Number Properties

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

On a number line, the distance from *A* to *B* is 4 and the distance from *B* to *C* is 5.

Quantity A	**Quantity B**
The distance from *A* to *C*	9

2.

a, *b*, *c*, and *d* are consecutive integers such that $a < b < c < d$

Quantity A	**Quantity B**
The average of *a*, *b*, *c*, and *d*	The average of *b* and *c*

3. *w*, *x*, *y*, and *z* are consecutive odd integers such that $w < x < y < z$. Which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ $wxyz$ is odd
☐ $w + x + y + z$ is odd
☐ $w + z = x + y$

4.

Quantity A	Quantity B
The sum of all the odd integers from 1 to 100, inclusive	The sum of all the even integers from 1 to 100, inclusive

5. If $a + b + c + d + e$ is odd, and $a, b, c, d,$ and e are integers, which of the following could be the number of integers among $a, b, c, d,$ and e that are even?

Indicate <u>all</u> such numbers.

☐ 0
☐ 1
☐ 2
☐ 3
☐ 4
☐ 5

6.

Quantity A	Quantity B
The least odd number greater than or equal to 5!	The greatest even number less than or equal to 6!

7. If set S consists of all positive integers that are multiples of both 2 and 7, how many numbers in set S are between 140 and 240, inclusive?

☐

8.

$$ab > 0$$
$$bc < 0$$

Quantity A	Quantity B
ac	0

9.

$$mn < 0$$

$$mp > 0$$

Q uantity A	**Q uantity B**
np	0

10.

$abc < 0$

$b^2c > 0$

Q uantity A	**Q uantity B**
ab	0

11.

a,b,and c are integers such that $a < b < c$

Q uantity A	**Q uantity B**
$\frac{a+b+c}{3}$	b

12.If $x^2 = y^2$,w hich of the follow ing m ust be true?

- ☐ $x = y$
- ☐ $x^2 - y^2 = 0$
- ☐ $|x| - |y| = 0$

13.If p and k are even,and q is odd,w hich of the follow ing cannot be even?

(A) pk

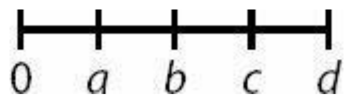
(B) pq

(C) $\frac{k}{p}$

(D) $\frac{qp}{k}$

(E) $\frac{q}{p}$

14.



Q uantity A	**Q uantity B**
$a \times c$	$b \times d$

15.If $a > b > c > d$ and $a = 2$,w hich of the follow ing m ust be negative?

(A) ab
(B) ac
(C) ad
(D) bd
(E) N one of the above.

16.If $y^2 = 4$ and $x^2y = 18$,$x + y$ could equal w hich of the follow ing values?

Indicate <u>tw o</u> such values.

☐ -5
☐ -1
☐ 1
☐ 5
☐ 6

17.

Q uantity A	**Q uantity B**
The rem ainder w hen 10^{11} is divided by 2	The rem ainder w hen 3^{13} is divided by 3

18.

q is odd

Q uantity A	**Q uantity B**
$(-1)^q$	$(-1)^{q+1}$

19.

n is a positive integer

Q uantity A	**Q uantity B**
$(-1)^{4n} \times (-1)^{202}$	$(3)^3 \times (-5)^5$

20.If n is the sm allest of three consecutive positive integers,w hich of the follow ing m ust be true?

(A) n is divisible
by 3 (B) n is even
(C) n is odd
(D) $(n)(n + 2)$ is even
(E) $n(n + 1)(n + 2)$ is divisible by 3

21.If x,y,and z are integers,$y + z = 13$,and $xz = 9$,w hich of the follow ing m ust be true?

(A) x is even

(B) $x = 3$ (C
) y is odd (D
) $y > 3$
(E) $z < x$

22.

$$abc > 0,$$
$$a < b,$$
$$\text{and } a^2(c) < 0$$

Q uantity A	**Quantity B**
ab	$b(ac)^2$

23.O n a num ber line, A is 6 units from B and B is 2 units from C .W hat is the distance betw een A and C ?

(A) 4
(B) 8
(C) 12
(D) 4 or 8
(E) 4,8,or 12

24.The average of 11 integers is 35.W hat is the sum of all the integers?

$\boxed{}$

25.W hat is the sum of all the integers from 1 to 80,inclusive?

(A) 3,200
(B) 3,210
(C) 3,230
(D) 3,240
(E) 3,450

26.The average of a set of 18 consecutive integers is 22.5.W hat is the sm allest integer in the set?

$\boxed{}$

27. p is the sum of all the integers from 1 to 150,inclusive. q is the sum of all the integers from 1 to 148,inclusive. W hat is the value of $p - q$?

$\boxed{}$

28.If m is the product of all the integers from 2 to 11,inclusive,and n is the product of all the integers from 4 to 11, inclusive,w hat is the value of $\frac{n}{m}$?

G ive your answ er as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

29.If $\sqrt{x}$ is an integer and $xy^2 = 36$,how m any values are possible for the integer y?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 8

30.

a,b,and c are positive even integers such that $8 > a > b > c$

Q uantity A	**Q uantity B**
The range of a,b,and c	The average of a,b,and c

31.If x is a non-zero integer and $0 < y < 1$,w hich of the follow ing m ust be greater than 1?

