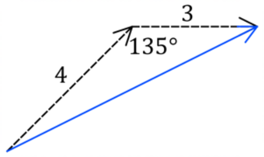
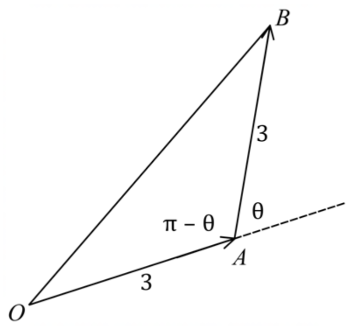
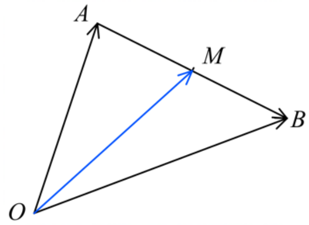
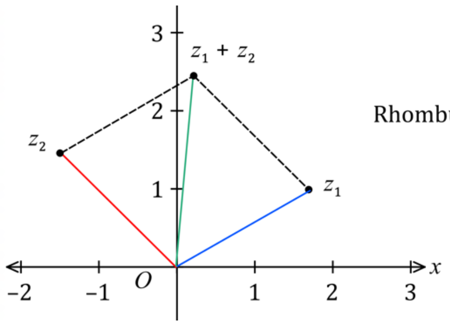
ACE Examination Paper 2
Year 12 Mathematics Extension 2 Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$zw = (3+4i)(1-i)$ $= 3 - 3i + 4i - 4i^2$ $= 7 + i$	1 Mark: D
2	$F_1 + F_2 + F_3 = 0$ $(\underset{\sim}{i} - 2\underset{\sim}{j}) + (3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}) + F_3 = 0$ $F_3 = -4\underset{\sim}{i} - \underset{\sim}{k}$	1 Mark: A
3	$\int \frac{1}{x^2 - 6x + 13}\,dx = \int \frac{dx}{(x-3)^2 + 2^2}$ $= \frac{1}{2}\tan^{-1}\frac{(x-3)}{2} + C$	1 Mark: C
4	Roots are $3+i, 3-i, \alpha$ and β Product of the roots $(3+i)(3-i)\alpha\beta = \frac{d}{a} = \frac{20}{1}$ $(9 - i^2)\alpha\beta = 20$ $10\alpha\beta = 20$ $\alpha\beta = 2$ (1) Sum of the roots $(3+i) + (3-i) + \alpha + \beta = -\frac{b}{a} = -\frac{-4}{1} = 4$ $\alpha + \beta = -2$ $\alpha = -2 - \beta$ (2) Substituting equation (2) into equation (1) $(-2-\beta)\beta = 2$ $\beta^2 + 2\beta + 2 = 0$ $\beta = -1 \pm i$ Hence $P(z) = [z - (-1+i)][z - (-1-i)][z - (3+i)][z - (3-i)\}$ $= (z^2 + 2z + 2)(z^2 - 6z + 10)$	1 Mark: B
5	Force $= \sqrt{3^2 + 4^2 - 2 \times 3 \times 4 \times \cos 135^\circ}$ $= \sqrt{25 + 12\sqrt{2}}$ 	1 Mark: B

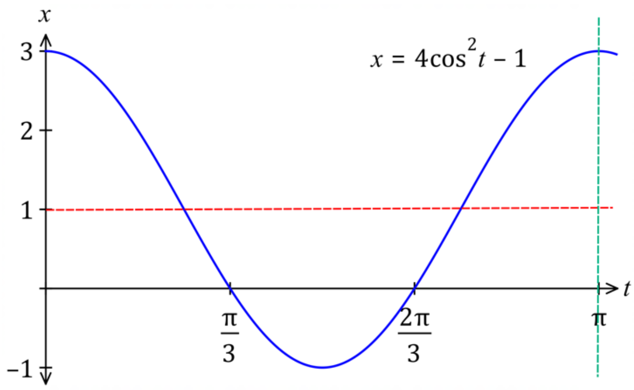
6	$z^{-1} = \frac{1}{2-3i} \times \frac{2+3i}{2+3i}$ $= \frac{2+3i}{4+9}$ $= \frac{1}{13}(2+3i)$	1 Mark: C
7	Let $u = 5 + \sin x$ $\frac{du}{dx} = \cos x$ $\int \frac{\sin x \cos x}{5+\sin x}\,dx = \int \frac{(u-5)\cos x}{u} \times \frac{du}{\cos x}$ $= \int 1 - \frac{5}{u}\,du$ $= u - 5\ln\lvert u\rvert + C$ $= 5 + \sin x - 5\ln\lvert 5+\sin x\rvert + C$ $= \sin x - 5\ln\lvert 5+\sin x\rvert + C$	1 Mark: D
8	$9n^2 - 4 = (3n+2)(3n-2)$ $n = 1 \quad 5 = 5 \times 1$ $n = 2 \quad 32 = 8 \times 4$ $n = 3 \quad 77 = 11 \times 7$ $n = 4 \quad 140 = 14 \times 10$ $\therefore$ There is only 1 value of n (n = 1) making $9n^2 - 4$ prime.	1 Mark: B
