Advanced Quant

The follow ing questions are *extrem ely* advanced for the G R E .W e have included them by popular dem and — students w ho are aim ing for perfect m ath G R E scores often w ish to practice on problem s that m ay even be harder than any they see on the real G R E .W e estim ate that a G R E test taker w ho does w ell on the e first m ath section and therefore is given a difficult second section m ight see one or tw o problem s,at m ost,of this level of difficulty.

If you are N O T aim ing for a perfect m ath score,w e absolutely recom m end that you skip these problem s!

If you are taking the G R E for business school or another quantitative program ,you m ay w ish to attem pt som e of these problem s.For instance,you m ight do one or tw o of these problem s — think of them as "brain teasers" — to cap off a study session from elsew here in the book.(For reference,getting 50% of these problem s correct w ould be a pretty incredible perform ance!)

Even if you *are* aim ing for a perfect m ath score,though,please m ake sure you are *flaw less* at the types of m ath problem s in the *rest* of this book before you w ork on these.Y ou w ill gain far m ore points by reducing silly m istakes (through practice,steady pacing,and good organization) on easy and m edium questions than by focusing on ultra-hard questions.

For m ore such problem s,visit the M anhattan Prep G R E blog for our w eekly C hallenge Problem .(A ccess to the archive of over 100 C hallenge Problem s is available to our course and G uided Self-Study students for free and to the public for a sm all fee.)

That said,attem pt these A dvanced Q uant problem s — if you dare!

1.The probability of rolling any num ber on a w eighted 6-sided die,w ith faces num bered 1 through 6,is directly proportional to the num ber rolled.W hat is the expected value of a single roll of the die?

(A) $4\frac{1}{6}$

(B) $4\frac{1}{3}$

(C) $4\frac{1}{2}$

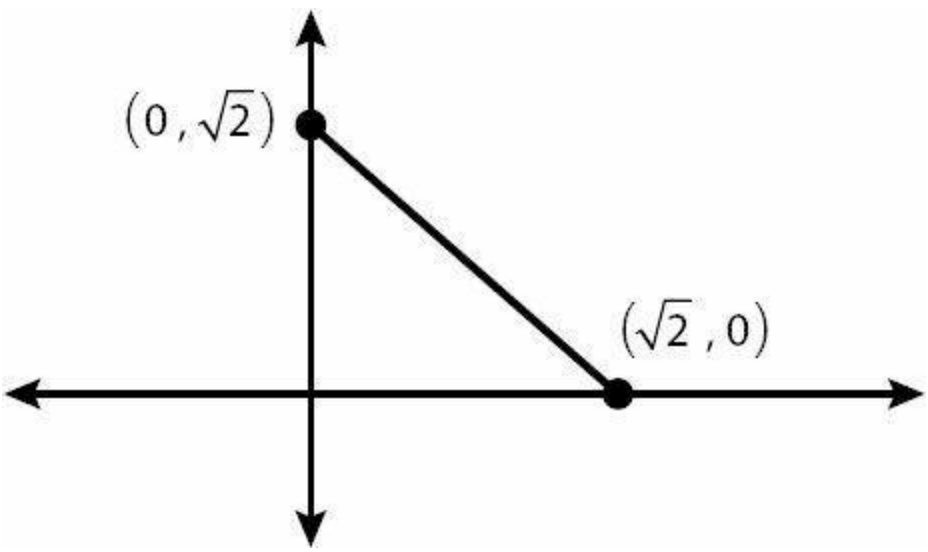
(D) $4\frac{2}{3}$

(E) $4\frac{5}{6}$

2.21 people per m inute enter a previously em pty train station beginning at 7:00:00 p.m .(7 o'clock and zero seconds). Every 9 m inutes beginning at 7:04:00 p.m .,a train com es and everyone w ho has entered the station in the last 9 m inutes gets on the train.If the last train com es at 8:25:00,w hat is the average num ber of people w ho get on each of the trains leaving from 7:00:00 to 8:25:00?

(A) 84
(B) 136.5
(C) 178.5
(D) 189
(E) 198.5

3.The random variable X has the follow ing continuous probability distribution in the range $0 \leq X \leq \sqrt{2}$,as show n in the coordinate plane w ith X on the horizontal axis:



The probability that $X < 0$ = the probability that $X > \sqrt{2} = 0$.

W hat is the m edian of X ?

(A) $\frac{\sqrt{2}-1}{2}$

(B) $\frac{\sqrt{2}}{4}$

(C) $\sqrt{2}-1$

(D) $\frac{\sqrt{2}+1}{4}$

(E) $\frac{\sqrt{2}}{2}$

4.

$x < 0$

Q uantity A	**Q uantity B**
$x^2 - 5x + 6$	$x^2 - 9x + 20$

5.If x is a positive integer,w hat is the units digit of $(24)^{5 + 2x}(36)^{6}(17)^{3}$?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 8

6.A rectangular solid is changed such that the w idth and length are increased by 1 inch apiece and the height is decreased by 9 inches.D espite these changes,the new rectangular solid has the sam e volum e as the original rectangular solid.If the w idth and length of the original rectangular solid are equal and the height of the new rectangular solid is 4 tim es the w idth of the original rectangular solid,w hat is the volum e of the rectangular solid?

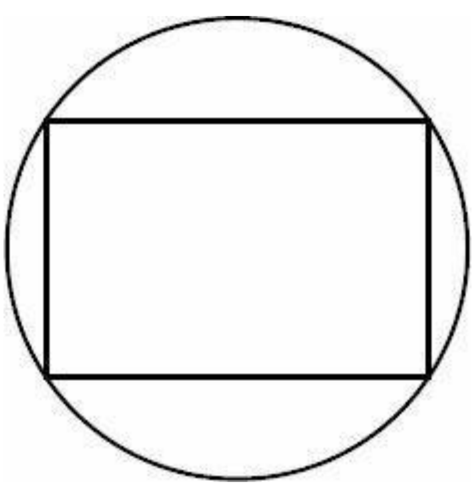
(A) 18
(B) 50
(C) 100
(D) 200
(E) 400

7.The sum of all solutions for x in the equation $x^2 - 8x + 21 = |x - 4| + 5$ is equal to:

(A) -7
(B) 7
(C) 10
(D) 12
(E) 14

8.In the figure show n,the circum ference of the circle is 10π.W hich of the follow ing is N O T a possible value for the area of the rectangle?

(A) 30
(B) 40
(C) $20\sqrt{2}$
(D) $30\sqrt{2}$
(E) $40\sqrt{2}$



9.The length of one edge of a cube equals 4.W hat is the distance betw een the center of the cube and one of its vertices?

(A) 2

(B) $2\sqrt{2}$

(C) $2\sqrt{3}$

(D) $4\sqrt{2}$

(E) $4\sqrt{3}$

10.If c is random ly chosen from the integers 20 to 99,inclusive,w hat is the probability that $c^3 - c$ is divisible by 12?

11.If x and y are positive integers greater than 1 such that $x - y$ and x/y are both even integers,w hich of the follow ing num bers m ust be non-prim e integers?

Indicate <u>all</u> such statem ents.

☐ x

☐ $x + y$

☐ y/x

12.The rem ainder w hen 120 is divided by single-digit integer m is positive,as is the rem ainder w hen 120 is divided by single-digit integer n.If $m > n$,w hat is the rem ainder w hen 120 is divided by $m - n$?

13.A circular m icrochip w ith a radius of 2.5 centim eters is m anufactured follow ing a blueprint scaled such that a measurem ent of 1 centim eter on the blueprint corresponds to a m easurem ent of 0.05 m illim eters on the microchip.W hat is the area of the blueprint,in square centim eters? (1 centim eter = 10 m illim eters)

π

14.

For a certain quantity of a gas,pressure P ,volum e V ,and tem perature T are related according to the form ula $PV = kT$,w here k is a constant.

<u>Q uantity A</u>	**<u>Q uantity B</u>**
The value of P if V = 20 and T = 32	The value of T if V = 10 and P = 78

15.

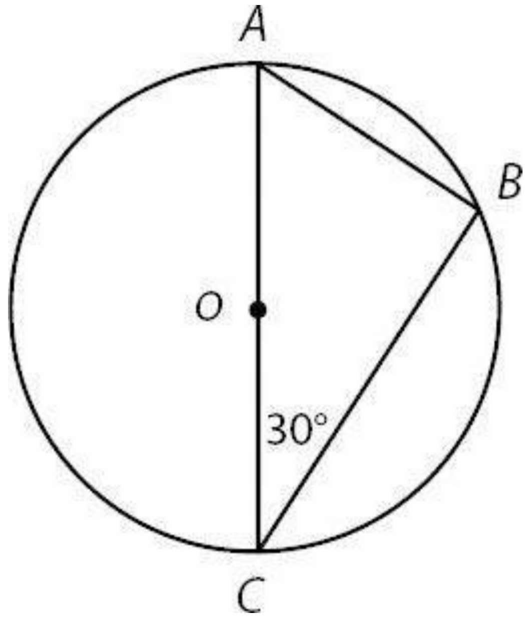


Figure not draw n to scale.

The circle w ith center *O* has a circum ference of $6\pi\sqrt{3}$.If *AC* is a diam eter of the circle,w hat is the length of line segm ent *BC* ?

(A) $\frac{3}{\sqrt{2}}$
(B) 3
(C) $3\sqrt{3}$
(D) 9
(E) $9\sqrt{3}$

16.A batch of w idgets costs *p* + 15 dollars for a com pany to produce and each batch sells for *p*(9 - *p*) dollars.For w hich of the follow ing values of *p* does the com pany m ake a profit?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

17.If *K* is the sum of the reciprocals of the consecutive integers from 41 to 60 inclusive,w hich of the follow ing is less than *K* ?

Indicate <u>all</u> such statem ents.

☐ 1/4
☐ 1/3
☐ 1/2

18.Triplets A dam ,B ruce,and C harlie enter a triathlon.There are nine com petitors in the triathlon.If every com petitor has an equal chance of w inning,and three m edals w ill be aw arded,w hat is the probability that at least tw o of the triplets w ill w in a m edal?

(A) 3/14
(B) 19/84
(C) 11/42
(D) 15/28
(E) 3/4

19.The expression $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}}$ extends to an infinite num ber of roots.W hich of the follow ing choices m ost closely approxim ates the value of this expression?

(A) $\sqrt{3}$
(B) 2
(C) 1+ $\sqrt{2}$
(D) 1+ $\sqrt{3}$
(E) $2\sqrt{3}$

20.H alf an hour after C ar A started traveling from N ew tow n to O ldtow n,a distance of 62 m iles,C ar B started traveling along the sam e road from O ldtow n to N ew tow n.The cars m et each other on the road 15 m inutes after C ar B started its trip.If C ar A traveled at a constant rate that w as 8 m iles per hour greater than C ar B 's constant rate,how m any m iles had C ar B driven w hen they m et?

(A) 14
(B) 12
(C) 10
(D) 9
(E) 8

21.

x and *y* are positive integers such that $x^2 5^y$ = 10,125

<u>Q uantity A</u>	<u>Q uantity B</u>
x^2	5^y

22.If $x = 2^b - (8^8 + 8^6)$,for w hich of the follow ing *b* values is *x* closest to zero?

(A) 20
(B) 24
(C) 25
(D) 30
(E) 42

23.If $k > 1$,w hich of the follow ing m ust be equal to $\dfrac{2}{\sqrt{k+1}+\sqrt{k-1}}$?

(A) 2
(B) $2\sqrt{2k}$
(C) $2\sqrt{k+1}+\sqrt{k-1}$
(D) $\dfrac{\sqrt{k+1}}{\sqrt{k-1}}$
(E) $\sqrt{k+1}-\sqrt{k-1}$

24.B ank account A contains exactly *x* dollars,an am ount that w ill decrease by 10% each m onth for the next tw o m onths.B ank account B contains exactly *y* dollars,an am ount that w ill increase by 20% each m onth for the next tw o m onths.If A and B contain the sam e am ount at the end of tw o m onths,w hat is the ratio of $\sqrt{x}$ to $\sqrt{y}$?

(A) 4 : 3
(B) 3 : 2
(C) 16 : 9
(D) 2 : 1
(E) 9 : 4

25.Let *a* be the sum of *x* consecutive positive integers.Let *b* be the sum of *y* consecutive positive integers.For w hich of the follow ing values of *x* and *y* is it N O T possible that *a* = *b*?

(A) *x* = 2;*y* = 6
(B) *x* = 3;*y* = 6
(C) *x* = 6;*y* = 4
(D) *x* = 6;*y* = 7
(E) *x* = 7;*y* = 5

26.

B ody M ass Index (B M I) is calculated by the form ula $\dfrac{703w}{h^2}$,w here *w* is w eight in pounds and *h* is height in inches.(12 inches = 1 foot.)

Q uantity A	**Q uantity B**
The num ber of pounds gained by a 6 foot,2 inch tall person w hose B M I increased by 1.0.	The num ber of pounds lost by a 5 foot,5 inch tall person w hose B M I decreased by 1.2.

27.B ag A contains 3 w hite and 3 red m arbles.B ag B contains 6 w hite and 3 red m arbles.O ne of the tw o bags w ill be chosen at random ,and then tw o m arbles w ill be draw n from that bag at random w ithout replacem ent.W hat is the probability that the tw o m arbles draw n w ill be the sam e color?

(A) 7/20
(B) 9/10
(C) 9/20
(D) 11/20
(E) 13/20

28.H ow m any positive four-digit integers contain the digit grouping “62” (in that order) at least once? For instance, 2628 and 6244 are tw o such integers to include,but 2268 and 5602 do not m eet the restrictions.

(A) 180
(B) 190
(C) 279
(D) 280
(E) 360

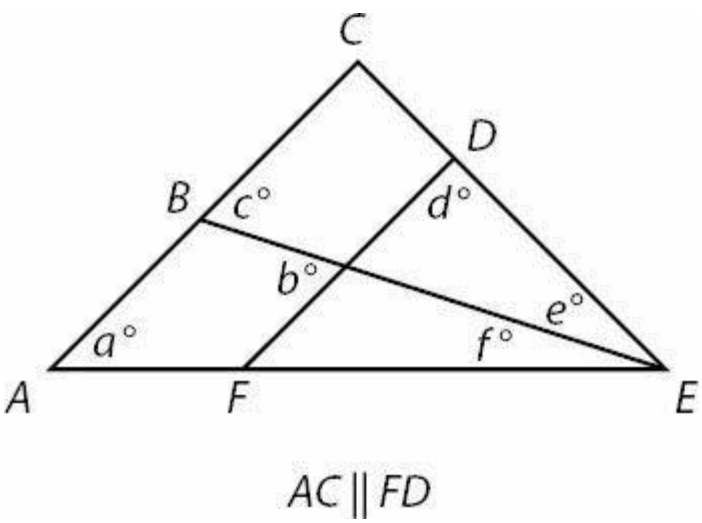
29.H ow m any 5 digit num bers that are divisible by 9 can be form ed using the digits 0,1,2,4,5,6 if repeats are not allow ed?

