# Polygons and Rectangular Solids

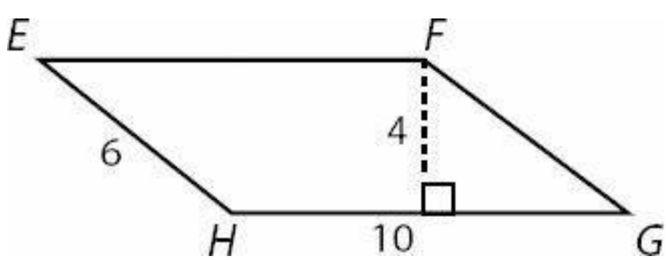
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

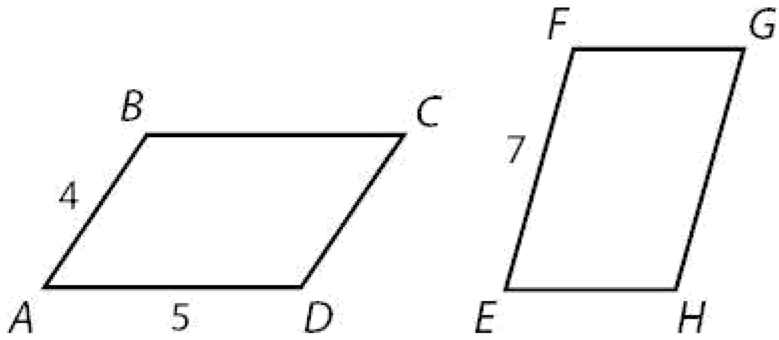
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



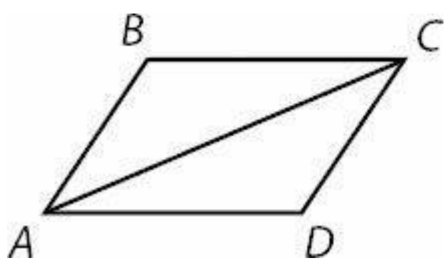

What is the area of parallelogram *EFGH*?

2.



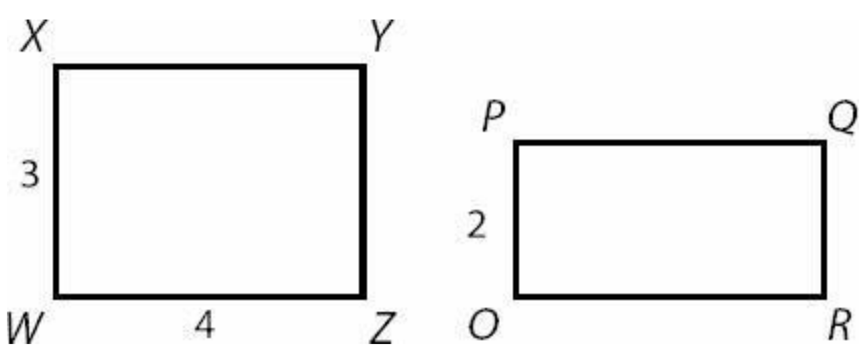

The two parallelograms pictured above have the same perimeter. What is the length of side *EH*?

3.



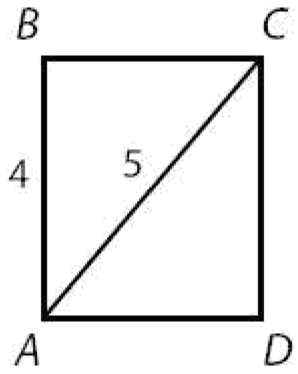

In Parallelogram *ABCD*, Triangle *ABC* has an area of 12. What is the area of Triangle *ACD*?

4.



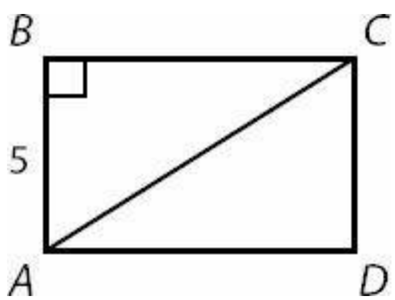

Rectangle *WXYZ* and Rectangle *OPQR* have equal areas. What is the length of side *PQ*?

5.



