Mixed Geometry

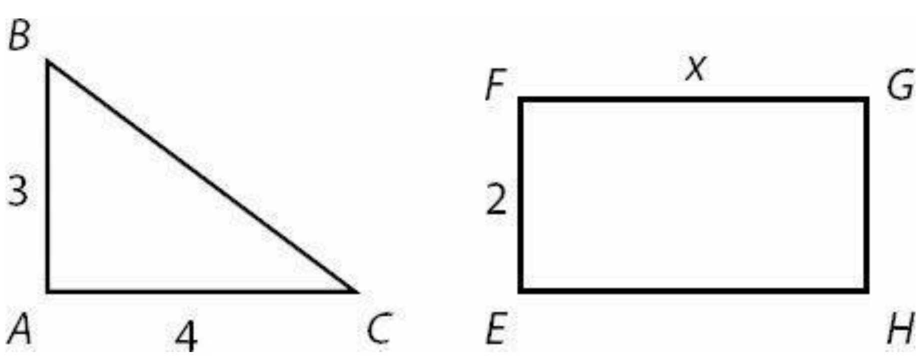
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

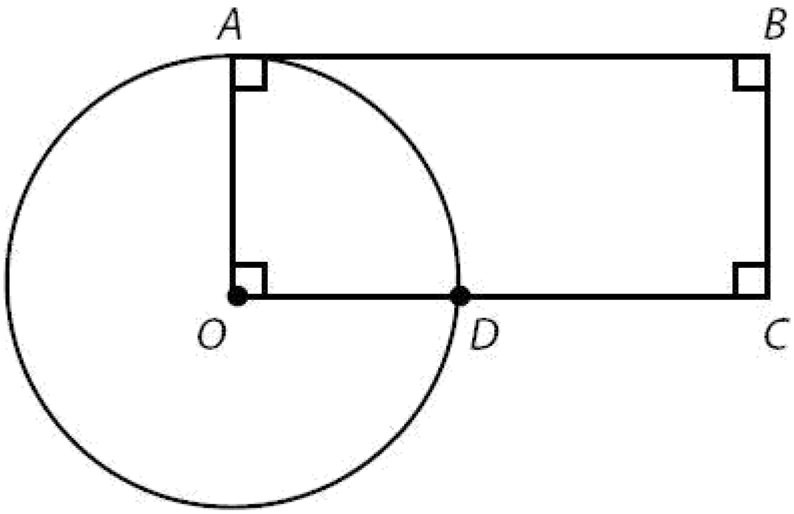
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



Right Triangle *ABC* and Rectangle *EFGH* have the same perimeter. What is the value of *x*?

2.



Point O is the center of the circle.

If the area of the circle is $36p$ and the area of the rectangle is 72,w hat is the length of DC ?

3.The center of a circle is (5,-2).(5,7) is outside the circle,and (1,-2) is inside the circle.If the radius,r,is an integer,how m any possible values are there for r?

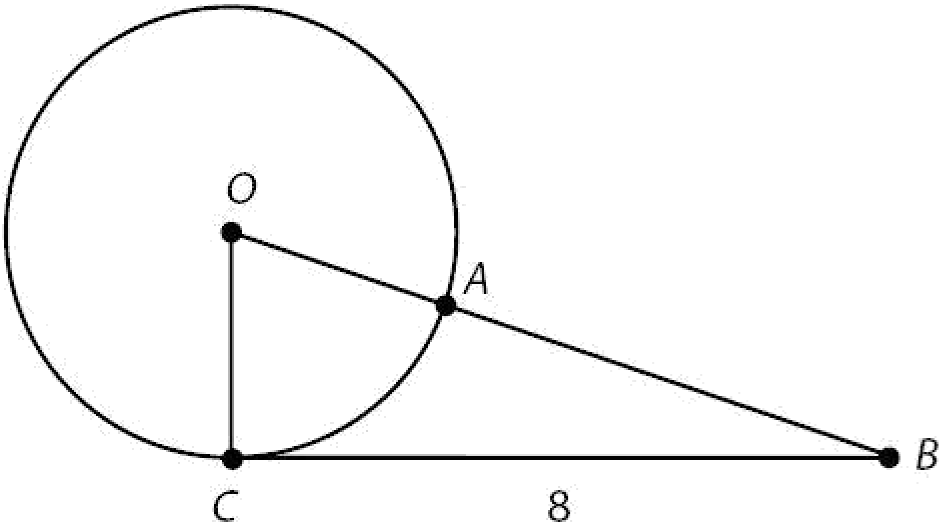
(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

4.

A square's perim eter in inches is equal to its area in square inches.
A circle's circum ference in inches is equal to its area in square inches.

Q uantity A	**Q uantity B**
The side length of the square.	The diam eter of the circle.

5.

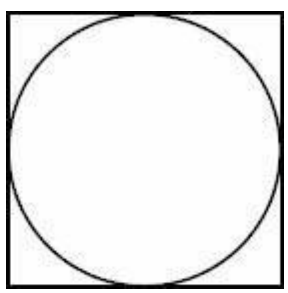


Im age N O T to scale

In the figure above,point *O* is the center of the circle,points *A* and *C* are located on the circle,and segm ent *BC* is tangent to the circle.If the area of triangle *O BC* line is 24,w hat is the length of *AB* ?

(A) 2
(B) 4
(C) 6
(D) 8
(E) 10

6.



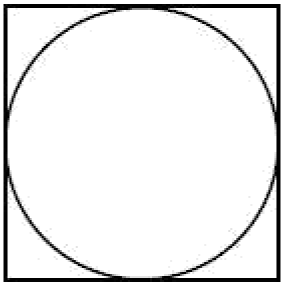
The circle is inscribed in the square.
The area of the circle is 25π.

Q uantity A	**Q uantity B**
The area of the square	50

7.If a circle is inscribed in a square w ith area 16,the area of the circle is equal to how m any π?

π

8.



If the circle is inscribed in the square above,and the area of the square is 50,w hat is the area of the circle?

