Carlingford High School Mathematics Assessment Task

Course: Preliminary Mathematics Advanced
Task: Preliminary Assessment Task 1 (Assignment)
Term 1 2020

Notification Date: 3/3/2020
Due Date: 17/3/2020
Weighting: 15%

Student Number:

Topics assessed

- F1-1: Algebraic techniques
- F1-2: Introduction to functions
- F1-3: Linear, quadratic and cubic functions
- F1-4: Further functions and relations

Outcomes assessed

- A student uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to solve, and where appropriate, compare alternative solutions to problems (MA11-1)
- A student uses the concepts of functions and relations to model, analyse and solve practical problems (MA11-2)
- A student uses appropriate technology to investigate, organise, model and interpret information in a range of contexts (MA11-8)
- A student uses reasoning to support conclusions which are appropriate to the context (MA11-9)

Nature of the task

- This assignment involves the use of technology, namely the graphing applications Geogebra or Desmos. Please refer to the guides *Hints for using Geogebra* and/or *Hints for using Desmos* on the *Google Classroom* set up for the assignment. The code is **v5pocsq**
- Sections of the assignment not involving graphing applications may be handwritten, and computer generated graphs with handwritten annotations may also be used to answer some questions.
- This assignment is to be submitted electronically to the *Google Classroom*, no later than 8:45am on the due date. A hard copy of the task is to be submitted to your teacher at the beginning of your Mathematics lesson on the same day.
- **Questions 1 and 3 contain options. If you do not use your assigned options, you may lose marks.**
- Questions regarding this task may be submitted to the *Google Classroom*.

Marking criteria

- Your task will be assessed against the four assessment outcomes. You are advised to refer to the outcomes while completing this task.
- You will receive a copy of the marking rubric that will be used to determine your mark for each question. The marking rubric will clarify the scope and expectations of each part of the task.

Feedback provided

- Your teacher will provide feedback outlining strengths and areas for improvement to build on knowledge, understanding and skills for future learning.

Assessment Guidelines/Policy

- Please refer to *Assessment Guidelines and Policies* from the *HSC Assessment Book 2020*. See your Deputy Principal if you do not have a copy.

Question 1

A rectangular sheet of cardboard has a perimeter of P cm ($P > 16$). Let w and x be the lengths of the sides.

a) Find an equation for w in terms of x and P.

Squares with 2 cm sides are cut out from each corner and the sides folded to form an open rectangular box.

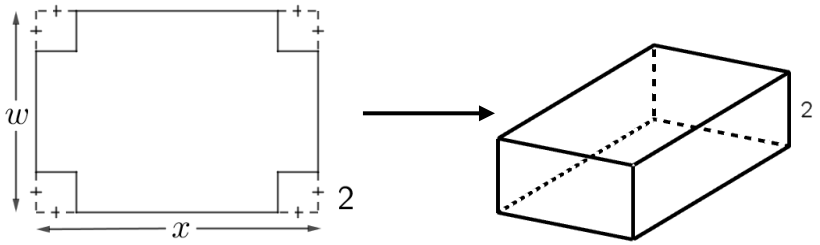

Fig 1. Not to scale.

b) Show that the volume V of the box is given by the equation $V(x) = -2x^2 + Px + 32 - 4P$.

For parts **c)** to **j)**, suppose that $P = 120$ cm.

c) Find an equation for V in the form $V(x) = a(x - h)^2 + k$.

d) Find the domain of $y = V(x)$ using interval notation.

e) Draw a careful one third page sketch of the parabola $y = V(x)$ over the domain found in part **d)**, showing all important features.

f) What does the turning point of this graph represent? What do you notice about the shape of the box for this value of x?

g) The side length of the squares cut from the corners is changed to 1 cm.
Find the new turning point and comment on any similarities or differences to your answer to part **f)**.

For parts **h)-k)**, let $s > 0$ be the side length of the corner squares cut from the sheet.

h) Explain why s must be less than 15 cm.

i) Write expressions for the length, width and height of the box in terms of s and x.

j) Using the graphing application Geogebra or Desmos, explore what happens to the volume V of the box when s varies. You may find the use of sliders helpful. Write a paragraph describing your findings, and include a link(s) to your graph or graphs.

k) Repeat part **j)** for a sheet with the **highlighted** perimeter $P = 66,\ 78,\ 84,\ 90$ cm.

