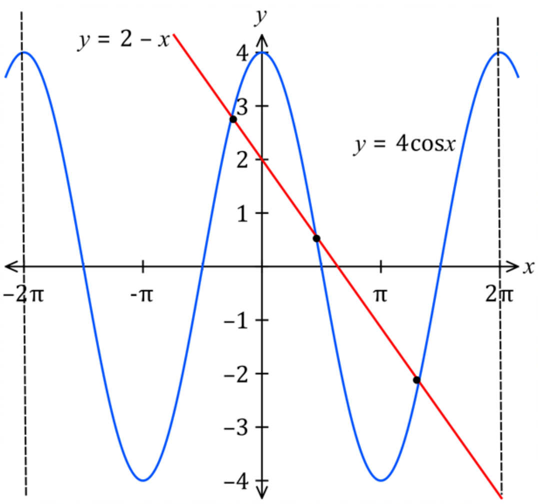
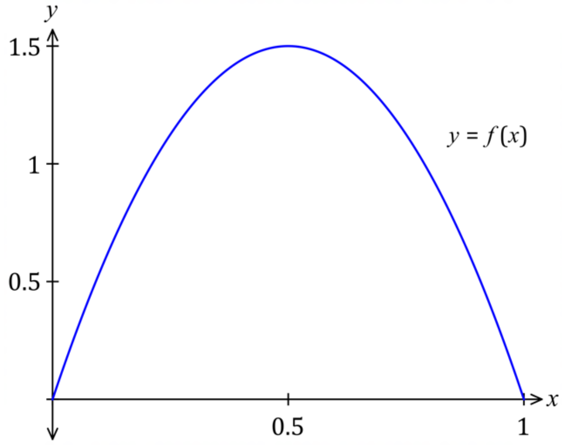
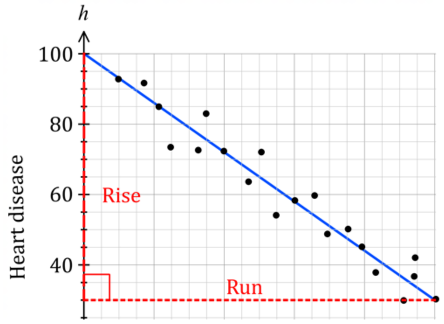
ACE Examination Paper 3
Year 12 Mathematics Advanced Yearly Examination
Worked solutions and marking guidelines

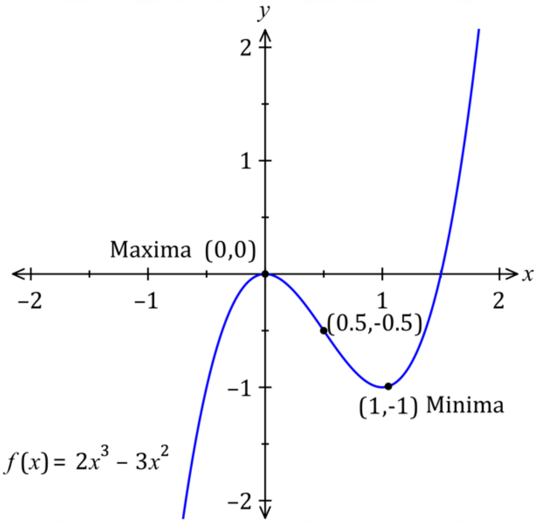
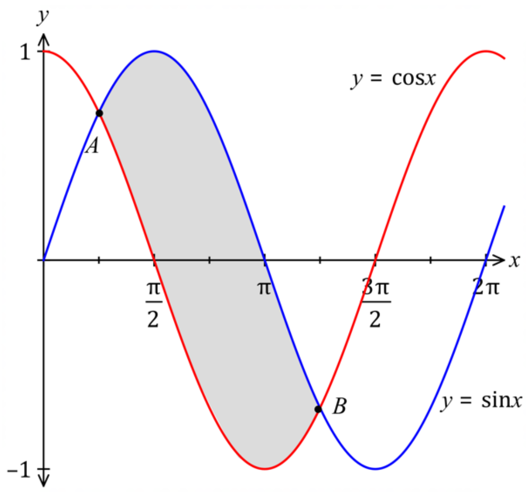
Section I		
	Solution	**Criteria**
1.	Cubic graph with x-intercepts at –2 and 2. Stationary point at –2. Test points into each equation. $y = -(x+2)^2(x-2)$	1 Mark: C
2.	$\int (4 + 2x + 3x^2)dx = 4x + x^2 + x^3 + C$	1 Mark: B
3.	$\sum_{n=1}^{5} (4n - 2) = 2 + 6 + 10 + 14 + 18$ AP with $a = 2$, $l = 18$ and $n = 5$ $S_n = \frac{n}{2}(a + l) = \frac{5}{2}(2 + 18) = 50$	1 Mark: C
4.	$y = (2 - x)^3 + 1$ $\frac{dy}{dx} = -3(2 - x)^2$ Gradient at the point when (1, 2) $m = -3(2 - 1) = -3$	1 Mark: A
5.	$\int_0^k \frac{1}{2}\sin x\, dx = \frac{1}{2}[-\cos x]_0^k = 1$ $-\frac{1}{2}(\cos k - \cos 0) = 1$ $\cos k = -1$ $k = \pi$	1 Mark: C
6.	Correlation coefficient of –0.5 is a negative linear relationship with medium strength.	1 Mark: A
