

# Carlingford High School

2015

## Mathematics HSC Course

### Year 12 Half Yearly Examination

Time allowed 2 hours

Name: .....

Teacher: (Please Circle)

Mr Cheng

Mrs Lobejko

Mr Gong

Mr White

Mr Wilson

#### General Instructions

- Start each question in a new booklet
- Do not write in columns
- Marks may be deducted for careless or badly arranged work
- Only calculators approved by the Board of Studies may be used
- All answers are to be completed in blue or black pen except graphs and diagrams
- No lending or borrowing

	MC	Q11	Q12	Q13	Q14	Q15	Total
	/10						/10
H3		/10	/10		/6	/3	/29
H4		/5		/5		/4	/14
H5				/9	/6	/6	/21
	/10	/15	/10	/14	/12	/13	/74

## Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space in the grid on the Answer Sheet on the last page.

1. If  $3^{3x-4} = 9$  then the solution is

A.  $x = \frac{7}{3}$

B.  $x = \frac{13}{3}$

C.  $x = 2$

D.  $x = \frac{3}{7}$

2. Find the value of  $x$  if:

$$\sqrt{75} + \sqrt{27} = \sqrt{x}$$

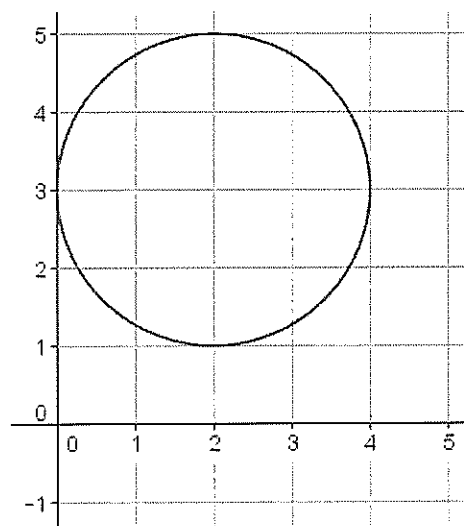
A. 102

B.  $\sqrt{3}$

C. 24

D. 192

3. Which equation corresponds to the graph below?



A.  $(x-2)^2 + (y-3)^2 = 4$

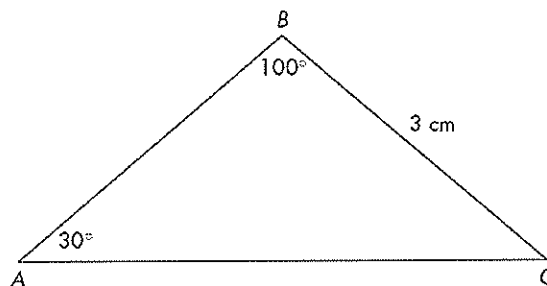
B.  $(x+2)^2 + (y+3)^2 = 4$

C.  $(x-2)^2 + (y-3)^2 = 2$

D.  $(x+2)^2 + (y+3)^2 = 2$

4. What is the exact value of  $\sin 120^\circ$ ?
- A.  $\frac{2}{\sqrt{3}}$   
 B.  $\frac{\sqrt{3}}{2}$   
 C.  $-\frac{1}{\sqrt{2}}$   
 D.  $-\frac{1}{\sqrt{3}}$
5. Which of the following equations has solutions  $x = \frac{2}{3}$  and  $x = -3$ ?
- A.  $x^2 - 7x - 6 = 0$   
 B.  $3x^2 + 7x - 6 = 0$   
 C.  $x^2 - x - 6 = 0$   
 D.  $3x^2 - x - 6 = 0$
6. What is the perpendicular distance of the point  $(2, -1)$  from the line  $y = 3x + 1$ ?
- A.  $\frac{6}{\sqrt{10}}$   
 B.  $\frac{6}{\sqrt{5}}$   
 C.  $\frac{8}{\sqrt{10}}$   
 D.  $\frac{8}{\sqrt{5}}$
7. The length of AC in the following triangle is:

- A. 4.32  
 B. 5.91  
 C. 6.00  
 D. 11.82



8. Using the substitution  $p = x^3$ , the solution for  $8x^6 - 35x^3 + 27 = 0$  is:

A.  $x = \pm 1, \pm \frac{3}{2}$

B.  $x = -1, -\frac{3}{2}$

C.  $x = 1, \frac{3}{2}$

D.  $x = \pm 1, \frac{3}{2}$

9. Simplify  $\frac{(2a^3b)^3}{(ab)^2}$

A.  $8a^7b$

B.  $8a^8b$

C.  $2a^7b$

D.  $2a^8b$

10. The linear function with equation  $4x - 2y + 3 = 0$  has

A. gradient = -2, y-intercept =  $-1\frac{1}{2}$

B. gradient =  $\frac{1}{2}$ , y-intercept =  $\frac{3}{4}$

C. gradient = 2, y-intercept =  $1\frac{1}{2}$

D. gradient = 4, y-intercept = 3

## Section II

Total marks (64)

Attempt Questions 11-14

Allow about 1 hour 45 minutes for this section

Answer all questions, starting each question on a **new** booklet with your name and question number at the top of the page.

**Question 11** Use a SEPARATE writing booklet (15 marks)

a. The third term of an arithmetic series is 32 and the sixth term is 17

i) Find the common difference. 2

ii) Find the sum of the first ten terms. 2

b. A student is supposed to read a book of 630 pages in 2 weeks. He actually read 5 pages the first day, 9 pages the second day, 13 the third and to continue increasing the number of pages by 4, every day.

i) How many pages did he read on the last day? 2

ii) What is the total number of pages he read during the 2 weeks? 2

iii) If he wanted to start with  $k$  pages find the value of  $k$  so he finished reading the book in 2 weeks. 2

c. Ezzat invests \$50 000 in an account which earns 8% interest, compounding annually. He intends to withdraw \$M at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for exactly 20 years, so that the account will then be empty.

i) Write an expression for the amount of money he has in the account immediately after he has made his first withdrawal? 1

ii) Write an expression in terms of M for the amount of money in the account immediately after his 20<sup>th</sup> withdrawal. 2

iii) Calculate the value of M which leaves his account empty after the 20<sup>th</sup> withdrawal. 2

**Question 12** Use a SEPARATE writing booklet (10 marks)

- a. Show that the equation of the locus of a point  $P(x,y)$ , which moves so  $PA=2PB$ , where  $A(-3,2)$ ,  $B(3,-4)$ , is a circle indicating the centre and radius. 3
- b. Given the parabola  $x^2 - 8x + 8y - 16 = 0$
- i) By completing the square, write the equation in the form  $(x-h)^2 = 4a(y-k)$ . 2
- ii) Find the coordinates of the vertex, focus and the equation of the directrix. 3
- iii) Draw a neat sketch of the parabola showing the above information and the y intercept. 2

**Question 13** Use a SEPARATE writing booklet (14 marks)

- a. Differentiate the following:
- i)  $y = 3x^2 + \frac{2}{x^2} - 4$  1
- ii)  $f(x) = \sqrt[3]{3x+2}$  2
- iii)  $y = \frac{3x-5}{5x+2}$  2
- b. Consider the curve given by  $y = x^3 - 6x^2 + 9x + 1$
- i) Find  $\frac{dy}{dx}$ . 1
- ii) Find the coordinates of the two stationary points. 2
- iii) Determine the nature of the stationary points. 2
- iv) Find any points of inflexion. 2
- v) Sketch the curve for  $x \geq 0$ . 2

**Question 14** Use a SEPARATE writing booklet (12 marks)

a. Find the primitive of

i)  $\int x^2(3x-1)dx$  2

ii)  $\int \frac{x+\sqrt{x}}{x\sqrt[3]{x^2}}dx$  2

b. Evaluate  $\int_0^1 (3x+2)^3 dx$  2

c. Consider the function  $y = \frac{4}{x^2+1}$

i) Copy and complete the following table. Answer to 2 dp where necessary 2

X	0	0.25	0.5	0.75	1
y					

ii) Apply Simpsons Rule with 5 function values to find an approximation for

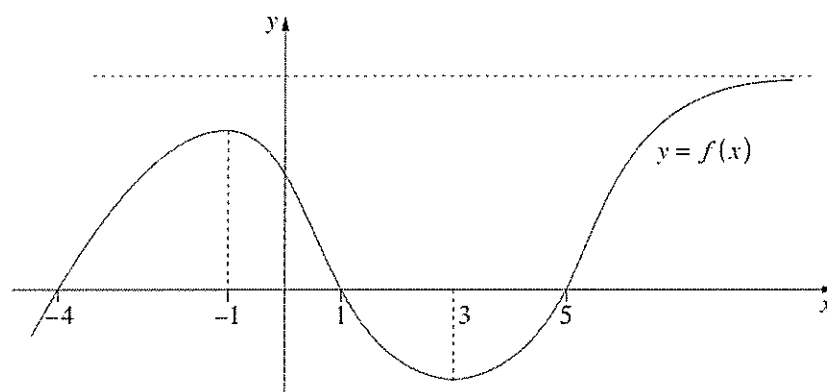
$\int_0^1 \frac{4}{x^2+1}dx$  2

Give your answer correct to one decimal place.

