

GMAT Problem Solving: Challenge

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1 Introduction

This document contains nothing but difficult GMAT Problem Solving questions—100 of them, to be exact. While students do not generally have the same issues with the Problem Solving question format as they do with Data Sufficiency, PS has its own challenges, as well. There are more complex word problems, and unlike DS questions, you are required to solve for a single number.

As in all of my GMAT preparation resources, you'll find these questions indexed by difficulty. That doesn't mean you should skip straight to the hardest questions, or even that you should start with the easier ones. On the GMAT itself, questions won't come labeled with their difficulty level, and despite the intent of the adaptive algorithm, they won't be precisely consistent in terms of difficulty either. Each question presents its own unique challenges, and the sooner you get accustomed to changing gears with every single question, the more time you'll have to prepare for that particular challenge of the exam.

For further, more specific practice, I have produced several other resources that may help you. There is one 100-question "Challenge" set that covers only Data Sufficiency questions, as well as several "Challenge" sets on topics such as Arithmetic, Algebra, Geometry, Number Properties, and Word Problems.

Also, The GMAT Math Bible has dozens of chapters covering the content you need to know for every type of GMAT problem. It's one thing to master an approach to Problem Solving, but that is only effective if you have already conquered the math basics. If you find you are struggling with the mechanics of these problems, your time is probably better spent with the GMAT Math Bible than in doing dozens and dozens of practice problems, hoping to pick up those skills along the way.

If you find yourself having problems with only the most difficult questions, you might try my "Extreme Challenge" set, which contains only 720 and higher level questions, many of which are Arithmetic-related.

As far as strategy is concerned, there are dozens of articles at GMAT HACKS to help you with your strategic approach to Arithmetic questions. Most importantly, you should make sure you understand every practice problem you do. It doesn't matter if you get it right the first time—what matters is whether you'll get it right the next time you see it, because the next time you see it could be on the GMAT.

With that in mind, carefully analyze the explanations. Redo questions that took you too long the first time around. Review questions over multiple sessions, rather than cramming for eight hours straight each Saturday. These basic study skills may not feel like the key to GMAT preparation, but they are the difference between those people who reach their score goals and those who never do.

Enough talking; there are 100 Problem Solving questions waiting inside. Get to work!

2 Difficulty Levels

In general, the level 5 questions in this guide are 560- to 620-level questions. The level 6 questions represent a broad range of difficulty from about 620 to 720, while the level 7 questions are higher still.

Moderately Difficult (5)

1, 3, 17, 23, 29, 30, 31, 40, 42, 43, 45, 48, 49, 51, 61, 78, 82, 83, 84, 96

Difficult (6)

2, 4, 5, 6, 8, 9, 12, 13, 15, 16, 20, 21, 24, 25, 26, 27, 28, 32, 34, 36, 37, 39, 41, 44, 46, 50, 52, 54, 55, 58, 60, 62, 63, 65, 66, 68, 71, 72, 73, 75, 76, 77, 80, 81, 85, 86, 87, 88, 89, 91, 93, 94, 97, 98, 99, 100

Very Difficult (7)

7, 10, 11, 14, 18, 19, 22, 33, 35, 38, 47, 53, 56, 57, 59, 64, 67, 69, 70, 74, 79, 90, 92, 95

3 Problem Solving

Note: this guide contains both an answer key (so you can quickly check your answers) and full explanations.

1. For which of the following values of m is $\frac{m-108}{m}$ NOT an integer?
(A) 6
(B) 8
(C) 9
(D) 12
(E) 18
2. In a certain company, the ratio of the number of salespeople to total employees is 1 to 3. If three of the salespeople were to leave the company and the company did not replace them, the ratio would be 1 to 4. How many total employees does the company have?
(A) 12
(B) 18
(C) 24
(D) 27
(E) 36
3. Which of the following is the value of $\sqrt{\sqrt{0.00000001}}$?
(A) 0.1
(B) 0.01
(C) 0.001
(D) 0.0001
(E) 0.00001
4. Tank A is $\frac{1}{3}$ full of water and tank B, which has three times the capacity of tank A, is $\frac{1}{2}$ full of water. If all of the water in tank A is poured into tank B, then tank B will be filled to what fraction of its capacity?
(A) $\frac{7}{18}$
(B) $\frac{5}{9}$
(C) $\frac{11}{18}$
(D) $\frac{2}{3}$
(E) $\frac{5}{6}$

3. *PROBLEM SOLVING*

5. What is the least positive integer that is divisible by each of the integers 2 through 7, inclusive?
- (A) 210
(B) 420
(C) 840
(D) 1,260
(E) 5,040
6. If $\frac{2}{3}$ of the air in a tank is removed with each stroke of a vacuum pump, how many strokes does it take before less than 1% of the original amount of air in the tank remains?
- (A) 2
(B) 3
(C) 4
(D) 5
(E) 6
7. Twin primes are defined as prime numbers that can be expressed as p and $(p + 2)$, and any number p that is a member of such a pair is considered to “have” a twin. For example, 3 and 5 are twin primes, and 3 has a twin. Each of the following prime numbers has a twin EXCEPT
- (A) 7
(B) 13
(C) 17
(D) 23
(E) 29
- vodka: 6
tequila: 5
rum: 5
8. The table above shows the number of specialty cocktails at a certain restaurant that include each ingredient. Although no cocktail includes all three ingredients, 3 cocktails include both tequila and rum, and 2 cocktails include both tequila and vodka. What is the maximum number of cocktails that could include both rum and vodka?
- (A) 1
(B) 2
(C) 3
(D) 4
(E) 5

3. *PROBLEM SOLVING*

9. How many different positive integers are factors of 484 ?
- (A) 6
 - (B) 8
 - (C) 9
 - (D) 11
 - (E) 12
10. A rectangular box has dimensions of 8 feet, 8 feet, and z feet. In terms of z , what is the greatest possible (straight-line) distance, in feet, between any two points on the box?
- (A) $8 + z$
 - (B) $8\sqrt{2} + z$
 - (C) $8z\sqrt{2}$
 - (D) $\sqrt{64 + z^2}$
 - (E) $\sqrt{128 + z^2}$
11. If p is a positive integer less than 75 and $\frac{3p}{84}$ is an integer, then p has how many different positive prime factors?
- (A) One
 - (B) Two
 - (C) Three
 - (D) Four
 - (E) Five
12. The probability is $\frac{1}{2}$ that a certain coin will turn up heads on any given toss. If the coin is to be tossed three times, what is the probability that on at least two of the tosses the coin will turn up tails?
- (A) $\frac{1}{8}$
 - (B) $\frac{3}{8}$
 - (C) $\frac{1}{2}$
 - (D) $\frac{3}{4}$
 - (E) $\frac{7}{8}$

3. *PROBLEM SOLVING*

13. As k increases from 149 to 151, which of the following must decrease?
- I. $150 - 2k$
 - II. $150 - \frac{1}{k}$
 - III. $\frac{150}{k^2} - k$
- (A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III
14. All of the bonds on a certain exchange are designated by a 3-letter, a 4-letter, or a 5-letter code that is created by using the 26 letters of the alphabet. Which of the following gives the maximum number of different bonds that can be designated with these codes?
- (A) $26(26^3 + 26^4)$
 - (B) $26(26^3 + 26^5)$
 - (C) $27(26^3 + 26^5)$
 - (D) $27(26^3) + 26^5$
 - (E) $26^3 + 27(26^5)$
15. At the rate of k knots per m minutes, how many knots does a ship travel in y hours?
- (A) $\frac{k}{ym}$
 - (B) $\frac{km}{y}$
 - (C) $\frac{60ky}{m}$
 - (D) $\frac{60km}{y}$
 - (E) $\frac{kmy}{60}$
16. Five drainage pipes, each draining water from a pool at the same constant rate, together can drain a certain pool in 12 days. How many additional pipes, each draining water at the same constant rate, will be needed to drain the pool in 4 days?
- (A) 6
 - (B) 9
 - (C) 10
 - (D) 12
 - (E) 15

3. *PROBLEM SOLVING*

17. $(2 + \sqrt{2})(\sqrt{3} - 2)(\sqrt{2} - 2)(2 + \sqrt{3}) =$
(A) $-\sqrt{6}$
(B) -2
(C) -1
(D) $\sqrt{3}$
(E) 2
18. If x is to be chosen at random from the set $\{1, 2, 3, 4\}$ and y is to be chosen at random from the set $\{4, 5, 6, 7\}$, what is the probability that xy will be even?
(A) $\frac{1}{8}$
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) $\frac{3}{4}$
(E) $\frac{7}{8}$
19. In a certain school district, 5 percent of the x students at School A are honor students, 20 percent of the y students at School B are honor students, and 14 percent of the z students at School C are honor students. If 8 percent of the total $x + y + z$ students are honor students, what is x in terms of y and z ?
(A) $y + 4z$
(B) $3y + 1.5z$
(C) $4y + 2z$
(D) $\frac{10y+7z}{4}$
(E) $\frac{20y+14z}{3}$
20. S is a set containing 8 different numbers. T is a set containing 6 different numbers, all of which are members of S . Which of the following statements CANNOT be true?
(A) The mean of S is greater than the mean of T .
(B) The range of S is equal to the range of T .
(C) The median of S is equal to the median of T .
(D) The mean of S is equal to the mean of T .
(E) The range of S is less than the range of T .
21. If $n = 8p$, where p is a prime number greater than 2, how many different positive even divisors does n have, including n ?
(A) Two
(B) Three
(C) Four
(D) Six
(E) Eight

3. *PROBLEM SOLVING*

22. For every integer m from 1 to 100, inclusive, the m th term of a certain sequence is given by $(-1)^m(2^{-m})$. If N is the sum of the first 100 terms in the sequence, then N is
- (A) less than -1
 - (B) between -1 and $-\frac{1}{2}$
 - (C) between $-\frac{1}{2}$ and 0
 - (D) between 0 and $\frac{1}{2}$
 - (E) greater than $\frac{1}{2}$
23. If set S consists of the first 10 positive multiples of 5, what is the positive difference between the average (arithmetic mean) of S and the median of S ?
- (A) 0
 - (B) 2.5
 - (C) 5
 - (D) 25
 - (E) 27.5
24. A certain die has 10 sides, and each side has a positive integer written on it. In a board game, a player's number of points for each turn is determined by rolling the die, then multiplying the resulting integer by the next greatest integer. If the possible number of points for any turn is between 10 and 180, then the least and greatest integers on the die could be
- (A) 2 and 10
 - (B) 3 and 12
 - (C) 3 and 13
 - (D) 4 and 13
 - (E) 4 and 14
25. Which of the following CANNOT be the median of the four positive integers a , b , c , and d , where $a < b < c < d$?
- (A) $\frac{a+c}{2}$
 - (B) $\frac{b+c}{2}$
 - (C) $\frac{a+d}{2}$
 - (D) $\frac{b+d}{2}$
 - (E) $\frac{c+d}{2}$

