Standard forms of Boolean Expressions

All Boolean expressions can be converted into either of two standard forms:

a. The Sum-Of-Products (SOP) form
b. The Product-Of-Sums (POS) form

Standardization makes the evaluation, simplification and implementation of Boolean expressions much more systematic and easier.

1.The Sum-Of-Products (SOP) form:

When two or more product terms are summed by Boolean addition, the resulting expression is a

Sum-Of-Products (SOP). Some examples are:

$AB + ABC$

$ABC + CDE + B'CD'$

$A+A'B'C+BCD'$ etc

2.The Product-Of-Sums (POS) form:

When two or more sum terms are multiplied, the resulting expression is a Product-Of-Sums (POS). Some examples are:

$(A' + B)(A+B'+C)$

$(A' + B'+ C')(C + D'+ E)(B'+C+D)$

$A'(A+B+C)(B'+C+D)$ etc

Standard Form

- Each term in the function may contain one, two, or any number of literals
  $F = y + xy + xyz$
- Either the ***sum of products*** or the ***products of sums, not both!***

F = AB + C(D + E) ➔ *Non Standard Form*

= AB + CD + CE ➔ *Standard Form*

Some Standard Functions:

$F_1 = y' + xy + x'yz'$ (sum of products)

$F2 = x(y' + z)(x' + y + z')$ (product of sums)

Boolean Function Representation:

Canonical Form:

Boolean functions expressed as a sum of minterms or product of maxterms are said to be in canonical form.

We can derive truth table from standard form.

We can represent Boolean Functions in Canonical Forms.

CANONICAL FORMS

- How to express a boolean function algebraically from a given truth table?

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$F(x,y,z)=?$

Summation of Minterms

- F(x,y,z)=?

X	Y	Z	Term	Designation	F
0	0	0	X'Y'Z'	m_0	0
0	0	1	X'Y'Z	m_1	1
0	1	0	X'YZ'	m_2	0
0	1	1	X'YZ	m_3	1
1	0	0	XY'Z'	m_4	1
1	0	1	XY'Z	m_5	1
1	1	0	XYZ'	m_6	0
1	1	1	XYZ	m_7	0

Summation of Minterms

- $F(x,y,z)= m_1+ m_3 + m_4 + m_5$

  $= X' Y' Z+X' YZ+XY'Z' +XY' Z$

  $=\sum(1, 3, 4, 5)$

X	Y	Z	Term	Designation	F
0	0	0	X'Y'Z'	m_0	0
0	**0**	**1**	**X'Y'Z**	m_1	**1**
0	1	0	X'YZ'	m_2	0
0	**1**	**1**	**X'YZ**	m_3	**1**
1	**0**	**0**	**XY'Z'**	m_4	**1**
1	**0**	**1**	**XY'Z**	m_5	**1**
1	1	0	XYZ'	m_6	0
1	1	1	XYZ	m_7	0

Summation of Minterms

- $F(x,y,z)=m_1+ m_3 + m_4 + m_5=X' Y' Z+X' YZ+XY'Z ' +XY' Z$
  $=\sum(1, 3, 4, 5)$
- $F' = m_0 + m_2 + m_6 + m_7 = X' Y'Z' +X'YZ' +XYZ' +XYZ$

X	Y	Z	Term	Designation	F
0	**0**	**0**	**X'Y'Z'**	**m_0**	**0**
0	0	1	X'Y'Z	m_1	1
0	**1**	**0**	**X'YZ'**	**m_2**	**0**
0	1	1	X'YZ	m_3	1
1	0	0	XY'Z'	m_4	1
1	0	1	XY'Z	M_5	1
1	**1**	**0**	**XYZ'**	**m_6**	**0**
1	**1**	**1**	**XYZ**	**m_7**	**0**

Multiplication of Maxterms

- F(x,y,z)=?

