A verages, W eighted A verages, M edian, and M ode

For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given), the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box, you are to enter your ow n answ er in the box. For questions follow ed by fraction-style num eric entry boxes, you are to enter your answ er in the form of a fraction. Y ou are not required to reduce fractions. For exam ple, if the answ er is 1/4, you m ay enter 25/100 or any equivalent fraction.

A ll num bers used are real num bers. A ll figures are assum ed to lie in a plane unless otherw ise indicated. G eom etric figures are not necessarily draw n to scale. Y ou should assum e, how ever, that lines that appear to be straight are actually straight, points on a line are in the order show n, and all geom etric objects are in the relative positions show n. C oordinate system s, such as *xy*-planes and num ber lines, as w ell as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* draw n to scale. A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1. H usain and D ino have an average of \$20 each. D ino w ins a cash prize, w hich raises their average to \$80. A ssum ing no other changes occurred, how m any dollars did D ino w in?

\$

2.

Janani is 6 centim eters taller than Preeti, w ho is 10 centim eters taller than R ey.

Q uantity A	**Q uantity B**
The average height of the three people	The m edian height of the three people

3. The average of Joelle’s five quiz scores is 88. W hat score does Joelle need to get on a sixth quiz to raise her average for all six quizzes to 90?

(A) 88

(B) 94
(C) 98
(D) 100
(E) 102

4.

The average of x and y is 55.The average of y and z is 75.

Q uantity A	**Q uantity B**
$z - x$	40

5.In C larice’s class,each test w eights her overall grade average three tim es as m uch as each quiz does.If C larice scored 88 and 94 on tw o quizzes,respectively,and she scored 90 on the only test,w hat is her current overall grade average?

6.W hat is the average of x,x - 6,and x + 12?

(A) x
(B) x + 2
(C) x + 9
(D) 3x + 6
(E) It cannot be determ ined from the inform ation given.

7.The average of four num bers is 12.If the set of num bers includes 9,11,and 12,w hat is the fourth num ber?

(A) 12
(B) 14
(C) 16
(D) 20
(E) 24

8.

For a set of 30 integers,the average is 30 and none of the integers are greater than 60.

Q uantity A	**Q uantity B**
The range of the set	30

9.If x is negative,w hat is the m edian of the list 20,x,7,11,3?

(A) 3
(B) 7
(C) 9
(D) 11
(E) 15.5

10.If the average of *n* and 11 is equal to 2*n*,then w hat is the average of *n* and $\frac{13}{3}$?

(A) 4
(B) 8
(C) 11
(D) 14
(E) 19

11.

Q uantity A	**Q uantity B**
The average of x - 3,x,x + 3,x + 4,and x + 11	The m edian of x - 3,x,x + 3,x + 4,and x + 11

12.John buys 5 books w ith an average price of $12.If John then buys another book w ith a price of $18,w hat is the average price of the 6 books?

(A) $12.50
(B) $13
(C) $13.50
(D) $14
(E) $15

13.Every w eek,R enee is paid 40 dollars per hour for the first 40 hours she w orks,and 80 dollars per hour for each hour she w orks after the first 40 hours.H ow m any hours w ould R enee have to w ork in one w eek to earn an average of 60 dollars per hour that w eek?

(A) 60
(B) 65
(C) 70
(D) 75
(E) 80

14.

A t a certain school,the 118 juniors have an average final exam score of 88 and the 100 seniors have an average final exam score of 92.

Q uantity A	**Q uantity B**
The average final exam score for all of the juniors and seniors com bined.	90

15.Last year a car dealership sold 640 cars over the entire year.This year,the dealership has sold an average of 32 cars per m onth for the first four m onths.W hat is the average num ber of cars sold per m onth over the entire 16-m onth period?

(A) 43
(B) 44
(C) 48
(D) 51
(E) 64

16.

Q uantity A

The average (arithm etic m ean) of *x*,*y*, and *z*

Q uantity B

The average (arithm etic m ean) of 0.5*x*,0.5*y*,and 0.5*z*

17.B alpreet’s quiz scores in English are 80,82,79 and 84.H er quiz scores in H istory are 90 and 71.W hat is the sum of the scores she w ould need to get on her next English quiz and her next H istory quiz to raise each class’ quiz score average to 85?

(A) 109
(B) 192
(C) 194
(D) 198
(E) 218

18.A aron’s first three quiz scores w ere 75,84,and 82.If his score on the fourth quiz reduced his average quiz score to 74,w hat w as his score on the fourth quiz?

19.Paco’s practice test scores are 650,700,630 and 640.W hat score on the 5th test w ould result in an average score of 660 for all 5 tests?

20.A quiz is scored from 0 to 110.JaeH a has 5 quiz scores: 90,95,88,84,92.W hat does the average on her next 2 quizzes need to be in order to bring her average for all 7 quizzes up to 95?

21.

The integer ages of the three children in the C hen fam ily range from 2 to 13,and no tw o children are the sam e age.

Q uantity A

The average age of all three children in the C hen fam ily

Q uantity B

10

22.

Four people have an average age of 18,and none of the people are older than 30.

Q uantity B

Q uantity A

The range of the four people's ages 25

23.

Set A consists of 5 num bers,w hich have an average value of 43.Set B consists of 5 num bers.

Q uantity A	**Q uantity B**
The value of *x* if the average of *x* and the 5 num bers in Set A is 46	The average of Set B if the average of the 10 num bers in Sets A and B com bined is 52

24.The average of 7 num bers is 12.The average of the 4 sm allest num bers in this set is 8,w hile the average of the 4 greatest num bers in this set is 20.H ow m uch greater is the sum of the 3 greatest num bers than the sum of the 3 sm allest num bers?

