GMAT Algebra: Challenge

Jeff Sackmann / GMAT HACKS

February 2008

Contents

1 Introduction

Ready for a challenge? This document contains nothing but difficult GMAT Algebra questions–100 of them, to be exact. They run the gamut from quadratics and inequalities to symbolism and sequences with several other topics thrown in for good measure.

As in all of my GMAT preparation resources, you'll find these questions indexed by difficulty. That doesn't mean you should skip straight to the hardest questions, or even that you should start with the easier ones. On the GMAT itself, questions won't come labeled with their difficulty level, and despite the intent of the adaptive algorithm, they won't be precisely consistent in terms of difficulty either. Each question presents its own unique challenges, and the sooner you get accustomed to changing gears with every single question, the more time you'll have to prepare for that particular challenge of the exam.

For further, more specific practice, I have produced several other resources that may help you. Another one of my 100-question practice sets, "Exponents and Roots," focuses entirely on those two categories. "Word Problems: Challenge" covers much of the same content, but contains only Word Problems, so you have to take an extra step or two before you even have a chance to do the algebra.

Also, The GMAT Math Bible has several chapters (along with focused practice) on Algebra and related issues, including individual chapters on fractions, decimals, simplifying expressions, linear equations, systems of equations, quadratic equations, inequalities, absolute value, exponents, roots, and more.

If you find yourself having problems with only the most difficult questions, you might try my "Extreme Challenge" set, which contains only 720 and higher level questions, many of which are Algebra-related.

You'll find articles at GMAT HACKS to help you with your strategic approach to Algebra questions. Most importantly, you should make sure you understand every practice problem you do. It doesn't matter if you get it right the first time–what matters is whether you'll get it right the next time you see it, because the next time you see it could be on the GMAT.

With that in mind, carefully analyze the explanations. Redo questions that took you too long the first time around. Review questions over multiple sessions, rather than cramming for eight hours straight each Saturday. These basic study skills may not feel like the key to GMAT preparation, but they are the difference between those people who reach their score goals and those who never do.

Enough talking; there are 100 Algebra questions waiting inside. Get to work!

2 Difficulty Levels

In general, the level 5 questions in this guide are 560- to 620-level questions. The level 6 questions representing a broad range of difficulty from about 620 to 720, while the level 7 questions are higher still.

Moderately Difficult (5)
PS
004, 007, 009, 013, 014, 015, 016, 017, 019, 035, 049, 050, 059
DS
064, 065, 066, 071, 072, 073, 075, 076, 077, 078, 080, 081, 082, 083, 085, 086, 087, 095, 096, 098, 099, 100

Difficult (6)
PS
002, 005, 006, 010, 011, 012, 018, 021, 024, 025, 026, 027, 028, 029, 030, 032, 033, 034, 036, 037, 038, 044, 045, 046, 047, 048, 051, 052, 056, 060
DS
061, 062, 063, 068, 069, 070, 074, 079, 084, 088, 090, 091, 092, 093, 094, 097

Very Difficult (7)
PS
001, 003, 008, 020, 022, 023, 031, 039, 040, 041, 042, 043, 053, 054, 055, 057, 058
DS
067, 089

3 Problem Solving

Note: this guide contains both an answer key (so you can quickly check your answers) and full explanations.

1. Which of the following is equal to x^k for all positive values of x and k ?
   (A) $(x^{\frac{k}{4}})^{\frac{3}{4}}$
   (B) $\frac{x^k}{x^{2k}}$
   (C) $x^{\frac{k}{2}} + x^{\frac{k}{2}}$
   (D) $(x^{\frac{k}{2}})^2$
   (E) $(x^k)^k$

2. For all positive integers m and n, the operation @ is defined by $m@n = \frac{m\hat{}2}{n-1}$. If $z@9 = 2$, then $z =$
   (A) 4
   (B) 5
   (C) 16
   (D) 17
   (E) 33

3. Hayden began walking from F to G, a distance of 40 miles, at the same time Ava began walking from G to F on the same road. If Hayden's walking speed was x miles per hour and Ava's was y miles per hour, how many miles away from F were they, in terms of x and y, when they met?
   (A) $\frac{40(x-y)}{x+y}$
   (B) $\frac{40x-y}{x+y}$
   (C) $\frac{x-y}{x+y}$
   (D) $\frac{40y}{x+y}$
   (E) $\frac{40x}{x+y}$

4. Working at the same constant rate, p bricklayers can lay a total of q bricks per hour. At this rate, how many bricks could 4 bricklayers lay in 8 hours?

(A) $\frac{p}{2q}$
(B) $\frac{2p}{q}$
(C) $\frac{32p}{q}$
(D) $\frac{2q}{p}$
(E) $\frac{32q}{p}$

5. As j increases from 135 to 136, which of the following must decrease?

I. $1 - j^2$
II. $j - j^2$
III. $\frac{1}{j^2}$

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) I, II and III

6. At the rate of y yards per m minutes, how many yards does a swimmer travel in x hours?

(A) $\frac{y}{mx}$
(B) $\frac{yx}{m}$
(C) $\frac{60y}{mx}$
(D) $\frac{60ym}{x}$
(E) $\frac{60xy}{m}$

7. In the formula $F = \frac{1}{(3k)^3}$, if k is halved, then F is multiplied by

(A) 27
(B) 8
(C) 1
(D) $\frac{1}{8}$
(E) $\frac{1}{27}$

v, w, x, y, z

8. An arithmetic sequence is a sequence in which each term after the first is equal to the sum of the preceding term and a constant. If the list of numbers shown above is an arithmetic sequence, which of the following must also be an arithmetic sequence?

I. $2v$, $2w$, $2x$, $2y$, $2z$
II. $v+2$, $w+2$, $x+2$, $y+2$, $z+2$
III. $\sqrt{v}$, $\sqrt{w}$, $\sqrt{x}$, $\sqrt{y}$, $\sqrt{z}$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

I. $a-14$, a, a, a, $a+14$
II. b, $b+1$, $b+2$, $b+3$, $b+4$
III. c, c, c, c, c

9. The data sets I, II, and III above are ordered from greatest standard deviation to least standard deviation in which of the following?

(A) I, II, III
(B) I, III, II
(C) II, III, I
(D) III, I, II
(E) III, II, I

10. Which of the following CANNOT be the median of the four consecutive positive integers w, x, y, and z, where $w < x < y < z$?

(A) $\frac{w+x}{2} - 1$
(B) $\frac{w+z}{2}$
(C) $\frac{x+y}{2}$
(D) $\frac{y+z}{2} - 1$
(E) $\frac{w+x+y+z}{4}$

11. If $\sqrt{3-4x} = 4 - \sqrt{3x}$, then $49x^2 =$

(A) $10x + 144$
(B) $10x - 169$
(C) $64x - 144$
(D) $64x + 169$
(E) $118x - 192$

12. A retail store that sells only shoes and accessories found that its revenues last month from those two categories could be expressed in the ratio $x : y$, respectively. If last month's total revenues were \$4,000, what was the difference between revenues from shoes and revenues from accessories?

(A) $\frac{x}{x+y}$
(B) $\frac{y}{x+y}$
(C) $\$4,000(\frac{x}{y})$
(D) $\$4,000(\frac{x}{x+y})$
(E) $\$4,000(\frac{x-y}{x+y})$

13. If $u > y$, $x > v$, $y > w$, and $y > x$, which of the following must be true?

I. $u > w$
II. $w > v$
III. $y > v$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

14. If $3 + \frac{2}{p} = 2 - \frac{3}{q}$ and $q = \frac{3}{2}$, then $p =$

(A) $-\frac{3}{2}$
(B) $-\frac{2}{3}$
(C) 0
(D) $\frac{2}{3}$
(E) $\frac{3}{2}$

15. The average (arithmetic mean) of 6 numbers is q. When one number is discarded, the average of the remaining numbers becomes $q - 2$. In terms of q, what is the discarded number?

(A) $q + 5$
(B) $q + 8$
(C) $q + 10$
(D) $q + 12$
(E) $q + 15$

16. All of the following equations have a root in common with $x^2 - 2x - 3 = 0$ EXCEPT
    (A) $x^2 - x - 6 = 0$
    (B) $x^2 + 5x + 4 = 0$
    (C) $x^2 + 6x + 5 = 0$
    (D) $x^2 + 2x - 3 = 0$
    (E) $x^2 - 4x + 3 = 0$

17. $(\frac{1}{p})^2 - (\frac{1}{p})(\frac{1}{q}) =$
    (A) $\frac{q-p}{p^2q}$
    (B) $\frac{p-q}{p^2q}$
    (C) $\frac{q-p}{q^2p}$
    (D) $\frac{q-p}{p^2-pq}$
    (E) $\frac{1}{p^2-pq}$

18. If it is true that $m^2 < 9$ and $m > -1$, which of the following must be true?
    (A) $m > -3$
    (B) $m > 1$
    (C) $m > 3$
    (D) $m < 1$
    (E) None of the above

19. If $x^4 = 3y^3$ and $2y = 6$, what is the value of $x^2 - y$?
    (A) 6
    (B) 12
    (C) 30
    (D) 78
    (E) 84

20. If k is a positive integer, and if the units' digit of k^2 is 1 and the units' digit of $(k+1)^2$ is 0, what is the units' digit of $(k+2)^2$?
    (A) 1
    (B) 3
    (C) 5
    (D) 7
    (E) 9

21. If $(r-8)$ is a factor of $r^2 - kr - 56$, then $k =$

(A) -15
(B) -10
(C) -1
(D) 1
(E) 10

22. For any integer n greater than 1, $n*$ denotes the product of all the integers from 1 to n, inclusive. How many prime numbers are there between $7*+2$ and $7*+7$, inclusive?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

23. When M is divided by S, the quotient is V and the remainder is 1. Which of the following expressions is equal to M ?

(A) SV
(B) $S+1$
(C) $SV+1$
(D) $S(V+1)$
(E) $S(V-1)$

24. If $s\&t = 2(s-t) + st$ for all integers s and t, then $3\&(-2) =$

(A) -16
(B) -4
(C) 2
(D) 4
(E) 10

25. A certain bakery sells six different-sized wedding cakes. Each cake costs x dollars more than the next one below it in size, and the price of the largest cake is \$24.50. If the sum of the prices of the six different cakes is \$109.50, what is the value of x?

