

K-Map

A	B	F
0	0	0
0	1	1
1	0	0
1	1	1

A different way to draw a truth table !

Take advantage of adjacency

$$F = A'B + AB = B$$

		B	
		0	1
A	0	0 $A'B'$	1 $A'B$
	1	0 AB'	1 AB

Keep common literal only!

Minimization with K-maps

1. Draw a K-map
2. Combine maximum number of 1's following rules:
 1. Only adjacent squares can be combined
 2. All 1's must be covered
 3. Covering rectangles must be of size 1,2,4,8, ... 2^n
3. Check if all covering are really needed
4. Read off the SOP expression

2-variable K-map

Given a function with 2 variables: $F(X,Y)$, the total number of minterms are equal to 4:

$$m_0, m_1, m_2, m_3$$

The size of the k-map is always equal to the total number of minterms.

Each entry of the k-map corresponds to one minterm for the function:

Row 0 represents: $X'Y'$, $X'Y$

Row 1 represents: XY' , XY

X \ Y	0	1
0	m_0	m_1
1	m_2	m_3

X \ Y	0	1
0	0	1
1	2	3

Example 1

For a given function $F(X,Y)$ with the following truth table, minimize it using k-maps

X	Y	F
0	0	0
0	1	0
1	0	1
1	1	1

		Y	
		0	1
X	0	0	0
	1	1	1

Combining all the 1's in only the adjacent squares

The final reduced expression is given by the common literals from the combination:

Therefore, since for the combination, Y has different values (0, 1), and X has a fixed value of 1,

The reduced function is: $F(X,Y) = X$

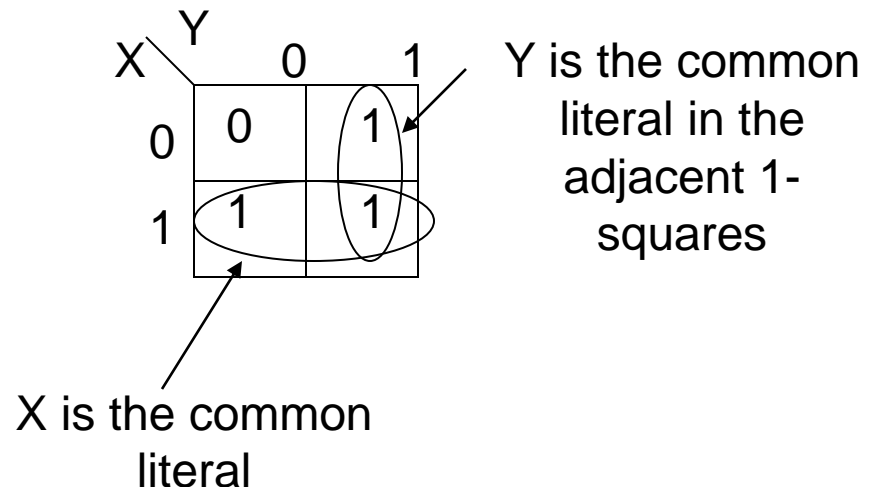
Example 2

Q. Simplify the function $F(X,Y) = \sum m(1,2,3)$

Sol. This function has 2 variables, and three 1-squares (three minterms where function is 1)

$$F = m_1 + m_2 + m_3$$

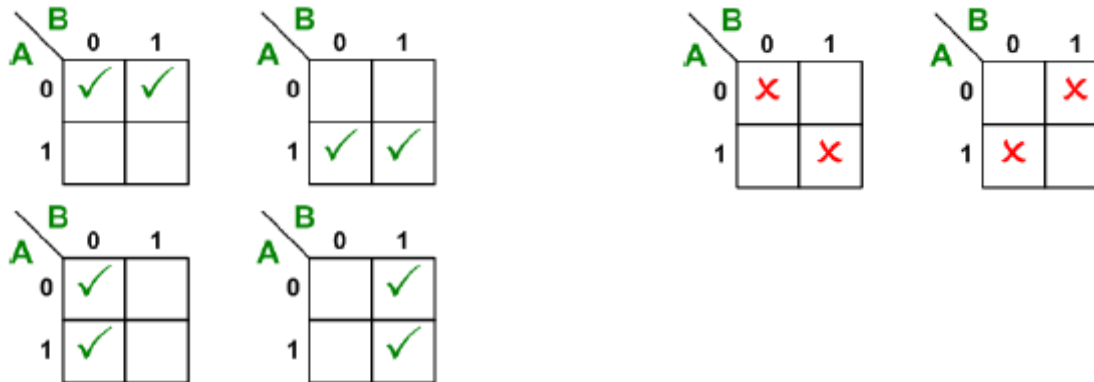
Note: The 1-squares can be combined more than once



Minimized expression: $F = X + Y$

2 variable K-Maps (Adjacency)

In an n -variable k -map, each square is adjacent to exactly n other squares



Q: What if you have 1 in all squares?