(A) 4
(B) 14
(C) 28
(D) 48
(E) 52

25.If the average of *a*,*b*,*c*,5,and 6 is 6,w hat is the average of *a*,*b*,*c*,and 13?

(A) 8
(B) 8.5
(C) 9
(D) 9.5
(E) It cannot be determ ined from the inform ation given.

26.The average (arithm etic m ean) of 8 num bers is 42.O ne of the num bers is rem oved from the set,and the resulting average (arithm etic m ean) of the rem aining num bers is 40.W hat num ber w as rem oved from the set?

(A) 26
(B) 28
(C) 50
(D) 54
(E) 56

27.The average of 13 num bers is 70.If the average of 10 of these num bers is 90,w hat is the average of the other 3 num bers?

(A) -130
(B) $\frac{10}{3}$
(C) 30
(D) 90
(E) 290

28.Tow n A has 6,000 citizens and an average (arithm etic m ean) of 2 radios per citizen.Tow n B has 10,000 citizens and an average (arithm etic m ean) of 4 radios per citizen.W hat is the average num ber of radios per citizen in both tow ns com bined?

G ive your answ er as a fraction.

29.

Joe’s quiz scores are 80,82,78,77,and 83.D ave’s quiz scores are 60,90,and 80.

Q uantity A	**Q uantity B**
The score Joe w ould need on his next quiz to increase his overall average score to 82.	The score D ave w ould need on his next quiz to increase his overall average score to 82.

30.Fiber X C ereal is 55% fiber.Fiber M ax C ereal is 70% fiber.Sheldon com bines an am ount of the tw o cereals in a single bow l of m ixed cereal that is 65% fiber.If the bow l contains 12 ounces of cereal,how m uch of the cereal, in ounces,is Fiber X ?

(A) 3
(B) 4
(C) 6
(D) 8
(E) 9

31.The average population in Tow n X w as recorded as 22,455 during the years 2000–2010,inclusive.H ow ever,an error w as later uncovered: the figure for 2009 w as erroneously recorded as 22,478,but should have been correctly recorded as 22,500.W hat is the average population in Tow n X during the years 2000–2010,inclusive, once the error is corrected?

L iquor B rand	**P ercent A lcohol**
D elicate Flow er	4%
Fine and D andy	6%
M onster Sm ash	12%

32.B ased on the chart above,w hich of the follow ing statem ents m ust be true?

Indicate <u>all</u> such statem ents.

☐ A cocktail m ade w ith one part Fine and D andy and tw o parts M onster Sm ash (and no other ingredients) w ill be m ore than 9% alcohol.
☐ A cocktail m ade w ith 11.5 gram s each of all three liquors (and no other ingredients) w ill be m ore than

7% alcohol.

☐ A cocktail m ade w ith one part D elicate Flow er,tw o parts Fine and D andy,and one part alcohol-free m ixer (and no other ingredients) w ill be m ore than 7% alcohol.

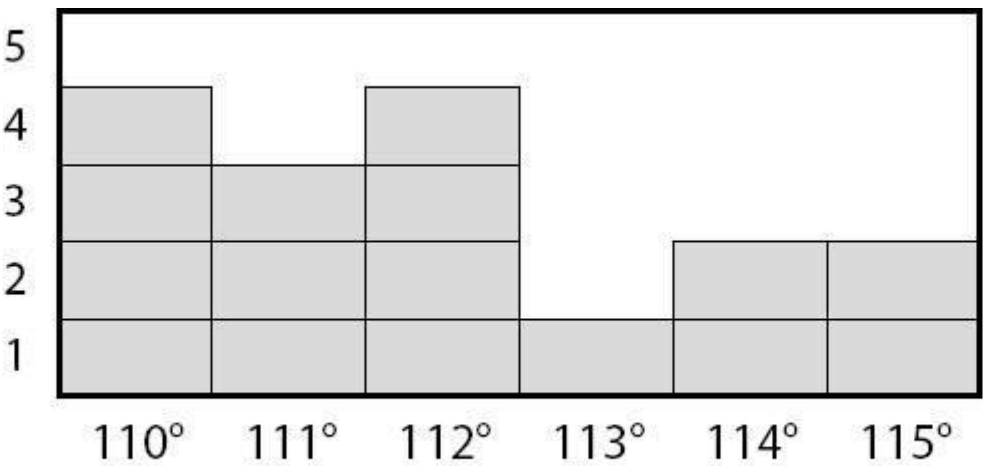
33.

$$S_n = 3n + 3$$

Sequence S is defined for all integers n such that $0 < n < 10{,}000$

<u>**Q uantity A**</u>	<u>**Q uantity B**</u>
The m edian of sequence *S*	The m ean of sequence *S*

34.The bar graph below displays the num ber of tem perature readings at each value from a sam ple,m easured in degrees Fahrenheit.W hat w as the average tem perature reading?



☐ degrees Fahrenheit

35.In a certain dance troupe,there are 55 w om en and 33 m en.If all of the w om en are 62 inches tall and all of the m en are 70 inches tall,w hat is the average height of the dancers in the troupe?

☐ inches

36.Set *A* : 1,3,5,7,9 Set *B* : 6,8,10,12,14

For the sets of num bers above,w hich of the follow ing statem ents are true?

Indicate <u>all</u> such statem ents.

☐ The m ean of Set *B* is greater than the m ean of Set *A* .
☐ The m edian of Set *B* is greater than the m edian of Set *A* .
☐ The standard deviation of Set *B* is greater than the standard deviation of Set *A* .
☐ The range of Set *B* is greater than the range of Set *A* .

