GMAT Geometry: Challenge

Jeff Sackmann / GMAT HACKS

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Contents

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1 Introduction

This document contains nothing but difficult GMAT Geometry questions–100 of them, to be exact. While the basic Geometry concepts–triangles, circles, the coordinate plane–are not that tricky, the GMAT has a nearly inexhaustible supply of questions that challenge your ability to use those fundamental rules.

As in all of my GMAT preparation resources, you'll find these questions indexed by difficulty. That doesn't mean you should skip straight to the hardest questions, or even that you should start with the easier ones. On the GMAT itself, questions won't come labeled with their difficulty level, and despite the intent of the adaptive algorithm, they won't be precisely consistent in terms of difficulty either. Each question presents its own unique challenges, and the sooner you get accustomed to changing gears with every single question, the more time you'll have to prepare for that particular challenge of the exam.

The GMAT Math Bible has several chapters (along with focused practice) on Geometry and related issues, including individual chapters on lines and angles, triangles, right triangles, quadrilaterals, circles, solids, polygons, and coordinate geometry. If you find you are struggling with the mechanics of these problems, your time is probably better spent with the GMAT Math Bible than in doing dozens and dozens of practice problems, hoping to pick up those skills along the way.

If you find yourself having problems with only the most difficult questions, you might try my "Extreme Challenge" set, which contains only 720 and higher level questions, many of which are Geometry-related.

As far as strategy is concerned, there are dozens of articles at GMAT HACKS to help you with your strategic approach to Arithmetic questions. Most importantly, you should make sure you understand every practice problem you do. It doesn't matter if you get it right the first time–what matters is whether you'll get it right the next time you see it, because the next time you see it could be on the GMAT.

With that in mind, carefully analyze the explanations. Redo questions that took you too long the first time around. Review questions over multiple sessions, rather than cramming for eight hours straight each Saturday. These basic study skills may not feel like the key to GMAT preparation, but they are the difference between those people who reach their score goals and those who never do.

Enough talking; there are 100 Geometry questions waiting inside. Get to work!

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2 Difficulty Levels

In general, the level 5 questions in this guide are 560- to 620-level questions. The level 6 questions represent a broad range of difficulty from about 620 to 720, while the level 7 questions are higher still.

Moderately Difficult (5)
PS
17, 21, 25, 31, 32, 40, 42, 46, 48, 49, 52
DS
58, 60, 61, 62, 63, 64, 65, 69, 70, 73, 75, 76, 80, 83, 84, 90, 92, 95, 98, 99

Difficult (6)
PS
1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15, 16, 18, 19, 20, 22, 24, 26, 27, 28, 29, 30, 33, 34, 36, 37, 38, 39, 43, 44, 45, 47, 50, 51, 53, 54
DS
57, 59, 66, 67, 68, 71, 72, 74, 77, 78, 79, 81, 82, 85, 87, 88, 89, 91, 93, 96, 97, 100

Very Difficult (7)
PS
7, 13, 14, 23, 35, 41, 55, 56
DS
86, 94

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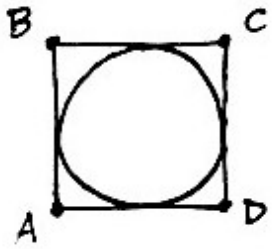
3 Problem Solving

Note: this guide contains both an answer key (so you can quickly check your answers) and full explanations.

1. Which of the following lists the number of points at which a circle can intersect two parallel lines?
   (A) 1 and 2 only
   (B) 2 and 4 only
   (C) 1, 2, and 4 only
   (D) 2, 3, and 4 only
   (E) 1, 2, 3, and 4

2. Coins are dropped into a rectangular toll box that is x feet long, y feet wide, and z feet deep. If the box is filled at the rate of approximately y cubic feet per hour, how many hours does it take to fill the box?
   (A) xz
   (B) $\frac{1}{xz}$
   (C) xyz
   (D) $\frac{1}{xyz}$
   (E) xy^2z



3. In the figure above, a circle is inscribed in square $ABCD$. If the area of the circle is 18π, approximately how much greater is the area of the square than the area of the circle?
   (A) 8
   (B) 12
   (C) 16
   (D) 20
   (E) 24

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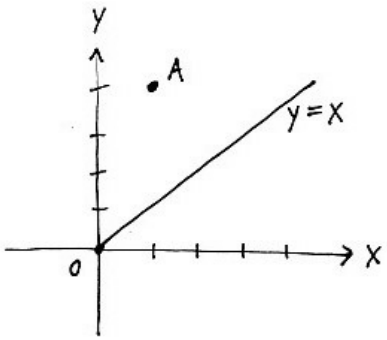
4. A fencing company is contracted to install fence along three sides of a yard with a perimeter of 44 feet. If the unfenced side is n feet long, and the company charges \$15 per foot of fencing installed, what will be the total cost of the job?

(A) $\$15(44 - n)$
(B) $\$15(22 - n)$
(C) $\$15(44) - n$
(D) $\$15(\frac{44}{n})$
(E) $\$15(\frac{22}{n})$

5. A closed cylindrical tank contains 45π cubic feet of water and is filled to half its capacity. When the tank is placed upright on its circular base on level ground, the height of the water in the tank is 5 feet. What is the radius of the tank's circular base?

(A) 3
(B) $2\sqrt{3}$
(C) $3\sqrt{2}$
(D) $3\sqrt{5}$
(E) 5



6. In the rectangular coordinate system above, the line $y = x$ is the perpendicular bisector of segment AB (not shown), and the x-axis is the perpendicular bisector of segment BC (not shown). If the coordinates of point A are $(1, 4)$, what are the coordinates of point C?

(A) $(-4, -1)$
(B) $(-1, 4)$
(C) $(4, -1)$
(D) $(1, -4)$
(E) $(4, 1)$

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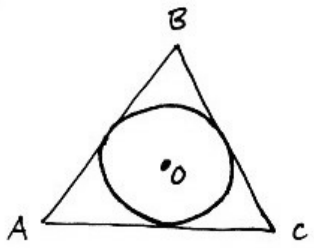

7. In the figure above, if the radius of circle O is r and $\triangle ABC$ is equilateral, what is the length of AB, in terms of r?

(A) $r\sqrt{2}$
(B) $r\sqrt{3}$
(C) $2r\sqrt{3}$
(D) $\frac{3}{2}r$
(E) $2r$

8. The perimeters of rectangular region R and square region S are equal. If the sides of R are in the ratio $3:1$, what is the ratio of the area of region R to the area of region S?

(A) $9:1$
(B) $9:4$
(C) $3:4$
(D) $2:3$
(E) $1:3$

9. A circular path forms the edge of a lake that has a diameter of 4 miles. Rodrigo ran once around the track at an average rate of 7 miles per hour. If y represents the number of hours it took Rodrigo to run completely around the lake, which of the following is true?

(A) $0.5 < y < 1.0$
(B) $1.5 < y < 2.0$
(C) $2.0 < y < 2.5$
(D) $2.5 < y < 3.0$
(E) $3.0 < y < 3.5$

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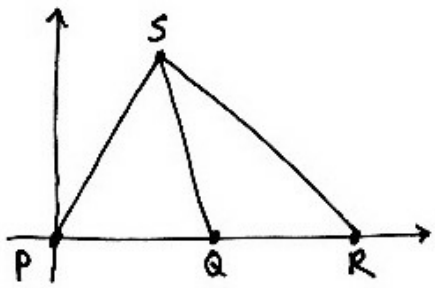
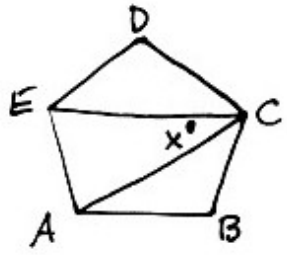


10. In the rectangular coordinate system above, the coordinates of point R are $(12, 0)$. If the area of $\triangle PQS$ is equal to the area of $\triangle QRS$, what are the coordinates of point Q?

(A) $(3, 0)$
(B) $(4, 0)$
(C) $(6, 0)$
(D) $(8, 0)$
(E) It cannot be determined from the information given.



11. If $ABCDE$ above is a regular pentagon, what is the value of x:

(A) 30
(B) 32
(C) 36
(D) 42
(E) 48

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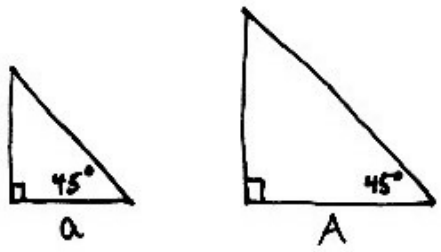

12. In the figures above, if the area of the triangle on the left is one-half the area of the triangle on the right, then in terms of a, $A =$

(A) $\frac{a}{4}$
(B) $\frac{a}{2}$
(C) $a\sqrt{2}$
(D) $\frac{a\sqrt{2}}{2}$
(E) $\frac{a^2}{2}$

13. The base of a hemisphere is inscribed in one face of a cube and the entire hemisphere in contained within the volume of the cube. What is the ratio of the radius of the hemisphere to the length of a side of the cube?

(A) $\sqrt{2} : \pi$
(B) $1 : 2$
(C) $2 : \pi$
(D) $1 : \sqrt{2}$
(E) $2 : 1$

14. On the rectangular coordinate plane, points P, Q, and R form a right triangle with two sides that are parallel to the x- and y-axes. If point P is located at $(-3, 2)$, point R is located at $(3, -6)$, and the 90-degree angle is at point Q, what is the perimeter of triangle PQR?

(A) 10
(B) 12
(C) 14
(D) 17
(E) 24

15. A tank in the shape of a right circular cylinder is $\frac{1}{2}$ full of oil and is resting upright on its base. If the capacity of the tank is 18 cubic meters and the height of the oil is 1 meter, what is the radius of the base of the cylinder, in meters?

(A) $\sqrt{3}$
(B) $\frac{3}{\sqrt{\pi}}$
(C) $\frac{2\pi}{3}$
(D) $3\sqrt{\frac{2}{\pi}}$
(E) $\frac{9}{\pi}$

16. On the xy-coordinate plane, point C is $(5, -2)$ and point D is $(-1, 2.5)$. The point on line segment CD that is twice as far from C as from D is

(A) $(1, -1)$
(B) $(1, 1)$
(C) $(2, 0.25)$
(D) $(3, 0.5)$
(E) $(3, 1)$

17. In isoceles triangle MNP, $MN = MP$, and $NP = 6$. Which of the following could be the length of side MN?

I. 2
II. 6
III. 12

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

18. In the xy-coordinate system, line segment AB is defined by the equation $x = 2y + 3$. If point A is located at (j, k) and point B is located at $(j + 4, k + v)$, then $v =$

(A) $\frac{1}{2}$
(B) $\frac{3}{2}$
(C) 2
(D) 4
(E) 8

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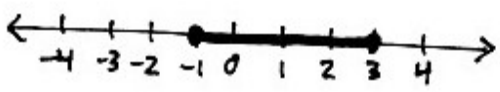
19. The length and width of a surface of a rectangular table have a ratio of $4:3$. If the surface of the table has a 45-inch diagonal, what is the perimeter of the table, in inches?

(A) 62
(B) 90
(C) 112
(D) 126
(E) 144

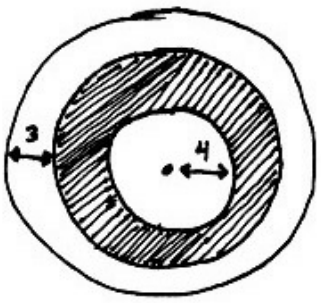


20. Which of the following inequalities is an algebraic expression for the shaded part of the number line above?

(A) $|x| \leq 1$
(B) $|x| \leq 3$
(C) $|x-2| \leq 1$
(D) $|x-1| \leq 2$
(E) $|x+1| \leq 2$

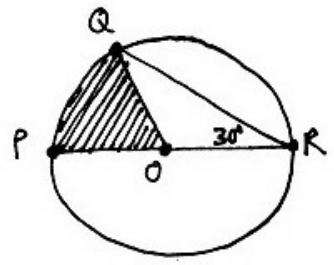
21. The length and width of a certain rectangular billboard are in a ratio of $3:2$. If the perimeter of the billboard is 32 feet, then its dimensions in feet are

(A) $\frac{9}{2}$ and $\frac{21}{2}$
(B) $\frac{32}{5}$ and $\frac{48}{5}$
(C) 6 and 9
(D) 12 and 18
(E) $\frac{64}{5}$ and $\frac{96}{5}$



22. The figure above shows three overlapping circular rings, each of which has its center at point O. If the radius of the largest of the three rings is 10, what is the area of the shaded region?

