M ath P ractice Section 2: M edium D ifficult

M ath P ractice Section: M
edium 20 Q uestions
35 M inutes

For questions in the Q uantitative C om parison form at ("Q uantity A " and "Q uantity B " given),the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box ☐,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes ☐/☐,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.

A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.

Set M consists of all the integers betw een -2 and 12,inclusive
Set N consists of all the integers betw een 9 and 15,inclusive

Q uantity A	**Q uantity B**
The sm allest integer in Set M that is also in Set N	9

2.

17% of p is equal to 18% of q,w here p and q are positive

Q uantity A	**Q uantity B**

p *q*

3.

C ircle *A* has area *a*

Sem icircle *B* has area $\frac{a}{2}$

Q uantity A	**Q uantity B**
The circum ference of C ircle *A*	Tw ice the perim eter of sem icircle *B*

4.

Q uantity A	**Q uantity B**
The standard deviation of the set 1,5,7,19	The standard deviation of the set 0,5,7,20

5.

A n isosceles triangle has a perim eter of 28.The shortest side has length 8.

Q uantity A	**Q uantity B**
The length of the longest side of the triangle	12

6.

$(3 - z)(z + 4) = 0$

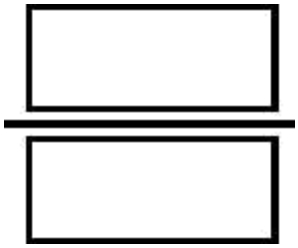
Q uantity A	**Q uantity B**
z	5

7.

$a > b > c >$
$d\ ab > 0$
$ad < 0$

Q uantity A	**Q uantity B**
ac	*cd*

8.If $12b = 2g$ and $4g - 3b = 63$,w hat is the value of $\frac{g}{b}$?
G ive your answ er as a fraction.

