2013
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

Name: ______________________ **Class: 12M___**

Teacher: **Mr Gong** **Mr Cheng** **Ms Strilakos**
Ms Nicolau **Ms Lobejko** **Ms Kellahan** **Mr White**

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write in blue or black pen.
- Pencil maybe used for diagrams only.
- Only Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks (100)

Section I

Total marks (**10**)

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II

Total marks (**90**)

- Attempt questions 11 – 16
- Answer in the booklet provided.
- Start a new booklet for each question
- All necessary working should be shown for every question
- Allow about 2 hours 45 minutes for this section

OUTCOME	MC	Q11	Q12	Q13	Q14	Q15	Q16	TOTAL
H1	/10							/10
H2						/7	/8	/15
H3		/6						/6
H4		/9	/5	/3				/17
H5			/10			/8	/7	/25
H6					/8			/8
H8					/7			/7
H9				/12				/12
TOTAL	/10	/15	/15	/15	/15	/15	/15	/100

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Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the **multiple choice answer sheet** at back of paper for Questions 1 – 10.

1. Convert the angle measurement 76° into radians correct to 3 significant figures.

   (A) 7.6

   (B) 1.33

   (C) 1.326

   (D) 0.422

2. Simplify the expression $\dfrac{x^2+4x+4}{x+2}$.

   (A) $x+2$

   (B) $\dfrac{x^2+2x+2}{x}$

   (C) $x-2$

   (D) $3x+4$

3. For the inequality $|2x-3| \le 4$ which solution is true?

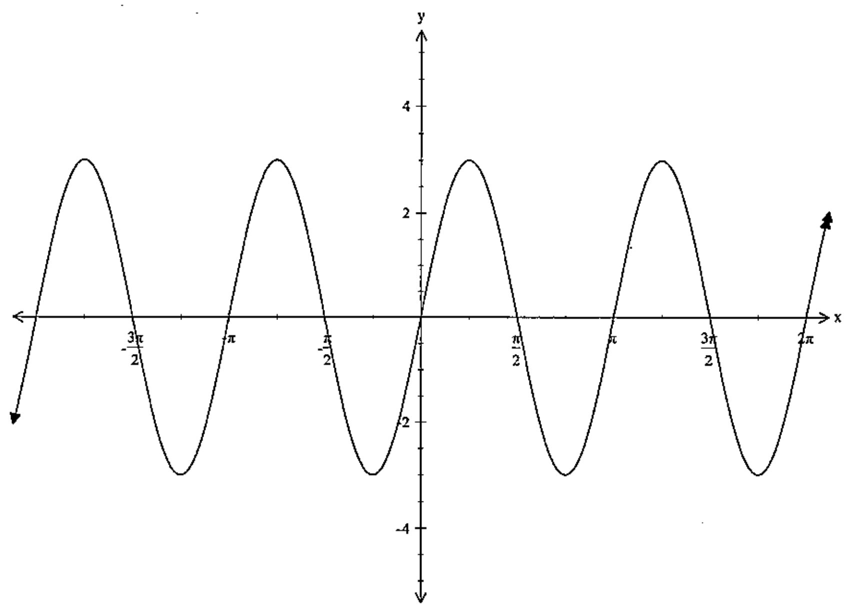
   (A) $\frac{1}{2} \le x \le 3\frac{1}{2}$

   (B) $x \le 3\frac{1}{2}$

   (C) $x \le -\frac{1}{2}$ or $x \ge 3\frac{1}{2}$

   (D) $-\frac{1}{2} \le x \le 3\frac{1}{2}$

4. Consider the graph of $f(x)$ shown:



Which of the following functions describes $f(x)$?

(A) $f(x) = 3\cos x$

(B) $f(x) = 3\cos 2x$

(C) $f(x) = 3\sin 2x$

(D) $f(x) = 3\sin x$

5. For the arithmetic series 3, 6, 9 … which expression could be used to evaluate S_{15}?

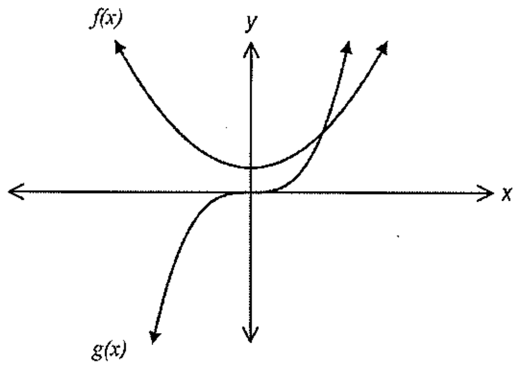
(A) $3 + 14(3)$

(B) $15[3 + 14(3)]$

(C) $\frac{15}{2}[3 + 14(3)]$

(D) $\frac{15}{2}[6 + 14(3)]$

6. Consider the functions $f(x)$ and $g(x)$ shown on the same pair of axes below:



Which statement is true?

(A) $f(x)$ and $g(x)$ are both even functions.

(B) $f(x)$ and $g(x)$ are both odd functions.

(C) $f(x)$ is an even function and $g(x)$ is an odd function.

(D) $f(x)$ and $g(x)$ are both neither odd nor even functions.

