T w o-V ariable W ord P roblem s

For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given),the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box ☐,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes $\frac{\square}{\square}$,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.

A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.There are five m ore com puters in the office than em ployees.If there are 10 em ployees in the office,w hat is the ratio of com puters to em ployees in the office?

(A) 2 : 3
(B) 2 : 5
(C) 3 : 2
(D) 3 : 5
(E) 5 : 2

2.Tw o parking lots can hold a total of 115 cars.The G reen lot can hold 35 few er cars than the R ed lot.H ow m any cars can the R ed lot hold?

(A) 35
(B) 40
(C) 70
(D) 75
(E) 80

3.Three friends sit dow n to eat 14 slices of pizza.If tw o of the friends eat the sam e num ber of slices,and the third eats tw o m ore slices than each of the other tw o,how m any slices are eaten by the third friend?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

4.

In 8 years,Polly's age (w hich is currently *p*) w ill be tw ice Q uan's age (w hich is currently *q*).

Q uantity A	**Q uantity B**
p - 8	2*q*

5.Pens cost 70 cents each,and pencils cost 40 cents each.If Iris spent $5.20 on 10 pens and pencils,how m any pencils did she purchase? ($1 = 100 cents)

(A) 4
(B) 6
(C) 8
(D) 10
(E) 13

6.Jack dow nloaded ten songs and tw o books for $48,Jill dow nloaded fifteen songs and one book for $44.H ow m uch did Jack spend on books,if all songs are the sam e price and all books are the sam e price?

(A) $14
(B) $20
(C) $28
(D) $29
(E) $30

7.M arisa has $40 m ore than B en,and B en has one-third as m uch m oney as M arisa.H ow m any dollars does B en have?

$ ☐

8.N orm an is 12 years older than M ichael.In 6 years,he w ill be tw ice as old as M ichael.H ow old is M ichael now ?

(A) 3
(B) 6
(C) 12
(D) 18
(E) 24

9.3 people split a $100 stereo but pay different am ounts.If A pays $5 less than B ,C pays m ore than $35,and all 3 people pay integer am ounts,w hat is the m ost A could pay?

(A) 29
(B) 29.5
(C) 30
(D) 33

(E) 34

10.K runchy K ustard sells only tw o kinds of doughnuts,glazed and cream -filled.A glazed doughnut has 200 calories, and a cream -filled doughnut has 360 calories.If M ichael ate 5 doughnuts totaling 1,640 calories,how m any w ere glazed?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

11.O lym pic lifting consists of tw o disciplines,the Snatch,and the C lean and Jerk.H alil's best Snatch and best C lean and Jerk sum to 295 kilogram s.If his best C lean and Jerk w as 25 kilogram s heavier than his best Snatch,w hat w as the w eight of his best C lean and Jerk?

(A) 135
(B) 142.5
(C) 145
(D) 147.5
(E) 160

12.The "aspect ratio" of a com puter m onitor is the ratio of the m onitor's w idth to its height.If a particular m onitor has an aspect ratio of 16 : 9,and a perim eter of 100 inches,how m any inches w ide is the m onitor?

(A) 18
(B) 25
(C) 32
(D) 36
(E) 64

13.C indy bought 48 containers of soda,all either 12-ounce cans or 20-ounce bottles.If the num ber of ounces she purchased in cans w as equal to the num ber of ounces she purchased in bottles,how m any bottles of soda did C indy buy?

(A) 18
(B) 21
(C) 24
(D) 27
(E) 30

14.R ed chips all have the sam e value as one another,blue chips all have the sam e value as one another,and yellow chips also all have the sam e value as one another.If the value of a red chip plus a blue chip is 4.25,the value of a blue chip plus a yellow chip is 2.75,and the value of a red chip plus a blue chip plus a yellow chip is 4.5,w hat is the value of a red chip plus a yellow chip?

(A) 0.25
(B) 2
(C) 2.25
(D) 2.75
(E) 3

15.Tw o runners' race tim es add to 170 seconds and one of the race tim es is ten seconds less than tw ice the other. W hat is the faster race tim e,in seconds?

(A) 40
(B) 50
(C) 60
(D) 70
(E) 110

16.B eth is tw elve years younger than A lan.In 20 years,B eth w ill be 80% of A lan's age.H ow old is B eth now ?

[ ] years old

17.R ey is 12 years younger than Sebastian.Five years ago,R ey w as half Sebastian's age.H ow old w ill Sebastian be next year?

(A) 15
(B) 20
(C) 25
(D) 30
(E) 35

18.D uring a sale,the local outlet of the C hasm sold three tim es as m any jeans as chinos.If they m ade tw ice as m uch profit for a pair of chinos as for a pair of jeans,and sold no other item s,w hat percent of their profits during the sale cam e from chinos?

(A) $16\frac{2}{3}\%$
(B) 20%
(C) 40%
(D) 60%
(E) $83\frac{1}{3}\%$

19.M arisol is tw ice as old as V ikram .Eight years ago,M arisol w as 6 years younger than three tim es V ikram 's age at that tim e.H ow old w ill M arisol be in 5 years?

[ ]

20.M ark is tw ice as old as V icky.Four years ago,M ark w as 6 years younger than three tim es V icky's age at that tim e. H ow old w ill M ark be in 2 years?

[ ]

21.The length of a rectangle is tw o m ore than tw ice the w idth,and the area of the rectangle is 40.W hat is the rectangle's perim eter?

[ ]

22.M arcy bought one pair of jeans at 70% off and one blouse at 40% off.If she paid $12 m ore for the blouse than for the jeans,and she spent a total of $84,w hat w as the original price of the jeans?