d. Find x in terms of t if  $\frac{d^2x}{dt^2} = 2$  and that at  $t = 0$ ,  $\frac{dx}{dt} = 0$ ,  $x = -1$ . 2

**Question 15** Use a SEPARATE writing booklet (13 marks)

a.

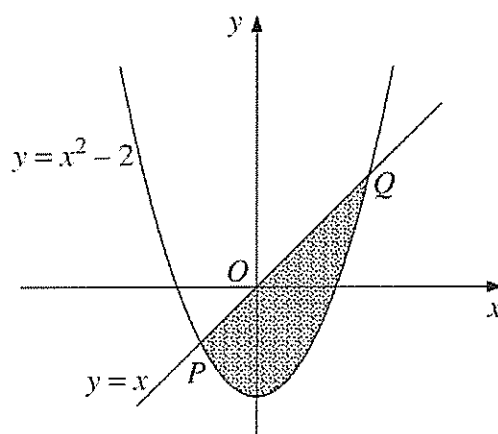


i) For which values of  $x$  is the derivative,  $f'(x)$ , negative? 1

ii) What happens to  $f'(x)$  for large values of  $x$ ? 1

iii) Sketch the graph  $y = f'(x)$ . 1

b. The diagram shows the graphs of  $y = x^2 - 2$  and  $y = x$ .



i) Find the  $x$  values of the points of intersection,  $P$  and  $Q$ . 1

ii) Calculate the area of the shaded region. 3

c) A piece of wire 60 cm long, is cut into two parts. One part forms a rectangle, whose length is five times its width and the other part forms a square.

i) Show that the sum of the two areas is  $A = 14x^2 - 90x + 225$ , where  $x$  is the width of the rectangle. 3

ii) Show that the minimum sum of the two areas is  $80\frac{5}{14} \text{ cm}^2$  3



STUDENT NAME .....

## Section I - ANSWER SHEET

10 marks

Attempt Questions 1 – 10

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

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	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Year 12 Half Yearly

1.  $3x - 4 = 2$   
 $x = 2$  C

2.  $5\sqrt{3} + 3\sqrt{3} = \sqrt{x}$   
 $8\sqrt{3} = \sqrt{x}$   
 $x = 64 \times 3$   
 $= 192$  D

3. A

4. B

5.  $(3x - 2)(x + 3) = 0$  B  
 $3x^2 + 7x - 6 = 0$

6.  $3x - y + 1 = 0$   
 $\frac{|3(2) - (-1) + 1|}{\sqrt{3^2 + 1^2}}$   
 $= \frac{8}{\sqrt{10}}$  C

7.  $\frac{AC}{\sin 60} = \frac{3}{\sin 30}$   
 $AC = \frac{3 \times 100}{\sin 30}$  B  
 $= 591$

8.  $8p^2 - 35p + 27 = 0$

C

9.  $\frac{8a^9b^2}{a^2b^2}$  A  
 $8a^7b$

10.  $2y = 4x + 3$  C  
 $yc \ 2x + \frac{3}{2}$

11. a)  $T_3 = 32$   
 $T_6 = 17$   
 $a + 2d = 32$   
 $a + 5d = 17$   
 $-3d = 15$   
 $d = -5$   
 $a = 42$

