UNSW Global
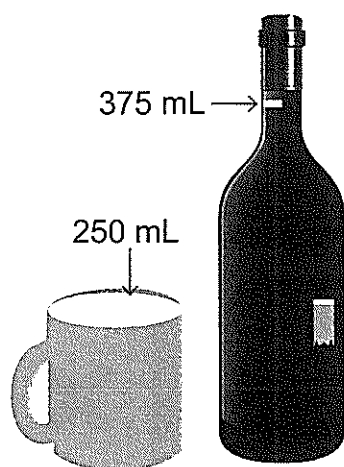## AUSTRALIA

PAPER
J

YEAR 12

2016
ICAS

International Competitions
and Assessments for Schools

MATHEMATICS

Educational Assessment Australia
eaa.unsw.edu.au

1. Ardi is making soup and needs 2 L of water. He has only this bottle and cup to measure the water.

375 mL →

250 mL

Which of these will measure exactly 2 L?

(A) 4 bottles and 2 cups
(B) 5 bottles and 1 cup
(C) 6 bottles
(D) 7 cups

2. Tom drew two circles. One has a diameter of 14 cm, the other has a diameter of 12 cm.

What is the difference in their areas, to the nearest whole number?

(A) 2 cm²
(B) 6 cm²
(C) 41 cm²
(D) 163 cm²

3. Lin wrote this number pattern.
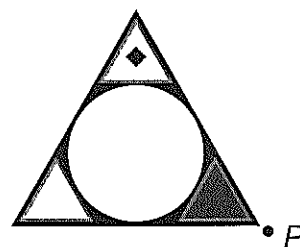
24, −12, 6, −3, ...

Which of these describes a way to find each term in the pattern?
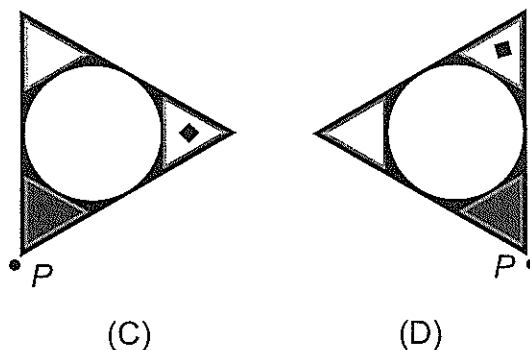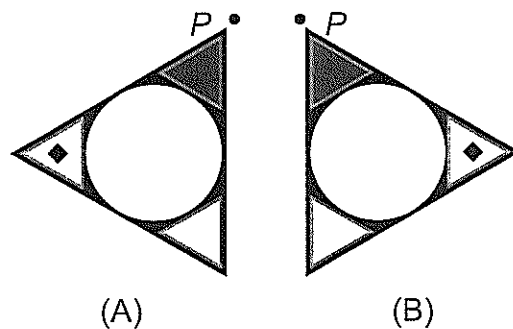
(A) divide the previous term by 0.5
(B) multiply the previous term by 0.5
(C) divide the previous term by −0.5
(D) multiply the previous term by −0.5
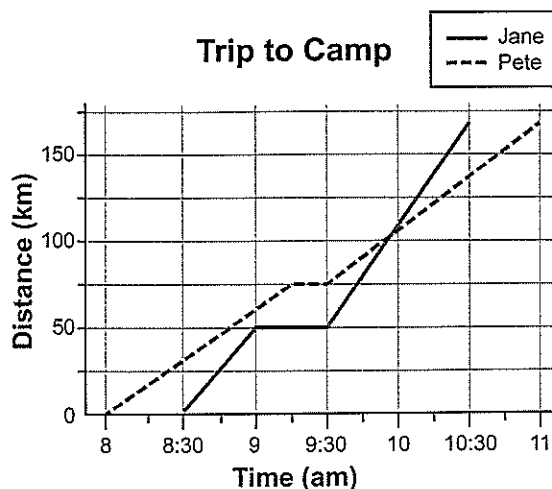
4. Jack drew this equilateral triangle.

P

He rotated it 90° clockwise about P.

Which of these is Jack's shape after the rotation?

(A)

(B)

(C)

(D)

5. Jane and Pete are travelling from school to camp. Pete travels on the bus while Jane travels in a car with her mother. Both groups stopped for morning tea.

