GMAT Number Properties: Challenge

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Contents

1 Introduction

This document contains nothing but difficult GMAT Number Properties questions–100 of them, to be exact. Number Properties is one of the most challenging topics on the test, since many questions can't be solved algebraically. There are plenty of techniques you can learn to make these questions more clear-cut (and those are discussed in the explanations), but most test-takers never learn them.

As in all of my GMAT preparation resources, you'll find these questions indexed by difficulty. That doesn't mean you should skip straight to the hardest questions, or even that you should start with the easier ones. On the GMAT itself, questions won't come labeled with their difficulty level, and despite the intent of the adaptive algorithm, they won't be precisely consistent in terms of difficulty either. Each question presents its own unique challenges, and the sooner you get accustomed to changing gears with every single question, the more time you'll have to prepare for that particular challenge of the exam.

For further, more specific practice, I have produced several other resources that may help you. I've also created a 100-question set called "Exponents and Roots," which covers exactly that material, including dozens of questions that force you to master every last exponent-related rule the GMAT will test you on. You'll also find many Number Properties questions in my general sets, "Problem Solving: Challenge" and "Data Sufficiency: Challenge."

Also, The GMAT Math Bible has several chapters (along with focused practice) on Number Properties and related issues, including individual chapters on prime numbers, factors, multiples, evens and odds, consecutive numbers, and more. If you find you are struggling with the mechanics of these problems, your time is probably better spent with the GMAT Math Bible than in doing dozens and dozens of practice problems, hoping to pick up those skills along the way.

If you find yourself having problems with only the most difficult questions, you might try my "Extreme Challenge" set, which contains only 720 and higher level questions, many of which are Number Properties-related.

As far as strategy is concerned, there are dozens of articles at GMAT HACKS to help you with your strategic approach to Arithmetic questions. Most importantly, you should make sure you understand every practice problem you do. It doesn't matter if you get it right the first time–what matters is whether you'll get it right the next time you see it, because the next time you see it could be on the GMAT.

With that in mind, carefully analyze the explanations. Redo questions that took you too long the first time around. Review questions over multiple sessions, rather than cramming for eight hours straight each Saturday. These basic study skills may not feel like the key to GMAT preparation, but they are the difference between those people who reach their score goals and those who never do.

Enough talking; there are 100 Number Properties questions waiting inside. Get to work!

2 Difficulty Levels

In general, the level 5 questions in this guide are 560- to 620-level questions. The level 6 questions representing a broad range of difficulty from about 620 to 720, while the level 7 questions are higher still.

Moderately Difficult (5)
PS
001, 002, 010, 035, 043
DS
051, 052, 053, 062, 067, 068, 070, 071, 073, 074, 075, 081, 083, 084, 086, 087, 090, 092, 094, 096, 097

Difficult (6)
PS
003, 005, 007, 009, 011, 012, 013, 014, 015, 016, 020, 021, 022, 024, 026, 027, 029, 032, 033, 034, 037, 038, 039, 040, 042, 044, 045, 046, 048
DS
049, 050, 054, 055, 056, 057, 058, 059, 061, 063, 064, 065, 066, 069, 072, 076, 077, 078, 079, 082, 085, 088, 089, 093, 095, 098, 099, 100

Very Difficult (7)
PS
004, 006, 008, 017, 018, 019, 023, 025, 028, 030, 031, 036, 041, 047
DS
060, 080, 091

3 Problem Solving

Note: this guide contains both an answer key (so you can quickly check your answers) and full explanations.

1. For which of the following values of n is $\frac{n+78}{n}$ an integer?
   (A) 9
   (B) 10
   (C) 11
   (D) 12
   (E) 13

2. What is the probability that a random number selected from the set of integers 101 through 300, inclusive, has a units digit of 6?
   (A) $\frac{20}{299}$
   (B) $\frac{20}{199}$
   (C) $\frac{3}{20}$
   (D) $\frac{2}{15}$
   (E) $\frac{1}{10}$

3. $\frac{24^3}{(6^2)(4^2)(2^2)}$
   (A) 2
   (B) 3
   (C) 6
   (D) 9
   (E) 12

4. If p is a positive integer and p^2 is divisible by 12, then the largest positive integer that must divide p^3 is
   (A) 2^3
   (B) 2^6
   (C) 3^3
   (D) 6^3
   (E) 12^2

5. What is the least positive integer that is divisible by each of the integers 1 through 6, inclusive?
   (A) 60
   (B) 120
   (C) 180
   (D) 240
   (E) 360

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6. Mersenne primes are prime numbers that can be written in the form $2^n - 1$, where n is itself a prime number. Which of the following is NOT a Mersenne prime?

(A) 3
(B) 7
(C) 31
(D) 127
(E) 511

7. How many different positive integers are factors of 225 ?

(A) 4
(B) 6
(C) 7
(D) 9
(E) 11

8. If n is a positive integer less than 200 and $\frac{5n}{42}$ is an integer, then n has how many different positive prime factors?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Seven

9. If the sum of n consecutive integers is 1, where $n > 1$, which of the following must be true?

I. n is an even number
II. n is an odd number
III. The average (arithmetic mean) of the n integers is 1

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III

10. If x is to be chosen at random from the set {1, 2, 3} and y is to be chosen at random from the set {4, 5, 6, 7}, what is the probability that xy will be odd?

(A) $\frac{1}{6}$
(B) $\frac{1}{3}$
(C) $\frac{1}{2}$
(D) $\frac{2}{3}$
(E) $\frac{5}{6}$

11. S is a set containing 8 different positive odd numbers. T is a set containing 7 different numbers, all of which are members of S. Which of the following statements CANNOT be true?

(A) The range of T is even.
(B) The mean of S is even.
(C) The mean of T is even.
(D) The range of S is equal to the range of T.
(E) The median of T is equal to the mean of T.

12. If $n = 2p$, where p is a prime number greater than 2, how many different positive even divisors does n have, including n ?

(A) One
(B) Two
(C) Three
(D) Four
(E) Six

13. If n is the average (arithmetic mean) of the first 10 positive multiples of 4 and if N is the median of the first 10 positive multiples of 4, what is the value of $N - n$?

(A) -4
(B) 0
(C) 4
(D) 20
(E) 22

14. If the positive integer p is odd and the positive integer q is prime, then pq CANNOT be a multiple of which of the following?

I. 8
II. 18
III. 38

(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

15. How many positive integers less than 30 are either a multiple of 2, an odd prime number, of the sum of a positive multiple of 2 and an odd prime?

(A) 29
(B) 28
(C) 27
(D) 25
(E) 23

16. Which of the following is NOT equal to the cube of an integer?

(A) $2^7 - 3$
(B) $6\sqrt{16} + 3$
(C) $3^3 \times 3$
(D) $\sqrt{25} + 3$
(E) $2^2 - 3$

17. If k is a positive integer, and if the units' digit of k^2 is 4 and the units' digit of $(k+1)^2$ is 1, what is the units' digit of $(k+2)^2$?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

18. For any integer m greater than 1, $\$m$ denotes the product of all the integers from 1 to m, inclusive. How many prime numbers are there between $\$7 + 2$ and $\$7 + 10$, inclusive?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

19. When A is divided by D, the quotient is 11 and the remainder is C. Which of the following expressions is equal to A ?

(A) $11D$
(B) $11 + C$
(C) $11D + C$
(D) $11(D + 1)$
(E) $11(D + C)$

20. For how many integers k is $k^2 = 2^k$?
   (A) None
   (B) One
   (C) Two
   (D) Three
   (E) More than three

21. What is the least positive integer that is divisible by each of the non-prime integers between 1 and 10, inclusive?
   (A) 120
   (B) 360
   (C) 864
   (D) 4,320
   (E) 17,280

22. The positive integer n is divisible by 16. If $\sqrt{n}$ is greater than 16, which of the following could be the value of $\frac{n}{16}$?
   (A) 13
   (B) 14
   (C) 15
   (D) 16
   (E) 17

23. The positive integer q is divisible by 15. If the product of q and the positive integer x is divisible by 20, which of the following must be a factor of x^2q ?
   (A) 25
   (B) 60
   (C) 75
   (D) 150
   (E) 300

24. If x is equal to the sum of the integers from 30 to 50, inclusive, and y is the number of even integers from 30 to 50, inclusive, what is the value of $x + y$?
   (A) 810
   (B) 811
   (C) 830
   (D) 850
   (E) 851

25. If s is the sum of consecutive even integers w, x, y, and z, where $w < x < y < z$, all of the following must be true EXCEPT

(A) $z - w = 3(y - x)$
(B) s is divisible by 8
(C) The average of w, x, y, and z is odd
(D) s is divisible by 4
(E) $w + x + 8 = y + z$

26. If x and y are integers and $x^2 + y^2$ is odd, which of the following must be even?

(A) x
(B) y
(C) $x + y$
(D) $xy + y$
(E) xy

27. If k, m and n are integers and $km - kn$ is odd, which of the following must be odd?

(A) k
(B) m
(C) n
(D) km
(E) kn

28. If n is a positive integer and n^2 is divisible by 96, then the largest positive integer that must divide n is

(A) 6
(B) 12
(C) 24
(D) 36
(E) 48

29. $\frac{(4^2)(3^2)(2^3)}{48^2}$

(A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) $\frac{1}{6}$
(D) $\frac{1}{9}$
(E) $\frac{1}{18}$

30. If two fair six-sided dice are thrown, what is the probability that the sum of the numbers showing on the dice is a multiple of 3 ?

(A) $\frac{1}{4}$
(B) $\frac{3}{11}$
(C) $\frac{5}{18}$
(D) $\frac{1}{3}$
(E) $\frac{4}{11}$

31. Exactly 36% of the numbers in set S are even multiples of 3. If 40% of the even integers in set S are not multiples of 3, what percent of the numbers in set S are not even integers?

