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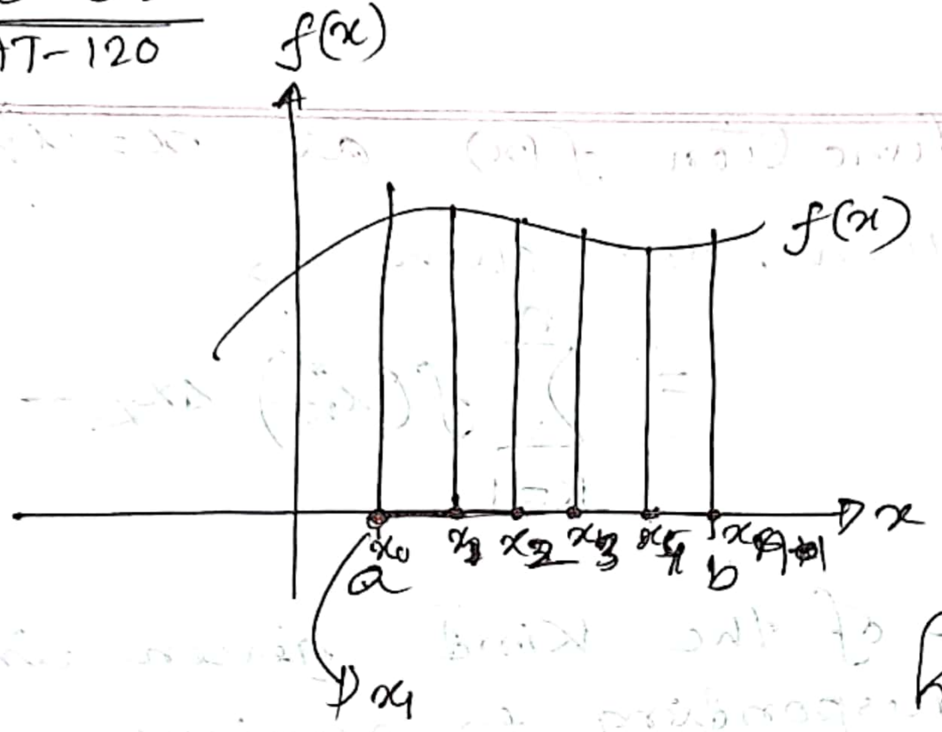

$$h = \frac{b-a}{n}$$

let $y=f(x)$ be a function on the closed interval $[a,b]$

1. Divide the interval $[a,b]$ into n- subintervals of equal length $\Delta x_K = x_K - x_{K-1}$, where
   $a = x_0 < x_1 < x_2 < \dots < x_n = b$ ⟶ (1)

2. choose a number x_K^* in each subintervals $[x_{K-1}, x_K]$, $K \in \overline{1,n}$, then the n numbers $x_1^*, x_2^*, \dots, x_n^*$ are called sample points in the intervals.

3. We calculate the value of the

function $f(x)$ at $x = x_k^*$

then, the sum is

$$= \sum_{k=1}^{n} f(x_k^*) \Delta x_k \rightarrow (2)$$

Sum of the kind given in (2) corresponding to various portions of $[a, b]$ are known as Riemann sum.

Since the sample points can be chosen arbitrarily, some common choices are given by

$$x_k^* = a + (k-1)\Delta x \rightarrow \text{left endpoint}$$

$$x_k^* = a + k\Delta x \rightarrow \text{Right endpoint}$$

$$x_k^* = a + \left(k - \frac{1}{2}\right)\Delta x \rightarrow \text{Mid point}$$

$\boxed{\text{Example-1}}$ Find the area ~~between the~~ enclosed by the function $f(x) = x^2 + 1$, on interval $[0, 2]$ and x axis using

i) left endpoint

ii) Right endpoint

iii) Midpoint

(when Number of subintervals is not mentioned)