(A) $\frac{25\pi}{4}$

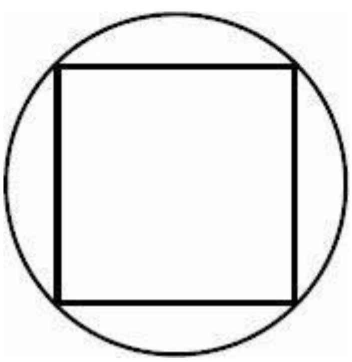
(B) $\frac{25\pi}{2}$

(C) 25π

(D) 50π

(E) $\frac{625\pi}{16}$

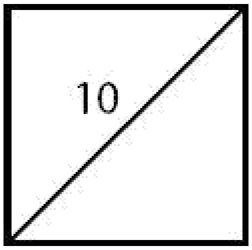
9.



In the figure above,a square is inscribed in a circle.If the area of the square is 4,w hat is the area of the circle?

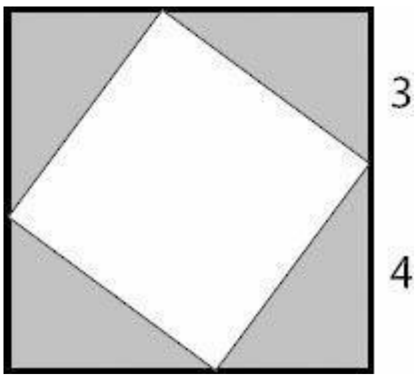
(A) π
(B) 2π
(C) 4π
(D) 6π
(E) 8π

10.



What is the area of the square in the figure above?

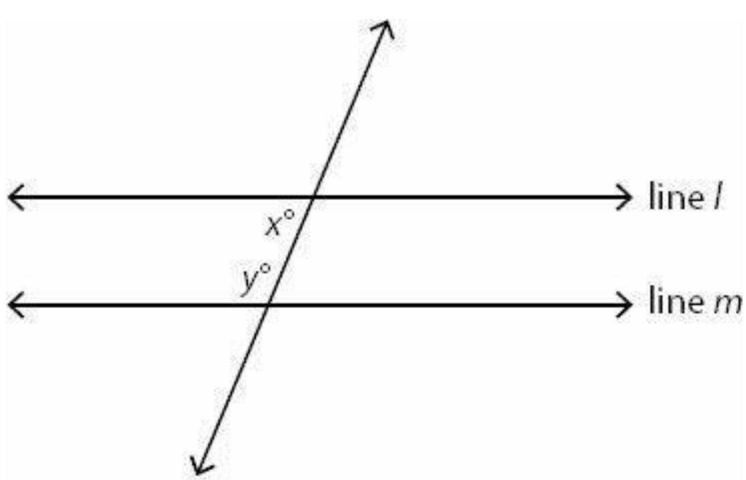
11.



In the 7-inch square above, another square is inscribed. What fraction of the larger square is shaded?

(A) 3/12
(B) 24/49
(C) 1/2
(D) 25/49
(E) 7/12

12.

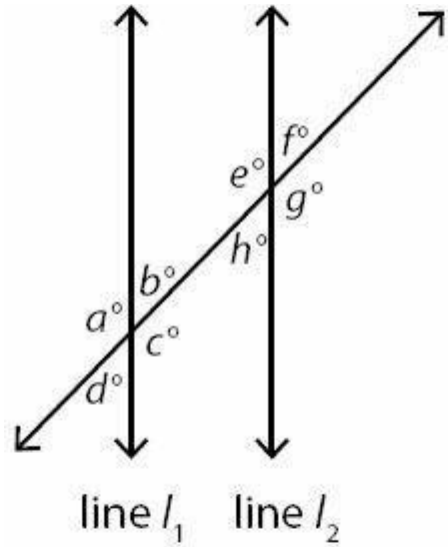


Lines *l* and *m* are parallel.

Q uantity A | **Q uantity B**

$x°$ | $180 - y°$

13.



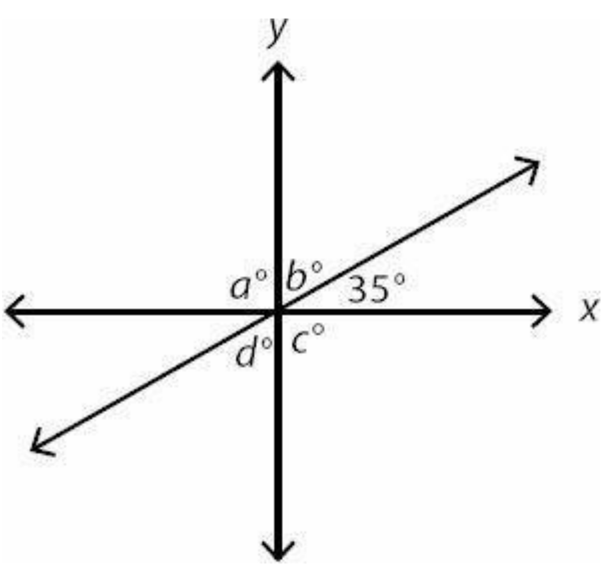
Lines l_1 and l_2 are parallel.

$a > 90$

Q uantity A | **Q uantity B**

$a + g + f$ | $e + b + h$

14.



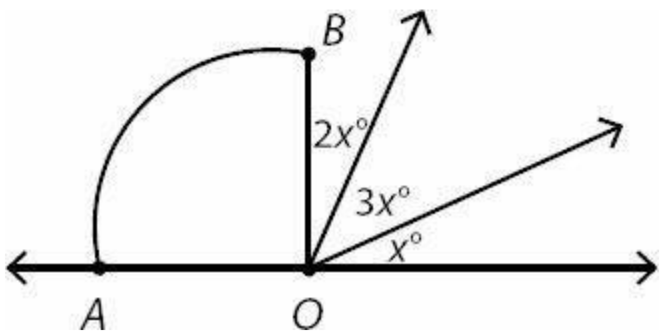
W hat is the value of $a + b + c + d$?

