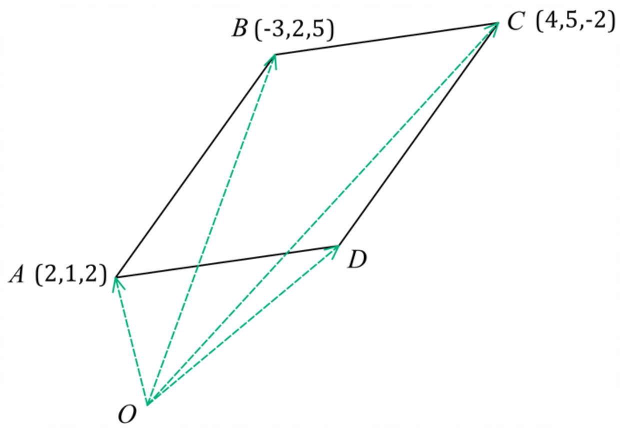
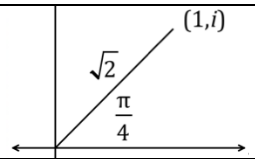
ACE Examination Paper 3
Year 12 Mathematics Extension 2 Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$\int \frac{(x-1)^2}{x}\,dx = \int \left(x - 2 + \frac{1}{x}\right) dx$ $= \frac{1}{2}x^2 - 2x + \ln\lvert x\rvert + C$	1 Mark: B
2	$\arg(z^7) = 7\arg(z)$ $= \frac{7\pi}{5} = -\frac{3\pi}{5}$	1 Mark: B
3	$x = 4\sin^2 t - 1$ $= 2(1-\cos 2t) - 1$ $= 1 - 2\cos 2t$ $\therefore$ Centre of motion is $x = 1$.	1 Mark: C
4	$u = x - 2$ $\frac{du}{dx} = 1$ or $du = dx$ When $x = 1$ then $u = -1$ and when $x = 3$ then $u = 1$. $\int_1^3 x(x-2)^5\,dx = \int_{-1}^{1} (u+2)u^5\,du$ $= \int_{-1}^{1} (u^6 + 2u^5)\,du$ $= \left[\frac{u^7}{7} + \frac{u^6}{3}\right]_{-1}^{1}$ $= \left[\frac{1}{7} + \frac{1}{3}\right] - \left[-\frac{1}{7} + \frac{1}{3}\right] = \frac{2}{7}$	1 Mark: B
5	$\overrightarrow{OP} = 3\underset{\sim}{i} + 6\underset{\sim}{j} - 2\underset{\sim}{k}$ $\quad \overrightarrow{OQ} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ $\lvert\overrightarrow{OP}\rvert = \sqrt{3^2 + 6^2 + (-2)^2} = 7$ $\lvert OQ\rvert = \sqrt{2^2 + (-2)^2 + 1^2} = 3$ $\overrightarrow{OP} \cdot \overrightarrow{OQ} = (3 \times 2) + (6 \times -2) + (-2 \times 1) = -8$ $\cos\angle POQ = \frac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{\lvert\overrightarrow{OP}\rvert\lvert\overrightarrow{OR}\rvert} = \frac{-8}{7 \times 3}$ $\angle POQ = 112.3926\ldots \approx 112.4°$	1 Mark: C

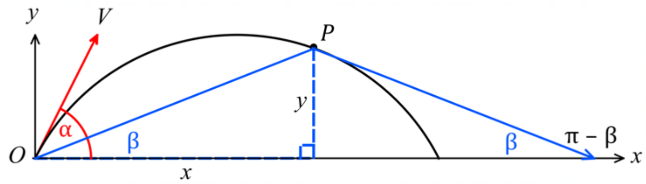
6	$z^2 + 6z + 10$ $z = \dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 10}}{2}$ $= \dfrac{-6 \pm \sqrt{-4}}{2} = \dfrac{-6 \pm 2i}{2} = -3 \pm i$ $= (z + 3 - i)(z + 3 + i)$	1 Mark: A
7	$F = ma$ $= mv\dfrac{dv}{dx}$ $= -2m(v + v^2)$ $\therefore \dfrac{dx}{dv} = -\dfrac{1}{2(1 + v)}$ $x = -\dfrac{1}{2}\int \dfrac{1}{1 + v}dv$	1 Mark: A
8	The converse of a statement 'If A then B' is 'If B then A'. The statements can be represented as: the converse of $A \Rightarrow B$ is $B \Rightarrow A$ or $A \Leftarrow B$. The converse of a true statement need not be true.	1 Mark: D
9	Vectors (A) (B) and (D) are opposite direction to $\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$ (A) $\sqrt{(-6)^2 + 12^2 + (-2)^2} = \sqrt{184}$ (B) $\sqrt{(-3)^2 + 6^2 + (-6)^2} = \sqrt{81}$ (D) $\sqrt{(-2)^2 + 4^2 + (-4)^2} = \sqrt{36} = 6$	1 Mark: D
