PHY111: Principles of Physics – I

Motion Along a Straight Line(Chapter 2)

Lec 5

Fahim Parvez (FAHIM)
Lecturer, MNS, BRCAU
Desk No. 5M61 (5^{th} Floor, MNS)
Email: parvez.fahim@bracu.ac.bd

Position and Displacement

Displacement

$$\Delta x = x_2 - x_1.$$

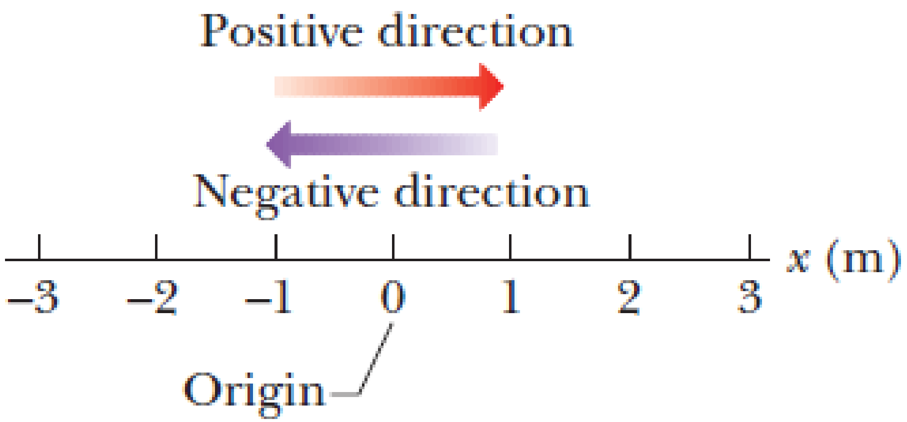


Figure 2-1 Position is determined on an axis that is marked in units of length (here meters) and that extends indefinitely in opposite directions. The axis name, here x, is always on the positive side of the origin.

Checkpoint 1

Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) -3 m, $+5$ m; (b) -3 m, -7 m; (c) 7 m, -3 m?