(A) x
(B) $\frac{x}{y}$
(C) xy^2
(D) x^2y
(E) $\frac{x^2}{y}$

32.

a,b,and c are consecutive integers such that $a < b < c < 4$

Q uantity A	**Q uantity B**
The range of a,b,and c	The average of a,b,and c

33.

$x = 2y = 4z$ and x,y,and z are integers

Quantity A	Quantity B
The average of x and $2y$	$4z + x - 2y$

34.

$\sqrt{xy}$ is a prime number,xy is even,and $x > 4y > 0$

Quantity A	Quantity B
y	1

35.

$abcd$ is even and positive,and abc is odd and positive

Quantity A	Quantity B
1	d

36.

$b - a < 0$ and $a + 2c < 0$

Quantity A	Quantity B
b	$-2c$

37.

In set N consisting of n integers,the average equals the median.

Quantity A	Quantity B
The remainder when n is divided by 2	The remainder when $n - 1$ is divided by 2

38.

x is even,$\sqrt{x}$ is a prime number,and $x + y = 11$

Quantity A	Quantity B
x	y

39.

The product of integers f,g,and h is even and the product of integers f and g is odd

Quantity A	Quantity B

The rem ainder w hen f is divided by 2 | The rem ainder w hen h is divided by 2

40.

$x, y,$ and z are integers
$xyz \geq 0$
$yz < 0$
$y < 0$

Quantity A	Quantity B
x	z

41.

$\sqrt{y} = 3$
$x^2 = 16$
$y - x > 10$

Quantity A	Quantity B
x	xy

42.If $\frac{17}{2^{10}5^{13}}$ is expressed as a term inating decim al,how m any zeroes are located to the right of the decim al point before the first non-zero digit?

(A) 10
(B) 12
(C) 13
(D) 15
(E) 17

43.If x is odd,all of the follow ing m ust be odd EXCEPT:

(A) $x^2 + 4x + 6$
(B) $x^3 + 5x + 3$
(C) $x^4 + 6x + 7$
(D) $x^5 + 7x + 1$
(E) $x^6 + 8x + 4$

44.

$x^2 > 25$ and $x + y < 0$

Quantity A	Quantity B
x	y

45.

The positive integer a is divisible by 2,and $0 < ab < 1$

Q uantity A	**Q uantity B**
b	$\frac{1}{2}$

46.

p and w are single-digit prim e num bers $p + w < 6$

p^2 is odd

Q uantity A	**Q uantity B**
3	w

47.If $\frac{x}{23}$ is a positive integer w ith a factor of 6,w hich of the follow ing statem ents m ust be true?

Indicate tw o such statem ents.

- ☐ $x > 23$
- ☐ x has at least 3 prim e factors
- ☐ x is odd
- ☐ x is prim e
- ☐ x is not divisible by 5

48.

$x^2 > y^2$ and $x > -|y|$

Q uantity A	**Q uantity B**
x	y

49.

The sum of four consecutive integers is -2

Q uantity A	**Q uantity B**
The sm allest of the four integers	-2

50.If g is an integer and x is a prim e num ber,w hich of the follow ing m ust be an integer?

Indicate all such expressions.

☐ $\dfrac{g^2x+5gx}{x}$

☐ $g^2-x^2\left(\dfrac{1}{3}\right)$

☐ $6\left(\dfrac{g}{2}\right)-100\left(\dfrac{g}{2}\right)^2=$

51.If $k=\dfrac{19!}{16!}$,w hich of the follow ing is the sm allest choice that does not have a prim e factor in com m on w ith *k*?

(A) 19
(B) 34
(C) 77
(D) 115
(E) 133

52.If $4^6 25^5 = 10^x + k$,and *x* is an integer,w hat is the m inim um positive value *k* could be?

(A) 0
(B) 30,000
(C) 30,000,000
(D) 10,000,000,000
(E) 30,000,000,000

53.Jose is m aking a necklace w ith beads in a repeating pattern of blue,red,green,orange,purple.If the 1st bead is blue,w hat color w ill the 234th bead be?

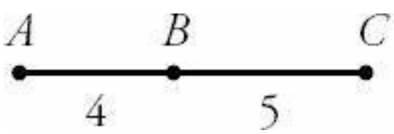
(A) blue
(B) red
(C) green
(D) orange
(E) purple

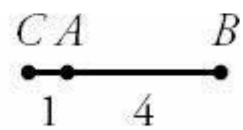
54.W hat is the units digit of 7^{94}?

55.W hat is the units digit of the sum $3^{47} + 5^{43} + 2^{12}$?

N um ber P roperties A nsw ers

1.**(D).**W henever a question looks this straightforw ard (4 + 5 = 9,so the quantities initially appear equal),be suspicious.D raw the num ber line described.If the points *A* ,*B* ,and *C* are in alphabetical order from left to right,then the distance from *A* to *C* w ill be 9.H ow ever,alphabetical order is not required.If the points are in the order *C* ,*A* ,and *B* from left to right,then the distance from *A* to *C* is 5 - 4 = 1.