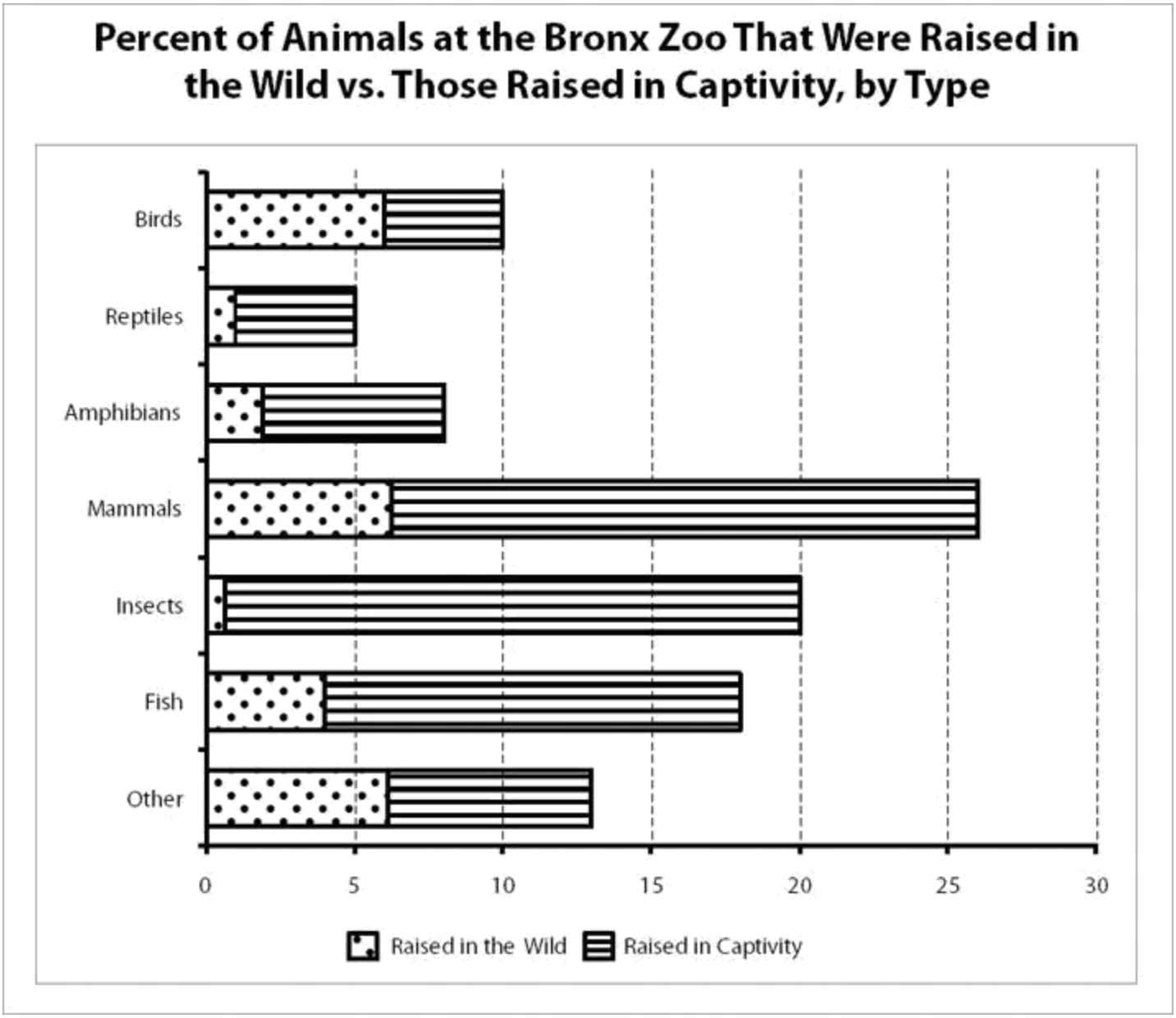


$9.81^3 + 27^4$ is equivalent to w hich of the follow ing expressions?

Indicate <u>all</u> such expressions.

- ☐ $3^7(2)$
- ☐ $3^{12}(2)$
- ☐ $9^6(2)$
- ☐ 912
- ☐ 324

Q uestions 10–12 are based on the follow ing chart.



10.A pproxim ately w hat percent of all the zoo's anim als are either m am m als that w ere raised in the w ild or am phibians raised in captivity?

(A) 8
(B) 12
(C) 18
(D) 34
(E) 100

11.If the B ronx Zoo donated all of its insects and fish to other zoos,approxim ately w hat percent of the anim als in the zoo w ould be birds raised in the w ild?

(A) 5
(B) 9
(C) 24
(D) 32
(E) 60

12.If the zoo currently has 80 total birds,w hat is the sm allest num ber of birds that could be added such that at least 20% of the anim als at the zoo w ould be birds?

(A) 10
(B) 80
(C) 100
(D) 125
(E) 200

13.Trail m ix is m ade by com bining 3 pounds of nuts that cost *x* dollars per pound w ith 1 pound of chocolate that costs *y* dollars per pound and 2 pounds of dried fruit that costs *z* dollars per pound.W hat is the cost in dollars per pound for the trail m ix?

(A) $\frac{3x+y+2z}{xyz}$

(B) $3x+y+2z$

(C) $\frac{3x+y+2z}{6}$

(D) $6(3x+y+2z)$

(E) $\frac{x}{3}+y+\frac{2}{z}$

14.If $z = 3^4$,then $\left(3^z\right)^z =$

(A) 3^{16}
(B) 3^{81}
(C) 3^{324}
(D) 3^{405}
(E) $3^{6,561}$

15.M aurice entered a num ber into his calculator and erroneously divided the num ber by 0.03 instead of 0.0003, resulting in an incorrect result.W hich of the follow ing is a single operation that M aurice could perform on his calculator to correct the error?

Indicate all such operations.

☐ M ultiply the incorrect product by 100
☐ D ivide the incorrect product by 100
☐ M ultiply the incorrect product by 0.01
☐ D ivide the incorrect product by 0.01

16.A com pany's annual expenses are com posed entirely of a fixed am ount in costs,plus a variable am ount that is directly proportional to the num ber of clients served.In 2009,the com pany served 450 clients and its total expense w as $830,000.In 2010,the com pany served 510 clients and its total expense w as $896,000.W hat is the com pany's fixed annual expense,in dollars?

(A) 1,844
(B) 1,757
(C) 335,000

(D) 485,000
(E) 830,000

17.W hich of the follow ing lines is perpendicular to $4x + 5y = 9$ on the *xy* plane?