(A) 20π
(B) 33π
(C) 49π
(D) 51π
(E) 84π



23. In circle O above, the length of PR is 8. What is the area of the shaded region?

(A) $\frac{3}{4}\pi$
(B) $\frac{4}{3}\pi$
(C) $\frac{8}{3}\pi$
(D) $\frac{9}{4}\pi$
(E) $\frac{32}{3}\pi$

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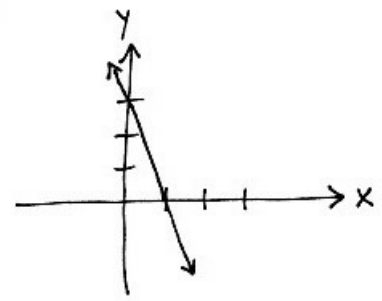
24. On the xy-coordinate plane, point A lies on the y-axis and point B lies on the x-axis. Points A, B, and C form a right triangle with a 90-degree angle at point C and an area of 30. If AC is parallel to the x-axis, and BC is parallel to the y-axis, which of the following could be the coordinates of point C?

(A) $(-6, -5)$
(B) $(-5, 12)$
(C) $(6, -9)$
(D) $(9, -8)$
(E) $(10, -3)$



25. In the coordinate system above, which of the following is the equation of line k?

(A) $x - y = 3$
(B) $x + y = 1$
(C) $3x - y = 1$
(D) $3x + y = 3$
(E) $x + 3y = 3$

26. On the xy-coordinate plane, line l_1 is defined by the equation $y = \frac{3}{2}x - 7$, and line l_2 is defined by the equation $y = -3x + \frac{1}{2}$. What is the y-coordinate of the point where lines l_1 and l_2 intersect?

(A) $-\frac{11}{2}$
(B) $-\frac{9}{2}$
(C) $-\frac{2}{3}$
(D) $\frac{5}{3}$
(E) $\frac{9}{4}$

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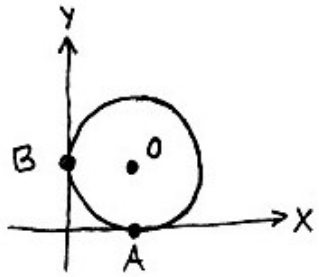

27. The circle with center O shown above is tangent to both axes at points A and B, respectively. If the area of circle O is 12π, what is the length of minor arc AB?

(A) $\frac{\pi}{\sqrt{2}}$
(B) $\pi\sqrt{2}$
(C) $\frac{2}{3}\pi\sqrt{3}$
(D) $\pi\sqrt{3}$
(E) $2\pi\sqrt{3}$

28. In the coordinate plane, which of the following lists the number of points (x, y) on the circumference a circle at which x or y could equal 0 ?

(A) 1 and 2 only
(B) 2 and 4 only
(C) 1, 2, and 4 only
(D) 2, 3, and 4 only
(E) 1, 2, 3, and 4

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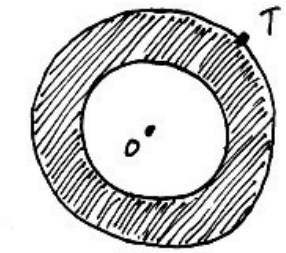
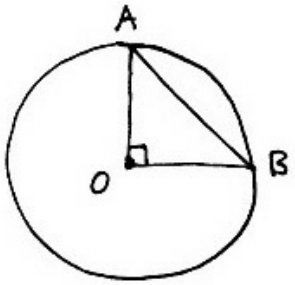


29. The shaded region in the figure above represents a circular tire. If the distance from point O to point T is 18 inches and the area of the shaded region is equal to the area of the unshaded region, what is the radius of the unshaded region, in inches?

(A) $\frac{9}{2}\sqrt{2}$
(B) $\frac{9}{2}\sqrt{3}$
(C) 9
(D) $9\sqrt{2}$
(E) $9\sqrt{3}$



30. The radius of circle O above is $2\sqrt{2}$. What is the perimeter of triangle OAB?

(A) 6
(B) 8
(C) $2\sqrt{2}+4$
(D) $4\sqrt{2}+4$
(E) $4\sqrt{2}+8$

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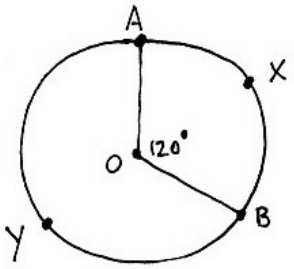
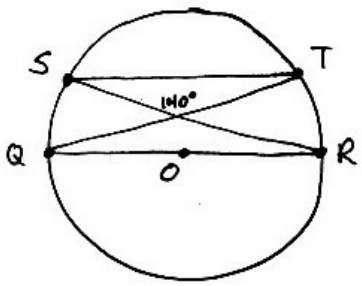


31. The circle above has center O and a radius of 6. How much greater is the length of arc AYB than the length of arc AXB?

(A) 2π
(B) 3π
(C) 4π
(D) 6π
(E) 8π

32. Points X, Y, and Z are located on the rectangular coordinate plane at points $(2, 3)$, $(-4, 3)$, and $(2, -3)$, respectively. What is the length of line segment YZ?

(A) 6
(B) $6\sqrt{2}$
(C) $6\sqrt{3}$
(D) 12
(E) $12\sqrt{2}$



33. In the circle above, ST is parallel to diameter QR, and the radius of the circle is 4.5. What is the length of minor arc QS?

(A) $\frac{\pi}{2}$
(B) π
(C) $\frac{3\pi}{2}$
(D) 2π
(E) $\frac{5\pi}{2}$

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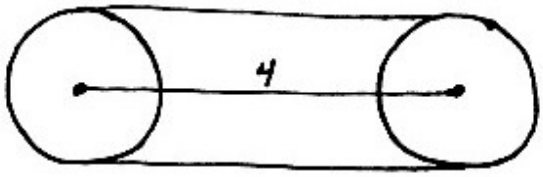

34. In the diagram above, a single string is drawn tightly around two circular spools, each of which have a diameter of 1 inch. If the distance between the centers of two two spools is 4 inches, what is the length of the string?

(A) $2 + 2\pi$
(B) $4 + \pi$
(C) $4 + 2\pi$
(D) $8 + \pi$
(E) $8 + 2\pi$

35. A piece of string 30 centimeters long is cut into 2 pieces. One of the pieces is used to form a square with side s. The other piece is used to form a circle. If no string is left over, which of the following represents the total area of the square and circular regions, in terms of s ?

(A) $4s + \frac{15-2s}{\pi}$
(B) $4s + \frac{30-4s}{\pi}$
(C) $s^2 - 2s + 15$
(D) $s^2 + \frac{(15-2s)^2}{\pi}$
(E) $s^2 + \frac{(30-4s)^2}{\pi}$

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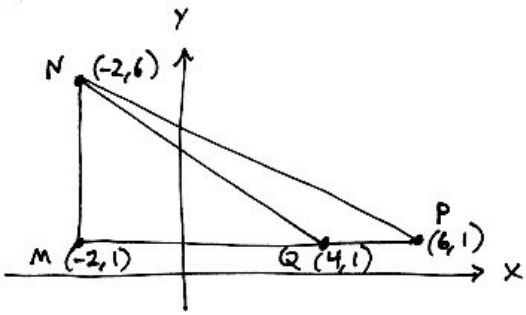
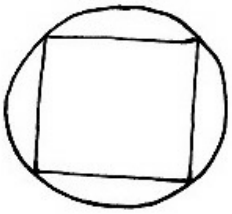


36. In the rectangular coordinate system above, the area of $\triangle MNP$ is how many times the area of $\triangle QNP$?

(A) $\frac{3}{2}$
(B) 2
(C) 3
(D) 4
(E) $\frac{9}{2}$



37. In the figure above, a square is inscribed in a circle. If the area of the circle is 18π, what is the area of the square?

(A) 12
(B) 18
(C) 36
(D) 54
(E) 72

38. A closed cylindrical tank contains 64π cubic feet of water and is filled to half its capacity. If, when the tank is placed upright on its circular base on level ground, the height of the water is equal to the radius of the base, what is the height of the tank?

(A) 2
(B) 4
(C) 8
(D) 12
(E) 16

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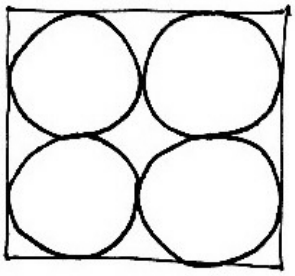
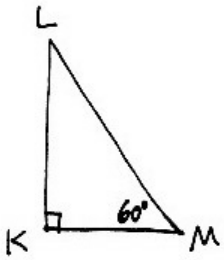


39. The figure above depicts the bases of four cans arranged in a square box with an area of 144 square centimeters. If the four cans have bases of equal size, what is the combined area of the bases of the four cans?

(A) 9π
(B) 27π
(C) 36π
(D) 48π
(E) 64π

40. A semicircle has a radius of 8. What is the approximate perimeter of the semicircle?

(A) 16
(B) 25
(C) 33
(D) 41
(E) 58



41. The perimeter of the triangle above is $6\sqrt{3} + 6$. What is the length of the hypotenuse of the triangle?

(A) 2
(B) 4
(C) $2\sqrt{3}$
(D) $3\sqrt{3}$
(E) $4\sqrt{3}$

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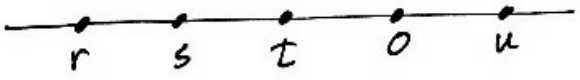
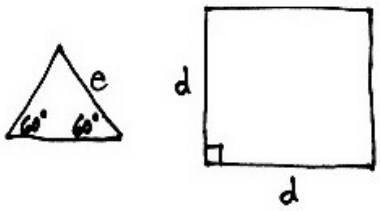


42. If r, s, t, and u are numbers on the number line shown above, which of the following quotients is least?

(A) $\frac{r}{s}$
(B) $\frac{r}{u}$
(C) $\frac{s}{t}$
(D) $\frac{t}{r}$
(E) $\frac{t}{u}$

43. On the xy-plane, each point on circle C has coordinates such that $x \geq 0$ and $y \geq 0$. If the center of circle C is the point $(3, 8)$, what is the maximum possible area of C?

(A) 6π
(B) 9π
(C) 24π
(D) 32π
(E) 64π



44. If the region on the right has twice the area of the region on the left, what is the ratio of $d : e$?

(A) $\sqrt[4]{3} : \sqrt{2}$
(B) $\sqrt[4]{3} : 2$
(C) $\sqrt{3} : \sqrt{2}$
(D) $\sqrt{3} : 2$
(E) $3 : 2$

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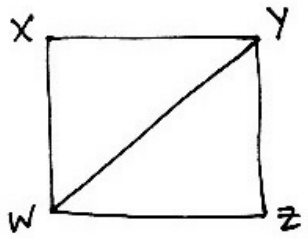
45. What is the greatest possible area of a square that is completely contained within a circle with radius 2, with one vertex at the center of the circle and one other vertex on the circle?

(A) $\frac{\sqrt{2}}{2}$
(B) $\sqrt{2}$
(C) 2
(D) $2\sqrt{2}$
(E) 4

46. The y-intercept of line k is negative. If the slope of line k is also negative, which of the following could be the x-intercept of line k?

I. -4
II. 0
III. 3

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III



47. In the figure above, $WXYZ$ is a square. If circle C (not shown) has a diameter equal to the length of WX, and circle D (not shown) has a diameter equal to the length of WY, the area of circle D is how many times the area of circle C?

(A) $\frac{\sqrt{2}}{2}$
(B) $\sqrt{2}$
(C) 2
(D) $2\sqrt{2}$
(E) 4

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48. A rectangular plot of land has a fence along three of its four sides. The unfenced side and the side opposite the unfenced side have a length that is three times the length of the other two sides. If the area of the plot is 432 square feet, what is the total length of the fence, in feet?

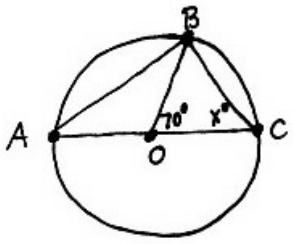
(A) 60
(B) 72
(C) 84
(D) 96
(E) 144

49. The area of a square panel is X square feet and the perimeter is Y square feet. If $X = Y + 5$, what is the perimeter of the panel?

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25

50. Square $ABCD$ is inscribed in circle C. If the length of line segment AB is 8, what is the length of minor arc AB?

(A) $\pi\sqrt{2}$
(B) 2π
(C) $2\pi\sqrt{2}$
(D) 8π
(E) $8\pi\sqrt{2}$



51. In the figure above, the vertices of triangle ABC lie on the circumference of circle O. What is the value of x?

(A) 35
(B) 45
(C) 55
(D) 65
(E) 70

52. A rectangular rug with side lengths of 2 feet and 6 feet is placed on a square floor that has an area of 64 square feet. If the surface of the rug does not extend beyond the area of the floor, what fraction of the area of the floor is not covered by the rug?