(i)  $S_{10} = \frac{10}{2} (2 \times 42 + 9 \times -5)$   
 $= 195$

b)  $T_{14} = 5 + 13 \times 4$   
 $= 57$

(ii)  $S_{14} = \frac{14}{2} (5 + 57)$   
 $= 434$

(iii)  $630 = \frac{14}{2} (2k + 13 \times 4)$   
 $= 14k + 364$   
 $14k = 266$   
 $k = 19$

$$c) (i) A_1 = 50000(1.08) - M$$

$$(ii) A_2 = (50000(1.08) - M)1.08 - M$$

$$= 50000(1.08)^2 - M(1+1.08)$$

$$A_3 = [50000(1.08)^2 - M(1+1.08)]1.08 - M$$

$$= 50000(1.08)^3 - M(1+1.08+1.08^2)$$

$$A_{20} = 50000(1.08)^{20} - M(1+1.08+1.08^2+\dots+1.08^{19})$$

$$(iii) A_{20} = 0$$

$$\therefore 50000(1.08)^{20} = M(1+1.08+1.08^2+\dots+1.08^{19})$$

$$= \frac{M(1.08^{20}-1)}{0.08}$$

$$M = \frac{50000(1.08)^{20}(0.08)}{1.08^{20}-1}$$

$$= \$5092.61$$

12. a)

$$\frac{PA}{PB} = 2 \Rightarrow \sqrt{(x+3)^2 + (y-2)^2} = 2\sqrt{(x-3)^2 + (y+4)^2}$$

$$(x+3)^2 + (y-2)^2 = 4[(x-3)^2 + (y+4)^2]$$

$$x^2 + 6x + 9 + y^2 - 4y + 4 = 4(x^2 - 6x + 9 + y^2 + 8y + 16)$$

$$= 4x^2 - 24x + 36 + 4y^2 + 32y + 64$$

$$3x^2 - 30x + 3y^2 + 36y = -87$$

$$x^2 - 10x + y^2 + 12y = -29$$

$$(x-5)^2 + (y+6)^2 = -29 + 25 + 36$$

$$= 32$$

$\therefore$  Circle centre  $(5, -6)$  radius  $\sqrt{32}$

$$b) (i) x^2 - 8x + 66 = 16 - 8y + 16$$

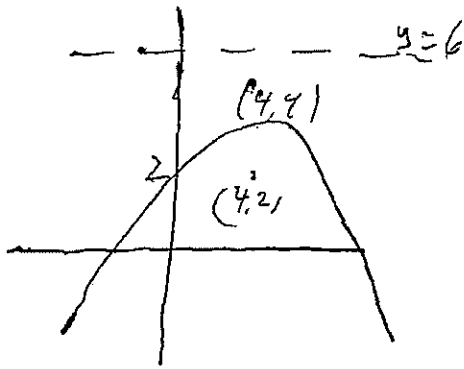
$$(x-4)^2 = -8(y-4)$$

$$(ii) V: (4, 4) \quad a = 2$$

$$F: (4, 2)$$

$$D: y = 6$$

(iii)



When  $x = 3$   $\frac{dy}{dx} = 6$   
 $\frac{d^2y}{dx^2} > 0$   
 $\therefore$  M. = T.P at  $(3, 1)$

(iv)  $6x - 12 = 0$   
 $x = 2$

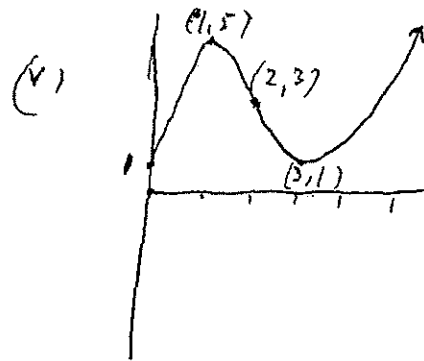
When  $x = 2$   $y = 3$

Test

$x$	1.5	2	2.5
$y''$	-2	0	3

- to + changes

$\therefore$  P of T at  $(2, 3)$



35 min

13. (a) (i)  $y' = 6x - \frac{4}{x^3}$

(ii)  $f(x) = (3x+2)^{1/3}$   
 $f'(x) = \frac{1}{3}(3x+2)^{-2/3} (3)$

$$= \frac{1}{\sqrt[3]{(3x+2)^2}}$$

(iii)  $y' = \frac{(5x+2)^3 - (3x-5)^5}{(5x+2)^2}$

$$= \frac{31}{(5x+2)^2}$$

(b) (i)  $\frac{dy}{dx} = 3x^2 - 12x + 9$

(ii)  $3x^2 - 12x + 9 = 0$   
 $x^2 - 4x + 3 = 0$   
 $(x-3)(x-1) = 0$   
 $x = 1, 3$