(A) 76%
(B) 60%
(C) 50%
(D) 40%
(E) 24%

32. The average (arithmetic mean) of the integers from 100 to 200, inclusive, is how much greater than the average of the integers from 20 to 100, inclusive?

(A) 50
(B) 60
(C) 90
(D) 100
(E) 150

33. S is a set containing 9 different positive odd primes. T is a set containing 8 different numbers, all of which are members of S. Which of the following statements CANNOT be true?

(A) The median of S is prime.
(B) The median of T is prime
(C) The median of S is equal to the median of T.
(D) The sum of the terms in S is prime.
(E) The sum of the terms in T is prime.

34. Of the three-digit integers greater than 600, how many have two digits that are equal to each other and the remaining digit different from the other two?

(A) 120
(B) 116
(C) 108
(D) 107
(E) 72

A = {2, 3, 5, 7}
B = {3, 4, 6, 8, 12}

35. Two integers will be randomly selected from the sets above, one integer from set A and one integer from set B. What is the probability that the sum of the two integers will be odd?

(A) 0.20
(B) 0.50
(C) 0.65
(D) 0.75
(E) 0.80

36. If $n = 2pq$, where p and q are distinct prime numbers greater than 2, how many different positive even divisors does n have, including n ?

(A) Two
(B) Three
(C) Four
(D) Six
(E) Eight

37. How many two-digit integers do not have any digits that are equal to each other?

(A) 90
(B) 81
(C) 80
(D) 75
(E) 72

38. If the sum of 5 consecutive integers is x, which of the following must be true?

I. x is an even number
II. x is an odd number
III. x is a multiple of 5

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III

39. For how many integers k is $k^3 = 3^k$?

(A) None
(B) One
(C) Two
(D) Three
(E) More than three

40. The product of the first seven positive multiples of three is closest to which of the following powers of 10?

(A) 10^9
(B) 10^8
(C) 10^7
(D) 10^6
(E) 10^5

41. A pair of prime numbers that can be expressed in the form $\{p, (p+6)\}$ is defined as a pair of "sexy primes." A "sexy triplet" is a group of three primes that can be expressed in the form $\{p, (p+6), (p+12)\}$. All of the following prime numbers are the middle term of a sexy triplet EXCEPT

(A) 11
(B) 13
(C) 17
(D) 19
(E) 23

42. How many positive integers less than 20 can be expressed as the sum of a positive multiple of 2 and a positive multiple of 3?

(A) 14
(B) 13
(C) 12
(D) 11
(E) 10

43. If the positive integer x is a multiple of 3 and the positive integer y is a multiple of 4, then xy must be a multiple of which of the following?

I. 6
II. 9
III. 12

(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

44. If set S consists of the first 10 positive multiples of 5 and set T consists of the next 10 positive multiples of 5, what is the positive difference between the average (arithmetic mean) of S and the median of T ?

(A) 5
(B) 25
(C) 47.5
(D) 50
(E) 52.5

45. How many different positive integers are factors of 294 ?

(A) 6
(B) 8
(C) 9
(D) 11
(E) 12

46. For which of the following values of x is $\frac{x+50}{2x}$ an integer?

(A) 25
(B) 50
(C) 100
(D) 250
(E) 500

47. If p is a positive integer and $\frac{10p}{96}$ is an integer, then the minimum number of prime factors p could have is

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

48. The average (arithmetic mean) of the even integers from 100 to 1000, inclusive, is how much greater than the average of the even integers from 10 to 100, inclusive?

(A) 495
(B) 500
(C) 545
(D) 550
(E) 900

4 Data Sufficiency

For all Data Sufficiency questions, the answer choices are as follows:

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
(C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient.
(E) Statements (1) and (2) TOGETHER are NOT sufficient.

49. If p is an integer, is q an integer?
(1) The average (arithmetic mean) of p, q, and r is p.
(2) r is an integer.

50. The number N is $5,32H$, the unit's digit being represented by H. What is the value of H?
(1) N is divisible by 3.
(2) N is divisible by 5.

51. If a, b, and c are three integers, are they consecutive integers?
(1) $b = 2c$
(2) $a = b - 1$

52. If p and x are integers, is x divisible by 5 ?
(1) p and x are consecutive integers.
(2) $p - 4$ is divisible by 5.

53. If $x = 0.abcd$, where a, b, c, and d each represent a nonzero digit of x, what is the value of x ?
(1) $\frac{a}{2} = b = 2c = 4d$
(2) The product of a and d is equal to the product of b and c.

54. If k is a nonzero integer, is $\frac{47-k}{k}$ an integer?
(1) k is not prime.
(2) $k < 4$

55. If t is an integer, is t odd?
(1) $\frac{t}{3}$ is not an integer.
(2) $\frac{t-1}{3}$ is not an integer.

56. Is s an odd integer?
    (1) $\sqrt{s}$ is not an even integer.
    (2) s^2 is not an even integer.

57. Can the positive integer k be expressed as the product of two integers, each of which is greater than 1?
    (1) k^2 has one more positive factor than k.
    (2) $11 < k < 19$

58. If n is an integer, then n is divisible by how many positive integers?
    (1) n and 3^4 are divisible by the same number of positive numbers.
    (2) n is the product of 2 and an odd prime number.

59. If y is an integer, is y^2 divisible by 15?
    (1) $\sqrt{y}$ is divisible by 3.
    (2) $\sqrt{y}$ is divisible by 5.

60. An infinite sequence of positive integers is called a "coprime sequence" if no term in the sequence shares a common divisor (except 1) with any other term in the sequence. If S is an infinite sequence of distinct positive integers, is S a coprime sequence?
    (1) An infinite number of integers in S are prime.
    (2) Each term in S has exactly two factors.

61. Is the positive integer n the sum of the positive prime numbers x and y?
    (1) $2 < x < y$
    (2) $n = 20$

62. If n is an integer, is $n - 2$ a prime number?
    (1) $n - 1$ is a prime number.
    (2) n is odd.

63. If n is an integer, is $\sqrt{n}$ a prime number?
    (1) $n - 2$ is a prime number.
    (2) n^2 is not a prime number.

64. If x is an integer between 2 and 100 and if $\sqrt{x}$ is also an integer, what is the value of x?
    (1) $\sqrt[3]{x}$ is an integer.
    (2) $2(\sqrt[3]{x}) = \sqrt{x}$

65. Is the positive integer p a multiple of 24?
 (1) p is a multiple of 2.
 (2) p is a multiple of 12.

66. Is the positive integer p a multiple of positive integer q?
 (1) When p is divided by q, the result is an integer.
 (2) When q is divided by p, the result is not an integer.

67. The number N is $5,7G6$, the ten's digit being represented by G. What is the value of G?
 (1) N is divisible by 4.
 (2) N is divisible by 9.

68. If p and q are integers, is $p+q$ divisible by 3?
 (1) $p-q$ is divisible by 3.
 (2) pq is divisible by 3.

69. If set S consists of the positive integers w, x, y, and z, is the range of the numbers in S greater than 6 ?
 (1) No two numbers in set S are consecutive.
 (2) None of the numbers in set S are multiples of 3.

70. If r and s are positive integers and $rs = 20$, what is the value of r ?
 (1) r and s are consecutive integers such that $r > s$.
 (2) r is prime.

71. How many integers n are there such than $v < n < w$?
 (1) $w - v = 6$
 (2) v and w are not integers.

72. If $x \neq 0$, what is the value of $\left(\frac{x^p}{x^q}\right)^2$?
 (1) p is a multiple of q.
 (2) q is a multiple of p.

73. If r and s are consecutive even integers, is r greater than s ?
 (1) $r+2$ and $s-2$ are consecutive even integers.
 (2) $r - 1 \neq s + 1$

74. If m and z are integers, is z divisible by 11 ?
 (1) m is a two-digit number with equal tens and units digits.
 (2) $m - z$ is divisible by 11.

75. Is the positive square root of x an integer?
    (1) x has exactly three factors.
    (2) $x < 16$

76. For a certain set of n numbers, where $n > 2$, is the average (arithmetic mean) equal to the median?
    (1) The range of the n numbers in the set is 14.
    (2) If the n numbers in the set are arranged in decreasing order, then the difference between any pair of successive numbers in the set is 2.

77. If k is a positive integer, is $\sqrt{k}$ an integer?
    (1) $k = m^2$, where m is an integer.
    (2) $\sqrt{k} = n$, where n^2 is an integer.

78. If p is an integer, is q an integer?
    (1) $\frac{q}{p}$ is an integer.
    (2) $\frac{p}{q}$ is an integer.

79. In the fraction $\frac{x}{y}$, where x and y are positive integers, what is the value of y ?
    (1) x is an even multiple of y.
    (2) $x - y = 2$

80. If x and y are integers, is $x^y < y^x$?
    (1) $x^y = 16$
    (2) x and y are consecutive even integers.

81. If $x = 0.jkmn$, where j, k, m, and n each represent a nonzero digit of x, what is the value of x ?
    (1) j, k, m, and n are prime numbers.
    (2) $j < m < k < n$

82. If z is a positive integer, is $\sqrt{z}$ an integer?
    (1) $\sqrt{9z}$ is an integer.
    (2) $\sqrt{8z}$ is not an integer.

83. If x and y are positive, is $\frac{x}{y}$ greater than 1 ?
    (1) x is not a factor of y.
    (2) y is not a factor of x.

84. Is the prime number p equal to 31 ?
   (1) p is equal to one less than twice the value of the square of an integer.
   (2) p is equal to four less than a multiple of five.