Trip to Camp   —— Jane   - - - Pete

How much longer than Jane did Pete spend travelling, not including the morning tea stop?

(A) 30 minutes
(B) 45 minutes
(C) 1 hour
(D) 1 hour 15 minutes

6. Two draught horses can pull three times the load that one draught horse can pull. Two oxen can pull double the load that two draught horses can pull.

How many times greater than the load that one draught horse can pull is the load that two oxen can pull?

(A) 2
(B) 3
(C) 5
(D) 6

7. The time, T hours, taken to fill this tank using the two pipes together is given by the formula:

$$\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$
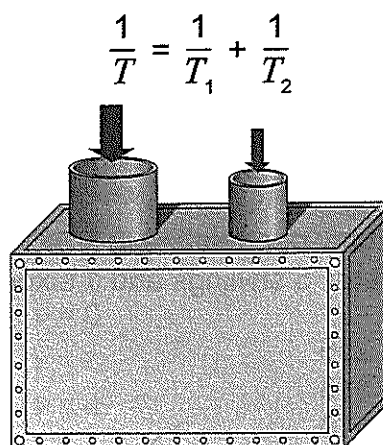
What is the value of T when $T_1 = \frac{2}{5}$ and $T_2 = \frac{2}{3}$?

(A) $\frac{1}{4}$

(B) $\frac{16}{15}$

(C) $3\frac{1}{15}$

(D) 4

8.
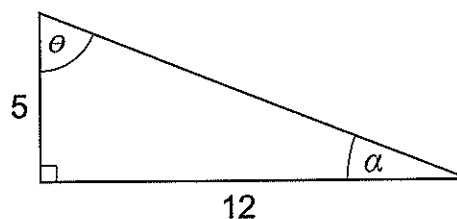
Which statement is correct?

(A) $\sin \alpha > \tan \theta$
(B) $\cos \alpha > \tan \theta$
(C) $\cos \alpha > \sin \theta$
(D) $\tan \alpha > \cos \theta$

9. In the early 20th century, Australian currency was organised as shown.

1 sixpence = 6 pennies
1 shilling = 2 sixpences
1 pound = 20 shillings
20 guineas = 21 pounds

How many pennies are in 2 guineas, 5 shillings and 1 sixpence?

(A) 256
(B) 318
(C) 570
(D) 906

10. In a school 60% of the students are girls. The proportion of girls who walk to school is 40%. The proportion of boys who walk to school is 20%.

What percentage of all the students walk to school?

(A) 30%
(B) 32%
(C) 36%
(D) 60%

11. Jia has a combination lock with 4 digits.

Each week Jia changes the code to her lock.

She always sets the code so that the digits increase in value from left to right and the 4-digit code is divisible by 3.

How many of Jia's codes are greater than 5000?

(A) 1
(B) 2
(C) 4
(D) 5

12. Sasi graphed two lines and shaded a region.
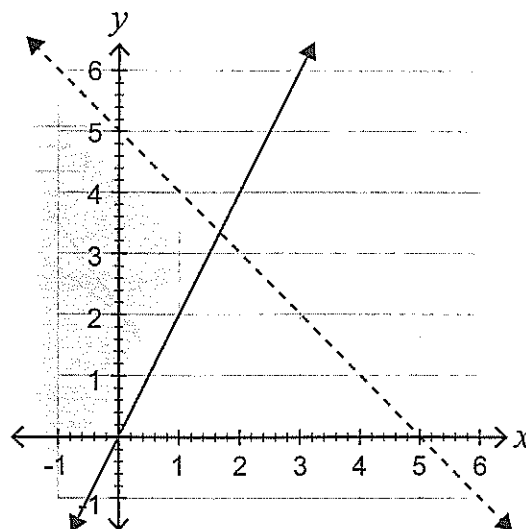
Which inequalities satisfy the shaded region?

(A) $y \geq 2x$ and $y < 5 - x$
(B) $y \leq 2x$ and $y > 5 - x$
(C) $y \geq 2x$ and $y < x + 5$
(D) $y \leq 2x$ and $y > x + 5$

13. Edward has a rectangular prism. He paints its surface completely.

5 cm

7 cm

11 cm

Then he cuts the prism into 1 cm cubes.

