GMAT Extreme Challenge (Quant)

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Contents

1 Introduction

This document contains nothing but the most difficult GMAT questions–100 of them, to be exact. Some content areas pop up quite a bit on these questions, including favorites such as combinations, permutations, probability, symbolism, sequences, and functions. There are a good number of Algebra concepts such as exponents, square roots, inequalities, and absolute value here, as well.

I've said more about the difficulty level of these questions, and what that means to you, in the next section.

For further, more specific practice, I have produced several other resources that may help you. I've created seven different "Challenge" sets; none of them are as consistently difficult as this set, but each one has a number of very challenging questions, and is packed with realistic questions in a certain content area. So, if you find yourself struggling with just one area, that is a logical next step in your preparation for the GMAT.

Also, The GMAT Math Bible has several chapters (along with focused practice) on the content covered here, including chapters on combinations, permutations, probability, sequences, functions, and more.

As far as strategy is concerned, there are dozens of articles at GMAT HACKS to help you with your strategic approach to Arithmetic questions. Most importantly, you should make sure you understand every practice problem you do. It doesn't matter if you get it right the first time–what matters is whether you'll get it right the next time you see it, because the next time you see it could be on the GMAT.

With that in mind, carefully analyze the explanations. Redo questions that took you too long the first time around. Review questions over multiple sessions, rather than cramming for eight hours straight each Saturday. These basic study skills may not feel like the key to GMAT preparation, but they are the difference between those people who reach their score goals and those who never do.

Enough talking; there are 100 extremely challenging GMAT questions waiting inside. Get to work!

2 Difficulty Level

In all of my other problem sets, this chapter has a list of question numbers, categorized by difficulty level. That's not necessary here, since virtually every problem in the entire set is among the hardest you are likely to see on the test.

It's important to recognize what that means. First of all, if you aren't already scoring at or above the 80th percentile on the Quantitative section, this set isn't likely to do you much good. You can't sit down at a piano and play a difficult piece until you've incrementally worked your way up to that level, and the same thing applies on the GMAT.

Second, realize that not every question will necessarily seem "extremely difficult" to you. In content areas such as probability, combinations, and permutations, just about every question is tough, in the sense that most test-takers get them wrong. However, if you are comfortable with the subject area, they may not seem difficult to you. Just as it will be on the test, you won't find these 100 questions to be of equal difficulty to you–some will be very hard, while others will be relatively easy.

Third, you may have seen questions from other sources that seem harder than these. It's possible that this set doesn't cover every single difficult question type, but for the most part, it is representative of the hardest questions you are likely to see on the GMAT. Other test-prep providers like to devise extremely long-winded, complicated problems to convince you to use more of their time or resources. While those questions might superficially cover the same topics as the GMAT does, they aren't realistic questions, and they aren't worth your time.

Finally, a word of caution. If you find that you are struggling with two-thirds or more of these questions, you should probably step back to less difficult questions. Doing these problems when you aren't ready for them is not a good use of your study time. Forcing yourself to memorize the process on 100 different 750-level questions will only get you a 750 if you've done the same thing for 700-level questions, 650-level questions, and so on. Many students try to skip the intermediate steps, and they end up with a test-day disaster. Don't let that happen to you!

3 Problem Solving

Note: this guide contains both an answer key (so you can quickly check your answers) and full explanations.

1. A firm is divided into four departments, each of which contains four people. If a project is to be assigned to a team of three people, none of which can be from the same department, what is the greatest number of distinct teams to which the project could be assigned?

   (A) 4^3
   (B) 4^4
   (C) 4^5
   (D) $6(4^4)$
   (E) $4(3^6)$

2. For a finite sequence of nonzero integers, the consecutiveness of the sequences is defined by the number of pairs of consecutive terms of the sequence for which the positive difference between the two consecutive terms is 1. What is the consecutiveness for the sequence $\{1, 4, 5, 6, -3, -4\}$?

   (A) One
   (B) Two
   (C) Three
   (D) Four
   (E) Five

3. A shipment consists of 1,800 parts, some of which are defective. If a part is chosen from the shipment at random, the probability of it being defective is $\frac{1}{9}$ the probability that it is not defective. How many of the parts in the shipment are defective?

   (A) 90
   (B) 120
   (C) 180
   (D) 200
   (E) 900

4. All of the citizens of a certain country have a five-character or six-character national identification code that is created using the 26 letters of the alphabet and the 10 digits from 0 to 9. Which of the following gives the maximum number of citizens who can be designated with these codes?

(A) $2(36^6)$
(B) $36(36^5)$
(C) $37(36^5)$
(D) $36(36^6)$
(E) $37(36^6)$

5. What is the greatest possible area of a triangular region with one side that corresponds with the diameter of a circle with radius 6, and the other vertex of the triangle on the circle?

(A) 24
(B) 36
(C) 40
(D) 48
(E) 72

6. When J is divided by P, the quotient is Q and the remainder is M. Which of the following expressions is equal to M ?

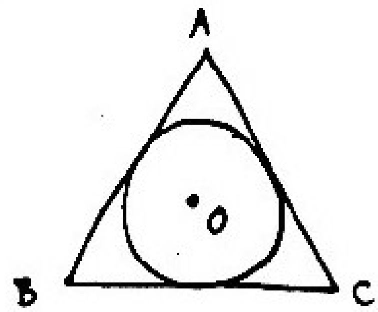
(A) $Q - J$
(B) $J - Q$
(C) $Q(J - P)$
(D) $Q - JP$
(E) $J - QP$

7. If $(2^{15})(25^8) = 5(10^m)$, what is the value of m?

(A) 7
(B) 8
(C) 15
(D) 16
(E) 23

8. Sequence S is defined as follows: $S_1 = 2$, $S_2 = 2^1$, $S_3 = 2^2$, ... , $S_n = 2^{n-1}$. What is the sum of the terms in sequence S when $n = 10$?

(A) 2^9
(B) 2^{10}
(C) 2^{16}
(D) 2^{35}
(E) 2^{37}



9. In the figure above, circle O is inscribed in equilateral triangle ABC. If the area of ABC is $24\sqrt{3}$, what is the area of circle O?

(A) $2\pi\sqrt{3}$
(B) 4π
(C) $4\pi\sqrt{3}$
(D) 8π
(E) 12π

10. If x is to be chosen at random from the set $\{1, 2, 3\}$, y is to be chosen at random from the set $\{4, 5, 6\}$, and z is to be chosen at random from the set $\{7, 8, 9, 10\}$, what is the probability that xyz will be even?

(A) $\frac{1}{9}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) $\frac{7}{9}$
(E) $\frac{8}{9}$

11. A research team is to consist of 3 scientists from Company A and 3 scientists from Company B. If the pool of available scientists from Company A is 5 and the pool of available scientists from Company B is 4 scientists, how many different research teams are possible?

(A) 84
(B) 40
(C) 30
(D) 20
(E) 12

12. For any integer n greater than 1, $n*$ denotes the product of all the integers from 1 to n, inclusive. How many multiples of 4 are there between $4*$ and $5*$, inclusive?

(A) 5
(B) 6
(C) 20
(D) 24
(E) 25

13. In how many arrangements can a teacher seat 3 girls and 3 boys in a row of 7 seats if the boys must occupy the first, fourth, and seventh seats, and one of the other seats must remain empty?

(A) 12
(B) 36
(C) 144
(D) 288
(E) 5,040

14. If a number between 0 and $\frac{1}{2}$ is selected at random, which of the following will the number least likely be between?

(A) $\frac{1}{3}$ and $\frac{1}{2}$
(B) $\frac{1}{6}$ and $\frac{1}{3}$
(C) $\frac{1}{9}$ and $\frac{1}{6}$
(D) $\frac{1}{12}$ and $\frac{1}{9}$
(E) 0 and $\frac{1}{12}$

15. A certain fish tank holds a total of 42 fish among four compartments. If the numbers of fish in the compartments are consecutive, what is the probability that a given fish is in the compartment that holds the most fish?

(A) $\frac{11}{42}$
(B) $\frac{5}{21}$
(C) $\frac{1}{4}$
(D) $\frac{3}{14}$
(E) $\frac{2}{7}$

16. A circular racetrack is 3 miles in length and has signs posted to indicate each $\frac{1}{10}$ mile increment. If a race car starts at a random location on the track and travels exactly one half mile, what is the probability that the car ends within a half mile of the sign indicating $2\frac{1}{2}$ miles?

(A) $\frac{1}{6}$
(B) $\frac{3}{10}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$
(E) $\frac{2}{3}$

17. If two fair six-sided dice are thrown, what is the probability that the sum of the numbers showing on the dice is 6 ?

(A) $\frac{1}{36}$
(B) $\frac{1}{12}$
(C) $\frac{5}{36}$
(D) $\frac{1}{6}$
(E) 1

18. When p is prime, $\widetilde{p} = p^2$. When p is not prime, $\widetilde{p}$ is equal to the largest prime factor of p. Which of the following is equal to $\widetilde{6} \times \widetilde{12}$?

(A) $\widetilde{2}$
(B) $\widetilde{3}$
(C) $\widetilde{6}$
(D) $\widetilde{9}$
(E) $\widetilde{72}$

19. In a certain lottery, the probability that a number between 12 and 20, inclusive, is drawn is $\frac{1}{6}$. If the probability that a number 12 or larger is drawn is $\frac{2}{3}$, what is the probability that a number less than or equal to 20 is drawn?

(A) $\frac{1}{18}$
(B) $\frac{1}{6}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$
(E) $\frac{5}{6}$

20. For every integer n from 1 to 9, inclusive, the nth term of a certain sequence is given by $(-1)^{n+1}(\frac{3}{2^n})$. If V is the sum of the first 9 terms in the sequence, then V is

(A) less than $\frac{1}{4}$
(B) between $\frac{1}{4}$ and $\frac{3}{4}$
(C) between $\frac{3}{4}$ and $\frac{5}{4}$
(D) between $\frac{5}{4}$ and $\frac{9}{4}$
(E) greater than $\frac{9}{4}$

21. A certain team has 12 members, including Joey. A three-member relay team will be selected as follows: one of the 12 members is to be chosen at random to run first, one of the remaining 11 members is to be chosen at random to run second, and one of the remaining 10 members is to be chosen at random to run third. What is the probability that Joey will be chosen to run second or third?

(A) $\frac{1}{1,320}$
(B) $\frac{1}{132}$
(C) $\frac{1}{110}$
(D) $\frac{1}{12}$
(E) $\frac{1}{6}$

22. Among a class of 60 people, 20 percent received an 'A' on their mid-term exam, 30 percent received an 'A' on their final exam, and 5 percent received an 'A' on both their mid-term exam and their final exam. If 1 person is to be randomly selected from the 60 people, what is the probability that the person selected will be one who received an 'A' on the mid-term exam but NOT on the final exam?

(A) $\frac{1}{20}$
(B) $\frac{3}{20}$
(C) $\frac{1}{4}$
(D) $\frac{1}{5}$
(E) $\frac{1}{6}$

23. Of the three-digit integers greater than 800, how many have two digits that are equal to each other and the remaining digit different from the other two?

(A) 36
(B) 51
(C) 53
(D) 54
(E) 60

24. A certain freshman class has 600 students and a certain sophomore class has 900 students. Among these students, there are 75 two-person sets of lab partners, each consisting of 1 freshman and 1 sophomore. If 1 student is to be selected at random from each class, what is the probability that the 2 students selected will be a set of lab partners?

(A) $\frac{1}{75}$
(B) $\frac{1}{96}$
(C) $\frac{1}{5{,}400}$
(D) $\frac{1}{7{,}200}$
(E) $\frac{1}{54{,}000}$

+	a	b	c
x	m	n	p
y	q	r	s
z	t	v	w

25. In the addition table above, each number in the table is the sum of the terms at the top of its column and the left of its row. If $x + c = 4$, $v - b = -2$, $a + z = 1$, knowing the value of which of the following variables would be sufficient to find the value of x ?

(A) n
(B) q
(C) s
(D) v
(E) w

26. A bag consists of 56 marbles, each of which is either green, white, or red. If the probability of selecting a green marble is twice the probability of selecting a red marble, and the probability of selecting a white marble is twice the probability of selecting a green marble, how many red marbles are in the bag?

(A) 7
(B) 8
(C) 14
(D) 16
(E) 28

27. If $3^{n-2} - 3^n = -(2^3)(3^8)$, what is the value of n?

(A) 8
(B) 10
(C) 12
(D) 13
(E) 14

28. A group of friends decide to order two appetizers from a certain menu. If the menu offers 8 different appetizers, how many different pairs of appetizers could the friends select?

(A) 64
(B) 56
(C) 32
(D) 28
(E) 16

29. A certain company designates each of its customers with a three- or four-letter code, where each letter is either A, B, C, D, E, or F. If the letters may be repeated and if the same letters used in a different order constitute a different code, how many different customers is it possible to uniquely designate with these codes?

