



Mathematical Conventions

for the Quantitative Reasoning
Measure of the GRE® revised
General Test

Overview

The mathematical symbols and terminology used in the Quantitative Reasoning measure of the test are conventional at the high school level, and most of these appear in the Math Review. Whenever nonstandard or special notation or terminology is used in a test question, it is explicitly introduced in the question. However, there are some assumptions about numbers and geometric figures that are particular to the test. These assumptions appear in the test at the beginning of the Quantitative Reasoning sections, and they are elaborated below.

Also, some notation and terminology, while standard at the high school level in many countries, may be different from those used in other countries or from those used at higher or lower levels of mathematics. Such notation and terminology are clarified below. Because it is impossible to ascertain which notation and terminology should be clarified for an individual test taker, more material than necessary may be included.

Finally, there are some guidelines for how certain information given in test questions should be interpreted and used in the context of answering the questions—information such as certain words, phrases, quantities, mathematical expressions, and displays of data. These guidelines appear at the end.

Numbers and quantities

- All numbers used in the test questions are real numbers. In particular, integers and both rational and irrational numbers are to be considered, but imaginary numbers are not. This is the main assumption regarding numbers. Also, all quantities are real numbers, although quantities may involve units of measurement.
- Numbers are expressed in base 10 unless otherwise noted, using the 10 digits 0 through 9 and a period to the right of the ones digit, or units digit, for the decimal point. Also, in numbers that are 1,000 or greater, commas are used to separate groups of three digits to the left of the decimal point.
- When a positive integer is described by the number of its digits, e.g., a two-digit integer, the digits that are counted include the ones digit and all the digits further to the left, where the left-most digit is not 0. For example, 5,000 is a four-digit integer, whereas 031 is not considered to be a three-digit integer.
- Some other conventions involving numbers: *one billion* means 1,000,000,000, or 10^9 (not 10^{12} , as in some countries); *one dozen* means 12; the Greek letter π represents the ratio of the circumference of a circle to its diameter and is approximately 3.14.
- When a positive number is to be rounded to a certain decimal place and the number is halfway between the two nearest possibilities, the number should be rounded to the greater possibility. For example, 23.5 rounded to the nearest integer is 24, and 123.985 rounded to the nearest 0.01 is 123.99. When the number to be rounded is negative, the number should be rounded to the lesser possibility. For example, -36.5 rounded to the nearest integer is -37 .
- Repeating decimals are sometimes written with a bar over the digits that repeat, as in $\frac{25}{12} = 2.08\overline{3}$ and $\frac{1}{7} = 0.14285\overline{7}$.
- If r , s , and t are integers and $rs = t$, then r and s are *factors*, or *divisors*, of t ; also, t is a *multiple* of r (and of s) and t is *divisible* by r (and by s). The factors of an integer include positive and negative integers. For example, -7 is a factor of 35, 8 is a factor of -40 , and the integer 4 has six factors: -4 , -2 , -1 , 1, 2, and 4. The terms *factor*, *divisor*, and *divisible* are used only when r , s , and t are integers. However, the term *multiple* can be used with any real numbers s and t provided r is an integer. For example, 1.2 is a multiple of 0.4, and -2π is a multiple of π .
- The *least common multiple* of two integers a and b is the least *positive* integer that is a multiple of both a and b . The *greatest common divisor* (or *greatest common factor*) of a and b is the greatest integer that is a divisor of both a and b .
- When an integer n is divided by a nonzero integer d resulting in a quotient q with remainder r , then $n = qd + r$, where $0 \leq r < |d|$. Furthermore, $r = 0$ if and only if n is a multiple of d . For example, when 20 is divided by 7, the quotient is 2 and the remainder is 6; when 21 is divided by 7, the quotient is 3 and the remainder is 0; and when -17 is divided by 7, the quotient is -3 and the remainder is 4.
- A *prime number* is an integer greater than 1 that has only two positive divisors: 1 and itself. The first five prime numbers are 2, 3, 5, 7, and 11. A *composite number* is an integer greater than 1 that is not a prime number. The first five composite numbers are 4, 6, 8, 9, and 10.
- Odd and even integers are not necessarily positive; for example, -7 is odd, and -18 and 0 are even.
- The integer 0 is neither positive nor negative.