85. Is the prime number x equal to 7 ?
   (1) 343 is a multiple of x.
   (2) 350 is a multiple of x.

86. If p is an integer, is p odd?
   (1) $\frac{p}{2}$ is not an odd integer.
   (2) $p - 2$ is an odd integer.

87. If $w + z = 36$, what is the value of $\frac{z}{w}$?
   (1) $wz = 180$.
   (2) w and z are both multiples of 6.

88. If p is an integer, then p is divisible by how many positive integers?
   (1) The only prime factors of p are 2, 3, and 5.
   (2) $p < 50$

89. What is the value of the integer z ?
   (1) z has exactly one prime factor.
   (2) $18 < z < 32$, and z has exactly three factors.

90. What is the value of the integer z ?
   (1) z is the largest prime factor of 323.
   (2) z is a prime factor of 133.

91. An infinite sequence of positive integers is called a "perfect sequence" if each term in the sequence is a perfect number, that is, if each term can be expressed as the sum of its divisors, excluding itself. For example, 6 is a perfect number, as its divisors, 1, 2, and 3, sum to 6. Is the infinite sequence S a perfect sequence?
   (1) Exactly one term in S is a prime number.
   (2) In sequence S, each term after the first in S has exactly 3 divisors.

92. What is the value of the two-digit integer p ?
   (1) p is divisible by 9.
   (2) The sum of the two digits is 9.

93. Is y an integer?
    (1) When y is divided by two, the result is the square of an integer.
    (2) When y is multiplied by two, the result is the square of an integer.

94. Is z an integer?
    (1) $2z$ is not an integer.
    (2) $\frac{z}{2}$ is not an integer.

95. If the units digit of integer n is greater than 2, what is the units digit of n ?
    (1) The units digit of n is greater than the units digit of n^2
    (2) The units digit of n is greater than the units digit of n^3

96. What is the value of the integer r ?
    (1) The only prime factors of r are 3 and 7.
    (2) Each of the integers 3, 7, and 21 are factors of r.

97. If x, y, and z are three integers, are they consecutive integers?
    (1) x, y, and z are prime numbers.
    (2) x, y, and z are odd.

98. Can the positive integer n be expressed as the product of two integers, each of which is greater than 1?
    (1) n is a multiple of 3.
    (2) n is a multiple of 4.

99. If y is an integer, is y^3 divisible by 8?
    (1) y is even.
    (2) $y^3 - y$ is even.

100. If z is an integer between 2 and 100 and if z is also the cube of an integer, what is the value of z?
    (1) $\frac{z}{4}$ is not an integer.
    (2) $\frac{z}{6}$ is not an integer.

5 Answer Key

For full explanations, see the next section.

1. E
2. E
3. C
4. D
5. A
6. E
7. D
8. B
9. A
10. B
11. C
12. B
13. B
14. A
15. B
16. C
17. A
18. A
19. C
20. C
21. B
22. E
23. B
24. E
25. B
26. E
27. A
28. C
29. A
30. D
31. D
32. C
33. E
34. D
35. C
36. C
37. B
38. C
39. B
40. C
41. D

42.	A
43.	D
44.	D
45.	E
46.	B
47.	B
48.	A
49.	C
50.	C
51.	E
52.	E
53.	A
54.	E
55.	E
56.	E
57.	A
58.	D
59.	C
60.	B
61.	E
62.	C
63.	E
64.	D
65.	E
66.	A
67.	C
68.	C
69.	C
70.	A
71.	C
72.	C
73.	D
74.	C
75.	A
76.	B
77.	A
78.	A
79.	C
80.	A
81.	C
82.	A
83.	E
84.	E
85.	A
86.	B
87.	E

88. C
89. B
90. A
91. D
92. E
93. A
94. A
95. C
96. E
97. D
98. B
99. A
100. A

6 Explanations

Each explanation includes reference to between 1 and 3 categories, as well as the difficulty level of each question.

1. E

Explanation: First, simplify the question. Another way of writing $\frac{n+78}{n}$ is $\frac{n}{n} + \frac{78}{n}$, or $1 + \frac{78}{n}$. Because 1 plus any integer is still an integer, the question is really whether $\frac{78}{n}$ is an integer. For $\frac{78}{n}$ to be an integer, n must be a factor of 78. It may take a few moments worth of arithmetic to test all the choices, but only one is a factor of 78: 13, choice (E).

2. E

Explanation: Probability is expressed by (number of desired outcomes) divided by (number of possible outcomes). In this case, the number of desired outcomes is the number of integers between 101 and 300 with a units digit of 6. The number of possible outcomes is the total number of integers in that set: 200. If the series of integers is written consecutively, every 10th number has a units digit of 6. Since there are 20 series of 10 integers between 101 and 300, inclusive, there must be 20 integers with a units digit of 6. Thus, the probability is $\frac{20}{200} = \frac{1}{10}$, choice (E).

3. C

Explanation: Start by breaking down each term to its prime factors:

$$\frac{\left[(2^3)(3)\right]^3}{(2^2)(3^2)(2^4)(2^2)}$$

Expand exponents and combine terms:

$$\frac{(2^9)(3^3)}{(2^8)(3^2)}$$

Cancel out all possible terms, and the result is $2 \times 3 = 6$, choice (C).

4. D

Explanation: If p is an integer, p^2 must not only be a multiple of 12, but a perfect square. The smallest perfect square that is a multiple of 12 is 36, so p^2 must be 36 or a multiple thereof. Thus, p must be 6 or a multiple of 6. It follows that p^3 must be at least 6^3, choice (D).

5. A

Explanation: A handy way to find the least common multiple of a series of numbers is to start with the largest number in the series, then multiply by other numbers in the series as necessary. In this case, start with 6. In order to be divisible by 6, of course, the number has to be 6 or a multiple thereof. Next comes 5: 6 isn't a multiple of 5, so multiply by 5, resulting in 30. Next, 4: 30 isn't a multiple of 4, but it is a multiple of 2. Multiply 30 by 2, and your result—60—is a multiple of 4. Next is 3: 60 is already a multiple of 3. Next, 2: 60 is already a multiple of 2. Finally, 1: 60, like any other integer, is a multiple of 1. 60, choice (A), is the smallest number that is divisible by each of the integers 1 through 6, inclusive.

6. E

Explanation: The trickiest part of this question is understanding the definition of a Mersenne prime. Once you figure out what a Mersenne prime is, it's just a matter of plugging in prime numbers for n and seeing which of the answer choices can be eliminated.

If $n = 2$, $2^n - 1 = 3$, so (A) isn't correct. If $n = 3$, $2^n - 1 = 7$, eliminating (B). If $n = 5$, $2^n - 1 = 31$, so (C) is wrong. If $n = 7$, $2^n - 1 = 127$, eliminating (D). That leaves only (E) which, for the record, is $2^n - 1$ where $n = 9$. It isn't necessary to know that $2^9 = 512$, but it will certainly come in handy to know powers of 2 up to 2^7 (128) or 2^8 (256).

7. D

Explanation: To find the number of factors of a number, start with 1 and work your way up. The first factor of 225 (or any other integer) is 1. When you find each factor, find the number that it can be multiplied by to return the integer—for example, in the case of 1, it pairs with 225. 2 is not a factor of 225, but 3 is. $\frac{225}{3} = 75$, so 75 is another factor. 4 is not a factor, but 5 is. $\frac{225}{5} = 45$, so 45 is also a factor. 9 is a factor, and $\frac{225}{9} = 25$. The only remaining factor is 15, which is the square root of 225, so there's no other distinct factor that pairs with 15. At this point, your scratchwork should show 9 factors—1, 225, 3, 75, 5, 45, 9, 25, and 15—making choice (D) correct.

8. B

Explanation: In order for $\frac{5n}{42}$ to be an integer, n must be a multiple of 42. (If 5 were divisible by 42, the expression could be simplified first, but since it isn't, we can ignore that step.) The question, then, boils down to how many different positive prime factors 42 has. This is a good time for a factor tree: $42 = 2(21) = 2(3)(7)$. Thus, n has three different prime factors, choice (B).

9. A

Explanation: First, try to figure out what type of set of consecutive integers could sum to 1. The most obvious group of integers is {0, 1}. No others are apparent: tacking on a number to the beginning or end of the list makes the sum too large or small; adding a number to the top and bottom, as in {-1, 0, 1, 2}, also makes the sum too small. It would appear that {0, 1} is the only possibility.

In that case, I is true, because $n = 2$. II, by the same token, is false. III is also false, as the average of {0, 1} is $\frac{1}{2}$, not 1. Thus, choice (A) is correct.

10. B

Explanation: Probability is given by (number of desired outcomes) divided by (number of possible outcomes). In this case, possible outcomes is the $3 \times 4 = 12$: because any number can be chosen from each of the two sets, the number of possibilities is the number of two-value sets. The only way for xy to be odd is if both x and y are odd. There are 2 possible odd values of x and 2 possible odd values of y, so the number of desired outcomes is $2 \times 2 = 4$. Finally, the probability is $\frac{4}{12} = \frac{1}{3}$, choice (B).

11. C

Explanation: Consider each answer choice in turn. (A) can be true: the largest and smallest numbers in T are each odd numbers (all the numbers are odd, so this must be the case), and an odd number minus another odd number is even, so (A) must be true. (B) can be true: the sum of the eight integers must be even (this is always the case with an even number of odd numbers), so it seems likely that if you divide that sum by eight, there must be a way to get an even result. (An example would be any eight consecutive odds, such as {1, 3, 5, 7, 9, 11, 13, 15}.) (C), however, cannot be true. The sum of seven odd numbers will always be odd, and an odd number divided by seven will never by even. Thus, the answer is (C).

12. B

Explanation: The question suggests that the answer will be the same regardless of the value of p, so we can choose an appropriate value for p. If $p = 3$, $n = 6$. n's divisors (factors) are 1, 2, 3, 6, two of which are even, choice (B).