Comment on any similarities or differences you observe to your answer in part **j)**.

Question 2

According to the Collins dictionary**,** a **spiral** is a shape which winds round and round, with each curve above or outside the previous one.

The **Fibonacci spiral** is drawn by connecting quarter circles such that each successive quadrant has radius equal to the next Fibonacci number. Fibonacci spirals occur in nature, for example in shells, flowers and pinecones.

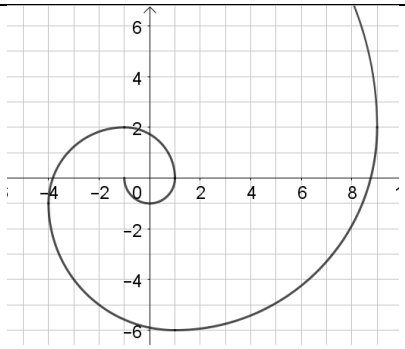

Fig 2. Fibonacci spiral

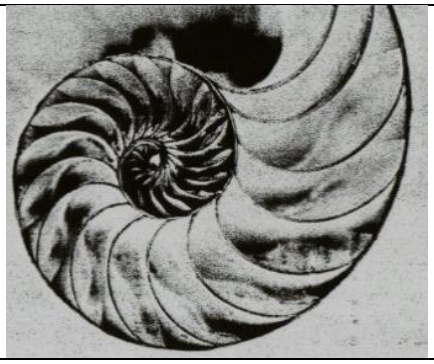

Fig 3. Nautilus shell

a) Is the Fibonacci spiral shown in Figure 2 a function? Explain your answer.

b) List the first 10 Fibonacci numbers and explain how the next Fibonacci number can be found.

c) A variation of the Fibonacci spiral involving semicircles is shown in Figure 4 below.
Write down the equations of the smallest 4 semicircles.

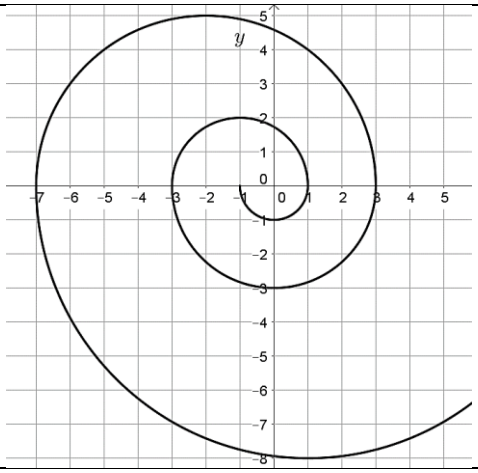

Fig. 4

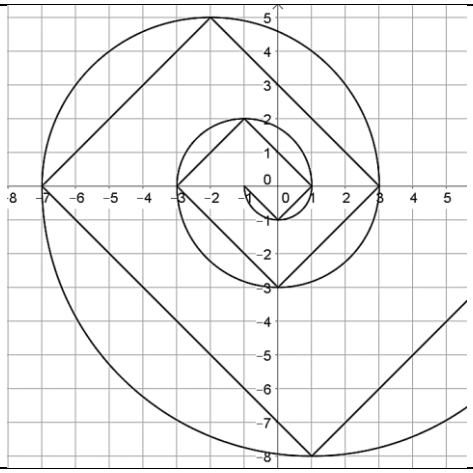

Fig. 5

d) What is the centre of the 6th semicircle? Explain how you found your answer.

Figure 5 shows an approximation to the spiral using straight line segments. **Pairs** of these segments can be defined by absolute value equations, with restricted domains.

e) Show that the smallest semicircle is approximated by a pair of line segments satisfying the equation $y = -1 + |x|$, for $-1 \leq x \leq 1$.

f) Find the absolute value equations that describe the next two pairs of lines, showing working to support your answer.

g) Your friend tells you that $y = -13 + |x|$ is the equation of the pair of line segments approximating the semicircle with radius 13. Are they correct? Explain your answer, and if they are wrong give the correct equation.