7.	$\frac{dy}{dx} = 3 - \frac{2}{x^2} = 3 - 2x^{-2}$ $y = 3x + 2x^{-1} + C$ Point (1, –2) satisfies the equation. $-2 = 3 \times 1 + 2 \times 1^{-1} + C$ $C = -7$ $\therefore y = 3x + \frac{2}{x} - 7$	1 Mark: D
8.	Stationary point $\frac{dy}{dx} = 0$, $\frac{dy}{dx} = 4x + a = 0$ or $a = -4x$ Stationary point at $x = -3$ $a = -4 \times -3 = 12$	1 Mark: D
9.	Amplitude $= 1$, Period $= \frac{2\pi}{3}$, Vertical shift $= -1$ Maximum value of $f(x) = \cos 3x - 1$ is 0 Minimum value $f(x) = \cos 3x - 1$ occurs when $x = \frac{2\pi}{9}$ $y = \cos 3x - 1 = \cos 3 \times \left(\frac{2\pi}{9}\right) - 1 = -\frac{3}{2}$ $\therefore -\frac{3}{2} \le y \le 0$	1 Mark: B
10.	Function is increasing when $f'(x) > 0$ (positive gradient). This occurs when x is between 6 and 8 or $6 < x < 8$.	1 Mark: D

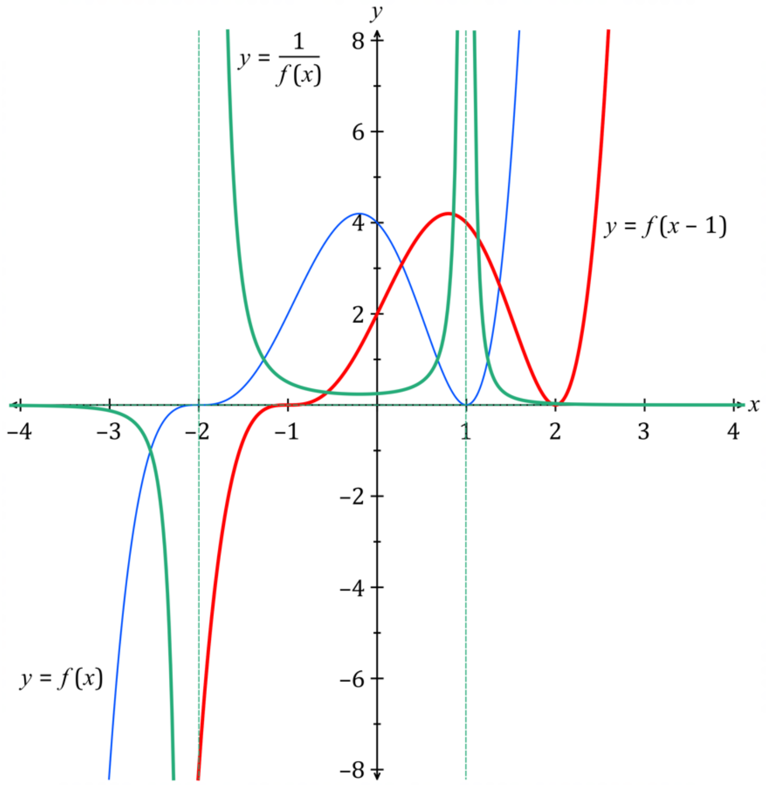
Section II		
11	$\int (3x+5)^2 dx = \frac{(3x+5)^3}{3 \times 3} + C$ $= \frac{1}{9}(3x+5)^3 + C$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
12(a)	Intersection value is 3.2464 (8% per year for 3 years) $FV = 3.2464 \ \times \$16\,000$ $= \$51\,942.40$ $\approx \$51\,942$ $\therefore$ Future value is \$15 942.	1 mark: Correct answer.
