D ivisibility and P rim es

For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given),the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box $\boxed{}$,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes $\frac{\boxed{}}{\boxed{}}$,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.

A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.For how m any positive integer values of x is $\frac{65}{x}$ an integer?

$\boxed{}$

2.If x is a num ber such that $0 < x \leq 20$,for how m any values of x is $\frac{20}{x}$ an integer?

(A) 4
(B) 6
(C) 8
(D) 10
(E) M ore than 10

3.

Quantity A	Quantity B
The num ber of even factors of 27	The num ber of even factors of 81

4.

Quantity A	Quantity B
The num ber of distinct factors of 10	The num ber of distinct prim e factors of 210

5.

Quantity A	Quantity B
The least com m on m ultiple of 22 and 6	The greatest com m on factor of 66 and 99

6.The num ber of students w ho attend a school could be divided am ong 10,12,or 16 buses,such that each bus transports an equal num ber of students.W hat is the m inim um num ber of students that could attend the school?

(A) 120
(B) 160
(C) 240
(D) 320
(E) 480

7.

Quantity A	Quantity B
The num ber of distinct prim e factors of 27	The num ber of distinct prim e factors of 18

8.

Quantity A	Quantity B
The num ber of distinct prim e factors of 31	The num ber of distinct prim e factors of 32

9.H ow m any factors greater than 1 do 120,210,and 270 have in com m on?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 30

10.C om pany H distributed $4,000 and 180 pencils evenly am ong its em ployees,w ith each em ployee getting an equal integer num ber of dollars and an equal integer num ber of pencils.W hat is the greatest num ber of em ployees that could w ork for C om pany H ?

(A) 9
(B) 10
(C) 20
(D) 40
(E) 180

11.*n* is divisible by 14 and 3.W hich of the follow ing statem ents m ust be true?

Indicate <u>all</u> such statem ents.

☐ 12 is a factor of *n*
☐ 21 is a factor of *n*
☐ *n* is a m ultiple of 42

12.Positive integers *a* and *b* each have exactly four factors.If *a* is a one-digit num ber and *b* = *a* + 9,w hat is the value of *a*?

13.R am on w ants to cut a rectangular board into identical square pieces.If the board is 18 inches by 30 inches,w hat is the least num ber of square pieces he can cut w ithout w asting any of the board?

(A) 4
(B) 6
(C) 9
(D) 12
(E) 15

14.If *n* is the product of 2,3,and a tw o-digit prim e num ber,how m any of its factors are greater than 6?

15.

m is a positive integer that has a factor of 8.

<u>Q uantity A</u>	**<u>Q uantity B</u>**
The rem ainder w hen *m* is divided by 6	The rem ainder w hen *m* is divided by 12

16.W hen the positive integer *x* is divided by 6,the rem ainder is 4.Each of the follow ing could also be an integer EX C EPT

(A) $\frac{x}{2}$

(B) $\frac{x}{3}$

(C) $\frac{x}{7}$

(D) $\frac{x}{11}$

(E) $\frac{x}{17}$

17.If $x^y = 64$ and x and y are positive integers,w hich of the follow ing could be the value of $x + y$?

Indicate <u>all</u> such values.

☐ 2
☐ 6
☐ 7
☐ 8
☐ 10
☐ 12

18.If k is a m ultiple of 24 but not a m ultiple of 16,w hich of the follow ing cannot be an integer?

(A) $\frac{k}{8}$

(B) $\frac{k}{9}$

(C) $\frac{k}{32}$

(D) $\frac{k}{36}$

(E) $\frac{k}{81}$

19.If $a = 16b$ and b is a prim e num ber greater than 2,how m any positive distinct factors does a have?

20.If *a* and *c* are positive integers and $4a + 3 = b$ and $4c + 1 = d$,w hich of the follow ing could be the value of $b + d$?

(A) 46
(B) 58
(C) 68
(D) 74
(E) 82

21.Each factor of 210 is inscribed on its ow n plastic ball,and all of the balls are placed in a jar.If a ball is random ly selected from the jar,w hat is the probability that the ball is inscribed w ith a m ultiple of 42?

(A) $\frac{1}{16}$
(B) $\frac{5}{42}$
(C) $\frac{1}{8}$
(D) $\frac{3}{16}$
(E) $\frac{1}{4}$

22.A t the C anterbury D og Fair,1/4 of the poodles are also show dogs and 1/7 of the show dogs are poodles.W hat is the least possible num ber of dogs at the fair?

23.A “prim e pow er” is an integer that has only one prim e factor.For exam ple,5 = 5,25 = 5 × 5,and 27 = 3 × 3 × 3 are all prim e pow ers,w hile 6 = 2 × 3 and 12 = 2 × 2 × 3 are not.W hich of the follow ing num bers is not a prim e pow er?

(A) 49
(B) 81
(C) 100
(D) 121
(E) 243

24.If *a* and *b* are integers such that $a > b > 1$,w hich of the follow ing cannot be a m ultiple of either *a* or *b*?

