

Carlingford High School



Mathematics

Year 10 Term 1 Examination

5.3 Course

2018

Name: Solih. Class: 5.3.

Circle your teacher's name: Mrs Lobejko Mrs Lego Miss Aung

Time allowed: 50 minutes

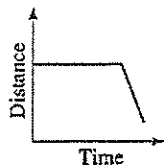
- Board approved calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or untidy work.
- Questions marked with an asterisk * are extension level questions.
- Complete the examination in blue or black pen.

Topic	Non-Linear Relationships	Quadratic Equations	Literacy	Total
Mark	/30	/16	/7	/53
Extension*	/5	/5		/10
Total	/35	/21	/7	/63

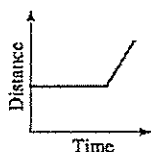
Section A: Non-Linear Relationships
(35 marks)

1. A man is driving his car along the highway at a constant speed. He then decreases his speed because road work is being carried out. Which graph below best represents the relationship between distance travelled and time taken?

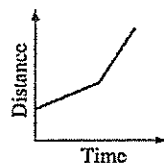
A.



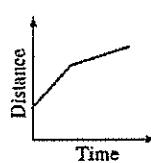
B.



B.



D.



2. The time (t) taken to drive from Sydney to Port Macquarie varies inversely with the average speed (s) of the car.

Given that the trip takes 4.5 hours travelling at an average speed of 90 km/h:

- a) Calculate the constant of variation. (2 marks)

$$t \propto \frac{k}{s}$$

$$t = \frac{k}{s}$$

$$\textcircled{1} \quad 4.5 = \frac{k}{90}$$

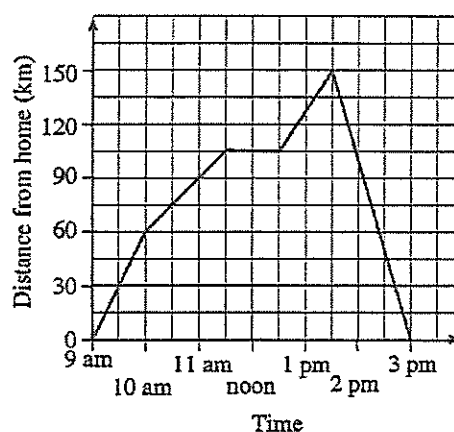
$$405 = k$$

$$\therefore k = 405 \quad \textcircled{1}$$

- b) Write a linear equation to describe this relationship. (1 mark)

$$t = \frac{405}{s}$$

3. The graph below shows Scott's distance from home at various times throughout the day.



- a) How far from home is Scott at 11 am? (1 mark)

90 km

- b) What is the furthest distance he reached from home? (1 mark)

150 km

- c) At what time(s) is Scott 75 km from home? (1 mark)

10:30 am and 2:15 pm

must have both

- d) Calculate his average speed for the entire journey. (1 mark)

$$s = \frac{300}{6}$$

$$= 50 \text{ km/h}$$

- e) Between what times did he stop for lunch? (1 mark)

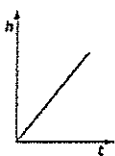
11:30 am and 12:30 pm

4. Water is poured into the container below at a steady rate.

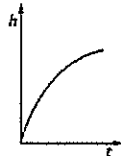


Which of the graphs below could represent the height of the water in the container as it is filled?

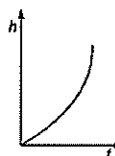
A.



B.



C.



D.



5. The value of E varies directly with the square of S .

It is known that $E = 20$ when $S = 10$. What is the value of E when $S = 40$?

A. 40

B. 80

C. 320

D. 400

6. Match each of the following graphs with its correct equation. (6 marks)

A. $y = 2x^2 - 3$

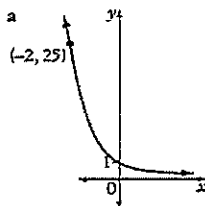
B. $y = -x^3 + 8$

C. $y = -(x - 4)^4$

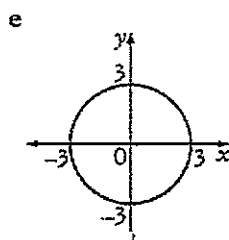
D. $y = 5^{-x}$

E. $y = -\frac{4}{x}$

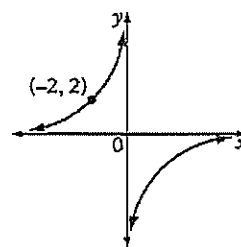
F. $x^2 + y^2 = 9$



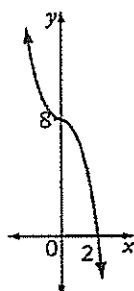
D



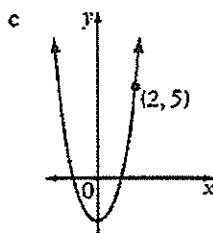
F



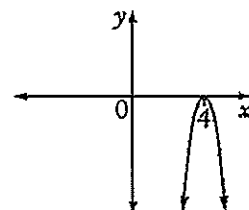
E



B



A



C

7. For the parabola with equation

$$y = x^2 - 6x + 8$$

a) find the x-intercepts. (2 marks)