How many of these cubes have paint on them?

(A) 385
(B) 334
(C) 250
(D) 145

14. Sara cut out these 8 coloured shapes from centimetre grid paper.

She joined all of the shapes together to form a right-angled isosceles triangle.

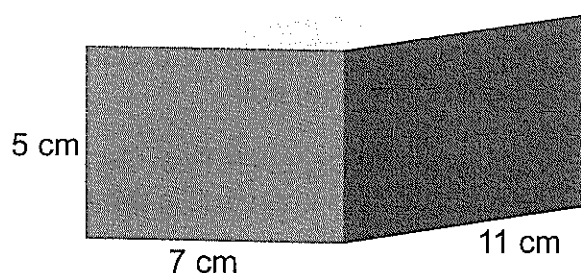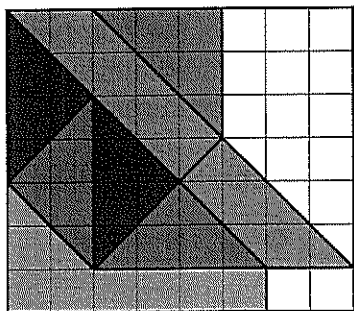What was the perimeter of Sara's triangle, correct to the nearest cm?

(A)   27 cm
(B)   31 cm
(C)   34 cm
(D)   36 cm

15. In this number pattern, the difference between consecutive numbers increases by a constant amount.

5, ___ , [?] , ___ , 19

What value must [?] be?

(A)   9.5
(B)   10
(C)   12
(D)   14.5

16. This is one of the formulae Einstein derived as part of his special theory of relativity.

$$T = \frac{t}{\sqrt{1 - \dfrac{v^2}{c^2}}} \text{, where } v \geq 0$$

Which of these values will result in $T > t$ given that T is finite?

(A) $v = 0$
(B) $v = c$
(C) $v < c$
(D) $v > c$

17. Lori wants to devise a method for finding the total shaded area in this figure.
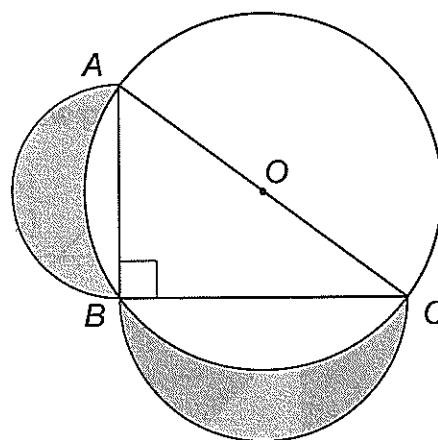
The areas of the regions are as follows:

A_1 : Area of semi-circle with diameter AC

A_2 : Area of semi-circle with diameter AB

A_3 : Area of semi-circle with diameter BC

A_4 : Area of triangle ABC

Which expression gives the shaded area?

(A) $A_2 + A_3 + A_4 - A_1$
(B) $A_1 + A_2 + A_3 - A_4$
(C) $A_1 - (A_2 + A_3)$
(D) $A_2 + A_3$

18. Eloise, Kurt and Cadel are completing laps on the same track.

| Athlete	Time for 1 lap
Eloise	72 seconds
Kurt	48 seconds
Cadel	42 seconds

They begin together at the start line and continue at this pace. They only stop when they all cross this line together.

How many laps did Kurt complete?

(A) 21
(B) 14
(C) 12
(D) 6

19. Anna has an open square prism with dimensions 6 cm × 6 cm × 14 cm. The prism is completely filled with water. Anna tilted the prism at 45° causing water to spill out.

6 cm

14 cm

45°

NOT TO SCALE
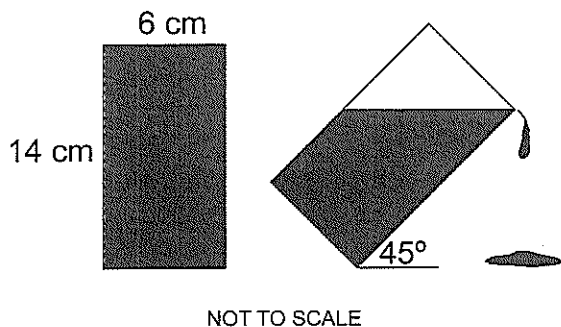
When the water stopped spilling out of the prism, Anna returned it to the upright position.

