Carlingford High School

2014

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 16

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

	Q1-10	Q11	Q12	Q13	Q14	Q15	Q16	Total
P3		/3		/2				/5
H1	/10							/10
H2						/2		/2
H3						/5		/5
H4			/10					/10
H5		/5		/13	/9	/8	/6	/41
H8					/6			/6
H9		/7	/5				/9	/21
Total	/10	/15	/15	/15	/15	/15	/15	/100

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. $\dfrac{x}{3}+\dfrac{3x-1}{2}$ can be simplified to

(A) $\dfrac{11x+3}{5}$

(B) $\dfrac{11x-3}{6}$

(C) $\dfrac{4x-3}{6}$

(D) $\dfrac{4x-1}{6}$

2. The point A has coordinates $(2,7)$ and B has coordinates $(-2,9)$.

What are the coordinates of the midpoint of the interval AB?

(A) $(0, 8)$

(B) $(-2, 1)$

(C) $(2, -1)$

(D) $(0, 3\frac{1}{2})$

3. Fully simplify the algebraic fraction: $\dfrac{x^3-8}{x^2-4}$.

(A) $\dfrac{x^2-2x+4}{x-2}$

(B) $x+2$

(C) $\dfrac{x^2+4x+4}{x+2}$

(D) $\dfrac{x^2+2x+4}{x+2}$

4. What is the derivative of $(3x^2+1)^4$?

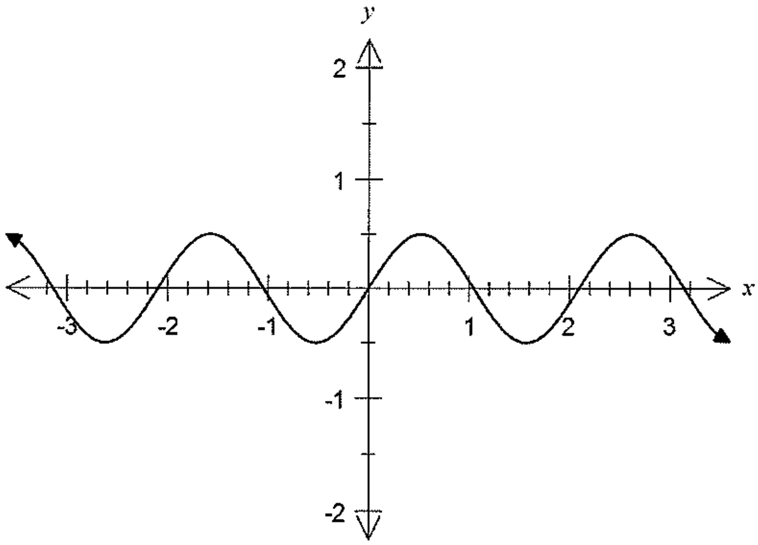
(A) $4(6x)^3$

(B) $6x(3x^2+1)^3$

(C) $24x(3x^2+1)^4$

(D) $24x(3x^2+1)^3$

5. What function would describe the graph shown, where x is in radians?



(A) $y = \frac{1}{2}\cos 3x$

(B) $y = \frac{1}{2}\sin 3x$

(C) $y = \frac{1}{2}\tan 3x$

(D) $y = \frac{1}{3}\sin 2x$

6. The quadratic function $3x^2 - 5x + 2$ has roots α and β.

Which of the following statements is true?

(A) $2\alpha\beta = -\frac{4}{3}$

(B) $\alpha^2 + \beta^2 = \frac{13}{9}$

(C) $2\alpha + 3\beta = \frac{25}{3}$

(D) $\alpha^2\beta^2 = \frac{2}{9}$

7. Consider the series $28, 7, \frac{7}{4}, \ldots$

Find the difference between S_5 and S_3.

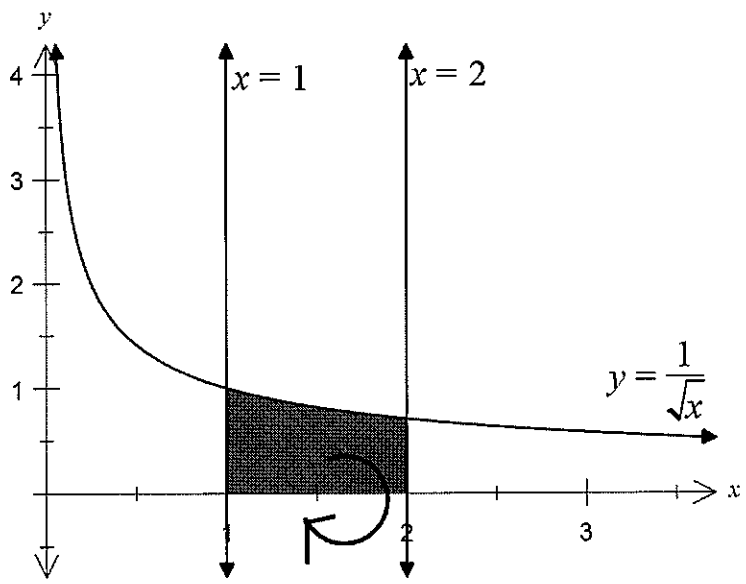
(A) $\frac{105}{64}$

(B) $\frac{231}{4}$

(C) $\frac{35}{64}$

(D) $\frac{231}{64}$

8. The region between the functions $y = \frac{1}{\sqrt{x}}$, $x = 1$ and $x = 2$ is rotated about the x-axis. Find the volume of the solid formed.



