Carlingford High School

Mathematics

Year 10 Yearly Examination
5.3 Course
2020

*Name:*__________________ *Teacher: Lobejko/Lego/Tang/Wilson*

Time allowed: 90 minutes *(Board approved calculators may be used)*

	Algebra &Surds	Equations &Logarithms	Non-Linear Graphs	Measurement	Data & Probability	Functions & Polynomials	Coordinate Geometry
Q1	/1						
Q2					/1		
Q3					/1		
Q4				/1			
Q5				/1			
Q6						/1	
Q7							/1
Q8					/1		
Q9	/14						
Q10		/19					
Q11			/14				
Q12				/13			
Q13					/12		
Q14						/13	
Q15							/7
Total	/15	/19	/14	/15	/15	/14	/8
%							

CIRCLE THE CORRECT ANSWER TO THE MULTIPLE CHOICE QUESTIONS: Q1 to Q8

Q1. What is $\frac{(4p)^2}{2p} \div p^3$ expressed in its simplest form?

(A) $\frac{8}{p^2}$

(B) $\frac{4}{p^2}$

(C) $2p^5$

(D) $8p^4$

Q2. Which of the statements correctly describes the data in this set?

20, 25, 25, 25, 45

(A) The range, mode, median and mean all have the same value.

(B) The mode exceeds the range.

(C) The range exceeds the median.

(D) The mean exceeds the median.

Q3. In a survey, a number of people entering a club were asked if they were members or non-members visiting.

The table below shows the results of the survey.

	Members	*Non-members*	*Total*
Female	85	60	145
Male	115	45	160
Total	200	105	305

A person is selected at random from the surveyed group.

What is the probability (to the nearest percent) that the person selected is a male and is a non-member of the club?

(A) 15

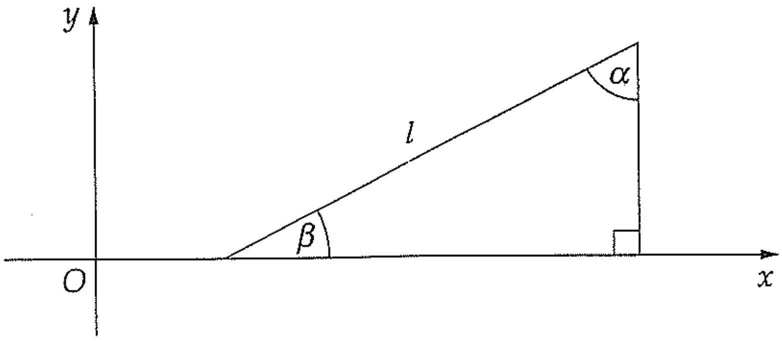
(B) 20

(C) 28

(D) 43

$\frac{45}{305} \times 100\%$

Q4.



Which of the following gives the gradient of line l?

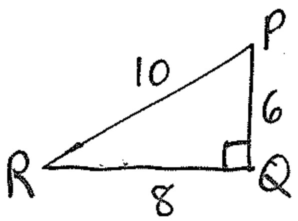
(A) $\tan \beta$ (B) $\tan \alpha$ (C) $\sin \beta$ (D) $\sin \alpha$

Q5.

Triangle PQR is right-angled with PR the hypotenuse.

If $\mathrm{Sin}R = 0.6$, what is the value of $\mathrm{Tan}P$?

(A) 0.75 (B) 0.8 (C) 1.25 (D) 1.33



$\mathrm{Tan}\,P = \frac{8}{6}$

$= 1.\dot{3}$

Q6.

Which inequality gives the domain of $y = \sqrt{2x-3}$?