3-variable K-maps

For 3-variable functions, the k-maps are larger and look different.

Total number of minterms that need to be accommodated in the k-map = 8

To maintain adjacency neighbors don't have more than 1 different bit

		BC			
		00	01	11	10
A	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

3-variable K-maps

BC		00	01	11	10
A	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

Note: You can only combine a power of 2 adjacent 1-squares. For e.g. 2, 4, 8, 16 squares. You cannot combine 3, 7 or 5 squares

BC		00	01	11	10
A	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

Minterms m_0 , m_2 , m_4 , m_6 can be combined as m_0 and m_2 are adjacent to each other, m_4 and m_6 are adjacent to each other

m_0 and m_4 are also adjacent to each other, m_2 and m_6 are also adjacent to each other

Example 1

Simplify $F = \sum m(1, 3, 4, 6)$ using K-map

		BC			
		00	01	11	10
A	0	0	1	1	2
	1	4	5	7	6
		1			1

Example 1

Simplify $F = \sum m(1, 3, 4, 6)$ using K-map

$$F = A'C + AC'$$

		B			
		BC		11	10
A	0	00	01	1	2
	1	4	5	7	6
		1			1

Example 2

Simplify $F = \sum m(0, 1, 2, 4, 6)$ using K-map

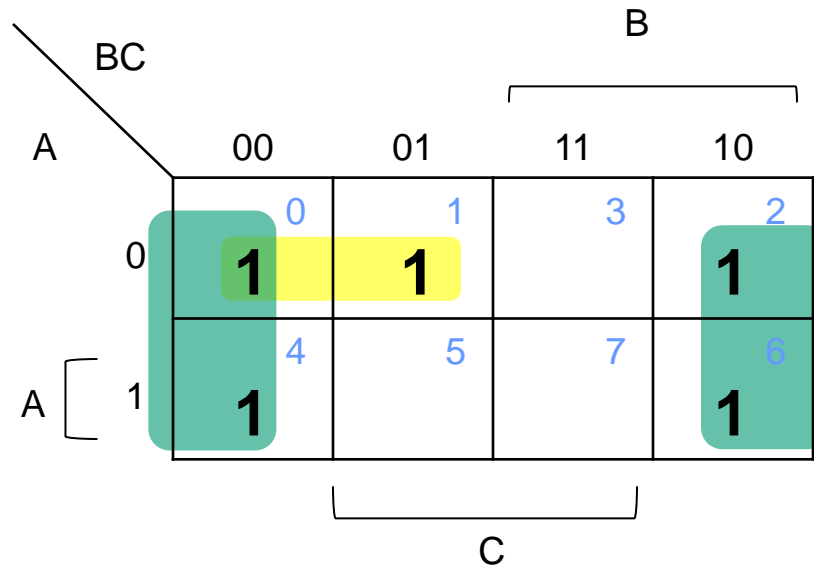
		BC			
		00	01	11	10
A	0	1 ⁰	1 ¹		1 ²
	1	1 ⁴			1 ⁶

Diagram illustrating the K-map for the function $F = \sum m(0, 1, 2, 4, 6)$. The K-map is a 2x4 grid with rows labeled A (0, 1) and columns labeled BC (00, 01, 11, 10). The minterms are marked with 1s in the cells corresponding to (A, BC) pairs (0, 00), (0, 01), (0, 10), (1, 00), and (1, 10). The cells are numbered 0 through 7 in blue. Brackets indicate groupings: a horizontal bracket above the top row is labeled B, and a horizontal bracket below the bottom row is labeled C.