Position and Displacement

Representation

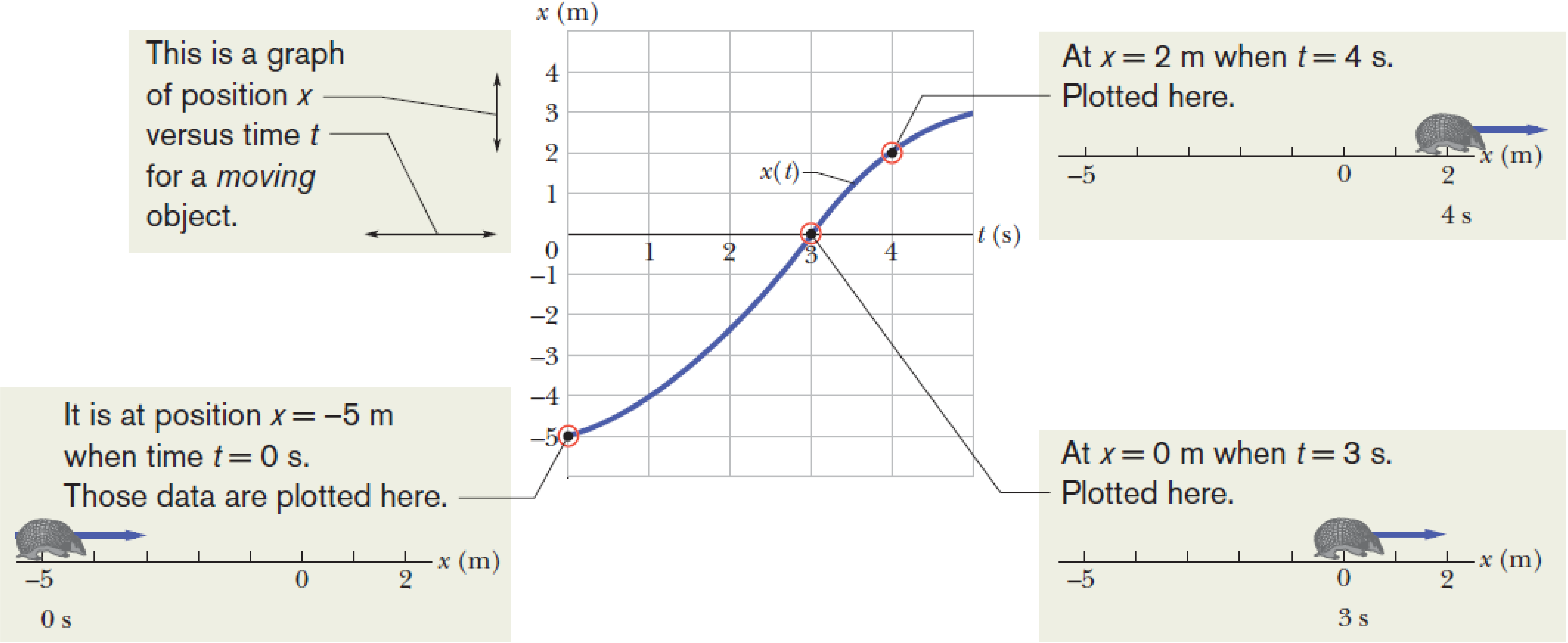


Figure 2-3 The graph of $x(t)$ for a moving armadillo. The path associated with the graph is also shown, at three times.

Velocity

Average Velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

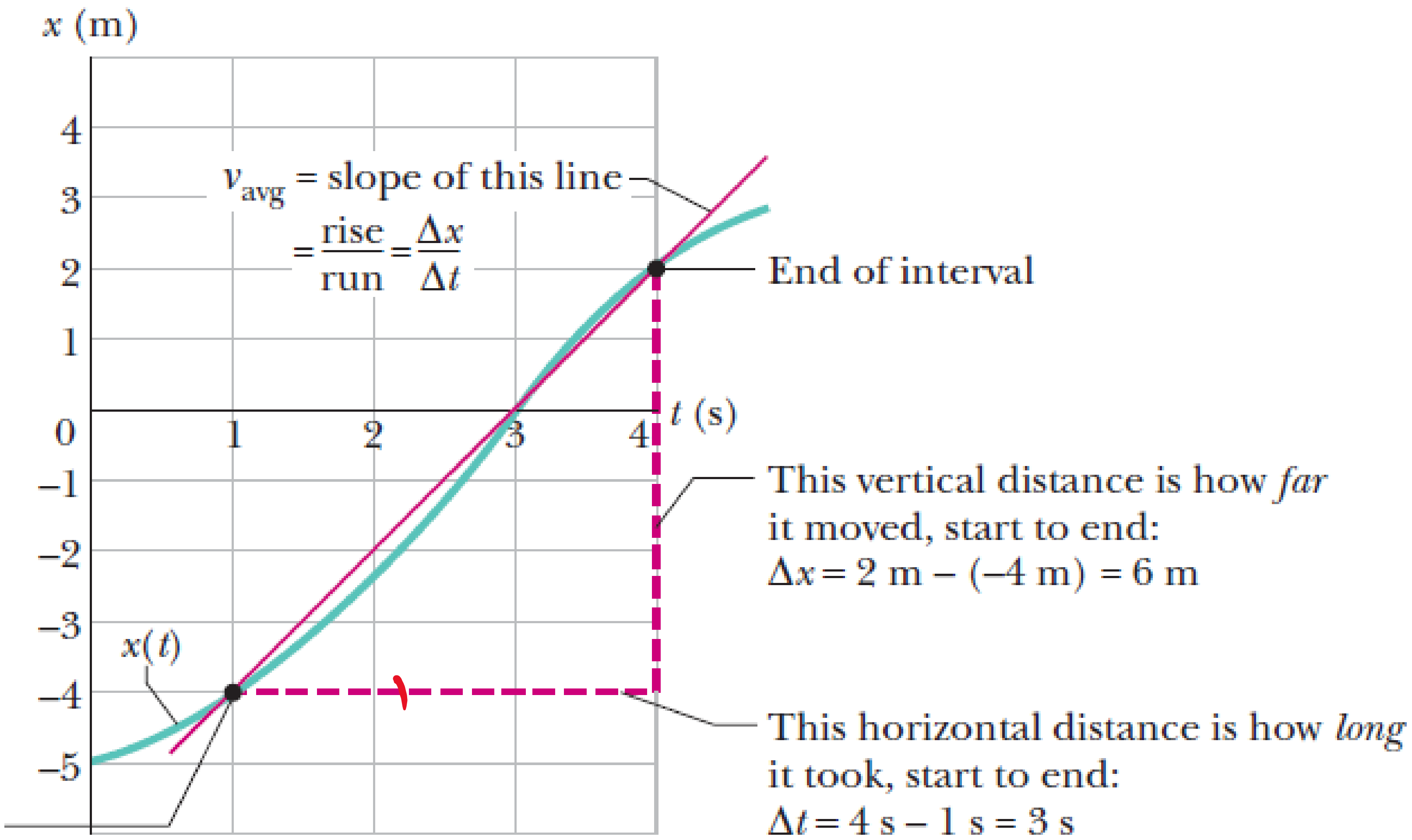
Slope of the line that connect two points in "x vs t" graph

