A C E
EXAM
PAPER

Student name: _____

PAPER 2

YEAR 12
YEARLY
EXAMINATION

Mathematics Standard 2

General Instructions

- Working time - 150 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 15 marks

- Attempt Questions 1-15
- Allow about 25 minutes for this section

Section II – 85 marks

- Attempt all questions in Section II
- Allow about 2 hours and 5 minutes for this section

Section I

15 marks
Attempt questions 1 - 15
Allow about 25 minutes for this section

Use the multiple-choice answer sheet for questions 1-15

1. Harry uses a 1.5 kilowatt per hour dishwasher for a total of 4 hours. He is charged at a rate of 25.72 cents per kilowatt. What is the cost of using the dishwasher?

   (A) \$0.32

   (B) \$1.03

   (C) \$1.54

   (D) \$6.00

2. Use the recurrence relation below to answer question two.

   Recurrence relation: $V_{n+1} = V_n(1 + r) + D$

   Where V_{n+1} is the value of the investment after $(n + 1)$ payments

   V_n is the value of the investment after n payments

   r is the rate of interest

   D is the payment per compounding period.

   Anthony makes an initial deposit of \$1000 on an investment at a rate of 5% p.a. compounding annually and an additional deposit of \$100 every year. How many years will it take for the Anthony's investment to accumulate to \$1472.88?

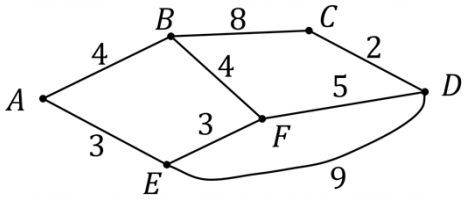
   (A) 2

   (B) 3

   (C) 4

   (D) 5

3. Prim's algorithm, beginning with vertex A, will be used to find the minimal spanning tree for the network below.



   Which vertex will be added last?

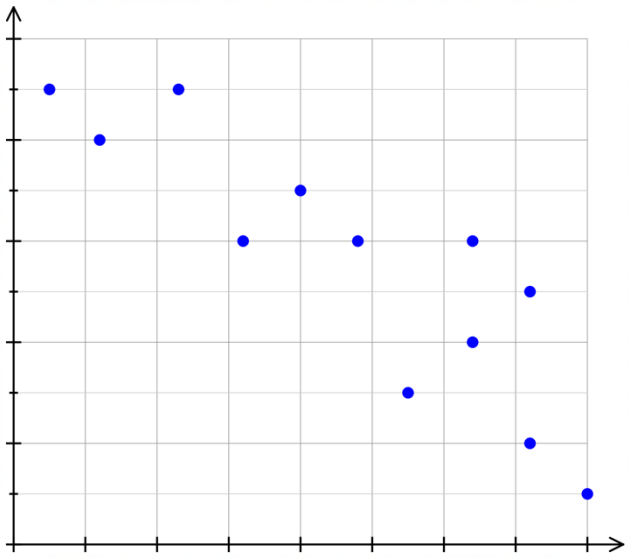
   (A) D

   (B) C

   (C) B

   (D) F

4.



What is the correlation between the variables in this scatterplot?

(A) Moderate negative

(B) Moderate positive

(C) Weak negative

(D) Weak Positive

5. The monthly repayments per $1000 on a bank home loan are shown in table below.

Term	8.00%	8.25%	8.50%
20 years	$8.36	$8.52	$8.68
25 years	$7.72	$7.88	$8.05

What is the monthly repayment for a loan of $320 000 at 8.25% p.a. interest rate for 25 years?

(A) $252.16

(B) $272.64

(C) $2521.60

(D) $2726.40

6. Millie's car uses 7.25 litres per 100 km. How many litres of petrol will her car use on a trip of 310 km from Bulahdelah to Wollongong?

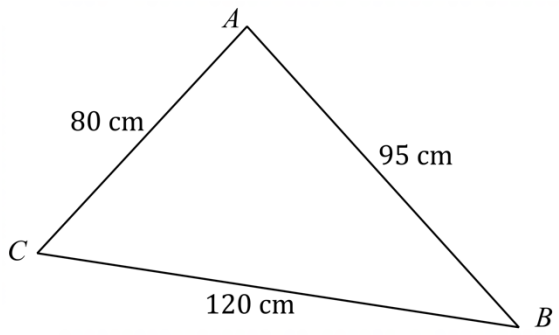
(A) 2.339 L

(B) 233.9 L

(C) 22.475 L

(D) 2247.5 L

7.