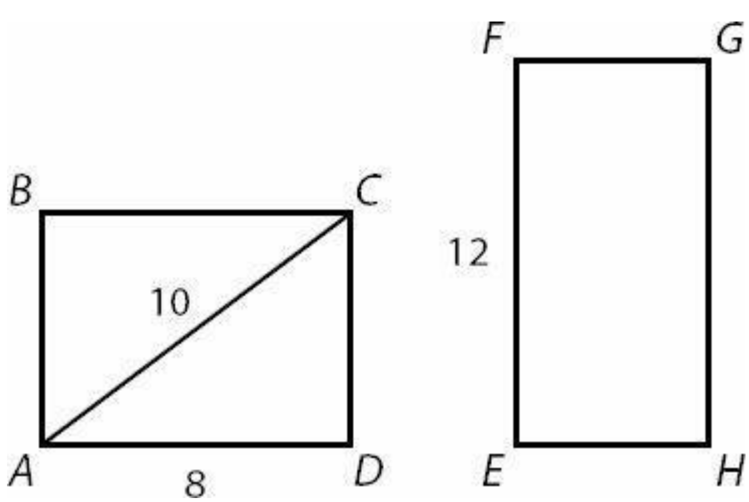

W hat is the area of R ectangle *ABC D* ?

6.



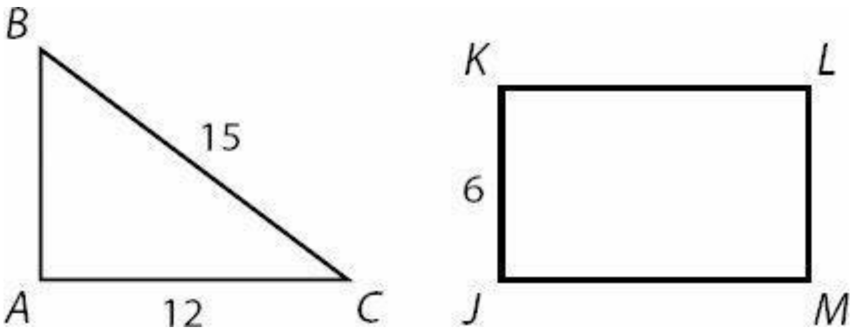

In R ectangle *ABC D* ,the area of Triangle *ABC* is 30.W hat is the length of diagonal *AC* ?

7.




R ectangles *ABC D* and *EFG H* have equal areas.W hat is the length of side *FG* ?

8.




Triangle *ABC* and Rectangle *JKLM* have equal areas. What is the perimeter of Rectangle *JKLM*?

9.

| **Quantity A** | **Quantity B** |
|---|---|
| The longest side of a rectangle with area 36 | 6 |

10.

| **Quantity A** | **Quantity B** |
|---|---|
| The area of a rectangle with perimeter 40 | 110 |

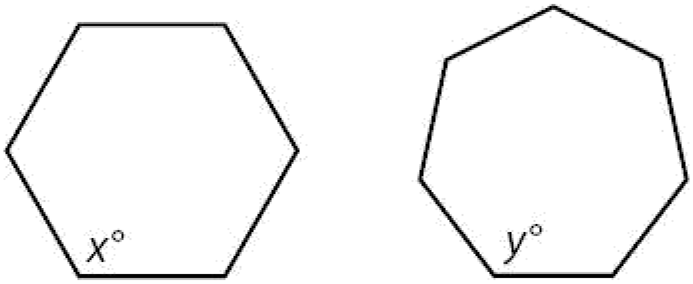
11. What is the area of a square with a diagonal measuring $4\sqrt{2}$ centimeters?

centimeters square

12.

| **Quantity A** | **Quantity B** |
|---|---|
| The area of a parallelogram with a base of length 4 and height of 3.5 | The area of a trapezoid with two parallel sides of lengths 5 and 9 and a height of 2 |

13.




**Q uantity A** | **Q uantity B**

$x$ | $y$

14.

A trapezoid has an area of 42 and a height that is less than or equal to 6.

**Q uantity A** | **Q uantity B**

The height of the trapezoid | The length of the longer base of the trapezoid

15.

The perim eter of square $W$ is 50% of the perim eter of square $D$ .

**Q uantity A** | **Q uantity B**

The ratio of the area of square $W$ to the area of square $D$ | $\frac{1}{4}$

16.A 10 by 15 inch rectangular picture is displayed in a 16 by 24 inch rectangular fram e.W hat is the area,in inches, of the part of the fram e not covered by the picture?

(A ) 150
(B ) 234
(C ) 244
(D ) 264
(E) 384

17.

A rectangular box has edges of length 2,3,and 4.

