PHY111: Principles of Physics – I

Lecture 1-3, Vector

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Vector

- Physics is the science of nature that explains how the universe works.
- To describe physical phenomena, we measure various quantities.
- These quantities are classified into two types:

1) Scalar Quantities:

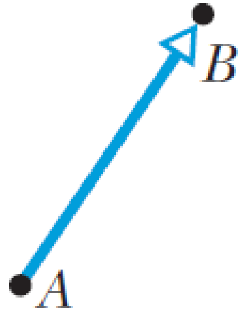
- Have magnitude only (no direction).
- Examples: Length, Temperature, Mass, Time.

2) Vector Quantities:

- Have both magnitude and direction.
- Require the language of vectors to describe.
- Examples: Velocity, Force, Acceleration, Momentum…

Geometrical Representation of Vector

Vectors are represented by arrows, which have an initial point and a terminal (final) point. In the figure below, point A is the initial point and point B is the terminal point of the vector. The length of the arrow represents the magnitude, and the direction is from A to B.



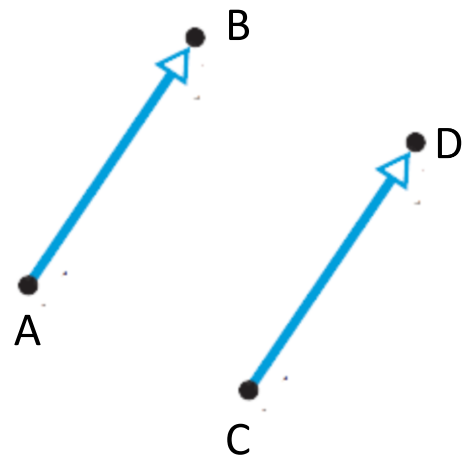
Notation of Vectors: $\overrightarrow{AB}$, $\vec{a}$, $\underline{a}$

Magnitude of a Vector: $|\overrightarrow{AB}|$, $|\overrightarrow{a}|$, a

Some Vectors

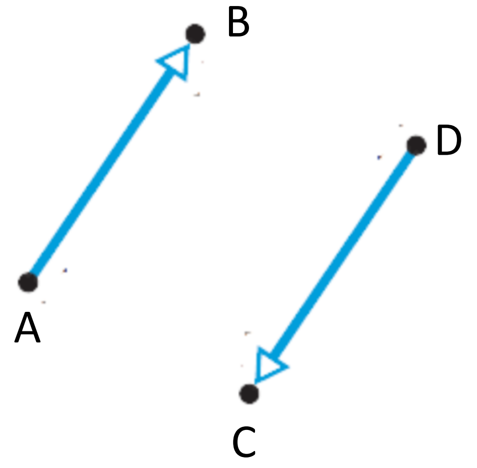
Same Vector: 1) Magnitude same
2) Parallel and direction same

$$\overrightarrow{AB} = \overrightarrow{CD} \quad and \quad |\overrightarrow{AB}| = |\overrightarrow{CD}|$$



Opposite Vector: 1) Magnitude same
2) Parallel and direction opposite

$$\overrightarrow{AB} = -\overrightarrow{DC} \quad and \quad |\overrightarrow{AB}| = |\overrightarrow{DC}|$$

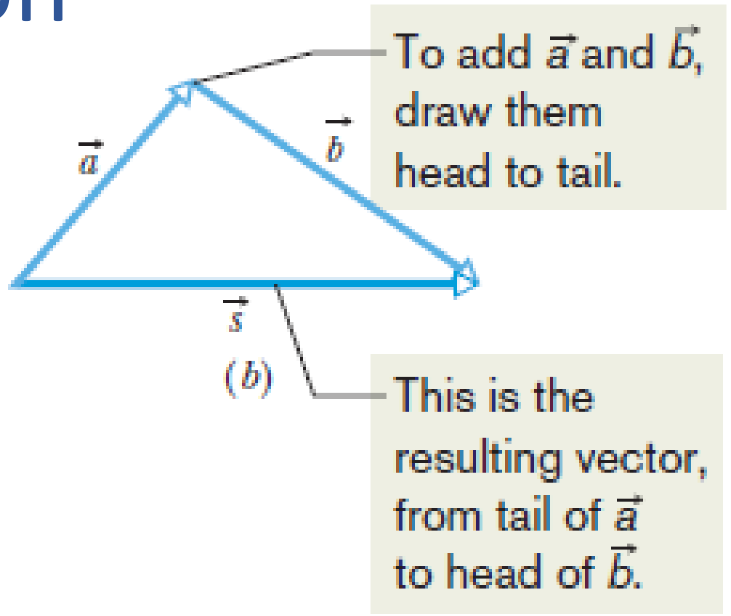


Unit Vector: 1) Magnitude 1
2) Directed in a particular direction

Notation: $\hat{a} = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

Vector Addition

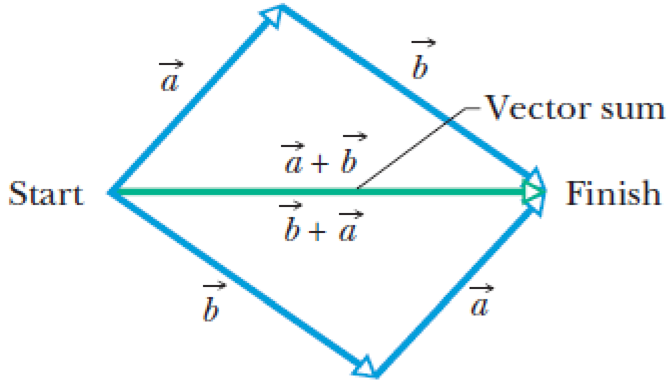
Two vectors and may be added geometrically by drawing them to a common scale and placing them head to tail. The vector connecting the tail of the first to the head of the second is the vector sum.



Properties of Vector Addition

Properties:

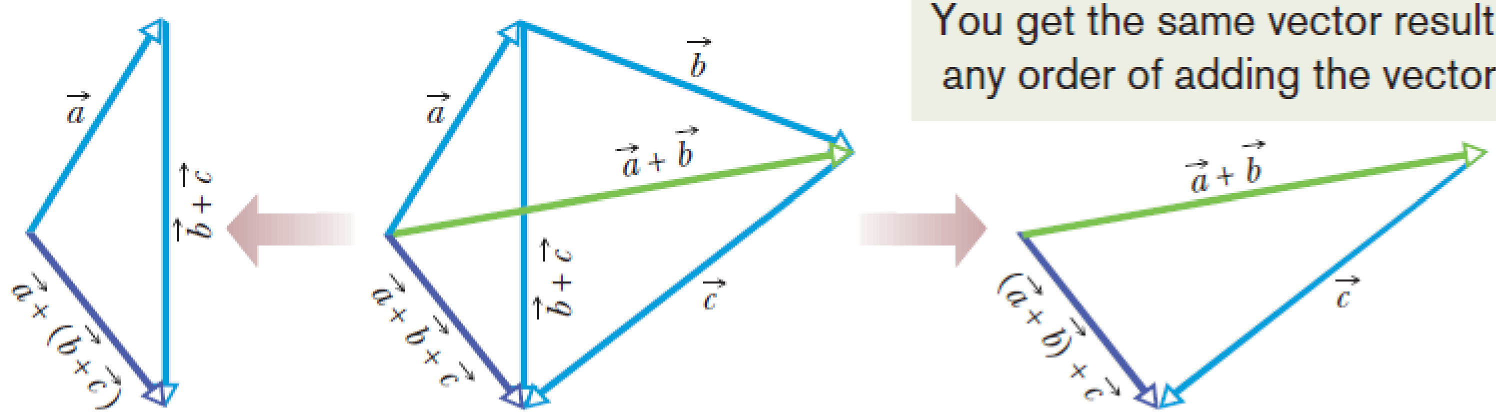
1) Commutative law: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



You get the same vector result for either order of adding vectors.

2) Associative law: $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

You get the same vector result for any order of adding the vectors.



Vector Subtraction

Subtraction of two vectors

$$\vec{d} = \vec{a} - \vec{b}$$

$$\vec{d} = \vec{a} + (-\vec{b})$$

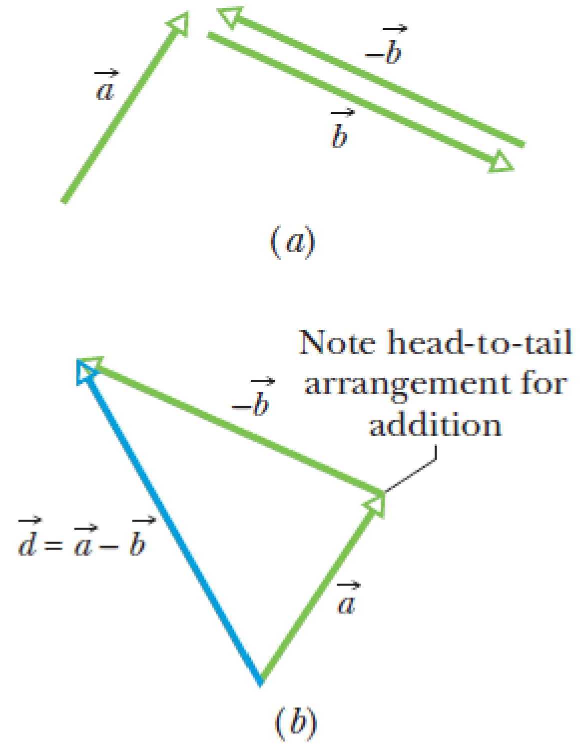
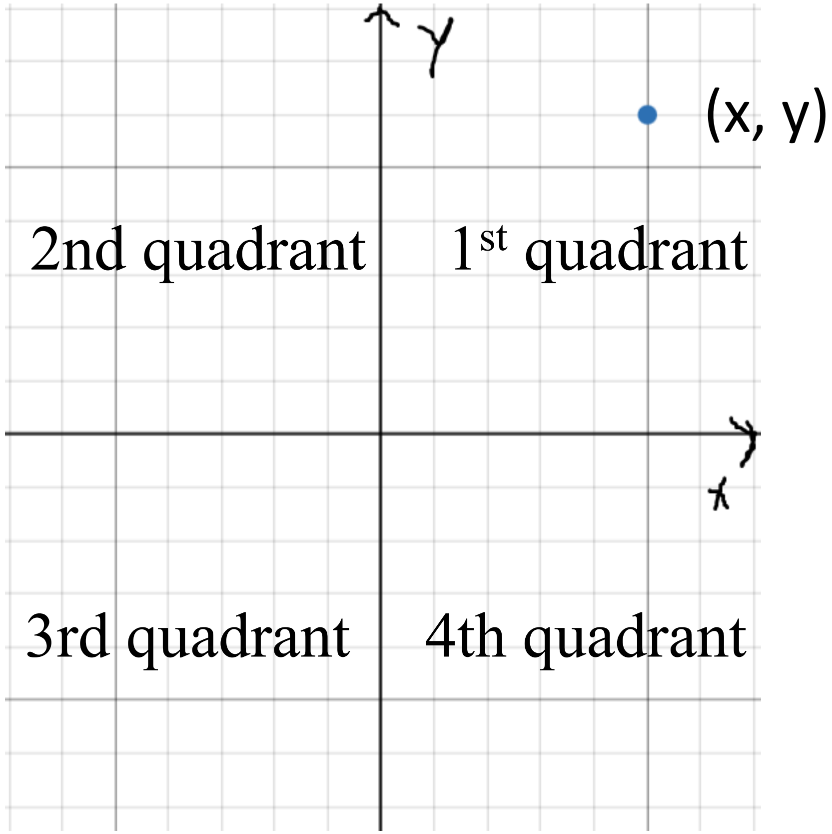


Figure 3-6 (a) Vectors $\vec{a}$, $\vec{b}$, and $-\vec{b}$. (b) To subtract vector $\vec{b}$ from vector $\vec{a}$, add vector $-\vec{b}$ to vector $\vec{a}$.

