# Carlingford High School

**2020** HIGHER SCHOOL CERTIFICATE
YEAR 11 COURSE
FINAL EXAMINATION

# Mathematics - Extension 1

**Student Number** ________________________________

**General Instructions**

- Reading time – 5 minutes
- Working time – 1 hour and 30 minutes
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided at the back of this paper

For Questions in Section II, show relevant mathematical reasoning and/or calculations

**Total marks : 50**

**Section I – 5 marks** (pages 3 – 4)

- Attempt Questions 1 – 5
- Allow about 10 minutes for this section

**Section II – 45 marks** (pages 5 – 9)

- Attempt Questions 6 – 9

Allow about 1 hour and 20 minutes for this section

| | Functions | Polynomials | Trigonometry | Inverse Trig | Combinatorics | Total |
|---|---|---|---|---|---|---|
| **1-5** | | | | | | **/5** |
| **6** | | **/4** | **/2** | | **/4** | **/10** |
| **7** | **/3** | **/2** | | | **/6** | **/11** |
| **8** | **/4** | | **/3** | **/3** | **/2** | **/12** |
| **9** | **/7** | | | **/3** | **/2** | **/12** |
| **Total** | **/14** | **/6** | **/5** | **/6** | **/14** | **/50** |
| | | | | | | **%** |

## Section I

**5 marks**
**Attempt Questions 1–5**
**Allow about 10 minutes for this section**

Use the multiple-choice answer sheet for Questions 1 – 5

1. How many distinct arrangements of the letters from the word **EXULTATIONS** are possible in which the vowels are all together?

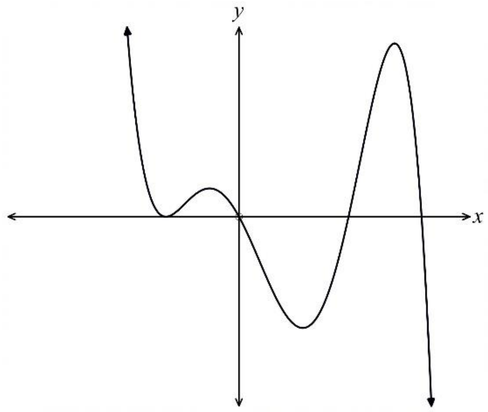
   (A) $\frac{7!\,5!}{2!}$

   (B) $\frac{11!}{2!}$

   (C) $7!\ 5!$

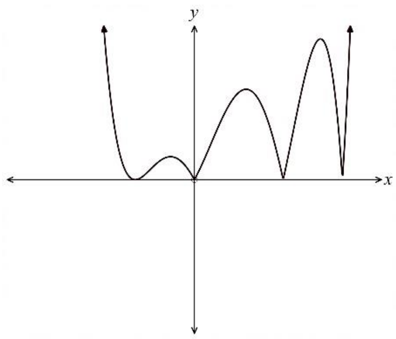
   (D) $11!$

2. The graph of $y = f(x)$ is shown below.

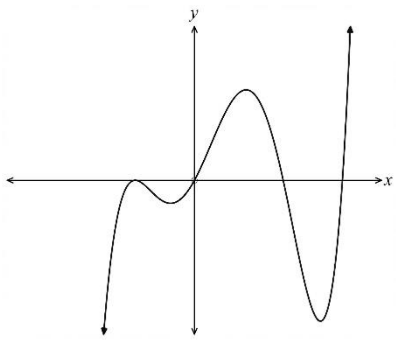



   Which graph shows $y = f(|x|)$ ?

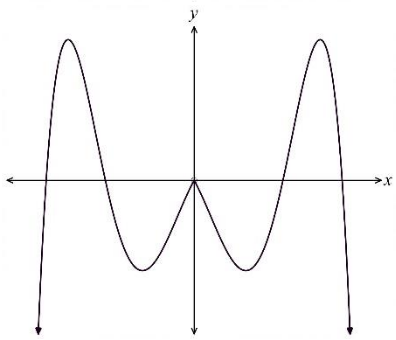
   (A)



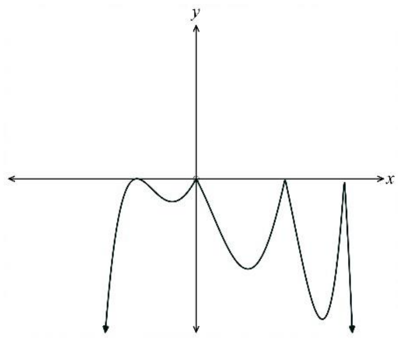

   (B)




   (C)