(A) 1512
(B) 1296
(C) 480
(D) 360
(E) 216

30. A certain board game consists of a circular path of 100 spaces colored in a repeating pattern of black, yellow, green, red, and blue. What is the probability that, in a random selection of seven consecutive spaces, 2 of the spaces are blue?

(A) $\frac{2}{25}$
(B) $\frac{1}{7}$
(C) $\frac{1}{5}$
(D) $\frac{2}{7}$
(E) $\frac{2}{5}$

$(-7, 5)$, $(1, -1)$, (v, w), $(v + k, w + m)$

31. In the xy-coordinate system, the four points listed above lie on the same line. What is the value of $\frac{m}{k}$?

(A) $-\frac{4}{3}$
(B) $-\frac{3}{4}$
(C) $-\frac{2}{3}$
(D) $\frac{3}{2}$
(E) $\frac{5}{7}$

32. A committee of four people is to be chosen from five married couples. What is the number of different committees that can be chosen if two people who are married to each other cannot both serve on the committee?

(A) 32
(B) 40
(C) 64
(D) 80
(E) 160

33. Among a population of 30,000 people, 45 percent live within 10 miles of their workplace, 78 percent live inside the city limits of Town T, and 33 percent live both within 10 miles of their workplace and inside the city limits of Town T. If 1 person is to be randomly selected from the 30,000 people, what is the probability that the person lives within 10 miles of their workplace but NOT inside the city limits of Town T?

(A) $\frac{3}{25}$
(B) $\frac{3}{10}$
(C) $\frac{33}{100}$
(D) $\frac{12}{25}$
(E) $\frac{9}{20}$

34. A certain company consists of 3 managers and 8 non-managers. How many different teams of 3 employees can be formed in which at least one member of the team is a manager and at least one member of the team is not a manager? (Two groups are considered different if at least one group member is different.)

(A) 84
(B) 108
(C) 135
(D) 270
(E) 990

35. The five sides of a pentagon have lengths of 2, 3, 4, 5, and 6 inches. Two pentagons are considered different only when the positions of the side lengths are different relative to each other. What is the total number of different possible pentagons that could be drawn using these five side lengths?

(A) 5
(B) 12
(C) 24
(D) 32
(E) 120

36. If two fair six-sided dice are thrown, what is the probability that the sum of the numbers showing on the dice is even?

(A) $\frac{15}{36}$
(B) $\frac{1}{2}$
(C) $\frac{17}{36}$
(D) $\frac{6}{11}$
(E) $\frac{5}{9}$

37. In a garden, 7 flowers are to be arranged around a circular walk. Two arrangements of the flowers are considered different only when the positions of the flowers are different relative to each other. What is the total number of different possible arrangements of the flowers?

(A) 7
(B) 120
(C) 720
(D) 840
(E) 5,040

38. In a certain factory, quality control staff inspect a sample of 2 wrenches from each box of 12 wrenches. How many different samples can be chosen for inspection from one box of wrenches?

(A) 6
(B) 24
(C) 36
(D) 66
(E) 72

39. If $f * g = \frac{f}{g} + \frac{f-g}{2}$ for all integers f and g, then $6 * (-3) =$

(A) 6.5
(B) 2.5
(C) -0.5
(D) -2.5
(E) -3.5

40. A certain class consists of 4 boys and 5 girls. How many different groups of 3 children can be formed in which at least one member of the group is a boy? (Two groups are considered different if at least one group member is different.)

(A) 74
(B) 94
(C) 144
(D) 164
(E) 224

41. In her company-sponsored retirement account, Kathleen is offered 8 stock funds and 4 bond funds to choose from. If Kathleen wishes to invest one-sixth of her retirment account in each of 3 of the stock funds and each of 3 of the bond funds, how many different ways can she allocate the resources in her retirement account?

(A) 56
(B) 154
(C) 168
(D) 224
(E) 924

42. If $j * k = j^2 - 2jk + k^2$ for all integers j and k, then $4 * 3 =$
(A) 49
(B) 13
(C) 1
(D) -1
(E) -14

43. A committee of three people is to be chosen from the president and vice president of four different companies. What is the number of different committees that can be chosen if two people who work for the same company cannot both serve on the committee?
(A) 16
(B) 24
(C) 28
(D) 32
(E) 40

44. For all integers n such that $n > 1$, the function $f(n)$ is defined to be the product of all the distinct prime factors of n. Which of the following values of n results in the smallest value of $f(n)$?
(A) 84
(B) 90
(C) 95
(D) 96
(E) 100

45. Every user of a certain website is required to choose a five- or six-character password that is created by using the 26 letters of the alphabet for every character except for the last, which must be one of the 10 digits from 0 to 9. Which of the following gives the maximum number of different passwords that can be created using these guidelines?
(A) $10(26^5)$
(B) $26(26^4)$
(C) $27(26^5)$
(D) $260(26^4)$
(E) $270(26^4)$

46. For all even integers n, $h(n)$ is defined to be the sum of the even integers between 2 and n, inclusive. What is the value of $\frac{h(18)}{h(10)}$?

(A) 1.8
(B) 3
(C) 6
(D) 18
(E) 60

47. For a finite sequence of nonzero numbers, the primeness of the sequence is defined the fraction of the terms in the sequence that are prime. What is the primeness of the sequence 1, 2, 4, 7, 11, 16 ?

(A) $\frac{1}{6}$
(B) $\frac{1}{3}$
(C) $\frac{1}{2}$
(D) $\frac{2}{3}$
(E) $\frac{5}{6}$

48. Of the three-digit integers greater than 750, how many have at least two digits that are equal to each other?

(A) 56
(B) 70
(C) 72
(D) 74
(E) 78

49. For every positive integer k, the kth term of a certain sequence is given by $\left|(-2)^{10-k}(\frac{1}{2^k})\right|$. What is the smallest value of k for which the corresponding term of the sequence is less than 1?

(A) 1
(B) 4
(C) 5
(D) 6
(E) 10

50. A sign will be painted using a certain palette consisting of 9 colors, including light blue. One of the 9 colors will be selected at random and used to paint the border of the sign, one of the remaining 8 colors will be selected at random and used to paint the background of the sign, and one of the remaining 7 colors will be selected at random and used to paint the text of the sign. What is the probability that light blue will be chosen for the border or the background?

(A) $\frac{2}{9}$
(B) $\frac{1}{9}$
(C) $\frac{1}{18}$
(D) $\frac{1}{72}$
(E) $\frac{1}{504}$

51. When x is even, $\lfloor x \rceil = \frac{a}{2} + 1$. When x is odd, $\lfloor x \rceil = 2a + 1$. Which of the following is equal to $\lfloor 7 \rceil \times \lfloor 4 \rceil$?

(A) $\lfloor 22 \rceil$
(B) $\lfloor 44 \rceil$
(C) $\lfloor 45 \rceil$
(D) $\lfloor 88 \rceil$
(E) $\lfloor 90 \rceil$

52. If the sum of 6 consecutive integers is x, which of the following must be true?

I. x is an even number
II. x is an odd number
III. x is a multiple of 3

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III

53. A certain university department consists of 5 tenured professors and 3 junior professors. If a hiring committee is to be formed that is made up of 3 tenured professors and 2 junior professors, how many different hiring committees are possible?

(A) 20
(B) 28
(C) 30
(D) 48
(E) 56

54. In how many ways can a choreographer arrange 4 female dancers and 1 male dancer in a row if the male dancer must be in the middle?
    (A) 24
    (B) 48
    (C) 60
    (D) 80
    (E) 120

55. If the operation @ is defined for all j and k by the equation $j@k = \frac{(j^2k^2)}{3}$, then $(3@-3)@3 =$
    (A) 3^{-5}
    (B) 3^{-3}
    (C) 3
    (D) 3^4
    (E) 3^7

56. A right circular cone is inscribed in a cube so that the base of the cone is inscribed in the base of the cube and the height of the cone is equal to the height of the cube. What is the ratio of the radius of the base of the cone to the height of the cone?
    (A) $1 : \pi$
    (B) $1 : 2$
    (C) $\pi : 4$
    (D) $\sqrt{2} : 1$
    (E) $2 : 1$

57. The function f is defined for all positive integers n by the following rule: $f(n)$ is the product of the distinct prime factors of n. If $f(n) < 100$ and n is not prime, what is the greatest possible value of $f(n)$?
    (A) 99
    (B) 95
    (C) 91
    (D) 87
    (E) 78

58. In the infinite sequence a_1, a_2, a_3, . . . , a_n, . . . , each term after the first is equal to half the value of the previous term. If $a_3 + a_6 = 72$, what is the value of a_1 ?

(A) 8
(B) 72
(C) 128
(D) 144
(E) 256

59. George owns twelve audio compact discs. If he takes two of them to play on an upcoming car trip, how many different pairs of compact discs could he choose?

(A) 144
(B) 132
(C) 72
(D) 66
(E) 36

60. If two of the four expressions $a + b$, $3a + b$, $a - b$, and $a - 3b$ are chosen at random, what is the probability that their product will be of the form of $a^2 + kb^2$, where k is a positive integer?

(A) $\frac{1}{12}$
(B) $\frac{1}{6}$
(C) $\frac{1}{4}$
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$

61. For each player's turn in a certain board game, a card is drawn. $\frac{3}{4}$ of the cards in the deck are marked with a circle, and the remaining cards are marked with a square. If five players draw a card and then return it to the deck, what is the probability that at least four of the cards drawn are marked with a square?

(A) $(\frac{1}{4})^3$
(B) $5(\frac{1}{4})^3$
(C) $\frac{3}{4}(\frac{1}{4})^4$
(D) $\frac{3}{2}(\frac{1}{4})^4$
(E) $(\frac{1}{4})^5$

62. In the rectangular coordinate plane, a rectangular region has vertices $(2.5, 4.5)$, $(2.5, -1.5)$, $(-7.5, 1.5)$, and $(-7.5, 4.5)$. If a point from within the rectangle is to be randomly selected such that both the x- and y-coordinates are integers, what is the probability that both coordinates of the selected point are negative?

(A) $\frac{1}{11}$
(B) $\frac{7}{60}$
(C) $\frac{2}{15}$
(D) $\frac{16}{77}$
(E) $\frac{4}{15}$

63. At a certain convention center, two meeting rooms are selected for an event. If there are 105 different possible selections of the two meeting rooms, how many different meeting rooms does the convention center have?

(A) 7
(B) 8
(C) 14
(D) 15
(E) 16

$A = \{2, 4, 6, 8\}$
$B = \{2, 4, 6, 8, 12\}$

64. Two integers will be randomly selected from the sets above, one integer from set A and one integer from set B. What is the probability that the product of the two integers will equal 24?

(A) 0.10
(B) 0.15
(C) 0.20
(D) 0.25
(E) 0.30

65. Each student at a certain university is given a four-character identification code, the first two characters of which are digits between 0 and 9, inclusive, and the last two characters of which are selected from the 26 letters of the alphabet. If characters may be repeated and the same characters used in a different order constitute a different code, how many different identification codes can be generated following these rules?

(A) 135,200
(B) 67,600
(C) 64,000
(D) 60,840
(E) 58,500

4 Data Sufficiency

For all Data Sufficiency questions, the answer choices are as follows:

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
(C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient.
(E) Statements (1) and (2) TOGETHER are NOT sufficient.

66. If m is a positive integer, is the value of $p+q$ at least twice the value of $3^m + 4^m$?
   (1) $p = 3^{m+1}$ and $q = 2^{2m+1}$
   (2) $m = 4$

67. If $\bigtriangledown$ represents one of the operations $+$, $-$, and $\times$, is $(a \bigtriangledown b) + (a \bigtriangledown c) = a \bigtriangledown (b + c)$ for all numbers a, b, and c ?
   (1) $\bigtriangledown$ represents subtraction.
   (2) $m \bigtriangledown 2$ is not equal to $2 \bigtriangledown m$ for some numbers m.

68. Is $|a| = b - c$?
   (1) $a + c \neq b$
   (2) $a < 0$

69. If p and q are positive integers, what is the remainder when 9^{p+q+1} is divided by 10?
   (1) $p = q$
   (2) $p + q = 4$

70. If a and b are positive integers and $a > b$, what is the remainder when $a^2 - 2ab + b^2$ is divided by 9 ?
   (1) The remainder when $a - b$ is divided by 3 is 2.
   (2) The remainder when $a - b$ is divided by 9 is 2.

71. In the xy-plane, at what point does the graph of $y = (x + k)^2$ intersect the x-axis?
   (1) The graph includes the point $(1, 9)$.
   (2) The graph includes the point $(-1, 1)$.

