Exponents and Roots

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
25^7	5^{15}

2.

$$216 = 2^x 3^y$$

x and *y* are integers.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
x	y

3.

<u>**Quantity A**</u>	<u>**Quantity B**</u>

$\sqrt{18}\sqrt{2}$	$\sqrt{6}$

4.

Q uantity A	**Q uantity B**
$\sqrt{3}+\sqrt{6}$	$\sqrt{9}$

5.

Q uantity A	**Q uantity B**
$\sqrt{7,777,777,777}$	88,000

6.If 5,000 = $2^x 5^y$ and x and y are integers,w hat is $x + y$?

$\boxed{}$

7.If $3^2 9^2 = 3^x$,w hat is x?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

8.

80 is divisible by 2^x.

Q uantity A	**Q uantity B**
x	3

9.

Q uantity A	**Q uantity B**
$(81)^2(900)^3$	270^6

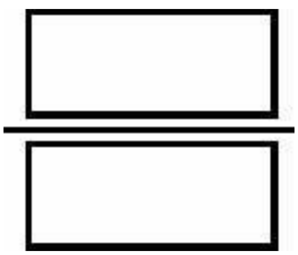
10.If $17\sqrt[3]{m} = 34$,w hat is $6\sqrt[3]{m}$?

$\boxed{}$

11. $\dfrac{1}{\dfrac{1}{\dfrac{1}{5^{-2}}}}$ is equivalent to:

(A) 1/25
(B) 1/5
(C) 1
(D) 5
(E) 25

12.If $77{,}742y^{11} = 4x^2$,w hat is $\dfrac{77{,}742y^{11}}{8x^2}$?



13. $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} =$

(A) $\sqrt{2}$
(B) 2
(C) $2\sqrt{2}$
(D) 4
(E) $4\sqrt{2}$

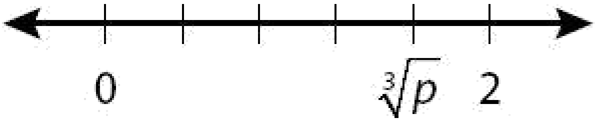
14.

Q uantity A	**Q uantity B**
$\dfrac{200}{\sqrt{200}}$	$\sqrt{200}$

15.For w hat positive integer is the square of the integer divided by the cube root of the integer equal to nine tim es the integer?

(A) 4
(B) 8
(C) 16
(D) 27
(E) 125

16.



If the hash m arks above are equally spaced,w hat is the value of *p*?

(A) 3/2 (B
) 8/5 (C)
24/15
(D) 512/125
(E) 625/256

17.W hat is the greatest prim e factor of $2^{99} - 2^{96}$?

18.If $2^k - 2^{k+1} + 2^{k-1} = 2^k m$ w hat is *m* ?

(A) -1
(B) -1/2
(C) 1/2
(D) 1
(E) 2

19.

Q uantity A	**Q uantity B**
$\frac{2}{9}(81)^{50}$	$\frac{(3^2)(9)^{99}}{2}$

20.If $5^{k+1} = 2{,}000$,w hat is $5^k + 1$?

(A) 399
(B) 401
(C) 1,996
(D) 2,000
(E) 2,001

21.If $3^{11} = 9^x$,w hat is the value of *x*?

22.If $x^7 = 2.5$,w hat is x^{14}?

23.If $\sqrt[5]{x^6} = x^{\frac{a}{b}}$,then the value of *a*/*b* =

24. $\dfrac{20^{-5}5^{10}8^{6}}{10^{8}25^{-2}} = ?$

(A) 1
(B) 4
(C) 5
(D) 6
(E) 10

25.If $\dfrac{5^7}{5^{-4}} = 5^a$ and $\dfrac{2^{-3}}{2^{-2}} = 2^b$ and $3^8(3) = 3^c$,w hat is the value of *a* + *b* + *c*?

26.If 12^x is odd and *x* is an integer,w hat is the value of x^{12}?

