# Lecture 4

## Boolean Algebra and Logic Simplification:



Introduced by George Boole in 1854.

- **Boolean:** which is always either true or false.
- **Variable:** a variable is a symbol used to represent a logical quantity. Any single variable can have a 1 or a 0 value. For example: A, B, C, X,Y…etc.
- **Complement:** the complement is the inverse of a variable and is indicated by a bar over the variable. For example, the complement of the variable A is $\overline{A}$. If A=1, then $\overline{A}$= 0. If A=0, then $\overline{A}$= 1.
- **Literal**: a variable or its complement.

**Laws of Boolean Algebra:**

1. **Commutative Laws:**
   a) The commutative law of addition for two variables is written as:
      A+B = B+A
   b) The commutative law of multiplication for two variables is written as:
      AB= BA
2. **Associative Laws:**
   a) The associative law of addition is written as follows for three variables:
      A+ (B+C) = (A+B)+C
   b) The associative law of multiplication is written as follows for three variables: A(BC) = (AB) C
3. **Distributive Laws: for three variables:**
   a) AND distributed over OR: A(B+C) = AB+ AC
   b) OR distributed over AND: A+ BC = (A+ B) (A+C)

**Rules of Boolean Algebra:**

1. $A+0=A$ 2. $A+1=1$ 3. $A.0=0$ 4. $A.1=A$ 5. $A+A=A$

6. $A+\overline{A}=1$ 7. $A.A=A$ 8. $A.\overline{A}=0$ 9. $\overline{\overline{(A)}}=A$ 10. $A+AB=A$

11. $A+\overline{A}B=A+B$ 12. $(A+B)(A+C)=A+BC$

* See the proofs of the above rules from book.

**Dual:** Operators ( AND/OR) and fixed values (1/0) will be changed but variables (A/B/C/X/Y/Z etc) will not change. NOT operator will not change, but AND will become OR and OR will become AND. For example: the dual of $A+1=1$ is

$A.0=0$, the dual of $A+B+C=0$ is $ABC=1$.

**Duality Principle:** It states that every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged. For example if $A+\overline{A}=1$ is true then $A.\overline{A}=0$ is true also.

**De- Morgan's theorems:**

i. $\overline{AB}=\overline{A}+\overline{B}$
ii. $\overline{A+B}=\overline{A}.\overline{B}$

* See the proofs of the above theorems from book.

**Example 1:** Proof that $A+\overline{A}C=A+C$

L.H.S= $A+\overline{A}C$

$=(A+\overline{A})(A+C)$ [using distributive law]

$=A+C$ [$A+\overline{A}=1$]

= R.H.S

So, L.H.S= R.H.S [proved]

Alternative approach:

Using truth table:

| A | C | $\bar{A}C$ | $A+\bar{A}C$ | A+C |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |

**Example 2:** Proof that X+ XY = X(X+Y)

L.H.S= X+XY

= (X+X) (X+Y) [∵ A+BC = (A+B) (A+C)]

= X (X+Y) [∵A+A= A]

= R.H.S

∴ L.H.S= R.H.S [proved]

**Example 3:** Proof that $X(\bar{X}+Y) = XY$

L.H.S = $X(\bar{X}+Y)$

= $X\bar{X} + XY$ [∵ A(B+C) = AB+ AC]

= 0 + XY

= XY

= R.H.S

∴ L.H.S= R.H.S [proved]