(A) 76
(B) 96
(C) 100
(D) 120
(E) 124

23.C ranw ell G olf C ourse offers tw o different pricing packages for golf lessons.U nder the “Sapphire” pricing plan, lessons can be bought for a flat rate of $80 per hour.U nder the “D iam ond” pricing plan,for an initial fee of $495,lessons can be bought for a rate of $15 per hour.If Jeanie buys the “D iam ond” pricing plan,how m any golf lessons does she need to take in order to have spent exactly 40% less than she w ould have under the “Sapphire” plan?

(A) 10
(B) 12
(C) 15
(D) 18
(E) 20

24.W all-to-w all carpeting is installed in a certain hallw ay.The carpeting costs $4.25 per square foot.If the perim eter of the hallw ay (in feet) is equal to 44% of the area of the hallw ay (in square feet) and the hallw ay is 50 feet long, how m uch did it cost to install the carpeting?

(A) $182.50
(B) $212.50
(C) $505.25
(D) $1,062.50
(E) $1,100.00

25.Jam al gets three m onthly credit card statem ents over the course of three m onths.If his average m onthly statem ent over these three m onths is $44 m ore than the m edian am ount,and the sum of the largest and the sm allest statem ent is $412,w hat is the total am ount that Jam al spent over these three m onths?

(A) $456
(B) $552
(C) $600
(D) $824
(E) $1,000

26.A certain kennel houses only collies,labs,and golden retrievers.If the ratio of collies to labs is 5 : 9,there are 66 golden retrievers,and 12 m ore golden retrievers than labs,w hat percent of the total num ber of dogs in the kennel are collies?

(A) 5%
(B) 9%
(C) 12%
(D) 20%
(E) 25%

27.If M ason is now tw ice as old as G unther w as 10 years ago,and *G* is G unther’s current age in years,w hich of the follow ing represents the sum of M ason and G unther’s ages 4 years from now ?

(A) $\frac{3G}{2}+3$
(B) 3*G* + 28 (C) 3*G* - 12 (D) 8 - *G*
(E) $14-\frac{3G}{2}$

28.A baker m akes a com bination of chocolate chip cookies and peanut butter cookies for a school bake sale.H is recipes only allow him to m ake chocolate chip cookies in batches of 7,and peanut butter cookies in batches of 6.If he m akes exactly 95 cookies for the bake sale,w hat is the m inim um num ber of chocolate chip cookies that he m akes?

(A) 7
(B) 14
(C) 21
(D) 28
(E) 35

29.Janie has 5 few er candies than M ark.If Janie gives M ark 5 candies,M ark w ill then have 4 tim es as m any candies as Janie.H ow m any candies does Janie have?

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25

30.If Standard Jeans cost $60 and D esigner Jeans cost 150% m ore,and 29 total pairs of jeans are sold for a total of $3,540,how m any pairs w ere D esigner Jeans?

(A) 2
(B) 9
(C) 18
(D) 20
(E) 23

31.Lou has three daughters: W en,M ildred,and Tyla.Three years ago,w hen Lou w as tw ice as old as Tyla,he w as thirty years older than M ildred.N ow ,he is forty-seven years older than W en.In four years,W en w ill be half as old as Tyla.W hat is Lou's,W en's,M ildred's and Tyla's com bined age?

(A) 138
(B) 144
(C) 154
(D) 166
(E) 181

32.A farm er has exactly 1,000 square feet of farm land,on w hich he can grow both soy and corn.Every square foot can produce either one pound of soy or three pounds of corn.If soy can be sold on the m arket for $12/pound for the first hundred pounds and then $6 per pound after that,and corn can be sold on the m arket for $10/pound, w hat is the num ber of square feet of farm land that the farm er should devote to soy to m ake a profit of exactly $13,080?

(A) 30
(B) 100
(C) 270
(D) 600
(E) 730

33.D w ayne planted 70 acres w ith tw o types of field beans,navy beans,and pinto beans.Each acre of navy beans yielded 27 bushels,and each acre of pinto beans yielded 36 bushels.If D w ayne grew tw ice as m any bushels of pinto beans as navy beans,how m any acres of pinto beans did he plant?

(A) 28
(B) 30
(C) 35
(D) 40
(E) 42

T w o-V ariable W ord P roblem s A nsw ers

1.**(C).**Let *c* = num ber of com puters.Let *e* = num ber of em ployees

*There are 5 m ore com puters than em ployees.*Y ou can translate that into an equation:

c = *e* + 5
If *e* = 10,then *c* = (10) +
5 *c* = 15

The ratio of com puters to em ployees is 15 : 10,w hich can be reduced to 3 : 2.

2.**(D).**Let *g* = the num ber of cars that the G reen lot can hold.Let *r* = the num ber of cars that the R ed lot can hold

The first tw o sentences can be translated into tw o equations:

g + *r* = 115
g = *r* - 35

Y ou w ant to solve for *r*,so you should substitute (*r* - 35) for *g* in the first equation:

(*r* - 35) + *r* = 115
2*r* - 35 = 115
2*r* = 150
r = 75

3.**(D).**Let *P* = the num ber of slices of pizza eaten by each of the tw o friends w ho eat the sam e am ount.Let *T* = the num ber of slices of pizza eaten by the third friend.

T = *P* + 2
P + *P* + *T* = 14

Substitute (*P* + 2) for *T* in the second equation:

P + *P* + (*P* + 2) =
14 3*P* + 2 = 14
3*P* = 12
P = 4

Y ou can use the value of *P* to solve for *T*:

T = *P* + 2 = 4 + 2 = 6

4.**(C).**This is an algebraic translation question,so you should start by translating the given inform ation into equations.

