Carlingford High School



Mathematics Year 10 5.3 Yearly Examination 2016

Name:			MANUAL AND TO THE STATE OF THE
	Please circle your	teacher's name.	
Ms Kellahan	Mrs Wilson/ Mrs Young	Mrs Lego	Mr Wilson

Time allowed: 1.5 hours

- Only board approved calculators may be used.
- Write in black pen. Diagrams and graphs maybe done in pencil.
- Show all necessary working.
- Attempt all questions.
- Marks may be deducted for careless or badly arranged work.
- Questions are worth 1 mark unless otherwise indicated in [].

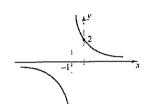
Topic	Mark
Non-linear Relationships	/9
Surface Area and Volume	/11
Trigonometric Functions	/11
Coordinate Geometry	/12
Data and Probability	/15
Inequations & Logarithms	/11
Geometric Figures	/8
Simultaneous Equations	/8
Circle Geometry	/11
Total	/96

Non-linear Relationships (9 marks)

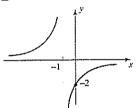
- 1. Which point does not lie on the graph of $v = 3 \times 4^x$? Circle your answer.
 - **A** $(-2, \frac{-3}{16})$ **B** (0, 3) **C** $(\frac{1}{2}, 6)$ **D** $(-1, \frac{3}{4})$
- The graph of $y = \frac{2}{x-1}$ is best represented by:

Circle your answer.

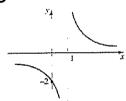
Α



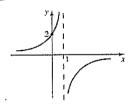
В



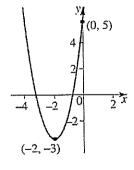
C



D



Find the equation of the graph below. [2]



- A pot 64 cm tall is positioned near a wall. The shape of the pot follows the curve $y = (x - 15)^2$ where y is the height of the pot and x cm is the distance of the pot from the wall.
 - a) How far is the base of the pot from the wall?

b) What is the shortest distance from the top of the vase to the wall? [2]

- c) If the pot is moved so the top just touches the wall, find the new distance from the wall to the base.
- d) Find the new equation that follows the shape of the pot.

Surface Area and Volume (11 marks)

1. A sphere has a surface area of 100 cm². The radius of the sphere is closest to:

Circle your answer.

A 2.82 cm

B 5.64 cm

C 10 cm

D 3.18 cm

2. A cone has a base radius of 10 cm and a height of 20 cm. Its volume is closest to:

Circle your answer.

A 500 cm³

B 1000 cm³

C 2000 cm³

D 5000 cm³

3. Find the total surface area of the pyramid a square-based pyramid with base length 10 m and vertical height 12 m. [3]

4. A spinning top is in the form of a hemisphere for its base with a cone on the top. The diameter of the base is 6.2 cm and the height of the cone above the centre of its base is 7.4 cm.

What is the volume of the toy?

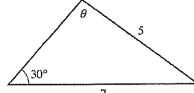
Round your answer to one decimal place.

[3]

5. Show that the *total surface area* of any solid hemisphere is 75% of the area of the full sphere. [3]

Trigonometric Functions (12 marks)

1. The size of angle marked θ to the nearest degree is:



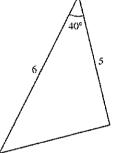
Circle your answer.

A 21°

- **B** 35°
- C 44°
- **D** 46°
- 2. The area of the triangle correct to 2 decimal places is:

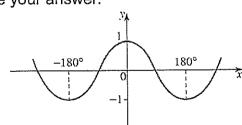
Circle your answer.

- **A** 8.03 units²
- **B** 9.64 units²
- **C** 11.49 units²
- **D** 15.00 units²



3. The following is a graph of:

Circle your answer.



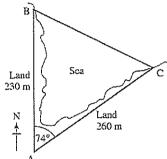
- A $y = \sin x$
- $\mathbf{B} \mathbf{y} = \cos \mathbf{x}$
- C y = tan x
- D none of these
- 4. The value of sin 43° is equal to:

Circle your answer.