Coordinate System in 2D

Cartesian Coordinate system:

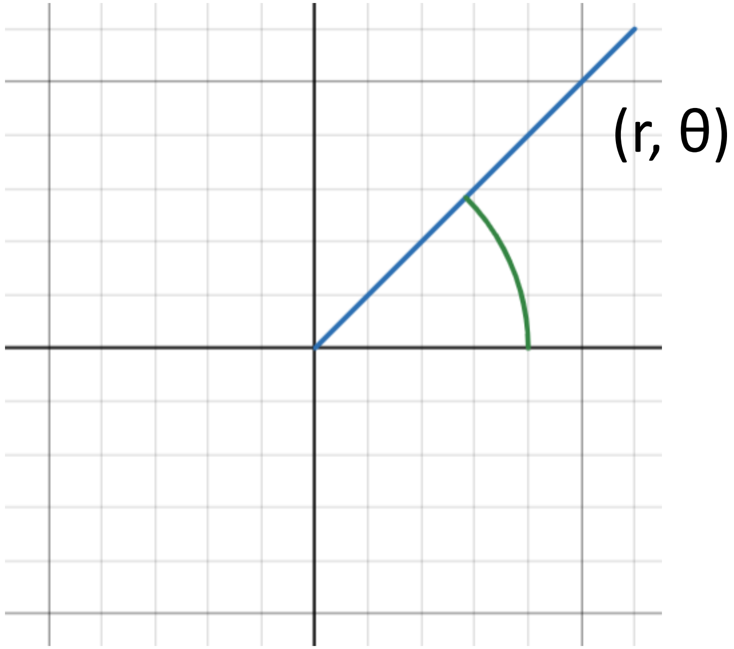
- Each point in 2D plane is represented by a ordered pair, (x, y).
- x is the perpendicular distance from the point to y-axis.
- y is the perpendicular distance from the point to x-axis.



Coordinate System in 2D

Polar coordinate system:

- Each point in 2D plane is represented by an ordered pair (r,θ).
- r is the distance from the origin to the point.
- θ is the angle measured anticlockwise from the positive x-axis



Cartesian to Polar / Polar to Cartesian transformation

Cartesian to Polar Coordinate:

$$r = \sqrt{x^2 + y^2}$$

$$tan\ \theta = \frac{y}{x}$$

Polar to Cartesian Coordinate:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Problem: 1) (x,y) = (1, 0.5), (1, -1)

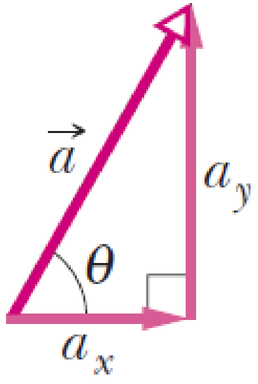
2) (r, θ) = (4, 45°)

Components of Vector

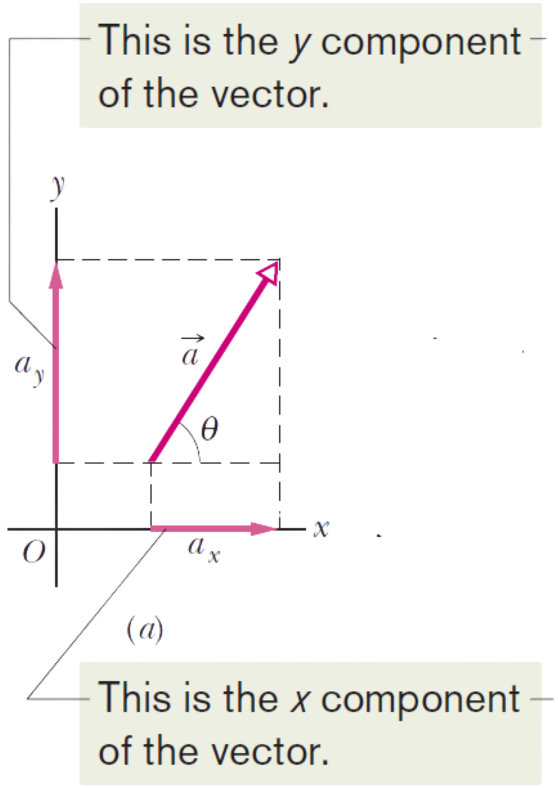
Let $\vec{a}$ be a vector. Its components along x-axis and y-axis are:

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta,$$

The components and the vector form a right triangle.



(c)



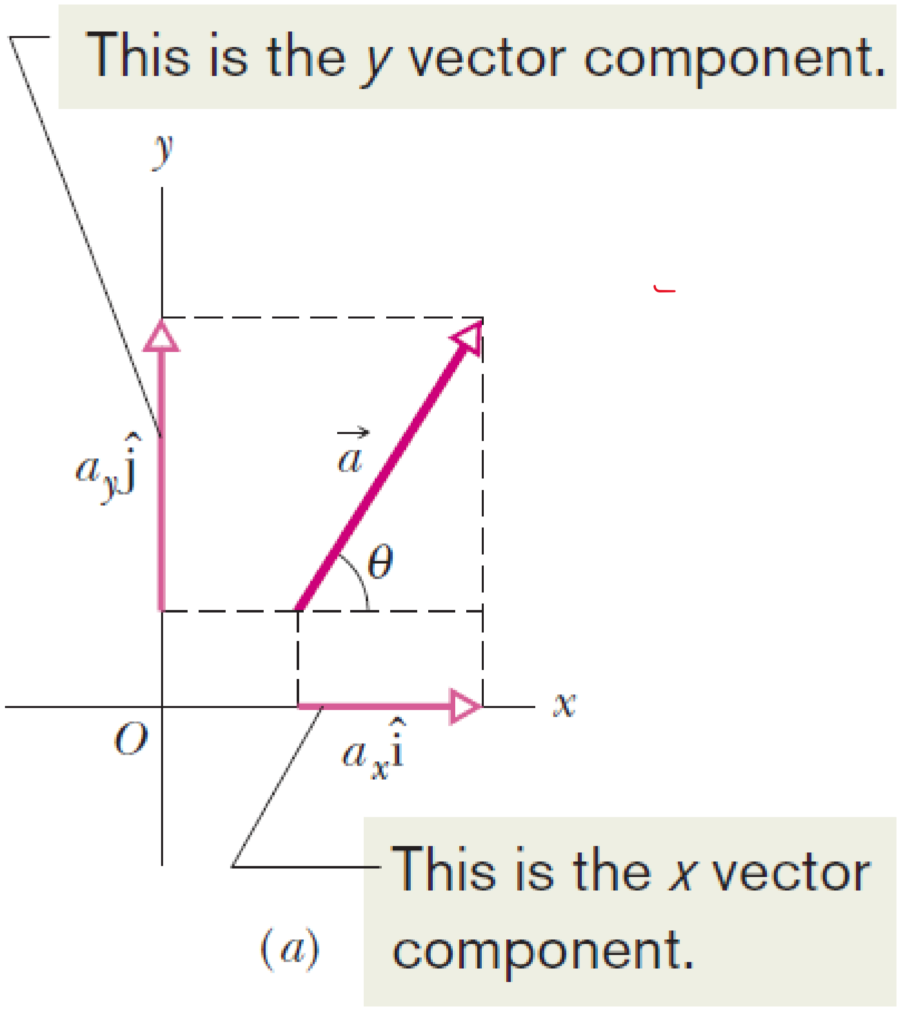
The angle θ is measured with respect to the x axis and in the anticlockwise direction.

Coordinate representation of Vector

$\hat{\imath}$ is the unit vector along x-axix and
$\hat{\jmath}$ is the unit vector along y axis

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

Problem:
1) If a = 1 and θ = 45°
the find $\vec{a}$.
2) If b = 1 and θ = 135°
the find $\vec{b}$.