What is the new depth of water in the prism?

(A) 3 cm
(B) 6 cm
(C) 8 cm
(D) 11 cm

20. At a beach shop, a pie sold for twice the price of an ice cream.

The graph shows the numbers of pies and ice creams sold on days with different maximum temperatures.
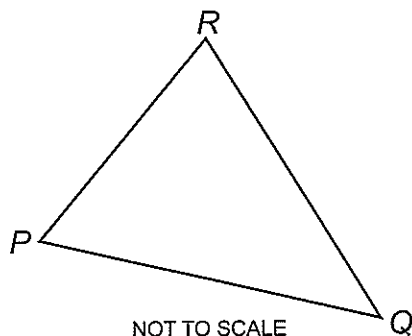
Beach Shop Sales
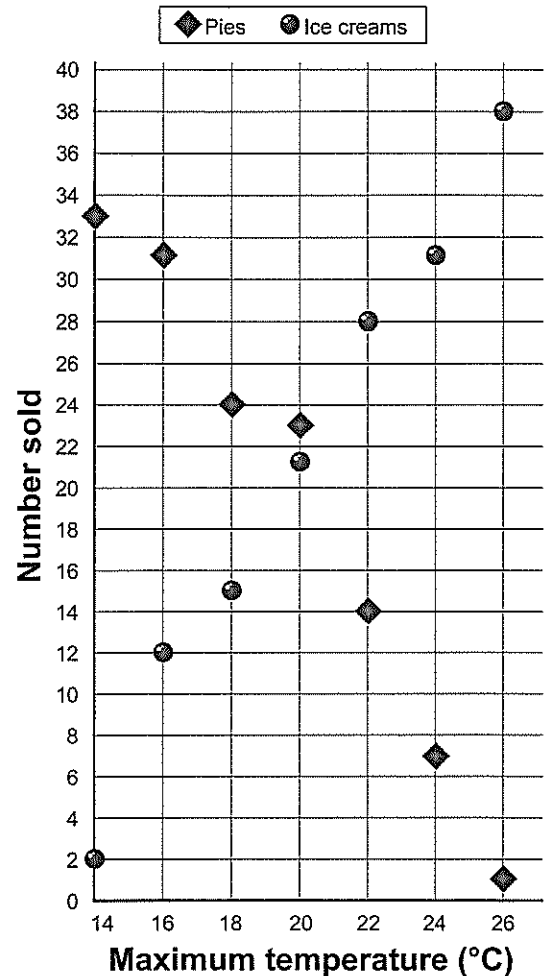
What was the maximum temperature on the day the total amount of money taken for pies and ice creams was the greatest?

(A) 14 °C
(B) 16 °C
(C) 20 °C
(D) 26 °C

21. The diagram shows triangle *PQR*.

NOT TO SCALE

$\angle P + \angle R = 107°$
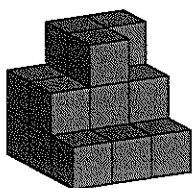
$\angle P + \angle Q = 118°$

$\angle R + \angle Q = 135°$

Which of these shows the angles in order of size?

(A) $\angle R, \angle Q, \angle P$
(B) $\angle Q, \angle P, \angle R$
(C) $\angle P, \angle Q, \angle R$
(D) $\angle P, \angle R, \angle Q$

22. Sam is using 27 small cubes to build a large cube.

Which two parts will complete the cube?

(A)

(B)

(C)

(D)

23. At Hayfield Senior High School

- 65% of students study History
- 40% of students study Geography
- 30 students study both subjects
- 10% of students study neither subject.

How many students are there at Hayfield Senior High?

(A)   95   (B)   200
(C)   300   (D)   600

24. The area of the curved surface of a coin is one-fifth of its total surface area.

radius

thickness

curved surface   NOT TO SCALE
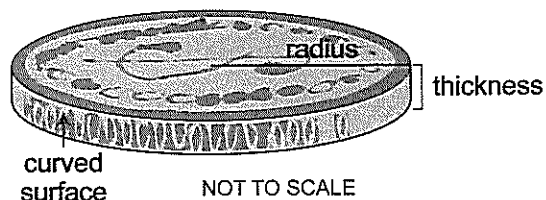
What percentage of the radius is the thickness of the coin?