R em em ber to add eight to both Polly and Q uan's ages,because they w ill *both* be eight years older in eight years!

$p + 8 = 2(q +$
8) $p + 8 = 2q +$
16 $p = 2q + 8$

Looking at the tw o colum ns,you can see that it w ould be helpful to m anipulate the equation one last tim e:

$p - 8 = 2q$

Y ou can see from that equation that the tw o colum ns are equal.The answ er is (C).

5.**(B).**M any questions that involve tw o unknow ns (e.g.,the num ber of pens and the num ber of pencils) can be translated either as one equation involving one variable,or tw o equations involving tw o variables each.

It's a bit m ore w ork to *translate* using just one variable than using tw o,but generally m uch less w ork to *solve* one equation w ith one variable than to solve a system of tw o equations w ith tw o variables.

W ith O ne V ariable

First,assign one variable to the pencils,and then define the pens in term s of the pencils:

N um ber of pencils = x
N um ber of pens = $10 - x$

Since this problem describes a real-life situation,it is not too difficult to w rite the form ula:

(C ost per pen × num ber of pens) + (cost per pencil × num ber of pencils) = total cost

Plugging in the num bers from the problem (70 cents per pen and 40 cents per pencil):

$70(10 - x) + 40x = 520$
$700 - 70x + 40x = 520$
$700 - 30x = 520$
180 =
$30x$ $x = 6$

W ith Tw o V ariables

First,assign one variable to the pencils,and another variable to the pens.

N um ber of pencils = x
N um ber of pens = y

$x + y = 10$
$70y + 40x = 520$

N ext,isolate y (from the first,sim pler equation) so you can substitute:

$y = 10 - x$

Substitute $10 - x$ for y:

$70(10 - x) + 40x = 520$
$700 - 70x + 40x = 520$
$700 - 30x = 520$
$180 =$
$30x$ $x = 6$

SH O R TC U T: This question allow s backsolving,choosing an answ er choice to see w hether it w orks.G enerally,you'll w ant to start w ith answ er choice (C);if it turns out to be too large,you can elim inate it and the tw o answ er choices larger than it;if it turns out to be too little,you can elim inate it and the tw o answ er choices less than it.(C) is not alw ays the m ost efficient answ er choice to test first,though.H ere,for instance,(D) and (E) are im plausibly large,so start w ith (B),the m iddle value of the rem aining answ ers.

B ut proceed w ithout that insight.A ssum e that Iris bought 8 pencils,and therefore 2 pens.8 × 40 + 2 × 70 = 320 + 140 = 460.That's 60 cents too little,so Iris m ust have bought few er pencils and m ore pens.Try 6 pencils and 4 pens. 6 × 40 + 4 × 70 = 240 + 280 = 520.(Y ou m ight also have noticed that every tim e Iris sw aps a pencil for a pen,she spends an extra 30 cents.)

6.**(C).**The equations are $10s + 2b = 48$ and $15s + b = 44$.The easiest next m ove w ould be to solve the second equation for b:

$b = 44 - 15s$

Substitute that into the first equation:

$10s + 2(44 - 15s) =$
$48\ 10s + 88 - 30s =$
$48\ -20s + 88 = 48$
$-20s = -$
$40\ s = 2$

Plug $s = 2$ back into w hichever equation you prefer to get that $b = 14$,and thus TW O books cost $28.

7.**20.**W rite both facts from the problem as sim ple equations:

$M = B + 40$

$B = \frac{1}{3} M$

Since you w ant the dollar am ount for B en,substitute for the other variable,M .Since M equals $B + 40$,w rite $B + 40$ in parentheses in place of M in the second equation:

$B = \frac{1}{3} (B + 40)$

N ow distribute:

$$B = \frac{B}{3} + \frac{40}{3}$$

Subtract $\frac{B}{3}$ from both sides (note that $\frac{B}{3}$ is the sam e as $\frac{1}{3}B$,so subtracting $\frac{1}{3}$ *B* from *B* w ill give you $\frac{2}{3}B$,or $\frac{2B}{3}$.

$$\frac{2B}{\quad} = \frac{40}{\quad}$$

$2B = 40$

$B = 20$

The final answ er is $20.

A lternatively,you could reason that if B en has one-third w hat M arisa does,then he's *m issing* tw o-thirds of her am ount.Since that tw o-thirds turns out to equal $40,then tw o-thirds of M arisa's m oney is $40.Thus,her total am ount is $60.D ivide by 3 to get B en's am ount,$20.

8.**(B).**Let *N* = N orm an's age now Let (*N* + 6) = N orm an's age in 6 years.

M = M ichael's age now (*M* + 6) = M ichael's age in 6 years.

Translate the first tw o sentences into equations.N ote that the second equation deals w ith N orm an and M ichael's ages in 6 years:

$N = M + 12$
$(N + 6) = 2(M + 6)$

Y ou w ant to solve for *N* ,so substitute (*M* + 12) for *N* in the second equation:

$(M + 12) + 6 = 2(M + 6)$
$M + 18 = 2M + 12$
$M + 6 = 2M$
$6 = M$

9.**(A).**This is a m axim ization question (w hat is the *m ost A* could pay).In order to solve m axim ization questions,you often have to m inim ize the other term s.In this case,you need to m inim ize *B* and *C* in order to m axim ize *A* .

The m inim um possible *C* is $36,leaving $64 for *A* and *B* to pay.Since *B* pays $5 m ore than *A* :

$B = A + 5$
$A + (A + 5) =$
$64\ 2A = 59$
$A = 29.5$

O f course,this isn't possible,because the three people have to pay integer values.So you need to m ove *C* up to $37,

leaving $63 for *A* and *B* to pay.