72. If W is the infinite sequence $W_1 = 5$, $W_2 = 55$, $W_3 = 555$, . . . , $W_k = 5(10^k) + W_{k-1}$, . . . , is every term in W divisible by the integer y ?
    (1) W_1 is divisible by y.
    (2) $y \neq 5$

73. A group of senators voted on three amendments, X, Y, and Z. 18 of the senators voted in favor of both X and Y, 15 voted in favor of both X and Z, and 20 voted in favor of both Y and Z. How many of the senators voted in favor of all three amendments?
    (1) Of the 18 senators who voted in favor of both X and Y, 11 voted in favor of Z.
    (2) Of the 15 senators who voted in favor of both X and Z, 4 did not vote in favor of Y.

74. If k is an integer, is $2^k + 3^k = m$?
    (1) $4^k + 9^k = m^2 - 12$
    (2) $k = 1$

75. If a is an integer and $a = \frac{|b|}{b}$, is $a = 1$?
    (1) $b > 0$
    (2) $a > -1$

76. If y is a positive integer, is $y^2 - y$ divisible by 4?
    (1) $y^2 + y$ is not divisible by 4.
    (2) $y^3 - y$ is divisible by 4.

77. Is $2a < a^2 + b$?
    (1) $0 < a < 2$
    (2) $-2 < b < 0$

78. In the xy-coordinate plane, line l_1 and line l_2 intersect at the point $(-2, -5)$. Is the product of their slopes negative?
    (1) The product of the x-intercepts of lines l_1 and l_2 is negative.
    (2) The product of the y-intercepts of lines l_1 and l_2 is positive.

79. If k is a positive integer and m is the product of the first 40 positive integers, what is the value of k ?
    (1) 10^k is a factor of m.
    (2) 10^k is a factor of n, where n is the product of the first 9 positive integers.

80. In the finite sequence of positive integers A_1, A_2, A_3, ..., A_{11}, each term after the second is the product of the two terms preceding it. If $A_4 = 18$, what is the value of A_{11}?

(1) $A_2 = 3$
(2) $A_6 = 1,944$

81. Is the integer p divisible by 6?

(1) $9p$ is divisible by 6.
(2) $8p$ is divisible by 6.

82. A chemist combined a milliliters of a solution that contained 20 percent substance S, by volume, with b milliliters of a solution that contained 8 percent substance S, by volume, to product c milliliters of a solution that was 10 percent substance S, by volume. What is the value of a?

(1) $b = 20$
(2) $c = 24$

83. An arithmetic progression is a sequence in which the difference of any two successive numbers of the sequence is a constant. For example, $\{4, 8, 12, 16\}$ is an arithmetic progression in which the difference of any two successive numbers is 4. Is the infinite sequence S an arithmetic progression?

(1) For any term n in S, $S_n = S_{n-1} + 2$
(2) Each term in S is a positive odd integer.

84. The sequence of s_1, s_2, s_3, . . . , s_n of n is such that $s_k = 2k - 1$ if k is odd and $s_k = -s_{k-1}$ if k is even. Is the sum of the terms in the sequence positive?

(1) The sum of the first 11 terms in the sequence is positive.
(2) n is odd.

85. If p, q, x, and y are positive, is the ratio of p to q equal to the ratio of x to y ?

(1) The ratio of p to y is equal to the ratio of q to x.
(2) The ratio of p to x is equal to the ratio of q to y.

86. If k is a positive integer and t is the remainder when $3 + 5k$ is divided by 4, what is the value of t?

(1) $k - 1$ is divisible by 4.
(2) $k > 25$

87. Each of the 30 students in a class is either male or female, and has blonde, brown, or red hair. If one student is to be randomly selected from the class, what is the probability that the student will either be female or have brown hair?

(1) The probability that the student will both be female and have brown hair is 0.1.
(2) The probability that the student will be female minus the probability that the student will have brown hair is 0.25.

88. Is $|a| - |b| \geq |a - b|$?

(1) $b > a$
(2) $a > 0$

89. Is $3^n > 1,000$?

(1) $3^{n-2} < 500$
(2) $3^n = 3^{n+1} - 486$

90. The positive integers r, s, and t are such that r is divisible by s and s is divisible by t. Is r even?

(1) st is odd.
(2) rt is even.

91. If $x < p < q < y$, is $|q - x| < |q - y|$?

(1) $|p - x| < |p - y|$
(2) $|q - x| < |p - y|$

92. The symbol $\triangledown$ represents one of the following operations: addition, subtraction, multiplication, or division. What is the value of $4 \triangledown 3$?

(1) $1 \triangledown 1 = 1$
(2) $2 \triangledown 2 = 1$

$s_1, s_2, s_3, \ . \ . \ . \ , s_{16}$

93. In the sequence shown, $s_n = s_{n-1} + k$, where $2 < n < 17$ and k is a nonzero constant. How many of the terms in the sequence are greater than 100?

(1) $s_1 = 5$
(2) $s_{16} = 185$

94. Is $\frac{125}{5^{n+1}} > 1$?

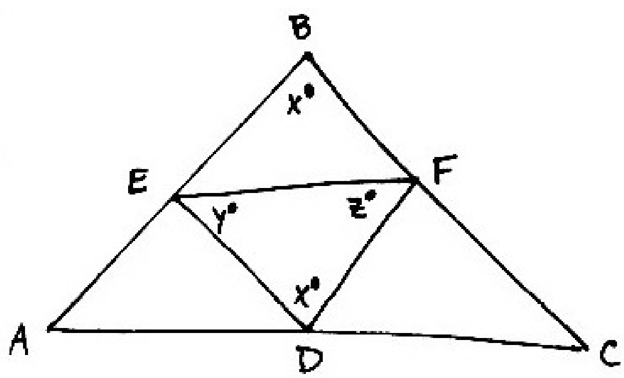
(1) $\frac{125}{5^{n+2}} < 1$
(2) $n < 0$

95. In the period between 1975 and 1985, the population of each of the five major cities in country C increased by at least 10,000 people. If the standard deviation of the populations in 1975 was 35,000 people, what was the standard deviation of the populations in 1985?

(1) The total increase in population of the five cities from 1975 to 1985 was 220,000 people.
(2) Each of the five cities increased in population by 12% in the period between 1975 and 1985.

96. An infinite sequence of positive integers is called a "beta sequence" if the number of odd integers in the sequence is finite. If S is an infinite sequence of positive integers, is S a beta sequence?

(1) The first ten integers in S are even.
(2) The difference between each successive pair of terms in S is a constant.



97. In the figure above, regions ABC and DEF are similar triangles. What is the value of y?

(1) $AB = BC$
(2) $z = 65$

98. Is $\frac{m}{n^2+1} > \frac{n}{m^2+2}$?

(1) $m = n$
(2) $n < 0$

99. If m and n are positive integers, what is the remainder when $m^2 - n^2$ is divided by 10?

(1) The remainder when m is divided by 10 is 3.
(2) The remainder when n is divided by 10 is 3.

100. If $\bigtriangledown$ represents one of the operations $+$, -, $\times$, and $\div$, is $m \bigtriangledown n = n \bigtriangledown m$ for all numbers m and n?

(1) $4 \bigtriangledown 2 \neq 8 \bigtriangledown 4$

(2) $\bigtriangledown$ represents subtraction.

5 Answer Key

For full explanations, see the next section.

1. B
2. C
3. C
4. C
5. B
6. E
7. C
8. B
9. D
10. E
11. B
12. E
13. C
14. D
15. E
16. C
17. C
18. B
19. D
20. C
21. E
22. B
23. C
24. D
25. E
26. B
27. B
28. D
29. A
30. E
31. B
32. D
33. A
34. B
35. C
36. B
37. C
38. D
39. B
40. A
41. D

42.	C
43.	D
44.	D
45.	E
46.	B
47.	C
48.	D
49.	D
50.	A
51.	D
52.	E
53.	C
54.	A
55.	E
56.	B
57.	B
58.	E
59.	D
60.	B
61.	A
62.	B
63.	D
64.	B
65.	B
66.	A
67.	D
68.	C
69.	D
70.	B
71.	C
72.	A
73.	D
74.	C
75.	D
76.	C
77.	C
78.	C
79.	B
80.	D
81.	C
82.	D
83.	A
84.	B
85.	B
86.	A
87.	E

88. C
89. B
90. B
91. E
92. B
93. C
94. B
95. B
96. C
97. C
98. C
99. C
100. B

6 Explanations

For a quick-reference answer key, see the previous section.

1. B

Explanation: The best way to solve permutations problems is usually to pretend as though we are assigning members one by one. This isn't a permutations problem–it's combinations, since the order doesn't matter–but coming up with the number of permutations is a good start. In this case, consider how many of the 16 people could be chosen for the first slot in the team of three people. There are no restrictions yet, so there are 16 possibilities.

After the first team member is assigned, not only can that person no longer be assigned, but no one on his or her team can be, as well. Thus, there are 12 possibilities for the second team member. By the same reasoning, there are 8 team members (anyone from the remaining two teams) that could be chosen for the third spot.

The total number of permutations is the product of those three numbers:

$16(12)(8) = (4^2)(4)(3)(4)(2) = 6(4^4)$

But, as I said, this isn't a permutations problem. Some teams are being double counted – a team of A, B, and C is the same as a team of C, B, and A, but those two are counted separately using the method above. So, find the number of permutations per team. A team of 3 could be arranged in any of 3! ways, or 6 ways. So, since each team represents 6 possible permutations, divide the number of permutations by 6 for the correct number of combinations:

$\frac{6(4^4)}{6} = 4^4$, choice (B).

2. C

Explanation: The verbiage is the biggest challenge here. In a sequence of 6 numbers, there are 5 sets of adjacent terms (1 and 4, 4 and 5, etc.). Check each one to see how many are consecutive:

1 and 4: no.
4 and 5: yes
5 and 6: yes
6 and -3: no
-3 and -4: yes

That's a total of 3, choice (C).

3. C

Explanation: If the probability of a part being defective is $\frac{1}{9}$ the probability of it not being defective, there are $\frac{1}{9}$ as many defective parts as non-defective parts. We can write that as an equation:

$d = \frac{1}{9}n$

Since all parts are either defective or non-defective, there's another equation, representing the total:

$n + d = 1,800$

Combine the two to solve for d:
$n + \frac{1}{9}n = 1,800$
$\frac{10}{9}n = 1,800$
$n = 1,800(\frac{9}{10}) = 180(9) = 1,620$
Use the value of n to solve for d:
$1,620 + d = 1,800$
$d = 180$, choice (C).

4. C
Explanation: For each character, there are 36 possibilities – any letter or single digit. The question doesn't say that digits can't be repeated, so we must assume that they may be repeated. The number of 5-digit codes is given as follows:
$36 \times 36 \times 36 \times 36 \times 36 = 36^5$
And the number of 6-digit codes is:
$36 \times 36 \times 36 \times 36 \times 36 \times 36 = 36^6$
The total number of codes, then, is:
$36^5 + 36^6 = 36^5(1) + 36^5(36) = 36^5(1 + 36) = (37)36^5$, choice (C).

5. B
Explanation: In calculating the area of a triangle, there are two variables that influence its size: base and height. Take the base as a given, since one side of the triangle is the diameter of the circle, which is double the radius, meaning that it has a length of 12.

Wherever we place the third vertex, the height of the triangle is a line running between that vertex and the diameter, and perpendicular to the diameter. The longest such line is a radius of the circle running from the center of the circle to the edge, so its length is 6. The area of the triangle, then, is:
$a = \frac{1}{2}bh = \frac{1}{2}(12)(6) = 36$, choice (B).

6. E
Explanation: The toughest part of this question is translating it into algebra. "J divided by P" is equivalent to $\frac{J}{P}$. If the quotient is Q, the result of $\frac{J}{P}$ is Q plus a remainder. If you know the quotient and remainder of a division problem, the answer is equivalent to $(quotient) + \frac{remainder}{denom}$, so in this case, the result of $\frac{J}{P}$ is $Q + \frac{M}{P}$. Thus, you have an equation; now you just have to solve it for M:
$\frac{J}{P} = Q + \frac{M}{P}$
$J = PQ + M$
$M = J - PQ$
That's equivalent to choice (E), $J - QP$, so that's the correct choice.

7. C
Explanation: Reduce each term to its prime factors:
$(2^{15})(5^2)^8 = 5((5 \times 2)^m)$
$(2^{15})(5^{16}) = 5(5^m)(2^m)$

Divide each side by 5:
$(2^{15})(5^{15}) = (5^m)(2^m)$
$m = 15$, choice (C).