(A)   25%   (B)   20%
(C)   12.5%   (D)   10%

25. This is the graph of $y = 2^x$.
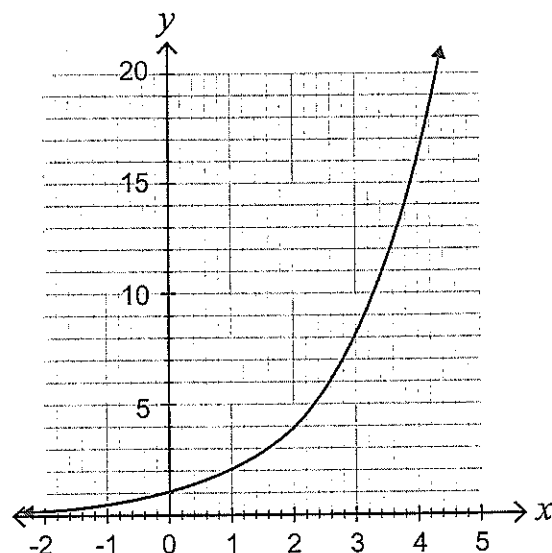
How many solutions are there for $2^x = x^2$?

(A)   1   (B)   2
(C)   3   (D)   4

26. In triangle *ABC*, *XB* = *XP*, *XA* = *XQ* and ∠*ACB* = 40°

NOT TO SCALE
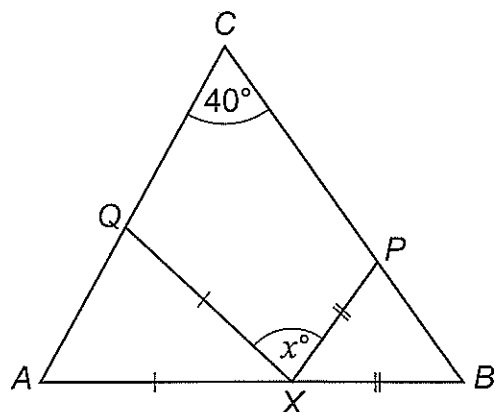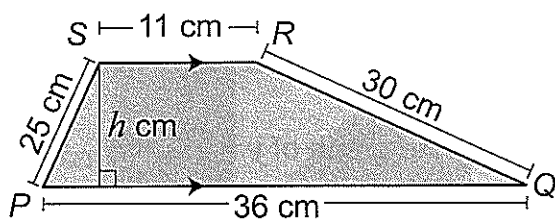
What is the value of x?

(A) 80
(B) 100
(C) 120
(D) 140

27. *PQRS* is a trapezium.

NOT TO SCALE

What is the value of h?

(A) 16
(B) 18
(C) 20
(D) 24

28. Jane is investigating the maximum number of regions formed by joining points on a circle.
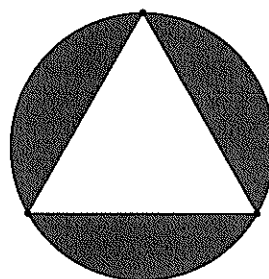
3 points
4 regions
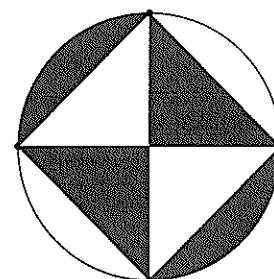
4 points
8 regions
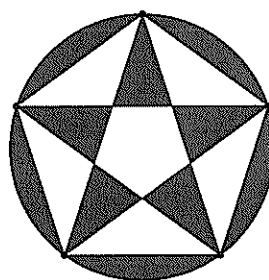
5 points
16 regions
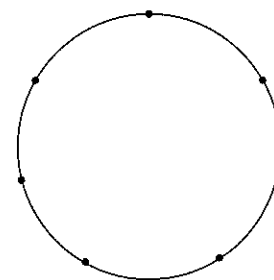
7 points
? regions

She used this formula to find the maximum number of regions, *R*, formed by joining *n* points, where *n* > 4.

$$R = \frac{n!}{4!(n-4)!} + \frac{n!}{2!(n-2)!} + 1$$

where

$4! = 4 \times 3 \times 2 \times 1$ and $2! = 2 \times 1$

What is the maximum number of regions formed by joining 7 points on a circle?