(A) $y = \frac{5}{4}x + 2$

(B) $y = -\frac{5}{4}x + 9$

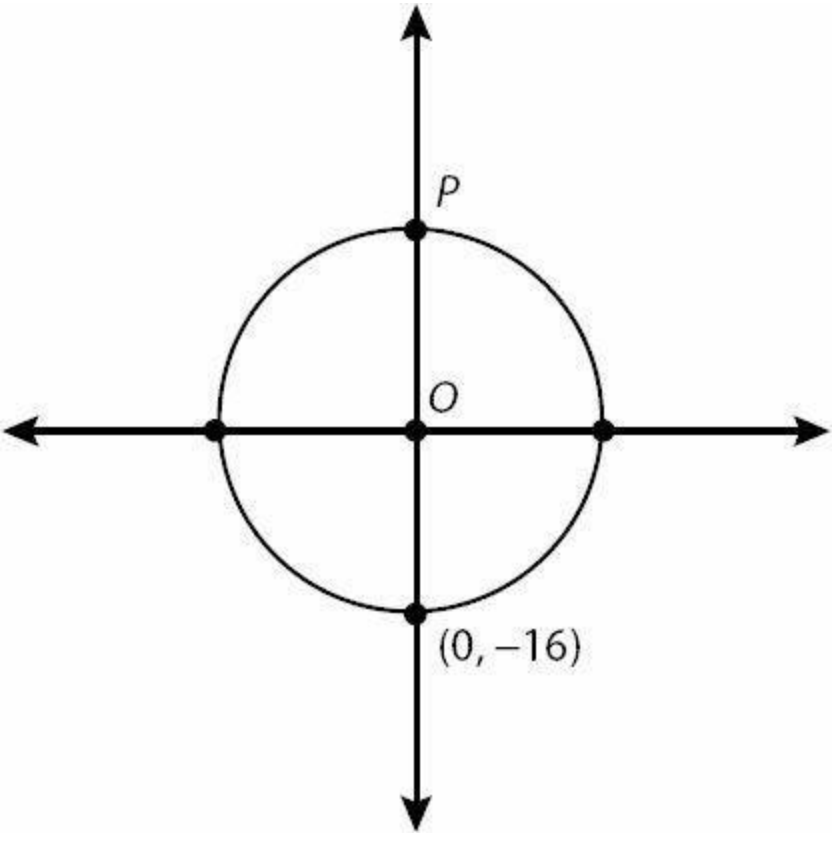
(C) $y = -4x + \frac{9}{5}$

(D) $y = \frac{4}{5}x - \frac{4}{5}$

(E) $y = -\frac{4}{5}x$

18.The tens digit is m issing from the three-digit num ber 8 __ 9.If the tens digit is to be random ly selected from the ten different digits from 0 to 9,w hat is the probability that the resulting three-digit num ber w ill be a m ultiple of 9?

(A) 0.1
(B) 0.2
(C) 0.4
(D) 0.9
(E) 1



19.In the figure above,the circle is centered at (0,0).W hat is the distance betw een point *P* and the point (-10,-8) (not show n on the graph)?

(A) 18
(B) 20
(C) 22
(D) 24
(E) 26

20.If $f(-0.5) = 0$,w hich of the follow ing could be $f(x)$?

(A) $2x + 2$
(B) $4x - 2$
(C) $4x^2 - 1$
(D) $x^2 - 1$
(E) $(-x)^2 - 2.5$

A nsw ers to M ath P ractice Section 2

1.**(C).**Set M consists of -2,-1,0,1,2,3,4,5,6,7,8,9,10,11,and 12.Q uantity A is the *least* of these integers that is also in Set N .The sm allest integer in Set N is 9,w hich is also in Set M ,so Q uantity A is 9.The tw o quantities are equal.

2.**(A).**A s algebra,"17% of *p* is equal to 18% of *q*" is:

$$\frac{17}{100}p = \frac{18}{100}q$$

Solve for *p*.The easiest w ay to do this is to first m ultiply both sides of the equation by 100,then divide both sides by 17:

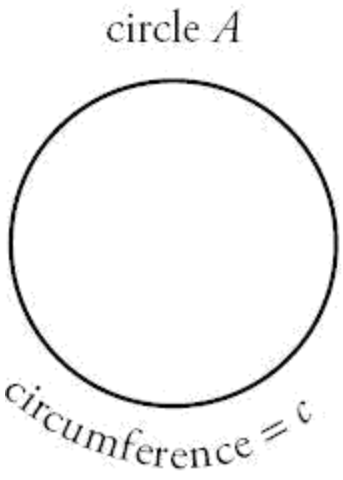
$$p = \frac{18}{17}q$$

Since $\frac{18}{17}$ is greater than 1 and both variables are positive,*p* is greater than *q*.