Direction and Magnitude of a Vector

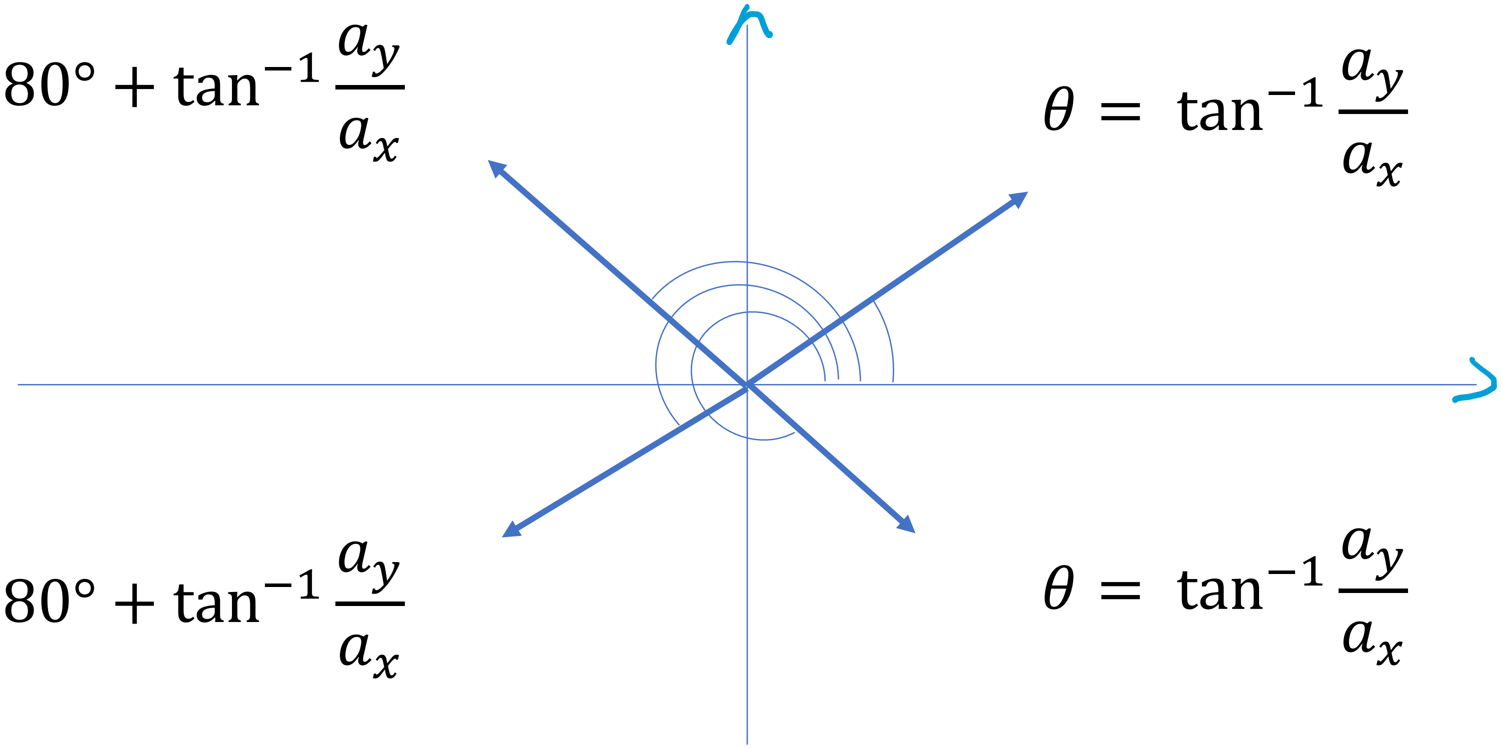
Magnitude a: $a = \sqrt{a_x^2 + a_y^2}$

Direction θ: $\tan\theta = \dfrac{a_y}{a_x}$

However, The angle depends on the orientation of a vector

$$\theta = 180° + \tan^{-1}\frac{a_y}{a_x}$$

$$\theta = \tan^{-1}\frac{a_y}{a_x}$$

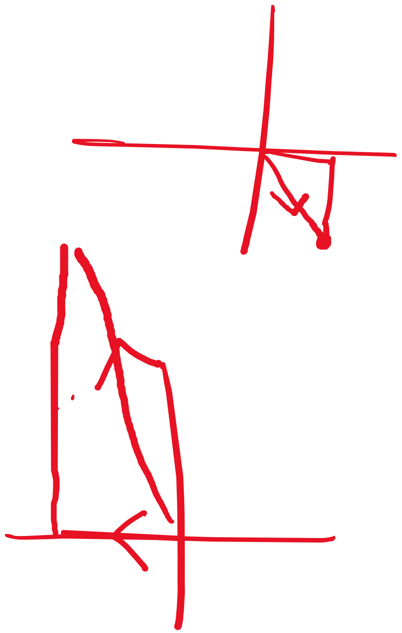


$$\theta = 180° + \tan^{-1}\frac{a_y}{a_x}$$

$$\theta = \tan^{-1}\frac{a_y}{a_x}$$

$$\vec{A} = \underline{2}\,\hat{i} - \underline{3}\,\hat{j}$$

$$\vec{B} = \underline{-3}\,\hat{i} + \underline{9}\,\hat{j}$$



$$|\vec{B}| = \sqrt{(-3)^2 + 9^2} =$$

Direction $\theta = 180 + \tan^{-1}\left(\frac{9}{-3}\right)$

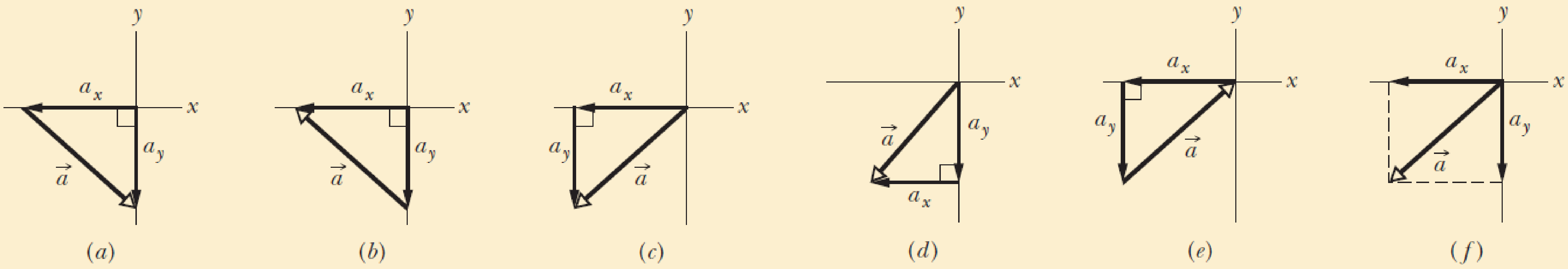
$$= 108.43^\circ$$

Checkpoint 1

The magnitudes of displacements $\vec{a}$ and $\vec{b}$ are 3 m and 4 m, respectively, and $\vec{c} = \vec{a} + \vec{b}$. Considering various orientations of $\vec{a}$ and $\vec{b}$, what are (a) the maximum possible magnitude for $\vec{c}$ and (b) the minimum possible magnitude?

Checkpoint 2

In the figure, which of the indicated methods for combining the x and y components of vector $\vec{a}$ are proper to determine that vector?



Sample Problem 3.02 Finding components, airpl

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. This means that the direction is not due north (directly toward the north) but is rotated 22° toward the east from due north. How far east and north is the airplane from the airport when sighted?

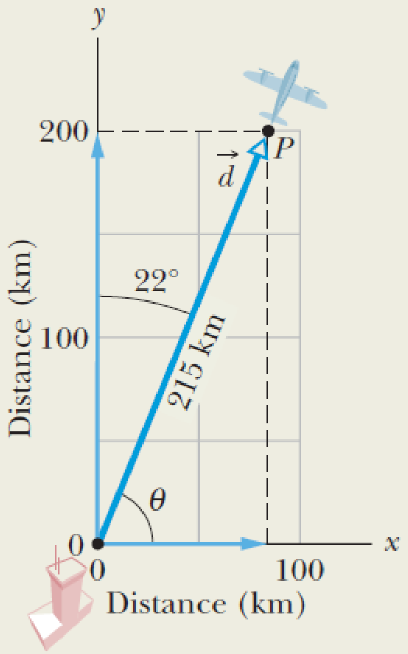


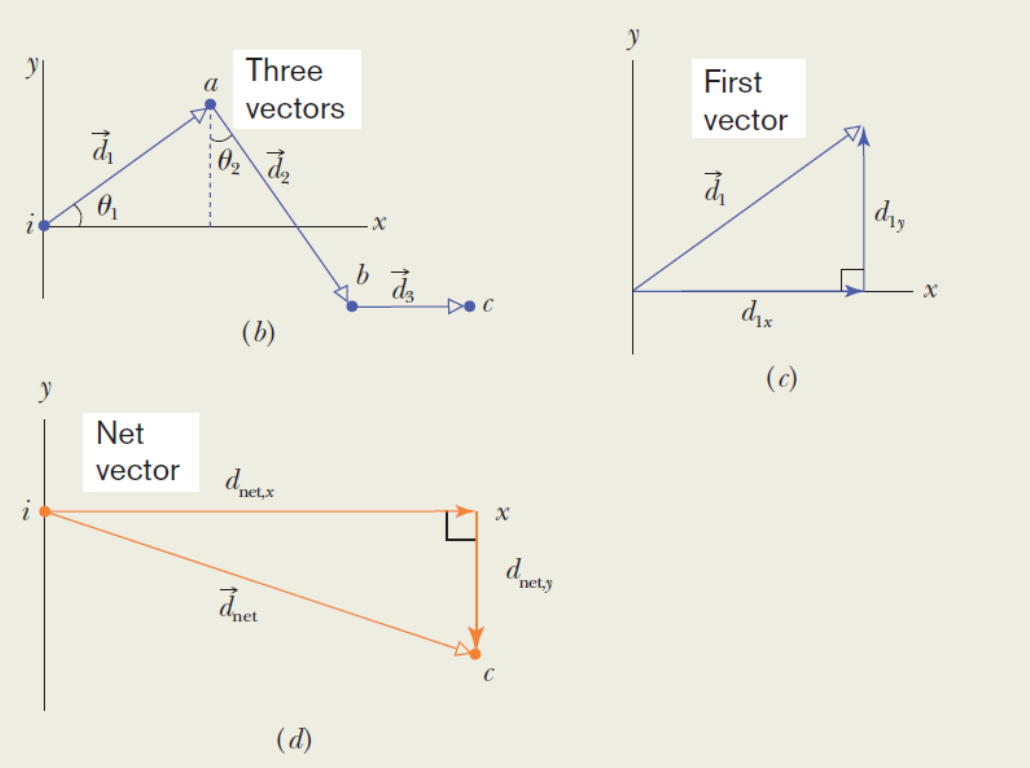
Figure 3-10 A plane takes off from an airport at the origin and is later sighted at *P*.

Sample Problem 3.03 Searching through a hedg

A hedge maze is a maze formed by tall rows of hedge. After entering, you search for the center point and then for the exit. Figure 3-16*a* shows the entrance to such a maze and the first two choices we make at the junctions we encounter in moving from point *i* to point *c*. We undergo three displacements as indicated in the overhead view of Fig. 3-16*b*:

$$d_1 = 6.00 \text{ m} \qquad \theta_1 = 40^\circ$$
$$d_2 = 8.00 \text{ m} \qquad \theta_2 = 30^\circ$$
$$d_3 = 5.00 \text{ m} \qquad \theta_3 = 0^\circ,$$

where the last segment is parallel to the superimposed x axis. When we reach point c, what are the magnitude and angle of our net displacement $\vec{d}_{net}$ from point i?



Sample Problem 3.04 Adding vectors, unit-

Figure 3-17*a* shows the following three vectors:

$$\vec{a} = (4.2\text{ m})\hat{i} - (1.5\text{ m})\hat{j},$$
$$\vec{b} = (-1.6\text{ m})\hat{i} + (2.9\text{ m})\hat{j},$$
and
$$\vec{c} = (-3.7\text{ m})\hat{j}.$$

What is their vector sum $\vec{r}$ which is also shown?