(A) 66
(B) 120
(C) 360
(D) 488
(E) 720

30.

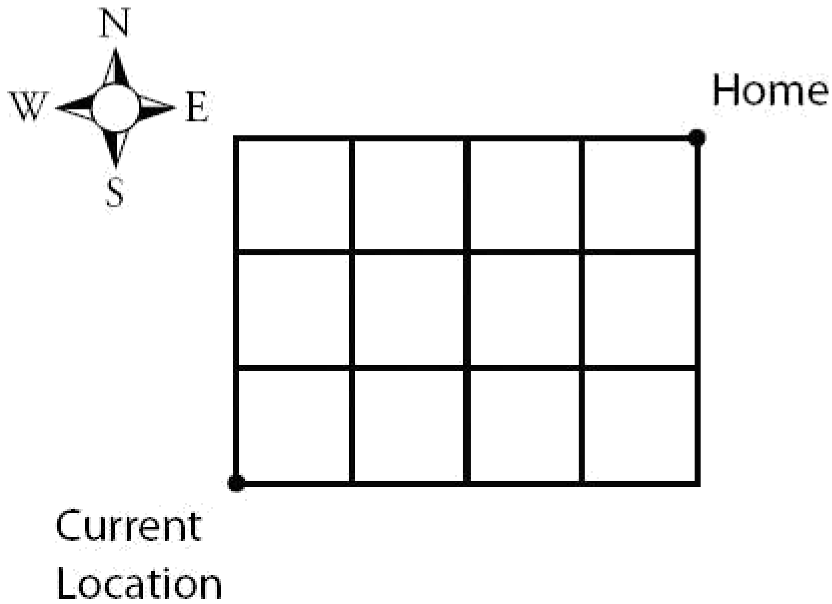
Q uantity A	**Q uantity B**
The average of all the m ultiples of 3 betw een 101 and 598	The average of all the m ultiples of 4 betw een 101 and 598

31.



Q uantity A	**Q uantity B**
$a + d - c - 90$	$90 - e - b - f$

32.



A m an w alks to his hom e from his current location on the rectangular grid show n.If he m ay choose to w alk north or east at any corner,but m ay never m ove south or w est,how m any different paths can the m an take to get hom e?

(A) 12
(B) 24
(C) 32
(D) 35
(E) 64

33.A bag contains 3 w hite,4 black,and 2 red m arbles.Tw o m arbles are draw n from the bag.W hat is the probability that the second ball draw n w ill be red if replacem ent is N O T allow ed?

(A) 1/36
(B) 1/12
(C) 7/36
(D) 2/9
(E) 7/9

34.

$x < 0$

Q uantity A	**Q uantity B**
$\left(\left(25^{x}\right)^{-2}\right)^{3}$	$\left(\left(5^{-3}\right)^{2}\right)^{-x}$

35.

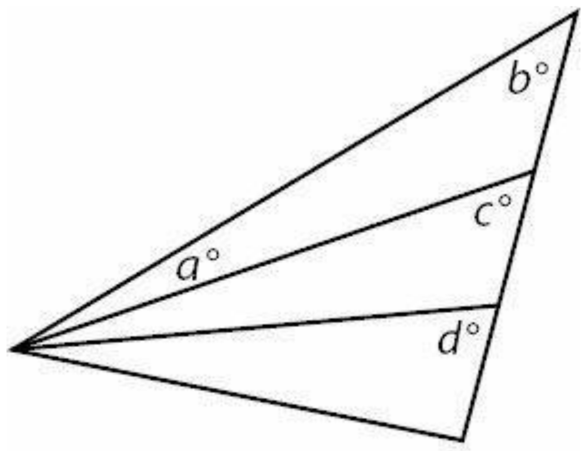
Q uantity A	**Q uantity B**
The sum of the m ultiples of 3 betw een -93 and 252,inclusive	9,162

36.

x is an integer

Q uantity A	Q uantity B
$(-1)^{X2} + (-1)^{X3} + (-1)^{X4}$	$(-1)^{X} + (-1)^{2x} + (-1)^{3x} + (-1)^{4x}$

37.



$d > c$

Q uantity A	Q uantity B
a	*d* - *b*

38.

Sequence *S* is such that $S_n = S_{n-1} + \frac{5}{2}$ and $S_1 = 1$

Sequence *A* is such that $A_n = A_{n-1} - 2.5$ and $A_1 = 36$

Q uantity A	Q uantity B
The sum of the term s in *S* from S_1 to S_{14}, inclusive	The sum of the term s in *A* from A_1 to A_{14}, inclusive

39.

Q uantity A	Q uantity B
$\dfrac{a^{64}-1}{(a^{32}+1)(a^{16}+1)(a^{8}+1)^{2}}$	1

40.

The circum ference of a circle is 7/8 the perim eter of a square.

Q uantity A	Q uantity B
The area of the square	The area of the circle

41.

P er Serving of:	C alories	C ost
Snack A	320	$1.50
Snack B	110	$0.45

C hoosing from the snacks in the table above,a group of people consum es 2,370 calories of snacks that cost a total of $10.65.

Q uantity A	**Q uantity B**
The num ber of servings of Snack A the group consum ed	5

42.

… In the sequence above,each term term

$a_1, a_2, a_3, \ldots, a_n,$

after the first is equal to the average of the preceding and the follow ing term .

Q uantity A	**Q uantity B**
$a_{51} - a_{48}$	$a_{37} - a_{34}$

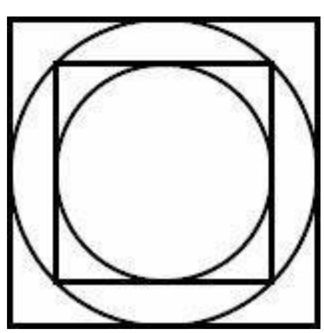
43.

The greatest com m on factor of 12*x* and 35*y* is

5*y* *x* and *y* are positive integers

Q uantity A	**Q uantity B**
The rem ainder w hen 12*x* is divided by 10	The greatest com m on factor of *x* and *y*

44.



Q uantity A	**Q uantity B**
The ratio of the area of the larger square to the area of the sm aller square	Tw ice the ratio of the area of the sm aller circle to the area of the larger circle

45.

$m = 2^{16}3^{17}4^{18}5^{19}$

$n = 2^{19}3^{18}4^{17}5^{16}$

Q uantity A	**Q uantity B**
W hen integer m is m ultiplied out,the num ber of zeroes at the end of m	W hen integer n is m ultiplied out,the num ber of zeroes at the end of n

46.

The sequence of num bers $a_1,a_2,a_3,\ldots,a_n,\ldots$ is defined by $a_n = 2^n - \frac{1}{2^{n-33}}$ for each integer $n \geq 1$.

Q uantity A	**Q uantity B**
The sum of the first 32 term s of this sequence	The sum of the first 31 term s of this sequence

47.Set S = {-1,4,30,-21}.If the m ean = 4,then the standard deviation,rounded to the nearest tenth,is equal to

48.Each of 100 balls has an integer value from 1 to 8,inclusive,painted on the side.The num ber n_x of balls representing integer x is given by the form ula $n_x = 18 - (x - 4)^2$.The interquartile range of the 100 integers is

(A) 1.5
(B) 2.0
(C) 2.5
(D) 3.0
(E) 3.5

49.W hen x is divided by 13 the answ er is y w ith a rem ainder of 3.W hen x is divided by 7 the answ er is z w ith a rem ainder of 3.If x,y,and z are all positive integers,w hat is the rem ainder of $\frac{yz}{13}$?

(A) 0
(B) 3
(C) 4
(D) 7
(E) 10

50.In a certain sequence,the term a_n is defined as the value of x that satisfies the equation 2 = (x/2) -a_{n-1}.If a_6 = 156,w hat is the value of a_2?

(A) 1
(B) 6
(C) 16

(D) 26
(E) 106

51.The operator ! is defined such that $a!b = a^{b} \times b^{-a}$

Quantity A	Quantity B
$(x!4) \div (4!x)$	$\frac{x^8}{16^x}$

52.What is the ratio of the sum of the even positive integers between 1 and 100 (inclusive) and the sum of the odd positive integers between 100 and 150?

(A) 102 to 125
(B) 50 to 51
(C) 51 to 56
(D) 202 to 251
(E) 2 to 3

53.For integer $n \geq 3$,a sequence is defined as $a_n = a_{n-1}^2 - a_{n-2}^2$ and $a_n > 0$ for all positive integer n.The first term ,a_1 is 2,and the fourth term is equal to the first term multiplied by the sum of the second and third terms.What is the third term ,a_3?

(A) 0
(B) 3
(C) 5
(D) 10
(E) 16

54.In a certain sequence,each term beyond the second term is equal to the average of the previous two terms.If a_1 and a_3 are positive integers,which of the following is NOT a possible value of a_5

(A) -9/4
(B) 0
(C) 9/4
(D) 75/8
(E) 41/2

55.The operator ? is defined by the following expression: $a?b = \left|\frac{a+1}{a}\right| - \frac{b+1}{b}$ where $ab \neq 0$.What is the sum of the solutions to the equation $x?2 = \frac{x?(-1)}{2}$?

(A) -1
(B) -0.75
(C) -0.25
(D) 0.25
(E) 0.75

56.X is a non-negative number and the square root of $(10 - 3X)$ is greater than X .

Q uantity A	**Q uantity B**
$\lvert X \rvert$	2

57.The area of an equilateral triangle is greater than $25\sqrt{3}$ but less than $36\sqrt{3}$

Q uantity A	**Q uantity B**
The length of one of the sides of the triangle	9

58.The inequality $|8 - 2x| < 3y - 9$ is equivalent to w hich of the follow ing?

(A) $2x < (17 - 3y)/2$
(B) $3y + 2x > 1$
(C) $6y - 2 < 2x$
(D) $1 - y < 2x < 17 + y$
(E) $3y - 1 > 2x > 17 - 3y$

59.In the sport of m ixed m artial arts (M M A),m ore than 30% of all fighters are skilled in both the M uy Thai and B razilian Jiu Jitsu styles of fighting.20% of the fighters w ho are not skilled in B razilian Jiu Jitsu are skilled in M uy Thai.60% of all fighters are skilled in B razilian Jiu Jitsu.

Q uantity A	**Q uantity B**
The percent of fighters w ho are skilled in M uy Thai	37%

60.The rate of data transfer,r,over a particular netw ork is directly proportional to the bandw idth,b,and inversely proportional to the square of the num ber of netw orked com puters,n.

Q uantity A	**Q uantity B**
The resulting rate of data transfer if the bandw idth is quadrupled and the num ber of netw orked com puters is m ore than tripled	$\frac{4}{9}r$

Advanced Quant Answers

1.**(B).**First,figure out the probability of each outcome.The die has six faces,numbered 1 through 6.Since the probability of rolling any particular number is directly proportional to that number,you can write each probability with an unknown multiplier *x* like so:

Probability of rolling a 1 = 1x = x
Probability of rolling a 2 = 2x
Probability of rolling a 3 = 3x
Probability of rolling a 4 = 4x
Probability of rolling a 5 = 5x
Probability of rolling a 6 = 6x

These are the only possible outcomes,so the probabilities must sum to 1:

$x + 2x + 3x + 4x + 5x + 6x$
$= 1$ $21x = 1$
$x = \frac{1}{21}$

Now you can find all the probabilities,since they are just multiples of *x*.

The expected value,or mean,of a roll of the die is found this way:

1. Multiply each outcome (1,2,3,4,5,and 6) by its corresponding probability.
2. Sum up all those products.

So the mean equals the following sum:

$$(1)\left(\frac{1}{21}\right)+(2)\left(\frac{2}{21}\right)+(3)\left(\frac{3}{21}\right)+(4)\left(\frac{4}{21}\right)+(5)\left(\frac{5}{21}\right)+(6)\left(\frac{6}{21}\right)$$

$$= (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2)\left(\frac{1}{21}\right)$$

$$= (1 + 4 + 9 + 16 + 25 + 36)\left(\frac{1}{21}\right)$$

$$= \frac{91}{21} = \frac{13}{3} = 4\frac{1}{3}$$

2.**(C).**From 7pm to 7:04,84 people enter the station (21 per minute).These 84 people will get on the 7:04 train.

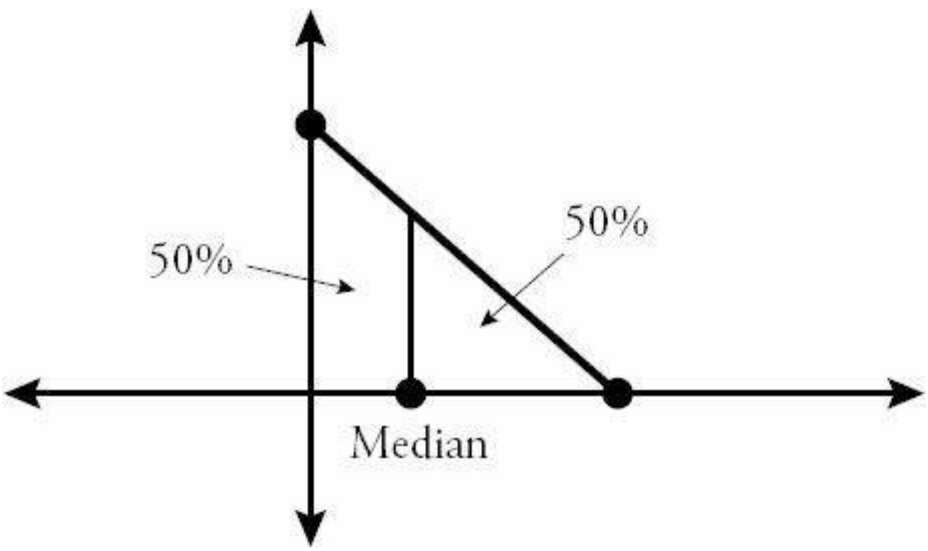
A fter that,for each 9 m inute period,9(21) = 189 people w ill enter the station and then get on a train.These trains w ill leave at 7:13,7:22,7:31,7:40,7:49,7:58,8:07,8:16,and 8:25.

Since 9 trains each have 189 people and the first train has 84 people,the average is:

$$\frac{9(189)+1(84)}{10}=178.5$$

N ote that the strange tim e form at (m inutes and seconds) doesn’t m ake the problem any harder — the problem is actually m ore clear if you know that the train com es at 7:04 and zero seconds,rather than 7:04 and 30 seconds,at w hich point m ore people w ould have entered the station.