9	$v = f(x)$ $a = v\frac{dv}{dx}$ $= f(x)f'(x)$	1 Mark: D
10	$\lvert\overrightarrow{QR}\rvert = \frac{1}{2}\lvert\overrightarrow{PQ}\rvert$ (Q divides PR into a ratio of 2 : 1) $\underset{\sim}{r} - \underset{\sim}{q} = \frac{1}{2}(\underset{\sim}{q} - \underset{\sim}{p})$ $\underset{\sim}{r} = \frac{1}{2}(\underset{\sim}{q} - \underset{\sim}{p}) + \underset{\sim}{q}$ $= \frac{3}{2}\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p}$	1 Mark: A

Section II		
	Solution	**Criteria**
11(a)(i)	$x^4+x^2-12=(x^2+4)(x^2-3)$	1 Mark: Correct answer.
11(a)(ii)	$x^4+x^2-12=(x^2+4)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$	1 Mark: Correct answer.
11(a)(iii)	$x^4+x^2-12=(x+2i)(x-2i)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$	1 Mark: Correct answer.
11(b)	Using De Moivre's theorem $z^n=\cos n\theta+i\sin n\theta \quad$ for $n=1,2,3,\ldots$ $z^6=\cos\left(6\times\frac{\pi}{6}\right)+i\sin\left(6\times\frac{\pi}{6}\right)$ $=\cos\pi+i\sin\pi$ $=-1$	2 Marks: Correct answer. 1 Mark: Uses De Moivre's theorem
11(c)(i)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ $=\left(-\underset{\sim}{i}+3\underset{\sim}{j}+4\underset{\sim}{k}\right)-\left(\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k}\right)$ $=-2\underset{\sim}{i}+\underset{\sim}{j}+2\underset{\sim}{k}$	1 Mark: Correct answer.
11(c)(ii)	$\left\|\overrightarrow{OA}\right\|=\sqrt{1^2+2^2+2^2}$ $=3$ $\left\|\overrightarrow{AB}\right\|=\sqrt{(-2)^2+1^2+2^2}$ $=3$ $\overrightarrow{OA}\cdot\overrightarrow{AB}=\left(1\times(-2)\right)+(2\times 1)+(2\times 2)$ $=4$ $\cos\theta=\frac{\overrightarrow{OA}\cdot\overrightarrow{AB}}{\left\|\overrightarrow{OA}\right\|\left\|\overrightarrow{AB}\right\|}=\frac{4}{3\times 3}$ $=\frac{4}{9}$	3 Marks: Correct answer. 2 Marks: Finds $\overrightarrow{OA}\cdot\overrightarrow{AB}$. 1 Mark: Finds $\left\|\overrightarrow{OA}\right\|$ and $\left\|\overrightarrow{AB}\right\|$.
