CARLINGFORD HIGH SCHOOL

Year 11 Mathematics Advanced

HSC Assessment Task 1
Term 4 2021

Time allowed: 55 minutes

Student Number: _____ **Solutions**

- ☐ Mrs Strilakos (11MAX1)
- ☐ Ms Bennett (11MAX2)
- ☐ Mr Cheng (11MAX3)
- ☐ Mrs Tang (11MAXA)
- ☐ Mr Fardouly (11MAA4)
- ☐ Mr Davis (11MAA5)
- ☐ Mrs Blakeley (11MAAB)
- ☐ Mr Wilson (11MAAC)
- ☐ Mr Gong (11MAAD)

MBla Exp & Logs Q1–4

GF Exp & Logs Q5–7a, b

AG Exp & Logs Q7 c, d
Disc stats Q1, 2

PW Disc stats Q3, 4

Instructions:

- All questions should be attempted
- Show ALL necessary working
- Marks may not be awarded for careless or badly arranged work
- Only NESA-approved calculators may be used
- Use black pen only (pencil may be used for diagrams only)

Topic	Question	Mark
Exponential & Logarithmic Functions	1	/3
	2	/2
	3	/2
	4	/3
	5	/2
	6	/4
	7	/7
	Total	/23
Descriptive Statistics	1	/2
	2	/4
	3	/6
	4	/5
	Total	/17
Exam Mark	/40	%

Exponential & Logarithmic Functions (23 marks)

Q1. Simplify the following:

a) $log_3 27$ [1]

$= log_3 3^3$

$= 3 \times log_3 3$

$= 3$

b) $log_a x^2 y + log_a yz - log_a xyz^2$ [2]

$= log_{10}\left(\frac{x^2 y^2 z}{xyz^2}\right)$

$= log_{10}\left(\frac{xy}{z}\right)$

Q2. If $x = log_{10} 3$ and $y = log_{10} 2$, express $log_{10} 3.6$ in terms of x and y. [2]

$log_{10} 3.6 = log_{10}\left(\frac{36}{10}\right)$

$= log_{10} 36 - log_{10} 10$

$= log_{10} 2^2 + log_{10} 3^2 - 1$

$= 2 log_{10} 2 + 2 log_{10} 3 - 1$

$= 2y + 2x - 1$

Q3. Solve $2log_{10} x = 3log_{10} 5 - log_{10} x$ [2]

$log_{10} x^2 = log_{10} 5^3 - log_{10} x$

$log_{10} x^2 = log_{10}\left(\frac{125}{x}\right)$

$x^2 = \frac{125}{x}$

$x^3 = 125$

$\therefore x = 5$

Q4. Solve $log_e 6 + log_e(x + 2) = log_e(x + 5) + log_e x$ **[3]**

$6(x+2) = x(x+5)$

$6x + 12 = x^2 + 5x$

$x^2 - x - 12 = 0$

$(x+3)(x-4) = 0$

$x = -3$ or 4

By substituting $x = -3$ or 4 into the original equation, we qualified the only solution is $x = 4$.

Q5. Differentiate $y = \frac{e^{x^2}}{x^2-1}$ **[2]**

Now $\frac{dy}{dx} = \frac{(x^2-1)(2xe^{x^2}) - e^{x^2}(2x)}{(x^2-1)^2}$

$= \frac{2xe^{x^2}(x^2-1-1)}{(x^2-1)^2}$

$= \frac{2xe^{x^2}(x^2-2)}{(x^2-1)^2}$

Q6. For the curve $y = -xe^{2x}$

a) Find the **exact value** of the gradient of the tangent to the curve at $x = -1$. **[2]**

Now $\frac{dy}{dx} = e^{2x}(-1) + (-x)(2e^{2x})$

$= e^{2x}(-1-2x)$

When $x = -1$ then

$\frac{dy}{dx} = e^{-2}(-1-2x-1)$

$= e^{-2}$ or $\frac{1}{e^2}$

b) Find the equation of the normal to the curve at $x = -1$. **[2]**

For normal: $\frac{1}{e^2} \times m_N = -1$ when $x=-1$ then $y = e^{-2}$

$\therefore m_N = -e^2$ $\therefore$ The point is $(-1, e^{-2})$

Now the equation of normal: $y - e^{-2} = -e^2(x+1)$

$y - e^{-2} = -e^2x - e^2$

$\therefore y = -e^2x + e^{-2} - e^2$

Q7. The number of bacteria in a culture is given by $N(t) = Ae^{kt}$.

