T riangles

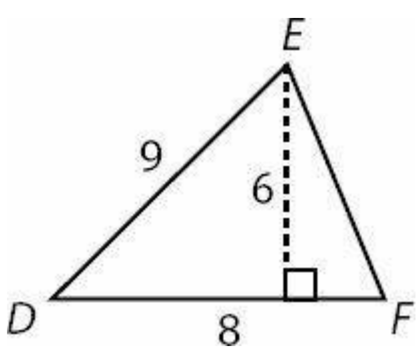
For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given),the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box ,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes ,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.

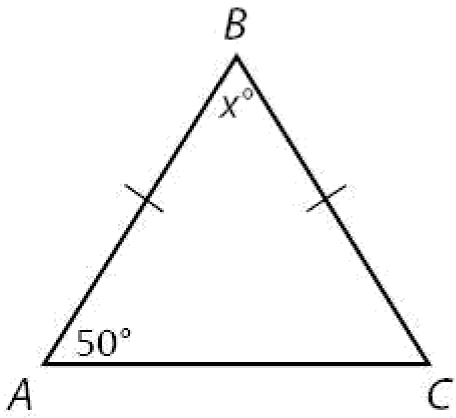
A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.W hat is the area of Triangle *D EF* ?



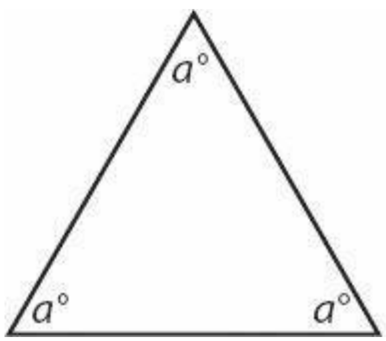
(A) 23
(B) 24
(C) 48
(D) 56
(E) 81

2.W hat is the value of *x*?



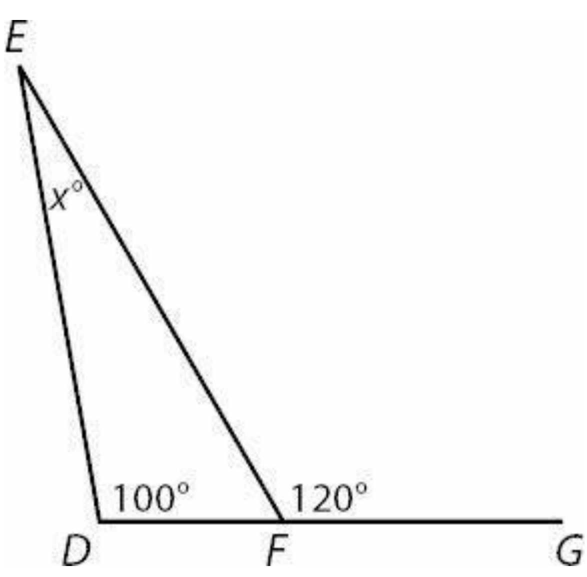
Not drawn to scale.

3.



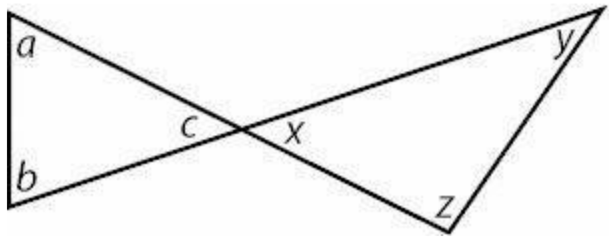
Q uantity A	**Q uantity B**
$2a + b$	$3a + \frac{b}{3}$

4.



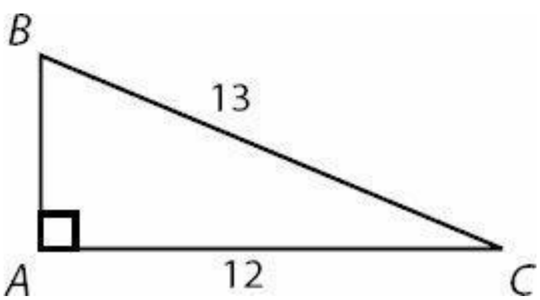
DFG is a straight line. What is the value of *x*?

5



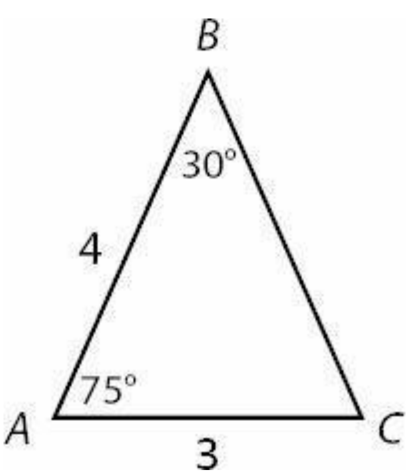
Quantity A	**Quantity B**
$a + b + x$	$c + y + z$

6.



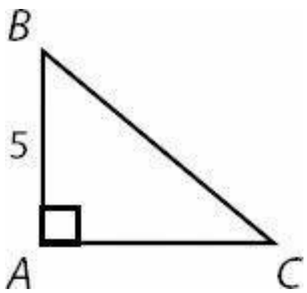
What is the area of right triangle *ABC* ?

7.



What is the perimeter of triangle *ABC* ?

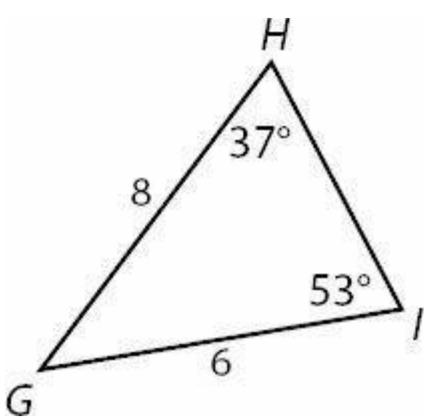
8.



The area of right triangle *ABC* is 15.W hat is the length of hypotenuse *BC* ?

(A) $\sqrt{34}$
(B) 6
(C) $\sqrt{51}$
(D) $\sqrt{61}$
(E) $\sqrt{71}$

9.

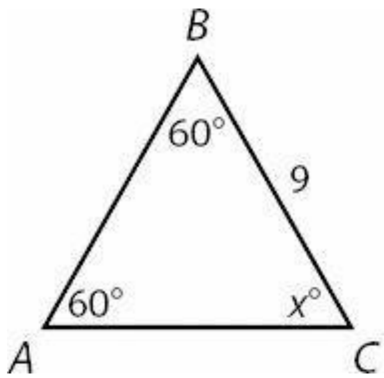


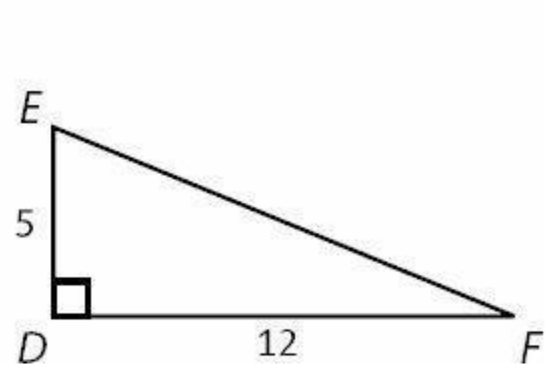
W hat is the length of side *H I*?

10.If the hypotenuse of an isosceles right triangle is 7 $\sqrt{2}$,w hat is the area of the triangle?

(A) 14
(B) 18
(C) 24.5
(D) 28
(E) 49

11.





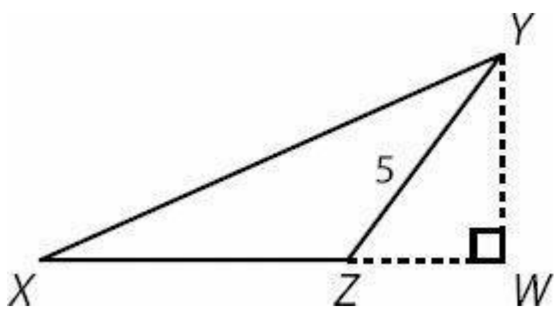
<u>Quantity A</u>

Perimeter of triangle *ABC*

<u>Quantity B</u>

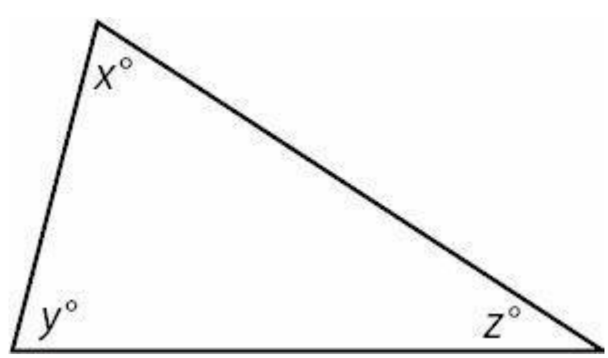
Perimeter of triangle *DEF*

12.



WZ has a length of 3 and *ZX* has a length of 6. What is the area of Triangle *XYZ*?

13.



In the figure shown, $z + x$ is 110 degrees.

<u>Quantity A</u>

x

<u>Quantity B</u>

y

14.

Isosceles triangle *ABC* has tw o sides w ith lengths 8 and 5.

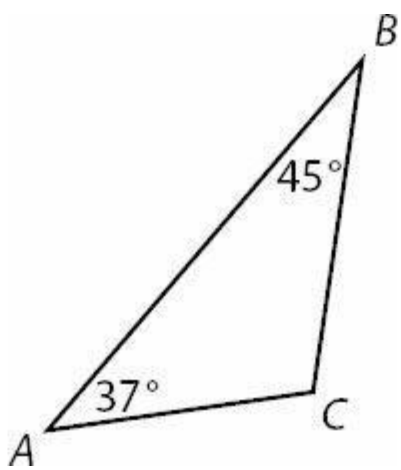
Q uantity A	**Q uantity B**
The length of the third side	8

15.

Isosceles triangle *ABC* has tw o sides w ith lengths 2 and 11.

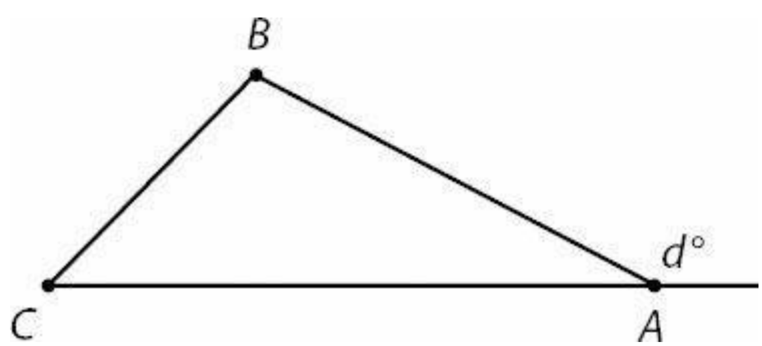
Q uantity A	**Q uantity B**
The length of the third side	11

16.



Q uantity A	**Q uantity B**
Side length *AC*	Side length *BC*

17.



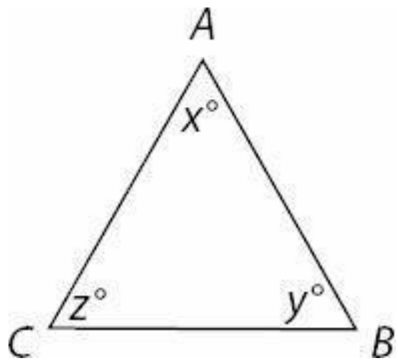
Q uantity A	**Q uantity B**
The sum of the m easures of angles *B* and *C*	*d*

18.

The sides of a right triangle are 3,4,and *z*.

Q uantity A	**Q uantity B**
z	5

19.



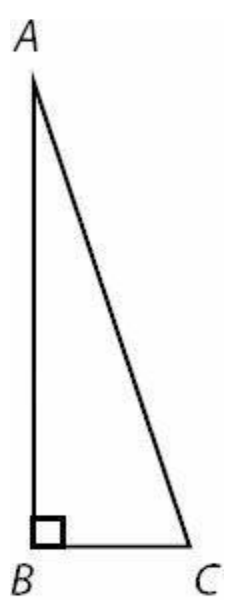
N ote: Figure N O T draw n to scale

$x > z$

$z > 60$

Q uantity A	**Q uantity B**
The length of side *AC*	The length of side *AB*

20.



