Circles and Cylinders

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

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1. A circle has an area of 16π. What is its circumference?

(A) 4π
(B) 8π
(C) 16π
(D) 32π
(E) It cannot be determined from the information given.

2. A circle has a circumference of 20π. What is its area?

(A) 10π
(B) 20π
(C) 40π
(D) 100π
(E) 400π

3. A circle has a circumference of 8. What is its area?

(A) $\frac{4}{\pi}$

(B) $\frac{4}{\pi^2}$

(C) $\frac{16}{\pi}$

(D) $\frac{16}{\pi^2}$

(E) 16π

4.A circle has a diam eter of 5.W hat is its area?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) $\frac{25\pi^2}{2}$

(D) 10π

(E) 25π

5.A circle's area equals its circum ference.W hat is its radius?

(A) 1
(B) 2
(C) 4
(D) 8
(E) 16

6.

C ircle *C* has a radius *r* such that $1 < r < 5$

Q uantity A	**Q uantity B**
The area of C ircle *C*	The circum ference of C ircle *C*

7.

Q uantity A	**Q uantity B**
The radius of a circle w ith area 36π	The radius of a circle w ith circum ference 12π

8.

Quantity A	**Quantity B**
The area of a circle with radius 4	The circumference of a circle with radius 6

9. A circle has radius $\frac{5}{3}$. What is its area?

(A) $\frac{\sqrt{5}\pi}{3}$

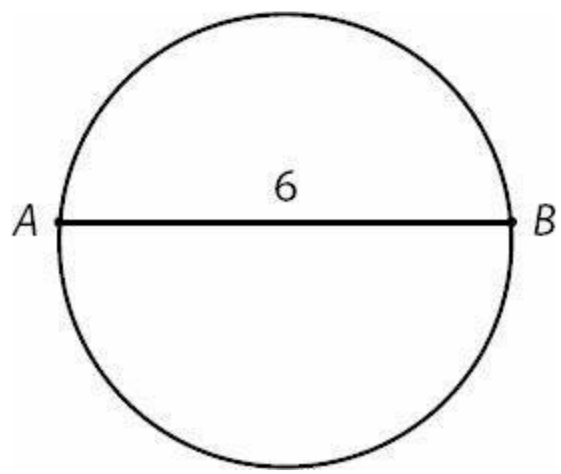
(B) $\frac{5\pi}{3}$

(C) $\frac{25\pi}{9}$

(D) $\frac{10\pi}{3}$

(E) $\frac{100\pi}{9}$

10.



AB is <u>not</u> a diameter of the circle

Quantity A	**Quantity B**
The area of the circle	9π

11. A circle has radius 0.01. What is its area?

(A) $\frac{\pi}{10}$

(B) $\frac{\pi}{100}$

(C) $\frac{\pi}{1,000}$

(D) $\frac{\pi}{10,000}$

(E) $\frac{\pi}{100,000}$

12. A circle has radius $\sqrt{x}$. What is its circumference?

(A) πx
(B) $2\pi x$
(C) $2\pi\sqrt{x}$
(D) $2\pi x^2$
(E) πx^2

13.

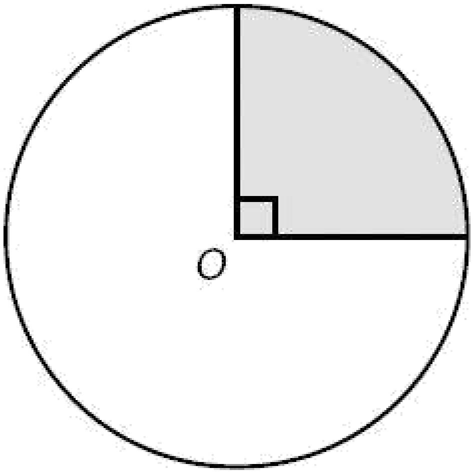
The circumference of a circle is greater than 7π.

Quantity A	**Quantity B**
The area of the circle	15π

14. A circle has an area of 4π. If the radius were doubled, the new area of the circle would be how many times the original area?