Question 3

a) Create your own spiral inspired by the Fibonacci numbers and submit a graph showing a continuous section of the spiral including **at least 5** y-intercepts. Your spiral should also satisfy the following guidelines.

- It must pass through the **highlighted** point $(1,1)$, $(-1,-1)$, $(1,3)$, $(-1,-3)$
- It may not cross its own path at any point.
- You may use segments which are
    - Straight
    - Circular arcs
    - Parabolic
    - [redacted]
    - A combination of the above

- Your graph should fill at least half an A4 page. Each piece of the spiral should be labelled with its Cartesian equation. It is **not** necessary to find the exact values of the coordinates of the points of intersection of the pieces.
- Your graph may be hand drawn on 5mm grid paper or generated using Geogebra or Desmos. If you chose to submit a hand drawn graph, you are strongly encouraged to use Geogebra or Desmos to model your spiral.

b) Explain how the spiral is related to the Fibonacci numbers, and how it would continue for y-values higher than those shown on your graph.

c) Briefly describe how you developed your spiral. Explain why you chose the functions which were included in your spiral and how you made any necessary adjustments to ensure that your graph passed through the point specified in part **a)**. You should include preliminary sketches or computer generated graphs to support your answer.

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Marking Rubric

Question	1 mark	2 marks	3 marks	Mark
1 a)	Correct expression, with working.			
1 b)	Full, correct derivation.			
1 c)	One error in working or equivalent.	Correct equation, with full working.		
1 d)	Correct answer			
1 e)	Graph has two features unlabelled or incorrect OR one error and is too small, rough or incorrect shape.	One feature unlabelled or incorrect or graph too small, rough or incorrect shape.	All features present and labelled, graph smoothly drawn of correct shape.	
1 f)	Correct answer to one part of question.	Correct answer to both parts of question.		
1 g)	Correct answer to one part of question.	Correct answer to both parts of question.		
1 h)	Correct answer			
1 i)	3 correct expressions			
1 j)	Evidence of some investigation and accurate description of observations or thorough investigation without correct conclusions.	Evidence of thorough investigation with appropriate use of technology and relevant conclusions.	Skilled use of technology to conduct thorough investigation, with link to graph(s). Well reasoned and supported conclusions which address all parts of question.	
1 k)	Evidence of some investigation and accurate description of observations or thorough investigation without correct conclusions.	Evidence of thorough investigation with appropriate use of technology and relevant conclusions.		
				/19
2 a)	Correct answer with explanation.			
2 b)	Single error in list or correct list with unclear or incorrect explanation.	Correct list and clear, correct explanation.		
2 c)	Two correct equations with working.	Three correct equations or one error consistently carried through. Relevant working shown.	Correct answer with correct working shown.	

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2 d)	Incorrect answer due to single error with clear explanation (or correct answer with some explanation/ working).	Correct answer with clear explanation/working.		
2 e)	Partial demonstration	Complete and clear demonstration		
2 f)	1 correct answer with evidence, or single error carried through both answers. No marks for answers only.)	Two correct answers with clear evidence. Use of technology for trial and error approach is satisfactory, if it is well documented.		
2 g)	Correct answer with partial explanation.	Correct answer with full explanation.		
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3 a)	No more than 3 separate errors.	One requirement unfulfilled **or** up to two segments of graph poorly drawn **or** given with incorrect equations.	Spiral fulfils all requirements. All segments are clearly labelled with correct equations and presentation is of a high standard.	
3 b)	Description of how to continue spiral clear **or** relationship to Fibonacci numbers clearly established.	Description of how to continue spiral clear. Relationship to Fibonacci numbers clearly established.	Continuation of spiral follows logically from the section shown, and is clearly described. Relationship to Fibonacci numbers significant and clearly established.	
3 c)	Reasonable description of process given, with some evidence of consideration of suitability of choices.	Description and supporting evidence demonstrate thorough consideration of guidelines and progress towards completion of task, **or** as for one mark with strong use of technology	Strong use of technology to solve problem. Description and supporting evidence demonstrate thorough consideration of guidelines and progress towards completion of task	
	Evidence of creative excellence in development and/or presentation of spiral.			
				/10
Total				**/43**

Student Number: ______________________________