10	Use the substitution $u = \sin x$ $\dfrac{du}{dx} = \cos x$ $du = \cos x dx$ When $u = 0$ then $u = 0$ and when $x = \frac{\pi}{2}$ then $u = 1$ $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1 - \sin^2 x}}dx = \int_0^1 \frac{1}{\sqrt{1 - u^2}}dx$ $= [\sin^{-1} u]_0^1$ $= \dfrac{\pi}{2}$	1 Mark: C

Section II

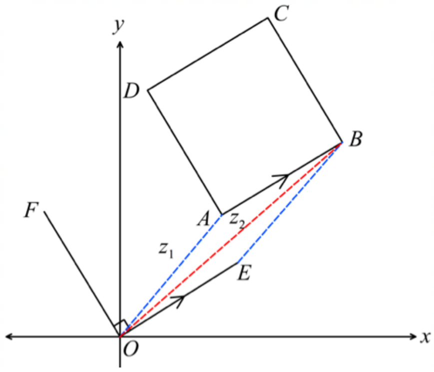
	Solution	Criteria
11(a) (i)	$\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{v} = \underset{\sim}{i} + 2\underset{\sim}{j} + m\underset{\sim}{k}$ and $\underset{\sim}{w} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ $\lvert\underset{\sim}{v}\rvert = \sqrt{1^2 + 2^2 + m^2} = 2\sqrt{3}$ $\sqrt{5 + m^2} = \sqrt{12}$ $5 + m^2 = 12$ $m^2 = 7$ $m = \pm\sqrt{7}$	2 Marks: Correct answer. 1 Mark: Shows understanding of the magnitude of a vector.
11(a) (ii)	Two vectors are perpendicular if and only if $\underset{\sim}{u}.\underset{\sim}{v} = 0$ $\underset{\sim}{u} \cdot \underset{\sim}{v} = (1 \times 1) + \big((-1) \times 2\big) + (2 \times m) = 0$ $1 - 2 + 2m = 0$ $m = \frac{1}{2}$	2 Marks: Correct answer. 1 Mark: Shows understanding when two vectors are perpendicular.
11(b) (i)	$\lvert z + w\rvert = \lvert 4 - 3i\rvert$ $= 5$	1 Mark: Correct answer.
11(b) (ii)	$z^2 - w^2 = (1 - i)^2 - (3 - 2i)^2$ $= -2i - (9 - 12i + 4i^2)$ $= -2i - 5 + 12i$ $= -5 + 10i$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
11(c)	 $\overrightarrow{OA} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$, $\overrightarrow{OB} = -3\underset{\sim}{i} + 2\underset{\sim}{j} + 5\underset{\sim}{k}$, $\overrightarrow{OC} = 4\underset{\sim}{i} + 5\underset{\sim}{j} - 2\underset{\sim}{k}$ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= -5\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ $\overrightarrow{DC} = \overrightarrow{AB}$ (opposite sides of a parallelogram are equal) $= -5\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ $\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{DC}$ $= 9\underset{\sim}{i} + 4\underset{\sim}{j} - 5\underset{\sim}{k}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds $\overrightarrow{AB}$ or shows some understanding.