X	Y	Z	Term	Designation	F
0	0	0	X + Y + Z	M_0	0
0	0	1	X + Y + Z’	M_1	1
0	1	0	X + Y’ + Z	M_2	0
0	1	1	X + Y’ + Z’	M_3	1
1	0	0	X’ + Y + Z	M_4	1
1	0	1	X’ + Y + Z’	M_5	1
1	1	0	X’ + Y’ + Z	M_6	0
1	1	1	X’ + Y’ + Z’	M_7	0

Multiplication of Max terms

By MinTerms, $F = m_1 + m_3 + m_4 + m_5 = X'Y'Z + X'YZ + XY'Z' + XY'Z$

$\mathbf{F' = m_0 + m_2 + m_6 + m_7 = X'Y'Z' + X'YZ' + XYZ' + XYZ}$

$\Rightarrow F = (X'Y'Z' + X'YZ' + XYZ' + XYZ)' = \mathbf{(X+Y+Z)(X+Y'+Z)}$
$\mathbf{(X'+Y'+Z)(X'+Y'+Z')} = M_0 M_2 M_6 M_7 = \prod(0, 2, 6, 7)$

X	Y	Z	Term	Designation	F
0	0	0	**X + Y + Z**	M_0	**0**
0	0	1	X + Y + Z'	M_1	1
0	1	0	**X + Y' + Z**	M_2	**0**
0	1	1	X + Y' + Z'	M_3	1
1	0	0	X' + Y + Z	M_4	1
1	0	1	X' + Y + Z'	M_5	1
1	1	0	**X' + Y' + Z**	M_6	**0**
1	1	1	**X' + Y' + Z'**	M_7	**0**

Conversion Between Canonical Forms

Boolean functions expressed as a *sum of minterms* or *product of maxterms* are called **canonical form**

By MinTerms, $F = m_1 + m_3 + m_4 + m_5 = \sum(1, 3, 4, 5)$ (function gives 1)

By MaxTerms, $F = M_0 M_2 M_6 M_7 = \prod(0, 2, 6, 7)$ (function gives 0)

X	Y	Z	MinTerms	**Designation**	MaxTerms	**Designation**	F
0	0	0	X'Y'Z'	m_0	X + Y + Z	M_0	0
0	0	1	X'Y'Z	m_1	X + Y + Z'	M_1	1
0	1	0	X'YZ'	m_2	X + Y' + Z	M_2	0
0	1	1	X'YZ	m_3	X + Y' + Z'	M_3	1
1	0	0	XY'Z'	m_4	X' + Y + Z	M_4	1
1	0	1	XY'Z	m_5	X' + Y + Z'	M_5	1
1	1	0	XYZ'	m_6	X' + Y' + Z	M_6	0
1	1	1	XYZ	m_7	X' + Y' + Z'	M_7	0

Function Expression in **Canonical Form**

Express the Boolean function ***F = A + B'C as a sum of minterms***

(All variables must be present in each term)

<u>Process: 1</u>

- In each term, for each missing variable ***P***, multiply ***(P+P')***, until all the variables appear in each term
- Finally, if same term appears multiple times, discard multiple copies

$$F = A + B'C = A(B+B') + B'C(A+A')$$

$$= AB + AB' + B'CA + B'CA'$$

$$= AB(C+C') + AB'(C+C') + B'CA + B'CA'$$

$$= ABC + ABC' + \mathbf{AB'C} + AB'C' + \mathbf{AB'C} + A'B'C$$

$$= ABC + ABC' + AB'C + AB'C' + A'B'C$$

$$= m7 + m6 + m5 + m4 + m1$$

Conversion to **Canonical Form**

Express the Boolean function $F = A + B'C$ ***as a sum of minterms.***

<u>Process: 2</u>

- Draw the Truth Table
- Read the minterms from the truth table

F = m1 + m4 + m5 + m6 + m7

Table 2.5
Truth Table for $F = A + B'C$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Conversion to **Canonical Form**

Express the Boolean function F = xy + x'z as a product of maxterms

<u>Process: 1</u>

- Convert the function into OR terms using the ***distributive law***

$$F = \mathbf{xy} + x'z$$
$$= (xy + x')(xy + z)$$
$$= (x + x')(y + x')(x + z)(y + z)$$
$$= (x' + y)(x + z)(y + z)$$

Distributive Law:

1. X(Y+Z) = XY + XZ
2. X+(YZ) = (X+Y) (X+Z)

- For each missing variable ***P***, add the ***PP'*** with each term, until all the variables appear in each term.