- A cos 47°
- B tan 43°
- **C** cos 43°
- **D** sin 47°

5. Solve the equation $\sqrt{2}\cos x - 1 = 0$ for $0 \le x \le 360^{\circ}$ [2]

6. A mini-triathlon requires the competitors to run 230 m directly north along a shore line from point A to point B. From point B they swim across to point C on the edge of shore. They cycle 260 m from point C directly to the starting point, A. The bearing of C from A has been recorded as 074°T.



a) Find the distance the competitors had to swim to the nearest metre. [2]

b) Find the bearing of C from B to the nearest degree. [3]

Coordinate Geometry (12 marks)

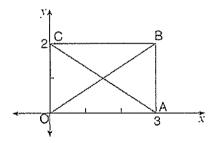
1. The midpoint of the interval joining the points (-1, -5) and (3, -5) is:

Circle your answer.

- **A** (2, -5)
- **B** (1, 5)
- **C** (1, -5)
- **D** (2, 0)
- 2. Determine the equation of the line that is perpendicular to the line whose equation is 2x+3y=6 and has the same *x*-intercept. [3]

3. Show that triangle PQR is right-angled, where the coordinates of the vertices are P (2, 6), Q (5, 7) and R (8, -2). [3]

4. Use the diagram below.



- a) If OABC is a rectangle, what are the coordinates of B?
- b) Find the length of OB and AC. What property of a rectangle have you proved? [2]
- c) Find the midpoint of OB and AC. What does your answer tell you about the diagonals of a rectangle? [2]

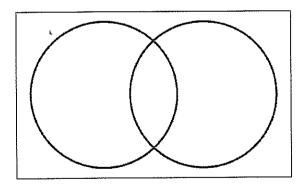
Data and Probability (15 marks)

1. Lewis records his marks in both English and History over a number of assignments.

English: 14, 18, 8, 20, 6, 15, 19, 10, 8, 17

History: 13, 16, 14, 15, 14, 13, 15

- a) Calculate the mean mark for each subject to 2 decimal places. [2]
- b) Calculate the standard deviation mark for each subject to 2 decimal places. [2]
- c) Comment on the distribution for each subject.
- 2. In a school of 340 students, 40% study Japanese, 75% study French and 20% study both.
 - a) Complete the Venn diagram below



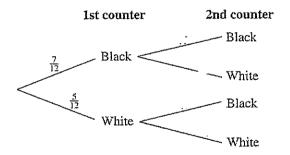
- **b)** Determine the probability that a student who is selected will study:
 - i. only one language
 - ii. at least one language
 - iii. a subject other than French or Japanese.

3. For the stem and leaf plot find the interquartile range.

Stem	
0	35669
1	01335
2	11489
3	00358
4	35669 01335 11489 00358 12345

[2]

- A bag contains 12 counters, 7 of which are black and the rest are white. A counter is selected from the bag and is not replaced in the bag before a second counter is selected.
 - a) Complete the probability tree to display all possible outcomes.



b) Find the probability that both counters selected are white.

- c) Find the probability that the counters selected are of different colours.
- d) Given the first counter chosen is white, find the probability that the next one chosen is black.

Inequations and Logarithms (11 marks)

1. Simplify log₂80 - log₂5.

Circle your answer.

A 4

B 6.299

C 8

- **D** 16
- If $\log\left(\frac{p}{q}\right) + \log\left(\frac{q}{p}\right) = \log(p+q)$, then:

Circle your answer.

- p q = 1
- В p + q = 0
- $p^2 q^2 = 1$
- p + q = 1
- Solve for x in each of the following. 3.
 - a) $log_3 x = 4$
 - b) $\log_{x} 64 = 3$
 - $\log_3 x \log_3 4 = \log_3 \left(\frac{1}{8}\right)$ [2]

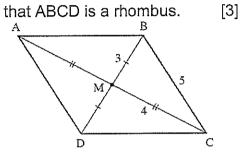
Solve $\frac{2k+1}{5} + \frac{3k-1}{2} \ge 13$ [2]

How many years will it take an investment of \$1000 to double in value if it receives a compound interest rate of 6% per annum? Use the formula $A = P(1+R)^n$, where P is the principal, A the total amount, R the compound interest rate and n the number of years of the investment. [3]

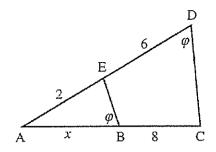
Use logs to determine the time.

