F unctions, F orm ulas, and Sequences

For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given),the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box ☐,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes ☐/☐,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.

A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.If $f(x) = x^2 + 1$,w hat is $f(2) + f(-2)$?

(A) 0
(B) 1
(C) 4
(D) 5
(E) 10

2.If $f(x) = 2x$ and $g(x) = x^3$,w hat is $f(g(-3))$?

(A) -6
(B) -27
(C) 54
(D) -54
(E) -216

3.If $h(x) = 2x^3 - 3$ and $h(m) = -19$,w hat is the value of m ?

(A) -3

(B) -2
(C) 2
(D) 6,856
(E) 6,862

4.If $f(x) = x - 3$ and $2[f(g)] = 14$,w hat is the value of $f(4g)$?

5.If $f(a,b) = a^2b^4$,and $f(m,n) = 5$,w hat is $f(3m,2n)$?

6.If $f(x) = x^2 - 1$,w hat is the value of $f(y) + f(-1)$?

(A) y^2 -
1 (B) y^2
(C) $y^2 + 1$
(D) $y^2 - 2y$
(E) $y^2 - 2y - 1$

7.If $g(x) = 3x - 3$,w hat is the value of $g(3) + g\left(-\frac{1}{3}\right)$?

8.If $f(x) = \frac{x}{2} - 1$,w hat is the value of $f(f(10))$?

9.If $h(x) = 5x^2 + x$,then $h(a + b) =$

(A) $5a^2 + 5b^2$
(B) $5a^3 + 5b^3$
(C) $5a^2 + 5b^2 + a + b$
(D) $5a^3 + 10ab + 5b^3$
(E) $5a^2 + 10ab + 5b^2 + a + b$

10.If $\sqrt{x} = 2x^2 + 2$,w hat is $\sqrt{4}$?

(A) $\sqrt{\sqrt{-1}}$
(B) $\sqrt{-2}$
(C) $\sqrt{\sqrt{2}}$
(D) $\sqrt{17}$
(E) $\sqrt{34}$

11.$\boxed{x}$ is defined as the least integer greater than *x* for all odd values of *x*,and the greatest integer less than *x* for all even values of *x*.W hat is $\boxed{-2} - \boxed{5}$?

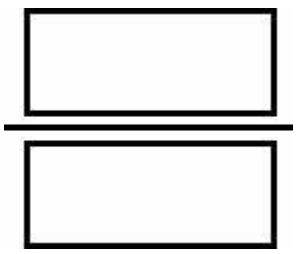
(A) -12
(B) -9
(C) -8
(D) -7
(E) 3

12.$g(x) = x^2 - 4$ and $g(c) = 12$.If $c < 0$,w hat is $g(c - 2)$?

13.If $h(x) = 2x - 1$ and $g(x) = x^2 - 3$,w hat is $h(g(5))$?

14.$h(x) = 2x - 1$ and $g(x) = x^2 - 3$.If $g(m) = 61$,w hat is $h(m)$ if $m > 0$?

15.If & *x*& is defined as one-half the square of *x*,w hat is the value of $\frac{\&4\&}{\&6\&}$?



16.If ~*x* = |14*x*|,w hich of the follow ing m ust be true?

Indicate all such answ ers.

- ☐ ~2 = ~(-2)
- ☐ ~3 + ~4 = ~7
- ☐ The m inim um possible value of ~*x* is zero

17.#*x* = the square of the num ber that is 2 less than *x*.W hat is the value of #5 - #(-1)?

18.If $g(x) = \frac{x^2(4x+9)}{(3x-3)(x+2)}$,w hich answ er represents all values of *x* for w hich *g(x*) is undefined?

(A) 0
(B) $-\frac{9}{4}$
(C) -2,1
(D) -2,0,1
(E) $-2, -\frac{9}{4}, 1$

19.If $f(x) = \frac{\sqrt{x-2}}{x}$ for all integer values of *x,*for how m any values of *x* is *f(x*) undefined?

(A) 0
(B) 1
(C) 2
(D) 3
(E) m ore than 3

20.

f(x) = 2*x* - 3
f(m) = -11

Q uantity A	**Q uantity B**
The value of *m*	H alf the value of *f(m*)

21.The price of a phone call consists of a standard connection fee,w hich does not change,plus a per-m inute charge. A 10-m inute call costs $2.90 and a 16-m inute call costs $4.40.H ow m uch does a 13-m inute call cost?

(A) $3.55
(B) $3.57
(C) $3.58
(D) $3.65

(E) $3.77

22.The first three term s in an arithm etic sequence are 30,33,and 36.W hat is the 80th term ?

23.The sequence S is defined as $S_n = 2(S_{n-1}) - 4$.If $S_1 = 6$,w hat is S_5?

(A) -20
(B) 16
(C) 20
(D) 24
(E) 36

24.The sequence S is defined as $S_n = 3(S_{n-1}) + 1$.If $S_1 = -2$,w hat is S_4?

(A) -39
(B) -41
(C) -43
(D) -45
(E) -47

25.If $S_n = S_{n-1} + S_{n-2} - 3$,then w hat is S_6 w hen $S_1 = 5$ and $S_2 = 0$?

(A) -6
(B) -5
(C) -3
(D) -1
(E) 1

26.If $S_n = S_{n-1} + S_{n-2} - 1$,then w hat is S_4 w hen $S_0 = -10$ and $S_2 = 0$?

(A) -3
(B) 0
(C) 9
(D) 10
(E) 14

27.If $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$,w hat is S_6 w hen $S_1 = 4$,$S_2 = 0$,and $S_4 = -4$?

(A) -2
(B) -12
(C) -16
(D) -20
(E) -24

28.The sequence P is defined as $P_n = 10(P_{n-1}) - 2$.If $P_1 = 2$,w hat is P_4?

29.The sequence *S* is defined as $S_{n-1} = \frac{1}{4}(S_n)$.If S_1 = -4,w hat is S_4?

(A) -256
(B) -64
(C) -1/16
(D) 1/16
(E) 256

30.In sequence A_n,A_1 = 45,and $A_n = A_{n-1} + 2$ for all integers $n > 1$.W hat is the sum of the first 100 term s in sequence A_n?

(A) 243
(B) 14,400
(C) 14,500
(D) 24,300
(E) 24,545

31.In a certain sequence,the term a_n is given by the form ula $a_n = a_{n-1} + 10$.W hat is the positive difference betw een the 10th term and the 15th term ?

(A) 5
(B) 10
(C) 25
(D) 50
(E) 100

32.In a certain sequence,the term a_n is given by the form ula $a_n = 10(a_{n-1})$.H ow m any tim es greater is a_{10} than a_8?

(A) 1
(B) 3
(C) 10
(D) 30
(E) 100

33.A shley and B eatrice received the sam e score on a physical fitness test.The scores for this test,*t*,are determ ined by the form ula $t = 3ps - 25m$ w here *s* and *p* are the num bers of sit-ups and push-ups the athlete can do in one m inute and *m* is the num ber of m inutes she takes to run a m ile.A shley did 10 sit-ups and 10 push-ups and ran an 8-m inute m ile.B eatrice did half as m any sit-ups and tw ice as m any push-ups.If both girls received the sam e overall score,how m any m inutes did it take B eatrice to run the m ile?

(A) 4
(B) 8
(C) 10
(D) 16
(E) 20

34.

The expression $a\{\}b$ is defined as $a\{\}b = (a - b)(a + b)$.

Q uantity A	**Q uantity B**

35.If 5||10 = 5 and 1||(-2) = 1,w hich of the follow ing could define the expression *a*||*b*?

(A) *b* - *a*

(B) $\frac{a^2 - b}{3}$

(C) -*ab*/4

(D) $\frac{b+15}{a}$

(E) *a* + *b* + 4

36.The m axim um height reached by a ball throw n straight up into the air can be determ ined by the form ula $16^2 + h = -t$ *vt* + *d*,w here *t* is the num ber of seconds since it w as throw n,*v* is the initial speed of the throw (in feet per second),*d* is the height (in feet) at w hich the ball w as released,and *h* is the height of ball *t* seconds after the throw .Tw o seconds after a ball is throw n,how high in the air is the ball if it w as released at a height of 6 feet and a speed of 80 feet per second?

(A) 96 feet
(B) 100 feet
(C) 102 feet
(D) 134 feet
(E) 230 feet

37.If $a\#b = a^2\sqrt{b} - a$,w here $b \geq 0$,w hat is the value of (-4)#4?

(A) -36
(B) -28
(C) 12
(D) 28
(E) 36

38.The expression *x*$*y* is defined as $\frac{x^2}{y}$,w here $y \neq 0$.W hat is the value of 9$(6$2)?