Ans

given Interval $= 2-0 = 2$

Divide this interval into n times then length of each subinterval

$$\Delta x = \frac{2}{n} = \frac{2}{n}$$

i) left endpoint

$$x_k^* = a + (k-1)\Delta x$$
$$= 0 + (k-1)\frac{2}{n}$$
$$= \frac{2(k-1)}{n}$$

then $\sum_{k=1}^{n} f(x_k^*)\Delta x$

$$= \sum_{k=1}^{n} f\left(\frac{2(k-1)}{n}\right)\cdot\frac{2}{n}$$

Formulas

1. $\sum_{k=1}^{n} k = 1+2+3+\dots+n = \frac{n(n+1)}{2}$

2. $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

3. $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+\dots+n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

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$$= \sum_{k=1}^{n} \left[\left\{ \frac{2(k-1)}{n} \right\}^2 + 1 \right] \frac{2}{n}$$

$$= \sum_{k=1}^{n} \frac{2}{n} \left[\frac{2^2}{n^2} (k^2 - 2k + 1) + 1 \right]$$

$$\cancel{= \sum_{k=1}^{n} \frac{2}{n} \times \left[\frac{2^2 k^2}{n^2} - \frac{2^3 k}{n^2} \right.}$$

$$= \frac{2^3}{n^3} \sum_{k=1}^{n} k^2 - \frac{2^4}{n^3} \sum_{k=1}^{n} k + \frac{2^3}{n^3} \sum_{k=1}^{n} 1 + \frac{2}{n}$$

$$= \frac{2^3}{n^3} \times \frac{n(n+1)(2n+1)}{6} - \frac{2^4}{n^3} \cdot \frac{n(n+1)}{2} + \frac{2^3}{n^3} n + \frac{2}{n} \cdot n$$

$$= \frac{2^3}{6} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) - \frac{2^3}{n} \left(1 + \frac{1}{n}\right) + \frac{2^3}{n^2} + 2$$

$\therefore$ Area, $A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x$

$$= \lim_{n \to \infty} \left[\frac{2^3}{6} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) - \frac{2^3}{n} \left(1 + \frac{1}{n}\right) + \frac{2^3}{n^3} + 2 \right]$$

$$= \cancel{\frac{2^3}{6}(1+0)(2+0) - \frac{2^3}{\infty}(1+0)}.$$

$$= \frac{2^3}{6}\left(1+\frac{1}{\infty}\right)\left(2+\frac{1}{\infty}\right) - \frac{2^3}{\infty}\left(1+\frac{1}{\infty}\right) + \frac{2^3}{\infty} + 2$$

$$= \frac{2^3}{6}(1+0)(2+0) - 0 + 0 - 2$$

$$= \frac{2^3}{6} \times 2 + 2$$

$$= \frac{2^3}{3} + 2 = \frac{2^3+6}{3} = \frac{14}{3}$$ Ans.

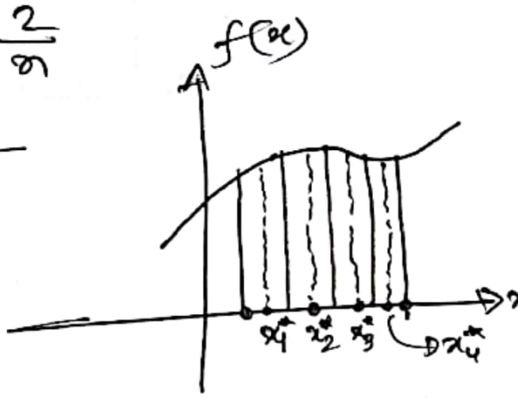
(iii) Mid point

$$x_k^* = a + \left(k - \frac{1}{2}\right)\Delta x$$

$$= 0 + \left(k - \frac{1}{2}\right)\frac{2}{n}$$

$$= \frac{(2k-1)2}{2n}$$

$$= \frac{2k-1}{n}$$



$$\therefore \sum_{k=1}^{n} f(x_k^*)\,\Delta x$$

$$= \sum_{k=1}^{n} \left\{ \left(\frac{2k-1}{n}\right)^2 + 1 \right\} \frac{2}{n}$$