(A) $a - 1$

(B) $b + 1$
(C) $b - 1$
(D) $a + b$
(E) ab

25.616 divided by 6 yields rem ainder p,and 525 divided by 11 yields rem ainder q.W hat is $p + q$?

26.If x is divisible by 18 and y is divisible by 12,w hich of the follow ing statem ents m ust be true?

Indicate all such statem ents.

☐ $x + y$ is divisible by 6
☐ xy is divisible by 48
☐ x/y is divisible by 6

27.If p is divisible by 7 and q is divisible by 6,pq m ust have at least how m any factors greater than 1?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 8

28.If r is divisible by 10 and s is divisible by 9,rs m ust have at least how m any factors?

(A) 2
(B) 4
(C) 12
(D) 14
(E) 16

29.If t is divisible by 12,w hat is the least possible integer value of a for w hich $\frac{t^2}{2^a}$ m ight not be an integer?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

30.If a,b,and c are m ultiples of 3 such that $a > b > c > 0$,w hich of the follow ing values m ust be divisible by 3?

Indicate all such values.

☐ $a + b + c$
☐ $a - b + c$
☐ $abc/9$

31.N ew cars leave a car factory in a repeating pattern of red,blue,black,and gray cars.If the first car to exit the factory w as red,w hat color is the 463rd car to exit the factory?

(A) red
(B) blue
(C) black
(D) gray
(E) It cannot be determ ined from the inform ation given.

32.Jason deposits m oney at a bank on a Tuesday and returns to the bank 100 days later to w ithdraw the m oney.O n w hat day of the w eek did Jason w ithdraw the m oney from the bank?

(A) M onday
(B) Tuesday
(C) W ednesday
(D) Thursday
(E) Friday

33.*x* and *h* are both positive integers.W hen *x* is divided by 7,the quotient is *h* w ith a rem ainder of 3.W hich of the follow ing could be the value of *x*?

(A) 7
(B) 21
(C) 50
(D) 52
(E) 57

34.*a*,*b*,*c*,and *d* are all positive integers.If $\frac{ab}{c+d} = 3.7$,w hich of the follow ing statem ents m ust be true?

Indicate <u>all</u> such statem ents.

☐ *ab* is divisible by 5.
☐ *c* + *d* is divisible by 5.
☐ If *c* is even,then *d* m ust be even.

35.W hen *x* is divided by 10,the quotient is *y* w ith a rem ainder of 4.If *x* and *y* are both positive integers,w hat is the rem ainder w hen *x* is divided by 5?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

36.W hat is the rem ainder w hen $13^{17} + 17^{13}$ is divided by 10?

37. a, b, c, and d are positive integers. If $\frac{a}{b}$ has a rem ainder of 9 and $\frac{c}{d}$ has a rem ainder of 10, w hat is the m inim um possible value for bd?

38. If n is an integer and n^3 is divisible by 24, w hat is the largest num ber that m ust be a factor of n?

(A) 1
(B) 2
(C) 6
(D) 8
(E) 12

39.

10! is divisible by 3^x5^y, w here x and y are positive integers.

Q uantity A	Q uantity B
The greatest possible value for x	Tw ice the greatest possible value for y

40.

Q uantity A	Q uantity B
The num ber of distinct prim e factors of 100,000	The num ber of distinct prim e factors of 99,000

41. For w hich tw o of the follow ing values is the product a m ultiple of 27?

Indicate tw o such values.

☐ 1
☐ 7
☐ 20
☐ 28
☐ 63
☐ 217
☐ 600
☐ 700

42. W hich of the follow ing values tim es 12 is not a m ultiple of 64?

Indicate all such values.

☐ 6^6
☐ 12^2
☐ 18^3
☐ 30^3
☐ 222

43.If $3^x(5^2)$ is divided by $3^5(5^3)$,the quotient term inates w ith one decim al digit.If $x > 0$,w hich of the follow ing statem ents m ust be true?

(A) x is even
(B) x is odd
(C) $x < 5$
(D) $x \geq 5$
(E) $x = 5$

44.<u>*abc*</u> is a three-digit num ber in w hich *a* is the hundreds digit,*b* is the tens digit,and *c* is the units digit.Let & (<u>*abc*</u>)& = $(2^a)(3^b)(5^c)$.For exam ple,& (203)& = $(2^2)(3^0)(5^3)$ = 500.For how m any three-digit num bers <u>*abc*</u> w ill the function & (<u>*abc*</u>)& yield a prim e num ber?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 9

D ivisibility and P rim es A nsw ers

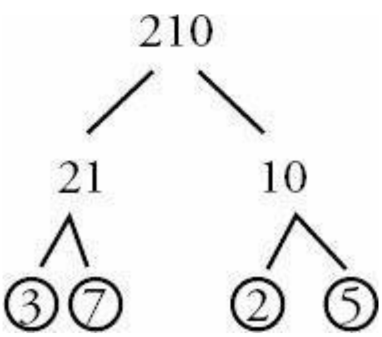
1.**4.**If *x* is a positive integer such that $\frac{65}{x}$ is also an integer,then *x* m ust be a factor of 65.The factors of 65 are 1,5, 13,and 65.Thus,there are 4 positive integer values of *x* such that $\frac{65}{x}$ is an integer.

