Carlingford High School Mathematics Assessment Task

Course: Preliminary Mathematics Extension 1
Task: Preliminary Assessment Task 1 (Assignment)

Notification Date: 7/4/2020
Due Date: 28/4/2020
Weighting: 30%

Topics assessed:

- ME-F1- Further Work With Functions

Outcomes assessed

- uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- manipulates algebraic expressions and graphical functions to solve problems ME11-2
- uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Nature of the task

This assignment requires you to investigate various elements of the first Extension 1 topic; "Further Functions". There are questions from three of the four areas we have covered: Inequalities, Graphing Techniques and Inverse Functions, and a question covering one important application area

There will be a Google Classroom set up for the assignment. All questions about the task should be posted there. You will be required to hand-write/draw your submission, scan and/or photograph your work, combine them into one document and upload it onto the Google Classroom, no later than **8:45am** on **Tuesday 28 April 2020**

Marking criteria

You will be assessed on how well you:

- select and use appropriate mathematical processes, technologies and language to investigate, organise and interpret calculations
- provide reasoning and justification related to the solved problems

Marking guidelines

- A marking rubric is attached to the assignment

Feedback provided

- Your teacher will provide feedback outlining strengths and areas for improvement to build on knowledge, understanding and skills for future learning

Assessment Guidelines/Policy

- Please refer to *Assessment Guidelines and Policies* from the *HSC Assessment Book 2020*. Contact your Deputy Principal if you do not have a copy

Inequalities (15 marks)

1. Solve $|2x+1| > \sqrt{2x+1}$

(a) by sketching and labelling the graphs of $y = |2x+1|$ and $y = \sqrt{2x+1}$, using an appropriate scale, and showing any points of intersection. (3 marks)

(b) algebraically, using $|2x+1| = \sqrt{(2x+1)^2}$, showing all necessary working. (3 marks)

2. (a) Show that $\frac{x+1}{x-1} = 1 + \frac{2}{x-1}$. (1 mark)

(b) Hence, solve $\frac{x+1}{x-1} \geq -2$, by sketching and labelling the graphs of $y = 1 + \frac{2}{x-1}$ and $y = -2$, showing any points of intersection. (3 marks)

(c) Solve $\frac{x+1}{x-1} \geq -2$, by multiplying both sides by $(x-1)^2$. (2 marks)

3. Show, using the *cases (critical points) method*, that $|x+1| + |x-1| < 2$ has no solution. (3 marks)

Graphing Techniques (18 marks)

4. Print the sheets of graphs given, and use these to answer the following questions.

(a) On diagram 1, sketch and clearly label the reciprocal function $y = \frac{1}{f(x)}$ showing all the main features. (2 marks)

(b) (i) On diagram 2, sketch and clearly label the function $y = |f(x)|$ showing all the main features. (2 marks)

(ii) On diagram 3, sketch and clearly label the function $y = f(|x|)$ showing all the main features. (2 marks)

(c) (i) On diagram 4, sketch and clearly label the function $y = \sqrt{f(x)}$ showing all the main features. (2 marks)

(ii) On diagram 5, sketch and clearly label the function $y^2 = f(x)$ showing all the main features. (2 marks)

5. Select your own function, $f(x) = (x-a)^2(x+b)$, where a and b are **different positive integers**, $a \leq 5$ and $b \leq 5$. Given $g(x) = \frac{1}{x}$

(a) Draw and clearly label sketches of $f(x)$ and $g(x)$ on two separate number planes. (2 marks)

(b) Use one of your number planes to draw and clearly label a sketch of $y = f(x) + g(x)$, using the *adding ordinates method*, showing all important features. (3 marks)

(c) Use the other number plane to draw and clearly label a sketch of $y = f(x) \cdot g(x)$, using the *multiplying ordinates method*, showing all important features. (3 marks)

Inverse Functions (12 marks)

6. Given the function $f(x) = 2x^2 - 12x + 15$,

(a) Write $2x^2 - 12x + 15$ in the form $a(x - b)^2 + c$, showing all necessary working. (2 marks)

(b) Find the domain and range of the function. (2 marks)

(c) What is the maximum possible restriction of the domain of $f(x)$ so that its inverse, $f^{-1}(x)$, will also be a function? (1 mark)

(d) Find $f^{-1}(x)$, showing all necessary working. (2 marks)

(e) What is the domain and range of $f^{-1}(x)$? (2 marks)

(f) On the same set of axes, and using the same scale for the x and the y axes, sketch and label the graphs of $y = f(x)$ (for your restricted domain), $y = x$ and $y = f^{-1}(x)$. (3 marks)

Application – The Simple Pendulum (5 marks)

7. The back-and-forth motion of a swing is an example of a **pendulum**. Pendulums are used in many engineered objects, such as clocks, metronomes, amusement park rides and earthquake seismometers, and help engineers use their understanding of motion, gravity, inertia and centripetal force to determine the force needed to propel an object into outer space, the braking power required to stop a vehicle at high speeds, and the optimal curve of a highway ramp.