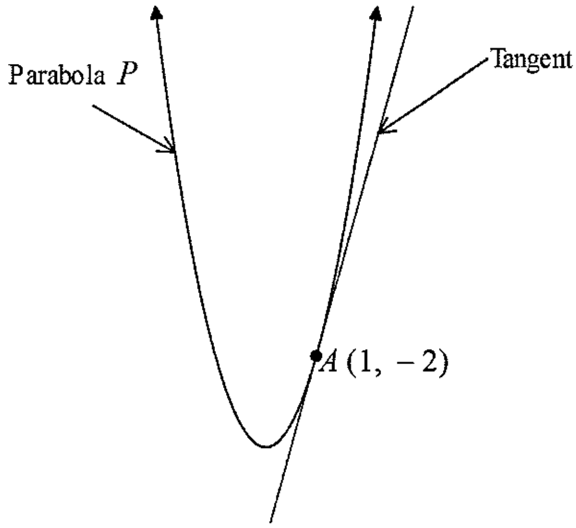
(A) $\pi \ln 2$ cubic units

(B) $\ln 2$ cubic units

(C) $2(\sqrt{2} - 1)$ cubic units

(D) $\ln \pi$ cubic units

9. The diagram shows the parabola P and it's tangent at the point $A(1, -2)$.



Which of the following equations might represent the normal to the parabola at the point A?

(A) $x - 3y + 5 = 0$

(B) $2x - 3y + 1 = 0$

(C) $x + 3y + 5 = 0$

(D) $x + 3y - 5 = 0$

10. For what domain and range is the function $y = \frac{1}{\sqrt{x-4}}$ defined?

(A) Domain: $x \geq 4$, Range: $y > 0$.

(B) Domain: $x > 4$, Range: $y > 0$.

(C) Domain: all real x, Range: all real y.

(D) Domain: $x < -2\ x > 2$, Range: $y < 0$.

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 writing booklet.

(a) Evaluate e^3 correct to 3 significant figures. **1**

(b) Solve these simultaneous equations: **2**

$$2x - y = -1$$
$$5x + 3y = 25$$

(c) Differentiate the following functions:

(i) $x^3 - 4x^2 + 2$ **1**

(ii) $2x \cos 3x$ **2**

(d) If $f'(x) = 6x^2 + 5x - 1$ and $f(-1) = 5$, find an expression for $f(x)$. **2**

(e) A particle moves so that its displacement from the origin is given by:

$x = -t^2 + 7t + 8$ (where x is displacement in metres and t is time in seconds)

(i) Show that the initial displacement of the particle is 8 metres. **1**

(ii) At what time will the particle be at the origin? **2**

(f) Consider the quadratic equation $3x^2 + kx + 5 = 0$:

(i) For what values of k does the equation have no real roots? **3**

(ii) Describe the graph of $y = 3x^2 + kx + 5$ when k takes the values found in part (i). **1**

End of Question 11

Question 12 (15 marks) Use the Question 12 writing booklet.

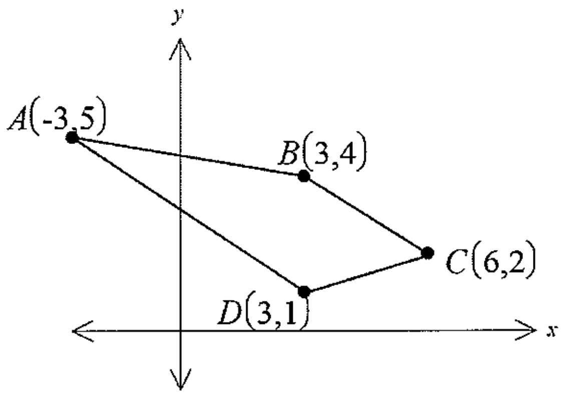
(a) A town called Benora is 15 kilometres away, on a bearing of 065° from another town called Andora. A third town, Calora is 42 kilometres East of Andora.

(i) Draw a diagram showing this information. **1**

(ii) Show that the distance from Benora to Calora is 29 kilometres, correct to the nearest kilometre. **2**

(iii) Find the bearing of Benora from Calora, correct to the nearest degree. **2**

(b) The points $A(-3, 5)$, $B(3, 4)$, $C(6, 2)$ and $D(3, 1)$ are the vertices of quadrilateral $ABCD$.