Not to scale

Find $\angle BAC$ to the nearest degree.

(A) 40°

(B) 42°

(C) 52°

(D) 86°

8. Isaac scored 81 in an assessment task. The mean for this task was 67 with a standard deviation of 7.0. What is Isaac's z-score?

(A) −2

(B) −1

(C) 1

(D) 2

9. The amount of money in a fund is given by $A = 600 \times 1.1^t$ where A is the amount of money and t is the time in years. What is the initial amount of money invested in the fund?

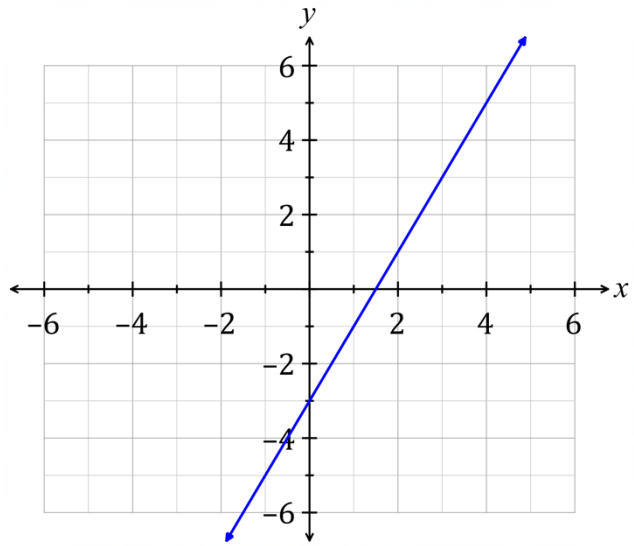
(A) $600

(B) $660

(C) $1000

(D) $1100

10.



The correct equation of line shown above is:

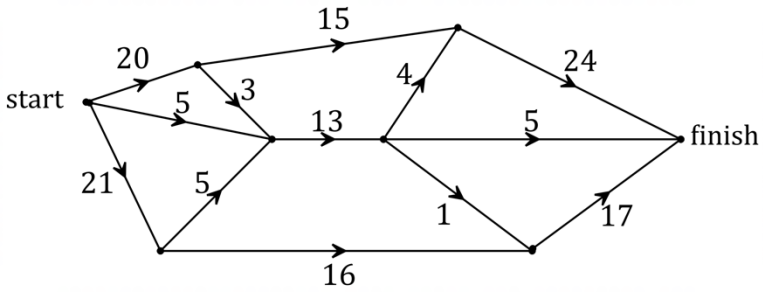
(A) $y = 2x - 3$

(B) $y = -2x - 3$

(C) $y = \frac{1}{2}x - 3$

(D) $y = -\frac{1}{2}x - 3$

11.



What is the maximum flow in the network diagram?

(A) 40

(B) 41

(C) 42

(D) 43

12. The cost $\$c$ of a birthday party is given by $c = 50n + 135$ where n is the number attending the party. If five people decide not to attend, by how much does the cost decrease?

(A) \$135

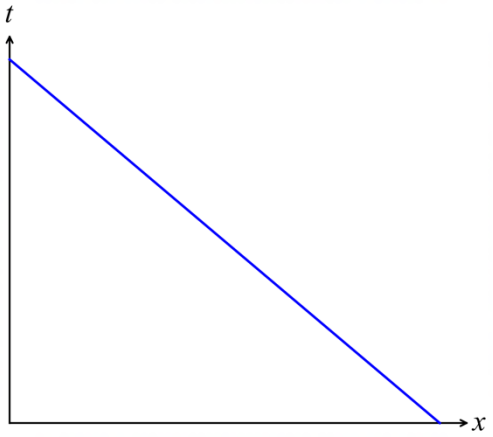
(B) \$185

(C) \$250

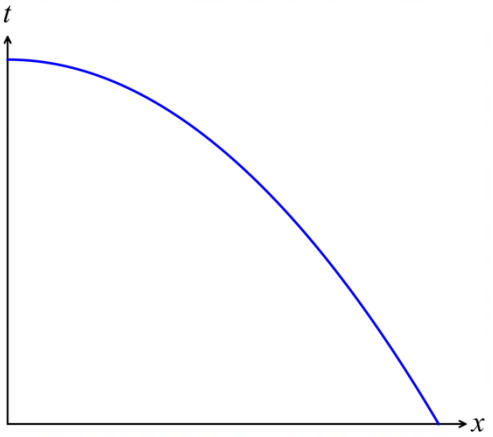
(D) \$385

13. The time (t) taken to write a report varies inversely with the number (x) of people writing the report. Which graph best represents this relationship?