2.**(C).**W hen integers are consecutive (or sim ply evenly spaced),the average equals the m edian.Since the m edian of this list is the average of the tw o m iddle num bers,Q uantity A and Q uantity B both equal the average of *b* and *c*. A lternatively,try this w ith real num bers.If the set is 2,3,4,5,both quantities equal 3.5.N o m atter w hat consecutive integers you choose,the tw o quantities are equal.

3.**I and III only.**This question tests your know ledge of the properties of odd num bers as w ell as of consecutives.

Statem ent I is TR U E,as m ultiplying only odd integers together (and no evens) w ill alw ays yield an odd answ er.

H ow ever,w hen adding,the rule is “an odd num ber of odds m akes an odd.” Sum m ing an even num ber of odds produces an even,so Statem ent II is FA LSE .

Statem ent III is TR U E .Since *w* ,*x*,*y*,and *z* are consecutive odd integers,you could define them all in term s of *w* :

$w = w$
$x = w + 2$
$y = w + 4$
$z = w + 6$

Thus,$w + z = w + (w + 6) = 2w + 6$
A nd $x + y = (w + 2) + (w + 4) = 2w + 6$

Therefore,$w + z = x + y$.A lternatively,try real num bers,such as 1,3,5,and 7.It is true that 1 + 7 = 3 + 5.This w ould hold true for any set of four consecutive,ordered odd num bers you select.

4.**(B).**N o m ath is required to solve this problem .N ote that the num bers from 1 to 100 include 50 even integers and 50 odd integers.The first few odds are 1,3,5,etc.The first few evens are 2,4,6,etc.Every even is 1 greater than its counterpart (2 is 1 greater than 1,4 is 1 greater than 3,6 is 1 greater than 5,etc.) N ot only is Q uantity B greater,it’s greater by precisely 50.

5.**I,III,and V only.**W hen adding integers,the rule is “an odd num ber of odds m akes an odd” (you can ignore the evens for purposes of evaluating only w hether the sum is even or odd — just count how m any odds are being added).If 5 integers sum to an odd,the possibilities are:

1 odd,4 evens
3 odds,2 evens
5 odds,0 evens

M ake sure to answ er for the num ber of *evens*.The answ ers are 0,2,and 4.

6.**(B).**W hen m ultiplying integers,just one even w ill m ake the product even.Thus all factorials greater than 1! (w hich is just 1) are even.

> $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.Since 120 is even,the "least odd num ber greater than or equal to 5!" is 121.
>
> $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.Since 720 is even,the "greatest even num ber less than or equal to 6!" is 720.

Q uantity B is greater.

7.**8.**A positive integer that is a m ultiple of both 2 and 7 is just a m ultiple of 14.Since 140 is a m ultiple of 14,start listing there and count the term s in the range: 140,154,168,182,196,210,224,238.

A lternatively,note that 140 is the 10th m ultiple of 14,and $240/14 \approx 17.143$ (use the calculator).Therefore,the 10th through the 17th m ultiples of 14,inclusive,are in this range.The num ber of term s is $17 - 10 + 1 = 8$ ("add one before you are done" for an inclusive list).

8.**(B).**If $ab > 0$,then *a* and *b* have the sam e sign.If $bc < 0$,then *b* and *c* have opposite signs.Therefore,*a* and *c* m ust have opposite signs.Therefore,*ac* is negative,and Q uantity B is greater.

If you find the logic difficult (*a* and *b* are sam e sign,*b* and *c* are opposite signs,therefore *a* and *c* are opposite signs), you could m ake a quick chart of the possibilities using plus and m inus signs:

a	*b*	*c*	
+	+	−	← first possibility, *a* and *c* have different signs
−	−	+	← second possibility, *a* and *c* have different signs

9.**(B).**If $mn < 0$,then *m* and *n* have opposite signs.If $mp > 0$,then *m* and *p* have the sam e sign.Therefore,*n* and *p* m ust have opposite signs.Therefore,*np* is negative,and Q uantity B is greater.

A lternatively,you could m ake a quick chart of the possibilities using plus and m inus signs:

m	*n*	*p*	
+	−	+	← first possibility, *n* and *p* have different signs
−	+	−	← second possibility, *n* and *p* have different signs

10.**(B).**If *abc* is negative,then either exactly 1 or all 3 of the values *a*,*b*,and *c* are negative:

−	−	−	← first possibility, all negative
−	+	+	← second possibility, 1 negative, 2 positives (order can vary)

If b^2c is positive, then c must be positive, since b^2 cannot be negative. If c is positive, eliminate the first possibility since all 3 variables cannot be negative. Thus, only one of *a, b,* and *c* are negative, but the one negative cannot be *c*. Either *a* or *b* is negative, and the other is positive. It doesn't matter which one of *a* or *b* is negative — that's enough to know that *ab* is negative and Quantity B is greater.