(i) Show that the equation of the line passing through B and C is $2x + 3y - 18 = 0$. **2**

(ii) Show that $AD \parallel BC$. **1**

(iii) Show that the perpendicular distance from point D to the line passing through B and C is $\frac{9\sqrt{13}}{13}$ units. **2**

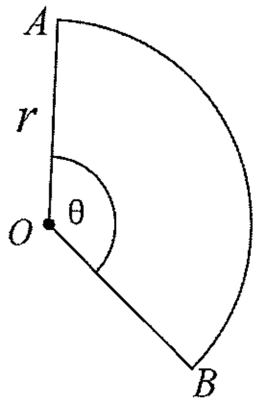
(iv) Show that $ABCD$ is a trapezium. **1**

(v) Given that the distance between B and C is $\sqrt{13}$ units, calculate the exact area of quadrilateral $ABCD$. **2**

Question 12 continues on page 9

Question 12 (continued)

(c) Tim was told that sector OAB has an area of $\frac{25\pi}{6}$ square units. The arc AB is $\frac{5\pi}{3}$ units long.



Tim was asked to find the exact values of r and θ.

His working out is shown below:

$$l = r\theta, \quad A = \frac{1}{2}r^2\theta$$

$$\therefore \quad \frac{1}{2}r^2\theta = \frac{25\pi}{6} \qquad (1)$$

$$r\theta = \frac{5\pi}{3} \qquad (2)$$

$$\frac{1}{2}r = \frac{5}{2} \qquad (3)$$

$$\therefore \quad r = 5 \text{ units}$$

(i) What operation did Tim perform on equations (1) and (2) to get to equation (3)? **1**

(ii) What is the value of θ? **1**

End of Question 12

Question 13 (15 marks) Use the Question 13 writing booklet.

(a) Robyn and Maria start jobs at the beginning of the same year. Robyn's salary is higher than Maria's. Both Robyn's and Maria's employers pay into their superannuation funds at the beginning of each month.

Robyn's employer deposits \$550 per month into her superannuation fund which earns interest at 0.5% per month. Maria's employer deposits \$520 per month into her superannuation fund which earns 0.6% per month.

(i) Show that the amount of interest that Robyn's superannuation earned in the first year was \$218.48. **3**

(ii) Let A_n represent the amount after n months. Show that the amount in Robyn's superannuation fund after n months is given by: **1**

$$A_n = 110550\left(1.005^n - 1\right)$$

(iii) After how many months will the amount in Maria's superannuation fund be greater than the amount in Robyn's? **3**

(b) *Temp4U* is an employment agency which specialises in contracting temporary employees. They have analysed the number of job applications received over the last five years. They found that the demand (D), measured in hundreds, for temporary employment at time (t years) is given by the function:

$$D(t) = 4\sin\left(\frac{\pi}{4}t\right) + 7$$

(i) Find all the times in the next 12 years where demand will be at its peak. **3**

(ii) State the amplitude and period of $D(t)$ and sketch its graph for the first twelve years. **3**

(c) Evaluate: $\lim\limits_{x \to \infty} \dfrac{x^4 + 3x^2 + 2}{5x^4 + 1}$. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 writing booklet.

(a) Use Simpson's rule to approximate the area between $y = \int_1^5 \frac{dx}{x^2+1}$, using 4 sub-intervals. **3**

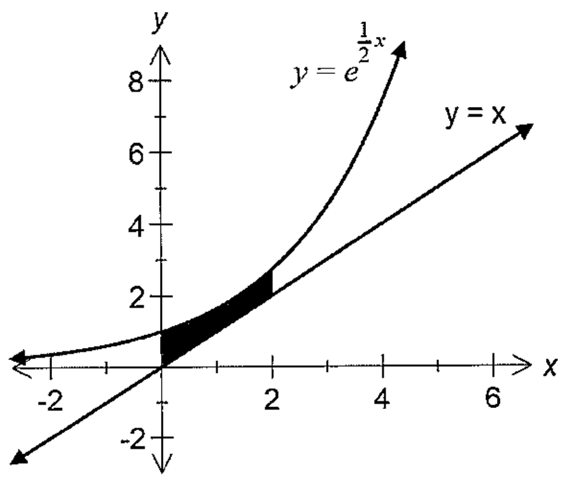
(b) A function is defined as $f(x) = x^3 - 3x^2$

(i) Find the coordinates of the stationary points, and determine their nature. **2**

(ii) Find the coordinates of any points of inflexion. **1**

(ii) Sketch the graph of $y = f(x)$, indicating clearly the stationary points, points of inflexion and x-intercepts. **2**

(c) The diagram shows the graphs of the functions $y = e^{\frac{1}{2}x}$ and $y = x$. The region between these 2 functions and the bounds $x = 0$ and $x = 2$ has been shaded. **3**



Calculate the exact area of the shaded region.

Question 14 continues on page 12

Question 14 (continued)

(d) For the parabola with equation $16y = x^2 - 4x - 12$:

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

End of Question 14

Question 15 (15 marks) Use the Question 15 writing booklet.