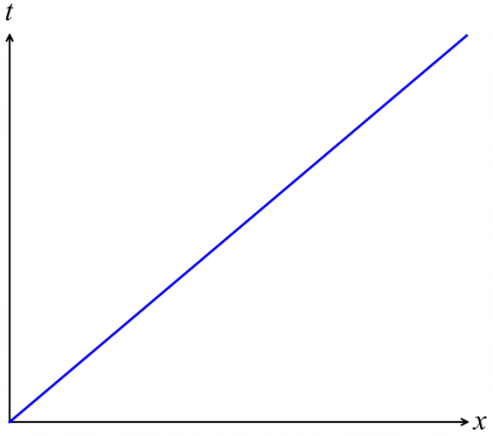
(A)



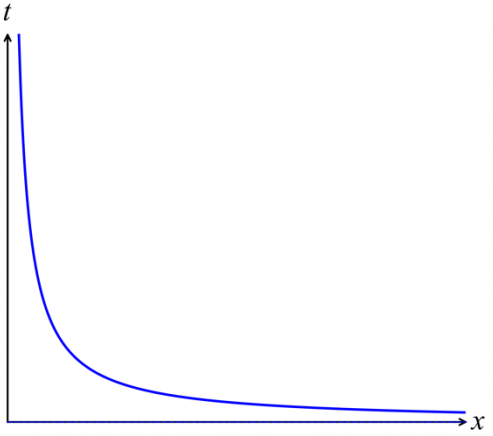
(B)



(C)



(D)



14. Amy scored 80 in a mathematics test. The mathematics test had a mean of 64 and a standard deviation of 8. A recent English test had a mean of 60 and a standard deviation of 11. What mark in the English test would have been equivalent to Amy's mathematics mark?

(A) 76

(B) 78

(C) 80

(D) 82

15. A credit card has a daily interest rate of 0.06% per day (no interest free period). Find the interest charged on \$1700 for 15 days. Answer correct to the nearest cent.

(A) \$1.02

(B) \$15.36

(C) \$102.00

(D) \$4074.15

Section II

85 marks
Attempt all questions
Allow about 2 hours and 5 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 16 (2 marks) **Marks**

A map has a scale of 1:400 000.

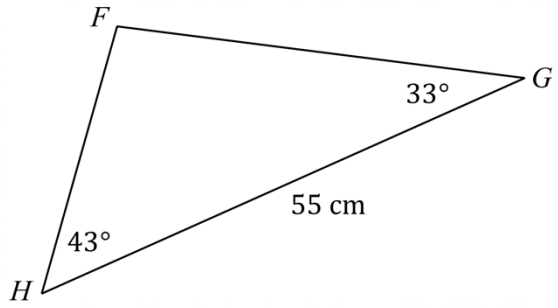
(a) Two towns are 2.5 cm apart on the map. **1**
What is the actual distance between the towns, in kilometres?

(b) The distance between two cities is 60 km. **1**
How far apart are the two cities on the map, in centimetres?

Question 17 (2 marks)

In a normally distributed set of scores, the mean is 74 and the standard deviation is 6. Approximately what percentage of the scores will lie between 62 and 86? **2**

Question 18 (3 marks) **Marks**



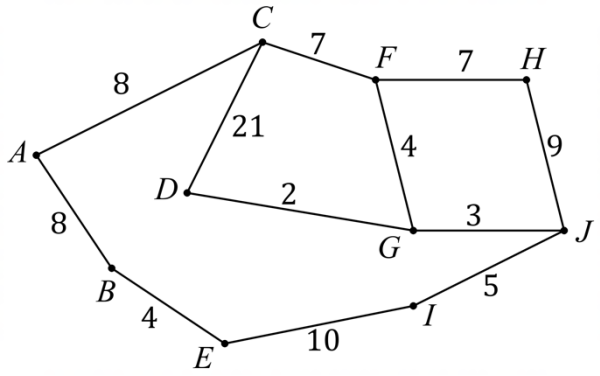
Not to scale

ΔFGH has $\angle FGH = 33°$ and $\angle GHF = 43°$. The length GH is 55 cm.