Mathematical expressions, symbols, and variables

- As is common in algebra, italic letters like x are used to denote numbers, constants, and variables. Letters are also used to label various objects, such as line ℓ , point P , function f , set S , list T , event E , random variable X , Brand X , City Y , and Company Z . The meaning of a letter is determined by the context.
- When numbers, constants, or variables are given, their possible values are all real numbers unless otherwise restricted. It is common to restrict the possible values in various ways. Here are some examples: n is a nonzero integer; $1 \leq x < \pi$; and T is the tens digits of a two-digit positive integer, so T is an integer from 1 to 9.
- Standard mathematical symbols at the high school level are used. These include the arithmetic operations $+$, $-$, \times , and \div , though multiplication is usually denoted by juxtaposition, often with parentheses, e.g., $2y$ and $(3)(4.5)$; and division is usually denoted with a horizontal fraction bar, e.g., $\frac{w}{3}$. Sometimes mixed numbers, or mixed fractions, are used, like $4\frac{3}{8}$ and $-10\frac{1}{2}$. These two numbers are equal to $\frac{35}{8}$ and $-\frac{21}{2}$, respectively. Exponents are also used, e.g., $2^{10} = 1,024$,

$$10^{-2} = \frac{1}{100}, \text{ and } x^0 = 1 \text{ for all nonzero numbers } x.$$

- Mathematical expressions are to be interpreted with respect to *order of operations*, which establishes which operations are performed before others in an expression. The order is as follows: parentheses; exponentiation; negation; multiplication and division (from left to right); addition and subtraction (from left to right). For example, the value of the expression $1 + 2 \times 4$ is 9, because the expression is evaluated by first multiplying 2 and 4 and then adding 1 to the result. Also, -3^2 means “the negative of ‘3 squared’ ” because exponentiation takes precedence over negation. Therefore, $-3^2 = -9$, but $(-3)^2 = 9$ because parentheses take precedence over exponentiation.

- Here are examples of other standard symbols with their meanings:

$x \leq y$	x is less than or equal to y
$x \neq y$	x and y are not equal
$x \approx y$	x and y are approximately equal
$ x $	the absolute value of x
\sqrt{x}	the nonnegative square root of x , where $x \geq 0$
$-\sqrt{x}$	the nonpositive square root of x , where $x \geq 0$
$n!$	the product of all positive integers less than or equal to n , where n is any positive integer and, as a special definition, $0! = 1$.
$\ell \parallel m$	lines ℓ and m are parallel
$\ell \perp m$	lines ℓ and m are perpendicular

- Because all numbers are assumed to be real, some expressions are not defined. For example, for every number x , the expression $\frac{x}{0}$ is not defined; if $x < 0$, then \sqrt{x} is not defined; and 0^0 is not defined.
- Sometimes special symbols or notation are introduced in a question. Here are two examples:

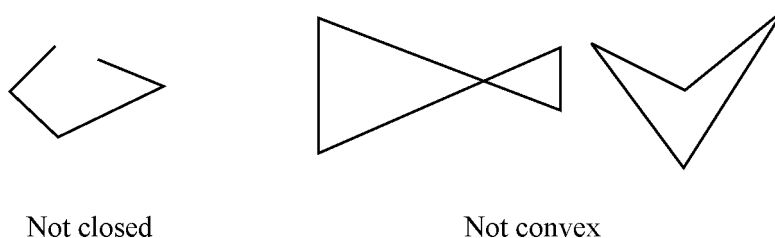
$$\text{The operation } \diamond \text{ is defined for all integers } r \text{ and } s \text{ by } r \diamond s = \frac{rs}{1+r^2}.$$

$$\text{The operation } \sim \text{ is defined for all nonzero numbers } x \text{ by } \sim x = -\frac{1}{x}.$$

- Sometimes juxtaposition of letters does *not* denote multiplication, as in “consider a three-digit integer denoted by XYZ , where X , Y , and Z are digits.” The meaning is taken from the context.
- Standard function notation is used in the test. For example, “the function g is defined for all $x \geq 0$ by $g(x) = 2x + \sqrt{x}$.” If the domain of a function f is not given explicitly, it is assumed to be the set of all real numbers x for which $f(x)$ is a real number. If f and g are two functions, then the *composition* of g with f is denoted by $g(f(x))$.