Geometric Figures (8 marks)

- The quadrilateral PQRS is a parallelogram. 1. If the adjacent sides are congruent, which of the following statements must be true?
 - Quadrilateral PQRS is a square.
 - Quadrilateral PQRS is a rectangle. В
 - Quadrilateral PQRS is a trapezium. С
 - Quadrilateral PQRS could be a D square or a rhombus.
- Prove that ABCD is a rhombus. 2.



a) Prove the triangles are similar.



[2]

b) Hence find *x* in the following diagram.[2]

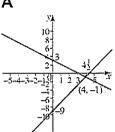
Simultaneous Equations (8 marks)

1. The graphical solution to the following pair of simultaneous equations is:

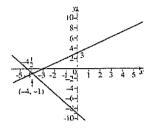
$$2x - y + 9 = 0$$

 $x + y + 3 = 0$

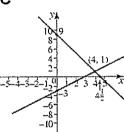
Α



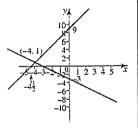
В



C



D



2. Use simultaneous equations to solve the following problem.

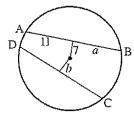
Kelly buys 3 chocolate bars and 2 packets of chips for \$12. Vanessa buys 4 packets of chips and 1 chocolate bar for \$9. What is the total cost for one chocolate bar and one packet of chips? [3]

3. a) Determine the intersection(s) of the straight line x + y = 5 and the circle $x^2 + y^2 = 25$ [3]

b) What is the relationship between the line and the circle?

Circle Geometry (11 marks)

1. If AB = CD, the values for a and b, respectively, in the figure below are:

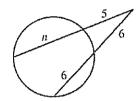


- **A** 7 and 77
- **B** 18 and 7
- C 11 and 7
- **D** 11 and 18

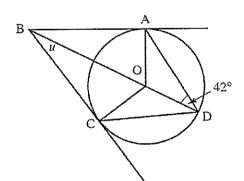
[2]

[3]

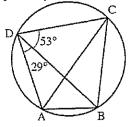
2. Find the value of *n* in the figure below. Give reasons.



3. Given that AB and BC are tangents to the circle, find the value of *u*. Give reasons.



4. ABCD is a cyclic quadrilateral.



a) Find the size of the angles CAB and CBA. Give reasons. [4]

b) Is CA a diameter? Justify your answer.

End of Exam Check your work!

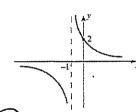
Non-linear Relationships (9 marks)

- 1. Which point does not lie on the graph of $v = 3 \times 4^x$? Circle your answer.
 - $(-2,\frac{-3}{16})$ **B** (0, 3)

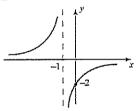
 - **C** $(\frac{1}{2}, 6)$ **D** $(-1, \frac{3}{4})$
- 2. The graph of $y = \frac{2}{x-1}$ is best represented by:

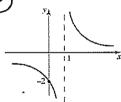
Circle your answer.

Α

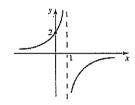


В





D



Find the equation of the graph below.

$$y = 2 \times c^{2} + 8 \times + 5$$

$$y = 2 \times c^{2} + 8 \times + 5$$

$$(0, 5)$$

$$-4 -2 = 0$$

$$(-2, -3)$$

 $y = a(x+2)^2 - 3$

At
$$(0,5)$$

 $5 = 0(0.+2)^2 - 3$

.'. Parabola
$$y = 2(x+2)^2 - 3$$
. (1)

- A pot 64 cm tall is positioned near a wall. The shape of the pot follows the curve $y = (x - 15)^2$ where y is the height of the pot and x cm is the distance of the pot from the wall.
 - a) How far is the base of the pot from the wall?