Average Speed

$$s_{avg} = \frac{\text{total distance}}{\Delta t}.$$

Velocity

Average Velocity



Instantaneous Velocity

- The instantaneous velocity (or simply velocity) v of a moving particle is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt},$$

where $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$.

$v = \frac{dx}{dt}$ = instantanious velocity.

- **Speed** is the magnitude of velocity

Acceleration

Average Acceleration

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t},$$

Instantaneous Acceleration

$$a = \frac{dv}{dt}.$$

Acceleration at a particular moment

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

$$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$a_{vag} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt}$$

Problem

•5 SSM The position of an object moving along an x axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds. Find the position of the object at the following values of t: (a) 1 s, (b) 2 s, (c) 3 s, and (d) 4 s. (e) What is the object's displacement between $t = 0$ and $t = 4$ s? (f) What is its average velocity for the time interval from $t = 2$ s to $t = 4$ s? (g) Graph x versus t for $0 \leq t \leq 4$ s and indicate how the answer for (f) can be found on the graph.

Problem

•15 GO (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what is its velocity at $t = 1$ s? (b) Is it moving in the positive or negative direction of x just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time t; if not, answer no. (f) Is there a time after $t = 3$ s when the particle is moving in the negative direction of x? If so, give the time t; if not, answer no.

BRAC UNIVERSITY
Inspiring Excellence

$$x = 4 - 12t + 3t^2$$

$$t_1 = 1$$

$$t_2 = 4$$

$$V_{vag} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{4 - (-8)}{4 - 1} = \frac{12}{3}$$

$$= 4 \frac{m}{s}$$

$$= 4 \frac{m}{s}$$

$$x_2(4) = 4 - 12 \times 4 + 3 \times 4^2$$

$$= 4$$

$$x_1(1) = 4 - 12 \times 1 + 3 \times 1^2 = -8$$

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$v = \underline{\frac{dx}{dt}} = \frac{d}{dt}\left(4 - 12t + 3t^2\right)$$

$$= \frac{d}{dt}(4) - \frac{d}{dt}(12t) + \frac{d}{dt}(3t^2)$$

$$= 0 - 12 \times 1 \times t^{1-1} + 3 \times 2 \times t^{2-1}$$

$$= -12 + 6t$$

(i) $\frac{d}{dt}(c) = 0$

(ii) $\frac{d}{dt}(c\,t^n)$

$$= c\,n\,t^{n-1}$$

$$v = -12 + 6t$$

$$v_1 = -12 + 6 \times 1 = -6\ ms^{-1}$$

$$v_2 = -12 + 6 \times 4 = 12\ ms^{-1}$$

$$a = \frac{dv}{dt}$$

$$= +6$$

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{12 - (-6)}{4 - 1}$$

$$= \frac{18}{3}$$

$$= 6\ \frac{ms^{-1}}{s}$$

$$= 6\ ms^{-2}$$

$t_1 = 1$

$t_2 = 4$

$v_1 = -6$

$v_2 = 12$

$a = 6$

$v = 0$

$v = -12 + 6t$

$\Rightarrow \underline{\underline{0}} = -12 + 6t$

$\Rightarrow 12 = 6t$

$\Rightarrow t = \frac{12}{6} = 2$

BRAC UNIVERSITY

Inspiring Excellence

$v = -12 + 6t$

$t_1 = 0 \qquad v = -12$

$t_1 = 1 \qquad v = -6$

$t_2 = 2 \qquad v = 0$

$t_3 = 3 \qquad v = 6$

Problem (HW)