(A) $x < \frac{3}{2}$

(B) $x > \frac{3}{2}$

(C) $x \leq \frac{3}{2}$

(D) $x \geq \frac{3}{2}$

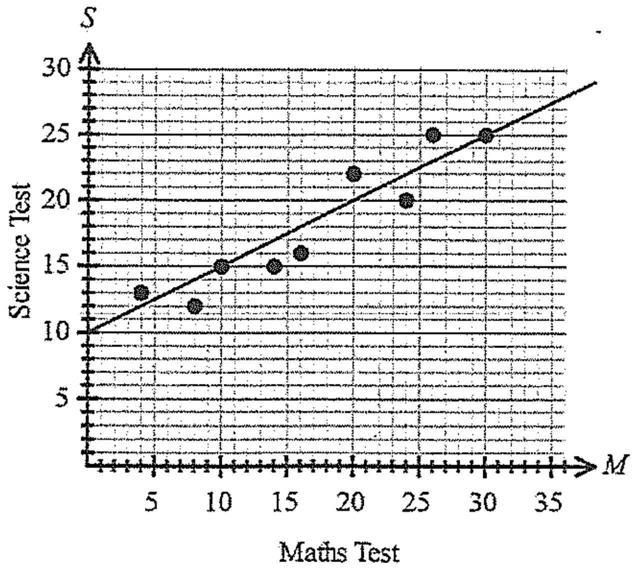
$2x - 3 \geq 0$

$2x \geq 3$

$x \geq \frac{3}{2}$

Q7.

A line of best fit is drawn below on the graph. Which equation best describes this line?



(A) $s = -\frac{m}{2} - 10$

(B) $s = \frac{m}{2} - 10$

(C) $s = \frac{m}{2} + 10$

(D) $s = 2m + 10$

Q8.

At a local shopping centre the management records the number of cars damaged per day in the car park over 100 days.

Number of Cars Damaged	0	1	2
Number of days	50	35	15

Based on this research, what is the expected number of cars that will be damaged on any given day in this car park?

(A) 0.325

(B) 0.65

(C) 60

(D) 260

Q9. (14 marks)

(a) Simplify [2]

i) $(x^2 + 3x - 3) + (-2x^2 - 4x + 5)$

$= -x^2 - x + 2$

ii) $4\sqrt{50} + \sqrt{2}$

$= 4\sqrt{25 \times 2} + \sqrt{2} = 20\sqrt{2} + \sqrt{2} = 21\sqrt{2}$

(b) Expand [2]

i) $(4x-3)^2 = 16x^2 - 24x + 9$

ii) $(\sqrt{6}-8)(\sqrt{2}+1) = \sqrt{12} + \sqrt{6} \underbrace{- 8\sqrt{2} - 8 = 2\sqrt{3}}_{\text{1 mark}} + \sqrt{6} - 8\sqrt{2} - 8$

(c) Factorise [3]

i) $2m^2 - m - 6 = (2m+3)(m-2)$

ii) $2x - xy + y^2 - 2y \begin{array}{l} = x(2-y) + y(y-2) \\ = x(2-y) - y(2-y) \end{array} \Big\}$ 1 mark

$= (x-y)(2-y) \leftarrow$ 1 mark

(d) Simplify [5]

i) $\frac{6x^2+10x+4}{x+1} = \frac{2(x^2+5x+2)}{(x+1)} = \frac{2\cancel{(x+1)}(3x+2)}{\cancel{(x+1)}}$

$= 2(3x+2)$

ii) $\frac{5}{n} - \frac{1}{n+3}$

$= \frac{5(n+3) - n}{n(n+3)}$

$= \frac{5n + 15 - n}{n(n+3)}$

$= \frac{4n+15}{n(n+3)}$

iii) $\frac{8m}{m^2-4} \div \frac{12}{3m-6}$

$= \frac{{}^{2}\cancel{8}m}{(m+2)\cancel{(m-2)}} \times \frac{\cancel{3}\cancel{(m-2)}}{\cancel{12}\ \cancel{3}} \leftarrow$ 1 mark

$= \frac{2m}{m+2} \leftarrow$ 1 mark

(e) Rationalise the denominator [2]

$$\frac{5\sqrt{3}}{2\sqrt{3}-4} \times \left(\frac{2\sqrt{3}+4}{2\sqrt{3}+4}\right) = \frac{30+20\sqrt{3}}{12-16} = \frac{10(3+2\sqrt{3})}{-4} = \frac{-5(3+2\sqrt{3})}{2}$$

1 mark 1 mark

Q10. (19 marks)

(a) Given $\frac{1}{u}+\frac{1}{v}=\frac{1}{t}$ find the value of v as a fraction, if $u=-1$ and $t=2$ [2]

$$\frac{1}{-1}+\frac{1}{V}=\frac{1}{2}$$

$$\frac{1}{V}=\frac{1}{2}+1$$ 1 mark

$$\frac{1}{V}=\frac{3}{2}$$

$$\therefore V=\frac{2}{3}$$ 1 mark

(b) Solve the following equations [3]

i) $(x+3)^2=16$

$x+3=\pm 4$

$x=\pm 4-3$

$=1$ OR -7

2 marks

ii) $log_4 x=-2$

$x=4^{-2}$

$x=\frac{1}{16}$

1 mark

(c) Solve $5^x=200$ showing working and writing the solution to two decimal places. [2]