$$x^2 - 6x + 8 = 0$$

$$(x - 4)(x - 2) = 0$$

$$x = 4 \text{ and } x = 2$$

$$\therefore (4, 0) \quad (2, 0)$$

b) find the y-intercept (1 mark)

$$y = 0 - 0 + 8$$

$$= 8$$

$$\therefore (0, 8)$$

c) find the equation of the axis of symmetry (1 mark)

$$x = \frac{-b}{2a}$$

$$= \frac{6}{2}$$

$$= 3$$

$$\therefore x = 3$$

d) find the co-ordinates of the vertex. (1 mark)

$$\text{when } x = 3$$

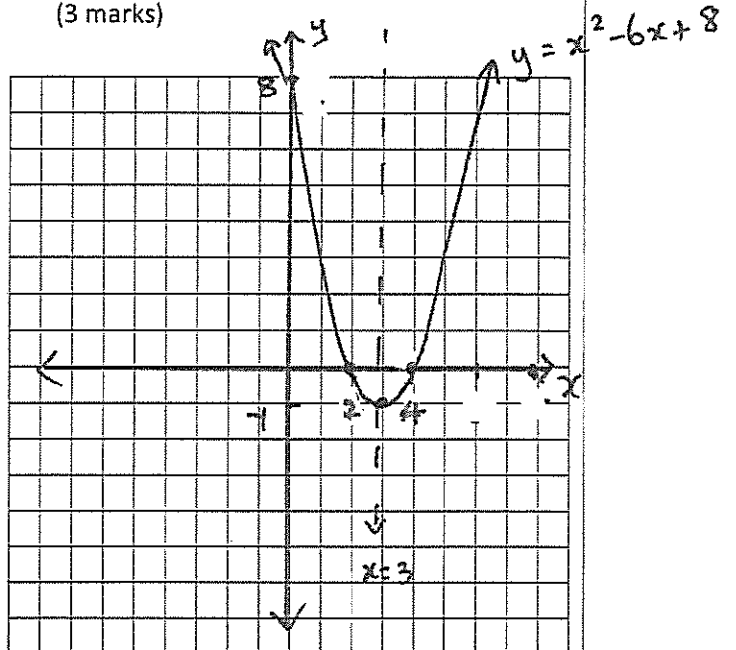
$$y = 3^2 - 6(3) + 8$$

$$= 9 - 18 + 8$$

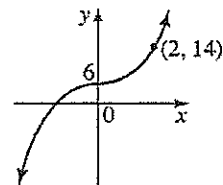
$$= -1$$

$$\therefore (3, -1)$$

e) Sketch the curve, clearly showing the intercepts, vertex and axis of symmetry (3 marks)



8. The curve below has an equation in the form $y = ax^3 + c$.



Find the values of a and c , and hence determine its equation. (2 marks)

$$y = ax^3 + c$$

$$y = ax^3 + 6$$

$$14 = a(2)^3 + 6$$

$$14 = 8a + 6$$

$$8 = 8a$$

$$1 = a$$

$$\therefore y = x^3 + 6$$

<p>*9. The quadratic function $y = 3x^2 + bx + c$ has a minimum value of -16 when $x = 3$. Find the values for b and c. (2 marks)</p> $y = 3x^2 + bx + c$ $-16 = 3(3)^2 + b(3) + c$ $-16 = 27 + 3b + c$ $-43 = 3b + c$ <p>axis of symmetry : $x = \frac{-b}{2a}$</p> $3 = \frac{-b}{6}$ $18 = -b$ $-18 = b$ $-43 = 3(-18) + c$ $11 = c$ <p>$\therefore b = -18$ and $c = 11$</p>	<p>*10. Does the point $(1, 1.5)$ lie on, inside or outside the circle with equation:</p> $x^2 + 2x + y^2 - 6y + 6 = 0?$ <p>(3 marks)</p> $x^2 + 2x + 1^2 + y^2 - 6y + (-3)^2 = -6 + 1 + (-3)^2$ $(x+1)^2 + (y-3)^2 = 4$ <p>\therefore centre $(-1, 3)$ $r = 2$</p> $d = \sqrt{(-1-1)^2 + (3-1.5)^2}$ $= \sqrt{4 + 2.25}$ $= \sqrt{6.25}$ $= 2.5$ <p>\therefore point lies outside the circle.</p>
<p>Q11. Find, algebraically, the points of intersection of : (3 marks)</p> $x - y = -5 \text{ and } x^2 + y^2 = 25$ $x = -5 + y$ $\therefore (-5 + y)^2 + y^2 = 25$ $25 - 10y + y^2 + y^2 = 25$ $2y^2 - 10y + 25 - 25 = 0$ $2y^2 - 10y = 0$ $2y(y - 5) = 0$ $y = 0 \quad y = 5$ $x = -5 \quad x = 0$ <p>$\therefore (-5, 0) \text{ and } (0, 5)$</p>	