$AC = 4\sqrt{10}$

BC is 1/3 the length of *AB*

Q uantity A | **Q uantity B**

The length of *AB* | 10

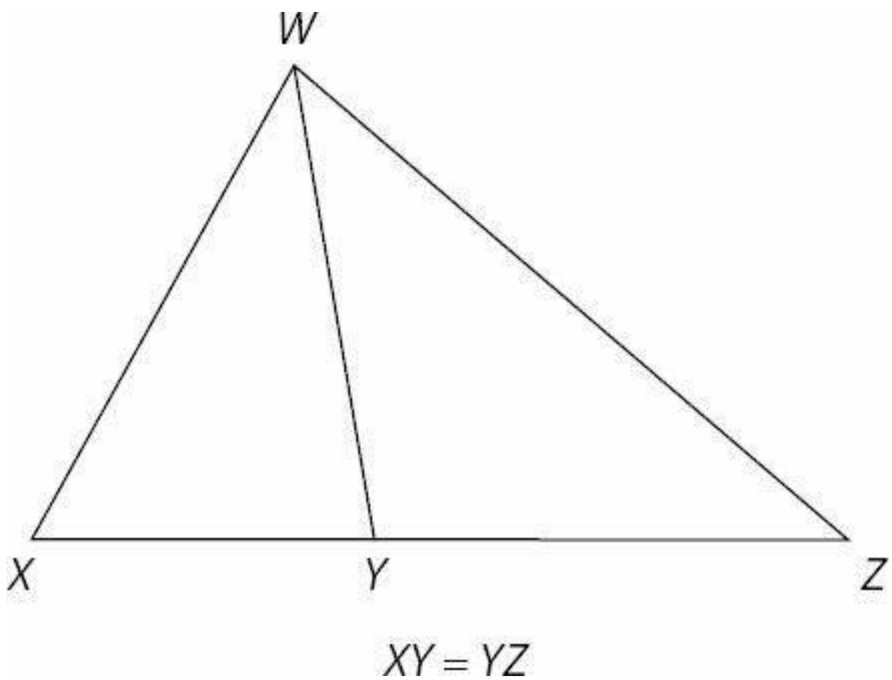
21.

A triangle has perim eter 24.

Q uantity A | **Q uantity B**

The area of the triangle | 20

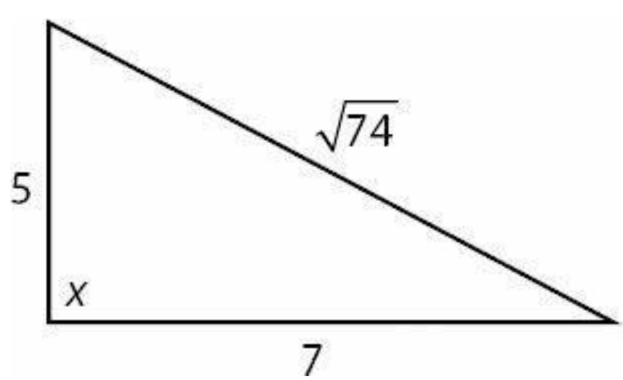
22.



Q uantity A | **Q uantity B**

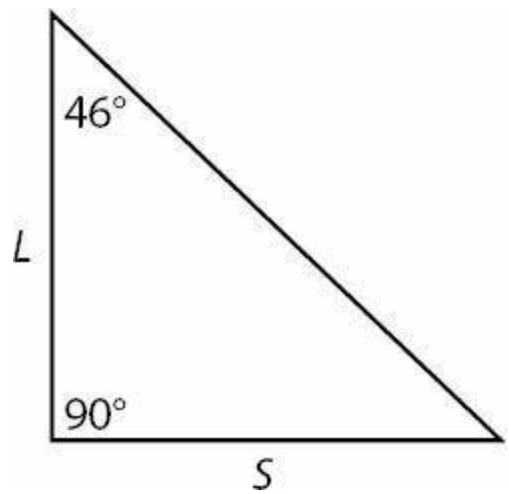
The area of *W YX* | The area of *ZYW*

23.



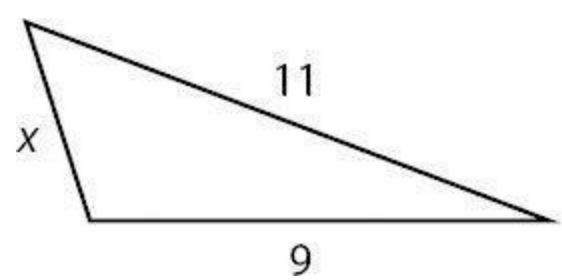
Q uantity A	**Q uantity B**
x	90

24.



Q uantity A	**Q uantity B**
$\frac{L}{S}$	1

25.



x is an integer

Q uantity A	**Q uantity B**
The num ber of possible values of x	17

26.If p is the perim eter of a triangle w ith one side of 6 and another side of 9,w hat is the range of possible values for p?

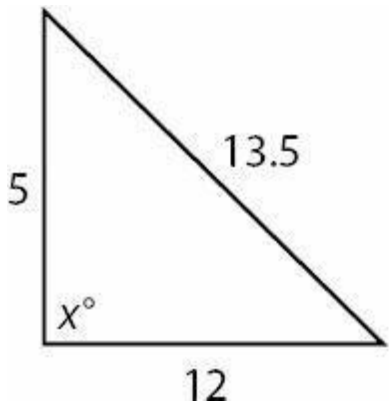
(A) $3 < p < 15$
(B) $15 < p < 24$
(C) $18 < p < 30$
(D) $18 < p < 42$
(E) $21 < p < 42$

27.

A right triangle has hypotenuse 8 and legs of 6 and *x.*

Quantity A	**Quantity B**
x	10

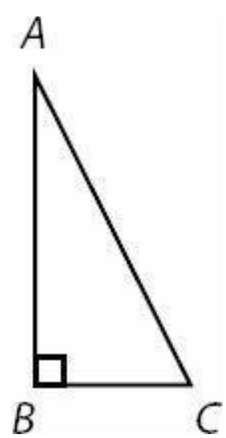
28.



Note: Figure NOT drawn to scale

Quantity A	**Quantity B**
x	90

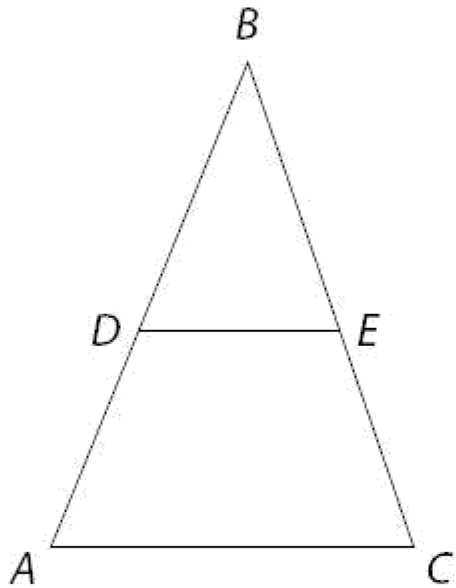
29.



The length of *BC* is equal to *x AB* is twice as long as *BC*

Quantity A	**Quantity B**
The length of *AC*	$x\sqrt{3}$

30.



D E is parallel to *AC*
BE = *EC*
AC = 14

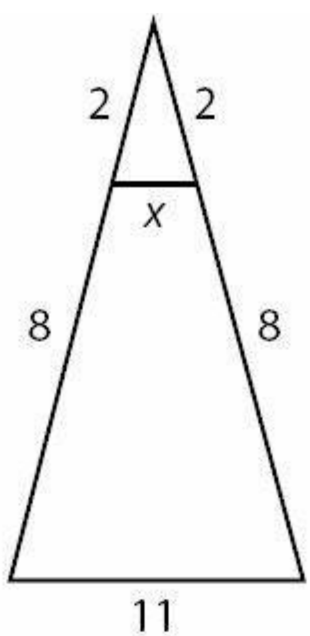
Q uantity A	**Q uantity B**
D E	7

31.

A triangle has sides of 8,*m* ,and *n.*

Q uantity A	**Q uantity B**
$|m - n|$	8

32.



Note: Figure NOT drawn to scale

If the line segm ent w ith length *x* is parallel to the line segm ent w ith length 11,w hat is the value of *x*?

(A) 1

(B) $\sqrt{2}$

(C) $\frac{11}{5}$

(D) $\frac{11}{4}$

(E) 5.5

33.

Tw o sides of a triangle have m easures 13 and 9.

Q uantity A	**Q uantity B**
The m easure of the third side of the triangle.	$\sqrt{226}$

34.W hat is the area of an equilateral triangle w ith side length 4?

(A) $2\sqrt{3}$

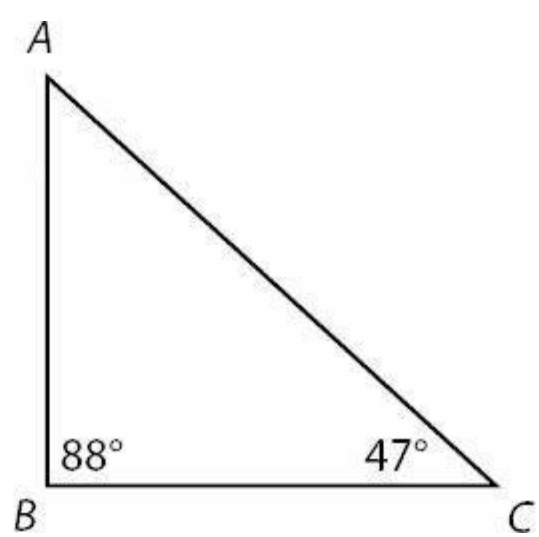
(B) 4.5

(C) $4\sqrt{2}$

(D) $4\sqrt{3}$

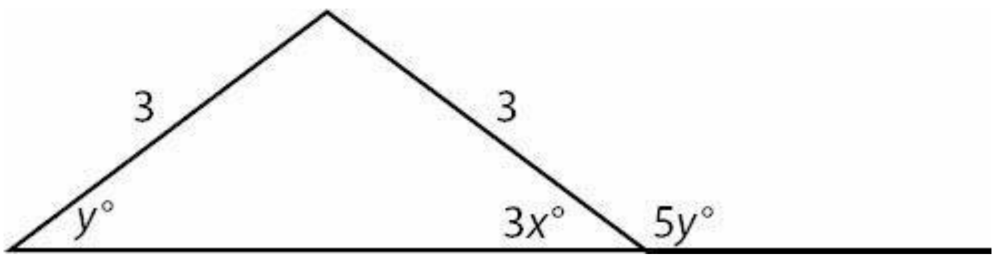
(E) 8

35.Triangle *ABC* is given below w ith angle m easures *B* and *C* given.



	Quantity A	Quantity B
	The length of side *AB*	The length of side *BC*

36.



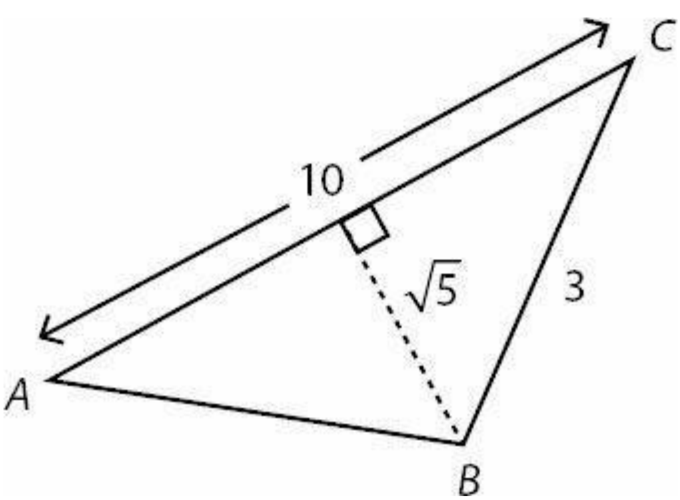
What is the value of *x* in the figure above?

(A) 5
(B) 10
(C) 18
(D) 30
(E) 54

37.An isosceles right triangle has an area of 50.What is the length of the hypotenuse?

(A) 5
(B) $5\sqrt{2}$
(C) $5\sqrt{3}$
(D) 10
(E) $10\sqrt{2}$

38.



Note: Figure NOT drawn to scale

In the figure above,what is the length of side *AB* ?

(A) 5

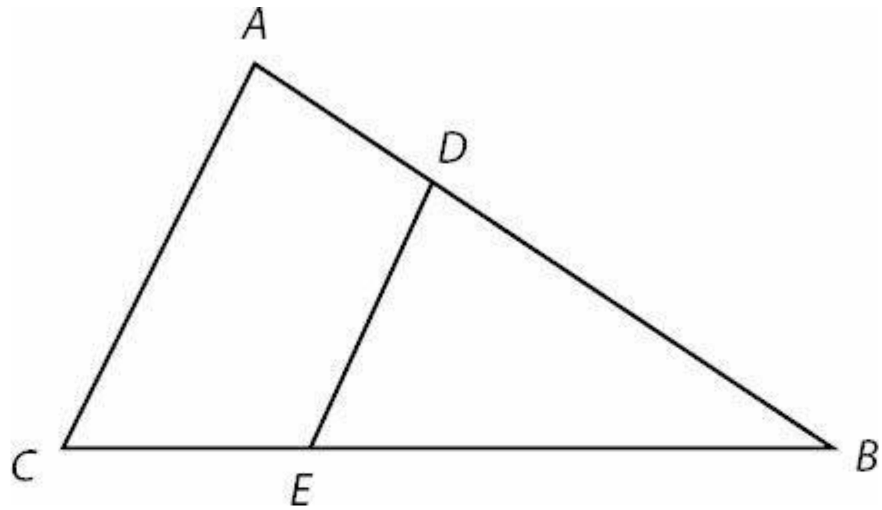
(B) $\sqrt{30}$

(C) $5\sqrt{2}$

(D) 8

(E) $\sqrt{69}$

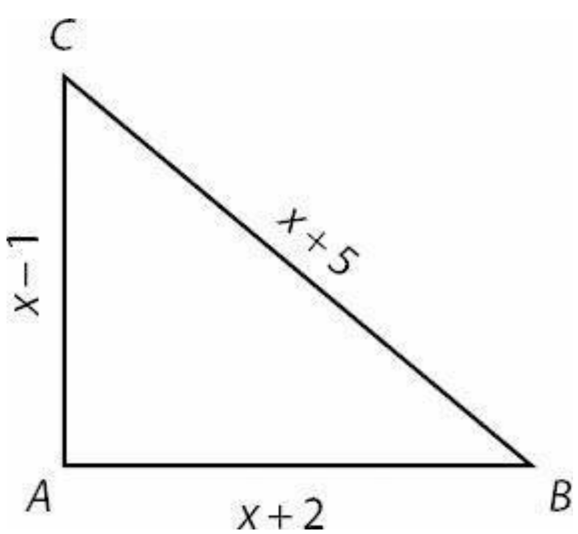
39.In the figure below ,*AC* is parallel to *D E* and the length of *D E* is equal to the length of *EB* .