⑥

$$= \frac{2}{n}\sum_{k=1}^{n}\left(\frac{4k^2-4k+1}{n^2}+1\right)$$

$$= \frac{2}{n}\times\frac{4}{n^2}\sum_{k=1}^{n}k^2 - \frac{8}{n^2}\sum_{k=1}^{n}k + \frac{2}{n^3}\sum_{k=1}^{n}1 + \frac{2}{n}\sum_{k=1}^{n}$$

$$= \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6} - \frac{8}{n^3}\,\frac{n(n+1)}{2} + \frac{2}{n^3}n + \frac{2}{n}n$$

$$= \frac{8}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - \frac{8}{2n}\left(1+\frac{1}{n}\right) + \frac{2}{n^2} + 2$$

$$\therefore \text{Area, } A = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k^*)\,\Delta x$$

$$= \lim_{n\to\infty}\left[\frac{8}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - \frac{8\left(1+\frac{1}{n}\right)}{2n} + \frac{2}{n^2} + 2\right]$$

$$= \frac{8}{6}\left(1+\frac{1}{\infty}\right)\left(2+\frac{1}{\infty}\right) - \frac{4\left(1+\frac{1}{\infty}\right)}{2\cdot\infty} + \frac{2}{\infty^2}$$

$$= \frac{8}{3} + 2$$

$$= \frac{8+6}{3} = \frac{14}{3}$$ Ans.

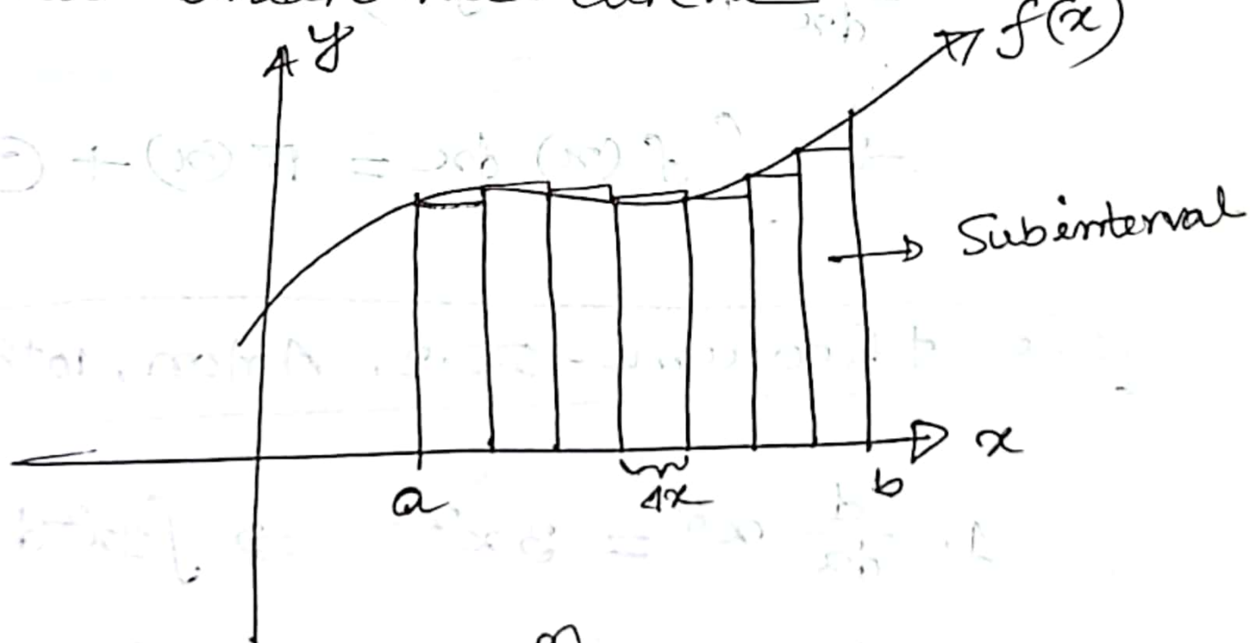
HW-1
Section, 5.4, H.A Book 10th edition
Q: 27–30
35–37

(ii) Done in class.