(A) 1.50
(B) 1.75
(C) 2.00
(D) 2.50
(E) 3.00

26. If $(5^{\sqrt{2}})^m = 25$, what is the value of m ?

(A) -2
(B) $-\sqrt{2}$
(C) $\sqrt{2}$
(D) 2
(E) $2\sqrt{2}$

27. If x and y are positive numbers and $x^3 + y^3 < 100$, then the greatest possible value of x is between

(A) 0 and 2
(B) 2 and 4
(C) 4 and 6
(D) 6 and 8
(E) 8 and 10

28. Which of the following is equal to the average (arithmetic mean) of $(x+3)^2$ and $(x-3)^2$?

(A) x^2
(B) $x^2 + 3$
(C) $x^2 + 9$
(D) $x^2 + 3x$
(E) $x^2 + 9x$

29. Which of the following is equal to x^{14} for all positive values of x ?

(A) $\frac{x^4}{x^{18}}$
(B) $(x^2)^{12}$
(C) $(x^7)^7$
(D) $(x^2)^7$
(E) $x^7 + x^7$

30. If $a = -1$ and $\frac{a-b}{c} = 1$, which of the following is NOT a possible value of b ?

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

31. If the operation # is defined for all f and g by the equation $f\#g = \frac{f^2 g}{2}$, then $3\#(-1\#-2) =$
    (A) $-\frac{9}{2}$
    (B) -1
    (C) 4
    (D) $\frac{9}{2}$
    (E) $\frac{81}{4}$

32. If $(4^x)(2^y) = 16$ and $(3^x)(3^y) = 27$, then $(x, y) =$
    (A) $(1, 2)$
    (B) $(2, 1)$
    (C) $(1, 1)$
    (D) $(2, 2)$
    (E) $(1, 3)$

33. If $\frac{x}{y} = \frac{4}{5}$, then $\frac{x}{x-y} =$
    (A) -4
    (B) $-\frac{1}{4}$
    (C) $\frac{1}{5}$
    (D) $\frac{1}{4}$
    (E) 4

34. An athlete swims S miles in H hours, then rides a bicycle B miles in four times the same number of hours. Which of the following represents the athlete's average speed, in miles per hour, for these two activities combined?
    (A) $\frac{S+B}{5H}$
    (B) $\frac{S+B}{4H}$
    (C) $\frac{S+B}{H}$
    (D) $\frac{5(S+B)}{H}$
    (E) $\frac{4(S+B)}{H}$

35. If y is z more than a, what is the value of $\frac{a}{y-z}$?
    (A) -2
    (B) -1
    (C) 0
    (D) 1
    (E) 2

4, k, 11, 3, m, 4

36. The arithmetic mean of the list of numbers above is 5. If k and m are integers and $k \neq m$, what is the median of the list?

(A) 3
(B) 3.5
(C) 4
(D) 4.5
(E) 5

37. If $p > 0$ and y is p percent of z, then, in terms of p, z is what percent of y ?

(A) $\frac{10{,}000}{p}$
(B) $\frac{10}{p}$
(C) $\frac{1}{p}$
(D) $\frac{1}{100p}$
(E) $100p$

38. A rectangular poster has an area of p inches and the longer of the dimensions is q inches. In terms of p and q, what is the difference between the lengths of the sides of the poster?

(A) $p - \frac{q}{p}$
(B) $\frac{p}{q}$
(C) $\frac{q}{p}$
(D) $q - \frac{p}{q}$
(E) $\frac{q}{p} - \frac{p}{q}$

39. If $x > y$, increasing the original price of an item by $x\%$ and then decreasing the new price by $y\%$ is equivalent to multiplying the original price by

(A) $\frac{1}{100(x-y)}$
(B) $\frac{1}{100}(x - y - \frac{xy}{100})$
(C) $\frac{x-y-xy}{100}$
(D) $1 + \frac{1}{100}(x - y + \frac{xy}{100})$
(E) $1 - \frac{x-y-xy}{100}$

40. At 1:00 PM, Train X departed from Station A on the road to Station B. At 1:30 PM, Train Y departed Station B on the same road for Station A. If Station A and Station B are p miles apart, Train X's speed is r miles per hour, and Train Y's speed is s miles per hour, how many hours after 1:00 PM, in terms of p, r, and s, do the two trains pass each other?

(A) $\frac{1}{2} + \frac{p-\frac{s}{2}}{r+s}$

(B) $\frac{p-\frac{s}{2}}{r+s}$

(C) $\frac{1}{2} + \frac{p-\frac{r}{2}}{r}$

(D) $\frac{p-\frac{r}{2}}{r+s}$

(E) $\frac{1}{2} + \frac{p-\frac{r}{2}}{r+s}$

41. An electronics salesman earned a $x\%$ commission on each of the m digital music player he sold in the month of March at the retail price of $\$p$. In April, he earned a $y\%$ commission on sales of the same item, and the price remained the sale. If $y > x$ and he sold q more digital music players in April than in March, how much more money did he earn selling digital music players in April than in March?

(A) $\frac{p}{100}[ym - x(m+q)]$

(B) $\frac{p}{100}[x(m+q) + ym]$

(C) $p[y(m-q) - xm]$

(D) $\frac{p}{100}[x(m+q) - ym]$

(E) $\frac{p}{100}[y(m+q) - xm]$

42. If x is equal to the sum of the even integers from m to n, inclusive, where m and n are positive even integers, which of the following represents the value of x in terms of m and n?

(A) $(\frac{m+n}{2})(\frac{n-m}{2}+1)$

(B) $3(m+n)$

(C) $\frac{n^2-m^2}{2}$

(D) $6(m+n)$

(E) $(\frac{m+n}{2})(\frac{n-m}{2})$

43. If $a = x^3y$, $b = x^2$, and $c = xy$, which of the following represents the value of $\frac{b^2c^2x^2}{a^3}$ in terms of x and y ?
    (A) xy
    (B) x
    (C) $\frac{x\hat{}2}{y}$
    (D) $\frac{1}{x}$
    (E) $\frac{1}{xy}$

44. For all numbers x and y, the operation & is defined by $x\&y = \frac{x-y}{x-1}$. If $(-3)\&q = 2$, then $q =$
    (A) -11
    (B) -8
    (C) -5
    (D) 5
    (E) 11

45. Positive integer z is 30 percent of 30 percent of positive integer y, and z percent of y equals 9. What is the value of y ?
    (A) 27
    (B) 50
    (C) 81
    (D) 90
    (E) 100

46. If $\sqrt{2x-3} = \sqrt{x} + 2$, then $x^2 =$
    (A) 49
    (B) $-7x$
    (C) $14x - 7$
    (D) $14x + 49$
    (E) $30x - 49$

47. If $n = 9\sqrt{\frac{1}{x^4}}$, what is the value of $\sqrt{n}$?
    (A) $\frac{1}{3x}$
    (B) $\frac{3}{x}$
    (C) $\frac{3}{x^2}$
    (D) $\frac{9}{x}$
    (E) $\frac{9}{x^2}$

48. On the number line, if $r > s$, if s is halfway between r and t, and t is halfway between r and u, then which of the following is equal to 1?

(A) $\frac{r-t}{s-u}$
(B) $\frac{r-t}{u-t}$
(C) $\frac{s-t}{r-u}$
(D) $\frac{u-t}{r-s}$
(E) $\frac{u-t}{t-r}$

49. If a, b, and c are nonzero integers and c is equal to the difference of a and b, all of the following could be true EXCEPT

(A) $a = b + c$
(B) $a = b - c$
(C) $b = a + c$
(D) $b = a - c$
(E) $c = a + b$

50. If $r + s + t = 3u$, then which of the following represents the average (arithmetic mean) of r, s, t, and u, in terms of u?

(A) $\frac{u}{4}$
(B) $\frac{u}{2}$
(C) u
(D) $\frac{3u}{4}$
(E) $4u$

51. At the rate of f floors per m minutes, how many floors does an elevator travel in s seconds?

(A) $\frac{fs}{60m}$
(B) $\frac{ms}{60f}$
(C) $\frac{fm}{s}$
(D) $\frac{fs}{m}$
(E) $\frac{60s}{fm}$

52. If $(3^{\sqrt{2}})^p = 3$, what is the value of p ?

(A) $2\sqrt{2}$
(B) $\sqrt{2}$
(C) $\frac{\sqrt{2}}{2}$
(D) $\frac{1}{2}$
(E) $\frac{\sqrt{2}}{4}$

v, w, x, y, z

53. A geometric sequence is a sequence in which each term after the first is equal to the product of the preceding term and a constant. If the list of numbers shown above is an geometric sequence, which of the following must also be a geometric sequence?

I. $2v$, $2w$, $2x$, $2y$, $2z$
II. $v+2$, $w+2$, $x+2$, $y+2$, $z+2$
III. $\sqrt{v}$, $\sqrt{w}$, $\sqrt{x}$, $\sqrt{y}$, $\sqrt{z}$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

54. When w is divided by x, the quotient is y and the remainder is z. Which of the following expressions is equal to y ?

(A) $w - z$
(B) $x(w - z)$
(C) $x(z - w)$
(D) $\frac{w-z}{x}$
(E) $\frac{z-w}{x}$

55. If the average of m and $(x + y)^2 = x^2 + y^2$, what is the value of m ?

(A) $x - y$
(B) $x + y$
(C) $(x - y)(x + y)$
(D) $(x - y)^2$
(E) $(x + y)^2$

56. If $b = -2$ and $\frac{a}{b-c} = 1$, which of the following is NOT a possible value of c ?

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

57. If $q > 0$ and m is $100q$ percent of n, then, in terms of q, n is what percent of m ?

(A) $\frac{10{,}000}{q}$
(B) $\frac{100}{q}$
(C) $\frac{1}{q}$
(D) $\frac{q}{100}$
(E) q

58. If the operation ^^ is defined for all x and y by the equation $x\hat{}\hat{}y = \frac{x^2y}{2}$, then $(2\hat{}\hat{} - 1)\hat{}\hat{}(-2\hat{}\hat{}1) =$

(A) -4
(B) -2
(C) 2
(D) 4
(E) 8

59. Positive integer y is 50 percent of 50 percent of 50 percent of positive integer x, and y percent of x equals 50. What is the value of x ?

(A) 50
(B) 100
(C) 200
(D) 1,000
(E) 2,000

60. If x and y are positive integers and $x^4 + y^4 < 10,000$, then the greatest possible value of x is between

(A) 0 and 3
(B) 3 and 6
(C) 6 and 9
(D) 9 and 12
(E) 12 and 15

4 Data Sufficiency

For all Data Sufficiency questions, the answer choices are as follows:

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
(C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient.
(E) Statements (1) and (2) TOGETHER are NOT sufficient.

61. Is x between 0 and 1?
   (1) $-x < x^3$
   (2) $x < x^2$

62. In the equation $x^2 - bx + c = 0$, b and c are constants. What is the value of b ?
   (1) 1 is a root of the equation $x^2 - bx + c = 0$.
   (2) $x + 1$ is a factor of $x^2 - bx + c = 0$.

63. What is the value of xy ?
   (1) $y = \frac{12}{x}$
   (2) $\frac{y}{12}$ is equal to the reciprocal of x.

64. If x, y, and z are positive numbers, is $x < y < z$?
   (1) $xy < yz$
   (2) $xz < yz$

65. If $xy = 8$, what is the value of $xy(x - y)$?
   (1) $x^2y = -64$
   (2) $xy^2 = -8$

66. If m, n, p, q, and r are different positive integers, which of the five integers is the median?
   (1) $n + r < m$
   (2) $n + r + m < q < p$

67. A Farey sequence of order n is the sequence of fractions between 0 and 1 which, when in lowest terms, have denominators less than or equal to n, arranged in order of increasing size. For example, the Farey sequence of order 3 is: $\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\}$. Is sequence S a Farey sequence?
(1) Sequence S has fewer than 10 elements.
(2) The second element of sequence S is $\frac{1}{5}$.

68. Is the positive integer p the sum of the positive prime numbers m and n?
(1) $m - n = 8$
(2) $p = 50$

69. If x is a positive integer and w is a negative integer, what is the value of xw?
(1) $x^w = \frac{1}{4}$
(2) $w = -2$

70. The number N is $3,1G5$, the ten's digit being represented by G. What is the value of G?
(1) N is divisible by 5.
(2) N is divisible by 9.