•16 The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph x versus t for the range -5 s to $+5$ s. (f) To shift the curve rightward on the graph, should we include the term $+20t$ or the term $-20t$ in $x(t)$? (g) Does that inclusion increase or decrease the value of x at which the particle momentarily stops?

Problem (HW)

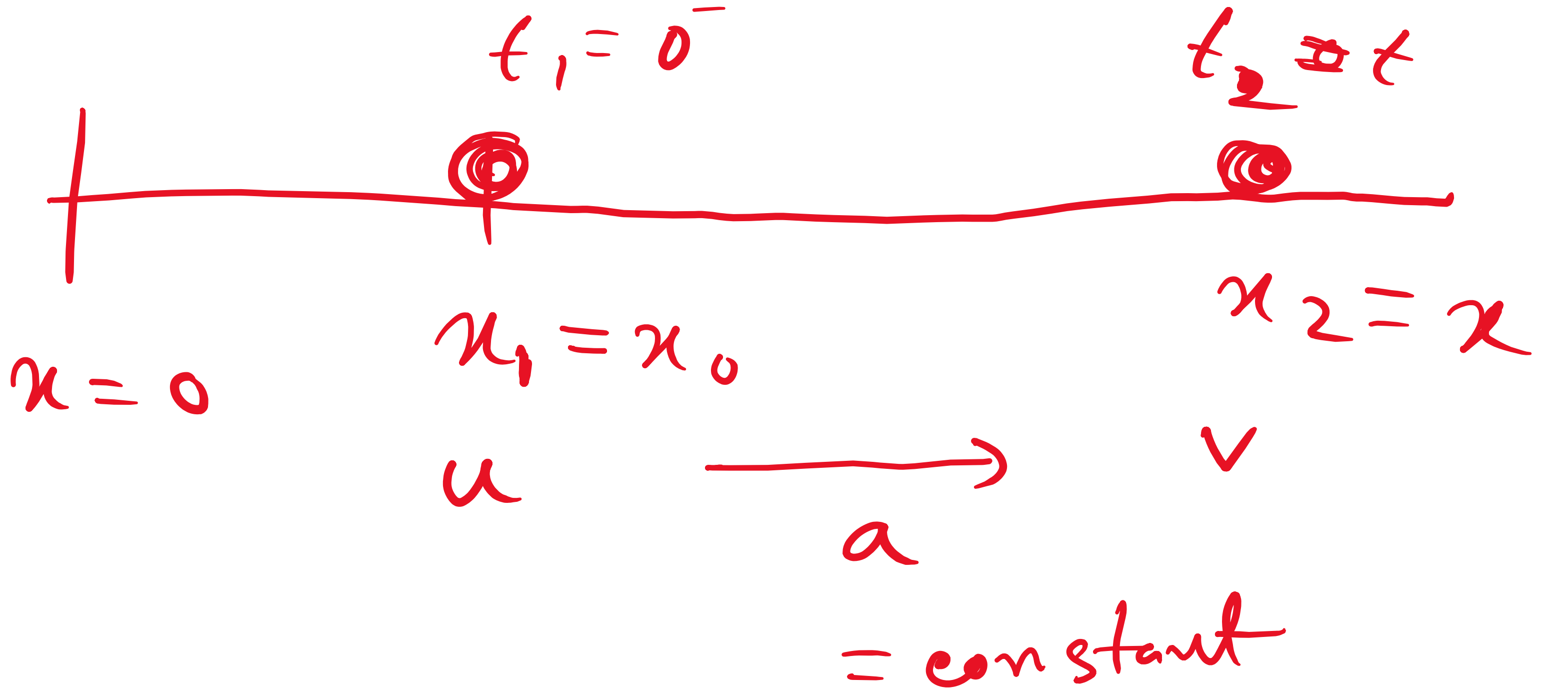
HW ••17 The position of a particle moving along the x axis is given in centimeters by $x = 9.75 + 1.50t^3$, where t is in seconds. Calculate (a) the average velocity during the time interval $t = 2.00$ s to $t = 3.00$ s; (b) the instantaneous velocity at $t = 2.00$ s; (c) the instantaneous velocity at $t = 3.00$ s; (d) the instantaneous velocity at $t = 2.50$ s; and (e) the instantaneous velocity when the particle is midway between its positions at $t = 2.00$ s and $t = 3.00$ s. (f) Graph x versus t and indicate your answers graphically.