(A) $\frac{3}{16}$
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) $\frac{13}{16}$
(E) $\frac{7}{8}$

53. For any square S in the xy-coordinate plane, the center of the square is defined as the point where the two diagonals of square S intersect. If the vertices of a certain square are $(-1, 7)$, $(5, 7)$, $(5, 2)$, and $(-1, 2)$, what are the coordinates of the center of the square?

(A) $(-1, 4.5)$
(B) $(1, 5)$
(C) $(2, 2)$
(D) $(2, 4.5)$
(E) $(3, 5)$

54. In the xy-plane, line l has a negative slope and a y-intercept of 2. If the triangle formed by l and the two axes has an area of 6, what is the x-intercept of line l?

(A) -6
(B) -3
(C) 2
(D) 3
(E) 6

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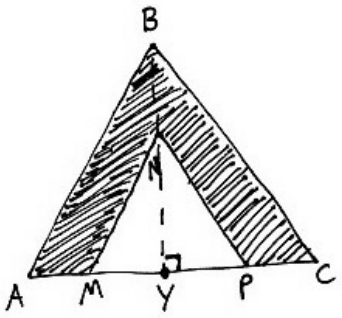

55. In the figure above, triangles ABC and MNP are both isoceles. AB is parallel to MN, BC is parallel to NP, the length of AC is 7 and the length of BY is 4. If the area of the unshaded region is equal to the area of the shaded region, what is the length of MP?

(A) $2\sqrt{2}$
(B) $2\sqrt{7}$
(C) $\frac{2\sqrt{3}}{3}$
(D) $\frac{7\sqrt{2}}{2}$
(E) $\frac{7\sqrt{3}}{3}$

56. If equilateral triangle MNP is inscribed in circle O with radius of 6, what is the length of minor arc MN?

(A) 2π
(B) 4π
(C) 6π
(D) 8π
(E) 12π

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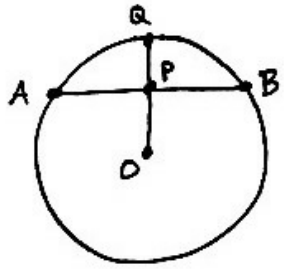
4 Data Sufficiency

For all Data Sufficiency questions, the answer choices are as follows:

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
(C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient.
(E) Statements (1) and (2) TOGETHER are NOT sufficient.

57. In the xy-plane, at what two points does the graph of $y = (x + a)(a + b)$ intersect the x-axis?
(1) The y-intercept of the graph is -4.
(2) $a + b = 3$



58. What is the length of chord AB in circle O above?
(1) $OQ = 5$ and $OP = 4$
(2) OQ is perpendicular to chord AB.

59. How many 360-degree rotations did a bicycle wheel make while rolling 50 meters in a straight line without slipping?
(1) Each 360-degree rotation took 3 seconds.
(2) The radius of the bicycle wheel was 0.3 meter.

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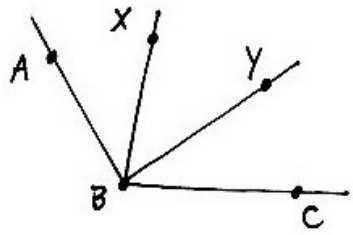
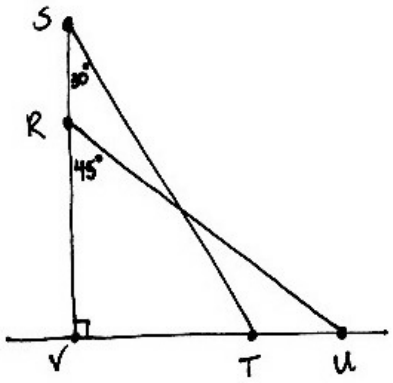


60. In the figure above, what is the measure of $\angle BYC$?
    (1) YC (not shown) is equal to BC, which is equal to BY.
    (2) XY (not shown) is equal to BX, which is equal to BY.

61. Is the perimeter of triangle T greater than the perimeter of square S?
    (1) The length of the longest side of T is twice the length of a side of S.
    (2) T is isoceles.



62. In the figure above, segments RU and ST represent two positions of the same support beam leaning against the side SV of a structure. The length UV is how much greater than the length TV?
    (1) The length of SV is 8 meters.
    (2) The length of RU is $8\sqrt{2}$ meters.

63. If O and P are each circular regions, what is the radius of region P?
    (1) The smaller region has an area that is equal to the area of a sector defined by a 135 degree central angle in the larger region.
    (2) The radius of region O is 2.5.

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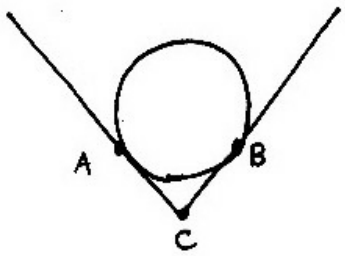
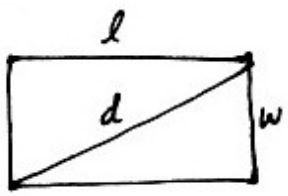
64. In the xy-plane, the slope of line k is $\frac{3}{2}$. Does line k pass through the point $(-1, -4)$?

(1) Line k passes through the point $(1, -1)$.
(2) Line k passes through the point $(3, 2)$.



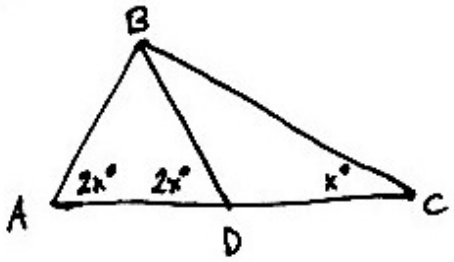
65. As shown in the graph above, the circular base of a large oak tree sits in a level field and touches two straight sides of a fence at points A and B. Point C shows where the two sides of the fence meet. How far from the center of the tree's base is point B ?

(1) The center of the base is 15 feet from point C.
(2) The base has an area of 180 square feet.



66. What is the length of side l in the rectangle above?

(1) $w = 4$ and $d = 8$
(2) The area of the rectangle is $16\sqrt{3}$.



67. In triangle ABC above, what is the measure of $\angle ABD$?

(1) $\angle DBC = 20$
(2) $DB = BC$

68. In $\triangle FGH$, if $FG = x$, $GH = y$, and $FH = z$, which of the three angles has the greatest degree measure?

(1) $x - y = y - z$
(2) $x > z$

69. What is the perimeter of right triangle ABC?

(1) $AB = 5$
(2) $AC = 5\sqrt{2}$

70. If P and Q are points in a plane and P lies inside the circle C with center O and radius 2, does Q lie inside circle C?

(1) The length of line segment OP is 1.
(2) OPQ is an equilateral triangle.

71. In the xy-plane, if line l has negative slope and passes through the point $(-4, q)$, is the x-intercept of line l positive?

(1) $q < 0$
(2) The slope of line l is -4

72. In the xy-plane, does the point $(-3, 3)$ lie on line k?

(1) The point $(3, -3)$ does not lie on line k.
(2) The slope of line k is -1.

73. The perimeter of the outside of truck tire R is $\frac{7}{8}$ the perimeter of the outside of truck tire T. What is the area of the outside of truck tire T?

(1) The diameter of the outside of truck tire R is 1.75 feet.
(2) The radius of the outside of truck tire T is 1 foot.

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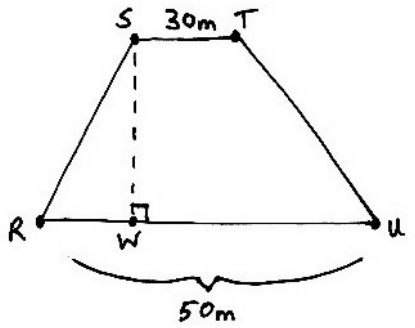
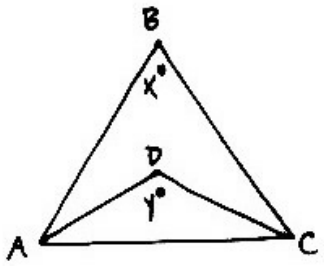


74. Quadrilateral $RSTU$ shown above is a site plan for the base of a hotel in which side RU is parallel to side ST and RU is longer than ST. What is the area of the base of the hotel?

(1) $RW = 9$ meters
(2) $RS = 15$ meters



75. In the figure above, what is the value of $x + y$?

(1) $\angle DCA = 35$
(2) $\angle CAB = 70$

76. Is x between -1.5 and 1.5 on the number line?

(1) x is to the left of 1 on the number line.
(2) x is to the right of -1 on the number line.

77. Lines m and n lie in the xy-plane and intersect at the point $(-2, 4)$. Is the slope of line m less than the slope of line n ?

(1) The x-intercept of line m is greater than the x-intercept of line n.
(2) The y-intercept of line n is greater than the y-intercept of line m.

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78. In the xy-plane, at what two points does the graph of $y = (x - k)(x + k)$ intersect the x-axis?

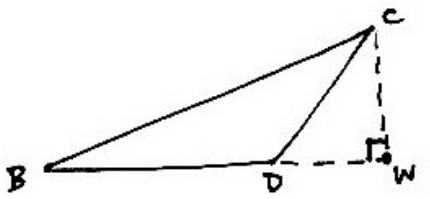
(1) $k^2 = 9$
(2) $k = 3$

79. A circle is inscribed in equilateral triangle XYZ, which is itself inscribed in a circle. What is the area of the smaller circle?

(1) The area of the larger circle is 24π.
(2) The radius of the larger circle is twice the radius of the smaller circle.

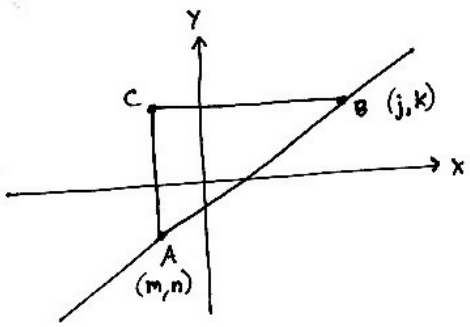
80. If m is the perimeter of rectangle N, what is the value of m?

(1) The area of rectangle N is 60.
(2) Each diagonal of rectangle N has length 13.



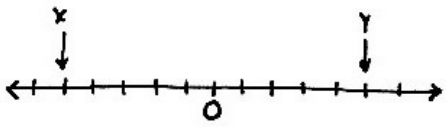
81. What is the area of triangular region BCD above?

(1) $BD = 2(DW)$
(2) The area of triangular region BWC is 22.



82. In the figure above, segments AC and BC are each parallel to one of the rectangular coordinate axes. Is the length of AC greater than the length of BC?

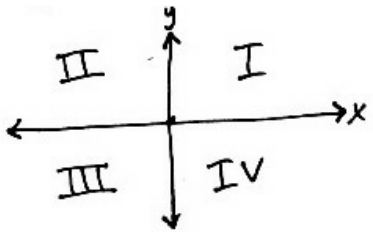
(1) $m = -1$ and $k = 3$
(2) The slope of segment AB is $\frac{4}{5}$



83. The successive marks on the number line above are equally spaced. What is the value of y?

(1) $x + y = 0$

(2) $y - x = \frac{3}{4}$



84. In the rectangular coordinate system shown above, does the line l intersect quadrant IV ?

(1) The slope of line l is between -1 and 1.

(2) Line l passes through the origin.

85. If m and n are lines in the xy-plane, is the product of the slopes of m and n equal to -1?

(1) m and n are perpendicular to each other.

(2) m and n intersect at point $(2, 2)$.

86. In the xy-plane, does the line with the equation $y = 2x - 4$ contain the point (a, b) ?

(1) $(2a - b - 4)(a + 5b + 2) = 0$

(2) $(4a + 3b - 1)(2a - b - 4) = 0$

87. In the xy-plane, what is the y-intercept of line l ?

(1) The x-intercept of line l is $\frac{3}{2}$.

(2) Line l is perpendicular to line k, which has a slope of -2.

88. If r and s are positive numbers, what are the coordinates of the midpoint of line segment MN in the xy-plane?

(1) The coordinates of M are $(r, 3 - s)$.

(2) The coordinates of N are $(3 - r, s)$.

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89. Circle O and line l lie in the xy-plane. If circle O has a center at $(3, 3)$ and a radius of 3, does line l intersect circle O?

