# Inequalities and A bsolute V alues

For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given),the answ er choices are alw ays as follow s:

(A ) Q uantity A is greater.
(B ) Q uantity B is greater.
(C ) The tw o quantities are equal.
(D ) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box ,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes ,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.
A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.

$$|3x - 18| = 9$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $x$ | 6 |

2.If $2z + 4 \geq -18$,w hich of the follow ing m ust be true?

(A ) $z \leq -11$
(B ) $z \leq 11$
(C ) $z \geq -11$
(D ) $z \geq -7$
(E) $z \geq 7$

3.

$$7y - 3 \leq 4y + 9$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| *y* | 4 |

4.

$$d + \frac{3}{2} < 8$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| 2*d* | 13 |

5.

$$\frac{4x}{7} \leq 15 + x$$

$$2y - 1.5 > 7$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| *x* | *y* |

6.

$$3|x - 4| = 16$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| *x* | $\frac{28}{3}$ |

7.If $b \neq 0$ and $\frac{a}{b} > 0$,then w hich of the follow ing m ust be true?

- ☐ $a > b$
- ☐ $b > 0$
- ☐ $ab > 0$

8.If $6 < 2x - 4 < 12$,w hich of the follow ing could be a value of *x*?

(A ) 4
(B ) 5
(C ) 7
(D ) 8
(E) 9

9.If $y < 0$ and $4x > y$,w hich of the follow ing could be equal to $\frac{x}{y}$?

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) 4

10.

$$|x + 6| = 3$$
$$|2y| = 6$$

| **Quantity A** | **Quantity B** |
|---|---|
| The greatest possible value for $x$ | The least possible value for $y$ |

11. If $|4y + 2| = 18$, which of the following could be the value of $y^2$?

Indicate two such values.

- ☐ 2
- ☐ 5
- ☐ 16
- ☐ 25
- ☐ 36

12.

$$3(x - 7) \geq 9$$
$$0.25y - 3 \leq 1$$

| **Quantity A** | **Quantity B** |
|---|---|
| $x$ | $y$ |

13. If $|1 - x| = 6$ and $|2y - 6| = 10$, which of the following could be the value of $xy$?

Indicate all such values.

- ☐ -40
- ☐ -14
- ☐ -10
- ☐ 56

14. If $2(x - 1)^3 + 3 \leq 19$, then the value of $x$ must be

(A) greater than or equal to

3 (B) less than or equal to 3

(C ) greater than or equal to -
3 (D ) less than or equal to -3
(E) less than -3 or greater than 3

15.If $3P < 51$ and $5P > 75$,w hat is the value of the integer $P$ ?

(A ) 15
(B ) 16
(C ) 24
(D ) 25
(E) 26

16.A bicycle w heel has spokes that go from a center point in the hub to equally spaced points on the rim of the w heel. If there are few er than six spokes,w hat is the sm allest possible angle betw een any tw o spokes?

(A ) 18 degrees
(B ) 30 degrees
(C ) 40 degrees
(D ) 60 degrees
(E) 72 degrees

17.

$$|-x| \geq 6$$

$xy^2 < 0$ w here $y$ is an integer.

| **Q uantity A** | **Q uantity B** |
|---|---|
| $x$ | -4 |

18.If $\frac{|x+4|}{2} > 5$ and $x < 0$,w hich of the follow ing could be the value of $x$?

Indicate <u>all</u> such values.

☐ -6
☐ -14
☐ -18

19.

$$|x^3| < 64$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $-x$ | $-\|x\|$ |

20.If $|0.1x - 3| \geq 1$,then $x$ could be w hich of the follow ing values?

Indicate <u>all</u> such values.

- ☐ 10
- ☐ 20
- ☐ 30
- ☐ 40
- ☐ 50
- ☐ 60

21.If $|3x + 7| \geq 2x + 12$,then

(A ) $x \leq \frac{-19}{5}$

(B ) $x \geq \frac{-19}{5}$

(C ) $x \geq 5$

(D ) $x \leq \frac{-19}{5}$ or $x \geq 5$

(E) $\frac{-19}{5} \leq x \leq 5$

22.

$|3 + 3x| < -2x$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $\|x\|$ | 4 |

23.If $|y| \leq -4x$ and $|3x - 4| = 2x + 6$,w hat is the value of $x$?

(A ) -3

(B ) -1/3

(C ) -2/5

(D ) 1/3

(E) 10

24.

$x$ is an integer such that $-x|x| > 4$.

| **Q uantity A** | **Q uantity B** |
|---|---|
| $x$ | 2 |

25.

$|x| < 1$ and $y > 0$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $\|x\| + y$ | $xy$ |

26.

$x$ and $y$ are positive num bers such that $x + y + z < 1$ and $xy = 1$

| Q uantity A | Q uantity B |
|---|---|
| $z$ | -1 |

27.

$|x| > |y|$ and $x + y > 0$

| Q uantity A | Q uantity B |
|---|---|
| $y$ | $x$ |

28.

$x$ and $y$ are integers such that $|x|(y) + 9 < 0$ and $|y| \leq 1$.

| Q uantity A | Q uantity B |
|---|---|
| $x$ | -9 |

29.If $x + y + z = 0$ and $z = 8$,w hich of the follow ing m ust be true?

(A ) $x < 0$
(B ) $y < 0$
(C ) $x - y < 0$
(D ) $z - y > 0$
(E) $x + y < 0$

30.

$p + |k| > |p| + k$

| Q uantity A | Q uantity B |
|---|---|
| $p$ | $k$ |

31.

$|x| + |y| > |x + z|$

| Q uantity A | Q uantity B |
|---|---|
| $y$ | $z$ |

32.

$b \neq 0$

$$\frac{|a|}{b} > 1$$

$$a + b < 0$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $a$ | 0 |

33.If $\frac{a}{b} > \frac{c}{d}$,w hich of the follow ing statem ents m ust be true?

Indicate <u>all</u> such statem ents.

- ☐ $\frac{a}{b} - \frac{c}{d} > 0$
- ☐ $ad < bc$
- ☐ $ad > bc$

34.If $^2$ < 0,w hich of the follow ing m ust be true? $f$ $g$

(A ) $f < 0$
(B ) $g < 0$
(C ) $fg < 0$
(D ) $fg > 0$
(E) $^2 < 0$ $f$

35.$\sqrt{96} < x\sqrt{6}$ and $\frac{x}{\sqrt{6}} < \sqrt{6}$ .If $x$ is an integer,w hich of the follow ing is the value of $x$?