**Q uantity A** | **Q uantity B**

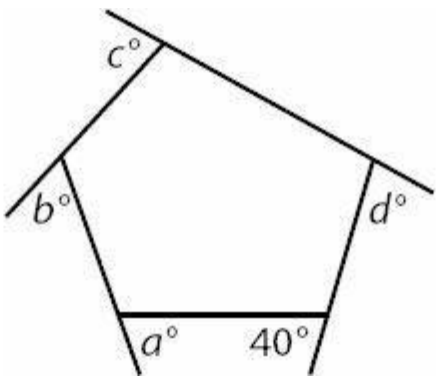
Tw ice the volum e of the box | The surface area of the box

18.A perfect cube has surface area 96.W hat is its volum e?

19.H ow m any 2 inch by 2 inch by 2 inch solid cubes can be cut from six solid cubes that are 1 foot on each side? (12 inches = 1 foot)

(A ) 8 (B
) 64 (C )
216
(D ) 1,296
(E) 1,728

20.



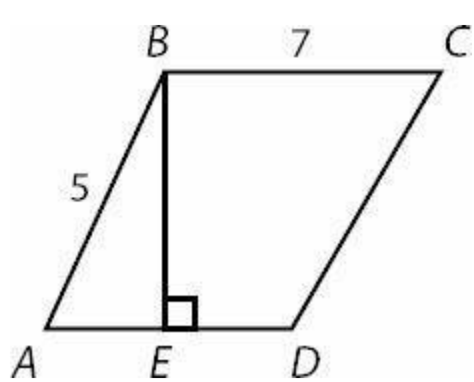

W hat is the value of $a + b + c + d$?

(A ) 240
(B ) 320
(C ) 360
(D ) 500
(E) 540

21.G arden A is a 225 m eter by 180 m eter rectangular vegetable garden,and G arden B is a rectangle w ith exactly half the length and w idth of G arden A .W hat is the ratio of the area of G arden A to the area of G arden B ?

(A ) 1 : 4
(B ) 1 : 2
(C ) 2 : 1
(D ) 4 : 1
(E) 8 : 1

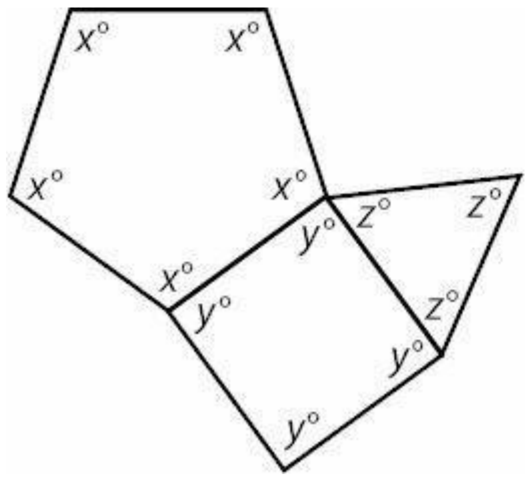
22.




In the trapezoid above,$AE = ED = 3$ and $BC$ is parallel to $AD$ .

| Q uantity A | Q uantity B |
|---|---|
| The area of the trapezoid | 35 |

23.




| Q uantity A | Q uantity B |
|---|---|
| The value of $x + y + z$ | 270 |

24.A rectangle has an area of $54\sqrt{2}$ and a length of 6.W hat is the perim eter of the rectangle?

(A) $15\sqrt{2}$
(B) $30\sqrt{2}$
(C) $6 + 9\sqrt{2}$
(D) $12 + 18\sqrt{2}$
(E) $18 + 12\sqrt{2}$

25.A 1 m eter by 1 m eter by 1 m eter sheet of paper is to be cut into 4 centim eter by 5 centim eter rectangles.H ow m any such rectangles can be cut from the sheet of paper? (1 m eter = 100 centim eters)

26.

A parallelogram has tw o sides w ith length 10 and tw o sides w ith length 5.

| Q uantity A | Q uantity B |
|---|---|
| The area of the parallelogram | 30 |

27.W hat is the area of a regular hexagon w ith side length 2?

(A) $2\sqrt{3}$

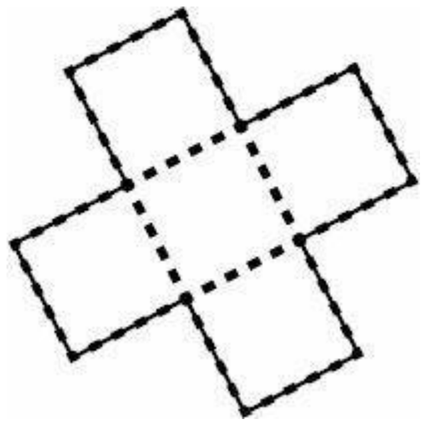
(B) $2\sqrt{6}$

(C) $6\sqrt{2}$

(D) $6\sqrt{3}$

(E) $12\sqrt{3}$

28.