Problem (HW)

HW **•20** (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when, if ever, is the particle's velocity zero? (b) When is its acceleration a zero? (c) For what time range (positive or negative) is a negative? (d) Positive? (e) Graph $x(t)$, $v(t)$, and $a(t)$.

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$



$$\text{Displacement} = x_2 - x_1 = x - x_0$$

$$a = \frac{dv}{dt}$$

$$\Rightarrow \int_u^v dv = \int_0^t a\,dt$$

$$\Rightarrow [v]_u^v = a \int_0^t dt$$

$$\Rightarrow [v]_u^v = a\,[t]_0^t$$

$$\Rightarrow v - u = a[t - 0]$$

$$\int c x^n \, dx$$

$$= c \int x^n \, dx$$

$$= c\,\frac{x^{n+1}}{n+1}$$

$$\Rightarrow \int \underline{dx} = \int x^0 \, dx$$

$$= \frac{x^{0+1}}{0+1} = x$$

$$\Rightarrow v - u = at$$

$$\Rightarrow \boxed{v = u + at}$$

$a = \text{constant}$

$$\Downarrow$$

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt}$$

$$* \; v = \frac{dx}{dt}$$

$$\Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt \Rightarrow [x]_{x_0}^{x} = \int_0^t (u + at)\,dt$$

$$\Rightarrow x - x_0 = \left[ut + \frac{at^2}{2}\right]_0^t$$

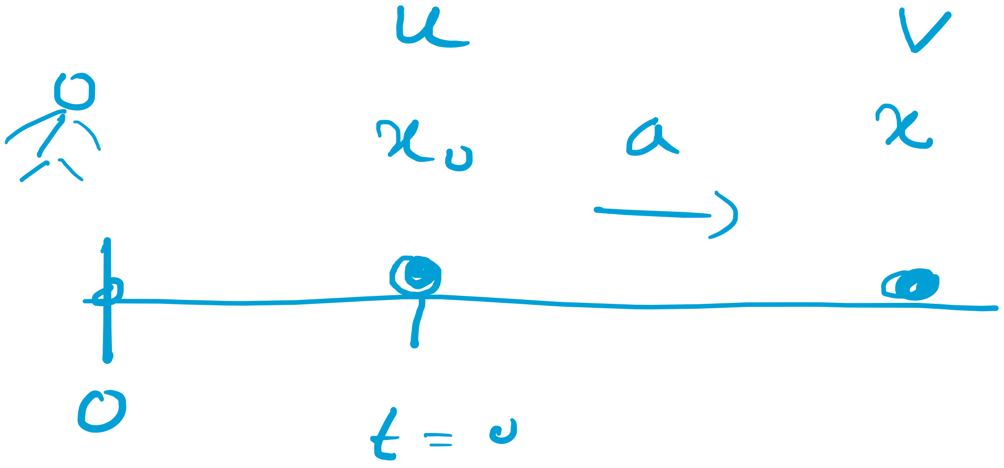
$$\Rightarrow x - x_0 = \left[ut + \frac{1}{2}at^2\right]_0^t$$

$$= \left(ut + \frac{1}{2}at^2\right) - \left(u \times 0 + \frac{1}{2}a \times 0^2\right)$$