3. *PROBLEM SOLVING*

26. The average (arithmetic mean) of n numbers is m . When one number is discarded, the average of the remaining numbers becomes p . In terms of m , n , and p , what is the discarded number?
- (A) $pn - mn - mp$
 - (B) $p(n - 1) - mn$
 - (C) $mn - pn + mp$
 - (D) $mn - p(n - 1)$
 - (E) $n(m + p)$
27. In a certain company, the ratio of the number of managers to the number of non-managers in any department must always be greater than 5 : 24. In the company, what is the maximum number of non-managers in a department that has 8 managers?
- (A) 36
 - (B) 37
 - (C) 38
 - (D) 39
 - (E) 40
28. If it is true that $n < q$ and $n > p$, which of the following must be true?
- (A) $n > -p$
 - (B) $n > -q$
 - (C) $n < -p$
 - (D) $-q < n < -p$
 - (E) None of the above
29. How many positive integers less than 30 are either an even prime number, a multiple of 3, or the sum of an even prime and a positive multiple of 3 ?
- (A) 22
 - (B) 21
 - (C) 20
 - (D) 19
 - (E) 18

3. *PROBLEM SOLVING*

30. An art dealer purchased a painting for \$360 and then offered the painting for sale for a price equal to his purchase price plus a markup that was 40% of his offered price. If the dealer sold the painting for 10% less than the price at which he offered it for sale, what was the dealer's profit?
- (A) \$180
(B) \$216
(C) \$240
(D) \$450
(E) \$540
31. $\sqrt{661}$ is between
- (A) 21 and 22
(B) 22 and 23
(C) 23 and 24
(D) 24 and 25
(E) 25 and 26
32. If $x^2 = 9y^2$, which of the following could be the value of $\frac{y}{x}$?
- (A) -3
(B) $-\frac{1}{3}$
(C) $-\frac{1}{9}$
(D) 1
(E) $\frac{1}{9}$
33. If p is a positive integer, and if the units' digit of p^2 is 1 and the units' digit of $(p+1)^2$ is 4, what is the units' digit of $(p+2)^2$?
- (A) 1
(B) 3
(C) 5
(D) 7
(E) 9
34. If $(s-5)$ is a factor of $s^2 - js + 25$, then $j =$
- (A) -10
(B) -5
(C) 0
(D) 5
(E) 10

3. *PROBLEM SOLVING*

35. For any integer p greater than 1, $*p*$ denotes the product of all the integers from 1 to p , inclusive. How many prime numbers are there between $*5*$ and $*5* + 7$, inclusive?
- (A) None
(B) One
(C) Two
(D) Three
(E) Four
36. It takes Elvys 25 minutes to drive from home to work at an average rate of 20 miles per hour. If Elvys drove the same route in 10 minutes, what would his average rate be, in miles per hour?
- (A) 25
(B) 35
(C) 40
(D) 50
(E) 60
37. When X is divided by Y , the quotient is 7 and the remainder is 2. Which of the following, in terms of Y , is the value of X ?
- (A) $7Y$
(B) $\frac{7}{2}Y$
(C) $7Y + 2$
(D) $7(Y + 2)$
(E) $7Y + \frac{2}{7}$
38. In how many arrangements can a teacher seat 2 girls and 4 boys in a row of 6 seats if the girls must occupy the second and fifth seats?
- (A) 720
(B) 48
(C) 36
(D) 24
(E) 8
39. If $\left(7^{\frac{3}{4}}\right)^n = 49$, what is the value of n ?
- (A) $\frac{3}{4}$
(B) $\frac{4}{3}$
(C) $\frac{3}{2}$
(D) 2
(E) $\frac{8}{3}$

3. *PROBLEM SOLVING*

40. If x and y are positive integers and $x^3 + y^3 < 1,000$, then the greatest possible value of x is between
- (A) 0 and 2
 - (B) 2 and 4
 - (C) 4 and 6
 - (D) 6 and 8
 - (E) 8 and 10
41. Which of the following is equal to the average (arithmetic mean) of $(2x + 1)^2$ and $(2x - 1)^2$?
- (A) $4x^2$
 - (B) $8x^2$
 - (C) $4x^2 + 1$
 - (D) $8x^2 + 1$
 - (E) $4x^2 + 2$
42. Of the following, which is greatest?
- (A) $\frac{1}{1.3}$
 - (B) 1.3^2
 - (C) 0.13
 - (D) $\frac{1.3}{13}$
 - (E) 1.3
43. The value of $\sqrt[3]{-101}$ is
- (A) between -9 and -10
 - (B) between -8 and -9
 - (C) between -4 and -5
 - (D) between -3 and -4
 - (E) undefined
44. Which of the following is equal to x^{12} for all positive values of x ?
- (A) $(x^4)^3$
 - (B) $(x^6)^6$
 - (C) $(x^3)^9$
 - (D) $\frac{x^4}{x^{16}}$
 - (E) $x^3 + x^9$

3. PROBLEM SOLVING

45. If $m = 2$ and $\frac{m-n}{p} = 1$, which of the following is NOT a possible value of n ?
- (A) -2
 - (B) -1
 - (C) 0
 - (D) 1
 - (E) 2
46. If $(2^x)(8^y) = 32$ and $(3^x)(9^y) = 81$, then $(x, y) =$
- (A) (1, 2)
 - (B) (2, 1)
 - (C) (1, 1)
 - (D) (2, 2)
 - (E) (1, 3)

+	a	b	c
x	-2	-4	4
y	6	p	5
7	q	1	-1

47. In the addition table above, each number in the table is the sum of the terms at the top of its column and the left of its row. What is the value of $p + q$?
- (A) -6
 - (B) 1
 - (C) 5
 - (D) 6
 - (E) 7
48. For how many integers n is $1^n = n^1$?
- (A) None
 - (B) One
 - (C) Two
 - (D) Three
 - (E) More than three

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49. If $\frac{x}{y} = \frac{2}{3}$, then $\frac{y}{x} - \frac{x}{y} =$
- (A) $\frac{5}{6}$
(B) $\frac{1}{3}$
(C) 0
(D) $-\frac{1}{3}$
(E) $-\frac{5}{6}$
50. A traveler drives J miles in H hours, then rides a train K miles in half the number of hours. Which of the following represents the traveler's average speed, in miles per hour, for the entire trip?
- (A) $\frac{J-K}{3H}$
(B) $\frac{3(J-K)}{2H}$
(C) $\frac{2(J+K)}{3H}$
(D) $\frac{3(J+K)}{2H}$
(E) $\frac{J+K}{2H}$
51. If a number between 0 and $\frac{1}{2}$ is selected at random, which of the following will the number most likely be between?
- (A) 0 and $\frac{3}{20}$
(B) $\frac{3}{20}$ and $\frac{1}{4}$
(C) $\frac{1}{4}$ and $\frac{1}{3}$
(D) $\frac{1}{3}$ and $\frac{2}{5}$
(E) $\frac{2}{5}$ and $\frac{1}{2}$
- 10, 3, k , m , 1, 7
52. The arithmetic mean of the list of numbers above is 5. If k and m are consecutive integers, what is the median of the list?
- (A) 3
(B) 3.5
(C) 4
(D) 4.5
(E) 5
53. If $xy = 1$, what is the value of $\frac{3(x+y)^2}{3x^2+y^2}$?
- (A) 2
(B) 3
(C) 6
(D) 9
(E) 27

3. *PROBLEM SOLVING*

54. Which of the following lists the number of points at which a circle can intersect a parallelogram?
- (A) 2, 4, and 8 only
 - (B) 2, 4, 6, and 8 only
 - (C) 1, 2, 3, 4, 6, and 8 only
 - (D) 2, 3, 4, 5, 6, 7, and 8 only
 - (E) 1, 2, 3, 4, 5, 6, 7, and 8
55. Logan and Hayley are planning to walk toward each other on the same road, Logan starting from A , Hayley starting from B . If the distance from A to B is 36 miles, Logan's walking speed is 4 miles per hour, and Hayley's walking speed is 3 miles per hour, how much longer, in hours, will it take Hayley than Logan to get to the midpoint between A and B ?
- (A) 1
 - (B) 1.5
 - (C) 2.5
 - (D) 3
 - (E) 4.5
56. The positive integer p is divisible by 11. If \sqrt{p} is an integer divisible by 3, which of the following must be a factor of $\frac{p}{11}$?
- (A) 9
 - (B) 14
 - (C) 15
 - (D) 33
 - (E) 99
57. A certain commodities exchange designates each commodity with a two- or three-character code, where each character is selected from the digits 0 through 9, inclusive, and the capital letters A through F, inclusive. If the characters may be repeated and if the same characters used in a different order constitute a different code, how many different commodities is it possible to uniquely designate with these codes?
- (A) 256
 - (B) 3360
 - (C) 3600
 - (D) 4096
 - (E) 4352

3. *PROBLEM SOLVING*

58. If p and q are integers are $pq - p^2$ is even, which of the following must also be even?
- (A) p
 - (B) q
 - (C) $p - q$
 - (D) pq
 - (E) $p - pq$
59. For every positive integer n , the function $f(n)$ is defined to be the product of all the prime numbers from 2 to n , inclusive. If p is the smallest prime factor of $f(50) + 1$, then p is
- (A) between 2 and 10
 - (B) between 10 and 20
 - (C) between 20 and 30
 - (D) between 30 and 40
 - (E) greater than 40
60. If n is divisible by 6, then the largest positive integer that must divide n^2 is
- (A) 6
 - (B) 12
 - (C) 24
 - (D) 36
 - (E) 144
61. $\frac{(9^2)(4^3)(2^3)}{72^2}$
- (A) 2^{-1}
 - (B) 2^{-2}
 - (C) 2^1
 - (D) 2^2
 - (E) 2^3
62. If $a \wedge b = ab - a(b - a)$ for all integers a and b , then $(-2) \wedge (-3) =$
- (A) -8
 - (B) -4
 - (C) 3
 - (D) 4
 - (E) 8

3. *PROBLEM SOLVING*

63. A certain club has 66 members, each of which is participating in exactly one of the six projects in which the club is currently involved. If the numbers of club members participating in the projects are consecutive even numbers, what is the probability that a given club member is participating in the project that the fewest club members are participating in?
- (A) $\frac{5}{66}$
(B) $\frac{1}{11}$
(C) $\frac{1}{9}$
(D) $\frac{4}{33}$
(E) $\frac{1}{6}$
64. A certain machine produces toy balls in an infinitely repeating cycle of blue, green, red, and yellow. If 10 consecutively produced balls are selected at random, what is the probability that exactly 3 of the balls selected are blue?
- (A) $\frac{3}{4}$
(B) $\frac{1}{2}$
(C) $\frac{3}{10}$
(D) $\frac{1}{4}$
(E) $\frac{1}{8}$
65. The formula $F = \frac{9}{5}C + 32$ gives the relationship between the temperature in degrees Fahrenheit, F , and the temperature given in degrees Celsius, C . What value of F is double the equivalent value of C ?
- (A) -80
(B) 0
(C) 80
(D) 160
(E) 320
66. In a certain game, amounts of money are represented by differently colored chips. If 2 blue chips equal 10 yellow chips and 3 yellow chips equal 20 red chips, how many blue chips are equivalent to 100 red chips?
- (A) 2
(B) 3
(C) 4
(D) 5
(E) 6