11. **(D).** Note that $\frac{a+b+c}{3}$ is just another way to express "the average of *a*, *b*, and *c*." The average of *a*, *b*, and *c* would equal *b* if the numbers were evenly spaced (such as 1, 2, 3, or 5, 7, 9), but that is not specified. For instance, the integers could be 1, 2, 57 and still satisfy the $a < b < c$ constraint. In that case, the average is 20, which is greater than $b = 2$.

The correct answer is (D).

12. **II and III only.** When you take the square root of $x^2 = y^2$, you do NOT get $x = y$. Actually, you get $|x| = |y|$. After all, if $x^2 = y^2$, the variables could represent 2 and -2, 5 and 5, -1 and -1, etc. The information about the signs of *x* and *y* is lost when the numbers are squared; thus, taking the square root results in absolute values, which allow both sign possibilities for *x* and *y*. Thus, statement I is not necessarily true.

From $x^2 = y^2$, simply subtract y^2 from both sides to yield statement II. If you can algebraically generate a statement from the original equation $x^2 = y^2$, that statement must be true.

To generate statement III, take the square root of both sides of $x^2 = y^2$ to get $|x| = |y|$, then subtract $|y|$ from both sides.

13. **(E).** "Cannot be even" means it must be either odd or a non-integer.

(A) *must be* even, as an even times any integer equals an even. (B) *must be* even, as an even times any integer equals an even. (C) *can be* even (for instance, if $k = 8$ and $p = 4$).
(D) *can be* even (for instance, if $p = 16$, $k = 2$, and $q = 1$).
(E) *cannot be* even, as an odd divided by an even is *never* an integer. CORRECT.

14. **(B).** The exact values of *a*, *b*, *c*, and *d* are unknown, as is whether they are evenly spaced (do NOT assume that they are, just because the figure looks that way). However, it is known that all of the variables are positive such that $0 < a < b < c < d$.

Because $a < b$ and $c < d$ and all the variables are positive, $a \times c < b \times d$. In words, the product of the two smaller numbers is less than the product of the two greater numbers. Quantity B is greater.

You could also try this with real numbers. You could try $a = 1$, $b = 2$, $c = 3$, and $d = 4$, or you could mix up the spacing, as in $a = 0.5$, $b = 7$, $c = 11$, $d = 45$. For any scenario that matches the conditions of the problem, Quantity B is greater.

15. **(E).** Don't assume the variables are integers or that they are equally spaced. It is possible that *b*, *c*, and *d* are all positive (integers or fractions), so it is not true that the products in choices (A) through (D) must be negative.

16.**-1,5.**From the first equation it seem s that y could equal either 2 or -2,but if $x^2y = 18$,then y m ust equal only 2 (otherw ise,x^2y w ould be negative).Still,the squared x indicates that x can equal 3 or -3.So the possibilities for $x + y$ are:

3 + 2 = 5 (-
3) + 2 = -1

17.**(C).**It is not necessary to calculate 10^{11} or 3^{13}.B ecause 10 is an even num ber,so is 10^{11},and 0 is the rem ainder w hen any even is divided by 2.Sim ilarly,3^{13} is a m ultiple of 3 (it has 3 am ong its prim e factors),and 0 is the rem ainder w hen any m ultiple of 3 is divided by 3.

18.**(B).**The negative base -1 to any odd pow er is -1,and the negative base -1 to any even pow er is 1.Since q is odd, Q uantity A = -1 and Q uantity B = 1.

19.**(A).**B efore doing any calculations on a problem w ith negative bases raised to integer exponents,check to see w hether one quantity is positive and one quantity is negative,in w hich case no further calculation is necessary.N ote that a negative base to an even exponent is positive,w hile a negative base to an odd exponent is negative.

Since n is an integer,4n is even.Thus,in Q uantity A ,$(-1)^{4n}$ and $(-1)^{202}$ are both positive,and Q uantity A is positive.In Q uantity B ,$(3)^3$ is positive but $(-5)^5$ is negative,and thus Q uantity B is negative.Since a positive is by definition greater than a negative,Q uantity A is greater.

20.**(E).**For three consecutive integers,the only possibilities are [odd,even,odd] or [even,odd,even].Since n could be odd or even,elim inate (B) and (C).C hoice (D) is true if n is even,but not if n is odd,so elim inate (D).In any set of three consecutive integers,one of the integers m ust be divisible by 3,but not necessarily n.Elim inate (A).

For the sam e reason,(E) m ust be true,as $n(n + 1)(n + 2)$ can be thought of as “the product of any three consecutive integers.” Since one of these integers m ust be divisible by 3,the product of those three num bers m ust also be divisible by 3.

21.**(D).**If $xz = 9$ and x and z m ust both be integers,then they are 1 and 9 (or -1 and -9) or 3 and 3 (or -3 and -3). Therefore,they are both odd.M ore generally,the product of tw o integers w ill only be odd if the com ponent integers them selves are both odd.B ecause z is odd,and $y + z$ equals 13 (an odd),y m ust be even.

Elim inate (A): x is N O T even.

Elim inate (B): x could be 3 but doesn’t have to be.

Elim inate (C): y is N O T odd.

Elim inate (E): z does not have to be less than x (for instance,they could both be 3).