(a) What is the size of $\angle HFG$? **1**

(b) What is the value of g, correct to one decimal place? **2**

Question 19 (3 marks)

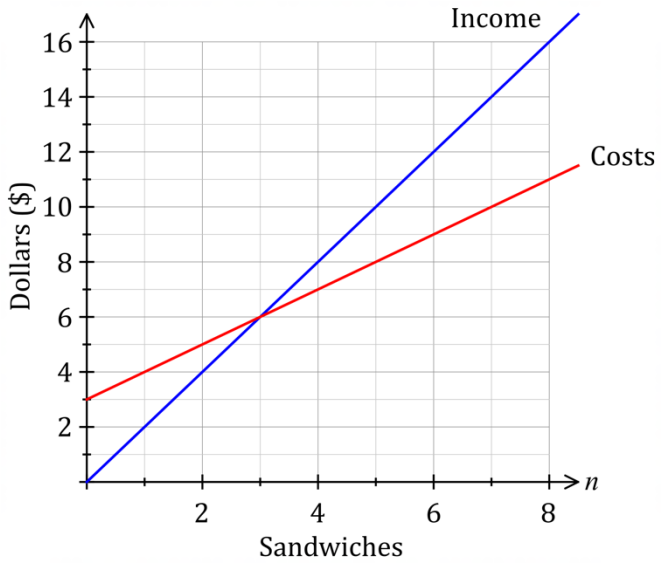


(a) List the vertices with an odd degree. **1**

(b) What is length of the shortest path from A to J? **2**

Question 20 (4 marks) **Marks**

The linear graphs below show the cost of making a sandwich and the income received from selling the sandwiches.



(a) Let the income received be $\$I$ and n the number of sandwiches sold. Write a formula for the income. **1**

(b) Let the costs of making a sandwich be $\$C$ and n the number of sandwiches sold. Write a formula for the costs. **1**

(c) What is the profit if 7 sandwiches are sold? **1**

(d) How many sandwiches are needed to be sold to break-even? **1**

Question 21 (5 marks) **Marks**

Activity	*Duration (min)*	*Immediate predecessors*
A	8	–
B	13	*A*
C	23	*A*
D	10	*B*
E	3	*C*
F	8	*D*
G	18	*E, F*

(a) Construct a network diagram using the activity chart. Show the earliest starting times (EST) and latest starting times (LST). **3**

(b) Find the critical path and minimum completion time. **2**

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Question 22 (3 marks)

Adam owns a credit card that has no annual fees and charges 15.7% p.a. simple interest on all purchases. The interest is charged from the day of purchase and includes the day of payment.

(a) Show that the daily interest rate is 0.0430%. **1**

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(b) On the 30th of March, Adam bought an entertainment unit for $1240 using his credit card. Adam paid his credit card account on the 10th of April. What was the total amount Adam paid for the entertainment unit, including interest? Answer correct to the nearest cent. **2**

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Question 23 (3 marks) **Marks**

The table below shows the present value of a $1 annuity.

Present value of $1					
Period	1%	2%	4%	6%	8%
1	0.9901	0.9804	0.9615	0.9434	0.9259
2	1.9704	1.9416	1.8861	1.8334	1.7833
3	2.9410	2.8839	2.7751	2.6730	2.5771
4	3.9020	3.8077	3.6299	3.4651	3.3121
5	4.8534	4.7135	4.4518	4.2124	3.9927

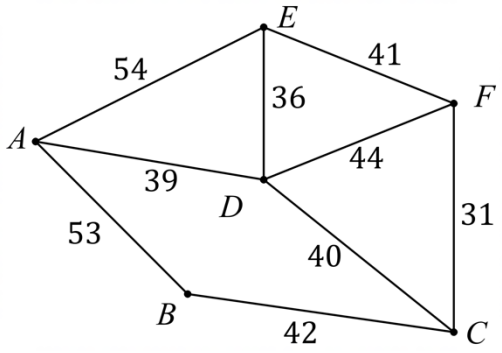
(a) What would be the present value of a $9000 per year annuity at 6% per annum for 5 years, with interest compounding annually? **1**

(b) An annuity of $6000 each three months is invested at 4% per annum, compounded quarterly for 1 year. What is the present value of the annuity? **1**

(c) What is the value of an annuity that would provide a present value of $43 230 after 3 years at 8% per annum compound interest? Answer to the nearest dollar. **1**

Question 24 (3 marks) **Marks**

There are five towns (*B*,*C*,*D*,*E* and *F*) that need to be linked by pipelines to a natural gas supply (*A*). The existing road links and the distance (in km) between the towns is shown in the network diagram below.