(A) 1/2
(B) 9/4
(C) 9/2
(D) 18
(E) 108

39.A m y deposited $1,000 into an account that earns 8% annual interest com pounded every 6 m onths.B ob deposited $1,000 into an account that earns 8% annual interest com pounded quarterly.If neither A m y nor B ob m akes any additional deposits or w ithdraw als,in 6 m onths how m uch m ore m oney w ill B ob have in his account than A m y?

(A) $40
(B) $8
(C) $4
(D) $0.40
(E) $0.04

40.The half-life of an isotope is the am ount of tim e required for 50% of a sam ple of the isotope to undergo radioactive decay.The half-life of the carbon-14 isotope is 5,730 years.H ow m any years m ust pass until a sam ple that starts out w ith 16,000 carbon-14 isotopes decays into a sam ple w ith only 500 carbon-14 isotopes?

(A) 180 years
(B) 1,146 years
(C) 5,730 years
(D) 28,650 years
(E) 183,360 years

41. $f(x) = \frac{2-x}{5}$ and $g(x) = 3x - 2$.If $f(g(x)) = 1$,w hat is the value of x?

(A) -5/3
(B) -1/3
(C) 2/3
(D) 1
(E) 5/3

42.$f(x) = 2x - 2$ and $g(x) = \frac{x}{2} + 2$.W hat is the value of $2f(g(2))$?

(A) 0
(B) 4
(C) 6
(D) 8
(E) 12

43.In a particular com petition,a skater's score,s,is calculated by the form ula $s = \frac{10(3t - 5f)}{m}$ w here t is the num ber of successful triple axels perform ed,f is the num ber of tim es the skater fell,and m is the length,in m inutes,of the perform ance.If Seiko received a score of 6 for a 5 m inute perform ance in w hich she had tw ice as m any successful triple axels as she had falls,how m any tim es did she fall?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

44.A n investor doubles his m oney every 8 years.A t 33 years old,he had 13 m illion dollars ($13m).H ow m uch m oney w ill he have w hen he retires at 65 years old?

(A) $26m
(B) $104m
(C) $130m
(D) $208m
(E) $260m

45.$a \sim b$ is defined as $a - \frac{5-b}{a}$.W hat is the value of $1 \sim ((-1) \sim 1)$?

(A) -9
(B) -1
(C) 1
(D) 2
(E) 5

$$\frac{50b-10a}{10+s}$$

46.A n archer's score is calculated by the form ula w here *b* is the num ber of bull's-eyes hit,*a* is the total num ber of arrow s shot,and *s* is the tim e in seconds it took the archer to shoot.B y how m any points w ould an archer w ho took 10 seconds to shoot 10 arrow s and hit all bull's-eyes beat an archer w ho shot tw ice as m any arrow s and hit half as m any bull's-eyes in 15 seconds?

(A) 2
(B) 7
(C) 10
(D) 18
(E) 20

47.Each term of a certain sequence is calculated by adding a particular constant to the previous term .The second term of this sequence is 27 and the fiftjh term is 84.W hat is the 1st term of this sequence?

(A) 20
(B) 15
(C) 13
(D) 12
(E) 8

48.If $a\#b=\frac{1}{2a-3b}$ and *a*@ *b* = 3*a* - 2*b*,w hat is the value of 1@ 2 - 3#4 ?

(A) -7/6
(B) -1
(C) -5/6
(D) 2/3
(E) 7/6

49.In a certain sequence,the term a_n is given by the form ula $a_n = a_{n-1} + 5$ w here $a_1 = 1$.W hat is the sum of the first 75 term s of this sequence?

(A) 10,150
(B) 11,375
(C) 12,500
(D) 13,950
(E) 15,375

50.In a certain sequence,the term a_n is given by the form ula $a_n = 2 \times a_{n-1}$ w here $a_1 = 1$.W hat is the positive difference betw een the sum of the first 10 term s of the sequence and the sum of the 11th and 12th term s of the sam e sequence?

(A) 1
(B) 1,024
(C) 1,025

(D) 2,048
(E) 2,049

51.

A n operation @ is defined by the equation $a@ b = (a - 1)(b - 2)$.

$x@ 5 = 3@ x$

Q uantity A	**Q uantity B**
The value of x	1

52.The w ait tim e in hours,w ,for a certain popular restaurant can be estim ated by the form ula $w = \frac{n + ks}{10}$ w here k is a constant,n is the num ber of parties w aiting ahead of you,and s is the size of your party.If a fam ily of 4 has a w ait tim e of 30 m inutes w hen 2 other parties are ahead of it,how long w ould a fam ily of 6 expect to w ait if there are 8 parties ahead of it?

(A) 45.5 m inutes
(B) 1 hour 15 m inutes
(C) 1 hour 25 m inutes
(D) 1 hour 45 m inutes
(E) 2 hours

53.A sequence is defined as $a_n = 5(a_{n-1}) - 3$ w here $a_4 = 32$.W hat is the first term of the sequence,a_1?

(A) 1 (B
) 7 (C)
16 (D)
128
(E) 157

54.

A certain sequence is defined by the form ula $a_n = a_{n-1} - 7$.

$a_7 = 7$

Q uantity A	**Q uantity B**
The value of a_1	-35

55.M onthly rent for units in a certain apartm ent building is determ ined by the form ula $k\left(\frac{5r^2 + 10t}{f + 5}\right)$ w here k is a constant,r and t are the num ber of bedroom s and bathroom s in the unit,respectively,and f is the floor num ber of the unit.A 2-bedroom ,2-bathroom unit on the first floor is going for $800/m onth.H ow m uch is the m onthly rent on a 3-bedroom unit w ith 1 bathroom on the 3rd floor?

(A) $825
(B) $875
(C) $900

(D) $925
(E) $1,000

56.

Q uantity A	Q uantity B
The sum of all the m ultiples of 3 betw een 250 and 350	9,990

57.Tow n A has a population of 160,000 and is grow ing at a rate of 20% annually.Tow n B has a population of 80,000 and is grow ing at a rate of 50% annually.

Q uantity A	Q uantity B
The num ber of years until Tow n B 's population is larger than that of Tow n A	3

58.If $f(x) = x^2$,w hat is $f(m + n) + f(m - n)$?

(A) $m^2 + n^2$
(B) $m^2 - n^2$
(C) $2m^2 + 2n^2$
(D) $2m^2 - 2n^2$
(E) m^2n^2

59.S_n is a sequence such that $S_n = (-1)^n$,w here $n \geq 1$.W hat is the sum of the first 20 term s in S_n?

60.If //*x*// m eans "the least integer greater than or equal to *x*," w hat is //-2.5// + //3.6//?

61.If $f(x,y) = x^2y$ and $f(a,b) = 6$,w hat is $f(2a,4b)$?

62.

$f(x) = m$ w here m is the num ber of distinct prim e factors of x.

Q uantity A	Q uantity B

$f(30)$ $f(64)$

63.In a particular sequence,S_n is equal to the units digit of 3^n w here n is a positive integer.If S_1 = 3,how m any of the first 75 term s of the sequence are equal to 9?

64.The sequence $a_1,a_2,a_3,\ldots,a_n$ is such that $a_n = 9 + a_{n-1}$ for all $n > 1$.If $a_1 = 11$,w hat is the value of a_{35}?

65.In sequence Q ,the first num ber is 3,and each subsequent num ber in the sequence is determ ined by doubling the previous num ber and then adding 2.H ow m any tim es does the digit 8 appear in the units digit of the first 10 term s of the sequence?

66.If $g(2m) = 2g(m)$ and $g(3) = 5.5$,w hat is $g(6)$?

67.For w hich of the follow ing functions $f(x)$ is $f(a + b) = f(a) + f(b)$?

(A) $f(x) = x^2$
(B) $f(x) = 5x$
(C) $f(x) = 2x + 1$
(D) $f(x) = \sqrt{x}$
(E) $f(x) = x - 2$

68.

Sam invests a principal of $10,000,w hich earns annually com pounded interest over a period of years.

Q uantity A	**Q uantity B**
The final value of the investm ent after 2 years at 8% interest,com pounded annually	The final value of the investm ent after 4 years at 4% interest,com pounded annually

69.The num ber of years it w ould take for the value of an investm ent to double,at 26% interest com pounded annually, is approxim ately

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

70.The interest rate,com pounded annually,that w ould bring a principal of $1,200 to a final value of $1,650 in 2 years is closest to

(A) 17%
(B) 18%
(C) 19%
(D) 20%
(E) 21%

71.A n investm ent is m ade at 12.5% annual sim ple interest.The num ber of years it w ill take for the cum ulative value of the interest to equal the original investm ent is

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

72.If $f(2a) = 2f(a)$ and $f(6) = 11$,w hat is $f(24)$?

(A) 22
(B) 24
(C) 44
(D) 66
(E) 88

73. $\frac{1}{2}f(x)=f\left(\frac{1}{2}x\right)$,w hich is true for all values of $f(x)$?