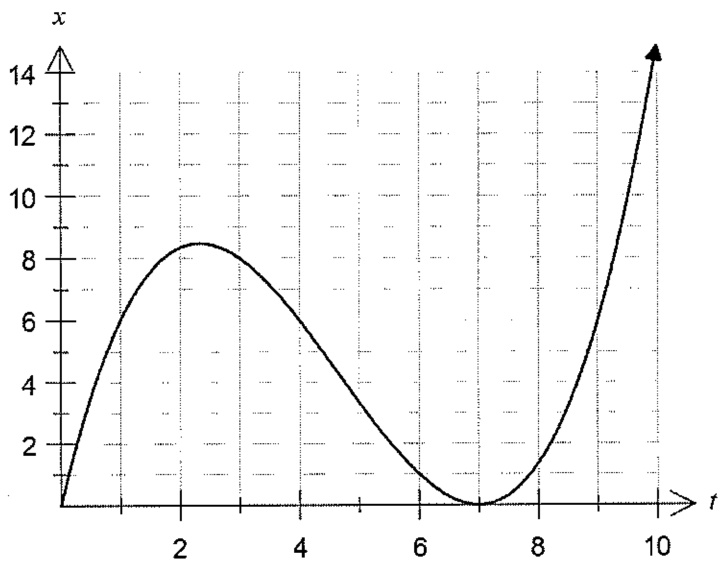
(a) The population of New South Wales in 2009 was 7.13 million. In 2012 the population had grown to approximately 7.3 million people.

(i) Assuming that the growth rate is proportional to the population, show that the annual growth rate is approximately 0.79%. **2**

(ii) Calculate the expected population of New South Wales in 2019 using this model. Give your answer rounded to the nearest hundred thousand. **1**

(iii) In what year will the population exceed 10 million? **2**

(b) The graph shows the displacement (x metres) of a particle at time (t seconds). The particle is moving horizontally.



(i) In the first 8 seconds, what is the particle's approximate maximum distance from the origin? **1**

(ii) Describe the motion of the particle at 4 seconds in terms of its displacement and velocity. **2**

(iii) Between which times is the particle's acceleration negative? What feature of the graph tells us this? **2**

Question 15 continues on page 14

Question 15 (continued)

(c) Show that the solutions to the equation: **3**

$$4\cos^2\theta = 6\sin\theta + 6 \text{ in the domain } 0^\circ \le \theta \le 2\pi \text{ are:}$$

$$\theta = \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}.$$

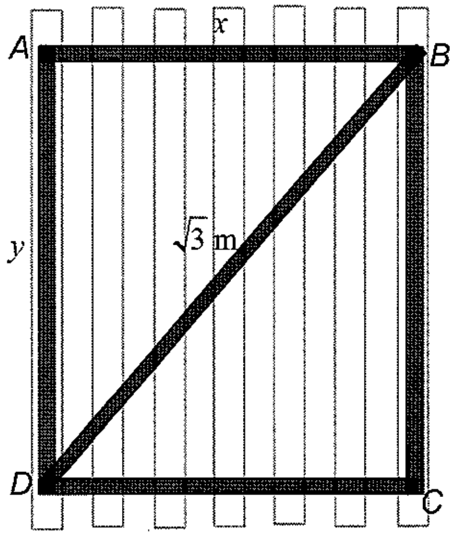
(d) Show that $\dfrac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$ for all integers $n \ge 1$. **2**

End of Question 15

Question 16 (15 marks) Use the Question 16 writing booklet.

(a) A swinging gate is to be constructed from timber palings. It will require a support frame using 5 pieces of timber: AB, AD, BD, BC and CD.

$AB \| CD$ and $AD \| BC$. $AB = CD = x$ metres. $AD = BC = y$ metres. BD is $\sqrt{3}$ metres long.



(i) Find an expression for y in terms of x. **1**

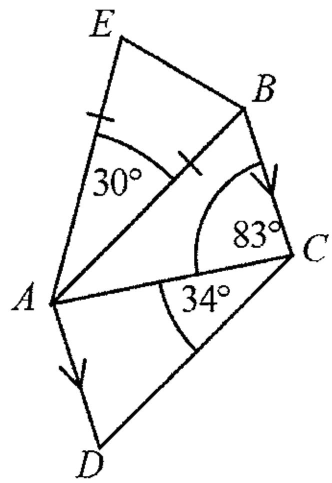
(ii) Show that the total length (L) of the timber pieces in the support frame is represented by $L = 2\left(x + \sqrt{3 - x^2} + \frac{\sqrt{3}}{2}\right)$. **1**

(iii) The gate will have its maximum strength when the length of its support frame is maximised. For what value of x will the gate have maximum strength? **4**

Question 16 continues on page 16

Question 16 (continued)

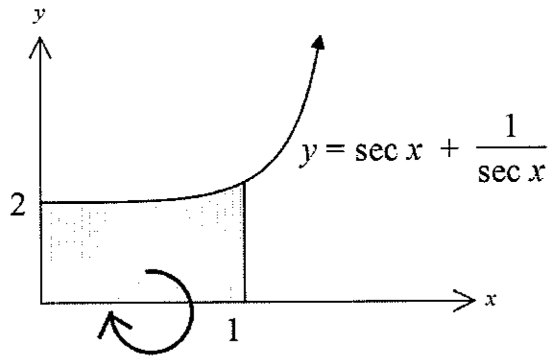
(b) In the diagram below: AD||BC , AE = AB, $\angle BAE = 30^\circ$, $\angle BCA = 83^\circ$, $\angle ACD = 34^\circ$, $\angle EBC = 138^\circ$.