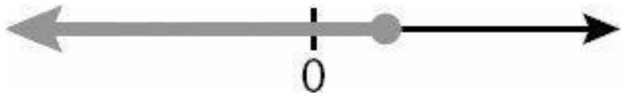
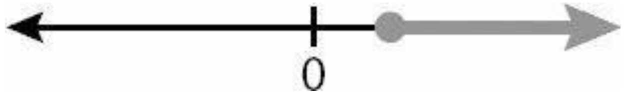
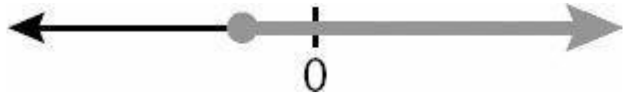
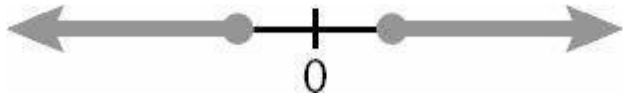
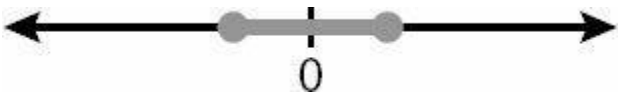
(A ) 2
(B ) 3
(C ) 4
(D ) 5
(E) 6

36.

$$|x|y > x|y|$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $(x + y)^2$ | $(x - y)^2$ |

37.W hich of the follow ing could be the graph of all values of $x$ that satisfy the inequality $4 - 11x \geq \frac{-2x+3}{2}$ ?


(A )

(B )

(C )

(D )

(E)

38.If $|x^2 - 6| = x$,w hich of the follow ing could be the value of $x$?

(A ) -2
(B ) 0
(C ) 1
(D ) 3
(E) 5

39.

$$-1 < a < 0 < |a| < b < 1$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $\left(\frac{a^2\sqrt{b}}{\sqrt{a}}\right)^2$ | $\frac{ab^5}{\left(\sqrt{b}\right)^4}$ |

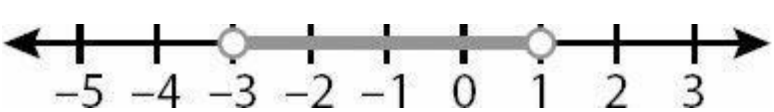
40.

$$x > |y| > z$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $x + y$ | $|y| + z$ |

41.The integers $k$,$l$,and $m$ are consecutive even integers betw een 23 and 33.W hich of the follow ing could be the average of $k$,$l$,and $m$ ?

(A ) 24
(B ) 25
(C ) 25.5
(D ) 28
(E) 32

42. 

The num ber line above represents w hich of the follow ing inequalities?

(A ) $x < 1$
(B ) $-6 < 2x < 2$
(C ) $-9 < 3x < 6$
(D ) $1 < 2x < 3$
(E) $x > -3$

43.For a jam balaya cook-off,there w ill be $x$ judges sitting in a single row of $x$ chairs.If $x$ is greater than 3 but no m ore than 6,w hich of the follow ing could be the num ber of possible seating arrangem ents for the judges?

Indicate <u>tw o</u> such num bers.

☐ 6
☐ 25
☐ 120
☐ 500
☐ 720

44.If $b \neq 0$,w hich of the follow ing inequalities m ust be equivalent to $\frac{a}{-3b} < c$?

Indicate <u>all</u> such inequalities.

☐ $\frac{a}{b} > -3c$
☐ $\frac{a}{-3} < bc$
☐ $a > -3bc$

45.

$$a - b > a + b + c$$

| **Quantity A** | **Quantity B** |
|---|---|
| $2b + c$ | $b + c$ |

46.

$$|x + y| = 10$$
$$x > 0$$
$$z < y - x$$

| **Quantity A** | **Quantity B** |
|---|---|
| $z$ | 10 |

47.

$$0 < a < \frac{b}{2} < 9$$

| **Q uantity A** | **Q uantity B** |
|---|---|
| 9 - $a$ | $\frac{b}{2} - a$ |

48.

For all values of the integer $p$ such that $1.9 < |p| < 5.3$, the function $f(p) = p^2$

| **Q uantity A** | **Q uantity B** |
|---|---|
| $f(p)$ for the greatest value of $p$ | $f(p)$ for the least value of $p$ |

49.If $\left|\frac{a}{b}\right|$ and $\left|\frac{x}{y}\right|$ are reciprocals and $\frac{a}{b}\left(\frac{x}{y}\right) < 0$ w hich of the follow ing m ust be true?

(A ) $ab < 0$

(B ) $\frac{a}{b}\left(\frac{x}{y}\right) < -1$

(C ) $\frac{a}{b} < 1$

(D ) $\frac{a}{b} = -\frac{y}{x}$

(E) $\frac{y}{x} > \frac{a}{b}$

50.If $mn < 0$ and $\frac{k}{m} + \frac{l}{n} < mn$ ,w hich of the follow ing m ust be true?

(A ) $km + ln < (mn)^2$
(B ) $kn + lm < 1$
(C ) $kn + lm > (mn)^2$
(D ) $k + l > mn$
(E) $kn > -lm$

51.W hich of the follow ing inequalities is equivalent to $|m + 2| < 3$ ?

(A ) $m < 5$
(B ) $m < 1$
(C ) $-5 < m < 5$

(D ) $m > -1$
(E) $-5 < m < 1$

52.If the reciprocal of the negative integer $X$ is greater than the sum of $Y$ and $Z$,then w hich of the follow ing m ust be true?

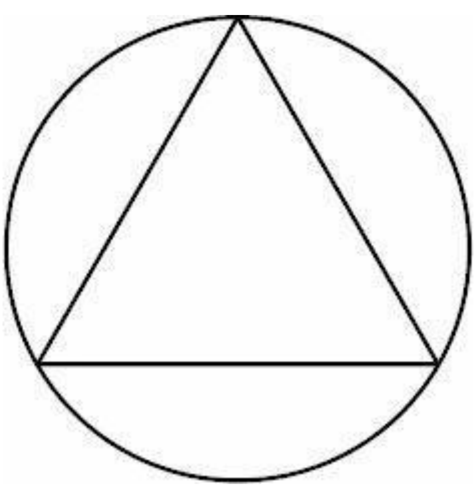
(A ) $X > Y + Z$
(B ) $Y$ and $Z$ are positive
(C ) $1 > X(Y + Z)$
(D ) $1 < XY + XZ$
(E) $\frac{1}{X} > Z - Y$

53.If $m + n - 2p < p + n + 4m$ ,w hich of the follow ing inequalities m ust be true?

(A ) $5m < 3p$
(B ) $p > -m$
(C ) $3m > 3p + 2n$
(D ) $p > 2$
(E) $n < p$

54.If $u$ and $-3v$ are greater than 0 and $\sqrt{u} < \sqrt{-3v}$,w hich of the follow ing cannot be true?

(A ) $u/3 < -v$
(B ) $u/v > -3$
(C ) $\sqrt{\frac{u}{-v}} < \sqrt{3}$
(D ) $u + 3v > 0$
(E) $u < -3v$



55.In the figure above,an equilateral triangle is inscribed in a circle.If the arc bounded by adjacent corners of the triangle is betw een 4 $\pi$ and 6 $\pi$ long,w hich of the follow ing could be the diam eter of the circle?