Example 2

Simplify $F = \sum m(0, 1, 2, 4, 6)$ using K-map

$$F = A' B' + C'$$



3 variable K-Maps (Adjacency)

A 3-variable map has 12 possible groups of 2 minterms

They become product terms with 2 literals

	00	01	11	10
0				
1				

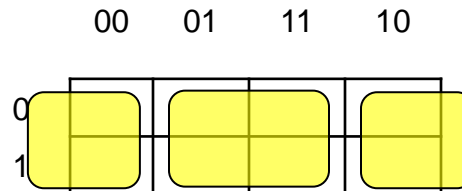
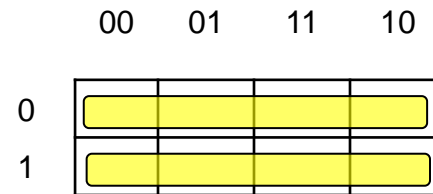
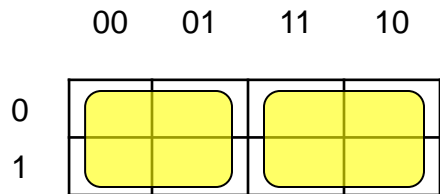
	00	01	11	10
0				
1				

	00	01	11	10
0				
1				

3 variable K-Maps (Adjacency)

A 3-variable map has 6 possible groups of 4 minterms

They become product terms with 1 literals



4-variable K-maps

A 4-variable function will consist of 16 minterms and therefore a size 16 k-map is needed

Each square is adjacent to 4 other squares

A square by itself will represent a minterm with 4 literals

Combining 2 squares will generate a 3-literal output

Combining 4 squares will generate a 2-literal output

Combining 8 squares will generate a 1-literal output

		CD			
		00	01	11	10
A	AB	C			
	00	m_0	m_1	m_3	m_2
	01	m_4	m_5	m_7	m_6
	11	m_{12}	m_{13}	m_{15}	m_{14}
	10	m_8	m_9	m_{11}	m_{10}

Diagram illustrating a 4-variable K-map (Karnaugh map) for variables A, B, C, and D. The map is a 4x4 grid of squares, each representing a minterm m_i . The rows are labeled by variables A and B (AB), and the columns are labeled by variables C and D (CD). The minterms are arranged in a Gray code sequence. The map shows the following minterms:

- Row 00 (A=0, B=0): m_0 (0000), m_1 (0001), m_3 (0011), m_2 (0010)
- Row 01 (A=0, B=1): m_4 (0100), m_5 (0101), m_7 (0111), m_6 (0110)
- Row 11 (A=1, B=1): m_{12} (1100), m_{13} (1101), m_{15} (1111), m_{14} (1110)
- Row 10 (A=1, B=0): m_8 (1000), m_9 (1001), m_{11} (1011), m_{10} (1010)

Brackets indicate the grouping of variables: C and D are grouped horizontally, and A and B are grouped vertically.

4-variable K-maps (Adjacency)

CD		00	01	11	10
AB					
00	m_0	m_1	m_3	m_2	
01	m_4	m_5	m_7	m_6	
11	m_{12}	m_{13}	m_{15}	m_{14}	
10	m_8	m_9	m_{11}	m_{10}	

Note: You can only combine a power of 2 adjacent 1-squares. For e.g. 2, 4, 8, 16 squares. You cannot combine 3, 7 or 5 squares

Right column and left column are adjacent; can be combined

Top row and bottom row are adjacent; can be combined

Many possible 2, 4, 8 groupings

Example

Minimize the function $F(A,B,C,D)=\sum m(1,3,5,6,7,8,9,11,14,15)$

A 4x4 Karnaugh map for the function $F(A,B,C,D)$. The rows are labeled AB (00, 01, 11, 10) and the columns are labeled CD (00, 01, 11, 10). The map contains 1s in the following cells: (00,01), (00,11), (01,01), (01,11), (01,10), (11,11), (11,10), (10,01), (10,11), (10,10). Four groups of 1s are highlighted with blue rounded rectangles and labeled with brackets: a vertical group for C (columns 01 and 11), a horizontal group for B (rows 01 and 11), a horizontal group for D (columns 00 and 01), and a vertical group for A (rows 10 and 11).