(i) Prove that $AB \parallel DC$. **2**

(ii) Prove that $\Delta ABC \equiv \Delta ACD$. **3**

(c) The area bounded by the function $y = \sec x + \dfrac{1}{\sec x}$, the y-axis and the line $x = 1$ is rotated about the x-axis.



(i) Show that $\left(\sec x + \dfrac{1}{\sec x}\right)^2 = \sec^2 x + \cos^2 x + 2$. **1**

(ii) Find the volume of the solid formed, given that: $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$. **3**

End of Paper

MATHEMATICS TRIAL 2014

MC: 1. B 2. A 3. D 4. D 5. B 6. B 7. C 8. A 9. C 10. B

QUESTION 11

a) $e^3 = 20.08553692$
$= 20.1$ (3 sf) P3 [1]

b) $2x - y = -1 \quad \ldots (1)$
$5x + 3y = 25 \quad \ldots (2)$
(1)×3 $6x - 3y = -3 \quad \ldots (3)$
(2)+(3) $11x = 22$
$x = 2$
Sub in (1) $4 - y = -1$
$y = 5$ P3 [2]

c) i) $y = x^3 - 4x^2 + 2$
$\frac{dy}{dx} = 3x^2 - 8x$ H5 [1]

ii) $y = 2x\cos 3x$
$\frac{dy}{dx} = vu' + uv'$
$= \cos 3x \cdot 2 + 2x(-3\sin 3x)$
$= 2\cos 3x - 6x\sin 3x$ [2] H5

d) $f'(x) = 6x^2 + 5x - 1.$
$f(x) = \frac{6x^3}{3} + \frac{5x^2}{2} - x + c$
At $x = -1$ $f(x) = 5$
$5 = 2(-1)^3 + \frac{5(-1)^2}{2} - (-1) + c$
$5 = -2 + \frac{5}{2} + 1 + c$
$c = 3\frac{1}{2}$
$\therefore f(x) = 2x^3 + \frac{5x^2}{2} - x + \frac{7}{2}$ H5 [2]

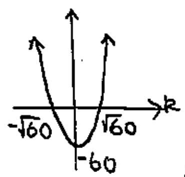
(e) $x = -t^2 + 7t + 8$

(i) At $t = 0$
$x = -(0)^2 + 7(0) + 8$
$x = 8$
$\therefore$ initial displacement is 8m. H9 [1]

(ii) At $x = 0$
$0 = -t^2 + 7t + 8$
$0 = t^2 - 7t - 8$
$0 = (t-8)(t+1)$
$\because t > 0$ particle will be at origin after 8s H9 [2]

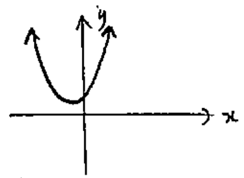
(f) i) $3x^2 + kx + 5 = 0$
For no real roots $\Delta < 0$
$\Delta = b^2 - 4ac$
$= k^2 - 4(3)(5)$
$= k^2 - 60$

$k^2 - 60 < 0$
$(k - \sqrt{60})(k + \sqrt{60}) < 0$



$-\sqrt{60} < k < \sqrt{60}$
$-2\sqrt{15} < k < 2\sqrt{15}$ H9 [3]

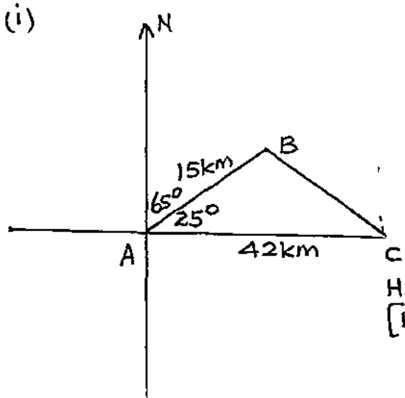
ii) The graph will be positive definite



$a > 0$ concave up
the parabola lies above the x axis H9 [1]

QUESTION 12

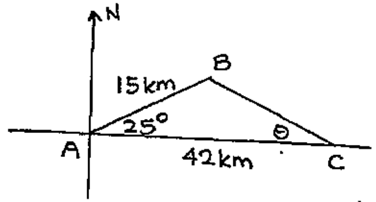
a) (i)



H4 [1]

(ii) $BC^2 = b^2 + c^2 - 2bc\cos A$
$= 42^2 + 15^2 - 2(42)(15)\cos 25^\circ$
$= 847.0521883$
$BC = 29.10416101$
$\therefore$ Benora is 29km from Calora H4 [2]

iii)



$\frac{\sin C}{AB} = \frac{\sin A}{BC}$
$\frac{\sin\theta}{15} = \frac{\sin 25^\circ}{29.10416101}$
$\sin\theta = 0.217813319$
$\theta = 12^\circ 34' 50.27''$