(a) Draw a minimum spanning tree that will ensure that all the towns are connected to the network, but that also minimises the amount of pipelines required. **2**

(b) What is the minimum length of pipeline to supply all the towns? **1**

Question 25 (1 mark)

The equation of least-squares line of best fit is given by $y = mx + c$ where

$$m = r\frac{s_y}{s_x} \text{ and } c = \bar{y} - m\bar{x}$$

What is the *y*-intercept of the least-squares line of best fit given $m = 0.6$, $\bar{x} = 50$ and $\bar{y} = 65$? **1**

Question 26 (3 marks) **Marks**

Sally's recent results in hospitality and timber are recorded in the table.

Course	*Class Mean*	*Class Standard Deviation*	*Sally's Result*
Hospitality	55	10	85
Timber	55	15	85

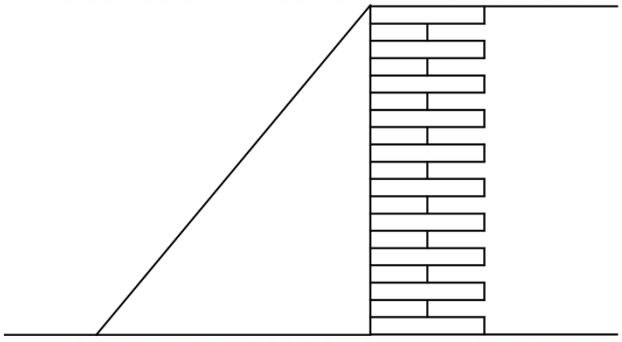
(a) What is Sally's z-score for timber? **1**

(b) Explain the z-score in timber in terms of the class mean and class standard deviation. **1**

(c) What hospitality mark would be equivalent to a z-score of –1? **1**

Question 27 (2 marks)

The angle of elevation to the top of a building from a car is 34°. **2**

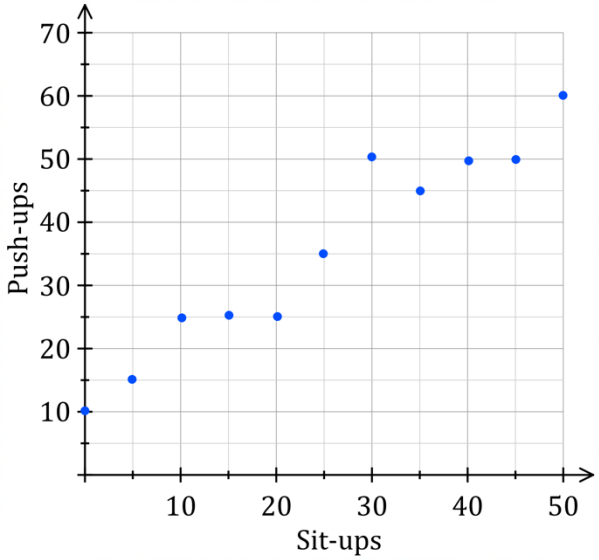


Not to scale

The approximate perpendicular height of the building is 30 metres. What is the distance, to nearest metre, from the car to the foot of the building?

Question 28 (5 marks) **Marks**

The scatterplot shows the number of sit-ups (s) and the number of push-ups (p) performed by ten students during a fitness test.



(a) Draw a line of best fit on the scatterplot. Find the gradient of this line. **2**

(b) Alyssa was absent for the push-up test. Predict her push-up result if she scored 36 on the sit-up test. **1**

(c) Calculate the value of the Pearson's correlation coefficient. Answer correct to two decimal places. **2**

Question 29 (1 mark)

Cooper is a hospital patient who is given 1.5 litres of fluid over 8 hours. **1**
What is the required drip rate in mL/h?

Question 30 (3 marks) **Marks**

A table for $200 000 at 7.25% p.a. reducible interest is shown below.