$A + (A + 5) =$
$63\ 2A = 58$
$A = 29$

The correct answ er is (A),or $29.

10.**(A).**M any questions that involve tw o unknow ns (e.g.,the num ber of glazed doughnuts and the num ber of cream - filled doughnuts) can be translated either as one equation involving one variable,or tw o equations involving tw o variables each.

It's a bit m ore w ork to *translate* using just one variable than using tw o,but generally m uch less w ork to *solve* one equation w ith one variable than to solve a system of tw o equations w ith tw o variables.

W ith O ne V ariable

M ichael ate 5 doughnuts.
N um ber of glazed = x
N um ber of cream -filled = $5 - x$

(num ber of glazed × calories per glazed) + (num ber of cream -filled × calories per cream -filled) = total calories

$200x + 360(5 - x) = 1{,}640$
$200x + 1{,}800 - 360x =$
$1{,}640 - 160x = -160$
$x = 1$

W ith Tw o V ariables

N um ber of glazed = x
N um ber of cream -filled = y

$x + y = 5$
$200x + 360y$ Isolate y in first equation to allow substitution.

$y = 5 - x$ Substitute $5 - x$ for y in second equation.

$200x + 360(5 - x) = 1{,}640$
$200x + 1{,}800 - 360x =$
$1{,}640 - 160x = 160$
$x = 1$

A lternative M ethod

This question allow s backsolving,choosing an answ er choice to see w hether it w orks.G enerally,you'll w ant to start w ith answ er choice (C);if it turns out to be too great,you can elim inate it and the tw o answ er choices less than it;if it

turns out to be too little,you can elim inate it and the tw o answ er choices less than it.

A ssum e that M ichael ate 3 glazed and therefore 2 cream -filled doughnuts.3(200) + 2(360) = 600 + 720 = 1,320. That's 320 calories too few .Y ou m ight notice that every tim e M ichael sw aps a glazed for a cream -filled,he consum es another 160 calories.Since 2(160) = 320,he needs to sw ap 2 of his 3 glazed for cream -filled.If you don't notice that just try (B) 2 glazed and 3 cream -filled,and see that yields only 1,480 calories.

11.**(E).**M any questions that involve tw o unknow ns (e.g.,the w eight of the Snatch and the w eight of the C lean and Jerk) can be translated either as one equation involving one variable,or tw o equations involving tw o variables each.

It's a bit m ore w ork to *translate* using just one variable than using tw o,but generally m uch less w ork to *solve* one equation w ith one variable than to solve a system of tw o equations w ith tw o variables.

W ith O ne V ariable

H alil's best Snatch and best C lean and Jerk sum to 295 kilogram s.

Weight of best C lean and Jerk = x
Weight of best Snatch = 295 - x

H is best C lean and Jerk w as 25 kilogram s heavier than his best Snatch.

$x = (295 - x) +$
$25\ x = 320 - x$
$2x = 320$
$x = 160$

W**ith Tw o V ariables**

Weight of best C lean and Jerk = x
Weight of best Snatch = y

$x + y = 295$ Isolate y to allow substitution.

$y = 295 - x$ Substitute 295 - x for y.

$x = (295 - x) +$
$25\ x = 320 - x$
$2x = 320$
$x = 160$

SH O R TC U T: Y ou could also subtract the difference (25) from the sum (295),then divide the result by 2.This w ill give you the w eight of the *lighter* lift.O r,you m ight notice sim ply that the C lean and Jerk m ust be m ore than half of the total of 295.H alf of 295 is 147.5,and only one answ er is greater than that.

12.**(C).**R ather than assigning separate variables to the w idth and height,define them both in term s of the sam e unknow n m ultiplier,based on the ratio given:

W idth = 16*m*
H eight = 9*m*

N ote that,since you w ant the w idth,Y O U A R E SO LV IN G FO R 16*m* ,N O T SIM PLY FO R *m* !

The perim eter of a rectangle = 2(length + w idth),or in this case 2(w idth + height)

100 = 2 × (16*m* +
9*m*) 100 = 50*m*
m = 2
16*m* = 32

SH O R TC U T: A nother m ethod depends on the sam e underlying logic,but forgoes the algebra.

Suppose the dim ensions w ere sim ply 16 inches and 9 inches.This w ould yield a perim eter of 50 inches.D ouble the w idth and height to double the perim eter.

13.**(A).**M any questions that involve tw o unknow ns (e.g.,the num ber of bottles and the num ber of cans) can be translated either as one equation involving one variable,or tw o equations involving tw o variables each.

W ith O ne V ariable

C indy bought 48 containers of soda
N um ber of bottles = *x*
N um ber of cans = 48 - *x*

The num ber of ounces she purchased in cans w as equal to the num ber of ounces she purchased in bottles
(ounces/can)(num ber of cans) = (ounces/bottle)(num ber of bottles)

12(48 - *x*) = 20*x*
576 - 12*x* = 20*x*
576 = 32*x*
x = 18

W ith Tw o V ariables

N um ber of bottles = *x*
N um ber of cans = *y*
C indy bought 48 containers of soda

x + *y* = 48

the num ber of ounces she purchased in cans w as equal to the num ber of ounces she purchased in bottles

(ounces/can)(num ber of cans) = (ounces/bottle)(num ber of bottle)

12*y* = 20*x* Isolate *y* in first equation to allow substitution.

$y = 48 - x$ Substitute into second equation.