8. B
Explanation: Since each term in the sequence is 2 to the power $n-1$, the entire sequence up to $n = 10$ looks like this:
$2 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9$
$2 + 2 = 2^2$, so the sum simplifies to:
$2^2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9$
$2^2 + 2^2 = 8 = 2^3$, so this is the same as:
$2^3 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9$
At this point, you should start to recognize a pattern. The two leftmost terms will add up to a term that is equal to the next term to the right. Eventually, you'll get to:
$2^9 + 2^9 = 2(2^9) = 2^{10}$, choice (B)

9. D
Explanation: To see the relationship between the two figures, draw two lines: one from the center of the circle downward, perpendicular to the base of the triangle; the other from the center of the circle to point B. The triangle formed by those two lines and the base of the triangle is a 30-60-90 triangle.

To use that information, you'll need to know the length of a side of the triangle. You'll need to use a 30-60-90 triangle to find that, too: if you draw a line down the middle of the triangle, that's the height, and it also forms a 30-60-90 triangle on either side. If a side of the triangle is s, the height is $\frac{1}{2}s\sqrt{3}$, so the area is:
$\frac{1}{2}(s)(\frac{1}{2}s\sqrt{3}) = 24\sqrt{3}$
$\frac{1}{4}s^2 = 24$
$s^2 = 96$
$s = \sqrt{96} = 4\sqrt{6}$
If a side of ABC is $4\sqrt{6}$, the longer leg of the 30-60-90 triangle is half that: $2\sqrt{6}$. That corresponds to the $x\sqrt{3}$ side of the triangle, while the radius is the x side, so solve for x:
$x\sqrt{3} = 2\sqrt{6}$
$x = 2\sqrt{2}$
If the radius of the circle is $2\sqrt{2}$, we can calculate the area:
$a = \pi r^2 = \pi(2\sqrt{2}) = 8\pi$, choice (D).

10. E
Explanation: If the product of three integers is to be even, at least one of the integers must be even. There are many combinations of evens and odds that result in at least one even integer out of the three; however, it's simpler to find the one combination that results in all odds: the one in which all three integers are odds.

To find the probability of an odd product, find the probability of each of the integers being odd:
x: $\frac{2}{3}$
y: $\frac{1}{3}$
z: $\frac{1}{2}$
The probability that all are odd is the product of those: $\frac{2}{3}(\frac{1}{3})(\frac{1}{2}) = \frac{1}{9}$
The probability that at least one is even is the opposite of $\frac{1}{9}$:
$1 - \frac{1}{9} = \frac{8}{9}$, choice (E).

11. B
Explanation: Find the number of possible 3-person terms from each of the two companies. The pool of available scientists from A is 5, so we'll need the combinations formula to find the number of possible 3-person subsets:
$\frac{n!}{k!(n-k)!} = \frac{5!}{3!2!} = \frac{5\times 4}{2\times 1} = 10$
For B, there are 4 scientists. Any 3-person team would exclude exactly one person. Since there are 4 different scientists who could be excluded, that's 4 possible 3-person teams.
Since the research team could be constructed of any pair of A teams and B teams, the answer is the product of our two combinations:
$10 \times 4 = 40$, choice (B).

12. E
Explanation: In this question, the symbol * is interchangeable with the concept of "factorial", usually abbreviated with "!". Thus, $4* = 4\times 3\times 2\times 1 = 24$, and $5* = 5\times 4\times 3\times 2\times 1 = 120$. The question boils down to: how many multiples of 4 are there between 24 and 120, inclusive.
For that, find the difference: $120 - 24 = 96$. Divide by 4: $\frac{96}{4} = 24$. Finally, add 1, because the endpoints are both multiples of 4, and the question uses the word "inclusive": $24 + 1 = 25$, choice (E).

13. C
Explanation: First determine the number of possible arrangements for the three boys in the three seats that the boys must occupy. There are 3 boys who could be placed in the first seat, 2 boys who could be placed in the fourth seat (since one boy is already in the first seat) and one boy could be placed in the 7th seat. That's a total of $3\times 2\times 1 = 6$ arrangements.
Next, find the number of arrangements for the remaining four seats. Here, there are four choices: any of the 3 girls, plus the possibility of an empty seat. So, for the first of those seats, there are 4 possibilities; for the second, there are 3, and so on. The total number of arrangements for those 4 seats is: $4\times 3\times 2\times 1 = 24$.
Since we have two separate permutations (6 arrangements of the boys and 24 arrangements of the girls and empty seat), the answer is the product of those: $6(24) = 144$, choice (C).

14. D

Explanation: To answer this question, find the smallest space between two endpoints. Think of it like a pie chart: if you randomly select a point on a pie chart, you are least likely to end up in the smallest sector of the chart.

Evalute each choice:

(A) $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$
(B) $\frac{1}{3} - \frac{1}{6} = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}$
(C) $\frac{1}{6} - \frac{1}{9} = \frac{3}{18} - \frac{2}{18} = \frac{1}{18}$
(D) $\frac{1}{9} - \frac{1}{12} = \frac{4}{36} - \frac{3}{36} = \frac{1}{36}$
(E) $\frac{1}{12}$

Each of the differences has the same numerator, so we can directly compare denominators. $\frac{1}{36}$ is the smallest.

15. E

Explanation: First, find the number of fish in the four compartments. If the number of fish in the compartment that holds the fewest fish is x, the other three compartments hold $x + 1$, $x + 2$, and $x + 3$ fish. Solve for x:

$x + x + 1 + x + 2 + x + 3 = 42$
$4x + 6 = 42$
$4x = 36$
$x = 9$

The compartment that holds the most fish, then, holds $x + 3 = 9 + 3 = 12$ fish. The probability any one of the 42 fish is in that compartment is:

$\frac{12}{42} = \frac{6}{21} = \frac{2}{7}$, choice (E).

16. C

Explanation: This question contains some language that makes it sound harder than it actually is. If the car starts at a random location and travels one half mile, it is still at a random location. That half mile is meaningless in terms of answering the question.

So, instead, find out the range of desired outcomes. If the car is between the mileposts indicating 2 miles and 3 miles (or 0 miles, since the track is 3 miles long), it is within a half mile of the $2\frac{1}{2}$ milepost. That's a desired range of 1 mile, out of a possible range of 3 miles–the length of the track. The probability, then, is $\frac{1}{3}$, choice (C).

17. C

Explanation: Since each die has six sides, there are 36 possible outcomes: the product of 6 possible outcomes for each die. The number of desired outcomes is the number of possible pairs that sum to 6:

1 and 5
2 and 4
3 and 3
4 and 2
5 and 1

That's five desired outcomes, so the probability is $\frac{5}{36}$, choice (C).

18. B

Explanation: Start by evaluating $\widetilde{6} \times \widetilde{12}$. Neither 6 nor 12 is prime, so each of the two terms is equal to the largest prime factor of the respective number. The only prime factors of each are 2 and 3, so the largest prime factors of each are 3. Thus:

$\widetilde{6} \times \widetilde{12} = 3 \times 3 = 9$

The brute force method here would be to go evaluate each of the five choices. That probably wouldn't take too long. Alternatively, realize that there are two possibilities. We know that $\widetilde{p}$ must either be a prime number squared, or the largest prime factor of a non-prime. 9 cannot be the largest prime factor of a non-prime because 9 isn't prime. Therefore, 9 must be the square of a prime, which it is: it's 3 squared. So, $9 = \widetilde{3}$, choice (B).

19. D

Explanation: The probability that a number larger than 20 is drawn is the difference between the probability that a number 12 or larger is drawn and the probability that a number between 12 and 20 is drawn: $\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$. The probability that a number less than or equal to 20 is drawn is the opposite of the probability that a number larger than 20 is drawn, so subtract that result from 1:

$1 - \frac{1}{2} = \frac{1}{2}$, choice (D).

20. C

Explanation: Given the complexity of this sequence, it's extremely unlikely that the GMAT expects you to find and then sum the first 9 terms of the sequence. It's more likely that you should find the first few terms and identify the pattern. Start by finding the first 3 terms:

$n = 1$

$(-1)^{1+1}(\frac{3}{2^1}) = 1(\frac{3}{2}) = \frac{3}{2}$

$n = 2$

$(-1)^{2+1}(\frac{3}{2^2}) = -1(\frac{3}{4}) = -\frac{3}{4}$

Sum: $\frac{3}{2} + (-\frac{3}{4}) = \frac{3}{4}$

$n = 3$

$(-1)^{3+1}(\frac{3}{2^3}) = 1(\frac{3}{8}) = \frac{3}{8}$

Sum: $\frac{3}{4} + \frac{3}{8} = \frac{9}{8}$

By this point, you may recognize that the terms of the sequence alternate between positives and negatives, and the absolute value of each one gets smaller. Thus, the effect of each subsequent term will be less and less. Further, each term will cancel out half of the effect of the previous term, so the sum will converge on some number in the range of those we've found: it must be bigger than $\frac{3}{4}$ but smaller than $\frac{9}{8}$. That range is included in (C).

21. E

Explanation: There's a lot of excess wording to this question when it's really a simple concept. Each of the team members has an equal chance to be selected to run first, second, or third, and (perhaps obviously) no team

member can be selected to run more than one of those. Therefore, from Joey's perspective, he has a $\frac{1}{12}$ chance of running first, a $\frac{1}{12}$ chance of running second, and a $\frac{1}{12}$ chance of running third. Since he can't run both second AND third, the chances that he'll run second OR third is the sum of those two probabilities:
$\frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$, choice (E).

22. B
Explanation: Since you're asked for a probability, work with percents, not the corrseponding numbers. (In other words, the fact that there are 60 people is irrelevant and potentially distracting.) If 20% received an 'A' on their mid-term and 5% received an 'A' on both, that means 15% received an 'A' on the midterm but not on the final. The corresponding fraction to 15% is:
$\frac{15}{100} = \frac{3}{20}$, choice (B).

23. C
Explanation: Consider the range of integers as two separate groups: the 800s and the 900s. They will have nearly identical properties, so focus on just one of the two to start with.

There are three ways that a three-digit number can have two digits equal to each other and a different third digit:
XXY
XYX
YXX

In the 900s, there are 9 numbers of the form XXY: they all start with 99, and there are 9 possibilities (0 through 8, inclusive) for the units digit. There are also 9 numbers of the form XYX: each starts and ends with 9, and there are 9 possibilities (again, 0 through 8 inclusive) for the tens digit. Finally, there are 9 numbers of the form YXX: each starts with 9, and the remaining two digits can be anything from 0 to 8 inclusive. Thus, there are 27 numbers of the form we're looking for in the 900s.

It stands to reason that there are also 27 numbers in this form in the 800s, as well. There are, but there's a final twist: the question asks for integers GREATER than 800, and 800 is itself one of those 27 numbers. Thus, there are only 26 numbers between 801 and 899 inclusive that fit this form. The total number, then, is $27 + 26 = 53$.

24. D
Explanation: If you begin by selecting a freshman, there are 75 desired outcomes: any one of the 75 freshmen who is part of one of the sets of lab partners. The probability of choosing one of those is $\frac{75}{600}$. Once a certain freshman is selected, there is only one desired outcome among the sophomores: that freshman's lab partner. The probability of selecting that particular sophomore is $\frac{1}{900}$. The probability of selecting a set of lab partners, then, is the product of those two fractions:
$\frac{75}{600} \times \frac{1}{900} = \frac{3}{24} \times \frac{1}{900} = \frac{1}{8} \times \frac{1}{900} = \frac{1}{7{,}200}$, choice (D).

25. E

Explanation: x is part of three equations in the addition table:

$m = x + a$

$n = x + b$

$p = x + c$

To find the value of x, we'll need the other two variables in one of those equations. The question gives us one such variable: if $x + c = 4$, then $p = 4$. The other two given equations provide us a variable as well:

$v - b = -2$ –> $b - 2 = v$ –> $z = -2$

$a + z = 1$ –> $t = 1$

Putting the pieces together: we know the value of z, so we can solve for the value of a:

$a - 2 = 1$

$a = 3$

We know now one of the two variables we need for two different ways to solve for x. Since we know a, finding m would finish the job; since we know p, c would finish the job. Consider each choice:

(A) Without b, n isn't enough.

(B) Since we know a, the value of q gives us y, but that doesn't help.

(C) We don't have y or c, so s doesn't help at all.

(D) v allows us to solve for b, but without n, that's not enough to find x.

(E) The value of w, plus the value of z, which we have, gives us c. Since we have p, we can combine c and p to find x. (E) is the correct choice.

26. B

Explanation: The question gives us three equations and three variables, one for each color of marble. Since there are 56 marbles, we know that:

$g + w + r = 56$

Further, the question gives us two relationships. If the probability of selecting a green is twice that of selecting a red, then there must be twice as many greens in the bag:

$g = 2r$

The same reasoning applies to the relationship between whites and greens:

$w = 2g$

If we translate the second of the three equations so we have r in terms of g, we can substitute each of the equations into the first:

$r = \frac{g}{2}$

$g + 2g + \frac{g}{2} = 56$

$\frac{7g}{2} = 56$

$7g = 112$

$g = 16$

Now solve for the number of red marbles:

$16 = 2r$

$r = 8$, choice (B).