(A) $f(x) = 2x +$
2 (B) $f(x) = 13x$
(C) $f(x) = x^2$
(D) $f(x) = x - 10$
(E) $f(x)=\sqrt{x-4}$

F unctions, F orm ulas, and Sequences A nsw ers

1.**(E).**The notation "*f* (*x*)" and "*f* (2)" indicates that you should plug 2 in for *x* in the given equation:

$f(2) = (2)^2 +$
$1\ f(2) = 5$

Likew ise,plug -2 in for *x*:

$f(-2) = (-2)^2 +$
$1\ f(-2) = 5$

N ow add: 5 + 5 = 10.

2.**(D).**Start w ith the innerm ost function,*g*(-3).The notation indicates that you should plug -3 in for *x* in the given equation:

$g(-3) = (-3)^3$
$g(-3) = -27$

N ow the problem reads "w hat is *f* (-27)?" Plug -27 in for *x* in the *f* (*x*) function:

$f(-27) = 2(-27) = -54$

3.**(B).**B e careful w ith the notation here.The problem indicates that *h(m)* = -19,*not* that *h*(-19) = som ething else.D o not plug -19 in for *x*;rather,plug *m* in for *x* and set the answ er equal to -19:

$2m^3 - 3 = -19$
$2m^3 = -16$
$m^3 = -8$
$m = -2$

4.**37.**B e careful w ith the notation here.2[*f* (*g*)] = 14 represents som e num ber (denoted by variable *g*) plugged into the function *f* (*g*),and then m ultiplied by 2,to yield the answ er 14.If 2[*f* (*g*)] = 14,then divide both sides by 2 to get *f* (*g)* = 7.

The m ain function is *f* (*x)* = *x* - 3.The notation *f* (*g*) indicates that you should plug *g* in for all instances of *x*: *f* (*g)* = *g* - 3.Y ou also determ ined that *f* (*g)* = 7,so set the tw o right-hand halves of the equations equal to each other: *g* - 3 =7. The value of *g* is therefore 10.

The question asks for the value of *f* (4*g*).Since *g* = 10,4*g* = 40,and *f* (40):

$f(40) = 40 - 3 = 37$

5.**720.**Plug *m* and *n* into the function in place of *a* and *b.*If *f* (*m* ,*n)* =

5,then: $m^2n^4 = 5$

This cannot be further sim plified,so continue to the second part of the problem : plug 3*m* and 2*n* into the function for *a* and *b*:

$f(3m, 2n) = (3m)^2(2n)^4 = 9m^2 16n^4 = 144m^2n^4$

Since $m^2n^4 = 5$,$144m^2n^4 = 144(5) = 720$.

6.**(A).**The question is asking you to plug *y* into the function,then plug -1 into the function,then add the tw o answ ers together.

$f(y) = y^2 - 1$ *f*
$(-1) = (-1)^2 - 1$
$f(-1) = 0$

Thus,$f(y) + f(-1) = y^2 - 1 + 0 = y^2 - 1$,or choice (A).

7.**2.**The question is asking you to plug 3 into the function,then plug $-\frac{1}{3}$ into the function,and then add the tw o answ ers together.

$$g(3) = 3(3) - 3$$
$$g(3) = 6$$
$$g\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) - 3$$
$$g\left(-\frac{1}{3}\right) = -1 - 3$$
$$g\left(-\frac{1}{3}\right) = -4$$

Thus, $g(3) + g\left(-\frac{1}{3}\right) = 6 + (-4) = 2.$.

8.**1.**W hen dealing w ith “nested” functions,tackle the innerm ost function first.

$$f(10) = \frac{10}{2} - 1 = 4$$

$$f(4) = \frac{4}{2} - 1 = 1$$

Thus, $f(f(10)) = 1$.

9. **(E).** The notation $h(a + b)$ indicates that you need to replace each x w ith the expression $(a + b)$

$h(a + b) = 5(a + b)^2 + (a + b)$
$h(a + b) = 5(a^2 + 2ab + b^2) + a + b$
$h(a + b) = 5a^2 + 10ab + 5b^2 + a + b$

This is equivalent to answ er choice (E).

10. **(A).** The question uses a m ade-up sym bol in place of the traditional notation $f(x)$. The question "If $\boxed{x} = 2x^2 + 2$,w hat is $\boxed{4}$?" is asking you to plug 4 into the function.

$$\boxed{4} = 2(4)^2 + 2$$
$$\boxed{4} = 34$$

D o not fall for trap answ er choice (E). The correct answ er is 34,w hich does not appear in the choices in that form .

Trap choice (E) is $\boxed{34}$,w hich equals $2(34)^2 + 2$;this is m uch larger than 34.

Y ou need to solve each answ er choice until you find one that equals 34. C hoice (A), $\boxed{\boxed{-1}}$,uses the function sym bol tw ice,w hich requires you to plug -1 into the function,then plug your answ er back into the function again:

$$\boxed{-1} = 2(-1)^2 + 2 = 4$$

$\boxed{4} = 2(4)^2 + 2 = 34$ (N ote: you do not need to com plete this m ath if you notice that $\boxed{4}$ m ust have the sam e value as the original $\boxed{4}$ in the question stem .)

Thus, $\boxed{\boxed{-1}} = 34$ and choice (A) is correct. It is not necessary to try the other answ er choices.

11. **(B).** This problem uses a m ade-up sym bol w hich is then defined verbally,rather than w ith a form ula. $\boxed{x}$ has tw o different definitions:

If x is odd, $\boxed{x}$ equals the least integer greater than x (for exam ple,if $x = 3$,then the "least integer greater than 3" is equal to 4)

If x is even, $\lfloor x \rfloor$ equals the greatest integer less than x (for exam ple,if $x = 6$,the “greatest integer less than x” is equal to 5.

Since -2 is even, $\lfloor -2 \rfloor$ = the greatest integer less than -2,or -3.

Since 5 is odd, $\lceil 5 \rceil$ = the least integer greater than 5,or 6.

Thus, $\lfloor -2 \rfloor - \lceil 5 \rceil$ = -3 - 6 = - 9.

12.**32.**For the function $g(x) = x^2 - 4$,plugging c in for x gives the answ er 12.Thus:

$c^2 - 4 = 12$
$c^2 = 16$
$c = 4$ or -4

The problem indicates that $c < 0$,so c m ust be -4.

The problem then asks for $g(c - 2)$.Since $c = -4$,$c - 2 = -6$.Plug -6 into the function:

$g(-6) = (-6)^2 - 4$
$g(-6) = 36 - 4 = 32$

13.**43.**The problem introduces tw o functions and asks for $h(g(5))$.W hen dealing w ith “nested” functions,begin w ith the innerm ost function.

$g(5) = 5^2 - 3 = 22$
$h(22) = 2(22) - 1 = 43$

Thus,$h(g(5)) = 43$.

14.**15.**The problem introduces tw o functions as w ell as the fact that $g(m) = 61$.First,solve for m :

$m^2 - 3 =$
$61\ m^2 = 64$
$m = 8$ or -8

The problem indicates that $m > 0$,so m m ust equal 8.The problem asks for $h(m)$.Since $m = 8$,find $h(8)$:

$h(8) = 2(8) -$
$1\ h(8) = 15$

$\frac{4}{9}$

15. **(or any equivalent fraction).**This function defines a m ade-up sym bol rather than using traditional function notation such as *f* (*x*).Since & *x*& is defined as “one-half the square of *x*”:

$$\&x\& = \frac{1}{2}x^2$$

The problem asks for & 4& divided by & 6& :

$$\&4\& = \frac{1}{2}(4)^2 = 8$$

$$\&6\& = \frac{1}{2}(6)^2 = 18$$

Therefore, $\frac{\&4\&}{\&6\&} = \frac{8}{18} = \frac{4}{9}$.

16.**I,II,and III.**This function defines a m ade-up sym bol: ~*x* is equivalent to |14*x*|.The question asks w hich statem ents m ust be true,so test each one.

~2 = ~(-2)
|14(2)|= |14(-2)|
|28|= |-28| 28 =
28

This statem ent m ust be TR U E .

Sim ilarly test the second statem ent:

~3 + ~4 = ~7 |14(3)|+
|14(4)|= |14(7)| 42 + 56
= 98 98 = 98

This statem ent m ust be TR U E .

Finally,the third statem ent is also true.Since ~*x* is equal to a statem ent inside an absolute value,this value can never be negative.If *x* = 0,then the value of |14*x*|is also 0.The m inim um possible value for ~*x* is 0.

17.**0.**This function defines a m ade-up sym bol,rather than using traditional notation such as *f* (*x*).First,translate the function:

#*x* = (*x* - 2)^2
#5 = (5 - 2)^2 = 9 and #(-1) = (-1 -2)^2 = 9.

#5 - #(-1) = 9 - 9 = 0.