7. $\frac{d}{dx}(\sin 2x) =$

(A) $2 \sin 2x$

(B) $2 \cos 2x$

(C) $2 \tan 2x$

(D) $\frac{1}{2} \cos 2x$

8. It is known that for a particular quadratic function, $\alpha + \beta = -\frac{5}{3}$ and $\alpha\beta = \frac{7}{3}$.
The quadratic function could be:

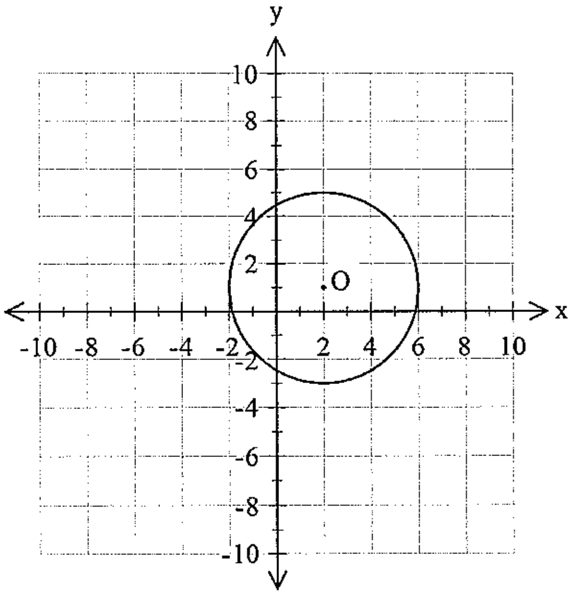
(A) $6x^2 + 10x + 14$

(B) $3x^2 - 5x + 7$

(C) $3x^2 + 5x - 7$

(D) $5x^2 - 7x + 3$

9. Which of the equations given below describes the circle centre O, shown?



(A) $x^2 - 4x + y^2 - 2y = 21$

(B) $x^2 + y^2 = 16$

(C) $(x-2)^2 + (y-1)^2 = 16$

(D) $(x+2)^2 + (y+1)^2 = 16$

10. The gradient of the normal to the curve $f(x) = 3x^3 - 4x + 2$ at the point (-1, 3) is:

(A) 5

(B) –5

(C) $-\frac{1}{5}$

(D) $-\frac{1}{3}$

End of Section I

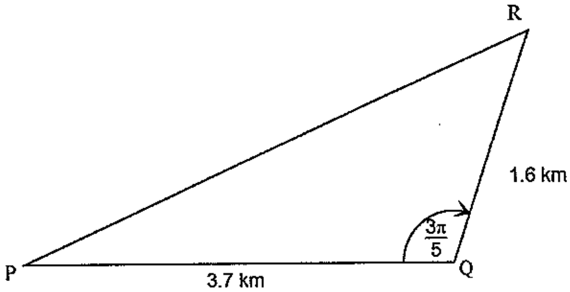
Section II

Total marks (90)
Attempt Questions 11-16
Start each question in a new booklet.
Allow about 2 hours 45 minutes for this section

Answer all questions, starting each question in a new booklet with your name and the question number on the front of the booklet. Do not work in columns.

Question 11 (15 marks) Use a new booklet. **Marks**

a) For triangle PQR:



i) Calculate its area correct to 2 decimal places. **1**

ii) Find the length of PR correct to 1 decimal place. **2**

iii) Find the size of $\angle R$, correct to 3 significant figures. **2**

b) For the function $y = 2\cos\frac{x}{3}$ state:

i) domain and range. **2**

ii) period and amplitude. **2**

c) Calculate the value of $\log_5 16$ correct to 2 decimal places. **2**

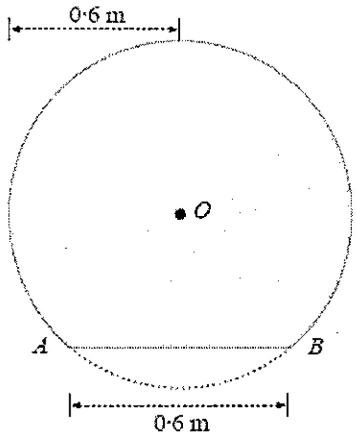
d) The limiting sum of a series $3 + x + x^2 + \ldots\ldots$ is 18. **2**
If $|x| < 1$, find the value of x.

e) Find the exact value of $\frac{d}{dx}\left(e^{3x^2+1}\right)$ when $x = 2$. **2**

End of Question 11

Question 12 (15 marks) Use a new booklet. **Marks**

a) Find the exact value of $\sin \dfrac{2\pi}{3}$ **1**

b) 

A table top is in the shape of a circle with a small segment removed as shown. The circle has a centre *O* and radius 0.6 metres. The length of the straight edge AB is also 0.6 metres.

i) Explain why $\angle AOB = \dfrac{\pi}{3}$ **1**

ii) Find the area of the table top. **3**

c) Differentiate with respect to *x* and simplify fully:

i) $\dfrac{\ln x}{x}$ **2**

ii) $5x\left(x^2 - 3\right)^7$ **3**

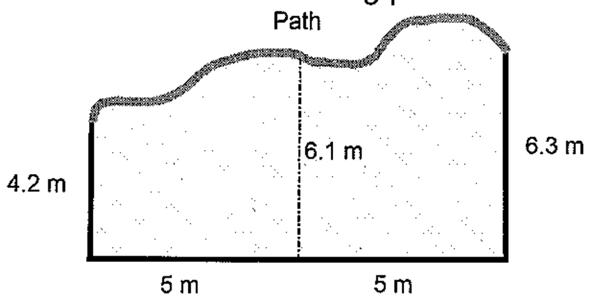
d) i) Find $\displaystyle\int 3x^2 + \sqrt{x}\ dx$ **2**

ii) Find $\displaystyle\int_0^2 \frac{5}{(x+2)^3}\,dx$ **3**

End of Question 12

Question 13 (15 marks) Use a new booklet. **Marks**

a) A man-made pond is planned for a recreation area. The pond will be enclosed by 3 straight concrete retainer walls and one irregular curved wall so that it can follow an existing path.