15.

A right isosceles triangle w ith a leg of length *f* has the sam e area as a square w ith a side of length 5.

Q uantity A	**Q uantity B**
f	*s*

16.



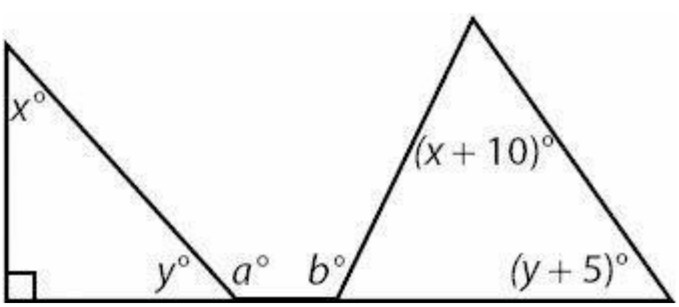
Sector *O AB* is a quarter-circle.

Q uantity A	**Q uantity B**
x	15

17. In the *xy*-plane, an equilateral triangle has vertices at (0,0) and (9,0). W hat could be the coordinates of the third vertex?

(A) (0,4.5)
(B) (4.5,4.5)
(C) $\left(\frac{9\sqrt{3}}{2}, \frac{9\sqrt{3}}{2}\right)$
(D) $(4.5,\ 9\sqrt{3})$
(E) $(4.5,\ \frac{9\sqrt{3}}{2})$

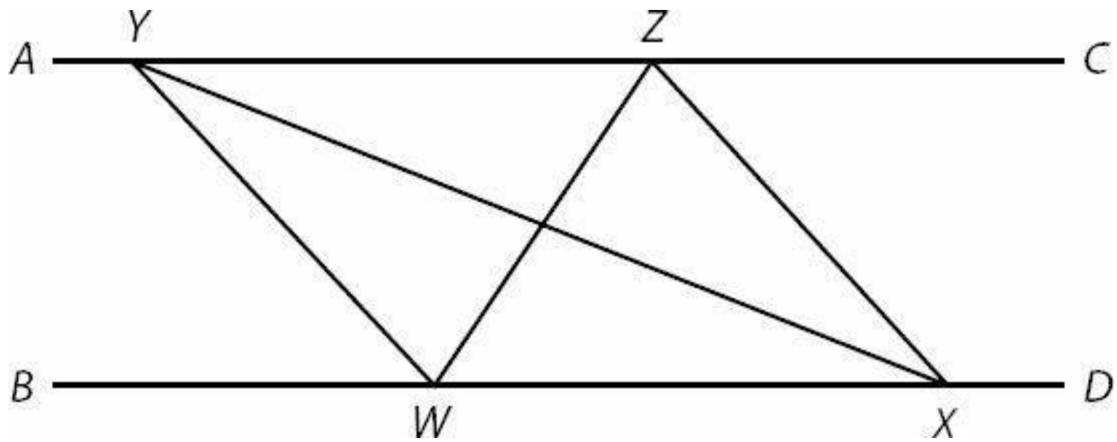
18.



W hat is *a* in term s of *b* and *y*?

(A) $b + y + 65$
(B) $b - y + 65$
(C) $b + y + 75$
(D) $b - 2y + 45$
(E) $b - y + 75$

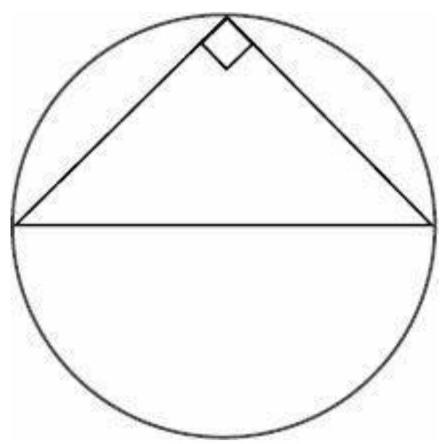
19.



In the figure above, line segm ents AC and BD are parallel.

Q uantity A	**Q uantity B**
The area of triangle WYX	The area of triangle WZX

20.



A right triangle is inscribed in a circle w ith an area of 16π centim eters^2 as show n above.

Q uantity A	**Q uantity B**
The hypotenuse of the triangle, in centim eters	8

21.A rectangular box has a length of 6 cm ,a w idth of 8 cm ,and a height of 10 cm .W hat is the length of the diagonal of the box,in cm ?

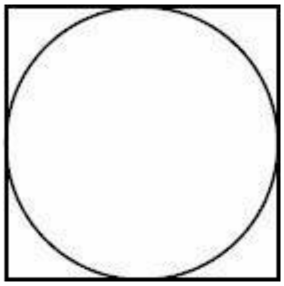
(A) 10
(B) 12
(C) $10\sqrt{2}$

(D) $10\sqrt{3}$
(E) 24

22.If the diagonal of a square garden is 20 feet long,w hat is the perim eter of the garden?

(A) $10\sqrt{2}$ feet
(B) $20\sqrt{2}$ feet
(C) 40 feet
(D) $40\sqrt{2}$ feet
(E) 80 feet

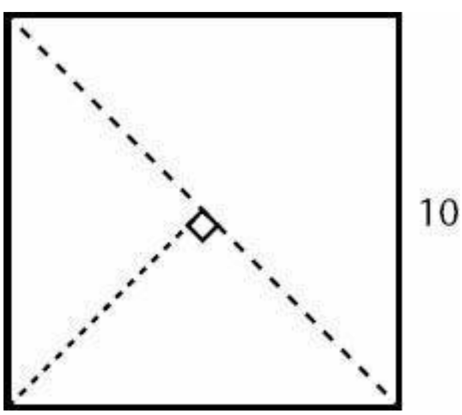
23.



In the figure above,if the diagonal of the square is 12,w hat is the radius of the circle?