The figure above is com posed of 5 squares of equal area,as indicated by the dotted lines.The total area of the figure is 45.

| Q uantity A | Q uantity B |
|---|---|
| The perim eter of the figure | 48 |

29.

| Q uantity A | Q uantity B |
|---|---|
| The largest possible area of a rhom bus w ith side 4. | The area of a square w ith side 4. |

30.A 2 foot by 2 foot by 2 foot solid cube is cut into 2 inch by 2 inch by 4 inch rectangular solids.W hat is the ratio of the total surface area of all the resulting sm aller rectangular solids to the surface area of the original cube? (1 foot = 12 inches)

(A ) 2 : 1
(B ) 4 : 1
(C ) 5 : 1
(D ) 8 : 1
(E) 10 : 1

31.If a cube has the sam e volum e (in cubic units) as surface area (in square units),w hat is the length of one side?

(A ) 1
(B ) 3
(C ) $\frac{5}{3}$

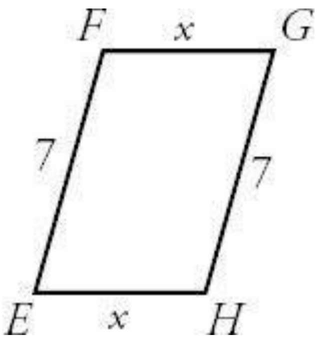
(D ) 6

(E) N o such cube is possible.

# P olygons and R ectangular Solids A nsw ers

1.**40.**The area of a parallelogram is base × height.In this parallelogram ,the base is 10 and the height is 4 (rem em ber, base and height need to be perpendicular).So the area is 10 × 4 = 40.

2.**2.**First find the perim eter of Parallelogram *ABC D* .Y ou know that 2 sides have a length of 4,and 2 sides have a length of 5.The perim eter is 2 × (4 + 5) = 18.That m eans Parallelogram *EFG H* also has a perim eter of 18.Y ou know side *G H* also has a length of 7.Y ou don’t know the lengths of the other 2 sides,but you know they have the sam e length,so for now say the length of each side is *x*.Y our parallelogram now looks like this:




So you know that 7 + *x* + 7 + *x* = 18 → 2*x* + 14 = 18 à 2*x* = 4 → *x* = 2

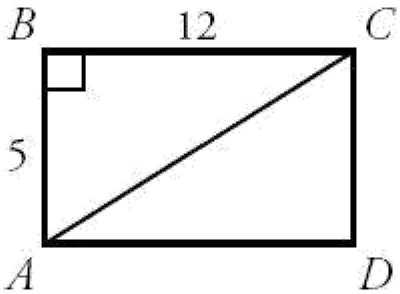
The length of side *EH* is 2.

3.**12.**O ne property that is true of any parallelogram is that the diagonal w ill split the parallelogram into tw o equal triangles.If Triangle *ABC* has an area of 12,then Triangle *AC D* m ust also have an area of 12.

4.**6.**Y ou can start by finding the area of R ectangle *W XYZ*.A rea of a rectangle is length × w idth,so the area of R ectangle *W XYZ* is 3 × 4 = 12.So R ectangle *O PQ R* also has an area of 12.Y ou know the length of side *O P* ,so that is the w idth of R ectangle *O PQ R* .So now you know the area,and you know the w idth,so you can solve for the length.l × 2 = 12 → l = 6.The length of side *PQ* is 6.

5.**12.**To find the area of R ectangle *ABC D* ,you need to know the length of *AD* or *BC* .In a rectangle,every internal angle is 90 degrees,so Triangle *ABD* is actually a right triangle.That m eans you can use the Pythagorean Theorem to find the length of side *AD* .A ctually,this right triangle is one of the Pythagorean Triplets— a 3-4-5 triangle.The length of side *AD* is 3.That m eans the area of R ectangle *ABC D* is 3 × 4 = 12.

6.**13.**Y ou know the area of Triangle *ABC* and the length of side *AB* .B ecause side *BC* is perpendicular to side *AB* ,you can use those as the base and height of Triangle *ABC* .So you know that 1/2(5) × (*BC* ) = 30.That m eans the length of side *BC* is 12.




N ow you can use the Pythagorean Theorem to find the length of diagonal *AC* ,w hich is the hypotenuse of right triangle *ABC* .Y ou can also recognize that this is a Pythagorean Triplet — a 5–12–13 triangle.The length of diagonal *AC* is 13.