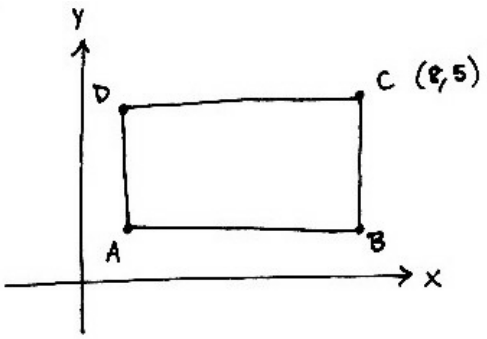
(1) The slope of line l is negative.
(2) The y-intercept of line l is 2.

90. What is the total surface area of rectangular solid S?

(1) The surface area of one of the faces of S is 24.
(2) The length of two of the edges of S are 4 and 8, respectively.

91. If $rs \neq 0$ and the points $(-r, s)$ and $(s, -r)$ lie in the same quadrant of the coordinate plane, is the point (a, b) in this same quadrant?

(1) $ab < 0$
(2) $sb > 0$



92. In the coordinate plane above, what is the area of rectangular region $ABCD$?

(1) A has coordinates $(1, 2)$.
(2) B has coordinates $(8, 2)$.

93. In the xy-plane, point A has coordinates (m, n) and point B has coordinates (p, q). What is the distance between A and B?

(1) $n - q = 6$
(2) $p - m = 6$

94. Lines r and s lie in the xy-plane. Is the y-intercept of line r less than the y-intercept of line s ?

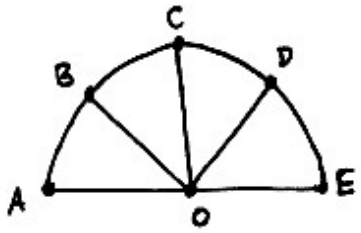
(1) At the intersection point of r and s, the x-coordinate and y-coordinate are both negative.
(2) The slope of line r is greater than the slope of line s.

95. Points A and B lie on the same plane as a circle with center O. If the radius of circle O is 10, do points A and B both lie within the area of circle O?

(1) The distance between points A and B is 21.
(2) The distance between points O and B is 11.

96. In the xy-plane, line l_1 passes through the point $(-1, 1)$ and line l_2 passes through the point $(3, 1)$. Are l_1 and l_2 perpendicular to each other?

(1) Lines l_1 and l_2 intersect at the point $(0, -1)$.
(2) The slope of line l_1 is -2.



97. In the figure above, O is the center of a semicircle, and points A, B, C, D, and E lie on the semicircle. If the degree measure of angle AOC is 85, what is the degree measure of angle AOB?

(1) The degree measure of angle BOD is 100 degrees.
(2) The degree measure of angle COE is 95 degrees.

98. In the xy-plane, the line l passes through the origin and through the point (c, d), where $cd \neq 0$. Is d positive?

(1) $cd < 0$
(2) The slope of line l is negative.

99. How many times did Joseph walk along a circular track in order to walk 300π meters?

(1) The circumference of the track is 80π meters.
(2) The diameter of the track is 80 meters.

100. In the xy-plane, if line l has negative slope and passes through the point $(-3, q)$, is the y-intercept of line l positive?

(1) The y-intercept of line l is equal to the x-intercept of line l.
(2) The slope of line l is -1.

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5 Answer Key

For full explanations, see the next section.

1. E
2. A
3. C
4. A
5. A
6. C
7. C
8. C
9. B
10. C
11. C
12. C
13. B
14. E
15. B
16. B
17. D
18. C
19. D
20. D
21. B
22. B
23. C
24. B
25. D
26. B
27. D
28. E
29. D
30. D
31. C
32. B
33. B
34. D
35. D
36. D
37. C
38. C
39. C
40. D
41. E

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42. B
43. B
44. A
45. C
46. A
47. C
48. A
49. D
50. C
51. C
52. D
53. D
54. E
55. D
56. B
57. C
58. C
59. B
60. A
61. A
62. D
63. E
64. D
65. B
66. A
67. A
68. C
69. E
70. B
71. A
72. C
73. D
74. C
75. E
76. C
77. D
78. D
79. A
80. C
81. C
82. B
83. B
84. E
85. A
86. C
87. C

88. C
89. E
90. E
91. A
92. A
93. C
94. C
95. D
96. A
97. E
98. E
99. D
100. E

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6 Explanations

For a quick-reference answer key, see the previous section.

1. E

Explanation: Start by considering the number of points at which a circle can intersect one line. They can intersect at one point–a point of tangency, where the line just touches the edge of the circle. It's also possible that a line intersects a circle twice–once at it passes through each side of the circle.

When a second line is introduced, there are more possibilities. If both lines intersect the circle twice, that's 4 points. If one line intersects the circle twice and the other is tangent with it, that's 3. If both lines are tangent, or if one line passes through the circle twice and the other doesn't touch the circle at all, that's 2. And if one of the lines is tangent to the circle and the other doesn't intersect the circle, that's 1. Choice (E) is correct.

2. A

Explanation: We're given all three of the dimensions of the box, so we can find the volume, which is the product of those three dimensions:

xyz

We're given the rate of cubic feet per hour; we're given cubic feet (xyz), and we're looking for hours. We can set up a rate equation to solve for hours:

$rate = \frac{ft^3}{hr}$

$y = \frac{xyz}{h}$

$yh = xyz$

$h = xz$, choice (A).

3. C

Explanation: When a circle is inscribed in a square, the length of the diameter is equal to the length of a side of the square. If the area of the circle is 18π, we can find the diameter:

$a = \pi r^2 = 18\pi$

$r^2 = 18$

$r = 3\sqrt{2}$

$d = 6\sqrt{2}$

Thus, a side of the square is $6\sqrt{2}$, and the area of the square is $(6\sqrt{2})^2 = 72$. To compare 72 and 18π, approximate $\pi = 3.1$, so $18\pi = 18(3.1) \approx 56$. The difference is about 16, choice (C).

4. A

Explanation: The total cost will be expressed as the product of cost per foot and total feet, and we know cost per foot, \$15. Total feet is the perimeter (44) excluding the unfenced side (n). Thus the total feet is $(44 - n)$. Total cost, then, is $\$15(44 - n)$, choice (A).

5. A

Explanation: Since we're given the height of the water in the tank at half full, and the volume at half full, we don't need to convert those numbers to the actual size of the tank. Think of the question as concerned a half-sized tank, one with a height of 5 and a volume of 45π cubic feet. Given the height and the volume of the tank, we can use the formula for the volume of a cylinder to find the radius of the base:

$v = \pi r^2 h$

$45\pi = \pi r^2(5)$

$9 = r^2$

$r = 3$, choice (A).

6. C

Explanation: While you can do this entire question mathematically, it's best to use a drawing as much as possible. But first, a little math: the slope of perpendicular lines are negative reciprocals of each other. The line $y = x$ has a slope of 1, so the slope of AB, it's perpendicular bisector, is -1. That means it moves to the right at the same rate it moves down. Other points on the line are, for instance, $(3, 2)$, $(2, 3)$, and $(1, 4)$. It intersects the line $y = x$ at the point $(2.5, 2.5)$. Since $y = x$ bisects AB, if the line moves over and down 1.5 each before intersecting $y = x$, it must move another 1.5 over and down before reaching point B, which is then at $(1, 4)$.

The second line is a little easier. BC is the perpendicular bisector of the x-axis, so it is a vertical line. B is 1 above the axis, so C must be 1 below the axis. The x-coordinate remains the same: C is located at $(4, -1)$, choice (C).

7. C

Explanation: To see the relationship between the two figures, start by drawing three lines outward from point O to the vertices of the circle. Those lines divide the circle into three equal parts – the center of the circle is the same as the center of the triangle, and since the triangle is equilateral, the three sectors are equal as well. Since a circle contains 360 degrees, each one of the sectors contains 120 degrees.

Next, draw a line down from point O to the base of the triangle. This splits the bottom sector in half. The resulting triangles are $30 : 60 : 90$ triangles. The short side, which corresponds to the 30 degree angle, is the radius of the circle. Since the side ratio of a $30 : 60 : 90$ triangle is $x : x\sqrt{3} : 2x$, we know that the sides of the triangle are $r : r\sqrt{3} : 2r$. $r\sqrt{3}$ is half of the base of the triangle, so each side of the triangle is double that: $2r\sqrt{3}$, choice (C).

8. C

Explanation: The perimeter of a rectangle is given by $p = 2l + 2w$, and the perimeter of a square is $p = 4s$. If the sides of R have a ratio of $3 : 1$, then $l = 3w$, so the perimeter is $2(3w) + 2w = 8w$. Thus, we can set the two perimeters equal to each other, and $4s = 8w$, or $s = 2w$. From here, it might

be easiest to pick numbers. Say the length of a side of the square is 6 and the width of the rectangle is 3. Since $l = 3w$, the length of the rectangle is 9.

The area of the square, then, is 36, and the area of the rectangle is $3(9) = 27$. The ratio, then, is $27 : 36 = 3 : 4$, choice (C).

9. B

Explanation: If the circular path has a diameter of 4 miles, then the path's circumference is 4π miles. Beyond that, this is a rate question:

$r = \frac{d}{t}$

$7 = \frac{4\pi}{y}$

$y = \frac{4\pi}{7} \approx \frac{12.4}{7} \approx 1.8$, choice (B).

10. C

Explanation: Since the bases of both triangles coincide with the x-axis, both heights are perpendicular to the same base, and both extend to point S. Thus, the heights of the two triangles are the same. So, if the areas are the same, their bases must be the same as well. The distance between P and R is 12, so if that is to be equally divided between PQ and QR, each of the bases must have a length of 6, meaning that Q is 6 to the right of the origin, at point $(6, 0)$, choice (C).

11. C

Explanation: In a regular pentagon, all the interior angles are equal, and all the sides are equal. Find the sum of the interior angles:

$sum = 180(n - 2) = 180(5 - 2) = 180(3) = 540$

Divide by 5 for the measure of each interior angle: $\frac{540}{5} = 108$.

Triangles ABC and CDE are each isoceles, since two of the sides are sides of the pentagon. Since vertices ABC and CDE have a degree measure of 108, the remaining degrees inside the two triangles are $180 - 108 = 72$ each, leaving 36 for each of the two other interior angles. Thus, two of the three angles that make up angle BCD are 36. If the sum of those three angles must be 108, the measure of x is $108 - 36 - 36 = 36$, choice (C).

12. C

Explanation: Since each triangle is a 45-45-90 triangle, the legs are equal. The legs are also the base and height, for the purposes of calculating the area. So, the area of the figure on the left is $\frac{1}{2}a^2$, and the one on the right is $\frac{1}{2}A^2$. The question tells us the relationship between those two:

$\frac{1}{2}a^2 = \frac{1}{2}(\frac{1}{2}A^2)$

Isolate A:

$a^2 = \frac{1}{2}A^2$

$a = \sqrt{\frac{1}{2}}A = \frac{A}{\sqrt{2}}$

$A = a\sqrt{2}$, choice (C).

13. B

Explanation: The GMAT will occasionally make questions look more complicated by introducing unfamiliar three-dimensional figures such as hemispheres, but the question almost always boils down to two-dimensional problems. In this case, we are concerned with the relationship of the radius of the hemisphere (which is the radius of the base of the hemisphere) to a length of the side of the cube (which is a length of a side of a base of a cube). In short, we're talking about a circle inscribed in a square.

In such a figure, the diameter of the circle is equal to each side of the square:
$d = s$

Since the diameter is twice the radius, we can substitute r for d:
$2r = s$
$\frac{r}{s} = \frac{1}{2}$, choice (B).

14. E

Explanation: The third point could be at one of two locations: either $(-3, -6)$ or $(3, 2)$. It doesn't matter which, so pick the one that will be easiest for you to work with. For now, let's work with $(3, 2)$. Given that point, PQ has a length of 6, from -3 to 3. QR has a length of 8, from 2 to -6. If the two legs of a right triangle are 6 and 8, the third is 10: this should be familiar as a multiple of the $3 : 4 : 5$ pythagorean triangle ratio. Thus, the sides are 6, 8, and 10, so the perimeter is 24. Choice (E) is correct.

15. B

Explanation: If the height of the oil is 1 meter and the tank is half full, the height of the tank is 2 meters. Given the capacity (the volume) and the height, we can solve for the radius:
$v = \pi r^2 h$
$18 = \pi r^2 (2)$
$r^2 = \frac{9}{\pi}$
$r = \sqrt{\frac{9}{\pi}} = \frac{\sqrt{9}}{\sqrt{\pi}} = \frac{3}{\sqrt{\pi}}$, choice (B).