(A) $3\sqrt{2}$
(B) 6
(C) $6\sqrt{2}$
(D) 9
(E) 18

24.



Julian takes a 10- by 10-inch square piece of paper and cuts it in half along the diagonal.H e then takes one of the halves and cuts it in half again from the corner to the m idpoint of the opposite side.A ll cuts are represented in the figure w ith dotted lines.W hat is the perim eter of one of the sm allest triangles,in inches?

(A) 10 (B)

$10\sqrt{2}$

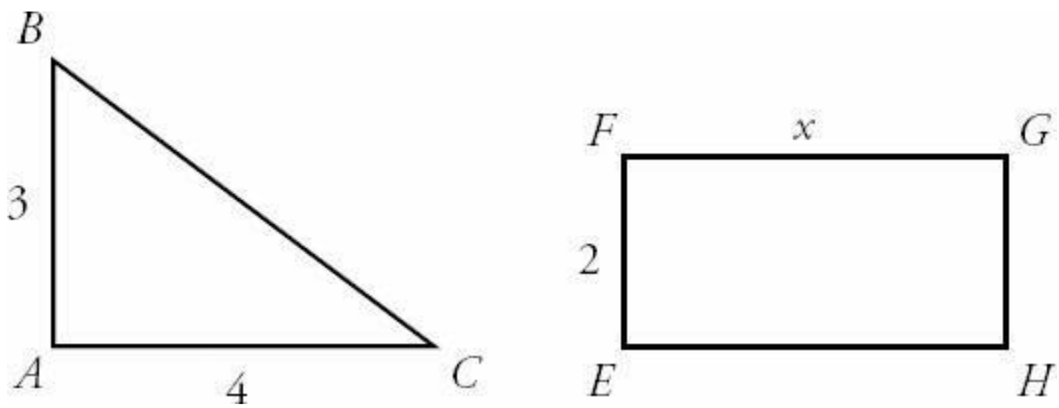
(C) 20

(D) 10 + $10\sqrt{2}$

(E) 10 + $20\sqrt{2}$

Mixed Geometry Answers

1.**4.**



Triangle *ABC* is a right triangle,so you can find the length of hypotenuse *BC* .This is a 3–4–5 triangle,so the length of side *BC* is 5.That means the perimeter of Triangle *ABC* is 3 + 4 + 5 = 12.

That means the perimeter of Rectangle *EFGH* is also 12.That means that 2 × (2 + x) = 12. So 4 + 2x = 12 → 2x = 8 → x = 4.

2.**6.**The area of this circle is 36π and the area of any circle is πr^2,so the radius of this circle is 6.Label both radii (*OA* and *OD*) as 6.Because *ABCO* is a rectangle,its area is base times height,where radius *OA* is the height.

> Area of a rectangle =
> bh 72 = b(6)
> b = 12

Since *OC* is a base of the rectangle,it is equal to 12.Subtract radius *OD* from base *OC* to get the length of segment *DC* : 12 - 6 = 6.

3.**(A).**This problem does not actually require any special formulas regarding circles.Calculate the distance between (5,-2) and (5,7).Since the x-coordinates are the same and 7 - (-2) = 9,the two points are 9 apart.Because (5,7) is outside the circle,the radius must be less than 9.

Similarly,(1,-2) is inside the circle.Calculate the distance between (5,-2) and (1,-2).Since the y-coordinates are the same,the distance is 5 - 1 = 4.Because (1,-2) is inside the circle,the radius must be more than 4.

The radius must be an integer that is greater than 4 and less than 9,so it can only be 5,6,7,or 8.Thus,there are 4 possible values for r.

4.**(C).**The perimeter of a square is 4s and the area of a square is s^2 (where s is a side length).If the square's perimeter equals its area,set the two expressions equal to each other and solve:

$4s = s^2$
$0 = s^2 - 4s$
$0 = s(s - 4)$
$s = 4$ or 0

Only $s = 4$ would result in an actual square, so $s = 0$ is not a valid solution.

The circumference of a circle is $2\pi r$ and the area of a circle is πr^2 (where r is the radius). If the circle's circumference equals its area, set the two expressions equal to each other and solve:

$2\pi r = \pi r^2$
$2r = r^2$
$0 = r^2 - 2r$
$0 = r(r - 2)$
$r = 2$ or 0

Only $r = 2$ would result in an actual circle, so $r = 0$ is not a valid solution.

If the radius of the circle is 2, then the diameter is 4. Thus, the two quantities are each equal to 4.

5. **(B).** Because BC is tangent to the circle, angle OCB is a right angle. Thus, radius OC is the height of the triangle. If the area of the triangle is 24, use the area formula for a triangle (and 8 as the base, from the figure) to determine the height:

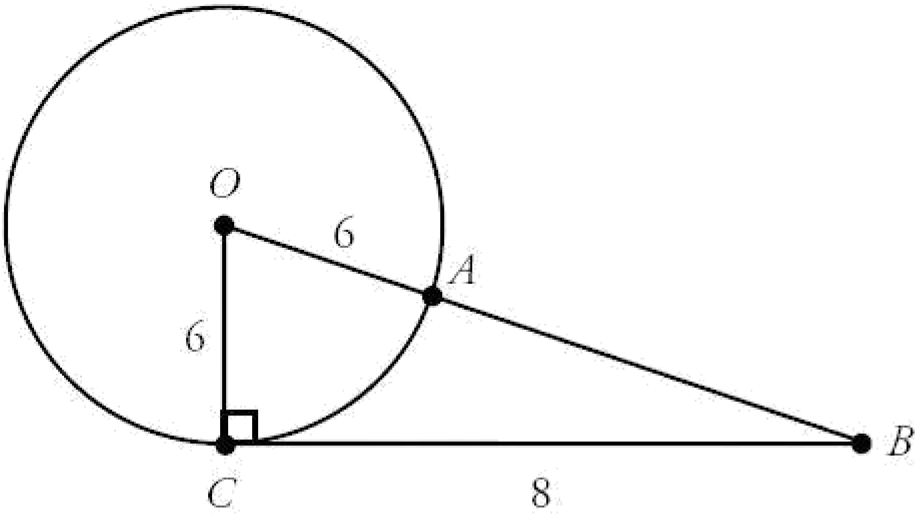
$A = \frac{bh}{2}$

$24 = \frac{8 \times OC}{2}$

$48 = 8 \times OC$

$6 = OC$

Thus, the radius of the circle is 6 (note that you have TWO radii on the diagram, OC and OA). Since you now have two sides of a right triangle, use the Pythagorean theorem to find the third:



Im age N O T to scale

$6^2 + 8^2 = (OB)^2$

$36 + 64 = (OB)^2$

$100 = (OB)^2$

$10 = OB$

(O f course,the 6–8–10 triangle is one of the special right triangles you should m em orize for the G R E!)