AB \ CD	00	01	11	10
00		1	1	
01		1	1	1
11			1	1
10	1	1	1	

$$F = CD + A'D + BC + AB'C'$$

Example

$$F(A,B,C,D) = \Sigma m(0,1,2,5,8,9,10)$$

CD AB		C=1			
		00	01	11	10
A=1	00	1	1		1
	01		1		
	11				
	10	1	1		1

B=1

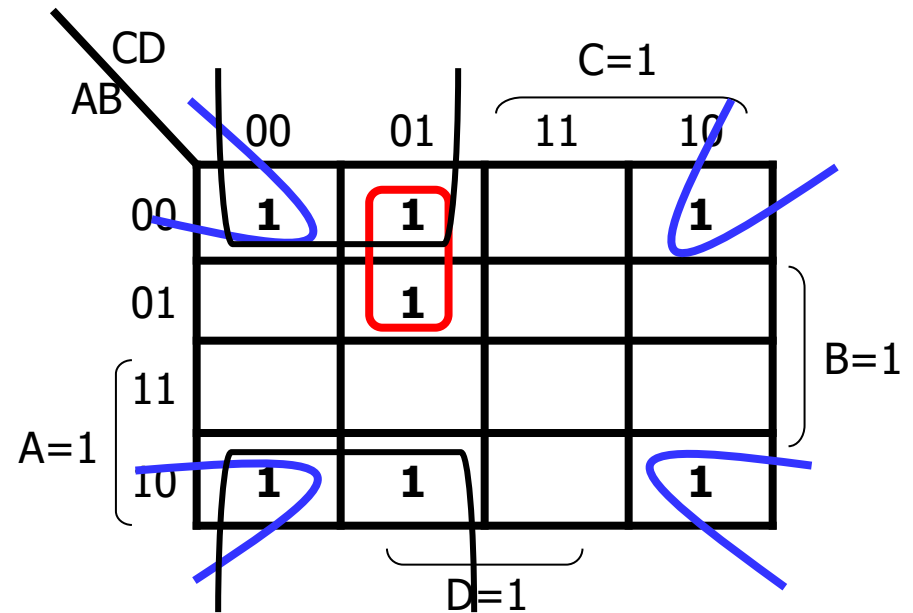
D=1

Example

$$F(A,B,C,D) = \sum m(0,1,2,5,8,9,10)$$

Solution:

$$F = B' D' + B' C' + A' C' D$$



Example (POS)

$$F(A,B,C,D) = \Sigma m(0,1,2,5,8,9,10)$$

Write F in the simplified product of sums (POS)

Two methods?

You already know one!

		CD			
		00	01	11	10
AB	00	1	1		1
	01		1		
	11				
	10	1	1		1

Annotations:

- $C=1$ (over 11 and 10 columns)
- $B=1$ (over 01 column)
- $A=1$ (over 11 and 10 rows)
- $D=1$ (under 00 and 10 columns)

Example (POS)

$$F(A,B,C,D) = \sum m(0,1,2,5,8,9,10)$$

Write F in the simplified product of sums (POS)

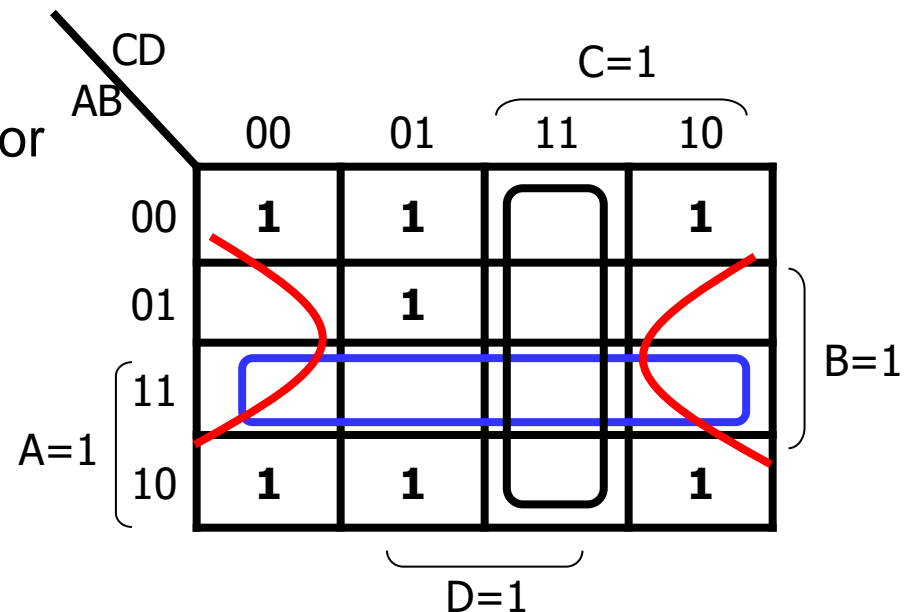
Method 2:

Follow same rule as before but for the ZEROS

$$F' = AB + CD + BD'$$

Therefore,

$$F'' = F = (A'+B')(C'+D')(B'+D)$$



Don't Cares

- In some cases, the output of the function (1 or 0) is not specified for certain input combinations either because
 - The input combination never occurs (Example BCD codes), or
 - We don't care about the output of this particular combination
- Such functions are called incompletely specified functions
- Unspecified minterms for these functions are called don't cares
- While minimizing a k-map with don't care minterms, their values can be selected to be either 1 or 0 depending on what is needed for achieving a minimized output.

Example

$$F = \sum m(1, 3, 7) + \sum d(0, 5)$$

Circle the x's that help get bigger groups of 1's (or 0's if POS).

Don't circle the x's that don't help.

		BC			
		00	01	11	10
A	0	⁰ X	¹ 1	³ 1	²
	1	⁴	⁵ X	⁷ 1	⁶

Diagram illustrating a 4-variable Karnaugh map for the function $F = \sum m(1, 3, 7) + \sum d(0, 5)$. The map is a 2x4 grid with rows labeled A (0, 1) and columns labeled BC (00, 01, 11, 10). The cells contain values: (0,00) is X, (0,01) is 1, (0,11) is 1, (0,10) is empty, (1,00) is empty, (1,01) is X, (1,11) is 1, and (1,10) is empty. Blue superscripted numbers 0 through 7 are placed above each cell. A bracket labeled 'B' is above the top row (A=0). A bracket labeled 'C' is below the bottom row (A=1).

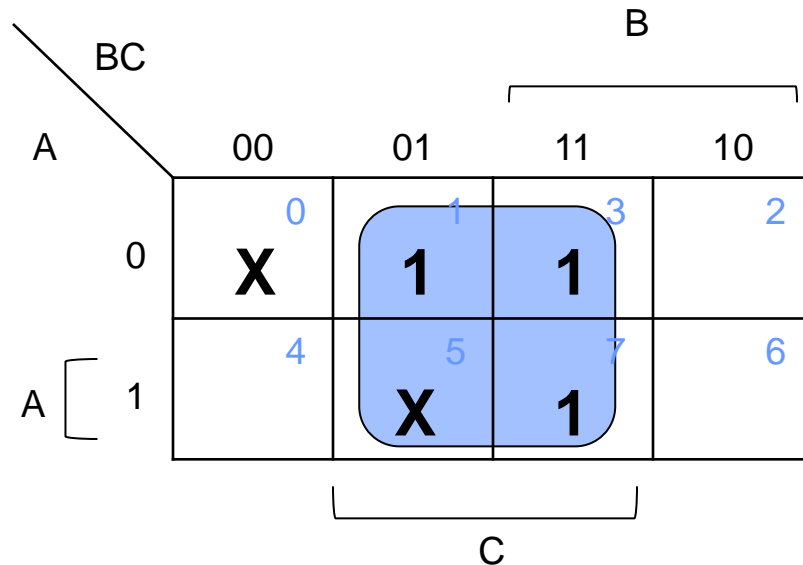
Example

$$F = \sum m(1, 3, 7) + \sum d(0, 5)$$

Circle the x's that help get bigger groups of 1's (or 0's if POS).

Don't circle the x's that don't help.

$$F = C$$



Example 2

$$F(A, B, C, D) = \sum m(1, 3, 7, 11, 15) + \sum d(0, 2, 5)$$

-Two possible solutions!

-Both acceptable.

-All 1's covered

		C				
		CD		01	11	10
A	00	X	1	1	X	B
	01	0	X	1	0	
	11	0	0	1	0	
	10	0	0	1	0	
				D		
				00	01	

(a) $F = CD + \bar{A} \bar{B}$

		C				
		CD		01	11	10
A	00	X	1	1	X	B
	01	0	X	1	0	
	11	0	0	1	0	
	10	0	0	1	0	
				D		
				00	01	

(b) $F = CD + \bar{A} D$

Src: Mano's Textbook