(A) 32   (B) 57
(C) 63   (D) 64

29. Which of these is closest to the number of digits in the number 2^{1000} when it is written out in full with no indices?

(A) 100
(B) 300
(C) 500
(D) 1000

30. Anna drew a decagon with equal sides.

NOT TO SCALE
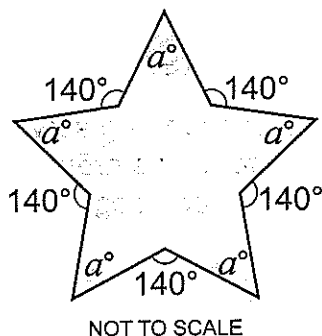
What is the value of a?

(A) 20
(B) 36
(C) 68
(D) 72

31. The diagram shows eight touching circles of equal radius.

The lines joining the centres of adjacent circles form an irregular hexagon.
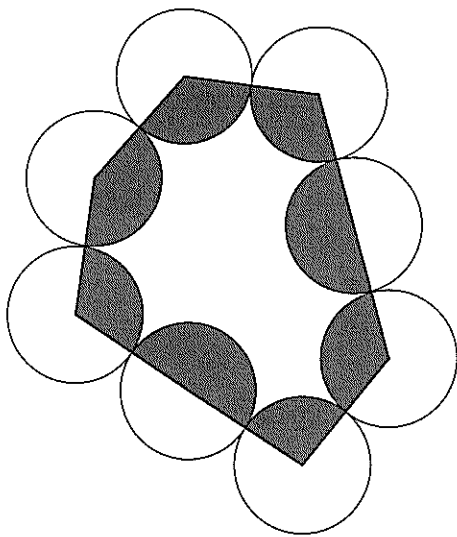
What is the ratio of the shaded area to the unshaded area of these circles?

(A) 1 : 6
(B) 1 : 3
(C) 3 : 8
(D) 3 : 5

32. Any number can be written as a product of its prime factors.

For example, $72 = 2 \times 2 \times 2 \times 3 \times 3$.

A square number is the product of m even and n odd prime numbers.

Which statement is true for all square numbers?

(A) m is even and n is odd.

(B) They are of the form $(m + n)^2$.

(C) They have a maximum of $\dfrac{n + 4}{2}$ different prime factors.

(D) They are divisible by 2^m.

33. A conical frustum is made by slicing the top off a cone, with the cut made parallel to the base.
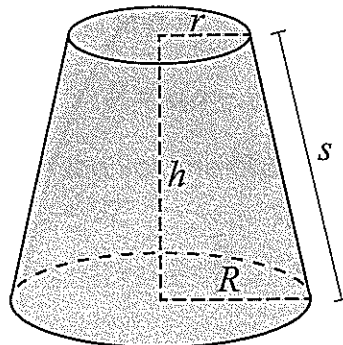
The radii at the top and bottom of the frustum are r and R respectively.

The perpendicular height is h and slant height is s. Also, $h = 2R = 3r$.

What is the value of $\dfrac{s}{h}$?

(A) $\dfrac{\sqrt{37}}{6}$

(B) $\sqrt{\dfrac{13}{3}}$

(C) $\dfrac{\sqrt{5}}{2}$

(D) $\dfrac{3}{2}$

34. Four students compared each of their scores to the mean of their scores.

- Abby scored 8 more than the mean.
- Bill scored 6 less than the mean.
- Cathy scored half the mean.
- David scored 82 which was above the mean.

What was the mean?

(A) 56
(B) 63
(C) 68
(D) 80

35. The entrance to a stadium is a parabolic arch, 6 m wide at the base.

A camera is mounted on one side of the arch as shown.

NOT TO SCALE

The height of the arch, h metres, can be modelled by the equation:

$$h = ax(b - x)$$

What is the maximum height of the arch, correct to one decimal place?

(A) 2.7 m
(B) 4.5 m
(C) 7.9 m
(D) 9.0 m

QUESTIONS 36 TO 40 ARE FREE RESPONSE.