3. *PROBLEM SOLVING*

67. What is the probability that, twelve seconds from a randomly selected starting time, a certain digital clock will display a number of seconds x such that $x < 10$?
- (A) $\frac{19}{60}$
(B) $\frac{3}{10}$
(C) $\frac{1}{5}$
(D) $\frac{11}{60}$
(E) $\frac{1}{6}$
68. There are ten players in a tennis league, and a pair of players is to be selected to play a match. At most, how many different pairs of players are possible?
- (A) 10
(B) 45
(C) 50
(D) 90
(E) 100
69. If two fair six-sided dice are thrown, what is the probability that the sum of the numbers showing on the dice is 11 ?
- (A) $\frac{11}{36}$
(B) $\frac{1}{9}$
(C) $\frac{1}{18}$
(D) $\frac{1}{36}$
(E) 0
70. A certain dice game can only be won if, when a player throws two fair six-sided dice, the number showing on one of the dice is a multiple of the number showing on the other. What is the probability that a player wins this game?
- (A) $\frac{11}{18}$
(B) $\frac{11}{36}$
(C) $\frac{1}{4}$
(D) $\frac{2}{9}$
(E) $\frac{7}{36}$

3. *PROBLEM SOLVING*

71. At a certain conference, 72% of the attendees registered at least two weeks in advance and paid their conference fee in full. If 10% of the attendees who paid their conference fee in full did not register at least two weeks in advance, what percent of conference attendees registered at least two weeks in advance?
- (A) 18.0%
(B) 62.0%
(C) 79.2%
(D) 80.0%
(E) 82.0%
72. A researcher computed the difference between the predicted numerical result and the actual numerical result for five different predictions, then calculated the mean, median, and standard deviation for that set of differences. (Each of the differences was greater than one.) If each of the differences were to be squared, which of these three statistics would change?
- (A) The mean only
(B) The standard deviation only
(C) The mean and the median
(D) The mean and the standard deviation
(E) The mean, median, and standard deviation
73. If the price of a certain bond on May 1st was $\frac{2}{3}$ the price of the bond on June 1st and the price of the bond on July 1st was 25% greater than the price of the bond on May 1st, then the price of the bond on June 1st was what percent of the average (arithmetic mean) price of the bond on May 1st and July 1st?
- (A) 50%
(B) 75%
(C) 120%
(D) $133\frac{1}{3}\%$
(E) 150%

3. *PROBLEM SOLVING*

74. A certain team of salespeople has 6 members, including Larry. One of the 6 members is to be chosen at random to be assigned Territory A, one of the remaining 5 members is to be chosen at random to be assigned Territory B, and one of the remaining 4 members is to be chosen at random to be assigned Territory C. What is the probability that Larry will be selected to be assigned either Territory B or Territory C?
- (A) $\frac{1}{30}$
(B) $\frac{1}{20}$
(C) $\frac{1}{12}$
(D) $\frac{1}{6}$
(E) $\frac{1}{3}$
75. A marketing firm found that, of 800 computer users surveyed, 280 were not familiar with either Website A or Website B, 220 were familiar only with Website A, and for every 3 computer users who were familiar only with Website B, one was familiar with both websites. How many of the 800 computer users were familiar with both websites?
- (A) 75
(B) 100
(C) 135
(D) 150
(E) 200
76. A closed cylindrical tank contains 27π cubic feet of water and is filled to half its capacity. When the tank is placed upright on its circular base on level ground, the height of the water in the tank is 3 feet. What is the circumference of the tank's circular base?
- (A) 3π
(B) 6π
(C) 9π
(D) 12π
(E) 27π

3. *PROBLEM SOLVING*

77. Among a class of 400 students, 80 percent studied at least 10 hours for the final exam, 25 percent received an 'A' on the final exam, and 20 percent of those who studied at least 10 hours received an 'A' on the final exam. If 1 student is to be randomly selected from the 400 students, what is the probability that the student selected will be one who studied at least 10 hours but did NOT received an 'A' on the final exam?
- (A) $\frac{1}{20}$
(B) $\frac{9}{100}$
(C) $\frac{1}{4}$
(D) $\frac{3}{5}$
(E) $\frac{16}{25}$
78. The average (arithmetic mean) of the even integers from 200 to 300, inclusive, is how much greater than the average of the even integers from 60 to 100, inclusive?
- (A) 140
(B) 150
(C) 160
(D) 170
(E) 200
79. Of the three-digit integers greater than 660, how many have two digits that are equal to each other and the remaining digit different from the other two?
- (A) 47
(B) 60
(C) 92
(D) 95
(E) 96
80. Positive integer m is 25 percent of 25 percent of positive integer n , and m percent of n equals 25. What is the value of n ?
- (A) 25
(B) 50
(C) 100
(D) 200
(E) 500

3. *PROBLEM SOLVING*

81. The product of the eight smallest two-digit integers is closest to which of the following powers of 10?
- (A) 10^9
 - (B) 10^8
 - (C) 10^7
 - (D) 10^6
 - (E) 10^5
82. If $n = \sqrt{\frac{1}{256}}$, what is the value of \sqrt{n} ?
- (A) $\frac{1}{2}$
 - (B) $\frac{1}{2\sqrt{2}}$
 - (C) $\frac{1}{4}$
 - (D) $\frac{1}{4\sqrt{2}}$
 - (E) $\frac{1}{8}$
83. If the positive integer x is a multiple of 4 and the positive integer y is a multiple of 8, then xy must be a multiple of which of the following?
- I. 4
 - II. 8
 - III. 12
- (A) I only
 - (B) III only
 - (C) I and II only
 - (D) I and III only
 - (E) I, II, and III
84. Last year Elaine spent 20% of her annual earnings on rent. This year she earned 15% more than last year and she spent 30% of her annual earnings on rent. The amount she spent on rent this year is what percent of the amount spent on rent last year?
- (A) 152.5
 - (B) 164.5
 - (C) 167.5
 - (D) 172.5
 - (E) 177.5

3. *PROBLEM SOLVING*

85. If $3^{2x+1} = 27^{x-3}$, then $x =$
- (A) -4
 - (B) -1
 - (C) 3
 - (D) 4
 - (E) 10
86. Which of these fractions has the greatest value?
- (A) $\frac{5}{2^2 3^2}$
 - (B) $\frac{11}{2^3 3^2}$
 - (C) $\frac{17}{2^2 3^3}$
 - (D) $\frac{31}{2^3 3^3}$
 - (E) $\frac{67}{2^4 3^3}$
87. On the number line, if $x > y$, if y is halfway between x and z , then $\frac{x-y}{x-z} =$
- (A) -2
 - (B) $-\frac{1}{2}$
 - (C) $\frac{1}{2}$
 - (D) 1
 - (E) 2
88. Which of the following could be the greatest common divisor of two prime numbers a and b , where $2 < a < b$?
- (A) 1
 - (B) $a - b$
 - (C) a
 - (D) b
 - (E) $a + b$
89. If x , y , and z are nonzero integers and $x - y = z$, which of the following is equal to 1?
- (A) $\frac{z-y}{x}$
 - (B) $\frac{z-x}{y}$
 - (C) $\frac{y-z}{x}$
 - (D) $\frac{x-z}{y}$
 - (E) $\frac{y-x}{z}$

3. *PROBLEM SOLVING*

90. $(\sqrt{7 + \sqrt{48}} + \sqrt{7 - \sqrt{48}})^2$
- (A) 14
 - (B) 16
 - (C) $2\sqrt{55}$
 - (D) $14 + 2\sqrt{50}$
 - (E) $16 + 2\sqrt{50}$
91. If $a > d$, $c > a$, $d > b$, and $a > 0$, which of the following must be true?
- I. b is negative
 - II. c is positive
 - III. d is positive
- (A) I only
 - (B) II only
 - (C) III only
 - (D) I and II
 - (E) II and III
92. When a is a multiple of 3, $\langle a \rangle = \frac{a}{3}$. Otherwise, $\langle a \rangle = 2a$. Which of the following is equal to $\langle 5 \rangle \times \langle 6 \rangle$?
- (A) $\langle 60 \rangle$
 - (B) $\langle 48 \rangle$
 - (C) $\langle 36 \rangle$
 - (D) $\langle 30 \rangle$
 - (E) $\langle 24 \rangle$
93. A class consists of 24 students. If a student is to be selected at random from the class, the probability that a woman will be selected is three times the probability that a man will be selected. How many women are in the class?
- (A) 6
 - (B) 8
 - (C) 16
 - (D) 18
 - (E) 20
94. Last year a certain bond yielded 5 percent of its face value in interest. If that interest was approximately 4 percent of the bond's selling price of \$7,500, what is the bond's face value?
- (A) \$6,000
 - (B) \$6,750
 - (C) \$7,425
 - (D) \$7,500
 - (E) \$9,375

3. *PROBLEM SOLVING*

95. For a finite sequence of nonzero integers, the number of variations in parity is defined as the number of pairs of consecutive terms of the sequence for which the sum of the two consecutive terms is odd. What is the number of variations in parity for the sequence 1, 4, 3, 5, 8, 6 ?
- (A) One
(B) Two
(C) Three
(D) Four
(E) Five
96. If $a = 3$ and $b = -2$, then $(b^2 - a)(x - y) - (a + b)(x + y) =$
- (A) $-2x$
(B) $-2y$
(C) $2x$
(D) $2y$
(E) $-2(x + y)$
97. What is the sum of the different positive prime factors of 720?
- (A) 7
(B) 8
(C) 10
(D) 16
(E) 17
98. If $x - y = z$, then which of the following represents the average (arithmetic mean) of x , y , and z , in terms of x ?
- (A) $\frac{x}{3}$
(B) $\frac{2x}{3}$
(C) x
(D) $2x$
(E) $3x$
- 8, x , y , 13, 3, 8
99. The arithmetic mean of the list of numbers above is 8. If x and y are integers and the range of the list is 10, all of the following could be the value of $x - y$ EXCEPT
- (A) 0
(B) 2
(C) 6
(D) 10
(E) 12

3. *PROBLEM SOLVING*

100. For all numbers j and k , the operation $*$ is defined by
 $j * k = \frac{j^3}{k^3}$. If $c * 2 = \frac{1}{8}$, then $c =$
- (A) -8
 - (B) -1
 - (C) 1
 - (D) 2
 - (E) 4

4 Answer Key

For full explanations, see the next section.