12(b)	Intersection value is 5.1010 (1% per month for 5 months) $FV = 5.1010 \times 2100$ $= \$10\,712.10$ $\approx \$10\,712$ $\therefore$ Future value is \$10 712.	1 mark: Correct answer.
13(a)	Number of boxes in each layer from the top are an AP: 6,7,8... $T_n = a(n-1)d$ $= 6 + (n-1) \times 1$ $= n + 5$ $\therefore$ Number of boxes in the bottom layer is $n + 5$.	2 Marks: Correct answer. 1 Mark: Recognises AP and uses nth term formula.
13(b)	Sum the boxes in each layer (a = 6 and l = $n + 5$) $S_n = \frac{n}{2}(a + l)$ $= \frac{n}{2}(6 + n + 5)$ $= \frac{1}{2}n(n + 11)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14	Use z-scores to compare results $z = \frac{x - \bar{x}}{s}$ $= \frac{66 - 82}{8}$ $= -2$ $z = \frac{x - \bar{x}}{s}$ $= \frac{61 - 71}{10}$ $= -1$ $\therefore$ Lex has improved as his z-score has increased.	2 Marks: Correct answer. 1 Mark: Finds the z-score or shows some understanding.
15	$a = 3t - 2$ $v = \frac{3t^2}{2} - 2t + C_1$ When t = 0 then v = 2 $2 = \frac{3 \times 0^2}{2} - 2 \times 0 + C_1$ $C_1 = 2$ $v = \frac{3t^2}{2} - 2t + 2$ $x = \frac{t^3}{2} - t^2 + 2t + C_2$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Integrates to find the velocity.

15	When $t = 0$ then $x = 4$ $4 = \frac{0^3}{2} - 0^2 + 2 \times 0 + C_2$ $C_2 = 4$ $x = \frac{t^3}{2} - t^2 + 2t + 4$ When $t = 5$ $x = \frac{5^3}{2} - 5^2 + 2 \times 5 + 4$ $= 51.5$ units $\therefore$ The particle is 51.5 units to the right after 5 seconds.	
16(a)	$\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int \frac{2x}{x^2+3}\,dx$ $= \frac{1}{2}\ln(x^2+3) + C$	2 Marks: Correct answer. 1 Mark: Recognises the log function as the primitive.
16(b)	$\int_0^{\frac{\pi}{3}} \cos 2x\, dx = \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{3}}$ $= \left[\frac{\sin\frac{2\pi}{3}}{2}\right] - \left[\frac{\sin 0}{2}\right]$ $= \frac{\sqrt{3}}{4}$	2 Marks: Correct answer. 1 Mark: Finds the primitive function or shows some understanding.
17	Draw graphs of $y = 4\cos x$ and $y = 2 - x$  Solutions are the points of intersection of the graphs $\therefore$There are 3 solutions to the $\cos x = 2 - x$.	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Draws one of the graphs.