37.Three people have $32,$72,and $98,respectively.If they pool their m oney then redistribute it am ong them ,w hat is the m axim um value for the m edian am ount of m oney?

(A) $72
(B) $85
(C) $98
(D) $101
(E) $202

38.

W eekly R evenue P er P roduct C ategory at O ffice Supply Store *X*

P roduct C ategory	W eekly R evenue in D ollars
Pens	164
Pencils	111
Legal Pads	199
Erasers	38
A verage of C ategories above	128

A ccording to the chart above,the average revenue per w eek per product category is $128.H ow ever,there is an error in the chart;the revenue for Pens is actually $176,not $164.W hat is the new ,correct average revenue per w eek per product category be,in dollars?

(A) 130
(B) 131
(C) 132
(D) 164
(E) 176

39.

A set of 7 integers has a range of 2,an average of 3,and a m ode of 3.

Q uantity A	**Q uantity B**
The third num ber in the set w hen the num bers are arranged in ascending order	The fifth num ber in the set w hen the num bers are arranged in ascending order

40.

Set *S* consists of the first 500 positive,even m ultiples of 7.

Q uantity A	**Q uantity B**
The average of the set	The m edian of the set

41.

The average of $3x$, x, and y is equal to $2x$

Quantity A	Quantity B
$2x$	y

42.The average age of the buildings on a certain city block is greater than 40 years old.If four of the buildings were built two years ago and none of the buildings are more than 80 years old,which of the following could be the number of buildings on the block?

Indicate all such numbers.

- ☐ 4
- ☐ 6
- ☐ 8
- ☐ 11
- ☐ 40

43.Four students contributed to a charity drive,and the average amounts contributed by each student was $20.If no student gave more than $25,what is the minimum amount that any student could have contributed?

$ ☐

44.

The average of 7 distinct integers is 12,and the least of these integers is -15.

Quantity A	Quantity B
The greatest that any of the integers could be	84

45.

Set N consists of the first 9 positive multiples of 3

Quantity A	Quantity B
The average of the first and last terms in the set	The average of the third and seventh terms in the set

46.The average of 15 consecutive integers is 88.What is the greatest of these integers?

☐

47.

The average of 5 integers is 10 and the range of the 5 integers is 10.

Quantity A	Quantity B
The median of the 5 integers	10

48.

3 numbers have a range of 2 and a median of 4.4

Quantity A	Quantity B
The greatest of the numbers	5.4

A verages, W eighted A verages, M edian, and M ode A nsw ers

1.**$120.**If the tw o people had an average of $20 each,they held a sum of 2($20) = $40.A fter D ino w ins a cash prize, the new sum is 40 + *p* and the new average is 80.Plug into the average form ula:

$$\text{A verage} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$80 = \frac{40 + p}{2}$$

$$160 = 40 + p$$

$$120 = p$$

D ino w on $120.

2.**(B).**Pick num bers that agree w ith the given height constraints.R ey is the shortest person,and if R ey is 100 cm tall, Preeti is 110 cm tall,and Janani is 116 cm tall.The average height is $\frac{100 + 110 + 116}{3} = 108.67$ (rounded to nearest 0.01).The m edian height is the m iddle height,w hich is 110.Q uantity B is greater.

A lternatively,note that Preeti's height is the m edian.Preeti's height is closer to Janani's than to R ey's.Since the average of Janani's and R ey's heights w ould be m idw ay betw een those heights,and Preeti's height is higher than that m iddle,the m edian is greater than the average.

3.**(D).**There are tw o w ays to solve this question.The first involves using the average form ula:

$$\text{A verage} = \frac{\text{Sum}}{\text{Number of Terms}}$$

If the average of Joelle's 5 quiz scores is 88,plug these num bers in and solve for the sum of the scores:

$$88 = \frac{\text{Sum}}{5}$$

$$88 \times 5 = \text{Sum}$$

$$440 = \text{Sum}$$

U se the average form ula again to solve for the sixth quiz score,*x*,keeping in m ind that the new average is 90 and the new num ber of quizzes is 6:

$$90 = \frac{440 + x}{6}$$

$90 \times 6 = 440 + x$
$540 = 440 +$
x $x = 100$

The other w ay to solve the question is by using the concept of residuals,or differences from the average.If 5 scores of 88 need to be brought up to the new average of 90,there are 5(2) = 10 points needed in the sum .So the sixth quiz score should be high enough to "give aw ay" those 10 points to the sum ,w hile still retaining 90 points for itself.That num ber is 100.

4.**(C).**Since the average of x and y is 55, $\frac{x+y}{2} = 55$,and $y + z = 150$.

Since the average of y and z is 75, $\frac{y+z}{2} = 75$,and $y + z = 150$.

Stack the tw o equations and subtract to cancel the y's and get $z - x$ directly:

$$
\begin{aligned}
z + y &= 150 \\
\underline{-(x + y} &\underline{= 110)} \\
z - x &= 40
\end{aligned}
$$

5.**90.4.**To account for the fact that tests w eight the grade three tim es as m uch as quizzes,include each test score as if it w ere three identical quizzes.So,2 quizzes and 1 test = 2 + 3 = 5 quizzes.

A verage = $\frac{\text{Sum}}{\text{Number of Terms}}$
A verage = (88 + 94 + 90 + 90 + 90)/5
A verage = 452/5 = 90.4

6.**(B).**The average form ula is just as easily applied to algebraic expressions as arithm etic ones:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$
$$\text{Average} = \frac{(x)+(x-6)+(x+12)}{3}$$
$$\text{Average} = \frac{3x+6}{3} = x+2$$

The correct answ er is (B).

7.**(C).**There are tw o w ays to solve this question.The first involves using the average form ula,plugging in 12 for the average and 4 for the num ber of term s.