Geometry

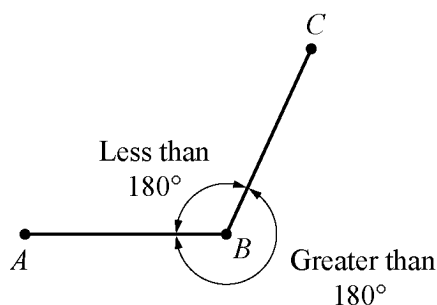
- In questions involving geometry, the conventions of plane (or Euclidean) geometry are followed, including the assumption that the sum of the measures of the interior angles of a triangle is 180 degrees.
- Lines are assumed to be “straight” lines that extend in both directions without end.
- Angle measures are in degrees and are assumed to be positive and less than or equal to 360 degrees.
- When a square, circle, polygon, or other closed geometric figure is described in words but not shown, the figure is assumed to enclose a convex region. It is also assumed that such a closed geometric figure is not just a single point or a line segment. For example, a quadrilateral **cannot** be any of the following:



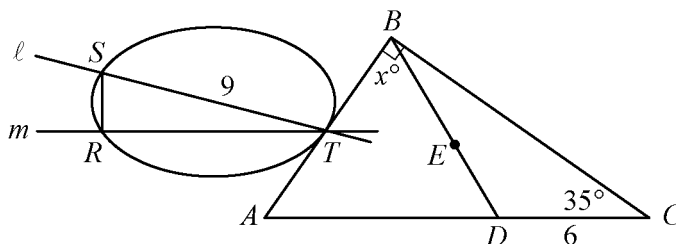
- The phrase *area of a rectangle* means the area of the region enclosed by the rectangle. The same terminology applies to circles, triangles, and other closed figures.
- The *distance between a point and a line* is the length of the perpendicular line segment from the point to the line, which is the shortest distance between the point and the line. Similarly, the *distance between two parallel lines* is the distance between a point on one line and the other line.
- In a geometric context, the phrase *similar triangles* (or other figures) means that the figures have the same shape. See the Geometry section of the Math Review for further explanation of the terms *similar* and *congruent*.

Geometric figures

- Geometric figures consist of points, lines, line segments, curves (such as circles), angles, and regions; also included are labels, and markings or shadings that identify these objects or their sizes. A point is indicated by a dot, a label or the intersection of two or more lines or curves. All figures are assumed to lie in a plane unless otherwise indicated.
- If points A , B , and C do not lie on the same line, then line segments AB and BC form two angles with vertex B —one angle with measure less than 180° and the other with measure greater than 180° , as shown below. Unless otherwise indicated, angle ABC , also denoted by angle B , refers to the *smaller* of the two angles.



- The notation AB may mean the line segment with endpoints A and B , or it may mean the length of the line segment. The meaning can be determined from the usage.
- Geometric figures **are not necessarily** drawn to scale. That is, you should **not** assume that quantities such as lengths and angle measures are as they appear in a figure. However, you should assume that lines shown as straight are actually straight, and when curves are shown, you should assume they are not straight. Also, assume that points on a line or a curve are in the order shown, points shown to be on opposite sides of a line or curve are so oriented, and more generally, assume all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities by sight or by measurement.
- To illustrate some of these conventions, consider the geometric figure below.



The following can be determined from the figure.