i) Use trapezoidal rule to approximate the area of the pond. **2**

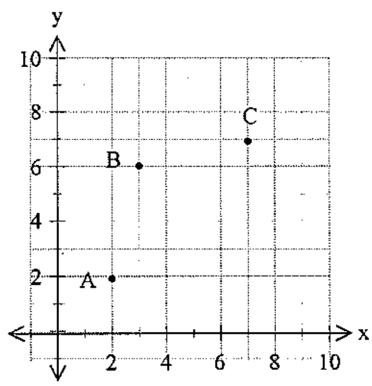
ii) The floor of the pond will be level and will allow a constant water depth of 1.1 metres. **1**

With water being pumped into the pond at a rate of 6.5 m^3 per hour, how long will it take to fill the pond from empty, correct to the nearest minute?

b) Sketch the region $y < |x + 2|$. **2**

c) The points A, B and C have coordinates (2, 2), (3, 6) and (7, 7) respectively.

Point D is a point on the number plane so that ABCD is a rhombus.



i) Show that the coordinates of point D are (6, 3) **1**

ii) Find the exact length of the diagonal AC. **1**

iii) Find the equation of the diagonal BD. **2**

iv) Explain the relationship between the gradients of AC and BD. **1**

v) Find the point of intersection of the diagonals AC and BD. **1**

vi) Find the area of ABCD. **2**

vii) If point E exists such that ABCE is a kite with twice the area of the rhombus ABCD, find the coordinates of E. **2**

End Question 13

Marks

Question 14 (15 marks) Use a new booklet.

a) A function is given by $f(x) = x^3 + 3x^2 - 9x + 3$.

i) $f(x)$ has 2 stationary points. One is at (1, -2). Find the coordinates of the other stationary point. **2**

ii) Determine the nature of both stationary points. **2**

iii) Show that the point (-1,14) is a point of inflexion. **2**

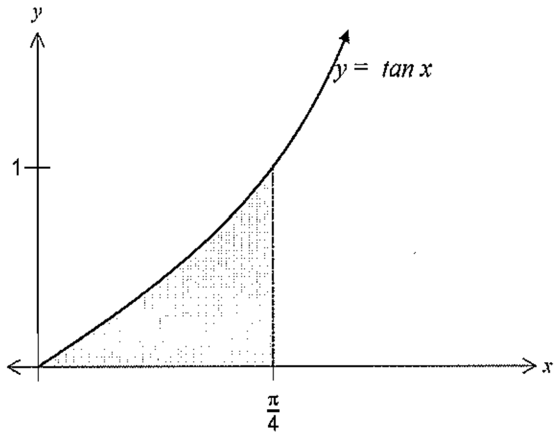
iv) Sketch $y = f(x)$ showing the stationary points and inflexion. **2**

b) An automatic air freshener dispenser releases a single spray every hour. The rate at which a single spray loses its scent over time is given by $\frac{dS}{dt} = -\frac{1}{2t+1}$, where S is the amount of scent still in the air at time t measured in hours.

i) Write an expression for S as a function of t given that a single spray contains 0.6 units of scent. **2**

ii) How long will it be before the scent from a single spray is completely gone? **2**

c) The region bounded by the curve $y = \tan x$ and the lines $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x - axis.



i) Show that the volume of the solid can be evaluated using the formula **1**

$$V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x - 1 \, dx .$$

ii) Evaluate the volume correct to 3 significant figures. **2**

End of Question 14

Question 15 (15 marks) Use a new booklet. **Marks**

a) At 7 pm on a Wednesday evening, Mr Morgan's water tank was full. The capacity of the tank was 3 000 litres. The tap on the tank was leaking such that the change in volume at any time (*t*) hours was proportional to the volume (*V*) of the tank. So, $\frac{dV}{dt} = -kV$.

i) Show that $V = V_0 e^{-kt}$ is a solution of this equation. **1**

ii) Given that the volume of the tank after 3 hours is 1 900 litres, show that $k = 0.1523$ correct to 4 decimal places. **2**

iii) By the time Mr Morgan discovered that the tank was leaking, there were only 250 litres of water remaining. At what time and on which day did Mr Morgan discover the leak (correct to the nearest minute)? **2**

b) The displacement of a particle at time (*t*) seconds is given by: **3**

$$x = 3e^{-2t} + 4e^{-t} + 2t\ .$$

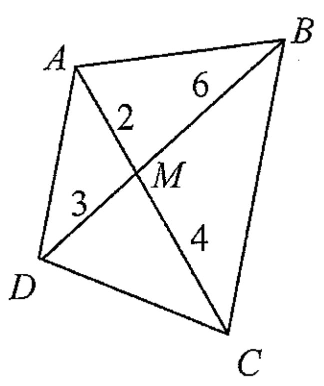
Find the exact time at which the particle comes to rest.

c) A point $P(x, y)$ moves such that its distance from the point (2, 3) and the line $y = -1$ is equal.

i) Describe the locus of the *P*. **1**

ii) Write the equation of the locus of *P*. **2**

d) In quadrilateral *ABCD*:



M is the point of intersection of the diagonals *AC* and *BD*.