$\log 5^x=\log 200$

$x\log 5=\log 200$

$\therefore x=\frac{\log 200}{\log 5}$

$=3.292\ldots$

$\doteqdot 3.29$

(d) Use the quadratic formula to solve for x in exact form. [2]

$$3x^2 - 8x - 4 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-4)}}{2(3)} \leftarrow \text{1 mark}$$

$$= \underbrace{\frac{8 \pm \sqrt{112}}{6} = \frac{8 \pm 4\sqrt{7}}{6}}_{\text{1 mark}} = \frac{4 \pm 2\sqrt{7}}{3}$$

(e) Solve the inequality $\frac{3}{5} - \frac{x}{4} \leq 1$ [2]

$$\left.\begin{aligned} \frac{12-5x}{20} &\leq 1 \\ 12 - 5x &\leq 20 \\ -5x &\leq 8 \end{aligned}\right\} \text{1 mark}$$

$$x \geq -\frac{8}{5} \leftarrow \text{1 mark}$$

(f) Solve the pair of simultaneous equations [2]

$2x + 5y = 8$ and $x - y - 4 = 0$

$$2x + 5y = 8$$
$$x - y = 4$$

$$2x + 5y = 8 \quad \text{——} ①$$
$$2x - 2y = 8 \quad \text{——} ②$$

$$① - ② \quad 7y = 0$$

$$\therefore y = 0 \checkmark \quad \text{1 mark}$$

$$x = 4 \checkmark \quad \text{1 mark}$$

(g) Given $log_a 2 = 0.431$ and $log_a 3 = 0.683$ evaluate (answer to 3 d.p.) [4]

i) $log_a 1.5$

$$= \log_a \frac{3}{2}$$

$$= \log_a 3 - \log_a 2 \leftarrow \text{1 mark}$$

$$= 0.683 - 0.431 = 0.252 \leftarrow \text{1 mark}$$

ii) $log_a \sqrt[3]{2}$

$$= \log_a 2^{\frac{1}{3}} \quad \} \text{1 mark}$$

$$= \frac{1}{3} \times \log_a 2 = \frac{1}{3} \times 0.431 = 0.144 \leftarrow \text{1 mark}$$

(h) Show $log_b\sqrt{\frac{a^5b^3}{c}} - 2log_b a + \frac{1}{2}log_b\frac{c}{a}$ equals $\frac{3}{2}$ [2]

$$= \frac{1}{2}\log_b \frac{a^5b^3}{c} - 2\log_b a + \frac{1}{2}\log_b c - \frac{1}{2}\log_b a$$

$$\left.\begin{aligned} &= \frac{1}{2}(\log_b a^5 + \log_b b^3 - \log_b c) + \frac{1}{2}\log_b c - \frac{5}{2}\log_b a \\ &= \frac{1}{2}(5\log_b a + 3 - \log_b c) + \frac{1}{2}\log_b c - \frac{5}{2}\log_b a \end{aligned}\right\} \text{1 mark}$$

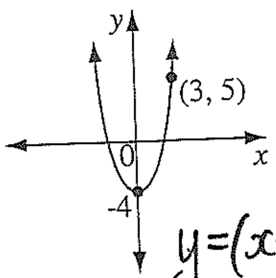
$$= \cancel{\frac{5}{2}\log_b a} + \frac{3}{2} - \cancel{\frac{1}{2}\log_b c} + \cancel{\frac{1}{2}\log_b c} - \cancel{\frac{5}{2}\log_b a}$$ ← 1 mark cancelling

$$= \frac{3}{2}$$

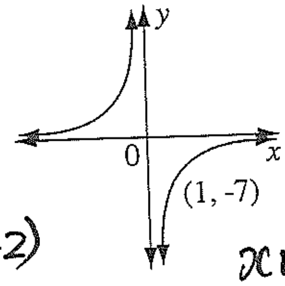
Q11. (14 marks)

(a) From the given equations write the appropriate equation next to the given graph. [6]

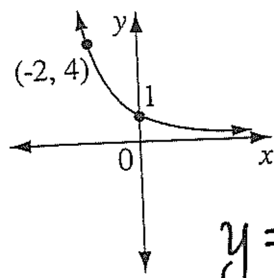
$y = (x+2)(x-2)$	$y = -2^x$	$y = -4 - x^2$
$y = 3x^3 - 2$	$x^2 + y^2 = 9$	$y = 2^{-x}$
$x^2 + y^2 = 81$	$y = -\frac{1}{4}x^3$	$y = 3 - (x+4)^2$