(A) 2
(B) 3
(C) 4
(D) 5
(E) It cannot be determined from the information given.

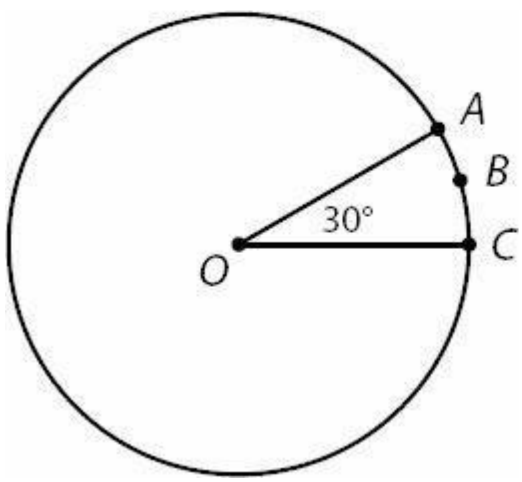
15.



In the figure above, point O is the center of the circle. If the radius of the circle is 8, what is the area of the shaded sector?

(A) 2π
(B) 4π
(C) 8π
(D) 16π
(E) 32π

16.



The radius of the circle with center O is 6.

Quantity A	**Quantity B**
The length of arc *ABC*	3

17. A sector of a circle has an arc length of 7π. If the diameter of the circle is 14, what is the measure of the central angle of the sector, in degrees?

(A) 45
(B) 60
(C) 90
(D) 120
(E) 180

18. A sector of a circle has a central angle of 270°. If the circle has a radius of 4, what is the area of the sector?

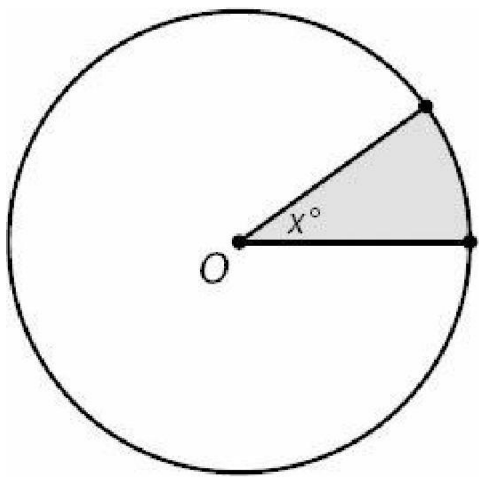
(A) 4π
(B) 8π
(C) 12π
(D) 16π
(E) 20π

19.

W ithin a circle w ith radius 12,a sector has an area of 24π.

Q uantity A	**Q uantity B**
The m easure of the central angle of the sector,in degrees	90

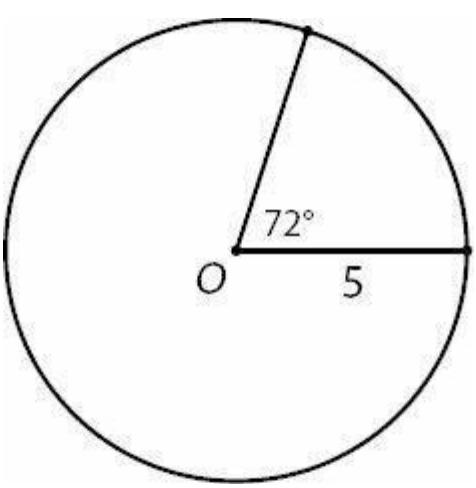
20.



The area of the shaded sector is $\frac{1}{10}$ of the area of the full circle.

Q uantity A	**Q uantity B**
$2x$	75

21.



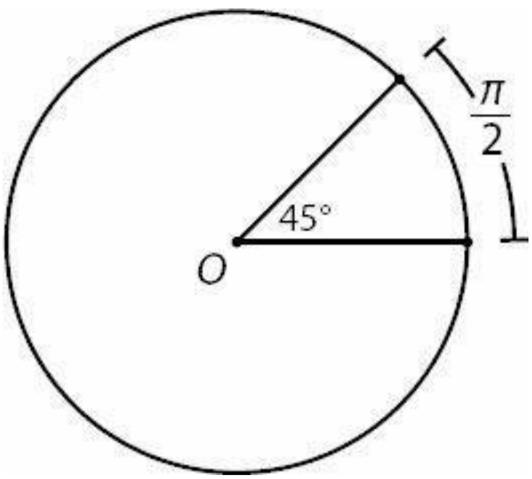
If O is the center of the circle,w hat is the perim eter of the sector w ith central angle 72°?