2.**(E).**N otice that the problem did N O T say that *x* had to be an integer.Therefore,the factors of 20 w ill w ork (1,2,4, 5,10,20),but so w ill 0.5,0.1,0.25,2.5,etc.It is possible to divide 20 into fractional parts— for instance,som ething 20 inches long could be divided evenly into quarter inches (there w ould be 80 of them ,as $\frac{20}{0.25} = 80$).There are an infinite num ber of *x* values that w ould w ork (it is possible to divide 20 into thousandths,m illionths,etc.),so the answ er is (E).It is very im portant on the G R E to notice w hether there is an integer constraint on a variable or not! A ny answ er like “M ore than 10” should be a clue that this problem m ay be less straightforw ard than it seem s.

3.**(C).**W hen counting factors,it helps to list them in pairs so you don’t m iss any.The factors of 27 are: 1 & 27,3 & 9.The factors of 81 are: 1 & 81,3 & 27,9 & 9.N either num ber has any even factors,so Q uantity A and Q uantity B are each 0 and therefore equal.

4.**(C).**The *factors* of 10 are 1 & 10,and 2 & 5.Since there are 4 factors,Q uantity A is 4.

The *prim e factors* of 210 are 2,3,5,and 7.



210 has 4 prim e factors,so Q uantity B is 4.Thus,the tw o quantities are equal.

5.**(A).**The least com m on m ultiple of 22 and 6 is 66.O ne w ay to find the least com m on m ultiple is to list the larger num ber’s m ultiples (it is m ore efficient to begin w ith the larger num ber) until you reach a m ultiple that the other num ber goes into.The m ultiples of 22 are 22,44,66,88,etc.The sm allest of these that 6 goes into is 66.

The greatest com m on factor of 66 and 99 is 33.O ne w ay to find the greatest com m on factor is to list all the factors of one of the num bers,and then pick the greatest one that also goes into the other num ber.For instance,the factors of 66 are 1 & 66,2 & 33,3 & 22,and 6 & 11.The greatest of these that also goes into 99 is 33.Thus,Q uantity A is greater.

6.**(C).**The num ber of students m ust be divisible by 10,12,and 16.So the question is really asking,“W hat is the least com m on m ultiple of 10,12,and 16?” Since all of the answ er choices end in 0,each is divisible by 10.Just use your calculator to test w hich choices are also divisible by 12 and 16.B ecause you are looking for the m inim um ,start by checking the sm allest choices.Since $\frac{120}{16}$ and $\frac{160}{12}$ are not integers,the sm allest choice that w orks is 240.

7.**(B).***D istinct* m eans *different from each other*.To find *distinct prim e factors*,m ake a prim e tree,and then disregard any repeated prim e factors.The integer 27 breaks dow n into 3 × 3 × 3.Thus,27 has only 1 *distinct* prim e factor.The integer 18 breaks dow n into 2 × 3 × 3.Thus,18 has 2 *distinct* prim e factors.

8.**(C).***D istinct* m eans *different from each other*.To find *distinct prim e factors*,you w ould generally m ake a prim e tree,and then disregard any repeated prim e factors.H ow ever,31 *is* prim e,so 31 is the only prim e factor of 31 and Q uantity A is 1.

A ny correct prim e tree you m ake for 32 w ill result in five 2’s,so 32 equals 2^5.Since this is the sam e prim e factor repeated five tim es,32 has only one *distinct* prim e factor.Q uantity B is 1,so the quantities are equal.

9.**(D).**Pick one of the num bers and list all of its factors on your paper.The factors of 120 are: 1 & 120,2 & 60,3 & 40,4 & 30,5 & 24,6 & 20,8 & 15,10 & 12.Since the problem specifically asks for factors “greater than 1,” elim inate 1 now .N ow cross off any factors that do N O T go into 210:

~~120~~, 2 & ~~60~~, 3 & ~~40~~, ~~4~~ & 30, 5 & ~~24~~, 6 & ~~20~~, ~~8~~ & 15, 10 & ~~12~~

N ow cross off any factors rem aining that do N O T go into 270.Interestingly,all of the rem aining factors (2,3,5,6, 10,15,30) *do* go into 270.This is 7 shared factors.

10.**(C).**In order to distribute $4,000 and 180 pencils evenly,the num ber of em ployees m ust be a factor of each of these tw o num bers.B ecause you are looking for the greatest num ber of em ployees possible,start by checking the greatest choices.