1. E
2. D
3. B
4. C
5. B
6. D
7. D
8. B
9. C
10. E
11. B
12. C
13. D
14. D
15. C
16. C
17. E
18. D
19. C
20. E
21. D
22. C
23. A
24. B
25. E
26. D
27. C
28. E
29. D
30. A
31. E
32. B
33. E
34. E
35. B
36. D
37. C
38. B
39. E
40. E
41. C

4. ANSWER KEY

- 42. B
- 43. C
- 44. A
- 45. E
- 46. B
- 47. E
- 48. B
- 49. A
- 50. C
- 51. A
- 52. D
- 53. D
- 54. E
- 55. B
- 56. A
- 57. E
- 58. E
- 59. E
- 60. D
- 61. E
- 62. D
- 63. B
- 64. B
- 65. E
- 66. B
- 67. E
- 68. B
- 69. C
- 70. A
- 71. D
- 72. E
- 73. D
- 74. E
- 75. A
- 76. B
- 77. E
- 78. D
- 79. D
- 80. D
- 81. A
- 82. C
- 83. C
- 84. D
- 85. E
- 86. C
- 87. C

4. *ANSWER KEY*

- 88. A
- 89. D
- 90. B
- 91. B
- 92. A
- 93. D
- 94. A
- 95. C
- 96. B
- 97. C
- 98. B
- 99. E
- 100. C

5 Explanations

For a quick-reference answer key, see the previous section.

1. E

Explanation: First, simplify the expression:

$$\frac{m-108}{m} = \frac{m}{m} - \frac{108}{m} = 1 - \frac{108}{m}$$

If the whole thing is not an integer, that means that $\frac{108}{m}$ must not be an integer, since 1 will always be an integer. Thus, the correct answer is the one that 108 is not evenly divisible by: 8. $\frac{108}{8} = 10\frac{1}{2}$, so choice (B) is correct.

2. D

Explanation: The current ratio is $\frac{1}{3}$. However, we can't subtract the three salespeople who leave from the top of a ratio. Instead, express the ratio as $\frac{1x}{3x}$, where x is a multiplier—if there are 10 salespeople and 30 total employees, for instance, $x = 10$.

Since salespeople are included in the total number of employees, we need to subtract from both the numerator and the denominator of the fraction:

$$\frac{x-3}{3x-3}$$

That fraction is equal to $\frac{1}{4}$, the new ratio of salespeople to total employees:

$$\frac{x-3}{3x-3} = \frac{1}{4}$$

$$3x - 3 = 4(x - 3)$$

$$3x - 3 = 4x - 12$$

$$9 = x$$

The question is asking for the total number of employees, which we initially expressed as $3x$. Since $x = 9$, $3x = 27$, choice (D).

3. B

Explanation: To avoid making a careless mistake counting zeroes, convert everything you can to exponents. $0.00000001 = 1 \times 10^{-8}$, and $\sqrt{x} = x^{\frac{1}{2}}$, so we can rewrite the expression as follows:

$$\left((1 \times 10^{-8})^{\frac{1}{2}} \right)^{\frac{1}{2}}$$

$$(1 \times 10^{-8})^{\frac{1}{4}}$$

$$1^{\frac{1}{4}} \times 10^{-8(\frac{1}{4})} = 1 \times 10^{-2} = 0.01, \text{ choice (B).}$$

4. C

Explanation: To give us a reference point for all of these different amounts, call the capacity of tank B x . If tank B has three times the capacity of tank A, A's capacity is $\frac{1}{3}x$.

If tank A is $\frac{1}{3}$ full of water, it contains $\frac{1}{3}(\frac{1}{3}x) = \frac{1}{9}x$ of water. If tank B is half full of water, it contains $\frac{1}{2}x$ of water. When all the water in A is poured into B, that means we're adding $\frac{1}{9}x + \frac{1}{2}x = \frac{2}{18}x + \frac{9}{18}x = \frac{11}{18}x$. Since the capacity of B is x , the fraction that B is filled to capacity is:

5. EXPLANATIONS

$$\frac{\frac{11}{18}x}{x} = \frac{11}{18}, \text{ choice (C).}$$

5. B

Explanation: Start with the largest numbers. Since 7, 6, and 5 have no common factors, the least positive integer divisible by all three is the product $7(6)(5)$. That number is even, but not divisible by 4; thus, the least common multiple of $7(6)(5)$ and 4 is $7(6)(5)(2)$, which is divisible by 4.

That product is divisible by 3 and 2, since 6 is divisible by 3 and 2. Thus, all that's left is finding $7(6)(5)(2) = 420$, choice (B).

6. D

Explanation: A different way of phrasing the question is this: After each stroke of a vacuum pump, $\frac{1}{3}$ of the air that was in the pump remains. So, after the first stroke, $\frac{1}{3}$ remains. After the second stroke, it's $\frac{1}{3}(\frac{1}{3}) = \frac{1}{9}$, which is about 11%. Keep going until you get under 1%:

Third stroke: $\frac{1}{9}(\frac{1}{3}) = \frac{1}{27}$, a bit less than $\frac{1}{25}$, which is 4%.

Fourth stroke: $\frac{1}{27}(\frac{1}{3}) = \frac{1}{81}$, greater than $\frac{1}{100}$, 1%.

Fifth stroke: $\frac{1}{81}(\frac{1}{3}) = \frac{1}{243}$, which is much smaller than $\frac{1}{100}$, so much smaller than 1%. Choice (D), then, is the answer.

7. D

Explanation: Consider each choice. If it has a twin, the number either 2 below or 2 greater than the number itself must be a prime.

(A) $7 - 2 = 5$, which is a prime.

(B) $13 - 2 = 11$, which is a prime.

(C) $17 + 2 = 19$, which is a prime.

(D) $23 - 2 = 21$, not a prime. $23 + 2 = 25$, also not a prime.

(E) $29 + 2 = 31$, which is a prime.

The only possible choice is (D), as 23 does not have a twin.

8. B

Explanation: There are 6 cocktails that contain vodka and 2 of those contain both tequila and vodka. Since no cocktail has all three ingredients, only 4 of those vodka cocktails can also contain rum.

By the same reasoning: 5 cocktails contain rum, 3 of those contain both tequila and rum, so only 2 of the rum cocktails can also contain vodka. That's the lower number, so that's the limit on the number of drinks that can contain both vodka and rum, so the correct choice is (B).

9. C

Explanation: Start with the easiest factors of 484, and work your way through all the possibilities:

$$484 = 1(484)$$

$$484 = 2(242)$$

$$484 = 4(121)$$

$$484 = 11(44)$$

5. EXPLANATIONS

$$484 = 22(22)$$

Thus, the factors are 1, 2, 4, 11, 22, 44, 121, 242, and 484, for a total of 9, choice (C).

10. E

Explanation: The greatest possible straight-line distance in a rectangular solid is formed as follows: take the diagonal of the base, and use that as one leg of a right triangle with the height of the solid to form another right triangle. The hypotenuse of that triangle is the greatest length.

It doesn't matter which side is which, so we can call 8 feet and 8 feet the length and width. The diagonal of the base, then, is the hypotenuse of a right triangle with legs 8 and 8, which has a length of $8\sqrt{2}$. The ultimate length we're looking for is the hypotenuse of a triangle with one leg of $8\sqrt{2}$ (the hypotenuse we just found) and the other leg of z , the height of the solid.

Using the pythagorean theorem, we can find the hypotenuse of that triangle:

$$z^2 + (8\sqrt{2})^2 = c^2$$

$$z^2 + 128 = c^2$$

$$c = \sqrt{128 + z^2}, \text{ choice (E).}$$

11. B

Explanation: First, simplify the expression $\frac{3p}{84}$: factor out a 3, and the result is $\frac{p}{28}$. Thus, since $\frac{p}{28}$ is an integer, p is a multiple of 28. Since p must be less than 75, find the multiples of 28 that are less than 75: 28 and 56. (The next multiple, 84, is greater than 75.)

Presumably, 28 and 56 have the same number of prime factors, so you can figure out the number of prime factors for either one. $28 = 4(7) = 2(2)(7)$, so there are two prime factors, choice (B).

12. C

Explanation: The words "at least" signal that you'll be working with more than one probability, then adding them together later. In this case, those multiple probabilities are the probability that two of the three tosses turn up tails, and the other is the probability is that all three of the tosses turn up tails.

The only ways the coin tosses can be arranged to make one of those things possible:

H T T

T H T

T T H

T T T

There are four possible outcomes, in the first three of which two tosses turn up tails, and in the last of which all three turn up tails. The total number of possibilities is the product of the number of possibilities for each toss:

$$(2)(2)(2) = 8$$

Probability is, as always, the number of desired outcomes divided by possible outcomes: $\frac{4}{8} = \frac{1}{2}$, choice (C).

5. EXPLANATIONS

13. D

Explanation: As k gets larger, $-k$ gets smaller, as does $\frac{1}{k}$, since a larger denominator means a smaller number. However, to combine the two, $-\frac{1}{k}$ gets larger as k gets larger.

I decreases. 150 doesn't change, but as k gets larger, $-k$ gets smaller, so $-2k$ gets smaller. Eliminate (B) and (E).

II increases. 150 doesn't change, but as k gets larger, $\frac{1}{k}$ gets smaller, so $-\frac{1}{k}$ gets larger. Eliminate (C).

III decreases. As k gets larger, k^2 increases, which means that $\frac{150}{k^2}$ decreases. Similarly, $-k$ decreases. So, the sum of $\frac{150}{k^2}$ and $-k$ decreases. (D) is the correct choice.

14. D

Explanation: Since the letters can be used in any order, and the order matters, it's a permutations problem. To find the number of 3-letter codes, find the number of letters (26) in each of the three positions in the code and multiply them together: $26(26)(26) = 26^3$.

The same logic applies to the four-letter and five-letter codes, so the sum of the codes is:

$$26^3 + 26^4 + 26^5$$

Since that doesn't look like any of the choices, start by factoring out 26^3 :

$$26^3(1 + 26 + 26^2)$$

$$26^3(27 + 26^2)$$

Still, it doesn't look like any of the choices, but if you multiply it out, we're finally there;

$$27(26^3) + 26^5, \text{ choice (D).}$$

15. C

Explanation: Since the rate is given in terms of minutes and the time is given in hours, you'll have to convert one to the other. Since there are 60 minutes in an hour, there are $60y$ minutes in y hours.

Now, you can plug in the given rate ($\frac{k}{m}$) and the given time ($60y$) into the rate formula to solve for the distance.

$$d = rt = \frac{k}{m}(60y) = \frac{60ky}{m}, \text{ choice (C).}$$

16. C

Explanation: If it takes five pipes 12 days to drain the pool, we can think of that as 60 pipe-days. It doesn't matter whether it's 60 pipes each working for one day or one pipe working for sixty days, it only matters that some number of pipes, together, drain the pool for 60 pipe-days.

To drain the pool in 4 days, it takes $\frac{60}{4} = 15$ pipes. Beware of (E): the question is asking for how many additional pipes (that is, over the original number of 5), not the number of pipes. Since we're looking for the additional number of pipes, the answer is $15 - 5 = 10$, choice (C).

