) $6x + y + 36 = 0$

6. Part of the graph of $y = f(x)$ is shown below.



Which statement is true?

(A) $f'(2) < f''(2) < f(2) < 2$

(B) $f'(2) < 2 < f(2) < f''(2)$

(C) $f''(2) < f'(2) < 2 < f(2)$

(D) $f''(2) < 2 < f'(2) < f(2)$

7. What is the size of the obtuse angle between the curves $f(x) = (x+2)^2 + 4$ and $g(x) = x^2 - 4$ at their point of intersection? Answer to the nearest degree.

(A) 17°

(B) 135°

(C) 146°

(D) 163°

8. A particle moves in a straight line. Its position at any time t is given by:

$x = 4\sin 2t + 3\cos 2t$

What is the acceleration in terms of x?

(A) $\ddot{x} = -4x$

(B) $\ddot{x} = -8x$

(C) $\ddot{x} = -16x^2$

(D) $\ddot{x} = -16\sin 2x - 12\cos 2x$

9. What is the domain of $f(x) = \sin^{-1}(4x)$?

(A) $-\pi \leq x \leq \pi$

(B) $-4 \leq x \leq 4$

(C) $-\frac{\pi}{8} \leq x \leq \frac{\pi}{8}$

(D) $-\frac{1}{4} \leq x \leq \frac{1}{4}$

10. A particle is moving in a straight line in simple harmonic motion. The displacement of the motion is given below.

$x = 6\cos^2 t - 1$ where x is displacement (metres) and t is the time (seconds)

Where is the centre of motion?

(A) $x = -1$

(B) $x = 0$

(C) $x = 1$

(D) $x = 2$

Section II

60 marks
Attempt questions 11 - 14
Allow about 1 hour and 45 minutes for this section
Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Use Newton's method to find a second approximation to the positive root of $x^4 - 5x^3 + 11x^2 - 12x + 4$. Take $x = 0.4$ as the first approximation. Answer correct to two decimal places. **2**

(b) A function is defined as $f(x) = \dfrac{5x+1}{x^2+x-2} - \dfrac{3}{x+2}$

(i) Show that $f(x) = \dfrac{2}{x-1}$ **1**

(ii) Find $f^{-1}(x)$ **2**

(iii) If $g(x) = x^2 + 5$ solve $f\big(g(x)\big) = 0.25$ **2**

(c) The letters of the word CURRICULUM are printed on separate cards which are held up by ten students.

(i) How many distinct arrangements of the cards are possible when the students stand in a straight line? **1**

(ii) How many distinct arrangements of the cards are possible when the students stand around in a circle? **1**

(iii) How many distinct arrangements of the cards can occur in a straight line if the two C's *cannot* be next to each other? **1**

(d) The point P (–2, 5) divides the interval joining A (–4, 1) and B (x, y) internally in the ratio 2 : 3. **2**

Find the coordinates of the point B.

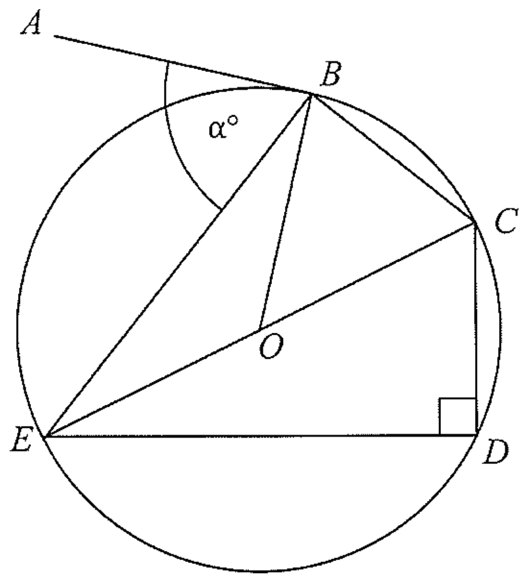
(e) **3**

The points B, C, D and E lie on the circle centre O.

AB is a tangent to the circle.

$\angle ABE = \alpha°$ and CE bisects $\angle BED$.