$$\boxed{x - x_0 = ut + \frac{1}{2}at^2}$$

$$\therefore v = u + at$$

$$\therefore x - x_0 = ut + \frac{1}{2}at^2$$



$a = \frac{dv}{dt}$, $v = \frac{dx}{dt}$

$x =$

$t = 0$ $t =$

$x_0 =$ x

u a v

$\therefore\ v = u + at$

$*\ x - x_0 = ut + \frac{1}{2}at^2$

$*\ v^2 = u^2 + 2a(x - x_0)$

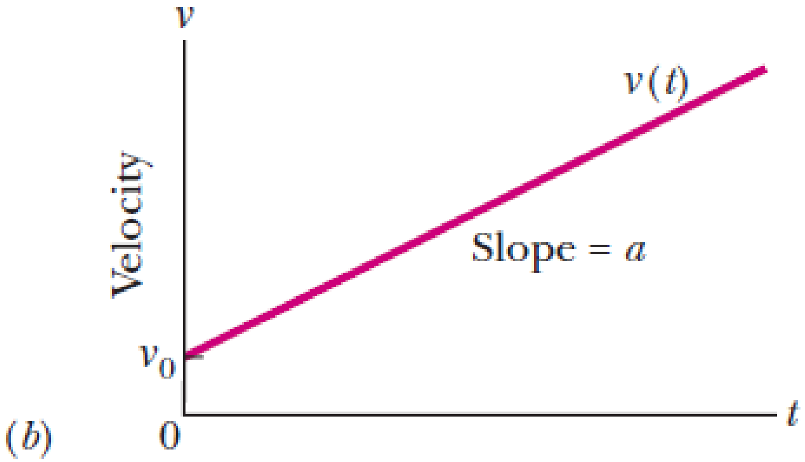
$*\ x - x_0 = \left(\frac{u+v}{2}\right)t$

Constant Acceleration

Acceleration/Average Acceleration

$$a = a_{avg} = \frac{v - v_0}{t - 0}.$$

or, $$\boxed{v = v_0 + at.}$$



Slope of the velocity graph is plotted on the acceleration graph.

Average Velocity

$$v_{avg} = \frac{x - x_0}{t - 0}$$

or, $$x = x_0 + v_{avg}t,$$

Constant Acceleration

Average Velocity

$$v_{avg} = \tfrac{1}{2}(v_0 + v).$$

Important Relation

1) $$x - x_0 = v_0 t + \tfrac{1}{2}at^2.$$

2) $$v^2 = v_0^2 + 2a(x - x_0).$$

3) $$x - x_0 = \tfrac{1}{2}(v_0 + v)t.$$

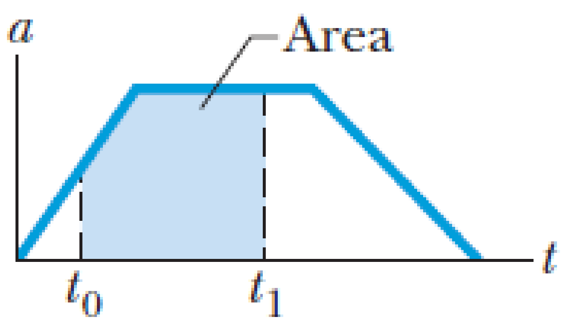
Constant Acceleration

•33 SSM ILW A car traveling 56.0 km/h is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later. (a) What is the magnitude of the car's constant acceleration before impact? (b) How fast is the car traveling at impact?

1D

If acceleration is not constant

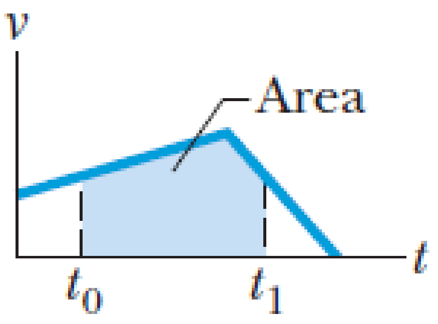
1) $$v_1 - v_0 = \int_{t_0}^{t_1} a\,dt.$$



(a)

This area gives the change in velocity.