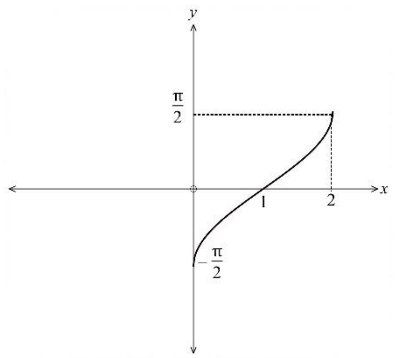



   (D)

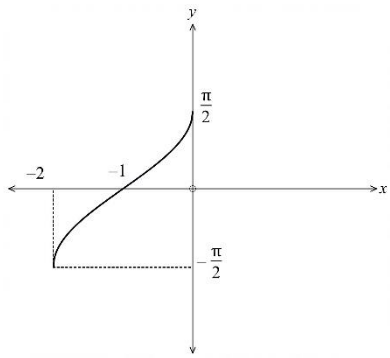


3. Which graph shows the curve $y = \sin^{-1}(x+1)$?

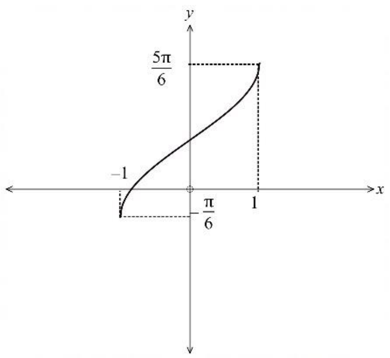
(A)



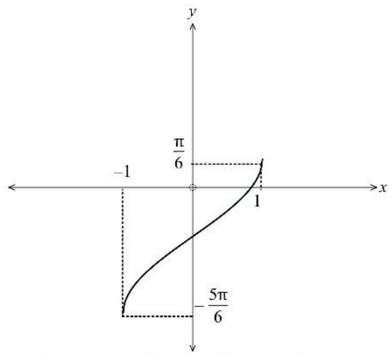

(B)




(C)




(D)




4. What is the exact value of cos (15°) ?

(A) $\frac{\sqrt{6}}{4}$

(B) $\frac{\sqrt{3}}{4}$

(C) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(D) $\frac{\sqrt{6}+\sqrt{2}}{4}$

5. Solve $|3x-4| \leq 16$.

(A) $-6\frac{2}{3} \leq x \leq 4$

(B) $-4 \leq x \leq 6\frac{2}{3}$

(C) $x \leq -4$ and $x \geq 6\frac{2}{3}$

(D) $x \leq -6\frac{2}{3}$ and $x \geq 4$

**Section II**

**45 marks**

**Attempt Questions 6 – 9.**

**Allow about 1 hour and 20 minutes for this section.**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 9 – 12, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 6** (10 marks) Use a new writing booklet.

(a) Use division of polynomials to find the remainder $R(x)$, when **2**

$P(x) = 4x^5 - 2x^3 + 2x^2 - 7$ is divided by $Q(x) = (2x^2 - 3)$.

(b) Find the values of $k$ and $m$ in the polynomial $P(x) = x^3 + kx + m$, given that $x + 3$ is a factor of $P(x)$ and the remainder when $P(x)$ is divided by $x + 1$ is 30. **2**

(c) Use the factorial definitions of $\binom{n}{r} = {}^n\mathbf{C}_r$ to show that $\binom{n}{n-1} = n$. **2**

(d) From a container holding four blue and three white cards, four cards are chosen and laid out in a row. **2**

How many different arrangements of colours of the four cards are possible?

(e) Show that $\sin\theta \sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{2}\cos\left(2\theta - \frac{\pi}{2}\right)$. **2**

**End of Question 6**

**Question 7** (11 marks) Use a new writing booklet.

(a) The polynomial $G(x) = 2x^3 - 3x^2 + 7x - 5$ has roots $\alpha$, $\beta$ and $\gamma$. **2**

Find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ .

(b) A car-pool has 15 vehicles of which five are vans and the remainder are sedans.

(i) In how many ways can a convoy of six vehicles be selected if there are no restrictions on the composition of the convoy? **1**