(E) $4000 could not be evenly distributed am ong 180 em ployees (although 180 pencils could). (D) $4,000 could be evenly divided am ong 40 people,but 180 pencils could not.
(C) is the greatest choice that w orks— $4,000 and 180 pencils could each be evenly distributed am ong 20 people.

A lternatively,find the greatest com m on factor (G C F) of the tw o num bers.Factor: 4,000 = 2 × 2 × 2 × 2 × 2 × 5 × 5 × 5 = $2^5 \times 5^3$ and 180 = 2 × 2 × 3 × 3 × 5 = $2^2 \times 3^2 \times 5$.These num bers have 2 × 2 × 5 in com m on,so 20 is the G C F. The correct answ er is (C).

11.**II and III only.**Since *n* is divisible by 14 and 3,*n* contains the prim e factors of both 14 and 3,w hich are 2,7,and 3.Thus,any num bers that can be constructed using only these prim e factors (no additional factors) are factors of *n*. Since 12 = 2 × 2 × 3,you C A N N O T m ake 12 by m ultiplying the prim e factors of *n* (you w ould need one m ore 2).
H ow ever,you C A N construct 21 by m ultiplying tw o of the know n prim e factors of *n* (7 × 3 = 21),so the second statem ent is true.Finally,*n* m ust be at least 42 (= 2 × 7 × 3,the *least com m on m ultiple* of 14 and 3),so *n* is definitely a m ultiple of 42.That is,*n* can only be 42,84,126,etc…

12.**6.**Start by considering integer *a*,w hich is the m ost constrained variable.It is a positive one-digit num ber (betw een 1 and 9,inclusive) and it has four factors.Prim e num bers have exactly tw o factors: them selves and one,so you only need to look at non-prim e one-digit positive integers.That's a short enough list:

> 1 has just one factor!
> 4 has 3 factors: 1,2,and 4
> 6 has 4 factors: 1,2,3,and 6
> 8 has 4 factors: 1,2,4,and 8
> 9 has 3 factors: 1,3,and 9

So the tw o possibilities for *a* are 6 and 8.N ow apply the tw o constraints for *b*.It is 9 greater than *a*,and it has exactly four factors.C heck the possibilities:

> If $a = 6$,then $b = 15$,w hich has 4 factors: 1,3,5,and 15.
> If $a = 8$,then $b = 17$,w hich is prim e,so it has only has 2 factors: 1 and 17.
> O nly $b = 15$ w orks,so *a* m ust be 6.

13.**(E).**C utting a rectangular board into square pieces m eans that R am on needs to cut pieces that are equal in length and w idth."W ithout w asting any of the board" m eans that he needs to choose a side length that divides evenly into both 18 and 30."The least num ber of square pieces" m eans that he needs to choose the largest possible squares.W ith these three stipulations,choose the largest integer that divides evenly into 18 and 30,or the greatest com m on factor,w hich is 6.This w ould give R am on 3 pieces going one w ay and 5 pieces going the other.H e w ould cut $3 \times 5 = 15$ squares of dim ension 6" × 6".N ote that this solution ignored squares w ith non-integer side length for the sake of convenience,a potentially dangerous thing to do.(A fter all,identical squares of 1.5" by 1.5" could be cut w ithout w asting any of the board.) H ow ever,to cut few er squares that are larger than 6" × 6",R am on could only cut 2 squares of 9" or 1 square of 18" from the 18" dim ension of the rectangle,neither of w hich w ould evenly divide the 30" dim ension of the rectangle. The com puted answ er is correct.

14.**4.**B ecause this is a num eric entry question,you can infer that the answ er w ill be the sam e regardless of w hich tw o-digit prim e you pick.So for the sake of sim plicity,pick the sm allest and m ost fam iliar tw o-digit prim e: 11.

If *n* is the product of 2,3,and 11,*n* equals 66 and its factors are:

Sm all	Large
1	66
2	33
3	22
6	11

There are four factors greater than 6: 11,22,33,and 66.

N otice that because the other given prim e factors of *n* (2 and 3) m ultiply to get exactly 6,you can only produce a factor greater than 6 by m ultiplying by the third factor,the "tw o-digit prim e num ber." The right-hand colum n represents that third factor m ultiplied by all of the other factors: 11×6,11×3,11×2,and 11×1.If you replace 11 w ith any other tw o-digit prim e,you w ill get the sam e result.(If you're not sure,try it!)

15.**(D).** Test values for m with the goal of proving (D). Because m has a factor of 8, m could equal 8, 16, 24, 32, 40, etc. If m is 24, both quantities are equal to 0. But if m is 32, Quantity A is 2 and Quantity B is 8.

16.**(B).** When dealing with remainder questions on the GRE, the best thing to do is test a few real numbers.

Multiples of 6 are 0, 6, 12, 18, 24, 30, 36, etc.

Numbers with a remainder of 4 when divided by 6 are those 4 greater than the multiples of 6:

x could be 4, 10, 16, 22, 28, 34, 40, etc.