$12(48 - x) = 20x$
$576 - 12x = 20x$
$576 = 32x$
$x = 18$

A lternative M ethod

This question allow s backsolving,choosing an answ er choice to see w hether it w orks.G enerally,you'll w ant to start w ith answ er choice (C);if it turns out to be too large,you can elim inate it and the tw o answ er choices larger than it;if it turns out to be too little,you can elim inate it and the tw o answ er choices less than it.There are exceptions,though. H ere,for instance,you notice that (C),(D) and (E) are im plausibly large.There m ust be few er bottles than cans,if the total volum es of the (large) bottles is to equal the total volum e of the (sm all) cans,w hich m eans that the answ er m ust be either (A) or (B).

Start w ith (B).A ssum e that C indy bought 21 bottles,and therefore 27 cans.21 × 20 = 420,27 × 12 = 324.These num bers aren't equal.C indy m ust have bought few er than 21 bottles,and m ore than 27 cans,and only (A) m atches that.

SH O R TC U T: B ecause (ounces/can)(num ber of cans) = (ounces/bottle)(num ber of bottles),the ratio of (num ber of bottles) to (num ber of cans) w ill be the inverse of the ratio of (ounces per bottle) to (ounces per can).So the ratio of bottles to cans is 12 : 20,or 3 : 5.This m eans that only 3/8 of all the containers are bottles,3/(5 + 3).3/8 of 48 is 18. This shortcut requires very little m ath,but a great deal of m athem atical sophistication.It com es in handy,but don't w orry if you didn't spot it.There are m any valid w ays to solve problem s like this one.

14.**(B).**First,w rite all the statem ents as equations:

$r + b = 4.25$
$b + y = 2.75$
$r + b + y = 4.5$

A lso w rite "$r + y$?" on your paper,as a rem inder of w hat you are looking for.

If $r + b = 4.25$,then $r + b + y = 4.5$ could be rew ritten as:

$4.25 + y =$
$4.5\ y = 0.25$

Since $b + y = 2.75$ and $y = 0.25$:

$b + 0.25 =$
$2.75\ b = 2.5$

Since $r + b = 4.25$ and $b = 2.5$:

$r + 2.5 = 4.25$

$r = 1.75$

Therefore, $r + y = 1.75 + 0.25 = 2$.

15. **(C).** C all the race tim es x and y. Since you are told a sum :

$$x + y = 170$$

O ne of the race tim es is ten seconds less than tw ice the other

$$x = 2y - 10$$

Since the second equation is already solved for x, plug $(2y - 10)$ in for x in the first equation:

$$2y - 10 + y = 170$$
$$3y - 10 = 170$$
$$3y = 180$$
$$y = 60$$

If $y = 60$ and the tim es add to 170, then $x = 110$.

N ote that you are asked for the *faster* race tim e — that m eans the sm aller num ber! The answ er is 60.

16. **28.** Since B eth is tw elve years younger than A lan:

$$B = A - 12$$

To translate, *In 20 years, Beth w ill be 80% of Alan's age*, m ake sure that B eth becom es $B + 20$ and A lan becom es $A + 20$ (if you w ill be doing other operations to these values, put parentheses around them to m ake sure the rules of PEM D A S are not violated):

$$B + 20 = 0.8(A + 20)$$
$$B + 20 = 0.8A + 16$$
$$B + 4 = 0.8A$$

Since the first equation is already solved for B , plug $(A - 12)$ into the sim plified version of the second equation in place of B .

$$B + 4 = 0.8A$$
$$(A - 12) + 4 = 0.8A$$
$$A - 8 = 0.8A$$
$$0.2A - 8 = 0$$
$$0.2A = 8$$
$$A = 40$$

A lan is 40. Since $B = A - 12$, B eth is 28. Since you are answ ering for B eth, the answ er is 28.

17. **(D).** M any questions that involve tw o unknow ns (e.g., R ey's age and Sebastian's age) can be translated either as one

equation involving one variable,or tw o equations involving tw o variables each.

It's a bit m ore w ork to *translate* using just one variable than using tw o,but generally m uch less w ork to *solve* one equation w ith one variable than to solve a system of tw o equations w ith tw o variables.

W ith O ne V ariable

Rey is 12 years younger than Sebastian

s = Sebastian's age N O W
s - 12 = R ey's age N O W

In problem s that involve values that change over tim e (ages,prices,num ber of em ployees,etc.),m ake sure to fix the variable to som e particular tim e.

H ow old w ill Sebastian be next year?

Solve for *s* + 1

Five years ago,Rey w as half Sebastian's age

Five years ago,R ey w as (*s* - 12) - 5,and Sebastian w as (*s* - 5),so…

(*s* - 12) - 5 = (*s* - 5)/2
s - 17 = (*s* - 5)/2

M ultiply both sides by 2 so you no longer have to deal w ith a fraction:

2(*s* - 17) = *s* -
5 2*s* - 34 = *s* -
5 2*s* = *s* + 29
s = 29
s + 1 = 30

W ith Tw o V ariables

r = R ey's age N O W
s = Sebastian's age N O W

H ow old w ill Sebastian be next year?

Solve for *s* + 1

Rey is 12 years younger than Sebastian

r = *s* - 12

Five years ago,Rey w as half Sebastian's age

$r - 5 = (s - 5)/2$

Substitute $s - 12$ for r.

$(s - 12) - 5 = (s - 5)/2$
$s - 17 = (s - 5)/2$

M ultiply both sides by 2 to clear the fraction.