N ote: Figure N O T draw n to scale

Q uantity A	**Q uantity B**
The length of side *AC*	The length of side *C B*

40.



In the right triangle above,w hat is the length of *AB* ?

(A) 9

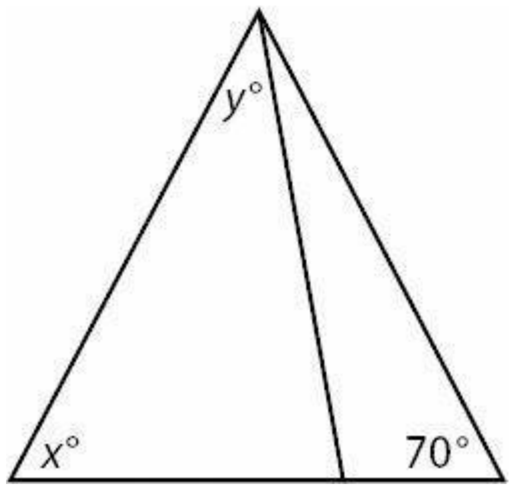
(B) 10

(C) 12

(D) 13

(E) 15

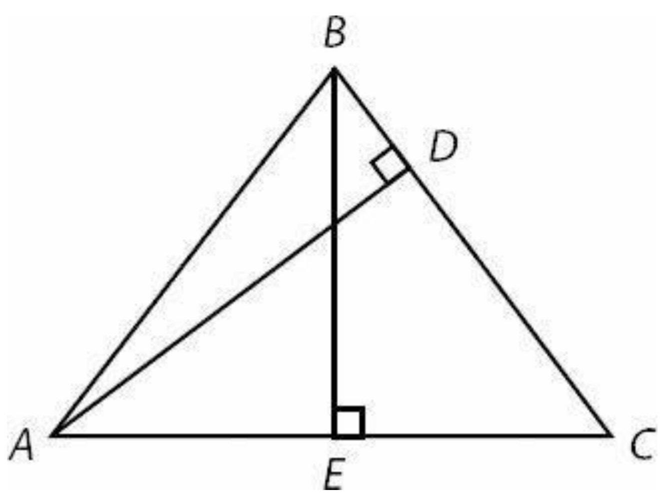
41.



Note: Figure NOT drawn to scale

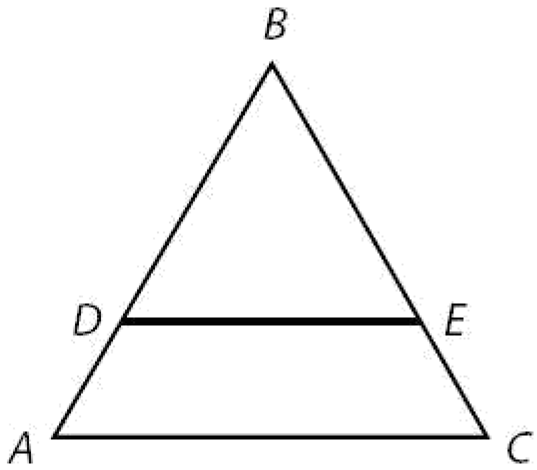
Quantity A	**Quantity B**
$x + y$	110

42.



Quantity A	**Quantity B**
The product of *BE* and *AC*	The product of *BC* and *AD*

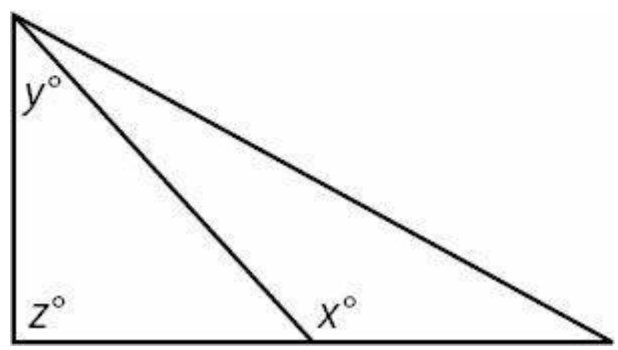
43.



In the figure above, DE and AC are parallel lines. If $AC = 10$, $DE = 6$, and $CE = 2$, what is the length of side BC?

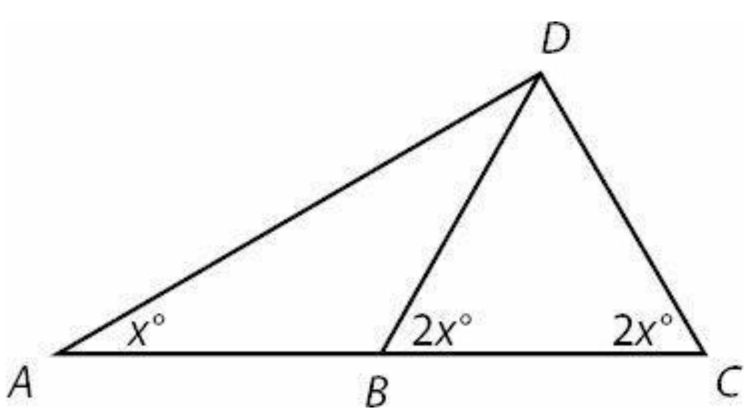
(A) 2
(B) 3
(C) 5
(D) 6
(E) 8

44.



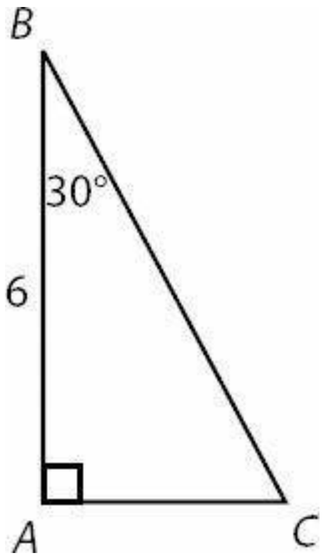
Quantity A	**Quantity B**
x	$y + z$

45.



Quantity A	Quantity B
The length of side *DC*	The length of side *AB*

46.



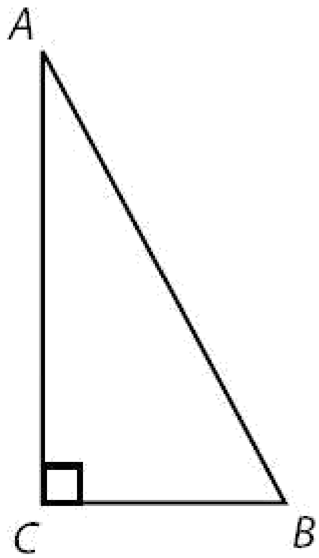
What is the perimeter of right triangle *ABC* above?

(A) $6 + 4\sqrt{3}$
(B) $6 + 6\sqrt{3}$
(C) $6 + 8\sqrt{3}$
(D) $9 + 6\sqrt{3}$
(E) $18 + 6\sqrt{3}$

47. A 10 foot ladder leans against a vertical wall and forms a 60 degree angle with the floor. Assuming the ground below the ladder is perfectly horizontal, how far above the ground is the top of the ladder?

(A) 5 feet
(B) $5\sqrt{3}$ feet
(C) 7.5 feet
(D) 10 feet
(E) $10\sqrt{3}$ feet

48.

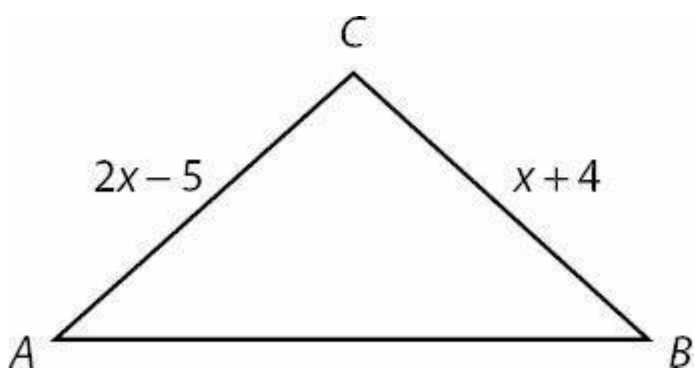


N ote: Figure N O T draw n to scale

Triangle *ABC* has area 36.If side *AC* is tw ice as long as side *C B* ,w hat is the length of side *AB* ?

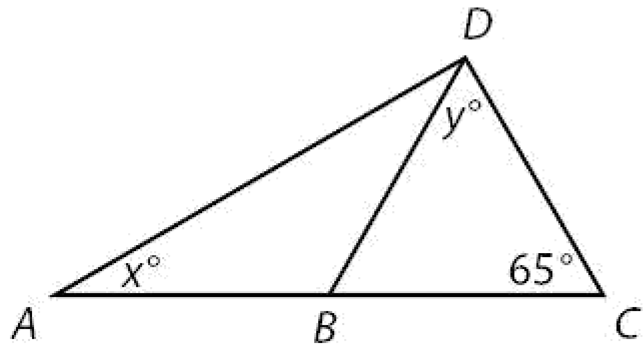
(A) 6
(B) 12
(C) $6\sqrt{5}$
(D) 18
(E) $12\sqrt{5}$

49.In the figure show n,the m easure of angle *A* is equal to the m easure of angle *B* .



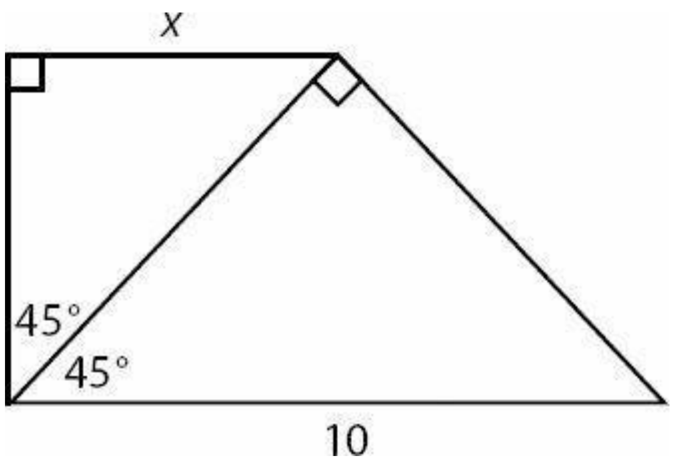
Q uantity A	**Q uantity B**
Side length *C B*	7

50.In the figure show n,side lengths *AB* ,*BD* ,and *D C* are all equal.



Q uantity A	**Q uantity B**
x	*y*

51.



In the figure show n,w hat is the value of *x*?

(A) 2.5

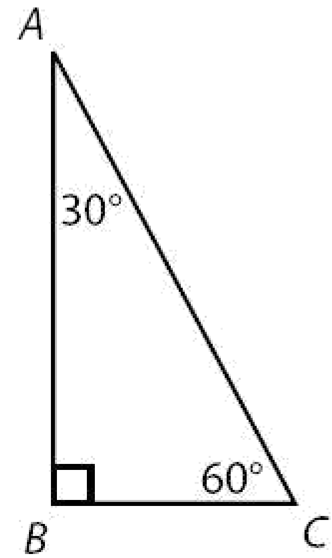
(B) $\frac{5}{\sqrt{2}}$

(C) 5

(D) $5\sqrt{2}$

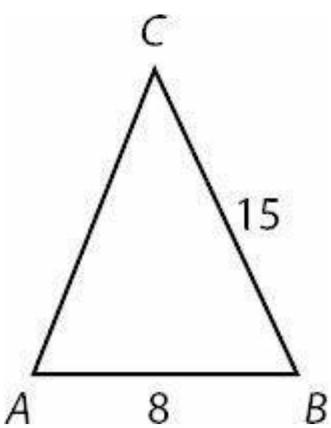
(E) $\frac{10}{\sqrt{2}}$

52.



Quantity A	Quantity B
The ratio of the length of side *BC* to the length of side *AB*	$\frac{10}{17}$

53.

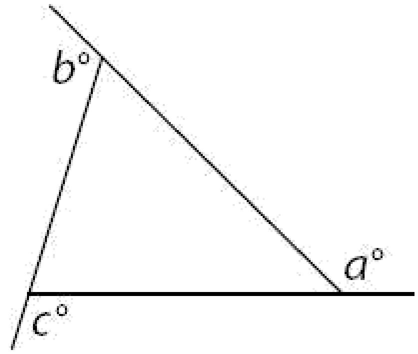


Note: Figure NOT drawn to scale

Which of the following statements individually provide sufficient information to calculate the area of triangle *ABC*?