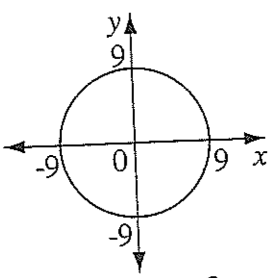
$y=(x+2)(x-2)$



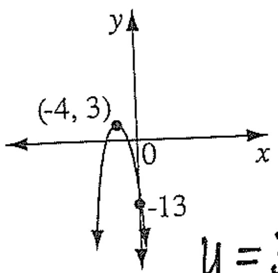
$xy = -7$



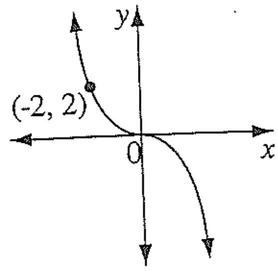
$y = 2^{-x}$



$x^2 + y^2 = 81$

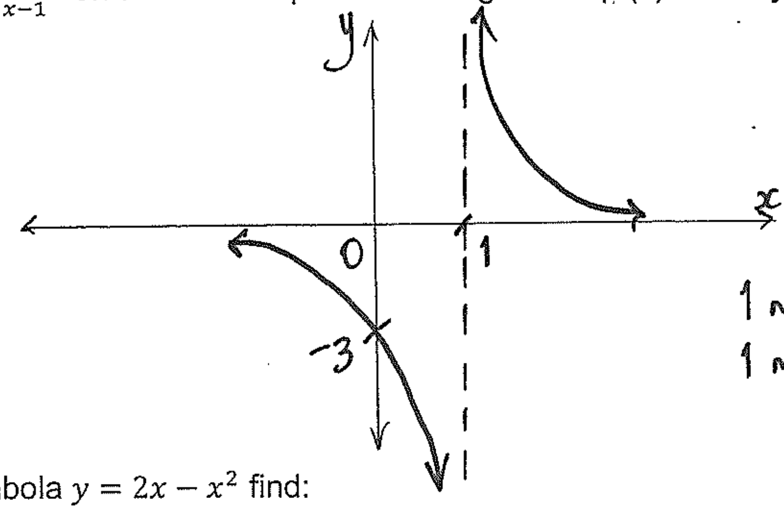


$y = 3-(x+4)^2$



$y = -\frac{1}{4}x^3$

(b) Sketch $y = \frac{3}{x-1}$ on the number plane showing intercept(s) and asymptote(s) [2]



1 mark for intercepts/asym

1 mark for shape

(c) For the parabola $y = 2x - x^2$ find: [4]

1 i) x *intecept(s)*

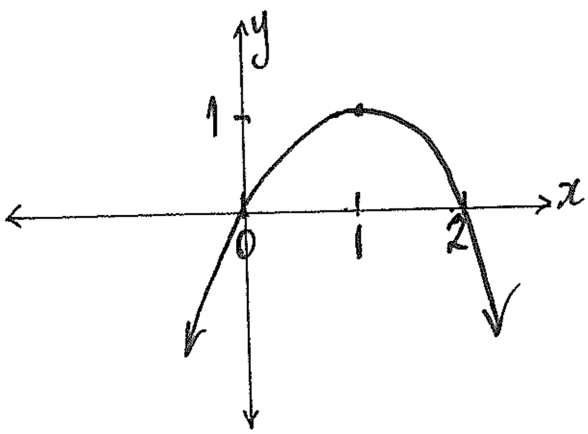
$$2x - x^2 = 0$$
$$x(2-x) = 0$$
$$x = 0, 2 \quad \text{OR} \quad (0,0)\ (2,0)$$

1 ii) y *intercept(s)*

When $x = 0,\ y = 0$ OR $(0,0)$

1 iii) *coordinates of the vertex* $(1,1)$

1 iv) Sketch



(d) Find the equation of the cubic curve that passes through (1,3) and cuts the y axis at -4. [2]

$$y = ax^3 - 4$$

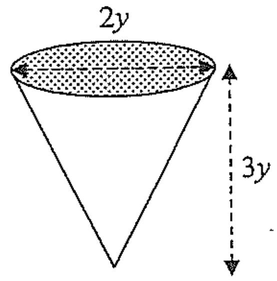
$$\left.\begin{aligned} \text{Sub }(1,3)\quad 3 &= a(1)^3 - 4 \\ a &= 7 \end{aligned}\right\} \text{1 mark}$$

$$\therefore\ y = 7x^3 - 4 \longleftarrow \text{1 mark}$$

Q12. (13 marks)