11(c)(iii)	$\left\|\overrightarrow{OB}\right\|=\sqrt{(-1)^2+3^2+4^2}$ $=\sqrt{26}$ In ΔOAB $\overrightarrow{OB}$ is the longest side. $\cos\theta=\frac{4}{9}$ $\sin\theta=\sqrt{1-\cos^2\theta}$ $=\sqrt{1-\left(\frac{4}{9}\right)^2}=\frac{\sqrt{65}}{9}$ $A=\frac{1}{2}ab\sin\theta$ $=\frac{1}{2}\times 3\times 3\times\sin(\pi-\theta)=\frac{9}{2}\sin\theta$ $=\frac{9}{2}\times\frac{\sqrt{65}}{9}$ $=\frac{\sqrt{65}}{2}$ square units  	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Shows some understanding.

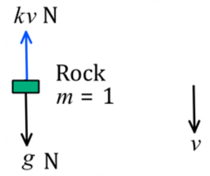
11(d)	Let $u = x, du = dx$ and $v = e^x, dv = e^x dx$ $\int xe^x\,dx = xe^x - \int e^x dx$ $= xe^x - e^x + C$	2 Marks: Correct answer. 1 Mark: Correctly applies integration by parts.
12(a) (i)	$i\bar{z} = \overline{\imath(3-\imath)}$ $= \overline{3\imath + 1}$ $= 1 - 3i$	1 Mark: Correct answer.
12(a) (ii)	$\frac{1}{z} = \frac{1}{3-i} \times \frac{3+i}{3+i}$ $= \frac{3+i}{9+i}$ $= \frac{3+i}{10}$	1 Mark: Correct answer.
12(b) (i)	$\underset{\sim}{u}.\underset{\sim}{v} = (5 \times (-2)) + (1 \times (-2)) + (3 \times 1)$ $= -9$ $\therefore$ Scalar product is –9.	1 Mark: Correct answer.
12(b) (ii)	Unit vector of $\underset{\sim}{v} = -2\underset{\sim}{\imath} - 2\underset{\sim}{\jmath} + \underset{\sim}{k}$ $\hat{\underset{\sim}{v}} = \frac{\underset{\sim}{v}}{\lvert\underset{\sim}{v}\rvert}$ $= \frac{-2\underset{\sim}{\imath} - 2\underset{\sim}{\jmath} + \underset{\sim}{k}}{\sqrt{(-2)^2 + (-2)^2 + 1}}$ $= \frac{1}{3}(-2\underset{\sim}{\imath} - 2\underset{\sim}{\jmath} + \underset{\sim}{k})$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
12(c) (i)	$v = 10 - x$ $v^2 = 100 - 20x + x^2$ $\frac{1}{2}v^2 = 50 - 10x + \frac{1}{2}x^2$ $a = \frac{d}{dx}\left(50 - 10x + \frac{1}{2}x^2\right)$ $= x - 10$	1 Mark: Correct answer.
12(c) (ii)	$\frac{dx}{dt} = 10 - x$ $\frac{dt}{dx} = \frac{1}{10-x}$ $t = -\ln(10-x) + C$ Initially $t = 0$ and $x = 0$ $0 = -\ln(10-0) + C$ $C = \ln 10$ $t = -\ln(10-x) + \ln 10$ $= \ln\left(\frac{10}{10-x}\right)$ $e^t = \frac{10}{10-x}$ $e^{-t} = \frac{10-x}{10}$ $x = 10 - 10e^{-t}$	2 Marks: Correct answer. 1 Mark: Finds t in terms of x.

12(c) (iii)	$\lim\limits_{t\to\infty}(10-10e^{-t}) = 10 - 10\times 0$ $= 10$ metres to the right	1 Mark: Correct answer.
12(d)	 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= (5\underset{\sim}{i} + \underset{\sim}{j} + 7\underset{\sim}{k}) - (2\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k})$ $= (3\underset{\sim}{i} + 5\underset{\sim}{j} + 2\underset{\sim}{k})$ Therefore $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$ $= (2\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}) + \frac{1}{2}(3\underset{\sim}{i} + 5\underset{\sim}{j} + 2\underset{\sim}{k})$ $= \frac{1}{2}(7\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k})$	2 Marks: Correct answer. 1 Mark: Finds $\overrightarrow{AB}$ or shows some understanding.