(A) $5 + 2\pi$
(B) $10 + 2\pi$
(C) $10 + 4\pi$
(D) $10 + 5\pi$
(E) $20 + 2\pi$

22.A sector of a circle has a radius of 8 and an area of 8π.W hat is the arc length of the sector?

(A) π
(B) 2π
(C) 4π
(D) 6π
(E) 8π

23.



If point O is the center of the circle in the figure above,w hat is the radius of the circle?

24.

Sector A and Sector B are sectors of tw o different circles.
Sector A has a radius of 4 and a central angle of 90°.
Sector B has a radius of 6 and a central angle of 45°.

Q uantity A	Q uantity B
The area of Sector A	The area of Sector B

25.W hat is the volum e of a right circular cylinder w ith a radius of 2 and a height of 4?

(A) 8π
(B) 12π
(C) 16π
(D) 32π
(E) 72π

26.W hat is the height of a right circular cylinder w ith radius 1 and volum e 16π?

27.

A right circular cylinder has volum e 24π.

Q uantity A	**Q uantity B**
The height of the cylinder	The radius of the cylinder

28.If a half-full 4-inch by 2-inch by 8-inch box of soym ilk is poured into a right circular cylindrical glass w ith radius 2 inches,how m any inches high w ill the soym ilk reach? (A ssum e that the capacity of the glass is greater than the volum e of the soym ilk.)

(A) 8

(B) 16

(C) $\frac{4}{\pi}$

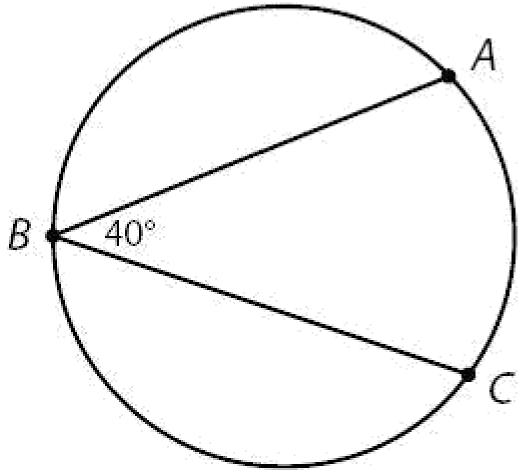
(D) $\frac{8}{\pi}$

(E) $\frac{16}{\pi}$

29.If a right circular cylinder's radius is halved and its height doubled,by w hat percent w ill the volum e increase or decrease?

(A) 50% decrease
(B) no change
(C) 25% increase
(D) 50% increase
(E) 100% increase

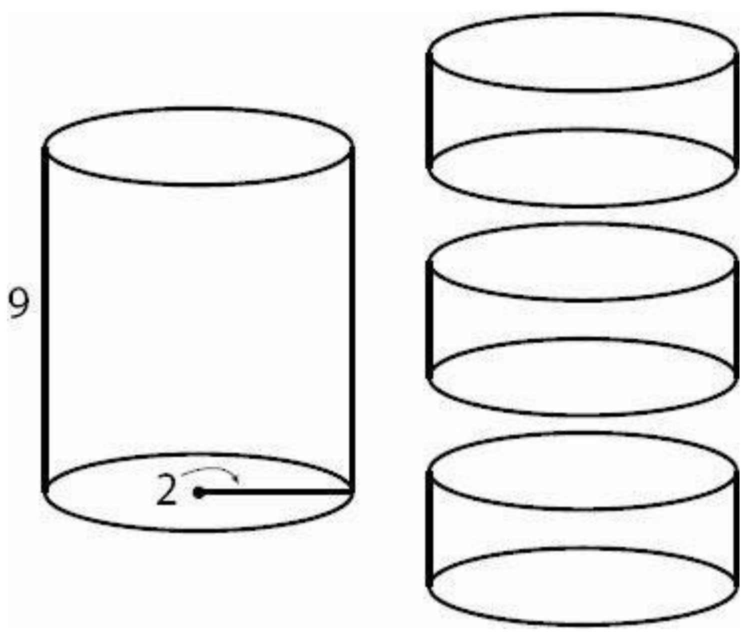
30.