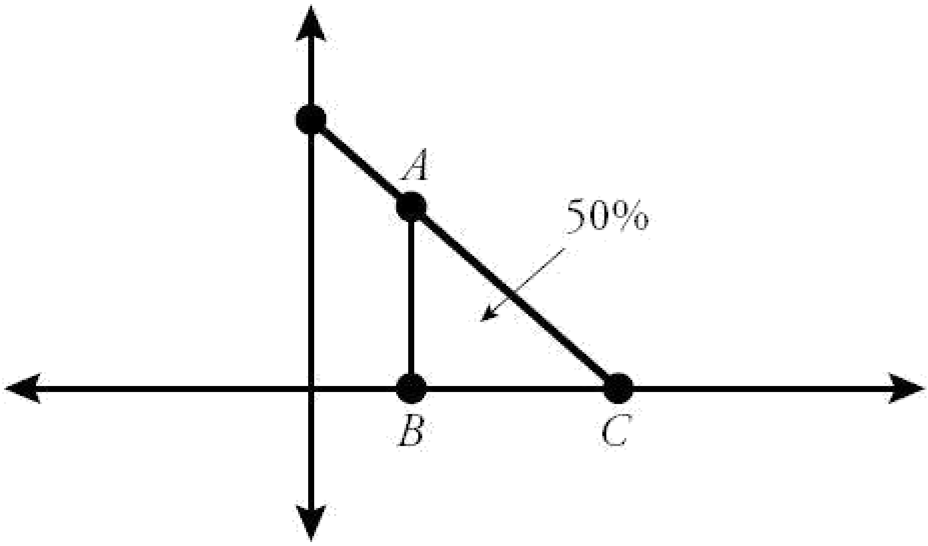
3.**(C).**A continuous probability distribution has a total area of 100% ,or 1,underneath the entire curve.The m edian of such a distribution splits the area into tw o equal halves,w ith 50% of the area to the left of the m edian and the other 50% to the right of the m edian:



In sim pler term s,the random variable *X* has a 50% chance of being above the m edian and a 50% chance of being below the m edian.Y ou can ignore the regions to the right or the left of this triangle,since the probability that *X* could fall in either of those regions is zero.So the question becom es this: w hat point on the *X* -axis w ill divide the large right triangle into tw o equal areas?

O ne shortcut is to note that the area of the large isosceles right triangle m ust be 1,w hich equals the total area under any probability distribution curve.Y ou can easily confirm this fact,though,by finding the area of this right triangle: $\frac{1}{2}bh=\frac{1}{2}\left(\sqrt{2}\right)\left(\sqrt{2}\right)=\frac{2}{2}=1$.

The quickest w ay to find the m edian is to consider the *sm all* isosceles right triangle, *ABC* as show n:



Triangle *ABC* m ust have an area of $\frac{1}{2}$.So w hat m ust be the length of each of its legs,*AB* and *BC* ? From the form ula $\frac{1}{2}bh = \frac{1}{2}$,and noting that the base *BC* equals the height *AB* ,you can see that the base *BC* m ust be 1 (the sam e as the height).Since the coordinates of point *C* are ($\sqrt{2}$,0),the coordinates of point *B* m ust be ($\sqrt{2}$,0).That is,the m edian is $\sqrt{2} - 1$.

4.**(B).**O ne w ay to solve is to set up an im plied equation or inequality,then m ake the sam e changes to both quantities, and finally com pare after sim plifying.

Q uantity A		**Q uantity B**	
$x^2 - 5x + 6$ $\underline{-(x^2 - 5x + 6)}$	?	$x^2 - 9x + 20$ $\underline{-(x^2 - 5x + 6)}$	
0	?	$-9x - (-5x) + 20 - 6$	N otice that x^2 is com m on to both quantities,so it can be ignored (i.e.it cancels).
0	?	$-9x + 5x + 14$	
0	<	$-4x + 14$	

B ecause *x* is negative,-4*x* + 14 = -4(*neg*) + 14 = *pos* + 14,w hich is greater than 0.

A nother w ay to solve is to factor and then com pare based on num ber properties.Q uantity A factors to(*x* - 2)(*x* - 3). Q uantity B factors to (*x* - 4)(*x* - 5).B ecause *x* is negative,“*x* m inus a positive num ber” is also negative.Each quantity is the product of tw o negative num bers,w hich is positive.

Quantity A : (*x* - 2)(*x* - 3) = (*neg*)(*neg*) = *pos*

Quantity B : (*x* - 4)(*x* - 5) = (m ore *neg*)(m ore *neg*) = m ore *pos*

Thus,Q uantity B is larger.

5.**(A).**24 to any pow er w ill end in the sam e units digit as 4 to the sam e pow er (it is alw ays true that,if you only need the last digit of the product,you only need the last digits of the num bers being m ultiplied).

4 to any pow er ends in either 4 or 6 ($4^1 = 4$,$4^2 = 16$,$4^3 = 64$,etc.) If the pow er is odd,the answ er w ill end in 4;if the pow er is even,the answ er w ill end in 6.

Since the exponent “5 + 2*x*” w ill be odd for any integer pow er of *x*,24^{5+2x} w ill end in 4.

36 to any pow er w ill end in the sam e units digit as 6 to the sam e pow er.Interestingly,pow ers of 6 alw ays end in 6,so 36^6 w ill end in 6.

17 to any pow er w ill end in the sam e units digit as 7 to the sam e pow er.W hile the units digits of the pow ers of 7 do indeed create a pattern,7^3 is just 343,w hich ends in 3.

Thus:

> 24^{5+2x} ends in 4
> 36^6 end in 6
> 7^3 ends in 3

M ultiplying three num bers that end in 4,6,and 3 w ill yield answ er that ends in 2,because (4)(6)(3) = 72,w hich ends in 2.

6.**(E).**The old solid has:

> W idth = *w*
> Length = *l*
> H eight = *h*

B ut then you are told that w idth and length are equal,so substitute right aw ay to reduce the num ber of variables:

The old solid has:

> W idth = *w*
> Length = *w*
> H eight = *h*

A fter the changes detailed in the problem ,the new solid has:

> W idth = *w* + 1
> Length = *w* + 1
> H eight = *h* - 9

Y ou are then told that the old and new solids have equal volum e.Since volum e = length × w idth × height:

$w^2h = (w + 1)^2(h - 9)$

B efore you get too far into sim plifying this,there is one m ore fact yet to be considered: the height of the new solid is four tim es the w idth of the original solid.Thus:

$h - 9 = 4w$

or

$h = 4w + 9$

Substitute into both spots in $w^2h = (w + 1)^2(h - 9)$ w here h appears:

$w^2(4w + 9) = (w + 1)^2(4w)$

D istribute $w^2(4w + 9)$ and FO IL $(w + 1)^2$:

$4w^3 + 9w^2 = (w + 1)^2(4w)$
$4w^3 + 9w^2 = (w^2 + 2w + 1)(4w)$
$4w^3 + 9w^2 = 4w^3 + 8w^2 + 4w$

Fortunately,you can now subtract $4w^3$ from both sides and sim plify from there:

$4w^3 + 9w^2 = 4w^3 + 8w^2 + 4w$
$9w^2 = 8w^2 + 4w$
$w^2 = 4w$
$w = 4$ (since w cannot be 0)

Y ou now need the *volum e* of the original solid.The old solid has:

W idth = w
Length = w
H eight = h

Y ou also know that $h = 4w + 9$.

Thus,w idth = 4,length = 4,and height = 4(4) + 9 = 25,and the volum e of the original solid is (4)(4)(25) = 400.

7.**(D).**Since the part of the equation inside the absolute value could have a positive value (in w hich case the absolute value is irrelevant) or a negative one,solve the equation tw ice,once for each scenario:

Scenario 1:	**Scenario 2:**
$x - 4 \geq 0$	$x - 4 \leq 0$
$x^2 - 8x + 21 = \lvert x - 4\rvert + 5$	$x^2 - 8x + 21 = \lvert x - 4\rvert + 5$
$x^2 - 8x + 21 = x - 4 + 5$	$x^2 - 8x + 21 = -(x - 4) + 5$

$$x^2 - 9x + 20 = 0$$
$$(x - 5)(x - 4) = 0$$
$$x = 5 \text{ or } 4$$

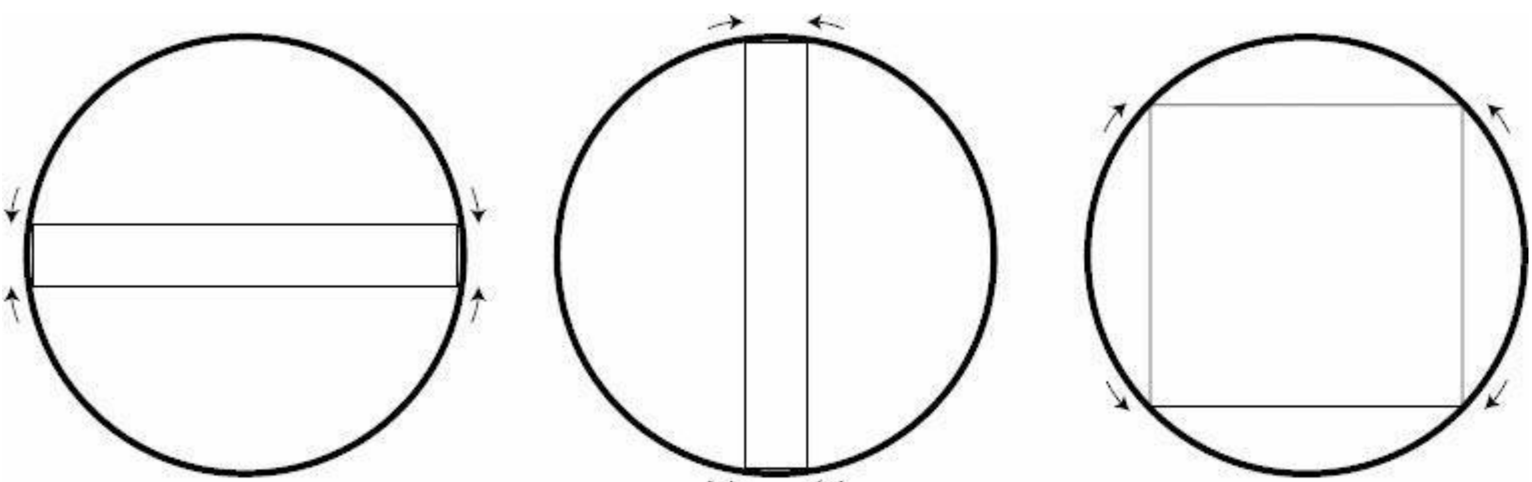
$$x^2 - 8x + 21 = -x + 4 + 5$$
$$x^2 - 7x + 21 = 0$$
$$(x - 4)(x - 3) = 0$$
$$x = 4 \text{ or } 3$$

Sum of the different solutions: 5 + 4 + 3 = 12.

The repeated solution (4 occurs tw ice) m ay sim ply be ignored — the w ork show s tw o different w ays of achieving the solution 4,but that is still just one solution to the equation.There are three total solutions that sum to 12.

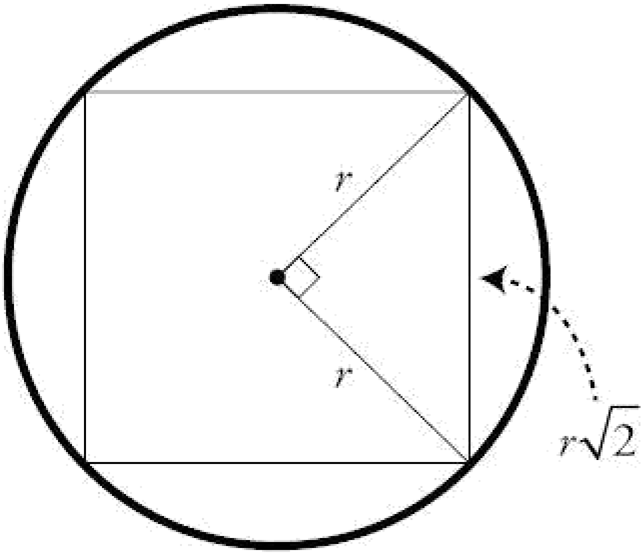
8.**(E).**This question asks you to determ ine w hich answ er choice lists an area for the inscribed rectangle that is not feasible.W hat m akes one possible area feasible and another one infeasible?

The inscribed rectangle can be stretched and pulled to extrem es: extrem ely long and thin,extrem ely tall and narrow , and som ew here in betw een:



Long and thin　　Tall and narrow　　In between

The “long and thin” and “tall and narrow ” rectangles w ill have a very sm all area,and the “in betw een” rectangle w ill have the largest possible area.In fact,the largest possible rectangle inscribed inside a circle w ill be a square:

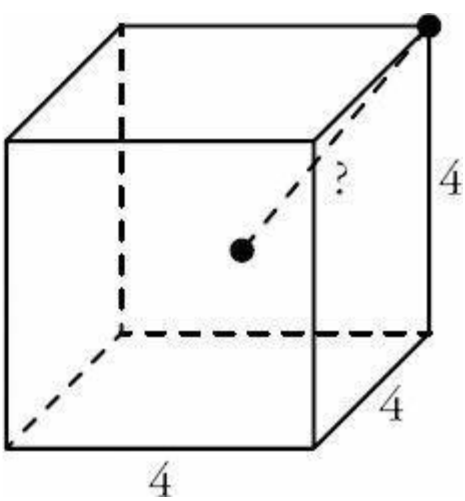


Square: maximal area

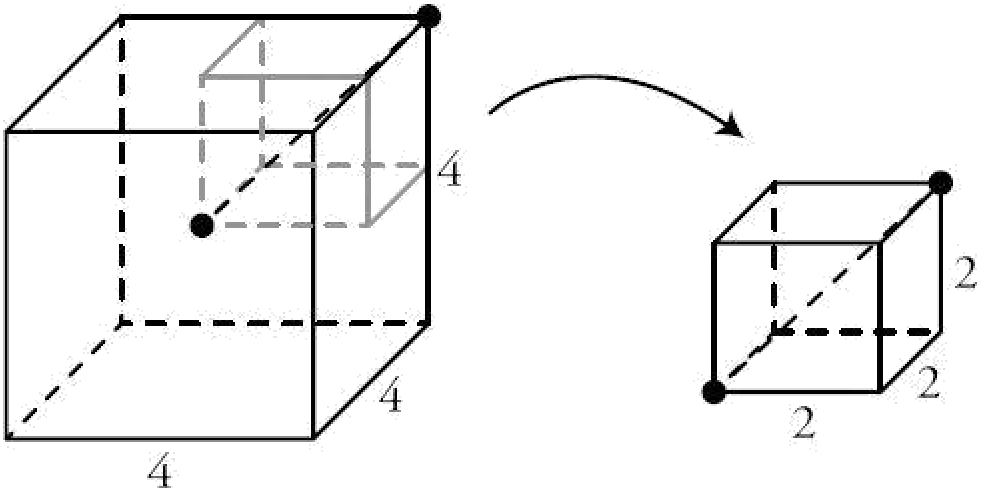
In this problem ,the circum ference = $10\pi = 2\pi r$.Thus $r = 5$,and the diagonal of the square is $2r = 10$.The square then has a side length of $5\sqrt{2}$ and an area of $(5\sqrt{2})^2 = 50$

O nly answ er choice (E) is larger than 50.The correct answ er is (E).