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$12 = \frac{\text{Sum}}{4}$$

$$48 = \text{Sum}$$

The sum of the three know n term s is 9 + 11 + 12 = 32.If the total of all four num bers m ust be 48,then the m issing fourth num ber is 48 - 32 = 16.

The other w ay to solve uses the concept of residuals,or differences from the average.The average of 12 is the “balance point” of the four num bers.The 9 is -3 from this balance point on the num ber line,11 is -1 from this balance point,and 12 is on this balance point,because it’s equal to the average.Thus,the three know n term s are w eighted -3 + (-1) = -4 from the average,so the fourth term s needs to be 12 + 4 = 16 in order for the set to balance.

8.**(D).**There are infinite possibilities for a set of 30 integers less than or equal to 60,w ith an average of 30.For instance:

Exam ple 1: The set consists of fifteen 0’s and fifteen 60’s
In this exam ple,the range is 60 and the average is 30.

Exam ple 2: The set consists of fifteen 14’s and fifteen 16’s.
In this exam ple,the range is 2 and the average is 30.

The range could be greater or less than 30.

9.**(B).**The easiest w ay to start thinking about a question like this is to plug in a value and see w hat happens.If x = -1, the list looks like this w hen ordered from least to greatest:

-1,3,7,11,20

The m edian is 7.B ecause any negative x you pick w ill be the least term in the list,the order of the list w on’t change, so the m edian w ill alw ays be 7.

10.**(A).**This question can be quickly solved w ith the average form ula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$2n = \frac{n+11}{2}$$

$$4n = n + 11$$

$$3n = 11$$

$$n = \frac{11}{3}$$

Since $n = 11/3$, the average of n and $\frac{13}{3}$ is:

$$\frac{\frac{11}{3}+\frac{13}{3}}{2}=\frac{\frac{24}{3}}{2}=\frac{8}{2}=4$$

Or just notice that the midpoint between $\frac{11}{3}$ and $\frac{13}{3}$ is $\frac{12}{3}$, just as 12 is the midpoint between 11 and 13. The average is $\frac{12}{3}=4$.

11. **(C).** To find the median of the numbers, notice that they are already in order from least to greatest: $x - 3, x, x + 3, x + 4, x + 11$

The median is the middle, or third, term: $x + 3$.

Now find the average of the numbers:

$$\frac{(x-3)+(x)+(x+3)+(x+4)+(x+11)}{5}=\frac{5x+15}{5}=x+3$$

The median and the mean are both ($x + 3$).

12. **(B).** First, calculate the cost of the first 5 books.

Sum = (Average cost)(Number of books) = (\$12)(5) = \$60

Total cost of all 6 books = \$60 + \$18 = \$78
Total number of books = 6

Average = \$78/6 = \$13 per book.

13. **(E).** Let h = number of hours Renee would have to work. The average rate Renee gets paid is equal to the total wages earned divided by the total number of hours worked. Rene earns \$40 per hour for the first 40 hours, so she makes 40 × 40 = \$1,600 in the first 40 hours. She also earns \$80 for every hour after 40 hours, for additional pay of \$80(h - 40). The total number of hours worked is h.

$$\frac{1{,}600+80(h-40)}{h}=60$$

$1{,}600 + 80h - 3{,}200 = 60h$

Now isolate h:

$80h - 1{,}600 = 60h$
$-1{,}600 = -20h$
$80 = h$

Y ou could also notice that 60 is exactly halfw ay betw een 40 and 80.Therefore,R enee needs to w ork an equal num ber of hours at $40 per hour and $80 per hour.If she w orks 40 hours at $40 per hour,she also needs to w ork 40 hours at $80 per hour.

14.**(B).**This is a w eighted average problem .B ecause the num ber of juniors is greater than the num ber of seniors,the overall average w ill be closer to the juniors' average than the seniors' average.Since 90 is halfw ay betw een 88 and 92, and the w eighted average w ill be closer to 88,Q uantity B is larger.

It is not necessary to do the m ath because this is a Q uantitative C om parison question w ith a very convenient num ber as Q uantity B .H ow ever,you can actually calculate the overall average by sum m ing up all 218 scores and dividing by the num ber of people: $\frac{118(88)+100(92)}{118+100} = 89.83...$

15.**(C).**The dealership sold 640 cars last year.This year,the dealership has sold 32 cars per m onth for the first 4 m onths of this year,w hich is a total of 4(32) = 128 cars.N ow ,if you're thinking that there's a difference betw een selling an average of 32 cars per m onth and selling exactly 32 cars per m onth,you're right.H ow ever,it w on't m ake a difference to the total sum of cars sold over the w hole period,w hich is all that is needed to calculate the average.

O ver the entire 16-m onth period,the dealership sold 640 + 128 = 768.N ow use the average form ula to calculate the answ er:

A verage num ber of cars sold per m onth = $\frac{\text{Sum (= \# of cars sold)}}{\text{Number of Terms (= \# of months)}}$

A verage num ber of cars sold per m onth = $\frac{\text{768 total cars}}{\text{16 months}}$ = 48 cars/m onth

16.**(D).**The average of *x*,*y*,and *z* is $\frac{x+y+z}{3}$.C alculated sim ilarly,the average of 0.5*x*,0.5*y*,and 0.5*z* is exactly half that.If the sum of the variables is positive,Q uantity A is greater.H ow ever,if the sum of the variables is negative, Q uantity B is greater.If the sum of the variables is zero,the tw o quantities are equal.

17.**(C).**For B alpreet to raise her English average to 85:

$\frac{80+82+79+84+x}{5}$ =
85 325 + x = 425
x = 100 +

For B alpreet to raise her history average to 85:

$$\frac{90+71+y}{3} = 85$$

$161 + y = 255$

$y = 94$

$x + y = 100 + 94 = 194$

The correct answ er is (C).