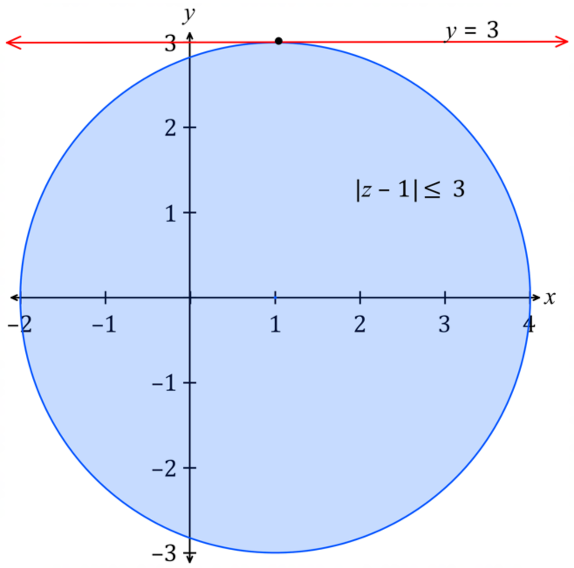
11(d)	Integration by parts $\int e^x \sin x\,dx = \int e^x \times \frac{d}{dx}(-\cos x)dx$ $= e^x(-\cos x) - \int e^x(-\cos x)\,dx$ $= -e^x\cos x + \int e^x \times \frac{d}{dx}(\sin x)dx$ $= -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$ $2\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x + C$ $\int e^x \sin x\,dx = \frac{e^x}{2}(\sin x - \cos x) + C$	3 Marks: Correct answer. 2 Marks: Finds the second application of integration by parts. 1 Mark: Correctly applies integration by parts.
12(a) (i)	Amplitude of motion is 3 cm (a = 3) Period of motion is 4 seconds $T = \frac{2\pi}{n} = 4$ $n = \frac{\pi}{2}$ $v^2 = n^2(a^2 - x^2)$ $= \left(\frac{\pi}{2}\right)^2 (3^2 - x^2)$ Maximum velocity occurs when x = 0 $v^2 = \left(\frac{\pi}{2}\right)^2 \times 3^2$ $v = \frac{3}{2}\pi$ cms^{-1}	2 Marks: Correct answer. 1 Mark: Finds the values of a and n or shows some understanding of the problem.
12(a) (ii)	Maximum acceleration is when x =–3 cm $\ddot{x} = -n^2x$ $= -\left(\frac{\pi}{2}\right)^2 \times -3$ $= \frac{3}{4}\pi^2$ cms^{-2}	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
12(a) (iii)	Displacement is $x = \pm 1$ cm $v^2 = \left(\frac{\pi}{2}\right)^2 (3^2 - (\pm 1)^2)$ $= \frac{8\pi^2}{4}$ $v = \pm\pi\sqrt{2}$ cms^{-1}	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
12(b) (i)	$z = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$  	1 Mark: Correct answer.
12(b) (ii)	Using De Moivre's theorem $z^n = \cos n\theta + i\sin n\theta$ for $n = 1, 2, 3, \ldots$ $z^{10} = \sqrt{2}^{10}\cos\left(10 \times \frac{\pi}{4}\right) + i\sin\left(10 \times \frac{\pi}{4}\right)$ $= 32\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2} = 32\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ $\therefore\ \lvert z^{10}\rvert = 32$ and $\arg(z^{10}) = \frac{\pi}{2}$	2 Marks: Correct answer. 1 Mark: Finds the modulus or the argument.

12(c) (i)	$t = \tan\frac{x}{2}$ $dt = \frac{1}{2}\sec^2\frac{x}{2}dx$ $dx = \frac{2}{1+t^2}dt$ When $x = 0$ then $t = 0$ and when $x = \frac{\pi}{2}$ then $t = 1$ $5 + 5\sin x - 3\cos x = \frac{5(1+t^2) + 10t - 3(1-t^2)}{1+t^2}$ $= \frac{8t^2 + 10t + 2}{1+t^2}$ $= \frac{2(4t^2 + 5t + 1)}{1+t^2}$ $\int_0^{\frac{\pi}{2}} \frac{1}{5 + 5\sin x - 3\cos x}dx = \int_0^1 \frac{1+t^2}{2(4t^2+5t+1)} \times \frac{2}{1+t^2}dt$ $= \int_0^1 \frac{1}{4t^2+5t+1}dt$	3 Marks: Correct answer. 2 Marks: Finds sin$x$ and cos$x$ in terms of t. 1 Mark: Shows some understanding.
12(c) (ii)	$\frac{1}{4t^2+5t+1} = \frac{1}{(4t+1)(t+1)} = \frac{1}{3}\left\{\frac{4}{4t+1} - \frac{1}{t+1}\right\}$ $\int_0^{\frac{\pi}{2}} \frac{1}{5 + 5\sin x - 3\cos x}dx = \int_0^1 \frac{1}{3}\left\{\frac{4}{4t+1} - \frac{1}{t+1}\right\}dt$ $= \frac{1}{3}\left[\ln\left(\frac{4t+1}{t+1}\right)\right]_0^1$ $= \frac{1}{3}\left(\ln\frac{5}{2} - \ln 1\right) = \frac{1}{3}\ln\frac{5}{2}$	2 Marks: Correct answer. 1 Mark: Correct expression for the integral in terms of t using partial fractions.
