CARLINGFORD HIGH SCHOOL

DEPARTMENT OF MATHEMATICS

Year 12 Mathematics

Half Yearly Examination 2018

Time allowed: 2 hours

Student Number: ______________________________

Instructions:

- Start a new booklet for each question
- Use black/blue pen. Pencil may be used for graphs and diagrams.
- Board approved calculators may be used
- Show all necessary working
- Marks may be deducted for illegible or badly set out work

	Question 1	**Question 2**	**Question 3**	**Question 4**	**Question 5**	**Question 6**	**Total**
Geometric Application of Calculus		/13			/10		**/23**
Integral Calculus			/12		/3	**/10**	**/25**
Series & Applications	/12		/3	/15		**/2**	**/32**
Total	**/12**	**/13**	**/15**	**/15**	**/13**	**/12**	**/80**

Question 1 (12 marks) **[START A NEW PAGE]**

(a) Consider the arithmetic sequence 5, 9, 13, …

(i) Find next term **1**

(ii) Is 2013 a term in the sequence? Explain your answer. **2**

(b) Find the sum of the first seven terms of the sequence **2**

5, 15, 45, 135,

(c) Find the limiting sum of the series: **2**

$$90+30+10+\ldots$$

(d) The infinite geometric series $x-\frac{x}{4}+\frac{x}{16}-\ldots$ has a limiting sum of $\frac{2}{5}$. **3**

Calculate the value of x.

(e) Which term of the sequence 24, 12, 6 …. is $\frac{3}{16}$? **2**

Question 2 (13 marks) [START A NEW PAGE]

(a) For the curve $y = 4 + 3x - x^3$

i) Find the first and second derivatives **2**

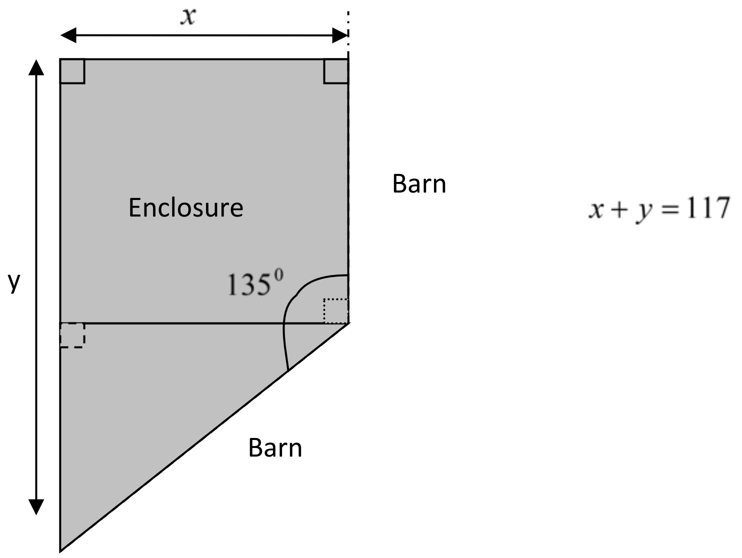
ii) any turning (stationary) points and their nature **2**

iii) any points of inflexion **1**

iv) the y-intercept **1**

v) sketch the curve showing the above **2**

(b) An enclosure is to be built adjoining a barn, as in the diagram. The walls of the barn meet at 135^0 and 117 metres of fencing is available for the enclosure, so that $x + y = 117$ where x and y are as shown in the diagram.



i) Show that the shaded area of the enclosure in square metres is given by

$$A = 117x - \frac{3}{2}x^2$$ **2**

ii) Show that the largest area of the enclosure occurs when $y = 2x$. **3**

Question 3 (15 marks) [START A NEW PAGE]

(a) Given $\dfrac{dv}{dt} = 2 - t$ and $v = 10$ when $t = 6$, find the function v in terms of t. **2**

(b) Find the following indefinite integrals

(i) $\displaystyle\int (\sqrt{x} - x^{-2})\ dx$ **2**

(ii) $\int (3x + 5)^7 dx$ 2

(c) Evaluate the following definite integrals in exact values.

(i) $\displaystyle\int_{-1}^{3} t(3t - 1)\ dt$ **3**

(ii) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x}}$ **3**

(d) The sum of the first two terms of a geometric series is 18 and the sum of the third and fourth terms of the series is 72. **3**

Show that there are two possible series which meet the criteria above and write down the first four terms of each series.