71. If x and y are integers, what is the value of y ?
(1) $2x = y$, and $x^3 = y^2$.
(2) $y - x = 4$, and $x^2 = 2y$.

72. What is the value of $a - b$?
(1) $a = 2(b + 5) - a$
(2) $(a - b)^3 = 125$

73. Is $z > y$?
(1) $xz > xy$
(2) x is negative.

74. If $y \neq 0$, does $x = 0$?
(1) $x^2y = x^3y$
(2) $\frac{x^2}{y} = x^3y$

75. Is $x < 0$?
(1) $x^2 > x^4$
(2) $x^2 > 0$

76. If m and n are positive integers and $mn = 36$, what is the value of m ?
    (1) n is the square of a prime number.
    (2) $m - n = 5$

77. What is the value of $x^2 + y^2$?
    (1) $x^2 + y^2 = 2xy + 1$
    (2) $x^2 + y^2 = 4 - 2xy$

78. What is the ratio of $x : y : z$?
    (1) $\frac{x}{y} = \frac{3}{2}$ and $xy = 15$
    (2) $\frac{y}{z} = \frac{2}{7}$ and $yz = 35$

79. If $x \neq 0$, what is the value of $\left(\frac{x^p}{y^q}\right)^5$?
    (1) $x = y$
    (2) $p = q$

80. If $xy < 5$, is $x < 1$?
    (1) $|y| > 5$
    (2) $\frac{x}{y} > 0$

81. If $\frac{r}{s} = \frac{3}{2}$, what is the value of $r + s$?
    (1) $s^2 = 4$
    (2) $r^2 = 9$

82. If x, y, and z are numbers, is $z = 20$?
    (1) $xy = z$
    (2) x is prime and y is the square of a prime.

83. Is x^3 greater than x ?
    (1) $x + x^3 > 0$
    (2) $x - x^3 < 0$

84. If p and x are integers, is x divisible by 5 ?
    (1) $x - 2 = 2p$
    (2) $2p - 3$ is divisible by 5.

85. Is $\frac{x}{p}(p^2 + q^2 + r^2) = xp + yq + zr$?
    (1) $\frac{z}{r} = \frac{x}{p}$
    (2) $\frac{x}{p} = \frac{y}{q}$

86. If a, b, and c are positive, is $a = \frac{b^2}{c}$?
   (1) $\frac{a}{b} = \frac{b}{c}$
   (2) $b = \sqrt{ac}$

87. If k is a positive integer, is $\sqrt{k}$ an integer?
   (1) $1 < \sqrt{k} < 3$
   (2) $k^2 < 16$

88. If p is an integer, is q an integer?
   (1) The average (arithmetic mean) of p and q is not an integer.
   (2) The average (arithmetic mean) of p, q, and $q + 4$ is p.

89. Is $\frac{1}{x-y} < y - x$?
   (1) $\frac{1}{x} < \frac{1}{y}$
   (2) $2x = 3y$

90. If x and y are nonzero integers, is $x^y < y^x$?
   (1) $x < y$
   (2) x and y are consecutive integers.

91. If z is a positive integer, is $\sqrt{z}$ an integer?
   (1) $\sqrt{xz}$ is an integer.
   (2) $x = z^2$

92. If $xy \neq 0$, is $\frac{1}{x} + \frac{1}{y} = 2$?
   (1) $x = y$
   (2) $x + y = 2xy$

93. If r and s are negative, is $\frac{r}{s}$ less than 1 ?
   (1) $r + 2s = \frac{s^2}{r}$
   (2) $\frac{r}{s}$ is 2 less than $\frac{s}{r}$

94. The symbol ■ represents one of the following operations: addition, subtraction, multiplication, or division. What is the value of 5■2 ?
   (1) 0■1 = 0
   (2) 1■1 = 1

95. Is y a negative number?
   (1) $y + 2$ is positive.
   (2) $y^2 > 10$

96. What is the average (arithmetic mean) of m and n ?

(1) The average (arithmetic mean) of $2m$ and n is 15.
(2) The average (arithmetic mean) of $2m$ and $2n$ is 22.

97. Is $stv = 1$?

(1) $s = v$, and s is the positive square root of $\frac{1}{t}$.
(2) $s = t = 1$.

98. If $xy \neq 0$, is $\frac{x}{y} < 0$?

(1) $xy^2 = 18$
(2) $x^2y = -12$

99. Is the range of the integers 5, 9, p, 6, 2, q greater than 7 ?

(1) The mean of the six integers is 5.5.
(2) $q < 9 < p$

100. Is $3^{x+2} < 9$?

(1) $3^x < 1$
(2) $x < 0$

5 Answer Key

For full explanations, see the next section.

1. D
2. A
3. E
4. E
5. E
6. E
7. B
8. D
9. A
10. A
11. B
12. E
13. E
14. B
15. C
16. D
17. A
18. A
19. A
20. A
21. D
22. A
23. C
24. D
25. D
26. C
27. C
28. C
29. D
30. B
31. A
32. A
33. A
34. A
35. D
36. C
37. A
38. D
39. B
40. E
41. E

42.	A
43.	E
44.	D
45.	E
46.	E
47.	B
48.	E
49.	E
50.	C
51.	A
52.	C
53.	E
54.	D
55.	D
56.	A
57.	B
58.	D
59.	C
60.	D
61.	B
62.	C
63.	D
64.	E
65.	D
66.	C
67.	C
68.	C
69.	A
70.	E
71.	A
72.	D
73.	C
74.	E
75.	E
76.	B
77.	E
78.	C
79.	C
80.	C
81.	E
82.	E
83.	B
84.	C
85.	C
86.	D
87.	C

88. B
89. E
90. E
91. C
92. B
93. D
94. E
95. C
96. B
97. A
98. C
99. B
100. D

6 Explanations

For quicker reference, there is an answer key in the section preceding this one..

1. D

Explanation: This is a tricky test of exponent rules, a topic that comes up in many forms on the GMAT. To determine the correct answer, you may have to evaluate each one:

(A) When given a power raised to a power, multiply powers: $x^{(\frac{k}{4})(\frac{3}{4})} = x^{\frac{3k}{16}}$

(B) A power over a power means you should subtract: $x^{k-2k} = x^{-k}$

(C) The only way to simplify this is to turn it into a multiplicative expression: $2(x^{\frac{k}{2}})$

(D) Multiply powers: $x^{(\frac{k}{2})(2)} = x^k$

(E) Again, multiply powers: $x^{k(k)} = x^{k^2}$

The only choice that is equivalent to x^k is choice (D).

2. A

Explanation: Given the rule for the operator @, plug in the values for m, n, and the solution. $m = z$, $n = 9$, and the solution is 2:

$\frac{z^2}{9-1} = 2$

$\frac{z^2}{8} = 2$

$z^2 = 16$

$z = 4$, choice (A). While the solution of $z^2 = 16$ could also be $z = -4$, the question limits the domain to positive integers for m and n.

3. E

Explanation: With variables in the question stem and answer choices, pick numbers. If $x = 6$ and $y = 4$, their combined rate is 10 mph, which means they met in 4 hours. After 4 hours, Hayden had walked $4(6) = 24$ miles from his starting point in F. Check each answer choice to see which one works out to 24 when $x = 6$ and $y = 4$:

(A): $\frac{40(6-4)}{6+4} = \frac{80}{10} = 8$. No.

(B): $\frac{40(6)-4}{6+4} = \frac{236}{10} = 23.6$. No.

(C): $\frac{6-4}{6+4} = \frac{2}{10} = \frac{1}{5}$. No.

(D): $\frac{40(4)}{6+4} = \frac{160}{10} = 16$. No.

(E): $\frac{60(4)}{6+4} = \frac{240}{10} = 24$. That's it.

4. E

Explanation: With variables in the question stem and answer choices, pick numbers. Say $p = 2$ and $q = 10$. If 2 bricklayers lay 10 bricks per hour, each bricklayer lays 5 per hour. At that rate, 4 bricklayers can lay $5(4) = 20$ per hour, which in 8 hours is a total of $8(20) = 160$ bricks. Plug in $p = 2$ and $q = 10$ to the answer choices to find the correct choice:

(A): $\frac{2}{(2)(10)} = \frac{1}{10}$. No.
(B): $\frac{2(2)}{10} = \frac{4}{10}$. No.
(C): $\frac{32(2)}{10} = \frac{32}{5}$. No.
(D): $\frac{2(10)}{2} = 10$. No.
(E): $\frac{32(10)}{2} = 160$. That's it.

5. E

Explanation: Notice, first, that the key terms in the roman numerals are j and j^2. As j increases from 135 to 136, j^2 increases as well, and because the squares of those numbers are so much larger than the numbers themselves, the increase in j^2 is much larger than the increase in j. So, to look at each of the roman numerals in turn:

I. As j increases, j^2 increases, but as j^2 increases, $-j^2$ decreases. Thus, $1-j^2$ decreases, so the answer must be (A), (C), (D), or (E).

II. As j increases, j goes up, but $-j^2$ goes down. Since the difference between 135^2 and 136^2 is bigger than that between 135 and 136, the decrease is larger than the increase. Thus, $j-j^2$ decreases between 135 and 136. So, the answer must be (C) or (E).

III. If the denominator gets bigger, the number gets smaller, so as j^2 increases, $\frac{1}{j^2}$ decreases. So, the answer must be (E), I, II, and III.

6. E

Explanation: This is a rate question in which you are given the rate and the time, and you're looking for the distance. The rate is $\frac{y}{m}$ (in yards per minute) and the time is x hours. In order to use both of those, you'll need to convert either minutes to hours or hours to minutes; it's probably easier to convert x hours to the proper number of minutes. Since 1 hour is the same as 60 minutes, x hours is equal to $60x$ minutes. Thus, we can set up the rate formula:

$r=\frac{d}{t}$
$\frac{y}{m} = \frac{d}{60x}$
Then cross-multiply:
$d = \frac{60xy}{m}$, choice (E).

7. B

Explanation: Note that k, the number that is halved, is not only raised to the third power, but it's in the denominator. If you halve the denominator of a fraction, you're doubling the fraction. If you halve something raised to the third power, you're actually multiplying the result by $\frac{1}{8}$. Since you're multiplying the result of the denominator by $\frac{1}{8}$, you're multiplying the result of the fraction by its reciprocal, 8, choice (B).

It may be easier to see this by picking numbers. First, say $k = 2$:
$F = \frac{1}{(3k)^3} = \frac{1}{(3\times2)^3} = \frac{1}{6^3} = \frac{1}{216}$
Now, take one-half of k, so $k = 1$:
$F = \frac{1}{(3k)^3} = \frac{1}{(3\times1)^3} = \frac{1}{3^3} = \frac{1}{27}$

$\frac{1}{216}$ is $\frac{1}{8}$ of $\frac{1}{27}$, so when k is halved, F is multiplied by 8. In this case, the numbers chosen are deliberately time-consuming to reward those test-takers who are able to solve the problem logically, as in the initial explanation.

8. D

Explanation: The best way to think of an arithmetic sequence is to picture a series of numbers on a number line. If five numbers make up an arithmetic sequence (such as 1, 2, 3, 4, and 5), they are equally spaced along the number line. If they are not equally spaced (as in, say, 2, 4, 8, 16, and 32), they are not an arithmetic sequence.

Since we know v, w, x, y, z is an arithmetic sequence, we know that those five numbers are equally spaced. Thus, if you double all of those numbers (as in I.) you have another series that is also an arithmetic sequence. By doubling all of the numbers, you doubled the space between them, but the space between them is still constant. For instance, if the five variables are 1, 2, 3, 4, and 5, the new sequence is 2, 4, 6, 8, and 10. Still equally spaced, so I must be an arithmetic sequence, narrowing our choices to (A) and (D).

II is also an arithmetic sequence: if you picture the five equally spaced numbers on a number line, adding two to each of them just shifts them all equal amounts in the same direction. If, again, the variables are 1, 2, 3, 4, and 5, the new sequence is 3, 4, 5, 6, and 7. Still equally spaced, so an arithmetic sequence, leaving us only with choice (D).