$\therefore$ Bearing is $270^\circ + 12^\circ 35'$ or $282^\circ 35'$ or 283° H4 [2]

b) i) B (3,4) C (6,2)
$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
$\frac{y-4}{x-3} = \frac{2-4}{6-3}$
$y - 4 = -\frac{2}{3}(x - 3)$
$3y - 12 = -2x + 6$
$2x + 3y - 18 = 0$ H4 [2]

ii) If AD ∥ BC $m_{AD} = m_{BC}$
A (−3,5) D (3,1)
$m_{AD} = \frac{1-5}{3--3}$
$= -\frac{2}{3}$
B(3,4) C(6,2)
$m_{BC} = \frac{2-4}{6-3}$
$= -\frac{2}{3}$
$= m_{AD}$
$\therefore$ AD ∥ BC H4 [1]

iii) $2x + 3y - 18 = 0$ (3,1)
⊥ $d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$
$= \frac{|2(3) + 3(1) - 18|}{\sqrt{2^2 + 3^2}}$
$= \frac{|-9|}{\sqrt{13}}$
$= \frac{9}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$
$= \frac{9\sqrt{13}}{13}$ units. H4 [2]

QUESTION 12 cont.

(iv) If ABCD is a trapezium it has ONE pair of parallel sides. AD ∥ BC

AB is not parallel to CD

(or show $m_{AB} \neq m_{CD}$)

(or show $BC \neq AD$)

from the diagram. ∴ ABCD is a trapezium. H9 (1)

v) using part (iii) $h = \frac{9\sqrt{13}}{13}$

$d_{AB} = \sqrt{(5-1)^2 + (-3-3)^2}$

$= \sqrt{52}$

$= 2\sqrt{13}$

$d_{BC} = \sqrt{13}$ (given)

$A = \frac{1}{2}h(a+b)$

$= \frac{1}{2} \cdot \frac{9\sqrt{13}}{13}(2\sqrt{13} + \sqrt{13})$

$= \frac{9\sqrt{13}(3\sqrt{13})}{26}$

$= \frac{351}{26}$

Area $= 13\frac{1}{2}$ units^2 H9 [2]

c) i) Tim divided equation 1 by equation 2. H9 [1]

ii) $l = r\theta$

$\frac{5\pi}{3} = 5\theta$

$\theta = \frac{\pi}{3}$ H9 [1]

QUESTION 13

a) i) $A_1 = 550(1.005)^{12}$

$A_2 = 550(1.005)^{11}$

$A_3 = 550(1.005)^{10}$

⁝

$A_{12} = 550(1.005)$

Total $= A_1 + A_2 + A_3 + \dots + A_{12}$

$= 550(1.005 + 1.005^2 + \dots 1.005^{12})$

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{1.005(1.005^{12} - 1)}{1.005 - 1}$

$= 12.39724018$

Total $= 550 \times S_n$

$= 6818.482101$

Interest $=$ total $- 550 \times 12$

$= \$218.48$ H5 [3]

ii) $A_n = 550 \times S_n$

$= \frac{550(1.005[1.005^n - 1])}{1.005 - 1}$

$= 110550[1.005^n - 1]$ H5 (1)

iii) For Maria

$A_n = 520 \times S_n$

$= \frac{520[1.006[1.006^n - 1]]}{1.006 - 1}$

$= 87186.\dot{6}[1.006^n - 1]$

Maria > Robyn (1)

$87186.\dot{6}(1.006^n - 1) > 110550(1.005^n - 1)$

$\frac{1.006^n - 1}{1.005^n - 1} > \frac{110550}{87186.\dot{6}}$

> 1.26796910

Guess and check

$n = 102$ LHS $= 1.267757392$

$n = 103$ LHS $= 1.268504921$

Maria has more money than Robyn after 103 months or 8 years and 7 months H5 [3]

b) $D(t) = 4\sin\left(\frac{\pi}{4}t\right) + 7$.

Max $D'(t) = 0$ $D''(t) < 0$

$D'(t) = \pi\cos\left(\frac{\pi}{4}t\right)$

$\therefore \cos\left(\frac{\pi}{4}t\right) = 0$

$\frac{\pi}{4}t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$

$t = 2, 4, 10 \dots$

$D''(t) = -\frac{\pi^2}{4}\sin\left(\frac{\pi}{4}t\right)$

$D''(2) = -\frac{\pi^2}{4}\sin\left(\frac{\pi}{2}\right)$

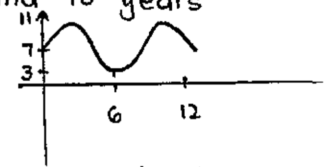
< 0 ∴ max. H5 [3]

$D''(4) = -\frac{\pi^2}{4}\sin\pi$

> 0 ∴ min

Maximum peaks occur after 2 and 10 years [3] H5

ii)



$a = 4$

$T = \frac{2\pi}{n}$

$T = 8$

(c) $\lim_{x\to\infty} \frac{x^4 + 3x^2 + 2}{5x^4 + 1} = \lim_{x\to\infty} \frac{\frac{x^4}{x^4} + \frac{3x^2}{x^4} + \frac{2}{x^4}}{\frac{5x^4}{x^4} + \frac{1}{x^4}}$