(A ) 6.5
(B ) 9
(C ) 11.9
(D ) 15
(E) 23.5

# Inequalities and A bsolute V alues A nsw ers

1.**(D ).**Since $3x - 18$ is inside an absolute value,it could be either positive or negative 9 to have an absolute value of 9. Thus,solve the equation tw ice,once as though $3x - 18$ is positive and once as though it is negative.

$$|3x - 18| = 9$$

$$\begin{array}{lll} (3x - 18) = +9 & \text{or} & (3x - 18) = -9 \\ 3x = 27 & & 3x = 9 \\ x = 9 & & x = 3 \end{array}$$

$$x = 9 \text{ or } 3$$

B ecause $x$ could be 9 or 3,$x$ could be greater or less than 6,so the correct answ er is (D ).

2.**(C ).**Solve the inequality algebraically:

$$2z + 4 \geq -18$$
$$2z \geq -22$$
$$z \geq -11$$

3.**(D ).**Solve the inequality algebraically:

$$7y - 3 \leq 4y + 9$$
$$3y - 3 \leq 9$$
$$3y \leq 12$$
$$y \leq 4$$

B ecause $y$ could be equal to 4 or greater than 4,the correct answ er is (D ).

4.**(B ).**Solve the inequality algebraically:

$$d + \frac{3}{2} < 8$$
$$d < 8 - 1.5$$
$$d < 6.5$$

Q uantity A is $2d$,so m ultiply both sides of the inequality by 2:

$$2d < 13$$

Q uantity B is equal to 13,w hile $2d$ is less than 13,so the correct answ er is (B ).

5.**(D ).**Solve algebraically for *x* and *y:*

$$\frac{4x}{7} \leq 15 + x$$
$$4x \leq 105 + 7x$$
$$-3x \leq 105$$
$$x \geq -35$$

(R em em ber to flip the inequality sign w hen dividing by -3!)

$$2y - 1.5 > 7$$
$$2y > 8.5$$
$$y > 4.25$$

K now ing that $x \geq -35$ and $y > 4.25$ is not enough to tell w hich is greater — the tw o ranges have a lot of overlap.For instance,*x* could be -30 and *y* could be 5 (Q uantity B is greater),or *x* could be 100 and *y* could be 20 (Q uantity A is greater).The correct answ er is (D ).

6.**(D ).**Solve the inequality by first dividing both sides by 3 to isolate the absolute value,then solving for the positive and negative possibilities of (*x* - 4).

$$3|x-4| = 16$$
$$|x-4| = \frac{16}{3}$$
$$(x-4) = \frac{16}{3} \quad \text{or} \quad (x-4) = -\frac{16}{3}$$
$$x - 4 = \frac{16}{3} \qquad x = -\frac{16}{3} + 4$$
$$x = \frac{16}{3} + 4 \qquad x = -\frac{16}{3} + \frac{12}{3}$$
$$x = \frac{16}{3} + \frac{12}{3} \qquad x = -\frac{4}{3}$$
$$x = \frac{28}{3}$$

*x* could be $\frac{28}{3}$ or $-\frac{4}{3}$,m aking the tw o quantities equal or Q uantity B greater,respectively.The correct answ er is (D ).

7.**III only.**If $\frac{a}{b} > 0$,then both *a* and *b* m ust have the sam e sign.That is,*a* and *b* are either both positive or both negative.Statem ent I could be true,but is not necessarily true.The relative values of *a* and *b* are not indicated by the inequality in the question stem .Statem ent II could be true,but is not necessarily true.If *a* w ere negative,*b* could be negative.Statem ent III m ust be true,as it indicates that *a* and *b* have the sam e sign.

8.**(C ).**W hen m anipulating a "three-sided" inequality,you m ust perform the sam e operations on all "sides." Therefore, the first step to sim plify this inequality w ould be to add 4 to all sides: 10 < 2*x* < 16.N ext,divide all sides by 2.The result is 5 < *x* < 8.The only answ er choice that fits w ithin the param eters of this inequality is 7.The correct answ er is (C ).

9.**(A ).**If *y* is negative,then dividing both sides of the second inequality by *y* yields $\frac{4x}{y} < 1$.R em em ber,you m ust sw itch the direction of the inequality sign w hen m ultiplying or dividing by a negative (w hether that negative is in num ber or variable form ).N ext,dividing both sides by 4 changes the inequality to $\frac{x}{y} < \frac{1}{4}$.The only answ er choice less than $\frac{1}{4}$ is 0.The correct answ er is (A ).

10.**(C ).**Solve each inequality,rem em bering that the phrase inside an absolute value can be positive or negative,so solve for each possibility:

|*x* + 6|= 3

| | | |
|---|---|---|
| (*x* + 6) = 3 | or | (*x* + 6) = -3 |
| *x* = -3 | | *x* + 6 = -3 |
| | | *x* = -9 |

*x* = -3 or -9

|2*y*|= 6

| | | |
|---|---|---|
| (2*y*) = 6 | or | (2*y*) = -6 |
| *y* = 3 | | *y* = -3 |

*y* = 3 or -3

The greatest possible value for *x* is -3.The least possible value for *y* is -3.The tw o quantities are equal,and the correct answ er is (C ).

11.**16,25.**Solve the inequality,rem em bering that 4*y* + 2 could be positive or negative,so solve for both possibilities:

|4*y* + 2|= 18

$(4y + 2) = 18$ or $(4y + 2) = -18$

$4y = 16$ $\quad$ $4y = -20$

$y = 4$ $\quad$ $y = -5$

$y = 4$ or $-5$

The value of $y^2$ could be 16 or 25.

12.**(D ).**Solve each inequality algebraically:

$3(x - 7) \geq 9$
$x - 7 \geq 3$
$x \geq 10$

$0.25y - 3 \leq 1$
$0.25y \leq$
$4\ y \leq 16$

Since the ranges for $x$ and $y$ overlap,either quantity could be greater.For instance,$x$ could be 11 and $y$ could be 15 ($y$ is greater),or $x$ could be 1,000 and $y$ could be -5 ($x$ is greater).The correct answ er is (D ).

13.-**40,-14,and 56 only.**Solve each absolute value:

$|1 - x| = 6$

$(1 - x) = 6$ or $(1 - x) = -6$

$-x = 5$ $\quad$ $-x = -7$

$x = -5$ $\quad$ $x = 7$

$x = -5$ or $7$

$|2y - 6| = 10$

$(2y - 6) = 10$ or $(2y - 6) = -10$

$2y = 16$ $\quad$ $2y = -4$

$y = 8$ $\quad$ $y = -2$

$y = 8$ or $-2$

Since $x = -5$ or 7 and $y = 8$ or -2,calculate all four possible com binations for $xy$:

$(-5)(8) = -40$

$(-5)(-2) = 10$
$(7)(8) = 56$
$(7)(-2) = -14$

Select -40,-14,and 56.(D o N O T pick -10,as *xy* could be 10,but not -10).