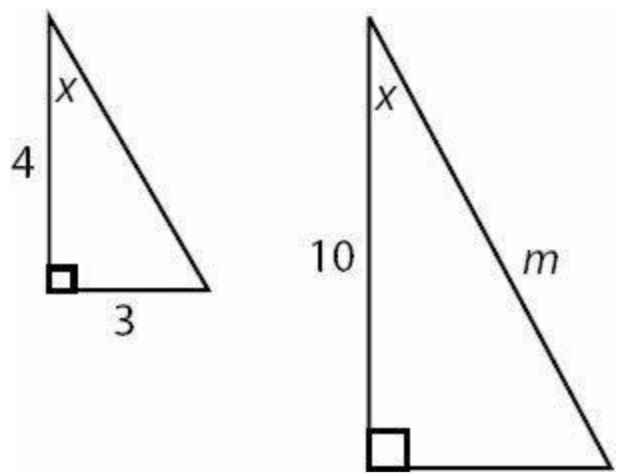
- ☐ Angle *B* equals 90.
- ☐ Side *AC* equals 17.
- ☐ *ABC* is a right triangle

54.



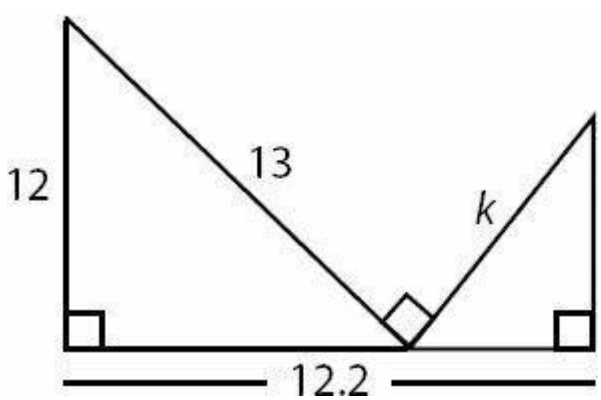
Quantity A	**Quantity B**
$a + b + c$	180

55.



Quantity A	**Quantity B**
m	15

56.

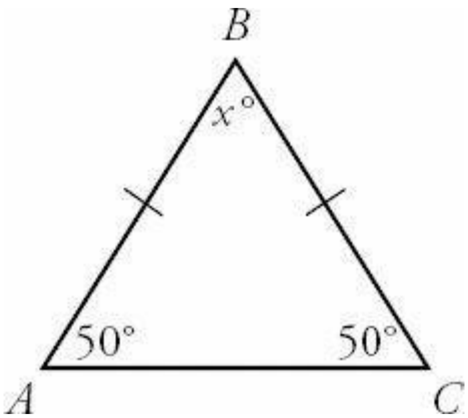


What is the length of hypotenuse k?

T riangles A nsw ers

1.**(B).**The area of a triangle is equal to $\frac{bh}{2}$.B ase and height m ust alw ays be perpendicular.U se 8 as the base and 6 as the height. $A = \frac{(8)(6)}{2} = 24$.

2.**80.**If you know the other 2 angles in a triangle,then you can find the third,because all 3 angles m ust add up to 180. In Triangle *ABC* ,sides *AB* and *BC* are equal.That m eans their opposite angles are also equal.That m eans that angle *AC B* is also 50°.

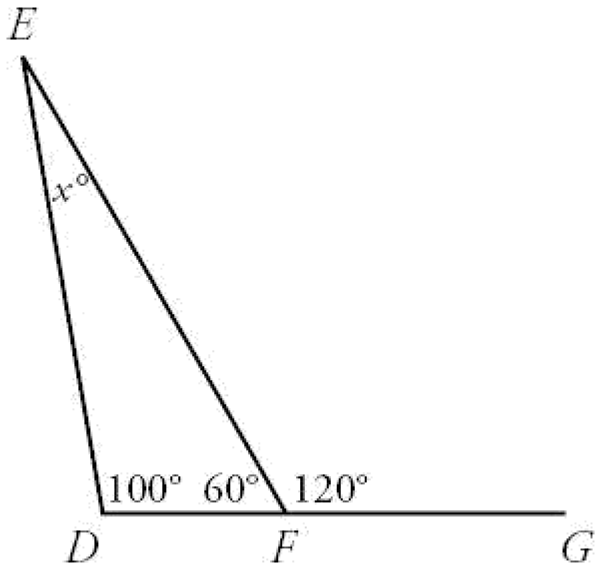


N ow that you know the other 2 angles,you can find angle *x*.Y ou know that 50 + 50 + *x* = 180,so *x* = 80.

3.**(C).**The three angles in a triangle m ust add up to 180°,so 3*a* = 180 and *a* = 60 (the triangle is equilateral).The four angles in a quadrilateral m ust add up to 360°,so 4*b* = 360 and *b* = 90 (the angles are right angles,so the figure is a rectangle).

Substitute the values of *a* and *b* into Q uantity A to get 2(60) + 90 = 210.Likew ise,substitute into Q uantity B to get $3(60) + \frac{90}{3} = 210$.The quantities are equal.

4.**20.**To find the value of *x*,you need to find the degree m easures of the other tw o angles in Triangle *D EF* .Y ou can m ake use of the fact that *D FG* is a straight line.Straight lines have a degree m easure of 180,so angle *D FE* + 120 = 180,w hich m eans angle *D FE* = 60.



N ow you can solve for *x*,because 100 + 60 + *x* = 180.Solving for *x*,you get *x* = 20.

5.**(C).**Since *c* and *x* are vertical angles,they are equal.So you can sw ap their positions in the quantities,to put all the angles in the sam e triangle together.

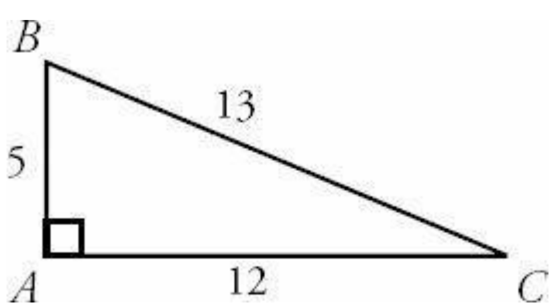
Q uantity A	Q uantity B
$a + b + c$	$x + y + z$

The three angles inside a triangle add up to 180°,so both sides are equal to 180.The quantities are equal.

6.**30.**To find the area,you need a base and a height.If you can find the length of side *AB* ,then *AB* can be the height and *AC* can be the base,because the tw o sides are perpendicular to each other.

Y ou can use the Pythagorean Theorem to find the length of side *AB* .$(a)^2 + (12)^2 = (13)^2$.$a^2 + 144 = 169$.$a^2 = 25$.$a = 5$.A lternatively,you could recognize that the triangle is a Pythagorean triplet 5–12–13.

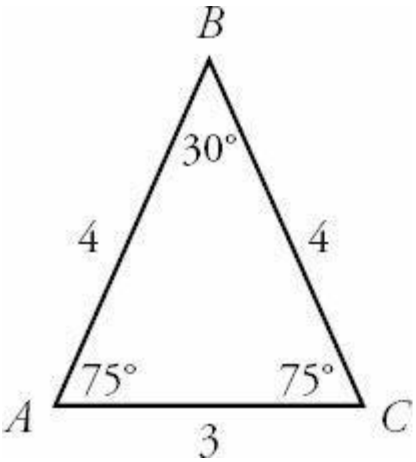
N ow that you know the length of side *AB* you can find the area. $\text{Area} = \frac{(12)(5)}{2} = 30.$.



7.**11.**To find the perim eter of Triangle *ABC* ,you need the lengths of all 3 sides.There is no im m ediately obvious w ay to find the length of side *BC* ,so let's see w hat inferences you can m ake from the inform ation the question gave you.

Y ou know the degree m easures of tw o of the angles in Triangle *ABC* ,so you can find the degree m easure of the third. Y ou'll label the third angle *x*.Y ou know that 30 + 75 + *x* = 180.Solving for *x* you find that *x* = 75.

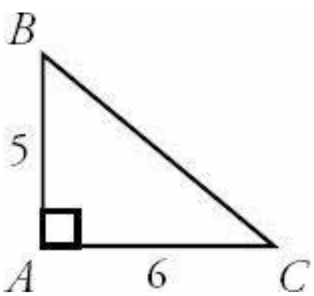
A ngle *BAC* and angle *BC A* are both 75,w hich m eans Triangle *ABC* is an isosceles triangle.If those tw o angles are equal,you know that their opposite sides are also equal.Side *AB* has a length of 4,so you know that *BC* also has a length of 4.



To find the perim eter,you add up the lengths of the three sides.4 + 4 + 3 = 11.

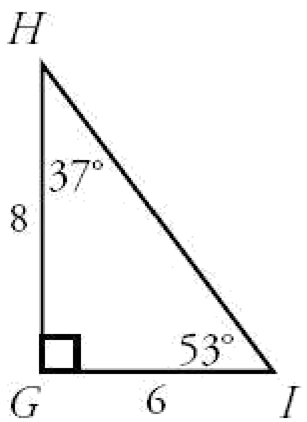
8.**(D).**To find the length of the hypotenuse,you need the lengths of the other tw o sides.Then you can use the Pythagorean Theorem to find the length of the hypotenuse.Y ou can use the area form ula to find the length of *AC* .

$\text{Area} = \frac{bh}{2}$,and you know the area and the height.So $15 = \frac{(\text{base})(5)}{2}$ .W hen you solve this equation,you find that the base = 6.



N ow you can use the Pythagorean Theorem .$(5)^2 + (6)^2 = c^2$.$25 + 36 = c^2$.$61 = c^2$.$\sqrt{61} = c$.Since 61 is not a perfect square,you know that *c* w ill be a decim al.61 is also prim e,so you cannot sim plify $\sqrt{61}$ any further.(It w ill be a little less than $\sqrt{64} = 8$.)

9.**10.**There is no im m ediately obvious w ay to find the length of side *H I*,so let’s see w hat you can infer from the picture.Y ou know tw o of the angles of Triangle *G H I*,so you can find the third.Y ou’ll label the third angle *x*.37 + 53 + *x* = 180.That m eans *x* = 90.So really your triangle looks like this:



Y ou should definitely redraw once you discover the triangle is a right triangle!

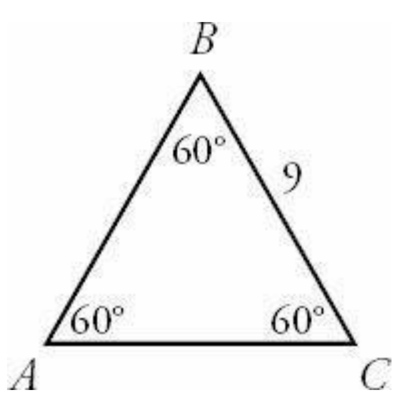
N ow that you know Triangle *G H I* is a right triangle,you can use the Pythagorean Theorem to find the length of *H I*.*H I* is the hypotenuse,so $(6)^2 + (8)^2 = c^2$.$36 + 64 = c^2$.$100 = c^2$.$10 = c$.The length of *H I* is 10.

A lternatively,you could have recognized the Pythagorean triplet.Triangle *G H I* is a 6–8–10 triangle.

10.**(C) 24.5.**A ll isosceles right triangles (or 45–45-90 triangles) have sides in the ratio 1 : 1 :$\sqrt{2}$.Thus,an isosceles right triangle w ith hypotenuse $7\sqrt{2}$ has all its sides in the ratio 7 : 7 :$7\sqrt{2}$.The base and height are each 7.From $A = \frac{bh}{2}$,you get $A = \frac{(7)(7)}{2} = 24.5$.

11.**(B).**To determ ine w hich triangle has the greater perim eter,you need to know the side lengths of all three sides of both triangles.B egin w ith Triangle *ABC* .

There's no im m ediate w ay to find the lengths of the m issing sides,so let's start by seeing w hat you can infer from the picture.Y ou know tw o of the angles,so you can find the third.Y ou'll label the unknow n angle *x*.$60 + 60 + x = 180$.$x = 60$.



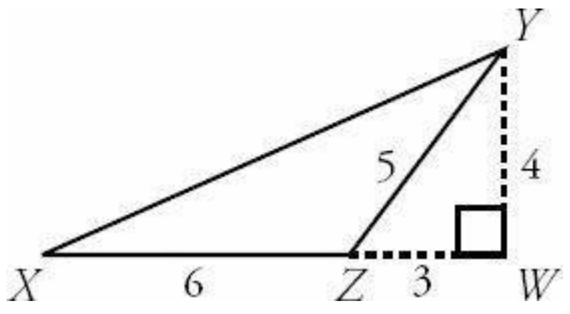
A ll three angles in Triangle *ABC* are 60°.If all three angles are equal,that m eans all three sides are equal in this equilateral triangle.So every side of Triangle *ABC* has a length of 9.That m eans the perim eter = 9 + 9 + 9 = 27.N ow look at Triangle *D EF* .Triangle *D EF* is a right triangle,so you can use the Pythagorean Theorem to find the length of side *EF* .*EF* is the hypotenuse,so $(5)^2 + (12)^2 = c^2$.$25 + 144 = c^2$.$169 = c^2$.$13 = c$.That m eans the perim eter is 5 + 12 + 13 = 30.A lternatively,5–12–13 is a Pythagorean triplet.

30 > 27, so Triangle *DEF* has a greater perimeter than Triangle *ABC*.

12. **12.** Start by filling in everything you know about Triangle *XYZ*.

To find the area of Triangle *XYZ*, you need a base and a height. If Side *XZ* is a base, then *YW* can act as a height. You can find the length of *YW* because Triangle *ZYW* is a right triangle, and you know the lengths of two of the sides. *YZ* is the hypotenuse, so $(a)^2 + (3)^2 = (5)^2$. $a^2 + 9 = 25$. $a^2 = 16$. $a = 4$.

Alternatively, you could recognize the Pythagorean triplet: *ZYW* is a 3–4–5 triangle.