12(e) (i)	$z_1 = \sqrt{3} + i$ $= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$ $= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $z_2 = -\sqrt{2} + \sqrt{2}i$ $= 2\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$ $= 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$	2 Marks: Correct answer. 1 Mark: Finds z_1 or z_2 or shows some understanding.
12(e) (ii)	 Rhombus is formed $\arg\left(\frac{z_2}{z_1}\right) - \arg(z_1 + z_2) = (\arg z_2 - \arg z_1) - \frac{1}{2}(\arg z_1 + \arg z_2)$ $= \left(\frac{3\pi}{4} - \frac{\pi}{6}\right) - \frac{1}{2}\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)$ $= \frac{\pi}{8}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

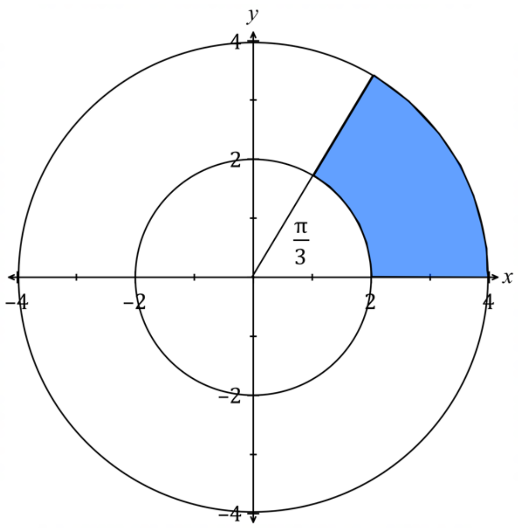
13(a) (i)	$\dfrac{5x^2-3x+1}{(x^2+1)(x-2)} = \dfrac{Ax+1}{(x^2+1)} + \dfrac{B}{(x-2)}$ Using partial fractions to find A and B $(Ax+1)(x-2) + B(x^2+1) = 5x^2-3x+1$ $(A+B)x^2 + (-2A+1)x + (-2+B) = 5x^2-3x+1$ $-2A+1=-3$ $A=2$ $-2+B=1$ $B=3$ $\therefore A=2$ and $B=3$	2 Marks: Correct answer. 1 Mark: Makes progress in finding A or B.
13(a) (ii)	$\displaystyle\int \frac{5x^2-3x+1}{(x^2+1)(x-2)}dx = \int \frac{2x+1}{(x^2+1)} + \frac{3}{(x-2)}dx$ $\displaystyle = \int \frac{2x}{(x^2+1)} + \frac{1}{(x^2+1)} + \frac{3}{(x-2)}dx$ $= \ln(x^2+1) + \tan^{-1}x + 3\ln\lvert x-2\rvert + C$	2 Marks: Correct answer. 1 Mark: Correctly finds one of the integrals.
13(b)	Step 1: To prove true for $n = 3$ $\dfrac{1}{3!} < \dfrac{1}{2^{3-1}}$ or $\dfrac{1}{6} < \dfrac{1}{4}$ Result is true for $n = 3$ Step 2: Assume true for $n = k$ $\dfrac{1}{k!} < \dfrac{1}{2^{k-1}}$ Step 3: To prove true for $n = k + 1$ $\dfrac{1}{(k+1)!} < \dfrac{1}{2^{(k+1)-1}} < \dfrac{1}{2^k}$ $\text{LHS} = \dfrac{1}{(k+1)!}$ $= \dfrac{1}{(k+1)k!}$ $< \dfrac{1}{(k+1)}\dfrac{1}{2^{k-1}}$ (assumption for $n = k$) $< \dfrac{1}{2} \times \dfrac{1}{2^{k-1}}$ ($k+1 > 2$ as $n \geq 3$) $= \dfrac{1}{2^k}$ $= \text{RHS}$ Step 4: True by induction	3 marks: Correct answer. 2 Marks: Proves the result true for $n = 3$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$ 1 Mark: Proves the result true for $n = 3$.