Example-2

calculate the Riemann sum of the function $f(x) = 3x^2+2x+3$ over the interval $[2,5]$ with 10 subintervals.
(i) Left end point (ii) Right end point
(iii) Midpoint.

(*) Area under the curve



$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

the expression $\int_a^b f(x)\,dx$ generally calculates the area under the curve $y=f(x)$ over the interval $[a,b]$. Here we integrate the function $f(x)$ with respect to x.

* **Indefinite Integral**: If the interval of ~~the~~ integration is not defined then we get an arbitrary constant after completing the integration. This type of integral is called indefinite ~~integration~~ integral.

$$\int f(x)\,dx = F(x) + C$$

The process of finding antiderivative is called ~~differen~~ antidefferential or Integ

$$\frac{d}{dx} F(x) = f(x)$$

$$\Rightarrow \int f(x)\,dx = F(x) + C$$

See theorem - 5.2.2, Anton, 10th edition

1. $\frac{d}{dx} x^3 = 3x^2 \Rightarrow \int 3x^2 dx = x^3 + C$

2. $\frac{d}{dx}(x^3+2) = 3x^2$ ~~$\Rightarrow \int (x^3+2)\,dx = x$~~
$\Rightarrow \int 3x^2 dx = x^3 + C$

* **Definite Integral:** If the interval of integration is defined then the arbitrary constant gets vanished. This type of integral is called definite integral.

$$\int_a^b f(x)\,dx$$

$a \rightarrow$ lower limit

$b \rightarrow$ upper limit.

* Integration formulas

$\int dx = x + c$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$, $n \neq -1$

$\int \cos x\,dx = \sin x + c$

$\int \sin x\,dx = -\cos x + c$

$\int \sec^2 x\,dx = \tan x + c$

$\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

$\int \sec x \tan x\,dx = \sec x + c$

$\int e^x dx = e^x + c$

$\int b^x dx = \frac{b^x}{\ln b} + c$

$\int \frac{1}{x}\,dx = \ln|x| + c$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$

$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}|x| + c$

$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \operatorname{cosec} x \, dx = \ln\left|\tan \frac{x}{2}\right| + C$$

$$\int \frac{dx}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \; ; \; \boxed{x = a\tan\theta}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C \; ; \; \cancel{x = a\sec\theta} \; ; \; a > x$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C \; ; \; \boxed{x = a\sin\theta}$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + C \; ; \; \boxed{x = a\sec\theta}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2+a^2}\right| + C$$

$$\int \sqrt{a^2-x^2} \, dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

HW-2

Q- 15-22, 25, 27-31

Section - 5.2, Howard Anton (10th edition)

* Integration by Substitution:

suppose that F is an antiderivative of f and that g is a differentiable function. the chain rule implies that the derivative of $F(g(x))$ can be expressed as

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x)) \cdot g'(x)$$

$$\Rightarrow \int F'(g(x))\, g'(x)\, dx = \int d\left[F(g(x))\right]$$

$$\Rightarrow \int F'(g(x))\, g'(x)\, dx = F(g(x)) + C$$

$$\Rightarrow \int f(g(x))\, g'(x)\, dx = F(g(x)) + C \longrightarrow (2)$$

where $f = F'$

let $g(x) = u \Rightarrow \frac{du}{dx} = g'(x) \Rightarrow g'(x)\,dx = du$

$$\int f(u)\, du = F(u) + C \longrightarrow (3)$$

the process of evaluating an integral of form (2) by converting it into form (3) with the substitution