The length of:
AM is 2 units
MC is 4 units
DM is 3 units
MB is 6 units

i) Show that $\Delta AMD \mathbin{|||} \Delta BMC$. **2**

ii) What type of quadrilateral is ABCD? Justify your answer. **2**

End of Question 15

Question 16 (15 marks) Use a new booklet. **Marks**

a) Find the shortest distance between the curve $y=x^2+3x+5$ and the line $y=3x-1$. **3**

b) Jack opened an investment account with an initial deposit of \$5000 on 1st February 2007. He did this so that he could provide his daughter with \$800 at the start of February each year for university text books. The account earned interest at a rate of 3% per annum, compounding annually.

The first \$800 withdrawal was made one year after the investment was set up.

i) Calculate the account balance immediately after the first withdrawal has been made. **1**

ii) Let A_n be the amount of money in the account after *n* years (when *n* withdrawals will have been made). **3**

Show that $A_n=\frac{1}{3}\left[80000-65000(1.03)^n\right]$.

iii) After withdrawing \$800 on 2nd February 2010, Jack's daughter advised him that she would need \$900 per year from 2011 onwards. **3**

How many more years of textbook fees can come out of the fund?

c) i) Show that $\frac{2}{x-a}+\frac{k}{x}+\frac{8}{x+a}=0$ can be expressed as a quadratic equation. **2**

ii) Hence find k if the roots are equal. **2**

iii) Using the larger of the value of k found in part (ii), find the root of the quadratic equation. **1**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Trial HSC Examination – Mathematics 2013

Section I – Multiple Choice Answer Sheet

Name______________________________.

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ☒ B ☒ C ○ D ○

Start Here ➔

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

CHS Mathematics Exams

2013
TRIAL HSC
EXAMINATION

Mathematics

SOLUTIONS

Trial HSC Examination – Mathematics 2013

Section I – Multiple Choice Answer Sheet

Name________________________________.

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	$2 + 4 =$	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

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correct

A ⊗ B ⊗ C ○ D ○

Start Here →

1.	A ○	B ●	C ○	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ○	C ●	D ○
5.	A ○	B ○	C ○	D ●
6.	A ○	B ○	C ●	D ○
7.	A ○	B ●	C ○	D ○
8.	A ●	B ○	C ○	D ○
9.	A ○	B ○	C ●	D ○
10.	A ○	B ○	C ●	D ○

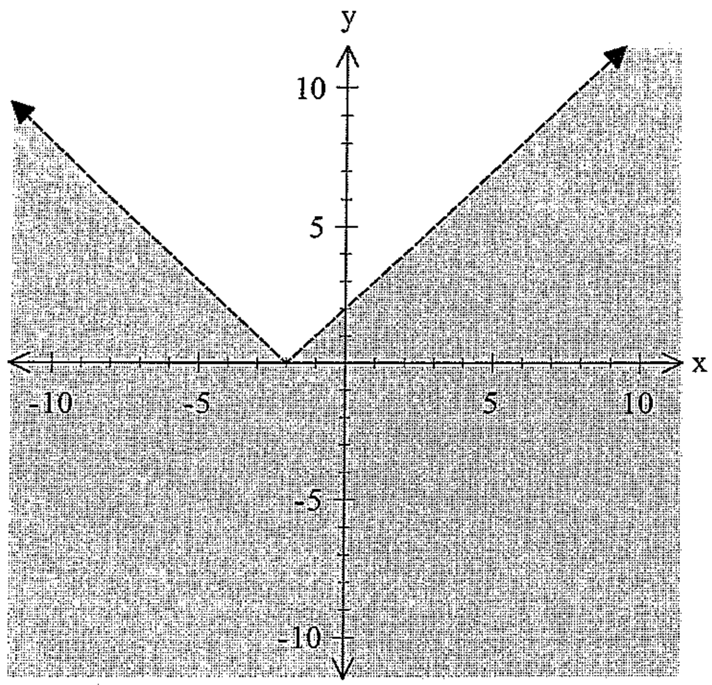
Question 11
Trial HSC Examination- Mathematics
2013

Part	Solution	Marks	Comment	outcome
(a)i	$Area = \frac{1}{2}ab\sin C$ $= \frac{1}{2} \times 3.7 \times 1.6 \times \sin\frac{3\pi}{5}$ $\approx 2.82\ \boldsymbol{km}^2$	 1		H4
(a) ii	$\overrightarrow{PR}^2 = 3.7^2 \times 1.6^2 - 2 \times 3.7 \times 1.6 \times \cos\frac{3\pi}{5}$ $= 19.90876121$ $\overrightarrow{PR} = \sqrt{19.90876121}$ $\approx 4.5\ \boldsymbol{km}$	1 1		H4
(a)iii	$\frac{\sin R}{3.7} = \frac{\sin\frac{3\pi}{5}}{4.461923488}$ $\sin R = 3.7 \times \frac{\sin\frac{3\pi}{5}}{4.461923488}$ $R = \sin^{-1}\left[3.7 \times \frac{\sin\frac{3\pi}{5}}{4.461923488}\right]$ ≈ 0.909 **radians**	1 1	Could use cosine rule instead. Could use approximated value of PR from part ii which gives an answer of 0.898 radians	H4
(b)	i) domain: all real x, range: $-2 \leq y \leq 2$ ii) period 6π; amplitude 2	2 2		H4