27. $\dfrac{200^{\frac{5}{2}}}{\sqrt{200}} = ?$

(A) 4 (B
) 40 (C)
400
(D) 4,000
(E) 40,000

28.

$$\frac{(10^3)(0.027)}{(900)(10^{-2})} = (3)(10^m)$$

Quantity A	Quantity B
The value of m	3

29. $\frac{1}{3}(10^6-10^4)=?$

(A) $33.\overline{3}$
(B) $3,333.\overline{3}$
(C) 33,000 (D) 330,000
(E) 333,333

30. Simplify: $\frac{2^2+2^2+2^3+2^4}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

(A) 2
(B) 4
(C) 8
(D) 16
(E) 32

31. $\frac{2^{-4}3^{-20}}{4^{-1}9^{-6}}=$

(A) $2^2 3^8$
(B) $2^1 3^{12}$
(C) $\frac{1}{2^2 3^8}$
(D) $\frac{1}{2^1 3^{12}}$
(E) $\frac{1}{2^2 3^{12}}$

32. If $\frac{0.000027\times10^x}{900\times10^{-4}}=0.03\times10^{11}$, what is the value of x?

(A) 13
(B) 14
(C) 15
(D) 16
(E) 17

33. $\left(\sqrt[2]{x}\right)\left(\sqrt[3]{x}\right)=$

(A) $\sqrt[5]{x}$

(B) $\sqrt[6]{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[5]{x^6}$

(E) $\sqrt[6]{x^5}$

34. $\left(\sqrt[3]{x^2}\right)\left(\sqrt[4]{x^5}\right)=$

(A) $\sqrt[7]{x^{10}}$

(B) $\sqrt[12]{x^{10}}$

(C) $\sqrt[12]{x^7}$

(D) $\sqrt[9]{x^{23}}$

(E) $\sqrt[12]{x^{23}}$

35.

$$n = 0.00025 \times 10^4 \text{ and } m = 0.005 \times 10^2$$

Q uantity A	**Q uantity B**
$\frac{n}{m}$	0.5

36. $\frac{40^{50} - 40^{48}}{2^{96}} \times 10^{-45} =$

(A) 20

(B) $10^3(1,599)$

(C) $10^2(1,601)$

(D) 200^6

(E) 200^{53}

37.W hich of the follow ing is equal to $x^{\frac{3}{2}}$??

(A) $x^2\sqrt{x}$

(B) $x\sqrt{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[3]{x}$

(E) $(x^3)^2$

38. $\sqrt{(360)(240)(3)(2)} =$

(A) 180
(B) 360
(C) 720
(D) 1,440
(E) 3,600

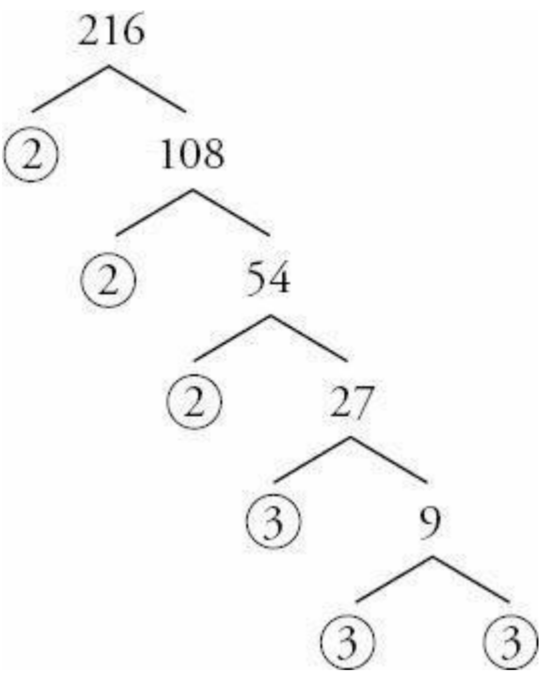
39. If $125^{14}48^{8}$ is w ritten out as an integer,how m any consecutive zeroes w ill that integer have at the end?

(A) 22
(B) 32
(C) 42
(D) 50
(E) 112

E xponents and R oots A nsw ers

1.**(B).**In problem s asking you to com pare or com bine exponents w ith different bases,convert to the sam e base w hen possible.Since 25 = 5^2,Q uantity A is equal to $(5^2)^7$.W hen raising a pow er to a pow er,m ultiply the exponents. Q uantity A is equal to 5^{14},so Q uantity B is larger.