$2s - 34 = s - 5$
$2s = s +$
$29\ s = 29$
$s + 1 = 30$

18.**(C).**If all the values given in a problem and its answ ers are *percents*,*ratios*,or *fractions of* som e unknow n,then the problem w ill probably be easiest to solve by stipulating values for the unknow ns.In this problem ,the tw o ratios given are 3 : 1 (jeans sold : chinos sold) and 2 : 1 (profits per pair of chinos : profits per pair of jeans).The easiest num ber to stipulate are:

3 pairs of jeans sold
1 pair of chinos sold
$2 profit/pair of chinos
$1 profit/pair of jeans

This yields $2 profit from the chinos,out of a total $5 in profit.2/5 = 40%

N otice that this relatively sim ple problem is surprisingly com plicated w ith variables.Even if you're clever enough to use just tw o variables rather than four,it's still a bit of w ork.

c = num ber of chinos sold
$3c$ = num ber of jeans sold
p = profit per jeans sold $2p$
= profit per chinos sold

solve for (profit for chinos)/[(profit for chinos) + (profit for jeans)]

$(c \times 2p)/(c \times 2p + 3cp)$
$2pc/(5pc) = 2/5 = 40\%$

If you use four variables,you'll alm ost certainly m ake a translation error.

19.**49.**W rite each sentence as its ow n equation:

Equation 1: $M = 2V$
Equation 2: $(M - 8) = 3(V - 8) - 6$

N ext,sim plify Equation 2 and substitute.Since Equation 1 is already solved for M ,the easiest w ay to substitute is to plug in $2V$ in place of M in Equation 2.

$2V - 8 = 3V - 24$ -
6 $2V - 8 = 3V - 30$
$2V + 22 = 3V$
$22 = V$

Thus, $M = 44$, and M arisol in 5 years, or $M + 5$, is equal to 49.

20. **30.** Translate the first tw o sentences in the problem into tw o separate equations:

Equation 1: $M = 2V$
Equation 2: $(M - 4) = 3(V - 4) - 6$

N ext, sim plify Equation 2 and substitute. Since Equation 1 is already solved for M, the easiest w ay to substitute is to plug in $2V$ in place of M in Equation 2.

$2V - 4 = 3V - 12$ -
6 $2V - 4 = 3V - 18$
$2V + 14 = 3V$
$14 = V$

Thus, $V = 14$, $M = 28$, and M ark in 2 years, or $M + 2$, is equal to 30.

21. **28.** C onvert this w ord problem into tw o equations w ith tw o variables. "The length is tw o m ore than tw ice the w idth" can be w ritten as:

$L = 2W + 2$

Since the area is 40 and area is length × w idth:

$LW = 40$

Since the first equation is already solved for L, plug $(2W + 2)$ in for L into the second equation:

$(2W + 2)W = 40$
$2W^2 + 2W = 40$

Since you now have a quadratic on your hands (you have both a W^2 and a W term), get all term s on one side to set them to zero:

$2W^2 + 2W - 40 = 0$

Sim plify as m uch as possible — in this case, divide the entire equation by 2 — before trying to factor:

$W^2 + W - 20 = 0$
$(W - 4)(W + 5) =$
0 $W = 4$ or -5

Since a w idth cannot be negative,the w idth = 4.Since $LW = 40$,the length m ust be 10.

Perim eter = $2L + 2W$
Perim eter = $2(10) + 2(4)$
Perim eter = 28

N ote that it m ight have been possible for you to puzzle out that the sides w ere 4 and 10 just by trying values.H ow ever, if you did this,you got lucky — no one said that the values even had to be integers! The ability to translate into equations and solve is very im portant for the G R E .

22.**(D).**To solve this problem ,establish the follow ing variables:

J = original jean price
B = original blouse price

Then establish a system of equations,keeping in m ind that “70% off” is the sam e as 100% - 70% = 30% ,or 0.3,of the original price:

$0.3J + 12 = 0.6B$
$0.3J + 0.6B = 84$

N ow sim ply use w hatever strategy you’re m ost com fortable w ith to solve a system of equations — for exam ple, aligning the equations and then subtracting them :

$0.3J + 12 = 0.6B$
$0.3J - 84 = -0.6B$
$0 + 96 = 1.2B$

$B = 96/1.2$
$B = 80$

A nd you can plug the price of the blouse back into the original equation to get the price of the jeans:

$0.3J + 12 = 0.6B$
$0.3J + 12 = 48$
$0.3J = 36$
$J = 120$

A lternatively,you could first figure out the price of the discounted jeans w ith this equation:

$x + (x + 12) = 84$
$2x + 12 = 84$
$2x = 72$
$x = 36$

Then plug that discounted price into the equation *discounted price = original price × (100% - percent discount):*

$36 = 0.3P$

360 = 3*P*
120 = *P*

23.**(C).**Start by assigning variables:

D = price under the D iam ond plan
S = price under the Sapphire plan
x = the num ber of lessons Jeanie takes

… then establish equations:

D = 495 +
15*x* *S* = 80*x*
0.6S = *D*

… then solve by substitution:

0.6*S* = 495 + 15*x*
0.6(80*x*) = 495 + 15*x*
48*x* = 495 + 15*x*
33*x* = 495
x = 15

24.**(D).**The equation for the perim eter of a space is 2*W* + 2*L* = *P* ,w here *W* is w idth and *L* is length.

The equation for the area is *A* = *W* × *L*.Thus,

0.44(*W* × *L*) = 2*W* + 2*L*
0.44(50*W*) = 2*W* +
2(50) 22*W* = 2*W* + 100
20*W* =
100 *W* = 5

If *W* = 5 and *L* = 50,then the total square footage of the room is 250,and the total cost is:

$4.25 × 250 = $1,062.50

25.**(B).**C all the sm allest statem ent *S*,the m iddle statem ent *M* ,and the largest statem ent *L*.