For the sake of completeness, consider III. If you take the square root of each number in an arithmetic sequence, you aren't left with another arithmetic sequence. If, one more time, you say the variables are 1, 2, 3, 4, and 5, the resulting square roots are 1, $\sqrt{2}$, $\sqrt{3}$, 2, and $\sqrt{5}$. You'll have to approximate, but the distance between 1 and $\sqrt{2}$ (about 0.4) is larger than the distance between $\sqrt{2}$ and $\sqrt{3}$ (about 0.3).

9. A

Explanation: The GMAT never expects you to actually solve for standard deviation; all you need is a working knowledge of the concept. Very approximately, the standard deviation of a set of numbers is the average distance of the numbers from the mean. Thus, III has no deviation at all (all the numbers are the mean). To rank the three sets, you only need to compare I and II. While three of the numbers in I are equal to the mean, the difference between a, $a - 14$, and $a + 14$ is sufficient to make the standard deviation for that set greater than that of II, which has more limited differences between the mean and the other terms.

Again, this is approximate, but to confirm that conclusion, the sum of the distances to the mean in I is $14 + 0 + 0 + 0 + 14 = 28$, while the sum of the distances to the mean in II is $2 + 1 + 0 + 1 + 2 = 6$. It isn't even close, so the greatest standard deviation belongs to I, followed by II, and finally by III, which we established has a standard deviation of zero. Choice (A) is correct.

10. A

Explanation: This is a good question on which to pick numbers. If the four variables are consecutive positive integers, it doesn't matter which ones you pick: the results will be the same for any set of consecutive positive integers. So, for simplicity's sake, let's choose $w = 2$, $x = 3$, $y = 4$, and $z = 5$. The median of the set is the average of the middle two numbers, or 3.5. Now plug in the variables to each of the five choices:

(A) $\frac{2+3}{2} - 1 = 1.5$ (looks like we've found an answer already)
(B) $\frac{2+5}{2} = 3.5$
(C) $\frac{3+4}{2} = 3.5$
(D) $\frac{4+5}{2} - 1 = 3.5$
(E) $\frac{2+3+4+5}{4} = 3.5$

Thus, the only answer that doesn't come out to 3.5 must be correct: choice (A).

11. B

Explanation: First, get rid of the radical signs by squaring both sides:
$\left(\sqrt{3-4x}\right)^2 = \left(4 - \sqrt{3x}\right)^2$
$3 - 4x = 16 - 2(4\sqrt{3x}) + 3x$
$8\sqrt{3x} = 13 + 7x$
Square both sides:
$192x = 169 + 182x + 49x^2$
$49x^2 = 10x - 169$, choice (B).

12. E

Explanation: Given variables in the question stem and the answer choices, pick numbers. Say $x = 3$ and $y = 1$. In that ratio, shoes account for $\frac{3}{4}$ of revenues, or \$3,000, leaving accessories with \$1,000 of revenues, or a difference of \$2,000. Plug in $x = 3$ and $y = 1$ to find which choice is correct.

(A): $\frac{3}{3+1} = \frac{3}{4}$. No.
(B): $\frac{1}{3+1} = \frac{1}{4}$. No.
(C): $\$4,000(\frac{3}{1}) = \$12,000$. No.
(D): $\$4,000(\frac{3}{3+1}) = \$3,000$. No – that's the total revenue from shoes.
(E): $\$4,000(\frac{3-1}{3+1}) = \$2,000$. That's it.

13. E

Explanation: Combine the inequalities to make further deductions. $u > y > w$, and $y > x > v$. No more deductions can be made: for instance, the relationships between w and x, as well as w and v remain unknown, except that all of these values are less than y. I must be true, as $u > y > w$. Eliminate (B) and (C). II is not necessarily true, as the relationship between w and v is not specified. Eliminate (D). III must be true, as $y > x > v$, so the correct answer is (E), I and III.

14. B

Explanation: If $q = \frac{3}{2}$, then $2 - \frac{3}{q} = 2 - \frac{3}{\frac{3}{2}} = 2 - 3(\frac{2}{3}) = 2 - 2 = 0$. Thus $3 + \frac{2}{p} = 0$, $\frac{2}{p} = -3$, $2 = -3p$, $p = -\frac{2}{3}$, choice (B).

15. C
Explanation: With variables in the answer choices, pick numbers. If $q = 4$, the original sum of the numbers is $6(4) = 24$. When one number is discarded, the average of the 5 remaining numbers is 2, for a sum of $5(2) = 10$. The discarded number was 14. Checking the answer choices with $q = 4$, the correct answer must be (C), $q + 10$, which is 14.

16. D
Explanation: First, find the roots of the equation in the question stem. It factors to $(x - 3)(x + 1) = 0$, so $x = 3$ or $x = -1$. On WOTF or "All of the following" questions, start with (E). (E) factors to $(x - 3)(x - 1)$ so it shares a root, eliminate it. (D) factors to $(x - 1)(x + 3)$, so it does not share a root – that's the answer. (C) factors to $(x + 5)(x + 1)$, so it shares a root, eliminate it. (B) factors to $(x + 4)(x + 1)$, so it shares a root, eliminate it. (A) factors to $(x - 3)(x + 2)$, so it shares a root, eliminate it.

17. A
Explanation: $(\frac{1}{p})^2 = \frac{1}{p^2}$.$(\frac{1}{p})(\frac{1}{q}) = \frac{1}{pq}$. The common denominator is p^2q, so $\frac{1}{p^2}$ becomes $\frac{q}{p^2q}$, and $\frac{1}{pq}$ becomes $\frac{p}{p^2q}$. The difference is $\frac{q-p}{p^2q}$, choice (A). This could also be achieved by picking numbers, say $p = 2$ and $q = 3$. $(\frac{1}{p})^2 = \frac{1}{4}$.$(\frac{1}{p})(\frac{1}{q}) = \frac{1}{6}$.$\frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \frac{1}{12}$. Check the choices to see which one works out to $\frac{1}{12}$:

(A): $\frac{3-2}{4(3)} = \frac{1}{12}$, looks good.
(B): $\frac{2-3}{4(3)} = -\frac{1}{12}$. No.
(C): $\frac{3-2}{9(2)} = \frac{1}{18}$. No.
(D): $\frac{3-2}{4-6} = -\frac{1}{2}$. No.
(E): $\frac{1}{4-6} = -\frac{1}{2}$. No.

18. A
Explanation: If $m^2 < 9$, then m is between -3 and 3. Thus, it must be greater than -3, choice (A). It isn't any more complicated than that, though the question tries to trick you into working harder by including the redundant bit of information that $m > -1$. (B) or (D) could be true, but they each leave out possible values of m, such as 0 (in (B)'s case) and 2 (in (D)'s case).

19. A
Explanation: This looks like the sort of question that a bit of adept equation shuffling could quickly solve, but it's easier to handle this one by simply solving for y from the second equation, then using that information to find x in the first equation.

If $2y = 6$, then $y = 3$.

If $y = 3$ and $x^4 = 3y^3$:
$x^4 = 3(3)^3 = 3^4$
$x = 3$
Thus, $x^2 - y = 3^2 - 3 = 9 - 3 = 6$, choice (A).

20. A
Explanation: If the units' digit of k^2 is 1, the units digit of k must be either 1 ($1^2 = 1$) or 9 ($9^2 = 81$, units' digit of 1). If the units' digit of $(k+1)^2$ is 0, $k+1$'s units' digit must be 0, as that's the only digit that, when squared, results in 0. Thus, k must be 9. So, $(k+2)^2 = (9+2)^2 = 11^2 = 121$, so the units' digit is 1, choice (A).

21. D
Explanation: To determine the coefficient of the middle term in a binomial (k, in this case), it's helpful to know the factorization of the binomial. The question gives you part of it, so you can set up the following equation:
$r^2 - kr - 56 = (r-8)(r-x)$
-56 is the product of -8 and x, so $x = 7$:
$(r-8)(r+7) = r^2 - r - 56$
r is the same as $1r$, so the value of k must be 1, choice (D).

22. A
Explanation: The symbol is equivalent to "!", the notation for factorial. Since the numbers we're working with are so large, the question can't expect that you find all of the numbers between $7*+2$ and $7*+7$ and then determine whether each one is prime. Instead, look for a shortcut. (If you don't find one, this is a great question to guess quickly and save some time on, because without a shortcut, you probably won't figure it out.)

First, think about the first number, $7*+2$. That's equal to $(7)(6)(5)(4)(3)(2)(1)+2$, or $2\left[(7)(6)(5)(4)(3)(1)+1\right]$. Because you can factor out a 2, you know that the number is even and, thus, not a prime. You can use the same reasoning for every other number in the list up to $7*+7$: if you were doing the last number, you could factor a 7 out of both $(7)(6)(5)(4)(3)(2)(1)$ and 7, leaving you with $7\left[(6)(5)(4)(3)(2)(1)+1\right]$, a multiple of 7, so a non-prime number. Thus, every number is the series is a mutliple of some number between 2 and 7, so none of them are prime, making (A) the correct choice.

23. C
Explanation: The algebraic equivalent of the division problem given in the question is as follows:
$\frac{M}{S} = V + \frac{1}{S}$
Once that is set up, you can solve for M:
$M = S(V + \frac{1}{S}) = SV + S(\frac{1}{S}) = SV + 1$, choice (C).

24. D

Explanation: Given the rule for operation &, plug in the values given for s and t. $s = 3$ and $t = -2$, so:
$2(s - t) + st$
$2(3 - (-2)) + 3(-2)$
$2(5) - 6$
$10 - 6 = 4$, choice (D).

25. D
Explanation: If the total of the six different cakes is \$109.50, the average price of the cakes is $\frac{109.5}{6} = 18.25$. (Yes, that math is a little cumbersome, but figure that 90 divided by 6 is 15, another 18 divided by 6 is 3 (for a sum of 18) and 1.5 divided by 6 is 0.25, for a total of 18.25.)

If the prices are equally spaced (the difference between each price is x), the average of the six prices is the same as the average of the highest and lowest prices. So, 18.25 is the average of 24.50 and the lowest price:
$\frac{24.5+x}{2} = 18.25$
$24.5 + x = 36.50$
$x = 12$

Now we know the difference in price between the lowest-priced and highest-price is $24.5 - 12 = 12.5$. If the difference in price between each cake is x, and there are six total cakes, that means there are 5 differences in price between the lowest-priced and highest-priced, which means that $5x = 12.5$, so $x = 2.5$, choice (D).

That method is fairly complicated. However, any simpler approach takes as much or (probably) more time. As an alternative, you could use the answer choices and figure out the prices of each of the cakes at each one of those values for x, then determine the sum of the prices. That wouldn't require the same degree of brainpower, but I'd be impressed if you could go through four or five choices in two minutes or less using that approach.

26. C
Explanation: When a number raised to a power is itself raised to a power, the resulting exponent is the product of the two exponents: in this case, $(5^{\sqrt{2}})^m = 5^{(\sqrt{2})m}$. Since $25 = 5^2$, we can set up an equation with a base of 5 on each side:
$5^{(\sqrt{2})m} = 5^2$
$(\sqrt{2})m = 2$
$m = \frac{2}{\sqrt{2}} = \frac{2(\sqrt{2})}{\sqrt{2}(\sqrt{2})} = \frac{2\sqrt{2}}{2} = \sqrt{2}$, choice (C).

27. C
Explanation:In order to find the greatest possible value of x, you'll need the greatest possible value of x^3. For that, you'll need the smallest possible value of y^3, which itself requires that you identify the smallest possible value of y. y must be positive so the smallest it can be is a little bit above zero, which means the smallest possible value of y^3 is also a little above zero. Thus, x^3 can be anything up to about 100. The value of x for which x^3 is closest to (but not

more than) 100 is 4, when $x^3 = 4^3 = 64$. Thus, x could be greater than 4, but not greater than 5, so the correct choice is (C), between 4 and 6.

