Carlingford High School

2017

TERM 2

Student Number

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Mathematics

- **General Instructions**
- Working time – 50 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 2 and 3, show relevant mathematical reasoning and/or calculations

Total Marks – 44

Section I Page 2

4 marks

- Multiple Choice
- Circle the correct answer

Section II Pages 3– 4

40 marks

- Attempt Questions 2 – 3

	Exponentials and Logarithms	Trigonometric Functions	Total
Q1	/2	/2	
Q2	/20		
Q3		/20	
Total			/44

QUESTION 1 (4 marks) CIRCLE THE CORRECT ANSWER

(a) Which expression is equivalent to $4 + \log_2 x$?

A. $\log_2 2x$

B. $\log_2 16x$

C. $4\log_2 2x$

D. $\log_2 8x$

(b) What is the solution to the equation $\log_2(x-2) = 5$?

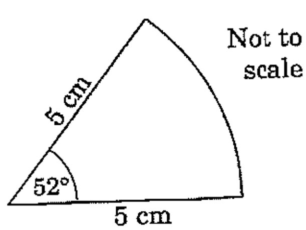
A. 8

B. 12

C. 27

D. 34

(c)



The area of this sector, correct to the nearest centimetre is:

A. 11cm^2
B. 5cm^2
C. 15cm^2
D. 12cm^2

(d) How many solutions does $(2sinx - 1)(tanx + 3) = 0$ have between 0 and 2π?

A. 1

B. 2

C. 3

D. 4

QUESTION 2 (20 marks)

(a) If $y = \frac{\log x}{\log 2}$, express x in terms of y. **1**

(b) Solve $\log_{10} x - \log_{10}(x - 1) = 1$ **2**

(c) Write log 2 + log 4 + log 8 +......log 512 in the form alog b where a and b are positive integers. **2**

(d) Differentiate with respect to x:

(i) $y = e^{4x}$ **1**

(ii) $y = \ln(x^3)$ **1**

(iii) $y = x^2 e^{3x}$ **2**

(e) Find the exact value for:

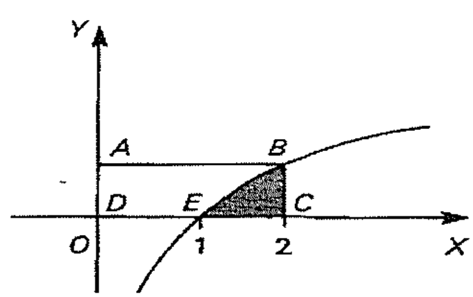
(i) $\int_0^2 e^{\frac{x}{2}}\, dx$ **2**

(ii) $\int_{-3}^{-1} \frac{2}{1-3x}\, dx$ **2**

(f) Find the equation of the tangent to the curve $y = \ln(2x - 5)$ at the point (3,0). **2**

(g) Use the trapezoidal rule with two strips to find the approximate area under the curve $y = e^x$ which is bounded by the x axis and the lines $x = 1$ and $x = 5$. **2**

(h) The diagram shows the graph of $\log_e x$.



(i) Find the co-ordinates of point A in exact form. **1**

(ii) Find the shaded area. **2**

QUESTION 3 (20 marks)

(a) Express 25° in radians in terms of π. **1**

(b) Find in degrees and minutes the angle subtended at the centre of a circle of radius 8cm by an arc length 6cm long. **2**

(c) Find solutions, in terms of π, for $2\sin x = 1$ for $0 \leq x \leq 2\pi$ **2**

(d) Consider the function $y = 3cos2x$

(i) Draw a neat sketch of $y = 3cos2x, -\pi \leq x \leq \pi$ **2**

(ii) Find the period of $y = 3cos2x$ **1**

(iii) Find the amplitude of $y = 3cos2x$ **1**

(e) Differentiate with respect to x:

(i) $y = (1 - cosx)^5$ **2**

(ii) $y = xsinx$ **2**

(f) Find the following indefinite integrals:

(i) $\int (sin2x + cos3x)\ dx$ **2**

(ii) $\int cos\pi x\ dx$ **2**

(g) (i) Draw a neat sketch of $y = secx$ **1**

(iii) The area $\int_0^{\frac{\pi}{6}} \sec x\ dx$ is rotated about the x axis. Find the volume so generated. **2**