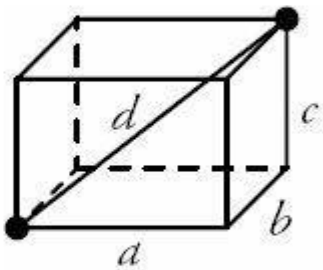
9.**(C).**First,you should represent the object w ith pictures,as is good practice w ith any 3-dim ensional situation.



The length of any side of the cube is 4,and you are asked for the distance betw een the center of the cube and any of its vertices (corners).If you chop up the cube into 8 sm aller cubes,you can see that the distance from the center of the $4 \times 4 \times 4$ cube to any corner is the diagonal of a $2 \times 2 \times 2$ cube.



Y ou can find the diagonal of a cube in a variety of w ays.Probably the fastest (besides applying a m em orized form ula) is to use the “super-Pythagorean” Theorem ,w hich extends to three dim ensions:



$$a^2 + b^2 + c^2 + = d^2$$

In the special case w hen the three sides of the box are equal,as they are in a cube,then you have this equation,letting s represent any side of the cube:

$$s^2 + s^2 + s^2 = d^2$$
$$3s^2 = d^2$$
$$s\sqrt{3} = d$$

Since $s = 2$,you know that $d = 2\sqrt{3}$.

10.**3/4.**Probability is (favorable outcom es)/(total # of possibilities).There are 99 – 20 + 1 = 80 possible values for c,so the unknow n is how m any of these c values yield a $c^3 - c$ that is divisible by 12.

The prim e factorization of 12 is 2 × 2 × 3.There are several w ays of thinking about this: num bers are divisible by 12 if they are divisible by 3 and by 2 tw ice,or if they are m ultiples of both 4 and 3,or if half of the num ber is an even m ultiple of 3,etc.

The expression involving c can be factored.

$$c^3 - c = c(c^2 - 1) = c(c - 1)(c + 1)$$

These are consecutive integers.It m ay help to put them in increasing order: $(c - 1)c(c + 1)$.Thus,this question has a lot to do w ith *C onsecutive Integers*,and not only because the integers 20 to 99 them selves are consecutive.

In any set of three consecutive integers,a m ultiple of 3 w ill be included.Thus,$(c - 1)c(c + 1)$ is alw ays divisible by 3 for any integer c.This takes care of part of the 12.So the question sim ply becom es "H ow m any of the possible $(c - 1) c(c + 1)$ values are divisible by 4?" Since the prim e factors of 4 are 2's,it m akes sense to think in term s of odds and evens.

$(c - 1)c(c + 1)$ could be (E)(O)(E),w hich is definitely divisible by 4,because the tw o evens w ould each provide at least one separate factor of 2.Thus,$c^3 - c$ is divisible by 12 w henever c is odd,w hich are the cases c = 21,23,25,… ,95, 97,99.That's ((99 – 21)/2) + 1 = (78/2) + 1 = 40 possibilities.

A lternatively,$(c - 1)c(c + 1)$ could be (O)(E)(O),w hich w ill only be divisible by 4 w hen the even term itself is a m ultiple of 4.Thus,$c^3 - c$ is also divisible by 12 w henever c is a m ultiple of 4,w hich are the cases c = 20,24,28,… , 92,96.That's ((96 - 20)/4) + 1 = (76/4) + 1 = 20 possibilities.

The probability is thus (40 + 20)/80 = 60/80 = 3/4.

11.**I and II only.**x cannot equal y,as that w ould m ake $x/y = 1 \neq$ even.So either $x > y$ or $y > x$.

x and y are both positive,and x/y is an integer,so $x > y$.

If $x - y$ is even,either x and y are both even,or they are both odd.

Since x/y = an even integer,$x = y \times$ an even integer.

O dd $\neq$ O dd × an even integer,so x and y can't be odd.

Even = Even × an even integer,so x and y m ust be even.

I.TR U E .x and y are both even,and x/y is an even integer.The sm allest value of x is 4,w hen y is 2,and $x/y = 4/2 = 2$. N o even num ber greater than 2 is prim e,so x can't be prim e.

II.TR U E .x and y are each positive even num bers and $x \neq y$.Thus,$x + y$ is even,and the sm allest possible value of $x + y = 4 + 2 = 6$.A ll even num bers greater than or equal to 6 are non-prim e.

III.FA LSE .It could be that $x = 4$ and $y = 2$,so $y/x = 1/2$,w hich is technically non-prim e,but is not an integer.In fact, if x/y = an even integer,y/x = 1/an even integer = positive fraction.

12.**0.**Since the rem ainder is defined as w hat is left over after one num ber is divided by another,it m akes sense that the leftover am ount w ould be positive.So w hy is this inform ation provided,if the rem ainder is "autom atically" positive? B ecause there is a third possibility: that the rem ainder is 0! So w hen you are told here that the rem ainder w hen 120 is divided by m is positive,you are really being told that $120/m$ does not have a rem ainder of 0.In other w ords,120 is not divisible by m ,or m is not a factor of 120.Sim ilarly,n is not a factor of 120.

A nother constraint on both m and n is that they are single-digit positive integers.So m and n are integers betw een 1 and 9, inclusive, that are not factors of 120. Only two such possibilities exist: 7 and 9.

Since $m > n$, $m = 9$ and $n = 7$. Thus, $m - n = 2$, and the remainder when 120 is divided by 2 is 0.

13. **225,000.** Microchip radius = (2.5 cm)(10 mm/cm) = 25 mm

Blueprint radius = 1 cm per every 0.05 mm on the microchip
= 10 mm per every 0.05 mm on the microchip
= (10 mm/0.05 mm on microchip)(25mm on microchip)
= (10 mm/0.05)(25)
= (1,000 mm/5)(25)
= (1,000 mm)(5)
= 5,000 mm
= (5,000 mm)(cm/10 mm)
= 500 cm

Blueprint area = $\pi \times r^2$
= $\pi \times (500 \text{ cm})^2$
= $250{,}000\pi \text{ cm}^2$

14. **(D).** If $PV = kT$, then $P = \frac{kT}{V}$. Quantity A is $P = \frac{k(32)}{(20)} = \frac{8}{5}k$.

If $PV = kT$, then $T = \frac{PV}{k}$. Quantity B is $T = \frac{(78)(10)}{k} = \frac{780}{k}$.

Don't rush to judgment, thinking that $780 > \frac{8}{5}$ means that Quantity B is greater. Notice that the k term is in the numerator of one quantity (so Quantity A increases with k) and the denominator of the other (so the larger k is, the smaller Quantity B is).

If $k = 1$, then Quantity B is greater $\left(780 > \frac{8}{5}\right)$. But if $k = 100$, Quantity A is greater (160 > 7.8). Thus, (D) is the answer.

15. **(D).** Some intuitive recollection of geometry rules and a picture drawn to scale can help you determine reasonable answer choices. If AC is a diameter of the circle, then Triangle ABC is a right triangle, with angle ABC = 90 degrees. The shortest side of a triangle is across from its smallest angle, and the longest side of a triangle is across from its largest angle. Therefore, $AC > BC > AB$.

The circumference of the circle = $\pi d = 6\pi\sqrt{3}$, so $d = 6\sqrt{3} \approx 6(1.7) = 10.2$. Thus, $AC \approx 10.2$ and $BC < 10.2$. But you can clearly see from the picture drawn to scale that BC is longer than half the diameter, so you can conservatively determine that $BC > 5.1$.

(A) $\frac{3}{\sqrt{2}} \approx \frac{3}{1.4} = \frac{30}{14} = 2\,{}^{1}\!/_{7}$ TOO LOW
(B) 3 TOO LOW
(C) $3\sqrt{3} \approx 3(1.7) = 5.1$ TOO LOW
(D) 9 OK
(E) $9\sqrt{3} \approx 9(1.7) = 15.3$ TOO HIGH

Alternatively, you could use rules of geometry to solve directly for the answer. Line *AC* passes through the center of the circle, so the inscribed Triangle *ABC* is a right triangle with angle *ABC* = 90°. Since angle *ACB* is 30°, angle *CAB* is 60°.

The sides in a 30–60–90 triangle have the ratio $1 : \sqrt{3} : 2$, so given any side, you can compute the other two sides.

First, use the circumference to solve for *AC* (the diameter):

$$6\pi\sqrt{3} = \pi d = \text{Circumference}$$
$$\frac{6\pi\sqrt{3}}{\pi} = d$$
$$6\sqrt{3} = d$$

Now you can use ratios (specifically, the unknown multiplier) to find *BC*.

	AB	BC	AC
Basic Ratio	$1x$	$\sqrt{3}\,x$	$2x$
Known Side			$6\sqrt{3}$
Unknown Multiplier			$x = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$
Computer Sides	$3\sqrt{3}$	$(\sqrt{3})(3\sqrt{3}) = 9$	$2(3\sqrt{3}) = 6\sqrt{3}$

Line segment *BC* has length 9.

16. **(B).** Profit equals revenue minus cost. The company's profit is:

$$p(9 - p) - (p + 15) = 9p - p^2 - p - 15 = -p^2 + 8p - 15$$

$= -(p^2 – 8p + 15)$
$= -(p – 5)(p – 3)$

Profit w ill be zero if $p = 5$ or $p = 3$,w hich elim inates answ ers (A) and (C).For $p > 5$,both $(p - 5)$ and $(p - 3)$ are positive.In that case,the profit is negative (i.e.,the com pany loses m oney).The profit is only positive if $(p - 5)$ and $(p - 3)$ have opposite signs,w hich occurs w hen $3 < p < 5$.

The correct answ er is (B).

17.**I and II only.**The sum (1/41 + 1/42 + 1/43 + 1/44 + … + 1/57 + 1/58 + 1/59 + 1/60) has 20 fractional term s.It w ould be nearly im possible to com pute if you had to find a com m on denom inator and solve w ithout a calculator and a lot of tim e.Instead,look at the m axim um and m inim um possible values for the sum .

Maxim um : The largest fraction in the sum is 1/41.*K* is definitely sm aller than 20 × 1/41,w hich is itself sm aller than 20 × 1/40 = 1/2.

Minim um : The sm allest fraction in the sum is 1/60.*K* is definitely larger than 20 × 1/60 = 1/3.

Therefore,$1/3 < K < 1/2$.

I.Y ES: $1/4 < 1/3 < K$
II.Y ES: $1/3 < K$
III.N O : $1/2 > K$

18.**(B).**First,m ake som e observations.W ith 9 com petitors and only 3 m edals aw arded,only 1/3 of the com petitors w ill w in overall.A lthough a sim plification,it is reasonable for each com petitor to see his or her chance of w inning a m edal as 1/3,or to expect to w in 1/3 of a m edal (pretending for a m om ent that m edals can be “shared”).

Y ou are asked for the probability *at least* 2 of the triplets w ill w in a m edal.In other w ords,you w ant 2/3 to 3/3 of the triplets to w in m edals,or for each triplet to w in 2/3 to 3/3 of a m edal.Since 2/3 and 3/3 are both greater than 1/3,you are looking for the probability that the triplets w ill w in m edals at a rate greater than that expected for com petitors overall.This w ould certainly be an unusual outcom e.Thus,the probability should be less than 1/2.Elim inate (D) and (E).Y ou could then at least m ake an educated guess from am ong the rem aining choices w ith at least a 1 in 3 shot at success.

To solve,use the probability form ula and com binatorics:

$$\text{Probability} = \frac{\text{specified outcome}}{\text{all possible outcomes}} = \frac{\text{\# of ways at least 2 triplets win medal}}{\text{\# of ways 3 medals can be awarded}}$$

First,find the total num ber of outcom es for the triathlon.There are nine com petitors;three w ill w in m edals and six w ill not.Set up an anagram grid w here Y represents a m edal,N no m edal:

com petitor:	C1	C2	C3	C4	C5	C6	C7	C8	C9
m edal:	Y	Y	Y	N	N	N	N	N	N

# of ways 3 medals can be awarded= $\frac{9!}{3!6!} = \frac{(9)(8)(7)}{(3)(2)(1)} = (3)(4)(7) = 84$

Now, you need to determine the number of instances when *at least two* brothers win a medal. Practically speaking, this could happen when (1) exactly three brothers win or (2) exactly two brothers win.

Start with *all three* triplets winning medals, where Y represents a medal:

triplet:	A	B	C		non-triplet:	C1	C2	C3	C4	C5	C6
medal:	Y	Y	Y		medal:	N	N	N	N	N	N

The number of ways this could happen is $\frac{3!}{3!} \times \frac{6!}{6!} = 1$. This makes sense, as there is only one instance in which all three triplets would win medals and all of the other competitors would not.

Next, calculate the instances when *exactly two* of the triplets win medals:

triplet:	A	B	C		non-triplet:	C1	C2	C3	C4	C5	C6
medal:	Y	Y	N		medal:	Y	N	N	N	N	N

Since both triplets and non-triplets win medals in this scenario, we need to consider possibilities for both sides of the grid. For the triplets, the number of ways that two could win medals is $\frac{3!}{2!1!} = 3$.

For the non-triplet competitors, the number of ways that one could win the remaining medal is $\frac{6!}{1!5!} = 6$.

Multiply these two numbers to get the total number of instances: 3 × 6 = 18.

The brothers win *at least two* medals in 18 + 1 = 19 cases. The total number of cases is 84, so the probability is 19/84.

The correct answer is (B).

19. **(B).** Since the answer asks for an approximation, you should use decimal approximations for all square roots in the question and answer choices.