16. B

Explanation: If a point is twice as far from C as from D, that must be true for each of the coordinates: the x-coordinate of the point must be twice as far from the x-coordinate of C as from the x-coordinate of D, and the same is true for y.

The x-coordinates are 5 and -1, 6 apart. The number that is twice as far from 5 as from -1 is +1. The y-coordinates are 4.5 apart, and the number that is twice as far from -2 as from 2.5 is +1. Thus, the coordinates of the point we're looking for are $(1, 1)$, choice (B).

Also, if you can make a drawing approximately to scale, you can probably eyeball it. The answer choices are reasonably far apart, and some of them aren't even on the line, so you can make a very good, probably correct, guess without any calculations at all.

17. D

Explanation: Consider each of the choices, keeping in mind the rules that govern the side lengths of a triangle: each side must be greater than the difference of the lengths of the other two sides, and less than the sum of the other two sides.

I. If $MN = 2$, then $MP = 2$. That doesn't work: NP can't be greater than the sum of those two sides, but it is 6, which is greater than 4.

II. If $MN = 6$, we have an equilateral triangle. That works.

III. If $MN = 12$, that works. 6 is less than the sum of 12 and 12, and is greater than the difference between 12 and 12. The correct choice is II and III, (D).

18. C

Explanation: The equation of a line holds true for any point along the line. So, if (j, k) is on the line $x = 2y + 3$, then it is true that $j = 2k + 3$. Applying the same reasoning to the other point:

$j + 4 = 2(k + v) + 3$

Substitute $2k + 3$ in for j in the second equation:

$2k + 3 + 4 = 2k + 2v + 3$

$4 = 2v$

$v = 2$, choice (C).

19. D

Explanation: The diagonal of a rectangle forms a right triangle with the rectangle's length and width. In that right triangle, if the legs have a ratio of 3 : 4, their ratio with the diagonal must be 3 : 4 : 5. (You can calculate that algebraically with the pythagorean theorem, but the 3 : 4 : 5 ratio comes up often enough on the GMAT that you should know it cold.)

If the diagonal is 45 inches, that's 9 times the 5 in the ratio. Thus, the amounts of the three side lengths are 9 times the ratio:

27 : 36 : 45

The perimeter is:

$p = 2l + 2w = 2(27) + 2(36) = 54 + 72 = 126$, choice (D).

20. D

Explanation: Without absolute value signs, the inequality that defines the shaded part of the line is $-1 \leq x \leq 3$, so we'll look for the choice that simplifies to that.

(A) If x is positive, $x \leq 1$, but if x is negative, $x \geq -1$. That inequality is $-1 \leq x \leq 1$.

(B) If x is positive, $x \leq 3$, but if x is negative, $x \geq -3$. That inequality is $-3 \leq x \leq 3$.

(C) If x is positive, $x - 2 \leq 1$, or $x \leq 3$ but if x is negative, $x - 2 \geq -1$, or $x \geq 1$ That inequality is $1 \leq x \leq 3$.

(D) If x is positive, $x - 1 \leq 2$, or $x \leq 3$ but if x is negative, $x - 1 \geq -2$, or $x \geq -1$ That inequality is $-1 \leq x \leq 3$, which is what we're looking for.

(E) If x is positive, $x + 1 \leq 2$, or $x \leq 1$ but if x is negative, $x + 1 \geq -2$, or $x \geq -3$ That inequality is $-3 \leq x \leq 1$.

(D) is the correct choice.

21. B

Explanation: While you could do this question algebraically, it's easier to simply check each choice. If the perimeter is 32 feet, the sum of the length and width is 16 feet.

(A) $\frac{9}{2} + \frac{21}{2} = \frac{30}{2} = 15$

(B) $\frac{32}{5} + \frac{48}{5} = \frac{80}{5} = 16$

(C) $6 + 9 = 15$

(D) $12 + 18 = 30$

(E) $\frac{64}{5} + \frac{96}{5} = \frac{160}{5} = 32$

Choice (B) is correct.

22. B

Explanation: If the radius of the largest ring is 10 and the distance between the edge of the largest and the next largest is 3, the radius of the middle-sized ring is 7. The area of that entire ring is $\pi r^2 = 49\pi$. The area of the shaded region, though, is the area of the middle-sized ring minus the area of the smallest ring. Since the radius of the smallest ring is 4, the area of that ring is 16π, which means that the area of the shaded region is $49\pi - 16\pi = 33\pi$, choice (B).

23. C

Explanation: It's a rarely tested rule, but when it comes up, there's no way around it: the measure of $\angle PRQ$ is half that of $\angle POQ$. The rule applies so long as R and Q are on the edge of the circle and O is the center of the circle.

So in this case, the shaded sector is defined by the angle 60 – double 30. Such a sector is $\frac{60}{360} = \frac{1}{6}$ of the circle. The diameter is given as 8, which means the radius is 4, so the area of the entire circle is 16π. The area of the sector, then, is $\frac{16\pi}{6} = \frac{8}{3}\pi$, choice (C).

24. B

Explanation: If A lies on the y-axis, then its x-coordinate is 0. Similarly, B's y-coordinate is 0. Thus, for BC and AC, each line's length is determined by difference between one of C's coordinates and one of A's or B's coordinates. For instance, since AC is parallel to the x-axis, A and C have the same y-coordinate. The difference between their x-coordinates, one of which is zero, is the length of AC. The same is true, only with opposite coordinates, for BC.

Thus, the coordinates of point C the base and height of the triangle. It doesn't matter whether they are negative or positive: on the coordinate plane, -5 is just as far away from 0 as +5 is. So, since $\frac{1}{2}bh = 30$, the product of the base and height – that is, the absolute value of the coordinates of C – must be 60. The only choice for which that is the case is (B).

25. D

Explanation: The two points on the graph that are easiest to recognize are the intercepts: $(1, 0)$ and $(0, 3)$. You already have the y-intercept (3), so if you calculate the slope, you can determine the equation of a line. The slope is as follows:

$\frac{y_2-y_1}{x_2-x_1} = \frac{3-0}{0-1} = -3$

So in the usual format, the equation of the line is:

$y = -3x + 3$

Add $3x$ to both sides, and you have $3x + y = 3$, choice (D).

26. B

Explanation: Given two linear equations and two variables, there is only one resulting pair of values for the two variables. That's the case here–given the two equations of the lines, treat them like a system of equations, and the values of x and y that result are the coordinates of the point of intersection.

Both equations offer a value for y in terms of x, so set those equal to each other:

$\frac{3}{2}x - 7 = -3x + \frac{1}{2}$

$3x - 14 = -6x + 1$

$9x = 15$

$x = \frac{15}{9} = \frac{5}{3}$

We're looking for the y-coordinate, so plug that back into one of the equations:

$y = -3(\frac{5}{3}) + \frac{1}{2} = -5 + \frac{1}{2} = -\frac{9}{2}$, choice (B).

27. D

Explanation: The only way that two perpendicular lines (such as the two axes) can be tangent to the same circle is if the arc they form represents exactly one quarter of the circle. More visually: radiuses OA and OB form a right angle, which means that the resulting sector of the circle is defined by a 90 degree angle – that's $\frac{1}{4}$ of the 360 degrees of the circle.

To find the arc, you need to divide the circumference by 4. For the circumference, you'll need the radius. Given the area, you can solve for the radius:

$12\pi = \pi r^2$

$r^2 = 12$

$r = 2\sqrt{3}$

$c = 2\pi r = 4\pi\sqrt{3}$

$\frac{1}{4}c = \frac{1}{4}(4\pi\sqrt{3}) = \pi\sqrt{3}$, choice (D).

28. E

Explanation: Put another way, the question is asking how many points at which a circle could intersect one of the two coordinate axes. Put another way still, we're wondering how many points at which a circle could intersect two perpendicular lines.

1 is possible: if one of the lines is tangent to the circle, and the other is outside the circle, that's only 1 point of intersection.

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2 is possible: if one of the lines passes through both sides of the circle, and the other line is outside the circle, that's 2. It's also possible, that both lines are tangent to different sides of the circle.

3 is also possible: if one of the lines passes through both sides of the circle and the other line is tangent to the circle, that's 3.

4 works as well: if both lines pass throug the circle twice, that's 4. That's true of any circle with a center at the origin (as well as many others). Choice (E) is correct.

29. D

Explanation: OT is a radius, so the area of the outer circle is $\pi(18^2)$. If the area of the unshaded and shaded regions are equal, each of the two areas is half of the area of the entire outer circle, or $\pi(18)(9)$. If that's the area of the inner circle, we can solve for its radius:

$\pi(18)(9) = \pi r^2$

$r^2 = 18(9)$

$r = 9\sqrt{2}$, choice (D).

30. D

Explanation: Since both OA and OB are radiuses of the circle, we know two sides of the triangle. And because angle AOB is 90, we know that those two radiuses are the legs of a $45 : 45 : 90$ triangle, which has a side ratio of $x : x : x\sqrt{2}$. Thus, the hypotenuse of the triangle, AB, has a length of $(2\sqrt{2})\sqrt{2} = 4$.

The sum of the three sides, then, is $2\sqrt{2} + 2\sqrt{2} + 4 = 4\sqrt{2} + 4$, choice (D).

31. C

Explanation: Arc AXB is defined by an angle of 120 degrees. That's $\frac{120}{360} = \frac{1}{3}$ of the circle. AYB is defined by 240 degrees $(360 - 120)$, which is $\frac{2}{3}$ of the circle. To calculate the length of each arc, multiply that fraction by the circumference of the circle, which is $2\pi r = 12\pi$.

$AXB = \frac{1}{3}(12\pi) = 4\pi$

$AYB = \frac{2}{3}(12\pi) = 8\pi$

The difference is 4π, choice (C).

32. B

Explanation: Since X and Y share a y-coordinate, they are parallel to the x-axis, and you can calculate their distance: $2 - (-4) = 6$. Similarly, X and Z are parallel to the y-axis, so you can calculate their distance as well: $3-(-3) = 6$. Since those two sides are each parallel to an axis, they are perpendicular to each other, so XYZ is a right triangle, with YZ as the hypotenuse. Since XY and XZ have the same distance, it's a 45-45-90 right triangle, meaning that the hypotenuse is $\sqrt{2}$ times the length of either leg. Thus, $YZ = 6\sqrt{2}$, choice (B).

33. B

Explanation: Since ST and QR are parallel, the triangle formed by Q, R, and the intersection of QT and SR is isoceles. Since one of the angles is 140, the other two angles are each 20.

Next, the minor arc QS is defined by the angle QOS (not shown in the diagram). Such an angle has twice the angle measure of the angle twice as far away from the edge of the circle, at QRS. So, the angle that defines the arc is 40. 40 is $\frac{1}{9}$ of the 360 degrees in the circle.

The circumference of the entire circle is $2\pi r = 2(4.5)\pi = 9\pi$. $\frac{1}{9}$ of that circumference is π, choice (B).

34. D

Explanation: To make the relationships clearer, draw a diameter vertically in each circle, perpendicular to the lines that connect the circles. Those sides are part of a rectangle that includes two lengths that are the same as the length of 4 that connects the two centers. So, the lines that run from the point of tangency of each circle to the other are 4 each.

The string runs around half of each circle, as well. If the diameter of each is one inch, the circumference of each spool is π. Thus, the sum of the two half-circumferences is also π. So, the part of the string that touches the spools is π in length, and the two parts that are not touching a spool are 4 each, for a total of $8 + \pi$, choice (D).

35. D

Explanation: Since the side of a square is s, the area of the square is s^2, eliminating (A) and (B). Finding the area of the circle is much more complicated.

The perimeter of the square is $4s$, so after forming the square, the remaining string is $30 - 4s$. That will be the circumference of the circle:

$c = 2\pi r = 30 - 4s$

$r = \frac{30-4s}{2\pi} = \frac{15-2s}{\pi}$

Given the radius, we can find the area:

$a = \pi r^2 = \pi(\frac{15-2s}{\pi})^2 =$

$\pi(\frac{15-2s}{\pi})(\frac{15-2s}{\pi}) =$

$\frac{(15-2s)^2}{\pi}$

Combine the two areas: $s^2 + \frac{(15-2s)^2}{\pi}$, choice (D).

36. D

Explanation: The area depends on two variables: the base and the height. Both of the triangles have the same height, so any difference in area is based on the difference in the length of their bases. $\triangle MNP$ has base of 8, while $\triangle QNP$ has a base of 2. Thus, one is 4 times greater than the other. Since that is the only variable in the area formula that changes, the area of the larger triangle is 4 times the area of the smaller triangle. Choice (D) is correct.