Prove that $\angle ECD = \angle ABE$.



Question 12 (15 marks) **Marks**

(a) Find $\int \frac{x}{\sqrt{1-49x^4}}dx$ **2**

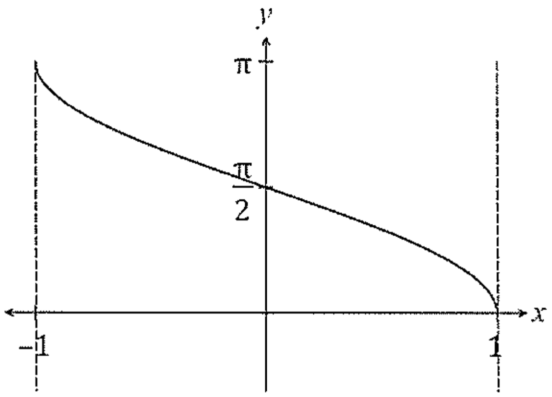
(b) Given that $(2+ax)^2(1+bx)^6 = 4 + 44x + 85x^2 + . . .$

where a and b are integers,

(i) Show that $4a + 24b = 44$ **3**

(ii) Find the possible values of a and b.

(c) The graph of $y = \cos^{-1}x$ $\{x : -1 \le x \le 1\}$ is shown below.



(i) Find the area bounded by the graph shown above, the x-axis and the line with the equation $x = -1$. **2**

(ii) Find the exact volume of the solid of revolution formed if the graph shown above is rotated about the y-axis. **2**

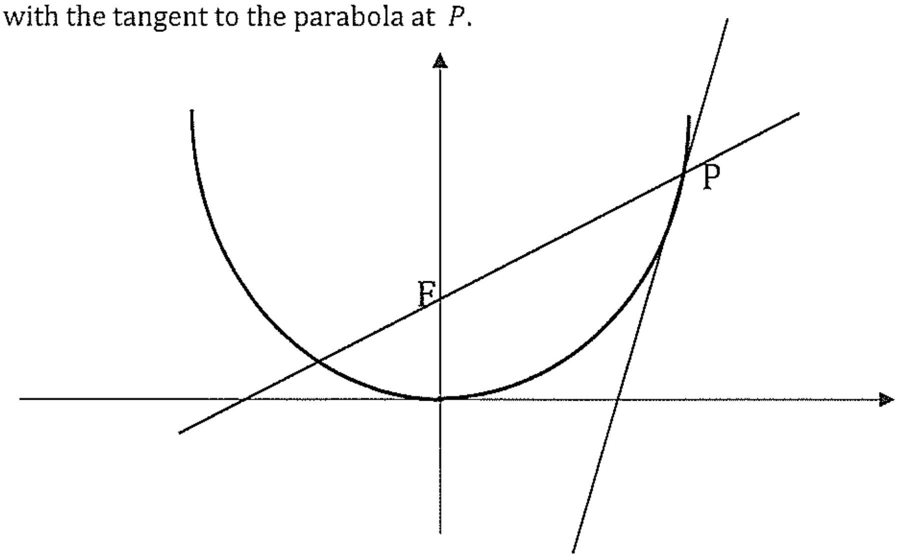
(d) Evaluate $\int_0^{15} \frac{x}{\sqrt{x+1}}\,dx$ using the substitution $u^2 = x+1$, $u > 0$ **2**

(e) The point $P(2p, p^2)$, where p is a parameter, lies on the parabola with Cartesian equation $x^2 = 4y$.

The point F is the focus of the parabola and O represents the origin.

The tangent to the parabola at P forms and angle θ with the positive x axis.

The straight line which passes through P and F forms an acute angle $\emptyset$ with the tangent to the parabola at P.