Section B: Quadratic Equations (21 marks)

1. Solve each of the following equations:
(10 marks)

a) $x^2 - 3x = 0$

$$x(x-3) = 0$$

$$x = 0 \quad \text{or} \quad x = 3$$

b) $x^2 - 13x + 42 = 0$

$$(x-6)(x-7) = 0$$

$$x = 6 \quad \text{or} \quad x = 7$$

c) $(x-4)^2 = 16$

$$x-4 = \pm 4$$

$$x = 0 \quad \text{or} \quad x = -8$$

d. $6x^2 - 13x - 5 = 0$

$$6x^2 + 2x - 15x - 5 = 0$$

$$2x(3x+1) - 5(3x+1) = 0$$

$$(3x+1)(2x-5) = 0$$

$$3x+1 = 0$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$2x-5 = 0$$

$$2x = 5$$

$$x = 2\frac{1}{2}$$

e) $\frac{x^2-12}{x} = 4$

$$x^2 - 12 = 4x$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6 \quad \text{or} \quad x = -2$$

2. By completing the square, find the solutions to the following equation, giving your answer in exact form. (2 marks)

$$3x^2 + 6x - 15 = 0$$

$$x^2 + 2x - 5 = 0$$

$$x^2 + 2x + (1)^2 = 5 + 1^2$$

$$(x+1)^2 = 6$$

$$x+1 = \pm\sqrt{6}$$

$$x = -1 \pm \sqrt{6}$$

*3. The sum of a number and its reciprocal is $2\frac{9}{10}$. What are the numbers? (3 marks)

$$x + \frac{1}{x} = 2\frac{9}{10}$$

$$x + \frac{1}{x} = \frac{29}{10}$$

$$x^2 + 1 = \frac{29x}{10}$$

$$10x^2 + 10 = 29x$$

$$10x^2 - 29x + 10 = 0$$

$$x = \frac{-(-29) \pm \sqrt{29^2 - 4 \times 10 \times 10}}{2 \times 10}$$

$$= \frac{29 \pm \sqrt{441}}{20}$$

$$= \frac{29 \pm 21}{20}$$

$$x = 2\frac{1}{2}, \quad \frac{2}{5}$$

4. Use the quadratic formula to solve the following equation, leaving your answer in simplest surd form. (2 marks)

$$2x^2 + 16x - 8 = 0$$

$$x = \frac{-16 \pm \sqrt{16^2 - 4 \times 2 \times -8}}{2 \times 2}$$

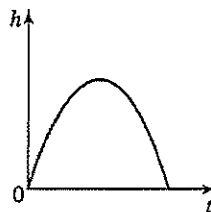
$$= \frac{-16 \pm \sqrt{320}}{4}$$

$$= \frac{-16 \pm 8\sqrt{5}}{4}$$

$$= \frac{-8(2 \pm \sqrt{5})}{4}$$

$$= -2(2 \pm \sqrt{5})$$

5. Daniel throws a stone vertically upwards as shown in the diagram below.



After t seconds its height h metres above the ground is given by the formula

$$h = 80t - 16t^2$$

- a) At what time(s) will the stone be 96 metres above the ground? (2 marks)

$$96 = 80t - 16t^2$$

$$16t^2 - 80t + 96 = 0$$

$$16(t^2 - 5t + 6) = 0$$

$$16(t-2)(t-3) = 0$$

$$t = 2, t = 3$$

\therefore at 2secs and 3secs

- *b) Can the stone ever reach a height of 128 metres? Justify your answer with appropriate calculations. (2 marks)

$$16t^2 - 80t + 128 = 0$$

$$t = \frac{80 \pm \sqrt{(-80)^2 - 4 \times 16 \times 128}}{2 \times 16}$$

$$= \frac{80 \pm \sqrt{-1792}}{32}$$

no solution.

\therefore will not reach 128m



Literacy: (7 marks)	Working space.
<p>Use the following words to complete the following sentences.</p> <p>Parabolas symmetrical vertex constants quadratic slope vertical concave horizontal</p> <p>1. The general form of a <u>quadratic</u> equation is $ax^2 + bx + c = 0$, where a, b and c are <u>constants</u></p> <p>2. Parabolas are <u>symmetrical</u> as they have an axis of symmetry.</p> <p>3. Parabolas with the shape U are said to be <u>concave</u> up.</p> <p>4. The turning point of a graph is called the <u>vertex</u>.</p> <p>5. The <u>slope</u> of a line indicates the speed at which an object is moving.</p> <p>6. A <u>horizontal</u> line indicates that an object is stationary.</p>	