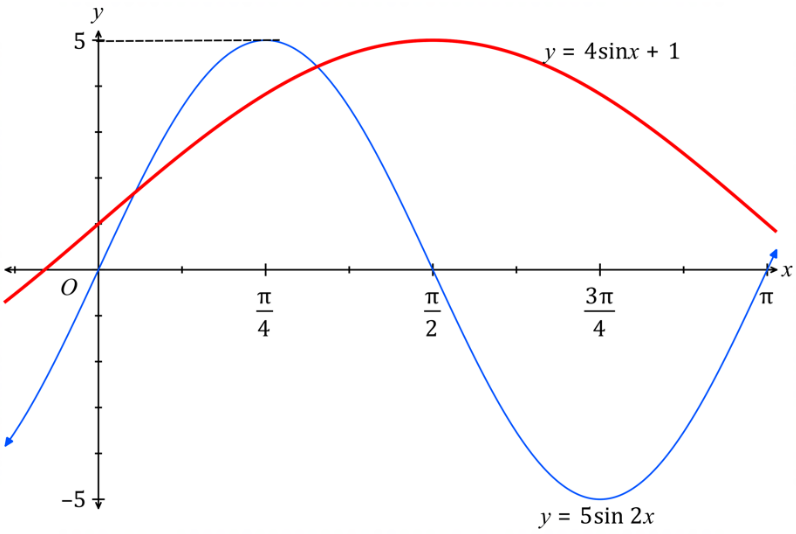
18(a)	$f(x) = 6x(1-x) \geq 0$ in $0 \leq x \leq 1$ (Function is quadratic, a negative coefficient, x-intercepts 0 and 1) $\therefore f(x) \geq 0$ for all x $\int_{-\infty}^{\infty} f(x)dx = \int_0^1 6x(1-x)dx$ $= \int_0^1 6x - 6x^2\, dx$ $= [3x^2 - 2x^3]_0^1$ $= 1$ $\therefore \int_{-\infty}^{\infty} f(x)dx = 1$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
18(b)	 Probability density function	2 Marks: Correct answer. 1 Mark: Draws the general shape of the function.
18(c)	$P(0 \leq X \leq 1) = 1$ (from the graph)	1 Mark: Correct answer.
18(d)	$P(0.5 \leq X \leq 1) = 0.5$ (Graph is symmetrical about $x = 0.5$)	1 Mark: Correct answer.
19(a)	$\frac{d}{dx}(e^x - 3)^4 = 4(e^x - 3)^3 e^x$ $= 4e^x(e^x - 3)^3$	1 Mark: Correct answer.
19(c)	$\frac{d}{dx} x\tan x = x\sec^2 x + \tan x$	1 Mark: Correct answer.
19(c)	$\frac{d}{dx}\ln(\cos x) = \frac{1}{\cos x} \times (-\sin x)$ $= -\tan x$	1 Mark: Correct answer.
20(a)	$T_n = ar^{n-1}$ $T_3 = ar^2 = 0.75$ (1) $T_7 = ar^6 = 12$ (2) Dividing the two equations $\frac{ar^6}{ar^2} = \frac{12}{0.75}$ $r^4 = 16$ $r = \pm 2$ $\therefore$ The common ratio is ± 2.	2 Marks: Correct answer. 1 Mark: Finds two equations using the nth term of a GP or shows some understanding.

20(b)	To find a substitute ± 2 for r into equation ② $T_7 = a \times (\pm 2)^6 = 12$ $a = \frac{12}{64} = \frac{3}{16}$ $\therefore$ First term is $\frac{3}{16}$	1 Mark: Correct answer.
20(c)	$T_n = ar^{n-1}$ $T_{10} = \frac{3}{16} \times (\pm 2)^9$ $= \pm 96$ $\therefore$ Tenth term is ± 96	1 Mark: Correct answer.
21(a)	$m = \frac{\text{Rise}}{\text{Run}}$ $= -\frac{70}{100}$ $= -0.7$ $\therefore$ Gradient is -0.7 	1 mark: Correct answer.
21(b)	y-intercept is 100 $y = mx + b$ $h = -0.7e + 100$	1 mark: Correct answer.
21(c)	Correlation coefficient is about –0.8	1 mark: Correct answer.
22	{55 000+(55 000+1650) +(55 000+1650+1650)+...} a = 55 000, n = 12 and d = 1650 $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{12} = \frac{12}{2}[2 \times 55\,000 + 11 \times 1650]$ $= 768\,900$ $\therefore$ Elijah would earn \$768 900 in twelve years of employment.	2 Marks: Correct answer. 1 Mark: Uses the formula for the sum of n terms of an AP with one correct value.
23(a)	$f(x) = 2x^3 - 3x^2 = 0$ $x^2(2x-3) = 0$ $\therefore x = 0$ or $x = \frac{3}{2}$	1 Mark: Correct answer.
23(b)	Turning points $f'(x) = 0$ $f'(x) = 6x^2 - 6x$ $6x(x-1) = 0$ $x = 0, x = 1$ $\therefore$ Turning points are $(0, 0)$ and $(1, -1)$ $f''(x) = 12x - 6$ At $(0, 0), f''(0) = -6 < 0$ Maxima At $(1, -1)$ $f''(1) = 6 > 0$ Minima	3 Marks: Correct answer. 2 Marks: Finds both of the turning points. 1 Mark: Finds the first derivative and equates it to zero.