18.**55.**To find A aron’s fourth quiz score:

$$\frac{75+84+82+x}{4} = 74$$

$241 + x = 296$

$x = 55$

19.**680.**To find the score Paco w ould need on his 5th test:

$$\frac{650+700+630+640+x}{5} = 660$$

$2{,}620 + x = 3{,}300$

$x = 680$

20.**108.**To find the score JaeH a w ould need to average on her next 2 quizzes to bring her total average up to a 95:

$$\frac{90+95+88+84+92+x+y}{7} = 95$$

$449 + x + y = 665$

$x + y = 216$

N ote that it is not necessary to determ ine *x* and *y* individually.Since the tw o new quiz scores sum to 216,their average is $\frac{216}{2} = 108$.

21.**(B).**Since there are only three children and the range is from 2 to 13,one child m ust be 2,one m ust be 13,and the other child’s age m ust fall som ew here in the m iddle.

If the average of the children’s ages w ere 10,as in Q uantity B ,the sum of the three ages w ould be 30.Subtract 2 and 13 from 30 to get that the third child w ould need to be 15.This is not possible,because the m iddle child cannot be older than the 13-year-old.Since this age is too great,the true average age m ust be less,and Q uantity B is greater.

A lternatively,since no tw o children are the sam e age,the m iddle child has a m axim um age of 12.If that child w ere 12, the average age w ould be $\frac{2+12+13}{3} = \frac{27}{3} = 9$.Thus,the true average age m ust be 9 or less,and Q uantity B is greater.

22.**(D).**If 4 people have an average age of 18,then the sum of their ages is 4 × 18 = 72.Since the question is about range,try to m inim ize and m axim ize the range.M inim izing the range is easy — if everyone w ere exactly 18,the average age w ould be 18 and the range w ould be 0.So clearly,the range can be sm aller than 25.

To m axim ize the range,m ake the oldest person the m axim um age of 30,and see w hether the youngest person could be just 1 year old w hile still obeying the other rules of the problem : the sum of the ages is 72 and,of course,no one can be a negative age.

O ne such set: 1,20,21,30

This is just one exam ple that w ould w ork.In this case,the range is 30 - 1 = 29,w hich is greater than 25.

The correct answ er is (D).

23.**(C).**If the average of the 5 num bers in Set A is 43,the sum of Set A is (5)(43) = 215.

For Q uantity A ,use the A verage Form ula.Sum all 6 num bers,and divide by 6:

$$\frac{\text{sum of the 5 numbers in Set A} + x}{6} = 46$$

$$\frac{215 + x}{6} =$$

46 215 + *x* =
276 *x* = 61

For Q uantity B ,use the A verage Form ula again:

$$\frac{\text{sum of Set A} + \text{sum of Set B}}{10} =$$

52 215 + sum of Set B = 520
sum of Set B = 305

The average of the 5 num bers in Set B is thus $\frac{305}{5}$ = 61.

A lternatively,you could note that each set of 5 num bers has the sam e “w eight” in the average of all 10 num bers.The average of Set A is 43,w hich is 52 - 43 = 9 below the average of all 10 num bers.The average of Set B m ust be 9 above the average of all 10 num bers: 52 + 9 = 61.

24.**(D).**U sing the average form ula, $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$,build three separate equations:

A ll 7 num bers:

$$12 = \frac{\text{Sum of all 7 numbers}}{7}$$

Sum of all 7 num bers = 84

The 4 sm allest num bers:

$$8 = \frac{\text{Sum of the 4 smallest numbers}}{4}$$

Sum of the 4 sm allest num bers = 32

The 4 greatest num bers:

$$20 = \frac{\text{Sum of the 4 greatest numbers}}{4}$$

Sum of the 4 greatest num bers = 80

There are only 7 num bers,yet inform ation is given about the 4 sm allest and the 4 greatest,w hich is a total of 8 num bers! The m iddle num ber has been counted tw ice— it is included in both the 4 greatest and the 4 sm allest.

The sum of all 7 num bers is 84,but the sum of the 4 greatest and 4 sm allest is 80 + 32 = 112.The difference can only be attributed to the double counting of the m iddle num ber in the set of 7: 112 - 84 = 28.

The m iddle num ber is 28,so subtract it from the sum of the 4 sm allest num bers to get the sum of the 3 sm allest num bers: 32 - 28 = 4.

N ow subtract the m iddle num ber from the sum of the 4 greatest num bers to get the sum of the 3 greatest num bers: 80 - 28 = 52.

The difference betw een the sum of the 3 greatest num bers and the sum of the 3 sm allest num bers is 52 - 4 = 48.

25.**(A).**Since $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$:

$$6 = \frac{a+b+c+5+6}{5}$$

$30 = a + b + c + 11$

$19 = a + b + c$

It is not necessary, or possible, to determine the values of a, b, and c individually. The second average includes all three variables, so the values will be summed again anyway.

$$\text{Average} = \frac{a+b+c+13}{4}$$

$$\text{Average} = \frac{19+13}{4}$$

$$\text{Average} = \frac{32}{4} = 8$$

26. **(E).** There are two ways to solve this question. The first involves using the average formula: Average = $\frac{\text{Sum}}{\text{Number of Terms}}$ or Sum = Average × Number of Terms.

If the average of 8 numbers is 42, the sum of all 8 numbers = 42 × 8 = 336.

After removing one number, the new average of the remaining 7 numbers is 40. So, the sum of the remaining 7 numbers = 40 × 7 = 280.