(A) $\sqrt{3} \approx$
1.7 (B) 2
(C) 1 + $\sqrt{2}$ ≈ 1 + 1.4 = 2.4

(D) 1 + $\sqrt{3}$ ≈ 1 + 1.7 = 2.7

(E) 2 $\sqrt{3}$ ≈ 2(1.7) = 3.4

N ote that there is a m inim um difference of 0.3 betw een answ er choices.This im plies that you m ust be *reasonably* careful w hen approxim ating,but w ill have no trouble choosing an answ er if you approxim ate every square root to the nearest tenth.

$$\sqrt{2} \approx 1.4$$
$$\sqrt{2+\sqrt{2}} \approx \sqrt{2+1.4} \approx \sqrt{3.4} \approx 1.8$$
$$\sqrt{2+\sqrt{2+\sqrt{2}}} \approx \sqrt{2+1.8} \approx \sqrt{3.8} \approx 1.9$$
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}} \approx \sqrt{2+1.9} \approx \sqrt{3.9} \approx 2$$
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}} \approx \sqrt{2+2} \approx \sqrt{4} = 2$$

A t this point,you can see that the expression is converging on 2.

A lternatively,an algebraic solution is possible if you recognize that the infinite expression is nested w ithin itself:

$$x = \sqrt{2+\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}\right)} = \sqrt{2+x}$$

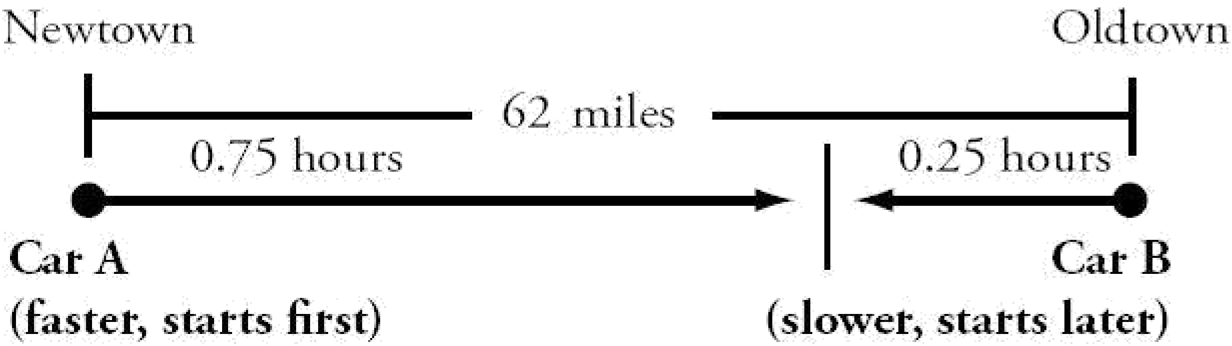
Y ou can solve for *x* as follow s:

$x = \sqrt{2+x}$
$x^2 = 2 + x$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$

This im plies that $x = 2$ or $x = -1$.Since *x* is the square root of a real,positive num ber,it m ust be positive,and you can conclude that $x = 2$.

The correct answ er is (B).

20.**(A).**D raw a diagram to illustrate the m om ent at w hich A and B pass each other m oving in opposite directions:



Y ou could test the answ er choices:

	B 's distance (m iles)	B 's rate (m ph) = D/T = $D/0.25$	A 's rate (m ph) = B 's rate + 8	A 's distance (m iles) = $R \times T$ = $R \times 0.75$	Total distance
(A)	**14**	**56**	**64**	**48**	**62**
(B)	12	48	56	42	54
(C)	10	40	48	36	46
(D)	9	36	44	33	42
(E)	8	32	40	30	38

O r you could solve algebraically,using an *RTD* chart.N ote that you m ust convert 15 m inutes to 1/4 (or 0.25) hours:

	R ate	**Tim e**	**D istance**
C ar A	(r + 8) m ph	0.75 hours	(0.75) (r + 8) m iles
C ar B	r m ph	0.25 hours	0.25r m iles
Total			62 m iles

Set up and solve an equation for the total distance:

$(0.75)(r + 8) + (0.25r) =$
$62\ 0.75r + 6 + 0.25r = 62$
$r = 56$

Therefore,C ar B traveled a distance of $0.25r = (0.25)(56) = 14$ m iles.

The correct answ er is (A).

21.**(D).**Factor 10,125 to its prim e factors: $10{,}125 = 3^4 5^3$.

So,$x^2 5^y = 3^4 5^3$.

In order to have 5^3 on the right side,there have to be three factors of 5 on the left side.A ll three could be in the 5^y term (i.e.,y could equal 3).O r,one of the 5’s could be in the 5^y term ,and tw o of the 5’s in the x^2 term ;i.e.y could equal 1 and x could have a single factor of 5.

In order to have 3^4 on the right side,x^2 m ust have $3^4 = (3^2)^2$ as a factor.In other w ords,x m ust have 3^2 as a factor, because 3^2 is certainly not a factor of 5.Thus,x is a m ultiple of 9.
The possibilities:

Q uantity A : x^2		Q uantity B : 5^y	C heck: The product m ust be 10,125	C heck: Q uantity A m ust be a perfect square	C heck: Q uantity B m ust be a pow er of 5
$x^2 = 9^2 = 81$	<	$5^y = 5^3 = 125$	(81)(125) = 10,125	yes	yes
$x^2 = (9 \times 5)^2 = 2{,}025$	>	$5^y = 5^1 = 5$	(2,025)(5) = 10,125	yes	yes

In one case,Q uantity A is greater.In the other,Q uantity B is greater.The correct answ er is (D).

22.**(B).**Testing the choices w ould be a natural w ay to solve this problem ,since the question doesn’t ask you to solve for b in general,but rather “for w hich of the follow ing is x closest to zero?” H ow ever,num bers betw een 2^{20} and 2^{42} are too large to plug and com pute.Y ou m ust m anipulate the term s w ith base 8 to see how they m ight balance w ith 2^b:

$x = 2^b - (8^8 + 8^6)$
$0 \approx 2^b - (8^8 + 8^6)$
$2^b \approx (8^8 + 8^6)$
$2^b \approx (8^6)(8^2 + 1)$
$2^b \approx ((2^3)^6) ((2^3)^2 +$
$1)$ $2^b \approx (2^{18})(2^6 + 1)$

Since 1 is very sm all in com parison to 2^6,w e can approxim ate $(2^6 + 1) \approx (2^6)$.Therefore,

$2^b \approx (2^{18})(2^6)$
$2^b \approx 2^{24}$
$b \approx 24$

The correct answ er is (B).

23.**(E).**Since there are variables in the answ er choices,w e should pick a num ber and test the choices.If $k = 2$,then $\frac{2}{\sqrt{k+1}+\sqrt{k-1}} = \frac{2}{\sqrt{3}+\sqrt{1}} \approx \frac{2}{1.7+1} = \frac{2}{2.7}$,w hich is less than 1.N ow test the answ er choices and try to m atch the target:

(A) 2 TO O H IG H

(B) $2\sqrt{2k} = 2\sqrt{4} = 4$ TOO HIGH

(C) $2\sqrt{k+1} + \sqrt{k-1} = 2\sqrt{3} + \sqrt{1} \approx 2(1.7) + 1 = 4.4$ TOO HIGH

(D) $\frac{\sqrt{k+1}}{\sqrt{k-1}} = \frac{\sqrt{3}}{\sqrt{1}} \approx 1.7$ TOO HIGH

(E) $\sqrt{k+1} - \sqrt{k-1} = \sqrt{3} - \sqrt{1} \approx 1.7 - 1 = 0.7$ OK

Alternatively, you could solve this problem algebraically. The expression given is of the form $\frac{2}{a+b}$, where $a = \sqrt{k+1}$ and $b = \sqrt{k-1}$.

You need to either simplify or cancel the denominator, as none of the answer choices have the denominator you start with, and most of the choices have no denominator at all. To be able to manipulate a denominator with radical signs, you must first try to eliminate the radical signs entirely, leaving only a^2 and b^2 in the denominator. To do so, multiply by a fraction that is a convenient form of 1:

$$\frac{2}{(a+b)} = \frac{2}{(a+b)} \times \frac{(a-b)}{(a-b)} = \frac{2(a-b)}{a^2 - b^2}$$

Notice the "difference of two squares" special product created in the denominator with your choice of (a - b).

Substituting for a and b,

$$\frac{2}{(\sqrt{k+1}+\sqrt{k-1})} \times \frac{(\sqrt{k+1}-\sqrt{k-1})}{(\sqrt{k+1}-\sqrt{k-1})} = \frac{2(\sqrt{k+1}-\sqrt{k-1})}{(k+1)-(k-1)} = \frac{2(\sqrt{k+1}-\sqrt{k-1})}{2} = \sqrt{k+1}-\sqrt{k-1}$$

The correct answer is (E).

24. **(A).** First, note the answer pairs (A) & (C) and (B) & (E), in which one ratio is the square of the other. This represents a likely trap in a problem that asks for the ratio of $\sqrt{x}$ to $\sqrt{y}$ rather than the more typical ratio of x to y. You can eliminate (D), as it is not paired with a trap answer and therefore probably not the correct answer. You should also suspect that the correct answer is (A) or (B), the "square root" answer choice in their respective pairs.

For problems involving successive changes in amounts — such as population-growth problems, or compound interest problems — it is helpful to make a table:

	Account A	**Account B**
Now	x	y
After 1 month	$\left(\frac{9}{10}\right)x$	$\left(\frac{12}{10}\right)y$

A fter 2 m onth	$\left(\frac{9}{10}\right)\left(\frac{9}{10}\right)x=\left(\frac{81}{100}\right)x$	$\left(\frac{12}{10}\right)\left(\frac{12}{10}\right)y=\left(\frac{144}{100}\right)y$

If the accounts have the sam e am ount of m oney after tw o m onths,then:

$$\left(\frac{81}{100}\right)x=\left(\frac{144}{100}\right)y$$

$$81x=144y$$

This can be solved for $\frac{\sqrt{x}}{\sqrt{y}}$:

$$\frac{x}{y}=\frac{144}{81}$$

$$\frac{\sqrt{x}}{\sqrt{y}}=\frac{\sqrt{144}}{\sqrt{81}}=\frac{12}{9}=\frac{4}{3}$$

The correct answ er is (A).

25.**(C).**C onsecutive integers have tw o key characteristics: they differ by a know n,constant value (i.e.,1),and they alternate odd,even,odd,even,etc.Y ou should use the odd/even property to evaluate these choices.This general approach is usually faster than considering specific values.It w orks particularly w ell for very general questions about w hether som ething C A N N O T or M U ST be true.

(A)	*a* = E + O = O ,or *a* = O + E = O	*b* = 3 pairs (E + O) = O ,or *b* = 3 pairs (O + E) = O	*a* could equal *b* (both O dd)
(B)	*a* = E + O + E = O ,or *a* = O + E + O = E	*b* = 3 pairs (E + O) = O ,or *b* = 3 pairs (O + E) = O	*a* could equal *b* (both O dd)
(C)	***a* = 3 pairs (E + O) = O ,or** ***a* = 3 pairs (O + E) = O**	***b* = 2 pairs (E + O) = E ,or** ***b* = 2 pairs (O + E) = E**	*a* ≠ *b* **(O dd ≠ E ven)**
(D)	*a* = 3 pairs (E + O) = O ,or *a* = 3 pairs (O + E) = O	*b* = O + 3 pairs (E + O) = E,or *b* = E + 3 pairs (O + E) = O	*a* could equal *b* (both O dd)
(E)	*a* = O + 3 pairs (E + O) = E,or *a* = E + 3 pairs (O + E) = O	*b* = O + 2 pairs (E + O) = O ,or *b* = E + 2 pairs (O + E) = E	*a* could equal *b* (both O dd or both Even)

The correct answ er is (C).

26.**(A).**First of all,note that the height of each person in question is fixed (no one grew taller or shorter);only w eights changed.Second,note that B M I is alw ays positive,and is proportional to w ;as w eight increases,B M I increases,and vice versa.So the language of the quantities — pounds gained … B M I increased” and “pounds lost … B M I decreased” — is aligned w ith this proportionality.B oth quantities are a positive num ber of pounds.

Since $\text{BMI} = \frac{703w}{h^2}$, change in $\text{BMI} = \frac{703w_{before}}{h^2} - \frac{703w_{after}}{h^2} = \frac{703}{h^2}\left(w_{before} - w_{after}\right)$.

To sim plify things,you can w rite this in term s of *BM I* and *w* ,the positive change in B M I and w eight,respectively:

$$\Delta BMI = \frac{703}{h^2}\Delta w$$

(The triangle sym bol indicating positive change in a quantity does not appear on the G R E — it is used here for convenience in notating an explanation.)

Since the quantities both refer to *w* ,rew rite the relationship as $\Delta w = \frac{h^2}{703}\Delta BMI$ .B oth *BM I* and *h* are given in each quantity,so *w* can be calculated and the relationship betw een the tw o quantities determ ined.(The answ er is definitely not (D).)

Q uantity A :

A 6'2" tall person is 6(12) + 2 = 74 inches tall.

$$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{74^2}{703}(1.0) = \frac{74^2}{703}$$

Q uantity B :

A 5'5" tall person is 5(12) + 5 = 65 inches tall.

$$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{65^2}{703}(1.2)$$

Since the 703 in the denom inator is com m on to both quantities,the com parison is really betw een 74^2 = 5,476 and 65^2(1.2) = 4,225(1.2) = 5,070.Q uantity A is greater.

27.**(C).**There are four different outcom es that can yield tw o balls of the sam e color: B ag A w ith w hite,B ag A w ith red,B ag B w ith w hite,or B ag B w ith red.The first decision that m ust be m ade is to choose a bag.B ecause the problem states that one of the tw o bags w ill be chosen at random ,you are no m ore likely to choose one bag than the other. Therefore,the probability of choosing B ag A ,P(A),and the probability of choosing B ag B ,P(B),m ust be the sam e, i.e.P(A) = P(B) = 1/2.

If B ag A is chosen,w hat is the probability of a m atched pair? First,com pute the probability of tw o w hites.The probability of the first w hite is 3/6 and the probability of the second w hite is 2/5,so the probability of a first A N D second w hite is (3/6)(2/5) = 1/5.Sim ilarly,the probability of tw o reds is (3/6)(2/5) = 1/5.If B ag A is chosen,you can obtain a m atch by either grabbing a pair of w hite O R a pair of red,so you m ust add their probabilities to get the total chance of a pair.This gives P(B ag A Pair) = 1/5 + 1/5 = 2/5.

Sim ilarly,if B ag B is chosen the probability of a pair of w hite m arbles is (6/9)(5/8) = 5/12 and the probability of a pair of red m arbles is (3/9)(2/8) = 1/12.Therefore,the probability of a pair is P(B ag B pair) = 5/12 + 1/12 = 6/12 = 1/2.The probability of choosing B ag A A N D a pair from B ag A is the product of the tw o events,(1/2)(2/5) = 1/5. Sim ilarly,the probability of choosing B ag B A N D a pair from B ag B is (1/2)(1/2) = 1/4.The total probability of choosing a pair w ill be the probability of choosing B ag A and a pair from B ag A O R choosing B ag B and a pair from bag B ,m eaning you m ust sum these tw o events.This gives: P(pair) = 1/5 + 1/4 = 4/20 + 5/20 = 9/20.