23(c)	Possible of inflexion $f''(x) = 12x - 6 = 0$ $12x = 6$ $x = 0.5$ When $x = 0.5, y = 2 \times 0.5^3 - 3 \times 0.5^2 = -0.5$ At $(0.5, -0.5)$ check for change in concavity $x = 0.4, f''(0.4) = 12 \times 0.4 - 6 = -1.2 < 0$ $x = 0.6, f''(0.6) = 12 \times 0.6 - 6 = 1.2 > 0$ $\therefore (0.5, -0.5)$ is a point of inflexion.	2 Marks: Correct answer. 1 Mark: Finds the point of inflexion.
23(d)		2 Marks: Correct answer. 1 Mark: Finds the general shape of the curve.
23(e)	$f(x) < 0$ when $x < 0$ or $0 < x < 1.5$ (see the graph)	1 Mark: Correct answer.
24(a)		2 Marks: Correct answer. 1 Mark: Draws one of the curves.
24(b)	$y = \sin x$ 1 $y = \cos x$ 2 Equation 1 divided by equation 2 $\frac{\sin x}{\cos x} = 1$ $\tan x = 1$ $x = \frac{\pi}{4}, \frac{5\pi}{4}, ..$ $\therefore A\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $B\left(\frac{5\pi}{4}, -\frac{1}{\sqrt{2}}\right)$	2 Marks: Correct answer. 1 Mark: Finds one value for x or shows some understanding.

24(c)	$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} [\sin x - \cos x]dx$ $= [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$ $= \left(-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}\right) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$ $= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$ $= 2\sqrt{2}$ square units	2 Marks: Correct answer. 1 Mark: Sets up the integral or shows some understanding.
25	$\frac{d}{dx}\ln_2 x^2 = \frac{2x}{x^2 \ln 2}$ $= \frac{2}{x\ln 2}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
26(a) 26(b)		2 Marks: Correct answer. 1 Mark: Draws the general shape of the curve or shows some understanding.
27	a =10 and S = 15 $S = \frac{a}{1-r}$ $15 = \frac{10}{1-r}$ $15 - 15r = 10$ $15r = 5$ $r = \frac{1}{3}$ $\therefore$ Common ratio is $\frac{1}{3}$.	2 Marks: Correct answer. 1 Mark: Substitutes either a =10 or S = 15 into limiting sum formula.
28(a)	A = 81.5 and B = 2.1 (Statistics mode on calculator) $y = mx + c = Bx + A$ $\therefore$ Equation of the least-squares line of best fit is $cost = 2.1 \times number + 81.5$	1 Mark: Correct answer.

28(b)	$cost = 2.1 \times number + 81.5$ $= 2.1 \times 48 + 81.5$ $= 182.3$ $\therefore$ Cost is \$182.30	1 Mark: Correct answer.
28(c)	$cost = 2.1 \times number + 81.5$ $249.50 = 2.1 \times number + 81.5$ $number = \dfrac{249.50 - 81.5}{2.1}$ $= 80$ $\therefore$ Number of meals is 80.	1 Mark: Correct answer.
29	$z = \dfrac{x - \bar{x}}{s}$ $= \dfrac{62 - 74}{6}$ $= -2$ $z = \dfrac{x - \bar{x}}{s}$ $= \dfrac{86 - 74}{6}$ $= 2$ $\therefore$ 95% of the scores lie between 62 and 86.	2 marks: Correct answer. 1 mark: Finds the z-score for 62 or 86.
30	$f(x) = \dfrac{\cos x}{2x + 2}$ $f'(x) = \dfrac{(2x+2)(-\sin x) - \cos x \times 2}{(2x+2)^2}$ $= \dfrac{(-2x-2)\sin x - 2\cos x}{(2x+2)^2}$	2 Marks: Correct answer. 1 Mark: Uses the quotient rule with at least one correct value.
31(a)	Almost certainly – 99.7% of the scores. 3 standard deviations above and below the mean. $4.50 - 3 \times 0.03 = 4.41$ cm $4.50 + 3 \times 0.03 = 4.59$ cm $\therefore$ Interval range is from 4.41 cm to 4.59 cm	1 mark: Correct answer.
31(b)	The manager is concerned because 4.62 cm is 4 standard deviations above the mean. This is extremely unlikely to occur and indicates the machine is not working correctly.	1 mark: Correct answer.