27. B

Explanation: Isolate n by isolating 3^n, which can be accomplished by separating 3^{n-2} into 3^n and 3^{-2}:

$3^n 3^{-2} - 3^n = -(2^3)(3^8)$

$3^n(3^{-2} - 1) = -(2^3)(3^8)$

$3^n(\frac{1}{9} - 1) = -(2^3)(3^8)$

$3^n(-\frac{8}{9}) = -(2^3)(3^8)$

$3^n = -(2^3)(3^8)(-\frac{9}{8}) = (2^3)(3^8)(3^2)(2^{-3}) = 3^{10}$

$n = 10$, choice (B).

28. D

Explanation: This is a combinations problem: we're selecting subsets of 2 from a larger set of 8, and the order of selection doesn't matter. Plug in 8 and 2 to the combinations formula:

$\frac{n!}{k!(n-k)!} = \frac{8!}{2!6!} = \frac{8\times7}{2\times1} = 4 \times 7 = 28$, choice (D).

29. A

Explanation: This is a permutations problem with two separate permutations to calculate. First, consider how many possible three-letter codes there are. Since letters can be repeated, there are 6 letters to choose from for the 3 places in the code, meaning that the number of permutations is:

$6 \times 6 \times 6 = 216$

For the four-letter code, the reasoning is similar, only there are four 6's instead of three:

$6 \times 6 \times 6 \times 6 = 1296$

The total number of possible codes, then, is the sum of those two:

$1296 + 216 = 1512$, choice (A).

30. E

Explanation: Regardless of which color you start with, a sequence of seven spaces will consist of each of the five colors at least once, and two of the colors twice. Which two colors repeat depends on which color you start with. Since that selection is random, the colors that repeat are random as well. Probability is desired outcomes divided by possible outcomes, and for a random starting point, there are 2 ways one 2 of the spaces are blue–if blue is the first color to repeat, or the second color to repeat. There are 5 possible outcomes, one for each of the colors that could be the starting point.

Thus, the probability is $\frac{2}{5}$, choice (E).

31. B

Explanation: The relationship between k and m is the slope of the line: between the points (v, w) and $(v + k, w + m)$, m is the change in y and k is the change in x. The slope, then, is $\frac{m}{k}$. In order to use that information, we need the slope. That's where the other two points come in.

Here's the slope formula:

$s = \frac{y_2 - y_1}{x_2 - x_1}$

Using $(-7, 5)$ as the first point and $(1, -1)$ as the second, that means the slope is:
$\frac{-1-5}{1-(-7)} = \frac{-6}{8} = -\frac{3}{4}$
Since the slope is $\frac{m}{k}$, that means our answer is choice (B).

32. D
Explanation: Start by treating this as a permutations problem. (It's not, but there's no way to use the combinations formula to include a restriction such as the one in this question.) Any of the 10 people can be the first person chosen. After the first person is chosen, there are 8 remaining possibilities–anyone except for the first person chosen and that person's spouse. After the second person is chosen, there are 6 remaining possibility–2 choices and their spouses are out of contention. Then, there are 4 possibilities for the 4th choice. If this were a permutations problem, the total number of permutations would be:
$10 \times 8 \times 6 \times 4$
However, the order doesn't matter. Thus, we've overcounted the number of possibilities. The difference between the combinations formula and the permutations formula is a $k!$ (where k is the size of the subset) in the denominator. Thus, in a case like this, divide the number of permutations by 4!, so the result is:
$\frac{10\times8\times6\times4}{4\times3\times2\times1} = 10 \times 8 = 80$, choice (D).

33. A
Explanation: Since you are given each set as a percent and you are looking for a probability, the actual numbers are irrelevant. Ignore the population of 30,000 people and focus on percents and fractions.
If 45 percent of the population lives within 10 miles of their workplace and 33 percent live within 10 miles of their workplace and inside the city limits, that means 12 percent lives within 10 miles of workplace but not inside the city lives. 12 percent is equal to the following probability in fraction form:
$\frac{12}{100} = \frac{3}{25}$, choice (A).

34. B
Explanation: There are two possible constructions of the team: there can be 2 managers are 1 non-manager, or 1 manager and 2 non-managers. We'll need to solve for the number of combinations for both.
First, 2 managers and 1 non-manager. If there are 3 managers, there are 3 ways of selecting 2 managers – any selection would leave exactly 1 manager out, so since are are 3 managers to possibly exclude, there are 3 combinations. For the 1 non-manager, there are 8 possibilities. Thus, the number of combinations for this team construction is $3 \times 8 = 24$.
Next, 1 manager and 2 non-managers. There are 3 managers, so 3 possibilities for the 1 manager. We'll need the combinations formula to find the number of pairs of non-managers that can be selected from a group of 8:
$\frac{n!}{k!(n-k)!} = \frac{8!}{2!6!} = \frac{8\times7}{2\times1} = 4 \times 7 = 28$

The total number of combinations for this team construction is $3 \times 28 = 84$
The total, then, is $24 + 84 = 108$, choice (B).

35. C
Explanation: Start by thinking of this as a typical permutations problem. How many ways could the numbers 2, 3, 4, 5, and 6 be arranged in different orders? The answer to that question would be 5!: the number permutations of 5 different objects. However, since "two pentagons are considered different only when the positions of the side lengths are different relative to each other," that eliminates some of those 5! permutations.

For instance, the following two arrangements of side lengths are the same:
2, 3, 4, 5, 6
6, 2, 3, 4, 5

In a typical permutations question, they would be counted as different. Here, the side lengths are the same relative to each other. In fact, for every possible pentagon, there are 5 "typical permutation" ways of counting it; in addition to the above, there are:
5, 6, 2, 3, 4
4, 5, 6, 2, 3
3, 4, 5, 6, 2

Thus, 5! overstates the number of possible pentagons by a factor of 5, so the total number of pentagons is:
$\frac{5!}{5} = 4! = 4 \times 3 \times 2 \times 1 = 24$, choice (C).

36. B
Explanation: The sum of two numbers will be even if both numbers are even or both numbers are odd. Further, the probability that any given dice roll will result in an even number is $\frac{1}{2}$, which is the same as the probability of getting an odd result.

So, the probability of both dice resulting in an even number (and thus, an even sum), is $\frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$. The probability that both dice result in an odd number (again, giving us an even sum) is the same. The probability of an even sum, then, is the total of those two probabilities: $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$, choice (B).

37. C
Explanation: Usually, the word "arrangement" signals that it is a permutations question. This example is no difference, except for the twist that the walk is circular. In a typical permutations problem (say, for instance, the flowers were to be arranged in a row), the answer would be 7!. However, when the flowers are arranged in a circle, some of the 7! permutations are redundant. Consider the following two permutations:
A B C D E F G
B C D E F G A

In a row, those are different. In a circle, they are arranged the same way relative to each other. Mathematically, that means that for every possible circular permutation, there are 7 row permutations. Thus, 7! is overcounting the number of circular permutations by a factor of 7, so the correct answer is:

$\frac{7!}{7} = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$, choice (C).

38. D

Explanation: This is a straightforward combinations problem: the order that the wrenches are selected doesn't matter. Use the combinations formula:

$\frac{n!}{k!(n-k)!} = \frac{12!}{2!10!} = \frac{12\times11}{2\times1} = 6 \times 11 = 66$, choice (D).

39. B

Explanation: Use the given expression and substitute 6 for f and -3 for g:

$f * g = \frac{f}{g} + \frac{f-g}{2} = \frac{6}{-3} + \frac{6-(-3)}{2} = -2 + \frac{9}{2} = \frac{-4}{2} + \frac{9}{2} = \frac{5}{2} = 2.5$, choice (B).

40. A

Explanation: There are three different possible boy/girl constructions of this group:

1 boy, 2 girls
2 boys, 1 girl
3 boys, 0 girls

We'll need to find the number of combinations for each one.

First, 1 boy, 2 girls. If there are 4 boys to choose from, the number of possibilities for the 1 boy is 4. Since there are 5 girls to choose from, we'll need to use the combinations formula to find the number of possible pairs of girls:

$\frac{n!}{k!(n-k)!} = \frac{5!}{2!3!} = \frac{5\times4}{2\times1} = 5 \times 2 = 10$

Thus, the number of possible 1 boy, 2 girl groups is $4 \times 10 = 40$

Next, 2 boys, 1 girl. There are 5 girls to choose from, so 5 possible 1-girl groups. Use the combinations formula for the number of possible pairs of boys:

$\frac{n!}{k!(n-k)!} = \frac{4!}{2!2!} = \frac{4\times3}{2\times1} = 6$

So, the number of groups like this is $5 \times 6 = 30$

Finally, 3-boy groups. There are 4 boys, so any 3-boy group would leave out exactly 1 boy. Since there are 4 different boys that could be excluded, the number of such groups is 4.

The total number of possible groups, then, is $40 + 30 + 4 = 74$, choice (A).

41. D

Explanation: Treat this as two separate combinations problems. The first consists of choosing 3 stock funds from a group of 8, and the second consists of choosing 3 bond funds from a group of 4. The first one involves applying the combinations formula:

$\frac{n!}{k!(n-k)!} = \frac{8!}{3!5!} = \frac{8\times7\times6}{3\times2\times1} = 8 \times 7 = 56$

The second doesn't require a formula at all. Each combination of three bond funds involves leaving out exactly one of the four funds. Thus, there are four possible groupings – one for each of the bond funds that could be left out.

To combine combinations, multiply:

$56 \times 4 = 224$, choice (D).

42. C
Explanation: Given the rule for $*$, plug in the given values for the variables, $j = 4$ and $k = 3$:
$j^2 - 2jk + k^2$
$(4)^2 - 2(4)(3) + (3)^2$
$16 - 24 + 9 = 1$, choice (C).

43. D
Explanation: Start by ignoring the fact that there are two different representatives of each company. If we were selecting 3 companies from a group of 4 companies, there would be 4 possible combinations: each one would exclude one of the 4 companies.

Of course, it isn't that easy. For each of the 4 company combination, there are a number of possible personnel choices. For each company that is chosen, there are 2 people to choose from. Thus, for any set of 3 companies, there are $2 \times 2 \times 2 = 8$ personnel choices. So, there are 4 possible company combinations, and 8 personnel choices for each of those combinations, so a total of $4(8) = 32$ combinations, choice (D).

44. D
Explanation: Don't get bogged down in the function language: the goal is to compare each of the 5 numbers by finding the product of the prime factors of each. That'll take some time, as it requires finding the distinct prime factors of each number:
(A) $84 = 2^2(3)(7)$, so $2 \times 3 \times 7 = 42$
(B) $90 = 2(3^2)(5)$, so $2 \times 3 \times 5 = 30$
(C) $95 = 5(19)$, so $5 \times 19 = 95$
(D) $96 = 2^5(3)$, so $2 \times 3 = 6$
(E) $100 = 2^2(5^2)$, so $2 \times 5 = 10$
The smallest product is that of the prime factors of 96, choice (D).

45. E
Explanation: Start by looking only at the five-character passwords. Any of 26 letters can be used for each of the first four characters, then any of the 10 digits can be used for the fifth. The number of possible five-character passwords is the product of those five numbers:
$26 \times 26 \times 26 \times 26 \times 10 = 10(26^4)$
The six-character passwords are based on a similar principle, only there are 5 26's instead of 4:
$26 \times 26 \times 26 \times 26 \times 26 \times 10 = 10(26^5)$
The total number of passwords is the sum of those two totals:
$10(26^4) + 10(26^5) =$
$10(26^4) + 10(26^4)(26) =$
$10(26^4)(1 + 26) =$

$10(27)(26^4) = 270(26^4)$, choice (E).

46. B

Explanation: To answer this question quickly, it's helpful to have a trick to find the sum of a set of consecutive integers (or evens, in this case). Since $h(18)$ represents the sum of the evens between 2 and 18, let's start with that example. The number of evens is found by taking the difference, dividing by 2, and, since the word "inclusive" is present, adding one:

$\frac{18-2}{2} + 1 = 9$

Next, find the average of the numbers. That's simply the average of the endpoints:

$\frac{18+2}{2} = 10$

The sum is equal to the product of the number of terms and the average of the terms:

$9(10) = 90$

Thus, $h(18) = 90$

Now, go through the same process for $h(10)$, the sum of evens from 2 to 10, inclusive:

$\frac{10-2}{2} + 1 = 5$

$\frac{10+2}{2} = 6$

$5(6) = 30$

The quotient, then, is:

$\frac{h(18)}{h(10)} = \frac{90}{30} = 3$, choice (B).