If the diam eter of the circle is 36,w hat is the length of arc *ABC* ?

(A) 8 (B
) 8π (C)
28π (D)
32π
(E) 56π

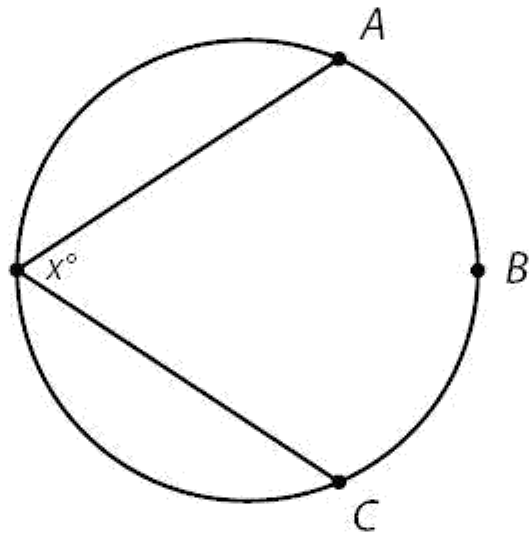
31.



If a solid right circular cylinder w ith height 9 and radius 2 is cut as show n into three new cylinders,each of equal and uniform height,how m uch new surface area is created?

(A) 4π
(B) 12π
(C) 16π
(D) 24π
(E) 36π

32.



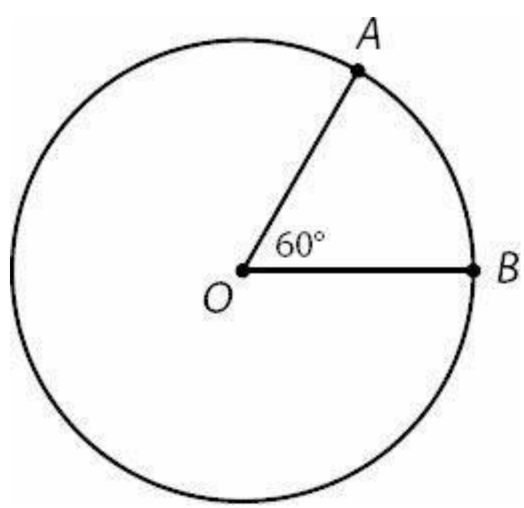
$x > 60°$

Q uantity A

Q uantity B

The ratio of the length of arc *ABC* to the circum ference of the circle 1/3

33.



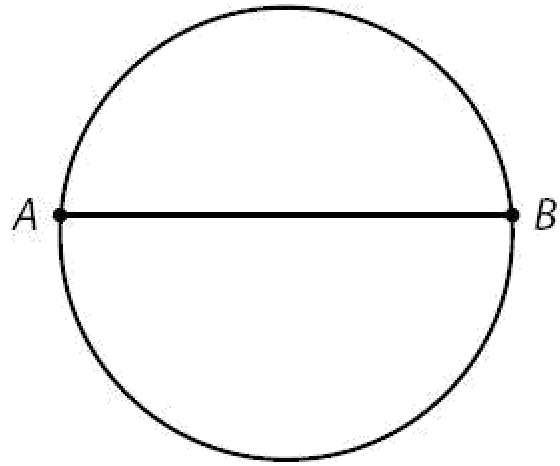
Point *O* is the center of the circle above.

Q uantity A

Q uantity B

The ratio of the length of m inor arc *AB* to m ajor arc *AB* 1/6

34.



The circle above has area 25.