The number that was removed accounts for the difference in these sums, so the number that was removed is 336 - 280 = 56.

The other way to solve the question is by using the concept of residuals, or differences from the average. A number was removed, and it caused the average of the remaining 7 numbers to drop by 2 (from 42 to 40). That requires a 7 × 2 = 14 point drop in the sum. The removed number must be 14 more than the preexisting average. That would be 42 + 14 = 56.

27. **(B).** $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$ or Sum = Average × Number of Terms.

The average of 13 numbers is 70, so:

Sum of all 13 terms = 70 × 13 = 910

The average of 10 of these numbers is 90, so:

Sum of 10 of these numbers = 90 × 10 = 900

Subtract to find the sum of “the other 3 numbers”: 910 - 900 = 10

Average of the other 3 numbers $\frac{\text{Sum}}{\text{Number of Terms}} = \frac{10}{3}$.

28. $\frac{13}{4}$ **(or any equivalent fraction).** To find this weighted average, you must find the sum of all the radios in Towns A and B, and divide by the total number of people in both towns:

$$\text{Average} = \frac{6{,}000(2)+10{,}000(4)}{16{,}000}$$

Cancel three zeros from each term:

$$\text{Average} = \frac{6(2)+10(4)}{16}$$

$$\text{Average} = \frac{52}{16}$$

This reduces to $\frac{13}{4}$, although you are not required to reduce.

29. **(B).** Average = $\frac{\text{Sum}}{\text{Number of Terms}}$. Set the unknown final test score as x for Joe and y for Dave.

Quantity A:

$$82 = \frac{80+82+78+77+83+x}{6}$$

$492 = 400 + x$
$x = 92$

Quantity B:

$$82 = \frac{60+90+80+y}{4}$$

$328 = 230 + y$
$y = 98$

30. **(B).** Use the weighted average formula to get the ratio of Fiber X to Fiber Max:

$\frac{0.55x+0.70m}{x+m} = 0.65$, where x is the amount of Fiber X and m is the amount of Fiber Max.

This is not that different from the regular average formula— on the top, there is the total amount of fiber (55% of Fiber X and 70% of Fiber Max), which is divided by the total amount of cereal ($x + m$) to get the average. Simplify by multiplying both sides by ($x + m$):

$0.55x + 0.70m = 0.65(x + m)$

$0.55x + 0.70m = 0.65x + 0.65m$

If you w ish,you can m ultiply both sides of the equation by 100 to elim inate all the decim als:

$55x + 70m = 65x +$
$65m$ $55x + 5m = 65x$
$5m = 10x$

$\frac{m}{x} = \frac{10}{5}$ or $\frac{2}{1}$

Since m and x are in a 2 to 1 ratio,2/3 of the total is m and 1/3 of the total is x.Since the total is 12 ounces,Fiber X accounts for $\frac{1}{3}(12) = 4$ ounces of the m ixed cereal.

O ne shortcut to this procedure is to note that the w eighted average (65%) is 10% aw ay from Fiber X 's percent and 5% aw ay from Fiber M ax's percent.Since 10 is tw ice as m uch as 5,the ratio of the tw o cereals is 2 to 1.H ow ever,it is a 2 to 1 ratio of Fiber M ax to Fiber X ,not the reverse! W hichever num ber is closer to the w eighted average (in this case,70% is closer to 65%) gets the larger of the ratio parts.Since the ratio is 2 to 1 (Fiber M ax to Fiber X),again, 1/3 of the cereal is Fiber X and $\frac{1}{3}(12) = 4$.

31.**22,457.**There is a sim ple shortcut for a change to an average.The figure for 2009 w as recorded as 22,478,but actually should have been recorded as 22,500.Thus,22 people in that year w ere not counted.Thus,the sum should have been 22 higher w hen the average w as originally calculated.

2000–2010,inclusive,is 11 years (subtract low from high and then add 1 to count an inclusive list of consecutive num bers).W hen taking an average,you divide the sum by the num ber of things being averaged (in this case,11).So the shortcut is to take the change to the sum and "spread it out" over all of the values being averaged by dividing the change by the num ber of things being averaged.

D ivide 22 by 11 to get 2.The average should have been 2 higher.Thus,the correct average for the 11 year period is 22,457.

A lternatively,the traditional m ethod: 22,455 × 11 years = 247,005,the sum of all 11 years' recorded populations. A dd the 22 uncounted people: the corrected sum w ould be 247,027.D ivide by 11 to get the real average: 22,457. (N ote that w hile the traditional m ethod is faster to explain,the shortcut is faster to actually execute!)

32.**I and II only.**This is a w eighted average problem .C onsider the statem ents individually:

I.TR U E .A cocktail m ade w ith one part Fine and D andy and tw o parts M onster Sm ash w ill have a percent alcohol equal to the average of 6% ,12% ,and 12% ,or $\frac{6\% + 12\% + 12\%}{3} = \frac{30\%}{3} = 10\%$.(C ount the M onster Sm ash tw ice since the cocktail contains tw ice as m uch of it.)

II.TR U E .The 11.5 gram s is irrelevant— all that m atters is that equal am ounts of each liquor w ere used,so there is no need to w eight the average.Take an ordinary average of 4,6,and 12 percent.The average is 7.333… % .

III.FA LSE .To find the percent alcohol for a cocktail m ade w ith one part D elicate Flow er (4% alcohol),tw o parts Fine and D andy (6% alcohol) and one part m ixer (0% alcohol),average 4,6,6,and 0.(C ount the 6 tw ice since tw o parts Fine and D andy w ent into the cocktail versus one part of each of the other com ponents.) The average of 4,6,6,and 0 is 4.If you got this w rong,you m ay have ignored the 0% alcohol m ixer.Y ou cannot ignore the effect of a zero in an average— a zero can often low er an average considerably.A lternatively,note that none of the com ponents of this cocktail have m ore than 6% alcohol.The resulting drink cannot have a greater concentration of alcohol than any of its com ponents.