28. C

Explanation: The simplest way to approach this is to multiply out the binomials, then add them:

$(x+3)^2 = (x+3)(x+3) = x^2 + 6x + 9$

$(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$

Thus, the average is the sum of the two divided by two:

$\frac{x^2+6x+9+(x^2-6x+9)}{2} = \frac{2x^2+18}{2} = x^2 + 9$, choice (C).

29. D

Explanation:This is an explicit test of exponent rules, a topic you must master to do well on GMAT math. Evaluate each choice individually:

(A) Given a power over a power, subtract the powers: $x^{4-18} = x^{-14}$

(B) A power to a power means you should multiply: $x^{2(12)} = x^{24}$

(C) Again, multiply powers: $x^{7(7)} = x^{49}$

(D) Multiply powers: $x^{2(7)} = x^{14}$

(E) There's no way to combine the powers: the only way to simplify this is to write as $2(x^7)$.

The only matching choice is (D).

30. B

Explanation: In an addition, subtraction, or multiplication problem, all values should be possible; in a division problem, always watch out for the possibility that the denominator is zero. In this case, c can't equal zero, so whatever value is impossible for b must be the one that corresponds to $c = 0$.

To simplify the expression, multiply by c:

$a - b = c$

If $a = -1$, then $-1 - b = c$, or $b = -c - 1$. If $c = 0$, then $b = -0 - 1 = -1$. So, since c cannot equal zero, b cannot equal -1, choice (B).

31. A

Explanation: Parentheses work the same way with unfamiliar operators as they do with more familiar ones: first evaluate $-1\# -2$, then use the result as the second term with 3. So, first, say $f = -1$ and $g = -2$:

$\frac{f^2g}{2} = \frac{(-1)^2(-2)}{2} = \frac{-2}{2} = -1$

Now, you're looking for the result of $3\# -1$, so evaluate $f = 3$ and $g = -1$:

$\frac{f^2g}{2} = \frac{(3)^2(-1)}{2} = \frac{-9}{2}$, choice (A).

32. A

Explanation: To work with all of these exponents, make all of the bases in each equation equal. First, with $(4^x)(2^y) = 16$:

$((2^2)^x)(2^y) = 2^4$

$(2^{2x})(2^y) = 2^4$

$2^{2x+y} = 2^4$
$2x + y = 4$
That's a lot of steps, but when you're comfortable with all of the exponent rules (which you should become, if you're not already), they'll come quickly, and you may be able to skip a few along the way. Now do the same with $(3^x)(3^y) = 27$:
$3^{x+y} = 3^3$
$x + y = 3$
Now you have two linear equations with two variables. Subtract the second from the first, and the result is $x = 1$, which means that y must equal 2. Thus, $(x, y) = (1, 2)$, choice (A).

33. A
Explanation: The simplest way to handle this question is to realize that, if $\frac{x}{x-y}$ always results in the same answer, you can pick any pair of values for x and y that fulfills the equation $\frac{x}{y} = \frac{4}{5}$. The easiest pair is $x = 4$, $y = 5$, which makes $\frac{x}{x-y} = \frac{4}{4-5} = \frac{4}{-1} = -4$, choice (A).

34. A
Explanation: Average speed is total distance over total time. The total distance in this case is the sum of S (distance covered swimming) and B (distance covered biking). The total time is the sum of H (hours spent swimming) and $4H$ (hours spent biking). Thus, total distance over total time is $\frac{S+B}{H+4H} = \frac{S+B}{5H}$, choice (A).

35. D
Explanation: First, set up the first statement algebraically. If y is z more than a, $y = a + z$. That doesn't appear to be directly applicable to the fraction given, so rewrite it with y and z on the same side of the equation by subtracting z from both sides:
$y - z = a$
Thus, the numerator and the denominator of $\frac{a}{y-z}$ are equal, so the fraction is equal to 1, choice (D).

36. C
Explanation: If the arithmetic mean of the list is 5, then:
$\frac{4+k+11+3+m+4}{6} = 5$
$22 + k + m = 6(5)$
$22 + k + m = 30$
$k + m = 8$
Obviously, there are multiple possible values of k and m, but since there can only be one correct answer, it appears that it doesn't matter which set of numbers you pick. Say that $k = 5$ and $m = 3$, that means that the set consists of 4,5,11,3,3, and 4. In order, that's $\{3, 3, 4, 4, 5, 11\}$. The two middle numbers are 4 and 4, so the median is 4, choice (C).

37. A

Explanation: The most important concept here is that "p percent" is equivalent to $\frac{p}{100}$. So, if y is p percent of z, then $y = (\frac{p}{100})z$. Is we call the number we're looking for x, then the second equation is $z = (\frac{x}{100})y$. Since we're looking for x, set that equation equal to x:

$z = \frac{xy}{100}$

$xy = 100z$

$x = \frac{100z}{y}$

Now, solve the other equation for $\frac{z}{y}$:

$y = \frac{pz}{100}$

$100y = pz$

$\frac{100}{p} = \frac{z}{y}$

Now, plug into the first equation:

$x = \frac{100z}{y} = 100(\frac{100}{p}) = \frac{10{,}000}{p}$, choice (A).

38. D

Explanation: The area of a rectangle is the product of it's length and width; let's say the longer dimension is the length (it doesn't really matter which is which), so in this case $a = lw$, or $p = q(w)$. Thus, the $w = \frac{p}{q}$. The difference between the lengths of the sides comes from subtracting the length of one side from the other; the two sides are q (given in the question) and $\frac{p}{q}$ (what we just figured out), so the difference is $q - \frac{p}{q}$, choice (D).

39. B

Explanation: $x\%$ is equal to $\frac{x}{100}$, so when you increase a price (call it p) by $x\%$, you're multiplying it by $(1+\frac{x}{100})$. So, after the first increase, the new price is $p(1+\frac{x}{100})$. Similarly, decreasing a price by $y\%$ is the same as multiplying it by $(1-\frac{y}{100})$, so the final price is $p(1+\frac{x}{100})(1-\frac{y}{100})$. Thus, we're multiplying the original price by

$(1+\frac{x}{100})(1-\frac{y}{100})$

$1+\frac{x}{100}-\frac{y}{100}-\frac{xy}{10{,}000}$

$1+\frac{1}{100}(x-y-\frac{xy}{100})$

40. E

Explanation: Unless you've got a half an hour to spare, this is a good opportunity to pick numbers. Pick them carefully—the right numbers can save you a lot of work. Let's say $p = 30$, $r = 10$, and $s = 15$. That way, Train X covers 5 miles in that first half hour between 1 and 1:30 PM, and the trains cover the remaining 25 miles in another hour. In other words, with those values for p, r, and s, the correct choice should equal 1.5 hours after 1:00 PM. Check each one:

(A):$\frac{1}{2}+\frac{30-\frac{15}{2}}{10+15} = \frac{1}{2}+\frac{22.5}{25} = 1.4$. No.

(B): $\frac{30-\frac{15}{2}}{10+15} = \frac{22.5}{25} = 0.9$. No.

(C): $\frac{1}{2}+\frac{30-\frac{10}{2}}{10} = \frac{1}{2}+\frac{25}{25} = 3$. No.

(D): $\frac{30-\frac{10}{2}}{10+15} = \frac{25}{25} = 1$. No.

(E): $\frac{1}{2} + \frac{30-\frac{10}{2}}{10+15} = \frac{1}{2} + \frac{25}{25} = 1.5$. That's it.

41. E

Explanation: With so many variables in the question stem and answer choices, picking numbers is the best option. Let's say $p = 100$, $x = 10$, $y = 20$, and $m = 15$ and $q = 5$. In that case, he earned \$10 on each of the 15 units he sold in March for a total commission of \$150. In April, he made \$20 per unit and sold 20 units, for a total of \$400. That's \$250 more in March than in April. Plug in those numbers into the answer choices to determine which choice is correct:

(A): $\frac{100}{100}[20(15) - 10(15+5)] = 300 - 200 = 100$. No.
(B): $\frac{100}{100}[10(15+5) + 20(15)] = 200 + 300 = 500$. No.
(C): $(100)[20(15-5) - 10(15)] = 100(200 - 150) = 5000$. No.
(D): $\frac{100}{100}[10(15+5) - 20(15)] = 200 - 300 = -100$. No.
(E): $\frac{100}{100}[20(15+5) - 10(15)] = 400 - 150 = 250$. That's it.

In case you were wondering, this is at the very edge of complexity you can ever expect out of a GMAT algebra problem. In fact, it may well cross the line. It isn't a problem to lose sleep over; at the same time, if you can do this, you will have no problem with the toughest of a whole class of problems the test might throw at you.

42. A

Explanation:The sum of a series of consecutive terms (evens, odds, integers, multiples, whatever) is equal to the product of the number of terms and the mean. In this case, the mean is the average of the outermost numbers, m and n: $\frac{m+n}{2}$. The number of terms is half the number of integers between m and n, plus one because the set of numbers includes both endpoints, m and n. So the number is half the difference plus one: $\frac{m-n}{2} + 1$.

Thus, the answer is the product of those two terms: $(\frac{m+n}{2})(\frac{m-n}{2} + 1)$, choice (A).

43. E

Explanation: Substite all of the values given into $\frac{b^2c^2x^2}{a^3}$:

$\frac{(x^2)^2(xy)^2x^2}{(x^3y)^3}$
$\frac{x^4x^2y^2x^2}{(x^3)^3y^3}$
$\frac{x^8y^2}{x^9y^3}$
$\frac{1}{xy}$, choice (E).

44. D

Explanation: Given the rule for the operator &, plug in the values for x, y, and solution. $x = -3$, $y = q$, and the solution is 2:

$\frac{-3-q}{-3-1} = 2$
$\frac{-3-q}{-4} = 2$
$-3 - q = -8$

$q = 5$, choice (D).

45. E

Explanation: First, set up the first equation given: if z is 30 percent of 30 percent of y, then:

$z = \frac{3}{10}(\frac{3}{10})y = \frac{9}{100}y$

Second, if z percent of y equals 9:

$(\frac{z}{100})y = 9$

The first equation gives you the value of z in terms of y, so you can combine the two equations by plugging that into the second equation:

$(\frac{\frac{9}{100}y}{100})y = 9$

$(\frac{9}{100}y)(y) = 900$

$y^2 = 900(\frac{100}{9})$

$y^2 = 100(100)$

$y = 100$, choice (E).

46. E

Explanation:To work toward the value of x, you must first get rid of the radical signs by squaring both sides of the equation:

$(\sqrt{2x-3})^2 = (\sqrt{x}+2)^2$

$2x - 3 = x + 2(2\sqrt{x}) + 4$

$4\sqrt{x} = x - 7$

Now, square both sides to get rid of the remaining radical:

$16x = x^2 - 14x + 49$

$x^2 = 30x - 49$, choice (E).

47. B

Explanation: Before you can take the square root of n, evaluate the expression inside the radical sign:

$\sqrt{\frac{1}{x^4}} = \frac{\sqrt{1}}{\sqrt{x^4}} = \frac{1}{x^2}$

So, $n = 9(\frac{1}{x^2}) = \frac{9}{x^2}$

Thus, $\sqrt{n} = \sqrt{\frac{9}{x^2}} = \frac{\sqrt{9}}{\sqrt{x^2}} = \frac{3}{x}$, choice (B).

48. E

Explanation: The easiest way to handle this question is to align the four variables on a number line, spaced in the manner the question instructs. If s is halfway between r and t, then it looks like this:

$t - s - r$

If t is halfway between r and u, then u is at the bottom of the number line, like this:

$u - -t - s - r$

So, for each answer choice, you can look at the distances between the variables on the number line. In order for any of the fractions to equal one, the numerator and denominator must be equal.