32	$A_1 = 25\,000(1.06)^1$ $A_2 = 25\,000(1.06)^2 + 25\,000(1.06)^1$ $A_{10} = 25\,000(1.06)^{10} + 25\,000(1.06)^9 + \cdots + 25\,000(1.06)^1$ $= 25\,000(1.06^{10} + 1.06^9 + \cdots + 1.06^1)$ GP with $a = 1.06$, $r = 1.06$ and $n = 10$ $A_{10} = 25\,000 \times \dfrac{1.06[1.06^{10} - 1]}{1.06 - 1}$ $= 349\,291.066\ldots$ $\approx \$349\,291$ $\therefore$ William's account balance after ten years is \$349 291.	3 Marks: Correct answer. 2 Marks: Finds the correct GP. 1 Mark: Shows some understanding.
33(a)	$V = \pi r^2\, h$ $200 = \pi r^2 \times h$ $h = \dfrac{200}{\pi r^2}$	1 Mark: Correct answer.

33(b)	$A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r \times \dfrac{200}{\pi r^2}$ $= 2\pi r^2 + \dfrac{400}{r}$	2 Marks: Correct answer. 1 Mark: Applies the surface area formula for a cylinder.
33(c)	$A = 2\pi r^2 + \dfrac{400}{r}$ $\dfrac{dA}{dr} = 4\pi r - 400r^{-2}$ $\dfrac{d^2A}{dr^2} = 4\pi + 800r^{-3}$ Minimum A occurs when $\dfrac{dA}{dr} = 0$ $4\pi r - 400r^{-2} = 0$ $4r(\pi - 100r^{-3}) = 0$ $r = 0$ (no can)or $r = \sqrt[3]{\dfrac{100}{\pi}} = 3.1692 \ldots \approx 3.17$ cm Check if a minimum When $r = \sqrt[3]{\dfrac{100}{\pi}}$ then $\dfrac{d^2A}{dr^2} = 4\pi + 800 \times \left(\sqrt[3]{\dfrac{100}{\pi}}\right)^{-3} = 12\pi > 0$ Minima $\therefore$ Radius of the can is 3.17 cm.	3 Marks: Correct answer. 2 Marks: Finds r = 3.17 cm 1 Mark: Calculates the first derivative or has some understanding of the problem.
34(a)	The highest y-value on the graph is 5 units. Thus, the amplitude (a) is 5. $\therefore\ a = 5$	1 Mark: Correct answer.
34(b)	From the graph, the period is π units. From the equation, the period is $\frac{2\pi}{b}$ $\dfrac{2\pi}{b} = \pi$ $b = 2$	1 Mark: Correct answer.
34(c)	The graph given to us has the equation $y = 5\sin 2x$. Now $y = 4\sin x + 1$ has an amplitude of 3 and period 2π. It is 'lifted up' by 1 unit.  	2 Marks: Correct answer. 1 Mark: Shows some understanding.

35	$y = \tan x$ $y' = \sec^2 x$ At the point where $x = \frac{\pi}{16}$ $y' = \sec^2 \frac{\pi}{16}$ $= \dfrac{1}{\cos^2 \frac{\pi}{16}}$ ≈ 1.04 $\therefore$ Gradient of the tangent is 1.04.	2 Marks: Correct answer. 1 Mark: Finds the first derivative.
36(a)	Initially $t = 0$ and $M = 2000$ $M = M_0 e^{kt}$ $2000 = M_0 e^{k \times 0}$ $M_0 = 2000$ Also $t = 5$ and $M = 3200$ $3200 = 2000 e^{k \times 5}$ $e^{5k} = \dfrac{3200}{2000}$ $5k \ln e = \ln 1.6$ $k = \dfrac{\ln 1.6}{5}$ $= 0.09400 \ldots$ ≈ 0.094 $\therefore M_0 = 2000$ and $k = 0.094$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
36(b)	We need to find M and $t = 10$ $M = 2000 e^{k \times 10}$ $= 5120$ $\therefore$ Predicted population after 10 years is 5120.	1 Mark: Correct answer.
36(c)	We need to find t when $M = 4000$ $4000 = 2000 e^{kt}$ $e^{kt} = 2$ $kt \ln e = \ln 2$ $t = \dfrac{\ln 2}{k}$ $= 7.3738 \ldots$ ≈ 7.4 years $\therefore$ It will take 7.4 years for production to double.	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.