18.**(C).**The term “undefined” refers to the circum stance w hen the solution is not a real num ber — for exam ple,w hen a value w ould ultim ately cause you to divide by 0,that situation is considered “undefined.” There aren’t m any circum stances that result in an undefined answ er.Essentially,you can’t take the square root of a negative,and you can’t divide by 0.There are no square roots in this problem ,but it’s possible that 0 could end up on the denom inator of the fraction.Set each of the term s in the denom inator equal to 0:

$3x - 3 = 0$
$3x = 3$
$x = 1$

$x + 2 =$
$0\ x = -2$

Thus,if $x = 1$ or $x = -2$,then you’d have to divide by 0,m aking $g(x)$ undefined.A ll other values are acceptable.

19.**(E).**The term “undefined” refers to the circum stance w hen the solution is not a real num ber — for exam ple,w hen a value w ould ultim ately cause you to divide by 0,that situation is considered “undefined.” There aren’t m any circum stances that result in an undefined answ er.Essentially,you can’t take the square root of a negative,and you can’t divide by 0.

Since you can’t divide by 0 and the bottom of the fraction is sim ply x,then x cannot be 0.So far,there is 1 “prohibited” value for x.

Since the top of the fraction is a square root and you can’t take the square root of a negative,you can conclude that the quantity inside the absolute value,$x - 2$,m ust be positive or zero:

$x - 2 \geq$
$0\ x \geq 2$

Therefore,there are an infinite num ber of values that x cannot be (1,0,-1,-2,-3,-4,-5…).

20.**(A).**The problem gives a function,$f(x) = 2x - 3$,and then indicates that,w hen m is plugged in to the function,the answ er is -11.Therefore:

$2m - 3 = -11$
$2m = -8$
$m = -4$

Q uantity A is equal to -4.The problem indicates that $f(m) = -11$,so Q uantity B is equal to $\frac{11}{2} = -5.5$.Q uantity A is larger.

21.**(D).**Since “the price of a phone call consists of a standard connection fee,w hich does not change,plus a per-m inute charge,” you can w rite a form ula,using variables for the unknow n inform ation.Let c equal the connection fee and r equal the per-m inute rate:

2.90 = c + r(10)
4.40 = c + r(16)

N ow ,either substitute and solve,or stack and com bine the equation.N ote that there is one c in each equation,so subtracting is likely going to be fastest:

4.40 = c + 16r
- (2.90 = c + 10r)
1.50 = 6r

r = 0.25

The calls cost 25 cents per m inute.N ote that m ost people w ill next plug r back into either equation to find c,but c isn't necessary to solve!

A 10-m inute call costs $2.90.That $2.90 already includes the basic connection fee (w hich never changes) as w ell as the per-m inute fee for 10 m inutes.The problem asks how m uch a 13-m inute call costs.A dd the cost for another 3 m inutes ($0.75) to the cost for a 10-m inute call ($2.90): 2.90 + 0.75 = $3.65.

In fact,if you notice earlier that both the 10-m inute and 16-m inute calls include the sam e connection fee (w hich never changes),you can use a shortcut to solve.The extra 6 m inutes for the 16-m inute call cost a total of $4.40 - $2.90 = $1.50.From there,you can calculate the cost per m inute (1.5 ÷ 6 = 0.25) or you can notice that 13 m inutes is halfw ay betw een 10 m inutes and 16 m inutes,so the cost for a 13-m inute call m ust also be halfw ay betw een the cost for a 10-m inute call and the cost for a 16-m inute call.A dd half of $1.50,or $0.75,to $2.90 to get $3.65.

22.**267.**W hile the sequence is clear (30,33,36,39,42,etc.),you don't have tim e to count to the 80th term .Instead, find a pattern.Each new term in the list adds 3 to the previous term ,so determ ine how m any tim es you need to add 3. (B y the w ay,the term "arithm etic sequence" m eans a sequence in w hich the sam e num ber is added or subtracted for each new term .)

Start w ith the first term ,30.To get from the first term to the second term ,you start w ith 30 and add 3 *once.*To get from the first term to the third term ,you start w ith 30 and add 3 *tw ice.*In other w ords,for the third term ,you add one few er instance of 3: tw ice rather than three tim es.To w rite this m athem atically,say: 30 + 3(n-1),w here n is the num ber of the term .(N ote: you don't need to w rite that,as long as you understand the pattern.)

To get to the 80th term ,then,start w ith 30 and add 3 exactly 79 tim es:

30 + (79 × 3) = 267

23.**(E).**The sequence $S_n = 2(S_{n-1}) - 4$ can be read as "to get any term in sequence S,double the previous term and subtract 4."

The problem gives S_1 (the first term) and asks for S_5 (the fifth term):

$$\frac{6}{S_1} \quad \frac{\quad}{S_2} \quad \frac{\quad}{S_3} \quad \frac{\quad}{S_4} \quad \frac{\quad}{S_5}$$

To get any term ,double the previous term and subtract 4.To get S_2,double S_1 (w hich is 6) and subtract 4: S_2 = 2(6) - 4 = 8.C ontinue doubling each term and subtracting 4 to get the subsequent term :

$$\underset{S_1}{6} \quad \underset{S_2}{8} \quad \underset{S_3}{12} \quad \underset{S_4}{20} \quad \underset{S_5}{36}$$

24.**(B).**The sequence $S_n = 3(S_n - 1) + 1$ can be read as “to get any term in sequence *S*,triple the previous term and add 1.”

The problem gives S_1 (the first term) and asks for S_4 (the fourth term):

$$\underset{S_1}{-2} \quad \underset{S_2}{\_\_} \quad \underset{S_3}{\_\_} \quad \underset{S_4}{\_\_}$$

To get any term ,triple the previous term and add 1.To get S_2,triple S_1 (w hich is -2) and add 1.Thus,S_2 = 3(-2) + 1 = -5.C ontinue tripling each term and adding 1 to get the subsequent term :

$$\underset{S_1}{-2} \quad \underset{S_2}{-5} \quad \underset{S_3}{-14} \quad \underset{S_4}{-41}$$

25.**(A).**The sequence $S_n = S_n - 1 + S_n - 2 - 3$ can be read as “to get any term in sequence *S*,add the tw o previous term s and subtract 3.”

The problem gives the first tw o term s and asks for the sixth term :

$$\underset{S_1}{5} \quad \underset{S_2}{0} \quad \underset{S_3}{\_\_} \quad \underset{S_4}{\_\_} \quad \underset{S_5}{\_\_} \quad \underset{S_6}{\_\_}$$

To get any term ,add the tw o previous term s and subtract 3.So the third term w ill equal 5 + 0 - 3 = 2.The fourth term w ill equal 0 + 2 - 3 = -1.The fifth term w ill equal 2 + (-1) - 3 = -2.The sixth term w ill equal -1 + (-2) - 3 = -6.

$$\underset{S_1}{5} \quad \underset{S_2}{0} \quad \underset{S_3}{2} \quad \underset{S_4}{-1} \quad \underset{S_5}{-2} \quad \underset{S_6}{-6}$$

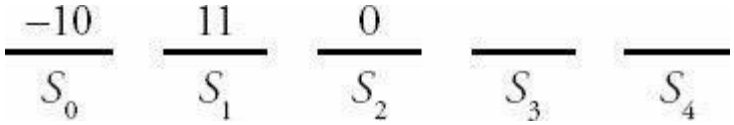
26.**(C).**The sequence $S_n = S_n - 1 + S_n - 2 - 1$ can be read as “to get any term in sequence *S*,add the tw o previous term s and subtract 1.”

The problem gives the zero-th term and the *second* term and asks for the fourth term :

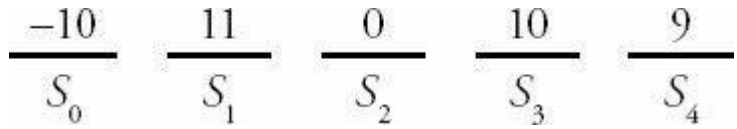
$$\underset{S_0}{-10} \quad \underset{S_1}{\_\_} \quad \underset{S_2}{0} \quad \underset{S_3}{\_\_} \quad \underset{S_4}{\_\_}$$

W ithin the sequence S_0 to S_2,the problem gives tw o values but not the third (S_1).W hat version of the form ula w ould include those three term s?

$S_2 = S_1 + S_0 - 1$
$0 = S_1 + (-10) - 1$
$0 = S_1 - 11$
$11 = S_1$

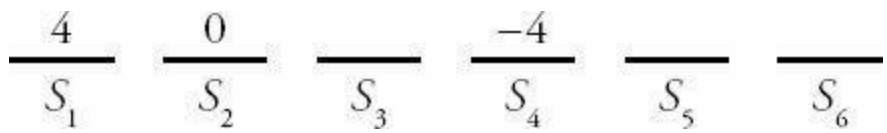


To get each subsequent term ,add the tw o previous term s and subtract 1.$S_3 = 0 + 11 - 1 = 10$.$S_4 = 10 + 0 - 1 = 9$.