Now you know that the area of Triangle *XYZ* is $\frac{bh}{2} = \frac{(6)(4)}{2} = 12$.

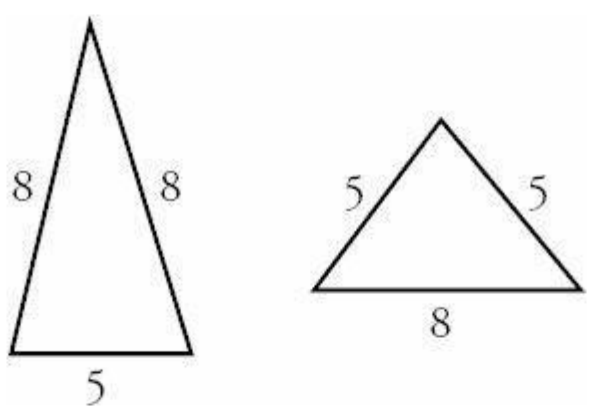
13. **(D).** The problem tells you that $z + x = 110$ degrees. Given that angles of a triangle must sum to 180 degrees, you also know that $x + y + z = 180$. Substitute 110 for $x + z$ on the left side:

$y + 110 = 180$

$y = 70$ degrees

The problem asks you to compare angles *x* and *y*. Although you have solved for *y*, you are still uncertain about the measure of angle *x* (all you know is that it must be greater than 0 and less than 110 degrees). Therefore, you cannot determine which quantity is greater.

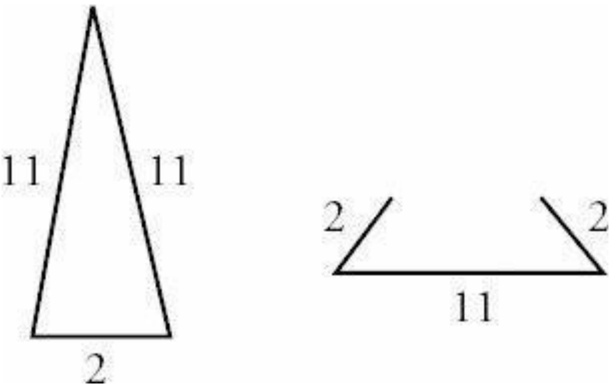
14. **(D).** An isosceles triangle has 2 equal sides, so this triangle must have a third side of either 8 or 5. Use the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to check whether both options are actually possible.



Since 8 - 5 = 3 and 8 + 5 = 13, the third side has to be greater than 3 and less than 13. Therefore, that side could indeed be either 5 or 8. You don't know which it is, though, so you cannot determine which quantity is greater.

15.**(C).**A n isosceles triangle has 2 equal sides,so this triangle m ust have a third side of either 2 or 11.B ecause one side is so long and the other so short,it is w orth testing via the Third Side R ule (any side of a triangle m ust be greater than the difference of the other tw o sides and less than their sum) to see w hether both possibilities are really possible.

From the Third Side R ule,a triangle w ith sides of 2 and 11 m ust have a third side greater than 11 - 2 = 9 and less than 11 + 2 = 13.Since 2 is not betw een 9 and 13,you sim ply cannot have a triangle w ith sides of length 2,2,and 11. H ow ever,you *can* have a third side of length 11: a 2–11–11 triangle is possible.So the third side m ust be 11.



Thus,the tw o quantities are equal.

16.**(A).**In this triangles problem ,you are asked to com pare the relative lengths of tw o sides of a triangle.In any triangle,the follow ing rule is true: the larger the angle,the longer the side opposite that angle.

Therefore,since angle *B* is larger than angle *A* ,the side opposite angle *B* m ust be longer than the side opposite angle *A* .Side length *AC* is longer than side length *BC* .Q uantity A is greater.

17.**(C).**B y definition,the exterior angle *d* is supplem entary to the adjacent interior angle and equal to the sum of the tw o non-adjacent angles.Thus,*d* = angle *B* + angle *C* .

A lternatively,try plugging in num bers.If *d* = 120,angle *A* equals 180 - 120 = 60.H ow m any degrees are left for angles *B* and *C* to share? 180 - 60 = 120,so angles *B* and *C* m ust add up to 120— the sam e as *d*.A s long as your exam ple obeys the rules of triangles (the 3 angles in the triangle add to 180) and straight lines (the 2 angles on the line also add to 180),your exam ple w ill show that *d* = angle *B* + angle *C* .The tw o quantities are equal.

18.**(D).**If the question had said the tw o LEG S of the triangle w ere 3 and 4,then the hypotenuse w ould be 5 (as you know from the 3–4–5 special right triangle).H ow ever,you can’t assum e that 3 and 4 are the legs.In fact,4 could be the *hypotenuse*.In that case,3 w ould still be a leg,and the length of the other leg w ould be $\sqrt{16-9}=\sqrt{7}$,as you could show from the Pythagorean theorem .Y ou don’t need to calculate this value— just recognize that it m ust be less than 4 (because 4 is the hypotenuse in this case).Since *z* could be either equal to or less than 5,you cannot determ ine w hich quantity is greater.

19.**(B).**Since *z* > 60 and *x* > *z*,*x* m ust also be greater than 60.Thus,*x* + *z* m ust be greater than 120,w hich leaves less than 60 degrees for the third angle *y*.Y ou can now order the angles by size: *y* < *z* < *x*.

The sm allest side is across from the sm allest angle,w hich is *y*,so the sm allest side m ust be *AC* .The m iddle side is across from the m iddle angle,w hich is *z*,so the m iddle side m ust be *AB* .A t this point,you know that the length of *AB* is greater than the length of *AC* .Q uantity B is greater.

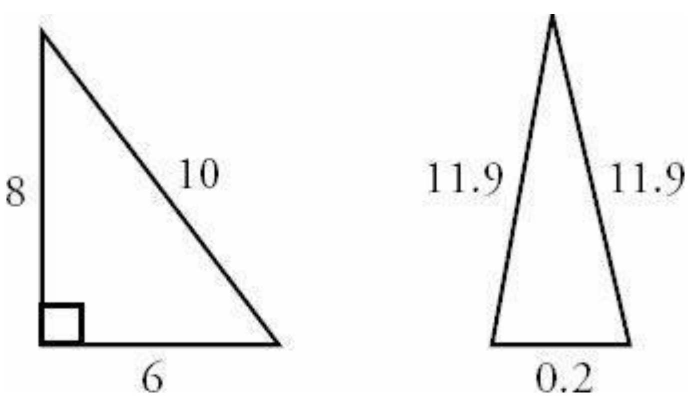
20.**(A).**Since *BC* is 1/3 the length of *AB* ,relate the tw o sides using a variable.The easiest w ay to do this is to label *AB* “3*x*” and label *BC* “*x*.” (C alling the sm aller side *x* lets you avoid using a fraction.)

N ow apply the Pythagorean Theorem :

$x^2 + (3x)^2 =$
$(4\sqrt{10})^2$ $x^2 + 9x^2 =$
160
$10x^2 = 160$
$10x^2 =$
16 $x = 4$

Y ou are looking for *AB* ,w hich equals 3*x* = 3(4) = 12.Q uantity A is greater.

21.**(D).**The perim eter of a triangle is not enough to find its area,and vice-versa.A triangle w ith perim eter 24 could be a right triangle w ith sides of 6,8,and 10,in w hich case the area w ould be $A = \frac{bh}{2} = \frac{6(8)}{2} = 24$,w hich is larger than 20.O r the triangle could have sides of 11.9,11.9,and 0.2,in w hich case the area w ould be incredibly sm all.



Y ou should note that you don’t have *com plete* freedom w ith the area.There is a m axim um area that a triangle w ith perim eter 24 can have— nam ely,the area of an equilateral triangle w ith side lengths of 8.Such a triangle w ould have an area of $\frac{s^2\sqrt{3}}{4} = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3} \approx 16(1.7)$,w hich is greater than 20.(Y ou can derive that form ula for the area of an equilateral triangle by dropping a height,m aking tw o 30–60–90 triangles.) B ut any positive area less than this m axim um is possible.Y ou can drastically shrink the area of a triangle w ith perim eter 24,m aking that area as close to zero as you w ish.Thus,you cannot determ ine w hich quantity is greater.

22.**(C).**The area of a triangle is equal to $\frac{bh}{2}$ .Y ou know that the tw o triangles have equal bases,since *XY* = *YZ*.They also have the sam e height,since they both have the sam e height as the larger triangle *W XZ*.The tw o quantities are equal.

23.**(C).**Y ou certainly cannot *assum e* that *x* = 90.B ut since you have three values for the sides of the triangle,you can test w hether the triangle is a right triangle by applying the Pythagorean Theorem to the three values and seeing w hether you get a true statem ent.

$5^2 + 7^2 = (\sqrt{74})^2$
$25 + 49 = 74$
$74 = 74$

Since 74 obviously equals 74,the Pythagorean theorem does apply to this triangle.So the triangle is a right triangle. N otice also that you used the side across from *x* as the hypotenuse.Thus,you can be sure that $x = 90$.The tw o quantities are equal.

24.**(B).**W ithin a single triangle,there is a direct relationship betw een the side length and the opposite angle.That is, the biggest side is opposite the biggest angle;the sm allest side is opposite the sm allest angle;and the m iddle side is opposite the m iddle angle.

Since the angles in a triangle sum to 180,the angle opposite *L* is 44.So *L* is sm aller than *S*,and $\frac{L}{S}$ is less than 1. Q uantity B is greater.

25.**(C).**From the Third Side R ule,any side of a triangle m ust be greater than the difference of the other tw o sides and less than their sum .Since 11 - 9 = 2 and 11 + 9 = 20,*x* m ust be betw een 2 and 20,*not* inclusive.N ote that *x* is an integer,so *x* m ust be betw een 3 and 19,*inclusive*.

N ow you can list the possibilities and count: *x* can be 3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,or 19.O r you can subtract the *inclusive* endpoints and “add 1 before you’re done:” 19 - 3 + 1 = 17.Either w ay,there are 17 total possibilities.The tw o quantities are equal.

26.**(C) 18 < p < 30.**From the Third Side R ule,any side of a triangle m ust be greater than the difference of the other tw o sides and less than their sum .Since 9 - 6 = 3 and 9 + 6 = 15,the unknow n third side m ust be betw een 3 and 15, *not* inclusive.To get the low er boundary for the perim eter,add the low er boundary of the third side to the other tw o sides: 3 + 6 + 9 = 18.To get the upper boundary for the perim eter,add the upper boundary for the third side to the other tw o sides: 15 + 6 + 9 = 30.Thus,*p* m ust be betw een 18 and 30,not inclusive— in other w ords,$18 < p < 30$.

27.**(B).**Y ou m ay have m em orized the 6–8–10 Pythagorean triple,a fact that this problem is trying to exploit— don’t be tricked into thinking that *x* equals 10! In a 6–8–10 triangle,10 w ould have to be the *hypotenuse*.In any right triangle,the hypotenuse m ust be the longest side.

Since the given triangle has 8 as the hypotenuse,the leg of length *x* m ust be less than 8.So *x* m ust also be less than 10. A t this point,you can safely choose (B).If you really w ant the actual value of *x*,sim ply apply the Pythagorean theorem :

$6^2 + x^2 = 8^2$
$36 + x^2 =$
$64\ x^2 = 28$
$x = \sqrt{28}$,w hich is betw een 5 and 6 (so it is less than 10).

Q uantity B is greater.

28. **(A).** One good approach here is to test the value in Quantity B. If angle x equals 90°, then you have a right triangle. Use the legs of 5 and 12 to find the hypotenuse. $5^2 + 12^2 = c^2$ will tell you that c equals 13 in this case. (Or, memorize the 5–12–13 Pythagorean triple, since it appears often on the GRE.)

Since the hypotenuse is slightly *longer* than 13, the angle across from the 13 must actually be slightly *larger* than 90°. Therefore, x is larger than 90. Quantity A is greater.

29. **(A).** Since the figure is a right triangle, set up a Pythagorean theorem to solve for the hypotenuse in terms of x:

$$x^2 + (2x)^2 = c^2$$
$$x^2 + 4x^2 = c^2$$
$$5x^2 = c^2$$
$$\sqrt{5x^2} = c$$
$$x\sqrt{5} = c$$

The hypotenuse is equal to $x\sqrt{5}$, which is greater than $x\sqrt{3}$ (as long as x is positive, which of course it is since x is a distance).

The "trick" in this problem is that if you accidentally simplify $(2x)^2$ as $2x^2$ rather than as $4x^2$, you end up with incorrect choice (C).

Quantity A is greater.

30. **(C).** If AC **=** 14 and is parallel to DE, then triangles DBE and ABC are similar.

Since $BE = EC$, $BC = 2BE$ (that is, the side of the big triangle is twice the side of the small triangle). Since the two triangles are similar, *all* the sides of the small triangle will equal half of the corresponding sides of the big triangle. Thus, $DE = 7$.

Alternatively, you can write out a proportion:

$$\frac{BE}{BC} = \frac{DE}{AC}$$

You don't know the exact lengths of BE and BC, but you do know that they are in a 1 to 2 ratio, which is all you need (even if you did know the exact lengths, you'd be able to reduce that fraction to 1/2 anyway).

$$\frac{1}{2} = \frac{DE}{14}$$

$DE = 7$

The tw o quantities are equal.

31.**(B).**From the Third Side R ule,any side of a triangle m ust be less than the sum of the other tw o sides and greater than their difference.