14.**(B ).** $2(x - 1)^3 + 3 \leq 19$
$2(x - 1)^3 \leq 16$
$(x - 1)^3 \leq 8$
Y ou can take the cube root of both sides of an inequality,because cubing a num ber,unlike squaring it,does not change its sign.

$x - 1 \leq$
$2\ x \leq 3$

This m atches the language in answ er choice (B ).

15.**(B ).**D ividing the first inequality by 3 results in $P < 17$.D ividing the second inequality by 5 results in $P > 15$. Therefore,$15 < P < 17$.B ecause $P$ is an integer,it m ust be 16.

16.**(E ).**In this scenario,if there are *n* spokes,there are *n* angles betw een them .Thus the m easure of the angle betw een spokes is $\frac{360}{n}$ .Since $n < 6$,you can rew rite this expression as $\frac{360}{\text{(less than 6)}}$.D ividing by a “less than” produces a “greater than” result.Therefore,$\frac{360}{\text{(less than 6)}}$ = greater than 60.The only answ er that is greater than 60 is (E).To verify,note that *n* can be at m ost 5,as *n* is an integer.B ecause there are 360 degrees in a circle,a w heel w ith 5 spokes w ould have $\frac{360}{\text{(less than 6)}}$ degrees betw een adjacent spokes.The correct answ er is (E).

17.**(B ).**First,solve the inequality for *x*,rem em bering the tw o cases you m ust consider w hen dealing w ith absolute value: $-x$ is positive and $-x$ is negative.

$|-x| \geq 6$

$+(-x) \geq 6$ or $-(-x) \geq 6$
$-x \geq 6$ $\quad x \geq 6$
$x \leq -6$

$x \leq -6$ or $x \geq 6$

B ecause $xy^2 < 0$,neither *x* nor *y* equals zero.A squared term cannot be negative,so $y^2$ m ust be positive.For $xy^2$ to be negative,*x* m ust be negative.This rules out the $x \geq 6$ range of solutions for *x*.Thus,$x \leq -6$ is the only range of valid solutions.Since all values less than or equal to -6 are less than -4,the correct answ er is (B ).

18.**III only.**Solve the absolute value inequality by first isolating the absolute value:

$$\frac{|x+4|}{2} > 5$$

$|x + 4| > 10$

If ($x$ + 4) is positive or zero,the absolute value bars do nothing and can be rem oved:

$x + 4 >$
$10$ $x > 6$

This is not a valid solution range,as the other inequality indicates that $x$ is negative.

Then solve for negative case.N ote that $|x + 4| > 10$ w hen ($x$ + 4) is m ore positive than 10 or *m ore negative* than -10.

$(x + 4) < -$
$10$ $x < -14$

A lternatively,using the identity that $|a| = -a$ w hen $a$ is negative:

$|x + 4| > 10$
$-(x + 4) > 10$ w hen ($x$ + 4) is
negative $-x - 4 > 10$
$-x > 14$

$x < -14$ (flip the inequality sign w hen m ultiplying both sides by -1.)

If $x < -14$,only -18 is a valid answ er.

19.**(D ).**First,solve the absolute value inequality,using the identity that $|a| = a$ w hen $a$ is positive or zero and $|a| = -a$ w hen $a$ is negative:

$$|x^3| < 64$$

$+(x^3) < 64$
$x < 4$

or

$-(x^3) < 64$
$x^3 > -64$ (Flip the inequality sign w hen m ultiplying by - 1.)
$x > -4$

$$-4 < x < 4$$

$x$ could be positive,negative,or zero.If $x$ is positive or zero,the tw o quantities are equal.If $x$ is negative,Q uantity A is greater.The correct answ er is (D ).

20.**10,20,40,50,60 only.**Solve the absolute value inequality,using the identity that $|a| = a$ w hen $a$ is positive or zero and $|a| = -a$ when $a$ is negative:

$|0.1x - 3| \geq 1$

$+(0.1x - 3) \geq 1$ or $-(0.1x - 3) \geq 1$

$0.1x - 3 \geq 1$

$0.1x \geq 4$

$x \geq 40$

$-0.1x + 3 \geq 1$

$-0.1x \geq -2$

$x \leq 20$ (Flip the inequality sign when dividing by -0.1)

Since $x \leq 20$ or $x \geq 40$, $x$ cannot equal 30, but it can be any of the other values from the choices.

Alternatively, plug the choices to test which values "work."

10: $|0.1(10) - 3| = |1 - 3| = |-2| = 2$, which is $\geq 1$. 20: $|0.1(20) - 3| = |2 - 3| = |-1| = 1$, which is $\geq 1$. 30: $|0.1(30) - 3| = |3 - 3| = |0| = 0$, which is NOT $\geq 1$. 40: $|0.1(40) - 3| = |4 - 3| = |1| = 1$, which is $\geq 1$. 50: $|0.1(50) - 3| = |5 - 3| = |2| = 2$, which is $\geq 1$. 60: $|0.1(60) - 3| = |6 - 3| = |3| = 3$, which is $\geq 1$.

21. **(D).** Solve $|3x + 7| \geq 2x + 12$, using the identity that $|a| = a$ when $a$ is positive or zero and $|a| = -a$ when $a$ is negative:

$+(3x + 7) \geq 2x + 12$ or $-(3x + 7) \geq 2x + 12$

$x + 7 \geq 12$

$x \geq 5$

$-3x - 7 \geq 2x + 12$

$-7 \geq 5x + 12$

$-19 \geq 5x$

$\frac{-19}{5} \geq x$

$x \leq \frac{-19}{5}$ or $x \geq 5$

22. **(B).** Solve the absolute value inequality, using the identity that $|a| = a$ when $a$ is positive or zero and $|a| = -a$ when $a$ is negative:

$|3 + 3x| < -2x$

$+(3 + 3x) < -2x$ or $-(3 + 3x) < -2x$

$3 + 5x < 0$

$5x < -3$

$x < -\frac{3}{5}$

$-3 - 3x < -2x$

$-3 < x$

$$-3 < x < -\frac{3}{5}$$

Since $x$ is betw een -3 and $-\frac{3}{5}$,its absolute value is betw een $\frac{3}{5}$ and 3.Thus,Q uantity A is less than Q uantity B .

23.**(C ).**The inequality is not strictly solvable,as it has tw o unknow ns.H ow ever,any absolute value cannot be negative.Putting $0 \leq |y|$and $|y| \leq -4x$ together,$0 \leq -4x$.D ividing both sides by -4 and flipping the inequality sign,this im plies that $0 \geq x$.

N ow solve the absolute value equation:

$$|3x - 4| = 2x + 6$$

| | | |
|---|---|---|
| $+(3x - 4) = 2x + 6$ | or | $-(3x - 4) = 2x + 6$ |
| $3x - 4 = 2x + 6$ | | $-3x + 4 = 2x + 6$ |
| $x - 4 = 6$ | | $4 = 5x + 6$ |
| $x = 10$ | | $-2 = 5x$ |

$$x = 10 \text{ or } -2/5$$

If $x = 10$ or $-2/5$,but $0 \geq x$,then $x$ can only be $-2/5$.