13(c) (i)	$1+i$ is a root that satisfies the equation. $P(z) = z^2 - (3-2i)z + (5-i)$ $P(1+i) = (1+i)^2 - (3-2i)(1+i) + (5-i)$ $= (1+2i-1) - (3+3i-2i+2) + 5 - i$ $= 2i - 5 - i + 5 - i$ $= 0$	2 Marks: Correct answer. 1 Mark: Substitutes $(1+i)$ into the polynomial equation.
13(c) (ii)	Let the other root be α. Sum of the roots. $\alpha + (1+i) = -\frac{b}{a}$ $= -\frac{-(3-2i)}{1}$ $\alpha = 3 - 2i - 1 - i$ $= 2 - 3i$	1 Mark: Correct answer.
13(d) (i)	$x = 4\cos^2 t - 1$ $x = 2(\cos 2t + 1) - 1$ $= 2\cos 2t + 1$ $\dot{x} = -4\sin 2t$ $\ddot{x} = -4 \times 2\cos 2t$ $= -4(x-1)$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
13(d) (ii)	$x = 4\cos^2 t - 1$ or $x = 2\cos 2t + 1$ Particle passes through O when $t = \frac{\pi}{3}, \frac{2\pi}{3}$ 	1 Mark: Correct answer.

13(c) (iii)	Velocity is increasing most rapidly when $\ddot{x}$ has the greatest positive value (or x takes the least value). Greatest value: $\ddot{x} = -8\cos 2t = 8$ when $t = \frac{\pi}{2}$ $\therefore$ Velocity is increasing most rapidly at $\frac{\pi}{2}$ seconds.	1 Mark: Correct answer.
14(a)	Let $u = 1 + e^x$ $du = e^x dx$ When $x = 0$ then $u = 2$ and when $x = 1$ then $u = 1 + e$ $\int_0^1 \frac{e^x}{(1+e^x)^2} dx = \int_2^{1+e} u^{-2} du = [-u^{-1}]_2^{1+e}$ $= -\frac{1}{1+e} + \frac{1}{2}$ $= \frac{e-1}{2(e+1)}$	2 Marks: Correct answer. 1 Mark: Uses an appropriate substitution and sets up the integration.
14(b) (i)	Integration by parts $I_n = \int_0^{\frac{\pi}{2}} \sin^{n-1} x \sin x dx$ $= -[\sin^{n-1} x \cos x]_0^{\frac{\pi}{2}} + (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x dx$ $I_n = (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x dx$	2 Marks: Correct answer. 1 Mark: Sets up the integration and shows some understanding.
14(b) (ii)	$I_n = (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x dx$ $= (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x (1 - \sin^2 x) dx$ $= (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x - \sin^2 x dx$ $= (n-1)[I_{n-2} - I_n]$ $= (n-1) I_{n-2} - nI_n + I_n$ $nI_n = (n-1) I_{n-2}$ $I_n = \frac{(n-1)}{n} I_{n-2}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution..
14(b) (iii)	$I_4 = \frac{(4-1)}{4} I_2$ $= \frac{3}{4} \times \frac{(2-1)}{2} I_0$ $= \frac{3}{8} \times \int_0^{\frac{\pi}{2}} 1 dx$ $= \frac{3\pi}{16}$	1 Mark: Correct answer.

14(c) (i)	Newton's second law $\ddot{y} = kv - g$ $\frac{dv}{dt} = kv - g$  	1 Mark: Correct answer.