(i) Find the gradient of FP in terms of p. **1**

(ii) Show that $\theta + \emptyset = \frac{\pi}{2}$ **3**

Question 13 (15 marks) **Marks**

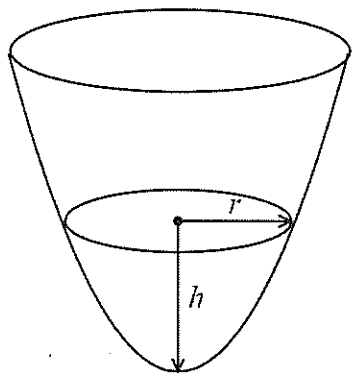
(a) If α, β and γ are the roots of $x^3 - 5x^2 + 7x + 5 = 0$ find:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha$ **1**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **1**

(b) Liquid is added to a glass container which has a parabolic shape and a circular cross section. **3**



When the radius of the top of the liquid is r cm, the depth of the liquid is h cm, such that

$$h = \frac{9r^2}{8} \text{ cm.}$$

The volume of the liquid at a given time is $V = \dfrac{9\pi r^4}{16}$ cm³.

The liquid is added at a rate of $\dfrac{45}{h}$ cm³ per second.

Find an expression for $\dfrac{dr}{dt}$ in terms of r and find its value when $h = 8$ cm.

(c) The number of people in a population at time t years is given by:

$N = 500 - 400e^{-0.1t}$

(i) Sketch the graph of N as a function of t, showing clearly the initial population and the limiting population size. **1**

(ii) Find the population size for which the rate of growth of the population is half the initial rate of growth. **2**

(d) (i) Show that $2\sin2x - 3\cos2x - 3\sin x + 3 = \sin x(6\sin x + 4\cos x - 3)$ **2**

(ii) Express $6\sin x + 4\cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **2**

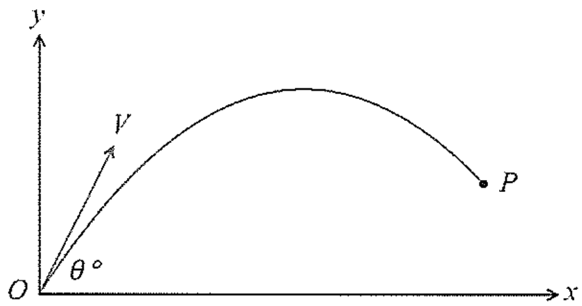
(iii) Hence, solve $2\sin2x - 3\cos2x - 3\sin x + 3 = 0$ for $0 \leq x < \pi$ **2**
Answer in radians correct to 3 significant figures.

Question 14 (15 marks) **Marks**

(a) Prove by mathematical induction that for $n > 0$, **3**

$$\frac{1}{3\times 4\times 5}+\frac{2}{4\times 5\times 6}+\cdots+\frac{n}{(n+2)(n+3)(n+4)}$$
$$=\frac{1}{6}-\frac{1}{n+3}+\frac{2}{(n+3)(n+4)}$$

(b) A projectile is fired from a point O with speed V ms^{-1} at an angle $\theta°$ above the horizontal, where $0 < \theta < \frac{\pi}{2}$. The projectile moves in a vertical plane under gravity where the acceleration due to gravity is 10 ms^{-2}. After 8 seconds the projectile hits the target P at a horizontal distance 288 metres from O and at a height 64 metres above O.



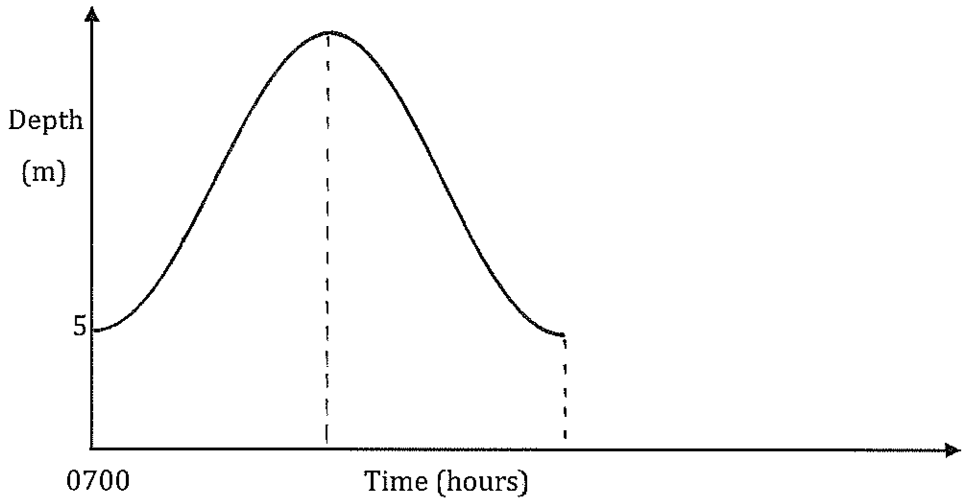
(i) Show that after t seconds, the horizontal and vertical displacements of the projectile are given by $x = Vt\cos\theta$ and $y = -5t^2 + Vt\sin\theta$. **2**