7.**4.**The first thing to notice in this problem is that you can find the length of side *C D* .Triangle *AC D* is a right triangle,and you know the lengths of tw o of the sides.Y ou can either use the Pythagorean Theorem or recognize that this is one of the Pythagorean Triplets— a 6–8–10 triangle.The length of side *C D* is 6.N ow you can find the area of R ectangle *ABC D* .Side *AD* is the length and side *C D* is the w idth.8 × 6 = 48.

That m eans that the area of R ectangle *EFG H* is also 48.Y ou can use the area and the length of side *EF* to solve for the length of side *FG* .12 × (*FG* ) = 48.The length of side *FG* is 4.

8.**30.**If you can find the length of side *AB* ,then you can find the area of Triangle *ABC* .Y ou can use the Pythagorean Theorem to find the length of side *AB* .$(12)^2 + (AB)^2 = (15)^2$ → $144 + (AB)^2 = 225$ → $(AB)^2 = 81$ → *AB* = 9.(A 9– *12–15 triangle is a 3–4–5 triangle,w ith all the m easurem ents tripled.)*

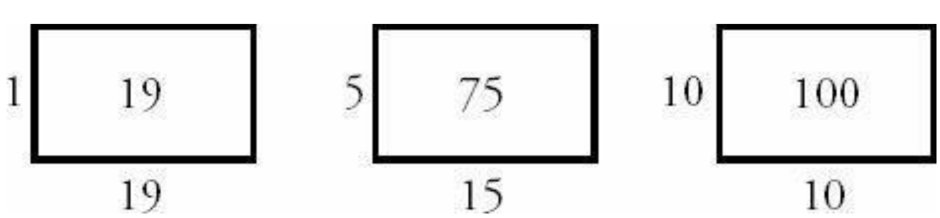
N ow that you know *AB* ,you can find the area of Triangle *ABC* .It's 1/2(12) × 9 = 54.

That m eans that R ectangle *JK LM* also has an area of 54.Y ou have one side of the rectangle,so you can solve for the other.6 × (*JM* ) = 54.So the length of side *JM* is 9.That m eans that the perim eter is 2 × (6 + 9) = 30.

9.**(D ).**A rectangle w ith area 36 could have length of 36 and w idth of 1,or length of 9 and w idth of 4,or an infinite num ber of other values,since the problem does not say that the side lengths m ust be integers.In every exam ple except one,though,the longer sides are longer than 6 (and the shorter sides are less than 6),and Q uantity A is greater.The only exception occurs if the rectangle is actually a square.If length is 6 and w idth is 6,the tw o quantities are equal.A square is definitely a type of rectangle! It is,in fact,an equilateral rectangle.This exception m eans the correct answ er is (D ).

N ote also that Q uantitative C om parisons are very often m ore interested in testing w eird cases and exceptions to rules than they are in testing your know ledge of straightforw ard cases.

10.**(B ).**W hile a rectangle w ith perim eter 40 could have m any different areas,all of these areas are less than 110:




H ow can you be sure this w ill alw ays be the case? It w ould be helpful to know the rule that the area of a rectangle w ith constant perim eter increases as length and w idth becom e m ore sim ilar,and is m axim ized w hen the rectangle is a square.Thus,the 10 by 10 version of the rectangle represents the m axim um possible area,w hich is still less than 110.

11.**16.**W hen a square is cut by a diagonal,tw o 45–45–90 triangles are created.U se the 45–45–90 form ula (sides in the ratio 1 : 1 :$\sqrt{2}$) to determ ine that the sides are equal to 4,and thus the area is 4 × 4 = 16.A lternatively,you could label each side of the square *x* (since they’re the sam e) and use the Pythagorean Theorem :

$$x^2 + x^2 = (4\sqrt{2})^2$$
$$2x^2 = 32$$
$$x^2 = 16$$
$$x = 4$$

Thus,area = 4 × 4 = 16.

12.**(C ).**The form ula for area of a parallelogram is *base* × *height*,so Q uantity A is 4 × 3.5 = 14.

The form ula for area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$ ,w here $b_1$ and $b_2$ are the lengths of the parallel sides,so Q uantity B is $\frac{(5+9)}{2} \times 2 = 14$ .

The tw o quantities are equal.

13.**(D ).**D o not assum e that any polygon is a regular figure unless you are told this.(For instance,if *every* angle in the hexagon w ere labeled w ith the sam e variable,you could be sure the hexagon w as regular).

U sing the form ula ($n$ - 2)(180) w here $n$ is the num ber of sides,you can calculate that the sum of the angles in the 6-sided figure is 720 and the sum of the angles in the 7-sided figure is 900.H ow ever,you do not know how those totals are distributed am ong the interior angles,so either $x$ or $y$ could be greater.