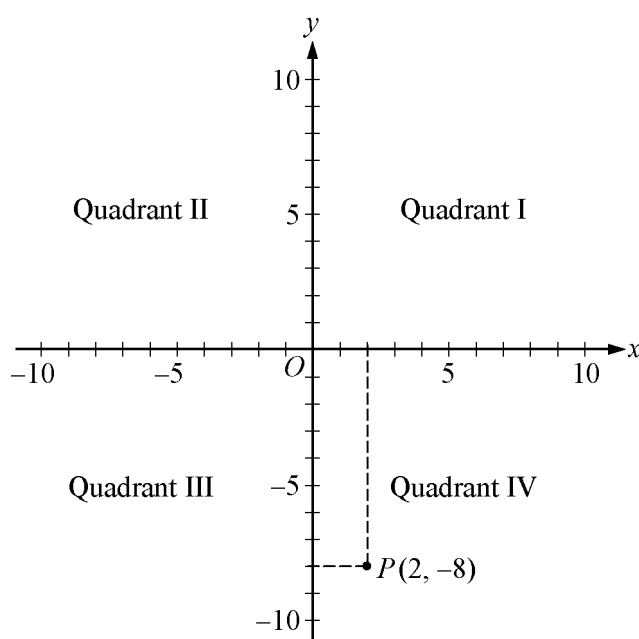
- ABD and DBC are triangles, and points R , S , and T lie on the closed curve.
- Points A , D , and C lie on a straight line, so ABC is a triangle with sides AB , BC , and AC .
- Point D is a distinct point between points A and C .
- Points A and S are on opposite sides of line m .
- Point E is on BD .
- $AD < AC$
- $ST = 9$, $DC = 6$, and the measure of angle C is 35 degrees.
- Angle ABC is a right angle, as indicated by the small square symbol at point B .
- The measure of angle ABD is x degrees, and $x < 90$.
- Line ℓ intersects the closed curve at points S and T , and the curve is tangent to AB at T .
- The area of the region enclosed by the curve is greater than the area of triangle RST .

The following **cannot** be determined from the figure.

- $AD > DC$
- The measures of angles BAD and BDA are equal.
- The measure of angle DBC is less than x degrees.
- The area of triangle ABD is greater than the area of triangle DBC .
- Angle SRT is a right angle.
- Line m is parallel to line AC .

Coordinate systems

- Coordinate systems, such as xy -planes and number lines, **are** drawn to scale. Therefore, you can read, estimate, or compare quantities in such figures by sight or by measurement, including geometric figures that appear in coordinate systems.
- The positive direction of a number line is to the right.
- As in geometry, distances in a coordinate system are nonnegative.
- The rectangular coordinate plane, or rectangular coordinate system, commonly known as the xy -plane, is shown below. The x -axis and y -axis intersect at the origin O , and they partition the plane into four quadrants. Each point in the xy -plane has coordinates (x, y) that give its location with respect to the axes; for example, the point $P(2, -8)$ is located 2 units to the right of the y -axis and 8 units below the x -axis. The units on the x -axis have the same length as the units on the y -axis, unless otherwise noted.



- Intermediate grid lines or tick marks in a coordinate system are evenly spaced unless otherwise noted.
- The term x -intercept refers to the x -coordinate of the point at which a graph in the xy -plane intersects the x -axis; it does not refer to the point itself. The term y -intercept is used analogously.

Sets, lists, and sequences

- Sets of numbers or other elements appear in some questions. Some sets are infinite, such as the set of integers; other sets are finite and may have all of their elements listed within curly brackets, such as the set $\{2, 4, 6, 8\}$. When the elements of a set are given, repetitions are *not* counted as additional elements and the order of the elements is *not* relevant. Elements are also called *members*. A set with one or more members is called *nonempty*; there is a set with no members, called the *empty set* and denoted by \emptyset . If A and B are sets, then the *intersection* of A and B , denoted by $A \cap B$, is the set of elements that are in both A and B , and the *union* of A and B , denoted by $A \cup B$, is the set of elements that are in either A or B or both. If all of the elements in A are also in B , then A is a *subset* of B . By convention, the empty set is a subset of every set. If A and B have no elements in common, they are called *disjoint* sets or *mutually exclusive* sets.