37. C

Explanation: In the figure, the diameter of the circle is the same as the diagonal of the square. From the area of the circle, we can find the diameter:

$\pi r^2 = 18\pi$

$r^2 = 18$

$r = 3\sqrt{2}$

$d = 2(3\sqrt{2}) = 6\sqrt{2}$

The diagonal of a square is the hypotenuse of a 45-45-90 right triangle formed with two of the square's sides. Using the ratio for 45-45-90 triangles, we know that the hypotenuse is $\sqrt{2}$ times the length of any side. Thus, $6\sqrt{2}$ is $\sqrt{2}$ times the length of a side, which makes the side of the square 6. If the side is 6, the area is $6^2 = 36$, choice (C).

38. C

Explanation: Ignore for now the fact that the tank is half full – do the calculations as if the tank were half the height, and were completely full. (Just remember to double the height before selecting an answer!)

Using the volume equation for a cylinder, we can calculate the height of the water:

$v = \pi r^2 h$

$64\pi = \pi x^2 x$ (x is both the height of the water and the radius of the base, which are equal)

$x^3 = 64$

$x = 4$

If the height of the water is 4, and the tank is half full, the height of the tank is 8, choice (C).

39. C

Explanation: Since the circles are all equal, if we draw two lines separating the box into four parts, each circle will fit perfectly in each of the smaller squares. Since the box has as an area of 144 and a side length of 12, each circle is inscribed in a square with a side length of 6. Thus, the diameter of each circle is 6, the radius is 3, and the area of each is $\pi r^2 = 9\pi$. Since there are four cans, that's four times the area: 36π, choice (C).

40. D

Explanation: The perimeter of a semicircle is the sum of the diameter (the base of the semicircle) and half the circumference of a circle of the same size. If the radius is 8, the diameter is 16. The circumference of a whole circle would be $2\pi r = 16\pi$, so the length of the circular part of the semicircle is half that, 8π. Approximate π as 3.1, so $8\pi \approx 25$. The total perimeter is $16 + 25 = 41$, choice (D).

41. E

Explanation: The triangle has angles of 30, 60, and 90, so the ratio of the side lengths is $x : x\sqrt{3} : 2x$. Thus, in terms of x, the length of the short

leg, the perimeter of the triangle is $x + x\sqrt{3} + 2x = 3x + x\sqrt{3}$. Use that to solve for x with the given perimeter:

$3x + x\sqrt{3} = 6\sqrt{3} + 6$

$x(3 + \sqrt{3}) = 6\sqrt{3} + 6$

$x = \frac{6\sqrt{3}+6}{3+\sqrt{3}}(\frac{3-\sqrt{3}}{3-\sqrt{3}}) =$

$\frac{18\sqrt{3}-18+18-6\sqrt{3}}{9-3} =$

$\frac{12\sqrt{3}}{6} = 2\sqrt{3}$

We're looking for the hypotenuse, which is $2x = 2(2\sqrt{3}) = 4\sqrt{3}$, choice (E).

42. B

Explanation: The least of the quotients must be negative, so u must be involved somehow, as the only way we can use these four numbers to generate a negative quotient is by including both a positive and negative number. That eliminates (A), (C), and (D). Now compare $\frac{r}{u}$ wth $\frac{t}{u}$.

r is less than t – for instance, $r = -5$ and $t = -2$. In the two quotients, the denominator is the same, so we can ignore that. We'll get the smallest result by having the smallest numerator, r. For instance, if $u = 3$, $\frac{r}{u} = -\frac{5}{3}$, while $\frac{t}{u} = -\frac{2}{3}$. Choice (B) is correct.

43. B

Explanation: Since the x-coordinate of the center of the circle is 3, the radius of the circle can be no more than 3. If the radius were greater than 3, the circle would extend to the left of the y-axis. Since the center has a y-coordinate of 8, we don't have any problem with a point 3 above or 3 below the center, so 3 is our radius. The area, then, is $\pi r^2 = 9\pi$, choice (B).

44. A

Explanation: The height of an equilateral triangle is the longer leg of a 30-60-90 triangle formed with half of the base and one of the sides. Thus, if the base is e, half the base is $\frac{1}{2}e$, and the height is $\frac{1}{2}e\sqrt{3}$. So, the area is:

$\frac{1}{2}(e)(\frac{1}{2}e\sqrt{3}) = \frac{1}{4}e^2\sqrt{3}$

The area of the square is easier: it's d^2.

The question tells us that $d^2 = 2(\frac{1}{4}e^2\sqrt{3})$, so we can go to work on that equation to isolate $\frac{d}{e}$, another form of the ratio $d : e$.

$d^2 = 2(\frac{1}{4}e^2\sqrt{3})$

$\frac{d^2}{e^2} = \frac{1}{2}\sqrt{3}$

$\frac{d}{e} = \sqrt{\frac{\sqrt{3}}{2}} = \frac{\sqrt{\sqrt{3}}}{\sqrt{2}} = \frac{\sqrt[4]{3}}{\sqrt{2}} = \sqrt[4]{3} : \sqrt{2}$, choice (A).

45. C

Explanation: The largest such square would be one where the diagonal is a radius: the radius connects two opposite points on the square. If the radius is 2, the diagonal is 2. Since the diagonal of a square is the hypotenuse of a 45-45-90 triangle formed by two sides of the square, we know the diagonal is $\sqrt{2}$ times the length of a side:

$s\sqrt{2} = 2$
$s = \frac{2}{\sqrt{2}}$
Square that for the area:
$s^2 = \frac{4}{2} = 2$, choice (C).

46. A

Explanation: If the y-intercept of the line is negative, it crosses the y-axis below the origin. If the slope is negative, it moves down and to the right, or up and to the left. It must move up to cross the x-axis, so it moves to the left, which means it crosses the x-axis to the left of the origin. The only one of the three roman numerals that are to the left of the origin is 1, so choice (A) is correct.

47. C

Explanation: Call the length of WX n. The diagonal of a square is the hypotenuse of a right triangle formed with two sides of the square, so the diagonal is $n\sqrt{2}$.

The diameter of circle C is n, meaning that the area is $\pi(\frac{n}{2})^2 = \frac{n^2\pi}{4}$. The diameter of D is $n\sqrt{2}$, so the area is $\pi(\frac{n\sqrt{2}}{2})^2 = \frac{n^2\pi}{2}$. That has half the denominator of the area of C, so it is twice as big. (C) is the correct choice.

48. A

Explanation: Call the longer side l and the shorter side w. $l = 3w$, and $lw = 432$. Substitute one equation into the other to solve for the side lengths:
$3w(w) = 432$
$3w^2 = 432$
$w^2 = 144$
$w = 12$
$l = 3w = 3(12) = 36$
The fence includes one long side (36) and two short sides (12 each), so the total length of fence is:
$36 + 12(2) = 60$, choice (A).

49. D

Explanation: $X = s^2$, where s is a side of the square, and $Y = 4s$. Substituting those expressions in terms of s into the equation given:
$s^2 = 4s + 5$
$s^2 - 4s - 5 = 0$
$(s - 5)(s + 1) = 0$
$s = 5$ or $s = -1$
The side must be positive, so $s = 5$. Thus, the perimeter is $4s = 4(5) = 20$, choice (D).

50. C

Explanation: The four points of an inscribed square break a circle into four parts, all equal. Thus, the arc AB is $\frac{1}{4}$ the circumference of the circle. To find that, we'll need the radius or diameter of the circle.

When a square is inscribed in a circle, the diagonal of the square coincides with the diameter of the circle. The diagonal of a square is the hypotenuse of a 45-45-90 triangle with two of the sides as legs, so we can use the 45-45-90 side ratio to find the length of the diagonal. If a side of the square is 8, the hypotenuse/diagonal/diameter is $8\sqrt{2}$. Thus, the circumference of the entire circle is $8\pi\sqrt{2}$, and the length of an arc that makes up $\frac{1}{4}$ of the circle is $2\pi\sqrt{2}$, choice (C).

51. C

Explanation: This question relies on a couple of rarely-tested rules. First, that $\angle BOC$ is double $\angle CAB$. This is true whenever you have two points on the edge of the circle (C and B, in this case), and two angles, one defined by the center of the the circle and one defined by a point formed by the diameter with one of the first two points. The short version relevant to this question is: the measure of $\angle CAB$ is 35, half of 70.

The other rule is that when a triangle is formed by a diameter and one other point on the circle, the vertex at the third point is a right angle. Therefore, $\angle ABC$ is 90. Now we have two of three angles is triangle ABC: 35, 90, and x:

$180 = 35 + 90 + x$

$x = 180 - 125 = 55$, choice (C).

52. D

Explanation: The area of the rug is 12 square feet. That leaves $64-12 = 52$ square feet of the floor not covered by the rug. That's $\frac{52}{64} = \frac{26}{32} = \frac{13}{16}$ of the floor that is not covered by the rug, choice (D).

53. D

Explanation: The two diagonals meet at the midpoint of each of the diagonals, at a point that is halfway between the left and right side of the square and halfway between the top and bottom of the square.

The left and right of the square are located at -1 and 5, so the middle of the square is 3 away from each, with an x-coordinate of 2. The top and bottom are at 2 and 7, so the middle is 2.5 away from each, with a y-coordinate of 4.5. The center, then, is $(2, 4.5)$, choice (D).

54. E

Explanation: The y-intercept gives us the height of the triangle. If the two axes are sides of the triangle, one of the vertices of the triangle is at the origin, $(0, 0)$. Thus, the height of the triangle is the distance from $(0, 0)$ to $(0, 2)$, 2. The base of the triangle is the distance between $(0, 0)$ and $(x, 0)$, where x is the x-intercept. To find that, use the information we have about the area of the triangle:

$a = \frac{1}{2}bh$

$6 = \frac{1}{2}b(2)$
$b = 6$

Thus, $x = 6$, and that's the location of the x-intercept, so (E) is the correct choice.

55. D

Explanation: Since all three sides of the two triangles are parallel to its corresponding side in the other triangle, the triangles are similar. Thus, the ratio of their sides is equal.

We're given enough information to find the area of ABC: the base is 7 and the height is 4, so the area is $\frac{1}{2}(4)(7) = 14$. If the shaded region and the unshaded region have the same area, that's 7 each, meaning that MNP has an area of 7.

We're looking for MP, the base, so call it x. Since the height of the larger triangle is $\frac{4}{7}$ the length of the base of that triangle, the same must hold true for the smaller triangle, so call NY $\frac{4}{7}x$. Finally, we can solve for x:

$7 = \frac{1}{2}x(\frac{4}{7}x)$
$7 = \frac{4}{14}x^2$
$x^2 = 7(\frac{14}{4}) = \frac{49}{2}$
$x = \frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$, choice (D).

56. B

Explanation: Since the triangle is equilateral, it breaks the circumference of the circle into three equal parts. If you draw the three radiuses OM, ON, and OP, each resulting sector is defined by a central angle of 120 degrees. Thus, the length of minor arc MN is $\frac{120}{360} = \frac{1}{3}$ the length of the circumference of the circle. (It doesn't matter how you label each of the vertices of the triangle; since it's equilateral, they are interchangeable.)

The radius of the circle is 6, so the circumference is $2\pi r = 12\pi$, so the length of the minor arc is $\frac{1}{3}(12\pi) = 4\pi$, choice (B).

57. C

Explanation: The graph intersects the x-axis at the point where $y = 0$. So, we're looking for the value of x when $(x + a)(x + b) = 0$. To break that down, we need to know the value of x when $x + a = 0$ or $x + b = 0$, or better yet: $x = -a$ or $x = -b$.

Statement (1) is insufficient. This tells us the graph includes the point $(0, -4)$, so $-4 = (0 + a)(0 + b)$, or $ab = -4$. That doesn't tell us the specific values of a and b.

Statement (2) is also insufficient: again, we know a relationship between a and b, but not their specific values.

Taken together, the statements are sufficient. If $ab = -4$ and $a + b = 3$, then a and b must be -1 and 4. We don't know which is which, but it doesn't matter: x has two possible values, each of which is the negative of one of those two. Choice (C) is correct.

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58. C

Explanation: Statement (1) is insufficient. We don't know anything about AB's position relative to OQ; remember that in Data Sufficiency, diagrams can be very misleading. While we know that A and B are on the circle and AB intersects OQ at point P, that's all we know.

Statement (2) is also insufficient. This gives us a clearer relationship between OQ and AB, but no distances, so no way to find the length of AB.

Taken together, the statements are sufficient. OA and OB are radiuses, and since $OQ = 5$, $OA = OB = 5$. Both OAP and OBP are right triangles, with a hypotenuse of 5 and one leg (OP) of 4. We can use the pythagorean theorem, or our knowledge of the common triplet $3 : 4 : 5$, to recognize that $AP = BP = 3$. Thus, $AB = 6$. Choice (C) is correct.