33.**(C).**Sequence *S* is an evenly spaced set,w hich can be seen by plugging in a few *n* values:

$S_1 = 3(1) + 3 = 6$
$S_2 = 3(2) + 3 = 9$
$S_3 = 3(3) + 3 = 12\ldots$

Term s increase by three every tim e *n* increases by 1;this m eets the definition of an evenly spaced set.For A N Y evenly spaced set,the m edian equals the m ean.

34.**112.**This is a w eighted average problem .Y ou C A N N O T sim ply average 110,111,112,113,114,and 115.Y ou m ust take into account how m any tim es each num ber appears.The chart is really another w ay of w riting:

110,110,110,110
111,111,111
112,112,112,112
113 114,114
115,115

In other w ords,the average tem perature reading is really an average of 16 num bers.The easiest w ay to do this is:

$$\frac{4(110)+3(111)+4(112)+1(113)+2(114)+2(115)}{16}$$

U se your calculator— the correct answ er is 112.

35.**65.**This is a w eighted average problem .Y ou C A N N O T sim ply average 62 and 70.Y ou m ust take into account how m any tim es each num ber appears (55 and 33 tim es,respectively).Y ou are actually averaging 88 num bers:

$$\frac{55(62)+33(70)}{88}=$$

U se your calculator.The correct answ er is 65.

36.**I and II only.**In both sets,the num bers are evenly spaced.M oreover,both sets are evenly spaced by the sam e am ount (adjacent term s increase by 2) and have the sam e num ber of term s (5 num bers in each set).The difference is that each term in Set B is 5 greater than the corresponding term in Set A (i.e.,6 - 1 = 5,8 - 3 = 5,etc.)

In evenly spaced sets,the m ean = m edian.A lso,if an evenly spaced set has an odd num ber of num bers,the m ean and m edian both equal the m iddle num ber.(W hen a set has an even num ber of num bers,the m ean and m edian both equal the average of the 2 m iddle num bers).

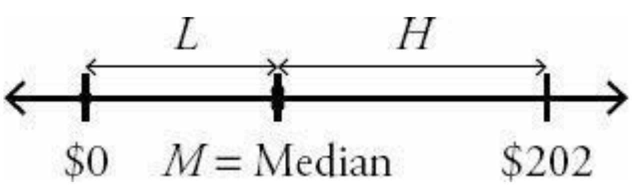
So,Set A has m ean and m edian of 5 and Set B has m ean/m edian = 10.Statem ent I and Statem ent II are true.

Since Sets A and B are equally spaced and have the sam e num ber of elem ents,their standard deviations are equal (that is,Set A is exactly as spread out from its ow n m ean as Set B is from its ow n m ean),so Statem ent III is false.

Since 9 - 1 = 8 and 14 - 6 = 8,the ranges are equal and Statem ent IV is false.

37.**(D).**The pool of m oney is \$32 + \$72 + \$98 = \$202.A fter the redistribution,each person w ill have an am ount betw een \$0 and \$202,inclusive.C all the am ounts *L*,*M* ,and *H* (low ,m edian,high).To m axim ize *M* ,m inim ize *L* and *H* .

The m inim um value for *H* is,in fact,*M* .The “highest” of the three values can actually be equal to the m edian (if *H* w ere low er than *M* ,the term order and therefore w hich num ber is the m edian w ould change,but if *H* = *M* ,*M* can still be the m edian).



Minim um *L* = \$0
Minim um *H* = *M*

M axim um *M* = Total pool of m oney - M inim um *L* - M inim um *H*
M = \$202 - \$0 -
M 2*M* = \$202
M = \$101

The correct answ er is (D).

38.**(B).**The chart provides the average and the num ber of product categories.If the incorrectly-calculated average w as \$128 for the 4 categories,then the sum w as 4 × 128 = \$512.Since the revenue for Pens w as actually \$176,not \$164, the sum should have been \$12 higher.Thus,the correct sum is \$524.D ivide by 4 to get \$131,the answ er.

A lternatively,notice that the \$128 average given in the question stem actually does a lot of w ork for you.If \$164 jum ps up to \$176,that’s an increase of \$12.D istributed over the four categories,it w ill bring the overall average up by \$3,from \$128 to \$131.

39.**(C).**A set of 7 integers w ith a range of 2 and an average of 3 could consist of only these possibilities:

Exam ple 1:2,2,2,3,4,4,4

Exam ple 2:2,2,3,3,3,4,4
Exam ple 3:2,3,3,3,3,3,4

H ow ever,the m ode of the set m ust be 3 for this set.The m ode is the m ost com m on num ber in the set.Exam ple 1 has tw o m odes: 2 and 4,so Exam ple 1 is invalid for this question.O nly Exam ples 2 and 3 rem ain.

In both valid exam ples,the third and fifth num bers in each set are 3.The tw o quantities are equal.

40.**(C).**The set begins 14,28,42,etc.H ow ever,the specific num bers— and even the num ber of elem ents— in the set are irrelevant,since the average equals the m edian for any evenly spaced set.C onsecutive m ultiples (in this case,of 14) are evenly spaced.

41.**(C).**W rite “the average of 3*x*,*x*,and *y* is equal to 2*x*” as an equation:

$$\frac{3x + x + y}{3} =$$

2*x* 4*x* + *y* = 6*x*
y = 2*x*

The tw o quantities are equal.