You could keep listing numbers, but this is probably enough to establish a pattern.

(A) $x/2 \longrightarrow$ ALL of the listed x values are divisible by 2. Eliminate (A).
(B) $x/3 \longrightarrow$ NONE of the listed x values are divisible by 3, but continue checking.
(C) $x/7 \longrightarrow$ 28 is divisible by 7.
(D) $x/11 \longrightarrow$ 22 is divisible by 11.
(E) $x/17 \longrightarrow$ 34 is divisible by 17.

The question is "Each of the following could also be an integer EXCEPT." Since four of the choices could be integers (B) must be the answer.

17.**III, IV, and IV only.** If $x^y = 64$ and x and y are positive integers, perhaps the most obvious possibility is that $x = 8$ and $y = 2$. However, "<u>all</u> such values" implies that other solutions are possible. One shortcut is noting that only an even base, when raised to a power, could equal 64. So you only have to worry about even possibilities for x. Here are all the possibilities:

$2^6 = 64 \longrightarrow x + y = 8$
$4^3 = 64 \longrightarrow x + y = 7$
$8^2 = 64 \longrightarrow x + y = 10$
$64^1 = 64 \longrightarrow x + y = 65$

The only possible values of $x + y$ listed among the choices are 7, 8, and 10.

18.**(C).** If k is a multiple of 24, it contains the prime factors of 24: 2, 2, 2, and 3. (It could also contain other prime factors, but you can only be sure of the prime factors contained in 24.)

If k were a multiple of 16, it would contain the prime factors of 16: 2, 2, 2, and 2.

Thus, if k is a multiple of 24 but NOT of 16, k must contain 2, 2, and 2, but NOT a fourth 2 (otherwise, it would be a multiple of 16).

Thus: k definitely has 2, 2, 2, and 3. It could have any other prime factors (including more 3's) EXCEPT for more 2's.

An answer choice in which the denominator contains more than three 2's would guarantee a non-integer result. Only choice (C) works. Since k has fewer 2's than 32, $k/32$ can never be an integer.

Alternatively, list multiples of 24 for k: 24, 48, 72, 96, 120, 144, 168, etc.

Then, eliminate multiples of 16 from this list: 24, ~~48~~, 72, ~~96~~, 120, ~~144~~, 168, etc.

A pattern emerges: k = (an odd integer) × 24.

(A) $k/8$ can be an integer, for example when k = 24. (B) $k/9$ can be an integer, for example when k = 72. (C) $k/32$ is correct by process of elimination.
(D) $k/36$ can be an integer, for example when k = 72.
(E) $k/81$ can be an integer, for example when k = 81 × 24.

19. **10.** Because this is a numeric entry question, there can be only one correct answer. So, plugging in any prime number greater than 2 for b must yield the same result. Try $b = 3$.

If $a = 16b$ and $b = 3$, then a is 48. The factors (NOT prime factors) of 48 are: 1 & 48, 2 & 24, 3 & 16, 4 & 12, and 6 & 8. There are 10 distinct factors.

20. **(C).** The two equations are already solved for b and d, and the question is about the value of $b + d$. So, stack the equations and add:

$$\begin{array}{l} 4a + 3 = b \\ \underline{4c + 1 = d} \\ 4a + 4c + 4 = b + d \end{array}$$

Because a and c are integers, $4a + 4c + 4$ is the sum of three multiples of 4, which is a multiple of 4 itself. Therefore, the other side of the equation, $b + d$, must also equal a multiple of 4.

You could also factor out the 4:

$$4a + 4c + 4$$
$$4(a + c + 1)$$

Since a and c are integers, $a + c + 1$ is an integer, so $4(a + c + 1)$ is definitely a multiple of 4, and $b + d$ is also a multiple of 4. Only choice (C) is a multiple of 4.

21. **(C).** The factors of 210 are as follows:

1 & 210
2 & 105
3 & 70
5 & 42
6 & 35
7 & 30

10 & 21
14 & 15

O ut of the list of 16 factors,there are tw o m ultiples of 42 (42 and 210).

Thus,the answ er is 2/16 or 1/8.

22.**10.**If 1/4 of the poodles are also show dogs,the num ber of poodles m ust be divisible by 4.(The num ber of dogs is necessarily an integer.) Since the least possible num ber is the goal,try an exam ple w ith 4 poodles.

If 1/7 of the show dogs are poodles,the num ber of show dogs m ust be divisible by 7.Since the least possible num ber is the goal,try an exam ple w ith 7 show dogs.

So far there are:

4 poodles,1 of w hich is a show dog
7 show dogs,1 of w hich is a poodle

N ote that the one poodle that is also a show dog *is the sam e dog* as the one show dog that is also a poodle! To get the total num ber of dogs,only count that dog *once,*not tw ice.In total:

3 poodles (non-show dogs)
1 dog that is both poodle and show dog
6 show dogs (non-poodles)

This equals 10 dogs in total.This exam ple m et all the constraints of the question w hile using m inim um values at each step,so this is the least possible num ber of dogs at the fair.