27.**(E).**The sequence $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$ can be read as “to get any term in sequence *S*,add the three previous term s and subtract 5.”

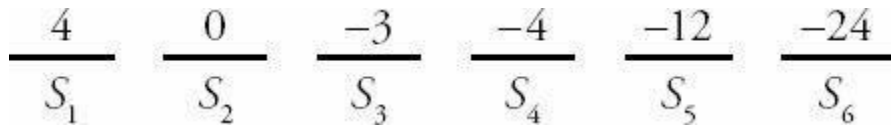
The problem gives the first,second,and *fourth* term s and asks for the sixth term :



W ithin the sequence S_1 to S_4,the problem gives three values but not the fourth (S_3).W hat version of the form ula w ould include those four term s?

$S_4 = S_3 + S_2 +$
$S_1 - 5 - 4 = S_3 + 4$
$+ 0 - 5 - 4 = S_3 - 1$
$-3 = S_3$

Fill in the new ly-calculated value.To find each subsequent value,continue to add the three previous term s and subtract 5.$S_5 = -4 + (-3) + 0 - 5 = -12$.$S_6 = -12 + (-4) + (-3) - 5 = -24$.



28.**1,778.**The sequence $P_n = 10(P_n - 1) - 2$ can be read as “to get any term in sequence *P* ,m ultiply the previous term by 10 and subtract 2.”

The problem gives the first term and asks for the fourth:

$$\frac{2}{P_1} \quad \frac{\quad}{P_2} \quad \frac{\quad}{P_3} \quad \frac{\quad}{P_4}$$

To get P_2,m ultiply 2 × 10,then subtract 2 to get 18.C ontinue this procedure to find each subsequent term (“to get any term in sequence *P* ,m ultiply the previous term by 10 and subtract 2.”) P_3 = 10(18) - 2 = 178.P_4 = 10(178) - 2 = 1,778.

$$\frac{2}{P_1} \quad \frac{18}{P_2} \quad \frac{178}{P_3} \quad \frac{1{,}778}{P_4}$$

29.**(A).**The sequence $S_{n-1} = \frac{1}{4}(S_n)$ can be read as “to get any term in sequence *S*,m ultiply the term *after* that term by $\frac{1}{4}$.” Since this form ula is “backw ards” (usually,you define later term s w ith regard to previous term s),you m ay w ish to solve the form ula for for S_n:

$$S_{n-1} = \frac{1}{4}(S_n)$$
$$4S_{n-1} = S_n$$
$$S_n = 4S_{n-1}$$

This can be read as “to get any term in sequence *S*,m ultiply the previous term by 4.”

The problem gives the first term and asks for the fourth:

$$\frac{-4}{S_1} \quad \frac{\quad}{S_2} \quad \frac{\quad}{S_3} \quad \frac{\quad}{S_4}$$

To get S_2, m ultiply the previous term by 4: (4)(-4) = -16.C ontinue this procedure to find each subsequent term .S_3 = (4)(-16) = -64.S_4 = (4)(-64) = -256.

$$\frac{-4}{S_1} \quad \frac{-16}{S_2} \quad \frac{-64}{S_3} \quad \frac{-256}{S_4}$$

30.**(B).**The first term of the sequence is 45,and each subsequent term is determ ined by adding 2.The problem asks for the sum of the first 100 term s,w hich cannot be calculated directly in the given tim e fram e;instead,find the pattern.The first few term s of the sequence are 45,47,49,51,… .

W hat’s the pattern? To get to the 2nd term ,start w ith 45 and add 2 once.To get to the 3rd term ,start w ith 45 and add 2 tw ice.To get to the 100th term ,then,start w ith 45 and add 2 ninety-nine tim es: 45 + (2)(99) = 243.

N ext,find the sum of all odd integers from 45 to 243,inclusive.To sum up any evenly-spaced set,m ultiply the average by the num ber of elem ents in the set.To get the average,average the first and last term s.Since $\frac{45+243}{2} = 144$,the average is 144.

To find the total num ber of elem ents in the set,subtract 243 - 45 = 198,then divide by 2 (count only the odd num bers, not the even ones).198/2 = 99 term s.N ow ,add 1 (to count both endpoints in a consecutive set,first subtract and then "add 1 before you're done").The list has 100 term s.

M ultiply the average and the num ber of term s:

144 × 100 = 14,400

31.**(D).**This is an arithm etic sequence w here the difference betw een successive term s is alw ays +10.The difference betw een,for exam ple,a_{10} and a_{11},is exactly 10,regardless of the actual values of the tw o term s.The difference betw een a_{10} and a_{12} is 10 + 10 = 20 or 10 × 2 = 20,because there are tw o "steps," or term s,to get from a_{10} to a_{12}. Starting from a_{10},there is a sequence of 5 term s to get to a_{15}.Therefore,the difference betw een a_{10} and a_{15} is 10 × 5 = 50.

32.**(E).**This is a geom etric sequence,in w hich every term is 10 tim es the term before.The problem does not provide the actual value of any term s in the sequence,but the sequence could be som ething like 10,100,1,000… ,or 35,350, 3,500… .Thus,any term is 100 tim es as large as the term that com es tw o before it.

A lternatively,try an algebraic approach.From the form ula,$a_9 = 10a_8$ and $a_{10} = 10a_9$.Substitute for a_9 in the second equation to give: $a_{10} = 10(10a_8) = 100a_8$.

33.**(B).**First,calculate A shley's score: $t = 3 \times 10 \times 10 - 25 \times 8 = 300 - 200 = 100$.If A shley and B eatrice scored the sam e score and B eatrice did half as m any sit-ups (5) and tw ice as m any push-ups (20): $t = 3 \times 5 \times 20 - 25m =$ 100.Solve for m : 300 - 25m = 100 and 25m = 200 so m = 8.

a

b

A lternatively,you could use logic.Since p and s are m ultiplied together in the score form ula,if you m ultiply s by $\frac{1}{2}$ and p by 2,the tw o girls w ill have the sam e overall value for 3ps.B eatrice w ill need the sam e m ile tim e,8 m inutes,in order to achieve the sam e overall score as A shley.

34.**(A).**This problem defines a function for the m ade-up sym bol {}.To calculate the value of Q uantity A ,follow the rules of PEM D A S.First calculate the expressions w ithin parentheses:

7{}6 = (7 - 6)(7 + 6) = 1 × 13 = 13
11{}11 = (11 - 11)(11 + 11) = 0

Then substitute these values back into the original expression:

(13){}(0) = (13 - 0)(13 + 0) = 13 × 13 = 169.Q uantity A is larger.

35.**(B).**This question provides tw o exam ples of the input and output into a m ade-up function,and asks w hich answ er choice could be that function.In other w ords,the function in w hich answ er choice gives the answ er 5 w hen you evaluate 5||10,and also gives the answ er 1 w hen you evaluate 1||(-2)?

O nly one answ er can w ork for both exam ples,so first test *a* 10 to determ ine w hether the function returns = 5 and *b* = 5.If it doesn't,cross off that choice.If it does,test *a* =hen you find an answ er that w orks for both, 1 and *b* = -2.W you can stop.

(A) D oes 10 - 5 = 5? Y ES.D oes (-2) -1 = 1? N o.C ross off answ er (A).

(B) D oes $\frac{5^2-10}{3}=5$? Y ES.D oes $\frac{1^2-(-2)}{3}=1$? Y ES.This is the correct answ er.

It is not necessary to test the rem aining answ ers;the w ork is show n below for com pleteness.

(C) D oes $\frac{-5\times10}{4}=5$ N O .C ross off answ er (C).

(D) D oes $\frac{10+15}{5}=5$? Y ES.D oes $\frac{-2+15}{1}=1$? N o.C ross off answ er (D).

(E) D oes 5 + 10 + 4 = 5? N O .C ross off
answ er (E). The answ er is (B).

36.**(C).**The question presents a form ula w ith a num ber of variables and also provides values for all but one of those variables (*h*,the height of the ball).Solve for *h* by plugging in the values given for the other variables: *t* = 2 seconds,*v* = 80 feet/second,*d* = 6 feet.

$h = -16(2)^2 + (80)(2) + 6 = 102$ feet

37.**(E).**This problem defines a function for the m ade-up sym bol #.In this problem *a* = (-4) and *b* = 4.Plug the values into the function: $(-4)^2\sqrt{4}-(-4)=16\times2+4=36$.D o not forget to keep the parentheses around the -4! A lso note that you take only the positive root of 4 (2) because the problem has been presented in the form of a real num ber underneath the square root sign.