2) $$x_1 - x_0 = \int_{t_0}^{t_1} v\,dt,$$



(b)

This area gives the change in position.

Problem: If $a(t)$ = 2t, at t = 0, v_0 = 10 m/s, x_0=0, Find the displacement at t =20 s.

1D

Free Fall of an object

1) $y = y_0 + v_0 t + \frac{1}{2}at^2$

$= y_0 + v_0 t - \frac{1}{2}gt^2$

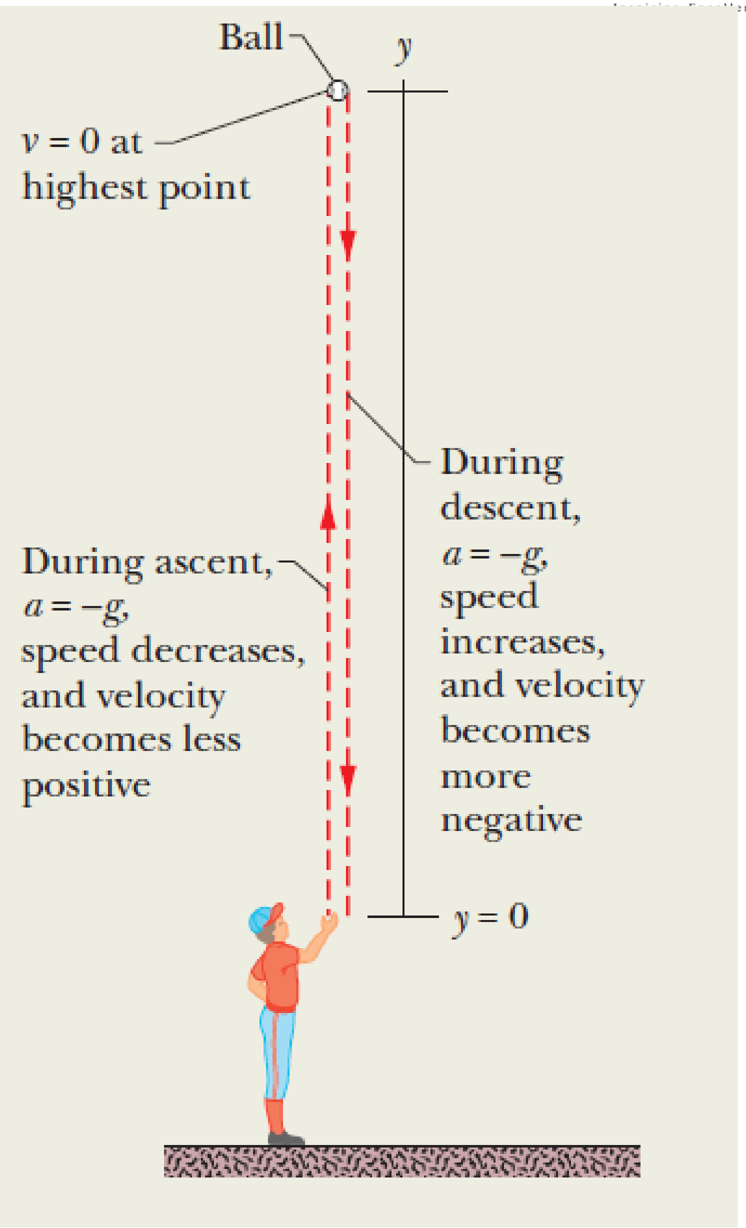
$\therefore a = -g$

2) $v = v_0 - gt$

3) $v^2 = v_0^2 - 2g(y - y_0)$

4) $y - y_0 = \frac{1}{2}(v + v_0)t$

At maximum height $v = 0$



Problems

BRAC UNIVERSITY
Inspiring Excellence

•48 A hoodlum throws a stone vertically downward with an initial speed of 12.0 m/s from the roof of a building, 30.0 m above the ground. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact?

•49 SSM A hot-air balloon is ascending at the rate of 12 m/s and is 80 m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? (b) With what speed does it hit the ground?