2.**(C).**M ake a prim e tree for 216:



216 = $2^3 3^3$,so x = 3 and y = 3.

3.**(A).**In Q uantity A ,$\sqrt{18}\sqrt{2} = \sqrt{36} = 6$.Since 6 is greater than $\sqrt{6}$,Q uantity A is larger.

4.**(A).**Y ou m ay N O T add $\sqrt{3}$ and $\sqrt{6}$ to get $\sqrt{9}$,but you can sim ply put each value in your calculator.$\sqrt{3}$ = 1.732… and $\sqrt{6}$ = 2.449… ,and their sum is about 4.18.Since Q uantity B is $\sqrt{9}$ = 3,Q uantity A is larger.

5.**(A).**Y ou have a calculator w ith a square root button,but 7,777,777,777 is too large for the calculator.

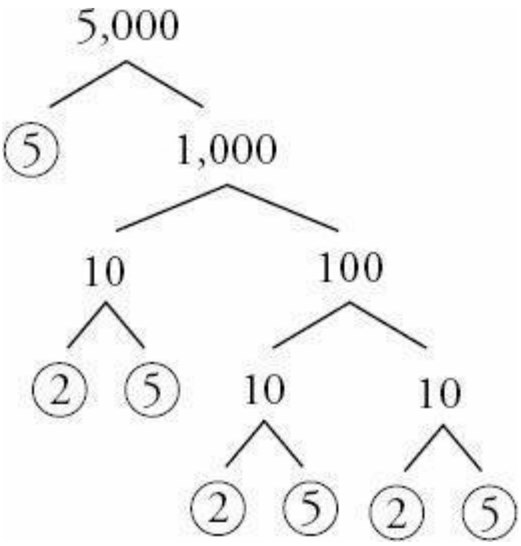
Y ou could also square both quantities,getting 7,777,777,777 in Q uantity A and $88{,}000^2$ in Q uantity B .H ow ever, $88{,}000^2$ is also too large for the calculator.H ere's a quick w orkaround.Square 88 in your calculator (there is no "squared" button on the G R E calculator — you have to type 88 × 88):

$88^2 = 7{,}744$

Therefore $88{,}000^2 = 7{,}744 \times 10^6 = 7{,}744{,}000{,}000$

Thus,Q uantity A is larger.

6.**7.**M ake a prim e tree for 5,000:



Y ou can see that 5,000 = $2^3 5^4$,therefore x = 3 and y = 4,and the answ er is 3 + 4 = 7.

7.**(E).**In problem s asking you to com pare or com bine exponents w ith different bases,convert to the sam e base w hen possible.Since 9 = 3^2:

$$3^2(3^2)^2 = 3^x$$

M ultiply exponents w hen raising a pow er to a pow er:

$$3^2 3^4 = 3^x$$

A dd exponents w hen m ultiplying w ith the sam e base:

$$3^6 = 3^x$$

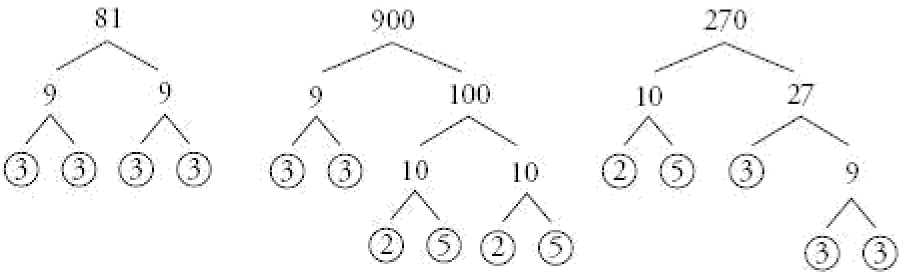
Therefore,x = 6.

8.**(D).**M ake a prim e tree for 80,or sim ply divide 80 by 2 in your calculator as m any tim es as you can before you get a non-integer answ er: 2 goes into 80 exactly 4 tim es.