Quantity A	**Quantity B**
The length of chord *AB*	10

C ircles and C ylinders A nsw ers

1.**(B).**Since the area form ula for a circle is $A = \pi r^2$:

$16\pi = \pi r^2$
$16 = r^2$
$4 = r$

Since the circum ference form ula is $C = 2\pi r$ and $r = 4$:

$C = 2\pi(4)$
$C = 8\pi$

2.**(D).**Since the circum ference form ula is $C = 2\pi r$:

$20\pi =$
$2\pi r\ 20 =$
$2r\ 10 = r$

Since the area form ula for a circle is $A = \pi r^2$ and $r = 10$:

$A = \pi(10)^2$
$A = 100\pi$

3.**(C).**Since the circum ference form ula is $C = 2\pi r$:

$8 = 2\pi r$

N ote that the circum ference is just 8,not 8π,so the radius is going to look a bit unusual.First,divide both sides by 2:

$4 = \pi r$
$\frac{4}{\pi} = r$

N ow ,plug the radius $\frac{4}{\pi}$ into the area form ula for a circle:

$$A = \pi\left(\frac{4}{\pi}\right)^2$$

$$A = \pi \times \frac{16}{\pi^2}$$

$$A = \frac{16}{\pi}$$

4.**(A).**If a circle's diam eter is 5,its radius is $\frac{5}{2}$.Plug this into the area form ula:

$$A = \pi\left(\frac{5}{2}\right)^2$$

$$A = \pi \times \frac{25}{4}$$

$$A = \frac{25\pi}{4}$$

5.**(B).**To find the radius that w ould m ake the area and the circum ference of a circle equal,sim ply set the area and circum ference form ulas equal to one another:

$\pi r^2 = 2\pi r$

Since both sides have both r and π,divide both sides by πr:

$r = 2$

6.**(D).**Picking num bers is the easiest w ay to prove (D).If you begin w ith a radius of 3,the area is 9π and the circum ference is 6π,so Q uantity A is greater.If you try a radius of 4,the area is 16π and the circum ference is 8π,so once again Q uantity A is greater.B ut if you try a radius of 2,both the area and the circum ference equal 4π.Therefore, Q uantity A is not alw ays greater,so the answ er is (D).N ote also that r is not required to be an integer.If you try a value close to the m inim um ,such as 1.1,Q uantity B w ould be greater.

7.**(C).**Since the area form ula for a circle is $A = \pi r^2$,calculate Q uantity A by plugging 36π into the form ula as the area:

$36\pi = \pi r^2$
$36 = r^2$
$6 = r$

Since the circum ference form ula for a circle is $C = 2\pi r$,calculate Q uantity B by plugging 12π into the form ula as the circum ference:

$$12\pi = 2\pi r$$
$$12 = 2r$$
$$6 = r$$

The tw o quantities are equal.In other w ords,a circle w ith area 36π w ill also have circum ference 12π.

8.**(A).**Since the area form ula for a circle is $A = \pi r^2$,calculate Q uantity A by plugging radius 4 into the form ula:

$$A = \pi(4)^2$$
$$A = 16\pi$$

Since the circum ference form ula for a circle is $C = 2\pi r$,calculate Q uantity B by plugging radius 6 into the form ula:

$$C = \pi 2(6)$$
$$C = 12\pi$$

Q uantity A is greater.

9.**(C).**Since the area form ula for a circle is $A = \pi r^2$,plug radius $\frac{5}{3}$ into the form ula:

$$A = \pi\left(\frac{5}{3}\right)^2$$

$$A = \frac{25\pi}{9}$$

10.**(A).**Since a diam eter is the longest straight line you can draw from one point on a circle to another (that is,a diam eter is the longest chord in a circle),the actual diam eter m ust be *greater* than 6.

IF the diam eter w ere 6,the radius w ould be 3,and the area w ould be:

$$A = \pi(3)^2$$
$$A = 9\pi$$

H ow ever,since the diam eter m ust be greater than 6,the area m ust be greater than 9π.D o N O T m ake the m istake of picking (D) for Q uantitative C om parison geom etry questions in w hich you cannot “solve.” There is often still a w ay to determ ine w hich quantity is greater.