Loan period in years	15	20	25	30
Monthly repayments	$1825.73	$1580.75	$1445.61	$1364.35

(a) Find the total amount that must be repaid if the loan is taken over 20 years. **1**

(b) How much extra is repaid if the loan is taken over 30 years rather 20 years? **2**

Question 31 (3 marks)

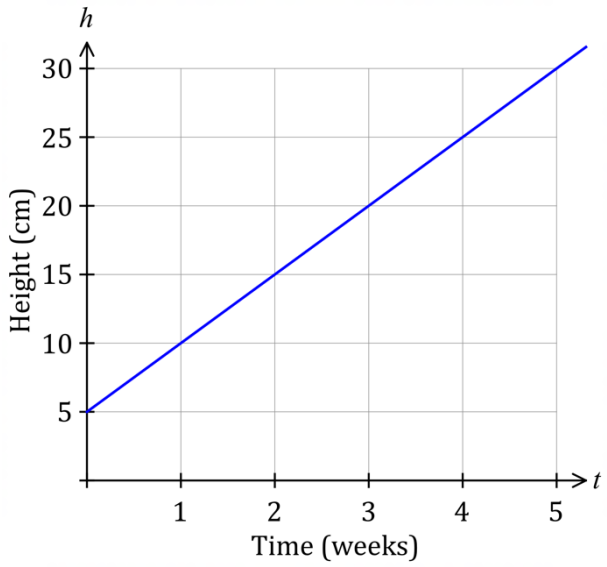
A group of year 12 students completed a survey regarding the number of hours of sleep per week. The results of the survey were normally distributed with 95% of the students indicating they had between 44 and 56 hours of sleep per week.

(a) Determine the mean number of hours of sleep per week from the group. **2**

(b) What was the standard deviation? **1**

Question 32 (3 marks) **Marks**

Stefan drew a graph of the height of a flowering shrub over five weeks.



(a) When was the initial height of the shrub? **1**

(b) Calculate the gradient of the line. **1**

(c) What is the equation of this line? **1**

Question 33 (2 marks)

The time taken (t, in hours) to complete a journey is inversely proportional to the speed (s, km/h). A car takes 4 hours to complete a journey at 65 km/h. Find the time taken to complete a journey if the car travels at 80 km/h. **2**

Question 34 (2 marks) **Marks**

The table below shows the future value of a $1 annuity.

Future value of $1					
Period	1%	2%	4%	6%	8%
4	4.0604	4.1216	4.2465	4.3746	4.5061
8	8.2857	8.5830	9.2142	9.8975	10.637
12	12.683	13.412	15.026	16.870	18.977
16	17.258	18.639	21.825	25.673	30.324
20	22.019	24.297	29.778	36.786	45.762

(a) What would be the future value of a $6000 per year annuity at 2% per annum for 20 years, with interest compounding annually? **1**

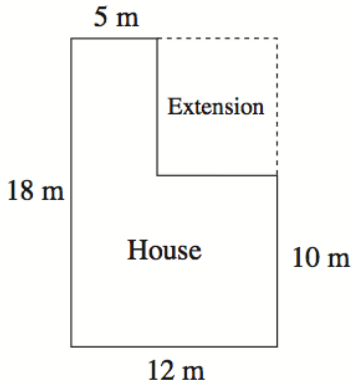
(b) An annuity of $5400 is invested every six months at 8% per annum, compounded biannually for 2 years. What is the future value of the annuity? **1**

Question 35 (2 marks)

The diagram opposite shows the plan of an extension to a house. **2**

A timber extension will cost $480 per square metre and a brick extension will cost $570 per square metre.

What will be the cost of building a brick extension?



Question 36 (4 marks) **Marks**

There are five motorways between five cities labelled A, B, C, D and E. The table below shows which cities are linked by the motorways and the length of each one in kilometres.

	A	B	C	D	E
A	–	–	–	22	46
B	–	–	43	19	–
C	–	43	–	7	–
D	22	19	7	–	–
E	46	–	–	–	–

(a) Represent the table shown above as a weighted network. **2**

(b) How would you travel from city E to city C? **1**

(c) What is the distance of the longest journey from city E to city C? **1**

Question 37 (3 marks) **Marks**

A truck is bought by a local council for \$120 000. It depreciates at 16% p.a.