14.**(B ).**The area form ula for a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$ .For a fixed area,the average of the bases is m inim ized w hen the height is m axim ized,and vice versa.If the area is 42 and the m axim um height is 6,then $\frac{(b_1 + b_2)}{2}$ is at least 7.Thus,the sum of the bases is at least 14.If tw o bases sum to 14,the longer base is greater than 7 (or else both bases are equal to 7).

Q uantity A is less than or equal to 6.
Q uantity B is greater than or equal to 7.

15.**(C ).**If one square has tw ice the perim eter,it has tw ice the side length,it w ill have four tim es the area. W hy is this? D oubling only the length doubles the area.Then,doubling the w idth doubles the area *again*.

You can also prove this with real numbers. Say square *W* has perimeter 8 and Square *D* has perimeter 16. Thus, square *W* has side 2 and Square *D* has side 4. The areas are 4 and 16, respectively. As a ratio, 4/16 reduces to 1/4.

16. **(B).** The area of the picture is 10 × 15 = 150. The area of the frame is 16 × 24 = 384. Subtract to get the answer: 384 - 150 = 234.

17. **(B).** The volume of a rectangular box is *length* × *width* × *height* = 2 × 3 × 4 = 24. Quantity A is double this volume, or 48.

The surface area of a rectangular box is 2(*length* × *width*) + 2(*width* × *height*) + 2(*length* × *height*) = 2(6) + 2(12) + 2(8) = 52.

Quantity B is greater.

18. **64.** The surface area of a cube is given by the formula Surface Area = $6(side)^2$. Or just think about it logically: since all the faces are the same, the total surface area is 6 times the surface area of a single face. Since the Surface Area = 96:

$$96 = 6(side)^2$$
$$16 = (side)^2$$
$$4 = side$$

The volume of a cube is Volume = $(side)^3 = 4^3 = 64$.

19. **(D).** Each large solid cube is 12 inches × 12 inches × 12 inches. Each dimension (length, width, and height) is to be cut identically at 2 inch increments, creating 6 smaller cubes in each dimension. Thus, 6 × 6 × 6 small cubes can be cut from each large cube. There are 6 large cubes to be cut this way, though, so the total number of small cubes that can be cut is 6(6 × 6 × 6) = 1,296.

20. **(B).** The interior figure shown is a pentagon, although an irregular one. The sum of the interior angles of any polygon can be determined using the formula $(n - 2)(180)$, where $n$ is the number of sides:

$$(5 - 2)(180) = (3)(180) = 540$$

Using the rule that angles forming a straight line sum to 180, the interior angles of the pentagon (starting at the top and going clockwise) are $180 - c$, $180 - d$, 140, $180 - a$, and $180 - b$. The sum of these angles can be set equal to 540.

$$540 = (180 - c) + (180 - d) + 140 + (180 - a) + (180 - b)$$
$$540 = 140 + 4(180) - a - b - c - d$$
$$540 - 140 - 720 = -(a + b + c + d)$$
$$-320 = -(a + b + c + d)$$

So, $a + b + c + d = 320$.

21. **(D).** Garden A has an area of 225 × 180 = 40,500. Garden B has an area of 112.5 × 90 = 10,125. The answer is 40,500/10,125, which reduces to 4/1, or a 4 : 1 ratio.

There is a more efficient solution,however.Halving only the length of a rectangle will divide the area by 2.Halving only the width will divide the area by 2.So halving both the length and width of the rectangle will divide the area by 4. The ratio is 4 : 1.

22.**(B ).**First,note that while the figure may *look* like parallelogram ,it is actually a trapezoid,as it has two parallel sides of unequal length (*AD* = 6 and *BC* = 7).A trapezoid has two parallel sides (the bases).The formula for the area of a trapezoid is $A=\frac{(b_1+b_2)}{2}\times h$,where $b_1$ and $b_2$ are the lengths of the parallel sides and $h$ is the height (*BE* in this figure).

Triangle *ABE* is a 3–4–5 special right triangle,so *BE* is 4.(You could also use the Pythagorean Theorem to determine this.)

Thus,the area is $\frac{(6+7)}{2}\times 4 = 26$.Quantity B is greater.

23.**(B ).**Each angle in the pentagon is labeled with the same variable,so this is a regular pentagon.Using the formula ($n$ - 2)(180),where n is the number of sides,the sum of all the interior angles of the pentagon is (3)(180) = 540 degrees.Divide by 5 to get $x$ = 108.

Now ,the quadrilateral.All four-sided figures have interior angles that sum to 360.If you didn't have that memorized, you could also use ($n$ - 2)(180) to determine this.Divide 360 by 4 to get $y$ = 90.

