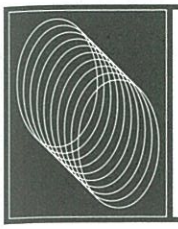


Topic 3
Conics

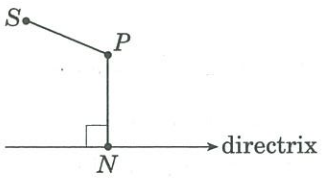
THEMES

1 **Definition of conics (in terms of eccentricity)**
2 **Important features of conics (foci, directrices, etc.)**
3 **Cartesian and parametric equations of conics**
4 **Tangents and normals to conics**

FORMULA TEST

1 A conic section is the locus of a variable point P whose distance PS from a fixed point S (the focus) is in a ☐ ratio to its distance PN from a fixed line (the directrix).

The ratio $\frac{PS}{PN}$ = ☐ is called the ☐.



2 For (i) a parabola, e ☐ 1
(ii) an ellipse, e ☐ 1
(iii) a hyperbola, e ☐ 1

Parabola

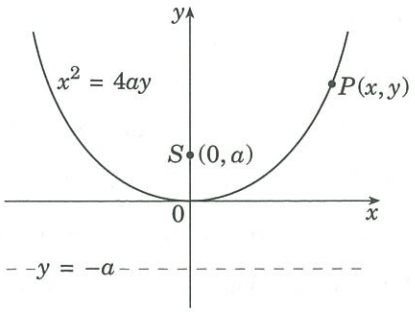
3 Coordinates of S are ☐, ☐.

4 Equation of directrix y = ☐.

5 Parametric coordinates of P are ☐, ☐ for parameter p.

6 The equation of a chord PQ through P and Q with parameters p, q: y = ☐

7 The equation of the tangent at P given that
(a) $P(x_1, y_1)$: ☐ **(b)** $P(2ap, ap^2)$: y = ☐



8 The equation of the normal at $P(2ap, ap^2)$ is ☐.

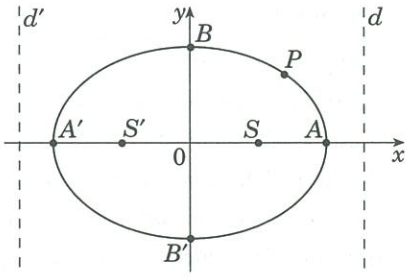
9 The equation of the chord of contacts of tangents from an external point (x_1, y_1) is ☐

The ellipse

10 The equation of the ellipse is ☐.

11 **(a)** The coordinates of the foci S, S' in terms of a, e are S ☐, ☐ and S' ☐, ☐.

(b) The coordinates of the vertices A, A' and B, B' are ☐, ☐, ☐, ☐, ☐, ☐.



12 The equations of directrices d, d' are x = ☐.

13 In terms of major/minor axis lengths, the eccentricity can be found from b^2 = ☐.

14 **(a)** The major axis length = ☐. **(b)** The minor axis length = ☐.

15 $SP + PS' =$ ☐

16 For parameter θ, the parametric equations are $x =$ ☐, $y =$ ☐.

17 The equation of the tangent at $P(x_1, y_1)$ is ☐.

18 The equation of the tangent at P(parameter θ) is ☐.

19 The equation of the normal at $P(x_1, y_1)$ is ☐.

20 The equation of the normal at P(parameter θ) is ☐.

[N.B. The equations 17–20 are best derived rather than committed to memory.]

The hyperbola

21 The equation of the hyperbola is ☐.

22 **(a)** The coordinates of the foci S, S' in terms of a, e are S ☐ and S' ☐.

(b) The coordinates of the vertices A, A' are ☐, ☐.

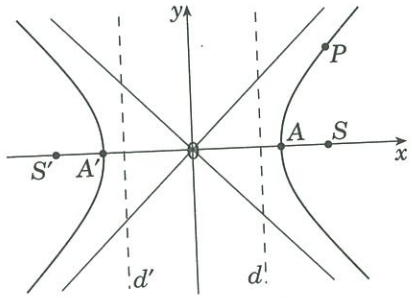
23 The equations of directrices d, d' are $x =$ ☐.