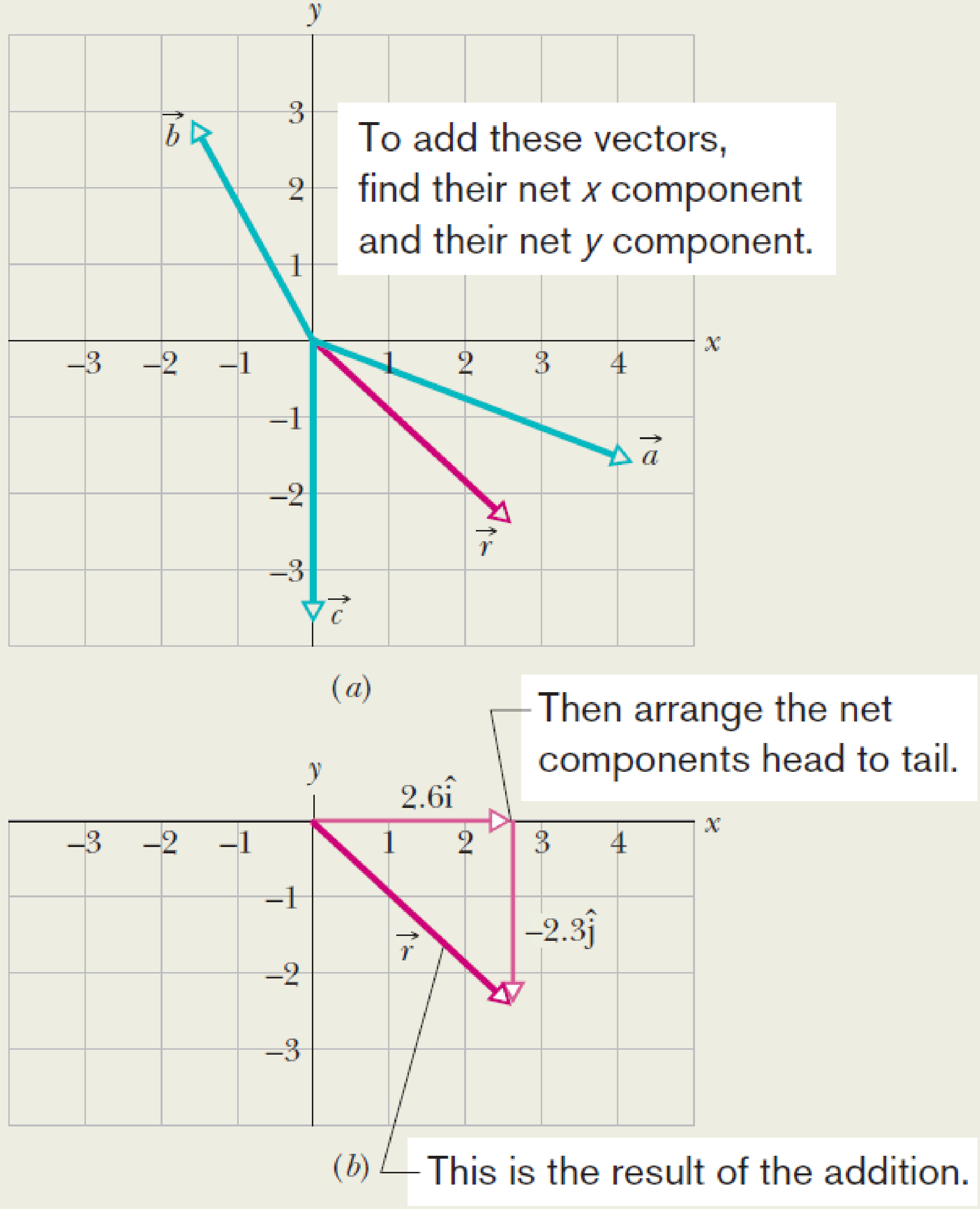


Figure 3-17 Vector $\vec{r}$ is the vector sum of the other three vectors.

Vector Addition:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} + (A_z + B_z) \hat{k}$$

Substruction;

$$\vec{A} - \vec{B} = (A_x - B_x) \hat{i} + (A_y - B_y) \hat{j} + (A_z - B_z) \hat{k}$$

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{B} = 0\hat{i} + 3\hat{j} + 9\hat{k}$

$\therefore$ Find a unit vector along $\vec{A} + \vec{B}$??

$$\hat{n} = \frac{\vec{A} + \vec{B}}{|\vec{A} + \vec{B}|}$$

$\therefore \vec{A} + \vec{B} = 2\hat{i} + 2\hat{j} + 10\hat{k}$

$|\vec{A} + \vec{B}| = \sqrt{2^2 + 2^2 + 10^2}$

$= \sqrt{108}$

$$= \frac{2\hat{i} + 2\hat{j} + 10\hat{k}}{\sqrt{108}}$$

$\vec{A} - \vec{B} = 2\hat{i} - 4\hat{j} - 8\hat{k}$

$$\vec{R} - \vec{A} + \vec{B} = 0 \Rightarrow \vec{R} = \vec{A} - \vec{B}$$

Find a unit vector along $\vec{R}$.

Projection of vector in 3D (Spherical Polar Coordinate)

If $\vec{a}$ and $\vec{b}$ are two vectors. If the angle between these two vectors is θ.

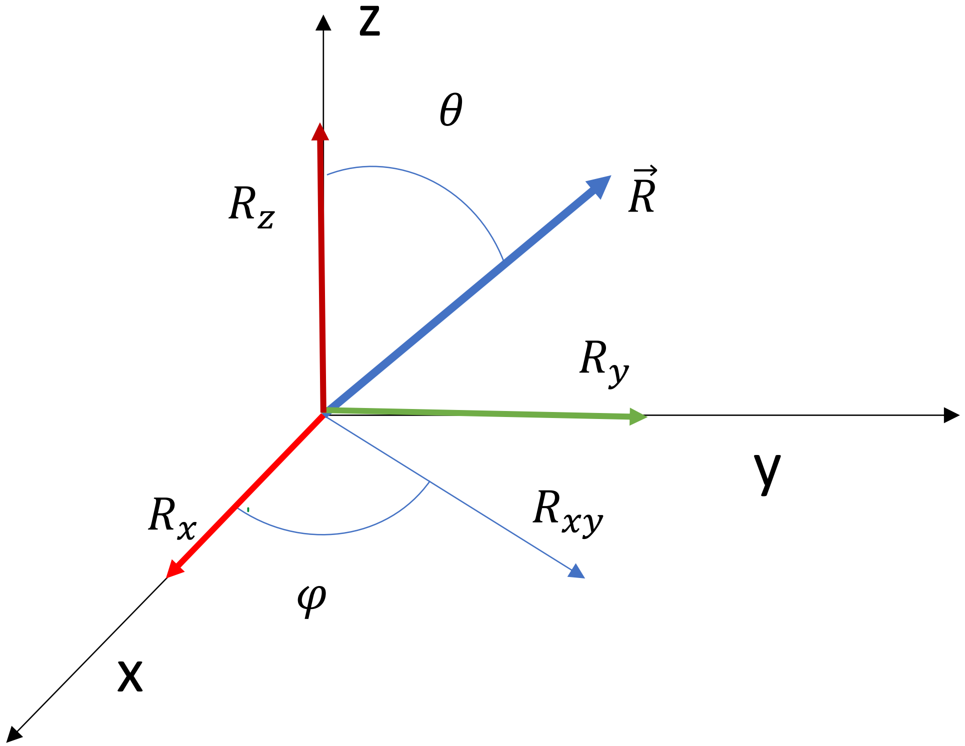
1) $R_z = R\ cos\theta$

2) $R_{xy} = R\ sin\theta$

3) $R_x = R_{xy}\cos\varphi = R\ sin\theta\ \cos\varphi$

4) $R_y = R_{xy}sin\varphi = R\ sin\theta\ sin\varphi$

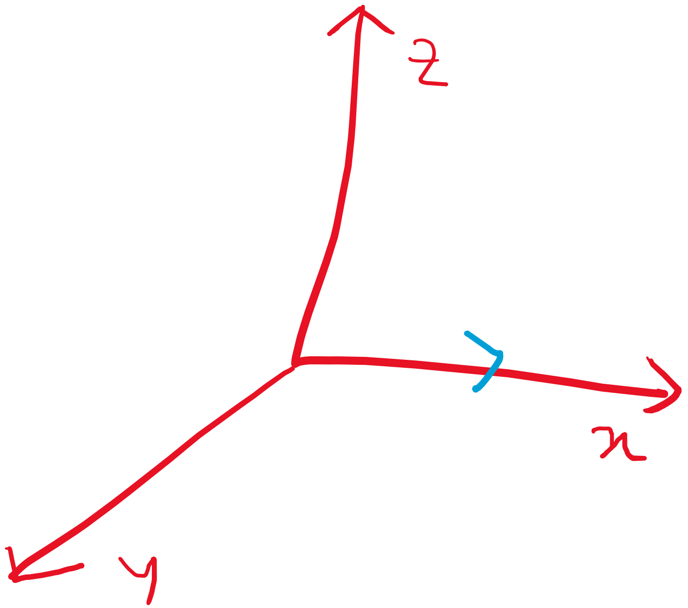
Therefore, $\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$



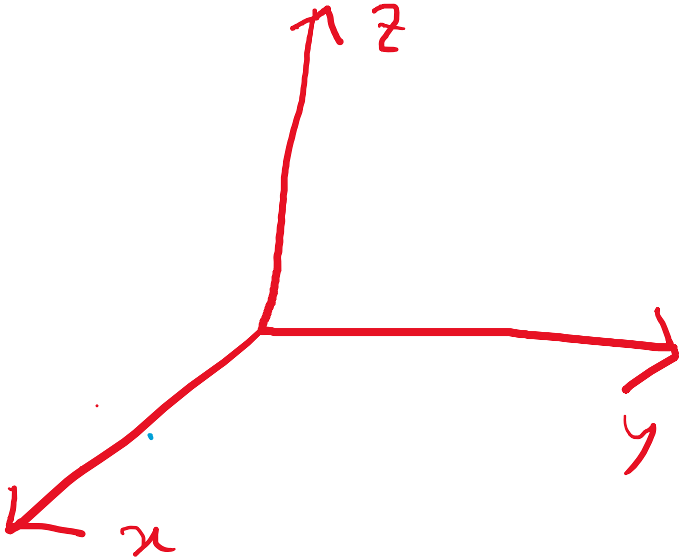
Problem 1: If R=4, θ = 30 degree, φ = 60 degree, Find $\vec{R}$.

Problem 2: If R=6, θ = 60 degree, φ = 120 degree, Find $\vec{R}$.

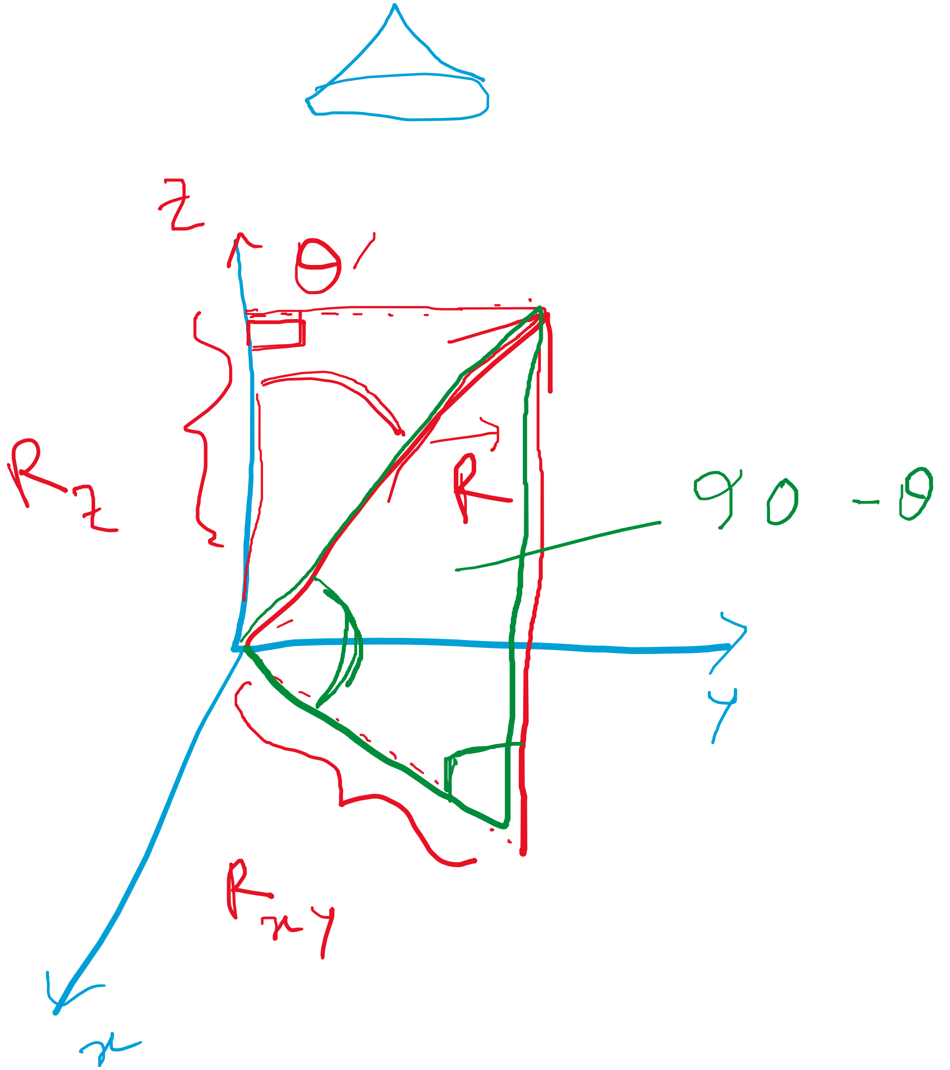
3D Coordinate



Left



Right Hand coordinate



$$\boxed{\begin{aligned}\cos\theta &= \frac{R_z}{R}\\ \Rightarrow R_z &= R\cos\theta\end{aligned}}$$