Question 11 Trial HSC Examination- Mathematics 2013				
Part	Solution	Marks	Comment	outcome
(c)	$\log_5 16 = \dfrac{\log_e 16}{\log_e 5}$ $\approx 1.72 \quad$ (2dp)	1 1		**H3**
(d)	$3 + x + x^2 + \ldots\ldots\ldots = 18$ $\therefore\ x + x^2 + \ldots\ldots\ldots = 15$ $a = x,\ r = x \qquad S_\infty = \dfrac{a}{1-r}$ $\dfrac{x}{1-x} = 15$ $x = 15 - 15x$ $16x = 15$ $x = \dfrac{15}{16}$	2	1 for setting up equation with limiting sum of 15. 1 for the correct value of x	H3
(e)	$\dfrac{d}{dx} e^{3x^2+1} = 6x\, e^{3x^2+1}$ when $x = 2,\ 6xe^{3x^2+1} = 12e^{13}$	1 1		H3
		/15		

Question 12	Trial HSC Examination- Mathematics	2013		
Part	**Solution**	**Marks**	**Comment**	**outcome**
(a)	$\sin\frac{2\pi}{3} = \sin(\pi - \frac{\pi}{3})$ $= \sin(\frac{\pi}{3})$ $= \frac{\sqrt{3}}{2}$	1		H4
(b) i)	Triangle AOB is equilateral therefore angle AOB is $\frac{\pi}{3}$	1		H4
ii)	Table top $= \pi r^2 - \frac{1}{2}r^2(\theta - \sin\theta)$ $= \pi(0.6)^2 - \frac{1}{2}(0.6)^2(\frac{\pi}{3} - \sin\frac{\pi}{3})$ $= 1.0983\ldots$ $= 1.1\text{m}^2$ (1dp)	2 1		H4
(c) i	Let $u = \ln x$ and $v = x$ then $u' = \frac{1}{x}$ and $v' = 1$ So, $\frac{vu' - uv'}{v^2} = \frac{x\left(\frac{1}{x}\right) - \ln x}{x^2}$ $= \frac{1 - \ln x}{x^2}$	1 1		H5
c) ii	let $u = 5x$ and $v = (x^2 - 3)^7$ then $u' = 5$ and $v' = 7(2x)(x^2 - 3)^6$ $= 14x(x^2 - 3)^6$ $vu' + uv' = 5(x^2 - 3)^7 + 5x(14x)(x^2 - 3)^6$ $= 5(x^2 - 3)^6[x^2 - 3 + 14x^2]$ $= 5(x^2 - 3)^6(15x^2 - 3)$ $= 15(x^2 - 3)^6(5x^2 - 1)$	1 1 1		H5

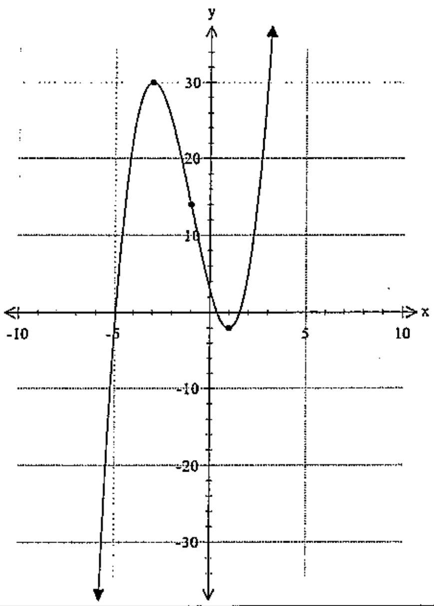
Question 12	Trial HSC Examination- Mathematics	2013		
Part	**Solution**	**Marks**	**Comment**	**outcome**
(d) i	$\int 3x^2 + \sqrt{x}\ dx = \int 3x^2 + x^{\frac{1}{2}}\, dx$			H5
	$= x^3 + \frac{2x^{\frac{3}{2}}}{3}$	1		
	$= x^3 + \frac{2\sqrt{x^3}}{3} + C$	1		
d) ii	$\int_0^2 \frac{5}{(x+2)^3}\, dx = 5\int_0^2 (x+2)^{-3}$	1		H5
	$= 5\left[\frac{(x+2)^{-2}}{-2}\right]_0^2$	1		
	$= -\frac{5}{2}\left[\frac{1}{(x+2)^2}\right]_0^2$			
	$= -\frac{5}{2}\left[\frac{1}{16} - \frac{1}{4}\right]$			
	$= \frac{15}{32}$	1		
		/15		

Question 13	Trial HSC Examination- Mathematics	2013		
Part	Solution	Marks	Comment	Outcome
(a)i	Area $= \frac{5}{2}(4.2 + 2 \times 6.1 + 6.3)$ $= 56.75\ m^2$	1 1		H4
(a) ii	Volume $= 56.75 \times 1.1$ $= 62.425\ \text{m}^3$ $62.425 \div 6.5 = 9$ hours 36 minutes.	1		H4
(b)		1 1	correct function graph correct region	H9
(c)i	By inspection, D has coordinates (6,3)	1		H9
(c) ii	$\overrightarrow{AC} = \sqrt{(7-2)^2 + (7-2)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}\ units$	1		H9
(c) iii	BD has $m = \frac{3-6}{6-3}$ $= -1$ $y - 6 = -1(x-3)$ $x + y - 9 = 0$	2	1 gradient 1 equation	H9
(c) iv	Diagonals in a rhombus intersect at right angles. Since they are perpendicular, the product of their gradients is -1.	1		H9