A t this point,only (D) rem ains,so it m ust be the answ er.To prove it,consider the constraint that lim its the value of y: $y + z = 13$.Since z could be -1,1,-3,3,-9,or 9,the m axim um possible value for z is 9,so y m ust be at least 4.A ll values that are at least 4 are also greater than 3,so (D) m ust be true.

22.**(B).**Since $a^2(c) < 0$ and it is not possible for a^2 to be negative,*c* must be negative.

If *abc* > 0 and *c* is negative,*ab* must be negative also,implying that *a* and *b* have opposite signs.Because *a* < *b*,*a* must be negative and *b* positive.

Thus,Quantity A is negative and Quantity B is pos × $(\text{neg} \times \text{neg})^2$ = pos × pos,which is positive.

23.**(D).**There is no rule that *A* ,*B* ,and *C* must occur in alphabetical order.If the points are ordered *A* ,*B* ,*C* ,the distance from *A* to *C* is 6 + 2 = 8.But if in the order *A* ,*C* ,*B* ,the distance from *A* to *C* is 6 – 2 = 4.Any other orders allowed by the relative distances (*C* ,*B* ,*A* ,for instance,or *B* ,*C* ,*A*) will also yield either 4 or 8.

24.**385.**To find the sum of a set of numbers,given the average and number of terms,use the average formula.Average $= \dfrac{\text{Sum}}{\text{Number of Terms}}$,so Sum = Average × Number of Terms = 35 × 11 = 385.

25.**(D).**To find the sum of a set of evenly-spaced numbers,multiply the median (which is also the average) by the number of terms in the set.The median of the numbers from 1 to 80 inclusive is 40.5 (the first 40 numbers are 1 through 40,and the second 40 numbers are 41 through 80,so the middle is 40.5).You can also use the formula $\dfrac{\text{First} + \text{Last}}{2}$ to calculate the median of an evenly-spaced set: $\dfrac{1+80}{2}$.Multiply 40.5 times 80 to get the answer:

26.**14.**In an evenly-spaced set,the average is also the median.Thus,22.5 is the median of the set.There are an even number of terms in the set,so the median is halfway between the two middle numbers.Thus,the 9th number is 22 and the 10th number is 23.Count backwards to get the answer,or write out the set:

First 9 integers (counting down): 22,21,20,19,18,17,16,15,14
Last 9 integers (counting up): 23,24,25,26,27,28,29,30,31

Alternatively,the 1st number is 8 “steps” down from the 9th number,so the smallest integer in the set is 22 - 8 = 14.

27.**299.***p* is a large number,but it consists entirely of *q* + 149 + 150.Thus,*p* - *q* is what’s left of *p* once the common terms are subtracted: 149 + 150 = 299.

28.$\mathbf{\dfrac{1}{6}}$.There is a trick to this problem — all of the integers in the product *n* will be canceled out by the same integers appearing in the product *m* :

$$\frac{n}{m} = \frac{\cancel{4\times5\times6\times7\times8\times9\times10\times11}}{2\times3\times\cancel{4\times5\times6\times7\times8\times9\times10\times11}} = \frac{1}{2\times3} = \frac{1}{6}$$

29.**(E).**If $\sqrt{x}$ is an integer,then *x* must be a perfect square.If *x* is a perfect square and $xy^2 = 36$,then *x* could actually equal *any* of the perfect square factors of 36,which are 1,4,9,or 36.(Only consider positive factors, because in order to have a valid square root,*x* must be positive.) Thus,y^2 could equal 36,9,4,or 1,respectively.

Of course, y^2 is positive, but y itself could be positive or negative. Thus $y = \pm 6, \pm 3, \pm 2$, or ± 1, for a total of 8 possible values.

30. **(C).** Integers a, b, and c must be 6, 4, and 2, respectively, as they are positive even integers less than 8 and ordered according to the given inequality. The range of a, b, and c is $6 - 2 = 4$. The average of a, b, and c is $\frac{6+4+2}{3} = \frac{12}{3} = 4$. The two quantities are equal.

31. **(E).** Find the choices that do not have to be greater than 1. Most obviously, x could be negative, which eliminates (A), (B), and (C). For choice (D), if $x^2 = 1$, that times the positive fraction y would be less than 1. In choice (E), x^2 must be positive and at least 1, so dividing by the positive fraction y increases the value.

32. **(D).** If the variables were also constrained to be positive, they would have to be 1, 2, and 3, making the quantities both equal to 2. However, the variables could be negative, for example, $a = -10$, $b = -9$, $c = -8$. The range of a, b, and c will always be 2 because the integers are consecutive, but the average can vary depending on the specific values.

33. **(C).** Since $x = 2y$, the average of x and $2y$ is $\frac{x+2y}{2} = \frac{x+x}{2} = x$. Similarly, $4z + x - 2y$ is $x + x - x = x$.

Alternatively, pick values, such as $x = 4$, $y = 2$, and $z = 1$.

Quantity A is the average of 4 and 2(2), which is equal to 4.

Quantity B is $4(1) + 4 - 2(2) = 4$.

The two quantities are equal for any set of values that conform to $x = 2y = 4z$, even negative test cases.