$$y = (x - 15)^2$$

. . Pot 16cm from wall

b) What is the shortest distance from the top of the vase to the wall? [2]

$$(x-15)^2 = 64$$

 $x-15 = \pm 8$
 $x = 15 \pm 8$

(I)

- Shortest distance is (\mathbf{I})
- c) If the pot is moved so the top just touches the wall, find the new distance from the wall to the base.

d) Find the new equation that follows the shape of the pot.

$$y = (x - 8)^2$$

Surface Area and Volume (11 marks)

1. A sphere has a surface area of 100 cm². The radius of the sphere is closest to:

Circle your answer.

(A) 2.82 cm

B 5.64 cm

C 10 cm

D 3.18 cm

2. A cone has a base radius of 10 cm and a height of 20 cm. Its volume is closest to:

Circle your answer.

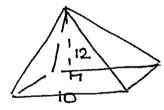
A 500 cm³

B 1000 cm³

©2000 cm³

D 5000 cm³

3. Find the total surface area of the pyramid a square-based pyramid with base length 10 m and vertical height 12 m. [3]



12 5

$$S = \sqrt{12^2 + 5^2}$$

 $S = 13$. 1

S.A. = $10 \times 10 + 4 \times \frac{1}{2} \times 10 \times 13$ (1) = 360 m^2 . (1)

must have units.

4. A spinning top is in the form of a hemisphere for its base with a cone on the top. The diameter of the base is 6.2 cm and the height of the cone above the centre of its base is 7.4 cm.

What is the volume of the toy?

Round your answer to one decimal place.

r = 3.1.

$$V = \frac{2}{3}\pi r^{3} + \frac{1}{3}\pi r^{2}h$$

$$= \frac{2}{3}\pi \times 3.1^{3} + \frac{1}{3}\pi \times 3.1^{2} \times 7.4 \text{ (2)}$$

$$= 62.4 \qquad 74.5$$

$$= 136.8645...$$

$$= 136.9 \text{ cm}^{3} \text{ (1)}$$

5. Show that the *total surface area* of any solid hemisphere is 75% of the area of the full sphere. [3]

SA sphere =
$$4\pi r^2$$

SA hemi = $\frac{1}{2} \times 4\pi r^2 + \pi r^2$
= $3\pi r^2$

$$\frac{SA_{H}}{SA_{S}} = \frac{3\pi r^{2}}{4\pi cr^{2}}$$

$$= \frac{3}{4}$$

$$= 75 \%$$

.'. SA of hemisphere is 75% of a sphere.

Trigonometric Functions (12 marks)

The size of angle marked θ to the nearest 1. degree is:

30°

Circle your answer.

A 21°

B 35°

D 46°

The area of the triangle correct to 2 decimal 2. places is:

Circle your answer.

8.03 units²

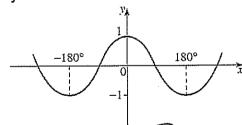
9.64 units²

11.49 units²

15.00 units²

The following is a graph of: 3.

Circle your answer.



 $A y = \sin x$

B) $v = \cos x$

C y = tan x

D none of these

The value of sin 43° is equal to: 4.

Circle your answer.

A)cos 47°

B tan 43°

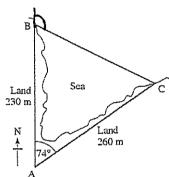
C cos 43°

D sin 47°

Solve the equation $\sqrt{2}\cos x - 1 = 0$ for $0 \le x \le 360^{\circ}$ [2]

> $\sqrt{2}\cos x - 1 = 0$ $cosx = \frac{1}{\sqrt{2}}$ x = 45, 360-45 $\therefore x = 45^{\circ}, 315^{\circ}$

6. A mini-triathlon requires the competitors to run 230 m directly north along a shore line from point A to point B. From point B they swim across to point C on the edge of shore. They cycle 260 m from point C directly to the starting point, A. The bearing of C from A has been recorded as 074°T.



a) Find the distance the competitors had to swim to the nearest metre.