23.**(C).**B reak dow n each of the num bers into its prim e factors.

(A) 49 = 7 × 7
(B) 81 = 3 × 3 × 3 × 3
(C) 100 = 2 × 2 × 5 ×
5 (D) 121 = 11 × 11
(E) 243 = 3 × 3 × 3 × 3 × 3

Since 100 has both 2 and 5 as prim e factors,it is not a prim e pow er.The correct answ er is (C).

24.**(C).**Since a positive m ultiple m ust be equal to or larger than the num ber it is a m ultiple of,answ er choice (C) cannot be a m ultiple of *a* or *b*,as it is sm aller than both integers *a* and *b*.

Y ou can also try testing num bers such that *a* is larger than *b*.

(A) If $a = 3$ and $b = 2$,$a - 1 = 2$,w hich is a m ultiple of *b*.
(B) If $a = 3$ and $b = 2$,$b + 1 = 3$,w hich is a m ultiple of *a*.
(C) Is the correct answ er by process of elim ination.
(D) If $a = 4$ and $b = 2$,$a + b = 6$,w hich is a m ultiple of *b*.
(E) If $a = 3$ and $b = 2$,$ab = 6$,w hich is a m ultiple of both *a* and *b*.

25.**12.**R em em ber,rem ainders are alw ays w hole num bers,so dividing 616 by 6 in your calculator w on't quite give you w hat you need.R ather,find the largest num ber less than 616 that 6 *does* go into (not 615,not 614,not 613 …).That num ber is 612.Since 616 - 612 = 4,the rem ainder *p* is equal to 4.

A lternatively,you could divide 616 by 6 in your calculator to get 102.66… .Since 6 goes into 616 precisely 102 w hole tim es,m ultiply 6 × 102 to get 612,then subtract from 616 to get rem ainder 4.

This second m ethod m ight be best for finding *q*.D ivide 525 by 11 to get 47.7272… .Since 47 × 11 = 517,the rem ainder is 525 - 517 = 8.

Therefore,*p* + *q* = 4 + 8 = 12.

26.**I only.**To solve this problem w ith exam ples,m ake a short list of possibilities for each of *x* and *y*:

x = 18,36,54…
y = 12,24,36…

N ow try to *disprove* the statem ents by trying several com binations of *x* and *y* above.In Statem ent I,*x* + *y* could be 18 + 12 = 30,54 + 12 = 66,36 + 24 = 60,or m any other com binations.Interestingly,all those com binations are m ultiples of 6.This m akes sense,as *x* and *y* individually are m ultiples of 6,so their sum is too.Statem ent I is true.

To test statem ent II,*xy* could be 18(12) = 216,w hich is N O T divisible by 48.Elim inate statem ent II.

A s for statem ent III,*x*/*y* could be 18/12,w hich is not even an integer (and therefore not divisible by 6),so III is not necessarily true.

27.**(D).**This problem is m ost easily solved w ith an exam ple.If *p* = 7 and *q* = 6,then *pg* = 42,w hich has the factors 1 & 42,2 & 21,3 & 14,and 6 & 7.That's 8 factors,but read carefully! The question asks how m any factors *greater than 1*,so the answ er is 7.N ote that choosing the sm allest possible exam ples (*p* = 7 and *q* = 6) w as the right m ove here,since the question asks "at least how m any factors… ?" If testing *p* = 70 and *q* = 36,m any,m any m ore factors w ould have resulted.The question asks for the m inim um .

28.**(C).**This problem is m ost easily solved w ith an exam ple.If *r* = 10 and *s* = 9,then *rs* = 90.The factors of 90 are 1 & 90,2 & 45,3 & 30,5 & 18,6 & 15,and 9 & 10.C ount to get a m inim um of 12 factors.

29.**(D).**If *t* is divisible by 12,then t^2 m ust be divisible by 144 or 2 × 2 × 2 × 2 × 3 × 3.Therefore,t^2 can be divided evenly by 2 at least four tim es,so *a* m ust be at least 5 before $\frac{t^2}{2^a}$ m ight not be an integer.

A lternatively,test values.If *t* = 12, $\frac{t^2}{2^a} = \frac{144}{2^a}$ .Plug in the choices as possible *a* values,starting w ith the sm allest choice and w orking up.

(A) Since $144/2^2$ = 36,elim inate.
(B) Since $144/2^3$ = 18,elim inate.

(C) Since $144/2^4 = 9$,elim inate.

(D) $144/2^5 = 4.5$.The first choice for w hich $\frac{t^2}{2^a}$ m ight not be an integer is (D).

30.**I,II,and III.**Since *a*,*b*,and *c* are all m ultiples of 3,$a = 3x$,$b = 3y$,$c = 3z$,w here $x > y > z > 0$ and all are integers. Substitute these new expressions into the statem ents.