Question 4 (15 marks) **[START A NEW PAGE]**

(a) After starting full-time work a man saves \$16 in the first week, \$20 in the second week, \$24 in the third week and continues to increase his savings each week by the same amount until the twelfth week.

i) Find the value of the first term (a) **1**

ii) Find the value of the common difference (d) **1**

iii) Write down the *n*th term **1**

iv) How much is saved in the twelfth week? **1**

v) How much is saved in total over the twelve weeks? **2**

(b) A person takes out a loan of \$20 000. The interest is calculated monthly at the rate of 1·5% per month, and is compounded each month. The person intends to repay the loan with interest in 36 equal monthly instalments of \$M.

i) How much does the farmer owe at the end of the first month in terms of M. **1**

ii) Write an expression involving M for the total amount owed by the person after 36 months. **2**

iii) Find the amount of each monthly instalment to the nearest cent. **2**

(c) A worker invests $\$P$ at the beginning of each month into a retirement fund that pays 6% p.a. compounded monthly, on the money invested, for 20 years.

(i) Show that after 2 months there is $\$P(1.005^2 + 1.005)$ in the fund. **1**

(ii) Write an expression for the amount after one year. **1**

(iii) The worker wishes to retire at the end of the 20 years with a lump sum of \$450 000. 2

What investment must the worker make at the beginning of each month?

Question 5 (13 marks) **[START A NEW PAGE]**

(a) A cylindrical can is to be constructed in such a way that the sum of its height and its diameter will be 18 cm.

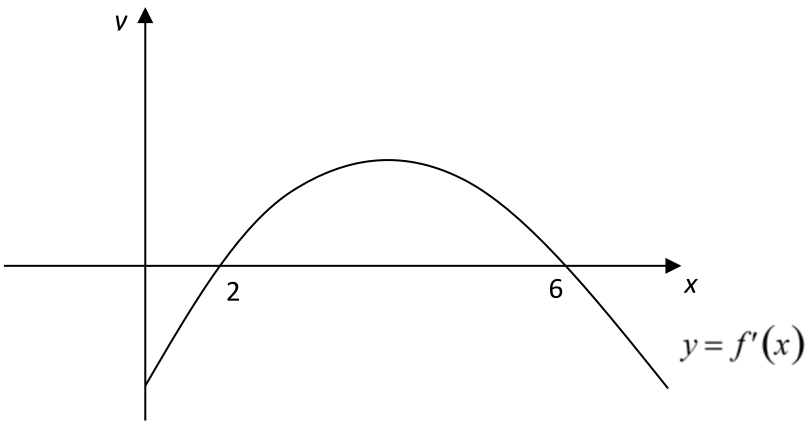
(i) Show that the volume can be expressed by $V = 18\pi r^2 - 2\pi r^3$, where r represents the radius. **2**

(ii) Hence, find the dimensions of the can that will make the volume a maximum. **3**

(iii) Calculate the maximum volume correct to the nearest cm^3. **1**

(b) Find the area of the region bounded by the curve $y = (x-1)(x-3)$, the x-axis and the lines $x = 2$ and $x = 4$ **3**

(c)



The diagram shows the graph of the gradient function of the curve $y = f(x)$.