13. B

Explanation: In any set of integers that are equally spaced (consecutive integers, consecutive evens or odds, or consecutive multiples, like in this question) the median and mean are equal. Thus, N and n are equal, so $N - n = 0$, choice (B).

14. A

Explanation: Because p is odd, p cannot be a multiple of 2. Because q is prime, q could be 2, but no other even number. Thus, pq can be a multiple of 2, but not a multiple of any power of 2, such as 4, 8, or 16. That means that I cannot be true. II could be true: if $p = 9$ and $q = 2$, $pq = 18$, a multiple of 18. III could also be true: if $p = 19$ and $q = 2$, $pq = 38$, a multiple of 38. Thus, the answer must be (A), I only.

15. B

Explanation: Work by process of elimination. There are 29 positive integers less than 30, and 14 of them are even (multiples of 2), leaving you with 15. Of those 15, 9 of them (3, 5, 7, 11, 13, 17, 19, 23, 29) are odd primes, leaving you with 6 numbers: 1, 9, 15, 21, 25, and 27. Of these, any number greater than one can be expressed as the sum of an even number and an odd prime: $9 = 7 + 2$, $15 = 13 + 2$, $21 = 19 + 2$, $25 = 23 + 2$, and $27 = 23 + 4$. That means that 28 numbers—each one of the original 29 except for 1—fits one of the categories given, making the correct choice (B).

16. C

Explanation: Evaluate each choice. (A) $= 128 - 3 = 125 = 5^3$. (B) $= 6(4) + 3 = 27 = 3^3$. (C) $= 27 \times 3 = 81$. Not a cube. (D) $= 5 + 3 = 2^3$. (E) $= 4 - 3 = 1 = 1^3$. (C) is the correct choice.

17. A

Explanation: If the units' digit of $k^2 = 4$, then k could have a units' digit of 2 or 8. If the units' digit of $(k+1)^2 = 1$, $(k+1)$ could have a units' digit of 1 or 9. Since $(k+1)$ must be 1 greater than k, the units' digit of k must be 8. Thus, the units' digit of $(k+2) = 0$, so $(k+2)^2$ has a units' digit of 0, choice (A).

18. A

Explanation: The notation $\$m$ is the same as factorial ($m!$) notation. Thus, $\$7 + 2 = (1)(2)(3)(4)(5)(6)(7) + 2$. The key to this question is recognizing that $\$7 + 2$ must be a multiple of 2, expressible like this: $2((3 \times 4 \times 5 \times 6 \times 7) + 1)$. In the case of $\$7 + 2$, you can factor out a 2; the same applies when adding 3, 4, 5, 6, or 7. If you can express an integer as the product of two integers (as in the case of $2((3 \times 4 \times 5 \times 6 \times 7) + 1)$), that integer is not prime: it has at least one factor aside from 1 and itself. (In that case, that's 2.)

When moving on to the range between $\$7 + 8$ and $\$7 + 10$, it gets a little trickier. When adding 8, 9, 10, the same method can be applied. In the case of $\$7 + 10$, you can factor out a two as follows: $(2)(3 \times 4 \times 5 \times 6 \times 7 + 1)$, so the number is a multiple of 2, so not a prime. The same process can be used with 8 or 9. Thus, none of the numbers in the range given are prime, choice (A).

19. C

Explanation: Another way of thinking about the question is as follows: D goes into A eleven times, with C left over. If there were no remainder, A would be equivalent to $11D$. As is, we have to account for the remainder, so $A = 11D + C$.

20. C

Explanation: The best way to think about this is to consider groups of integers. k could not be negative: if it were, k^2 would be positive integer and 2^k would be a positive fraction. If $k = 0$, $k^2 = 0$ and $2^k = 1$. If $k = 1$, $k^2 = 1$ and $2^k = 2$. If $k = 2$, $k^2 = 4$ and $2^k = 4$, so there's at least one possibility. It is also the case that $4^2 = 2^4$. There are no others, so the only possibilities are $k = 2$ and $k = 4$, so choice (C) is correct.

21. B

Explanation: First, list the numbers included in the set: 1, 4, 6, 8, 9, and 10. Starting from the largest numbers: in order for an integer to be divisible by 9 and 10, it must be 90 or a multiple thereof. 90 isn't divisible by 8, but it is divisible by 2: in other words, in order to be divisible by 8, 90 needs to be multiplied by 4, resulting in 360. 360 is a multiple of 6, 4, and 1, so 360, choice (B), is the least integer that is divisible by all of the numbers in our set.

22. E

Explanation: If $\sqrt{n} > 16$, then $n > 16^2$. (We can only square both sides of the inequality because we know both are positive.) If $n > 16^2$, then $\frac{n}{16} > \frac{16^2}{16}$, or $n > 16$. If n must be larger than 16, the only possible answer is (E).

23. B

Explanation: If q is divisible by 15, it must have factors of 3 and 5 (and perhaps others, but we have no way of knowing). If xq is divisible by 20, either x or q must be divisible by 4, in addition to q being divisible by 5 (which we already knew). It's possible than $x = 4$ and $q = 15$, or $x = 1$ and $q = 60$; either one fulfills the requirements of the question. Thus, we don't know much about x. When we look for factors of x^2q, it's better to think of that as $x(xq)$. We know little about x, other than the fact it's an integer, but we know that xq must have factors of 3, 4, and 5. Thus, xq must be at least 60, so no matter what x is, x^2q is a multiple of 60, making (B) the correct answer.

24. E

Explanation: Rather than simply adding up the integers from 30 to 50, look for a shortcut. Think of that sum as 10 groups of 80: $30 + 50$, $31 + 49$ ... to $39 + 41$, leaving only 40. Thus, the sum is $10(80) + 40 = 840$. The number of evens from 30 to 50, inclusive, is 11, so $x + y = 851$.

25. B

Explanation: Because there's not a pattern among the answer choices, you'll have to evaluate each on independently. (A) must be true: if the four numbers are consecutive evens in the order given, $z - w = 6$, and $y - x = 2$. (B) is the correct choice: for no set of values for the four variables will the sum be a multiple of 8. The most obvious example is $2 + 4 + 6 + 8 = 20$. (C) must be true: the average will always be the mean of the middle two numbers, and since those two numbers are even numbers separated by two, the mean will be the odd number in between them. (D) must be true: we've already seen that s could be 20; if you add 2 to each number, $s = 28$; if you continue, s increases by multiples of 8, to $s = 36$ then $s = 44$. In each case, s is divisible by 4. (E) must be true: $w = y - 4$ and $x = z - 4$, so the equation could be written as follows: $(y - 4) + (z - 4) + 8 = y + z$, or $y + z = y + z$.

26. E

Explanation: If a perfect square is even, its square root will also be even. The same is true for odds. If $x^2 + y^2$ is odd, either x^2 or y^2 (but not both) must be odd. The same, then, holds true for x and y. One of the two variables must be odd, the other even. That eliminates (A) and (B). (C) is also wrong. $x + y$ must be odd: $odd + even = odd$. We can also eliminate (D): xy must be even

(*even* $\times$ *odd* = *even*), but we don't know whether y is even or odd. (E), then, must be correct. xy (*even* $\times$ *odd* or *odd* $\times$ *even*) must be even.

27. A
Explanation: If $km - kn$ is odd, one of the terms must be even, and the other odd. The only way for one of them to be odd is for both variables in the term to be odd. Thus, k must be odd. One of m and n must be odd, the other even, but it's not possible from the information given to know which one. Thus, k must be odd, choice (A).

28. C
Explanation: The prime factorization of 96 is $(2^5)(3)$, so n^2 must share those prime factors. However, that can't be all: 96 isn't a perfect square, so if n is an integer and n^2 is a perfect square, n^2 must be bigger. The way to determine the smallest possible value of n^2 is to take the prime factorization of 96 and increase the number of each prime factor included so that all of the powers are even. In other words, 2^5 isn't a perfect square, but 2^6 is. 3 isn't a perfect square, but 3^2 is. So, the smallest possible value of n^2 is $96(6) = (2^6)(3^2)$. If that's n^2, n must be no smaller than $(2^3)(3) = 24$, choice (C).

29. A
Explanation: Prime factorizations will make simplifying this fraction much easier. Start by reducing each term to its prime factors:
$\frac{(2^4)(3^2)(2^3)}{[(2^4)(3)]^2}$
Combine terms and distribute exponents:
$\frac{(2^7)(3^2)}{(2^8)(3^2)}$
Cancel out all possible terms, and the result is $\frac{1}{2}$, choice (A).

30. D
Explanation: There are 4 different multiples of 3 that can result from the sum of two six-sided dice: 3, 6, 9, and 12. You'll need to find the probability of each one of those coming up. There are a total of 6×6 possible results; how many result in a multiple of 3? There are two ways to get a sum of 3: if the first die comes up 1 and the other 2, or if the first comes up 2 and the other one. Do the same with the other multiples of 3:
6: 1, 5; 2, 4; 3, 3; 4, 2; 5, 1 (5 total)
9: 3, 6; 4, 5; 5, 4; 6, 3 (4 total)
12: 6, 6 (1 total)
There are $2 + 5 + 4 + 1 = 12$ different ways for the two dice to sum to a multiple of three, so the probability is $\frac{12}{36} = \frac{1}{3}$.

31. D
Explanation: If 40% of the even integers in set S are not multiples of 3, 60% of the even integers in set S are multiples of 3. Put another way, 60% of the even integers (even multiples of 3) in set S represent 36% of the total numbers

in the set. Converting that to numbers we can more easily work with: if there are 100 total numbers in the set, 36 of them are even multiples of 3. 36 is 60% of the even integers; 36 = 60% of 60, so there are 60 even integers. That means that 60% of the numbers in the set are even integers; thus, 40% are not, choice (D).

