# CARLINGFORD HIGH SCHOOL

## DEPARTMENT OF MATHEMATICS

## Year 12 Mathematics 2U

## Term2 Assessment Task 2014

**Time allowed: 55 minutes**

**Name:** _____ **Class:** _____ **Teacher**_____

**White / Lobejko / Fardouly / Lego / Wilson**

**Instructions:**

- All questions should be attempted.
- Show ALL necessary working on your own paper.
- Marks may not be awarded for careless or badly arranged work.
- Only board-approved calculators may be used.
- Start each question on a new page and only write on one side of each sheet of paper.

| | Q1 | Q2 | Q3 | Q4 | Q5 | TOTAL |
|---|---|---|---|---|---|---|
| H3 | /10 | /9 | | | | /19 |
| H5 | | | | /8 | /8 | /16 |
| H8 | | | /8 | | | /8 |
| TOTAL | /10 | /9 | /8 | /8 | /8 | /43 |

**QUESTION 1 ( 10 MARKS )** **MARKS**

a) Solve $\log(x-2)+\log(x-5)=\log(x+3)$ 3

b) If $log_a 3 = 0.12$ *and* $log_a 4 = 0.21$ find $log_a 36$. 2

c) Simplify $log_3\sqrt{3}+log_3\frac{1}{9}$. 2

d) Differentiate and simplify:

(i) $y=\ln(4-x^2)$ 1

(ii) $f(x)=\ln(\frac{2x+1}{x-3})$ 2

**QUESTION 2 ( 9 MARKS )**

a) Find

(i) $\int \frac{2x+1}{2x^2+2x-5}dx$ 2

(ii) $\int 5e^{1-x}dx$ 1

(iii) $\int_1^2 e^{3x}-ex\,dx$ 2

b) (i) If $y=\frac{e^x}{x}$, find $\frac{dy}{dx}$. 1

(ii) Hence, show that the equation of the normal to the curve $y=\frac{e^x}{x}$ at $x=-1$ is $ex-2y-\frac{2}{e}+e=0$. 3

**QUESTION 3 ( 8 MARKS )**

a) Calculate the volume formed by rotating the area bounded by the curve $y=\frac{1}{\sqrt{2x}}$, $x=e$, $x=3e^2$ and the $x$ axis about the $x$ axis. Give your answer correct to 3 significant figures. 3

b) (i) Sketch $y=\ln(x-1)$. 1

(ii) For $y=\ln(x-1)$ make $x$ the subject 1

(iii) Hence, or otherwise find the exact area between $y=\ln(x-1)$, the $x$ axis and $x=2$ and $x=4$. 3

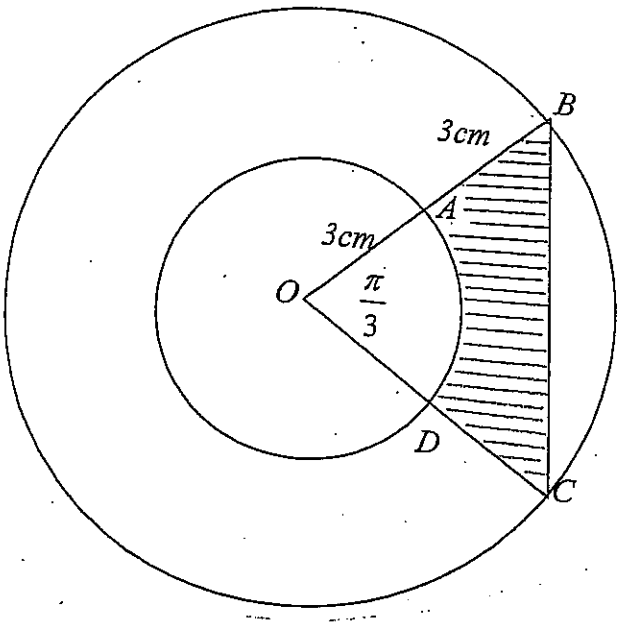
**QUESTION 4 ( 8 MARKS )**

a) Express $280^o$ in radians in exact form. 1

b) State the exact value of $cos\frac{7\pi}{6}$. 1

c) Solve $1+tanx=0\ for\ 0\le x\le 2\pi$. 2

d)




NOT TO SCALE

In the diagram, O is the centre of the two circles. OA=AB=3cm.
The angle BOC subtended at the centre is $\frac{\pi}{3}$.
Giving answers in exact form:

(i) Find the length of the arc BC. 1

(ii) Calculate the area of the sector AOD. 1

(iii) Calculate the area of the triangle BOC. 1

(iv) Find the shaded area, correct to 2 decimal places. 1

**QUESTION 5 ( 8 MARKS )**

a) Simplify $\frac{\sin(\pi-\theta)}{\tan(2\pi-\theta)}$. 2

b) (i) State the period and amplitude of $y = -4sin\frac{x}{2}$. 1

(ii) Sketch $y = -4sin\frac{x}{2}$ $for$ $-2\pi \leq x \leq 2\pi$ 2

(iii) From your graph how many solutions exist for $-4sin\frac{x}{2} = -1$. 1

(iv) Calculate the values of $x$ which satisfy the equation $-4sin\frac{x}{2} = -1$ for $-2\pi \leq x \leq 2\pi$. Give your answer(s) in radians correct to 2 decimal places. 2

# MATHEMATICS – TERM 2 2014

1. a) $\log[(x-2)(x-5)] = \log(x+3)$

$\therefore (x-2)(x-5) = x+3$ (1)

$x^2 - 7x + 10 = x+3$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$x = 1, 7$ (1)