That doesn't m ean x *is 4,how ever!* The problem did N O T say "80 is *equal to* 2^x".R ather,it said "divisible by."

80 is divisible by 2^4,and therefore also by 2^3,2^2,2^1,and even 2^0 (anything to the 0th pow er = 1).Thus,x could be 0, 1,2,3,or 4,and could therefore be sm aller than,equal to,or greater than 3.

9.**(B).**B reak dow n 81,900,and 270 into their prim e factors:



81 = 3^4,900 = $2^2 3^2 5^2$,and 270 = $2^1 3^3 5^1$.Therefore:

Quantity A = $(3^4)^2(2^2 3^2 5^2)^3 = (3^8)(2^6 3^6 5^6) = 2^6 3^{14} 5^6$

Quantity B = $(2^1 3^3 5^1)^6 = 2^6 3^{18} 5^6$

Since Q uantity A and Q uantity B both have 2^6 and 5^6,focus on 3^{14} vs.3^{18}.Q uantity B is larger.

10.**12.**This question looks m uch m ore com plicated than it really is — note that you are not asked for *m* itself,but rather for $\sqrt[3]{m}$.Just think of $\sqrt[3]{m}$ as a very fancy variable that you don't have to break dow n:

$$17\sqrt[3]{m} = 34$$

$$\sqrt[3]{m} = \frac{34}{17}$$

$$\sqrt[3]{m} = 2$$

Therefore,$6\sqrt[3]{m}$.

11.**(E).**O ne quick trick to sim plifying efficiently here is know ing that a negative exponent in the denom inator turns into a positive exponent in the num erator.In other w ords,the low erm ost portion of the fraction,$\frac{1}{5^{-2}}$,is sim ply equal to 5^2.

$$\frac{1}{\frac{1}{\frac{1}{5^{-2}}}} = \frac{1}{\frac{1}{5^2}}$$

D ividing by $\frac{1}{5^{-2}}$ is the sam e as m ultiplying by the reciprocal,w hich w ill leave 5^2 in the num erator:

$$\frac{1}{\frac{1}{5^2}} = 1 \times \frac{5^2}{1} = 5^2$$

The answer is simply 5^2, or 25.

12. **1/2.** This question looks much more complicated than it really is! Since $77{,}742y^{11} = 4x^2$, simply substitute $4x^2$ for $7{,}742y^{11}$ in the numerator:

$$\frac{4x^2}{8x^2} = \frac{1}{2}$$

13. **(B).** To begin solving, start at the "inner core" — that is, the physically smallest root sign:

$$\begin{aligned}
&\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} = \\
&\sqrt{2+\sqrt{2+\sqrt{2+2}}} = \\
&\sqrt{2+\sqrt{2+2}} = \\
&\sqrt{2+2} = 2
\end{aligned}$$

14. **(C).** Quantity A has a root sign on the bottom of a fraction. When you see this, rationalize the denominator by multiplying the fraction by the denominator over the denominator (so that the number you are multiplying by is equal to 1, so you are not changing the value of the fraction):

$$\frac{200}{\sqrt{200}}\left(\frac{\sqrt{200}}{\sqrt{200}}\right) = \frac{200\sqrt{200}}{200} = \sqrt{200}$$

(Please note that $\sqrt{200} \times \sqrt{200}$ is simply 200. There is no need to multiply out to get $\sqrt{40{,}000} = \sqrt{200} \times \sqrt{200} = 200$. Instead, simply think of the root signs canceling out.)
The two quantities are the same.