If 3000 bacteria increase to 9000 after 6 hours.

a) What is the value of A? **[1]**

When $t=0$, $N=3000$ then $Ae^{K\times0} = 3000$ $\therefore A = 3000$

b) Find the exact value of k. **[2]**

When $t=6$ & $N=9000$ then $9000 = 3000e^{6K}$

$3 = e^{6K}$

$\log_e 3 = 6K$ $\therefore K = \frac{\log_e 3}{6}$

c) Find the number of bacteria present after 15 hours, correct to the nearest hundred. **[2]**

When $t=15$ then $N = 3000e^{\frac{15\times\log_e 3}{6}}$

$= 46765.3718\ldots$

$\therefore N \doteq 46800$

d) Find the **rate of change** (bacteria/hour) after 15 hours. **[2]**

$\because N = Ae^{kt}$ then $\frac{dN}{dt} = AKe^{kt} = \frac{3000}{6}\times\log_e 3\, e^{\frac{t}{6}\log_e 3}$

$= 500\log_e 3\, e^{\frac{t}{6}\times\log_e 3}$

When $t=15$ then $\frac{dN}{dt} = 500\log_e 3\, e^{\frac{15}{6}\times\log_e 3}$

$= 8562.835358\ldots$

$\therefore$ Rate of change $\doteq$ 8563 bacteria/hour.

Descriptive Statistics (18 marks)

Q1. Use **two** of the following words to describe the type of each data.

categorical **continuous** **discrete** **nominal** **numerical** **ordinal**

a) The listing of the top 10 music downloads for the week. **[1]**

ordinal

b) The number of spectators attending a state origin game. **[1]**

discrete

Q2. A teacher has organised a stem-and-leaf plot for the results of a class test out of 100 marks.

Stem	Leaf
3	3 8
4	4 6 9
5	3 5 8
6	8 8
7	2 7 8
8	2 2 4 4 5 6

Key: 3|8 = 38 out of 100

a) Calculate the mean, correct 1 decimal place. **[1]**

From calculator, $\bar{x} = 65.36842105\ldots$

$\therefore \bar{x} \doteq 65.4$

b) Calculate the standard deviation, correct to 1 decimal place. **[1]**

From calculator, $\sigma = 17.20223525\ldots$

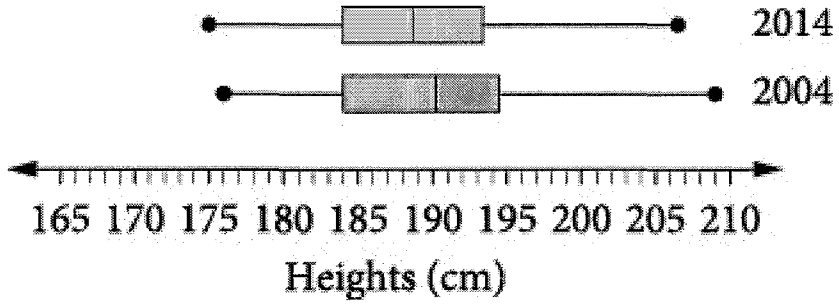
$\therefore \sigma \doteq 17.2$

c) Jill was absent on the day of the test. When she returned to school and sat the test, she scored 100 out of 100.

When this additional score is included, **describe** how the mean and standard deviation have changed, and explain the reasons why? **[2]**

The mean and standard deviation both increased, because the score 100 is much higher than the mean. Also it is an extreme value.

Q3. The parallel box plots below represent the heights of NRL players for the years 2004 and 2014.



a) i) For the 2004 data, write the **lower quartile**, **upper quartile** and **interquartile range**. **[2]**

Lower quartile = 184

Upper quartile = 194

IQR = 194 − 184 = 10

ii) Determine whether the 2004 data includes an **outlier.**

Justify your answer with calculations. **[2]**

Lower fence = 184 − 1.5 × 10 = 169

Upper fence = 194 + 1.5 × 10 = 209

Thus no heights are less than 169cm nor larger than 209cm

Therefore, there are no outliers present.