24.**(B ).**If $-x|x| \geq 4$,$-x|x|$is positive.B ecause $|x|$is positive by definition,$-x|x|$is positive only w hen $-x$ is also positive. This occurs w hen $x$ is negative.For exam ple,$x = -2$ is one solution allow ed by the inequality: $-x|x| = -(-2) \times |-2| = 2 \times 2 = 4$.

So,Q uantity A can be -2,-3,-4,-5,-6,etc.The m axim um value of Q uantity A is less than 2,so Q uantity B is greater.

25.**(A ).**The inequality $|x| < 1$ allow s $x$ to be either a positive or negative fraction (or zero).Interpreting the absolute value sign,it is equivalent to $-1 < x < 1$.A s indicated,$y$ is positive.

W hen $x$ is a negative fraction,

Q uantity A : $|x| + y$ = positive fraction + positive = positive

Q uantity B : $xy$ = negative fraction × positive = negative

Q uantity A is greater in these cases.

W hen $x$ is zero,

Q uantity A : $|x| + y$ = 0 + positive = positive

Q uantity B : $xy = 0 \times \text{positive} = 0$

Q uantity A is greater in this case.

W hen $x$ is a positive fraction,

Q uantity A : $|x| + y$ = positive fraction + $y$ = greater than $y$

Q uantity B : $xy$ = positive fraction × $y$ = less than $y$

Q uantity A is greater in these cases.

In all cases,Q uantity A is greater.

26.**(B ).**Solve the inequality for $z$.

$x + y + z < 1$
$z < 1 - (x + y)$

B ased on the facts that $x$ and $y$ are positive and $xy = 1$,either $x$ and $y$ both equal 1 or they are reciprocals (e.g.,2 and $\frac{1}{2}$,3 and $\frac{1}{3}$,4 and $\frac{1}{4}$,etc.).Thus,the m inim um value of $x + y$ is 2.Plugging into the inequality for $z$:

$z < 1 - (x + y)$
$z < 1$ - (at least
2) $z <$ at m ost -1

B ecause $z$ cannot equal -1 ($z$ is less than -1) Q uantity B is greater.

27.**(B ).**In general,there are four cases for the signs of $x$ and $y$,som e of w hich can be ruled out by the constraints of this question.

| $x$ | $y$ | $x + y > 0$ |
|---|---|---|
| pos | pos | true |
| pos | neg | true w hen $\|x\| > \|y\|$ |
| neg | pos | false w hen $\|x\| > \|y\|$ |
| neg | neg | false |

So only the first tw o cases need to be considered for this question.

If $x$ and $y$ are both positive,$|x| > |y|$just m eans that $x > y$.

If $x$ is positive and $y$ is negative,$x > y$ sim ply because positive > negative.

In both cases, $x > y$. Quantity B is greater.

28. **(D).** If $y$ is an integer and $|y| \leq 1$, then $y = -1, 0,$ or $1$. The other inequality can be simplified from $|x|(y) + 9 < 0$ to $|x|(y) < -9$. In words, $|x|(y)$ is negative. Because $|x|$ cannot be negative by definition, $y$ must be negative, so only $y = -1$ is possible.

If $y = -1$, then $|x|(y) = |x|(-1) = -|x| < -9$. So, $-|x| = -10, -11, -12, -13$, etc.

Thus, $x = \pm 10, \pm 11, \pm 12, \pm 13$, etc. Some of these $x$ values are greater than -9 and some are less than -9.

29. **(E).** If $x + y + z = 0$ and $z = 8$, then $x + y = -8$. It is definitely true that $-8 < 0$, so $x + y < 0$ must be true.

Alternatively, find a counterexample to disprove the other choices.

(A) $x$ could be positive: $x = 5$ and $y = -13$ make $x + y = 5 + (-13) = -8$.

(B) $y$ could be positive: $x = -13$ and $y = 5$ make $x + y = -13 + 5 = -8$.

(C) $x - y$ could be positive: $x = 5$ and $y = -13$ make $x - y = 5 - (-13) = 18$ and $x + y + z = 5 + (-13) + 8 = 0$.

(D) $z - y$ could be positive: $z = 8$ and $y = -13$ make $z - y = 8 - (-13) = 21$ and $x = 5$ would make the sum $x + y + z = 5 + (-13) + 8 = 0$.

(E) $x + y$ CANNOT be positive or zero, as $x + y + z$ would then be at least 8, not equal to 0.

30. **(A).** In general, there are four cases for the signs of $p$ and $k$, some of which can be ruled out by the constraints of this question.

| $p$ | $k$ | $p + \|k\| > \|p\| + k$ |
|---|---|---|
| pos | pos | Not in this case: For positive numbers, absolute value "does nothing," so both sides are equal to $p + k$. |
| pos | neg | True for this case: $p$ + (a positive absolute value) is greater than $p$ + (a negative value). |
| neg | pos | Not in this case: $k$ + (a negative value) is less than $k$ + (a positive absolute value). |
| neg | neg | Possible in this case: It depends on relative values. Both sides are a positive plus a negative. |

Additionally, check whether $p$ or $k$ could be zero.

If $p = 0$, $p + |k| > |p| + k$ is equivalent to $|k| > k$. This is true when $k$ is negative.

If $k = 0$, $p + |k| > |p| + k$ is equivalent to $p > |p|$. This is not true for any $p$ value.

So, there are three possible cases for $p$ and $k$ values. For the second one, use the identity that $|a| = -a$ when $a$ is negative.

| $p$ | $k$ | Interpret: |
|---|---|---|
| pos | neg | $p$ = pos > neg = $k$<br>$p > k$ |
| neg | neg | $p + \|k\| > \|p\| + k$<br>$p + -(k) > -(p) + k$<br>$p - k > -p + k$<br>$2p - k > k$<br>$2p > 2k$<br>$p > k$ |
| 0 | neg | $p$ = 0 > neg = $k$<br>$p > k$ |

In all the cases that are valid according to the constraint inequality, $p$ is greater than $k$. Quantity A is greater.

31. **(D).** Given only one inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D).

For example, $x = 2$, $y = 5$, and $z = 3$.

Check that $|x| + |y| > |x + z|$: $|2| + |5| > |2 + 3|$ is $7 > 5$, which is true.

In this case, $y > z$.

Try to find another example such that $y < z$. Always consider negatives in inequalities and absolute value questions.

Consider another example: $x = 2$, $y = -5$ and $z = 3$.

Check that $|x| + |y| > |x + z|$: $|2| + |-5| > |2 + 3|$ is $7 > 5$, which is true.

In this case, $z > y$.

Either statement could be greater. It cannot be determined from the information given.

32. **(B).** If $\frac{|a|}{b}$ is greater than 1, then it is positive. Because $|a|$ is nonnegative by definition, $b$ would have to be positive. Thus, if you cross multiply, you do not have to flip the sign of the inequality:

$$\frac{|a|}{b} > 1$$

$|a| > b$

To summarize, $b > 0$ and $|a| > b$. Putting this together, $|a| > b > 0$.