34. **(B).** If $\sqrt{xy}$ is a prime number, $\sqrt{xy}$ could be 2, 3, 5, 7, 11, 13, etc. Square these possibilities to get a list of possibilities for xy: 4, 9, 25, 49, 121, 169, etc. However, xy is even, so xy must equal 4.

Finally, $x > 4y > 0$, which implies that both x and y are positive. Solve $xy = 4$ for x, then substitute to eliminate the variable x and solve for y.

If $xy = 4$, then $x = \frac{4}{y}$.

If $x > 4y$, then $\frac{4}{y} > 4y$.

Because y is positive, you can multiply both sides of the inequality by y and you don't have to flip the sign of the inequality: $4 > 4y^2$

Finally, divide both sides of the inequality by 4: $1 > y^2$

Thus,y is a positive fraction less than 1 (you already know $y > 0$).Q uantity B is greater.

35.**(D).**Since *abc* is odd and *abcd* is even,*d* has to be even — if it is an integer.For instance,it could be the case that $a = 1$,$b = 1$,$c = 1$,and $d = 2$.In this case,*d* w ould be greater than 1.H ow ever,*d* could be a fraction — for exam ple, *abcd* could equal $3 \times 3 \times 3 \times \frac{2}{3} = 18$.In this case,*d* w ould be $\frac{2}{3}$,w hich is less than 1.

36.**(B).**Solve both inequalities for *a*.A dd *a* to both sides of $b - a < 0$ to get $b < a$.From $a + 2c < 0$,subtract $2c$ on both sides to get $a < -2c$.Put these together: $b < a < -2c$.Thus,*b* m ust be less than $-2c$.Q uantity B is greater.

37.**(D).**“The rem ainder w hen … divided by 2” is a fancy w ay of asking w hether a integer is odd or even (evens yield rem ainder 0 w hen divided by 2;odds yield rem ainder 1 w hen divided by 2).

If *n* is even,Q uantity A is 0 and Q uantity B is 1.

If *n* is odd,Q uantity A is 1 and Q uantity B is 0.

In this problem ,*n* is the num ber of num bers in the set.So,could a set in w hich the average equals the m edian have either an even or an odd num ber of num bers? A bsolutely.In fact,it is true of *any* evenly spaced set (and of som e other sets) that the average equals the m edian.For instance:

1,2,3,4	average and m edian are both 2.5	$n = 4$	→ Q uantity B is greater
1,2,3	average and m edian are both 2	$n = 3$	→ Q uantity A is greater

The correct answ er is (D).

38.**(B).**If $\sqrt{x}$ is a prim e num ber,$x = (\sqrt{x})(\sqrt{x})$ is the square of a prim e num ber.Squaring a num ber does not change w hether it is odd or even (the square of an odd num ber is odd and the square of an even num ber is even).Since *x* is even,it m ust be the square of the only even prim e num ber.Thus,$\sqrt{x} = 2$ and $x = 4$.Since $x + y = 11$,$y = 9$ and Q uantity B is greater.

39.**(A).**If *fg* is odd and both *f* and *g* are integers,both *f* and *g* are odd.The rem ainder w hen odd *f* is divided by 2 is 1. Since *fgh* is even and *f* and *g* are odd,integer *h* m ust be even.Thus,w hen *h* is divided by 2,the rem ainder is 0.Q uantity A is greater.

40.**(B).**If $yz < 0$ and $y < 0$,*z* m ust be positive and *y* negative.If $xyz \geq 0$ and $yz < 0$,then *x* m ust be negative or 0. Q uantity A is at m ost 0,w hile Q uantity B is positive.

41.**(A).**If $\sqrt{y} = 3$,$y = 9$.If $x^2 = 16$,then $x = 4$ or -4.Since $y - x > 10$ and $y = 9$:

$9 - x >$
$10 - x > 1$
$x < -1$

This rules out the $x = 4$ possibility.Thus,x equals -4 and $xy = (-4)(9) = -36$.Since -4 is greater than -36,Q uantity A is greater.

42.**(A).**D ecim al placem ent can be determ ined by how m any tim es a num ber is m ultiplied or divided by 10. M ultiplying m oves the decim al to the right,and dividing m oves the decim al to the left.Look for pow ers of 10 in the given fraction,rem em bering that $10 = 2 \times 5$.

$$\frac{17}{2^{10}5^{13}} = \frac{17}{2^{10}5^{10}5^{3}} = \frac{17}{(2\times5)^{10}5^{3}} = \frac{17}{10^{10}5^{3}} = \frac{17/125}{10^{10}} = \frac{0.136}{10^{10}}$$

There are no zeros to the right of the decim al point before the first non-zero digit in 0.136.H ow ever,dividing by 10^{10} w ill m ove the decim al to the left 10 places,resulting in 10 zeros betw een the decim al and the “136” part of the num ber.