28.**(C).**There are three different cases in w hich you m ust count: 62_ _,_62_,and _ _62.In the case of 62_ _,any digits from 00 to 99 w ill w ork,w hich gives you 100 num bers.In the case of _62_,you have 9 choices for the first digit as you are allow ed to use any num ber from 1–9 inclusive,but not zero because you m ust m eet the requirem ent of using a four digit positive integer.For the last digit you still allow any num ber from 0–9,w hich is 10 choices.Thus, by the fundam ental counting principle,for _62_ you have (9)(10) = 90 choices.For the case of _ _62 you again have 1–9 inclusive for the first digit and 0–9 inclusive for the second digit for a total of 90 choices.H ow ever,in this case you are double counting one num ber,since 6262 already appeared in the 62_ _ case.Therefore there are only 89 new num bers that m eet the criteria.Since you could create the case 62_ _ O R _62_ O R _ _62,you m ust add the num ber of possibilities together for each case to achieve the total.This gives you 100 + 90 + 89 = 279.

29.**(B).**In order for a num ber to be divisible by 9 the sum of the digits m ust be a m ultiple of 9.The low est num ber that can be m ade by sum m ing 5 of the digits is given by 0 + 1 + 2 + 4 + 5 = 12 and the highest num ber that can be m ade is 1 + 2 + 4 + 5 + 6 = 18.The only num ber in this range that sum s to a m ultiple of 9 is 18,and thus the only possible com bination of num bers you can use is {1,2,4,5,6}.In other w ords,no com bination of num bers using the num ber 0 w ill ever yield a m ultiple of 9.The question can now be rephrased as,“H ow m any different 5 digit num bers can be m ade using the digits {1,2,4,5,6} w ithout repetition?” as these num bers w ill alw ays sum to 18 and thus w ill alw ays be divisible by 9.In this case,the answ er is sim ply 5!,as there are 5 choices for the first num ber,4 for the second,3 for the third,and so on.Thus,there are 5! = 120 possible 5 digit num bers that are divisible by 9.

30.**(B).**This problem is greatly sim plified if you realize that you do not need to sum all the m ultiples of 4 (or of 3) in the given range and divide by the num ber of such m ultiples,using the typical average form ula.The average for a set of evenly spaced integers is equal to the average of the first and last term .

Q uantity A : The m ultiples of 3 betw een 101 and 598 are 102,105,108,… ,591,594,597.The average of the w hole set is $\frac{102+597}{2}=\frac{699}{2}=349.5$

Q uantity B : The m ultiples of 4 betw een 101 and 598 are 104,108,112,… ,588,592,596.The average of the w hole set is $\frac{104+596}{2}=\frac{700}{2}=350$

31.**(C).**Set up an im plied inequality and perform identical operations on each quantity,grouping variables.

Q uantity A		**Q uantity B**
$a + d - c - 90$	?	$90 - e - b - f$
$a + d - c$	?	$180 - e - b - f$
$a + d - c + e + b + f$	?	180
$(a + d + e + f) + (b - c)$	?	180

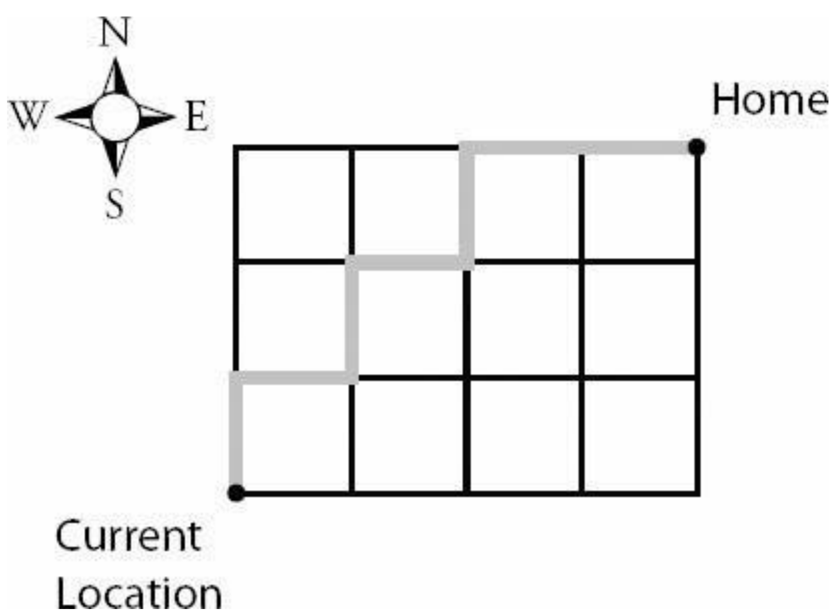
In the last step above, only the order of the variables was changed, and parentheses added to group certain terms. Notice that the angle at point *C* and point *D* is the same, as *AC* and *FD* are parallel lines intersected by the transversal *CE*. So, the first set of parentheses holds the sum of the interior angles of the biggest triangle *ACE*, which is 180.

Also because *AC* and *FD* are parallel lines intersected by transversal *BE*, $b = c$, so $b - c = 0$ in the second set of parentheses.

Quantity A		**Quantity B**
$(a + d + e + f) + (b - c)$	?	180
$(180) + (0)$	=	180

Thus, the quantities each equal 180 and the correct answer is (C).

32. **(D).** Given that the man can only move north and east, he must advance exactly 7 blocks from his current location to get home regardless of which path he takes. Of these 7 blocks, 4 must be moving east and 3 must be moving north. An example path is given below:



The problem can then be rephrased as follows: "Of the 7 steps, when does the man choose to go east and when does he choose to go north?" Labeling each step as N for north and E for east, you can see the problem as the number of unique rearrangements of NNNEEEE (for example, this arrangement corresponds to going north 3 times and then east 4 times straight to home). This is given by $\frac{total!}{repeats!} = \frac{7!}{3!4!} = 35$.

33. **(D).** Denote red as *R*, white as *W*, and black as *B*. There are exactly three ways in which the selections may occur where you get red as the second marble, either *RR*, *WR*, or *BR*. Since you may have any of these options the problem is an OR, so calculate the probability of each event and then add them together. First, for *RR*, you have the probability of the first red as (2/9) and the second red as (1/8), yielding a probability of red AND red as (2/9)(1/8) = 1/36. Similarly the probability of first white AND second red is (3/9)(2/8) = 1/12. Finally, the probability of first black and second red is (4/9)(2/8) = 1/9. Thus, the total probability of the second marble being red is P(*RR*) + P(*WR*) + P(*BR*) = 1/36 + 1/12 + 1/9 = 1/36 + 3/36 + 4/36 = 8/36 = 2/9. Of course, an easier way to solve this problem is to consider that the first draw is completely irrelevant, so you may consider the second draw alone. For the second draw, there are 2 red marbles out of a total of 9 marbles, giving you 2/9. Keep in mind that even though there are often difficult solution methods, sometimes a clever insight can greatly simplify the problem.

34.**(A).**Sim plify both quantities,rem em bering that a pow er to a pow er m eans you m ultiply the exponents.A lso,25 is 5 squared,so you can substitute,putting both quantities in term s of a base of 5.

Quantity A : $((25^{x})^{-2})^{3} = 25^{-6x} = (5^{2})^{-6x} = 5^{-12x}$

Quantity B : $((5^{-3})^{2})^{-x} = 5^{6x}$

Typically,w hen you are com paring exponents w ith the sam e base,the one w ith the larger exponent is greater.It m ight be tem pting to conclude that 6*x* > -12*x*,but be careful w ith negative variables.

If x = -1,Q uantity A = 5^{12} and Q uantity B = 5^{-6},or $\frac{1}{5^{6}}$.In this case,Q uantity A is m uch larger.

If $x = -\frac{1}{2}$,Q uantity A = 5^{6} and Q uantity B = 5^{-3},or $\frac{1}{5^{3}} = \frac{1}{125}$.A gain,Q uantity A is m uch larger.

If x = -10,Q uantity A = 5^{120} and Q uantity B = 5^{-60}.Y ou can see that,the m ore negative *x* gets,the larger the difference betw een Q uantity A and Q uantity B becom es.Q uantity A w ill alw ays be larger.

A nother w ay to look at it:

Q uantity A : $5^{-12x} = 5^{-12 \times negative} = 5^{negative}$

Q uantity B : $5^{6x} = 5^{6 \times negative} = 5^{negative}$

Even if $|x|$is a tiny fraction,i.e.you are taking som e high order root of 5 such as $\sqrt[8]{5}$ or $\sqrt[100]{5}$,these quantities w ould approach 1 such that

Quantity A : $51^{positive} > 1$

Quantity B : $51^{negative} < 1$

Since Q uantity A is greater than 1 and Q uantity B is less than 1,Q uantity A is larger.

35.**(A).**First,notice that -93 and 252 are both m ultiples of 3 and “inclusive” m eans they should be included in the sum w ith all the m ultiples of 3 in betw een them .Listing and adding the num bers w ould be tim e consum ing and error prone,so som e strategies are useful.

Q uantity A : (-93) + (-90) + (-87) + … + (-6) + (-3) + 0 + 3 + 6 + … + 87 + 90 + 93 + 96 + 99 + … 246 + 249 + 252

Since 252/3 = 84,252 is the 84th positive m ultiple of 3.B y the sam e logic,m inus 93 is the 31st negative m ultiple of 3 (because -93/3 = -31).So the sum in question is 3 tim es the sum of the integers from -31 to +84,inclusive.

Q uantity A : $3 \times [(-31) + (-30) + (-29) + \ldots + (-2) + (-1) + 0 + 1 + 2 + \ldots + 29 + 30 + 31 + 32 + 33 + \ldots\ 82 + 83 + 84]$

N otice that all of the negative integers have an additive inverse elsew here in the sum that cancels them out.For exam ple,(-31) + 31 = 0,and (-30) + 30 = 0,etc.So the sum in question is really 3 tim es the sum of just the integers from 32 to 84,inclusive.

Q uantity A : $3 \times [32 + 33 + \ldots\ 82 + 83 + 84]$

N ow ,apply the form ula for sum m ing consecutive integers: $\frac{First + Last}{2} \times Number\ of\ terms$

The num ber of term s is Last - First + 1 = 84 - 32 + 1 = 53.

Q uantity A : $3 \times [32 + 33 + \ldots\ 82 + 83 + 84] = 3\times\left[\frac{32+84}{2}\times 53\right] = 3\times\left[\frac{116}{2}\times 53\right] = 3 \times [58 \times 53]$
$= 3 \times 3{,}074 = 9{,}222$

Thus,Q uantity A is larger.

36.**(B).**W hen you see a negative base raised to an integer pow er,the question is about positives and negatives: $(-1)^{odd} = -1$ and $(-1)^{even} = +1$.

If *x* is even,all of the exponents in this question are even.

Q uantity A : $(-1)^{even2} + (-1)^{even3} + (-1)^{even4} = (-1)^{even} + (-1)^{even} + (-1)^{even} = 1 + 1 + 1 = 3$

Q uantity B : $(-1)^{even} + (-1)^{2\times even} + (-1)^{3\times even} + (-1)^{4\times even} = 1 + 1 + 1 + 1 = 4$

If *x* is odd,*som e* of the exponents in this question are odd.

Q uantity A : $(-1)^{odd2} + (-1)^{odd3} + (-1)^{odd4} = (-1)^{odd} + (-1)^{odd} + (-1)^{odd} = (-1) + (-1) + (-1) = -3$

Q uantity B : $(-1)^{odd} + (-1)^{2\times odd} + (-1)^{3\times odd} + (-1)^{4\times odd} = (-1)^{odd} + (-1)^{even} + (-1)^{odd} + (-1)^{even}$
$= (-1) + 1 + (-1) + 1$
$= 0$

In both cases,Q uantity B is greater than Q uantity A .

37.**(B).**B ecause an exterior angle of a triangle is equal to the sum of the tw o opposite interior angles of the triangle (in this case,the top sm all triangle),$c = a + b$.

Since $d > c$ and you can substitute $a + b$ for c,you know :

$$d > a + b$$

Subtract b from both sides:

$$d - b > a$$

Thus,Q uantity B is larger.

38.**(B).**M any students find sequence notation intim idating,but it doesn't have to be.Let's rephrase each sequence in norm al language.

$S_n = S_{n-1} + \frac{5}{2}$ is just saying that *every term in S is equal to the term before it,plus 5/2*.

$A_n = A_{n-1} - 2.5$ is just saying that *every term in A is equal to the term before it,m inus 2.5.*

O f course,5/2 and 2.5 are equal,w hich is a good clue that there's probably som e sim ple w ay to solve this problem w ithout actually sum m ing up a sequence.

Since $S_1 = 1$ and every term in S is just 2.5 greater than the term before it,Sequence S begins like this: 1,3.5,6,8.5, 11,13.5,16,18.5,21,23.5,…

Since $A_1 = 36$ and every term in A is just 2.5 less than the term before it,Sequence A begins like this: 36,33.5,31, 28.5,26,23.5,21,18.5…

A t this point,it looks as though the tw o sequences are going to have a lot of term s in com m on! R em em ber,any com m on elem ents appearing in both Q uantity A and Q uantity B can just be canceled out.

Y ou could just w rite out all 14 term s for each colum n,or you could "skip up" to S_{14} by noting that S_{14} is just going to be S_1 plus 2.5,thirteen tim es (since it takes thirteen "jum ps" to get from 1 to 14).Take the first term ,1,plus 13(2.5) = 32.5 to get $S_{14} = 33.5$.

Y ou can also "skip up" to the final term in A .To get to A_{14},take A_1 and subtract 2.5 thirteen tim es (since it takes thirteen "jum ps" to get from 1 to 14).Take the first term ,36,m inus 13(2.5) = 32.5 to get $A_{14} = 3.5$.

So,Q uantity A looks like this: The sum of 1,3.5,6,… ,28.5,31,33.5

A nd Q uantity B looks like this: The sum of 36,33.5,31,… ,8.5,6,3.5

That is,all the term s from 3.5 to 33.5,inclusive,are held in com m on by both sets,so you can safely subtract them out H ere's w hat's left.

Quantity A : 1

Quantity B : 36

Q uantity B is greater.