When  $x = 1$   $y = 5$

$x = 3$   $y = 1$

$\therefore$  SP's at  $(1, 5)$  and  $(3, 1)$

(iii)  $\frac{d^2y}{dx^2} = 6x - 12$

When  $x = 1$   $\frac{d^2y}{dx^2} = -6$   
 $< 0$

$\therefore$  Max T.P at  $(1, 5)$

$$14. a) (i) \int 3x^3 - x^2 dx$$

$$= \frac{3x^4}{4} - \frac{x^3}{3} + C$$

$$(ii) \int \frac{x + x^{\frac{1}{2}}}{x^{\frac{5}{2}}} dx$$

$$= \int x^{-\frac{3}{2}} + x^{-\frac{7}{2}} dx$$

$$= 3x^{\frac{1}{2}} - \frac{6}{5}x^{-\frac{1}{2}} + C$$

$$= 3\sqrt{x} - \frac{6}{5\sqrt{x}} + C$$

$$b) \frac{(3x+2)^4}{3+4} \Big|_0^1$$

$$\frac{(3x+2)^4}{12} \Big|_0^1$$

$$= \frac{5^4}{12} - \frac{2^4}{12}$$

$$= \frac{609}{12}$$

$$c) (i) \begin{array}{ccccc} x & 0 & 0.25 & 0.5 & 0.75 & 1 \\ y & 4 & 3.76 & 3.2 & 2.56 & 2 \end{array}$$

$$(ii) \frac{1}{12} (4+2 + 4(3.76) + 2(3.2) + 4(2.56))$$

$$= 3.1$$

$$(iii) \frac{dx}{dt} = 2t + c$$

$$\text{When } t=0 \quad \frac{dx}{dt} = 0 \therefore c=0$$

$$\frac{dx}{dt} = 2t$$

$$x = t^2 + K$$

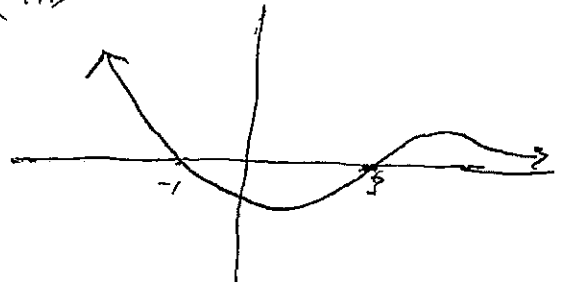
$$\text{When } t=0 \quad x=-1 \therefore K=-1$$

$$\therefore x = t^2 - 1$$

$$15. a) (i) -1 < x < 3$$

$$(ii) \text{ Approaches } \infty$$

$$(iii)$$



$$b) (i) x^2 - 2 = x$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, -1$$

$$(ii) \int_{-1}^2 x - (x^2 - 2) dx$$

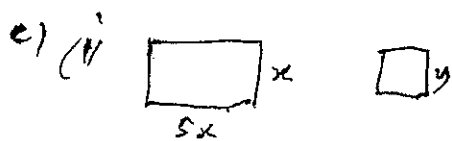
$$= \int_{-1}^2 x - x^2 + 2 dx$$

$$= \left[ \frac{x^2}{2} - \frac{x^3}{3} + 2x \right]_{-1}^2$$

$$= \left( \frac{2^2}{2} - \frac{2^3}{3} + 2(2) \right) - \left( \frac{(-1)^2}{2} - \frac{(-1)^3}{3} + 2(-1) \right)$$

$$= \left( 2 - \frac{8}{3} + 4 \right) - \left( \frac{1}{2} + \frac{1}{3} - 2 \right)$$

$$= \frac{9}{2} \text{ units}^2$$



$$12x + 4y = 60$$

$$3x + y = 15$$

$$y = 15 - 3x$$

$$\begin{aligned} S &= 5x^2 + y^2 \\ &= 5x^2 + (15 - 3x)^2 \\ &= 5x^2 + 225 - 90x + 9x^2 \end{aligned}$$

$$= 14x^2 - 90x + 225$$

(ii)  $S' = 28x - 90$

when  $S' = 0$

$$28x - 90 = 0$$

$$x = \frac{45}{14}$$

$S'' = 28$

$$> 0$$

$\therefore$  Minimum

$$S = 14\left(\frac{45}{14}\right)^2 - 90\left(\frac{45}{14}\right) + 225$$

$$= 80\frac{5}{14} \text{ cm}^2$$