Carlingford High School

2018

Year 12 Extension 1 Mathematics

Term 2 Examination 2018

Time allowed 50 min

Student number……………………………………

General Instructions

o   Do not write in columns

o   Marks may be deducted for careless or badly arranged work

o   Only calculators approved by the Board of Studies may be used

o   All answers are to be completed in black pen except graphs and diagrams

o   No lending or borrowing

Q1 Trigonometric Functions	Q2 Inverse Functions	Q3 Integration Techniques	Total
/14	/13	/14	/41

Question 1   Trigonometric Functions   Marks

a)   Two circles have radii 4 centimetres and 7 centimetres respectively. If their centres   3
are 10 centimetres apart, find the area common to both circles.

b)   Prove that $\cot\theta - 2\cot 2\theta = \tan\theta$.   3

c)   i.   Express $\cos\theta - \sin\theta$ in the form $A\cos(\theta + \alpha)$ in which A is a positive   2
number.

     ii.   Hence or otherwise solve the equation $\cos\theta - \sin\theta = 1$ for $0^0 \le \theta \le 360^0$,   2
Giving your answers correct to the nearest degree.

d)   Use the substitution $t = \tan\dfrac{\theta}{2}$, or otherwise to solve the equation:

     $3\sin\theta - 2\cos\theta = 3$ for $0^0 \le \theta \le 360^0$ 4

     Give your answers correct to the nearest minute where necessary.

Question 2   Inverse Functions   Marks

a)   What is the domain of the function $f(x) = 2\sin^{-1}\left(\dfrac{x}{2}\right)$ 1

b)   Consider the function $f(x) = \dfrac{x}{x+3}$.

     i.   Show that $f'(x) > 0$ for all x in the domain.   2

     ii.   State the equation of the horizontal asymptote of $y = f(x)$.   1

     iii.   Without using any further calculus, sketch the graph of $y = f(x)$.   2

     iv.   Explain why $f(x)$ has an inverse function $f^{-1}(x)$.   1

     v.   Find an expression for the inverse function $f^{-1}(x)$.   2

     vi.   What is the domain of $f^{-1}(x)$?   1

c)   Evaluate $\displaystyle\int_{\sqrt{2}}^{\sqrt{3}} \dfrac{dx}{\sqrt{4-x^2}}$ 3

Question 3 Integration Techniques   Marks

a)   Find $\int_0^\pi \cos^2 3x \, dx$ 3

b)   Using the substitution $u = 3x^3 + 1$ or otherwise, evaluate the integral

   $\int_0^1 x^2 \sqrt{3x^3 + 1} \, dx$ 3

c)   Evaluate $\int_0^{\frac{\pi}{4}} \sin x \cos^2 x \, dx$ 3

d)   i.   Use the substitution $t = \tan \dfrac{x}{2}$, to show that $\operatorname{cosec} x + \cot x = \cot \dfrac{x}{2}$ 2

   ii. Hence evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\operatorname{cosec} x + \cot x \right) dx$ 3

End of Exam