Question 13	Trial HSC Examination- Mathematics	2013		
Part	Solution	Marks	Comment	Outcome
(c) v	Diagonals in a rhombus bisect. Finding the midpoint of either diagonal will give us the point of intersection of both. $M=\left(\frac{7+2}{2},\frac{7+2}{2}\right)$ $=(4.5,4.5)$	1		H9
(c) vi	*Area of a rhombus* $=\frac{1}{2}$ *the product of the diagonals* $\overrightarrow{AC}=5\sqrt{2}$ $\overrightarrow{BD}=\sqrt{(6-3)^2+(3-6)^2}$ $=\sqrt{18}$ $=3\sqrt{2}$ *units* $Area=\frac{1}{2}\times 5\sqrt{2}\times 3\sqrt{2}$ $=15$ *units*^{2}	1 1		H9

Question 13	Trial HSC Examination- Mathematics	2013		
Part	**Solution**	**Marks**	**Comment**	**Outcome**
(c)vii	For the area of ABCE to be double the area of ABCD, the length BE must be twice the length BD, since area of a kite is also $A=\frac{1}{2}xy$. So, $\overrightarrow{BE}=6\sqrt{2}$ Using distance $\overrightarrow{BE}^2=(x-3)^2+(y-6)^2$ $\therefore\quad (6\sqrt{2})^2=(x-3)^2+(y-6)^2$ formula $(x-3)^2+(y-6)^2=72$ ① We also know that E must lie on the line BD since the diagonals of a kite must bisect at right angles. So E must satisfy the equation of BD. $x+y-9=0$ ② $y=9-x$ *rearranging* ② $\therefore\quad (x-3)^2+(9-x-6)^2=72$ *substituting* ② *into* ① $(x-3)^2+(3-x)^2=72$ $x^2-6x+9+9-6x+x^2=72$ $2x^2-12x-54=0$ $x^2-6x-27=0$ $(x-9)(x+3)=0$ $x=9$ *or* $x=-3$ *when* $x=9$, $y=9-9=0:E=(9,0)$ *when* $x=-3$, $y=9+3=12:E=(-3,12)$	1 1	1 mark for equation (1) 1 mark for finding a correct point.	
		/15		

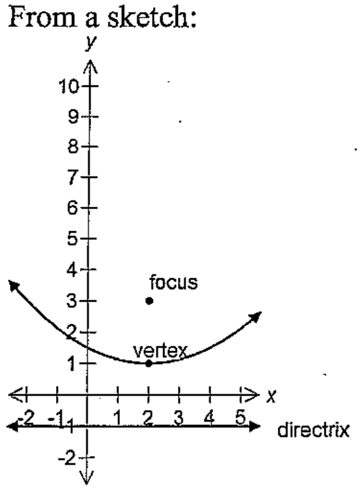
Question 14	Trial HSC Examination- Mathematics	2013		
Part	Solution	Marks	Comment	Outcome
(a) i	$f(x) = x^3 + 3x^2 - 9x + 3$ Stationary points exist when $f'(x) = 0$. Let $f'(x) = 3x^2 + 6x - 9$ $3x^2 + 6x - 9 = 0$ $3(x^2 + 2x - 3) = 0$ $3(x + 3)(x - 1) = 0$ $\therefore \quad x = -3 \text{ or } x = 1$ We know that there is a stationary point at (1, -2) $When\ x = -3,\ y = 3^3 + 3(3)^2 - 9(3) + 3$ $= 30$ So the other stationary point is at (-3, 30).	1 1		H8
a) ii	To determine nature of stationary points we can look at $f''(x)$. $f''(x) = 6x + 6$ At the point (1, -2): $f''(x) = 6(1) + 6$ $= 12$ > 0 $\therefore minimum\ turning\ point$ At the point (-3, 30): $f''(x) = 6(-3) + 6$ $= -12$ < 0 $\therefore maximum\ turning\ point$	1 1		H8

Question 14	Trial HSC Examination- Mathematics	2013		
Part	Solution	Marks	Comment	Outcome
aiii	Possible inflexion at $f''(x) = 0$ $let\ 6x + 6 = 0$ $6x = -6$ $x = -1$ Sub $x = -1$ into $f(x)$ $y = 14$ Test concavity either side of $x = -1$ When $x = 0$, $f''(x) = 6(0)+6$ $= 6$ >0 When $x = -2$, $f''(x) = 6(-2)+6$ $= -12+6$ $= -6$ <0 $\therefore$ Concavity changes and there is an inflexion at the point $(-1,14)$.	1 1	concavity test	H8
aiv		1 1	for correct direction marking correct points	H8