Since the hypotenuse OB is equal to 10 and the radius OA is equal to 6,subtract to get the length of AB .The answ er is 10 - 6 = 4.

6.**(A).**The area of the circle = $25\pi = \pi r^2$,so the radius is 5 and therefore the diam eter of the circle is 10.The diam eter of the circle is equal to the side of the square (the circle and square are “equally tall”),so the area of the square is 10 × 10 = 100.

A lternatively,the area of the circle is 25π,w hich is approxim ately 25(3.14),or greater than 75.The square is clearly larger than the circle,so the area of the square is greater than 75,w hich is greater than 50.

7.**4.**If the square has area 16,its sides equal 4.If the square is 4 “tall,” so is the circle.That is,the side of the square is equal to the diam eter of the circle.Since the diam eter of the circle is 4,the radius is 2.For the circle,area $A = \pi r^2 = \pi(2)^2$ or 4π.Since the question asks “how m any π?” and π is already w ritten next to the box,type only 4 in the box.

8.**(B).**If the area of the square is 50,the sides of the square are $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$.

If the square is $5\sqrt{2}$ “tall”,so is the circle.That is,the side of the square is equal to the diam eter of the circle.Since the circle diam eter is $5\sqrt{2}$,the radius is $\frac{5\sqrt{2}}{2}$.U sing the form ula for the area of a circle,$A = \pi r^2$:

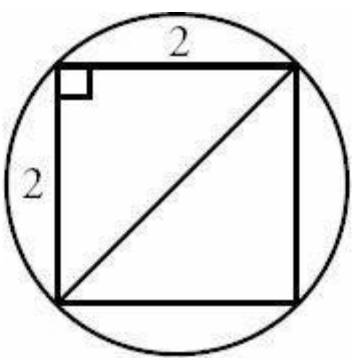
$$A = \pi\left(\frac{5\sqrt{2}}{2}\right)^2$$

$$A = \pi\left(\frac{25 \times 2}{4}\right)$$

$$A = \frac{25\pi}{2}$$

N ote that even if you got a bit lost in the m ath,you could estim ate quite reliably! The square is clearly a bit larger than the circle,so the circle area should be a bit less than 50.Put all the answ ers in your calculator,using 3.14 as an approxim ate value for π,and you w ill quickly see that choice (A) = 19.625,w hich is too sm all,and choice (B) = 39.25,w hile the other three choices are m uch too large (larger than the square!)

9.**(B).**If the area of the square is 4,then the side length is 2.To find the area of the circle,you need the circle's radius,w hich is not obvious yet.D raw a diagonal in the square— this line segm ent is also a diam eter of the circle. Then use the Pythagorean theorem (or the 45–45–90 angle form ula) to find the diagonal length:



$2^2 + 2^2 = d^2$ (w here d is the diagonal of the square and the diam eter of the circle) $8 = d^2$

$\sqrt{8} = d$

B ecause $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$ is the diam eter of the circle,the radius is $1\sqrt{2}$ or just $\sqrt{2}$.The area of the circle is:

$A = \pi r^2$

$A = \pi(\sqrt{2})^2$

$A = 2\pi$

N ote that even if you got a bit lost in this problem ,you could just use com m on sense to estim ate.The circle is obviously larger than the square,so the answ er should be som ew hat larger than 4.Plug in π = 3.14 using your calculator to see w hich choices are reasonable.C hoice (A) is too sm all.C hoice (B) is about 6.28.C hoice (C) is *tw ice* as big,and (D) and (E) are even larger.O nly (B) is reasonable.

10.**50.**O ne w ay to solve this problem is by using the Pythagorean theorem .A ll sides of a square are equal to s,so:

$s^2 + s^2 = 10^2$

$2s^2 = 100$

$s^2 = 50$

N ote that you *could* solve for *s* ($s = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$),but the area of the square is s^2,w hich is already calculated above.The area of the square is 50.

11.**(B).**Each of the shaded triangles is a 3–4–5 Pythagorean triple.(O r,just note that each shaded triangle has legs of 3 and 4;the Pythagorean theorem w ill tell you that each hypotenuse = 5).

Since each hypotenuse is also a side of the square,the square has area 5 × 5 = 25.

The larger square (the overall figure) has area 7 × 7 = 49.

Subtract to find the area of the shaded region: 49 - 25 = 24.

The fraction of the larger square that is shaded is therefore 24/49.

12.**(C).**W hen tw o parallel lines are cut by a transversal,sam e-side interior angles are supplem entary.Thus,$x + y = 180$,and $x = 180 - y$.

13.**(A).**W hile the exact m easures of any of the angles are not given,w hen parallel lines are cut by a transversal,only tw o angle m easures are created: all the “big” angles are the sam e,and all the “sm all” angles are the sam e.Since $a > 90$ (i.e.,the picture is,indeed,the w ay it looks) and $a = c = e = g$,infer that *a*,*c*,*e*,and *g* are all the sam e “big” angle m easure,w hich is greater than 90.

Sim ilarly,$b = d = f = h$,so these are the sam e “sm all” angle m easure,w hich is less than 90.