(A) $r - t$ is a little less than $s - u$

(B) $r-t$ and $u-t$ are equal, but $u-t$ is negative, while $r-t$ is positive.
(C) $s-t$ is much smaller than $r-u$
(D) $u-t$ is negative, while $r-s$ is positive
(E) $u-t$ and $t-r$ are both negative, and of equal length, so (E) is the correct choice.

49. E
Explanation: If c is equal to the difference of a and b, there are two possible equations that would represent that:
$c = a - b$ or $c = b - a$
First, take the first one and see if it can be manipulated to look like any of the choices:
$c = a - b$
$c + b = a$, eliminate (A)
$b = a - c$, eliminate (D)
Now, look at the other possible equation:
$c = b - a$
$b = c + a$, eliminate (C)
$a = b - c$, eliminate (B)
The only choice left is (E), so that must be correct.

50. C
Explanation: The mean of the four variables is $\frac{r+s+t+u}{4}$. We can plug in the equation given:
$\frac{(3u)+u}{4} = \frac{4u}{4} = u$, choice (C).

51. A
Explanation: You're given a rate and a time, and you're looking for distance. This is clearly a job for the rate formula. Since the rate is in terms of minutes and the time is in seconds, you'll need to convert one or the other; it's probably easier to convert s seconds to minutes than the rate to floors per second. Since 1 minute equals 60 seconds, s seconds equals $\frac{s}{60}$ minutes. Now we can plug our rate and time into the rate formula:
$r = \frac{d}{t}$
$\frac{f}{m} = \frac{d}{\frac{s}{60}}$
Now, cross-multiply:
$dm = \frac{fs}{60}$
$d = \frac{fs}{60m}$, choice (A).

52. C
Explanation: When a number raised to a power is itself raised to a power, the resulting exponent is the product of the two exponents: in this case, $(3^{\sqrt{2}})^p = 3^{(\sqrt{2})p}$. Since $3 = 3^1$, we can set up an equation with a base of 3 on each side:
$3^{(\sqrt{2})p} = 3^1$
$(\sqrt{2})p = 1$

$p = \frac{1}{\sqrt{2}} = \frac{1(\sqrt{2})}{(\sqrt{2})\sqrt{2}} = \frac{\sqrt{2}}{2}$, choice (C).

53. E

Explanation: If you can grasp the concept of a geometric sequence in the two minutes you're given on the GMAT, you're in good shape: this is about as hard as algebra questions will ever get. To get started, try to think of an example sequence that would qualify as geometric: a simple one is 1, 2, 4, 8, and 16, where the constant is 2. ("Each term after the first is equal to the product of the preceding term and a constant.")

I. is also a geometric sequence: if you double all of the terms in the sequence above, the result is 2, 4, 8, 16, 32; still, the numbers are separated by the same constant: 2.

II. is not a geometric sequence: the resulting sequence is 3, 4, 6, 10, and 18. The constant needed to get from 3 to 4 is $\frac{4}{3}$, but the constant needed to get from 4 to 6 is $\frac{3}{2}$. Since those numbers aren't the same, the constant isn't the same (it isn't a constant at all!), so it's not a geometric sequence.

III. is a geometric sequence: the resulting sequence is 1, $\sqrt{2}$, 2, $2\sqrt{2}$, 4. To turn each number into the next (e.g. $\sqrt{2}$ into 2), you need to multiply by the same number: $\sqrt{2}$. Since that multiplier is consistent (it is a constant, this time), the sequence is a geometric one. Thus, the correct choice is (E), I and III.

54. D

Explanation:The algebraic equivalent of the division problem above is as follows:

$\frac{w}{x} = y + \frac{z}{x}$

To find y, solve that equation for y:

$y = \frac{w}{x} - \frac{z}{x} = \frac{w-z}{x}$, choice (D).

55. D

Explanation: First, simplify $(x+y)^2$: $(x+y)^2 = (x+y)(x+y) = x^2+2xy+y^2$. Now, set up the average as the sum of terms over the number of terms (2):

$\frac{m+(x^2+2xy+y^2)}{2} = x^2 + y^2$

$m + (x^2 + 2xy + y^2) = 2(x^2 + y^2)$

$m + (x^2 + 2xy + y^2) = 2x^2 + 2y^2$

$m = 2x^2 + 2y^2 - x^2 - 2xy - y^2$

$m = x^2 + y^2 - 2xy = (x - y)(x - y) = (x - y)^2$, choice (D).

56. A

Explanation: When you're given a fraction, the denominator cannot equal zero unless the fraction is undefined. Since this particular fraction equals 1, it's not undefined, so the denominator can't equal zero. Thus, $b - c \neq 0$. Since $b = -2$:

$-2 - c \neq 0$

$-c \neq 2$

$c \neq -2$, choice (A).

57. B

Explanation: $100q$ percent is equivalent to $\frac{100q}{100} = q$, so if m is $100q$ percent of n, $m = qn$. If we call the number that we're looking for x, the question is: $n = (\frac{x}{100})m$. Since we're looking for x, solve the second equation for x:

$n = \frac{xm}{100}$

$100n = xm$

$x = \frac{100n}{m}$

Now, we need to find $\frac{n}{m}$ from the first equation, $m = qn$:

$m = qn$

$\frac{n}{m} = \frac{1}{q}$

Plug that back into the first equation:

$x = \frac{100n}{m} = 100(\frac{1}{q}) = \frac{100}{q}$, choice (B).

58. D

Explanation: Parentheses work the same way with unfamiliar operators as they do with more familiar ones: first evaluate the expressions inside each pair of parentheses:

$(2\hat{}\hat{} - 1) = \frac{2^2(-1)}{2} = \frac{-4}{2} = -2$

$(-2\hat{}\hat{}1) = \frac{(-2)^2(1)}{2} = \frac{4}{2} = 2$

Now, you're left with $(-2)\hat{}\hat{}(2)$:

$(-2\hat{}\hat{}2) = \frac{(-2)^2(2)}{2} = 4$, choice (D).

59. C

Explanation: First, set up the first part as an equation: if y is 50 percent of 50 percent of 50 percent of x, then:

$y = (\frac{1}{2})(\frac{1}{2})(\frac{1}{2})x = \frac{1}{8}x$

The second part, y percent of x equals 50, sets up like this:

$(\frac{y}{100})x = 50$

$\frac{xy}{100} = 50$

Finally, to solve for x, combine the equations. Since $y = \frac{1}{8}x$, plug that into the second equation:

$x(\frac{1}{8}x) = 5,000$

$x^2 = 40,000$

$x = 200$, choice (C).

60. D

Explanation: To determine the largest possible value of x, you'll need to determine the smallest possible value of y. If y is a positive integer, then y could be 1, making $\mathrm{y}^4 = 1$. Thus, x^4 could be a little less than 9,999, and the sum $x^4 + y^4$ would still be less than 10,000. So, you need to find the approximate number that, raised to the fourth power, is equal to 9,999. That sounds like a tall order until you realize that you're just approximating, and that $10,000 = 10^4$. So, 9,999 is approximately 10 raised to the fourth power, so the greatest possible value of x is a little bit less than 10, or choice (D), between 9 and 12.

61. B

Explanation: If you elect to pick numbers on a question like this, remember that when comparing expressions such as x, $-x$, x^2, and x^3 (without coefficients), you should be testing one number each in the areas less than -1, between -1 and 0, between 0 and 1, and greater than 1.

Statement (1) is insufficient: the statement is true if x is a positive number, but that could be a positive number between 0 and 1 or a positive number greater than 1.

Statement (2) is sufficient: x is always less than x^2, except for when x is a fraction. When x is a number between 0 and 1, x^2 is less than x. So, the answer to the question is "no," and the correct choice is (B).

62. C

Explanation: To find the value of b, you'll need to know the factors of the equation, e.g. $(x-4)(x+3)$. Statement (1) is insufficient: it gives you one of the two factors. If 1 is a "root" of the equation, one of the factors is $x-1$.

Statement (2) is also insufficient: it gives you another factor: $x+1$.

Taken together, the statements are sufficient. You know that $x^2 - bx + c = (x+1)(x-1) = x^2 - 0x - 1$, so $b = 0$. Choice (C) is correct.

63. D

Explanation: Statement (1) is sufficient: multiply both sides of the equation by x, and the result is $xy = 12$. Statment (2) is also sufficient: it gives you the equation

$\frac{y}{12} = \frac{1}{x}$

Which, cross-multiplied, gives you this:

$xy = 12$

Both statements are sufficient independent of each other, so the correct choice is (D).

64. E

Explanation: Statement (1) is insufficient. Since we know y is positive, we can divide both sides of the inequality by y, and the result is $x < z$. That partially answers the question, but without knowing how y relates to x and z, we don't know enough.

Statement (2) is also insufficient: similarly, we can divide both sides of the inequality by z, leaving us with $x < y$. That also partially answers the question.

Taken together, we know that x is less than both y and z, but without knowing whether z is greater than y, we don't have enough information to answer the question. Choice (E) is correct.

65. D

Explanation: To answer the question, we could find the value of $(x - y)$ (since we already have xy) or we could simply solve for both x and y.

Statement (1) is sufficient. Since $xy = 8$, we can rearrange the terms on the left side of the equation and solve for x:

$x^2y = -64$
$(xy)x = -64$
$8x = -64$
$x = -8$

If $x = -8$ and $xy = 8$, we know that $y = -1$. Given the value of both variables, we can find the value of $xy(x - y)$.

Statement (2) is also sufficient, for similar reasons. We can rearrange the left side again:

$xy^2 = -8$
$(xy)y = -8$
$8y = -8$
$y = -1$

With the value of y, we can solve for x. Choice (D) is correct.

66. C

Explanation: Statement (1) is insufficient: if $n + r$ is less than m and all the numbers are positive, we know that both n and r are less than m, but we know nothing about how p and q compare to those three numbers.

Statement (2) is also insufficient: by similar reasoning, we know that n, r, and m are all smaller than q and p. The median, then, must be one of the first three numbers, but we don't know which one is the largest of the three.

Taken together, we have enough information. (2) tells us that the median must be the largest of n, r, and m, and (1) tells us that m is greater than n and r. Thus, m must be the median. Choice (C) is correct.

67. C

Explanation: Statement (1) is insufficient: S could be anything, from a random non-Farey sequence to the given sample Farey sequence of order 3. Statement (2) is also insufficient: a Farey sequence of order 5 could have $\frac{1}{5}$ as its second element, but there are also an infinite variety of other sequences that could have $\frac{1}{5}$ as their second element.

Taken together, the statements are sufficient. The Farey sequence of order 5 would look like this: $\{0, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, 1\}$ Since that has 11 terms, the only Farey sequence that has $\frac{1}{5}$ as its second element has more than 10 elements. Thus, if S has $\frac{1}{5}$ as its second element and has fewer than 10 elements, it is not a Farey sequence. (C) is the correct choice.

68. C

Explanation: To answer this question, you at the very least need the value of p; it would help to know something about m and n as well. Statement (1) is insufficient; there are many possible prime number pairs that are 8 apart, and without knowing the value of p, we have no idea whether their sum is equal to p.

Statement (2) is insufficient: it's helpful to know that $p = 50$, but m and n could be 3 and 47, or they could be 3 and 5. In one case, the sum is equal to p; in the other, it isn't.

Taken together, the statements are sufficient. The only pair of numbers that are 8 apart and sum to 50 are 21 and 29. 21 isn't a prime number, so whatever the values of m and n, they aren't 21 and 29. Thus, so long as m and n are 8 apart, they don't sum to 50, so their sum is not equal to p. (C) is the correct choice.