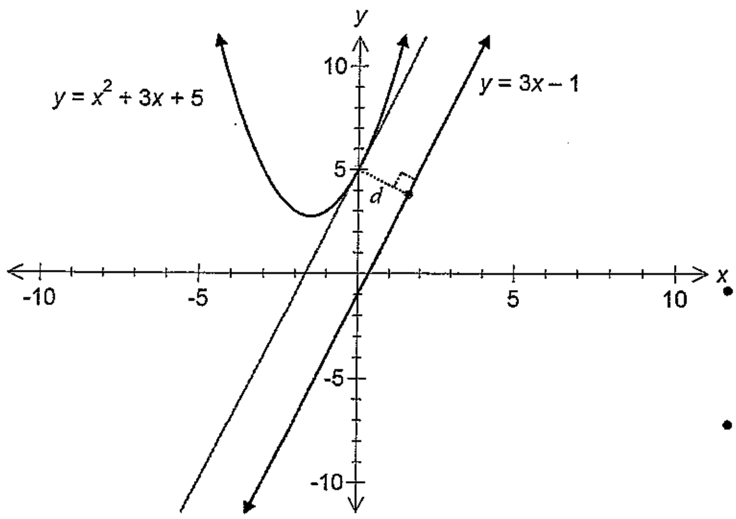
Question 14	Trial HSC Examination- Mathematics	2013		
Part	**Solution**	**Marks**	**Comment**	**Outcome**
(b)i	$\frac{dS}{dt} = -\frac{1}{2t+1}$ $S = \int -\frac{1}{2t+1}\,dt$ $= -\frac{1}{2}\ln(2t+1) + C$ When $t = 0$, S = 0.6 units $\therefore\ 0.6 = -\frac{1}{2}\ln(2(0)+1) + C =$ $0.6 = -\frac{1}{2}\ln 1 + C$ $0.6 = 0 + C$ $C = 0.6$ $\therefore\ S = 0.6 - \frac{1}{2}\ln(2t+1)$	 1 1		H6
(b) ii	Scent will be gone when S = 0. $0.6 - \frac{1}{2}\ln(2t+1) = 0$ $0.6 = \frac{1}{2}\ln(2t+1)$ $1.2 = \ln(2t+1)$ $2t + 1 = e^{1.2}$ $2t = e^{1.2} - 1$ $t = \frac{e^{1.2} - 1}{2}$ ≈ 1.16 *hours or* 1 *hour and 10 minutes*	 1 1		H6

Question 14	Trial HSC Examination- Mathematics	2013		
Part	**Solution**	**Marks**	**Comment**	**Outcome**
(c) i	Volume of a solid rotated about the x-axis is given by $V=\pi\int y^2\,dx$ Since $y=\tan x$, $y^2=\tan^2 x$. We know that: $\sec^2 x = 1 + \tan^2 x$ $\therefore\ \tan^2 x = \sec^2 x - 1$ So: $V=\pi\int y^2\,dx$ $=\pi\int_0^{\frac{\pi}{4}} \sec^2 x - 1\,dx$	1	using trig identity	H8
(c) ii	$V=\pi\int_0^{\frac{\pi}{4}} \sec^2 x - 1\,dx$ $=\pi\left[\tan x - x\right]_0^{\frac{\pi}{4}}$ $=\pi\left[\left(1-\frac{\pi}{4}\right)-(0-0)\right]$ $=\pi-\frac{\pi^2}{4}$ ≈ 0.674 *cubic units*	1 1	integration	H8

Question 15	Trial HSC Examination- Mathematics	2013	
Part	**Solution**	**Marks**	**Comment Outcome**
(a)i	If $V = V_0 e^{-kt}$ then by differentiation: $\frac{dv}{dt} = -kV_0 e^{-kt}$ But $V_0 e^{-kt}$ can be replaced with v, giving: $\frac{dV}{dt} = -kV$ $\therefore V = V_0 e^{-kt}$ is a solution of the equation.	1	H5
(a)ii	$V = V_0 e^{-kt}$ $1900 = 3000e^{-3k}$ $\frac{1900}{3000} = e^{-3k}$ $-3\mathbf{k} = \ln\left(\frac{1900}{3000}\right)$ $\mathbf{k} = \frac{\ln\left(\frac{1900}{3000}\right)}{-3}$ ≈ 0.1523	1 1	H5
(a)iii	$250 = 3000\, e^{-0.1523\, t}$ $\frac{250}{3000} = e^{-0.1523\, t}$ $-0.1523\, t = \ln\left(\frac{250}{3000}\right)$ $t = \frac{\ln\left(\frac{250}{3000}\right)}{-0.1523}$ $\approx$ 16 *hours and 19 minutes* *So Mr Morgan discovered the leak at 11:19 am on Thursday*	1 1	H5

Question 15	Trial HSC Examination- Mathematics	2013	
Part	Solution	Marks	Comment Outcome
b)	$$x = 3e^{-2t} + 4e^{-t} + 2t$$ *The particle will be at rest when* $\dot{x}$ *equals zero.* $$\dot{x} = -6e^{-2t} - 4e^{-t} + 2$$ $Let\ -6e^{-2t} - 4e^{-t} + 2t = 0$ $3e^{-2t} + 2e^{-t} - t = 0$ $\frac{(3e^{-t}+3)(3e^{-t}-1)}{3} = 0$ $(e^{-t}+1)(3e^{-t}-1) = 0$ $e^{-t} = -1$ *has no solution* *or* $3e^{-t} = 1$ $e^{-t} = \frac{1}{3}$ $t = -\ln\frac{1}{3}$ $t = \ln 3$	1 1 1	H5
(c)i	The locus of point P is a parabola with focus (2, 3) and directrix y=-1.	1	H5
(c)ii	From a sketch:  we can see that the parabola will be concave up. Therefore it is of the form: $(x-h)^2 = 4a(y-k)$ The focal length (a) is 2 units. The coordinates of the vertex (h,k) are (2, 1). ∴The equation of the locus is $(x-2)^2 = 8(y-1)$	1 1	H2 Or general form

Question 15	Trial HSC Examination- Mathematics	2013	
Part	Solution	Marks	Comment Outcome
(d)i	*In* ΔAMD *and* ΔBMC: $\angle AMD = \angle BMC$ (*vertically opposite angles are equal*) $\frac{AM}{MC} = \frac{2}{4} = \frac{1}{2}$ $\frac{MD}{BM} = \frac{3}{6} = \frac{1}{2}$ $\therefore \frac{AM}{MC} = \frac{MD}{BM}$ $\therefore \Delta AMD \parallel\mid \Delta BMC$ (*two pairs of corresponding sides in proportion and included angles equal*)	1 1	H2
(d) ii	*Since* $\Delta AMD \parallel\mid \Delta BMC$, $\angle DAM = \angle BCM$ (*corresponding angles in similar triangles equal*) $\therefore$ AD || BC (*alternate angles on parallel lines are equal*) *It can be shown that AB is not parallel to DC since alternate angles on those lines are not equal.* $\therefore$ *ABCD is a trapezium (one pair of parallel opposite sides)*	1 1	H2