Q uantity A is the sum of tw o “big” angles and one “sm all.”
Q uantity B is the sum of one “big” angle and tw o “sm all.”
Q uantity A is greater.

If you w ish to try this w ith real num bers,plug in $a = 100$ (for exam ple),and you w ill see that *a*,*c*,*e*,and *g* are all equal to 100,and *b*,*d*,*f*,and *h* are all equal to 80,so Q uantity A w ould equal 280 and Q uantity B w ould equal 260.For any exam ple w ith $a > 90$,Q uantity A w ill be larger.

14.**290.**A ngles that “go around in a circle” sum to 360 degrees.It m ay be tem pting to sim ply subtract 35 from 360 and answ er 325,but don’t overlook the unlabeled angle,w hich is opposite and therefore equal to 35°.So,subtract 35 + 35 = 70 from 360 to get the answ er,290.

15.**(A).**In the right isoceles triangle,the base and height are the perpendicular sides,w hich are each of length *f*.Thus, A rea = $\frac{f^2}{2}$ .The square,of course,has area s^2 .Thus, $\frac{f^2}{2} = s^2$,or $f^2 = 2s^2$ .Since *f* and *s* are definitely positive,*f* is larger than *s*.

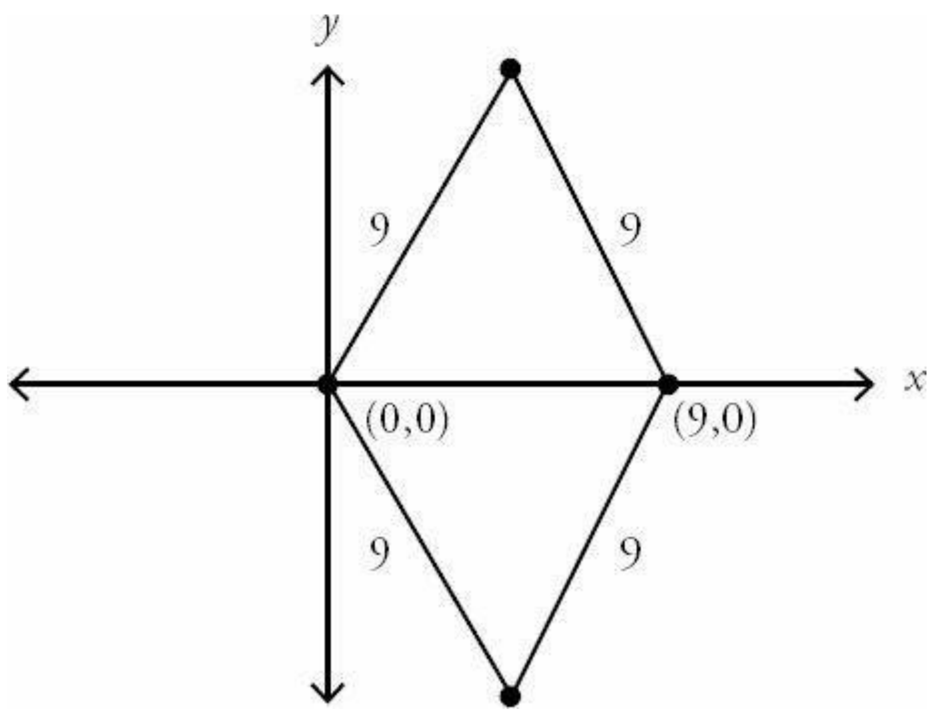
A lternatively,you could envision that if the areas w ere equal,the triangle sides w ould have to be m uch longer.A fter all,an isosceles right triangle is only half of a square.

16.**(C).**If sector *O AB* is a quarter-circle,then the angle at *O* m easures 90°.Thus,since angles that m ake up a straight line m ust sum to 180,$2x + 3x + x$ m ust sum to 90:

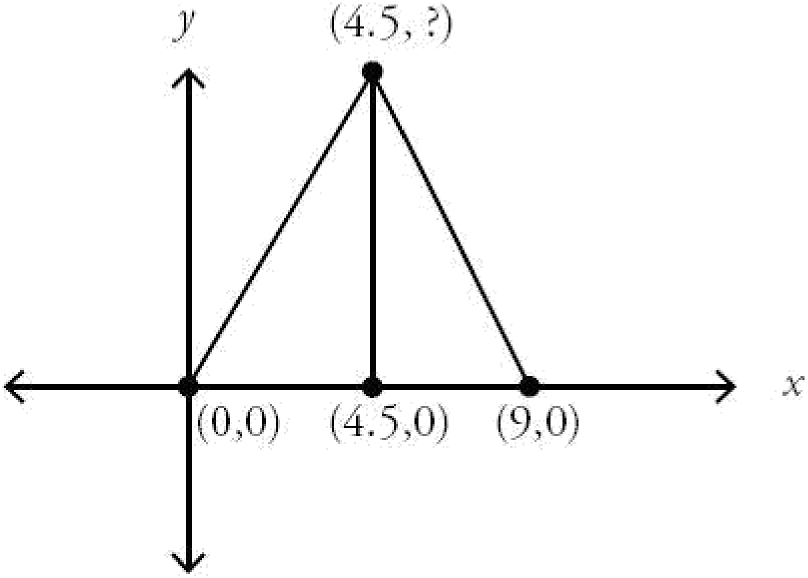
$2x + 3x + x = 90$
$6x = 90$
$x = 15$

The tw o quantities are equal.

17.**(E).**G iven the tw o vertices at (0,0) and (9,0),there are only tw o possible locations for the third coordinate: (9,0)



A ll of the answ er choices are positive,so focus on the “upper” option.Since equilateral triangles are sym m etrical,the *x*-coordinate of the third vertex w ill be halfw ay betw een 0 and 9,or 4.5.Thus,the answ er m ust be (B),(D),or (E).
D raw the height of the triangle:



The height of an equilateral triangle is given by $h = \frac{s\sqrt{3}}{2}$ .A lternatively,note that the height cuts the 60–60–60 triangle into tw o 30–60–90 triangles.U se the side length ratios for a 30–60–90 triangle (1 :$\sqrt{3}$: 2) to determ ine that,since the side across from the 30-degree angle is equal to 4.5,the side across from the 60-degree angle w ill be equal to 4.5 $\sqrt{3}$ or $\frac{9\sqrt{3}}{2}$.