13(a)	Step 1: To prove true for $n = 1$ $T_1 = 3.2^1 + 1 = 7$ Result is true for $n = 1$ Step 2: Assume true for $n = k$ $T_k = 3.2^k + 1$ Step 3: To prove true for $n = k + 1$ $T_{k+1} = 3.2^{k+1} + 1$ given $T_{k+1} = 2T_k - 1$ LHS $= 2T_k - 1$ $= 2(3.2^k + 1) - 1$ $= 3.2^{k+1} + 1$ $=$ RHS Step 4: True by induction	3 Marks: Correct answer. 2 Marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$. 1 Mark: Proves the result true for $n = 1$.

13(b)	Use the substitution $x = 2\sin\theta$ $\frac{dx}{d\theta} = 2\cos\theta$ $dx = 2\cos\theta d\theta$ When $x = 0$ then $\theta = 0$ and when $x = \sqrt{2}$ then $\theta = \frac{\pi}{4}$ $\int_0^{\sqrt{2}} \sqrt{4 - x^2}dx = \int_0^{\frac{\pi}{4}} 2\cos\theta \times 2\cos\theta d\theta$ $= 4\int_0^{\frac{\pi}{4}} \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta$ $= 4\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}$ $= 4\left(\frac{\pi}{8} + \frac{1}{4}\right)$ $= \frac{\pi}{2} + 1$	3 Marks: Correct answer. 2 Marks: Finds the primitive function. 1 Mark: Correctly expresses the integral in terms of θ.
13(c)	 Position at P $\tan\beta = \frac{y}{x}$ $= \frac{-\frac{1}{2}gt^2 + Vt\sin\alpha}{Vt\cos\alpha}$ $= \tan\alpha - \frac{g}{2V\cos\alpha}t$ (1) Velocity at P $\tan(\pi - \beta) = \frac{\dot{y}}{\dot{x}}$ $-\tan\beta = \frac{-gt + V\sin\alpha}{V\cos\alpha}$ $\tan\beta = \frac{g}{V\cos\alpha}t - \tan\alpha$ (2) Equating equations (1) and (2) $\tan\alpha - \frac{g}{2V\cos\alpha}t = \frac{g}{V\cos\alpha}t - \tan\alpha$ $2\tan\alpha = \frac{3}{2}\frac{g}{V\cos\alpha}t$ $\therefore t = \frac{4V\sin\alpha}{3g}$	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Finds an expression for $\tan\beta$. 1 Mark: Uses the equations of motion appropriately.

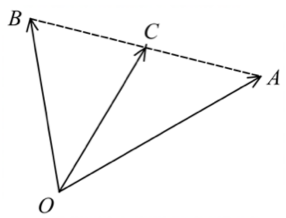
13(d) (i)	$5x^3 - 3x^2 + 2x - 1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)\,x^2$ $= Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$ Therefore $(A+C)\,x^3 = 5x^3$ ① $(B+D)\,x^2 = -3x^2$ ② $Ax = 2x$ ③ $B = -1$ ④ Hence $A = 2$, $B = -1$, $C = 3$ and $D = -2$	2 Marks: Correct answer. 1 Mark: Finds two of the pronumerals or shows some understanding.
13(d) (ii)	$\int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2}dx = \int \left(\frac{2}{x} - \frac{1}{x^2} + \frac{3x-2}{x^2+1}\right)dx$ $= \int \left(\frac{2}{x} - \frac{1}{x^2} + \frac{3x}{x^2+1} - \frac{2}{x^2+1}\right)dx$ $= 2\ln x + \frac{1}{x} + \frac{3}{2}\ln(x^2+1) - 2\tan^{-1}x + C$	2 Marks: Correct answer. 1 Mark: Correctly finds one of the integrals.
13(e)	$z.\bar{z} = (x+iy)(x-iy)$ $= x^2 - i^2y^2$ $= x^2 + y^2$ $= \left(\sqrt{x^2+y^2}\right)^2$ $= \lvert z\rvert^2$	2 Marks: Correct answer. 1 Mark: Expands $z.\bar{z}$ or shows some understanding.