(ii) Find the exact values for V and θ. **3**

(iii) Find the velocity of the projectile at the moment of impact with the target P, giving the speed correct to the nearest integer and the angle to the horizontal correct to the nearest degree. **3**

(c) The level of the sea in a harbour is assumed to rise and fall with simple harmonic motion. On a certain day low tide occurs at 07.00 hours when the depth of the sea will be 5 metres. The next high tide will occur at 13.15 hours when the depth of the sea will be 17 metres.

A ship wishes to enter the harbour that day and needs a minimum sea depth of 6.5 metres.



(i) Find the amplitude, centre of motion and period of the curve describing the tide's motion. **1**

(ii) Hence find the equation for the depth of water y, at any time t, where t is the number of hours after low tide. **1**

(ii) Calculate, to the nearest minute, the earliest time it can enter the harbour on this day and the time by which it must leave. **2**

End of paper

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180^\circ$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

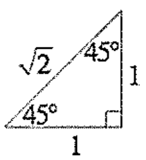
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \sec\theta = \frac{1}{\cos\theta}$$

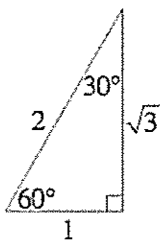
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios





Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

***n*th term of an arithmetic series**

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

***n*th term of a geometric series**

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Mathematics (continued)

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\dfrac{dy}{dx} = F'(u)\dfrac{du}{dx}$

If $y = e^{f(x)}$, then $\dfrac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\dfrac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\dfrac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\dfrac{b}{a}$ $\qquad$ $\alpha\beta = \dfrac{c}{a}$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$\text{Area} = \dfrac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$$\sin(\theta+\phi) = \sin\theta\cos\phi+\cos\theta\sin\phi$$

$$\cos(\theta+\phi) = \cos\theta\cos\phi-\sin\theta\sin\phi$$

$$\tan(\theta+\phi) = \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$$

***t* formulae**

If $t = \tan\frac{\theta}{2}$, then

$$\sin\theta = \frac{2t}{1+t^2}$$

$$\cos\theta = \frac{1-t^2}{1+t^2}$$

$$\tan\theta = \frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta = a,\quad \theta = n\pi+(-1)^n\sin^{-1}a$

$\cos\theta = a,\quad \theta = 2n\pi\pm\cos^{-1}a$

$\tan\theta = a,\quad \theta = n\pi+\tan^{-1}a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2 = 4ay$,

$$x = 2at,\quad y = at^2$$

At $(2at,\ at^2)$,

tangent: $y = tx - at^2$

normal: $x + ty = at^3 + 2at$

At $(x_1,\ y_1)$,

tangent: $xx_1 = 2a(y+y_1)$

normal: $y - y_1 = -\frac{2a}{x_1}(x-x_1)$

Chord of contact from $(x_0,\ y_0)$: $xx_0 = 2a(y+y_0)$

Acceleration

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$$x = b + a\cos(nt+\alpha)$$

$$\ddot{x} = -n^2(x-b)$$

Further integrals

$$\int\frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}+C$$

$$\int\frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

Sum and product of roots of a cubic equation

$$\alpha+\beta+\gamma = -\frac{b}{a}$$

$$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

Estimation of roots of a polynomial equation

Newton's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k} = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

Year 12 Mathematics Extension 1 Section I - Answer Sheet

Student Number ____________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○