(a) Calculate the value of the truck after 3 years using the declining balance formula. Answer correct to 2 decimal places. **1**

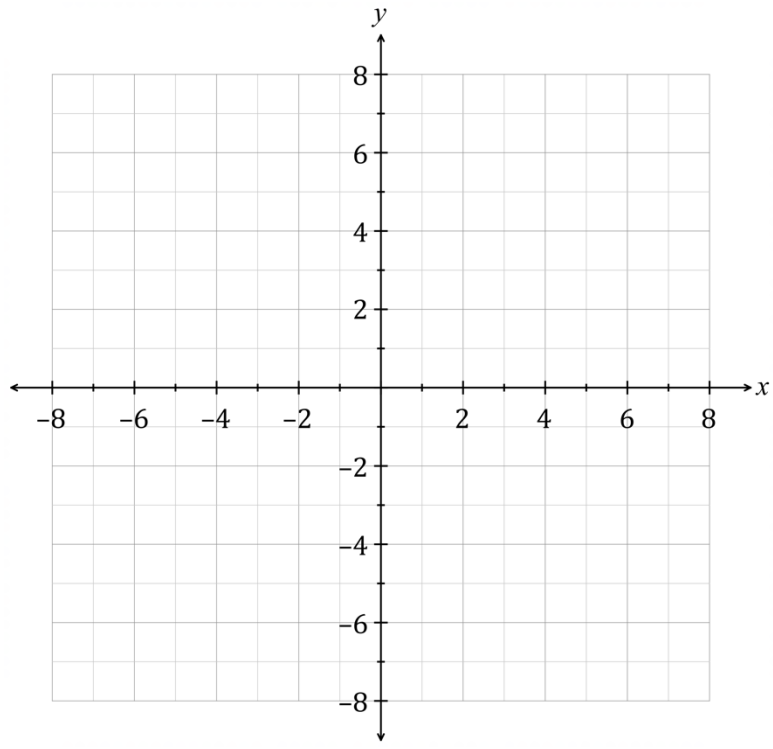
(b) What is the percentage loss in value of the truck after 3 years? Answer correct to the nearest whole number. **2**

Question 38 (3 marks)

Draw the graphs of the following pairs of equations to find their simultaneous solution. **3**

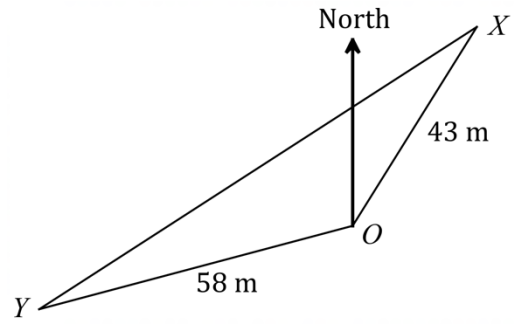
$5x + 2y = 16$

$x - y = -1$



Question 39 (3 marks) **Marks**

XOY represents a triangular area of land. The bearing of *X* from *O* is 028° and the bearing of *Y* from *O* is 244°. The distance *YO* is 58 m and the distance *XO* is 43 m



Not to scale

(a) Find $\angle XOY$. **1**

(b) Calculate the area of the land to the nearest square metre. **1**

(c) What is the length of X*Y* correct to one decimal place? **1**

Question 40 (2 marks)

The ages of the residents who live in Hudson Creek are normally distributed. The mean age is 56 years and the standard deviation is 14. What percentage of the residents are younger than 70? **2**

Question 41 (3 marks) **Marks**

Archie borrowed \$372 000 at 8% p.a. reducible interest. The interest is charged monthly and the monthly repayment is \$2840. The table shows the amounts owing during the first month.

Months (n)	*Principal (P)*	*Interest (I)*	*P + I*	*P + I - R*
1	\$372 000.00	\$2480.00	\$374 480.00	\$371 640.00
2				
3				

(a) What is the principal at the beginning of the second month? **1**

(b) How much is owed at the end of the second month? **1**

(c) How much is owed at the end of the third month? **1**

Question 42 (2 marks)

An estimate of a person's maximum heart rate (MHR) is given by the formula: **2**

$\text{MHR} = 220 - \text{AGE (years)}$ where MHR is measured in beats per minute and AGE is measured in years.

It is estimated that a healthy person should have a heart rate of 55% of their maximum rate when beginning to exercise. Holly is a healthy 17 years, 6 months old girl. What is an estimate of her heart rate, in beats per minute, when she begins exercising?