This height is the *y*-coordinate of the third vertex.The answ er is (4.5, $\frac{9\sqrt{3}}{2}$).

18.**(E).**A n exterior angle of a triangle is equal to the sum of the tw o opposite interior angles.From the left triangle,$a = x + 90$.From the right triangle,$b = (x + 10) + (y + 5) = x + y + 15$.

A lternatively,you could use the facts that the interior angles of a triangle sum to 180,as do angles that form a straight line.From the left triangle,$x + y + 90$ and $y + a$ both equal 180,so $x + y + 90 = y + a$,or $x + 90 = a$.From the right triangle,$180 = (x + 10) + (y + 5) + (180 - b)$,or $b = x + y + 15$.

The question asks for *a* in term s of *b* and *y*,so *x* is the variable that needs to be elim inated.D o so by solving one equation for *x*,and substituting this expression for *x* in the other equation.

From the right triangle: $b = x + y + \longrightarrow x = b - y - 15$

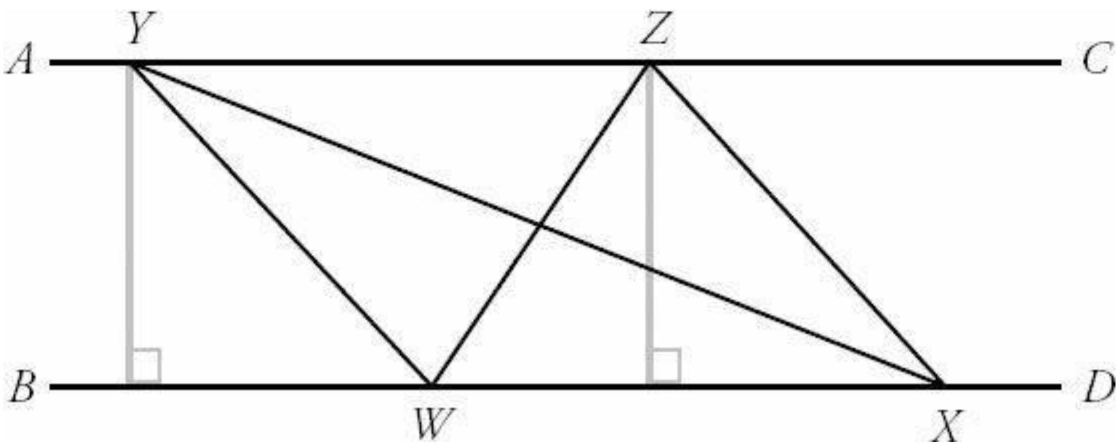
From the left triangle:

$a = x + 90$
$a = (b - y - 15) + 90$
$a = b - y + 75$

19.**(C).**B oth triangles,*W YX* and *W ZX* ,share a com m on base of segm ent *W X* .A s the area of a triangle is given by

A rea = (1/2)(base)(height)

and both triangles have equal bases,you can determ ine w hich has a greater area by determ ining w hich has a greater height.The height is a perpendicular line draw n from the highest point on the triangle to the base.In this case,the heights w ould be given in gray below :



B y the definition of parallel lines,*AC* and *BD* are uniform distance apart.Therefore,the heights show n are the sam e. B ecause these triangles have equal bases and heights,they m ust have equal area.

20.**(C).**To solve this problem ,recall that a triangle inscribed in a sem i-circle w ill be a right triangle *if and only if* one side of the triangle is the diam eter (i.e.,the center of the circle m ust lie on one side of the triangle).B ecause this is a right triangle,the hypotenuse m ust be the diam eter of the circle.

To find the diam eter of the circle,recall the form ula for area: A rea = πr^2.

$$16\pi \text{ cm}^2 = \pi r^2$$
$$16 \text{ cm}^2 = r^2$$
$$r = 4 \text{ cm}$$

G iven that diam eter is tw ice the radius,the diam eter (i.e.,the hypotenuse of the triangle) is 8 cm .Q uantity A is 8, m aking the tw o quantities equal.

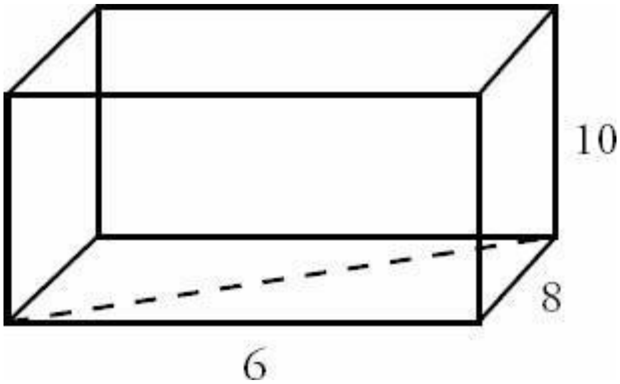
21.**(C).**The fastest approach to solving this problem is to use the “Super Pythagorean Theorem ,” w hich states that the diagonal of any rectangular box is given by:

$$d^2 = l^2 + w^2 + h^2$$

W here *l*,*w* ,and *h* are the length,w idth,and height of the box,respectively.Plugging in yields

$$d^2 = 6^2 + 8^2 + 10^2$$
$$d^2 = 36 + 64 + 100$$
$$d^2 = 200$$
$$d = 10\sqrt{2}$$