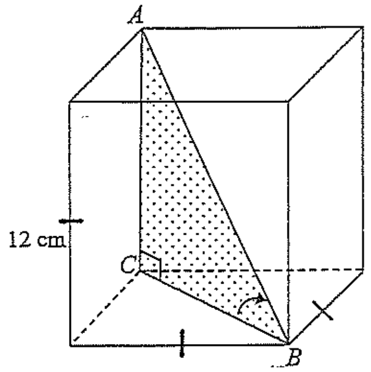
(a) In terms of y and π write a simplified expression for the volume of this cone. [2]

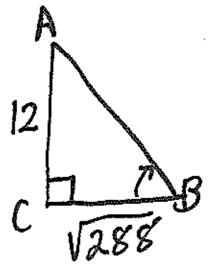
radius = y



$$V = \pi r^2 h \times \frac{1}{3}$$
$$= \frac{1}{3} \times \pi \times y^2 \times 3y \leftarrow \text{1 mark}$$
$$= \pi y^3 \leftarrow \text{1 mark}$$

(b) A right triangle ABC sits inside a cube of side $12cm$, as shown.
Find the size of $Angle\ ABC$ to the nearest degree. [2]

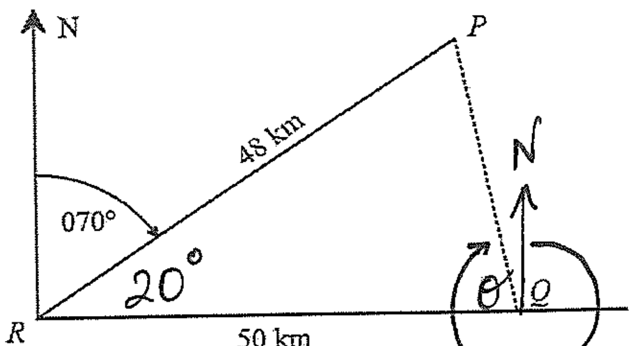




$$\tan B = \frac{12}{\sqrt{288}}$$
$$\therefore \angle B = 35^\circ \leftarrow \text{1 mark}$$

$CB = \sqrt{288}$
Using Pythagoras — 1 mark

(c) The diagram shows a town Q which is $50km$ due east of town R.
The town P is $48km$ from R on a bearing of 070° [4]



i) Show that the distance PQ is $17km$, to the nearest km.

$$PQ^2 = 48^2 + 50^2 - 2(48)(50) \times \cos 20^\circ \leftarrow \text{1 mark}$$
$$= 293.4754\ldots$$

$$\therefore PQ = 17.13\ldots \leftarrow \text{1 mark}$$
$$\doteqdot 17 \text{ km}$$

ii) What is the bearing of P from Q?