-END OF PAPER-

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180^\circ$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

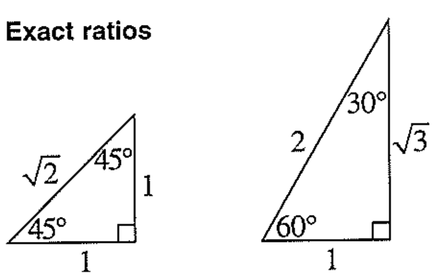
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point–gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

*n*th term of an arithmetic series

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

*n*th term of a geometric series

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Mathematics

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\frac{dy}{dx} = F'(u)\frac{du}{dx}$

If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\frac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\frac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\frac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

Carlingford High School 2017
Term 2 HSC Mathematics Exam
Answers

Q2 a-d AG

Q2 e-g LW

2h + Q3 a-c GF

3 d-e KC

Q3 f-g PW

Question 1

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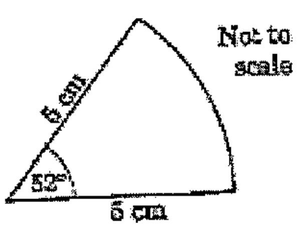

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(d) How many solutions does $(2sinx-1)(tanx+3)=0$ have between 0 and 2π?

A. 1

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C. 3

D. 4

QUESTION 2

a) $y = \dfrac{\log x}{\log 2}$

$\log x = y \log 2$

$\log x = \log 2^y$

$x = 2^y$ (1)

b) $\log_{10} x - \log_{10}(x-1) = 1$

$\log_{10} \dfrac{x}{x-1} = \log_{10} 10$

$\dfrac{x}{x-1} = 10$

$x = 10x - 10$

$9x = 10$

$x = \dfrac{10}{9}$ (2)

c) $\log 2 + \log 4 + \log 8 + \cdots + \log 512$

$= \log 2 + 2\log 2 + 3\log 2 + \cdots + 9\log 2$

$= 45 \log 2$ (2)

d) i) $y = e^{4x}$

$\dfrac{dy}{dx} = 4e^{4x}$ (1)

ii) $y = \ln x^3$

$= 3\ln x$

$\dfrac{dy}{dx} = \dfrac{3}{x}$ (1)

iii) $y = x^2 e^{3x}$

$u = x^2$, $u' = 2x$, $v = e^{3x}$, $v' = 3e^{3x}$

$\dfrac{dy}{dx} = vu' + uv'$

$= e^{3x} \cdot 2x + x^2 \cdot 3e^{3x}$

$= xe^{3x}(2 + 3x)$ (2)

e) i) $\int_0^2 e^{\frac{x}{2}}\,dx$

$= 2\int_0^2 \frac{1}{2} e^{\frac{x}{2}}\,dx$

$= 2\left[e^{\frac{x}{2}}\right]_0^2$

$= 2(e^1 - e^0)$

$= 2(e - 1)$ (2)

ii) $\int_{-3}^{-1} \dfrac{2}{1-3x}\,dx$

$= \dfrac{2}{-3}\int_{-3}^{-1} \dfrac{-3}{1-3x}\,dx$

$= -\dfrac{2}{3}\left[\ln(1-3x)\right]_{-3}^{-1}$

$= -\dfrac{2}{3}(\ln 4 - \ln 10)$

$= -\dfrac{2}{3} \ln \dfrac{2}{5}$ or $\dfrac{2}{3} \ln \dfrac{5}{2}$ (2)

f) $y = \ln(2x-5)$ at $(3,0)$

$\dfrac{dy}{dx} = \dfrac{2}{2x-5}$

$m = \dfrac{2}{2\times 3 - 5}$

$m = \cancel{4}\ 2$

$y - y_1 = m(x - x_1)$

$y - 0 = \cancel{3}\,2(x-3)$

$y = \cancel{3x-9}$ $y = 2x - 6$ (2)

g)

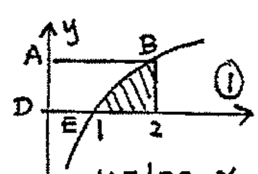
$A = \dfrac{h}{2}\left(f(1) + 2f(3) + f(5)\right)$