32. C

Explanation: When dealing with a series of consecutive integers, the mean is equal to the median. Since the middle number of the series from 100 to 200, inclusive, is 150, that's the mean. By the same reasoning, the mean of integers from 20 to 100 is 60. The difference, then, is $150 - 60 = 90$, choice (C).

33. E

Explanation: Examine each answer in turn. (A) must be true: the median of a set that contains an odd number of terms must be one of those terms. So if each term is a prime number, the median must be a prime number.

(B) can be true: the median is not one of the terms (since there are an even number of terms), but if the middle terms are, for instance, 11 and 23, the median would be 17, a prime number.

(C) could be true: using the example from (B), if the one number if S that isn't in T was 17, the medians would be the same. The sets, then, could be as follows. $T = \{3, 5, 7, 11, 23, 29, 31, 37\}$ and S = $\{3, 5, 7, 11, 17, 23, 29, 31, 37\}$.

(D) could be true: the sum of 9 odd primes must be odd, and given the wide range of possible primes, it seems likely that there is a possible prime result. As it turns out, the sum of the first 9 odd primes is 113, which is prime.

(E) cannot be true. The sum of eight odds, whether they are prime or not, is even, and an even number (bigger than two, as this sum must be) cannot be prime. (E) is the correct choice.

34. D

Explanation: To get us started, look at a set of 100 numbers. As an example, take 700 to 799. There are three ways that a three-digit number can have two digits equal to each other: the first two could be equal, the first and third could be equal, or the second and third could be equal. For the series from 700 to 799, let's look at each one of those.

If the first two are equal, that means the first two digits must be 7 and 7. There are ten possible third digits, but only nine of them count toward our total because 777 violates the rule that the third digit must be different. That's nine possibilities. Next, if the first and third digits are equal, we're looking for numbers in the form 7x7—again, ten possibilities for x, but we can't count 777. There's another nine. Finally, the third form is 7xx; there are 10 possibilities for x, but we can't count 7, so there's 9 more. That's a total of 27 possibilities from 700 to 799.

There are four sets of one hundred that are relevant to this question: 600 to 699, 700 to 799, 800 to 899, and 900 to 999. Thus, the total is $27(4) = 108$.

However, we're looking for numbers greater than 600—not including 600. Since 600 fits the requirements, we've counted that number, so our answer is $108-1=107$, choice (D).

35. C

Explanation: Probability is given by (number of desired outcomes) / (number of possible outcomes). In this case, possible outcomes is the $4 \times 5 = 20$: because any number can be chosen from each of the two sets, the number of possibilities is the number of two-value sets. The number of desired outcomes is trickier. There are two ways for the sum of the two integers to be odd: one must be odd and one must be even, so in the first case, the number from A is even and the number from B is odd; in the other, the odd and even numbers are reversed.

The number of possibilities for the first case—selecting an even from A and an odd from B—is $1(1) = 1$, because there is only one even in A and only one odd in B.

36. C

Explanation: The question suggests that the answer will be the same regardless of which numbers are chosen for p and q, so assign values to those variables to make the question more accessible. If p and q are 3 and 5, respectively, $n = 2(3)(5) = 30$. 30's divisors are 1, 2, 3, 5, 6, 10, 15, and 30, 4 of which are even, choice (C).

37. B

Explanation: First, recognize that there are 90 two-digit numbers. There are 9 two-digit numbers in which the digits are equal: 11, 22...99. Thus, the number of two-digit integers that do not have equal digits is $90 - 9 = 81$, choice (B).

38. C

Explanation: Because the roman numerals are concerned with the evenness or oddness of the sum of the integers, it's important to consider that when thinking through the possible characteristics of x. An obvious example for the 5 consecutive integers is $\{1, 2, 3, 4, 5\}$, which sums to 15, an odd number. However, it sums to an odd number only because the first and last numbers is the set are odd; if you start with an even, as in $\{2, 3, 4, 5, 6\}$, the sum is 20, an even.

Because of those two examples, we can eliminate I and II, as we've devised a counterexample to each. III is a bit trickier. Cast the consecutive integers in terms of a variable, where n is the first number in the list: $\{n, n+1, n+2, n+3, n+4\}$. In that case, the sum is $5n + 10$. Because n must be an integer, $5n$ must be a multiple of 5. Since 10 is a multiple of 5, as well, $5n + 10$ must be a multiple of 5, making III true. You may have noticed that proving III is unimportant in this question: once I and II are disproved, the only possible answer choice is (C), III only. Of course, that's fastest: now you're prepared just in case you have to evaluate a roman numeral like III, as well.

39. B

Explanation: The best way to think about this is to consider groups of integers. k could not be negative: if it were, k^3 would be positive integer and 3^k would be a positive fraction. If $k = 0$, $k^3 = 0$ and $3^k = 1$. If $k = 1$, $k^3 = 1$ and $3^k = 3$. If $k = 2$, $k^3 = 8$ and $3^k = 9$. If $k = 3$, $k^3 = 27$ and $3^k = 27$, so there's at least one possibility. For any integer larger than 3, k^2 will be smaller than 2^k: for instance, if $k = 4$, $4^3 < 3^4$. Thus, the only possibility is $k = 3$, so choice (B) is correct.

40. C

Explanation: To find the product of $3 \times 6 \times 9 \times 12 \times 15 \times 18 \times 21$, it'll be much easier to find a way to estimate than to multiply all of those numbers. Since our end goal is a power of 10, look for groups of numbers within those seven that have a product near a power of ten. When the question uses the word "closest," remember that approximation will make the question go much faster. For instance, $6 \times 15 = 90$, which we can call 10^2. 9×12 is a little greater than 100, so call that 10^2. 18×21 is a little less than 400, so $18 \times 21 \times 3$ is a little less than 1200, or about 10^3. We've now included all seven numbers, so we need to multiple $(10^2)(10^2)(10^3) = 10^7$, choice (C).

41. D

Explanation: The most difficult aspect of this question is making sense of the definition. Once you grasp the notion that a sexy triplet is just a group of three prime numbers each separated by six, it's just a matter of going through the choices one by one and finding which one doesn't fit the definition.

If 11, choice (A), is the middle term, the other terms are 5, and 17, both primes, so (A) is wrong. If 13 is the middle term, 7 and 19 are the other terms. They are both primes, so (B) is incorrect. If 17 is the middle term, the other primes are 11 and 23, both primes, making (C) incorrect. If 19 is the middle term, 13 and 25 are the other terms: 25 is not prime, so (D) is the correct answer. No need to check (E); for the sake of completeness, though, the other terms in that triplet are 17 and 29, both primes.

42. A

Explanation: Positive multiples of 2 are even numbers; the relevant multiples of 3 are 3, 6, 9, 12, 15, and 18. No number smaller than 5 can be expressed as the sum of one and the other, as the smallest options are 2 and 3. Rather than going through every number between 5 and 19, look for patterns. There are 8 odd numbers between 5 and 19, inclusive, and each of them can be expressed as the sum of an even number and 3, so those 8 must be counted. The smallest even number that could be counted is 8 (2 + 6), and by the same reasoning, every even number between 8 and 18,inclusive, must be counted, adding 6 more to our total. That's $6 + 8 = 14$ total numbers, choice (A).

43. D

Explanation: If x is a multiple of 3 and y is a multiple of 4, xy must be at least 12 or some multiple of 12. Obviously III is correct; less obviously, if xy is a multiple of 12, it must also be a multiple of 6, numeral I. Depending on which multiple of 12 xy is, it could be a multiple of 9, but it might not be.

44. D

Explanation: When a set consists of consecutive integers, consecutive evens or odds, or consecutive multiples (as do the sets in this question), the median and mean are equal. It takes much less calculation to come up with the median, so let's use that as a shortcut. The first 10 positive multiples of 5 are {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}, so the middle numbers are 25 and 30, making the median 27.5. The next 10 are {55, 60, 65, 70, 75, 80, 85, 90, 95, 100}, so the middle numbers are 75 and 80, making the median 77.5. That's a difference of $77.5 - 27.5 = 50$. Alternatively, you can recognize that each number in T is exactly 50 greater than a number in S, so the difference in means must be 50, as well.

45. E

Explanation: To find the number of factors of a number, start with 1 and work your way up. The first factor of 294 is 1. Since $\frac{294}{1} = 294$, 294 is also a factor. (Of course: an integer is always a factor of itself.) Because 294 is even, 2 is a factor, which means $\frac{294}{2} = 147$ is, as well. The digits of 294 sum to 15 (a multiple of 3), so 3 is a multiple of 294. $\frac{294}{3} = 98$ is, as well. Because 2 and 3 are both factors, 6 is a factor, and $\frac{294}{6} = 49$ is as well. Because 49 is a factor, 7 is a factor, as is $\frac{294}{7} = 42$. The next and final factors are 14 and 21: both identifiable as factors because they are factors of 42. The complete list—1, 294, 2, 147, 3, 98, 6, 49, 7, 42, 14, and 21—numbers 12, choice (E).

46. B

Explanation: First, simplify the question. Another way of writing $\frac{x+50}{2x}$ is $\frac{x}{2x} + \frac{50}{2x}$, or $\frac{1}{2} + \frac{25}{x}$. In order for $\frac{1}{2} + \frac{25}{x}$ to be an integer, $\frac{25}{x}$ must be $\frac{1}{2}$, or some integer $+\frac{1}{2}$. (C), (D), and (E) aren't possible: $\frac{25}{100}$ is less than $\frac{1}{2}$, and the larger the denominator, the smaller the result will be. (A), $x = 25$, makes $\frac{25}{x} = 1$, which does not result in an integer. Thus, the only value that makes the expression return an integer is (B), $x = 50$.