14(a) (i)	$a^2 + b^2 - 2ab = (a-b)^2 \geq 0$ $\therefore a^2 + b^2 \geq 2ab$	1 Mark: Correct answer.
14(a) (ii)	Using the result in part (a) $a^2 + b^2 \geq 2ab$ $a^2 + d^2 \geq 2ad$ $b^2 + d^2 \geq 2bd$ $a^2 + c^2 \geq 2ac$ $b^2 + c^2 \geq 2bc$ $c^2 + d^2 \geq 2cd$ Adding all the above inequations $3(a^2 + b^2 + c^2 + d^2) \geq 2(ab + ac + ad + bc + bd + cd)$	2 Marks: Correct answer. 1 Mark: Uses the result in part (a).
14(a) (iii)	$a^2 + b^2 + c^2 + d^2$ $= (a+b+c+d)^2 - 2(ab + ac + ad + bc + bd + cd$ ① Multiply equation ① by 3 and use $a + b + c + d = 1$ $3(a^2 + b^2 + c^2 + d^2) = 3 - 6(ab + ac + ad + bc + bd + cd)$ Now using the result in part (b) $2(ab + ac + ad + bc + bd + cd)$ $\leq 3 - 6(ab + ac + ad + bc + bd + cd)$ $8(ab + ac + ad + bc + bd + cd) \leq 3$ $(ab + ac + ad + bc + bd + cd) \leq \frac{3}{8}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Writes the sum of the squares in terms of the products taken two at a time.

14(b) (i)	 $OE//AB$ and $OE = AB$ $\therefore OABE$ is a parallelogram Let w be the vector that correspond to point E. $w + z_1 = z_2$ $\therefore w = z_2 - z_1$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(b) (ii)	$OF \perp OE$ and $OF = OE$ $\therefore F$ corresponds to $i(z_2 - z_1)$ (multiplying a complex number by i corresponds to an anticlockwise rotation about the origin through 90°)	1 Mark: Correct answer.
14(b) (iii)	Since $AD//OF$ and $AD = OF$ Point D corresponds to the complex number: $z_1 + i(z_2 - z_1) = z_1(1 - i) + iz_2$	1 Mark: Correct answer.
14(c)	$\underset{\sim}{i}$ component $4 + \lambda = 0$ $\lambda = -4$ $\underset{\sim}{j}$ component $10 - 4 \times 5 = a$ $a = -10$ $\underset{\sim}{k}$ component $-1 - 4 \times (-3) = b$ $\therefore a = -10$ and $b = 11$.	2 Marks: Correct answer. 1 Mark: Finds λ or shows some understanding.
14(d) (i)	$I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx \quad n \geq 2$ $= \int_0^{\frac{\pi}{2}} -\sin^{n-1} x \frac{d\cos x}{dx} dx$ $= -[\sin^{n-1} x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \frac{d(\sin^{n-1} x)}{dx} dx$ $= 0 + \int_0^{\frac{\pi}{2}} \cos x (n-1) \sin^{n-2} x \cos x dx$ $= (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x dx$	2 Marks: Correct answer. 1 Mark: Correctly applies integration by parts.

14(d) (ii)	$I_n = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}x \cos^2 x dx$ $= (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}x(1-\sin^2 x)\ dx$ $= (n-1)\int_0^{\frac{\pi}{2}} (\sin^{n-2}x - \sin^n x)\, dx$ $= (n-1)I_{n-2} - (n-1)\, I_n$ $nI_n = (n-1)\, I_{n-2}$ $I_n = \frac{n-1}{n}\, I_{n-2}$	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.
14(d) (iii)	$I_4 = \frac{3}{4}\, I_2$ $= \frac{3}{4} \times \frac{1}{2}\, I_0$ $= \frac{3}{8} \times \int_0^{\frac{\pi}{2}} dx$ $= \frac{3\pi}{16}$	2 Marks: Correct answer. 1 Mark: Applies the recurrence relation to find an expression for I_4.
15(a)	$\lvert z-1\rvert \le 3$ represents a region with centre $(1, 0)$ and radius less than or equal to 3. $\mathrm{Im}(z) \ge 3$ represents a region above the horizontal line $y = 3$. The point (1,3) is where the two inequalities hold.  	3 Marks: Correct answer. 2 Marks: Correctly graphs one inequality. 1 Mark: Makes some progress.