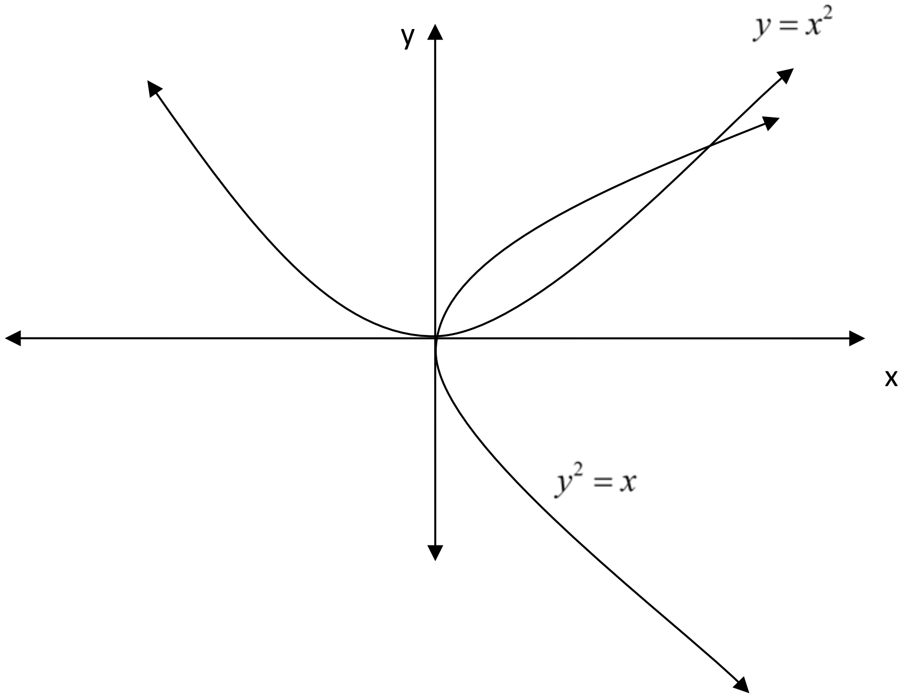
(i) For what values of x does $y = f(x)$ have a local minimum? Justify your answer. **2**

(ii) Draw a possible sketch of the curve $y = f(x)$. **2**

Question 6 (12 marks) **[START A NEW PAGE]**

(a) Evaluate $\sum_{n=3}^{5}\left(n^2 + 3\right)$ **2**

(b)

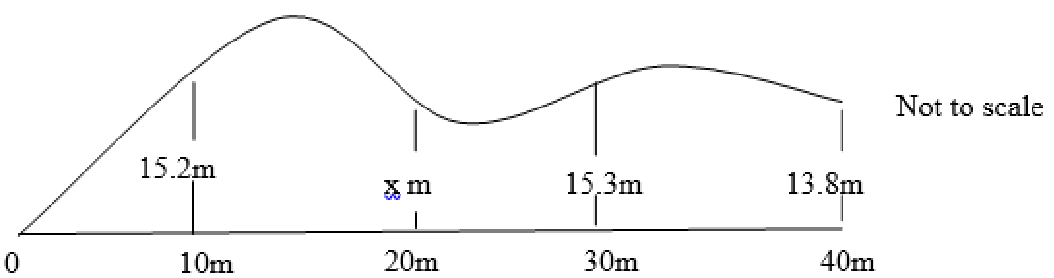


(i) Find the points of intersection of the two curves. **2**

(ii) Find the area between the curves. **2**

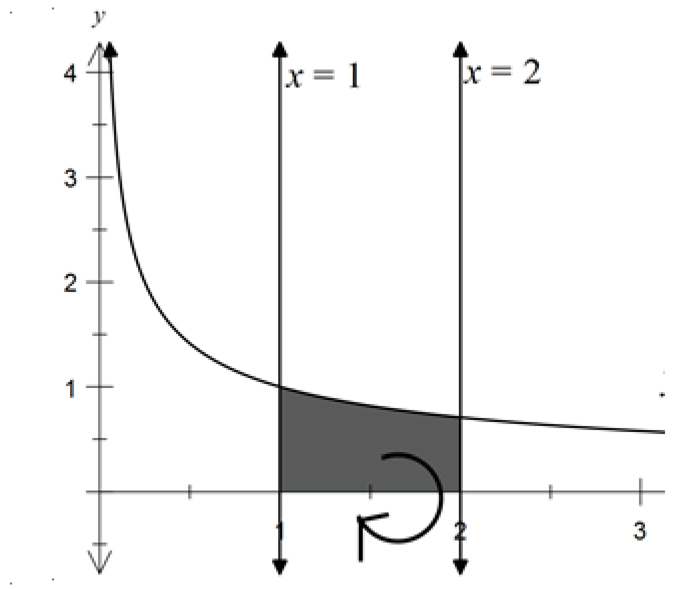
(c)

The following diagram represents a cross-section through a lake. The depth of the lake is marked every 10 metres.



Using Simpson's with five function values John found an approximation for the area of the lake to be 546 m^2. Find the value of x? **3**

(d) The region between the functions $y = \frac{1}{x}$, $x = 1$ and $x = 2$ is rotated about the x-axis. Find the volume of the solid formed. 3



END OF EXAM