14(c) (ii)	$\frac{dt}{dv} = \frac{1}{kv - g}$ $\int \frac{dt}{dv} dv = \int \frac{1}{kv - g} dv$ $t = \frac{1}{k}\ln(kv - g) + C$ Initial conditions $t = 0$ and $v = 0$ $0 = \frac{1}{k}\ln(-g) + C$ $C = -\frac{1}{k}\ln g$ $t = \frac{1}{k}\ln(kv - g) - \frac{1}{k}\ln g$ $= \frac{1}{k}\ln\left(\frac{kv - g}{g}\right)$ $kt = \ln\left(\frac{kv - g}{g}\right)$ $e^{kt} = \frac{kv - g}{g}$ $= \frac{kv}{g} - 1$ $v = \frac{g}{v}\left(e^{kt} + 1\right)$	3 Marks: Correct answer. 2 Marks: Correctly substitutes the initial conditions into the expression for t. 1 Mark: Finds the correction expression for t.
14(c) (iii)	$\frac{dv}{dt} = v\frac{dv}{dy}$ $kv - g = v\frac{dv}{dy}$ $\frac{dv}{dy} = \frac{kv - g}{v}$ $\frac{dy}{dv} = \frac{v}{kv - g}$ $= \frac{1}{k} \times \frac{kv}{kv - g}$ $= \frac{1}{k} \times \frac{kv - g + g}{kv - g}$ $= \frac{1}{k} \times \left(1 + \frac{g}{kv - g}\right)$ $ky = \left(v + \frac{g}{k}\ln(kv - g)\right) + C$ Initially $y = 0$ and $v = 0$ $0 = 0 + \frac{g}{k}\ln(-g) + C$ $C = \frac{g}{k}\ln g$ $ky = v + \frac{g}{k}\ln(kv - g) + \frac{g}{k}\ln g$ $ky = v + \frac{g}{k}(\ln(kv - g) + \ln g)$ $ky = v + \frac{g}{k}\ln(kgv - g^2)$	3 Marks: Correct answer. 2 Marks: Finds the correction expression for ky. 1 Mark: Uses results for part (b) to determine an expression for $\frac{dv}{dy}$

14(d)	Let $u = \tan^{-1}x$ $\frac{du}{dx} = \frac{1}{1+x^2}$ $\int \frac{\ln(\tan^{-1}x)}{1+x^2}dx = \int \ln u du$ $= u\ln u - \int 1du$ $= u\ln u - u + C$ $= \tan^{-1}x\ln(\tan^{-1}x) - \tan^{-1}x + C$	2 Marks: Correct answer. 1 Mark: Uses an appropriate substitution and sets up the integration.
15(a)	$t = \tan\frac{x}{2}$ $dt = \frac{1}{2}\sec^2\frac{x}{2}dx$ $dx = \frac{2}{1+t^2}dt$ When x = 0 then t = 0 and when $x = \frac{\pi}{2}$ then t = 1 $2 + 2\sin x - \cos x = \frac{2(1+t^2) + 4t - (1-t^2)}{1+t^2}$ $= \frac{3t^2+4t+1}{1+t^2}$ $= \frac{(3t+1)(t+1)}{1+t^2}$ $\int_0^{\frac{\pi}{2}} \frac{1}{2+2\sin x - \cos x}dx = \int_0^1 \frac{1+t^2}{(3t+1)(t+1)} \times \frac{2}{1+t^2}dt$ $= \int_0^1 \frac{2}{(3t+1)(t+1)}dt$ $= \int_0^1 \left[\frac{3}{3t+1} - \frac{1}{t+1}\right]dt$ $= [\ln(3t+1) - \ln(t+1)]_0^1$ $= 2\ln 2 - \ln 2$ $= \ln 2$	4 Marks: Correct answer. 3 Marks: Correct expression for the integral in terms of t using partial fractions. 2 Marks: Finds the value of $2 + 2\sin x - \cos x$ in terms of t and changes the limits. 1 Mark: Sets up the integration using t formulas.