An experiment was conducted to explore the relationship between the length of a pendulum, l metres, and the period of the pendulum swing, P seconds. For each pendulum length, the time taken for the first 30 oscillations was recorded, and the average period for one oscillation determined.

The results are summarised in the following table:

Length of pendulum in metres	l	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Period in seconds	P	0.6	0.9	1.1	1.3	1.4	1.6	1.7

(a) Using the value ranges $0 \leq l \leq 1.0$ and $0 \leq P \leq 2.0$, and a scale of $1cm = 0.1m$ on both axes, plot each point and draw a smooth curve to represent the relationship between the two variables. (2 marks)

(b) On the same axes, graph and label the inverse, $f^{-1}(x)$, of this function, by reflecting each point in the line $y = x$. (2 marks)

(c) Explain what the inverse function represents for this experiment. (1 mark)

END OF ASSESSMENT TASK ☺

Question 4 (Graphing Techniques) Worksheet

Diagram 1

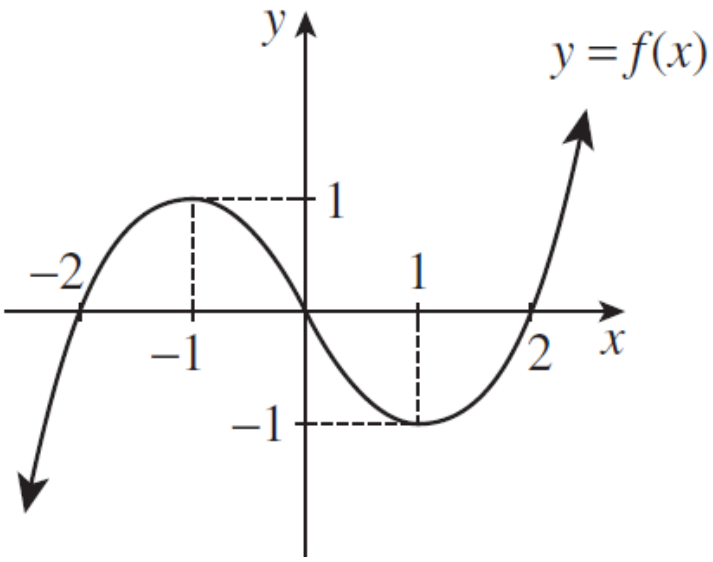

Diagram 2

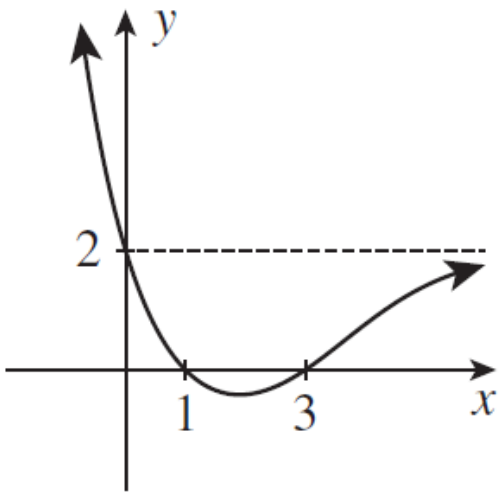

Diagram 3

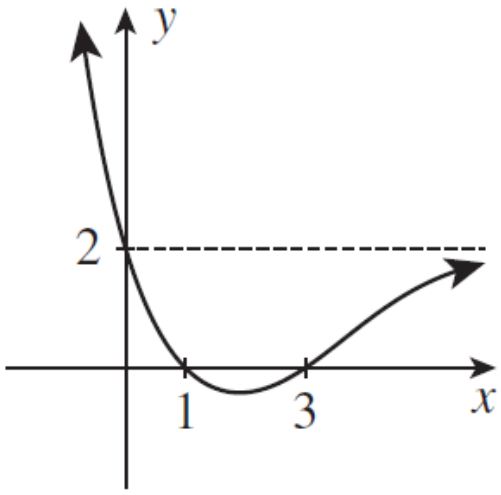

Diagram 4

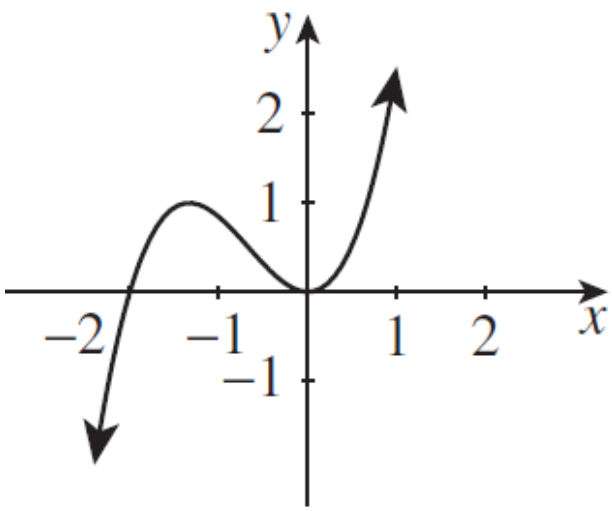

Diagram 5