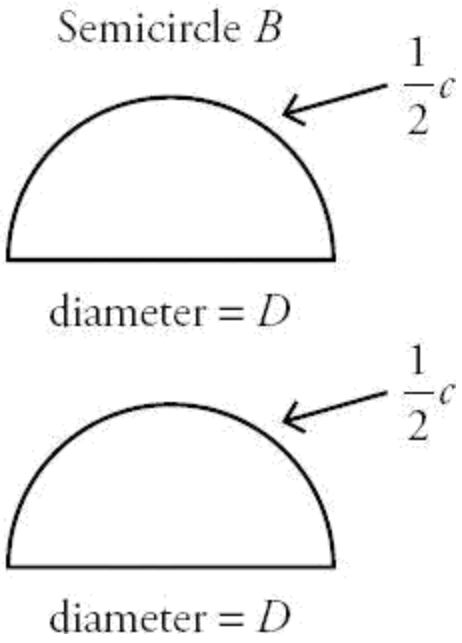
(N ote that it w as necessary to know that both variables w ere positive! If they w ere negative, $p = \frac{18}{17}q$ w ould im ply that *p* is m ore negative than *q*,so *q* w ould have been greater than *p*.W ithout inform ation about sign,the answ er w ould have been (D).)

3.**(B).**If the given sem icircle has half the area of the circle,then the Sem icircle *B* is sim ply equal to half of C ircle *A* . H ow ever,that does *not* m ean that the sem icircle has half the perim eter.O bserve:

Quantity A



Quantity B



The sem icircle is draw n tw ice,as Q uantity B refers to “tw ice the perim eter of Sem icircle *B* .” N ote that Q uantity A is equal to the circum ference *c*,w hile Q uantity B is equal to this sam e circum ference,plus tw ice the length of the diam eter.Q uantity B is greater.

4.**(B).**Standard deviation m easures the variance from the m ean;the m ore spread out a set is,the higher the deviation. The set in Q uantity B is the sam e as the one in Q uantity A ,but w ith the sm allest num ber *even sm aller* and the largest num ber *even larger*,so the set in Q uantity B is m ore spread out,and has a greater standard deviation.

5.**(D).**A n isosceles triangle has tw o sides that are equal and a third side that is a different length.The isosceles triangle in this question has a perim eter of 28 and shortest side of length 8.N ow ,suppose that the shortest side is the one that is repeated,such that the triangle has tw o sides of length 8 and one other side of length *x*.This w ould m ean:

8 + 8 + *x* = Perim eter
16 + *x* =
28 *x* = 12

So,this triangle w ould have lengths 8,8,and 12 as the three legs.Test this triangle via the Third Side R ule: the length of any side of a triangle m ust be greater than the difference betw een the other tw o sides and less than the sum of the other tw o sides.The third side (*x*) m ust be greater than 8 - 8 = 0 and less than 8 + 8 = 16.Since 12 is betw een 0 and 16,this is a legal triangle.

O n the other hand,consider the possibility that the other side,*x*,is repeated and the length 8 is used only once.In this case:

x + *x* + 8 =
28 2*x* = 20
x = 10

The sides of this triangle are 10,10,and 8.Test this triangle via the Third Side R ule: the third side (8) m ust be greater than 10 - 10 = 0 and less than 10 + 10 = 20.Since 8 is betw een 0 and 20,this is a legal triangle.

For one triangle,the quantities in Q uantity A and Q uantity B w ould be equal,but for the other,Q uantity B w ould be greater than Q uantity A .Therefore,the relationship cannot be determ ined from the given inform ation.

6.**(B).**(3 - *z*)(*z* + 4) = 0,so either (3 - *z*) or (*z* + 4) m ust equal 0:

3 - *z* =
0 *z* = 3

O R

z + 4 =
0 *z* = -4

z is either 3 or -4.Either w ay,Q uantity B is greater.

7.**(D).**If *ad* < 0,*a* and *d* have opposite signs.B ecause *a* > *d*,*a* m ust be positive and *d* m ust be negative.Sim ilarly,if *ab* > 0,*a* and *b* have the sam e sign,so *a* and *b* are both positive.The rem aining variable *c* can be positive,0,or negative and still fall betw een *b* and *d*.If c is 0,the tw o quantities are equal.If *c* is positive,Q uantity A is positive and Q uantity B is negative.If *c* is negative,Q uantity B is positive and Q uantity A is negative.The relationship cannot be determ ined from the inform ation given.