15. **(D).** To solve this question, you need to write the information from the question as an equation. Call "the square of the integer" x^2, "the cube root of the integer" $\sqrt[3]{x}$, and "nine times the integer" $9x$:

$$\frac{x^2}{\sqrt[3]{x}} = 9x$$

There are a few ways to proceed from here, but it might be most helpful to convert $\sqrt[3]{x}$ into its other form, $x^{\frac{1}{3}}$, and then subtract the exponents on the left side of the equation (alw ays subtract exponents,of course,w hen dividing w ith the sam e base):

$$\frac{x^2}{x^{\frac{1}{3}}} = 9x$$

$$x^{2-\frac{1}{3}} = 9x$$

$$x^{\frac{5}{3}} = 9x$$

A good next m ove w ould be to raise both sides to the 3rd pow er:

$$\left(x^{\frac{5}{3}}\right)^3 = (9x)^3$$
$$x^5 = 9^3x^3$$

N ow sim ply divide both sides by x^3:

$$x^2 = 9^3$$

Since 9 is really just 3^2:

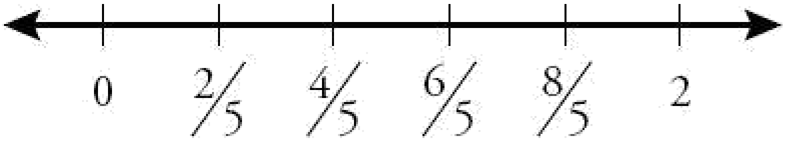
$$x^2 = (3^2)^3$$
$$x^2 = 3^6$$
$$\sqrt{x^2} = \sqrt{3^6}$$
$$x = 3^3$$
$$x = 27$$

A lternatively,you could try the answ ers.For instance,for choice (E):

$$\frac{27^2}{\sqrt[3]{27}} = 9(27)$$
$$\frac{27^2}{3} = 9(27)$$
$$27^2 = 9(27)(3)$$
$$27 = 9(3)$$

Thus,choice (E) is correct.W hile this backsolving approach w orks,it m ay be a bit slow er than the algebra approach.

16.**(D).**To determ ine the distance betw een hash m arks,divide 2 (the distance from 0 to 2) by 5 (the num ber of segm ents the num ber line has been divided into).The result is 2/5.Therefore:



N ote that 2 is equal to 10/5,so you can see that the num ber line is labeled correctly.

Since $\sqrt[3]{p}$ m arks the sam e hash m ark on the num ber line as 8/5:

$$\sqrt[3]{p} = \frac{8}{5}$$

$$p = \left(\frac{8}{5}\right)^3$$

$$p = \frac{512}{125}$$

The answ er is (D).W atch out for trap answ er choice (B),w hich represents $\sqrt[3]{p}$,not *p.*

17.**7**.Y ou cannot subtract $2^{99} - 2^{96}$ to get 2^3! Y ou cannot directly com bine,even w ith the sam e base,w hen adding or subtracting.(A s it turns out,the difference betw een 2^{99} and 2^{96} is m uch,m uch larger than 2^3.) Instead,factor out the largest num ber 2^{99} and 2^{96} have in com m on:

$$2^{99} - 2^{96} = 2^{96}(2^3 - 1) = 2^{96}(7)$$

Since $2^{99} - 2^{96}$ is equal to $2^{96}7^1$,its greatest prim e factor is 7.

18.**(B).**First,break dow n 2^{k+1} as 2^k2^1 and 2^{k-1} as 2^k2^{-1}:

$$2^k - 2^k2^1 + 2^k2^{-1} = 2^km$$

Factor out 2^k from the left,then cancel 2^k from both sides:

$$2^k(1 - 2^1 + 2^{-1}) = 2^k m$$

$$1 - 2^1 + 2^{-1} = m$$

$$1 - 2 + \frac{1}{2} = m$$

$$-\frac{1}{2} = m$$

19.**(B).**A good w ay to begin com paring these quantities is to look for sim ilarities — specifically,81^{50} and 9^{99} can each be broken dow n to pow ers of 3,as $81 = 3^4$ and $9 = 3^2$:

Quantity A : $\frac{2}{9}(3^4)^{50} = \frac{2}{9}(3^{200})$

Quantity B : $\frac{(3^2)(3^2)^{99}}{2} = \frac{(3^2)(3^{198})}{2} = \frac{3^{200}}{2}$ or $\frac{1}{2}(3^{200})$

Since 3^{200} is the sam e on both sides,ignore it (or elim inate it by dividing both quantities by 3^{200}).Since 1/2 is greater than 2/9,Q uantity B is larger.