$u = g(x)$ and $du = g'(x)\,dx$

is called the method of u-substitu

(35) $\int \frac{e^x}{1+e^{2x}}\,dx$

$= \int \frac{e^x}{1+(e^x)^2}\,dx$

let $e^x = t$

$\frac{d\,e^x}{dx} = \frac{dt}{dx}$

$\Rightarrow e^x = \frac{dt}{dx}$

$\therefore e^x dx = dt$

$= \int \frac{dt}{1+t^2}$

$= \tan^{-1} t + c$

$= \tan^{-1}(e^x) + c$ Ans.

(42) $\int \frac{\cos 4\theta\, d\theta}{(1+2\sin 4\theta)^4}$

let $1+2\sin 4\theta = t$

$\Rightarrow 2\cos 4\theta \cdot 4\,d\theta = dt$

$\Rightarrow \cos 4\theta\, d\theta = \frac{dt}{8}$

$= \int \frac{\frac{dt}{8}}{t^4}$

$= \frac{1}{8}\int \frac{1}{t^4}\,dt$

$$= \frac{1}{8}\int t^{-4}\,dt$$

$$= \frac{1}{8}\,\frac{t^{-4+1}}{-4+1} + c$$

$$= \frac{1}{8}\times\frac{1}{-3}\,t^{-3} + c$$

$$= -\frac{1}{24t^3} + c \quad \text{Ans}$$

(44) $\int \tan^3 5x \,\sec^2 5x \, dx$

$$= \int t^3 \frac{dt}{5}$$

$$= \frac{1}{5}\int t^3\,dt$$

$$= \frac{1}{5}\,\frac{t^4}{4} + c$$

$$= \frac{1}{20} t^4 + c \quad \text{Ans.}$$

let $\tan 5x = t$

$\Rightarrow \frac{d}{dx}\tan 5x = \frac{dt}{dx}$

$\Rightarrow \sec^2 5x\cdot(5) = \frac{dt}{dx}$

$\therefore \sec^2 5x\,dx = \frac{dt}{5}$

(45) $\int \frac{\tan x}{\ln(\cos x)}\,dx$

$$= \int \frac{-dt}{t} = -\ln|t| + c$$

let $\ln(\cos x) = t$

$\frac{1}{\cos x}\cdot(-\sin x)\,dx = dt$

$\Rightarrow -\tan x\,dx = dt$

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* $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx$
* $\int \frac{dx}{x(1+\ln x)}$
* $\int \frac{dx}{x^2-x+1}$
* $\int \frac{x^2 dx}{\sqrt{1-x^6}}$
* $\int \frac{\cos x \, dx}{3-\cos^2 x}$
* $\int \frac{1+\tan\frac{x}{2}}{1+\sin x} dx$

HW-3

5.3 → 15–56

Howard Anton (10th edition)

* Evaluating Definite Integrals by substitution: (5.9) H.A.

HW-4

5.9 → 5–22, 32, 33, 35–40

H. Anton (10th)

40/5.9 $\int_{\pi^2}^{4\pi^2} \frac{1}{\sqrt{x}} \sin\sqrt{x} \, dx$ using Method of substitution

Ans: Here, let $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{dx}{\sqrt{x}} = 2$

	Old (x)	New (t)
L.L	π^2	π
UL	$4\pi^2$	2π

$$= \int_{\pi}^{2\pi} \sin t \cdot 2\,dt$$

$$= 2\int_{\pi}^{2\pi} \sin t\,dt$$

$$= 2\left[-\cos t\right]_{\pi}^{2\pi}$$

$$= 2\left[-\cos 2\pi + \cos\pi\right]$$

$$= 2\left(-1 + (-1)\right)$$

$$= 2 \times 2 = 4 \cdot \text{Ans.}$$