Since one side equals 8,the other tw o sides m ust differ by less than 8.Thus:

$|m - n| < 8$

The absolute value signs are not totally necessary here,in fact— all they do is guarantee that you're taking the positive difference of *m* and *n* (it doesn't m atter w hich one is longer).

Since $|m - n|$m ust be less than 8,Q uantity B is greater.

32.**(C).**The tw o triangles— the sm all triangle and the large triangle (w hich encom passes the sm all triangle)— share one angle,the angle at the top of both triangles.

Furtherm ore,because the parallel lines create equal angles w hen cut by transversals (in this case,the sides of the triangle),the other tw o angles of both of the triangles are also in equal m easure.Therefore,the tw o triangles are sim ilar.

B ecause the tw o triangles are sim ilar,they are in proportion to one another.The sm all triangle has tw o sides of 2, w hile the large triangle has corresponding sides of 10 (N O T 8! The lengths labeled 8 are N O T the sides of an actual triangle— the large triangle has tw o sides of 8 + 2 = 10).So the tw o triangles are in a 2 : 10 proportion to one another. R educe 2 : 10 to 1 : 5 and w rite a proportion:

$$\frac{1}{5} = \frac{x}{11}$$

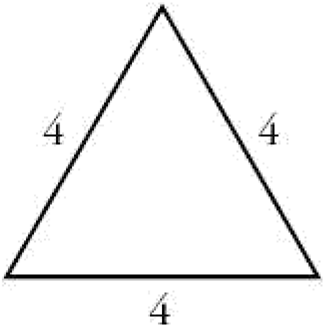
$$5x = 11$$

$$x = \frac{11}{5}$$

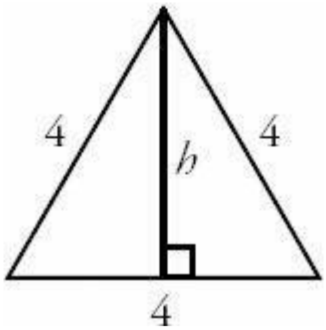
33.**(D).**From the Third Side R ule,a triangle w ith sides of 13 and 9 m ust have a third side greater than 13 - 9 = 4 and less than 13 + 9 = 22.

U se the square root button on the calculator to see that $\sqrt{226} \approx 15.033$,or note that since $\sqrt{225} = 15$, $\sqrt{226}$ m ust be just a little m ore than 15 (but certainly less than 16).Since the third side's length is betw een 4 and 22,it could be m ore or less than $\sqrt{226}$.Y ou cannot determ ine w hich quantity is greater.

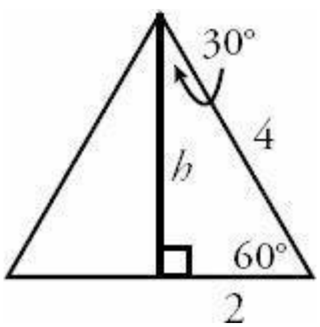
34.**(D).**A n equilateral triangle w ith side length 4 can be draw n as:



In order to find the area,recall that the area of a triangle is given by the form ula $A=\frac{bh}{2}$.The base of the triangle is already know n to be 4,so you m ust find the height in order to solve for area.The height is the straight line from the highest point on the triangle dropped dow n perpendicular to the base:



The angle opposite *h* m ust be 60°,since it is one of the three angles of the original equilateral triangle.Thus,the triangle form ed by *h* is a 30–60–90 triangle as show n below in red.



U sing the properties of 30–60–90 triangles,you know that *h* is equal to the shortest side m ultiplied by $\sqrt{3}$.Thus, $h= 2\sqrt{3}$ and the area is given by

$$A=\frac{bh}{2}=\frac{4\times 2\sqrt{3}}{2}=4\sqrt{3}$$

The shortcut form ula for the area of an equilateral triangle is $A=\frac{s^2\sqrt{3}}{4}$,w hich can be derived by the sam e logic as show n above.

35.**(A).**Label angle *A* w ith the variable *x*.Since,the three angles of a triangle m ust add up to 180 degrees,you know that

$x + 88 + 47 = 180$
$x + 135 = 180$
$x = 45$

Therefore,angle *A* has a m easure of 45 degrees.N ow ,although you do not know the lengths of the sides,the largest side is opposite the largest angle,and the sm allest side is opposite the sm allest angle.B ecause angle *C* is larger than angle *A* ,you know that the side opposite angle *C* is longer than the side opposite angle *A* .In other w ords,side *AB* is longer than side *BC* .Q uantity A is greater.

36.**(B).**Y ou are asked for the value of *x*.Since there are tw o unknow ns,look for tw o equations to help you solve.The first equation com es from the fact that 3*x* and 5*y* m ake a straight line,so they m ust add to 180:

$3x + 5y = 180$

The second equation com es from the isosceles triangle theorem ,w hich states that angles across from sides of a triangle w ith equal length are equal.In this case,the tw o sides w ith length 3 are equal,so the angles across from them (*y* and 3*x*) m ust also be equal:

$y = 3x$

Substitute for *y* in the first equation:

$3x + 5(3x) = 180$
$3x + 15x = 180$
$18x = 180$
$x = 10$

37.**(E).**Y ou are told that the area is 50,so $\frac{bh}{2}$ = 50.In an isosceles right triangle,base = height,so you can substitute another *b* in for *h*:

$\frac{b^2}{2} = 50$
$b^2 = 100$
$b = 10$

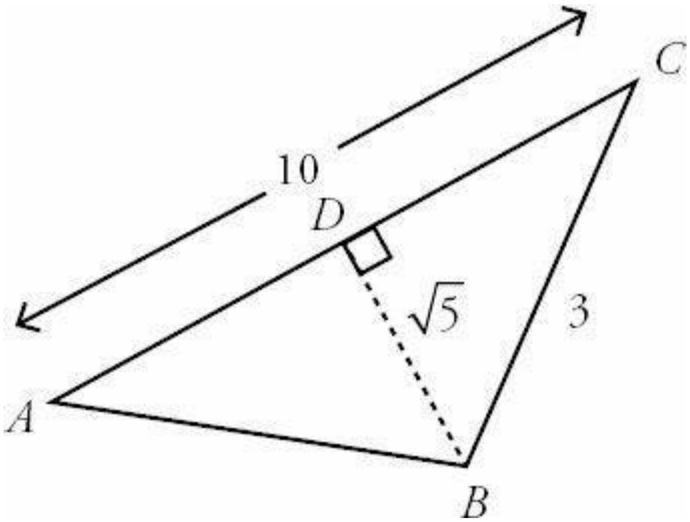
A n isosceles right triangle follow s the 45–45–90 triangle form ula,so the hypotenuse is $10\sqrt{2}$.

A lternatively,use the Pythagorean Theorem to find the hypotenuse:

$10^2 + 10^2 = c^2$
$200 = c^2$

Thus, $c = \sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$.

38. **(E).** You are asked for the length of segment *AB*. For convenience, put the letter *D* on the point at the right angle between *A* and *C*, as shown:



Solve this multi-step problem by working backwards from your goal. To find the length of *AB*, you can use the Pythagorean theorem on triangle *ADB*, since angle *ADB* must be a right angle. In order to use the Pythagorean theorem, you need the lengths of the two legs. *BD* is known, so you just need *AD*. Since *AD* and *DC* add up to a line segment of length 10, you know that $AD = 10 - DC$.

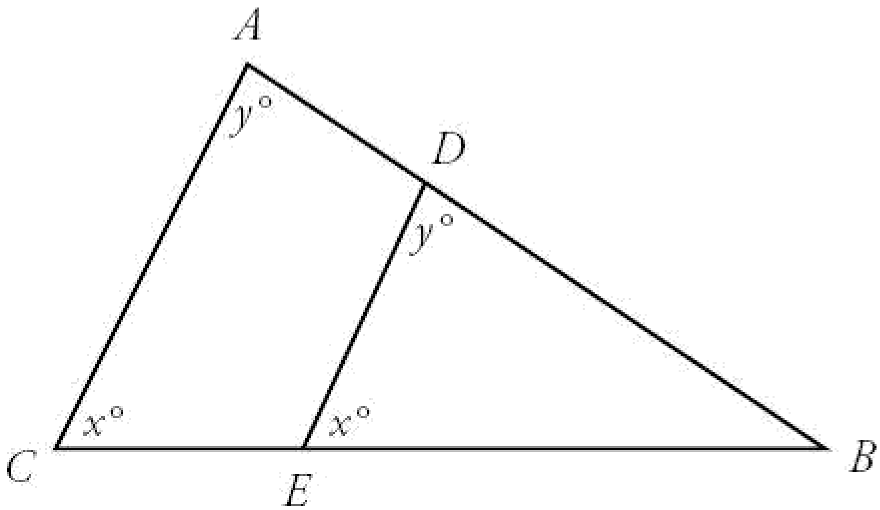
Finally, to find *DC*, apply the Pythagorean theorem to triangle *BDC*:

$$(\sqrt{5})^2 + (DC)^2 = 3^2$$
$$5 + (DC)^2 = 9$$
$$(DC)^2 = 4$$
$$DC = 2$$

So $AD = 10 - DC = 10 - 2 = 8$. Apply the Pythagorean theorem to *ADB*:

$$(\sqrt{5})^2 + 8^2 = (AB)^2$$
$$5 + 64 = (AB)^2$$
$$69 = (AB)^2$$
$$AB = \sqrt{69}$$

39. **(C).** You are asked to compare the lengths of *AC* and *CB*. Call angle *DEB* $x°$. *DE* and *AC* are parallel, and they are both cut by transversal *CB*. So angles *DEB* and *ACB* are corresponding angles— they have the same measure, and angle *ACB* will also be $x°$. Similarly, if angle *EDB* is labeled as $y°$, then angle *A* will also be $y°$. At this point, the diagram looks like this:



N ow you have tw o triangles w ith all three angles the sam e,*AC B* and *D EB* (angle *B* w ill obviously be the sam e in both triangles).So these triangles are sim ilar,and the sides of *AC B* w ill be in the sam e proportions as the corresponding sides of *D EB* .

Y ou are told that the length of *D E* is equal to the length of *EB* .(W hen the figure is not to scale,don't trust your eyes — trust w hat the problem tells you!) To m aintain sim ilarity,the corresponding sides on the larger triangle (*AC* and *C B*) m ust also be equal.Thus,although you do not know the lengths of *AC* and *C B* ,you know they m ust be the sam e.The tw o quantities are equal.

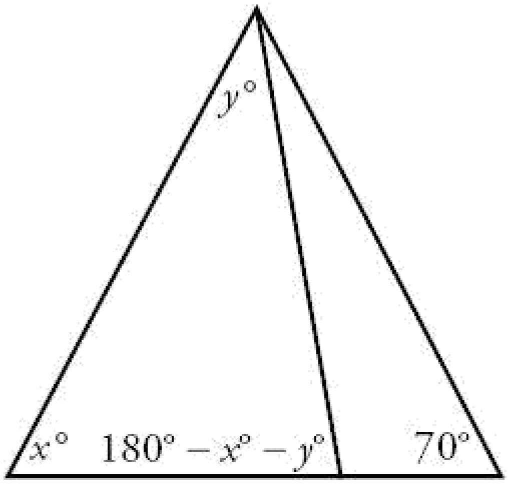
40.**(C).**B ecause you are told that this is a right triangle,you can use the Pythagorean theorem to solve for the lengths of the sides.The Pythagorean theorem states that $a^2 + b^2 = c^2$ w here *c* is the hypotenuse and *a* and *b* are the legs of a right triangle.In this case,$x + 5$ m ust be the length of the hypotenuse (the longest side),because $x + 5$ is definitely greater than both $x + 2$ and $x - 1$.Plug the expressions into the theorem and sim plify:

$(x - 1)^2 + (x + 2)^2 = (x + 5)^2$
$(x^2 - 2x + 1) + (x^2 + 4x + 4) = x^2 + 10x +$
25 $2x^2 + 2x + 5 = x^2 + 10x + 25$
$x^2 - 8x - 20 = 0$
$(x - 10)(x + 2) = 0$
$x = 10$ or $x = -2$

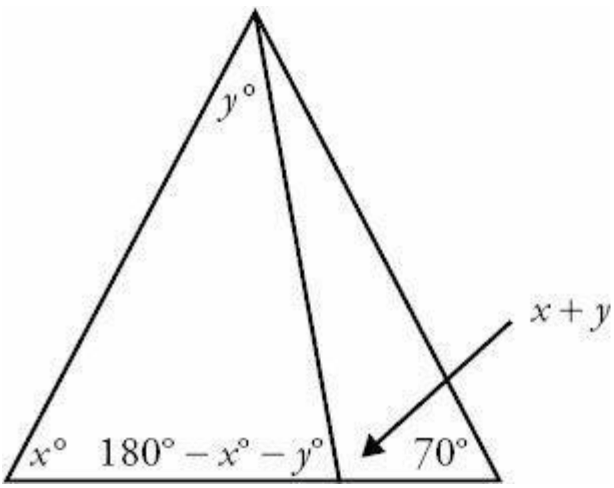
H ow ever,$x = -2$ is not an option;side lengths can't be negative.So *x* m ust equal 10.This is N O T the final answ er, how ever.Y ou are ultim ately asked for side length *AB* :

$AB = x + 2 = 10 + 2 = 12.$

41.**(B).**Y ou are asked to com pare $x + y$ w ith 110.To do so,fill in the m issing angles on the triangles.In the triangle on the left,all three angles m ust add up to 180 degrees.Therefore,the m issing angle m ust be $(180 - x - y)$,as show n here:



N ow consider the angle next to the one you just solved for.These tw o angles add up to 180,form ing a straight line.So the adjacent angle m ust be $x + y$.