38.**(B).**This problem defines a function for the m ade-up sym bol $.O rder of operation rules (PEM D A S) stay the sam e even w hen the problem uses m ade-up sym bols.First,calculate the value of the expression in parentheses,6$2.
Plug *x* = 6 and *y* = 2 into the function: $6^2/2 = 36/2 = 18$.R eplace 6$2 w ith 18 in the original expression to give 9$18. A gain,plug *x* = 9 and *y* = 18 into the function: $9\$18 = 9^2/18 = 81/18 = 9/2$.

39.**(D).**B oth A m y and B ob start w ith $1,000 and earn 8% interest annually;the difference is in how often this interest is com pounded.A m y's interest is com pounded tw ice a year at 4% each tim e (8% annual interest com pounded 2 tim es a year m eans that she gets half the interest,or 4% ,every six m onths).B ob's interest is com pounded four tim es a year at 2% (8% divided by 4 tim es per year) each tim e.A fter 6 m onths,A m y has $1,000 × 1.04 = $1,040.00 (1 interest

paym ent at 4%) and B ob has \$1,000 × $(1.02)^2$ = \$1,040.40 (2 interest paym ents at 2%).The difference is \$1,040.40 - \$1,040.00 = \$0.40.

For B ob's interest,you can also calculate the tw o separate paym ents.A fter three m onths,B ob w ill have \$1,000 × 1.02 = \$1,020.00.A fter six m onths,B ob w ill have \$1,020 × 1.02 = \$1,040.40.

40.**(D).**A fter each half-life,the sam ple is left w ith half of the isotopes it started w ith in the previous period.A fter one half-life,the sam ple goes from 16,000 isotopes to 8,000.A fter tw o half-lives,it goes from 8,000 to 4,000.
C ontinue this pattern to determ ine the total num ber of half-lives that have passed: 4,000 becom es 2,000 after 3 half-lives,2,000 becom es 1,000 after 4 half-lives,1,000 becom es 500 after 5 half-lives.The sam ple w ill have 500 isotopes after 5 half-lives.Thus,m ultiply 5 tim es the half-life,or 5 × 5730 = 28,650 years.

N ote that the answ er choices are very spread apart.O nce you have determ ined that 5 half-lives have passes,you can estim ate: 5 × 5000 = 25,000 years;answ er (D) is the only possible answ er.

41.**(B).**The question is asking you to substitute the expression for *g(x*) into the function for *f* (*x*),and set the answ er equal to 1.Since *g(x)* = 3*x* - 2,substitute the expression 3*x* - 2 in for *x* in the expression for *f* (*x*):

$$f(g(x)) = \frac{2-g(x)}{5} = \frac{2-(3x-2)}{5} = \frac{4-3x}{5}$$

Since *f(g(x*)) = 1,solve the equation $\frac{4-3x}{5} = 1$:

$$\begin{aligned} 4-3x &= 5 \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$

42.**(D).**Start w ith the innerm ost portion of 2*f* (*g*(2)):

g(2) = 2/2 + 2 = 3

Since *g*(2) = 3,now the expression is 2*f* (3).First evaluate *f* (3): 2(3) - 2 = 4.

Therefore 2*f* (3) = 2 × 4 = 8.

43.**(B).**If Seiko had tw ice as m any triple axels as falls then *t* = 2*f*.Substitute 2*f* in for *t*,along w ith the other given inform ation:

$$6 = \frac{10(3(2f) - 5f)}{5}$$

$$6 = \frac{10(6f - 5f)}{5}$$

$$6 = \frac{10f}{5}$$

$$30 = 10f$$

$$f = 3$$

N ote that you w ant to substitute 2*f* in for *t*,and not the reverse,because the problem asks for the value of *f*.

44.**(D).**The investor's am ount of m oney doubles every 8 years.C alculate the am ount of m oney for each 8-year period:

A ge	$ (m illions)
33	13
33 + 8 = 41	13 × 2= 26
41 + 8 = 49	26 × 2= 52
49 + 8 = 57	52 × 2 = 104
57 + 8 = 65	104 × 2= 208

A t age 65,the investor w ill have $208 m illion.

45.**(B).**This problem defines a function for the m ade-up sym bol ~.Follow PEM D A S order and start w ith the inner m ost parentheses,(-1)~1.In this case *a* = -1 and *b* = 1: $-1 - \left(\frac{5-1}{-1}\right) = -1 - \left(\frac{4}{-1}\right) = -1 + 4 = 3$.(N ote: it's im portant to insert the parentheses! R eplace (-1)~1 w ith 3 and evaluate the next function: 1~3.N ow *a* = 1 and *b* = 3:

$$1 - \left(\frac{5-3}{1}\right) = 1 - \left(\frac{2}{1}\right) = 1 - 2 = -1$$

46.**(D).**C alculate each of the archer's scores by plugging in the appropriate values for *b*,*a*,and *s*.For the first archer, *b* = *a* = *s* = 10 and the score is $\frac{(50 \times 10) - (10 \times 10)}{10 + 10} = \frac{400}{20} = 20$.For the second archer,*b* = half of 10 = 5, *a* = tw ice as m any as 10 = 20,and *s* = 15.The score for the second archer is $\frac{(50 \times 5) - (10 \times 20)}{10 + 15} = \frac{50}{25} = 2$.

The difference in scores is 20 - 2 = 18.

47.**(E).**Let *k* equal the constant added to a term to get the next term .If the second term = 27,then the third term = 27 + *k*,the 4th term = 27 + 2*k*,and the 5th term = 27 + 3*k*.The 5th term equals 84,so create an equation:

27 + 3*k* = 84
3*k* = 57
k = 19

To find the first term ,subtract *k* from the second term .The first term = 27 - 19 = 8.

48.**(C).**This problem defines functions for the m ade-up sym bols # and @ .Substitute *a* = 1 and *b* = 2 into the function for *a*@ *b*: 3(1) - 2(2) = -1.Substitute *a* = 3 and *b* = 4 into the function for *a*#*b*:

$\frac{1}{2(3)-3(4)} = \frac{1}{-6} = -\frac{1}{6}$.Subtract: $(-1) - \left(-\frac{1}{6}\right) = -1 + \frac{1}{6} = -\frac{5}{6}$.

49.**(D).**This is an arithm etic sequence: each new num ber is created by adding 5 to the previous num ber in the sequence.C alculate the first few term s of the sequence: 1,6,11,16,21,and so on.A rithm etic sequences can be w ritten in this form : $a_n = a_1 + k(n - 1)$,w here *k* is the added constant and *n* is the num ber of the desired term .In this case,the function is: $a_n = 1 + 5(n - 1)$.The 75th term of this sequence is $a_{75} = 1 + 5(74) = 371$.

To find the sum of an arithm etic sequence,m ultiply the average value of the term s by the num ber of term s.The average of any evenly-spaced set is equal to the m idpoint betw een the first and last term s.The average of the 1st and 75th term s is $\frac{1+371}{2} = 186$.There are 75 term s.Therefore,the sum of the first 75 term s = 186 × 75 = 13,950.

50.**(E).**This is a geom etric sequence: each new num ber is created by m ultiplying the previous num ber by 2.C alculate the first few term s of the series to find the pattern: 1,2,4,8,16,and so on.G eom etric sequences can be w ritten in this form : $a_n = r^{n-1}$,w here *r* is the m ultiplied constant and *n* is the num ber of the desired term .In this case,the function is $a_n = 2^{n-1}$.

The question asks for the difference betw een the sum of the first 10 term s and the sum of the 11th and 12th term s. W hile there is a clever pattern at play,it is hard to spot.If you don't see the pattern,one w ay to solve is to use the calculator to add the first ten term s: 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1,023.

The 11th + 12th term s = 1,024 + 2,048 = 3,072

Subtract to get 2,049.

A lternatively,you m ay see the pattern in the first few term s (1,2,4,8,16…): Every term is equal to 1 m ore than the sum of the ones before it.For exam ple,1 + 2 = 3 and the next term is 4.1 + 2 + 4 = 7 and the next term is 8.Thus,the sum of the first 10 term s of the sequence is 1 less than the 11th term .The 11th term = 2^{10} = 1,024,so the sum of the first 10 term s = 1,023.and the difference betw een the 10th and 11th term s equals 1.A dd the value of the 12th term or 1 + 2,048 = 2,049.

51.**(B).**This problem defines a function for the m ade-up sym bol @ .U se the definition of the new sym bol to rew rite the equation $x@ 5 = 3@ x$ w ithout the @ operator.

For $x@ 5$,$a = x$ and $b = 5$: $x@ 5 = (x - 1)(5 - 2) = 3x - 3$.
For $3@ x$,$a = 3$ and $b = x$: $3@ x = (3 - 1)(x - 2) = 2x - 4$.

Equating these tw o expressions gives us:

$3x - 3 = 2x -$
$4 x = -1$

Q uantity B is larger.