69. A

Explanation: Statement (1) is sufficient: If $x^2 = \frac{1}{4}$, x is a positive integer, and w is a negative integer, there are two possible values for x and w:

$x = 2$ and $w = -2$: $2^{-2} = \frac{1}{4}$

$x = 4$ and $w = -1$: $4^{-1} = \frac{1}{4}$

In the first case, $xw = 2(-2) = -4$; in the second, $xw = 4(-1) = -4$. Since the product xw is always -4, we can answer the question.

Statement (2) is insufficient: we have no way of determining the value of x. Choice (A) is correct.

70. E

Explanation: Statement (1) is insufficient: in fact, it doesn't give you any useful information at all. If the final digit of the number is 5, you already know that the number is divisible by 5. Not only can you eliminate (A) and (D), but since there's no way that (1) will help you answer the question, you can eliminate (C) as well.

Statement (2) is also insufficient. Since N is divisible by 9, the sum of the digits of N must also be divisible by 9. Thus, $3+1+5+G = 9+G$ is divisible by 9, which means G could be either 0 or 9. Since (1) won't help you answer the question, you don't need to consider the statements together; (E) is the correct choice.

71. A

Explanation: Statement (1) is sufficient: it gives you two equations and two variables. If the presence of the exponents in the second equation makes you nervous, go ahead and solve the system:

$x^3 = y^2$

$x^3 = (2x)^2$

$x^3 = 4x^2$

$x = 4$

Thus, if $2x = y$, $y = 8$.

Statement (2) is insufficient. While there are two equations and two variables, the exponent creates a problem this time:

$y - x = 4$

$y = x + 4$

Then plug that into the second equation:
$x^2 = 2y$
$x^2 = 2(x + 4)$
$x^2 = 2x + 8$
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x = 4$ or $x = -2$. Since there are two possible answers (each of the values of x lead to a different value of y). Choice (A) is correct.

72. D
Explanation: To answer the question, you'll need either the value of $a - b$ or the value of both a and b. Statement (1) is sufficient: when you see the same variable on both sides of the equation, odds are the statement is hiding something, so you need to simplify the equation:
$a = 2(b + 5) - a$
$2a = 2b + 10$
$2a - 2b = 10$
$a - b = 5$
Statement (2) is also sufficient. 125 is 5^3, so if $(a - b)^3 = 125$, $a - b = 5$. Choice (D) is correct.

73. C
Explanation: Statement (1) is insufficient. Since we don't know whether x is positive or negative, there are two possible ways to simplify the inequality:
If x is positive, $z > y$
If x is negative, $z < y$
Depending on x's sign, the answer could be "yes" or "no."
Statement (2) is also insufficient on its own: it tells you nothing about z or y.
Taken together, you have enough information. Given that x is negative, you know the result of (1) when simplified is $z < y$. That's the opposite of the question asked, so the answer to the question is "no," and choice (C) is correct.

74. E
Explanation: In each of the statements, both variables are on both sides of the equation, so the first thing to do when confronting each one is to simplify.
Statement (1) is insufficient: divide both sides by x^2y and the result is $1 = x$. It's also possible that $x = 0$: anytime every term in an equation is multiplied by the same variable, that variable could be zero. So x must be 0 or 1.
Statement (2) is also insufficient. Again, x could be zero, as every term is multiplied by x. Simplify the equation:
$\frac{x^2}{y} = x^3y$
$\frac{1}{y} = xy$
$x = \frac{1}{y^2}$
Aside from the possibility that x is zero, x could be anything: it depends on the value of y.

Taken together, the statements are still insufficient. In both cases, x could be zero, but it's also the case that x could be 1; we established that for (1), and in (2), $x = 1$ when $y = 1$. Choice (E) is correct.

75. E

Explanation: Statement (1) is insufficient; first, simplify it. Because x^2 must be positive (any number squared is positive), divide both sides by x^2:

$\frac{x^2}{x^2} > \frac{x^4}{x^2}$

$1 > x^2$

Since x^2 must be positive, it's between 0 and 1, which means that x could be any number between -1 and 1. For instance, if $x = -\frac{1}{2}$, $x^2 = \frac{1}{4}$. And if $x = \frac{1}{2}$, $x^2 = \frac{1}{4}$ as well.

Statement (2) is also insufficient: in fact, this statement tells you nothing you didn't already know. Any number squared is positive, so of course $x^2 > 0$. Not only is this statement insufficient, it's completely worthless: it won't help you use the information in (1) to answer the question, so the correct choice must be (E).

76. B

Explanation: There are many possible pairs of numbers that multiply to 36. Statement (1) is insufficient: n could be either 4 (2 squared) or 9 (3 squared); depending on which one it is, m could be either 9 or 4, respectively.

Statement (2) is sufficient: there is only one pair of numbers that multiply to 36 for which m is 5 greater than n: $m = 9$ and $n = 4$. Even without solving for those two values, you may realize that, of all the pairs of numbers whose product is 36, only one of them can be separated by 5.

77. E

Explanation: This question rigorously tests your familiarity with common binomials. The only way to do anything with statement (1) is to subtract $2xy$ from both sides:

$x^2 - 2xy + y^2 = 1$

$(x - y)(x - y) = 1$

$x - y = 1$ or $x - y = -1$

There are an infinite number of possible solutions for x and y, so there's no way to determine a specific value for $x^2 + y^2$.

Statement (2) is insufficient for similar reasons:

$x^2 + y^2 = 4 - 2xy$

$x^2 + 2xy + y^2 = 4$

$(x + y)(x + y) = 4$

$x + y = 2 or x + y = -2$

Again, there are an infinite number of possibilities for x and y.

Taken together, you still don't have enough information. If you had exactly two equations (such as $x - y = 1$ and $x + y = 2$), you could solve, but you have two different pairs of possible equations, which is insufficient to find the values of the variables. Choice (E) is correct.

78 C

Explanation: To find a relationship between three numbers, you'll need two different relationships between two of the numbers; for instance, the relationship between x and y and the relationship between y and z.

Statement (1) gives you the relationship (ratio, fraction, whatever you prefer to call it) between x and y, and also gives you the product of the two numbers (which is useless for the purposes of determining a ratio). It is insufficient, because we need to have some knowledge of how x or y relates to z.

Statement (2) is structured the same way, and is also insufficient. We're given the relationship between y and z, but nothing relating either variable to x.

Taken together, we have enough information. Given the ratio $x : y$ and the ratio $y : z$, we can combine them to find the ratio $x : y : z$. Choice (C) is correct.

79. C

Explanation: Before looking at the statements, it's worth considering how you might simplify the expression, but since the bases and the powers are different in the numerator and denominator, there's no way to do so.

Statement (1) is insufficient: if $x = y$, you can simplify as follows:

$\left(\frac{x^p}{y^q}\right)^5 = \left(\frac{x^p}{x^q}\right)^5 = (x^{p-q})^5$

Without $p - q$, you can't answer the question.

Statement (2) is also insufficient: there's no way to simplify the expression without knowing something of the relationship between x and y.

Taken together, the statements are sufficient. Pick up where you left off with (1):

$(x^{p-q})^5 = (x^{p-p})^5 = (x^0)^5 = 1^5 = 1$

Choice (C) is correct.

80. C

Explanation: Statement (1) tells you than y is either greater than 5 or less than -5. If $y > 5$ and $xy < 5$, x must be less than 1. However, if y is less than -5, x could be any positive number (-5 times a positive will always be negative, and thus less than 5) or x could be a negative number greater than -1.

Statement (2) simply tells you that the two variables have the same signs; with no more information about y, you can't determine how large or small x must be.

Taken together, the statements are sufficient. Note that in (1), the only way in which x could be greater than 1 is when y is less than -5 and x was a positive number. If the two variables must have the same signs, that scenario is impossible. Thus, we're left only with the possibilities that $y > 5$ (in which case x is less than 1) or $y < -5$ and x is between -1 and 0. In either case, x is less than 1, so the correct choice is (C).

81. E

Explanation: Statement (1) is insufficient: if $s^2 = 4$, then $s = 2$ or $s = -2$. If $\frac{r}{s} = \frac{3}{2}$, if $s = 2$ then $r = 3$. If $s = -2$ then $r = -3$. In the first case $r+s = 5$, in the second, $r + s = -5$.

Statement (2) is also insufficient: in this case we're given the possibiliities that $r = 3$ or $r = -3$; the results are the same.

Taken together, the statements are still insufficient. The possibilities still exist that $r + s = 5$ and $r + s = -5$, so we don't have a clear answer to the question. (E) is the correct choice.

82. E

Explanation: Statement (1) is clearly insufficient: if $z = 20$, then $xy = 20$, and we know nothing about x or y. Statement (2) is also insufficient: the question is asking about z, and the statement only offers information about x and y.

Taken together, the statements are still insufficient. It is possible that $z = 20$; if $x = 5$ (a prime) and $y = 4$ (the square of a prime), then $xy = 20$, so according to (1), $z = 20$ as well. However, there are an infinite number of other possibilities for xy, such as $x = 3$ and $y = 9$, for $z = xy = 27$. Since we don't have any more restrictions on the values of x, y, or z, the correct choice is (E).

83. B

Explanation: When you're dealing with terms like x, x^2, and x^3, remember that there are four important types of numbers to test: positive numbers greater than 1, negative numbers less than 1, positive fractions (between 0 and 1) and negative fractions (between 0 and -1). Those rules come in much handier on this question once you've simplified the statements: in each case, subtract one of the terms from the left side so that you have a comparison, not a more complicated inequality.

Statement (1) simplifies to $x^3 > -x$. That is true when x is any positive number (x^3 is positive, $-x$ is not), but is not true when x is negative (x^3 is negative, $-x$ is positive). If x is positive, we don't know whether x^3 is greater than x: if x is less than 1, x is greater than x^3; if x is greater than 1, x^3 is greater than x. (1), then, is insufficient.

Statement (2) simplifies to $x^3 > x$. That's what the question is asking, so if it is true, the answer to the question must be yes. The correct choice is (B).

84. C

Explanation: Statement (1) is insufficient. We're interested in x, so rewrite the question with x on one side:

$x = 2p + 2$

We don't know anything about p, so we don't know whether $2p+2$ is divisible by 5.

Statement (2) is also insufficient: the question is about x, and it contains no information about x.

Taken together, the statements are sufficient. If $2p-3$ is divisible by 5, that means that $2p$ is 3 greater than a multiple of 5. Thus, $2p+2$ is 2 greater than 3 greater than a multiple of 5. Put more simply, it's 5 greater than a multiple of 5. Any number that is 5 greater than a multiple of 5 is itself a multiple of 5, so $2p+2$ is a multiple of 5. Since $x=2p+2$, x is a multiple of 5, as well.

85. C

Explanation: Another way to write the question is to multiply $\frac{x}{p}$ through the parentheses on the left side:

$\left(\frac{x}{p}\right)p^2+\left(\frac{x}{p}\right)q^2+\left(\frac{x}{p}\right)r^2=xp+\left(\frac{x}{p}\right)q^2+\left(\frac{x}{p}\right)r^2$

There are still plenty of issues to be resolved, but it's another way to look at the question that might come in handy as the statements offer more information.

Statement (1) is insufficient, but it gets the left side of the equation closer to the right side of the equation:

$xp+\left(\frac{x}{p}\right)q^2+\left(\frac{x}{p}\right)r^2=xp+\left(\frac{x}{p}\right)q^2+\left(\frac{z}{r}\right)r^2=xp+\left(\frac{x}{p}\right)q^2+zr$

Since we don't know whether $\left(\frac{x}{p}\right)q^2=yq$, we can't answer the question.