17. E

5. EXPLANATIONS

Explanation: The problem does everything possible to disguise it, but this is really just two sets of differences of squares. Rewrite each term so that the radical comes first:

$$(\sqrt{2} + 2)(\sqrt{3} - 2)(\sqrt{2} - 2)(\sqrt{3} + 2)$$

Rearrange them so that the $\sqrt{2}$'s and $\sqrt{3}$'s are near each other:

$$(\sqrt{2} + 2)(\sqrt{2} - 2)(\sqrt{3} + 2)(\sqrt{3} - 2)$$

Now, solve the first pair and the second pair. Just as $(x+y)(x-y) = x^2 - y^2$, you can use the shortcut on these:

$$((\sqrt{2})^2 - 2^2)((\sqrt{3})^2 - 2^2)$$

$$(2 - 4)(3 - 4) = (-2)(-1) = 2, \text{ choice (E).}$$

18. D

Explanation: For the product of two numbers to be even, either one of the two numbers must be even or both of the numbers must be even. It would be simpler, then, to look for the opposite: the probability that the product is odd. The only way the product of two numbers is odd is if both numbers are odd.

So, the probability that x is odd is $\frac{1}{2}$, since 2 of the 4 numbers in the first set are odd. Similarly, the probability that y is odd is also $\frac{1}{2}$. The probability that both are odd is $(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$. Since we're looking for the probability that xy is NOT odd, subtract that from 1:

$$1 - \frac{1}{4} = \frac{3}{4}, \text{ choice (D).}$$

19. C

Explanation: There are two ways of expressing the total number of honor students. First, by finding the number at each school: for instance, 5 percent of the x students at School A is $0.05x$. For all three:

$$0.05x + 0.2y + 0.14z$$

Then, we're given the average rate, which we can also express algebraically:

$$0.08(x + y + z)$$

Those two are equal to each other, so to find the value of x in terms of y and z , we can set them equal to each other and solve for x :

$$0.05x + 0.2y + 0.14z = 0.08(x + y + z)$$

$$5x + 20y + 14z = 8x + 8y + 8z$$

$$12y + 6z = 3x$$

$$x = 4y + 2z, \text{ choice (C).}$$

20. E

Explanation: Jump right into the choices, and evaluate each one:

(A) This is possible, if the terms in T are the six largest terms in S .

(B) This is also possible, if the largest and smallest terms in S are among those in T .

(C) This is also possible, for instance if S is $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and T is $\{1, 2, 3, 6, 7, 8\}$, in which case the median of each set is 4.5.

(D) Using the same examples as in (C), the means are equal as well.

5. EXPLANATIONS

(E) This is correct: if every number in T is also in S , the ranges could be equal, and the range of T could be smaller (see (A)), but the range of T cannot be any greater than that of S .

21. D

Explanation: This looks very complicated, but it doesn't have to be. Because the question can have only one answer, every possible value of p will give you the same result. Thus, pick an easy number: say, $p = 3$. If $p = 3$, $n = 8p = 8(3) = 24$. 24's divisors are 1, 2, 3, 4, 6, 8, 12, and 24, which include 6 even divisors, choice (D).

22. C

Explanation: First, when you see a sequence question asking you to sum the first 100 terms, recognize that you aren't really going to sum that many terms. After the fourth term or so, the pattern should become clear, and your work should be very close to finished.

In this case, start by evaluating the first three or four terms, and look for a pattern:

$$\begin{aligned}(-1)^1(2^{-1}) &= -1\left(\frac{1}{2}\right) = -\frac{1}{2} \\(-1)^2(2^{-2}) &= 1\left(\frac{1}{4}\right) = \frac{1}{4} \\(-1)^3(2^{-3}) &= -1\left(\frac{1}{8}\right) = -\frac{1}{8} \\(-1)^4(2^{-4}) &= 1\left(\frac{1}{16}\right) = \frac{1}{16}\end{aligned}$$

By now, the pattern should become apparent. The sign goes back and forth between negative and positive, and the absolute value of the number keeps getting smaller. This is why the final 96 terms don't matter very much: compared to the first two or three, they are too small to have much of an effect on the total.

Now, think about what will happen when you add all of these terms. Add the terms you've figured out, one by one, again looking for a pattern:

$$\begin{aligned}-\frac{1}{2} + \frac{1}{4} &= -\frac{1}{4} \\-\frac{1}{4} + -\frac{1}{8} &= -\frac{3}{8} \\-\frac{3}{8} + \frac{1}{16} &= -\frac{5}{16}\end{aligned}$$

If you visualize this succession on a number line, you'll find that the sum zigzags. However, because the absolute value of the terms keep getting smaller, those remaining 96 terms aren't going to change the total much: they are just going to move the total back and forth, in smaller and smaller amounts, around our answer of $-\frac{5}{16}$. Thus, since the answer choices offer us ranges, we can confidently select (C), the range in which all three of our intermediate sums fit.

23. A

Explanation: You can answer this question in about fifteen seconds if you know the appropriate shortcut. Anytime you have a set of numbers that are equally spaced, whether consecutive integers, consecutive evens, or consecutive multiples of 5 (like these), the mean and the median are equal. You may want to try a few sets to prove it to yourself; it's always the case.

5. EXPLANATIONS

Thus, whatever the mean and median are of S , they are equal. So, the difference between the mean and median is 0, choice (A).

24. B

Explanation: The best way to handle this question is by using the answer choices to guess and check. For instance, the lower of the two numbers in the correct choice must fit the guidelines given in the question: when you multiply it by a number one greater than itself, the product must be greater than 10.

So, if you try (A), $2(3) = 6$, it doesn't work. (B), though, works: $3(4) = 12$, which is between 10 and 180.

If 3 works, 4 works, so the lower limit isn't going to help us much. On the high end, start by trying 13, as it appears in two choices. $13(14) = 182$, which is greater than 180, so it can't be right. Taken one step further, if 13(14) is too big, 14(15) must be too big as well. The only remaining choice, once we eliminate the ones containing 2, 13, and 14, is (B).

25. E

Explanation: The median must be somewhere between b and c , the two middle numbers. Any of the first four answer choices could be that midpoint, depending on how the numbers are spaced.

However, (E) could not, so it's the correct choice. If d is greater than c , the average of c and d must be greater than c . Since the median must be less than c , (E) cannot be the median.

26. D

Explanation: With variables in the answer choices, pick numbers. Say $n = 3$, $m = 4$, and $p = 5$. Thus the original set of 3 numbers has an average of 4, which means the sum of that group was 12. After one number was discarded, the set contains 2 numbers with an average of 5; the sum of the new group is 10, so the discarded number was 2. Which answer choice, using these numbers, works out to 2?

(A): $(5)3 - (4)3 - (4)5 = 15 - 12 - 20 = -17$. No.

(B): $5(3 - 1) - (4)3 = 10 - 12 = -2$. No.

(C): $(4)3 - (5)3 + (4)5 = 12 - 15 + 20 = 17$. No.

(D): $4(3) - 5(3 - 1) = 12 - 10 = 2$. Looks good.

(E): $3(4 + 5) = 27$. No.

27. C

Explanation: Write the question algebraically as follows, setting the ratio on one side of the equation and the actual numbers of managers and non-managers on the other:

$$\frac{5}{24} < \frac{8}{x}$$

Simplify to find the greatest value for x :

$$5x < 8(24)$$

$$x < \frac{8(24)}{5} = \frac{192}{5} = 38.4$$

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Since there are no partial non-managers, the greatest number of non-managers is 38, choice (C).

28. E

Explanation: To combine the two inequalities in the question, if n is less than q and greater than p :

$$p < n < q$$

Since the answer choices all deal in negatives, it may be useful to multiply each term by -1 , reversing the signs as necessary:

$$-p > -n > -q$$

Now consider each choice:

(A) needn't be true: if $n = 1$ and $p = -2$, $-p$ is greater than n .

(B) needn't be true: if $n = -2$ and $q = -1$, $-q$ is greater than n .

(C) needn't be true: if $n = 2$ and $p = 1$, n is greater than $-p$.

(D) needn't be true: in (B), we saw that n could be less than $-q$.

(E) is all that's left, so it must be correct.

29. D

Explanation: To make matters simpler, there's only one even prime to consider: 2. Of the 29 integers less than 30, 9 of them are multiples of three: {3, 6, 9, 12, 15, 18, 21, 24, 27}. Add two to each one, and you get 9 more numbers: {5, 8, 11, 14, 17, 20, 23, 26, 29}. Add 2, the sole even prime, and that's a total of $9 + 9 + 1 = 19$ numbers, choice (D).

30. A

Explanation: The dealer's offered price, which we'll call s , is equal to 360 + 40% of his offered price, $0.4s$. So:

$$s = 360 + 0.4s$$

$$360 = 0.6s$$

$$s = 360\left(\frac{10}{6}\right) = 60(10) = 600$$

The offered price is 600, so 10% less than that is $600 - 60 = 540$. Profit is the difference between the sale price and the purchase price:

$$540 - 360 = 180, \text{ choice (A).}$$

31. E

Explanation: Rather than trying to determine the value of $\sqrt{661}$, find the squares of the numbers that differentiate the answer choices. Start in the middle to save time: if the resulting number is too small, move up; if it's too big, move down.

$$24^2 = 576. \text{ Move up:}$$

$$25^2 = 625. \text{ 661 is larger, so } \sqrt{661} \text{ must be between 25 and 26, choice (E).}$$

32. B

Explanation: Solve the equation for $\frac{y}{x}$:

$$x^2 = 9y^2$$

$$1 = \frac{9y^2}{x^2}$$

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$$\frac{1}{9} = \frac{y^2}{x^2}$$

$$\frac{y}{x} = \pm \frac{1}{3}$$

There are two possible answers, but only one of them is represented among the choices: $-\frac{1}{3}$, (B).

33. E

Explanation: If the units' digit of p^2 is 1, the units' digit of p must be 1 ($p^2 = 1$) or 9 ($p^2 = 81$). If $p = 1$, $(p + 1)^2 = (1 + 1)^2 = 4$, so p must be 1. If $p = 1$, $(p + 2)^2 = (1 + 2)^2 = 3^2 = 9$, choice (E).

34. E

Explanation: If $(s - 5)$ is one of the factors of $s^2 - js + 25$, we can determine the other factor. $s^2 - js + 25 = (s - 5)(s + x)$, and since $-5x = 25$, $x = -5$. Knowing the other factor, we can determine the value of j :

$$(s - 5)(s - 5) = s^2 - 10s + 25$$

$j = 10$, choice (E).

35. B

Explanation: The symbol here is equivalent to factorial notation: $*5* = 5(4)(3)(2)(1) = 120$. In other words, we want to know how many prime numbers are in the set $\{120, 121, 122, 123, 124, 125, 126, 127\}$. First, you can eliminate the even numbers, since none of those are prime.

Next, eliminate 121, which is 11 squared. 123 is a multiple of 3, and 125 is a multiple of 5. Thus, the only number left to check is 127. You can establish that any number is prime if it has no prime factors between 1 and its square root. The square root of 127 is a little less than 12, so we have to check whether 127 is divisible by 2, 3, 5, 7, or 11. It isn't, so 127 must be prime. It's the only one, making (B) the correct choice.

36. D

Explanation: The question gives two different pairs of times and rates, one of which contains the variable we're looking for. In both cases, the distance is the same. Since $d = rt$, and the distance in both cases is the same, the product of the rate and the time must be the same, as well.

We can set that up in equation form:

$$20(25) = r(10)$$

$$2(25) = r$$

$r = 50$, choice (D).