$$a^{2} = 260^{2} + 230^{2} - 2 \times 260 \times 230$$

$$\times \cos 74^{\circ}$$

$$a = \sqrt{87533.77224}$$

1.a=296m

. distance between competitors is 296m.

b) Find the bearing of C from B to the nearest degree. [3]

$$\frac{\sin \beta}{260} = \frac{\sin 74}{296}$$
= $\frac{260 \times \sin 74}{296}$

1.B = 58°

. The bearing of C from B is 180-58=122°

Coordinate Geometry (12 marks)

 The midpoint of the interval joining the points (-1, -5) and (3, -5) is:

Circle your answer.

- **A** (2, -5)
- B (1, 5)
- (C) (1, -5)
 - **D** (2, 0)
- 2. Determine the equation of the line that is perpendicular to the line whose equation is 2x+3y=6 and has the same x-intercept. [3]

$$2x + 3y = 6$$

$$3y = -2x + 6$$

$$y = -\frac{2x}{3} + 2$$

$$m_1 = -\frac{2}{3}$$
 intercept
$$0 = -\frac{2x}{3} + 2$$

$$m_2 = \frac{3}{2}$$

$$x = 3$$

Equation of line: $m_2 = \frac{3}{2}$ (3,0)

$$y - 0 = \frac{3}{2} (x - 3)$$
 (1)

$$y = \frac{3}{2}(x-3)$$

$$y = \frac{3}{2}x - \frac{9}{2}$$

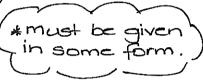
3x - 2y - 9 = 0

3. Show that triangle PQR is right-angled, where the coordinates of the vertices are P (2, 6), Q (5, 7) and R (8, -2). [3]

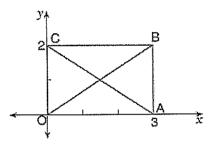
$$m_{pq} = \frac{7-6}{5-2}$$
 $m_{qq} = \frac{-2-7}{8-5}$ $= \frac{1}{3}$ $= -3$ (1)

:. PQ L QR. ()*

.'. ΔPQR is a right angled triangle.



4. Use the diagram below.



a) If OABC is a rectangle, what are the coordinates of B?

b) Find the length of OB and AC. What property of a rectangle have you proved? [2]

OB =
$$\sqrt{3^2 + 2^2} = \sqrt{13}$$

AC = $\sqrt{3^2 + 2^2} = \sqrt{13}$

.'. Diagonals are equal. $^{ extstyle extstyl$

c) Find the midpoint of OB and AC. What does your answer tell you about the diagonals of a rectangle? [2]

$$M_{OB} = \left(\frac{3+0}{2}, \frac{2+0}{2}\right) = \left(\frac{11}{2}, 1\right)$$

$$M_{AC} = \left(\frac{0+3}{2}, \frac{0+2}{3}\right) = \left(\frac{11}{2}, 1\right)$$

. The diagonals bisect (

Data and Probability (15 marks)

Lewis records his marks in both English and History over a number of assignments.

English: 14, 18, 8, 20, 6, 15, 19, 10, 8, 17

History: 13, 16, 14, 15, 14, 13, 15

a) Calculate the mean mark for each subject to 2 decimal places.

E $\bar{x} = 13.50$

1 each

H $\bar{x} = 14.29$

b) Calculate the standard deviation mark for each subject to 2 decimal places. [2]

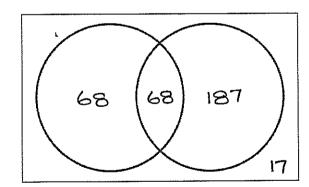
H 4n = 1.03

leach

c) Comment on the distribution for each subject.

Lewis has a higher mean and lower standard deviation in history so has done better. and been more consistent.

- In a school of 340 students, 40% study Japanese, 75% study French and 20% study both.
 - a) Complete the Venn diagram below



or as م/ م Dr decimal

b) Determine the probability that a student who is selected will study:

i. only one language

 $P(\text{only one}) = \frac{255}{340} = \frac{3}{4}$

ii. at least one language

 $P(at | east 1) = 1 - \frac{17}{340} = \frac{19}{20}$

iii. a subject other than French or Japanese.

$$P(\text{neither}) = \frac{17}{340}$$
$$= \frac{1}{20}$$

For the stem and leaf plot find the interquartile range.