A lternatively,you could notice also that $x + y$ is the exterior angle to the triangle on the left,so it m ust be the sum of the tw o non-adjacent angles (nam ely,x and y).

N ow ,the three angles of a triangle m ust add up to 180,and no angle can equal 0.So any *tw o* angles in a triangle m ust add up to *less* than 180.C onsider the triangle on the right side,w hich contains angles of $x + y$ and 70.Then their sum is less than 180.

$$(x + y) + 70 < 180$$

Subtract 70 from both sides:

$$x + y < 110$$

Q uantity B is greater.

42.**(C).**First determ ine how Q uantities A and B relate to the triangle.For instance,exam ine Q uantity A ,the product of *BE* and *AC* .N otice that *BE* is the height of the triangle,w hile *AC* is the base.This product should rem ind you of the form ula for area: $A = \frac{bh}{2}$.

With $b = AC$ and $h = BD$,and m oving the 2,you

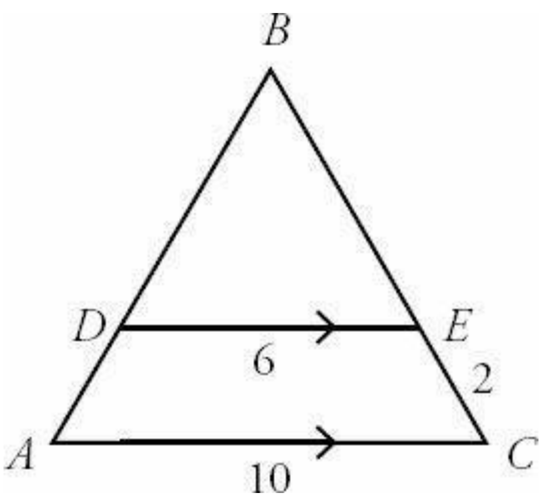
get $2 \times$ A rea = $(AC)(BD)$

W hat about BC and AD ? W ell,you can consider BC the base— and if you do so,then AD is the height to that base.So you can put $b = AC$ and $h = BD$ into the area form ula,yielding

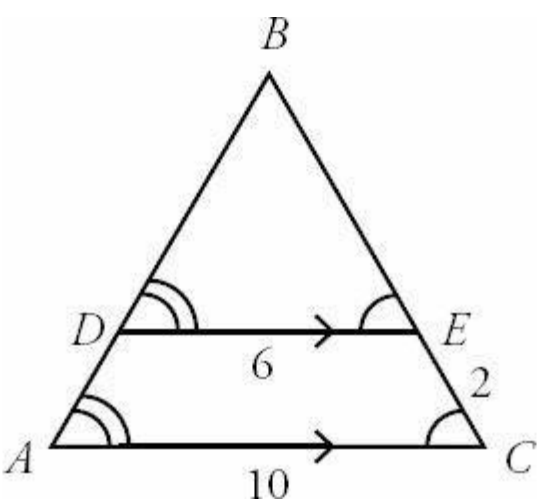
$2 \times$ A rea = $(BC)(AD)$

B oth Q uantity A and Q uantity B are tw ice the area of the triangle.The tw o quantities are equal.

43.**(C)** First,label the diagram w ith the inform ation given:



w here the arrow s represent parallel lines.B ecause DE and AC are parallel,angle DEB m ust be the sam e as angle ACB , as they are form ed by the sam e transversal (line segm ent BC).Sim ilarly,angle BDE m ust be the sam e as angle BAC .



Triangle BDE has all the sam e angles as triangle BAC (angle B is shared),so the tw o triangles are sim ilar.The ratio of any tw o corresponding sides on sim ilar triangles is the sam e,w hichever pair of sides you pick.Y ou can find this constant ratio from DE and AC: $\frac{DE}{AC} = \frac{6}{10} = \frac{3}{5}$ .So all other corresponding sides m ust also be in the ratio of 3 to 5.A ssign x to the unknow n length of BE .Then BC w ill be $x + 2$.A pply the ratio above:

$$\frac{BE}{BC} = \frac{x}{x+2} = \frac{3}{5}$$

C ross m ultiply and solve for *x:*

$5x = 3(x + 2)$
$5x = 3x + 6$
$2x = 6$
$x = 3$

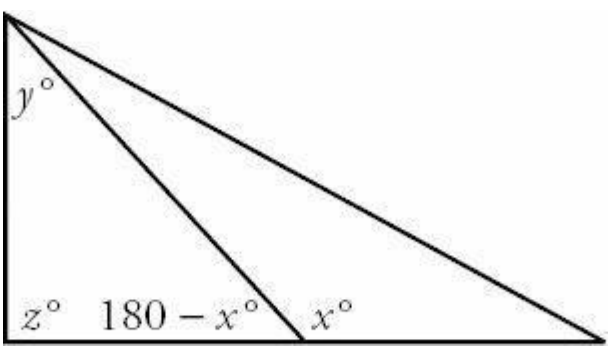
R em em ber that *x* is the length of side *BE* ,but you w ant the length of *BC* .

$BC = x + 2 = 3 + 2 = 5$

44.**(C).**Y ou can solve this problem in tw o w ays.The quick w ay is to notice that *x* is the exterior angle to the sm aller triangle on the left (w hich contains *y* and *z*).Since *y* and *z* are the non-adjacent interior angles,you can im m ediately apply the rule that the exterior angle (*x*) is equal to the sum of the tw o non-adjacent interior angles (*y* and *z*).

The longer w ay is to derive that relationship,essentially,from tw o rules: (1) the three angles in a triangle add up to 180,and (2) the tw o angles form ed by a segm ent running into a straight line (such as *x* and its unlabeled neighbor) also add up to 180.

First,label that m issing angle as 180 - *x* as show n:

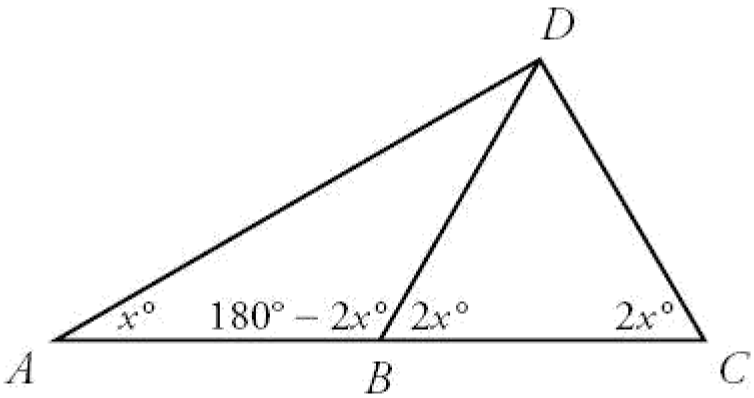


N ow apply the rule that the three angles in a triangle add up to 180:

y + *z* + (180 - *x*) =
180 *y* + *z* + 180 - *x* =
180 *y* + *z* - *x* = 0
y + *z* = *x*

The tw o quantities are equal.

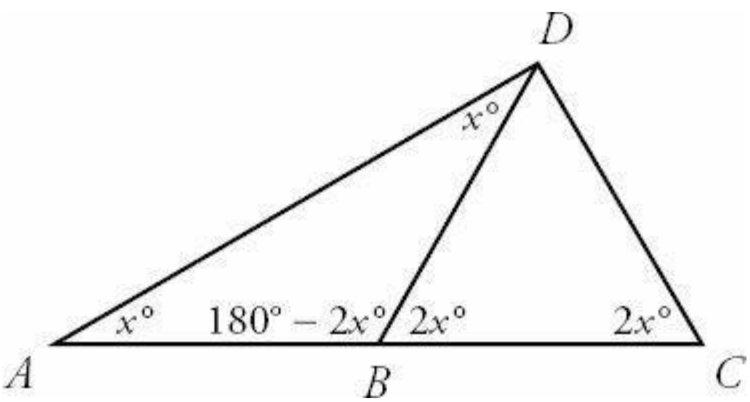
45.**(C).**To com pare *D C* and *AB* ,first solve for the unlabeled angles in the diagram .The tw o angles at point *B* m ake a straight line,so they add up to 180,and the unlabeled angle is 180 - 2*x*,as show n:



N ow m ake the angles of triangle *ABD* add up to 180:

> (A ngle *A*) + (A ngle *B*) + (A ngle *D*) =
> 180 *x* + (180 - 2*x*) + (A ngle *D*) = 180
> *x* + 180 - 2*x* + (A ngle *D*) =
> 180 -*x* + (A ngle *D*) = 0
> A ngle *D* = *x*

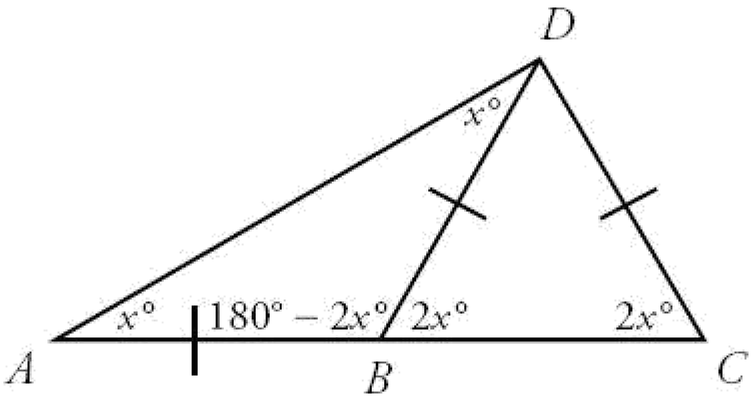
Therefore,the figure becom es



(Y ou could have also gotten here if you noticed that angle *D BC* (equal to 2*x*) is the exterior angle to the triangle on the left,and so it equals the sum of the tw o non-adjacent angles in that triangle.O ne of those angles,nam ely angle *A* , is *x*,so the other one m ust be *x* as w ell.)

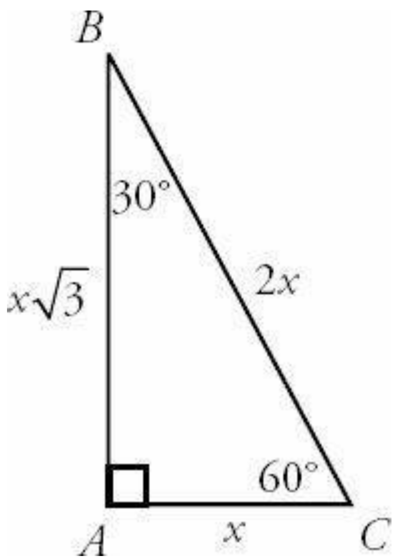
N ow apply the properties of isosceles triangles.The tw o angles labeled *x* are equal,so the triangle that contains them (triangle *ABD*) is isosceles,and the sides opposite those equal angles are also equal.Put a slash through those sides (*AB* and *D B*) to m ark them as the sam e length.

Likew ise,the tw o angles labeled 2*x* are equal,so the triangle that contains them (triangle *D BC*) is isosceles,and the sides opposite those angles (*D B* and *D C*) are equal.A dding one m ore slash through *D C* ,you get



Thus,sides *AB* and *D C* have the sam e length.The tw o quantities are equal.

46.**(B).**To com pute the perim eter of this triangle,you need the lengths of all three sides.B ecause *A* is a right angle and angle *B* is 30°,right triangle *ABC* is a 30–60–90 triangle.For any 30–60–90 triangle,the sides are in these proportions:



N ow you m ust m atch up this universal 30–60–90 triangle to your given triangle,so that you can find *x* in this particular case.The only labeled side in the given triangle (6) m atches the $x\sqrt{3}$ side in the universal triangle (they're both opposite the 60° angle),so set them equal to each other:

$$6 = x\sqrt{3}$$

$$x = \frac{6}{\sqrt{3}}$$

R ationalize the denom inator by m ultiplying by $\frac{\sqrt{3}}{\sqrt{3}}$ (w hich does not change the value of *x*,as $\frac{\sqrt{3}}{\sqrt{3}}$ is just a form of 1):

$$x = \frac{6}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$x = \frac{6\sqrt{3}}{3}$$

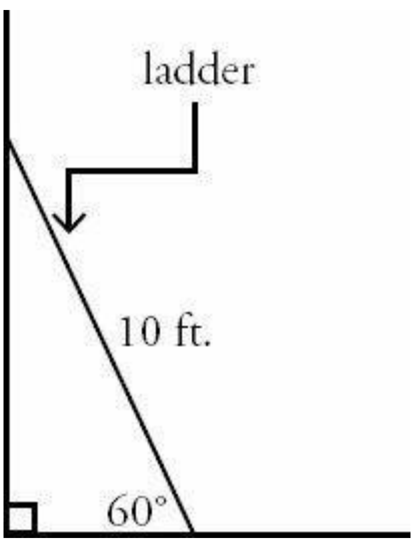
$$x = 2\sqrt{3}$$

N ow figure out all the sides in your given triangle.The length of side *AC* (*x*) is $2\sqrt{3}$,the length of side *AB* is given by 6,and the length of side *BC* is $2x = 2(2\sqrt{3}) = 4\sqrt{3}$.