$= \frac{1+0+0}{5+0}$

$= \frac{1}{5}$

(2) P: [2]

QUESTION 14

a)i) $y = \int_1^5 \frac{dx}{x^2+1}$

x	1	2	3	4	5
y	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$	$\frac{1}{26}$

$A_1 = \frac{1}{3}\left[f(1) + 4f(2) + f(3)\right]$
$= \frac{1}{3}\left[\frac{1}{2} + 4\times\frac{1}{5} + \frac{1}{10}\right]$
$= \frac{7}{15}$

$A_2 = \frac{1}{3}\left[f(3) + 4f(4) + f(5)\right]$
$= \frac{1}{3}\left[\frac{1}{10} + 4\times\frac{1}{17} + \frac{1}{26}\right]$
$= \frac{413}{3315}$

$A_1 + A_2 = \frac{392}{663}\,u^2 \quad (\doteqdot 0.591\,u^2)$ H8 [3]

b)i) $f(x) = x^3 - 3x^2$
$f'(x) = x^2(x-3)$
$f'(x) = 3x^2 - 6x$
$f''(x) = 6x - 6$

Let $f'(x) = 0$
$0 = 3x^2 - 6x$
$0 = 3x(x-2)$
$x = 0, 2$

At $x=0$ $y=0$ $f''(0) = -6$ max
At $x=2$ $y=-4$ $f''(2) = 6$ min

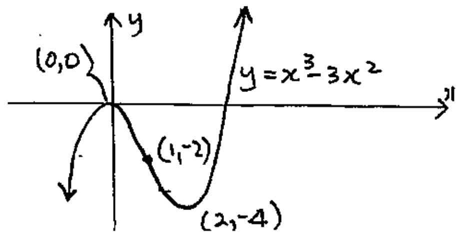
Stat points $(0,0)$ maximum
$(2,-4)$ minimum H5 [5]

ii) Point of inflexion $f''(x) = 0$
$6x - 6 = 0$
$x = 1 \quad y = -2$

Check for concavity change

x	1^-	1	1^+
$f''(x)$	$-$	0	$+$

Point of inflexion at $(1,-2)$

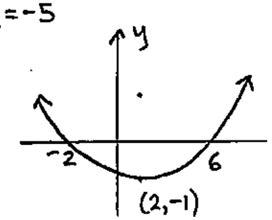


c) $A = \int_0^2 \left(e^{\frac{1}{2}x} - x\right) dx$
$= 2\int_0^2 \left(\frac{1}{2}e^{\frac{1}{2}x} - \frac{1}{2}x\right) dx$
$= \left[2e^{\frac{1}{2}x} - \frac{x^2}{2}\right]_0^2$
$= (2e - 2) - (2 - 0)$
Area $= 2e - 4$ units^2
$= 2(e-2)\,u^2$ H8 [3]

d) $16y = x^2 - 4x - 12$
$16y = x^2 - 4x + 4 - 16$

$16y + 16 = (x-2)^2$
$16(y+1) = (x-2)^2$

i) vertex $(2,-1)$
ii) focus $(2,3)$ (note $a=4$)
iii) directrix $y = -5$



H5 [4]

QUESTION 15

a)i) $P = Ae^{kt}$
$7.3 = 7.13e^{3k}$
$\ln\frac{7.3}{7.13} = 3k\ln e$
$k = \left[\ln\frac{7.3}{7.13}\right] \div 3$
$k = 0.007854371$
$\doteqdot 0.0079$ or 0.79% H3 [2]

ii)
$P = 7.3e^{10k}$
$= 7.3e^{10\times0.007854371}$
$= 7.712596851$
Population is 7.7 million H3 [1]

iii) $7.13e^{kt} > 10$
$kt\ln e > \ln\left(\frac{10}{7.13}\right)$
$t > \ln\left(\frac{10}{7.13}\right) \div 0.00785$
> 43.06823023 H3 [2]

In 2052 the population will exceed 10 million.

b)i) About 8.5m H5 [1]

ii) At 4s the particle is 6m from origin and travelling with negative velocity towards origin [2] H5

iii) After about 4.7s there is a point of inflexion so acceleration is negative between 0s and 4.7s. H5 [2]
graph is concave down

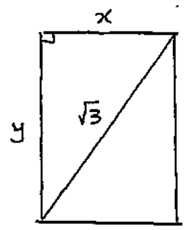
c) $4\cos^2\theta = 6\sin\theta + 6$
$4(1 - \sin^2\theta) = 6\sin\theta + 6$
$0 = 4\sin^2\theta + 6\sin\theta + 2$
$0 = 2\sin^2\theta + 3\sin\theta + 1$
$0 = (2\sin\theta + 1)(\sin\theta + 1)$
$\sin\theta = -\frac{1}{2}$
$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin\theta = -1$
$\theta = \frac{3\pi}{2}$ H5 [3]

d) $\frac{1}{\sqrt{n}+\sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$

$\frac{1}{\sqrt{n}+\sqrt{n+1}} \times \frac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}} = \frac{\sqrt{n}-\sqrt{n+1}}{(\sqrt{n})^2 - (\sqrt{n+1})^2}$
$= \frac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)}$
$= \frac{\sqrt{n}-\sqrt{n+1}}{-1}$
$= \sqrt{n+1} - \sqrt{n}$ H2 [2]