47. C

Explanation: The term "primeness" is the only challenging part of this problem; get past the wording, and you'll see that all you have to do is identify which of the 6 numbers are prime. 2, 7, and 11 are the only primes in the sequence. The fraction of the terms that are prime is $\frac{3}{6} = \frac{1}{2}$, choice (C).

48. D

Explanation: Start with the numbers from 751 up to 799. There are three ways that numbers will have at least two digits equal to each other:

XXY

XYX

YXX

In the form XXY, those are numbers 770 to 779, inclusive: 10 numbers.

In the form XYX, those are numbers such as 757. There are five numbers greater than 750, but we've already counted 777, so there are 4 to add.

In the form YXX, those are numbers such as 755. There are five numbers greater than 750, but we've already counted 777, so there are 4 to add.

In the 800s, the reasoning is the same, but the numbers are different.

XXY: 880 through 889 inclusive: 10 numbers.

XYX: 808 through 898 inclusive, except for 888, which is already counted: 9 numbers.

YXX: 800 through 899 inclusive, except for 888, which is already counted: 9 numbers.

There is the same number of possibilities in the 900s as in the 800s.

700s: $10 + 4 + 4 = 18$

800s: $10 + 9 + 9 = 28$

900s: 28

Total: $18 + 28 + 28 = 74$, choice (D).

49. D

Explanation: The expression for each term is a monster; you don't have much of an alternative to jumping in and trying a few values of k. Since (B), (C), and (D) are bunched together, odds are the correct answer is one of those. Try (C):

$\left|(-2)^{10-5}(\frac{1}{2^5})\right| = \left|(-2)^5(\frac{1}{32})\right| = \left|(-32)(\frac{1}{32})\right| = |(-1)| = 1$

That's not less than one, so it's clearly not correct. We're looking for a term that is less than 1, so the corresponding value of k is probably one greater or one smaller. To get a result less than one, we'll want the denominator to be larger, so we want 2^k to be larger, which means we need k to be larger. Thus, 6 is a reasonable choice. Test it to confirm that the result is less than 1:

$\left|(-2)^{10-6}(\frac{1}{2^6})\right| = \left|(-2)^4(\frac{1}{64})\right| = \left|(16)(\frac{1}{64})\right| = \left|(\frac{1}{4})\right| = \frac{1}{4}$

That's less than 1. Not only that, we know that 5 is not less than 1. It's a reasonable assumption, based on the work we've done, that as k gets smaller, the expression results in a larger number, so 6 would appear to be the smallest value of k that has a result less than 1. Choice (D) is correct.

50. A

Explanation: It's easy to get distracted by all of the wording in this question, but if you can make it through, you'll see that each color has an equal chance of being selected for any of the three parts of the sign. Therefore, light blue (like any other color) has a $\frac{1}{9}$ chance of being selected for any of the three roles. More specifically, it has a $\frac{1}{9}$ chance of being selected for the border and a $\frac{1}{9}$ chance of being selected for the background. The probability that it will be selected for one or the other, then, is the sum of those two:

$\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$, choice (A).

51. D

Explanation: This is a rare type of symbolism question in which you don't always apply the same rule with the same symbol. Since 7 is odd, we apply the relevant rule:

$\lfloor x \rceil = 2a + 1$

$\lfloor 7 \rceil = 2(7) + 1 = 15$

Since 4 is even, we apply the other rule:

$\lfloor x \rceil = \frac{a}{2} + 1$

$\lfloor 4 \rceil = \frac{4}{2} + 1 = 2 + 1 = 3$

Thus, $\lfloor 7 \rceil \times \lfloor 4 \rceil = 15(3) = 45$

45 isn't the answer–45 must be equal to one of the answers, however. We could go evalute each of the choices, but rather than solving 5 equations, let's solve 2. Either 45 is equal to $\frac{a}{2}+1$, where a is even, or it's even to $2a+1$, where a is odd. Consider each possibility:

$45 = \frac{a}{2}+1$
$\frac{a}{2} = 44$
$a = 88$
a is even, so this works. $45 = \lfloor 88 \rceil$, choice (D).
For the sake of completeness, look at the other option:
$45 = 2a+1$
$44 = 2a$
$a = 22$
$\lfloor 22 \rceil$ is among the answers, but that rule only applies when a is odd, so it doesn't apply to 22. Choice (D) is correct.

52. E
Explanation: The best way to think about this sequence of six numbers is a sum, starting with n: $n+(n+1)+(n+2)+(n+3)+(n+4)+(n+5) = 6n+15$. No matter what number you start with, $6n$ is an even multiple of 3, so since 15 is an odd multiple of three, the sum must be odd and a multiple of three. Thus, roman numerals II and III are true, while I is false. Choice (E) is correct.

53. C
Explanation: Start by finding the number of possibly junior professor groups and tenured professor groups. If there are 5 tenured professors, we need the combinations formula to determine the number of possible 3-person teams:
$\frac{n!}{k!(n-k)!} = \frac{5!}{3!2!} = \frac{5\times 4}{2\times 1} = 10$
Since there are 3 junior professors, any selection of 2 junior professors will exclude exactly one. There are 3 different junior professors, so there are 3 different ways to exclude one junior professor.

We're could put together a pair from any of the 3-person tenured teams and any of the 2-person junior teams, so the resulting number of combinations is the product of the two sub-group combinations:
$10 \times 3 = 30$, choice (C).

54. A
Explanation: Since there is only one possible dancer to place in the middle position, we can ignore him. The number of ways the dancers can be arranged depends only on how many ways the four female dancers can be arranged. Any of the four dancers can be placed in the first spot on the far left. Once that dancer is placed, there are 3 to choose from for the next spot, then 2 for the next spot, and finally only one possibility for the final spot. The number of possible arrangements, then, is the product of those numbers: $4\times 3\times 2\times 1 = 24$, choice (A).

55. E

Explanation: First, evaluate the expression inside the parentheses. If $j = 3$ and $k = -3$, then $\frac{(j^2k^2)}{3} = \frac{9(9)}{3} = \frac{3^4}{3} = 3^3$. (We don't have to be working with exponents here, but since that's the form of the answer choices, it'll end up being most efficient that way.)

Now, $j = 3^3$ and $k = 3$. Thus, $\frac{(j^2k^2)}{3} = \frac{(3^3)^2(3^2)}{3} = \frac{3^6 3^2}{3} = \frac{3^8}{3} = 3^7$, choice (E).

56. B

Explanation: Because the cone is inscribed in a cube, each dimension of the cone is closely related to the length of a side of the cube. For instance, because the base of the cone largely coincides with the base of the cube and the heights are equal, the height of the cone is s, where s is a side of the cube.

The radius of the base of the cone takes a bit more work. Forget about all three dimensions for now: the base of the cone is a circle inscribed in a square–the base of the cube. Here, the length of a side is equal to the diameter of the circle. Since the diameter is twice the radius:

$s = d = 2r$

$r = \frac{s}{2}$

Now we can relate the height and the radius of the cone in terms of s:

$r : h$

$\frac{s}{2} : s$

$\frac{1}{2} : 1$

$1 : 2$, choice (B).

57. B

Explanation: Another way to put this question is: $f(n)$ is the product of distinct prime factors, so the prime factorization of $f(n)$ must contain only one of each prime factor. For instance, 9 could not be $f(n)$, because $9 = 3^2$. 10, however, could be $f(n)$, since $10 = 2 \times 5$ – only one of each prime number.

So, consider each choice, starting with the largest number:

$99 = 9 \times 11 = 3^2 \times 11$ – two 3's, so it doesn't work.

$95 = 5 \times 19$ – one of each. It works.

There's no need to go any further, since even if the other 3 chocies are acceptable values of $f(n)$, they are smaller than 95. Choice (B) is correct.

58. E

Explanation: Each term can be written in terms of any other term in the sequence. For instance:

$a_2 = \frac{1}{2}a_1$

$a_3 = \frac{1}{2}a_2 = \frac{1}{2}(\frac{1}{2}a_1)$

And so on, until:

$a_6 = \frac{1}{2}a_5 = \frac{1}{2}(\frac{1}{2}a_4) = \frac{1}{2}(\frac{1}{2}(\frac{1}{2}a_3)) = \frac{1}{2}(\frac{1}{2}(\frac{1}{2}(\frac{1}{2}a_2))) = \frac{1}{2}(\frac{1}{2}(\frac{1}{2}(\frac{1}{2}(\frac{1}{2}a_1))))$

or, in more convenient form:

$a_6 = \frac{1}{32}a_1$

So if $a_3 + a_6 = 72$, then:

$\frac{1}{4}a_1 + \frac{1}{32}a_1 = 72$
$8a_1 + a_1 = 72(32)$
$9a_1 = 72(32)$
$a_1 = 8(32) = 256$, choice (E).

59. D
Explanation: This is a combinations problem: we're selecting a subset of 2 from a larger set of 12, and the order of selection doesn't matter. Plug in 12 and 2 to the combinations formula:
$\frac{n!}{k!(n-k)!} = \frac{12!}{2!10!} = \frac{12\times11}{2\times1} = 6 \times 11 = 66$, choice (D).

60. B
Explanation: First, determine how many possible combinations there are of the four expressions. Since there are four expressions and we'll choose two of them, we can use the combinations formula:
$c = \frac{n!}{k!(n-k)!} = \frac{4!}{2!2!} = \frac{4\times3}{2} = 6$
Now determine how many of those combinations fit the requirements. The product must result in an a^2, which rules out $3a+b$, since $3a$ times a will result in $3a^2$. The product must also result in a positive k, since the form is $+k$ and k must be positive. That rules out any combination where one of the b terms is positive and the other is negative. The only remaining pair is:
$(a-b)(a-3b)$
That's just one, so the probability of choosing that one is $\frac{1}{6}$, choice (B).

61. A
Explanation: There are only a few ways that at least four of the cards will be marked with a square. First, all five could be marked with a square. Second, exactly four could be marked with a square. There are five ways for that to happen: any of the five cards could be marked with a circle, and the remaining cards marked with a square. The probability of our desired outcome is the sum of all of those possibilities.
That's a total of six distinct outcomes:
SSSSS
CSSSS
SCSSS
SSCSS
SSSCS
SSSSC
The five-squares outcome has a probability as follows:
$(\frac{1}{4})^5$
since the probability of selecting a square is $\frac{1}{4}$, and that must happen five consecutive times.
The probability of any of the other five is:
$\frac{3}{4}(\frac{1}{4})^4$
since the probability of selecting a circle is $\frac{3}{4}$, the probability of selecting each of the squares is $\frac{1}{4}$.

Since there are five ways to select one circle and four squares, multiply that probability by 5:

$5(\frac{3}{4}(\frac{1}{4})^4)$

Then, to consider the five-squares outcome as well, add that probability:

$5(\frac{3}{4}(\frac{1}{4})^4) + (\frac{1}{4})^5 =$

$\frac{15}{4}(\frac{1}{4})^4 + \frac{1}{4}(\frac{1}{4})^5 =$

$(\frac{15}{4} + \frac{1}{4})(\frac{1}{4})^4 =$

$4(\frac{1}{4})^4 = (\frac{1}{4})^3$, choice (A).

62. B

Explanation: First, we are only considering integer coordinates. So, while the width of the rectangle extends from -7.5 to 2.5, we're only considering the range from -7 to 2, which includes 10 points. The height extends from -1.5 to 4.5, we're only concerned with -1 to 4, a total of 6 points. Thus, there are a total of 60 points that have integer coordinates and are within the rectangle.

Next, determine how many have two negative coordinates. Those must have x values between -7 and -1, inclusive, and a y value of -1. That's 7 x values and only one possible y value, for a total of 7 acceptable points. The probability of selecting an acceptable point, then, is $\frac{7}{60}$, choice (B).

63. D

Explanation: This is a combinations question in which we are given the number of combinations. On the GMAT, if this occurs, it is extremely likely that the value of k (in this case, the subset of desired meeting rooms) is 2. Set up the equation:

$c = \frac{n!}{k!(n-k)!}$

$105 = \frac{n!}{2!(n-2)!}$

Recognize that $n! = n(n-1)(n-2)!$, so the equation can be simplified:

$105 = \frac{n(n-1)}{2}$

$210 = n^2 - n$

$n^2 - n - 210 = 0$

$(n + 14)(n - 15) = 0$

$n = 15$ or $n = -14$

There must be a positive number of meeting rooms, so $n = 15$, choice (D).

64. B

Explanation: To find the probability, you'll need the number of desired outcomes (pairs of numbers from the sets whose product is 24) and the number of possible outcomes. The possible outcomes are the number of possible pairs of numbers from the two sets, which is the product of the number of terms in each set: $4(5) = 20$.

The desired outcomes are as follows, with numbers from A and B, respectively:

2 and 12

4 and 6

6 and 4
Since there are 3 such pairs, the probability is $\frac{3}{20} = 0.15$, choice (B).