15(b) (i)	Now $(a-b)^2 \geq 0$ $a^2+b^2 > 2ab$ 1 Also $(a-c)^2 \geq 0$ $a^2+c^2 > 2ac$ 2 Also $(b-c)^2 \geq 0$ $b^2+c^2 > 2bc$ 3 Equations 1 + 2 + 3 $2(a^2+b^2+c^2) \geq 2(ab+ac+bc)$ $a^2+b^2+c^2 \geq (ab+ac+bc)$ $a^2+b^2+c^2+2(a^2+b^2+c^2)$ $\geq 2(ab+ac+bc)+\text{ab}+ac+bc$ $a^2+b^2+c^2+2(a^2+b^2+c^2) \geq 3(ab+ac+bc)$ $(a+b+c)^2 \geq 3(ab+ac+bc)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution such as using $a^2+b^2 > 2ab$
15(b) (ii)	$a^2+b^2+c^2 \geq (ab+ac+bc)$ Let $a = xy$, $b = xz$ and $c = yz$ $x^2y^2+x^2z^2+y^2z^2 \geq (xyxz+xyyz+xzyz)$ $\geq xyz(x+y+z)$ Replace x= a, $y = b$ and $z = c$ $a^2b^2+a^2c^2+b^2c^2 \geq abc(a+b+c)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
15(c) (i)		2 Marks: Correct answer. 1 Mark: Correctly graphs one of the inequalities or shows some understanding.
15(c) (ii)	Area $= \dfrac{1}{2}(4^2-2^2)\dfrac{\pi}{3}$ $= 2\pi$ square units	1 Mark: Correct answer.
15(d)	We have to prove that the negation is true. For all real numbers x we have $x^2 \neq -1$ Clearly, for any real number x, we have $x^2 \geq 0$ and so $x^2 \neq -1$. $\therefore$ There exists some real number x such that $x^2 = -1$ is false	2 Marks: Correct answer. 1 Mark: Shows some understanding.
15(e) (i)	Simple harmonic motion occurs when $\ddot{x} = -n^2(x-b)$ $x = 1+\sqrt{3}\cos 4t + \sin 4t$ $\dot{x} = -\sqrt{3} \times 4\sin 4t + 4\cos 4t$ $\ddot{x} = -\sqrt{3} \times 4^2\cos 4t - 4^2\sin 4t$ $= -4^2(\sqrt{3}\cos 4t + \sin 4t)$ $\ddot{x} = -4^2(x-1)$ $\therefore$ SHM about the position $x = -1$ ($n = 4$ and $b = 1$)	1 Mark: Correct answer.

15(e) (ii)	$x = 1 + \sqrt{3}\cos 4t + \sin 4t$ $= 1 + 2\sin\frac{\pi}{3}\cos 4t + 2\cos\frac{\pi}{3}\sin 4t$ $= 1 + 2\left[\sin 4t\cos\frac{\pi}{3} + \cos 4t\sin\frac{\pi}{3}\right]$ $= 1 + 2\sin\left(4t + \frac{\pi}{3}\right)$ (in the form $x = b + a\sin(nt + \alpha)$) $\therefore$ Amplitude is 2	1 Mark: Correct answer.
15(e) (iii)	Maximum speed at $\ddot{x} = 0$ or $x = 0$ (centre of motion) $\ddot{x} = -4^2\left(\sqrt{3}\cos 4t + \sin 4t\right) = 0$ $\sqrt{3}\cos 4t + \sin 4t = 0$ $\frac{\sin 4t}{\cos 4t} = -\sqrt{3}$ $\tan 4t = -\sqrt{3}$ $4t = \frac{2\pi}{3}, \frac{5\pi}{3}, \ldots$ $t = \frac{\pi}{6}, \frac{5\pi}{12}, \ldots$ $\therefore$ Particle first reaches maximum speed at $t = \frac{\pi}{6}$	2 Marks: Correct answer. 1 Mark: Makes some progress.
16(a) (i)	$\underset{\sim}{i}$ component $6 + \lambda = 0$ $\lambda = -6$ $\underset{\sim}{j}$ component $19 - 6 \times 4 = a$ $a = -5$ $\underset{\sim}{k}$ component $-1 - 6 \times (-2) = b$ $\therefore$ $a = -5$ and $b = 11$.	2 Marks: Correct answer. 1 Mark: Finds λ or shows some understanding.