20.**(B).**The key to solving this problem is realizing that you can split 5^{k+1} into 5^k5^1.(Exponents are added w hen m ultiplying w ith the sam e base,so the process can also be reversed;thus,any expression w ith the form x^{a+b} can be split into x^ax^b.)

$5^{k+1} = 2{,}000$

$5^k5^1 = 2{,}000$

N ow divide both sides by 5:

$5^k = 400$

So,$5^k + 1 = 401$.

N otice that you can't solve for *k* itself — *k* is not an integer,since 400 is not a "norm al" pow er of 5.B ut you don't need to solve for *k*.Y ou just need 5^k.

21.**5.5**.B egin by converting 9 to a pow er of 3:

$3^{11} = (3^2)^x$

$3^{11} = 3^{2x}$

Thus,$11 = 2x$ and $x = 5.5$.

22.**6.25**.It is not necessary to find *x* to solve this problem .Sim ply square both sides:

$x^7 = 2.5\ (x^7)^2$

$= (2.5)^2\ x^{14} =$

6.25

23.**6/5.**Just as a square root is the sam e as a 1/2 exponent,so too is a fifth root the sam e as a 1/5 exponent.Thus:

$$\sqrt[5]{x^6} = (x^6)^{\frac{1}{5}} = x^{\frac{6}{5}}$$

Since $x^{\frac{6}{5}} = x^{\frac{a}{b}}$, *a*/*b* = 6/5.

24.**(C).**Since $20^{-5} = \frac{1}{20^5}$ and $25^{-2} = \frac{1}{25^2}$, one quick shortcut is to convert any term w ith a negative exponent to one w ith a positive exponent by m oving it from the num erator to the denom inator or vice-versa:

$$\frac{20^{-5}5^{10}8^6}{10^8 25^{-2}} = \frac{5^{10}8^6 25^2}{20^5 10^8}$$

Then,convert the non-prim e term s to prim es:

$$\frac{5^{10}8^6 25^2}{20^5 10^8} = \frac{5^{10}(2^3)^6(5^2)^2}{(2^2 5^1)^5(2^1 5^1)^8} = \frac{5^{10}2^{18}5^4}{2^{10}5^5 2^8 5^8} = \frac{2^{18}5^{14}}{2^{18}5^{13}} = 5$$

25.**19.**To solve this problem ,you need to know that w hen dividing w ith the sam e base,you subtract the exponents, and w hen m ultiplying w ith the sam e base,you add the exponents.Thus:

$$\frac{5^7}{5^{-4}} = 5^{7-(-4)} = 5^{11}, \text{ so } a = 11.$$

$$\frac{2^{-3}}{2^{-2}} = 2^{-3-(-2)} = 2^{-1}, \text{ so } b = -1.$$

$$3^8(3) = 3^8(3^1) = 3^9, \text{ so } c = 9.$$

Therefore,*a* + *b* + *c* = 11 + (-1) = 9 = 19.

26.**0.**This is a bit of a trick question.12^x is odd? H ow strange! 12^1 is 12,12^2 is 144 … it soon becom es easy to see that every “norm al” pow er of 12 is going to be even.(A n even num ber such as 12 m ultiplied by itself any num ber of tim es w ill yield an even answ er.) These norm al pow ers are 12 raised to a positive integer.W hat about negative integer exponents? They are all fractions of this form :$\frac{1}{12^{\text{positive integer}}}$.

The only w ay for 12^x to be odd is for *x* to equal 0.A ny nonzero num ber to the 0^{th} pow er = 1.Since *x* = 0 and the question asks for x^{12},the answ er is 0.

27.**(E).**To solve this problem ,you need to know that a square root is the sam e as a 1/2 exponent:

$$\frac{200^{\frac{5}{2}}}{\sqrt{200}}=\frac{200^{\frac{5}{2}}}{200^{\frac{1}{2}}}=200^{\frac{5}{2}-\frac{1}{2}}=200^{\frac{4}{2}}=200^2=40,000$$

28.**(B).**Since $(10^3)(0.027)$ is sim ply 27 and $(900)(10^{-2})$ is sim ply 9:

$$\frac{27}{9}=(3)(10^m)$$

$$3=3(10^m)$$

$$1=10^m$$

Y ou m ight be a little confused at this point as to how 10^m can equal 1.H ow ever,you can still answ er the question correctly.If *m* w ere 3,as in Q uantity B ,10^m w ould equal 1,000.H ow ever,10^m actually equals 1.So *m* m ust be sm aller than 3.