QUESTION 16



i) $x^2+y^2=3$

$y^2=3-x^2$

$y=\sqrt{3-x^2}$ H5 [1]

ii) $L = 2x+2y+\sqrt{3}$

$= 2x + 2\sqrt{3-x^2}+\sqrt{3}$

$= 2\left(x+\sqrt{3-x^2}+\frac{\sqrt{3}}{2}\right)$ H5 [1]

iii) $L = 2x+2\sqrt{3-x^2}+\sqrt{3}$

$\frac{dL}{dx} = 2 + 2.\frac{1}{2}(3-x^2)^{-1/2}. -2x$

$= 2 - \frac{2x}{\sqrt{3-x^2}}$

let $\frac{dL}{dx}=0$

$\frac{2-2x}{\sqrt{3-x^2}}=0$

$1 = \frac{x}{\sqrt{3-x^2}}$

$\sqrt{3-x^2} = x$

$3-x^2=x^2$

$2x^2=3$

$x^2=\frac{3}{2}$

$x=\pm\sqrt{\frac{3}{2}}$

Check for max$^{\underline{m}}$ or min$^{\underline{m}}$.

$\frac{dL}{dx} = -2x(3-x^2)^{-\frac{1}{2}}$.

$u=-2x$

$u'=-2$

$v=(3-x^2)^{-1/2}$

$v' = -\frac{1}{2}(3-x^2)^{-\frac{3}{2}}. -2x.$

$= x(3-x^2)^{-3/2}$

$\frac{d^2L}{dx^2} = (3-x^2)^{-\frac{1}{2}}.(-2) + -2x.x(3-x^2)^{-3/2}$

$= \frac{-2}{\sqrt{3-x^2}} - \frac{2x^2}{\sqrt{(3-x^2)^3}}$

OR test using 1st derivative

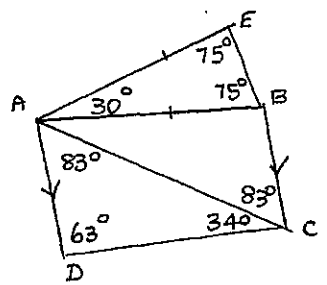
$= \frac{-2(\sqrt{3-x^2})^2-2x^2}{(\sqrt{3-x^2})^3}$

$= \frac{-6+2x^2-2x^2}{(\sqrt{3-x^2})^3}$

$= \frac{-6}{(\sqrt{3-x^2})^3}$

which is negative for $x=\frac{3}{2}$

$\therefore$ max$^{\underline{m}}$ at $x=\frac{3}{2}$ H5 [4]



ΔAEB is isosceles

$\therefore \angle AEB = \angle EBA$

base <s of isos Δ =

$\angle AEB+\angle EBA+30^\circ=180^\circ$ (< sum of Δ)

$\angle AEB=\angle EBA=75^\circ$

$\angle EBC=138^\circ$ (given)

$\angle ABC=138^\circ-75^\circ$

$=63^\circ$

$\angle ABC+\angle BCA+\angle BAC=180^\circ$ (< sum of Δ)

$63^\circ+83^\circ+\angle BAC=180^\circ$

$\angle BAC=34^\circ$

$\therefore \angle BAC=\angle DCA$

$\therefore$ AB ∥ DC as alternate <s are = H9 [2]

(ii) In ΔABC and ΔACD

$\angle ACB=\angle DAC$ (alt <s = AD ∥ CB)

$\angle BAC=\angle DCA$ (alt <s = AB ∥ CD)

AC is common

$\Delta ABC \equiv \Delta ACD$ (AAS) H9 [3]

c)(i) $y=\sec x+\frac{1}{\sec x}$

$\left(\sec x+\frac{1}{\sec x}\right)^2 = \sec^2 x + 2\sec x.\frac{1}{\sec x} + \frac{1}{\sec^2 x}$

$= \sec^2 x+\cos^2 x+2.$ H9 [1]

(ii) $\cos^2 x = \frac{1}{2}(\cos 2x+1)$

$V=\pi\int_a^b y^2\,dx$

$=\pi\int_0^1(\sec^2 x+\cos^2 x+2)\,dx$

$=\pi\int_0^1(\sec^2 x+\frac{1}{2}\cos 2x+\frac{1}{2}+2)\,dx$

$=\pi\left[\tan x+\frac{1}{4}\sin 2x+\frac{1}{2}x+2x\right]_0^1$

$=\pi\left[\left(\tan 1+\frac{1}{4}\sin 2+\frac{5}{2}\right)-\left(\tan 0+\frac{1}{4}\sin 0+0\right)\right]$

$=\pi\ (4.284732081)$

$= 13.46088283$

$\doteqdot 13.5\,u$ H9 [3]