43.**(C).**For even and odd questions,you can either think it out logically,or plug in a num ber.Since one choice requires raising the num ber to the 6th pow er,pick som ething sm all! Plug in $x = 1$:

(A) $x^2 + 4x + 6 = 1 + 4 + 6 = 11$
(B) $x^3 + 5x + 3 = 1 + 5 + 3 = 9$
(C) $x^4 + 6x + 7 = 1 + 6 + 7 = 14$
(D) $x^5 + 7x + 1 = 1 + 7 + 1 = 9$
(E) $x^6 + 8x + 4 = 1 + 8 + 4 = 13$

For the logic approach,rem em ber that an odd num ber raised to an integer pow er is alw ays odd,an odd num ber m ultiplied by an odd num ber is alw ays odd,and an odd num ber m ultiplied by an even num ber is alw ays even:

(A) $x^2 + 4x + 6$ = odd + even + even = odd
(B) $x^3 + 5x + 3$ = odd + odd + odd = odd (C
) $x^4 + 6x + 7$ = odd + even + odd = even (D
) $x^5 + 7x + 1$ = odd + odd + odd = odd
(E) $x^6 + 8x + 4$ = odd + even + even = odd

44.**(D).**If $x^2 > 25$,then $x > 5$ O R $x < -5$.For instance,x could be 6 or -6.

If $x = 6$:

$6 + y <$
$0\ y < -6$

x is greater than y.

If $x = -6$:

$-6 + y <$
$0\ y < 6$

y could be less than *x* (e.g.,*y* = -7) or greater than *x* (e.g.,*y* = 4).

45.**(B).**If the positive integer *a* is divisible by 2,it is a positive even integer.Thus,the m inim um value for *a* is 2. Thus,since *ab* < 1,*b* m ust be less than $\frac{1}{2}$.

46.**(A).**If the sum of tw o prim es is less than 6,either the num bers are 2 and 3 (the tw o sm allest unique prim es),or both num bers are 2 (just because the variables are different letters doesn’t m ean that *p* cannot equal *w*).B oth num bers cannot equal 3,though,or *p* + *w* w ould be too great.If p^2 is odd,*p* is odd,and therefore *p* = 3,so *w* can only be 2.

47.**I and II only.**If $\frac{x}{23}$ is divisible by 6,*x* m ust have factors of 2 and 3,as w ell as 23.A nother w ay to w rite this is $\frac{x/23}{6}$ = positive integer,so $\frac{x}{23}$ = 6 × positive integer,and finally *x* = 2 × 3 × 23 × positive integer.

I.TR U E .*x* m ust be greater than 23,as it is 23 × positive values.
II.TR U E .*x* has at least 3 prim e factors,nam ely,2,3,and 23.
III.False.B ecause *x* has a factor of 2,it is not odd.
IV .False.B ecause it has m ore than one prim e factor,*x* is definitely *not* prim e.
V .M aybe.W hile *x* m ust have factors of 2,3,and 23,it could also have other prim e factors.For instance,*x* could be 2(3)(23),or it could be a very large num ber w ith m ore factors,such as $2^2 3^2 23^4 5^{11} 13^2$.Thus,*x* m ay or m ay not be divisible by 5.

48.**(A).**If $x^2 > y^2$,*x* m ust have a greater absolute value than *y*.For instance:

	x	*y*
Exam ple 1	3	2
Exam ple 2	-3	2
Exam ple 3	3	-2
Exam ple 4	-3	-2

If *x* > -|*y*|m ust also be true,w hich of the exam ples continue to be valid?

	x	*y*	*x* > -|*y*|?	
Exam ple 1	3	2	3> -|2|	TR U E
Exam ple 2	-3	2	-3 > -|2|	FA LSE
Exam ple 3	3	-2	3 > -|-2|	TR U E
Exam ple 4	-3	-2	-3> -|-2|	FA LSE

O nly Exam ple 1 and Exam ple 3 rem ain.

x	*y*

Exam ple 1	3	2
Exam ple 3	3	-2

Thus,either x and y are both positive and x has a larger absolute value (Q uantity A is greater) or x is positive and y is negative (Q uantity A is greater).In either case,Q uantity A is greater.

49.**(C).**W rite an equation: $x + (x + 1) + (x + 2) + (x + 3)= -2$.

$4x + 6 = -2$
$4x = -8$
$x = -2$

Thus,the integers are -2,-1,0,and 1.The sm allest of the four integers equals -2.

50.**I and III only.**In Statem ent I,x can be factored out and elim inated: $\frac{g^2x+5gx}{x}=\frac{x(g^2+5g)}{x}= g^2+5g$ Since g is an integer,so is $g^2 + 5g$.

In Statem ent II,g^2 is certainly an integer,but $x^2\left(\frac{1}{3}\right)$ is only an integer if $x = 3$ (since 3 is the only prim e num ber divisible by 3),so statem ent II does not have to be an integer.

W hen Statem ent III is sim plified, $6\left(\frac{g}{2}\right)-100\left(\frac{g}{2}\right)^2=3g-\frac{100g^2}{4}= 3g-25g^2$ results;since g is an integer,$3g – 25g^2$ is also an integer,albeit a negative one.

51.**(C).**$\frac{19!}{16!}=\frac{19\times18\times17\times\cancel{16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}{\cancel{16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}$ $= 19 \times 18 \times 17$.N o need to m ultiply these out,because the question is only about com m on prim e factors.If $k = 19 \times 18 \times 17$,$k$'s prim e factors are 19,2,3,3, and 17 (19 and 17 are prim e,and 18 breaks dow n into prim e factors 2,3,and 3).