Statement (2) is also insufficient, for a similar reason:

$xp+\left(\frac{x}{p}\right)q^2+\left(\frac{x}{p}\right)r^2=xp+\left(\frac{y}{q}\right)q^2+\left(\frac{x}{p}\right)r^2=xp+yq+\left(\frac{x}{p}\right)r^2$

Again, we're missing information for one of the three terms.

Taken together, we can answer the question. We can simplify the left side of the initial equation:

$xp+\left(\frac{x}{p}\right)q^2+\left(\frac{x}{p}\right)r^2=xp+\left(\frac{y}{q}\right)q^2+\left(\frac{z}{r}\right)r^2=xp+yq+zr$

That's the same as the right side of the initial equation, so the answer to the question is "yes." Choice (C) is correct.

86. D

Explanation: Before launching into the statements, it might be useful to rearrange the equation in the question a little bit:

$a=\frac{b^2}{c}$

$ac=b^2$

$c=\frac{b^2}{a}$

Any one of those forms could come in handy for recognizing the relationship between the statements and the question.

Statement (1) is sufficient: multiply both sides by b, and the result is:

$b\left(\frac{a}{b}\right)=b\left(\frac{b}{c}\right)$

$a=\frac{b^2}{c}$

In other words, the statement tells us this equation is correct, so the answer to the question is "yes."

Statement (2) is also sufficient. Square both sides:

$(b)^2=(\sqrt{ac})^2$

$b^2=ac$

We established at the outset that that is the same as the equation we're trying to confirm, so this is also enough information. Choice (D) is correct.

87. C

Explanation: Another way of thinking about the question is: is k the square of an integer? Statement (1) is insufficient: to simplify, square each part of the inequality:

$(1)^2 < \left(\sqrt{k}\right)^2 < (3)^2$

$1 < k < 9$

k could be any integer between 1 and 9, not including the endpoints. If $k = 4$, $\sqrt{k}$ is an integer. If k is any of the other possible numbers, $\sqrt{k}$ is not an integer.

Statement (2) is also insufficient: if $k^2 < 16$, then k is less than 4. k could be 1, which is the square of an integer, or it could be 2 or 3, which are not.

Taken together, the statements are sufficient. (2) limits the possibilities for k to 1, 2, and 3, while (1) rules out $k = 1$. Thus, k must be 2 or 3, neither of which is a perfect square, so the answer is "no." Choice (C) is correct.

88. B

Explanation: Statement (1) is not sufficient. If the average of p and q is not an integer, q could be an integer, as in $p = 2$ and $q = 3$, or it could not be an integer, as in $p = 2$, $q = 2.5$. In both cases, the average is a non-integer.

Statement (2) is sufficient: set up the equation and simplify;

$\frac{p+q+(q+4)}{3} = p$

$p + 2q + 4 = 3p$

$2q = 2p - 2$

$q = p - 2$

If p is an integer and q is two less than p, q must also be an integer. (B) is the correct choice.

89. E

Explanation: $x - y$ and $y - x$ will always have opposite signs; for proof, consider that $x - y = -1(y - x)$. So, when $x - y$ is negative (and, by extension, $\frac{1}{x-y}$ is negative), $y - x$ will be positive and the answer will be yes. If $x - y$ is positive, the opposite is true. In other words, the object in this question is determining whether $x - y$ is positive or negative.

Statement (1) is insufficient. The statement is true if x is negative and y is positive, in which case $x < y$ and $x - y$ is negative. However, if both x and y are positive, then x must be greater than y, as in $\frac{1}{3} > \frac{1}{2}$.

Statement (2) is also insufficient: if x and y are positive, x must be greater than y (as in $x = 3$ and $y = 2$), but if x and y are both negative, x must be less than y (as in $x = -3$ and $y = -2$).

Taken together, the statements are still sufficient. Consider the examples given for (2): the values $x = 3$ and $y = 2$ satisfy (1) as well. However, the values $x = -3$ and $y = -2$ are not possible: $\frac{1}{-3}$ is greater than $\frac{1}{-2}$. The other possibility discussed with (1), in which the variables have opposite signs, is not possible in the equation given in (2). Thus, choice (C) is correct.

90. E

Explanation: Statement (1) is insufficient: to consider just two possible pairs of values, if $x = 1$ and $y = 2$, then $x^y < y^x$: $1^2 < 2^1$. But, if $x = 3$ and $y = 4$, then $x^y > y^x$: $3^4 > 4^3$.

Statement (2) is also insufficient: take any pair of consecutive integers and see whether they make the answer "yes" or "no." Now reverse the integers (if $x = 1$ and $y = 2$, then it could be true than $x = 2$ and $y = 1$), and you'll get the opposite result.

Taken together, the statements are still insufficient. Both of the examples given for (1) are consecutive integers, so they satisfy both statements. However, they generate contradictory answers, so the correct choice is (E).

91. C

Explanation: Another way of phrasing the question is: is z the square of an integer.

Statement (1) is insufficient: it relies on having some sort of relationship between x and z, and we know nothing about x.

Statement (2) is also insufficient: again, because we know nothing about x, it doesn't tell us anything about z, or $\sqrt{z}$, which is what we're looking for.

Taken together, the statements are sufficient. To combine the statements, plug the second equation into the first one:

$\sqrt{xz} = \sqrt{(z^2)z} = \sqrt{z^3} = z\sqrt{z} = \text{integer}$

The only way $z\sqrt{z}$ is an integer is if $\sqrt{z}$ is an integer. For instance, if $\sqrt{z}$ is not an integer (say, $z = 5$), then $z\sqrt{z} = 5\sqrt{5}$, which is not an integer. The same logic applies to any non-square. So, if $z\sqrt{z}$ is an integer, z must be a perfect square, so $\sqrt{z}$ must be an integer. Choice (C) is correct.

92. B

Explanation: Given that both of the statements contain equations with both x and y, it may be useful to simplify the question before going any further.

$\frac{1}{x} + \frac{1}{y} = 2$

$\frac{y}{xy} + \frac{x}{xy} = 2$

$x + y = 2xy$

Statement (1) is insufficient. Substituting x for y in the last of the simplified equations:

$x + y = 2xy$

$x + x = 2x^2$

$2x = 2x^2$

$1 = x$

Since you don't know that $x = 1$, you can't answer the question.

Statement (2) is sufficient: it's exactly what the simplified question looks like. If $x + y = 2xy$, the answer is "yes," so (B) is the correct choice.

93. D

Explanation: First, get a handle on what the question is asking. If both r and s are negative, r must be greater than s for $\frac{r}{s}$ to be less than 1, as in the example $r = -2$, $s = -3$. Because $\frac{r}{s}$ will be greater than 1 if r is less than s, the question is essentially asking when r is greater than s.

Statement (1) is sufficient: first, simplify:

$r + 2s = \frac{s^2}{r}$

$r^2 + 2rs = s^2$

Since both r and s are negative, $2rs$ must be positive. Thus, s^2 is greater than r^2. If s^2 is greater than r^2 and both variables are negative, r must be greater than s. For illustration, again consider the examples $r = -2$ and $s = -3$; if $s^2 = 9$ and $r^2 = 4$ and both variables are the negative roots, r is greater than s.

Statement (2) is also sufficient, and can be shown by similar reasoning:

$\frac{r}{s} = \frac{s}{r} - 2$

$r^2 = s^2 - 2rs$

$r^2 + 2rs = s^2$

That's what we deduced from the first statement, so the rest of the logic is identical. (D) is the correct choice.

94. E

Explanation: To answer the question, you need to know which operator the symbol represents. Statement (1) is insufficient: it could be multiplication ($0 \times 1 = 0$) or division ($\frac{0}{1} = 0$). Statement (2) is also insufficient: it could be multiplication or division as well.

Taken together, the statements are still insufficient: given that neither statement eliminates multiplication or division as a possible operator, the value of $5 \blacksquare 2$ could be $5 \times 2 = 10$ or $\frac{5}{2}$.

95. C

Explanation: Statement (1) is insufficient: translated into algebra, it looks like this:

$y + 2 > 0$

$y > -2$

If y is greater than -2, it may be negative, but it also may be positive.

Statement (2) is also insufficient: in order for y^2 to be greater than 10, y must either be greater than $\sqrt{10}$ (about 3.2) or less than $-\sqrt{10}$. In the first case, it's positive; in the second, it's negative.

Taken together, the statements are sufficient. Given the range of possible values in (2), only the positive answers (those greater than $\sqrt{10}$) are also greater than -2, as (1) requires them to be. Thus, y must be greater than $\sqrt{10}$, so the answer is "no." Choice (C) is correct.

96. B

Explanation: To find the average of two numbers, you need the sum. Statement (1) is insufficient: the resulting equation

$\frac{2m+n}{2} = 15$

can't be simplified to give you $m + n$.

Statement (2) is sufficient: its equation can be simplified:

$\frac{2m+2n}{2} = 22$
$m + n = 22$
The average is 11, and choice (B) is correct.

97. A
Explanation: Statement (1) is sufficient: given two identities, take the time to figure out what they're telling you. The second looks like this:
$s = \sqrt{\frac{1}{t}}$
$s^2 = \frac{1}{t}$
$t = \frac{1}{s^2}$
Substitute the given equations for t and v, and the result is:
$stv = 1$
$s(\frac{1}{s^2})(s) = 1$
$\frac{s^2}{s^2} = 1$
$1 = 1$
We know that $1 = 1$, so the answer to the question is "yes."
Statement (2) is insufficient: we don't know anything about v, without which information we can't answer the question. (A) is the correct choice.

98. C
Explanation: If $\frac{x}{y}$ is to be less than zero, the two variables must have different signs: if a positive is divided by a negative (or vice versa), the result is negative, or less than zero.
Statement (1) is insufficient: If $x(y^2) = 18$, then x and y^2 must have the same sign, so that their product is positive. Since any number squared is positive, y^2 must be positive, so x must be positive as well. However, we don't know whether y is positive or not, so we don't know whether $\frac{x}{y}$ is negative.
Statement (2) is also insufficient. By similar reasoning: either x^2 or y must be negative, while the other is positive. x^2 must be positive, so y must be negative. However, we don't know whether x is positive, so we can't answer the question.
Taken together, the statements are sufficient. (1) tells us that x is positive; (2) tells us that y is negative; combined, we know that $\frac{x}{y} = \frac{\text{positive}}{\text{negative}} = \text{negative}$. Choice (C) is correct.

99. B
Explanation: Indepedent of p and q, the set of integers listed has a range of 7. If the range is to be greater than 7, either p or q must be less than 2 (the smallest given number) or larger than 9 (the largest given number).
Statement (1) is insufficient. To establish that, solve for $p + q$:
$\frac{5+9+p+6+2+q}{6} = 5.5$
$22 + p + q = 33$
$p + q = 11$
It's possible that the range is greater than 7; for instance, if $p = 10$ and $q = 1$. However, if $p = 5$ and $q = 6$, the range is equal to, not greater than, 7.

Statement (2) is sufficient: if p is greater than 9, the range of the set is greater than 7. The smallest number is 2 (or less, if q is less than 2), so the range is $p - 2$, and since p is greater than 9, that difference is greater than 7. (B) is the correct choice.

100. D

Explanation: First, simplify the question:
$3^{x+2} < 9$
$3^{x+2} < 3^2$
$x + 2 < 2$
$x < 0$
In other words, the question is asking whether x is negative.
Statement (1) is sufficient: simplify this as well:
$3^x < 1$
$3^x < 3^0$
$x < 0$
Statement (2) is sufficient as well, as it tells you directly that x is negative. Choice (D) is correct.