52.**(B).**Start by solving for the constant,k.A fam ily of 4 ($s = 4$) has a w ait tim e of 30 m inutes ($w = 0.5$ hours — don't forget that w is in hours!) w hen 2 parties are ahead of it ($n = 2$).Plug these values into the form ula: $0.5 = \frac{2 + 4k}{10}$.Solve for k:

$5 = 2 + 4k$
$3 = 4k$
$k = \frac{3}{4}$

To solve for the w ait tim e of the fam ily of 6 w ith 8 parties ahead of it,plug these values into the form ula along w ith $k = \frac{3}{4}$: $w = \frac{8 + \left(\frac{3}{4}\right)6}{10} = 1.25$ hours.The answ er choices are show n in hours and m inutes.0.25 hours is equal to 0.25 × 60 m inutes = 15 m inutes.The answ er is 1 hour 15 m inutes.

53.**(A).**The given sequence can be read as,"To get any term in a,m ultiply the previous term by 5 and then subtract 3." The problem indicates that $a_4 = 32$ and asks for the value of a_1.

$\underline{\quad A_1 \quad} \quad \underline{\quad A_2 \quad} \quad \underline{\quad A_3 \quad} \quad \underline{\quad 32 \quad A_4 \quad}$

Start w ith a_4 and find a_3:

$32 = 5a_3 - 3$
$35 = 5a_3$
$7 = a_3$

U se a_3 to find a_2:

$7 = 5a_2 - 3$

$10 = 5a_2$

$2 = a_2$

U se a_2 to find a_1:

$2 = 5a_1 - 3$

$5 = 5a_1$

$1 = a_1$

54.**(A).**The sequence $a_n = a_n - 1 - 7$ can be read as “to get any term in sequence *a*,subtract 7 from the previous term .” The problem provides the 7th term ;plug the term into the function in order to determ ine the pattern.N ote that Q uantity A asks for the value of a_1,so try to find the 6th term :

$7 = a_6 - 7$

$a_6 = 14$

In other w ords,each previous term w ill be 7 larger than the subsequent term .Therefore,$a_7 = 7$,$a_6 = 14$,$a_5 = 21$,and so on.The term a_1,then,is larger than the starting point,7,and m ust also be larger than the negative value in Q uantity B .Q uantity A is larger.N ote that the value in Q uantity B is the result of incorrectly *subtracting* 7 six tim es,rather than adding it.

55.**(A).**First,solve for the constant *k* using the price inform ation of the 2-bedroom ,2-bath unit ($m = 800$,$r = t = 2$ and $f = 1$):

$$800 = k\left(\frac{5(2)^2 + 10(2)}{1+5}\right)$$

$$800 = k\left(\frac{20+20}{6}\right)$$

$$800 = k\left(\frac{20}{3}\right)$$

$$800\left(\frac{3}{20}\right) = k$$

$$40(3) = k$$

$$120 = k$$

N ext,solve for the rent on the 3-bedroom ,1-bath unit on the 3rd floor ($r = 3$,$t = 1$,and $f = 3$):

$$m = 120\left(\frac{5(3)^2 + 10(1)}{3+5}\right)$$

$$m = 120\left(\frac{45+10}{8}\right)$$

$$m = 120\left(\frac{55}{8}\right)$$

$$m = 15(55)$$

$$m = 825$$

56.**(B).**First,find the sm allest m ultiple of 3 in this range: 250 is not a m ultiple of 3 (2 + 5 + 0 = 7,w hich is not a m ultiple of 3).The sm allest m ultiple of 3 in this range is 252 (2 + 5 + 2 = 9,w hich is a m ultiple of 3).N ext,find the largest m ultiple of 3 in this range.350 is not a m ultiple of 3 (3 + 5 + 0 = 8);the largest m ultiple of 3 in this range is 348.

The sum of an evenly-spaced set of num bers equals the average value m ultiplied by the num ber of term s.The average value is the m idpoint betw een 252 and 348: (252 + 348) ÷ 2 = 300.To find the num ber of term s,first subtract 348 - 252 = 96.This figure represents all num bers betw een 348 and 252,inclusive.To count only the m ultiples of 3,divide 96 by the 3: 96/3 = 32.Finally,“add 1 before you’re done” because you do w ant to count both end points of the range: 32 + 1 = 33.

The sum is 300 × 33 = 9,900.Since 9,900 is sm aller than 9,990,Q uantity B is larger.

57.**(A).**Set up a table and calculate the population of each tow n after every year;use the calculator to calculate Tow n A ’s population.If you feel com fortable m ultiplying by 1.5 yourself,you do not need to use the calculator for Tow n B . Instead,add 50% each tim e (e.g.,from 80,000,add 50% or 40,000 to get 120,000).

	Tow n A	**Tow n B**
N ow	160,000	80,000
Y ear 1	160,000(1.2) = 192,000	80,000 + 40,000 = 120,000
Y ear 2	192,000(1.2) = 230,400	120,000 + 60,000 = 180,000
Y ear 3	230,400(1.2) = 276,480	180,000 + 90,000 = 270,000

N ote that,after three years,Tow n A still has m ore people than Tow n B .It w ill take longer than 3 years,then,for Tow n B to surpass Tow n A ,so Q uantity A is larger.

58.**(C).**The problem provides the function $f(x) = x^2$ and asks you to evaluate $f(m + n) + f(m - n)$.Plug into this function tw ice — first,to insert $m + n$ in place of x,and then to insert $m - n$ in place of x.

$f(m + n) = (m + n)^2 = m^2 + 2mn + n^2$

$f(m - n) = (m - n)^2 = m^2 - 2mn + n^2$

N ow add the tw o:

$$(m^2 + 2mn + n^2) + (m^2 - 2mn + n^2) = 2m^2 + 2n^2$$

59.**0.**A dding 20 individual term s w ould take quite a long tim e.Look for a pattern.The first several term s in $S_n = (-1)^n$,w here $n \geq 1$:

$$S_1 = (-1)^1 = -1$$
$$S_2 = (-1)^2 = 1$$
$$S_3 = (-1)^3 = -1$$
$$S_4 = (-1)^4 = 1$$

The term s alternate -1,1,-1,1,and so on.If the term s are added,every pair of -1 and 1 w ill add to zero;in other w ords,for an even num ber of term s,the sum w ill be zero.20 is an even num ber,so the first 20 term s add to zero.

60.**2.**The “least integer greater than or equal to” language is tricky because of the w ords “least” and “greater” in the sam e sentence.First,is the problem asking for a num ber that is larger or sm aller than the starting num ber? “G reater than or equal to” indicates that the resulting num ber w ill be larger than the starting num ber.R ephrase this as “w hat is the next largest integer?”

The next largest integer starting from -2.5 is -2.(R em em ber that,for negative num bers,larger m eans “closer to 0.”)

The next largest integer starting from 3.6 is 4.

$$-2 + 4 = 2$$

61.**96.**The problem provides the function $f(x,y) = x^2y$ and also the fact that w hen a and b are plugged in for x and y, the answ er is 6.In other w ords:

$$f(x,y) = x^2y$$
$$f(a,b) = a^2b = 6$$

The problem asks for the value of $f(2a,4b)$.First,plug $2a$ in for x and $4b$ in for y:

$$f(2a,4b) = (2a)^2(4b)$$
$$f(2a,4b) = 4a^2(4b)\ f$$
$$(2a,4b) = 16a^2b$$

The problem already provides the value for the variables: $a^2b = 6$.Therefore,$16a^2b = 16(6) = 96$.

62.**(A).**The problem indicates that $f(x) = m$ w here m is the num ber of distinct (or different) prim e factors of x.For exam ple,if $x = 6$,6 has tw o distinct prim e factors: 2 and 3.Therefore,the corresponding answ er (m value) w ould be 2.

For Q uantity A *f* (30): 30 has 3 distinct prim e factors (2,3,and 5),so *f* (30) = 3.

For Q uantity B *f* (64): 64 is m ade of the prim e factors 2,2,2,2,2,and 2).This is only one distinct prim e factor,so *f* (64) = 1.Q uantity A is larger.

63.**19.**It's too m uch w ork to w rite out the first 75 term s of a sequence,so there m ust be som e kind of pattern.Figure it out (the units digit is bolded):

3^1 = **3**
3^2 = **9**
3^3 = 2**7**
3^4 = 8**1**
3^5 = 24**3**
3^6 = 72**9**

The pattern is 3,9,7,1 and repeats every 4 term s.(N ote: you can m em orize this pattern or recreate it w hen you need it.For the num bers 0 to 9,there are no m ore than 4 units digits in the pattern;if you rem em ber this,you only need to test the first 4 term s.)