Now ,the triangle.It is equilateral,so $z$ = 60.(The sum of angle measures in a triangle is always 180;if the angles are equal,they will each equal 60.)

Thus,$x + y + z$ = 108 + 90 + 60 = 258.Quantity B is greater.

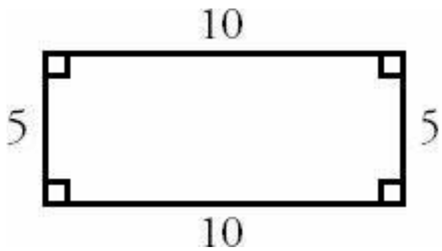
24.**(D ).**Since the area of a rectangle is *length* × *width*:

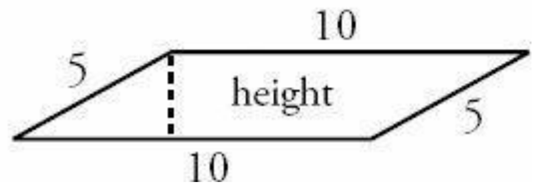
$$54\sqrt{2} = 6 \times width$$
$$9\sqrt{2} = width$$

Since perimeter is 2*length* + 2*width*,the perimeter of the rectangle is $2(6) + 2(9\sqrt{2}) = 12 + 18\sqrt{2}$,or choice (D ).

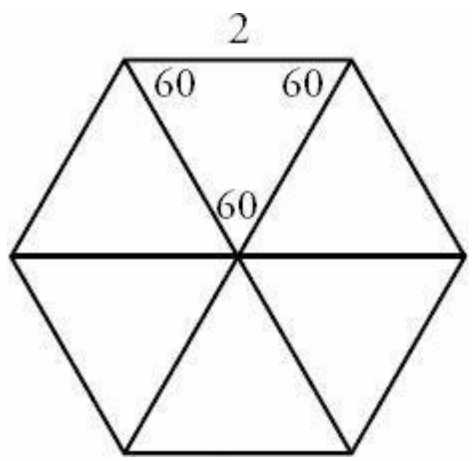
25.**500.**Since the sheet of paper is measured in meters and the small rectangles in centimeters,first convert the measures of the sheet of paper to centimeters.The large sheet of paper measures 100 cm by 100 cm .The most efficient way to cut 4 cm by 5 cm rectangles is to cut vertically every 4 cm and horizontally every 5 cm (or vice versa; the idea is that all the small rectangles should be oriented the same direction on the larger sheet).Doing so creates a grid of $\frac{100}{4}\times\frac{100}{5} = 25\times 20 = 500$ small rectangles.

26.**(D ).**The form ula for the area of a parallelogram is *base* × *height*,w here height is the perpendicular distance betw een the parallel bases,not necessarily the other side of the parallelogram .H ow ever,if the parallelogram is actually a rectangle,the height IS the other side of the parallelogram ,and is thereby m axim ized.So,if the parallelogram is actually a rectangle,the area w ould be equal to 50,but if the parallelogram has m ore extrem e angle m easures,the height could be very,very sm all,m aking the area m uch less than 30.




27.**(D ).**D ivide the hexagon w ith three diagonals (running through the center) to get six triangles.Since the sum of the angles in any polygon is (*n* - 2)(180),the sum for a hexagon is 720.D ivide by 6 to get that each angle is 120.W hen you divide the hexagon into triangles,you split each 120 to m ake tw o 60 degree angles for each triangle.A ny triangle that has tw o angles of 60 m ust have a third angle of 60 as w ell,since triangles alw ays sum to 180.Thus,all six triangles are equilateral.Therefore,all three sides of each triangle are equal to 2.

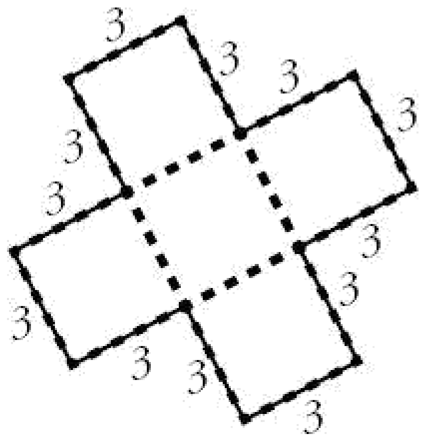



For any equilateral triangle,the height equals half the side tim es $\sqrt{3}$.Therefore,the height is sim ply $\sqrt{3}$.Since *A* = $\frac{bh}{2}$,the area of each equilateral is $\frac{2\sqrt{3}}{2} = \sqrt{3}$.