24 In terms of major/minor axis lengths, the eccentricity can be found from $b^2 =$ ☐.

25 **(a)** The transverse axis length = ☐.

(b) The conjugate axis length = ☐.

26 The equations of the asymptotes are $y =$ ☐.



27 $|SP - P'S| =$ ☐

28 For parameter θ, the parametric equations are $x =$ ☐, $y =$ ☐.

29 The equation of the tangent at $P(x_1, y_1)$ is ☐.

30 The equation of the tangent at P(parameter θ) is ☐.

31 The equation of the normal at $P(x_1, y_1)$ is ☐.

32 The equation of the normal at P(parameter θ) is ☐.

[N.B. The equations 29–32 are best derived rather than committed to memory.]

The rectangular hyperbola

33 For a hyperbola with rectangular asymptotes $y =$ ☐, the equation of the hyperbola (rectangular) is ☐.

34 For the special hyperbola $xy = k^2$, k a constant, then the

(a) parametric equations are $x =$ ☐, $y =$ ☐

(b) equation of the chord PQ (with parameters p, q) is ☐

(c) equation of the tangent at P (parameter p) is ☐

(d) equation of the normal at P (parameter p) is ☐

(e) equation of the chord from $P(x_1, y_1)$ to $Q(x_2, y_2)$ is ☐

(f) equation of the tangent at $P(x_1, y_1)$ is ☐

[N.B. The equations 34(b) – (f) are best derived rather than committed to memory.]

ANSWERS TO FORMULA TEST

1 constant ratio;
$\frac{PS}{PN} = e$, the eccentricity

2 (i) $e = 1$
(ii) $e < 1$
(iii) $e > 1$

Parabola

3 $(0, a)$

4 $y = -a$

5 $\left(2ap, ap^2\right)$

6 $y = \left(\frac{p+q}{2}\right)x - apq$

7 **(a)** $xx_1 = 2a\left(y + y_1\right)$
(b) $y = px - ap^2$

8 $x + py = 2ap + ap^3$

9 $xx_1 = 2a\left(y + y_1\right)$

The ellipse

10 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

11 **(a)** $S(ae, 0)$; $S'(-ae, 0)$
(b) $A(a, 0)$; $A'(-a, 0)$
$B(0, b)$; $B'(0, -b)$

12 $x = \pm\frac{a}{e}$

13 $b^2 = a^2\left(1 - e^2\right)$, $e < 1$

14 **(a)** major axis $= 2a$
(b) minor axis $= 2b$

15 $SP + PS' = 2a$

16 $x = a\cos\theta$, $y = b\sin\theta$

17 $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

18 $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

19 $\frac{xa^2}{x_1} - \frac{yb^2}{y_1} = a^2 - b^2$

20 $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$

The hyperbola

21 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

22 **(a)** $S(ae, 0)$; $S'(-ae, 0)$
(b) $A(a, 0)$; $A'(-a, 0)$

23 $x = \pm\frac{a}{e}$

24 $b^2 = a^2\left(e^2 - 1\right)$, $e > 1$

25 **(a)** transverse axis $= 2a$
(b) conjugate axis $= 2b$

26 $y = \pm\frac{b}{a}x$

27 $|SP - P'S| = 2a$

28 $x = a\sec\theta$, $y = b\tan\theta$

29 $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

30 $\frac{x}{a}\sec\theta - \frac{x}{b}\tan\theta = 1$

31 $\frac{xa^2}{x_1} + \frac{yb^2}{y_1} = a^2 + b^2$

32 $xa\cos\theta + yb\cot\theta = a^2 + b^2$

The rectangular hyperbola

33 $y = \pm x$; $x^2 - y^2 = a^2$

34 **(a)** $x = kt$, $y = \frac{k}{t}$
(b) $x + pqy = k\left(p + q\right)$
(c) $x + p^2y = 2kp$
(d) $p^2x - y = \frac{k}{p}\left(p^4 - 1\right)$
(e) $k^2x + x_1x_2y = k^2\left(x_1 + x_2\right)$
(f) $xy_1 + x_1y = 2k^2$