A lternatively,pick num bers.If *a* = 4,*b* = 3,*c* = 2,and *d* = -1,then all the criteria of the problem are fulfilled,and Q uantity A is greater.B ut if *a* = 4,*b* = 3,*c* = -5,and *d* = -10,then all the criteria of the problem are still fulfilled,but Q uantity B is greater.

8. $\frac{6}{1}$ **(or any equivalent fraction).**Solve one equation for a single variable,and substitute into the other equation:

Eq.(1): 12*b* = 2*g*	Eq.(2): 4*g* - 3*b* = 63
12*b* = 2*g*	
6*b* = *g*	Isolate *g* in Eq.(1).D ivide by 2.
4(6*b*) - 3*b* = 63	Substitute (6*b*) for *g* in Eq.(2).
24*b* - 3*b* = 63	Solve for *b*.Sim plify.
21*b* = 63	C om bine like term s.
b = 3	D ivide by 21.
12(3) = 2*g*	Substitute (3) for *b* in Eq.(1).Solve for *g*.
36 = 2*g*	Sim plify.
g = 18	D ivide by 2.

b = 3 and *g* = 18,so $\frac{g}{b} = 6$.

9.**II and III only.**To sim plify $81^3 + 27^4$,note that both bases are pow ers of 3.R ew rite the bases and com bine.

$81^3 + 27^4 =$
$(34)^3 + (3^3)^4 =$
$3^{12} + 3^{12} =$
$3^{12}(1 + 1) =$
$3^{12}(2)$

Since $3^{12}(2)$ appears in the choices,this is one answ er.H ow ever,this is an “indicate all” question,so you should check w hether any other choices are equivalent.O ne other choice,$9^6(2)$,also qualifies,since $9^6(2) = (3^2)^6(2) = 3^{12}(2)$.

10.**(B).**26% of the anim als are m am m als,and about a quarter *of those* w ere raised in the w ild: $\frac{1}{4}$ of 26% = about 6.5% .8% of all the anim als are am phibians,and about three quarters *of those* w ere raised in captivity: $\frac{3}{4}$ of 8% = about 6 % .In total,these tw o categories account for about 12% of all the zoo’s anim als.

11.**(B).**To solve this question,im agine that there w ere originally 100 anim als in the zoo.If the zoo gives aw ay all the insects and fish,then there are 38 few er anim als (20 + 18) in the zoo,or 62.B ut there are still 10 birds,w hich now m ake up about 16% of the zoo’s anim als (use your calculator to find this if you don’t feel com fortable estim ating).O f those,a little m ore than half w ere raised in the w ild.A m ong the choices,only 9% is a little m ore than half of 16% .

12.**(C).**If the zoo has 80 birds,w hich m ake up 10% of the total num ber of anim als at the zoo,then there are 800 anim als total.To correctly calculate how m any birds m ust be added,realize that any birds added increases not only the subtotal of 80 birds but also the total of 800 anim als.If adding new anim als (rather than trading reptiles for birds,for exam ple),you cannot sim ply double the num ber of birds to double the percent of the anim als that are birds!

Thus,use the follow ing inequality: 80

$$\frac{80 + x}{800 + x} \geq \frac{20}{100}$$

$100(80 + x) \geq 20(800 + x)$

$8{,}000 + 100x \geq 160{,}000\ 20x$
$80x \geq 8{,}000$
$x \geq 100$

A t least 100 birds m ust be added such that at least 20% of the anim als at the zoo w ould be birds (check: There w ould be 180 birds am ong 900 anim als,or 20% of the total).

13.**(C).**This question is a tricky one,because even though it never uses the w ord *average* or the w ord *ratio*,it’s m ore or less a com bined ratio and averages question.The trail m ix is nuts,chocolate,and dried fruit in a ratio of 3 : 1 : 2. For every 6 pounds of trail m ix,there are 3 pounds of nuts,1 pound of chocolate,and 2 pounds of dried fruit.

The cost of 6 pounds of trail m ix is $3x + y + 2z$.H ow ever,to solve for the cost of one pound,divide by 6.Y ou could also think of this as a kind of average:

A verage = (Sum)/(# of term s) = (3x + y + 2z)/6,w here each “term ” is a pound.