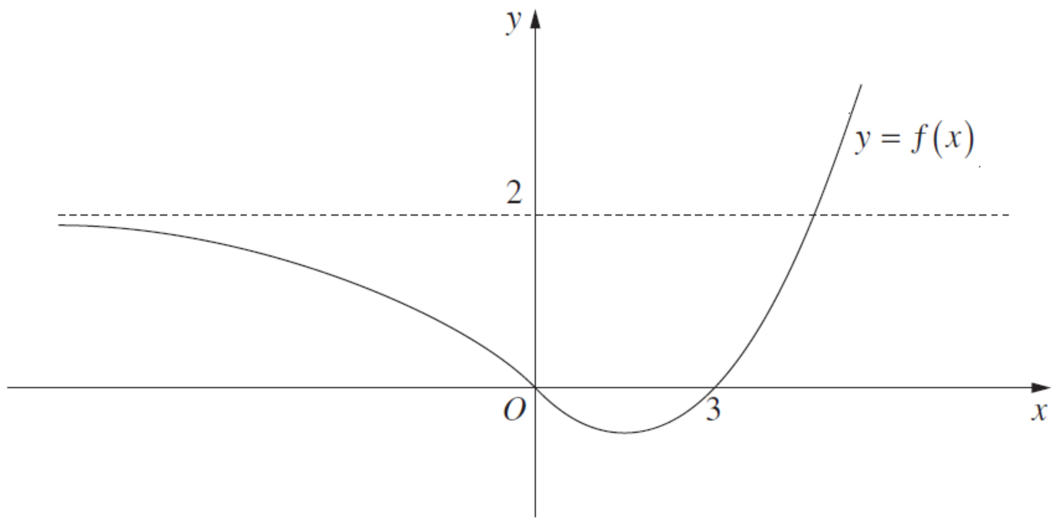
(ii) If the vehicles are selected at random, what is the probability that the convoy includes at least four vans? **2**

(c) A group of students gather for a year meeting.

(i) How many students must be present to guarantee that at least five are born in the same month? **1**

(ii) If there are 56 students present, show that there must be two months in the year which contain at least ten birthdays in total. **2**

(d)




The diagram above shows the graph of the function $y = f(x)$. The graph has a horizontal asymptote at $y = 2$. Draw a one-third page sketch of $y = \frac{1}{f(x)}$ **3**

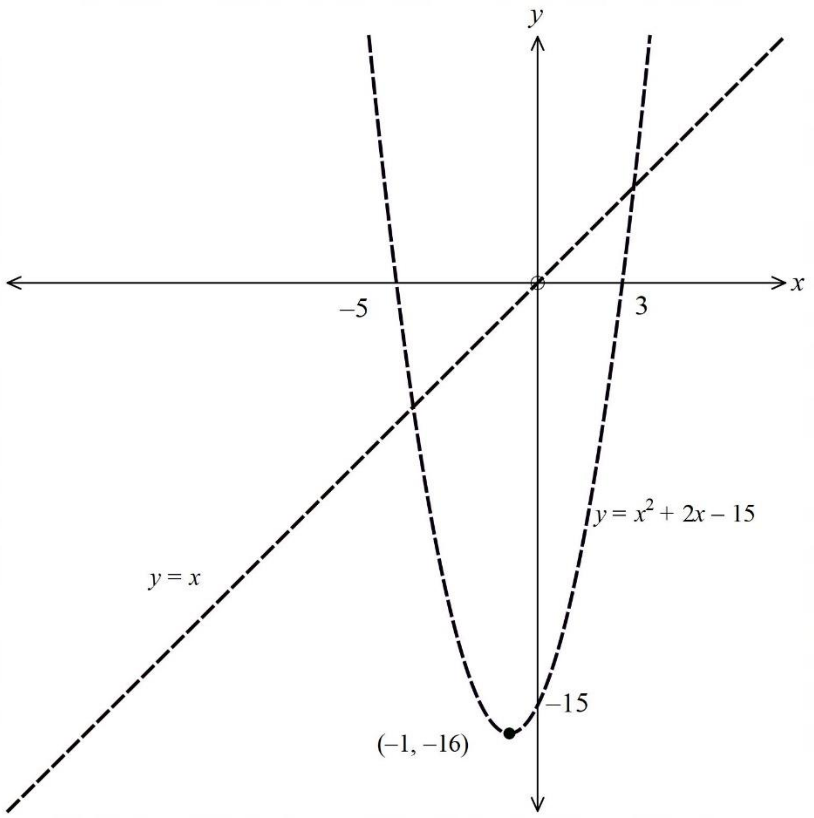
**End of Question 7**

**Question 8** (12 marks) Use a new writing booklet.

(a) Write the expansion of $(1 + 2y)^6$ . **2**

(b) A function is defined as $f(x) = x^2 + 2x - 15$.

The graphs of $y = f(x)$ and of the line $y = x$ are shown on the diagram below.