A s it turns out,the only w ay 10^m can equal 1 is if *m* = 0.A ny nonzero num ber to the 0th pow er is equal to 1.

29.**(D).**Y ou C A N N O T sim ply subtract 10^6 = 10^4 to get $10^{\ 2}$.This is because you cannot do any operation directly to the exponents w hen subtracting w ith the sam e base.R ather,you m ust factor out the largest pow er of 10 each term has in com m on:

$$\frac{1}{3}\left[10^4(10^2-1)\right]=\frac{1}{3}\left[10^4(99)\right]=\frac{1}{3}[990,000]=330,000$$

30.**(D).**Y ou could factor 2^2 out of the top,but the num bers are sm all enough you m ight as w ell just say that the num erator is 4 + 4 + 8 + 16 = 32.

FO IL the denom inator:

$$(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$$
$$\sqrt{25}+\sqrt{3}\sqrt{5}-\sqrt{3}\sqrt{5}-\sqrt{9}$$
$$\sqrt{25}-\sqrt{9}$$

5 - 3 = 2

32/2 = 16 is your final answ er.

31.**(C).**Since $2^{-4}=\frac{1}{2^4},\ 3^{-20}=\frac{1}{3^{20}}$,etc.,one quick shortcut here is to note that $\frac{2^{-4}3^{-20}}{4^{-1}9^{-6}}=\frac{4^1 9^6}{2^4 3^{20}}$,and solve from there:

$$\frac{4^1 9^6}{2^4 3^{20}} = \frac{(2^2)^1 (3^2)^6}{2^4 3^{20}} = \frac{2^2 3^{12}}{2^4 3^{20}} = \frac{1}{2^2 3^8}$$

32.**(A).**O ne good approach is to convert 0.000027,900,and 0.03 to pow ers of 10:

$$\frac{27 \times 10^{-6} \times 10^x}{9 \times 10^2 \times 10^{-4}} = 3 \times 10^{-2} \times 10^{11}$$

N ow com bine the exponents on the term s w ith base 10:

$$\frac{27 \times 10^{-6+x}}{9 \times 10^{-2}} = 3 \times 10^9$$

Since 27/9 = 3,cancel the 3 from both sides,then com bine pow ers of 10:

$$\frac{10^{-6+x}}{10^{-2}} = 10^9$$

$10^{-6+x-(-2)} = 10^9$

$10^{-4+x} = 10^9$

Y ou can now see that -4 + x = 9,so x = 13.

33.**(E).**A good first step is to convert to fractional exponents.Since a square root is the sam e as a 1/2 exponent and a cube root is the sam e as a 1/3 exponent:

$$x^{\frac{1}{2}} x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\left(\frac{3}{6}+\frac{2}{6}\right)} = x^{\frac{5}{6}} = \sqrt[6]{x^5}$$

34.**(E).**A good first step is to convert to fractional exponents.Since a cube root is the sam e as a 1/3 exponent and a 4th root is the sam e as a 1/4 exponent:

$$(x^2)^{\frac{1}{3}} (x^5)^{\frac{1}{4}} = \left(x^{\frac{2}{3}}\right)\left(x^{\frac{5}{4}}\right) = x^{\left(\frac{2}{3}+\frac{5}{4}\right)} = x^{\left(\frac{8}{12}+\frac{15}{12}\right)} = x^{\frac{23}{12}} = \sqrt[12]{x^{23}}$$

35.**(A).**To sim plify 0.00025 × 10^4,sim ply m ove the decim al in 0.00025 four places to the right to get 2.5.To sim plify 0.005 × 10^2,m ove the decim al in 0.005 tw o places to the right to get 0.5.Thus,n = 2.5,m = 0.5,and n/m =2.5/0.5 = 5.