39.**(B).**This problem depends on know ing how to factor the "difference of squares." The basic form ula that you learned in the M anhattan Prep's *Algebra G RE* ® *Strategy G uide* is this:

$x^2 - y^2 = (x - y)(x + y)$

The im portant part about learning this form ula is that *anything* can be "subbed in" for *x* and *y*.A nother w ay to think about it is that tw o perfect squares can be "subbed in" for x^2 and y^2.For instance,a^{64} and 1 can serve as x^2 and y^2 (rem em ber,1 is a perfect square — if it helps,think of it as 1^2).

So,you can factor $a^{64} - 1$ in the num erator according to the pattern above:

$$\text{Quantity A:} \quad \frac{\left(a^{32}+1\right)\left(a^{32}-1\right)}{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)^{2}}$$

In order to do this,you also need to know how to take the square root of a^{64}.To take the square root of any num ber w ith an even exponent,just cut the exponent in half.D o not take the square root of the exponent itself! So,$\sqrt{a^{64}} = a^{32}$,not a^8.

N ow ,notice that $a^{32} - 1$ also m atches the pattern (a perfect square m inus a perfect square).$a^{32} + 1$,how ever,cannot be factored.Let's factor $a^{32} - 1$ only:

$$\text{Quantity A:} \quad \frac{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{16}-1\right)}{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)^{2}}$$

Looking good.B ut w ait! $a^{16} - 1$ A LSO m atches the pattern! Y ou can factor again:

$$\text{Quantity A:} \quad \frac{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)\left(a^{8}-1\right)}{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)^{2}}$$

Y ou actually could factor $a^8 - 1$,but nothing on the bottom is going to cancel w ith term s broken dow n further on top. Instead,cancel com m on term s on top and bottom .

Q uantity A : $\frac{\cancel{(a^{32}+1)}\cancel{(a^{16}+1)}(a^8+1)(a^8-1)}{\cancel{(a^{32}+1)}\cancel{(a^{16}+1)}(a^8+1)^2} = \frac{(a^8+1)(a^8-1)}{(a^8+1)^2} = \frac{\cancel{(a^8+1)}(a^8-1)}{\cancel{(a^8+1)}(a^8+1)} = \frac{(a^8-1)}{(a^8+1)}$

W hatever a^8 is,the num erator of Q uantity A is less than the denom inator of Q uantity A (w hich is also definitely positive,since $a^8 \geq 0$).Thus,Q uantity A < 1.The correct answ er is (B).

40.**(A).**This problem introduces a square and a circle,and stating that the circum ference of the circle is 7/8 the perim eter of the square.

This is license to plug in.Since both a square and a circle are regular figures — that is,all squares are in the sam e proportion as all other squares,and all circles are in the sam e proportion as all other circles — you can be certain that plugging only *one* set of values w ill give the sam e result you'd get from plugging *any* set of values.B ecause the figures are regular and related in a know n w ay (circum ference = 7/8 × square perim eter),there is no need to repeatedly try different values as is often necessary on Q uantitative C om parisons.

Y ou could say the radius of the circle is 2,so the circum ference is 4π.Then,the perim eter of the square is $(8/7)(4\pi) = 32\pi/7$.This isn't ideal,because then you are stuck w ith π in the calculations for the square,w here it is unnecessarily aw kw ard.

It is best to pick values for the square.If the side of the square is 2,the perim eter is 4(2) = 8 and the area is (2)(2) = 4.Then,circum ference of the circle is (7/8)(8) = 7.Since circum ference is $2\pi r = 7$,the radius of the circle is

$$r = \frac{7}{2\pi}$$

U sing these num bers:

Q uantity A : The area of the square = 4

Q uantity B : The area of the circle = $\pi r^2 = \pi\left(\frac{7}{2\pi}\right)^2 = \pi\left(\frac{49}{4\pi^2}\right) = \frac{49}{4\pi} \approx 3.9$

(U se the calculator and the approxim ation 3.14 for π to determ ine that Q uantity A is larger.)

41.**(C).**Y ou can set up and sim plify tw o equations,one for the calories consum ed and the other for cost,using variables *A* and *B* for the num ber of servings of Snacks *A* and *B* ,respectively.

C alories:
320*A* + 110*B* = 2,370
32*A* + 11*B* = 237 {divided by 10}

C ost:
$1.50*A* + $0.45*B* = $10.65
150*A* + 45*B* = 1,065 {m ultiply by 100 to elim inate decim als}

$30A + 9B = 213$ {divided by 5}

So you now have a system of tw o equations and tw o variables,w hich could be solved for A and B .B ut even in their sim plified form ,these tw o equations have aw kw ard coefficients that w ill m ake solving m essy.

Since this is a Q uantitative C om parison question,it w ould be sm arter to "cheat off of the easy statem ent." That m eans to plug in the 5 from Q uantity B as a possible num ber of servings of Snack A ,and see w hat happens.

Plug $A = 5$ in to each equation.

C alories: $32(5) + 11B = 237$,so $160 + 11B = 237$,and $11B = 77$.Therefore $B = 7$.

C ost: $30(5) + 9B = 213$,so $150 + 9B = 213$,and $9B = 63$.Therefore $B = 7$.

This show s that $A = 5$ and $B = 7$ is the solution you w ould get by solving the system of equations yourself.

Thus,Q uantity A and Q uantity B are both 5.

42.**(C).**In this recursive function,each term is dependent on tw o others:

$a_2 = (a_1 + a_3)/2$
$a_3 = (a_2 + a_4)/2$
$a_4 = (a_3 + a_5)/2$

… and so on.W ithout actual num bers to plug in,it w ill be difficult to com pare Q uantity A and Q uantity B .

Y ou could try to put all a_n in term s of a_1 algebraically,and hope to find a pattern.If you haven't already,go ahead,try it! It's a m ess.

The best w ay to m ake sense of the sequence definition is to list som e (random ly m ade-up) actual num bers that follow the sequence rules.

If $a_1 = 1$ and $a_2 = 3$,you can extrapolate that the sequence is: 1,3,5,7,9,11,etc.
If $a_1 = -100$ and $a_2 = 50$,you can continue the sequence: -100,50,200,350,500,650,etc.
If $a_1 = 0$ and $a_2 = -4$,the sequence is: 0,-4,-8,-12,-16,-20,etc.

N o m atter the value of a_1 and a_2,the pattern is the sam e.A fter the first term ,each term in the sequence is equal to the preceding term plus som e constant.The constant in the test sequences w as equal to $a_2 - a_1$,according to the num bers you started w ith.

N ow you can m ore easily put all a_n in term s of a_1:

$a_1 = a_1$
$a_2 = a_1 + c$

$a_3 = a_2 + c = a_1 +$
$2c$ $a_4 = a_1 + 3c$
$a_5 = a_1 + 4c$
…
$a_n = a_1 + (n - 1)c$

Thus:

Q uantity A	**Q uantity B**
$a_{51} - a_{48} = (a_1 + 50c) - (a_1 + 47c) = 3c$	$a_{37} - a_{34} = (a_1 + 36c) - (a_1 + 33c) = 3c$

The tw o quantities are the sam e.

43.**(B).**The term 12*x* has prim e factors 2,2,3,and *x* (actually,you don't know w hether *x* is prim e,but since you don't know anything else about it right now ,leave it as *x*).

The term 35*y* has prim e factors 5,7,and *y*.(A gain,you don't know w hether *y* is prim e,but you can't do anything m ore w ith it right now .)

Y ou are told that the greatest factor held in com m on betw een 12*x* and 35*y* is 5*y*.Therefore,12*x* and 35*y* each contain both 5 and *y*.O f course,you already knew that 35*y* contained both 5 and *y*,but you have definitely just learned som ething new about 12*x* - it also contains 5*y*.Y ou also now know that *y* C A N N O T contain 2 and/or 3,since,if it did, the greatest com m on factor w ould also contain the 2 and/or 3 (since 12*x* and 35*y* w ould then *both* contain the 2 and/or 3).Sim ilarly,*x* cannot contain a 7 — if it did,the 7 w ould appear in the greatest com m on factor (w hich it does not).

Thus,so far,you know :

12*x* contains 2,2,3,5,*y*,and possibly other factors but N O
T a 7 35*y* contains 5,7,*y* and *y* does N O T contain 2 or 3

Take a look at som e exam ples.If $x = 55$ and $y = 11$,then *x* correctly contains both 5 and *y*,and the G C F of 12(55) and 35(11) w ould indeed be 5*y*,or 55.A lternatively,if $x = 5$ and $y = 1$,then the G C F of 12(5) and 35(1) w ould again be 5*y*,w hich in this case w ould be 5.

In both exam ples,the rem ainder w hen 12*x* is divided by 10 is 0 and thus Q uantity A is equal to 0.Y ou can be certain that this w ill alw ays be true because 12*x* definitely contains both 2 and 5.A ny integer w ith 2 and 5 in its prim e factors w ill alw ays be a m ultiple of 10.

In the first exam ple,$x = 55$ and $y = 11$,the G C F of *x* and *y* is 11.In the second exam ple,$x = 5$ and $y = 1$,the G C F of *x* and *y* is 1.Since *x* and *y* w ill alw ays be integers,their greatest com m on factor w ill alw ays be 1 or m ore.Thus,Q uantity B is larger.

44.**(A).**O ne good w ay to w ork through this problem is to pick a num ber,ideally starting w ith the innerm ost shape,the sm all circle.Let's say this circle has radius 1 and diam eter 2,w hich w ould also m ake the side of the sm aller square equal to 2.

If the sm all square has side 2,its diagonal w ould be $2\sqrt{2}$ (based on the 45-45-90 triangle ratios,or you could do the Pythagorean Theorem using the legs of 2 and 2).If the diagonal is $2\sqrt{2}$,then the diam eter of the larger circle is also $2\sqrt{2}$ (and the radius of the larger circle is one-half of that,or $\sqrt{2}$),m aking the side of the larger square also equal to $2\sqrt{2}$.Therefore:

Sm all circle: radius = 1,area = π

Large circle: radius = $\sqrt{2}$,area = 2π

Sm all square: side = 2,area = 4

Large square: side = $2\sqrt{2}$,area = 8

Thus,the large circle has tw ice the area of the sm all circle,and the large square has tw ice the area of the sm all square. This w ill w ork for any num bers you choose.In fact,you m ay w ish to m em orize this as a shortcut: if a circle is inscribed in a square that is inscribed in a circle,the large circle has tw ice the area of the sm all circle;sim ilarly,if a square is inscribed in a circle that is inscribed in a square,the large square has tw ice the area of the sm all square.

In Q uantity A ,the ratio of the area of the larger square to the sm aller square is 2/1 = 2.

In Q uantity B ,tw ice the ratio of the area of the sm aller circle to the area of the larger circle = 2(1/2) = 1.

45.**(A).**This problem is m uch easier than it looks! O f course,the integers are m uch too large to fit in your calculator. H ow ever,all you need to know is that a pair consisting of one 2 and one 5 m ultiplies to 10 and therefore adds a zero to the end of a num ber.For instance,a num ber w ith tw o 2's and tw o 5's in its prim e factors w ill end w ith tw o zeroes, because the num ber is a m ultiple of 100.

Q uantity A has 19 5's and m any m ore 2's (since 2^{16} and 4^{18} together is obviously m ore than 19 2's — if you really w ant to know ,it's 2^{16} and $(2^2)^{18}$,or 2^{16} and 2^{36},or 2^{52},or 52 2's).Since you need *pairs* m ade up of one 2 and one 5, you can m ake exactly 19 pairs (the leftover 2's don't m atter),and the num ber ends in 19 zeroes.
Q uantity B has 16 5's and m any m ore 2's (specifically,there are 53 2's,but you should be able to tell at a glance that there are obviously m ore than 16 2's,so you don't need to calculate this).Since you need *pairs* m ade up of one 2 and one 5,you can m ake exactly 16 pairs (the leftover 2's don't m atter),and the num ber ends in 16 zeroes.Thus,Q uantity A is larger.

46.**(A).**C alculate several term s of the sequence defined by $a_n = 2^n - \frac{1}{2^{n-33}}$ and look for a pattern.

$$a_1 = 2^1 - \frac{1}{2^{-32}} = 2^1 - 2^{32}$$

$$a_2 = 2^2 - \frac{1}{2^{-31}} = 2^2 - 2^{31}$$

$$\ldots$$

$$a_{16} = 2^{16} - \frac{1}{2^{-17}} = 2^{16} - 2^{17}$$

$$a_{17} = 2^{17} - \frac{1}{2^{-16}} = 2^{17} - 2^{16}$$

$$\ldots$$

$$a_{31} = 2^{31} - \frac{1}{2^{-2}} = 2^{31} - 2^2$$

$$a_{32} = 2^{32} - \frac{1}{2^{-1}} = 2^{32} - 2^1$$

N otice that the 16th and 17th term s (the tw o m iddle term s in a set of 32 term s) are arithm etic inverses,that is,their sum is zero.Likew ise,the 1st and 32nd term s sum to zero,as do the 2nd and 31st term s.In the first 32 term s of the sequence,there are 16 pairs that each sum to zero.Thus,Q uantity A is zero.

For the sum of the first 31 term s,you could either

1. Subtract a^{32} from the sum of the first 32 term s: 0 - (2^{32} - 2^1) = 2^1 - 2^{32} = 2 - (a very large num ber) = negative,or
2. R ealize that in the first 31 term s,all term s except $a1$ can be paired such that the pair sum s to zero,so the sum of the first 19 term s = $a1$ = 2^1 - 2^{32} = 2 - (a very large num ber) = negative.

Thus,Q uantity B is negative,w hich is less than zero.Q uantity A is larger.

47.**36.6.**To calculate first find the squared differences betw een the average and the term s.For exam ple,the difference betw een the average,4,and the first term ,-1,is 5.5^2 = 25.D o this for all four term s:

-1: $(4 - (-1))^2 = 5^2 = 25$
4: $(4 - (0))^2 = 4^2 = 16$
30: $(4 - (30))^2 = (-26)^2 = 676$
-21: $(4 - (-21))^2 = (25)^2 = 625$

A dd all of these term s together: 25 + 16 + 625 + 676 = 1,342.

Take the square root (use the calculator!): Square root of 1342 = 36.633.

The question asked you to round to the nearest tenth,so the answ er is 36.6.

N ote: Thus far,all of the questions w e've seen on the real G RE dance around the issue of calculating standard deviation — no question has actually asked you to calculate it.H ow ever,w e've included this exam ple here,just in case the G RE ups the ante on us in future.

48.**(C).**B efore figuring out how m any balls you have of each integer value,consider w hat the question is asking: the "interquartile range" of a group of 100 integers.To find this range,split the 100 integers into tw o groups,a low er 50 and an upper 50.Then find the m edian of each of those groups.The m edian of the low er group is the first quartile (Q_1),w hile the m edian of the upper group is the third quartile (Q_3).Finally,Q_3-Q_1 is the interquartile range.