In order for $a + b$ to be negative, $a$ must be more negative than $b$ is positive. For example, $a = -4$ and $b = 2$ agree with all the constraints so far. Note that $a$ cannot be zero (because $\frac{|a|}{b} = 0$ in this case, not > 1) and $a$ cannot be positive (because $a + b > 0$ in this case, not < 0).

Therefore, $a < 0$. Quantity B is greater.

33. **I only.**

Statement I: TRUE. Subtract $\frac{c}{d}$ from both sides of the inequality $\frac{a}{b} > \frac{c}{d}$, and you will get $\frac{a}{b} - \frac{c}{d} > 0$. It must be true.

Statement II: Maybe. This is only true if *b* and *d* have opposite signs, because it is the result of multiplying both sides by *bd* and flipping the inequality sign, which you would only do when *bd* is negative.

Statement III: Maybe. This is only true if *b* and *d* have the same sign, because it is the result of multiplying both sides by *bd* without flipping the inequality sign, which is only acceptable when *bd* is positive.

34. **(B).** Neither $f$ nor $g$ can be zero, or $f^2g$ would be zero. The square of either a positive or negative base is always positive, so $f^2$ is positive. In order for $f^2g < 0$ to be true, $g$ must be negative. Therefore, the correct answer is (B). Answer choices (A), (B), and (C) are not correct because $f$ could be either positive or negative. Answer choice (E) directly contradicts the truth that $f^2$ is positive.

35. **(D).** Solve the first inequality:

$$\sqrt{96} < x\sqrt{6}$$

$$\frac{\sqrt{96}}{\sqrt{6}} < x$$

$$\sqrt{16} < x$$

$$4 < x$$

Solve the second inequality:

$$\frac{x}{\sqrt{6}} < \sqrt{6}$$

$$x < \sqrt{6}\sqrt{6}$$

$$x < \sqrt{36}$$

$$x < 6$$

C om bining the tw o inequalities,4 < $x$ < 6 so $x$ m ust be 5.The correct answ er is (D ).

36.**(B ).**In general,there are four cases for the signs of $x$ and $y$,som e of w hich can be ruled out by the constraint in the question stem .U se the identity that |$a$|= $a$ w hen $a$ is positive or zero and |$a$|= -$a$ w hen $a$ is negative:

| $x$ | $y$ | $\lvert x \rvert y > x \lvert y \rvert$ is equivalent to: | True or False? |
|---|---|---|---|
| pos | pos | $xy > xy$ | False: $xy = xy$ |
| pos | neg | $xy > x(-y)$ | False: $xy$ is negative,and $-xy$ is positive. |
| neg | pos | $(-x)y > xy$ | True: $xy$ is negative,and $-xy$ is positive. |
| neg | neg | $(-x)y > x(-y)$ | False: $-xy = -xy$ |

N ote that if either $x$ or $y$ equals 0,that case w ould also fail the constraint.

The only valid case is w hen $x$ is negative and $y$ is positive.

Q uantity A : $(x + y)^2 = x^2 + 2xy + y^2$
Q uantity B : $(x - y)^2 = x^2 - 2xy + y^2$

Ignore (or subtract) $x^2 + y^2$ as it is com m on to both quantities.Thus,

Q uantity A : $2xy$ = 2(negative)(positive) = negative
Q uantity B : $-2xy$ = -2(negative)(positive) = positive

Q uantity B is greater.

37.**(A ).**First,solve $4 - 11x \geq \frac{-2x+3}{2}$ for $x$:

$4 - 11x \geq \frac{-2x+3}{2}$

$8 - 22x \geq -2x + 3$

$5 - 22 \geq -2x$

$5 \geq 20x$

$$\frac{5}{20} \geq x$$
$$\frac{1}{4} \geq x$$

Thus,the correct choice should show the black line beginning to the right of zero (in the positive zone),and continuing indefinitely into the negative zone.Even w ithout actual values (other than zero) m arked on the graphs,only (A ) m eets these criteria.

38.**(D ).**W hile you could set $x^2 - 6$ equal to both *x* and -*x* and then solve both equations (there is a positive and negative case because of the absolute value),it is probably easier for m ost people to plug in the answ ers:

| | *x* | $x^2 - 6$ | $x^2 - 6$ |
|---|---|---|---|
| (A ) | -2 | $(-2)^2 - 6 = 4 - 6 = -2$ | 2 |
| (B ) | 0 | $(0)^2 - 6 = 0 - 6 = -6$ | 6 |
| (C ) | 1 | $(1)^2 - 6 = 1 - 6 = 5$ | 5 |
| **(D )** | **3** | $\mathbf{(3)^2 - 6 = 9 - 6 = 3}$ | **3** |
| (E) | 5 | $(5)^2 - 6 = 25 - 6 = 19$ | 19 |

N ote that only *x* = 3 w orks.W hile this chart show s the results of trying every choice,note that if you w ere doing this on your ow n,you could stop as soon as you got a choice that w orked.

39.**(A ).**From $-1 < a < 0 < |a| < b < 1$,the follow ing can be determ ined:

*a* is a negative fraction,
*b* is a positive fraction,and
*b* is m ore positive than *a* is negative.(i.e.,$|b| > |a|$,or *b* is farther from 0 on the num ber line than *a* is.)

U sing exponent rules,sim plify the quantities.

Q uantity A : $$\left(\frac{a^2\sqrt{b}}{\sqrt{a}}\right)^2 = \frac{\left(a^2\right)^2\left(\sqrt{b}\right)^2}{\left(\sqrt{a}\right)^2} = \frac{a^4 b}{a} = a^3 b$$

Q uantity B : $$\frac{ab^5}{\left(\sqrt{b}\right)^4} = \frac{ab^5}{\left(b^{1/2}\right)^4} = \frac{ab^5}{b^{\frac{1}{2}\times 4}} = \frac{ab^5}{b^2} = ab^3$$

D ividing both quantities by *b* w ould be acceptable,as *b* is positive and doing so w on't flip the relative sizes of the quantities. It would be nice to cancel $a$'s, too, but it is problematic that $a$ is negative. Dividing both quantities by $a^2$ would be okay, though, as $a^2$ is positive.

Divide both quantities by $a^2b$.

Quantity A : $\frac{a^3b}{a^2b} = a$

Quantity B : $\frac{ab^3}{a^2b} = \frac{b^2}{a}$

Just to make the quantities more similar in form, divide again by $b$, which is positive.

Quantity A : $\frac{a}{b}$

Quantity B : $\frac{b}{a}$

Both quantities are negative, as $a$ and $b$ have opposite signs. Remember that $b$ is more positive than $a$ is negative. (i.e., $|b| > |a|$, or $b$ is farther from 0 on the number line than $a$ is.) Thus, each fraction can be compared to -1.

Quantity A : $\frac{a}{b}$ is less negative than -1. That is, $-1 < \frac{a}{b}$.