$$\cos(90-\theta) = \frac{R_{xy}}{R}$$

$$\Rightarrow \sin\theta =$$

$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

$$R_z = \underline{R}\cos\theta$$

$$R_{xy} = R\sin\theta$$

$$R_x = \underline{R}\sin\theta\cos\phi$$

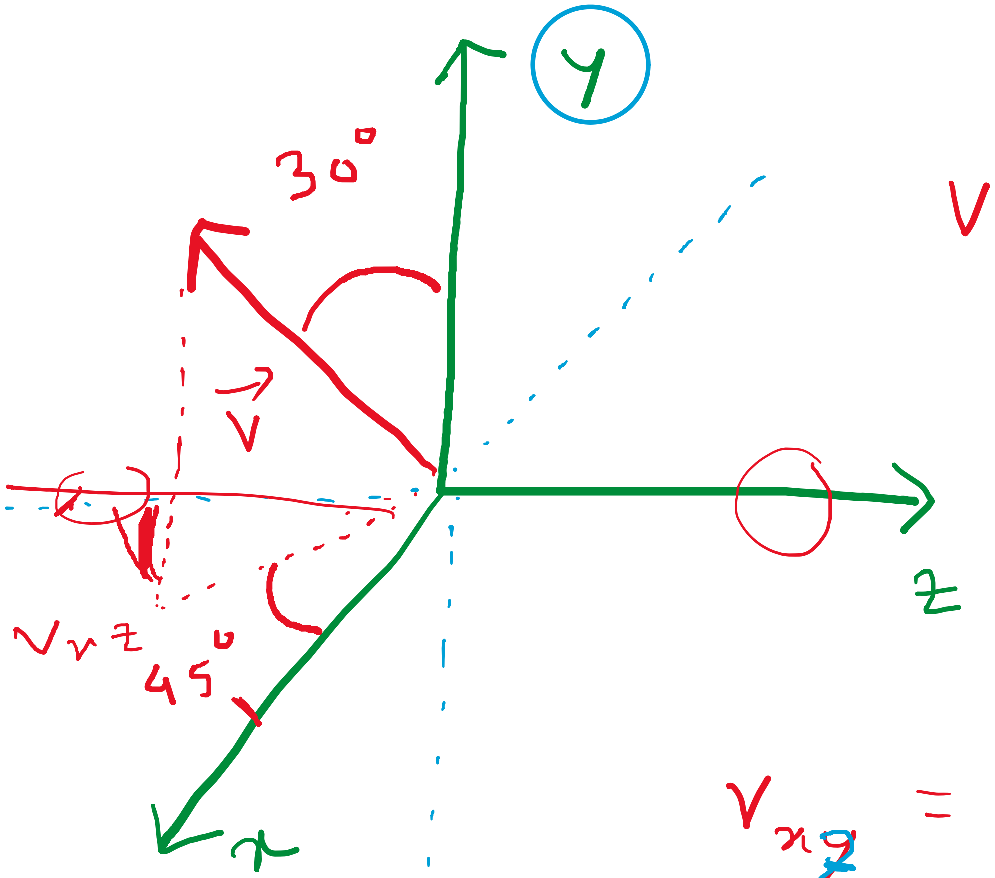
$$R_y = \underline{R}\sin\theta\sin\phi$$

Magnitude:

$$\underline{|\vec{R}|} = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \quad /$$

$$= \underline{R}$$



$V = 6$ unit

$\vec{V} = V_x\,\hat{i} + V_y\,\hat{j} + V_z\,\hat{k}$

$V_y = V\cos\theta = 6\cos 30^\circ = 3\sqrt{3}$

$V_{xz} = V\sin 30^\circ = 6\sin 30^\circ = 3$

$\underline{V_x} = V_{xz}\cos \underline{45}^\circ = 3 \times \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$\underline{V_z} = -V_{xz}\sin 45^\circ = -\frac{3}{\sqrt{2}}$

$\vec{V} = \frac{3}{\sqrt{2}}\hat{i} + 3\sqrt{3}\,\hat{j} - \frac{3}{\sqrt{2}}\hat{k}$

Scalar Product or Dot Product

$$\vec{a} \cdot \vec{b} = ab \cos \phi,$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.$$

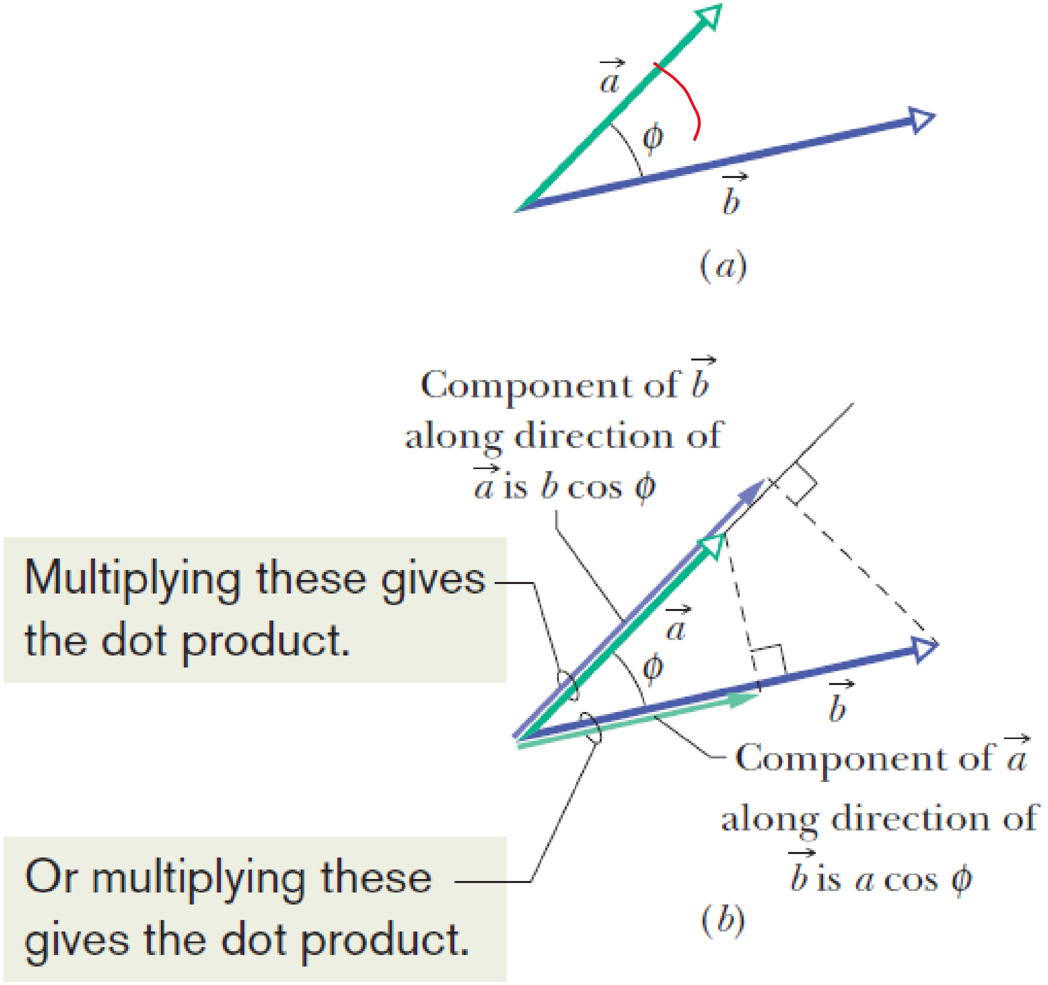
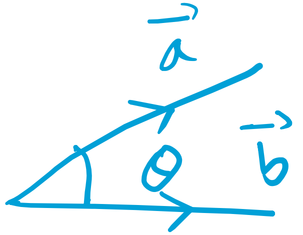


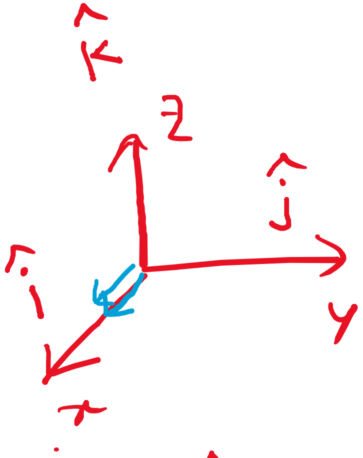
Figure 3-18 (*a*) Two vectors $\vec{a}$ and $\vec{b}$, with an angle ϕ between them. (*b*) Each vector has a component along the direction of the other vector.

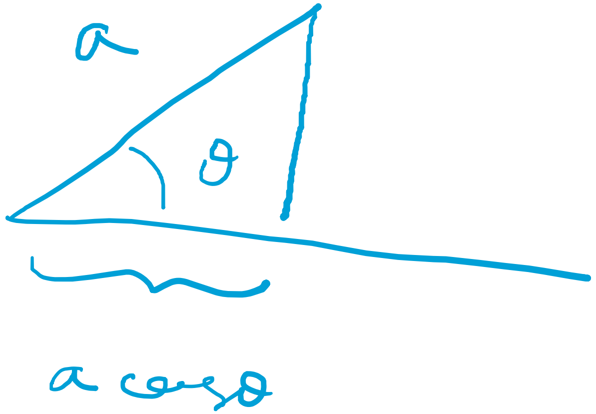
Definition: Scalar product / Dot product



$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$= b\,(a \cos\theta)$$





$$\hat{i} \cdot \hat{i} = 1 \times 1 \times \cos 0^{\circ} = 1$$

$\hat{i}\cdot\hat{j} = 1\times 1\times \cos 90^\circ = 0$

$\hat{i}\cdot\hat{k} = 0$

$\hat{j}\cdot\hat{i} = 0$

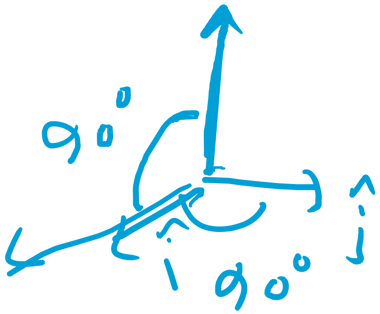
$\hat{j}\cdot\hat{j} = 1$

$\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{i} = 0$

$\hat{k}\cdot\hat{j} = 0$

$\hat{k}\cdot\hat{k} = 1$



$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$$

$$= a_x \hat{i} \cdot b_x \hat{i} + a_x \hat{i} \cdot b_y \hat{j} + a_x \hat{i} \cdot b_z \hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$\vec{a} = 2\hat{i} + 2\hat{j} + 0\hat{k}$

$\vec{b} = 3\hat{i} - 5\hat{k}$

$\therefore$ calculate $\vec{a} \cdot \vec{b}$.