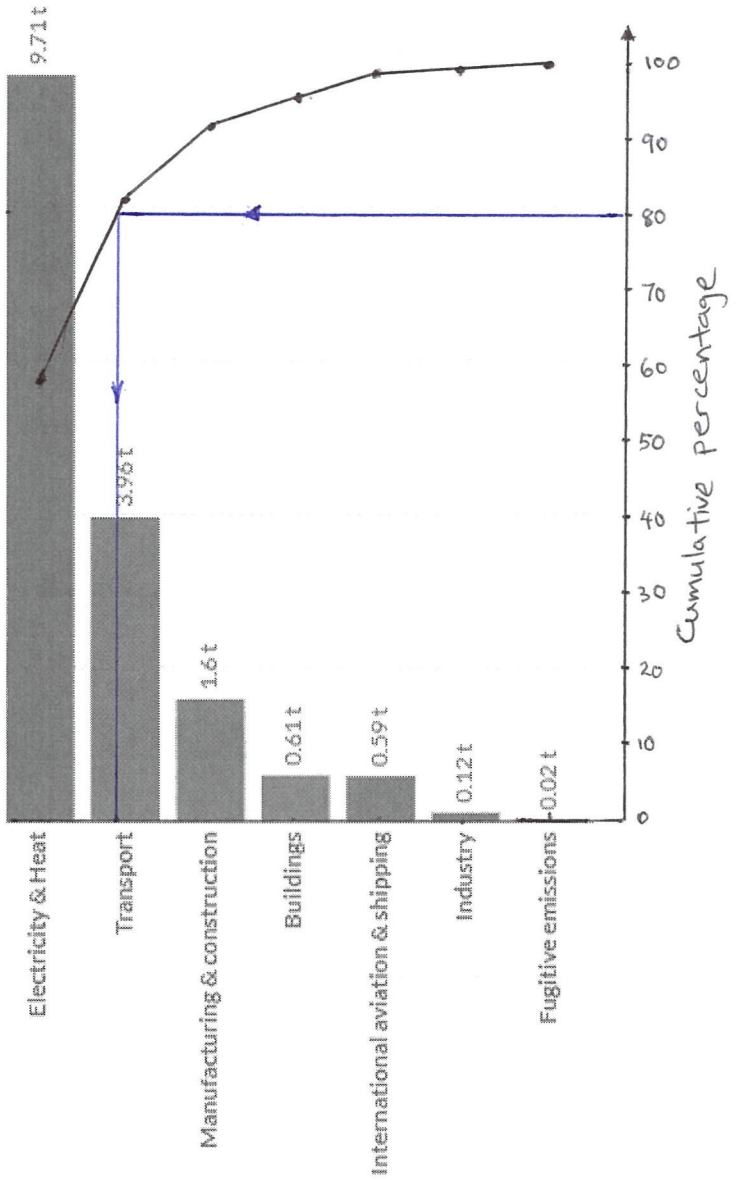
b) Use the **interquartile range** and **median** for both 2004 and 2014 to **compare** the two data sets. **[2]**

For 2004: median = 190 cm, IQR = 10

For 2014: median = 188.5 cm, IQR = 193 − 184 = 9

Looking at the median and IQR values, there was little difference in the 10 year period.

Q4. The following graph shows the amount of per capita carbon dioxide (CO_2) emissions by sector in Australia in 2016, measured in tonnes of carbon dioxide.



a) Complete the **cumulative percentage** column in the table on the next page. **[1]**

b) Use a ruler and appropriate scale to complete the **Pareto Chart** using the above graph. **[2]**

c) Use the Pareto principle to determine which sectors the government should focus on to decrease emissions.

Show on the Pareto chart how you obtained this information. **[2]**

From the graph, based on Pareto principle, the government should focus on decreasing emissions with transport, Electricity and heat.

Sector	Emissions (CO_2) (tonnes)	Percentage	Cumulative Percentage
Electricity and Heat	9.71	58.5	58.5
Transport	3.96	23.8	82.3
Manufacturing and Construction	1.6	9.6	91.9
Buildings	0.61	3.7	95.6
International Aviation and Shipping	0.59	3.6	99.2
Industry	0.12	0.7	99.9
Fugitive Emissions	0.02	0.1	100
Total	**16.61**	**100**	

End of Exam:)