But $x > 5$

$\therefore x = 7$ (1)

b) $\log_a 36 = \log_a(9 \times 4)$

$= \log_a 9 + \log_a 4$ (1)

$= 2\log_a 3 + \log_a 4$

$= 0.45$ (1)

c) $\log_3 3^{\frac{1}{2}} + \log_3 3^{-2}$ (1)

$= \frac{1}{2}\log_3 3 - 2\log_3 3$

$= -1\frac{1}{2}$ (1)

d) (i) $\frac{dy}{dx} = \frac{-2x}{4-x^2}$ (1)

(ii) $f(x) = \ln(2x+1) - \ln(x-3)$

$f'(x) = \frac{2}{2x+1} - \frac{1}{x-3}$ (1)

$= \frac{2(x-3) - (2x+1)}{(2x+1)(x-3)}$

$= \frac{-7}{}$ (1)

2. a) (i) $\frac{1}{2}\int \frac{4x+2}{2x^2+2x-5}\,dx$ (1)

$= \frac{1}{2}\log(2x^2+2x-5) + c$

(ii) $-5e^{1-x} + c$

(iii) $\frac{1}{3}e^{3x} - \frac{ex^2}{2}\Big]_1^2$ (1)

$= \left(\frac{1}{3}e^6 - 2e\right) - \left(\frac{1}{3}e^3 - \frac{e}{2}\right)$

$= \frac{1}{3}e^6 - \frac{1}{3}e^3 - \frac{3e}{2}$ (1)

b) (i) $\frac{dy}{dx} = \frac{xe^x - e^x}{x^2}$ (1)

$= \frac{e^x(x-1)}{x^2}$

(ii) When $x = -1$

$\frac{dy}{dx} = \frac{-2}{e}$ (1)

For normal $m = \frac{e}{2}$

$\therefore y + \frac{1}{e} = \frac{e}{2}(x+1)$ (1)

$2y + \frac{2}{e} = ex + e$ (1)

$ex - 2y + e - \frac{2}{e} = 0$

3 a) $V = \pi \int_e^{3e^2} \frac{1}{2x}\, dx$ ①

$= \frac{\pi}{2} \left[ \log x \right]_e^{3e^2}$ ①

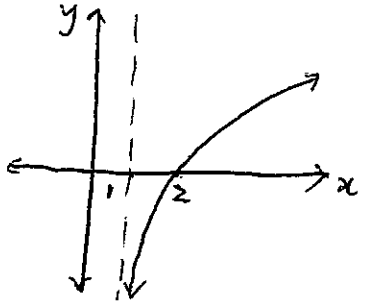
$= \frac{\pi}{2} (\log 3e^2 - \log e)$

$= \frac{\pi}{2} (\log 3 + 2\log e - \log e)$

$= \frac{\pi}{2} (\log 3 + 1)$

$= 3.30 \text{ units}^3$ ①

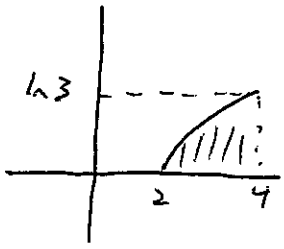
b) (i)




(ii) $x - 1 = e^y$

$x = e^y + 1$

(iii)




$A = 4\ln 3 - \int_0^{\ln 3} e^y + 1\, dy$ ①

$= 4\ln 3 - \left[ e^y + y \right]_0^{\ln 3}$ ①

$= 4\ln 3 - \left[ (e^{\ln 3} + \ln 3) - (1) \right]$

$= 4\ln 3 - 3 - \ln 3 + 1$

$= 3\ln 3 - 2 \text{ units}^2$ ①

4. a) $\frac{280\pi}{180} = \frac{14\pi}{9}$

b) $-\frac{\sqrt{3}}{2}$

c) $\tan x = -1$

① $x = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$ $\quad x = 180 - 45, 360 - 45$

① $= \frac{3\pi}{4}, \frac{7\pi}{4}$ $\quad$ or $= 135^\circ, 315^\circ$

$= \frac{3\pi}{4}, \frac{7\pi}{4}$

d (i) $A\overset{\frown}{C}BC = 6 \times \frac{\pi}{3}$

$= 2\pi \text{ cm}$

(ii) $A = \frac{1}{2} \times 3^2 \times \frac{\pi}{3}$

$= \frac{3\pi}{2} \text{ cm}^2$

(iii) $A = \frac{1}{2} \times 6 \times 6 \times \sin \frac{\pi}{3}$

$= 9\sqrt{3} \text{ cm}^2$

(iv) $A = 9\sqrt{3} - \frac{3\pi}{2}$

$= 10.88 \text{ cm}^2$

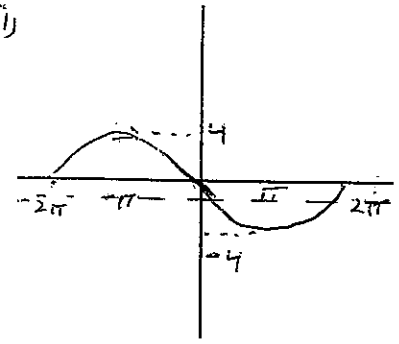
5 a) $\dfrac{\sin\theta}{-\tan\theta}$ ①

$= -\sin\theta \times \dfrac{\cos\theta}{\sin\theta}$

$= -\cos\theta$ ①

(b)(i) Period $= 4\pi$
Amplitude $= 4$ } ①

(ii)




① for shape
① for critical points

(iii) 2

(iv) $\sin\dfrac{x}{2} = \dfrac{1}{4}$

$\dfrac{x}{2} = 0.2527, \pi - 0.2527$ ①

$= 0.2527, 2.8889$

$x = 0.51, 5.78$ ①