Statem ent I: $a + b + c = 3x + 3y + 3z = 3(x + y + z)$.Since $(x + y + z)$ is an integer,this num ber m ust be divisible by 3.

Statem ent II: $a - b + c = 3x - 3y + 3z = 3(x - y + z)$.Since $(x + y + z)$ is an integer,this num ber m ust be divisible by 3.

Statem ent III: $abc/9 = (3x3y3z)/9 = (27xyz)/9 = 3xyz$.Since xyz is an integer,this num ber m ust be divisible by 3.

31.**(C).**Pattern problem s on the G R E often include a very large series of item s that w ould be im possible (or at least unw ise) to w rite out on paper.Instead,this problem requires you to recognize and exploit the pattern.In this case, after every 4th car,the color pattern repeats.B y dividing 463 by 4,you find that there w ill be 115 cycles through the 4 colors of cars— red,blue,black,gray— for a total of 460 cars to exit the factory.The key to solving these problem s is the rem ainder.B ecause there are 463 - 460 = 3 cars rem aining,the first such car w ill be red,the second w ill be blue, and the third w ill be black.

32.**(D).**This is a pattern problem .A n efficient m ethod is to recognize that the 7th day after the initial deposit w ould be Tuesday,as w ould the 14th day,the 21st day,etc.D ivide 100 by 7 to get 14 full w eeks com prising 98 days,plus 2 days left over.For the tw o leftover days,think about w hen they w ould fall.The first day after the deposit w ould be a W ednesday,as w ould the first day after w aiting 98 days.The second day after the deposit w ould be a Thursday,and so w ould the 100th day.

33.**(D).**D ivision problem s can be interpreted as follow s: dividend **=** divisor × quotient + rem ainder.This problem is dividing *x* by 7,or distributing *x* item s equally to 7 groups.A fter the item s are distributed am ong the 7 groups,there are 3 things left over,the rem ainder.This m eans that the value of *x* m ust be som e num ber that is 3 larger than a m ultiple of 7,such as 3,10,17,24,etc.The only answ er choice that is 3 larger than a m ultiple of 7 is 52.

34.**II and III only.**Start by rearranging the equation:$\frac{ab}{c+d} = \frac{37}{10}$ is equivalent to $10ab = 37(c + d)$.R em em ber that all four variables are positive integers.B ecause 37 and 10 have no shared factors,$(c + d)$ m ust be a m ultiple of 10 and *ab* m ust be a m ultiple of 37,in order to m ake the equation balance.

I.C ould be true.A ll the requirem ents are m et if $ab = (5)(37)$ and $(c + d) = 50$,so *ab* could be divisible by 5.B ut all the requirem ents are m et if $ab = 37$ and $(c + d) = 10$,in w hich case *ab* is not divisible by 5.

II.M U ST be true.B ecause $(c + d)$ m ust be divisible by 10,it m ust be divisible by both 5 and 2.

III.M U ST be true.B ecause $(c + d)$ m ust be divisible by 10,it m ust be divisible by both 5 and 2.Thus,$(c + d)$ m ust be even,so if *c* w ere even,*d* w ould have to be even,too.

35.**(E).**This is a bit of a trick question— any num ber that yields rem ainder 4 w hen divided by 10 w ill also yield rem ainder 4 w hen divided by 5.This is because the rem ainder 4 is less than both divisors,and all m ultiples of 10 are also m ultiples of 5.For exam ple,14 yields rem ainder 4 w hen divided either by 10 or by 5.This also w orks for 24,34, 44,54,etc.

36.**0.**The rem ainder w hen dividing an integer by 10 alw ays equals the units digit.Y ou can also ignore all but the units digits,so the question can be rephrased as: *W hat is the units digit of* $3^{17} + 7^{13}$*?*

The pattern for the units digits of 3 is [3,9,7,1].Every fourth term is the sam e.The 17th pow er is 1 past the end of the repeat: 17 - 16 = 1.Thus,3^{17} m ust end in 3.

The pattern for the units digits of 7 is [7,9,3,1].Every fourth term is the sam e.The 13th pow er is 1 past the end of the repeat: 13 - 12 = 1.Thus,7^{13} m ust end in 7.The sum of these units digits is 3 + 7 = 10.Thus,the units digit is 0.

37.**110.**W hen dividing,the rem ainder is alw ays less than the divisor.If you divided *a* by *b* to get a rem ainder of 9, then *b* m ust have been greater than 9.Sim ilarly,*d* m ust be greater than 10.Since *b* and *d* are integers,the sm allest they could be is 10 and 11,respectively.

Thus,the m inim um that *bd* could be is 10 × 11 = 110.

A s an exam ple,try *a* = 19,*b* = 10,*c* = 21,and *d* = 11 (generate *a* by adding rem ainder 9 to the value of *b,*and generate *c* by adding rem ainder 10 to the value of *d.*)

It is not possible to generate an exam ple in w hich *any* of the four num bers are sm aller.The least possible value of *bd* is 110.