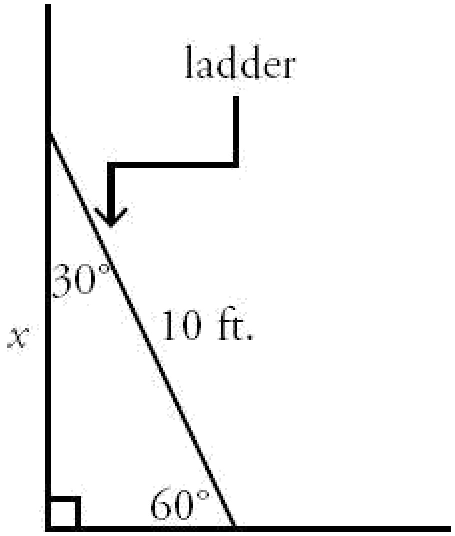
Finally,add up all the sides to get the perim eter:

$$\text{Perimeter} = 6 + 2\sqrt{3} + 4\sqrt{3}$$
$$\text{Perimeter} = 6 + 6\sqrt{3}$$

47.**(B).**First,draw a diagram and label all the givens:

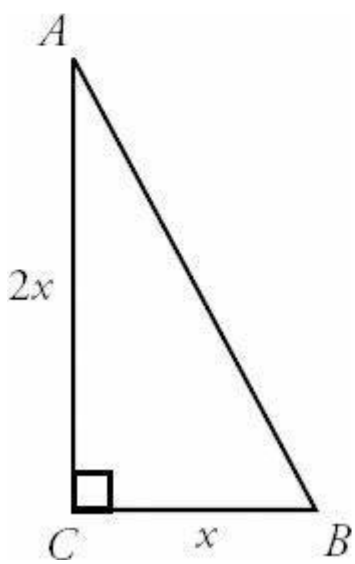


Since the w all is vertical and the floor is perfectly horizontal,the angle w here they m eet is 90°.So the triangle is 30–60–90.Y ou are asked to find the vertical distance from the top of the ladder to the floor,so represent this length as *x*.



In any 30–60–90 triangle,the short leg (opposite the 30° angle) is the hypotenuse divided by 2,m aking the floor-side 10 ÷ 2 = 5 feet.The longer leg (opposite the 60° angle) is $\sqrt{3}$ tim es the short leg.So $x = 5\sqrt{3}$ feet.

48.**(C).**D raw a diagram and label the sides of the triangle w ith the inform ation given.Since *AC* is tw ice as long as *C B* , label *C B* as *x* and *AC* as 2*x*,as show n



Y ou are given the area of the triangle,and you can use *x* as the base and 2*x* as the height in the form ula for area (w hich equals $\frac{bh}{2}$).Plug in and solve for *x*:

$$36 = \frac{x(2x)}{2} = \frac{2x^2}{2}$$

$$36 = x^2$$

$$x = 6$$

So *C B* = 6 and *AC* = (2)(6) = 12.U se the Pythagorean Theorem to find *AB* :

$$(AB)^2 = 6^2 + 12^2$$

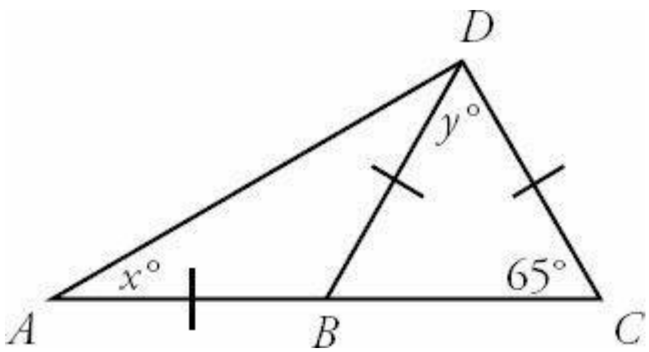
$(AB)^2 = 36 +$
$144\ (AB)^2 = 180$
$AB = 6\sqrt{5}$

49.**(A).**In order to m ake the com parison,you need the length of *C B* .Since angles *A* and *B* are equal,the triangle is isosceles,and the sides opposite those angles (*AC* and *C B*) m ust also be equal.W rite the equation:

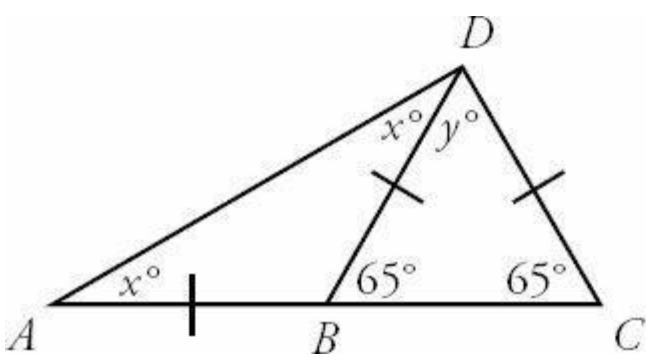
$2x - 5 = x +$
$4\ x - 5 = 4$
$x = 9$

C B is therefore equal to 9 + 4 = 13.Q uantity A is greater.

50.**(B).**R edraw the figure and label the equal sides:



The tw o sm all triangles (on the left and on the right) are each isosceles,because they each contain tw o equal sides. From the isosceles triangle theorem ,the angles opposite equal sides are also equal.Fill in m ore angles on the figure:



The three angles in the triangle on the right m ust add up to 180 degrees:

$65 + 65 + y = 180$
$130 + y =$
$180\ y = 50$

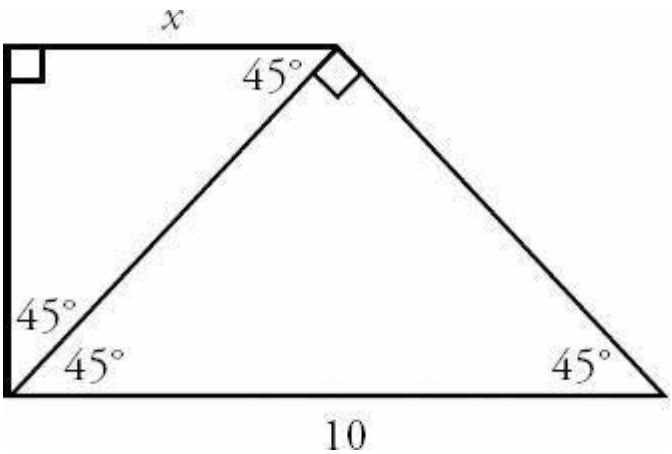
The tw o angles at point *B* m ake a straight line,so they add up to 180 degrees.So the unlabeled angle m ust be 180 - 65 = 115 degrees.

Finally,the three angles in the triangle on the left m ust sum to 180 degrees:

$x + x + 115 =$
$180\ 2x = 65$
$x = 32.5$

So y is greater than x. Quantity B is greater.

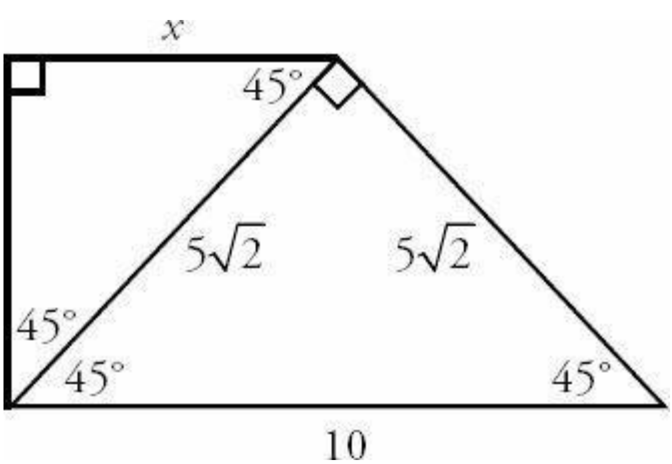
51. **(C).** Redraw the figure and label all angles, applying the rule that the angles in a triangle add up to 180:



You can now see that you have two separate 45–45–90 triangles. In a 45–45–90 triangle, the sides are in a $1 : 1 : \sqrt{2}$ ratio. Thus, the length of each leg equals the length of the hypotenuse divided by $\sqrt{2}$. The hypotenuse of the larger triangle is 10, so each leg of that triangle is $\frac{10}{\sqrt{2}}$. Rationalize the denominator by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$:

$$\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Add more labels to your figure:



Now you know that the hypotenuse of the smaller triangle is $5\sqrt{2}$. Apply the 45–45–90 triangle ratio once more ($1 : 1 : \sqrt{2}$) to see that $x = 5$.

52.**(B).**The sides of a 30–60–90 triangle are alw ays in a 1:$\sqrt{3}$: 2 ratio.Since *BC* is across from the 30° angle and *AB* is across from the 60° angle,Q uantity A is equal to $\frac{1}{\sqrt{3}}$.

In the calculator,this is 0.578… .U se the calculator to see that Q uantity B = 10/17 = 0.588… and is slightly larger than Q uantity A .

A lternatively,set the tw o fractions next to each other and cross m ultiply to com pare them :

$$\frac{1}{\sqrt{3}} \ ? \ \frac{10}{17}$$
$$17 \ ? \ 10\sqrt{3}$$
$$1.7 \ ? \ \sqrt{3}$$
$$1.7^2 \ ? \ 3$$

1.7^2 = 2.89.N ote that 1.7 is a good approxim ation of $\sqrt{3}$,but$\sqrt{3}$ is actually a bit bigger.

Q uantity B is larger.

53.**I and II only.**If A ngle *B* = 90°,then 8 and 15 are the base and height,and you can calculate the area.Statem ent I is sufficient.

If side *AC* = 17,you can plug 8,15,and 17 into the Pythagorean theorem to see w hether you get a true statem ent.U se 17 as the hypotenuse in the Pythagorean theorem because 17 is the longest side:

$$8^2 + 15^2 = 17^2$$
$$64 + 225 = 289$$
$$289 = 289$$

Since this is obviously true,the triangle is a right triangle w ith the right angle at *B* .If A ngle *B* = 90°,then 8 and 15 are the base and height,and you can calculate the area.(8–15–17 is a Pythagorean triplet,so if you had that fact m em orized,you could skip the step above.) Statem ent II is sufficient.

K now ing that *ABC* is a right triangle (Statem ent III) is *not* sufficient to calculate the area because you don’t know w hich angle is the right angle.A triangle w ith sides of 8 and 15 could have hypotenuse 17,as you’ve already seen,but another scenario is possible: perhaps 15 is the hypotenuse.In this case,the third side is sm aller than 15,and the area is sm aller than in the 8–15–17 scenario.

54.**(A).**The three interior angles of the triangle add up to 180.Try an exam ple: say each interior angle is 60°.In that case,*a*,*b*,and *c* w ould each equal 120° (since tw o angles that m ake up a straight line add to 180),and Q uantity A w ould equal 360.

Y ou can prove this result in general by expressing each interior angle in term s of *a*,*b*,and *c*,and then setting their sum equal to 180:

$(180 - a) + (180 - b) + (180 - c) =$
$180\ 540 - a - b - c = 180$
$360 = a + b + c$

Q uantity A is greater.

55.**(B).**Since both triangles have a 90° angle and an angle *x*,the third angle of each is the sam e as w ell (because the three angles in each triangle add up to 180).A ll the corresponding angles are equal,so the triangles are sim ilar,and the ratio of corresponding sides is constant.

The sm aller triangle is a 3–4–5 Pythagorean triple (the m issing hypotenuse is 5).Set up a proportion that includes tw o pairs of corresponding sides.The w ords “4 is to 10 as 5 is to *m* ” becom e this equation:

$$\frac{4}{10} = \frac{5}{m}$$

$4m = 50$

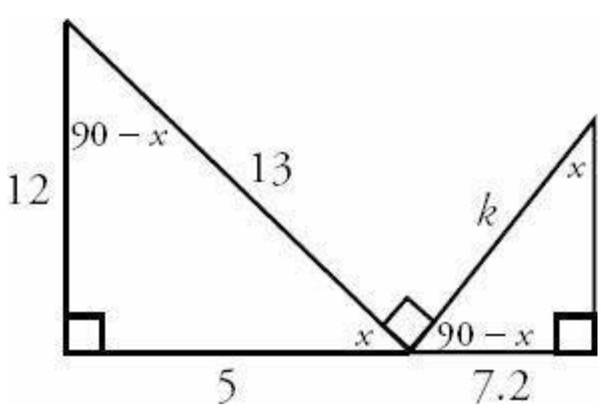
$m = 12.5$

Q uantity B is larger.

56.**7.8.**B egin by noting that the triangle on the left is a 5–12–13 Pythagorean triple,so the bottom side is 5.Subtract 12.2 - 5 = 7.2 to get the bottom side of the triangle on the right.

N ext,the tw o unm arked angles that “touch” at the m iddle m ust add up to 90,because they form a straight line together w ith the right angle of 90° betw een them ,and all three angles m ust add up to 180.M ark the angle on the left *x*.The angle on the right m ust then be 90 - *x*.

N ow the other angles that are still unm arked can be labeled in term s of *x*.U sing the rule that the angles in a triangle add up to 180,the angle betw een 12 and 13 m ust be 90 - *x*,w hile the last angle on the right m ust be *x*,as show n:



Since each triangle has angles of 90,*x*,and 90 - *x*,the triangles are sim ilar.This observation is the key to the problem . N ow you can m ake a proportion,carefully tracking w hich side corresponds to w hich.7.2 corresponds to 12,since each side is across from angle *x*.Likew ise,*k* corresponds to 13,since each side is the hypotenuse.W rite the equation and solve for *k*:

$$\frac{7.2}{12} = \frac{k}{13}$$

$$\frac{(13)7.2}{12} = k$$

$$k = 7.8$$