CARLINGFORD HS ADVENTURE IN LEARNING

Carlingford High School

2020 YEAR 12 ASSESSMENT TASK 3

Mathematics Advanced

STUDENT NUMBER: SOLUTIONS

Teacher: (Please Circle) 12MAA A (Ms Bennett) 12MAAB (Mr Wilson)

12MAA_1 (Ms Strilakos) 12MAA_2 (Mr Gong) 12MAA_3 (Mr Cheng) 12MAA_4 (Ms Blakeley)

General Instructions

- Working time - **55 minutes**
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 45

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 10 minutes for this section

Section II – 35 marks (pages 5 – 12)

- Attempt Questions 11 – 19
- Allow about 45 minutes for this section

TOPIC	MARKS	
Financial Mathematics **Questions: 1 – 3, 13 – 16**	**/18**	
Applications of Differentiation **Questions: 4 – 7, 11 – 12**	**/17**	
Integral Calculus **Questions: 8 – 10, 17 – 19**	**/15**	
TOTAL	**/45**	%

Section I
10 marks
Attempt Questions 1 - 10.
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1. The first three terms of an arithmetic series are 3, 7 and 11.

   What is the 15^{th} term of this series?

   A 59

   B 63

   C 465

   D 495

2. The limiting sum of an infinite geometric series exists when:

   A $r > 1$

   B $|r| > 1$

   C $|r| < 1$

   D $r < 1$

3. How many terms are there in the geometric series 3, 6, 12, 24, ..., 384 ?

   A 7

   B 8

   C 9

   D 10

4. For what value(s) of x does the graph of $f(x) = \frac{1}{3}x^3 - x^2 + 3$ have a horizontal tangent?

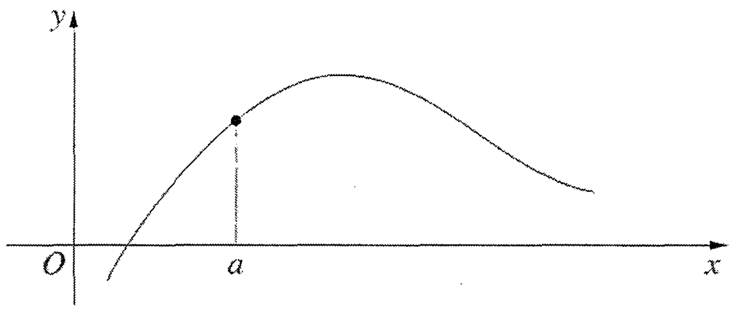
   A 0

   B 0 and 3

   C 2

   D 0 and 2

5. The diagram shows the graph $y = f(x)$.



Which of the following statements is true?

A $f'(a) > 0$ and $f''(a) < 0$

B $f'(a) > 0$ and $f''(a) > 0$

C $f'(a) < 0$ and $f''(a) < 0$

D $f'(a) < 0$ and $f''(a) > 0$

6. A particle is moving in a straight line so that its displacement is given by $x = 30 + 25t - 5t^2$, where x is in metres and t is in seconds.

What is the initial velocity of the particle?

A -5 m s^{-1}

B 10 m s^{-1}

C 25 m s^{-1}

D 30 m s^{-1}

7. A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres.

At a certain time, $\dot{x} = -3 \text{ m s}^{-1}$ *and* $\ddot{x} = 2 \text{ m s}^{-1}$.

Which statement describes the motion of the particle at that time?

A The particle is moving to the right with increasing speed.

B The particle is moving to the left with increasing speed.

C The particle is moving to the right with decreasing speed.

D The particle is moving to the left with decreasing speed.

8. Find $\int e^{3x}\,dx$

A $e^{3x} + C$

B $3e^{3x} + C$

C $\dfrac{e^{3x}}{3} + C$

D $\dfrac{e^{3x+1}}{3x+1} + C$

9. Find $\int \sin(6x)\,dx$

A $\dfrac{1}{6}\cos(6x) + C$

B $6\cos(6x) + C$

C $-6\cos(6x) + C$

D $-\dfrac{1}{6}\cos(6x) + C$

10. A particle is moving in a straight line with its velocity given by $\dot{x} = 6t^2 - 4$, where x is in metres and t is in seconds.