Quantity B : $\frac{b}{a}$ is more negative than -1. That is, $\frac{b}{a} < -1$. Quantity A is greater.

40. **(D).** Given only a compound inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D). Always consider negatives in inequalities and absolute value questions.

For example, $x = 10$, $y = -9$, and $z = 8$.

Check that $x > |y| > z$: $10 > |-9| > 8$, which is true.

In this case, $x + y = 10 + (-9) = 1$ and $|y| + z = 9 + 8 = 17$. Quantity B is greater.

Try to find another example such that Quantity A is greater.

For example, $x = 2$, $y = 1$, and $z = -3$.

C heck that $x > |y| > z$: $2 > |1| > -3$,w hich is true.

In this case,$x + y = 2 + 1 = 3$ and $|y| + z = 1 + (-3) = -2$.Q uantity A is greater.

Either statem ent could be greater.It cannot be determ ined from the inform ation given.

41.**(D ).**The values for $k$,$l$,and $m$ ,respectively,could be any of the follow ing three sets:

Set 1: 24,26,and 28
Set 2: 26,28,and 30
Set 3: 28,30,and 32

For evenly spaced sets w ith an odd num ber of term s,the average is the m iddle value.Therefore,the average of $k$,$l$,and $m$ could be 26,28,or 30.O nly answ er choice (D ) m atches one of these possibilities.

42.**(B ).**The num ber line indicates a range betw een,but not including,-3 and 1.H ow ever,$-3 < x < 1$ is not a given option.H ow ever,answ er choice (B ) gives the inequality $-6 < 2x < 2$.D ividing all three sides of this inequality by 2 yields $-3 < x < 1$.

43.**120 and 720 only.**If $x$ is "greater than 3 but no m ore than 6," then $x$ is 4,5,or 6.If there are 4 judges sitting in 4 seats,they can be arranged $4! = 4 \times 3 \times 2 \times 1 = 24$ w ays.If there are 5 judges sitting in 5 seats,they can be arranged $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ w ays.If there are 6 judges sitting in 6 seats,they can be arranged $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ w ays.Thus,24,120,and 720 are all possible answ ers.O nly 120 and 720 appear in the choices.

44.**I only.**For this problem ,you need to know that m ultiplying or dividing an inequality by a negative requires you to flip the inequality sign.Thus,m ultiplying or dividing an inequality by a variable should *not* be done unless you know *w hether* to flip the inequality sign (i.e.,w hether the variable represents a positive or negative num ber).

Statem ent I: TR U E .M ultiply both sides of the original inequality by -3 and flip the inequality sign.

Statem ent II: M aybe.M ultiply both sides of the original inequality by $b$ to get Statem ent II,but only if $b$ is positive.If $b$ is negative,the direction of the inequality sign w ould have to be changed.

Statem ent III: M aybe.M ultiplying both sides of the original inequality by $-3b$ could lead to Statem ent III,but because the inequality sign flipped,this is only true if $-3b$ is negative (i.e.,if $b$ is positive).

45.**(D ).**There are three variables in the original question,but not all of them are relevant.Sim plify the original constraint:

$a - b > a + b + c$
$-b > b + 4$
$0 > 2b + c$

So,$2b + c$ is negative.

N ext,notice that $b + c$ is com m on to both quantities,so subtracting it from both w ill not change their relative values:

Quantity A : $2b + c - (b + c) = 2b + c - b - c = b$
Quantity B : $b + c - (b + c) = 0$

This question is really about the sign of $b$!

Based on the constraint that $2b + c$ is negative, $b$ could be positive or negative.

If $b = 2$ and $c = -6$, $2b + c = 4 - 6 = -2$, which is negative. In this case, Quantity A is greater.

If $b = -2$ and $c = 1$, $2b + c = -4 + 1 = -3$, which is negative. In this case, Quantity B is greater.

The correct answer is (D).

46.**(B).** From $z < y - x$, the value of $z$ depends on $x$ and $y$. So, solve for $x$ and $y$ as much as possible. There are two cases for the absolute value equation: $|x + y| = 10$ means that $(x + y) = \pm 10$. Consider these two cases separately

The positive case:

$x + y = 10$, so $y = 10 - x$.
Substitute into $z < y - x$, getting $z < (10 - x) - x$, or $z < 10 - 2x$. Because $x$ is at least zero, $10 - 2x \leq 10$.
Putting the inequalities together, $z < 10 - 2x \leq 10$. Thus, $z < 10$.

The negative case:

$x + y = -10$, so $y = -10 - x$.
Substitute into $z < y - x$, getting $z < (-10 - x) - x$, or $z < -10 - 2x$. Because $x$ is at least zero, $-10 - 2x \leq -10$.
Putting the inequalities together, $z < -10 - 2x \leq -10$. Thus, $z < -10$.

In both cases, 10 is greater than $z$. The correct answer is (B).

47.**(A).** The variable $a$ is common to both quantities, and adding it to both quantities to cancel will not change the relative values of the quantities.

Quantity A : $(9 - a) + a = 9$

Quantity B : $\left(\frac{b}{2} - a\right) + a = \frac{b}{2}$

According to the given constraint, $\frac{b}{2} < 9$, so Quantity A is greater. The correct answer is (A).

48.**(C).** If $p$ is an integer such that $1.9 < |p| < 5.3$, $p$ could be 2, 3, 4, or 5, as well as -2, -3, -4, -5. The greatest value of $p$ is 5,for w hich the value of $f(p) = 5^2 = 25$.The least value of $p$ is -5,for w hich the value of $f(p) = (-5)^2 = 25$.

49.**(D ).**If $\frac{a}{b}\left(\frac{x}{y}\right) < 0$,then the tw o fractions have opposite signs.Therefore,by the definition of reciprocals, $\frac{a}{b}$ m ust be the negative inverse of $\frac{x}{y}$,no m atter w hich one of the fractions is positive.In equation form ,this m eans $\frac{a}{b} = -\frac{y}{x}$,w hich is choice (D ).The other choices are possible but not certain.

50.**(C ).**In order to get $m$ and $n$ out of the denom inators of the fractions on the left side of the inequality,m ultiply both sides of the inequality by $mn$.The result is $kn + lm > (mn)^2$.The direction of the inequality sign changes because $mn$ is negative.This is an exact m atch w ith (C ),w hich m ust be the correct answ er.

51.**(E ).**W hen dealing w ith absolute values,alw ays consider tw o cases.

The first case is w hen the expression w ithin the absolute value signs is positive.If $m + 2 > 0$,then $|m + 2| = m + 2$,and therefore $m + 2 < 3$.Subtracting 2 from both sides,this inequality becom es $m < 1$.

The second case is w hen $m + 2 < 0$,so that $|m + 2| = -(m + 2)$,and therefore $-(m + 2) < 3$.D ivide both sides by -1 to get $m + 2 > -3$,rem em bering to flip the inequality sign.Subtracting 2 from both sides,this inequality becom es $m > -5$

C om bining these tw o inequalities,the result is $-5 < m < 1$.