Question 16	Trial HSC Examination- Mathematics	2013	
Part	Solution	Marks	Comment Outcome
(a)	The shortest distance between the curve and the line will be the perpendicular distance at a point where the the tangent to the curve is parallel to the line.  The gradient of the line $y = 3x - 1$ is 3. The gradient function for the curve $y = x^2 + 3x + 5$ is $\frac{dy}{dx} = 2x + 3$ We need $\frac{dy}{dx} = 3$. So: $2x + 3 = 3$ $2x = 0$ $x = 0$ *when* $x = 0$, $y = 5$ $\therefore$ The curve's tangent is parallel to the line at the point (0,5) on the curve.	1 1	H2

Question 16	Trial HSC Examination- Mathematics		2013
Part	**Solution**	**Marks**	**Comment Outcome**
(a) cont'd	Using perpendicular distance formula: $d = \left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\rvert$ $= \left\lvert \frac{3 \times 0 + -1 \times 5 - 1}{\sqrt{3^2 + (-1)^2}} \right\rvert$ $= \left\lvert \frac{-6}{\sqrt{10}} \right\rvert$ $= \frac{6}{\sqrt{10}}$ $= \frac{3\sqrt{10}}{5}$ *units*	1	
(b)i	$A_1 = 5000 \times 1.03 - 800$ $= \$4350$	1	H5
(b) ii	$A_2 = A_1 \times 1.03 - 800$ $= (5000(1.03) - 800) \times 1.03 - 800$ $= 5000(1.03)^2 - 800(1.03) - 800$ $A_3 = A_2 \times 1.03 - 800$ $= 5000(1.03)^3 - 800(1.03)^2 - 800(1.03) - 800$ $A_n = 5000(1.03)^n - 800\left(1 + 1.03 + 1.03^2 + \ldots + 1.03^{n-1}\right)$ $1 + 1.03 + 1.03^2 + \ldots + 1.03^{n-1}$ is a geometric series with $a = 1, r = 1.03$ and n terms. $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$ $= \frac{1\left(1.03^n - 1\right)}{1.03 - 1}$ $= \frac{1.03^n - 1}{0.03}$ $= \frac{100\left(1.03^n - 1\right)}{3}$ (continues next page)	1	H5

Question 16	Trial HSC Examination- Mathematics	2013	
Part	**Solution**	**Marks**	**Comment Outcome**
	$\therefore A_n = 5000(1.03)^n - 800\left[\frac{100(1.03^n - 1)}{3}\right]$ $= \frac{1}{3}\left[15000(1.03)^n - 80000(1.03)^n + 80000\right]$ $= \frac{1}{3}\left[80000 - 65000(1.03)^n\right]$	1 1	
(b)iii	At the end of 3 years (after the 3rd withdrawal) the amount of money in the account is: $A_3 = \frac{1}{3}\left[80000 - 65000(1.03)^3\right]$ $= \$2990.915$ With the same interest rate, but \$900 withdrawals, A_n changes to $A_n = 2990.915(1.03)^n - 900\left(\frac{100(1.03)^n - 1}{3}\right)$ $= \frac{1}{3}\left[8972.745(1.03)^n - 90000(1.03)^n + 90000\right]$ $= 30000 - 274009.085(1.03)^n$ To find when the fund will run out, we look at when $A_n = 0$ $30000 - 27009.085(1.03)^n = 0$ $30000 = 27009.085(1.03)^n$ $\frac{30000}{27009.085} = 1.03^n$ $n = \log_{1.03}\left(\frac{30000}{27009.085}\right)$ ≈ 3.55 *years* So there will only be 3 more years in which the full \$900 can be withdrawn.	1 1 1	H5

Question 16	Trial HSC Examination- Mathematics	2013	
Part	Solution	Marks	Comment Outcome
(c) i	$\frac{2}{x-a}+\frac{k}{x}+\frac{8}{x+a}=0$ $\frac{2x(x+a)+k(x-a)(x+a)+8x(x-a)}{x(x-a)(x+a)}=0$ $\frac{2x^2+2ax+kx^2-ka^2+8x^2-8ax}{x(x-a)(x+a)}=0$ $\frac{(10+k)x^2+(-6a)x-(ka^2)}{x(x-a)(x+a)}=0$ $(10+k)x^2+(-6a)x-(ka^2)=0$	1 1	H2
(c) ii	For the roots to be equal, $\Delta=0$, i.e., $36a^2+4(10+k)(ka^2)=0 \qquad \div 4a^2$ $9+(10+k)k=0$ $k^2+10k+9=0$ $(k+9)(k+1)=0$ $k=-1,-9$	1 1	H2
(c) iii	Using, $k=-1$, the larger value: $(10+k)x^2+(-6a)x-(ka^2)=0$ $(10-1)x^2+(-6a)x-(-1a^2)=0$ $9x^2-6ax+a^2=0$ $(3x-a)(3x-a)=0$ $\therefore x=\frac{a}{3},\frac{a}{3}$	 1	H2
		/15	