59. B

Explanation: One rotation would mean that the wheel had traveled the length of its circumference; 50 meters, then, is some number of circumferences. That number of circumferences is the number of rotations it took.

Statement (1) is insufficient: this is irrelevant, focusing on rate instead of the circumference of the wheel.

Statement (2) is sufficient. If we know the radius, we can determine the circumference of the wheel, and how many circumferences add up to 50 meters. Choice (B) is correct.

60. A

Explanation: Statement (1) is sufficient. If $BY = BC = YC$, then BYC is an equilateral triangle, and all of the angles, including $\angle BYC$, are equal to 60.

Statement (2) is insufficient. This gives us the same set of facts about BXY, so we know it is equilateral. However, we're interested in $\angle BYC$, which isn't part of that triangle. Choice (A) is correct.

61. A

Explanation: Statement (1) is sufficient. Say that the length of a side of S is x, which makes the longest side of T $2x$. The perimeter of S, then, is $4x$. It's possible that the two other sides of T are just a little shorter than $2x$, say $1.75x$ each. In that case, the perimeter of T is $5.5x$. It's also possible that the two sides are just barely long enough to make a side of $2x$ possible. Since any side of a triangle has to be shorter than the sum of the other two sides, the shortest possible other sides of T would sum to a little more than $2x$, say $2.05x$. That gives us a perimeter of $4.05x$. Either way, the perimeter of T is greater than the perimeter of S.

Statement (2) is insufficient. This gives us the relationship between sides of T, but nothing to relate T to S. Choice (A) is correct.

62. D

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Explanation: Since RU and ST represent the same support beam, $RU = ST$. Given the angles shown in the figure, we know that STV is a 30-60-90 triangle, and that RUV is a 45-45-90 triangle. Since the hypotenuses are the same, we know the relationship between every single line in the figure. To find the difference in measurement between UV and TV, we just need one actual measurement to go with all of those ratios.

Statement (1) is sufficient. It gives us a measurement, so we should recognize that's enough. Given SV, we can find the other two sides of triangle STV. With the hypotenuse of that triangle, which is equal to RU, we can find the other sides of triangle RUV. With all of those sides, we can find both UV and TV.

Statement (2) is also sufficient. In this case, we start with the hypotenuse of one of the triangles, which we know is equal to the hypotenuse of the other triangle. With both hypotenuses, we can use the ratios of the 30-60-90 and 45-45-90 triangles to find all of the measurements of the sides in the figure. Choice (D) is correct.

63. E

Explanation: Statement (1) is insufficient. It does give us the ratio of the area of the smaller region to the area of the larger region: 135 : 360, or 9 : 24. But without knowing whether P is the larger or smaller region, and with no idea of the actual sizes, we can't answer the question.

Statement (2) is also insufficient. This tells us nothing about P.

Taken together, the statements are still insufficient. We don't know whether O or P is the larger region, so while we have the radius of one of the two, and the ratio between them, the answer differs depending on whether O or P is the larger region. Choice (E) is correct.

64. D

Explanation: Given the slope of the line, all we need to know whether the line passes through a specific point is one other point on the line.

Statements (1) and (2) both give us that, so each statement is sufficient on its own. Given two points on a line, we can find the slope,and with the slope, we can determine the equation of a line. From there, we can check to see whether any point is or is not on the line. Choice (D) is correct.

65. B

Explanation: Statement (1) is insufficient. How the distance from the center to C relates to the size of the circle depends on the angle at point C, which we don't know anything about.

Statement (2) is sufficient. The distance from the center of the base to B is the radius. If we know the area of the base, we can find the radius. Choice (B) is correct.

66. A

Explanation: Statement (1) is sufficient. The length, width, and diagonal form a right triangle, so:

$l^2 + w^2 = d^2$

$l^2 + 4^2 = 8^2$

We can solve for l.

Statement (2) is insufficient. Given the area, we know the product lw, but not the exact measure of either. Choice (A) is correct.

67. A

Explanation: Statement (1) is sufficient. $\angle BDC$ is the important one to focus on. It is complementary with $\angle ADB$, so it is equal to $180 - 2x$. Also, it is one of the three angles of DBC. Since an angle that is complementary to one of the angles in a triangle (as $\angle ADB$ is) is equal to the sum of the other two angles,

$\angle ADB = \angle DBC + \angle DCB$

$2x = x + y$

$y = x$

So, $\angle DBC = 20 = x$. Thus, $2x = 40$, so two of the angles in triangle ABD are 40, leaving 100 degrees for the measure of $\angle ABD$.

Statement (2) is insufficient. We're looking for an angle measure, and we're given the relationship between sides. This statement is another way of telling us that $\angle DBC = \angle DCB$, but it doesn't give us any numbers to work with. Choice (A) is correct.

68. C

Explanation: The angle with the greatest degree measure is the one that corresponds with the longest side. It doesn't matter in Data Sufficiency exactly which side corresponds with which angle; suffice it to say that you could figure it out. If you can find the longest side, you can answer the question.

Statement (1) is insufficient. Another way of looking at the equation is as:

$x + z = 2y$

If all three of the sides are different, then either x or z must be greater than y, but we don't know which one. The equation works for either of the following:

$3 + 5 = 2(4)$ (z is largest)

$5 + 3 = 2(4)$ (x is largest)

Statement (2) is also sufficient. It gives us no information about how y relates to the lengths of the other two sides.

Taken together, the statements are sufficient. (1) tells us that either x or z must be greater than y, but not which one. If $x > z$, as (2) tells us, x must be the largest, making FG the longest side. Choice (C) is correct.

69. E

Explanation: Statements (1) and (2) are each sufficient on their own. Given only one side of a right triangle, we can't infer the other two sides, both of which are needed to find the perimeter.

Taken together, the statements are still insufficient. If AB and AC are the legs, the hypotenuse will be larger than $5\sqrt{2}$. If AC is the hypotenuse, the third side is 5. Those two scenarios give us different resulting perimeters, so the correct choice is (E).

70. B

Explanation: Statement (1) is insufficient. We're interested in the location of Q, and (1) tells us nothing about it.

Statement (2) is sufficient. If OPQ is equilateral, then $OP = OQ$. Since P lies within the circle, OP is less than 2 (the radius of the circle). Thus OQ is less than 2 as well, meaning that Q is less than 2 from the center of a circle with radius 2, and that it lies within the circle. Choice (B) is correct.

71. A

Explanation: Statement (1) is sufficient. We know l passes through a point in quadrant III. If the slope is negative, it moves down and to the right, or up and to the left, from that point. Thus, as it moves up and to the left from that point, it will eventually cross the x-axis to the left of the origin, so the x-intercept is negative.

Statement (2) is insufficient. It doesn't matter what the exact value of the negative slope is. If the point $(-4, q)$ has a sufficiently large positive q value (say, 100), the x-intercept will be positive even with a steep negative slope. Choice (A) is correct.

72. C

Explanation: Statement (1) is insufficient. There are an infinite number of lines that do not run through $(3, -3)$, some of those run through $(-3, 3)$, while many more do not.

Statement (2) is also insufficient. We don't know anything about the line except for the slope, so there are again an infinite number of possibilities.

Taken together, the statements are sufficient. There is only one line that has a slope of -1 and passes through the point $(-3, 3)$. That line does pass through $(3, -3)$ as well: given those two points, you can calculate the slope:

$m = \frac{-3-3}{3-(-3)} = \frac{-6}{6} = -1$

So, if $(3, -3)$ does not lie on the line, the line cannot be the one line that has a slope of -1 and passes through the point $(-3, 3)$. The answer is "no," and the correct choice is (C).

73. D

Explanation: We're given the relationship between the perimeters of two circles. From that, we can find the relationship between their radiuses. Basically, once we know the relationship between one measure of a circle, we know them all.

Statement (1) is sufficient. Given the diameter of R, we can find the diameter of T, and from the diameter of T, the radius of T, then the area of T.

Statement (2) is also sufficient. Given the radius of T, we can find the area of T. Choice (D) is correct.

74. C

Explanation: To find the area of a trapezoid, we need the measure of both bases (which we have: 30 and 50) and the height, which is represented here as SW.

Statement (1) is insufficient. It gives us no clue as to the height of the figure.

Statement (2) is also insufficient. It is the hypotenuse of right triangle RSW, but we don't know either of the other measurements of the triangle.

Taken together, the statements are sufficient. Given RW and RS, two sides of a right triangle, we can find the length of SW, the height of the trapezoid. Choice (C) is correct.

75. E

Explanation: Statement (1) is insufficient. It gives us only one of the angles in triangle DCA, and to find y, we'd need two. Not only that, it gives us no clue as the measure of x, which is in a triangle with no clear relation to DCA.

Statement (2) is also insufficient. This one gives us only one of the angles in ABC, and no way to find anything about DCA.

Taken together, the statements are still insufficient. Despite the fact that the triangles share a side, there is no known relationship between their angles, and since we don't know whether the triangles are isoceles, knowing one angle in each is not enough to find the third angle. Choice (E) is correct.

76. C

Explanation: Statements about a variable's location on the number line can almost always be simplified to inequalities. For instance, this question asks:

Is $-1.5 < x < 1.5$?

Statement (1) is insufficient. It says: $x < 1$. It could be greater than -1.5, and thus between -1.5 and 1.5, but it could also be less than -1.5.

Statement (2) is also insufficient. It says: $x > -1$. This could be less than 1.5, but it could also be greater than 1.5.

Taken together, the statements are sufficient. The two inequalities combined are: $-1 < x < 1$. A number within that range must also be within the wider range of $-1.5 < x < 1.5$. Choice (C) is correct.

77. D

Explanation: Statement (1) is sufficient. Draw two lines, both of which pass through the point $(-2, 4)$. No matter how you draw them, the one that intersects the x-axis further to the left has the greater slope. Thus, n has the greater slope.

Statement (2) is also sufficient. You can consider the same lines you drew for (1). No matter how you draw them, the one that intersects the y-axis at

the higher point has the higher slope. Thus, again, n has the greater slope. Choice (D) is correct.

78. D

Explanation: When a graph intersects the x-axis, $y = 0$. So, $y = (x - k)(x + k)$ intersects the axis when $0 = (x - k)(x + k)$. In other words, when $x - k = 0$ or $x + k = 0$, or simpler still: when $x + k$ or $x = -k$. To find the values of x, all we need is the value of k.

Statement (1) is sufficient. While k could be 3 or -3, it doesn't matter: if $k = 3$, $x = 3$ or $x = -3$; if $k = -3$, $x = -3$ or $x = 3$.

Statement (2) is sufficient, and simpler: with the value of k, we can find the values of x. Choice (D) is correct.

79. A

Explanation: The math involved in relating circles and inscribed/circumscribed equilateral triangles is complicated; there's no need to use it here. Suffice it to say that they are related in predictable ways: as with circles and inscribed/circumscribed squares, when you know the size of one of the figures, you can find the size of the other.

Statement (1) is sufficient. From the area of the larger circle, you can find the radius of the larger circle. That radius is also a line going from the center of the triangle to a vertex of the triangle; it's the hypotenuse of a 30-60-90 triangle that includes half of the base of the triangle and a third side from the center of the triangle/circle down to the base. That third side is also the radius of the smaller circle. Using the 30-60-90 ratios, you can find the relationship between the radiuses, and thus find the radius of the smaller circle and then the area.

Statement (2) is insufficient. Ignoring all of the relationships between the figures, recognize that it gives you no concrete measurements. Without at least one of those, you can't find the radius of either circle, or any other measurement. Choice (A) is correct.

80. C

Explanation: $m = 2l + 2w$, where l is the length of the rectangle and w is the width of the rectangle.

Statement (1) is insufficient. If $lw = 60$, we have one equation with two variables.

Statement (2) is insufficient. The diagonal is the hypotenuse of a right triangle formed by two of the legs of the rectangle, so $l^2 + w^2 = 13^2$. That's another two-variable equation, so it's not enough to find each of the variables on its own.

Taken together, the statements are sufficient. The math, while not too difficult, is time-consuming, so consider that you have two variables and two equations. Technically, you need to have two linear equations; solving these two equations will give you two sets of answers, but in just about every GMAT geometry problem of this sort, one of the answers will be negative, and since this is geometry, there are no negative lengths. Choice (C) is correct.

81. C

Explanation: The base of the triangle is BD and the height is CW, so if we can find those two measurements, we can solve for the area.

Statement (1) is insufficient. It gives us the relationship between the bases of triangles BCD, BCW, and DCW; BCD has $\frac{2}{3}$ the base of BCW. Still, we know nothing about the height.