1 mark

$$\frac{\sin\theta}{48} = \frac{\sin 20^\circ}{17.13}$$

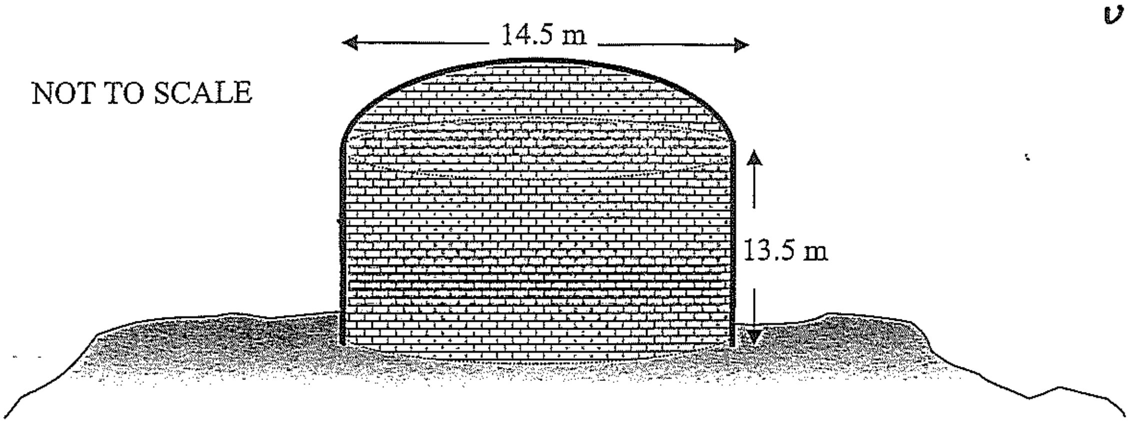
$$\sin\theta = 0.9583...$$

$$\theta = 73^\circ$$

$$\text{Bearing} = 270^\circ + 73^\circ = 343^\circ$$

(d) The diagram shows an ancient Greek building with diameter 14.5 *metres*. [5]
The height of the lower cylindrical section is 13.5 *metres*.
The cylinder is surmounted by a hemispherical dome.

1 mark for units



i) Calculate the volume of the building to the nearest cubic metre.

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \pi r^2 h$$

$$= \frac{1}{2}\left(\frac{4}{3}\pi \times 7.25^3\right) + \pi \times 7.25^2 \times 13.5$$

$$= 798.128... + 2229.254...$$

$$= 3027.382..$$

$$= 3027 \text{ m}^3$$

ii) Calculate the surface area of the building not including the circular base. (correct to 3 significant figures)

$$SA = \frac{1}{2}(4\pi r^2) + 2\pi r h$$

$$= \frac{1}{2}(4 \times \pi \times 7.25^2) + 2 \times \pi \times 7.25 \times 13.5$$

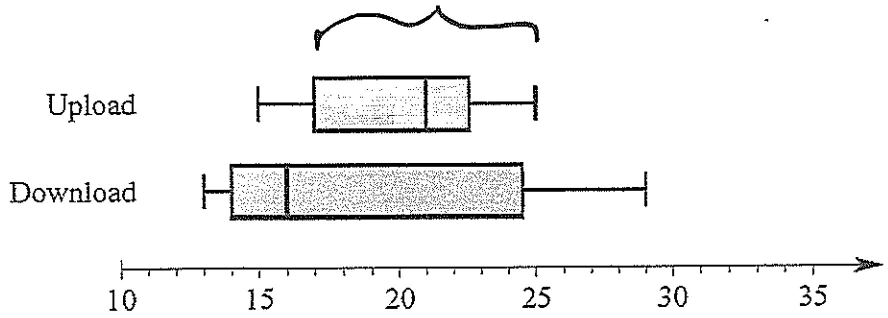
$$= 330.259.. + 614.9667....$$

$$= 945.225...$$

$$\doteq 945 \text{ m}^2$$

Q13. (12 marks)

(a) The parallel box and whisker plots show the amount of data (in MG) uploaded and downloaded daily.



i) Compare the two sets of data referring to the median and IQR. [2]

Median : Upload = 21 , Download = 16

IQR : Upload = 22.5 − 17 = 5.5 Download = 24.5 − 14 = 10.5

Median for upload is higher

IQR for download is greater ∴ less consistent amounts.

ii) Describe the shape of the *Download* distribution [1]

Positively skewed

iii) What percentage of the *Upload* was between 17 and 25? 75% [1]

(b) Two exams have a mean of 60%. Test A has a standard deviation of 12%. Test B has a standard deviation of 20%. Answer the following questions. [4]

i) In which test is a result of 80% stronger? Test A

ii) In which test would there more likely to be an outlier? Test B

iii) If 2% needed to be added to each of the scores in Test A, how would this affect the mean and standard deviation.(use the words *increased, decreased, unchanged*)

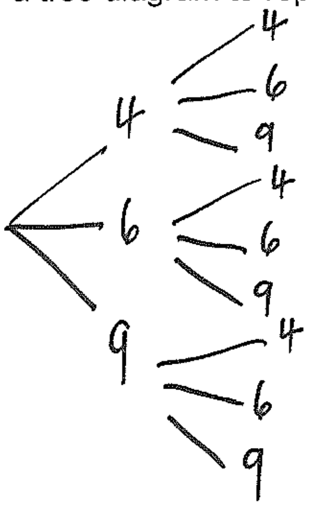
Mean would increase by 2% ← 1 mark

SD would be unchanged ← 1 mark

(c) A two digit number is formed using the digits 4, 6 and 9. [4]

The same digit can be repeated.

i) Draw a tree diagram to represent all the possible outcomes.



ii) How many in the sample space? 9

iii) What is the probability of forming a number where both digits are the same?