Write your answer in the boxes provided on the ANSWER SHEET and fill in the ovals that match your answer.

36. Pete runs a dog-walking service. He charges $36 to walk a group of dogs and $20 to walk one dog.
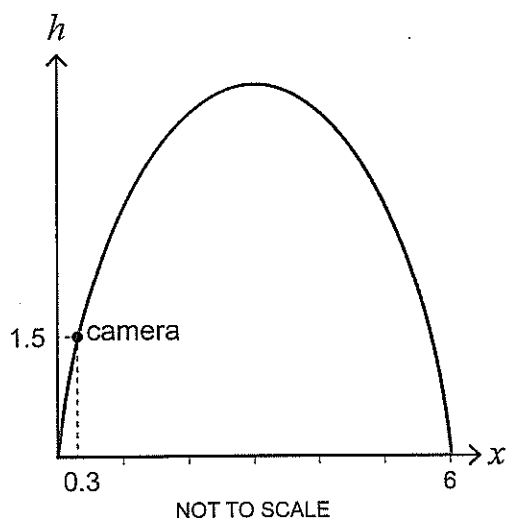
His income for the year was $5760.

The group walks earned Pete $1800 more than the individual walks.

How many dog walks did Pete get paid for that year?

37. A factory mixes cordial with water in a barrel in the ratio of 2 : 5.

50 L of cordial was accidentally added to the barrel and the ratio of cordial to water in the mixture changed to 12 : 25.

Water was then added to the mixture to restore the original ratio.

How many litres of water were in the final mixture?

38. $AC = BC = x$ cm, $AB = PB$, $PC \parallel AB$ and $\angle ACB = 90°$.

NOT TO SCALE
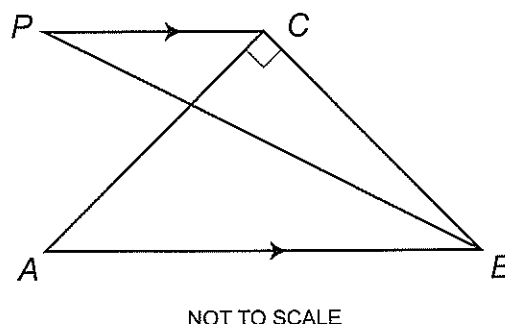
What is the size of $\angle PBC$, in degrees?

39. In Sydney, 4% of the population suffer from numeria. The test to detect this disease is 90% accurate.

Sarah tests positive for numeria. She drew up this table to organise the information.

	Test accurate	Test inaccurate
Numeria		
No numeria		

Given the diagnosis, what is the probability that Sarah actually has numeria?

Give your answer to the nearest per cent.

40. Cai's factory produces identical rectangular sheets of cardboard with dimensions a cm by b cm.

Four identical squares of side length x cm are cut from the corners and the remaining cardboard is folded to make an open box. The variables a, b and x are integers and $0 < 2x < a < b < 30$.
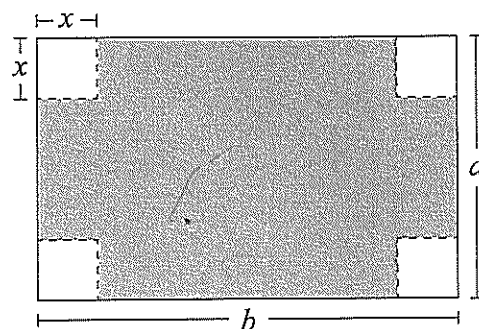
When squares of side length 3 cm are cut out, the volume is 819 cm³.

Cai used this formula to calculate the fixed values of a and b.

$$V = x(a - 2x)(b - 2x)$$

What is the volume of the box, in cm³, when $x = 5$?

PAPER J

+ × − =

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AUSTRALIA

HOW TO FILL OUT THIS SHEET:

USE A 2B OR B PENCIL

- Print your details clearly in the boxes provided.
- Make sure you fill in only one oval in each column.
- Rub out all mistakes completely.