36.**(B).**Since you cannot directly com bine exponential term s w ith the sam e base w hen you are adding and subtracting, you w ill need to factor out the top of the fraction:

$$\frac{40^{50} - 40^{48}}{2^{96}} \times 10^{-45}$$

$$\frac{40^{48}(40^2 - 1)}{2^{96}} \times 10^{-45}$$

$$\frac{40^{48}(1599)}{2^{96}} \times 10^{-45}$$

N ow ,your goal should be to elim inate the fraction entirely by getting the denom inator to cancel out.O ne good w ay to do this is to break up the 40,so as to isolate som e 2's that w ill allow 2^{96} to be canceled out.(N ote that it w ould also be possible to break 40 into 8 and 5,but in this particular problem ,it seem s w ise to leave the 10 intact so it can ultim ately com bine w ith the 10^{-45}).

$$\frac{(4 \times 10)^{48}(1599)}{2^{96}} \times 10^{-45}$$

$$\frac{4^{48}10^{48}(1599)}{2^{96}} \times 10^{-45}$$

$$\frac{(2^2)^{48}10^{48}(1599)}{2^{96}} \times 10^{-45}$$

$$\frac{2^{96}10^{48}(1599)}{2^{96}} \times 10^{-45}$$

2^{96} cancels! C om bine 10^{48} and 10^{-45} for the final answ er.

$10^{48}(1,599) \times 10^{-45}$ $10^{3}(1,599)$

37.**(B)**.Since a one-half exponent is the sam e as a square root,$x^{\frac{3}{2}}$ could also be w ritten as $\sqrt{x^3}$.This,how ever,does not appear in the choices.N ote,how ever,that $\sqrt{x^3}$ can be sim plified a bit:

$$\sqrt{x^2 \times x}$$

$$\sqrt{x^2} \times \sqrt{x}$$

$$x\sqrt{x}$$

This m atches choice (B).A lternatively,convert the answ er choices.For instance,in incorrect choice (A), $x^2\sqrt{x} = x^2x^{\frac{1}{2}} = x^{\frac{5}{2}}$.Since this is not equal to $x^{\frac{3}{2}}$,elim inate (A).C orrect choice (B) can be converted as such:

$x\sqrt{x} = x^1 x^{\frac{1}{2}} = x^{\frac{3}{2}}$.

38.**(C).**The G R E calculator has a square root button,but it w on’t w ork on num bers as large as 360 × 240 × 3 × 2. Thus,you w ant to break this num ber dow n enough to pull out som e perfect squares.Y ou can already see that 360 is just 36 × 10.N ow break it dow n a bit m ore:

$$\sqrt{(36)(10)(24)(10)(3)(2)}$$

N ote that the tw o 10’s inside can m ake 100,w hich is a perfect square:

$$\sqrt{(36)(100)(24)(3)(2)}$$

M ultiply (24)(3)(2) to see if you get a perfect square.Y ou do! It’s 144:

$$\sqrt{(36)(100)(144)}$$

Since the operation inside the root sign is m ultiplication,it is allow able to break up the root sign into three separate root signs,as such:

$$\sqrt{36}\sqrt{100}\sqrt{144}$$

The answ er is 6 × 10 × 12 = 720.

39.**(B).**Exponents questions are usually about prim es,because you alw ays w ant to create com m on bases,and the easiest com m on bases are prim es.In order to answ er this question,you have to understand w hat creates zeroes at the end of a num ber.

10 = 5 × 2
40 = 8 × 5 × 2 100 = 10 ×
10 = 2 × 5 × 2 × 5
1,000 = 10 × 10 × 10 = 2 × 5 × 2 × 5 × 2 × 5

W hat you’ll notice is that zeroes are created by 10’s,each of w hich is created by one 2 and one 5.So to answ er this question,you sim ply need to w ork out how m any pairs of 2’s and 5’s are in the expression:

$$125^{14}48^{8} = (5^{3})^{14} \times (2^{4} \times 3)^{8} = 5^{42} \times 2^{32} \times 3^{8}$$

Even though there are 42 pow ers of 5,there are only 32 pow ers of 2,so you can only m ake 32 pairs of one 5 and one 2.The answ er is (B).