The m edian of a group of 50 integers is the average (arithm etic m ean) of the 25th and the 26th integers w hen ordered from sm allest to largest.O ut of the ordered list of 100 integers from sm allest to largest,then,find #25 and #26 and average them to get the first quartile.Likew ise,find #75 and #76 and average them to get the third quartile.Then perform the subtraction.

Each ball has an integer value painted on the side — either 1,2,3,4,5,6,7,or 8.Figure out how m any balls there are for each integer by applying the given form ula,starting w ith the low est integer in the list (1) and going up from there.

N um ber of balls labeled num ber 1 = 18 - $(1 - 4)^2$ = 18 - (-3)2 = 18 - 9 = 9 balls.These represent balls #1 through #9.

N um ber of balls labeled num ber 2 = 18 - $(2 - 4)^2$ = 18 - $(-2)^2$ = 18 - 4 = 14 balls,representing balls #10 through #23. B e careful w hen counting;the 14th ball is #23,not #24,because #10 is the first,#11 is the second,and so on.

N um ber of balls labeled num ber 3 = 18 - $(3 - 4)^2$ = 18 - $(-1)^2$ = 18 - 1 = 17 balls,representing #24 through #40.

A t this point,you can tell that balls #25 and #26 both have a 3 on them .So the first quartile Q_1 is the average of 3 and 3,nam ely 3.N ow keep going!

N um ber of balls labeled num ber 4 = 18 - $(4 - 4)^2$ = 18 - $(0)^2$ = 18 balls,representing balls #41 through #58.

N um ber of balls labeled num ber 5 = 18 - $(5 - 4)^2$ = 18 - $(1)^2$ = 18 - 1 = 17 balls,representing #59 through #75.

Y ou can stop here.B all #75 has a 5 on it (in fact,the last 5),w hile ball #76 m ust have a 6 on it (since 6 is the next integer in the list).Thus,the third quartile Q_1 is the average of 5 and 6,or 5.5.N otice that you have to count carefully — if you are off by even just one either w ay,you'll get a different num ber for the third quartile.

Finally,Q_3-Q_1 = 5.5 - 3 = 2.5,the interquartile range of this list of integers.

49.**(A).**Setting up the inform ation in the question in the form of an equation,you see that:

$x/13 = y + 3/13$

$x = 13y + 3$

and

$x/7 = z + 3/7$

$x = 7z + 3$

Setting the tw o values for x equal to one another you see that

$$13y + 3 = 7z + 3$$
$$13y = 7z$$

B ecause y and z m ust be w hole num bers,y m ust have 7 as a factor and z m ust have 13 as a factor.y and z can share an unlim ited num ber of factors,but y m ust have a 7 in its prim e box and z m ust have a 13 in its prim e box.

The question now asks w hat is the rem ainder of $\frac{yz}{13}$.Since 13 is in the num erator,it can be canceled out of the fraction,leaving a 1 in the denom inator and resulting in a w hole num ber w hich has a rem ainder of 0.

50.**(B).**First solve for x in the equation to reveal the recursive form ula for calculating a_n: $2 = (x/2) - a_{n-1}$

$4 = x - 2(a_{(n-1)})$

$x = 4 + 2(a_{(n-1)})$

Since “the term a_n is defined as the value of x that satisfies the equation,” substitute a_n for x to get the real form ula you are being asked to use:

$$a_n = 4 + 2(a_{(n-1)})$$

Since you have a_6 and w ant to calculate a previous term ,a_4,it m ay be useful to rew rite the equation in a form that allow s you to solve for the previous term .That is,solve for $(a_{(n-1)})$:

$$a_{(n-1)} = (a_n - 4)/2$$

N ow let a_b be a_n and a_5 be $a_{(n-1)}$:

$a_5 = (156 - 4)/2 = 76$

$a_4 = (76 - 4)/2 = 36$

$a_3 = (36 - 4)/2 = 16$

A nd $a_2 = (16 - 4)/2 = 6$

51.**(C).**C om pute the expressions for each of the term s:

$x!4 = x^4 \times 4^{-x}$ and $4!x = 4^x \times x^{-4}$

D ividing the first by the second yields

$$\frac{x^4 4^{-x}}{4^x x^{-4}} = \frac{x^4}{x^{-4}} \times \frac{4^{-x}}{4^x} = x^8 4^{-2x}$$

There are a num ber of w ays w e could w rite $x^8 4^{-2x}$:

$$x^8 4^{-2x} = \frac{x^8}{4^{2x}} = \frac{x^8}{16^x}$$

The tw o quantities are equal.

52.**(A).**First,calculate the sum of the even integers betw een 1 and 100 (2,4,6… 98,100).Y ou can think of this list as 50 even integers to be sum m ed or,m ore usefully,you can think of it as 25 integer pairs each of w hich sum to 102 (2 + 100,4 + 98,… ,50 + 52).The sum of these 25 pairs is sim ply 25 × 102.This is an alternate approach to the usual form ula that tells you to com pute the average value tim es the num ber of term s.N ext,calculate the sum of the odd integers betw een 100 and 150 (101,103,105,… ,147,149).Like before,you can turn this list into pairs,though you need to be careful because there are an odd num ber of integers in this list and the m iddle num ber (125) w ill not get a pair: (101 + 149,103 + 147,… ,123 + 127).There are 12 pairs that each add to 250 and one leftover integer, 125.The sum is 12 × 250 + 125 or 12.5 × 250.The ratio be asked for in the question is (25 × 102) to (12.5 × 250). Y ou can divide both sides by 25 to yield the ratio 102 to 125.

53.**(C).**The problem gives tw o w ays to calculate the fourth term : (1) the definition of the sequence tells you that $a_4 = a_3^2 - a_2^2$ and (2) you are told that $a_4 = a_1(a_2 + a_3) = 2(a_2 + a_3)$.Setting these tw o equal gives $a_3^2 - a_2^2 = 2(a_2 + a_3)$. Factor the left side: $(a_3 + a_2)(a_3 - a_2) = 2(a_2 + a_3)$.Since $a_n > 0$ for all possible n's,you know that $(a_3 + a_2)$ does not equal 0 and you can divide both sides by it: $a_3 - a_2 = 2$ and $a_3 = a_2 + 2$.U sing the definition of a_3,you know $a_3 = a_2^2 - a_1^2 = a_2^2 - 4$.Substituting for a_3 yields: $a_2 + 2 = a_2^2 - 4$ and $a_2^2 - a_2 - 6 = 0$.Factor and solve: $(a_2 - 3)(a_2 + 2) = 0$;$a_2 = 3$ or -2.a_n m ust be positive,so $a_2 = 3$ and $a_3 = a_2 + 2 = 3 + 2 = 5$.

54.**(D).**Since a_1 and a_3 are integers,a_2 m ust also be an integer: $a_3 = (a_1 + a_2)/2$ or IN T = (IN T + a_2)/2 so 2(IN T) = IN T + a_2 and a_2 = 2(IN T) - IN T w hich is itself an integer.a_4 w ill be the average of tw o integers.If $a_2 + a_3$ is even,a_4 w ill be an integer.If $a_2 + a_3$ is odd,a_4 w ill be a decim al ending in 0.5.If a_4 is an integer,a_5 can be an integer or can be a decim al ending in 0.5.If a_4 is a decim al ending in 0.5,a_5 m ust be a decim al ending in 0.25 or 0.75.a_5 cannot be a decim al ending in 0.375 such as 75/8 = 9.375.N ote that a_5 can be negative: Even if a_1 and a_3 are positive,that does not rule out the possibility that a_2 (and subsequent term s) could be negative.

55.**(D).**U se the definition of ? to rew rite the equation: $\left|\frac{x+1}{x}\right| - \frac{2+1}{2} = \frac{1}{2}\left(\left|\frac{x+1}{x}\right| - \frac{-1+1}{-1}\right)$.Sim plifying yields: $\left|\frac{x+1}{x}\right| - \frac{3}{2} = \frac{1}{2}\left|\frac{x+1}{x}\right|$. Let $z = \left|\frac{x+1}{x}\right|$.Substitute z into the equation: $z - 3/2 = (1/2)z$ or $z = 3$.To solve $\left|\frac{x+1}{x}\right| = 3$,take tw o cases

(1) $\frac{x+1}{x} > 0.$ $\frac{x+1}{x}$ = 3 or x = 0.5.

(2) $\frac{x+1}{x} < 0.$ $\frac{x+1}{x}$ = -3 or x = -0.25.The sum of the solutions is 0.5 + (-0.25) = 0.25.

56.**(B).**Expressed algebraically, $\sqrt{10-3X} > X$.B ecause both sides of this inequality are non-negative,you can square both sides to result in the follow ing:

$10 - 3X > X^2$
$0 > X^2 + 3X - 10$
$0 > (X + 5)(X - 2)$

N ow ,because the product of (X + 5) and (X - 2) is negative,you can deduce that the larger of the tw o expressions,(X + 5),m ust be positive and the sm aller expression,(X - 2),m ust be negative.Therefore,X > -5 and X < 2.C om bining these yields -5 < X < 2.

H ow ever,because the question indicates that X is non-negative,X m ust be 0 or greater.Therefore,0 $\leq$ X < 2.The absolute value sign in Q uantity A doesn’t change anything — X is still greater than or equal to zero and less than 2,and Q uantity B is larger.
A lternatively,plug the value from Q uantity B into $\sqrt{10-3X} > X$:

$$\sqrt{10-3(2)} > 2$$
$$\sqrt{4} > 2$$
$$2 > 2$$

This is FA LSE,so X cannot be 2.

N ow ,plug in a sm aller or larger value to determ ine w hether X needs to be greater than or less than 2.If x = 1:

$$\sqrt{10-3(1)} > 1$$
$$\sqrt{7} > 1$$

$\sqrt{7}$ is betw een 2 and 3,so this is true.

Trying values w ill show that only values greater than or equal to zero and less than 2 m ake the statem ent true,so Q uantity A m ust be sm aller than 2.

57.**(A).**The area of an equilateral triangle is $\frac{b^2\sqrt{3}}{4}$ w here b is the length of one side.Since this area is betw een 25

$\sqrt{3}$ and $36\sqrt{3}$,you can substitute to get $25\sqrt{3} < \frac{b^2\sqrt{3}}{4} < 36\sqrt{3}$.D ividing all sides by $\sqrt{3}$ yields 25 < $b^2/4$ < 36.M ultiplying all sides by 4 yields 100 < b^2 < 144,and taking the square root of all sides,one gets 10 < *b* < 12.Since every possibility for *b* is greater than 9,Q uantity A is larger.

58.**(E).**W hen dealing w ith absolute values,you m ust typically consider tw o outcom es.First determ ine the outcom e if the expression w ithin the absolute value sign is positive.So,if 8 - 2*x* > 0,then |8 - 2*x*|= 8 - 2*x*,and therefore 8 - 2*x* < 3*y* - 9 or 2*x* > 17 - 3*y*.

Y ou also m ust determ ine the outcom e if the expression w ithin the absolute value sign is negative.So,if 8 - 2*x* < 0, then |8 - 2*x*|= 2*x* - 8,and therefore 2*x* - 8 < 3*y* - 9 or 2*x* < 3*y* - 1.C om bining these tw o inequalities,one arrives at 3*y* - 1 > 2*x* > 17 - 3*y*.

N ow a quick sanity check to m ake sure the inequality m akes sense: 3*y* - 9 m ust be greater than 0 or the absolute value could not be less than 3*y* - 9.So *y* > 3.This m eans 17 - 3*y* < 8,and 3*y* - 1 > 8,so there is definitely room for 2*x* to fit betw een those values.If the potential values of 17 - 3*y* and 3*y* - 1 had overlapped,this w ould be an indication either that a m istake had been m ade or that the problem required further investigation to refine the result.A s it is,(E) w ill w ork as an answ er for this problem .

59.**(A).**This is an overlapping set problem .M atrix 1 show s an initial setup for a double-set m atrix.The colum ns are headed “Skilled in B JJ” and “N ot Skilled in B JJ.” The row s are headed “Skilled in M uy Thai” and “N ot Skilled in M uy Thai.” There is also a total row and a total colum n.

W hen dealing w ith overlapping sets,consider w hether the question is giving inform ation regarding the population as a w hole or regarding a subset of the population.W hile the first statem ent (“30% of all fighters”) refers to the w hole population,the second statem ent (“20% of the fighters w ho are not skilled in B razilian Jiu Jitsu”) refers to a subset of the population,in this case the 40% w ho are not skilled in B razilian Jiu Jitsu.Thus,8% are skilled in M uy Thai but not in B razilian Jiu Jitsu,as seen in M atrix 1.

M atrix 1

	Skilled in B JJ	**N ot Skilled in B JJ**	**Total**
Skilled in M uy Thai	> 30	8	
N ot Skilled in M uy Thai			
Total	60	40	100

M atrix 2 show s how to fill out additional cells.N otably,there are som e ranges of values that are possible for the cells in the first colum n.These ranges are lim ited by 0 on the low end and 60 on the high end.

M atrix 2

	Skilled in B JJ	**N ot Skilled in B JJ**	**Total**
Skilled in M uy Thai	> 30 but≤60	8	
N ot Skilled in M uy Thai	≤0 but < 30	32	
Total	60	40	100

M atrix 3 show s the ranges of values that are possible for the percentage of people skilled in M uy Thai and the percentage of people not skilled in M uy Thai.Particularly,the percent of fighters w ho are skilled in M uy Thai is greater than 38 but less than or equal to 68.Thus,Q uantity A is larger.

M atrix 3	Skilled in B JJ	N ot Skilled in B JJ	Total
Skilled in M uy Thai	> 30 but ≤ 60	8	> 38 but ≤ 68
N ot Skilled in M uy Thai	≤ 0 but < 30	32	≤ 32 but < 62
Total	60	40	100

60.**(B).**This is a good exam ple of a problem w here one can use the idea of extrem e values.Y ou can express this situation w ith the equation $r = \frac{kb}{n^2}$,w here *k* is a constant.Q uadrupling *b* and m ore than tripling *n* yields the follow ing equation: $r_1 = \frac{k \times 4b}{(\text{"greater than } 3n\text{"})^2}$,w here r_1 represents the new rate of data transfer. Squaring a value that is greater than 3 produces a value that is greater than 9,allow ing one to rew rite the equation as $r_1 = \frac{k \times 4b}{\text{"greater than } 9n^2\text{"}}$.R earranging this equation yields $r_1 = \frac{4}{\text{"greater than 9"}} \times \frac{k \times b}{n^2}$ divided by a value greater than 9 is a value that is less than $\frac{4}{9}$.Thus,Q uantity B is greater.