$= \dfrac{2}{2}\left(e^1 + 2e^3 + e^5\right)$

Area $= (e + 2e^3 + e^5)$ units^2

191.303 (2)

h)



$y = \log_e x$

$\therefore x = e^y$

i) $A = (0, \ln 2)$ (1)

ii) Find unshaded area ABED

$\int_0^{\ln 2} e^y\,dy$

$= \left[e^y\right]_0^{\ln 2}$

$= e^{\ln 2} - e^0$

$= 2 - 1$

$= 1$ (2)

$\therefore$ Area $= (2\ln 2 - 1)$ units^2

QUESTION 3

a) $25^\circ = 25 \times \frac{\pi}{180}^c$

$= \frac{5\pi}{36}^c$ 1

b) $L = \frac{\theta}{360} \times 2\pi r$

$6 = \frac{\theta}{360} \times 2\pi \times 8$

$\theta = \frac{6 \times 360}{16\pi}$

$= 42.97183463^\circ$

$\theta \doteqdot 42^\circ 58'$

OR $L = r\theta$

$\theta = \frac{6}{8}$

$\theta = \frac{6}{8} \times \frac{180}{\pi}$

$\theta = 42^\circ 58'$ 2

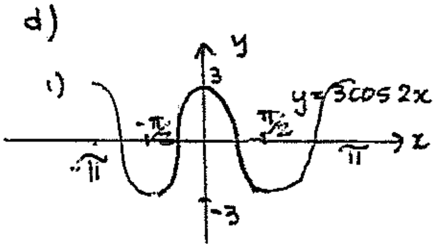
c) $2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = 30^\circ, 150^\circ$

$x = \frac{\pi}{6}^c \quad \frac{5\pi}{6}^c$ 2

d)

i) 

2

ii) period $= \pi$ 1

iii) amplitude $= 3$ 1

e) i) $y = (1 - \cos x)^5$

$\frac{dy}{dx} = 5(1-\cos x)^4(+\sin x)$

$= 5\sin x(1-\cos x)^4$ 2

ii) $y = x\sin x$

$\frac{dy}{dx} = vu' + uv'$

$= \sin x . 1 + x\cos x$

$= \sin x + x\cos x$ 2

$u = x$
$u' = 1$
$v = \sin x$
$v' = \cos x$

f) i) $\int (\sin 2x + \cos 3x)\ dx$

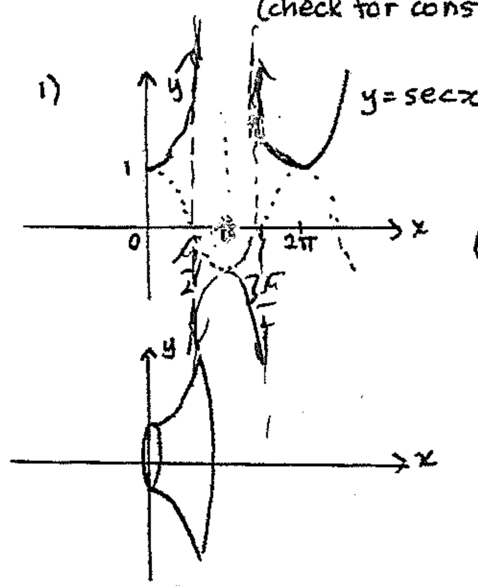
$= -\frac{\cos 2x}{2} + \frac{\sin 3x}{3} + c$

$= \frac{\sin 3x}{3} - \frac{\cos 2x}{2} + c$ 2

ii) $\int \cos \pi x\ dx$

$= \frac{\sin \pi x}{\pi} + c$ 2

(check for constant)

g) i) 

1

$V = \pi \int_0^{\frac{\pi}{6}} \sec^2 x\ dx$

$= \pi \left[\tan x\right]_0^{\frac{\pi}{6}}$

$= \pi\left(\tan \frac{\pi}{6} - \tan 0\right)$ 2

$= \pi \times \frac{1}{\sqrt{3}}$

Volume $= \frac{\sqrt{3}\pi}{3}$ units^3

(Accept $1.813799364\ u^3$).