52.**(D ).**The inequality described in the question is $0 > \frac{1}{X} > Y + Z$.M ultiplying both sides of this inequality by $X$ ,the result is $0 < 1 < XY + XZ$.N otice that the direction of the inequality sign m ust change because $X$ is negative.

(A ) M aybe true: true only if $X$ equals -1.
(B ) M aybe true: either $Y$ or $Z$ or both can be negative.
(C ) False: the direction of the inequality sign is opposite the correct direction determ ined above. (D ) TR U E .It is a proper rephrasing of the original inequality.
(E) M aybe true: it is not a correct rephrasing of the original inequality.

53.**(B ).**The given inequality can be sim plified as follow s:

$m + n - 2p < p + n + 4m$
$m - 2p < p + 4m$
$-3p < 3m$
$p > -m$ (R em em ber to flip the inequality sign w hen dividing by -3.)

The correct answ er is (B ).

54.**(D ).**W hen the G R E w rites a root sign,the question w riters are indicating a nonnegative root only.Therefore both sides of this inequality are positive.Thus,you can square both sides w ithout changing the direction of the inequality sign.So $u < -3v$.N ow evaluate each answ er choice:

(A ) M ust be true.D ivide both sides of $u < -3v$ by 3.

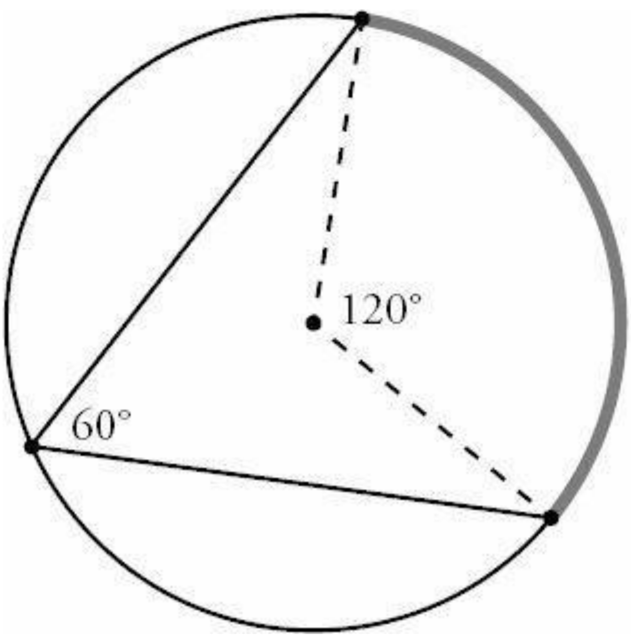
(B ) M ust be true.It is given that $-3v > 0$ and therefore, $v < 0$.Then,w hen dividing both sides of $u < -3v$ by $v$,you m ust flip the inequality sign and get $u/v > -3$.

(C ) M ust be true.This is the result after dividing both sides of the original inequality by $\sqrt{-v}$.

(D ) C A N N O T be true.A dding $3v$ to both sides of $u < -3v$ results in $u + 3v < 0$,not $u + 3v > 0$.

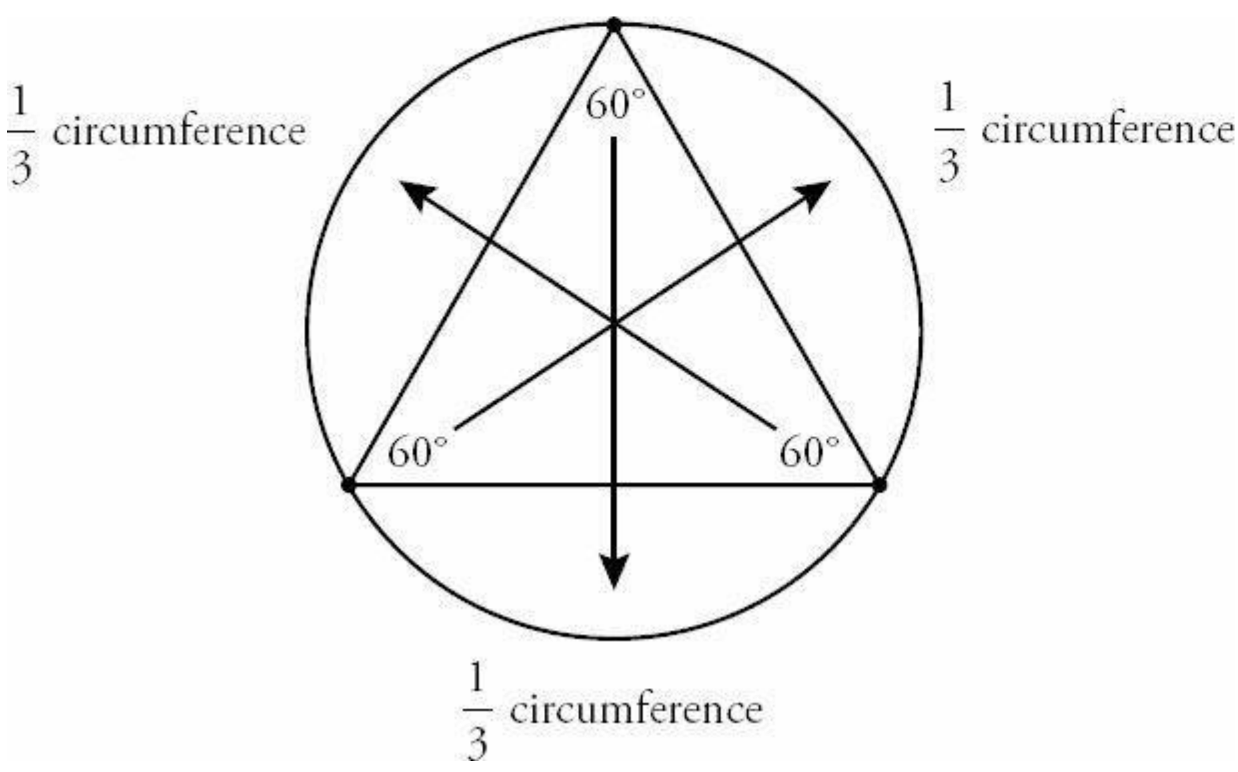
(E) M ust be true.This is the result of squaring both sides of the original inequality.

55.**(D ).**Since each of the three arcs corresponds to one of the 60 degree angles of the equilateral triangle,each arc represents 1/3 of the circum ference of the circle.The diagram below illustrates this for just one of the three angles in the triangle:




$$\frac{120°}{360°} = \frac{1}{3}$$

The sam e is true for each of the three angles:



Since each of the three arcs is betw een $4\pi$ and $6\pi$,triple these values to determ ine that the circum ference of the circle is betw een $12\pi$ and $18\pi$.B ecause circum ference equals $\pi$ tim es the diam eter,the diam eter of this circle m ust be betw een 12 and 18.O nly choice (D ) is in this range.