42.**8,11,40.**B ecause A verage = $\frac{\text{Sum}}{\text{Number of Terms}}$,this question about averages depends both on *x*,the total num ber of buildings on the block,and on the sum of the building ages.The 4 buildings that are 2 years old have a total age of 4(2),and the (*x* - 4) other buildings have a total age of (*x* - 4)(no m ore than 80).

A verage age = $\frac{4(2)+(x-4)(\text{no more than } 80)}{x}$

H aving m any 80 year old buildings on the block w ould raise the average m uch closer to 80.(For instance,if there w ere a m illion 80-year-old buildings and four 2-year-old buildings,the average w ould be very close to 80 years old.) So,there is som e m inim um num ber of older buildings that could raise the average above 40.

Ignore the “greater than” 40 years old constraint on the average building age for a m om ent.W hat is the m inim um *x* needed to be to m ake the average age exactly 40 w hen the age of the other buildings is m axim ized at 80?

40*x* = 8 + (*x* - 4)
(80) 40*x* = 8 + 80*x*
- 320 -40*x* = -312
x = $\frac{312}{40} = 7.8$

B ecause there can’t be a partial building and the age of the buildings can’t be greater than 80,*x* m ust be at least 8 to bring the average age up over 40.(Y ou w ould need even m ore buildings to bring the average above 40 if those older buildings w ere only betw een 50 and 70 years old,for exam ple.)

A lternatively,test the answ er choices.Try the first choice,4 buildings.Since 4 of the buildings on the block are only 2 years old,this choice can’t w ork— the average age of the buildings w ould be 2.

Try the second choice.W ith 6 total buildings,there w ould be four 2-year-old buildings,plus tw o others.To m axim ize the average age,m axim ize the ages of the tw o other buildings by m aking them both 80 years old.

$$\frac{4(2)+2(80)}{6} = 28$$

Since the average is less than 40 years old,this choice is not correct.

Try the third choice.W ith 8 total buildings,there w ould be the four 2-year-old buildings,plus four others.To m axim ize the average age,m axim ize the ages of the four other buildings by m aking them each 80 years old:

$$\frac{4(2)+4(80)}{8} = 41$$

Since the average age is greater than 40 years old,this choice is correct.Since the other,greater choices allow the possibility of even m ore 80-year-old buildings,increasing the average age further,those choices are also correct.

43.**$5.**The average of four values is $20.Thus,the sum of the four values is $80.To determ ine the m inim um contribution one student could have given,m axim ize the contributions of the other three students.If the three other students each gave the m axim um of $25,the fourth student w ould only have to give $5 to m ake the sum equal to $80.

44.**(A).**If the average of 7 integers is 12,then their sum m ust be 7 × 12 = 84.To m axim ize the largest of the num bers,m inim ize the others.

The sm allest num ber is -15.The integers are distinct (that is,different from each other),so the m inim um values for the sm allest 6 integers are -15,-14,-13,-12,-11,and -10.To find the m axim um value for the 7th integer,sum -15,-14,-13,-12,-11,-10,and *x*,w hile setting that sum equal to 84:

$- 15 + (-14) + (-13) + (-12) + (-11) + (-10) + x =$
$84 -75 + x = 84$
$x = 159$

Q uantity A is greater.

45.**(C).**In an evenly spaced set,the m iddle num ber is also the average.N um bers equally spaced on opposite sides of the m iddle w ill also average to the average of the w hole set.Thus,the answ er is (C).H ere is the set w ritten out,w ith the m edian underlined:

3 6 9 12 $\underline{15}$ 18 21 24 27

In an evenly spaced set,the m edian is equal to the average.Thus,15 is the average.It is also the case that 12 and 18 average to 15.So do 9 and 21.So do 6 and 24.A nd so do 3 and 27.

B oth quantities are equal to 15.

46.**95.**In any evenly spaced set,the average equals the m edian.Thus,88 is the m iddle num ber in the set.Since the set has 15 elem ents,the 8th elem ent is the m iddle one.

Low est 7 integers:	81	82	83	84	85	86	87
M iddle integers:	88						
G reatest 7 integers:	89	90	91	92	93	94	95

The largest integer in the list is 95.If you w ere confident about the process,you could skip listing the integers. Instead you could reason that to go from 8th integer to the 15th integer,you m ust add 7: 88 + 7 = 95.

47.**(D).**If the average of 5 integers is 10,their sum m ust be 50.The range is given as 10.Try to m ake tw o exam ples w here this is true,but w here the m edians are as different as possible.

Exam ple 1: 5,10,10,10,15

In this case,Q uantity A is equal to Q uantity B .

Is there a list such that the average is still 10 and the range is still 10,but the m edian is som ething else? Try adjusting the three m iddle num bers w hile keeping 5 as the least integer and 15 as the greatest integer.To adjust the num bers w ithout disturbing the average,anything you subtract from one num ber should be added to another num ber,so the sum stays constant.

Exam ple 2: 5,6,12,12,15

H ere,4 w as subtracted from the second term ,and then 2 w as added to each of the third and fourth term s.The average and range still each equal 10,but now the m edian is 12.The answ er is (D).

48.**(D).**If the set has an odd num ber of term s,then the m edian is the m iddle num ber.So,the m iddle num ber is 4.4. The set has a range of 2.The other tw o num bers could be 2 apart and also equally distributed around 4.4:

Exam ple 1: 3.4,4.4,5.4

H ere,the tw o quantities are equal.

O r,the tw o other num bers could be 2 apart but both a bit higher,or both a bit low er.

Exam ple 2: 4.3,4.4,6.3

Exam ple 3: 2.5,4.4,4.5

Thus,Q uantity A could be equal to,less than,or greater than Q uantity B .The correct answ er is (D).