65. B
Explanation: Since characters may be repeated, there are 10 possibilities for each of the first two characters (the digits from 0 to 9) and 26 possibilities for each of the last two characters. The total number of permutations, then, is the product of those four numbers of possibilities:
$10 \times 10 \times 26 \times 26 = 100 \times 676 = 67,600$, choice (B).

66. A
Explanation: Start by translating the question:
$p + q \geq 2(3^m + 4^m)$?
Statement (1) is sufficient. You can plug those two equations into the left side of the equation so that the only remaining variable is m:
$3^{m+1} + 2^{2m+1} \geq 2(3^m + 4^m)$?
Simplify:
$3^m(3) + 2^{2m}(2) \geq 2(3^m) + 2(2^{2m})$?
$3(3^m) \geq 2(3^m)$?
$3 \geq 2$?
The answer is "yes."
Statement (2) is not sufficient. On its own, it allows us to evaluate the right side of the inequality, but not the left. Choice (A) is correct.

67. D
Explanation: It'll save you some time to recognize that the two statements say the same thing. There are only three possible operators that the symbol could stand for, and of the three, only subtraction means that $m \bigtriangledown 2 \neq 2 \bigtriangledown m$. Thus, both statements say that the symbol stands for subtraction.
Further, since you only have to concern yourself with the one operator, you don't even have to determine whether the answer is "yes" or "no." With only one operator, there is only one possible answer. Regardless of which answer it is, the statements are each sufficient, and choice (D) is correct.

68. C
Explanation: There are two ways for the equation in the question to be true. If a is positive, then it is true if $a = b - c$. If a is negative, then it is true if $-a = b - c$, or put another way, $a = c - b$. To answer the question, we need to know whether a is positive or negative, and if the corresponding equation is true.
Statement (1) is insufficient. $a + c \neq b$ is the same as $a \neq b - c$, which means that, if a is positive, the answer is "no." However, it doesn't tell us what the answer is if a is negative.
Statement (2) is also insufficient: it gives us the sign of a, but nothing about how it relates to b and c.

Taken together, the statements are sufficient. Since we know a is negative, we know the question asks, "Is $a = c - b$?" (1) tells us that that is not true, so the answer is "no." Choice (C) is correct.

69. D

Explanation: The remainder when a number is divided by 10 is the units digit of the number. 9 to an integer power will always have one of two units digits: 1 or 9. For instance, here are the first 4 integer powers of 9:

9
81
729
6,561

You don't even need to do the math to find 729 or 6,561: the units digit is simply the product of two units digits so if you know $9^2 = 81$, you can multiple 1 times 9 to find the units digit of 9^3. Long story short, all odd powers of 9 have a units digit of 9, and all even powers have a units digit of 1. To answer the question, we only need to know whether the exponent is even or odd.

Statement (1) is sufficient. Whether or not p and q are even or odd, if they are the same, their sum is even. Thus, $p + q + 1$ is an odd number, so the units digit of 9^{p+q+1} is 9.

Statement (2) is also sufficient. Here, we know that the exponent is 5. Even if we haven't worked out the pattern discussed above, recognize that we can find the units digit of 9^5. Choice (D) is correct.

70. B

Explanation: First, recognize that $a^2 - 2ab + b^2 = (a - b)^2$.

Statement (1) is insufficient. A number that has a remainder of 2 when divided by 3 can be written as follows:

$3i + 2$

So, if $a - b = 3i + 2$, then $a^2 - 2ab + b^2 = (3i + 2)^2 = 9i^2 + 12i + 4$

$9i^2$ must be a multiple of 9, but we don't know whether $12i$ is a multiple of 9. Depending on the value of i, it could be, but it could also have a remainder. Without knowing how $12i$ relates to 9, we can't answer the question.

Statement (2) is sufficient. A number that has a remainder of 2 when divided by 9 is written like this:

$9i + 2$

So now, $a^2 - 2ab + b^2 = (9i + 2)^2 = 81i^2 + 36i + 4$

$81i^2$ must be a multiple of 9, as must $36i$. So, the remainder when $a^2 - 2ab + b^2$ is divided by 9 is 4. Choice (B) is correct.

71. C

Explanation: To answer the question, you'll need to know the value of x when $y = 0$. In other words, you'll be working with the equation:

$0 = (x + k)^2$
$0 = x + k$
$x = -k$

If you can find the value of k, you can answer the question.

Statement (1) is insufficient. Plug in the point's values for x and y:

$9 = (1 + k)^2$

$1 + k = 3$ or $1 + k = -3$

$k = 2$ or $k = -4$

Statement (2) is also insufficient. Follow the same procedure:

$1 = (-1 + k)^2$

$-1 + k = 1$ or $-1 + k = -1$

$k = 2$ or $k = 0$

Taken together, the statements are sufficient. The only value of k that satisfies both statements is $k = 2$, which means that $x = -2$, so the point at which the graph intersects the x-axis is $(-2, 0)$. Choice (C) is correct.

72. A

Explanation: Statement (1) is sufficient. If 5 is divisible by y, y must be either 1 or 5. All of the other terms in the sequence are also divisible by 1 and 5.

Statement (2) is not sufficient. We don't know anything about y from this statement alone, so for all we know, it could be 79, which most of the terms in the sequence are not divisible by. Choice (A) is correct.

73. D

Explanation: Statement (1) is sufficient. If 11 senators who voted for both X and Y voted for Z, that's 11 senators who voted for all three. Any senator who did not vote for both X and Y did not vote for all three, so 11 is the total we're looking for.

Statement (2) is also sufficient. If 4 of those who voted for X and Z did not vote for Y, 11 did. By the same reasoning as (1), those 11 voted for all 3, and any senator who did not vote for both X and Z did not vote for all three, so we don't need to concern ourselves with them. Choice (D) is correct.

74. C

Explanation: Statement (1) is insufficient, but it's worth working through the algebra to check. Recognize that most of the terms in the equation in (1) are squares of the corresponding terms in the question. To compare more carefully, square the equation in the question:

$(2^k + 3^k)^2 = m^2$?

$2^{2k} + 2(2^k 3^k) + 3^{2k} = m^2$?

$4^k + 9^k = m^2 - 2(6^k)$?

That's very close to (1), just that it includes $2(6^k)$ instead of 12. Since we don't know the value of k, we can't answer the question yet.

Statement (2) is also insufficient. While we're given k, we don't know m, and we have no way of finding it.

Taken together, the statements are sufficient. Since $k = 1$, $2(6^k) = 2(6) = 12$, so (1) is equivalent to the equation in the question, indicating that the answer is "yes." Choice (C) is correct.

75. D
Explanation: When b is positive, $|b| = b$; when b is negative, $|b|$ and b just have opposite signs. Thus, the only two possible values of a are 1 (when $|b| = b$) and -1 (when they are not equal).
Statement (1) is sufficient: if b is positive, $|b| = b$ and $a = 1$.
Statement (2) is also sufficient: there are only two possible values of a, and if it is larger than -1, it must be 1. Choice (D) is correct.

76. C
Explanation: $y^2 - y = y(y-1)$, which is the product of two consecutive numbers. One must be even while the other is odd. The product is divisible by 4 if and only if the even number is divisible by 4.
Statement (1) is insufficient. $y^2 + y = y(y+1)$, which is also the product of two consecutive numbers: the larger of the two in the question, and the one larger than that. If that product is not divisible by 4, then the even number of the two of those is not divisible by 4. If $y-1$ and $y+1$ are even, and $y+1$ is not divisible by 4, then $y-1$ must be divisible by 4, meaning that $y(y-1)$ must be divisible by 4; but if y is even, then it is not divisible by 4, meaning that $y(y-1)$ is not divisible by 4.
Statement (2) is also insufficient. $y^3 - y = y(y^2 - 1) = y(y+1)(y-1)$, the product of three consecutive integers, two of which are the two in the question. If that product is divisible by 4, then either y is divisible by 4 (and even) or both $y-1$ and $y+1$ are even. In the first case, $y(y-1)$ is divisible by 4. In the second, it's possible that $y-1$ is not divisible by 4, meaning that $y(y-1)$ is not, as well.
Taken together, the statements are still insufficient. If $y(y+1)(y-1)$ is divisible by 4 and $y(y+1)$ is not, that means that $y-1$ must be divisible by 4. Because $y(y+1)$ is not divisible by 4, we can rule out the possibility in (2) that y is divisible by 4, meaning that $y-1$ and $y+1$ are both even. If $y+1$ was divisible by 4, $y(y+1)$ would be divisible by 4; since it is not, $y-1$ must be divisible by 4. Thus, $y(y-1)$ must be divisible by 4. Choice (C) is correct.

77. C
Explanation: Statement (1) is insufficient. At its simplest level, we can rule this out because we don't know anything about b. As it happens, when a is between 0 and 2, $2a$ is always greater than a^2, but if b is large enough, then $a^2 + b$ would be greater than $2a$.
Statement (2) is also insufficient: in this case, we don't have any information about a.
Taken together, the statements are sufficient. Since b is negative, $a^2 + b$ is less than a^2. Since when a is between 0 and 2, a^2 is less than $2a$, $a^2 + b$ must be less than $2a$ as well. Choice (C) is correct.

78. C
Explanation: Statement (1) is insufficient. If the product of the x-intercepts is negative, one must be positive and the other negative. The line

with a positive x-intercept must have a positive slope, since it must go up and to the right in order to intersect the x-axis to the right of the origin. The line with a negative x-intercept could have a positive or negative slope, depending on whether the x-intercept is to the right or left of -2.

Statement (2) is also insufficient. If the product of the y-intercepts is positive, then either both y-intercepts are negative or both are positive. If both are positive, both slopes must be positive, since the lines must move up and to the right to intersect the y-axis above the origin. If both are negative, both slopes could be negative, or the two slopes could have different signs.

Taken together, the statements are sufficient. The line that (1) tells us must have a positive x-intercept must have a negative y-intercept, since it has a positive slope. Thus, according to (2), the other line must have a negative y-intercept as well. A line that has a negative x-intercept and a negative y-intercept must have a negative slope, so we know that the two slopes have different signs, meaning that their product is negative. Choice (C) is correct.

79. B

Explanation: Statement (1) is insufficient. We don't need to find the value of m, but it would help to know how many 10s are included among its factors. To determine that, identify the factors that share factors with 10, specifically those with 5's as factors:

5, 10, 15, 20, 25, 30, 35, 40

Since 40! includes the product of those 8 numbers, it includes all of the 5's that are factors of those numbers. Each of those numbers has one 5 as a factor, except for 25, which has two. Thus, one of 40!'s factors is 5^9. There are 20 even numbers among 40!'s factors, so 2^9 must be a factor as well, meaning that 10^9 is a factor. If 10^9 is a factor, there are many possible factors in the form 10^k: k could be any integer from 1 to 9, inclusive.

Statement (2) is sufficient. n is a much smaller number, and among the first 9 integers, there is only one multiple of 5. Thus, 9!'s factors only include 5^1, not 5 to any higher power. 9!'s factors also include a 2, so its factors include 10^1, but not 10 to any higher power (that would require more 5's). Thus, $k = 1$. Choice (B) is correct.

80. D

Explanation: Statement (1) is sufficient. If we know the value of A_2 and A_4, we can find the value of A_3, since the 2nd and 3rd term have a product equal to the 4th term. Once we have the 3rd and 4th terms, we can find the 5th term, and so on up to the 11th term.

Statement (2) is also sufficient. In this case, having the 4th and 6th terms allow us to find the 5th term. With the 5th and 6th, we can find the 7th, and so on, up to the 11th term. Choice (D) is correct.

81. C

Explanation: Statement (1) is insufficient. If $9p$ is divisible by 6, then $9p$ must have as factors all of the prime factors of 6: 2 and 3. 9 has 3 as a factor, but not 2, so p must have 2 as a factor. p could have 3 as a factor as well, but we don't know that.

Statement (2) is also insufficient. Using similar reasoning, $8p$ must have as factors both 2 and 3. 8 has 2 as a factor, but not 3, so p must have 3 as a factor. We don't know whether p is divisible by 2.

Taken together, the statements are sufficient. (1) tells us that p is divisible by 2; (2) tells us it is divisible by 3. If it is divisible by both, it must be divisible by 6. Choice (C) is correct.

82. D

Explanation: The question gives us two equations. First, a milliliters of solution plus b milliliters of solution is a total of c milliliters of solution, so:

$a + b = c$

Also, we know the amounts of substance S in each solution, so:

$0.2a + 0.08b = 0.1c$

That's two equations with three variables. To answer the question, we need a third distinct linear equation.

Both statements (1) and (2) are sufficient on their own: each offers a third equation, so we can solve for the value of a. Choice (D) is correct.