Which statement below is correct?

A $x = 2t^3 - 4t + C$

B $x = 2t^3 - 4t$

C $\ddot{x} = 2t^3 - 4t$

D $\ddot{x} = 12t - 4$

Section II
40 marks
Attempt Questions 11 – 22.
Allow about 40 minutes for this section.

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (8 marks)

The curve $y = x^3 + bx + c$ has a minimum stationary point at $M(1, 5)$.

(a) Find the values of the constants b and c. **2**

$y' = 3x^2 + b$

$y' = 0$ at $(1,5)$: $0 = 3(1)^2 + b$

$b = -3$

$\therefore y = x^3 - 3x + c$

sub $(1,5)$: $5 = (1)^3 - 3(1) + c$

$c = 7$

$\therefore y = x^3 - 3x + 7$

and $b = -3$, $c = 7$

(b) Find the coordinates of the other stationary point N and determine its nature. **2**

$y' = 3x^2 - 3$

$= 3(x^2 - 1)$

Solve $y' = 0$

$0 = 3(x^2 - 1)$

$= 3(x+1)(x-1)$

$x = 1, -1$

When $x = -1$, $y = (-1)^3 - 3(-1) + 7$

$= 9$

Check nature:

x	-2	-1	0
y'	>0	0	<0

$\therefore (-1, 9)$ maximum

Question 11 continues on page 6

Question 11 (continued)

(c) Find the point of inflection. **2**

$y'' = 6x$

Solve $y'' = 0$

$6x = 0$

$x = 0$

When $x = 0$: $y = (0)^3 - 3(0) + 7$

$= 7$

Check concavity change:

x	-1	0	1
y''	<0	0	>0

$\therefore (0,7)$ is a point of inflection

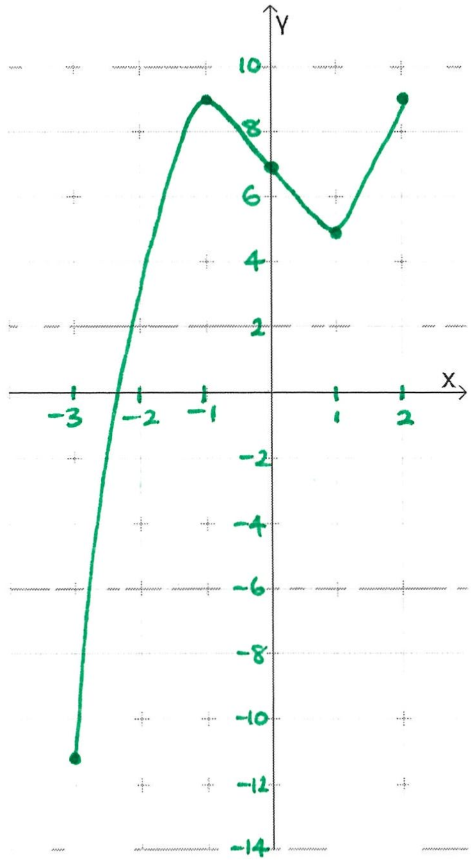
(d) Sketch the curve for $-3 \le x \le 2$ **2**

when $x = 2$: $y = (2)^3 - 3(2) + 7$

$= 9$

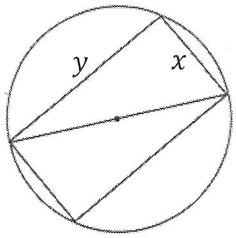
when $x = -3$: $y = (-3)^3 - 3(-3) + 7$

$= -11$



Question 12 (5 marks)

A rectangle is cut from a circular disc of radius 6 cm.