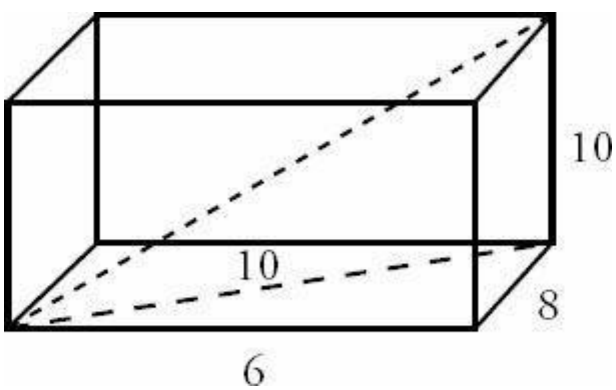
A lternatively,one could avoid the Super Pythagorean Theorem by applying the norm al Pythagorean theorem tw ice.To find the diagonal of the box,you m ust first find the diagonal of one of the sides.C hoosing this side as the base,



w here the dashed line represents the diagonal of the base.A pplying the Pythagorean theorem :

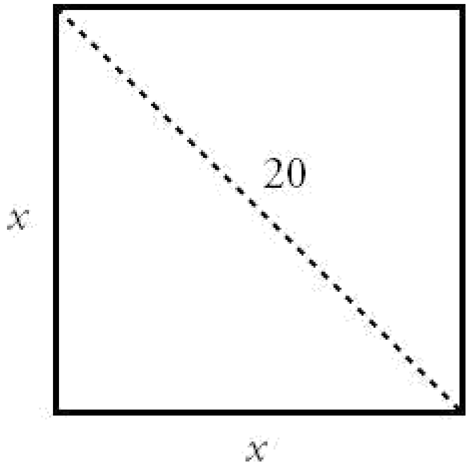
$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = 10$$

From here,draw the diagonal of the box and apply the Pythagorean theorem again as show n.



$$d^2 = 10^2 + 10^2$$
$$d^2 = 100 + 100$$
$$d^2 = 200$$
$$d = 10\sqrt{2}$$

22.**(D).**D raw the follow ing figure to represent the square garden.



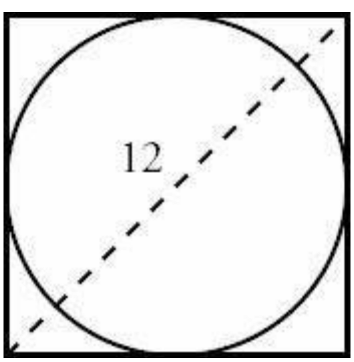
Label the diagonal w ith the given length of 20 and the sides w ith the variable x.The diagonal form s the hypotenuse of a right triangle w ith legs of length x.U sing Pythagorean theorem ,solve for the length of the legs as

$$x^2 + x^2 = 20^2$$
$$2x^2 = 400$$
$$x^2 = 200$$
$$x = 10\sqrt{2}$$

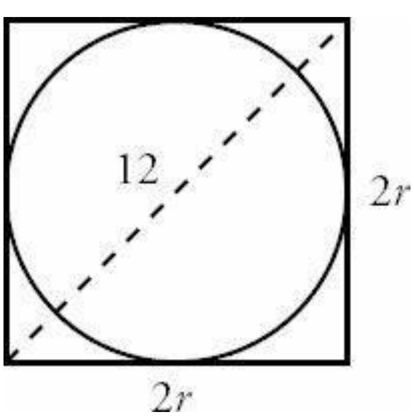
The perim eter of a square is 4 × (side length). Perim eter = 4 × ($10\sqrt{2}$)

Perim eter = $40\sqrt{2}$

23.**(A).**B egin by diagram m ing the figure,labeling the diagonal of the circle as 12.



From here,recognize that the square is as “tall” as the circle,or the side length of the square equals the diam eter of the circle,w hich is $2r$.



By the Pythagorean theorem :

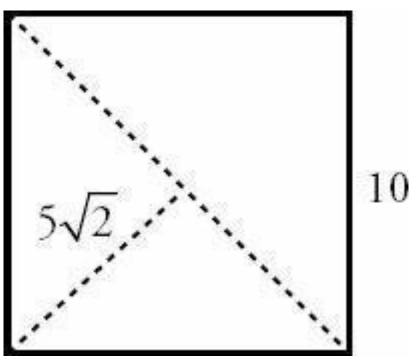
$(2r)^2 + (2r)^2 =$
12^2 $4r^2 + 4r^2 =$
144 $8r^2 = 144$
$r^2 = 18$ r
$= 3\sqrt{2}$

24.**(D).**In order to compute the perimeter of one of the smaller triangles,first compute the length of the diagonal. For a square with side length 10 inches,the length of the diagonal can be computed by the Pythagorean theorem :

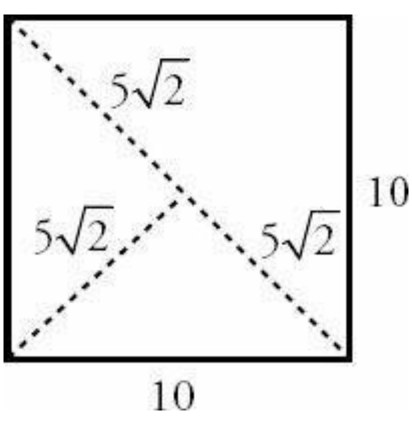
$(\text{diagonal})^2 = (\text{side})^2 + (\text{side})^2$
$(\text{diagonal})^2 = 10^2 + 10^2$
$(\text{diagonal})^2 = 200$
$\text{diagonal} = 10\sqrt{2}$

Alternatively,recognize that the diagonal of a square is always $\sqrt{2}$ times the side length.

The second cut goes from the corner to the midpoint of the diagonal,so that slice is half as long as the diagonal of the square: $\frac{10\sqrt{2}}{2} = 5\sqrt{2}$ .This can be seen as



Similarly,because the remaining line in each of the smaller triangles is half of a diagonal,each is of length $5\sqrt{2}$ inches.



A dding up the lengths of the sides,the perim eter of the sm allest triangle is

Perim eter = 10 + $5\sqrt{2}$ +
$5\sqrt{2}$ Perim eter = 10 + $10\sqrt{2}$