Since there are six such triangles,the answ er is $6\sqrt{3}$.

28.**(B ).**If a figure w ith area of 45 is com posed of 5 equal squares,sim ply divide to get that the area of each square is 9 and thus the side of each square is 3.

D on’t m ake the m istake of adding up EV ER Y side of every square to get the perim eter— m ake sure you only count lengths that are actually part of the perim eter of the overall figure.(N ote that the central square does not have any lengths that are part of the perim eter).

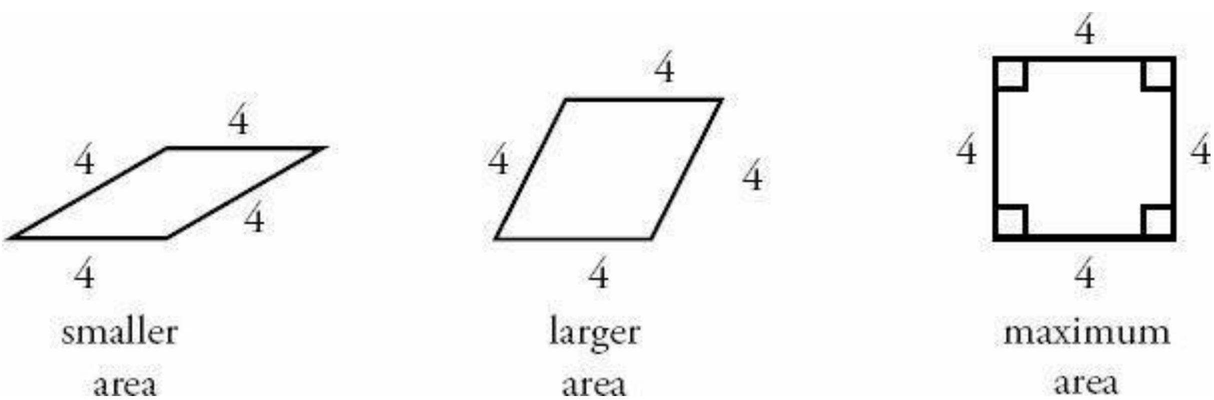



The perim eter is m ade of 12 segm ents,each w ith length 3.The perim eter is 36.

Incorrect choice (A ) com es from reasoning that 5 squares have 20 total sides,each of length 3,and thus the com bined length w ould be 60.Y ou also cannot just subtract the four dotted line lengths,as each of these w as actually counted tw ice,as part of the central square and one of the others.This m istake w ould incorrectly yield choice (C ).The best approach here is to m ake a quick sketch of the figure,label the sketch w ith w hat you know ,and count up the perim eter.

29.**(C ).**First,note that Q uantity B is sim ply 16.

For Q uantity A ,a rhom bus is a parallelogram w ith 4 sides of equal length.A square is a type of rhom bus (specifically, it is a rhom bus that has four equal angles).The rhom bus w ith the largest possible area w ould be a square— a square identical to the one described in Q uantity B .




30.**(E ).**To find the surface area of the original cube,first convert the side lengths to inches (it is N O T okay to find surface area or volum e and then convert using 1 foot = 12 inches;this is only true for straight-line distances).The equation for surface area is $6s^2$.So,the surface area of the large original cube is 6(24 inches)$^2$ = 3,456 square inches.

Each large solid cube is 24 inches × 24 inches × 24 inches.To cut the large cube into 2 inch by 2 inch by 4 inch rectangular solids,tw o dim ensions (length and w idth,say) w ill be sliced every 2 inches,w hile one dim ension (height, say) w ill be sliced every 4 inches.Thus, $\frac{24}{2}\times\frac{24}{2}\times\frac{24}{4}=12\times12\times6=864$ sm all rectangular solids can be cut from the large cube.

The equation for the surface area of a rectangular solid is: 2*lw* + 2*w h* + 2*lh*.In this case,that is 2(2 × 2) + 2(2 × 4) + 2(2 × 4) = 8 + 16 + 16 = 40 square inches per sm all rectangular solid.There are 864 sm all rectangular solids,so the total surface area is:

$40 \times 864 = 34{,}560$ square inches.

Finally,the ratio of the total surface area of all the resulting sm aller rectangular solids to the surface area of the original cube is the ratio of 34,560 to 3,456.This ratio reduces to 10 to 1.

31.**(D )**.To solve this question,you need the equations for the volum e and the surface area of a cube:

V olum e = $s^3$ Surface area = $6s^2$

If a cube has the sam e volum e as surface area,set these equal:

$s^3 = 6s^2$

$s = 6$