47. B

Explanation: First, simplify $\frac{10p}{96}$: divide the top and bottom by 2, and the result is $\frac{5p}{48}$. In order for $\frac{5p}{48}$ to be an integer, p must be at least 48. Because we're looking for the minimum number of prime numbers p could have, we can focus on the smallest possible value of p. To find the number of prime factors, use a factor tree: $48 = (6)(8) = (2)(3)(2)(2)(2)$. Thus, p has a minimum of two different prime factors, choice (B).

48. A

Explanation: When dealing with a series of consecutive integers, the mean is equal to the median. Since the middle number of the series from 100 to 1000 is 550, that's the mean. By the same reasoning, the mean of the integers from 10 to 100 is 55. The difference, then, is $550 - 55 = 495$, choice (A).

49. C

Explanation: If the average of three numbers is one of the numbers, that means the numbers are equally spaced. Mathematically, if $\frac{p+q+r}{3} = p$:

$p + q + r = 3p$

$q + r = 2p$

$q - p = p - r$

In other words, the difference between p and q is equal to the difference between p and r. That doesn't tell you if q is an integer, because you don't know anything about r.

Statement (2) is also insufficient: simply knowing that r is an integer isn't enough to tell you anything about q.

Taken together, the statements are sufficient. If r is an integer, the difference between p and r is an integer: integer - integer = integer. Thus, the difference between p and q is an integer, and since p is an integer, q must be an integer as well. Choice (C) is correct.

50. C

Explanation: Statement (1) is insufficient: if a number is divisible by 3, the sum of the digits must be a multiple of 3. $5 + 3 + 2 = 10$, so if H is 2, 5, or 8, the sum of N's digits would be 12, 15, or 18, a multiple of 3. Statement (2) is also insufficient: a number is a multiple of 5 anytime the unit's digit is 0 or 5, so H could be 0 or 5.

Taken together, the statements are sufficient. The only value of H that satisfies both statements is 5, which makes the number a multiple of both 3 and 5.

51. E

Explanation: Statement (1) is insufficient: since we know nothing about a, we only know the relationship between two of the three variables. Statement (2) is similar: we know that a and b are consecutive, but nothing about c. Taken together, the statements are still insufficient. From (1), b and c could be 2 and 1, 4 and 2, or 8 and 4; in the second case, $a = 3$, making the answer "yes." In the other cases (as well as several that are not listed), the answer is "no," so the correct answer is (E).

52. E

Explanation: Statement (1) is insufficient: without knowing whether p or x is a multiple of 5 (or anything else about them), knowing that they are consecutive doesn't help. Statement (2) is also insufficient, as it says nothing about x.

Taken together, the statements are still insufficient. If $p - 4$ is divisible by 5, then $p + 1$ (which is 5 greater than $p - 4$) is also divisible by 5. If p and x

are consecutive, then x is either $p+1$ or $p-1$. If $x=p+1$, we know it is a multiple of 5; if $x=p-1$, we know that x is not a multiple of 5. The correct choice is (E).

53. A

Explanation: Statement (1) is sufficient because there is only one set of values for the four integers that fulfills the equations. Because a, b, c, and d must be between 1 and 9 (the question indicates that they are nonzero digits), it must be the case that $a=8$, $b=4$, $c=2$, and $d=1$.

Statement (2) is insufficient: a, b, c, and d could be 1, 2, 4, and 8 or 1, 2, 3, and 6.

54. E

Explanation: First, rephrase the question. $\frac{47-k}{k}$ can be written as $\frac{47}{k}-\frac{k}{k}$, or $\frac{47}{k}-1$. For $\frac{47}{k}-1$ to be an integer, $\frac{47}{k}$ must be an integer, and the only way for that to occur is if k is a positive or negative factor of 47. The question, then, can be cast as follows: Is k a factor of 47?

Statement (1) is insufficient: there are many non-prime values of k that make the answer "no," but if $k=1$, the answer is "yes." Statement (2) is also insufficient: if $k=1$, the answer is "yes," while if $k=2$, the answer is "no." Taken together, the statements are still insufficient. In both cases, k could be 1, in which case the answer is "yes." It's also possible that k is a negative number, such as -2, in which case k is not a factor, and the answer is "no." The correct choice is (E).

55. E

Explanation: Statement (1) is insufficient: if $\frac{t}{3}$ is not an integer, that means t is not a multiple of 3. Thus, t could be any non-multiple of 3, such as 1, 2, 4, or 5. In some of those cases, t is odd; in others, it's even.

Statement (2) is also insufficient: if $\frac{t-1}{3}$ is not an integer, that means $t-1$ is not a multiple of 3. Thus, $t-1$ could be any non-multiples of 3, such as 1, 2, 4, or 5, meaning that t could be any of the following: 2, 3, 5, or 6. Some of those are odd, some are even.

Taken together, the statements are insufficient. Looking at the lists of possibilities generated by each statement, there are a couple in common: 2 and 5. If t could be either 2 or 5, it could be odd (a "yes" answer) but it could also be even (a "no") answer. Choice (E) is correct.

56. E

Explanation: Statement (1) is insufficient. If rs is not an even integer, it could be an odd integer or a non-integer. If $\sqrt{s}=3$, $s=9$, but if $\sqrt{s}=\sqrt{2}$, $s=2$. Statement (2) is also insufficient: again, if s^2 is not even, it could be odd or a non-integer. If $s^2=81$, $s=9$, but if $s^2=3$, $s=\sqrt{3}$.

Taken together, the statements are still insufficient. As we've seen, s could be 9 (making the answer "yes") but it could also be a non-integer such as $\sqrt{3}$, in which case $\sqrt{s}=\sqrt{3}$ and $s^2=3$. The correct answer is (E).

57. A

Explanation: The only integers that can be expressed as the product of two integers, each greater than 1, are non-primes. In other words, the question is asking whether k is a non-prime. It would be equally valid to recast the question as "Is k prime?" because it doesn't matter whether the answer is "yes" or "no," simply whether the statements are sufficient.

Statement (1) is sufficient. The only types of numbers k such that k^2 has exactly one more positive factor than k are primes. Prime numbers have two factors and their squares have three. If k had more than two factors, the number of factors would increase by more than 1 when squared. Thus, k must be prime, answering the question.

Statement (2) is insufficient: k could be 13 or 17 (prime numbers) or 12, 14, 15, 16, or 18 (non-primes).

58. D

Explanation: Statement (1) is sufficient: if we wanted to spend the time, we could figure out just how many numbers 3^4 is divisible by. Once we did that, we'd know how many numbers n is divisible by. Statement (2) is also sufficient: any number n that is the product of 2 and a prime number has exactly 4 factors: 1, itself, 2, and the prime number.

59. C

Explanation: In order for y^2 to be divisible by 15, y must also be divisible by 15. Statement (1) indicates that y is divisible by 3; if the square root of y is divisible by 3, y must be divisible by 9, and is also divisible by 3. That knowledge might come in handy, but it isn't enough to answer the question. Statement (2) is similar: by the same reasoning, we know that y is divisible by 25 (and 5), also insufficient on its own.

Taken together, the statements are sufficient. If y is divisible by both 3 and 5, it must be divisible by 15, so the correct choice is (C).

60. B

Explanation: The most challenging part of this problem is understanding the nature of the question. Basically, we need to find out whether there are any two numbers in S that share a factor other than 1.

Statement (1) is insufficient: while the infinite number of primes in S do not (by definition) share any factors aside from 1, we don't know whether there are other (non-prime) integers in the sequence that might share factors with each other, or with the primes.

Statement (2) is sufficient: the question tells us that each term in the sequence is distinct, and (2) indicates that each term is prime. Only prime numbers have exactly two factors.

61. E

Explanation: Statement (1) is not sufficient, if only because we don't yet know anything about n. $x+y$ could be an integer, but without further knowledge the statement doesn't tell us much of anything. Statement (2) also isn't very helpful. 20 could be expressed as the sum of two primes (13 and 7 or 17 and 3) but there are many other options: 1 and 19, 2 and 18, and many others.

Taken together, the statements are still insufficient. x and y could be 7 and 13, respectively, in which case the answer is "yes"; x and y could also be 3 and 5, in which case the answer is "no."

62. C

Explanation: Statement (1) is insufficient. If $n = 4$, $n-1 = 3$ and $n-2 = 2$, a prime number. If $n = 6$, $n-1 = 5$ (a prime), but $n-2 = 4$, not a prime. Statement (2) is also insufficient: if $n = 5$, $n-2 = 3$, a prime number, but if $n = 11$, $n-2 = 9$, not a prime.

Taken together, the statements are sufficient. In order for $n-2$ to be prime when $n-1$ is prime, $n-1$ and $n-2$ must be consecutive primes: in other words, they must be the only two consecutive primes: 3 and 2, respectively, in which case $n = 4$. Since n is odd, n cannot equal 4, so that rules out the one value of n for which the answer is "yes." The correct choice is (C).

63. E

Explanation: Another way to think about the question is this: Is n the square of a prime? Keep in mind that $\sqrt{n}$ may not be an integer at all.

Statement (1) is insufficient: if $n = 9$, $n-2 = 7$, and the answer is "yes." If $n = 13$, $n-2 = 11$ and the answer is "no." Statement (2) is not only insufficient, it is useless: since we know n is an integer, it is a given that n^2 is not a prime. Because (2) is useless, it can add nothing to Statement (1), ensuring that the correct choice is (E).

64. D

Explanation: If $\sqrt{x}$ is an integer, that means that x is a perfect square. Statement (1) is sufficient. There is only one number between 2 and 100 that is both a square and a cube of an integer, and that number is 64. Statement (2) is also sufficient. To simplify, square each side: $[2(\sqrt[3]{x})]^2 = x$. Then cube each side: $64x^2 = x^3$. Divide by x^2, and $x = 64$. It's the only possible value for x, so the statement is sufficient.