This is choice (C).A lternatively,pick num bers.For exam ple:

x = 6
y = 5
z = 2

In this exam ple,3 lbs.of nuts that cost x = 6 dollars per pound plus 1 lb.chocolate that costs y = 5 dollars per pound plus 2 lbs.dried fruit that costs z = 2 dollars per pound w ould cost:

3(6) + 1(5) + 2(2) = 27

Thus,6 pounds of trail m ix (3 lbs.nuts + 1 lb.chocolate + 2 lbs.dried fruit) w ould cost $27.So,1 pound w ould cost one-sixth of that: 27/6 or 9/2 dollars,w hich is $4.50.

N ow ,plug x = 6,y = 5,and z = 2 into the choices to see w hich answ er yields $4.50.O nly (C) w orks.

14.**(E).**Since $3^4 = 81$,$z = 81$.So,$(3^z)^z = (3^{81})^{81} = 3^{81\times 81} = 3^{6,561}$.

15.**I and IV only.**Since 0.03 is 100 tim es greater than 0.0003,w hen M aurice accidentally divided by 0.03 instead of 0.0003,he divided by a num ber 100 tim es too big.Thus,m ultiplying by 100 w ill correct the error.Thus,Statem ent I is correct.

H ow ever,dividing by any quantity is the sam e as m ultiplying by its reciprocal.So,m ultiplying by 100 is the sam e as dividing by 0.01.Thus,Statem ent IV is also correct.

A lternatively,pick a num ber.D ivide by both 0.03 and 0.0003,and then check each answ er to see w hich correct the error.For instance,suppose the original num ber w ere 12.

12 divided by 0.03 = 400 ← IN C O R R EC T R ESU LT
12 divided by 0.0003 = 40,000 ← C O R R EC T R ESU LT

N ow ,perform the operation in each answ er choice on the incorrect product,400,to see w hich operations turn that product into 40,000.O perations I and IV w ork.

16.**(C).**B egin by constructing a function describing the situation in the problem .U sing E for expenses,x for the num ber of clients,c for the expense per client,and f for fixed costs:

$E(x) = xc + f$

In w ords,expense as a function of the num ber of clients equals the num ber of clients m ultiplied by the variable cost per client,plus the fixed cost.

In 2009,the com pany served 450 clients and its total expense w as $830,000.Thus:

830,000 = 450*c* + *f*

In 2010,the com pany served 510 clients and its total expense w as $896,000.Thus:

896,000 = 510*c* + *f*

Since it is easier to isolate *f* than *c* in each equation,get *f* by itself for each equation and then set the opposite sides equal:

830,000 = 450*c* + *f*
f = 830,000 - 450*c*

896,000 = 510*c* + *f*
f = 896,000 - 510*c*

830,000 - 450*c* = 896,000 - 510*c*
830,000 + 60*c* = 896,000
60*c* = 66,000
c = 1,100

Plug *c* = 1,100 into either equation to find *f*:

f = 830,000 -
450(1,100) *f* = 335,000

A lternatively,subtract $896,000 - $830,000 to get $66,000,w hich m ust be the cost difference betw een serving 450 clients and serving 510 clients (a difference of 60 clients).D ivide $66,000 by 60 clients to get $1,100,the variable cost per client.Then,m ultiply $1,100 × 450 = $495,000 to get the variable cost of serving 450 clients,not counting the fixed cost.Finally,subtract this figure from the total cost of serving 450 clients to get the fixed cost.$830,000 - $495,000 = $335,000.The num bers should look fam iliar;the point is that you can “reason through it” w ithout strictly setting up equations.

17.**(A).**First,algebraically m anipulate 4*x* + 5*y* = 9 into *y* = *m x* + *b* form at,w here *m* is the slope and *b* is the *y*-intercept.

4*x* + 5*y* = 9
5*y* = –4*x* + 9

$$y = -\frac{4}{5}x + \frac{9}{5}$$

Since $m = -\frac{4}{5}$,the slope is $-\frac{4}{5}$.Perpendicular lines have negative reciprocal slopes.Thus,the correct answ er has a slope of $\frac{5}{4}$.O nly choice (A) qualifies.