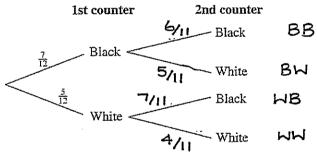
Stem	Leaf
0	35669
1	0-1335
2	11489
3	0035•8
4	12345

- Q1 = 10.5 Q= 36.5

(1)

[2]

- A bag contains 12 counters, 7 of which are black and the rest are white. A counter is selected from the bag and is not replaced in the bag before a second counter is selected.
 - a) Complete the probability tree to display all possible outcomes.



b) Find the probability that both counters selected are white.

$$P(WW) = \frac{5}{12} \times \frac{4}{17}$$

c) Find the probability that the counters

selected are of different colours. P(different) = $\frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$ = 35/66

d) Given the first counter chosen is white, find the probability that the next one chosen is black.

$$P(2nd Black) = \frac{7}{11}$$
.

Inequations and Logarithms (11 marks)

1. Simplify $\log_2 80 - \log_2 5$.

Circle your answer.

A)4

B 6.299

C 8

- **D** 16
- 2. If $\log\left(\frac{p}{q}\right) + \log\left(\frac{q}{p}\right) = \log(p+q)$, then:

Circle your answer.

- **A** p q = 1
- **B** p + q = 0
- **C** $p^2 q^2 = 1$
- (D) p+q=1
- 3. Solve for x in each of the following.
 - a) $\log_3 x = 4$

b) $\log_x 64 = 3$

$$x^3 = 64$$

c) $\log_3 x - \log_3 4 = \log_3 \left(\frac{1}{8}\right)$ [2]

$$\log_3 \frac{2c}{4} = \log_3 \frac{1}{8}$$

Equating:

$$\therefore \mathfrak{L} = \frac{1}{2}$$

(1

4. Solve
$$\frac{2k+1}{5} + \frac{3k-1}{2} \ge 13$$
 [2]

$$4k+2+15k-5>130$$
 ①
$$19k-3>130$$

$$19k>133$$

5. How many years will it take an investment of \$1000 to double in value if it receives a compound interest rate of 6% per annum? Use the formula $A = P(1+R)^n$, where P is the principal, A the total amount, R the compound interest rate and n the number of years of the investment. Use logs to determine the time. [3]

$$2000 = 1000 (1+00)^n$$
 $2 = 1.06^n$

$$\ln 2 = \ln 1.06^{n}$$

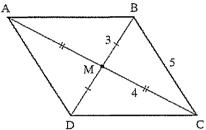
$$n = \frac{\ln 2}{\ln 1.06}$$

$$= 11.8956...$$

.'. It takes 12 years (1) to double the investment

Geometric Figures (8 marks)

- The quadrilateral PQRS is a parallelogram.
 If the adjacent sides are congruent, which of the following statements must be true?
 - A Quadrilateral PQRS is a square.
 - B Quadrilateral PQRS is a rectangle.
 - C Quadrilateral PQRS is a trapezium.
 - Quadrilateral PQRS could be a square or a rhombus.
- 2. Prove that ABCD is a rhombus. [3]



AM = MC (given) BM = MD (given)

. . diagonals bisect each other.

Since $3^2 + 4^2 = 5^2$ (Pythagoras) the $\angle BMC = 90^\circ$ (1)

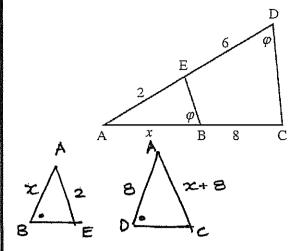
diagonal bisect each other at right angles and ABCD is a rhombus.

(1)

or prove all sides are equal

3. a) Prove the triangles are similar.

[2]



In AABE and AADC:

LABE = LADC (given)

LA is common.