From your know ledge of m edians,you should know that *M* is the sam e as the m edian,since there are only three values.The equation for averages is:

$$\frac{\text{Sum of \#'s}}{\text{Number of \#'s}} = \text{A verage}$$

Incorporate the equation for averages into the follow ing equation:

$$\frac{S+M+L}{3} = M + 44$$

$S + M + L = 3M + 132$
$S + L = 2M + 132$

W hile you don't know the actual quantities of *S* and *L*,you know their sum :

412 = 2*M* + 132
280 = 2*M*
140 = *M*

Finally,add *M* to the sum of *S* and *L*:

140 + 412 = 552

26.**(D).**Start by assigning variables:

C = N um ber of collies
L = N um ber of labs
G = N um ber of golden retrievers

… and w ork from w hat you know .

G = 66
L = 66 - 12
L = 54

R atios w ork like fractions,and you can set them up accordingly.

$$\frac{5}{9} = \frac{C}{54}$$

C ross m ultiplying and sim plifying,you get:

C = 30

N ow take the num ber of collies and express it as a percentage of the total num ber of dogs:

Total # of D ogs = 30 + 54 + 66 = 150

$$\frac{30}{150} \times 100 = 20\%$$

27.**(C).**The sum of M ason and G unther's ages 4 years from now requires adding 4 to B O TH ages.

First equation (w hat you are looking for):

$(M + 4) + (G + 4) =$
$? \; M + G + 8 = ?$

Since M ason is tw ice as old as G unther w as 10 years ago,put $(G - 10)$ in parentheses and build the second equation from there (the parentheses are crucial).

Second equation:

$M = 2(G - 10)$
$M = 2G - 20$

N ote that the answ er choices ask for the sum of the ages 4 years from now ,in term s of G ,so substitute for M (the variable you substitute for is the one that drops out).

Substituting from the second equation into the first:

$(2G - 20) + G + 8 =$
$? \; 3G - 12 = ?$

This m atches choice (C).

A lternatively,you could w rite the second equation,$M = 2(G - 10)$,and then com e up w ith tw o values that “w ork” in this equation for M and G .The easiest w ay to do this is to m ake up G ,w hich w ill then tell you M .For instance,set $G = 12$ (use any num ber you w ant,as long as it’s over 10,since the problem strongly im plies that G unther has been alive for m ore than 10 years).

$M = 2(12 - 10)$
$M = 4$

If G unther is 12,then M ason is 4.In four years,they w ill be 16 and 8,respectively.A dd these together to get 24.

N ow ,plug $G = 12$ into each answ er choice to see w hich yields the correct answ er (for this exam ple),24.O nly choice (C) w orks.

28.**(E).**The equation for the situation described is $7x + 6y = 95$,w here x stands for the num ber of batches of chocolate chip cookies and y stands for the num ber of batches of peanut butter cookies.

It looks as though this equation is not solvable,because you have tw o variables and only one equation.H ow ever,since the baker can only m ake w hole batches,x and y m ust be integers,w hich really lim its the possibilities.

Furtherm ore,you w ant the *m inim um* num ber of chocolate chip cookies the baker could have m ade.So,try 1 for x and see if you get an integer for y (use your calculator w hen needed!):

$7(1) + 6y = 95$
$6y = 88 \; y$
$= 14.6…$

Since this did not result in an integer num ber of batches of peanut butter cookies,this situation doesn't w ork.Try 2,3, 4,etc.for *x*.(D on't try values out of order — rem em ber,there m ight be m ore than one *x* value that w orks,but you need to be sure that you have the sm allest one!)

Y ou w ill see that the sm allest value that w orks for *x* is 5:

$7(5) + 6y = 95$
$6y = 60$
$y = 10$

R em em ber that you need the m inim um num ber of chocolate chip *cookies*,not *batches of cookies.*Since the m inim um num ber of batches is 5 and there are 7 cookies per batch,the m inim um num ber of chocolate chip cookies is 35.

29.**(B).**First,translate the problem into tw o equations,w riting "Janie after she gave M ark 5 candies" as (J - 5) and "M ark after receiving 5 m ore candies" as (M + 5).

$J = M - 5$
$4(J - 5) = M + 5$

Since $J = M - 5$,plug $M - 5$ in for J in the second equation:

$4(M - 5 - 5) = M +$
5 $4(M - 10) = M +$
5 $4M - 40 = M + 5$
$4M = M + 45$
$3M = 45$
$M = 15$

If $M = 15$,then,since Janie has 5 few er candies,$J = 10$.

A lternatively,you could backsolve from the answ ers.Start w ith choice (C).If Janie had 15 candies,M ark w ould have 20.If Janie gave M ark 5,she w ould have 10 and he w ould have 25.Since 25 is N O T 4 tim es 10,this answ er is not correct.Since you w ant M ark to have m ore and Janie to have less,you m ight intuit that you should try a sm aller answ er.Try choice (B).If Janie had 10 candies,M ark w ould have 15.If Janie then gave M ark 5,she w ould have 5 and he w ould have 20.Since 20 is 4 tim es m ore than 5,this is the answ er.

30.**(D).**This is an algebra question w ith tw o unknow ns: the num ber of Standard jeans sold and the num ber of D esigner jeans sold.

First,you're told that 29 pairs of jeans are sold altogether: $s + d = 29$

Y ou also know that the cost of Standard Jeans is $60 and the cost of D esigner Jeans is $150 (B e careful! The question doesn't say D esigner Jeans cost 150% *as m uch* as Standard Jeans,but 150% *m ore*.If one thing costs 100% m ore than another thing,it's tw ice as m uch,so som ething that costs 150% m ore is 2.5 tim es as m uch).Y ou've been given the total cost of the jeans,so you can w rite a second equation:

$60s + 150d = 3{,}540$

N ow ,look at your tw o equations:

$s + d = 29$
$60s + 150d = 3{,}540$

The easiest w ay to solve from here is to m ultiply the top equation by 60,then com bine the tw o through elim ination.