$\vec{a} \cdot \vec{b} = 2 \times 3 + 2 \times 0 + 0 \times -5$

$= 6$

$\vec{a} \cdot \vec{b} = 2 \times 3 + 2 \times -5$

$= -4$

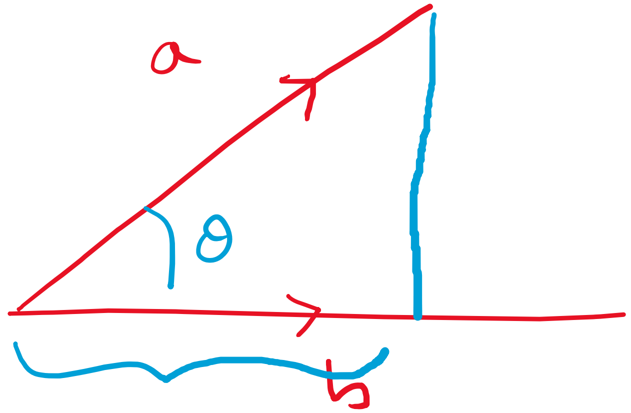
$\vec{a} \cdot \vec{b} = 2 \times 3\hat{i} + 2 \times 0\hat{j} + 0 \times -5\hat{k}$

$= 6\hat{i} + \cdots$

Use of dot product :

$\vec{a} = 2\hat{i} - 3\hat{j}$

$\vec{b} = \hat{i} + 4\hat{j}$



Find the component / projection of $\underline{\vec{a}}$ along $\underline{\vec{b}}$.

$\therefore$ Projection / component $= a\cos\underline{\theta}$

$=$

We know,

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\Rightarrow a\cos\theta = \frac{\vec{a}\cdot\vec{b}}{b}$$

Projection of $\vec{a}$ along $\vec{b}$ $= a\cos\theta$

$$= \frac{\vec{a}\cdot\vec{b}}{b} = \frac{-10}{\sqrt{17}} =$$

$$\vec{a}\cdot\vec{b} = 2-12 = -10$$

$$b = |\vec{b}| = \sqrt{1^2+4^2} = \sqrt{17}$$

(*) Find the angle between the vectors $\underline{\vec{a}}$ and $\underline{\vec{b}}$:

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\Rightarrow \cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab}$$

$$\therefore \theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{ab}\right)$$

$$= \cos^{-1}\left(\frac{-10}{\sqrt{17}\sqrt{13}}\right) = 132.3^{0}$$

$$\vec{a}\cdot\vec{b} = -10$$

$$b = \sqrt{17}$$

$$a = \sqrt{2^2+(-3)^2} = \sqrt{13}$$

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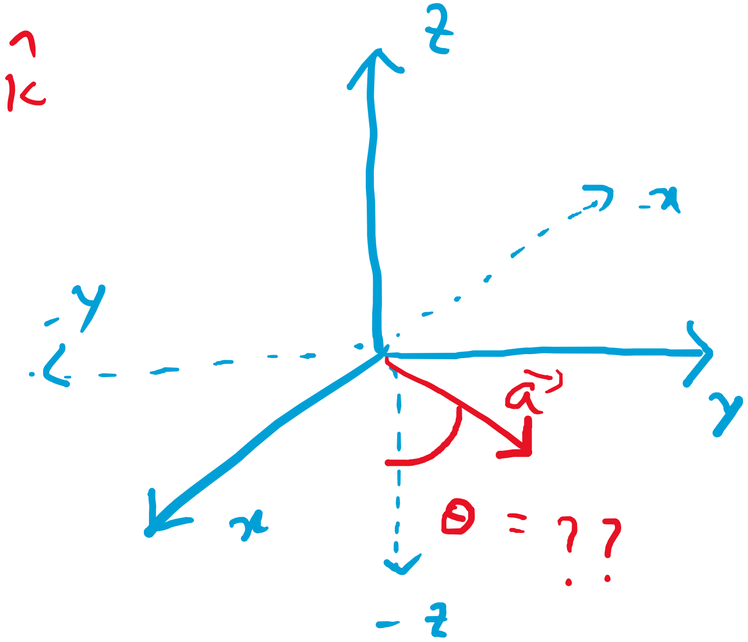
(*) Find the angle between vector $\vec{a}$ and negative direction of z axis.?

$\vec{a}$, $\vec{c} = -\hat{k} = -2\hat{k} = -3\hat{k}$

$$\therefore \theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{c}}{a\,c}\right)$$

$$= \cos^{-1}\left(\frac{0}{\sqrt{13}\times 1}\right) = 90^\circ$$



Vector Product or Cross Product

If $\vec{a}$ and $\vec{b}$ are two vectors. Then their cross product is defined as:

$$\vec{a} \times \vec{b} = a\, b \sin\theta\, \hat{n}$$

$$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{\mathrm{i}} + (a_z b_x - b_z a_x)\hat{\mathrm{j}} + (a_x b_y - b_x a_y)\hat{\mathrm{k}}.$$

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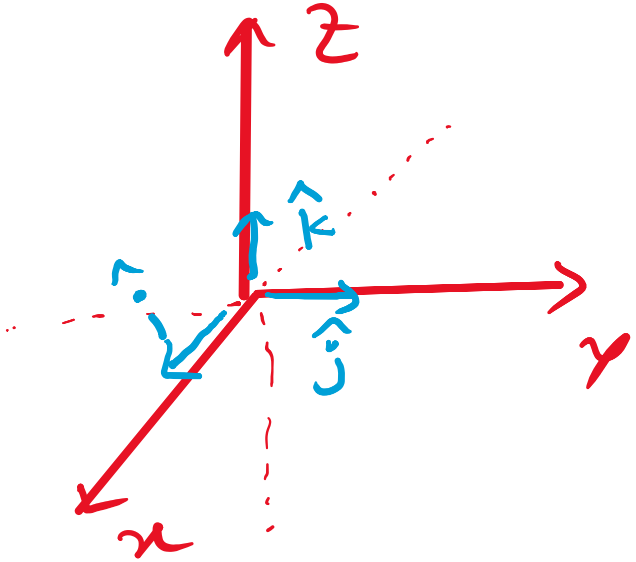
$$\hat{i} \times \hat{i} = 1 \cdot 1 \cdot \sin 0 = \underline{0}$$

$$\hat{i} \times \hat{j} = 1 \cdot 1 \cdot \sin 90^\circ \; \hat{k} = \hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$



$$\boxed{\begin{array}{l|l|l} \hat{i} \times \hat{j} = \hat{k} & \hat{i} \times \hat{k} = -\hat{j} & \hat{i} \times \hat{i} = \underline{0} \\ \hat{j} \times \hat{k} = \hat{i} & \hat{k} \times \hat{j} = -\hat{i} & \hat{j} \times \hat{j} = \underline{0} \\ \hat{k} \times \hat{i} = \hat{j} & \hat{j} \times \hat{i} = -\hat{k} & \hat{k} \times \hat{k} = \underline{0} \end{array}}$$

$$\left.\begin{array}{l} \hat{i} \wedge \hat{j} = \\ \hat{j} \wedge \hat{k} = \\ \hat{k} \wedge \hat{i} = \end{array}\right\} 90^\circ$$

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$$\vec{a} \times \vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) \times (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$

$$= a_x\hat{i} \times b_x\hat{i} + a_x\hat{i} \times b_y\hat{j} + a_x\hat{i} \times b_z\hat{k}$$

$$+ \cdots + \cdots + \cdots$$

$$+ \cdots + \cdots + \cdots$$

$$= a_x b_x\,\hat{i} \times \hat{i} + a_x b_y\,\hat{i} \times \hat{j} + a_x b_y\,\hat{i} \times \hat{k}$$

$$+ \cdots$$

$$= 0 + a_x b_y\,\hat{k} + a_x b_y(-\hat{j}) + \cdots$$

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$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i}\left(a_y b_z - b_y a_z\right) - \hat{j}\left(a_x b_z - b_x a_z\right)$$

$$+ \hat{k}\left(a_x b_y - b_x a_y\right)$$

$$\vec{a} = 2\hat{i} - 4\hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} - 2\hat{k}$$

$$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -2 \\ 2 & -4 & 3 \end{vmatrix} = -8\hat{i} - 13\hat{j} - 12\hat{k}$$

$$= \hat{i}\left(0 \times 3 - (-2) \times (-4)\right) - \hat{j}\left(3 \times 3 - 2 \times (-2)\right)$$

$$+ \hat{k}\left(3 \times -4 - 2 \times 0\right)$$

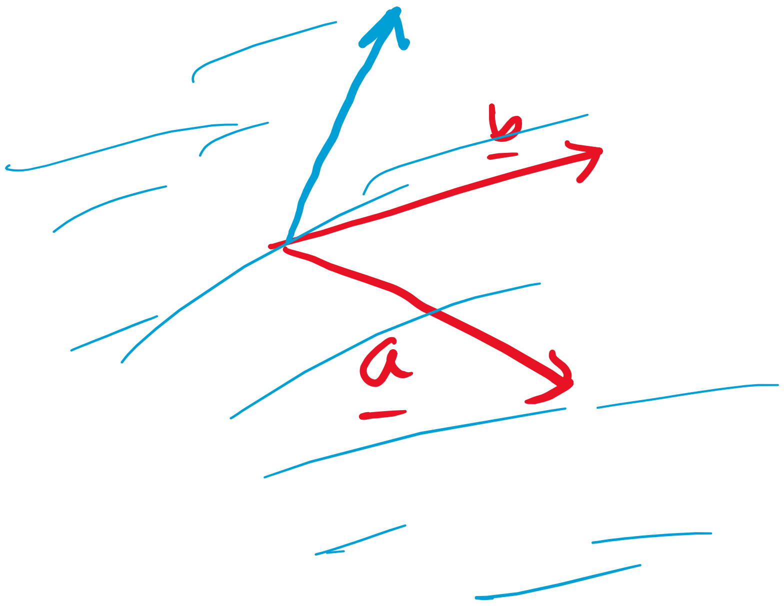
$$=$$

Magnitude of $|\vec{b} \times \vec{a}| = ??$

$$|\vec{b} \times \vec{a}| = \sqrt{(-8)^2 + (-13)^2 + (-12)^2} = \sqrt{(\quad)}$$

$\therefore$ Find a unit vector perpendicular to the plane formed by vectors $\vec{a}$ and $\vec{b}$.