.'. DABE III DADC (matching angles equal.) (1)

b) Hence find x in the following diagram.[2]

$$\frac{x}{8} = \frac{2}{x+8}$$
 (sides in same ratio) (

 $x^{2}+8x=16$ $x^{2}+8x-16=0$ $x = -8 \pm \sqrt{64-4\times1\times-16}$ $= -8 \pm \sqrt{128}$ $= -8 \pm 8\sqrt{2}$ $= -4 \pm 4\sqrt{2}$ $= 4\sqrt{2}-4 \text{ or } -4-4\sqrt{2}$ not valid <0 <0 $<x = 4\sqrt{2}-4 \text{ or } 1.656854249$

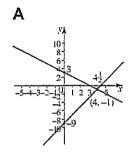
(rounded correctly)

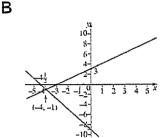
Simultaneous Equations (8 marks)

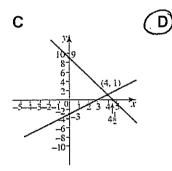
1. The graphical solution to the following pair of simultaneous equations is:

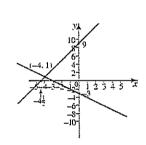
$$2x - y + 9 = 0$$

 $x + y + 3 = 0$









2. Use simultaneous equations to solve the following problem.

Kelly buys 3 chocolate bars and 2 packets of chips for \$12. Vanessa buys 4 packets of chips and 1 chocolate bar for \$9. What is the total cost for one chocolate bar and one packet of chips? [3]

Let chocolate bar be x. Let chips be y.

$$3x + 2y = 12$$
 ①



8.

From 2 x = 9-4y 3

Sub 3 into 1

$$3(9-4y) + 2y = 12$$

 $27-12y + 2y = 12$
 $-10y = -15$
 $y = 1.5$

$$1.1 \times = 9 - 4 \times 1.5$$

... A chocolate bar is \$1.50 and a packet of chips (1) is \$3.

a) Determine the intersection(s) of the straight line x + y = 5 and the circle x² + y² = 25 [3]

$$x=5-y$$

Sub (1) into $x^2+y^2=25$
 $(5-y)^2+y^2=25$ (1)
 $25-10y+y^2+y^2=25$

$$25 - 10y + y^{2} + y^{2} = 25$$

$$2y^{2} - 10y = 0$$

$$2y(y - 5) = 0$$

$$x=5$$
 for $x=0$

.'. Points of intersection are (5,0) and (0,5) (1

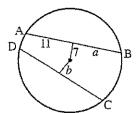
b) What is the relationship between the line and the circle?

The line is a secont as it cuts the circle twice.

Accepted "chord":

Circle Geometry (11 marks)

If AB = CD, the values for a and b, respectively, in the figure below are:



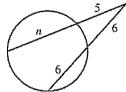
- A 7 and 77
- **B** 18 and 7
- (C)11 and 7
- **D** 11 and 18

[2]

[3]

Find the value of *n* in the figure below. 2. Give reasons.

 $(n+5) \times 5 = 12 \times 6$

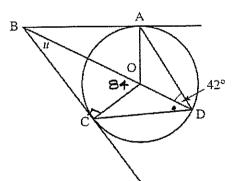


5n + 25 = 72

5n = 47

(intercepts of seconts)

Given that AB and BC are tangents to the 3. circle, find the value of u. Give reasons.



L CD0 = 420

LBOC=84° (angle at centre)

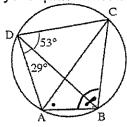


twice the circumference) LBCO = 90° (tangent perpendicular to radius).

... u = 180 - 90 - 84 (angle sum

End of Exam Check your work!

4. ABCD is a cyclic quadrilateral.



a) Find the size of the angles CAB and CBA. Give reasons.

LCAB = 53° (angles subtended by equal chords)

LCBA = 180-53-29° = 98° (opposite angles in cyclic .ir quadrilateral).

b) Is CA a diameter? Justify your answer.

No CA is not a diameter as 4BA ≠90°.