15(b) (i)	Given a > 0 and $b > 0$ $\left(\sqrt{a}-\sqrt{b}\right)^2 \ge 0$ $a + b - 2\sqrt{ab} \ge 0$ $\frac{a+b}{2} \ge \sqrt{ab}$	1 Mark: Correct answer.

15(b) (ii)	Using the result in part(a) $\left(\frac{a+b}{2}\right)^2 \geq \left(\sqrt{ab}\right)^2$ $\frac{(a+b)^2}{4} \geq ab$ $\frac{(a+b)}{ab} \geq \frac{4}{(a+b)}$ $\frac{1}{a}+\frac{1}{b} \geq \frac{4}{a+b}$ $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right) \geq 4$	2 Marks: Correct answer. 1 Mark: Uses the result in part (a) and makes some progress.
15(c) (i)	$(x+iy)^2 = -3-4i$ $x^2-y^2+2ixy = -3-4i$ Equating the real and imaginary parts $x^2-y^2=-3$ ① $2xy=-4$ ② From equation ② $y=-\frac{2}{x}$ and substitute into equation ① $x^2-\frac{4}{x^2}=-3$ $x^4+3x^2-4=0$ $(x^2+4)(x^2-1)=0$ $x^2=1$ $x=\pm 1$ $\therefore$ When $x = 1$ then $y = -2$ and when $x = -1$ then $y = 2$ Also $[\pm(1-2i)]^2=-3-4i$ $\sqrt{-3-4i}=\pm(1-2i)$	2 Marks: Correct answer. 1 Mark: Finds one pair of integers or equates the real and imaginary parts.
15(c) (ii)	Using $\sqrt{-3-4i}=\pm(1-2i)$ $z=\frac{3\pm\sqrt{9-4(3+i)}}{2}$ $=\frac{3\pm\sqrt{-3-4i}}{2}$ $=\frac{3\pm(1-2i)}{2}$ $z=2-i$ or $z=1+i$	2 Marks: Correct answer. 1 Mark: Uses the result from part (a) and shows some understanding.
15(d) (i)	$\overrightarrow{EB}=\overrightarrow{AO}$ $=-\overrightarrow{OA}$ $=-3\underset{\sim}{i}$ $\overrightarrow{EF}=\overrightarrow{OC}$ $=4\underset{\sim}{k}$ $\overrightarrow{OE}=\overrightarrow{OB}+\overrightarrow{BE}$ $=5\underset{\sim}{j}+\overrightarrow{OA}$ $=5\underset{\sim}{j}+3\underset{\sim}{i}$ $=3\underset{\sim}{i}+5\underset{\sim}{j}$ $\overrightarrow{OF}=\overrightarrow{OE}+\overrightarrow{EF}$ $=3\underset{\sim}{i}+5\underset{\sim}{j}+4\underset{\sim}{k}$	2 Marks: Correct answer. 1 Mark: Finds two of the vectors.

15(d) (ii)	$\overrightarrow{OM} = \overrightarrow{OF} + \overrightarrow{FM}$ $= \overrightarrow{OE} + \overrightarrow{EF} + \overrightarrow{FM}$ $= 3\underset{\sim}{\imath} + 5\underset{\sim}{\jmath} + 4\underset{\sim}{k} + \frac{1}{2}(-\overrightarrow{GF})$ $= 3\underset{\sim}{\imath} + 5\underset{\sim}{\jmath} + 4\underset{\sim}{k} + \frac{1}{2}(-\overrightarrow{OA})$ $= 3\underset{\sim}{\imath} + 5\underset{\sim}{\jmath} + 4\underset{\sim}{k} + \frac{1}{2}(-3\underset{\sim}{\imath})$ $= \frac{3}{2}\underset{\sim}{\imath} + 5\underset{\sim}{\jmath} + 4\underset{\sim}{k}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
16(a) (i)	Resolving forces $m\ddot{x} = F - kv^2$ $\ddot{x} = \frac{1}{m}(F - kv^2)$	2 Marks: Correct answer. 1 Mark: Resolves forces.