(a) Show that the formula for the area of the rectangle is $A = x\sqrt{144 - x^2}$. **2**

radius = 6cm $\rightarrow$ diameter = 12cm

$y^2 = 12^2 - x^2$

$y = \sqrt{144 - x^2}$

Area $= xy$

$= x\sqrt{144 - x^2}$

(b) Find the area of the largest possible rectangle that can be produced. **3**

$A = x(144 - x^2)^{1/2}$

$A' = (144 - x^2)^{1/2} \cdot 1 + x \cdot \frac{1}{2}(144 - x^2)^{-1/2} \cdot -2x$

$= \sqrt{144 - x^2} - \dfrac{x^2}{\sqrt{144 - x^2}}$

Solve $A' = 0$

$\sqrt{144 - x^2} = \dfrac{x^2}{\sqrt{144 - x^2}}$

$144 - x^2 = x^2$

$144 = 2x^2$

$x^2 = 72$

$x = \sqrt{72}$ $\quad (x \neq -\sqrt{72})$

Check maximum :

x	8	$\sqrt{72}$	9
A'	>0	0	<0

$\therefore$ maximum

Area $= x\sqrt{144 - x^2}$

$= \sqrt{72}\,[144 - (\sqrt{72})^2]$

$= \sqrt{72} \times \sqrt{72}$

$= 72 \text{ m}^2$

Question 13 (2 marks)

Evaluate $\sum_{2}^{5}(n^2+n)$

$= 2^2+2 + 3^2+3 + 4^2+4 + 5^2+5$

$= 68$

Question 14 (3 marks)

Find the sum of the first 50 terms of an arithmetic series, given that the fifteenth term is 34 and the sum of the first eight terms is 20.

T_{15} : $a + 14d = 34 \quad \ldots (1)$

S_8 : $\frac{8}{2}(2a+7d) = 20$

$2a + 7d = 5 \quad \ldots (2)$

$2\times(1)$: $2a + 28d = 68 \quad \ldots (3)$

$(3)-(2)$: $21d = 63$

$d = 3$

sub $d = 3$ into (1) : $a + 14(3) = 34$

$a = -8$

S_{50} : $\frac{50}{2}(2(-8)+49(3)) = 3\,275$

Question 15 (3 marks)

Find the limiting sum of the geometric series $3 + 3(\sqrt{2}-1) + 3(\sqrt{2}-1)^2 + \ldots$
Express your answer with a rational denominator.

$r = \sqrt{2}-1$

$S_\infty = \frac{a}{1-r}$

$= \frac{3}{1-(\sqrt{2}-1)}$

$= \frac{3}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$

$= \frac{6+3\sqrt{2}}{2}$

Question 16 (7 marks)

Kathryn borrows \$280 000 to buy a house. She takes a loan out over a period of 25 years at an interest rate of 6% per annum reducible, compounded monthly. Kathryn makes monthly repayments with interest being calculated just before each payment is made. Let A_n be the balance owing after the nth repayment. Her repayments are \$M per month.

(a) Show that $A_3 = 280\ 000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)$ 6% p.a = 0.5% per month **1**

$A_1 = 280\ 000 \times 1.005 - M$

$A_2 = (280\ 000 \times 1.005 - M)1.005 - M$

$= 280\ 000 \times 1.005^2 - 1.005M - M$

$A_3 = (280\ 000 \times 1.005^2 - 1.005M - M)1.005 - M$

$= 280\ 000 \times 1.005^3 - 1.005M^2 - 1.005M - M$

$= 280\ 000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)$

(b) Find an expression for A_n and use it to calculate her monthly repayments, if she pays off the loan in 25 years. **3**

25 years = 300 months

$A_n = 280\ 000 \times 1.005^n - M(1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1})$

$A_{300} = 0$

$0 = 280\ 000 \times 1.005^{300} - M(1 + 1.005 + 1.005^2 + \dots + 1.005^{299})$

$280\ 000 \times 1.005^{300} = M\left(\frac{1(1.005^{300} - 1)}{1.005 - 1}\right)$

$M = \dfrac{280\ 000 \times 1.005^{300}}{\left(\frac{1.005^{300} - 1}{1.005 - 1}\right)}$

$= \$1804.04$

Question 16 continues on page 10

Question ~~11~~ 16 (continued)

(c) If Kathryn had made monthly repayments of $2 000, how many monthly repayments would it have taken to pay off the loan? **3**

$A_n = 280000 \times 1.005^n - M(1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1})$

$0 = 280000 \times 1.005^n - 2000\left(\frac{1(1.005^n - 1)}{1.005 - 1}\right)$

$0 = 1400 \times 1.005^n - 2000 \times 1.005^n + 2000$

$2000 = 600 \times 1.005^n$

$1.005^n = \frac{10}{3}$

$n \log 1.005 = \log\left(\frac{10}{3}\right)$

$n = \frac{\log\left(\frac{10}{3}\right)}{\log 1.005}$

$= 241.396\dots$

$\therefore$ 242 repayments

Question 17 (7 marks)