83. A

Explanation: Statement (1) is sufficient: it defines the sequence as one in which each term is 2 more than the last. Based on the definition given for an arithmetic sequence, that's what we're looking for: the difference between each term is a constant.

Statement (2) is insufficient. It's possible that the resulting sequence would be an arithmetic progression, as in $\{1, 5, 9, 13...\}$, but it's also possible that it is not, as in $\{1, 3, 7, 17, 21...\}$. Choice (A) is correct.

84. B

Explanation: As is often the case in abstract sequence problems, it's helpful to work out a few of the terms to get a more concrete idea of what the sequence consists of.

$k = 1$, $s_k = 2(1) - 1 = 1$

$k = 2$, $s_k = -s_1 = -1$

$k = 3$, $s_k = 2(3) - 1 = 5$

$k = 4$, $s_k = -5$

As should become clear by now, when you sum the terms of the sequence, each pair of terms cancels each other out. So, when there is an even number of terms in the sequence, the sum of the terms is 0. The only time the sum is not 0 is when there is an odd number of terms: in that case, the sum of terms is equal to the last term.

Statement (1) is insufficient. In fact, we can figure that out without any further information. Given the definition of the sequence, we know that the sum of the first 11 terms is positive.

Statement (2) is sufficient. If the number of terms in the sequence is odd, the terms will not cancel each other out: the sum of the first n terms will be equal to the final term, s_n, which must be positive, since n must be positive. Choice (B) is correct.

85. B

Explanation: Algebraically, the question asks:
$\frac{p}{q} = \frac{x}{y}$?
or:
$py = qx$?
Statement (1) is insufficient. Simplified, this says:
$\frac{p}{y} = \frac{q}{x}$
$px = qy$
That's one equation with four variables; it certainly doesn't tell us enough to determine whether a separate equation with the same four variables is also true.
Statement (2) is sufficient. This one simplifies to:
$\frac{p}{x} = \frac{q}{y}$
$py = qx$
That's the same equation the question is asking about, so now we can confirm that the equation is true, and the answer to the question is "yes." Choice (B) is correct.

86. A

Explanation: When looking for the remainder, ignore constants: in this case, the remainder when $3 + 5k$ is divided by 4 is dependent only on the value of $5k$. $3 + 5k$ has a different remainder than $5k$ does, but for a data sufficiency question, it doesn't matter: if we know one, we know the other.

Statement (1) is sufficient. If $k - 1$ is divisible by 4, then the remainder when k is divided by 4 is 1. Another way to put that is that $k = 4i + 1$, where i is an integer. Thus, $5k = 5(4i + 1) = 20i + 5$. $20i$ is divisible by 4, and 5 has a remainder of 1 when divided by 4, so the remainder when $5k$ is divided by 4 is 1.

Statement (2) is insufficient. Given a range of values for k, there are many possible answers. If $k = 1$, the remainder is 0; if $k = 2$, the remainder is 1. Choice (A) is correct.

87. E

Explanation: Judging only from the question, it seems unlikely that we'll be able to answer this question: given males, females, and three colors of hair, there are six possible gender/hair color combinations, and we're looking for the sum of the probabilities of 4 of them: female/blonde, female/brown, female/red, and male/brown.

Statement (1) is insufficient. This gives us the probability of one of the four we're looking for, but no information aabout the other three.

Statement (2) is also insufficient: this provides the difference between two probabilities, neither of which is a specific probability that we're looking for. We could recast the question as the sum of the probabilities that a student is female or a student is male with brown hair, but still, the two variables in the statement don't correspond with the ones we're looking for.

Taken together, the statements are still insufficient. Consider a couple of possibilities. If the probability that a student is female is 0.5, then the probability that a student has brown hair is 0.25. Since 0.1 of the students are females with brown hair, that means 0.15 are males with brown hair, so the total of the four probabilities we're looking for is 0.65.

But try a different starting value. If the probability a student is female is 0.6, the probability that a student has brown hair is 0.35. That means 0.25 are males with brown hair, so the total of the four probabilities is 0.85. Choice (E) is correct.

88. C

Explanation: Statement (1) is insufficient. If $b > a$, then $a - b$ is less than 0, which means that $|a - b| = -(a - b) = b - a$. Thus, we can simplify the question to:

$|a| - |b| \geq b - a$?

How we evaluate the left side depends on two things: whether a is positive, and whether b is positive. Given that $b > a$, there are three possibilities:

- both are positive
  - $|a| - |b| = a - b$
  - Is $a - b \geq b - a$? No.
- both are negative
  - $|a| - |b| = -a - (-b) = b - a$
  - Is $b - a \geq b - a$? Yes.
- b is positive and a is negative
  - $|a| - |b| = -a - (b)$
  - Is $-a - b \geq b - a$?
  - Is $-b \geq b$? Since b is positive, no.

Statement (2) is also insufficient. We already know that if a is positive and b is positive, the question simplifies to:

Is $a - b \geq b - a$?

However, without knowing whether b is greater than a, we can't answer the question; if a is greater, the answer is "yes."

Taken together, the statements are sufficient. Of the three possibilities offered by (1), only the first is an option for (2), so we know the question simplifies to:

Is $a - b \geq b - a$?

Since we know that $b > a$, we know the answer is "no." Choice (C) is correct.

89. B

Explanation: Statement (1) is insufficient. Simplify to isolate 3^n:

$3^n 3^{-2} < 500$

$\frac{3^n}{9} < 500$
$3^n < 4,500$
That could be greater than 1,000, but we don't know.

Statement (2) is sufficient. Without doing any math, you could recognize that (2) is a one-variable equation, so you can solve for the value of n. Given that value, you can find the value of 3^n and compare it to 1,000.

Algebraically, isolate 3^n:
$3^n = 3^n(3) - 486$
$3^n - 3^n(3) = -486$
$3^n(1-3) = -486$
$3^n = \frac{-486}{-2} = 243$
Choice (B) is correct.

90. B

Explanation: Another way to phrase the information in the question is: r is a multiple of s and s is a multiple of t. We can also deduce that r is a multiple of t. We would know that r is even if either s or t or both were even; even if s and t were both odd, it's possible that r is even.

Statement (1) is insufficient. If st is odd, both s and t are odd. They could be, for instance, 3 and 9, which means r could be an odd number, such as 27, or it could be an even, such as 18.

Statement (2) is sufficient. If rt is even, one or the other or both must be even; if r is even, we've answered the question; if t is even, r must be even because it is a multiple of t. Choice (B) is correct.

91. E

Explanation: Since all of the expressions are contained within absolute value signs, we're talking about distances here. It may be easier to think of the four variables as landmarks along a highway, and the question as asking whether x or y is farther from q.

Statement (1) is insufficient. If y is farther from p than x is from p, we know that p is to the left of the midpoint between x and y. q could also be to the left of the midpoint (in which case it is closer to x), but it could just as easily be to the right of the midpoint.

Statement (2) is also insufficient. We know that $|p-y| > |q-y|$, which tells us that both $|q-y|$ and $|q-x|$ are less than $|p-y|$, but not which is greater.

Taken together, the statements are still insufficient. We have several pieces of information:
$|p-x| < |p-y|$
$|q-x| < |p-y|$
$|q-y| < |p-y|$
$|p-x| < |q-x|$
To put what we can in the correct order:
$|p-x| < |q-x| < |p-y|$
and
$|q-y| < |p-y|$

Still, we have no way to deduce the relationship between $|q-x|$ and $|q-y|$. Choice (E) is correct.

92. B

Explanation: To answer the question, we need to know which operator the symbol represents.

Statement (1) is insufficient: the operator could be multiplication ($1 \times 1 = 1$) or division ($\frac{1}{1} = 1$). Statement (2) is sufficient: the only operator for which $2 \triangledown 2 = 1$ is division: $\frac{2}{2} = 1$. Choice (B) is correct.

93. C

Explanation: A sequence like the one described in the question is an "arithmetic" sequence: the difference between each set of consecutive terms is the same constant. For instance, if that constant (k) is 1, the sequence could be {5, 6, 7, ...}.

Statement (1) is insufficient. We know the first term in the sequence, but we don't know the constant. If the constant is 1, none of the 16 terms will be greater than 100; if the constant is 10, many of them will be.

Statement (2) is also insufficient. Again, we know one term in the sequence, but can't deduce the constant. If $k = 1$, all of the terms in the sequence are greater than 100; if the constant if 10, many fewer are.

Taken together, the statements are sufficient. The difference between s_1 and s_{16} is $15k$ – the difference between each pair of consecutive terms is k, and there are 15 sets of consecutive terms separating the two outermost terms in the sequence. So,

$185 = 5 + 15k$

$15k = 180$

$k = 12$

Since we know the constant, we can figure out each term of the sequence, and then determine how many of them are greater than 100. Choice (C) is correct.

94. B

Explanation: First, simplify the question:

$125 > 5^{n+1}$?

$5^3 > 5^{n+1}$?

$3 > n + 1$?

$2 > n$?

Statement (1) is insufficient. Follow the same procedure to simplify:

$125 < 5^{n+2}$

$5^3 < 5^{n+2}$

$3 < n + 2$

$1 < n$

If n is greater than 1, we don't know whether it is less than 2.

Statement (2) is sufficient. If n is less than 0, it must be less than 2. Choice (B) is correct.

95. B

Explanation: Statement (1) is insufficient. Given the total population increase, we don't know how the increase was distributed among the five cities. If a disproportionate amount of the increase happened to the largest city, the standard deviation increased; if most occurred in the smallest cities, the standard devation decreased.

Statement (2) is sufficient. If each of the 5 cities increased by the same percentage, the standard deviation increased by that percentage as well. The math to prove it is quite involved, and not necessary for the GMAT – suffice it to say that when all the terms in a set increase or decrease by the same relative (percentage, not actual number) amount, the standard deviation changes by that amount, as well. Choice (B) is correct.

96. C

Explanation: Statement (1) is insufficient: we don't know anything about the infinite number of terms after the first 10.

Statement (2) is also insufficient. If the difference between each pair of terms is a constant, the numbers could be all even (if the first term is an even and the constant is even), all odd (if the first term is odd and the constant is even), or a mix (if the constant is odd).

Taken together, the statements are sufficient. If the first 10 integers are even, the constant must be even – the difference between any two even numbers is itself even. If you keep adding that even constant to even numbers, the result will always be even. Thus, every term in S is even, meaning that the number of odds is finite. Choice (C) is correct.

97. C

Explanation: Since the triangles are similar, we know that angle BCA is equal to DEF, and angle BAC is equal to DFE.

Statement (1) is insufficient. If AB=BC, then ABC is isoceles, and angles y and z are equal. The measure of those angles, however, depends on the measure of angle x, which we don't know.

Statement (2) is also insufficient. While $x + y + z = 180$, we need two of the angles, or at least another equation like the one in (1), to find the value of y.

Taken together, the statements are sufficient. (1) tells us that $y = z$, so if $z = 65$, then $y = 65$. Choice (C) is correct.

98. C

Explanation: Since any number squared must be positive, and the sum of two positives must be positive, we can multiply each side of the inequality in the question by the two denominators without worrying that we are multiplying by a negative. The result is:

$m(m^2 + 2) > n(n^2 + 1)$?

$m^3 + 2m > n^3 + n$?

Statement (1) is insufficient. If $m = n$, then $m^3 = n^3$, so we can cancel out those two terms in the question, leaving us with:

$2m > n$?

Or:

$2m > m$?

The answer to that question depends on whether m is positive or negative. If positive, $2m$ is greater; if negative, m is greater.

Statement (2) is also insufficient. This doesn't tell us anything about m, so while we can determine that $n^3 + n$ is negative, we don't know how it compares to the other half of the inequality.

Taken together, the statements are sufficient. We know from (1) that the question is equivalent to $2m > m$? If n is negative, m is negative, so $2m$ is less than m. Choice (C) is correct.

99. C

Explanation: Statements (1) and (2) on their own are not sufficient: each one only tells you about one of the two variables in the expression.

Taken together, the statements are sufficient. The remainder of a number when divided by 10 is the same as the number's units digit. Thus, m's units digit is 3, so the units digit of m^2 is 9. The same applies to n. So, $m^2 - n^2$ is the difference between two numbers with the same units digits. When the numbers are subtracted, the units digits result in a difference of 0, so $m^2 - n^2$, when divided by 10, has a remainder of 0 – the units digit is 0. Choice (C) is correct.

100. B

Explanation: $m \bigtriangledown n = n \bigtriangledown m$ only when the operation in question in addition or multiplication.

Statement (1) is insufficient: $4 \bigtriangledown 2 = 8 \bigtriangledown 4$ only when the operation is division, so since the opposite is true, the operation could be addition, subtraction, or multiplication.

Statement (2) is sufficient: if the symbol is subtraction, it is not addition or multiplication, so the answer is "no." Choice (B) is correct.