C hoices (A) is w rong because 19 has a prim e factor in com m on w ith k (that factor is 19 itself).C hoice (B) is w rong because 34 has a prim e factor of 17,as does k.C hoice (C) is correct.77 has prim e factors 7 and 11 and does not have a prim e factor in com m on w ith k.N ote that 115 also does not share any prim e factors w ith k,but the questions asks for the *sm allest* choice that w orks.

52.**(E).**Since $4^6 25^5$ is too big for the calculator,try another strategy: note that $4 \times 25 = 100$,and the other side of the equation involves a pow er of 10.Separate out “pairs” of 4 and 25 on the left:

$4^6 25^5 = 10^x + k$
$4^1(4^5 25^5) = 10^x + k$
$4^1(4 \times 25)^5 = 10^x + k$

$4^1(100)^5 \quad = 10^x + k$

Thus, the left side of the equation is $4(100^5)$, or $4(10^{10})$, or 40,000,000,000. Thus:

40,000,000,000 = [a power of 10] + *k*

To minimize *k* while keeping it positive, maximize the power of 10 while keeping it less than $4^6 25^5$. The greatest power of 10 that is less than 40,000,000,000 is 10,000,000,000, or 10^{10}. Thus:

40,000,000,000 = 10,000,000,000 + *k*
30,000,000,000 = *k*

53. **(D).** Since the pattern has 5 elements, find the remainder when 234 is divided by 5, which is just the units digit in this case. Alternatively, 5 goes evenly into 230, and since 234 - 230 = 4, the remainder is 4. The 4th color in the pattern, orange, is the answer.

54. **9.** The units digits of 7 to positive integers create a repeating pattern (this works for digits other than 7 also). By multiplying 7 by itself repeatedly in the calculator, you can generate the pattern:

7^1 = **7**
7^2 = 4**9**
7^3 = 34**3**
7^4 = 2,40**1**
7^5 = 16,80**7**
7^6 = 117,64**9**
7^7 = 823,54**3**
7^8 = 5,764,80**1**
Pattern: 7,9,3,1

The pattern for the units digits of 7 to a power is 7,9,3,1. (You should know that none of the patterns ever have more than 4 elements before repeating, so you don't actually have to multiply out 8 or more times, as shown above.)

To find the 94th item in a pattern of 4 repeating items, find the remainder when 94 is divided by 4. 94 divided by 4 in the calculator is 23.5. Ignore the decimal and multiply 4 × 23 to find the largest number (less than 94) that 4 *does* go into: it's 92. 94 divided by 4 gives remainder 2 (since 94 – 92 = 2).

Thus, to get to the units digit of 7^{94}, the pattern (7,9,3,1) repeats 23 times, and then continues 2 more elements into the pattern. The second element in the pattern is 9.

55. **8.** For any digit, that digit to increasing positive integer powers will end in units digits that create a repeating pattern:

3^1 = **3**
3^2 = **9**
3^3 = 2**7**

3^4 = 8**1**
3^5 = 24**3**
3^6 = 72**9**
3^7 = 2,18**7**
3^8 = 6,56**1**
Pattern: 3,9,7,1

3^{47} w ill end in the digit that is the 47th item in the pattern 3,9,7,1.Since it’s a pattern of 4 elem ents,find the largest m ultiple of 4 that is still less than 47: it’s 44.Since 47 - 44 = 3,the rem ainder w hen 47 is divided by 4 is 3.Thus,3^{47} w ill end in the units digit that is the 3rd item in the pattern.Thus,3^{47} ends in 7.

5^1 = **5**
5^2 = 2**5**
5^3 = 12**5**

… this one’s a “freebie,” as 5 to any pow er ends in 5 and 5^{43} m ust end in 5.

2^1 = **2**
2^2 = **4**
2^3 = **8**
2^4 = 1**6**
2^5 = 3**2**
2^6 = 6**4**
2^7 = 12**8**
2^8 = 25**6**
2^9 = 51**2**
2^{10} = 1,02**4**
Pattern: 2,4,8,6

2^{12} w ill end in the digit that is the 12th item in the pattern 2,4,8,6.Since 4 goes into 12 evenly,2^{12} w ill end in the last item in the pattern,w hich is 6.(Y ou also could just keep going w ith the pattern above: 2^{11} is 2,048,and 2^{12} = 4,096,w hich ends in 6).Thus,2^{12} ends in 6.

Thus,3^{47},5^{43},and 2^{12} are very large num bers that end in 7,5,and 6,respectively.Im agine that you w ere adding those very large num bers by hand (the xxxx just indicates the unknow n and unim portant parts of these num bers):

```
  xxxxxxx7
  xxxxxxx5
+ xxxxxxx6
```

To begin adding these num bers,you w ould add 7 + 5 + 6 = 18.Y ou w ould put 8 in the units place of your answ er,and carry the 1:

```
         1
  xxxxxxx7
```

xxxxxxx5
±
xxxxxxx6 8

The units digit of the answ er is 8.