37. C

Explanation: To write the equation algebraically:

$$\frac{X}{Y} = 7 + \frac{2}{Y}$$

Multiply both sides by Y to get X on its own:

$$X = 7Y + 2, \text{ choice (C).}$$

38. B

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Explanation: The question establishes that the arrangement must be B-G-B-B-G-B. For the first seat, there are four possible boys. For the second seat, there are two possible girls. For the third seat, there are three possible boys (one of them is already seated). Continuing that logic, here are the possible numbers for each seat:

4-2-3-2-1-1

To find the answer, multiply those numbers: $4(2)(3)(2)(1)(1) = 48$, choice (B).

39. E

Explanation: Simplify both sides so that they are expressed as 7 raised to an exponent:

$$\left(7^{\frac{3}{4}}\right)^n = 49$$

$$7^{\frac{3}{4}(n)} = 7^2$$

$$7^{\frac{3n}{4}} = 7^2$$

$$\frac{3n}{4} = 2$$

$$3n = 8$$

$$n = \frac{8}{3}, \text{ choice (E).}$$

40. E

Explanation: x will be the largest when y is the smallest, so solve for x when $y = 1$, the smallest possible value for y :

$$x^3 + 1^3 < 1,000$$

$$x^3 < 999$$

$1000 = 10^3$, so 999 is a little less than 10^3 , so x must be between 8 and 10, choice (E).

41. C

Explanation: This requires a fair amount of algebra: jump right in and solve.

$$\frac{(2x+1)^2 + (2x-1)^2}{2} = \frac{4x^2 + 4x + 1 + 4x^2 - 4x + 1}{2} = \frac{8x^2 + 2}{2} = 4x^2 + 1, \text{ choice (C).}$$

42. B

Explanation: Since 1.3 is one of the choices, and since it appears in some form in every choice, compare each choice to 1.3.

(A) is less than 1.3: the reciprocal of any number greater than 1 is itself less than one.

(B) is greater than 1.3: any number greater than 1 gets greater still when squared.

(C) is much smaller than 1.3.

(D) is much smaller: $\frac{1.3}{13} = \frac{1}{10}$.

(E) is 1.3.

The only choice greater than 1.3 is (B), so (B) is the correct choice.

43. C

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Explanation: The third root of a negative number is defined, so you can eliminate (E) immediately; if it were an even-numbered root (such as a square root), (E) would be correct.

Rather than trying to determine the value of $\sqrt[3]{-101}$, better to work from the answer choices and see what might work. $-3^3 = -27$, which is likely too large. $-4^3 = -64$, which is still larger but close. $-5^3 = -125$, which is on the other side of -101, so the correct choice is (C): $\sqrt[3]{-101}$ is between -4 and -5.

44. A

Explanation: Using the exponent rules that you should have memorized and internalized by now, check each of the choices:

(A) $(x^4)^3 = x^{4(3)} = x^{12}$

(B) $(x^6)^6 = x^{6(6)} = x^{36}$

(C) $(x^3)^9 = x^{3(9)} = x^{27}$

(D) $\frac{x^4}{x^{16}} = x^{4-16} = x^{-12}$

(E) $x^3 + x^9 = x^3(1 + x^6)$

Only (A) matches what we're looking for, so it is the correct choice.

45. E

Explanation: If a fraction is equal to 1, the numerator and denominator must be the same; they can be any number except for zero. (If $p = 0$, the fraction is undefined.) So, $m - n \neq 0$. Substitute $m = 2$:

$$2 - n \neq 0$$

$$n \neq 2, \text{ choice (E).}$$

46. B

Explanation: The question gives you two equations and two variables. Before you can solve, you need to simplify each one:

$$(2^x)(8^y) = 32$$

$$(2^x)(2^{3y}) = 32$$

$$2^{x+3y} = 2^5$$

$$x + 3y = 5$$

$$(3^x)(9^y) = 81$$

$$(3^x)(3^{2y}) = 81$$

$$3^{x+2y} = 3^4$$

$$x + 2y = 4$$

To combine the two equations, subtract the second from the first, which results in $y = 1$. Plug that into the second equation:

$$x + 2(1) = 4$$

$$x = 4 - 2 = 2$$

$$\text{So, } (x, y) = (2, 1), \text{ choice (B).}$$

47. E

Explanation: To find $p + q$, we'll need to figure out what the addition table tells us about them. Since $p = y + b$ (the sum of the labels on p 's row

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and column) and $q = a + 7$, $p + q = a + b + y + 7$. That's not much help; we're looking for an integer, so we'll need to find out something about the remaining variables.

We can solve for b : 1, the center of the bottom row, is the sum of b and 7, so b must be -6. -4, in the center of the top row, is the sum of b and x , and since $b = -6$, $x = 2$. That doesn't help us directly, but since -2 (upper left) is the sum of a and x , -2 is the sum of a and 2, so $a = -4$. Finally, the sum of a and y is 6, and since $a = -4$, $-4 + y = 6$, so $y = 10$.

Finally, we know that $b = -6$, $a = -4$, and $y = 10$, so we can plug those values into our initial equation:

$$p + q = a + b + y + 7$$

$$p + q = -4 + (-6) + 10 + 7 = 7, \text{ choice (E).}$$

48. B

Explanation: No matter what the value of n , $1^n = 1$; one raised to any power is 1. Therefore, you need to figure out how many values of n make $n^1 = 1$. There's only one: when $n = 1$, $1^1 = 1$, so the correct choice is (B).

49. A

Explanation: If $\frac{x}{y} = \frac{2}{3}$, $\frac{y}{x} = \frac{3}{2}$. So, $\frac{y}{x} - \frac{x}{y} = \frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6}$, choice (A).

50. C

Explanation: Average speed is given by the total distance divided by total time. Total distance here is the sum of the distance driven (J) plus distance ridden on the train (K). Total time is the time driven (H) plus time on the train ($\frac{H}{2}$). So, total distance over total time is:

$$\frac{J+K}{H+\frac{H}{2}} = \frac{J+K}{\frac{3H}{2}} = (J+K)\left(\frac{2}{3H}\right) = \frac{2(J+K)}{3H}, \text{ choice (C).}$$

51. A

Explanation: Since the number is selected at random, it is most likely to occur between the two numbers that are farthest apart. Thus, we need to compare the ranges of each of the five choices. Since nearly all of the fractions are common, it may be easiest to work with their decimal equivalencies:

(A) $0.15 - 0 = 0.15$

(B) $0.25 - 0.15 = 0.1$

(C) $0.33 - 0.25 = 0.08$

(D) $0.4 - 0.33 = 0.07$

(E) $0.5 - 0.4 = 0.1$

The largest range is (A), so that's the correct choice. Note that $\frac{1}{3}$ isn't exactly 0.33, but for the purposes of this question, it's close enough to establish that the relevant ranges are smaller than that of (A).

52. D

Explanation: Since we're expected to solve for one number, we don't need to worry about the broad range of possible values for k and m . It appears that,

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regardless of which pair of values we choose (so long as they fit the constraints given), we'll come up with the same answer.

If the average of the six numbers is 5, we can set up an equation to solve for $k + m$:

$$\frac{10+3+k+m+1+7}{6} = 5$$

$$21 + k + m = 30$$

$$k + m = 9$$

Say that $k = 5$ and $m = 4$, then the set consists of $\{1, 3, 4, 5, 7, 10\}$. Since there are an even number of terms, the median is the average of the middle two numbers, 4 and 5. That average is 4.5, choice (D).

53. D

Explanation: Since the base of the numerator and denominator is the same, we can reduce the expression to that base with one exponent:

$$3^{((x+y)^2 - (x^2 + y^2))}$$

To simplify that exponent, multiply out the square of $(x + y)$, which results in the following:

$$x^2 + 2xy + y^2 - x^2 - y^2 = 2xy$$

Since $xy = 1$, $2xy = 2$, meaning that the expression is $3^2 = 9$, choice (D).

54. E

Explanation: A parallelogram and circle can be tangent at exactly one point, so eliminate (A), (B), and (D). That leaves only the question of whether they can intersect at exactly 7 points. It can: since a parallelogram needn't have sides of equal length (as a square does), one side can be tangent to the circle (creating one point of intersection), while the parallel side can extend beyond the circle (creating two points of intersection). Choice (E) is correct.

55. B

Explanation: If the distance between A and B is 36 miles, the midpoint is 18 miles from each. Logan, walking at 4 mph, will cover that distance in $\frac{18}{4} = 4.5$ hours. Hayley, walking at 3 mph, will cover that distance in 6 hours. The difference is 1.5 hours, choice (B).

56. A

Explanation: If \sqrt{p} is divisible by 3, p itself must be divisible by 9. If p is divisible by both 9 and 11, p must be a multiple of 99, which we can write as $99(\text{integer})$. Thus, $\frac{p}{11} = \frac{99(\text{integer})}{11} = 9(\text{integer})$, which is equivalent to a multiple of 9. Therefore, 9 must be a factor of p ; any of the other numbers could be factors, but might not be, so choice (A) is correct.

57. E

Explanation: With 10 digits and 6 letters, there are 16 possibilities for each character. The number of two-character codes is the product of the number of characters that are possible in each of the two positions: $16 \times 16 = 16^2$. The same logic applies to the three-character codes: the total number is 16^3 .

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The final challenge is finding the actual value of $16^2 + 16^3$. It's easier if you factor it:

$$16^2 + 16^3 = 16^2(1 + 16) = 16^2(17) = 256(17)$$

Finally, it takes a bit of heavy lifting, but $256 \times 17 = 4352$, choice (E).

58. E

Explanation: If $pq - p^2$ is even, either both terms are even or both terms are odd. If both terms are even, then p is even, in which case pq is even regardless of whether q is even or odd. If both terms are odd, p is odd, which means q must also be odd. So, consider each choice:

(A) p could be even or odd.

(B) q could be even or odd.

(C) When p is even, q could be even or odd, meaning $p - q$ could be even or odd.

(D) When p is odd, q is also odd, in which case pq is odd.

(E) When p is even, p and pq are both even, so $p - pq$ is even. When p is odd, p and pq are both odd, so $p - pq$ is even. (E) is correct.

59. E

Explanation: Since $f(50)$ is the product of all prime numbers less than 50, $f(50)$ is a multiple of all of those prime numbers. $f(50) + 1$, then, is not a multiple of any of those prime numbers. For instance, if $f(50)$ is a multiple of 23, the next multiple of 23 is $f(50) + 23$. $f(50) + 1$ is not. That reasoning applies to all the prime numbers up to 50, not just 23.

So, the smallest prime factor of $f(50) + 1$ must be larger than any prime number less than 50. Among the answer choices, it must be greater than 40, choice (E).

60. D

Explanation: If n is divisible by 6, then n^2 must be divisible by $6^2 = 36$. So, since n is a multiple of 36, the largest positive integer that must divide n^2 is 36, choice (D).