- Lists of numbers or other elements are also used in the test. When the elements of a list are given, repetitions *are* counted as additional elements and the order of the elements *is* relevant. For example, the list 3, 1, 2, 3, 3 contains five numbers, and the first, fourth, and last numbers in the list are each 3.
- The terms *data set* and *set of data* are not sets in the mathematical sense given above. Rather they refer to a list of data because there may be repetitions in the data, and if there are repetitions, they would be relevant.
- Sequences are lists that often have an infinite number of elements, or terms. The terms of a sequence are often represented by a fixed letter along with a subscript that indicates the order of a term in the sequence. For example, $a_1, a_2, a_3, \dots, a_n, \dots$ represents an infinite sequence in which the first term is a_1 , the second term is a_2 , and more generally, the n th term is a_n for every positive integer n . Sometimes the n th term of a sequence is given by a formula, such as $b_n = 2^n + 1$. Sometimes the first few terms of a sequence are given explicitly, as in the following sequence of consecutive even negative integers: $-2, -4, -6, -8, -10, \dots$
- Sets of consecutive integers are sometimes described by indicating the first and last integer, as in “the integers from 0 to 9, inclusive.” This phrase refers to 10 integers, with or without “inclusive” at the end. Thus, the phrase “during the years from 1985 to 2005” refers to 21 years.

Data and statistics

- Numerical data are sometimes given in lists and sometimes displayed in other ways, such as in tables, bar graphs, or circle graphs. Various statistics, or measures of data, appear in questions: measures of central tendency—mean, median, and mode; measures of position—quartiles and percentiles; and measures of dispersion—standard deviation, range, and interquartile range.
- The term *average* is used in two ways, with and without the qualification “(arithmetic mean).” For a list of data, the *average (arithmetic mean)* of the data is the sum of the data divided by the number of data. The term *average* does not refer to either *median* or *mode* in the test. Without the qualification of “arithmetic mean,” *average* can refer to a rate or the ratio of one quantity to another, as in “average number of miles per hour” or “average weight per truckload.”
- When *mean* is used in the context of data, it means *arithmetic mean*.
- The *median* of an odd number of data is the middle number when the data are listed in increasing order; the *median* of an even number of data is the arithmetic mean of the two middle numbers when the data are listed in increasing order.
- For a list of data, the *mode* of the data is the most frequently occurring number in the list. Thus, there may be more than one mode for a list of data.
- For data listed in increasing order, the *first quartile*, *second quartile*, and *third quartile* of the data are three numbers that divide the data into four groups that are roughly equal in size. The first group of numbers is from the least number up to the first quartile. The second group is from the first quartile up to the second quartile, which is also the median of the data. The third group is from the second quartile up to the third quartile, and the fourth group is from the third quartile up to the greatest number. Note that the four groups themselves are sometimes referred to as quartiles—*first quartile*, *second quartile*, *third quartile*, and *fourth quartile*. The latter usage is clarified by the word “in” as in the phrase “the cow’s weight is *in* the third quartile of the herd.”
- For data listed in increasing order, the *percentiles* of the data are 99 numbers that divide the data into 100 groups that are roughly equal in size. The 25th percentile equals the first quartile; the 50th percentile equals the second quartile, or median; and the 75th percentile equals the third quartile.
- For a list of data, where the arithmetic mean is denoted by m , the *standard deviation* of the data refers to the nonnegative square root of the mean of the squared differences between m and each of

the data. This statistic is also known as the *population standard deviation* and is not to be confused with the *sample standard deviation*.

- For a list of data, the *range* of the data is the greatest number in the list minus the least number. The *interquartile range* of the data is the third quartile minus the first quartile.

Data distributions and probability distributions

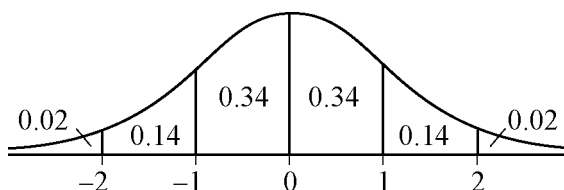
- Some questions display data in *frequency distributions*, where discrete data values are repeated with various frequencies, or where preestablished intervals of possible values are assigned frequencies corresponding to the numbers of data in the intervals. For example, the lifetimes, rounded to the nearest hour, of 300 lightbulbs could be in the following 10 intervals: 501–550 hours, 551–600 hours, 601–650 hours, . . . , 951–1,000 hours; consequently, each of the intervals would have a number, or frequency, of lifetimes, and the sum of the 10 frequencies is 300.
- Questions may involve *relative frequency distributions*, where each frequency of a frequency distribution is divided by the total number of data in the distribution, resulting in a relative frequency. In the example above, the 10 frequencies of the 10 intervals would each be divided by 300, yielding 10 relative frequencies.
- Some questions describe *probability experiments*, or *random experiments*, that have a finite number of possible *outcomes*. In a random experiment, any particular set of outcomes is called an *event*, and every event E has a *probability*, denoted by $P(E)$, where $0 \leq P(E) \leq 1$. If each outcome of an experiment is equally likely, then the probability of an event E is defined as the following ratio:

$$P(E) = \frac{\text{The number of outcomes in the event } E}{\text{The number of possible outcomes in the experiment}}.$$

- If E and F are two events in an experiment, then “ E and F ” is an event, which is the set of outcomes that are in the intersection of events E and F . Another event is “ E or F ,” which is the set of outcomes that are in the union of events E and F .
- If E and F are two events and E and F are mutually exclusive, then $P(E \text{ and } F) = 0$.
- If E and F are two events such that the occurrence of either event does not affect the occurrence of the other, then E and F are said to be *independent* events. Events E and F are independent if and only if $P(E \text{ and } F) = P(E)P(F)$.
- A *random variable* is a variable that represents values resulting from a random experiment. The values of the random variable may be the actual outcomes of the experiment if the outcomes are numerical, or the random variable may be related to the outcomes more indirectly. In either case, random variables can be used to describe events in terms of numbers.
- A random variable from an experiment with only a finite number of possible outcomes also has only a finite number of values and is called a *discrete random variable*. When the values of a random variable form a continuous interval of real numbers, such as all of the numbers between 0 and 2, the random variable is called a *continuous random variable*.
- Every value of a discrete random variable X , say $X = a$, has a probability denoted by $P(a)$. A histogram (or a table) showing all of the values of X and their probabilities $P(X)$ is called the *probability distribution* of X . The *mean of the random variable* X is the sum of the products $X P(X)$ for all values of X .
- The mean of a random variable X is also called the *expected value* of X or the *mean of the probability distribution* of X .
- For a continuous random variable X , every interval of values, say $a \leq X \leq b$, has a probability, which is denoted by $P(a \leq X \leq b)$. The *probability distribution* of X is a curve in the XY -plane

that mimics the tops of the bars of a histogram, only smoother. The curve is the graph of a function f whose values are nonnegative and whose graph is therefore above the X -axis. The curve $y = f(X)$ is related to the probability of each interval $a \leq X \leq b$ in the following way: $P(a \leq X \leq b)$ is equal to the area of the region that is below the curve, above the X -axis, and between the vertical lines $X = a$ and $X = b$. The area of the entire region under the curve is 1.

- The *mean of a continuous random variable X* is the point m on the X -axis at which region under the distribution curve would perfectly balance if a fulcrum were placed at $X = m$. The *median of X* is the point M on the X -axis at which the line $X = M$ divides the region under the distribution curve into two regions of equal area.
- The *standard deviation of a random variable X* is a measure of dispersion, which indicates how spread out the probability distribution of X is from its mean. It is also called the *standard deviation of the probability distribution of X* .
- The most important probability distribution is the *normal distribution*, whose distribution curve is shaped like a bell. A random variable X with this distribution is called *normally distributed*. The curve is symmetric about the line $X = m$, where m is the mean as well as the median. The right and left tails of the distribution become ever closer to the X -axis but never touch it.
- The *standard normal distribution* has mean 0 and standard deviation 1. The following figure shows the distribution, including approximate probabilities corresponding to the six intervals shown.