$\frac{3}{9} = \frac{1}{3}$

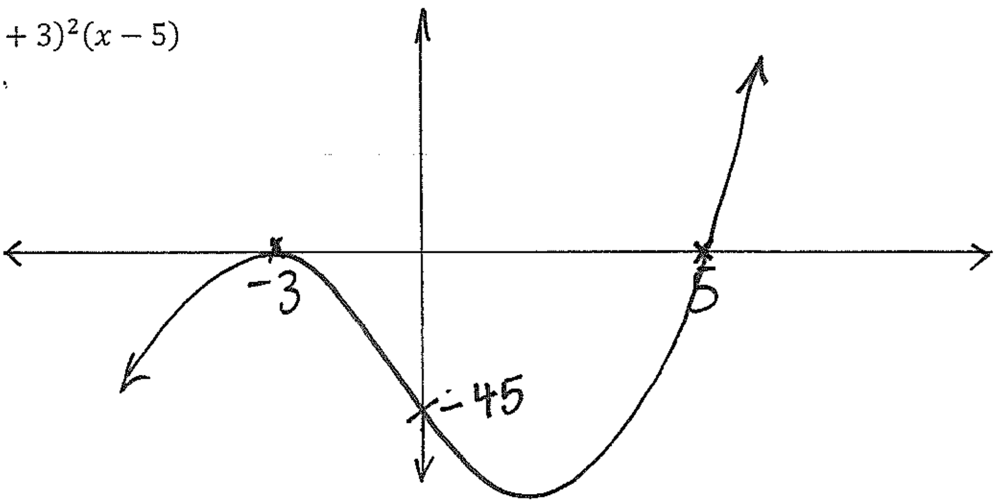
iv) If no repeats where allowed, what is the probability of forming a number greater than 90?

$\frac{2}{6} = \frac{1}{3}$

Q14. (13 marks)

(a) Sketch the curve clearly showing the x intercepts. [2]

$y = (x + 3)^2(x - 5)$



(b) Factorise the polynomial $P(x) = 2x^3 - 7x^2 - 3x + 18$ into its linear factors. [3]

$P(2) = 2(2)^3 - 7(2)^2 - 3(2) + 18$

$= 0$

$\therefore (x-2)$ is a factor ← 1 mark

$$\begin{array}{r l}
 & 2x^2 - 3x - 9 \quad \leftarrow \text{1 mark} \\
x-2 \,\big) & 2x^3 - 7x^2 - 3x + 18 \\
 & 2x^3 - 4x^2 \\
\hline
 & -3x^2 - 3x \\
 & -3x^2 + 6x \\
\hline
 & -9x + 18 \\
 & -9x + 18 \\
\hline
 & 0
\end{array}$$

$2x^2 - 3x - 9 = (2x+3)(x-3)$

$P(x) = (x-2)(2x+3)(x-3)$

1 mark

(c) State the domain and range of the function $f(x) = x^2 - 7$ [2]

Domain : all real x

Range : $y \geq -7$

(d) If $f(x) = x + \frac{3}{x}$ find: [4]

i) $f(6) = 6 + \frac{3}{6}$

$= 6\frac{1}{2}$ ← 1 mark

ii) $f(6) - f(-1)$

$= 6\frac{1}{2} - (-1 + \frac{3}{-1})$

$= 6\frac{1}{2} + 1 + 3$

$= 10\frac{1}{2}$ ← 1 mark

ii) Show $f\left(\frac{1}{p}\right) = \frac{1+3p^2}{p}$

2 marks

$f(\frac{1}{p}) = \frac{1}{p} + \frac{3}{\frac{1}{p}}$ ← 1 mark

$= \frac{1}{p} + 3p$ ← 1 mark.