$60s + 150d = 3{,}540$
$-60s + 60d = 1{,}740$
$90d = 1{,}800$

Therefore,$d = 20$.

31.**(A).**The key to this tricky-sounding problem is setting up variables correctly and ensuring that you subtract or add appropriately for these variables w hen representing their ages at different points in tim e.

L = Lou's age now
W = W en's age now
M = M ildred's age now
T = Tyla's age now

R epresent the second sentence of the problem :

Equation 1: $(L - 3) = 2(T - 3)$
Equation 2: $(L - 3) = (M - 3) + 30$

N ext:

Equation 3: $L = W + 47$
Equation 4: $(W + 4) = (T + 4)/2$

In order to solve this problem effectively,look for w ays that you can get tw o of the equations to have the sam e tw o variables in them .If you have tw o equations w ith only tw o variables,you can solve for both of those variables. Equation 4 has a W and a T;the only other equation w ith a T is Equation 1.If you substitute the L in Equation 1 w ith the W from Equation 3,you w ill have tw o equations w ith just W 's and T's.

Equation 1: $(L - 3) = 2(T - 3)$

$(W + 47) - 3 = 2(T - 3)$
$W + 44 = 2T - 6$
$W + 50 = 2T$

Equation 4: $(W + 4) = (T + 4)/2$

$2W + 8 = T + 4$
$2W + 4 = T$

N ow com bine the equations to solve for W .

W + 50 = 2(2*W* + 4)
W + 50 = 4*W* + 8
W + 42 = 4*W*
42 = 3*W*
14 = *W*

N ow that you know W en's age,you can solve for the rest.

Equation 3: *L* = *W* + 47

L = 14 + 47
L = 61

Equation 1: (*L* - 3) = 2(*T* - 3)

(61 - 3) = 2(*T* - 3)
58 = 2*T* - 6
64 = 2*T*
32 = *T*

Equation 2: (*L* - 3) = (*M* - 3) + 30

61 - 3) = (*M* - 3) +
30 58 = *M* + 27
31 = *M*

N ow that you know that *L* = 61,*W* = 14,*M* = 31,and *T* = 32,add them together to find the answ er.

61 + 14 + 31 + 32 = 138

32.**(C).**First,assign variables:

S = num ber of square feet devoted to soy
C = num ber of square feet devoted to corn

… and set up the equation:

[(12)100 + (6)(*S* - 100)] + [(10)(3)(*C*)] = 13,080

This m eans that the farm er gets $12 per pound for the first 100 pounds,or (12)100,and then $6 per pound for each pound after 100,or (6)(*S* - 100).H e also gets $10 per pound for corn,but you have to also account for the fact that he can grow 3 pounds of corn per acre,not just one.H ence,(10)(3)(*C*).

Sim plify:

[1,200 + 6*S* - 600] + [30*C*] =
13,080 600 + 6*S* + 30*C* = 13,080

$6S + 30C = 12{,}480$
$S + 5C = 2{,}080$

Y ou also know that:

$S + C = 1{,}000$
$S = 1{,}000 - C$

C om bining,you get:

$(1{,}000 - C) + 5C = 2{,}080$
$1{,}000 + 4C = 2{,}080$
$4C = 1{,}080$
$C = 270$
$S = 1{,}000 -$
$270\ S = 730$

33.**(E).**This question is difficult to translate.B egin by finding tw o things that are equal,and build an equation around that equality.*D w ayne grew tw ice as m any bushels of pinto beans as navy beans*:

2(bushels of navy beans) = (bushels of pinto beans)

B reak that dow n further:

bushels of navy beans = acres of navy beans × bushels per acre of navy beans
bushels of pinto beans = acres of pinto beans × bushels per acre of pinto beans

So:

2(acres of navy beans × bushels per acre of navy beans) =
(acres of pinto beans × bushels per acre of pinto beans)

Each acre of navy beans yielded 27 bushels,and each acre of pinto beans yielded 36 bushels

2 × 27 × (acres of navy beans) = 36 × (acres of pinto beans)

Y ou can finish your translation w ith one variable or w ith tw o.

<u>O ne V ariable</u>

N um ber of acres planted w ith pinto beans = p
N um ber of acres planted w ith navy beans = $70 - p$

$2 \times 27(70 - p) =$
$36p\ 54(70 - p) =$
$36p\ 3{,}780 - 54p =$
$36p\ 3{,}780 = 90p$
$p = 42$

Tw o V ariables

N um ber of acres planted w ith pinto beans = p
N um ber of acres planted w ith navy beans = n

$2 \times 27n = 36p$

$n + p = 70$

Isolate n to allow substitution:

$n = 70 - p$

Substitute:

$2 \times 27(70 - p) =$
$36p\ 54(70 - p) =$
$36p\ 3{,}780 - 54p =$
$36p\ 3{,}780 = 90p$
$p = 42$

SH O R TC U T: N otice that the ratio of (bushels of pinto beans produced) to (bushels of navy beans produced) is 2 : 1, greater than the ratio of (bushels of pinto beans produced per acre) to (bushels of navy beans produced per acre),36 : 27.This m eans that a greater num ber of acres m ust have been planted w ith pinto beans than w ith navy beans,so the answ er m ust be m ore than half of the 70 total,so either (D) or (E).Y ou could sim ply backsolve the easier answ er choice,(D).

40 acres of pinto beans × 36 bushels per acre = 1,440 bushels

30 acres of navy beans × 27 bushels per acre = 810 bushels

1440 isn’t quite tw ice 810,so 40 isn’t quite enough.The answ er m ust be (E).