16(a) (ii)	$v\frac{dv}{dx} = \frac{1}{m}(F - kv^2)$ $\frac{dv}{dx} = \frac{(F - kv^2)}{mv}$ $\frac{dx}{dv} = \frac{mv}{(F - kv^2)}$ $\int_{x_1}^{x_2} 1dx = \int_{v_1}^{v_2} \frac{mv}{(F - kv^2)} dv$ (Distance travelled is $x_2 - x_1$) $x_2 - x_1 = -\frac{m}{2k}[\ln(F - kv^2)]_{v_1}^{v_2}$ $= \frac{m}{2k}[\ln(F - kv^2)]_{v_2}^{v_1}$ $= \frac{m}{2k}[\ln(F - k{v_1}^2) - \ln(F - k{v_2}^2)]$ $x = \frac{m}{2k}\ln\left(\frac{F - k{v_1}^2}{F - k{v_2}^2}\right)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds an expression for $\frac{dx}{dv}$ or has some understanding of the problem.
16(b) (i)	Step 1: To prove the statement true for *n* = 1 $\sum_{i=1}^{1} 1^2 = \frac{1}{6} + \frac{1^2}{2} + \frac{1^3}{3} = 1$ Result true for *n* = 1 Step 2: Assume the result true for *n* = *k* $\sum_{i=1}^{k} i^2 = \frac{k}{6} + \frac{k^2}{2} + \frac{k^3}{3}$ Step 3: To prove the result true for *n* = *k* + 1 $\sum_{i=1}^{k+1} i^2 = \frac{k+1}{6} + \frac{(k+1)^2}{2} + \frac{(k+1)^3}{3}$	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Proves the result true for *n* = 1 and attempts to use the result of *n* = *k* to prove the result for *n* = *k* + 1. 1 Mark: Proves the result true for *n* = 1.

	$\text{LHS} = \sum_{r=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$ $= \frac{k}{6} + \frac{k^2}{2} + \frac{k^3}{3} + (k+1)^2$ $= \frac{k + 3k^2 + 2k^3 + 6(k^2 + 2k + 1)}{6}$ $= \frac{2k^3 + 9k^2 + 13k + 6}{6}$ $\text{RHS} = \frac{k+1}{6} + \frac{(k+1)^2}{2} + \frac{(k+1)^3}{3}$ $= \frac{(k+1)\big(1 + 3(k+1) + 2(k^2 + 2k + 1)\big)}{6}$ $= \frac{(k+1)(1 + 3k + 3 + 2k^2 + 4k + 2)}{6}$ $= \frac{(k+1)(2k^2 + 7k + 2)}{6}$ $= \frac{2k^3 + 9k^2 + 13k + 6}{6}$ LHS = RHS Result is true for $n = k + 1$ Step 4: Result true by the principle of mathematical induction.	
16(b) (ii)	$\lim_{n\to\infty} \frac{1^2 + 2^2 + 3^2 + \cdots + n^2}{n^3} = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} i^2}{n^3}$ $= \lim_{n\to\infty} \frac{1}{n^3}\left(\frac{n}{6} + \frac{n^2}{2} + \frac{n^3}{3}\right)$ $= \lim_{n\to\infty} \left(\frac{1}{6n^2} + \frac{1}{2n} + \frac{1}{3}\right) = \frac{1}{3}$	1 Mark: Correct answer.
16(c) (i)	$\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$ $\overrightarrow{OC} + \overrightarrow{CA} = \overrightarrow{OA}$ Now $\overrightarrow{BC} = \overrightarrow{CA}$ $\therefore \overrightarrow{OC} - \overrightarrow{OB} = \overrightarrow{OA} - \overrightarrow{OC}$  $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$ $= \frac{1}{2}\left(3\underset{\sim}{i} + 4\underset{\sim}{k} + 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right) = \frac{1}{2}\left(5\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k}\right)$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
16(c) (ii)	$\overrightarrow{AC} = \frac{4}{3}\overrightarrow{AB}$ $\overrightarrow{OC} - \overrightarrow{OA} = \frac{4}{3}(\overrightarrow{OB} - \overrightarrow{OA})$ $\overrightarrow{OC} = \frac{4}{3}(\overrightarrow{OB} - \overrightarrow{OA}) + \overrightarrow{OA}$ $= \frac{4}{3}\left(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k} - 3\underset{\sim}{i} - 4\underset{\sim}{k}\right) + 3\underset{\sim}{i} + 4\underset{\sim}{k}$ $= \frac{4}{3}\left(-\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k}\right) + \frac{1}{3}\left(9\underset{\sim}{i} + 12\underset{\sim}{k}\right) = \frac{1}{3}\left(5\underset{\sim}{i} - 8\underset{\sim}{j}\right)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Sets up the equation with position vectors.