$$F = (x' + y)(x + z)(y + z) = (\underline{x' + y} + zz')(\underline{x + z} + yy')(\underline{y + z} + xx')$$
$$= (x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)(x + y + z)(x' + y + z)$$
$$\mathbf{= (x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)}$$

Conversion to **Canonical Form**

Express the Boolean function F = xy + x'z as a product of maxterms

Process: 2

- Draw the Truth Table
- Read the maxterms from the truth table

$F = M_0 M_2 M_4 M_5$

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Determining standard expressions from a truth table:

Example 1: $F(A,B,C) = \sum (1,4,5,6,7)$

$$= A'B'C + AB'C' + AB'C + ABC' + ABC$$

$$= A'B'C + AB'(C'+C) + AB(C'+C)$$

$$= A'B'C + AB' + AB \; [\because C'+C=1]$$

$$= A'B'C + A(B+B')$$

$$= A'B'C + A \; [\because B'+B=1]$$

$$= (A+A')(A+B'C) \; [\text{Applying distributive law}]$$

$$= A+B'C \; [\because A'+A=1]$$

Ans: $A+B'C$

Example2:

$F(A,B,C)= \Pi\ (0,2,3)$

$= (A+B+C)(A+B'+C)(A+B'+C')$

$= (AA+AB'+AC+AB+BB'+BC+AC+B'C+CC)(A+B'+C')$

$=(A+AB'+AB+AC+0+BC+B'C+C)\ (A+B'+C')\ [\because BB'=0, A+A=A, AA=A]$

$= [A+A(B'+B)+AC+C(B+B')+C]\ (A+B'+C')$

$= (A+A+AC+C+C)\ (A+B'+C')\ [\because B'+B=1]$

$= (A+AC+C)\ (A+B'+C')\ [\because A+A=A]$

$= (A+C)\ (A+B'+C')\ [\because A+AC=A]$

$= AA+AB'+AC'+AC+B'C+CC'$

$= A+AB'+A+B'C\ [\because C+C'=1, CC'=0]$

$=A+A+B'C\ [\because A+AB'=A]$

$= A+B'C\ [\because A+A=A]$

Ans: $A+B'C$

Conversion between minterm and maxterm:

- Sum of Product (SOP) : related to minterm
- Product of Sum (POS) : related to maxterm

F= A'BC'+A'BC+ABC'+ABC

= ∑ (2,3,6,7)

$$= m_2 + m_3 + m_6 + m_7$$

$$F' = m_0 + m_1 + m_4 + m_5$$

$$(F')' = (m_0 + m_1 + m_4 + m_5)'$$

$$F = M_0 M_1 M_4 M_5$$

= Π (0,1,4,5)

= (A+B+C) (A+B+C') (A'+B+C)(A'+B+C')

Converting SOP to POS:

Example: F= A+B'C

= A(B'C'+B'C+BC'+BC) +B'C(A+A')

= AB'C'+AB'C+ABC'+ABC+AB'C+A'B'C

$= \sum (4,5,6,7,1)$

$= m_4 + m_5 + m_6 + m_7 + m_1$

$$\therefore \quad F' = m_0 + m_2 + m_3$$

$$(F')' = (m_0 + m_2 + m_3)'$$

$$F = M_0 M_2 M_3$$

= (A+B+C)(A+B'+C)(A+B'+C') [Ans]

Converting POS to SOP:

Example: $F = (A+B'+C)(A'+B)$

$= AA'+AB+A'B'+BB'+A'C+BC$

$= AB+A'B'+A'C+BC \; [\because AA'=0, BB'=0]$

$= AB(C+C')+A'B'(C+C')+A'C(B+B')+BC(A+A')$

$= ABC+ABC'+A'B'C+A'B'C'+A'BC+A'B'C+ABC+A'BC$

$= ABC+ABC'+A'B'C+A'B'C'+A'BC \; [\because A+A=A]$

$= AB+A'B'+A'BC$

$= AB+A'(B'+BC)$

$= AB+A'[(B'+B)(B'+C)]$ [Applying distributive law]

$= AB+A'(B'+C)$

$= AB+A'B'+A'C$ [Ans]

- F(x,y,z)= 1 means

$$= \sum (0,1,2,3,4,5,6,7)$$

$$= \Pi \text{ (NIL)}$$