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$



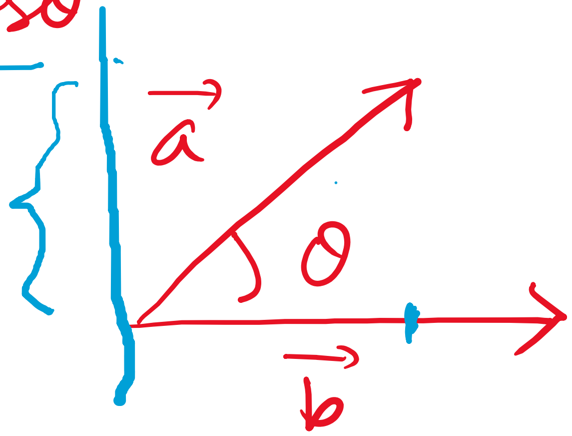
$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

$$\vec{a} \times \vec{b} = ab \sin\theta \, \hat{n}$$

$$|\bar{a} \times \bar{b}| = ab \sin\theta$$

(*)

* Component of $\vec{a}$ along $\vec{b}$ = $a\cos\theta$



* Component of $\vec{a}$ perpendicular to the vector $\vec{b}$ (*)

$= a\sin\theta$

$$\vec{a} \times \vec{b} = ab\sin\theta\,\hat{n}$$

$$|\bar{a} \times \bar{b}| = ab\sin\theta$$

$$\therefore\ a\sin\theta = \frac{|\vec{a} \times \vec{b}|}{b}$$

$$a\sin\theta \neq \frac{\vec{a} \times \vec{b}}{a}$$

Projection of a vector

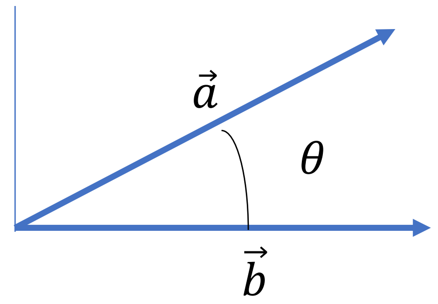
If $\vec{a}$ and $\vec{b}$ are two vectors. If the angle between these two vectors is θ.

Component

Projection of $\vec{a}$ along $\vec{b}$ is $= a\,cos\theta$

$$= a\,\frac{\vec{a}\cdot\vec{b}}{a\,b} = \frac{\vec{a}\cdot\vec{b}}{b}$$

$$\vec{a}\cdot\vec{b} = ab\,cos\theta$$



||

Projection of $\vec{a}$ perpendicular to $\vec{b}$ is $= a\,sin\theta$

$$= \cancel{a}\,\frac{|\vec{a}\times\vec{b}|}{\cancel{a}\,b} = \frac{|\vec{a}\times\vec{b}|}{b}$$

$$|\vec{a}\times\vec{b}| = ab\,sin\theta$$

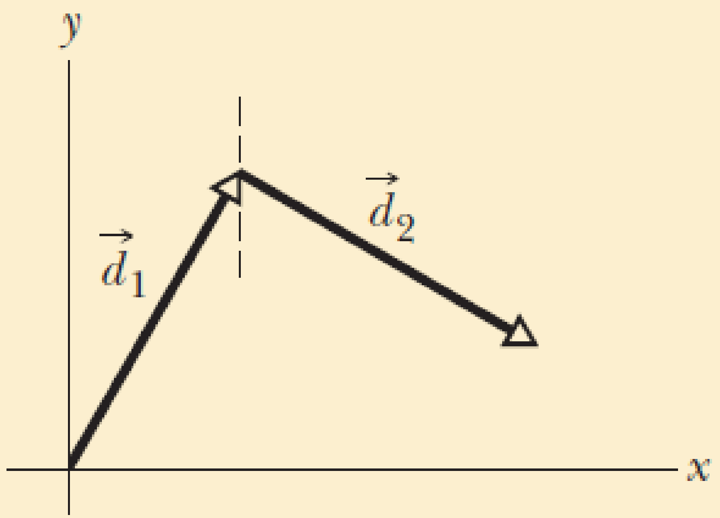
Problem: $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$, and $\vec{b} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$,

i) Find the projection of $\vec{a}$ along $\vec{b}$ and projection $\vec{b}$ along $\vec{a}$.

ii) Find the projection of $\vec{a}$ perpendicular to $\vec{b}$ and projection $\vec{b}$ perpendicular $\vec{a}$.

Checkpoint 3

(a) In the figure here, what are the signs of the x components of $\vec{d}_1$ and $\vec{d}_2$? (b) What are the signs of the y components of $\vec{d}_1$ and $\vec{d}_2$? (c) What are the signs of the x and y components of $\vec{d}_1 + \vec{d}_2$?



Sample Problem 3.05 Angle between two vector

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$? (*Caution:* Although many of the following steps can be bypassed with a vector-capable calculator, you will learn more about scalar products if, at least here, you use these steps.)

Checkpoint 4

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if $\vec{C} \cdot \vec{D}$ equals (a) zero, (b) 12 units, and (c) -12 units?

Sample Problem 3.07 Cross product, unit-

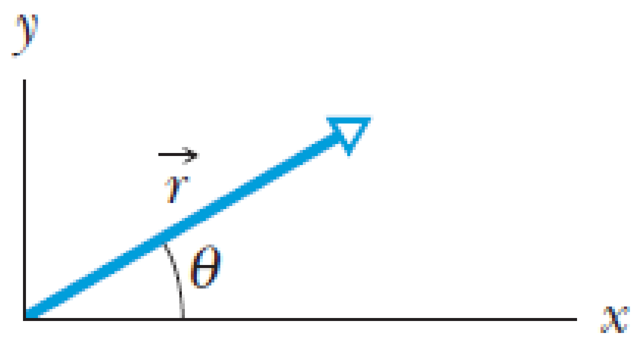
If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

Checkpoint 5

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if the magnitude of the vector product $\vec{C} \times \vec{D}$ is (a) zero and (b) 12 units?

Problems

•2 A displacement vector $\vec{r}$ in the xy plane is 15 m long and directed at angle $\theta = 30°$ in Fig. 3-26. Determine (a) the x component and (b) the y component of the vector.



HW •3 SSM The x component of vector $\vec{A}$ is -25.0 m and the y component is $+40.0$ m. (a) What is the magnitude of $\vec{A}$? (b) What is the angle between the direction of $\vec{A}$ and the positive direction of x?

Problems

HW **•9** Two vectors are given by

$$\vec{a} = (4.0\text{ m})\hat{\mathrm{i}} - (3.0\text{ m})\hat{\mathrm{j}} + (1.0\text{ m})\hat{\mathrm{k}}$$

and $$\vec{b} = (-1.0\text{ m})\hat{\mathrm{i}} + (1.0\text{ m})\hat{\mathrm{j}} + (4.0\text{ m})\hat{\mathrm{k}}.$$

In unit-vector notation, find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$.

HW **•11** SSM (a) In unit-vector notation, what is the sum $\vec{a} + \vec{b}$ if $\vec{a} = (4.0\text{ m})\hat{\mathrm{i}} + (3.0\text{ m})\hat{\mathrm{j}}$ and $\vec{b} = (-13.0\text{ m})\hat{\mathrm{i}} + (7.0\text{ m})\hat{\mathrm{j}}$? What are the (b) magnitude and (c) direction of $\vec{a} + \vec{b}$?

Problems

HW **•12** A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Sketch the vector diagram and determine (a) the magnitude and (b) the angle of the car's total displacement from its starting point.

HW **•15** SSM ILW WWW The two vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-28 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30°$ and $\theta_2 = 105°$. Find the (a) x and (b) y components of their vector sum $\vec{r}$, (c) the magnitude of $\vec{r}$, and (d) the angle $\vec{r}$ makes with the positive direction of the x axis.

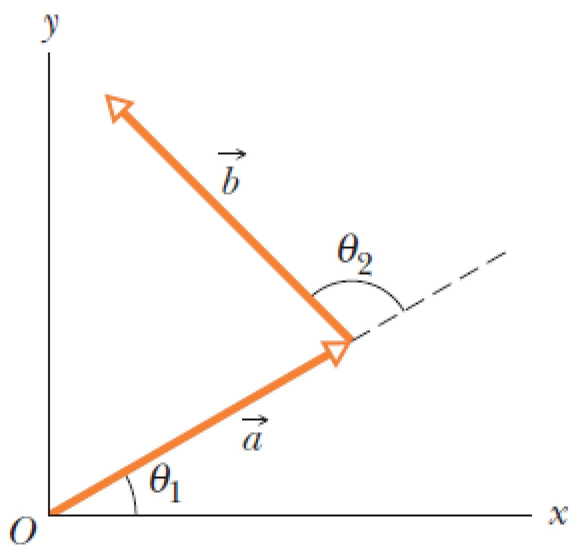


Figure 3-28 Problem 15.

Problems

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HW

•17 GO ILW Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ each have a magnitude of 50 m and lie in an xy plane. Their directions relative to the positive direction of the x axis are 30°, 195°, and 315°, respectively. What are (a) the magnitude and (b) the angle of the vector $\vec{a} + \vec{b} + \vec{c}$, and (c) the magnitude and (d) the angle of $\vec{a} - \vec{b} + \vec{c}$? What are the (e) magnitude and (f) angle of a fourth vector $\vec{d}$ such that $(\vec{a} + \vec{b}) - (\vec{c} + \vec{d}) = 0$?

Problems

HW **••23** If $\vec{B}$ is added to $\vec{C} = 3.0\hat{i} + 4.0\hat{j}$, the result is a vector in the positive direction of the y axis, with a magnitude equal to that of $\vec{C}$. What is the magnitude of $\vec{B}$?

HW **••30** GO Here are two vectors:

$$\vec{a} = (4.0\text{ m})\hat{i} - (3.0\text{ m})\hat{j} \quad \text{and} \quad \vec{b} = (6.0\text{ m})\hat{i} + (8.0\text{ m})\hat{j}.$$

What are (a) the magnitude and (b) the angle (relative to $\hat{i}$) of $\vec{a}$? What are (c) the magnitude and (d) the angle of $\vec{b}$? What are (e) the magnitude and (f) the angle of $\vec{a} + \vec{b}$; (g) the magnitude and (h) the angle of $\vec{b} - \vec{a}$; and (i) the magnitude and (j) the angle of $\vec{a} - \vec{b}$? (k) What is the angle between the directions of $\vec{b} - \vec{a}$ and $\vec{a} - \vec{b}$?

Problems

HW **•34** Two vectors are presented as $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of $\vec{a}$ along the direction of $\vec{b}$. (*Hint:* For (d), consider Eq. 3-20 and Fig. 3-18.)