Graphical representations of data

- Graphical data presentations, such as bar graphs, circles graphs, and line graphs, **are** drawn to scale; therefore, you can read, estimate, or compare data values by sight or by measurement.
- Standard conventions apply to graphs of data unless otherwise indicated. For example, a circle graph represents 100 percent of the data indicated in the graph's title, and the areas of the individual sectors are proportional to the percents they represent. Scales, grid lines, dots, bars, shadings, solid and dashed lines, legends, etc., are used on graphs to indicate the data. Sometimes, scales that do not begin at 0 are used, and sometimes broken scales are used.
- In Venn diagrams, various sets of objects are represented by circular regions and by regions formed by intersections of the circles. In some Venn diagrams, all of the circles are inside a rectangular region that represents a universal set. A number placed in a region is the number of elements in the subset represented by the smallest region containing the number, unless otherwise noted. Sometimes a number is placed above a circle to indicate the number of elements in the entire circle.

Miscellaneous Guidelines for Interpreting and Using Information in Test Questions

- Numbers given in a question are to be used as exact numbers, even though in some real-life settings they are likely to have been rounded. For example, if a question states that "30 percent of the company's profit was from health products," then 30 is to be used as an exact number; it is not to be used as a rounded number obtained from, say, 29 or 30.1.
- An integer that is given as the number of certain objects, whether in a real-life or pure-math setting, is to be taken as the total number of such objects. For example, if a question states that "a bag contains 50 marbles, and 23 of the marbles are red," then 50 is to be taken as the total number of marbles in the bag and 23 is to be taken as the total number of red marbles in the bag, so that the

other 27 marbles are not red. Fractions and percents are understood in a similar way, so “one-fifth, or 20 percent, of the 50 marbles in the bag are green” means 10 marbles in the bag are green and 40 marbles are not green.

- When a multiple-choice question asks for an approximate quantity without stipulating a degree of approximation, the correct answer is the choice that is closest in value to the quantity that can be computed from the information given.
- Unless otherwise indicated, the phrase “difference between two quantities” is assumed to mean “positive difference,” that is, the greater quantity minus the lesser quantity. For example, “for which two consecutive years was the difference in annual rainfall least?” means “for which two consecutive years was the *absolute value of the difference* in annual rainfall least?”
- When the term *profit* is used in a question, it refers to *gross profit*, which is the sales revenue minus the cost of production. The profit does not involve any other amounts unless they are explicitly given.
- The common meaning of terms such as *months* and *years* and other everyday terms are assumed in questions where the terms appear.
- In questions involving real-life scenarios in which a variable is given to represent a number of existing objects or another nonnegative amount, the context implies that the variable is greater than 0. For example, “Jane sold x rugs and deposited her profit of y dollars into her savings account” implies that x and y are greater than 0.
- Some quantities may involve units, such as inches, pounds, and Celsius degrees, while other quantities are pure numbers. Any units of measurement, such as English units or metric units, may be used. However, if an answer to a question requires converting one unit of measurement to another, then the relationship between the units is given in the question, unless the relationship is a common one, such as the relationships between minutes and hours, dollars and cents, and metric units like centimeters and meters.
- In any question, there may be some information that is not needed for obtaining the correct answer.
- When reading questions, do not introduce unwarranted assumptions. For example, if a question describes a trip that begins and ends at certain times, the intended answer will assume that the times are unaffected by crossing time zones or by changes to the local time for daylight savings, unless those matters are explicitly mentioned. As another example, do not consider sales taxes on purchases unless explicitly mentioned.
- The display of data in a Data Interpretation set of questions is the same for each question in the set. Also, the display may contain more than one graph or table. Each question will refer to some of the data, but it may happen that some part of the data will have no question that refers to it.
- In a Data Interpretation set of questions, each question should be considered separately from the others. No information except what is given in the display of data should be carried over from one question to another.
- In many questions, mathematical expressions and words appear together in a phrase. In such a phrase, each mathematical expression should be interpreted *separately* from the words before it is interpreted *along with* the words. For example, if n is an integer, then the phrase “the sum of the first two consecutive integers greater than $n + 6$ ” means $(n + 7) + (n + 8)$; it does not mean “the sum of the first two consecutive integers greater than n ” plus 6, or $(n + 1) + (n + 2) + 6$. That is, the expression $n + 6$ should be interpreted first, separately from the words. However, in a phrase like “the function g is defined for all $x \geq 0$,” the phrase “for all $x \geq 0$ ” is a mathematical shorthand for “for all numbers x such that $x \geq 0$.”