(i) Determine what restriction would need to be put on the domain of $f(x)$ if it is to have an inverse function $f^{-1}(x)$. **1**

(ii) Determine the equation of the inverse function $f^{-1}(x)$. **2**

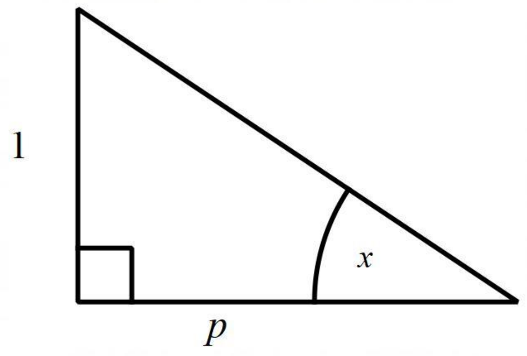
(iii) Draw a sketch showing the graph of $y = f^{-1}(x)$. **1**

**Question 8 continues on page 8**

**Question 8 continued**

(c) (i) Evaluate $\sin^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$. **1**

(ii) Using the triangle below, or otherwise, simplify $\sin^{-1}(\cos(x))$. **2**




(d) (i) Show that $\sin X - 2\cos X = \frac{2t^2+2t-2}{1+t^2}$, where $t = \tan\frac{X}{2}$. **1**

(ii) Hence, or otherwise, solve $\sin X - 2\cos X = 0$ for $0^{\circ} \leq X \leq 360^{\circ}$. **2**

Answer to the nearest degree.

**End of Question 8**

**Question 9** (12 marks) Use a new writing booklet.

(a) Show that : $\sin^{-1}(-x) + \cos^{-1}(-x) + \tan^{-1}(\tan x) = x + \frac{\pi}{2}$. **3**

(b) Solve the inequality **3**

$$\frac{2x}{(x+3)(x-2)} \leq 1$$

(c) Find and simplify the constant term of **2**

$$\left(\frac{x}{2} + \frac{2}{x^2}\right)^{12}$$

(d) A curve has parametric equations $x = 4\sin t + 3$ and $y = 4\cos t - 1$.

(i) Eliminate the parameter $t$ and show that the curve is a circle and give its Cartesian equation. **2**

(ii) Show that two of the points of intersection of the circle with the parabola $y = x^2 - 6x - 8$ lie on a diameter of the circle. **2**

**End of Paper**

# Mathematics Advanced
# Mathematics Extension 1
# Mathematics Extension 2

REFERENCE SHEET

## Measurement

### Length

$$l = \frac{\theta}{360} \times 2\pi r$$

### Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

### Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

### Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

## Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

### Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

## Financial Mathematics

$$A = P(1+r)^n$$

### Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

## Logarithmic and Exponential Functions

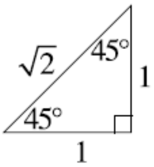
$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

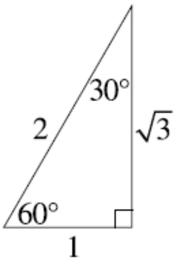
## Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$




$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$




$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

### Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

### Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

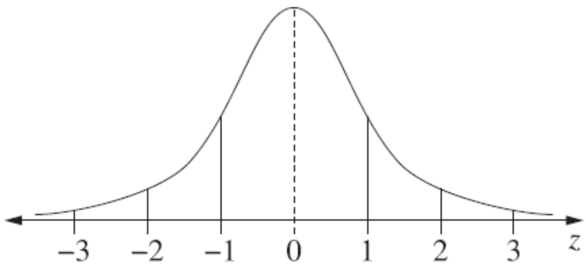
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

## Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

### Normal distribution




- approximately 68% of scores have $z$-scores between –1 and 1
- approximately 95% of scores have $z$-scores between –2 and 2
- approximately 99.7% of scores have $z$-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

### Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

### Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

### Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

## Differential Calculus

| Function | Derivative |
| --- | --- |
| $y = f(x)^n$ | $\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$ |
| $y = uv$ | $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ |
| $y = g(u)$ where $u = f(x)$ | $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ |
| $y = \frac{u}{v}$ | $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ |
| $y = \sin f(x)$ | $\frac{dy}{dx} = f'(x)\cos f(x)$ |
| $y = \cos f(x)$ | $\frac{dy}{dx} = -f'(x)\sin f(x)$ |
| $y = \tan f(x)$ | $\frac{dy}{dx} = f'(x)\sec^2 f(x)$ |
| $y = e^{f(x)}$ | $\frac{dy}{dx} = f'(x)e^{f(x)}$ |
| $y = \ln f(x)$ | $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$ |
| $y = a^{f(x)}$ | $\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$ |
| $y = \log_a f(x)$ | $\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$ |
| $y = \sin^{-1} f(x)$ | $\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$ |
| $y = \cos^{-1} f(x)$ | $\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$ |
| $y = \tan^{-1} f(x)$ | $\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$ |

## Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

### Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

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### Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

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### Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

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### Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

## Year 11 Final Examination 2020

## Mathematics Extension 1 Course

Student Number ____________________________

## Section I – Multiple Choice Answer Sheet

**Allow about 10 minutes for this section**

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○