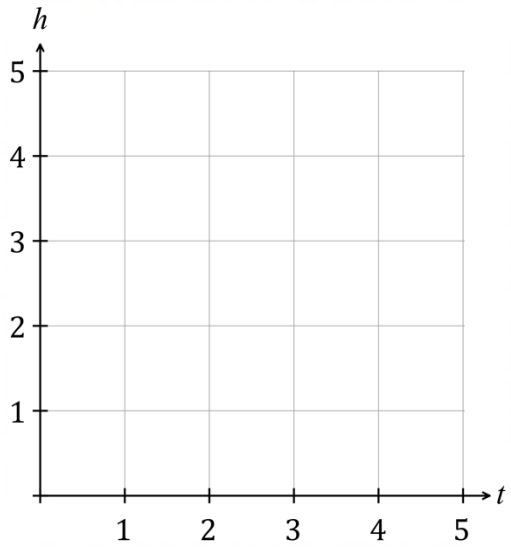
Question 43 (4 marks) **Marks**

Adam throws a ball and it takes 4 seconds to reach the ground. The height it reaches is given by the formula: $h = -t^2 + 4t$

(a) Complete the following table of values. **1**

t	0	1	2	3	4
h					

(b) Draw the graph of $h = -t^2 + 4t$ using the number plane below. **1**

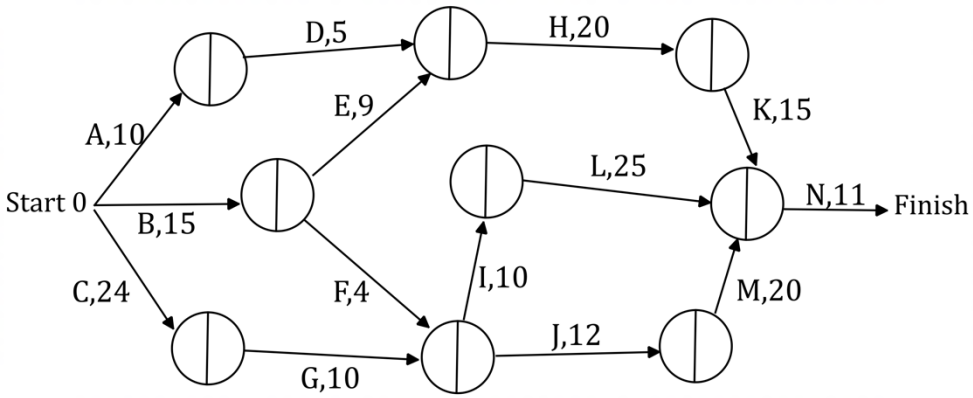


(c) What is the maximum height reached by the ball? **1**

(d) When is the maximum height reached? **1**

Question 44 (4 marks) **Marks**

The network diagram for a project is shown below. The duration for each activity is in days.



(a) Write the earliest starting times (EST) and latest starting times (LST) on the above network diagram. **3**

(b) What is the minimum completion time for the project? **1**

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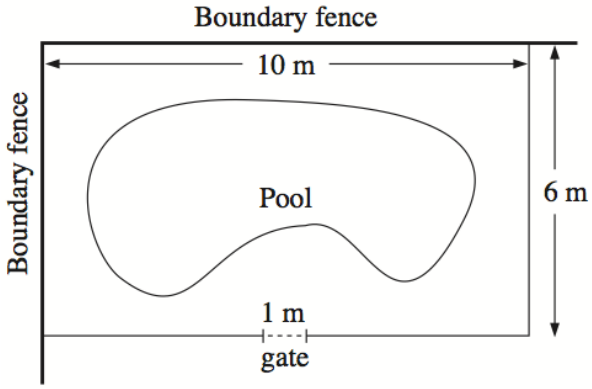
Question 45 (2 marks)

A plan of a swimming pool is shown below. **2**



The boundary fences of this pool are already in place. Fencing costs \$73·50 per metre. The gate costs \$255. What is the cost of completing the pool enclosure.

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End of paper

NSW Education Standards Authority

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Standard 1
Mathematics Standard 2

REFERENCE SHEET

Measurement

Precision

Absolute error = $\frac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement – absolute error

Length, area, surface area and volume

$$l = \frac{\theta}{360} \times 2\pi r$$

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(x + y\right)$$

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV\left(1 + r\right)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0\left(1 - r\right)^n$$

Statistical Analysis

$$z = \frac{x - \bar{x}}{s}$$

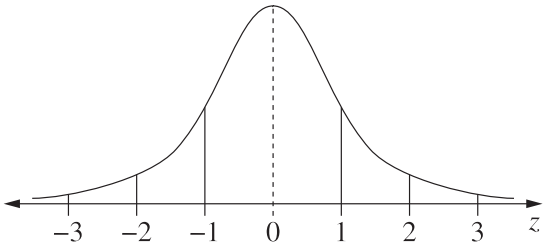
An outlier is a score

less than $Q_1 - 1.5 \times IQR$

or

more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3