EXAMPLE 1: Debbie Bach

FIRST NAME: DEBBIE
LAST NAME: BACH

EXAMPLE 2: Chan Ai Beng

FIRST NAME: CHAN
LAST NAME: AI BENG

EXAMPLE 3: Jamal bin Abas

FIRST NAME: JAMAL BIN
LAST NAME: ABAS

FIRST NAME to appear on certificate

LAST NAME to appear on certificate

Are you male or female?
○ Male   ○ Female

Does anyone in your home usually speak a language other than English?
○ Yes   ○ No

School name: _____

Town / suburb: _____

Today's date: ___ / ___ / ___

Postcode: _____

DATE OF BIRTH

Day	Month	Year
⓪ ⓪	⓪ ⓪	⓪ ⓪
① ①	① ①	① ①
② ②	② ②	② ②
③ ③	③ ③	③ ③
④	④ ④	④ ④
⑤	⑤ ⑤	⑤ ⑤
⑥	⑥ ⑥	⑥ ⑥
⑦	⑦ ⑦	⑦ ⑦
⑧	⑧ ⑧	⑧ ⑧
⑨	⑨ ⑨	⑨ ⑨

CLASS (optional)

Ⓐ Ⓚ
Ⓑ Ⓛ
Ⓒ Ⓜ
Ⓓ Ⓝ
Ⓔ Ⓞ
Ⓕ Ⓟ
Ⓖ Ⓠ
Ⓗ Ⓡ
Ⓘ Ⓢ
Ⓙ Ⓣ

TO ANSWER THE QUESTIONS

MULTIPLE CHOICE
Questions 1 to 35

Example: 4 + 6 =

(A)   2
(B)   9
(C)   10
(D)   24

The answer is 10, so fill in the oval ©, as shown.

Ⓐ   Ⓑ   ●   Ⓓ

FREE RESPONSE
Questions 36 to 40

Example: 6 + 6 =

- The answer is 12, so WRITE your answer in the boxes.
- Write only ONE digit in each box, as shown, and fill in the correct ovals, as shown.

USE A 2B OR B PENCIL

START

1 Ⓐ Ⓑ Ⓒ Ⓓ   11 Ⓐ Ⓑ Ⓒ Ⓓ   21 Ⓐ Ⓑ Ⓒ Ⓓ   31 Ⓐ Ⓑ Ⓒ Ⓓ

2 Ⓐ Ⓑ Ⓒ Ⓓ   12 Ⓐ Ⓑ Ⓒ Ⓓ   22 Ⓐ Ⓑ Ⓒ Ⓓ   32 Ⓐ Ⓑ Ⓒ Ⓓ

3 Ⓐ Ⓑ Ⓒ Ⓓ   13 Ⓐ Ⓑ Ⓒ Ⓓ   23 Ⓐ Ⓑ Ⓒ Ⓓ   33 Ⓐ Ⓑ Ⓒ Ⓓ

4 Ⓐ Ⓑ Ⓒ Ⓓ   14 Ⓐ Ⓑ Ⓒ Ⓓ   24 Ⓐ Ⓑ Ⓒ Ⓓ   34 Ⓐ Ⓑ Ⓒ Ⓓ

5 Ⓐ Ⓑ Ⓒ Ⓓ   15 Ⓐ Ⓑ Ⓒ Ⓓ   25 Ⓐ Ⓑ Ⓒ Ⓓ   35 Ⓐ Ⓑ Ⓒ Ⓓ

6 Ⓐ Ⓑ Ⓒ Ⓓ   16 Ⓐ Ⓑ Ⓒ Ⓓ   26 Ⓐ Ⓑ Ⓒ Ⓓ

7 Ⓐ Ⓑ Ⓒ Ⓓ   17 Ⓐ Ⓑ Ⓒ Ⓓ   27 Ⓐ Ⓑ Ⓒ Ⓓ

8 Ⓐ Ⓑ Ⓒ Ⓓ   18 Ⓐ Ⓑ Ⓒ Ⓓ   28 Ⓐ Ⓑ Ⓒ Ⓓ

9 Ⓐ Ⓑ Ⓒ Ⓓ   19 Ⓐ Ⓑ Ⓒ Ⓓ   29 Ⓐ Ⓑ Ⓒ Ⓓ

10 Ⓐ Ⓑ Ⓒ Ⓓ   20 Ⓐ Ⓑ Ⓒ Ⓓ   30 Ⓐ Ⓑ Ⓒ Ⓓ

36   37   38   39   40

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