65. E

Explanation: Statement (1) is insufficient: p could be any even number. If $p = 2$, the answer is "no," if $p = 48$, the answer is "yes." Statement (2) is also insufficient: if $p = 12$, the answer is "no," while if $p = 48$, the answer is "yes."

Taken together, the statements are insufficient. If p is a multiple of 12, we know that p must be even (a multiple of 2) so the statements together tell us nothing that Statement (2) didn't provide on its own.

66. A

Explanation: In order that p be a multiple of q, p divided by q must be an integer. That's the definition of "multiple," so when Statement (1) tells us the answer so directly, we know it's sufficient. The same is not the case for Statement (2): if $p = 2$ and $q = 3$, the result is not an integer, and p is not a multiple of q. If $p = 4$ and $q = 2$, the result is not an integer, and p is a multiple of q.

67. C

Explanation: Statement (1) is not sufficient: if a number is a multiple of 4, the last two digits make up a multiple of 4. Thus, G must be 1, 3, 5, 7, or 9. For instance, if $G = 1$, the last two digits of 5,716 are 16, which is a multiple of 4. Statement (2) is also insufficient. For a number to be a multiple of 9, the digits must add up to a multiple of 9. $5 + 7 + 6 = 18$, so if $G = 0$, the sum of the digits is 18, a multiple of 9. G could also be 9, in which case the sum is 27.

Taken together, the statements are sufficient. (1) tells us G must be 1, 3, 5, 7, or 9. (2) indicates that G must be 0 or 9. The only number that satisfies both statements is $G = 9$.

68. C

Explanation: Statement (1) is insufficient: if $p = 3$ and $q = 0$, the answer is "yes," while if $p = 4$ and $q = 1$, the answer is "no." Statement (2) is also insufficient: if $p = 3$ and $q = 3$, the answer is "yes," while if $p = 3$ and $q = 1$, the answer is "no."

Taken together, the statements are sufficient. If pq is divisible by 3 and both are integers, at least one of the integers must be divisible by 3. If one of the numbers is divisible by 3 and the difference between p and q is divisible by 3, the other number must be divisible by 3 as well. The sum of two multiples of 3 is also a multiple of 3, so $p + q$ must be divisible by 3. The answer is "yes," and the correct choice is (C).

69. C

Explanation: Statement (1) is insufficient: if each of the numbers is separated by 2, the four integers could be 1, 3, 5, and 7, with a range of exactly 6. Of course, there are many configurations that would allow for a range much greater than that. Statement (2) is also insufficient: the numbers could be 1, 2, 4, and 5, with a range of less than 6. Again, it's easy to imagine a set of four numbers with a range much greater than 6.

Taken together, the statements are sufficient. As we saw with (1), the only way for the range to be 6 or less without including any consecutive integers is to have a series of consecutive odds or evens. However, any such series will have a multiple of 3 for every third member: 2, 4, 6, 8 has 6; 7, 9, 11, 13 has 9; the same is true for any such grouping. Thus, in order to avoid multiples of 3 as well as consecutive integers, two of the numbers must be separated further, forcing the range to be wider than 6. The correct choice is (C).

70. A

Explanation: Statement (1) is sufficient. The question gives us one equation for r and s; (1) provides another: $r - 1 = s$. Given two linear equations with two variables, we can solve for each. Statement (2) is insufficient: there are two different prime numbers that are divisible by 20: 2 and 5. Since r could be either, that's not enough information.

71. C

Explanation: Statement (1) is insufficient: w and v are 7 and 1, respectively, there are 5 possible integer values for n. If w and v are 7.5 and 1.5, respectively, there are 6 possible integer values for n. In more general terms, the answer depends on whether v and w are integers as well as the space between them. Statement (2) is insufficient: it doesn't indicate the difference between v and w.

Taken together, the statements are sufficient. Since we know the two variables are non-integers, and that the difference between them is 6, we can deduce that there are 6 possible integer values for n.

72. C

Explanation: Another way to write the expression in the question is: $(x^{p-q})^2$. In other words, we need the values of p and q, or the difference between them. Statement (1) is insufficient: there are an infinite number of possibilities for values and differences. The same is true of Statement (2).

Taken together, the statements are sufficient. If p is a multiple of q and q is a multiple of p, p and q must be equal. The difference between them, then, is 0, so $x^{p-q} = x^0 = 1$.

73. D

Explanation: Statement (1) is sufficient: if $r + 2$ and $s - 2$ are consecutive evens, r could be 2 less than s ($r = 4$ and $s = 6$, for instance), or r could be 6 less than s ($r = 2$ and $s = 8$, for instance). But because the question specifies that r and s are consecutive, r must be 2 less than s, directly answering the question.

Statement (2) is also sufficient. Given that r and s are consecutive, r could be 2 greater than s ($r = 4$ and $s = 2$, for example), or r could be 2 less than s ($r = 8$ and $s = 10$, for example). If $r - 1$ is not equal to $s + 1$, that eliminates the first case, so r must be 2 less than s, again directly answering the question.

74. C

Explanation: Statement (1) is insufficient: it tells you nothing about z, though it does tell you that m is divisible by 11. (Any two-digit number with the same tens and units digits is a multiple of 11.) Statement (2) is also insufficient, since you don't know anything about m.

Taken together, the statements are sufficient. You can write m as $11i$, where i is some integer. (This is because m is divisible by 11.) You also write $m - z$ as $11j$, where j is some other integer. So, to express it as an equation:

$m - z = m - z$

$11j = 11i - z$

$z = 11i - 11j$

$z = 11(i - j)$

Since $i - j$ is an integer, $z = 11(integer)$, so z is divisible by 11. Choice (C) is correct.

75. A

Explanation: The question is essentially asking: is x the square of an integer? Statement (1) is sufficient: the only integers that have exactly three factors are the squares of prime numbers. Thus, x is the square of a prime number, and thus it's the square of an integer.

Statement (2) is insufficient: x could be any number less than 16; if $x = 9$, the answer is "yes," while if $x = 8$, the answer is "no."

76. B

Explanation: In a set of numbers, the mean is equal to the median if the numbers are equally spaced, as in consecutive integers, consecutive evens, or consecutive multiples of 5. Statement (1) is insufficient: if the range is 14, the numbers could be consecutive, or they could be randomly distributed in that range. Statement (2) is sufficient: this is another way of saying that the numbers are either consecutive evens or consecutive odds. Either way, they are equally spaced, so the mean and the median are equal. Choice (B) is correct.

77. A

Explanation: The question is asking whether k is the square of an integer. Statement (1) is sufficient: if m is an integer, then m^2 is the square of an integer. Since $k = m^2$, k must be a square of an integer as well. Statement (2) is insufficient: if n^2 is an integer, then n is the square root of an integer. Thus, $\sqrt{k}$ is the square root of an integer, meaning only that k is an integer, which we already knew. Choice (A) is correct.

78. A

Explanation: Statement (1) is sufficient. To say that $\frac{q}{p}$ is an integer is equivalent to saying that q is divisible by p. If p is an integer, then q must be as well. Statement (2) is insufficient: if $\frac{p}{q}$ is an integer, that means p is divisible by q. q could be an integer, but it could be any number of non-integers: if $p = 4$, for instance, q could be 2, or it could be 0.5. Choice (A) is correct.

79. C

Explanation: Be careful of a common mistake here: because they refer to "the fraction $\frac{x}{y}$" doesn't mean that the fraction is an integer, e.g., that x is a multiple of y. Statement (1) is insufficient: while it establishes that $\frac{x}{y}$ is an integer, it gives you no information about the values of the numbers.

Statement (2) is also insufficient: x and y could be any pair of integers separated by two, such as 1 and 3, 2 and 4, or 3 and 5.

Taken together, the statements are sufficient. Of the pairs of numbers separated by two, the only ones that satisfy (1) are 2 and 4. 4 is an even multiple of 2, so x must be 4 and y must be 2. The only other set of numbers separated by two where one of the numbers is a multiple of the other is 3 and 1, and 3 isn't even. Choice (C) is correct.

80. A

Explanation: Statement (1) is sufficient: if $x^y = 16$, then x and y could be any of the following pairs: {16, 1}, {4, 2}, or {2, 4}. To look at each one:

$16^1 > 1^{16}$

$4^2 = 2^4$

$2^4 = 4^2$

In none of the three cases is $x^y < y^x$, so the answer must be "no."

Statement (2) is insufficient: we've established that when x and y are 2 and 4, the answer is no. However, if x and y are 0 and 2:

$0^2 < 2^0$

So the answer could be "yes." Choice (A) is correct.

81. C

Explanation: Statement (1) is insufficient: because there are only four prime numbers that are single-digit numbers, the four variables must be 2, 3, 5, and 7, but we don't know which is which. Statement (2) is also insufficient: we only know which variables are larger than the others. Taken together, we have all the information we need: knowing the identities and the orders of the four digits, j, k, m, and n must be 2, 5, 3, and 7, respectively.

82. A

Explanation: Statement (1) is sufficient. $\sqrt{9z} = 3\sqrt{z}$, so in order for that number to be an integer, $\sqrt{z}$ must either be an integer or an integer divided by 3. Since z itself is an integer, $\sqrt{z}$ could be an integer (an integer squared is an integer), but it could not be an integer divided by 3: an integer divided by 3, such as $\frac{2}{3}$ results in an integer divided by 9 (like $\frac{4}{9}$) when squared. So, $\sqrt{z}$ must be an integer.