$= \frac{1+3p^2}{p}$

(e) If $f(x) = \sqrt{(5-x)} - 1$ find the inverse function $f^{-1}(x)$ [2]

$y = \sqrt{(5-x)} - 1$

$x = \sqrt{(5-y)} - 1$

$x+1 = \sqrt{(5-y)}$

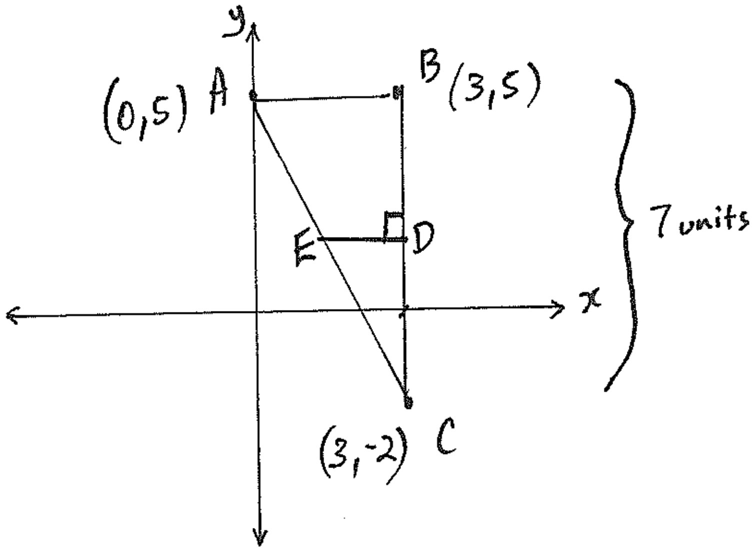
$(x+1)^2 = (5-y)$

$x^2 + 2x + 1 = 5 - y$

$y = -x^2 - 2x + 4$

Q15. (7 marks)

(a) Given $A\ (0,5),\ B(3,5)\ and\ C(3,-2)$, use the number plane and answer the following questions.



i) Find the equation of AC in general form. [2]

$M_{AC} = \frac{5--2}{0-3} = \frac{7}{-3}$

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{7}{3}(x - 0)$

$3y - 15 = -7x$

$7x + 3y - 15 = 0$

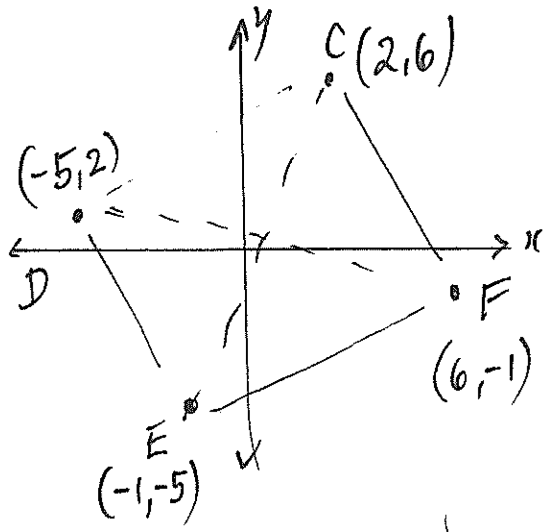
ii) What is the equation of the perpendicular bisector of BC? (label this interval DE) [1]

$y = 1\frac{1}{2}$

(b) A quadrilateral has vertices $C\ (2,6),\ D\ (-5,2),\ E\ (-1,-5)\ and\ F\ (6,-1)$.

Show that the quadrilateral is a square by using the properties of the diagonals. [4]

Square : diagonals bisect each other at 90°
diagonals are equal



$$M_{EC} = \frac{-5-6}{-1-2} = \frac{11}{3}$$

$$M_{DF} = \frac{-1-2}{6--5} = \frac{-3}{11}$$

$$\frac{11}{3} \times -\frac{3}{11} = -1$$

$\therefore EC \perp DF$

1 mark

Midpoint of EC $= \left(\frac{2+-1}{2}, \frac{6+-5}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$

Midpoint of DF $= \left(\frac{-5+6}{2}, \frac{2-1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$

1 mark

$\therefore$ diagonals bisect each other at right angles

$$d_{EC} = \sqrt{(2--1)^2+(6--5)^2} = \sqrt{9+121} = \sqrt{130}$$

$$d_{DF} = \sqrt{(6--5)^2+(-1-2)^2} = \sqrt{121+9} = \sqrt{130}$$

1 mark

$\therefore EC = DF$

$\therefore$ diagonals are equal in length.

END OF EXAM

1 mark conclusion

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi r h$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Trigonometric Functions

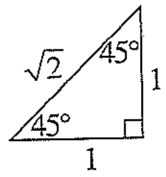
$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

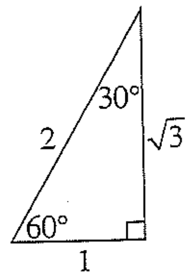
$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$





Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Logarithmic and Exponential Functions

$$\log_a x = \frac{\log_b x}{\log_b a}$$