61. E

Explanation: Since the numerator consists of 9's, 4's, and 2's, it will be easiest to simplify the fraction if you reduce 72 to its component 9's, 4's, and 2's:

$$\frac{(9^2)(4^3)(2^3)}{72^2} = \frac{9 \times 9 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2}{72 \times 72} = \frac{9 \times 9 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2}{9 \times 4 \times 2 \times 9 \times 4 \times 2}$$

Cross out all the numbers that occur in the numerator and denominator, and you're left with:

$$4 \times 2 = 8 = 2^3, \text{ choice (E).}$$

62. D

Explanation: Simply plug in the given values for a and b : $a = -2$ and $b = -3$:

$$ab - a(b - a)$$

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$$\begin{aligned} & -2(-3) - -2(-3 - -2) \\ & 6 - (-2(-1)) \\ & 6 - 2 = 4, \text{ choice (D).} \end{aligned}$$

63. B

Explanation: If the numbers of club members in each of the six projects are consecutive even numbers, they can be written as follows:

$$x, x + 2, x + 4, x + 6, x + 8, x + 10$$

The sum of the club members is 66, and is also the sum of those six expressions:

$$\begin{aligned} 66 &= x + (x + 2) + (x + 4) + (x + 6) + (x + 8) + (x + 10) \\ 66 &= 6x + 30 \\ 6x &= 36 \\ x &= 6 \end{aligned}$$

The smallest number of club members working on one of the projects is x , which we know is 6. The probability that one of the 66 members is among those 6 is:

$$\frac{6}{66} = \frac{1}{11}, \text{ choice (B).}$$

64. B

Explanation: The question tells us that the sequence must be blue-green-red-yellow, or BGRY. However, since the consecutively produced balls are chosen at random, we don't know which of the colors our particular 10 start with.

For instance, our 10 could be BGRY BGRY BG, in which case 3 of the balls are blue. But if the first of our 10 is red, the sequence is RY BGRY BGRY, in which case only 2 are blue. There are only four possibilities, and we've already considered two of them, so see what happens when you start with the other two colors:

Green: GRY BGRY BGR (2 are blue)

Yellow: Y BGRY BGRY B (3 are blue)

So, in two of the four cases, 3 of the balls are blue. That's a probability of $\frac{2}{4} = \frac{1}{2}$, choice (B).

65. E

Explanation: We're given one equation expressing the relationship between F and C , two ways of writing the same temperature in two different formats. The question gives us another equation: $F = 2C$. The answer, F , is the value of F that makes both of those equations true. So, plug in the second equation into the first:

$$\begin{aligned} 2C &= \frac{9}{5}C + 32 \\ \frac{10}{5}C &= \frac{9}{5}C + 32 \\ \frac{1}{5}C &= 32 \\ C &= 32(5) = 160 \end{aligned}$$

If the Celsius temperature is 160, then the corresponding Fahrenheit temperature is given by the second equation:

$$F = 2(160) = 320, \text{ choice (E).}$$

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66. B

Explanation: To find the exchange rate between blue chips and red chips, we'll need to find the ratio of blue chips to red chips. To find that, rewrite the two given ratios so that the number of yellow chips is equal in both:

$$\frac{\text{blue}}{\text{yellow}} = \frac{2}{10} = \frac{6}{30}$$
$$\frac{\text{yellow}}{\text{red}} = \frac{3}{20} = \frac{30}{200}$$

So, 6 blue chips is equivalent to 30 yellow chips, which is equivalent to 200 red chips. We now have a ratio between blue and red:

$$\frac{\text{blue}}{\text{red}} = \frac{6}{200} = \frac{3}{100}$$

Finally, 100 red chips are equivalent to 3 blue chips, choice (B).

67. E

Explanation: As it turns out, adding or subtracting twelve seconds from the start time doesn't make much of a difference. The question is the same as one asking: what is the probability that a digital clock is displaying a number of seconds x such that $x < 10$? There's a ten-second window that makes this true whether we work with those twelve seconds or not.

So, if there are sixty possible second numbers that could be displayed (anything from 0 to 59, inclusive), the probability that one of ten possible second numbers (between 0 and 9, inclusive), is $\frac{10}{60} = \frac{1}{6}$, choice (E).

68. B

Explanation: This is a combinations problem: order doesn't matter, since a match between players A and B is the same as a match between players B and A.

To answer the question, use the combinations formula, where the set is 10 players and the desired subset is 2 players:

$$\frac{n!}{k!(n-k)!} = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10(9)}{2} = 45, \text{ choice (B).}$$

69. C

Explanation: The only way the sum of the numbers is 11 is if the dice show 5 and 6. However, there are two ways for that to happen. Call one of the dice A and the other B. A could show 5 and B could show 6, or A could show 6 and B could show 5.

The total number of possible results is the product of the number of possible results on each die, or $6(6) = 36$. Thus, the probability is the number of results that gives us 11 over the number of possible results, or $\frac{2}{36} = \frac{1}{18}$, choice (C).

70. A

Explanation: It's easiest to think of this problem by envisioning first rolling one of the dice, then rolling the other. If the first die results in a 1, there are 6 ways that one of the dice is a multiple of the other: any number (including 1 itself), is a multiple of 1.

If the first die is a 2, there are 4 winning outcomes: 1, 2, 4, and 6.

If the first die is a 3, there are 3 winning outcomes: 1, 3, and 6.

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If the first die is a 4, there are 3 winning outcomes: 1, 2, and 4.

If the first die is a 5, there are 2 winning outcomes: 1 and 5.

If the first die is a 6, there are 4 winning outcomes: 1, 2, 3, and 6.

The total number of possibilities is $6(6) = 36$, and the total number of winning outcomes is $6 + 4 + 3 + 3 + 2 + 4 = 22$. Thus, the probability of winning is $\frac{22}{36} = \frac{11}{18}$, choice (A).

71. D

Explanation: If 10% of the attendees who paid their conference fee in full did not register at least two weeks in advance, 90% of the attendees who paid their conference fee in full DID register at least two weeks in advance. The question tells us that those 90% of the attendees who paid their conference fee in full represent 72% of the attendees of the conference.

The question, then, is "What percent of 90% is 72%?" That's algebra:

$$0.9x = 0.72$$

$$x = \frac{.72}{.9} = \frac{72}{90} = \frac{8}{10} = 80\%, \text{ choice (D).}$$

72. E

Explanation: Because the question points out that each of the differences was greater than one, we know that, when squared, each difference gets bigger. Thus, the mean and median both increase: the mean because the sum of the terms increases; the median because, whatever the middle term (or two middle terms) was, it is now bigger.

The standard deviation changes as well. When squaring numbers larger than 1, the degree of dispersion becomes larger. Take, for instance, 2 and 3. Initially, the difference between them is 1. But when squared, they become 4 and 9, 5 apart. Because they are more dispersed, the standard deviation is larger, so the correct answer is (E).

73. D

Explanation: Call the price of the bond on June 1st x . That means the price on May 1st was $\frac{2}{3}x$. The price on July 1st, then, was $(\frac{5}{4})(\frac{2}{3}x) = \frac{5}{6}x$. The average price of the bond on May 1st and July 1st, then, is:

$$\frac{\frac{5}{6}x + \frac{2}{3}x}{2} = \frac{\frac{5}{6}x + \frac{4}{6}x}{2} = \frac{\frac{9}{6}x}{2} = (\frac{3}{2}x)(\frac{1}{2}) = \frac{3}{4}x.$$

So, to rephrase the question: " x (the price of the bond on June 1st) is what percentage of $\frac{3}{4}x$ (the average of May 1st and July 1st)? To find that, just divide one by the other:

$$\frac{x}{\frac{3}{4}x} = \frac{1}{\frac{3}{4}} = \frac{4}{3}, \text{ choice (D).}$$

74. E

Explanation: Despite the complicated setup, the question is actually quite simple. The probability that any team member is assigned to any given territory is $\frac{1}{6}$. Since Larry can't be assigned to both B and C, the sum of the probabilities is $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$, choice (E).

75. A

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Explanation: This question is tailor-made for the overlapping sets equation:

$$T = G1 + G2 - B + N$$

The total is 800 users; 280 are familiar with neither website, so $N = 280$. If we say that $3x$ users are familiar only with B and x are familiar with both ($B = x$), $x + 3x$ are familiar with B ($G2 = 4x$). 220 are only familiar with A, so the number familiar with A is 220 plus the number familiar with both: $220 + x$. Plug in the values and solve:

$$800 = 220 + x + 4x - x + 280$$

$$300 = 4x$$

$$x = 75$$

We're looking for the overlap, which is given by x , so the correct answer is 75, choice (A).

76. B

Explanation: While the measurements of the tank and water aren't the same (the water only fills half the tank), we can ignore the size of the tank: we're given the height of the water and the volume of the water.

Volume of a cylinder is $\pi r^2 h$, so here, where the volume is 27π and the height is 3, we can solve for r :

$$\pi r^2(3) = 27\pi$$

$$r^2(3) = 27$$

$$r^2 = 9$$

$$r = 3$$

Circumference is $2\pi r$, so $c = 2\pi(3) = 6\pi$, choice (B).

77. E

Explanation: If 80 percent of the students studied at least 10 hours, that's 320 of the 400 students. If 25% of the students got an A, that's 100 of the 400 students. If 20 percent of those who studied at least 10 hours (320 of the 400) got an A, that's 64 students. So, if 320 students studied at least 10 hours but only 64 of those got an A, that's $320 - 64 = 256$ students who studied for 10 hours but did not get an A.

The probability that a randomly selected student studied for 10 hours but did not get an A is the number of students for whom those qualifications are true (256) divided by the total number of students (400):

$$\frac{256}{400} = \frac{64}{100} = \frac{16}{25}, \text{ choice (E).}$$

78. D

Explanation: The average of a series of consecutive integers (or consecutive evens, or an set of equally spaced numbers) is equal to the average of the endpoints. So, the average of the evens from 200 to 300 is equal to the average of 200 and 300, 250. Similarly, the average of all the evens between 60 and 100 is 80. 250 is 170 greater than 80, so (D) is the correct choice.

79. D

5. EXPLANATIONS

Explanation: The first step is to figure out how many qualifying numbers are in each set of 100, e.g. 700 to 799, or 800 to 899. Take, for example, the set from 700 to 799:

9 of the numbers between 770 and 779 qualify: all of the numbers except for 777. 9 numbers start with a 7 and end with a 7, but don't have a 7 in the middle, such as 707 and 787. Finally, 9 numbers have matching tens and units digits, but not all three digits in common, including 711 and 722. So, between 700 and 799, there are a total of 27 qualifying numbers.

Thus, between 700 and 999, there are $3(27) = 81$ numbers. That leaves only the numbers between 661 and 699. There are 8 qualifying numbers between 661 and 699, 3 numbers that have 6 as the units digit (676, 686, 696), and 3 numbers that have matching tens and units digits (677, 688, 699), so the total is $81 + 8 + 3 + 3 = 95$, choice (D).

80. D

Explanation: Rather than working with the decimal equivalent of 25%, 0.25, work with fractions. So, if $m = 25\%(25\%)(n)$, that's the same as $m = \frac{1}{4}(\frac{1}{4})(n) = \frac{1}{16}n$.