16(a) (ii)	$\overrightarrow{OP} = (6 + \lambda)\underset{\sim}{i} + (19 + 4\lambda)\underset{\sim}{j} + (-1 - 2\lambda)\underset{\sim}{k}$ Direction vector of l_1: $\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$ $\overrightarrow{OP}$ and l_1 are perpendicular $(6 + \lambda)\underset{\sim}{i} + (19 + 4\lambda)\underset{\sim}{j} + (-1 - 2\lambda)\underset{\sim}{k} \cdot (\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}) = 0$ Hence $6 + \lambda + (19 + 4\lambda)4 + (-1 - 2\lambda) - 2 = 0$ $6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$ $21\lambda + 84 = 0$ $\lambda = -4$ Therefore $\overrightarrow{OP} = (6 - 4)\underset{\sim}{i} + (19 + 4 \times (-4))\underset{\sim}{j} + (-1 - 2 \times (-4))\underset{\sim}{k}$ $= 2\underset{\sim}{i} + 3\underset{\sim}{j} + 7\underset{\sim}{k}$	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Applies the statement for perpendicular vectors. 1 Mark: Shows some understanding.

16(b) (i)	$z^5 + z - 1 = 0$ $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^5 + \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) - 1 = 0$ $\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} + \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - 1 = 0$ $\frac{1}{2} - i\frac{\sqrt{3}}{2} + \frac{1}{2} + i\frac{\sqrt{3}}{2} - 1 = 0$ $0 = 0$ $\therefore a = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ is a root of the equation $z^5 + z - 1 = 0$	2 Marks: Correct answer. 1 Mark: Substitutes a into the equation and uses De Moivre's theorem.
16(b) (ii)	Monic cubic equation is the root of $z^5 + z - 1 = 0$ $z^5 + z - 1 = 0$ has real coefficients with a as a root. Complex roots occur in conjugate pairs or $\bar{a}$ is a root. $(z-a)(z-\bar{a}) = z^2 - (a+\bar{a})z + a\bar{a}$ $= z^2 - 2\cos\frac{\pi}{3}z + 1$ $= z^2 - z + 1$ Therefore $z^2 + z - 1 = (z^2 - z + 1)(z^3 + az^2 + bz - 1)$ $= z^5 + az^4 + bz^3 - z^2 - z^4 - az^3 - bz^2 + z + z^3 + az^2 + bz - 1$ $= z^5 + (a-1)z^4 + (b-a+1)z^3 + (a-b-1)z^2 + (b+1)z - 1$ Equating the coefficients of z^4: $a - 1 = 0, a = 1$ Equating the coefficients of z: $b + 1 = 1, b = 0$ $\therefore$ Monic cubic equation $z^3 + z^2 - 1 = 0$	2 Marks: Correct answer. 1 Mark: Uses the conjugate root theorem or makes some progress towards the solution.
16(c)	Step 1: To prove true for $n = 1$ LHS $= \tan\left[(2\times 1 - 1)\frac{\pi}{4}\right] = 1$ RHS $= (-1)^{1+1} = 1$ Result is true for $n = 1$ Step 2: Assume true for $n = k$ $\tan\left[(2k-1)\frac{\pi}{4}\right] = (-1)^{k+1}$ Step 3: To prove true for $n = k + 1$ $\tan\left[(2(k+1)-1)\frac{\pi}{4}\right] = (-1)^{k+1+1}$ LHS $= \tan\left[(2(k+1)-1)\frac{\pi}{4}\right]$ $= \tan\left[(2k-1+2)\frac{\pi}{4}\right]$ $= \tan\left[\left(2k-1\right)\frac{\pi}{4} + \frac{\pi}{2}\right)\right]$ $= -\cot\left[2k-1)\frac{\pi}{4}\right]$ $= -\left\{\tan\left[(2k-1)\frac{\pi}{4}\right]\right\}^{-1}$ $= -1 \times \{(-1)^{k+1}\}^{-1}$ $= -1 \times \{(-1)^{-1}\}^{k+1}$ $= -1 \times (-1)^{k+1}$ $= (-1)^{k+1+1}$ $=$ RHS Step 4: True by induction	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$ 1 Mark: Proves the result true for $n = 1$