11.**(D).**Since the form ula for the area of a circle is $A = \pi r^2$,plug radius 0.01 into the form ula.H ow ever,since the answ ers are in fraction form at,it is probably easier to convert to fraction form now rather than at the end.Since

$0.01 = \frac{1}{100}$ (you can verify this in your calculator):

$$A = \pi\left(\frac{1}{100}\right)^2$$

$$A = \pi\left(\frac{1}{10,000}\right)$$

$$A = \frac{\pi}{10,000}$$

12.**(C).**Since the circum ference form ula for a circle is $C = 2\pi r$,plug $\sqrt{x}$ in for the radius:

$$C = 2\pi\sqrt{x}$$

This expression does not sim plify — no m ore w ork is required.N ote that incorrect answ er choice (A) is the result of accidentally using the area form ula rather than the circum ference form ula.

13.**(D).**The circum ference is "greater than 7π." D o not m ake the m istake of thinking that it has to be at least 8π! There is N O rule that the num ber before the π m ust be an integer.If the circum ference of the circle w ere 8π,the radius w ould be 4 and the area therefore 16π,m aking Q uantity A greater.

H ow ever,the circum ference of the circle could be 7.5π,in w hich case the radius w ould be 3.75 and the area w ould be 14.0625π,m aking Q uantity B greater.Thus,the answ er is (D).

14.**(C).**To begin,find the original radius of the circle: A rea = πr^2 = 4π,so r = 2.O nce doubled,the new radius is 4.A circle w ith a radius of 4 has an area of 16π.The new area of 16π is 4 tim es the old area of 4π.

15.**(D).**If the sector has a central angle of 90°,then the sector is 1/4 of the circle,because $\frac{90}{360} = \frac{1}{4}$.To find the area of the sector,first find the area of the w hole circle.The radius is 8,w hich m eans the full circle area is $\pi(8)^2$ = 64π.If the circle area is 64π,then the sector's area is 1/4 × 64π = 16π.

16.**(A).**If the sector has a central angle of 30°,then it is 1/12th of the circle,because $\frac{30}{360} = \frac{1}{12}$.To find the arc length of the sector,first find the circum ference of the entire circle.The radius of the circle is 6,so the circum ference is 2π(6) = 12π.That m eans that the arc length of the sector is (1/12)(12π) = π.Since π is about 3.14, Q uantity A is greater.

17.**(E).**To find the central angle of the sector,first determ ine w hat fraction of the full circle the sector represents. The diam eter of the circle is 14,so the circum ference is π(14) = 14π.Since the arc length is

7π,the sector is 1/2 the full circle.That m eans that the central angle of the sector is 1/2 of 360°,or 180°.

18.**(C).**The sector is 3/4 of the circle,because $\frac{270}{360}=\frac{3}{4}$.To find the area of the sector,first find the area of the w hole circle.The radius of the circle is 4,so the area is $\pi(4)^2 = 16\pi$.That m eans the area of the sector is $(3/4)(16\pi) = 12\pi$.

19.**(B).**First find the area of the w hole circle.The radius is 12,w hich m eans the area is $\pi(12)^2 = 144\pi$.Since the sector has an area of 24π and $\frac{24\pi}{144\pi}=\frac{1}{6}$,the sector is 1/6th of the entire circle.That m eans that the central angle is 1/6th of 360,or 60°.

20.**(B).**If the area of the sector is $\frac{1}{10}$ of the area of the full circle,then the central angle is $\frac{1}{10}$ of the degree m easure of the full circle,or $\frac{1}{10}$ of 360 = 36 = *x*.Thus,Q uantity A = 2(36) = 72.

21.**(B).**To find the perim eter of a sector,you need the radius of the circle and the arc length of the sector.B egin by determ ining w hat fraction of the circle the sector is.The central angle of the sector is 72°,so the sector is $\frac{72}{360}=\frac{1}{5}$ of the circle.The radius is 5,so the circum ference of the circle is $2\pi(5) = 10\pi$.The arc length of the sector is 1/5 of the circum ference: $(1/5)(10\pi) = 2\pi$.The perim eter of the sector is sim ply this 2π,plus the tw o radii that m ake up the straight parts of the sector: $10 + 2\pi$.