Statement (2) is insufficient. $\sqrt{8z} = \sqrt{8}\sqrt{z} = 2\sqrt{2}\sqrt{z}$. If $\sqrt{z}$ is an integer, $\sqrt{8z}$ will not be an integer. There are also many non-integer values for $\sqrt{z}$ that make $\sqrt{8z}$ a non-integer, such as $\sqrt{3}$. Choice (A) is correct.

83. E

Explanation: For $\frac{x}{y}$ to be greater than 1 when both numbers are positive, x must be greater than y. Statement (1) is not sufficient: if x is not a factor of y, x could be greater or smaller than y. The same reasoning makes (2) insufficient: simply knowing whether the numbers are factors or multiples of each other doesn't determine which one is greater. Putting the two together only tells you whether $\frac{x}{y}$ or $\frac{y}{x}$ is an integer, which is irrelevant to the question at hand. (E) is the correct choice.

84. E

Explanation: Statement (1) is insufficient: while 31 is, in fact, equal to one less than twice the value of the square of an integer (4), it's not the only prime that's true of. For instance, one less than twice the value of the square of 2 is 7, and one less than twice the value of the square of 6 is 71.

Statement (2) is also insufficient: there are many prime numbers that are four less than a multiple of five (or, the same thing: one more than a multiple of five) such as 11, 41, and 71. Taken together, the statements are still insufficient, because even with both of these requirements fulfilled, p could be either 31 or 71. The correct choice is (E).

85. A

Explanation: Statement (1) is sufficient. To determine whether x is 7, you need to know what the prime factors of 343 are. To do that without a lot of long division, find a simple number near 343: 350 works great. You probably know that $350 = 7(50)$, which means that 7 less than 350 is 7(49). Thus, $343 = 7(49) = 7(7)(7)$. The only prime factor of 343 is 7, so x must be 7.

Statement (2) is insufficient: $350 = 7(50) = 7(2)(5)$, so x could be 7, 2, or 5. Choice (A) is correct.

86. B

Explanation: Statement (1) is insufficient: if p is an integer and $\frac{p}{2}$ is not odd, that leaves two possibilities: either $\frac{p}{2}$ is not an integer (for instance, when $p = 1$), or $\frac{p}{2}$ is an even integer, as it is when $p = 4$. In the first case, p is odd; in the second, p is even, so you can't answer the question.

Statement (2) is sufficient: another way to write it is:

$p - 2 = odd$

$p = odd + 2$

An odd integer plus an even integer (2) is an odd integer, so p must be odd. Choice (B) is correct.

87. E

Explanation: Statement (1) is insufficient: having the equations $w + z = 36$ and $wz = 180$ is enough to determine what the two numbers are, but not enough to determine which is which. One must be 6 and the other must be 30, but depending on which is which, $\frac{z}{w}$ can have two different results.

Statement (2) is also insufficient: on its own, it leaves several possibilities for w and z (6 and 30, 12 and 24, 18 and 18).

Taken together, the statements are still insufficient. Both statements are true if the variables represent 6 and 30, but as discussed above, the answer is different depending on which of the variables is 6 and which of the variables is 30. Choice (E) is correct.

88. C

Explanation: Statement (1) is insufficient: p could be 30, or it could be any multiple of 30. To take just a couple of examples, the number of factors of 300 is much greater than the number of factors of 30. Statement (2) is also insufficient: p could be any integer less than 50.

Taken together, the statements are sufficient. There is only one multiple of 30 that is less than 50 ($p = 30$), so we can figure out how many numbers p is divisible by simply by counting the factors of 30.

89. B

Explanation: Statement (1) is insufficient: if z has one prime factor, that means z is either a prime number or a prime raised to an integer power. That leaves an infinite number of possibilities for the value of z.

Statement (2) is sufficient. If z has exactly three factors, that means it's the square of a prime. There's only one square of a prime between 18 and 32: that's 25. Choice (B) is correct.

90. A

Explanation: Statement (1) looks like it'll take a long time to figure out, but you'd be wasting your time to do so. Of any number, there's only one largest prime factor. Thus, if we know z is the largest prime factor of 323, we know we could figure out what it is, even if we don't know exactly what that number is. So, it's sufficient.

Statement (2) is insufficient. This one we'll have to figure out. 133 is 7 away from 140, so since $140 = 20(7)$, $133 = 19(7)$. Thus, z could be either 7 or 19, since both numbers are prime. Choice (A) is correct.

91. D

Explanation: The trickiest part of this question is figuring out the pattern: what types of numbers are perfect numbers? You won't be able to figure out much under time constraints, but testing out a few numbers would tell you that not many numbers are "perfect."

Statement (1) is sufficient: no prime number is perfect, as a prime number has only two divisors, and the factor other than itself is 1. The sum of a prime number's divisors (excluding itself) is 1, not the prime number! The answer to the question, then, is "no."

Statement (2) is also sufficient. Any number with three divisors is the square of a prime: 4, 9, 25, etc. The sum of the divisors of the square of a prime number x (excluding itself) is always $1 + \sqrt{x}$, and there is no prime number x for which $1 + \sqrt{x} = x$, so the answer must be "no."

92. E

Explanation: Statement (1) is insufficient: there are several two-digit integers that are divisible by 9. Statement (2) is not only insufficient, it says the same thing: if the digits of a number add up to 9, that means the number is divisible by 9. Since (1) is insufficient, (2) is as well.

Further, when the two statements say the same thing, they will never be sufficient when taken together. Thus, without considering any further, we can eliminate (C) and choose (E), the correct answer.

93. A

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Explanation: Statement (1) is sufficient. To write this as an equation:
$\frac{y}{2} = \text{integer}^2$
$y = 2(\text{integer}^2)$
An integer squared will always equal an integer, and double that number will always be an integer. Thus, y must be an integer.
Statement (2) is not sufficient. Following the same logic:
$2y = integer^2$
$y = \frac{\text{integer}^2}{2}$
If the integer is odd, then the square of the integer is also odd, and the square divided by 2 will be a non-integer. On the other hand, if the integer is even, the square will be even, making the square divided by two an integer. Choice (A) is correct.

94. A
Explanation: Statement (1) is sufficient. If $2z$ is not an integer, then, algebraically: $2z \neq$ integer, or $z \neq \frac{\text{integer}}{2}$. Any integer can be written as an integer over 2 (for instance, $1 = \frac{2}{2}$, $7 = \frac{14}{2}$, $8 = \frac{16}{2}$), so the fact that z isn't an integer divided by 2 is equivalent to the fact that z isn't an integer. Thus, the answer is "no."
Statement (2) is insufficient: if $\frac{z}{2} \neq$ integer, then $z \neq 2$(integer), which means that z isn't an even integer. That means z could either be an odd integer or a non-integer, which isn't enough information to determine whether it's an integer. Choice (A) is correct.

95. C
Explanation: Statement (1) is insufficient: if the units digit of n is greater than the units digit of n^2, the units digit of n could be 8 (units digit of the square is 4) or 9 (units digit of the square is 1).
Statement (2) is also insufficient: if the units digit of n is greater than the units digit of n^3, the units digit of n could be 7 (units digit of the cube is 3) or 8 (units digit of the cube is 2).
Taken together, the statements are sufficient. The only value for the units digit of n that satisfies both statements is 8, for which the units digit of the square is 4, and the units digit of the cube is 2. Choice (C) is correct.

96. E
Explanation: Statement (1) is insufficient. Knowing the prime factors of a number gives you the numbers smallest possible value (in this case, 21), but also allows for an infinite number of possibilities. For instance, r could be 3(7)(7) or 3(3)(7), along with any other product of some number of 3's and some number of 7's.
Statement (2) is also insufficient: if 3, 7, and 21 are factors of r, r could be 21, but it could also be any multiple of 21, such as 42, 63, or 210.
Taken together, the statements are still insufficient. While r could be 21, it could also be 63 (equal to 3(3)(7)), which satisfies both statements. The correct choice is (E).

97. D

Explanation: Statement (1) is sufficient: there are no three prime numbers that are consecutive integers. Since the answer is "no," the statement is sufficient. Statement (2) is also sufficient: if the three integers are odd, they cannot be consecutive. Both statements are sufficient independent of each other, so the correct answer is (D).

98. B

Explanation: The only integers that can be expressed as the product of two integers, each greater than 1, are non-primes. In other words, the question is asking whether k is a non-prime. It would be equally valid to recast the question as "Is k prime?" because it doesn't matter whether the answer is "yes" or "no," simply whether the statements are sufficient.

Statement (1) is insufficient: if $n = 3$, it is prime; if n is any other multiple of 3, it is not. Statement (2) is sufficient: the smallest value of n is 4, which is not prime, and any other multiple of 4 is also not prime.

99. A

Explanation: As always, consider ways to simplify the question before moving to the statements. If y is even, y^3 will always be divisible by 8: if y is even, it is a multiple of 2, so y^3 must be $(2x)(2x)(2x) = 8x^3$, where x is some integer, making y^3 a multiple of 8. If y is odd, y^3 will be odd, and thus not a multiple of 8.

Statement (1) answers the question directly: since we know that for any even integer y, y^3 is divisible by 8, the answer is yes, and this statement is sufficient. Statement (2) is insufficient. If y is even, y^3 is even, so $y^3 - y = even - even = even$. If y is odd, y^3 is odd, so $y^3 - y = odd - odd = even$. Knowing that the difference is even doesn't tell us whether y is even or odd, so the statement is insufficient.

100. A

Explanation: If z is the cube of an integer, and between 2 and 100, z must be 8, 27, or 64, the cube of 2, 3, or 4, respectively. Statement (1) is sufficient: if $\frac{z}{4}$ is not an integer, z cannot be 8 or 64, so it must be 27. Statement (2) is insufficient: it doesn't rule out any of the three possibilities, none of which are divisible by 6