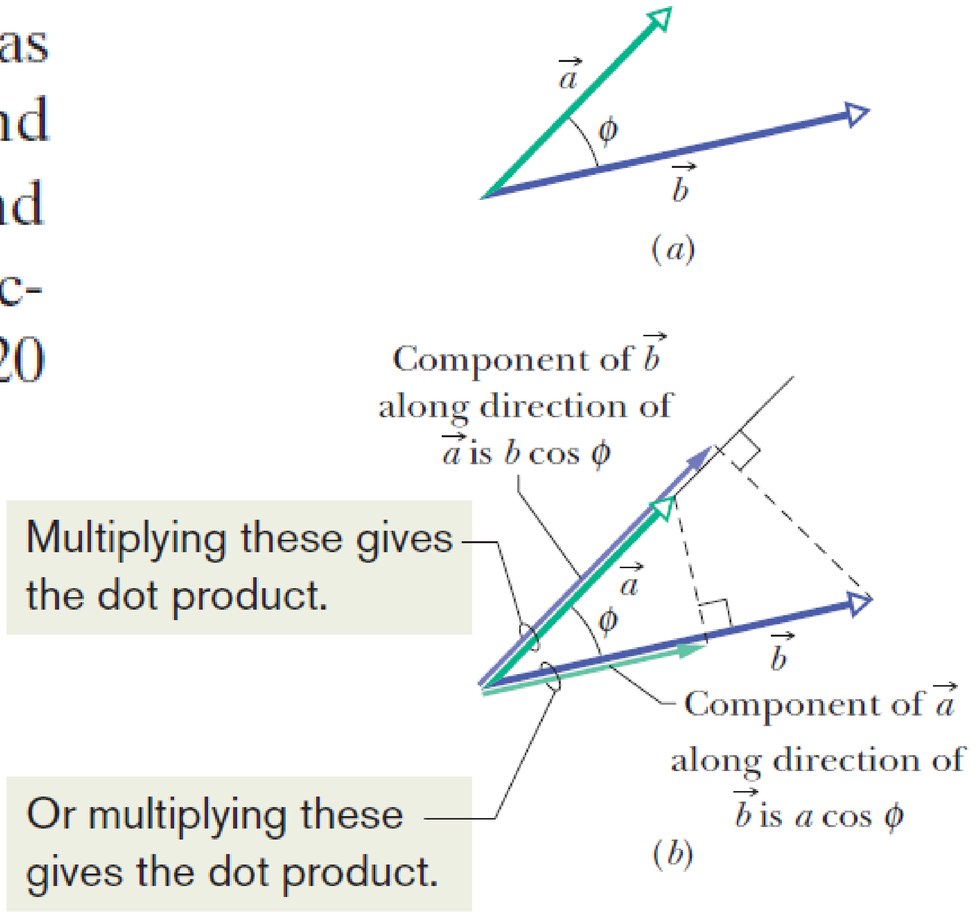

Figure 3-18 (*a*) Two vectors $\vec{a}$ and $\vec{b}$, with an angle ϕ between them. (*b*) Each vector has a component along the direction of the other vector.

Problems

HW **•35** Two vectors, $\vec{r}$ and $\vec{s}$, lie in the xy plane. Their magnitudes are 4.50 and 7.30 units, respectively, and their directions are 320° and 85.0°, respectively, as measured counterclockwise from the positive x axis. What are the values of (a) $\vec{r} \cdot \vec{s}$ and (b) $\vec{r} \times \vec{s}$?

HW **•36** If $\vec{d}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{d}_2 = -5\hat{i} + 2\hat{j} - \hat{k}$, then what is $(\vec{d}_1 + \vec{d}_2) \cdot (\vec{d}_1 \times 4\vec{d}_2)$?

•37 Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$, $\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a} \cdot (\vec{b} \times \vec{c})$, (b) $\vec{a} \cdot (\vec{b} + \vec{c})$, and (c) $\vec{a} \times (\vec{b} + \vec{c})$.

Problems

HW

••43 SSM ILW The three vectors in Fig. 3-33 have magnitudes $a = 3.00$ m, $b = 4.00$ m, and $c = 10.0$ m and angle $\theta = 30.0°$. What are (a) the x component and (b) the y component of $\vec{a}$; (c) the x component and (d) the y component of $\vec{b}$; and (e) the x component and (f) the y component of $\vec{c}$? If $\vec{c} = p\vec{a} + q\vec{b}$, what are the values of (g) p and (h) q?

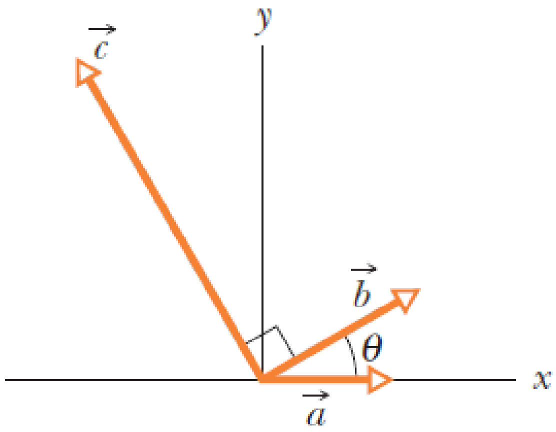

Figure 3-33 Problem 43.

Problems

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50 Vector $\vec{d}_1$ is in the negative direction of a y axis, and vector $\vec{d}_2$ is in the positive direction of an x axis. What are the directions of (a) $\vec{d}_2/4$ and (b) $\vec{d}_1/(-4)$? What are the magnitudes of products (c) $\vec{d}_1 \cdot \vec{d}_2$ and (d) $\vec{d}_1 \cdot (\vec{d}_2/4)$? What is the direction of the vector resulting from (e) $\vec{d}_1 \times \vec{d}_2$ and (f) $\vec{d}_2 \times \vec{d}_1$? What is the magnitude of the vector product in (g) part (e) and (h) part (f)? What are the (i) magnitude and (j) direction of $\vec{d}_1 \times (\vec{d}_2/4)$?

Problems

HW **52** Here are three displacements, each measured in meters: $\vec{d}_1 = 4.0\hat{i} + 5.0\hat{j} - 6.0\hat{k}$, $\vec{d}_2 = -1.0\hat{i} + 2.0\hat{j} + 3.0\hat{k}$, and $\vec{d}_3 = 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$. (a) What is $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3$? (b) What is the angle between $\vec{r}$ and the positive z axis? (c) What is the component of $\vec{d}_1$ along the direction of $\vec{d}_2$? (d) What is the component of $\vec{d}_1$ that is perpendicular to the direction of $\vec{d}_2$ and in the plane of $\vec{d}_1$ and $\vec{d}_2$? (*Hint:* For (c), consider Eq. 3-20 and Fig. 3-18; for (d), consider Eq. 3-24.)

Problems

HW **60** If $\vec{a} - \vec{b} = 2\vec{c}$, $\vec{a} + \vec{b} = 4\vec{c}$, and $\vec{c} = 3\hat{i} + 4\hat{j}$, then what are (a) $\vec{a}$ and (b) $\vec{b}$?

HW **61** (a) In unit-vector notation, what is $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ if $\vec{a} = 5.0\hat{i} + 4.0\hat{j} - 6.0\hat{k}$, $\vec{b} = -2.0\hat{i} + 2.0\hat{j} + 3.0\hat{k}$, and $\vec{c} = 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$? (b) Calculate the angle between $\vec{r}$ and the positive z axis. (c) What is the component of $\vec{a}$ along the direction of $\vec{b}$? (d) What is the component of $\vec{a}$ perpendicular to the direction of $\vec{b}$ but in the plane of $\vec{a}$ and $\vec{b}$? (*Hint:* For (c), see Eq. 3-20 and Fig. 3-18; for (d), see Eq. 3-24.)

Problems

HW **63** Here are three vectors in meters:

$$\vec{d}_1 = -3.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$$

$$\vec{d}_2 = -2.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$$

$$\vec{d}_3 = 2.0\hat{i} + 3.0\hat{j} + 1.0\hat{k}.$$

What results from (a) $\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3)$, (b) $\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3)$, and (c) $\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3)$?

HW **67** Let $\hat{i}$ be directed to the east, $\hat{j}$ be directed to the north, and $\hat{k}$ be directed upward. What are the values of products (a) $\hat{i} \cdot \hat{k}$, (b) $(-\hat{k}) \cdot (-\hat{j})$, and (c) $\hat{j} \cdot (-\hat{j})$? What are the directions (such as east or down) of products (d) $\hat{k} \times \hat{j}$, (e) $(-\hat{i}) \times (-\hat{j})$, and (f) $(-\hat{k}) \times (-\hat{j})$?

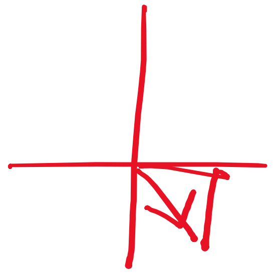
Problems

HW **73** Two vectors are given by $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of $\vec{a}$ along the direction of $\vec{b}$.

$\vec{a} = (2\hat{i} + 3\hat{j})\,m$ $\qquad$ $\vec{b} = (-4\hat{i} + 5\hat{j})\,m$

1 (a) $\vec{a} - \vec{b} - \vec{r} = 0$

$\Rightarrow \vec{r} = \vec{a} - \bar{b} = 6\hat{i} - 2\hat{j})\,m$



$|\vec{r}| = \sqrt{6^2 + (-2)^2} = \sqrt{40}\ m$

$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{-2^m}{6^m}\right) = -\tan\left(\frac{1}{3}\right)$

$= -$

(b) projection of $\vec{a}$ along $\vec{b}$

$$= a \cos\theta$$

$$= \cancel{a} \frac{\vec{a} \cdot \vec{b}}{\cancel{a}\, b} = \frac{\vec{a} \cdot \vec{b}}{b}$$

$$= \frac{7\, m^2}{\sqrt{41}\, m} = \frac{7}{\sqrt{41}}\, m.$$

$$\underline{\vec{a} \cdot \vec{b}} = (2 \times -4 + 3 \times 5)$$

$$= 7\, m^2$$

$$b = \sqrt{(-4)^2 + 5^2}$$

$$= \sqrt{41}\, m$$

(c)

$$\vec{c} = \vec{r} + 4\hat{k}\,m$$

$$= (6\hat{i} - 2\hat{j})m + 4\hat{k}\,m$$

$$= (6\hat{i} - 2\hat{j} + 4\hat{k})\,m$$

Assume: $\vec{d} = -\hat{j}$

Angle between $\vec{c}$ and $\vec{d}$ is θ.

$$\therefore \theta = \cos^{-1}\left(\frac{\vec{c} \cdot \vec{d}}{c\;d}\right) = \cos^{-1}\left(\frac{2}{\sqrt{6^2+2^2+4^2}\;\sqrt{1}}\right)$$

$$=$$

(a) $\vec{b} \cdot (\vec{c} \times \vec{a})$

$$\vec{c} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -2 & 4 \\ 2 & 3 & 0 \end{vmatrix} = (-12\hat{i} + 8\hat{j} + 22\hat{k})\, m^2$$

$$\vec{b} \cdot (\vec{c} \times \vec{a}) = (-4\hat{i} + 5\hat{j})\, m \cdot (-12\hat{i} + 8\hat{j} + 22\hat{k})\, m^2$$

$$= (48 + 40)\, m^3$$

End of Chapter 3 (Vector)