18.**(A).**If the tens digit is to be random ly selected from the digits 0 to 9,there are ten possibilities for the com pleted num ber.U sing your calculator,divide each by 9 to see w hich ones are m ultiples of 9:

809 ← not a m ultiple of 9
819 ← M U LTIPLE O F 9
829 ← not a m ultiple of 9
839 ← not a m ultiple of 9
849 ← not a m ultiple of 9
859 ← not a m ultiple of 9
869 ← not a m ultiple of 9
879 ← not a m ultiple of 9
889 ← not a m ultiple of 9
899 ← not a m ultiple of 9

The answ er is 1/10,or 0.1.

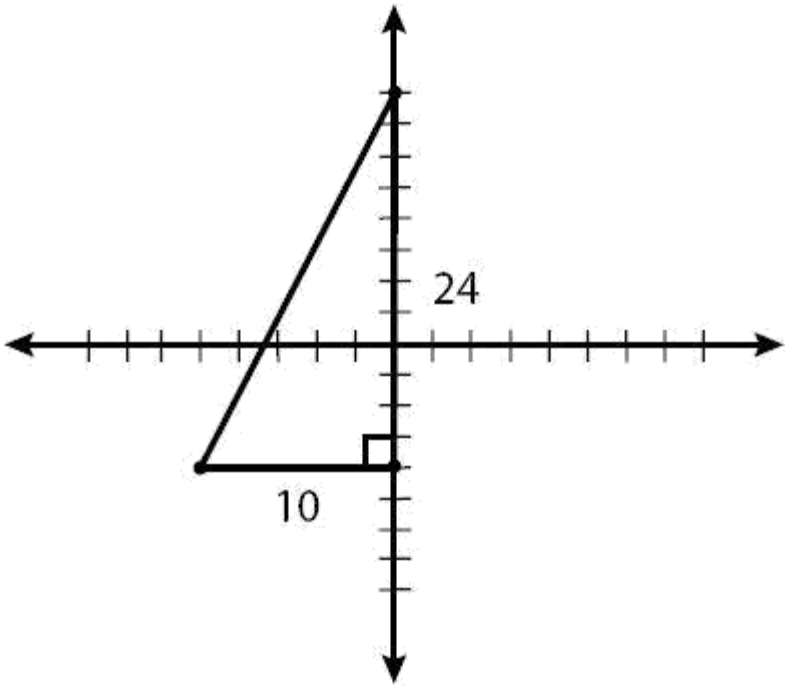
A lternatively,a num ber is divisible by 9 if the sum of its digits is a m ultiple of 9.The existing digits sum to 8 + 9 = 17,so the addition of 0 through 9 m eans that the sum of all three digits could be 17 through 26,inclusive.O nly one m ultiple of 9 (i.e.,18) is found in this range.

19.**(E).**B ecause the circle is centered at (0,0) and passes through (0,-16),the radius of the circle is 16.Point *P* lies on the circle and the *y*-axis,so it lies exactly one radius above the origin.Point *P* 's coordinates are therefore (0,16). To find the distance betw een (0,16) and (-10,-8),either use the distance form ula,or draw a graph and m ake a right triangle on w hich you can use the Pythagorean theorem .

From the distance form ula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(-10-0)^2 + (-8-16)^2}$$
$$d = \sqrt{(-10)^2 + (-24)^2}$$
$$d = \sqrt{676}$$
$$d = 26$$

To use the triangle m ethod,plot (0,16) and (-10,-8),then drop a line dow n from (0,16) to m ake a right triangle.To do so,you w ill need to add the third point (0,-8).



U se the coordinates to determ ine the lengths of the legs,then use the Pythagorean theorem (the hypotenuse is *d*):

$24^2 + 10^2 = d^2$
$576 + 100 = d^2$
$676 = d^2$
$d = 26$

20.**(C).**If $f(-0.5) = 0$,then the answ er is 0 w hen $x = -0.5$.For each choice,plug in -0.5 for x.O nly if the result is 0 could the choice be $f(x)$.

(A) $2x + 2 = 2(-0.5) + 2 = -1 + 2 = 1$
(B) $4x - 2 = 4(-0.5) - 2 = -2 - 2 = -4$
(C) C O R R EC T.$4x^2 - 1 = 4(-0.5)^2 - 1 = 4(0.25) - 1 =$
$1 - 1 = 0$ (D) $x^2 - 1 = (-0.5)^2 - 1 = 0.25 - 1 = -0.75$
(E) $(-x)^2 - 2.5 = (-(-0.5))^2 - 2.5 = (0.5)^2 - 2.5 = 0.25 - 2.5 = -2.25$