The 2nd term in the pattern of 4 term s is equal to 9;for each set of 4 term s,then,there w ill be one num ber w ith a units digit of 9.Since 75 divided by 4 = 18 rem ainder 3,the entire pattern w ill repeat 18 tim es,for a total of 18 term s w ith a units digit of 9,follow ed by 3 extra term s.The three extra term s (3,9,7) include one extra 9.Therefore,a units digit of 9 appears 18 + 1 = 19 tim es in this sequence.

64.**317.**Each term in the sequence is 9 greater than the previous term .To m ake this obvious,you m ay w ant to w rite a few term s of the sequence: 11,20,29,38,etc.

a_{35} com es 34 term s after a_1 in the sequence.In other w ords,a_{35} is 34 × 9 = 306 greater than a_1.

Thus,a_{35} = 11 + 306 = 317.

65.**9.**A fter the first term in the sequence,every term has a units digit of 8:

$Q_1 = 3$

$Q_2 = 2(3) + 2 = 8$
$Q_3 = 2(8) + 2 = 18$
$Q_4 = 2(18) + 2 = 38$
$Q_5 = 2(38) + 2 = 78$
…

So 8 w ill be the units digit nine out of the first ten tim es.

66.**11.**This question concerns som e function for w hich the full form ula is not given.The problem indicates that $g(2m) = 2g(m)$.In other w ords,this function is such that plugging in 2*m* is the sam e as plugging in *m* and then m ultiplying by 2.Plug $g(3)$ into the equation for $g(m)$:

$g(6) = 2g(3)$
$g(6) = 2(5.5)$
$g(6) = 11$

67.**(B).**The question asks w hich of the functions in the answ er choices is such that perform ing the function on $a + b$ yields the sam e answ er as perform ing the function to a and b individually and then adding those answ ers together.

The correct answ er should be such that $f(a + b) = f(a) + f(b)$ is true for any values of a and b.Test som e num bers,for exam ple $a = 2$ and $b = 3$.

	$f(a + b) = f(5)$	$f(a) = f(2)$	$f(b) = f(3)$	D oes $f(a + b) = f(a) + f(b)$?
(A)	$f(5) = 5^2 = 25$	$f(2) = 2^2 = 4$	$f(3) = 3^2 = 9$	N o
(B)	$f(5) = 5(5) = 25$	$f(2) = 5(2) = 10$	$f(3) = 5(3) = 15$	Y es
(C)	$f(5) = 2(5) + 1 = 11$	$f(2) = 2(2) + 1 = 5$	$f(3) = 2(3) + 1 = 7$	N o
(D)	$f(5) = \sqrt{5}$	$f(2) = \sqrt{2}$	$f(3) = \sqrt{3}$	N o
(E)	$f(5) = 5 - 2 = 3$	$f(2) = 2 - 2 = 0$	$f(3) = 3 - 2 = 1$	N o

A lternatively,use logic — for w hat kinds of operations are perform ing the operation on tw o num bers and then adding them together the sam e as adding the original num bers together and then perform ing the operation? M ultiplication or division w ould w ork,but squaring,square-rooting,adding,or subtracting w ould not.The correct function can contain O N LY m ultiplication and/or division.

68.**(B).**Y ou can solve this problem by applying the com pound interest form ula:

$$V = P\left(1 + \frac{r}{100}\right)^t$$

Since the principal P is the sam e in both cases,you can leave it out and just com pare the rest.

Q uantity A : $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{8}{100}\right)^2 = (1.08)^2 = 1.08 \times 1.08 = 1.1664$

Q uantity B : $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{4}{100}\right)^4 = (1.04)^4 = 1.04 \times 1.04 \times 1.04 \times 1.04 \approx 1.1699$

Q uantity B is larger.

A lternatively,you can use logic.N otice that the *sim ple* interest in each case w ould be the sam e: 2 years of 8% *sim ple* interest (of an unchanging principal) is equal to 4 years of 4% *sim ple* interest of the sam e principal.N ow go back to the com pounded w orld.If the sim ple interest scenarios are the sam e,then it w ill alw ays be true that the com pounded scenario w ith *m ore frequent* com pounding w ill give you a larger principal in the end,because you're earning "interest

on the interest" m ore often.

2 periods of 8% com pounded interest	<	4 periods of 4% com pounded interest	<	8 periods of 2% com pounded interest	<	16 periods of 1% com pounded interest

The differences are sm all but real.

69.**(B).**Start w ith $1,and m ultiply by $\left(1+\frac{26}{100}\right)$ = 1.26 for each year that passes.In order for the am ount to double,it w ould have to reach $2.

End of Y ear 1: $1 × 1.26 = $1.26
End of Y ear 2: $1.26 × 1.26 = $1.5876
End of Y ear 3: $1.5876 × 1.26 = $2.000376 ≈ $2.00

It takes 3 years for the investm ent to double in value.If you m istakenly thought in term s of *sim ple* interest,you m ight think it w ould take about 4 years (since 26% is just a tiny bit m ore than 25% = 1/4).In the com pounded case,you're earning "interest on the interest," though,so the investm ent grow s m ore quickly.

70.**(A).**Y ou can try the answ er choices to find the answ er.Start w ith either choice (B) or choice (D).A n 18% interest rate corresponds to m ultiplying by 1.18.C om pounding over tw o years m eans m ultiplying by 1.18 tw ice:

$1,200 × 1.18 × 1.18 = $1,670.88.

This is close to $1,650 but 17% could be closer.Try answ er choice (A):

$1,200 × 1.17 × 1.17 = $1,642.68

Since this result is in fact closer to $1,650,the interest rate m ust be closer to 17% than to 18% .

A lternatively,use the com pound interest form ula and solve for the m issing interest rate:

$$V = P\left(1+\frac{r}{100}\right)^t$$

$$1{,}650 = 1{,}200\left(1+\frac{r}{100}\right)^2$$

$$\frac{1{,}650}{1{,}200} = \left(1+\frac{r}{100}\right)^2$$

$$\sqrt{1.375} = 1+\frac{r}{100}$$

$$1.172 = 1+\frac{r}{100}$$

$$(0.172)100 = r$$

$$r \approx 17$$

71.**(E).**“Sim ple” interest m eans that the interest is calculated based on the initial am ount every tim e;the interest earned is not included in future calculations.Each year,the investm ent pays 12.5% ,or 1/8,of the original investm ent as sim ple interest.A s a result,it w ill take exactly 8 years for the cum ulative interest to add up to the original investm ent.

B e careful not to apply the com pound interest form ula here.If the 12.5% interest is in fact com pounded annually,it w ill take only about 6 years for the investm ent to double in value.

72.**(C).**This question concerns som e function for w hich the full form ula is not provided.The problem indicates that *f* (2*a)* = 2*f* (*a*).In other w ords,this function is such that plugging in 2*a* is the sam e as plugging in *a* and then m ultiplying by 2.Plug *f* (6) = 11 into the equation *f* (2*a)* = 2*f* (*a*):

f (2(6)) = 2(11)
f(12) = 22

U se the sam e process a second tim e.If *a* = 12 and *f* (12) = 22:

f (2(12)) = 2(22)
f (24) = 44

A lternatively,use logic.W hen you plug in 2*a*,you’ll get the sam e answ er as w hen you plug in *a* and then m ultiply by 2. Plugging in 24 is the sam e as plugging in 6 a total of 4 tim es,and w ill give you an answ er 4 tim es as big as plugging in 6.Since plugging in 6 yields 11,plugging in 24 yields 44.

73.**(B).**The question is asking “For w hich function is perform ing the function on *x* and TH EN m ultiplying by 1/2 the equivalent of perform ing the function on 1/2 of *x*?”

The fastest m ethod is to use logic: since the order of operations says that order does not m atter w ith m ultiplication and division but D O ES m atter betw een m ultiplication and addition/subtraction,or m ultiplication and exponents,you

need a function that has only m ultiplication and/or division.O nly answ er choice (B) qualifies.

A lternatively,try each choice.

	$\frac{1}{2}f(x)$	$f\left(\frac{1}{2}x\right)$	equal?
(A)	$f\left(\frac{1}{2}x\right)$	$\frac{1}{2}(2x+2)=x+1$	N o
(B)	$\frac{1}{2}(13x)=\frac{13x}{2}$	$13\left(\frac{1}{2}x\right)=\frac{13x}{2}$	Y es
(C)	$\frac{1}{2}x^2$	$\left(\frac{1}{2}x\right)^2=\frac{1}{4}x^2$	N o
(D)	$\frac{1}{2}(x-10)=\frac{x}{2}-5$	$\frac{1}{2}x-10=\frac{x}{2}-10$	N o
(E)	$\frac{1}{2}\sqrt{x-4}$	$\sqrt{\frac{1}{2}x-4}$	N o

If you aren’t sure w hether the tw o term s in choice (E) are equal,try plugging in a real num ber for *x*.If *x* = 8,then the left-hand value becom es 1 and the right-hand value becom es the square root of 0.The tw o values are *not* the sam e.