Statement (2) is also insufficient: we don't know anything about the relationship between BCD and BWC.

Taken together, the statements are sufficient. BCD and BWD have the same height, so if the base of BCD is $\frac{2}{3}$ the base of BWC, the area of BCD is also $\frac{2}{3}$ the area of BWC. We know the area of BWC is 22, the area of BCD is $\frac{2}{3}(22) = \frac{44}{3}$, choice (C).

82. B

Explanation: Statement (1) is insufficient. Given the values of m and k, we can determine the precise coordinates of C, which are (m, k), but those two values don't tell us anything about the length of either AC or BC.

Statement (2) is sufficient. A line with a slope of 1 moves to the right at the same rate that it moves up; if AB had a slope of 1, AC (the amount the line moves up) would be equal to BC (the amount the line moves over). A slope of $\frac{4}{5}$ indicates that the slope is flatter than a slope of 1: for every 4 it moves up, it moves 5 over. That gives you the ratio of $AC : BC$, which is $4 : 5$. Thus, BC is greater. Choice (B) is correct.

83. B

Explanation: First, note that y is 5 spaces to the right of 0, and x is 5 spaces to the left of zero.

Statement (1) is insufficient, and redundant: since we know the marks are equally spaced, x and y have the same absolute value; when they are added, they must add to zero.

Statement (2) is sufficient. Since y is 5 spaces to the right of zero, we can say that $y = 5s$, where s is the length of one space. Similarly, $x = -5s$. So, if $y - x = \frac{3}{4}$, then:

$5s - (-5s) = \frac{3}{4}$
$10s = \frac{3}{4}$
$5s = \frac{3}{8}$
$y = \frac{3}{8}$

Choice (B) is correct.

84. E

Explanation: Statement (1) is insufficient. A line with negative slope must pass through quadrants II and IV; it will also pass through I or III, but not both. (Or neither, if it passes through the origin.) A line with positive slope must pass through I and III, but only one of II and IV (unless, again, it passes through the origin, in which case neither of II and IV). The statement

doesn't tell us whether the slope is positive or negative, so we have no idea what quadrants it must pass through, and which ones it could.

Statement (2) is also insufficient. Again, the answer depends on the sign of the slope, which we have no information about.

Taken together, the statements are still insufficient. If the slope is positive, the line passes through I and III, but not II and IV; if the slope is negative, the line passes through II and IV, but not I or III. Choice (E) is correct.

85. A

Explanation: If the product of two slopes is -1, the slopes are negative reciprocals of each other–for instance, 2 and $-\frac{1}{2}$, or $\frac{1}{3}$ and -3. If the slopes of two lines are negative reciprocals of each other, the lines are perpendicular.

Statement (1) is sufficient: if the lines are perpendicular, their slopes are negative reciprocals of each other, so the product of the slopes is -1.

Statement (2) is insufficient: we don't know anything about the slopes of the two lines, only one point that both lines pass through. Choice (A) is correct.

86. C

Explanation: If the line with the equation $y = 2x - 4$ contains the point (a, b), then a and b can be substituted into the equation: it must be true that $b = 2a - 4$, or $2a - 4 - b = 0$.

Statement (1) is insufficient. One of the two expressions must equal zero: either

$2a - b - 4 = 0$

or

$a + 5b + 2 = 0$

If the first is true, the answer is "yes," if the second is true, the answer is probably "no."

Statement (2) is insufficient, for the same reason. One of the two expressions must equal zero; if the first equals zero, the answer is "no," but if the second does, the answer is "yes."

Taken together, the statements are sufficient. The only way both equations are true is if $2a - b - 4 = 0$, and if that is the case, we know that $y = 2x - 4$ contains the point (a, b). Choice (C) is correct.

87. C

Explanation: To find the y-intercept of a line, you need either the equation of the line, or one point and the slope of the line (which would, in turn, allow you to find the equation of the line).

Statement (1) is insufficient. This gives you one point on the line: $(\frac{3}{2}, 0)$. That's not enough to find another specific point.

Statement (2) is also insufficient. Perpendicular lines have slopes that are negative reciprocals of each other, so line l must have a slope of $\frac{1}{2}$. Without a point, though, that isn't enough.

Taken together, the statements are sufficient. (1) gives us one point on the line, and (2) gives us the slope. From that, we can find any point we'd like

on the line, including the point at which $x = 0$, the y-intercept. Choice (C) is correct.

88. C

Explanation: The midpoint of a line segment is determined by averaging the x-coordinates of the endpoints and averaging the y-coordinates of the endpoints.

Statements (1) and (2) are each insufficient on their own, since each only gives us coordinates of one of the two endpoints. We need both.

Taken together, the statements are sufficient. The x-coordinate of the midpoint is the average of r and $3 - r$:

$\frac{r+(3-r)}{2} = \frac{3}{2}$

And the y-coordinate is the average of $3 - s$ and s:

$\frac{(3-s)+s}{2} = \frac{3}{2}$

Thus, the midpoint is $(\frac{3}{2}, \frac{3}{2})$. Choice (C) is correct.

89. E

Explanation: Statement (1) is insufficient: we know nothing about the location of the line relative to the circle; it could run right through the middle of it or it could miss the circle by a huge amount.

Statement (2) is also insufficient. The circle is tangent to the y-axis at $(0, 3)$, above the y-intercept of the line. If the line has a steep negative slope, such as -5, then the line doesn't intersect the circle. If the slope is flatter, such as $-\frac{1}{3}$, or is positive, the line intersects the circle.

Taken together, the statements are still insufficient. In (2), we looked at two possible negative slopes that give contradictory answers, so even if we know the slope is negative and the y-intercept of the line is 2, we don't know whether the line intersects the circle. Choice (E) is correct.

90. E

Explanation: The formula for the surface area of a rectangular solid is $2lw + 2lh + 2wh$, where l is length, w is width, and h is height.

Statement (1) is insufficient. 24 could be lw, lh, or wh; and regardless of which one it is, we don't know anything about the third side, the other that is not one of the sides of the face that has an area of 24.

Statement (2) is also insufficient: we need all three dimensions of the solid to find its surface area.

Taken together, the statements are still insufficient. Either 4 or 8 must be one of the sides of the face with a surface area of 24; if it is 4, the other side of that face is 6, meaning the dimensions are 4, 6, and 8. If it is 8, the other side is 3, meaning the dimensions are 3, 4, and 8. Those two sets of dimensions result in different total surface areas, so the correct choice is (E).

91. A

Explanation: The question takes a little bit of puzzling out, but it's worth it. Within any quadrant, all of the x-coordinates have the same sign, and all of

the y-coordinates do as well. So, if $(-r, s)$ and $(s, -r)$ lie in the same quadrant, $-r$ and s have the same sign. Or: r and s have different signs. Thus, $(-r, s)$ and $(s, -r)$ are in one of the two quadrants where the coordinates have the same sign: I (upper right) or III (lower left).

Statement (1) is sufficient. If a and b have different signs, we don't know whether they are in quadrant II or quadrant IV, but we know they aren't in I or III, which is where $(-r, s)$ and $(s, -r)$ are located.

Statement (2) is insufficient. We know that s and b have the same sign, so $(-r, s)$ and (a, b) have the same signs on their y-coordinates. However, we don't know anything about a, so we can't answer the question. Choice (A) is correct.

92. A

Explanation: Statement (1) is sufficient. To find the area of a rectangle, you need both of its dimensions: the length and width. Given A's and C's coordinates, we can find the width: the distance between 1 and 8. We can also find the length: the distance between 2 and 5.

Statement (2) is insufficient. We already know the x-coordinate of the right leg of the rectangle, so all it offers us is the length of the rectangle. Choice (A) is correct.

93. C

Explanation: If you know the distance formula, it comes in handy here:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Or for this example:
$d = \sqrt{(p - m)^2 + (q - n)^2}$
If you don't know the distance formula, it's okay: the concept behind the distance formula is exactly the same as the pythagorean theorem. If you draw a right triangle with sides parallel to the axes and a hypotenuse connecting points A and B, you can use the pythagorean theorem to find the length of AB, and the math is identical.

Statements (1) and (2) are each insufficient on their own: they give you the distance between one of the two coordinates, but not both.

Taken together, the statements are sufficient. (2) gives us $p - m$, and (1) gives us the negative version of $q - n$. Plug them in to the distance formula, and you can answer the question:
$d = \sqrt{(6)^2 + (-6)^2} = \sqrt{36 + 36} = 6\sqrt{2}$
Choice (C) is correct.

94. C

Explanation: Statement (1) is insufficient. If the lines intersect in quadrant III, one of the y-intercepts is greater than the other, but we don't know which line is which, so we can't know which has the greater y-intercept.

Statement (2) is also insufficient. If the lines intersect to the left of the y-axis, then r has a greater y-intercept. But, if the lines intersect to the right of the axis, s has the greater y-intercept.

Taken together, the statements are sufficient. Since (1) tells us that the lines intersect to the left of the y-axis, we can look at the options discussed in (2) and recognize that r must have the greater y-intercept. It doesn't matter whether both slopes are negative, both are positive, or there is one of each. Choice (C) is correct.

95. D

Explanation: Statement (1) is sufficient. While we know nothing about the specific locations of A or B relative to the circle, we know the diameter of O is 20. If two points are 21 apart, there is no way both are inside the area of the circle. If one is on the circle or inside the circle, the other one must not be. Since the question is asking whether both are within the area of the circle, we can answer, "no."

Statement (2) is also sufficient. O is the center of the circle, which has a radius of 10, meaning that any point within 10 of O is inside the circle, and any point more than 10 away from O is not. Since B is not within the area of the circle, we know that the answer is "no." Choice (D) is correct.

96. A

Explanation: If two lines are perpendicular, the slopes of the lines are negative reciprocals of each other. Short of a very elaborate alternative, the way to determine whether two lines on the coordinate plane are perpendicular is to focus on that relationship between their slopes.

Statement (1) is sufficient. The question gives us one point on each of the two lines, and by telling us where the lines intersect, (1) gives us another point on each line. With two points on each line, we can calculate the slope of each line and determine whether they are negative reciprocals of each other.

Statement (2) is insufficient. We only have the one point on line l_2, so we can't calculate the slope of that line. Choice (A) is correct.

97. E

Explanation: If AOC is 85, then COE is 95, and we're left looking for the rest of the specifics.

Statement (1) is insufficient. Since BOD is 100, that means AOB and DOE sum to 80, but we don't know much each angle occupies.

Statement (2) is not only insufficient, it's redundant: the question already gives us this information, albeit not in the same form.

Taken together, the statements are insufficient. (2) is redundant, so if (1) is insufficient, there is no way for the two statements to combine to answer the question. Choice (E) is correct.

98. E

Explanation: Statement (1) is insufficient: if the product of the coordinates is negative, one of the coordinates is negative and one of the coordinates is positive, but we don't know which one is which.

Statement (2) is also insufficient. If the slope is negative and the line passes through the origin, the line travels entirely through quadrant II (the lower right) and quadrant IV (upper left). In II, the x-coordinate is positive and the y-coordinate is negative; in IV, it's the other way around. In short, for every point on the line, there is one positive and one negative coordinate, so like in (1), $cd < 0$. Again, we don't know which coordinate is which.

Taken together, the statements are still insufficient. Each one tells us the same thing: that one coordinate is negative while the other is positive. Choice (E) is correct.

99. D

Explanation: The length of the circular track is the circumference, so 300π meters is some number of circumferences. Call that number x:

$xc = 300\pi$

$x(2\pi r) = 300\pi$

$xr = 150$

Statement (1) is sufficient. We can use the first of the formulas to solve for x:

$xc = 300\pi$

$x(80\pi) = 300\pi$

$x = \frac{300}{80} = \frac{30}{8} = \frac{15}{4}$

Statement (2) is also sufficient. In this case, we can deduce that the radius is 40 and use the last of the equations we started with:

$xr = 150$

$40x = 150$

$x = \frac{150}{40} = \frac{15}{4}$

Choice (D) is correct.

100. E

Explanation: Statement (1) is insufficient. A line with a slope of -1 will always have equal x- and y-intercepts, but they could be positive or negative. For instance, l could pass through the points $(-1, 0)$ and $(0, -1)$; it could also pass through $(1, 0)$ and $(0, 1)$.

Statement (2) is also insufficient. In (1), we deduced that the line has a slope of -1, but it was still insufficient: the intercepts could be positive or negative.

Taken together, the statements are still insufficient. (2) doesn't add anything we didn't already know from (1), so if (1) is insufficient, the statements are insufficient together. (E) is the correct choice.