Find:

(a) $\int (x^3 - 3x^2 + 4)\ dx$ **1**

$= \frac{x^4}{4} - x^3 + 4x + C$

(b) $\int \left(\frac{1}{x^3} + \sqrt{x}\right) dx$ **2**

$= \int (x^{-3} + x^{\frac{1}{2}})\ dx$

$= \frac{x^{-2}}{-2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{-1}{2x^2} + \frac{2}{3} x\sqrt{x} + C$

$\left(-\frac{1}{2x^2} + \frac{2}{3} x^{3/2} + C\right)$

(c) $\int x(x^2 + 1)^3\, dx$ **2**

$= \frac{1}{2} \int 2x (x^2+1)^3\ dx$

$= \frac{1}{2}\left[\frac{1}{4} (x^2+1)^4\right] + C$

$= \frac{1}{8} (x^2+1)^4 + C$

(d) $\int \frac{x+3}{x^2 + 6x + 2}\, dx$ **2**

$= \frac{1}{2} \int \frac{2(x+3)}{x^2+6x+2}\ dx$

$= \frac{1}{2} \ln |x^2+6x+2| + C$

Question 18 (2 marks)

Evaluate $\int_{-1}^{3} \sqrt{1+x^2}\ dx$ using the trapezoidal rule with 4 subintervals. Give your answer correct to one decimal place.

$$A = \frac{3-(-1)}{2(4)} \left\{ f(-1) + 2\left[f(0) + f(1) + f(2) \right] + f(3) \right\}$$

$$= \frac{1}{2}\left[\sqrt{1+(-1)^2} + 2\left(\sqrt{1+(0)^2} + \sqrt{1+(1)^2} + \sqrt{1+(2)^2}\right) + \sqrt{1+(3)^2} \right]$$

$$= \frac{1}{2}\left[\sqrt{2} + 2\left(\sqrt{1} + \sqrt{2} + \sqrt{5}\right) + \sqrt{10} \right]$$

$$= 6.938\ldots$$

$$= 6.9 \text{ (1dp)}$$

Question 19 (3 marks)

A curve has $\frac{dy}{dx} = \cos 4x$ and passes through the point $\left(\pi, \frac{\pi}{4}\right)$.

Find the equation of the curve.

$$y = \frac{1}{4}\sin 4x + C$$

sub $\left(\pi, \frac{\pi}{4}\right)$: $\quad \frac{\pi}{4} = \frac{1}{4}\sin 4(\pi) + C$

$$C = \frac{\pi}{4}$$

$$\therefore \quad y = \frac{1}{4}\sin 4x + \frac{\pi}{4}$$

END OF PAPER

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2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

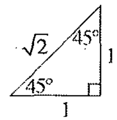
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



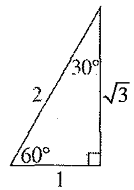
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A\sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

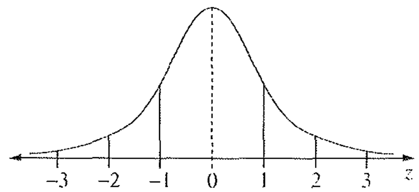
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x-\mu}{\sigma}$$

An outlier is a score less than $Q_1 - 1.5 \times IQR$ or more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X-\mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = |x\underset{\sim}{i} + y\underset{\sim}{j}| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos\theta + i\sin\theta)]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

STUDENT NUMBER: SOLUTIONS

Carlingford High School

2020 YEAR 12 ASSESSMENT TASK 3

Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ (correct) C ○ D ○

1.	A ●	B ○	C ○	D ○
2.	A ○	B ○	C ●	D ○
3.	A ○	B ●	C ○	D ○
4.	A ○	B ○	C ○	D ●
5.	A ●	B ○	C ○	D ○
6.	A ○	B ○	C ●	D ○
7.	A ○	B ○	C ○	D ●
8.	A ○	B ○	C ●	D ○
9.	A ○	B ○	C ○	D ●
10.	A ●	B ○	C ○	D ○