The other equation can be written as follows:

$$\frac{m}{100}(n) = 25$$

To combine the equations, plug in the value of m from the first equation in to the second one:

$$\frac{\frac{1}{16}n}{100}(n) = 25$$

$$\frac{1}{16}n^2 = 2500$$

$$n^2 = 2500(16)$$

$$n = \sqrt{2500(16)} = \sqrt{25(100)(16)} = 5(10)(4) = 200, \text{ choice (D).}$$

81. A

Explanation: There's no way that the GMAT expects you to precisely calculate the product of the eight smallest two-digit integers. Instead, estimate:

$$(10)(11)(12)(13)(14)(15)(16)(17)$$

10(11) is a little greater than 100.

(14)(15) is close to 200.

Thus, if you subtract one from one of the numbers and add one to the other number, you'll still be near 200:

$$(13)(16) \approx 200$$

$$(12)(17) \approx 200$$

Thus, those four pairs of numbers come out to about $(100)(200)(200)(200) = 2^3 100^4$. Since we rounded down, 2^3 can be rounded up to 10, our approximation is $10(100^4) = 10(10^8) = 10^9$, choice (A).

82. C

Explanation: First, simplify the value of n .

$$n = \sqrt{\frac{1}{256}} = \frac{\sqrt{1}}{\sqrt{256}} = \frac{1}{16}$$

$$\sqrt{n} = \sqrt{\frac{1}{16}} = \frac{\sqrt{1}}{\sqrt{16}} = \frac{1}{4}, \text{ choice (C).}$$

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83. C

Explanation: If x is a multiple of 4, it must be at least 4. If y is a multiple of 8, it must be at least 8. Multiply them together, and the number is either 32 or a multiple thereof; by extension, it's still a multiple of 4 and 8.

However, if neither x nor y is necessarily a multiple of 3, we don't know whether xy is a multiple of 12. Thus, xy must only be a multiple of I and II, choice (C).

84. D

Explanation: Working with all of these percents, it's quickest to pick a number to start with. With percents, that number is usually 100.

If last year's earnings were \$100, Elaine spend \$20 on rent. If she earned 15% more this year, she earned \$115. 30% of \$115 is \$34.50. The question is asking what percent \$34.50 is of \$20:

$$\frac{34.5}{20} = \frac{172.5}{100} = 172.5\%, \text{ choice (D).}$$

85. E

Explanation: To work with exponents on both sides of an equation, first make the bases equal:

$$3^{2x+1} = (3^3)^{x-3}$$

$$3^{2x+1} = 3^{3x-9}$$

$$2x + 1 = 3x - 9$$

$$x = 10, \text{ choice (E).}$$

86. C

Explanation: To compare fractions, find a convenient common denominator. Since all of the denominators are a combination of 2 raised to a power and 3 raised to a power, the GMAT has done much of the work for you. Don't multiply those out: just multiply the smaller fractions by whatever number of 2's and 3's are necessary to make the denominators equal. The largest is $2^4 3^3$, so make the others equal to that:

$$(A) \frac{5}{2^2 3^2} \left(\frac{2^2 3}{2^2 3} \right) = \frac{5(2^2 3)}{2^4 3^3} = \frac{60}{2^4 3^3}$$

$$(B) \frac{11}{2^3 3^2} \left(\frac{2(3)}{2(3)} \right) = \frac{11(2)(3)}{2^4 3^3} = \frac{66}{2^4 3^3}$$

$$(C) \frac{17}{2^2 3^3} \left(\frac{2^2}{2^2} \right) = \frac{17(2^2)}{2^4 3^3} = \frac{68}{2^4 3^3}$$

$$(D) \frac{31}{2^3 3^3} \left(\frac{2}{2} \right) = \frac{31(2)}{2^4 3^3} = \frac{62}{2^4 3^3}$$

$$(E) \frac{67}{2^4 3^3}$$

With the denominators equal, you can directly compare the numerators. 68 is the largest of them, so (C) is the largest of the fractions.

87. C

Explanation: It's useful to visualize these variables on a number line. If $x > y$, it looks like this:

$$y - - - x$$

If y is halfway between x and z , z must be less than y , like this:

$$z - - - y - - - x$$

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So, if $x - y = 2$ (for instance), $y - z$ must be 2 as well. More directly, if $x - y = 2$, $x - z$ must be double that: 4. So, $\frac{x-y}{x-z} = \frac{2}{4} = \frac{1}{2}$, choice (C).

88. A

Explanation: The greatest common divisor of two numbers must be less than or equal to the smaller of the two numbers. However, since the numbers are prime, that eliminates cases such as $a = 3$, $b = 6$, where the smaller number is a factor of the larger one. Two prime numbers, by definition, share no factors, so the greatest common divisor is 1, choice (A).

89. D

Explanation: For a fraction to equal 1, the numerator and denominator must be equal. Since $x - y = z$, a fraction with $x - y$ on the top or bottom with z in the other place would suffice, but no choice fits that description.

Instead, rearrange the equation:

$$x = z + y$$

There are no matches for those two, either, so try once more:

$$x - z = y$$

Finally, choice (D) matches: the numerator is $x - z$ and the denominator is y . Since they are equal, the fraction is equal to 1.

90. B

Explanation: This looks very complicated, but remember that every tricky GMAT question will eventually reduce itself to something much more straightforward. In fact, given how complicated this looks, the odds are disproportionately high that it will reduce to something very simple. If you had to guess, (A) and (B) are by far the best choices.

To do the math: since this is just a binomial, multiply it out:

$$\left(\sqrt{7 + \sqrt{48}}\right)^2 + 2(\sqrt{7 + \sqrt{48}})(\sqrt{7 - \sqrt{48}}) + \left(\sqrt{7 - \sqrt{48}}\right)^2$$

The two outer terms are simple, just a radical squared, while the middle term is 2 times the difference of squares:

$$7 + \sqrt{48} + 2\sqrt{(7 + \sqrt{48})(7 - \sqrt{48})} + 7 - \sqrt{48}$$

$$7 + 2\sqrt{7^2 - (\sqrt{48})^2} + 7$$

$$7 + 2\sqrt{49 - 48} + 7$$

$$7 + 2\sqrt{1} + 7$$

$$7 + 2 + 7 = 16, \text{ choice (B).}$$

91. B

Explanation: Combine the inequalities to make further deductions. $c > a > 0$; $a > d > b$. No more deductions can be made: for instance, there is no stated relationship between either d and 0 or b and 0, which is very important in this question as the romans make statements regarding values being positive or negative. I is not necessarily true: there is no stated relationship between b and 0. Eliminate (A) and (D). II must be true: $c > a > 0$. Eliminate (C). III

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is not necessarily true: there is no stated relationship between d and 0. The correct answer must be (B), II only.

92. A

Explanation: First, solve for $\langle 5 \rangle \times \langle 6 \rangle$. Since 5 is not a multiple of 3, $\langle 5 \rangle = 2(5) = 10$. Since 6 is a multiple of 3, $\langle 6 \rangle = \frac{6}{3} = 2$. The product, then, is 20.

You could now go through all five choices, but there's a more efficient way. Depending on the value of a , 20 must be equal to either $\frac{a}{3}$ or $2a$. So:

$$\frac{a}{3} = 20$$

$$a = 60$$

$$2a = 20$$

$$a = 10$$

Only one of those values of a is present, so a must be 60, choice (A). To confirm, since 60 is a multiple of 3, $\langle 60 \rangle = \frac{60}{3} = 20$.

93. D

Explanation: Don't let the word "probability" scare you: this is a ratio question. If the probability that a woman is selected is three times the probability that a man is selected, then the ratio of women to men is 3 : 1. We are looking for the relationship between the number of women (what we're looking for) and the total number (what we have, 24). That's 3 : (3 + 1), or 3 : 4. We can set up an equation:

$$\frac{x}{24} = \frac{3}{4}$$

$$4x = 3(24)$$

$$x = \frac{3(24)}{4} = 3(6) = 18, \text{ choice (D).}$$

94. A

Explanation: The problem offers enough information to find the amount of interest. If that's 4% of the selling price of \$7,500:

$$i = 0.04(7,500) = 4(75) = 300$$

That interest is also 5% of the face value, which is the number we're looking for. Again, we can set up an equation:

$$300 = 0.05f$$

$$f = \frac{300}{0.05} = \frac{30,000}{5} = 6,000, \text{ choice (A).}$$

95. C

Explanation: For the sum of a pair of integers to be odd, one of the numbers must be even and the other odd. To find the number of variations in parity, look at each one of the pairs of consecutive numbers:

(1, 4): yes, a variation in parity

(4, 3): yes

(3, 5): no

(5, 8): yes

(8, 6): no

That's three variations in parity, choice (C).

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96. B

Explanation: Since we're given the values of a and b , plug them into the expression:

$$\begin{aligned}((-2)^2 - 3)(x - y) - (3 + (-2))(x + y) &= \\(4 - 3)(x - y) - (3 - 2)(x + y) &= \\(1)(x - y) - (1)(x + y) &= \\x - y - (x + y) &= \\x - y - x - y &= \\0 - 2y = -2y, \text{ choice (B).}\end{aligned}$$

97. C

Explanation: Start by finding the prime factorization of 720:

$$720 = 8(90) = (2)(2)(2)(6)(15) = (2)(2)(2)(2)(3)(3)(5)$$

The different prime factors are 2, 3, and 5, so the sum of those factors is $2 + 3 + 5 = 10$, choice (C).

98. B

Explanation: The average of the three variables is $\frac{x+y+z}{3}$, but in order to solve for it in terms of x , we need to substitute something in for $y + z$. To do that, we'll have to rearrange the equation given:

$$x - y = z$$

$$x = y + z$$

Now, to substitute:

$$\frac{x+y+z}{3} = \frac{x+x}{3} = \frac{2x}{3}, \text{ choice (B).}$$

99. E

Explanation: If the range of the list is 10, neither x nor y can be larger or smaller than the current endpoints, 3 and 13, which are already 10 apart. Thus, if x and y are integers, they must each be between 3 and 13, inclusive.

If the average of the six numbers is 8, we can set up an equation:

$$\frac{8+x+y+13+3+8}{6} = 8$$

$$32 + x + y = 48$$

$$x + y = 16$$

If one of the numbers is 13, the other is 3, which is acceptable, and makes $x - y = 10$, choice (D). Since the larger number can't get any larger, and the smaller number can't get any smaller without violating the rule that the range must be 10, (E) is impossible. If the difference between x and y were 12, the range would be 12, which the question tell us it's not.

Incidentally, you could figure this out without solving for $x + y$, as we did here. Just use this same thought process: the difference between x and y can't be any greater than the difference between the largest and smallest numbers (the range), and if the range is 10, $x - y$ can't be greater than 10.

100. C

Explanation: Plug c and 2 in for j and k :

$$\frac{j^3}{k^3} = \frac{c^3}{2^3} = \frac{1}{8}$$

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$$\begin{aligned}\frac{c^3}{8} &= \frac{1}{8} \\ 8c^3 &= 8 \\ c^3 &= 1 \\ c &= 1, \text{ choice (C).}\end{aligned}$$