38.**(C).**Start by considering the relationship betw een *n* and n^3.B ecause *n* is an integer,for every prim e factor *n* has, n^3 m ust have three of them .Thus,n^3 m ust have prim e num bers in m ultiples of 3.If n^3 has one prim e factor of 3,it m ust actually have tw o m ore,because n^3's prim e factors can only com e in triples.

The question says that n^3 is divisible by 24,so n^3's prim e factors m ust include at least three 2's and a 3.B ut since n^3 is a cube,it m ust contain at least three 3's.Therefore *n* m ust contain at least one 2 and one 3,or 2 × 3 = 6.

39.**(C).**First,expand 10! as 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1.

(D o N O T m ultiply all of those num bers together to get 3,628,800— it's true that 3,628,800 is the value of 10!,but analysis of the prim e factors of 10! is easier in the current form .)

N ote that 10! is divisible by 3^x5^y,and the quantities concern the greatest possible values of *x* and *y*,w hich is equivalent to asking,"W hat is the m axim um num ber of tim es you can divide 3 and 5,respectively,out of 10! w hile still getting an integer answ er?"

In the product 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1,only the m ultiples of 3 have 3 in their prim e factors,and only the m ultiples of 5 have 5 in their prim e factors.H ere are all the prim es contained in 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 and therefore in 10!:

10 = 5 × 2

$9 = 3 \times 3$
$8 = 2 \times 2 \times 2$
$7 = 7$
$6 = 2 \times 3$
$5 = 5$
$4 = 2 \times 2$
$3 = 3$
$2 = 2$
1 = no prim es

There are four 3’s and tw o 5’s total.The m axim um values are $x = 4$ and $y = 2$.Therefore,Q uantity A and Q uantity B are each 4,so the correct answ er is (C).

40.**(B).**Since only the num ber of *distinct* prim e factors m atter,not w hat they are or how m any tim es they are present, you can tell on sight that Q uantity A has only 2 distinct prim e factors,because 100,000 is a pow er of 10.(A ny prim e tree for 10,100,or 1,000,etc.w ill contain only the prim e factors 2 and 5,occurring in pairs.)

In Q uantity B ,99,000 breaks dow n as 99 × 1,000.Since 1,000 also contains 2’s and 5’s,and 99 contains even m ore factors (specifically 3,3,and 11),Q uantity B has m ore distinct prim e factors.It is not necessary to m ake prim e trees for each num ber.

41.**V and V II only.**For tw o num bers to have a product that is a m ultiple of 27,the tw o num bers need to have at least three 3’s am ong their com bined prim e factors,since 27 = 33.O nly 63 and 600 are m ultiples of 3,so the other choices could be elim inated very quickly if you see that.There’s no need to actually m ultiply the num bers together. Since 63 is 3 × 3 × 7 and 600 is 3 × 200,their product w ill have the three 3’s required for a m ultiple of 27.

42.**III,IV ,and V only.**B ecause $64 = 2^6$,m ultiples of 64 w ould have at least six 2’s am ong their prim e factors.

Since 12 (w hich is 2 × 2 × 3) has tw o 2’s already,a num ber that could be m ultiplied by 12 to generate a m ultiple of 64 w ould need to have,at m inim um ,the *other* four 2’s needed to generate a m ultiple of 64.

Since you w ant the choices that don’t m ultiply w ith 12 to generate a m ultiple of 64,select only the choices that have *four or few er 2’s* w ithin their prim e factors.

6^6	$= (2 \times 3)^6$	six 2’s	INCORRECT
12^2	$= (2^2 \times 3)^2$	four 2’s	INCORRECT
18^3	$= (2 \times 3^2)^3$	three 2’s	CORRECT
30^3	$= (2 \times 3 \times 5)^3$	three 2’s	CORRECT
222	$= (2 \times 3 \times 37)$	one 2	CORRECT

43.**(D).**W hen a non-m ultiple of 3 is divided by 3,the quotient does not term inate (for instance,1/3 = 0.333…).

Since $3^x(5^2)/3^5(5^3)$ does N O T repeat forever,*x* m ust be large enough to cancel out the 3^5 in the denom inator.Thus,*x* m ust be at least 5.N ote that the question asks w hat M U ST be true.C hoice (D) m ust be true.C hoice (E),$x = 5$, represents one value that w ould w ork,but this choice does not *have* to be true.

44.**(B).**Since a prim e num ber has only tw o factors,1 and itself,$(2^{a})(3^{b})(5^{c})$ cannot be prim e unless the digits *a*,*b* and *c* are such that tw o of the digits are 0 and the third is 1.For instance,$(2^{0})(3^{1})(5^{0}) = (1)(3)(1) = 3$ is prim e.Thus, the only three values of $\underline{abc}$ that w ould result in a prim e num ber & ($\underline{abc}$)& are 100,010,and 001.H ow ever,only one of those three num bers (100) is a three digit num ber.