22.**(B).**C om pare the given area of the sector to the calculated area of the circle.The radius of the circle is 8,so the area of the circle is $\pi(8)^2 = 64\pi$.The area of the sector is 8π,or $\frac{8\pi}{64\pi}=\frac{1}{8}$ of the circle.The radius is 8,so the circum ference of the w hole circle is $2\pi(8) = 16\pi$.Since the sector is 1/8 of the circle,the arc length is $(1/8)(16\pi) = 2\pi$.

23.**2.**If the sector has a central angle of 45°,then the sector is $\frac{45}{360}=\frac{1}{8}$ of the circle.Thus,the arc length of the sector is 1/8 of the circum ference of the circle,or stated differently,the circum ference is 8 tim es the arc length of the sector.The circum ference is $8(\pi/2) = 4\pi$.From the circum ference form ula,$4\pi = 2\pi r$,so $r = 2$.The radius of the circle is 2.

24.**(B).**Sector *A* is $\frac{90}{360}=\frac{1}{4}$ of the circle w ith radius 4.The area of this circle is $\pi(4)^2 = 16\pi$,so the area of Sector *A* is 1/4 of 16π,or 4π.

Sector *B* is $\frac{45}{360}=\frac{1}{8}$ of the circle w ith radius 6.The area of this circle is $\pi(6)^2 = 36\pi$,so the area of Sector *B* is 1/8 of 36π or 4.5π.

4.5π is greater than 4π,so the area of Sector *B* is greater than the area of Sector *A* .

25.**(C).**U se the form ula for volum e of a right circular cylinder,$V = \pi r^2 h$.(This form ula is easy to m em orize as it is sim ply the area of a circle,m ultiplied by height).$V = \pi(2)^2(4) = 16\pi$.

26.**16.**From the form ula for volum e of a right circular cylinder,$V = \pi r^2 h$:

$$16\pi = \pi(1)^2 h$$
$$16 = h$$

27.**(D).**Plugging into the form ula for volum e of a right circular cylinder,$V = 24\pi = \pi r^2 h$.H ow ever,there are m any com binations of r and h that w ould m ake the volum e 24π.For instance,$r = 1$ and $h = 24$,or $r = 4$ and $h = 1.5$.K eep in m ind that the radius and height don't even have to be integers,so there truly are an infinite num ber of possibilities, som e for w hich h is greater and som e for w hich r is greater.

28.**(D).**A box is a rectangular solid w hose volum e form ula is sim ply V = length × w idth × height.So,the volum e of the box is 4 inches × 2 inches × 8 inches = 64 inches^3.Since the box is half full,there are 32 inches^3 of soym ilk.This volum e w ill not change w hen the soym ilk is poured from the box into the cylinder.The form ula for the volum e of a cylinder is $V = \pi r^2 h$,so:

$32 = \pi(2)^2 h$,w here r and h are in units of inches.

$$\frac{32}{4\pi} = h$$

$$\frac{8}{\pi} = h$$

The height is $8/\pi$ inches.N ote that the height is "w eird" (divided by π) because the volum e of the cylinder did *not* have a π.

29.**(A).**A ccording to the form ula for the volum e of a right circular cylinder,the original volum e is $V = \pi r^2 h$.To halve the radius,sim ply replace r w ith $r/2$.To double the height,sim ply replace h w ith $2h$.The only caveat: be sure to use parentheses!

$$V = \pi\left(\frac{r}{2}\right)^2(2h) = \frac{2\pi r^2 h}{2^2} = \frac{\pi r^2 h}{2}$$

Thus,the volum e,w hich w as once $\pi r^2 h$,is now $\frac{\pi r^2 h}{2}$.In other w ords,it has been cut in half,or reduced by 50% .

A lternatively,plug in num bers.If the cylinder originally had radius 2 and height 1,the volum e w ould be $V = \pi(2)^2(1) = 4\pi$.If the radius w ere halved to becom e 1 and the height w ere doubled to becom e 2,the volum e w ould be $V = \pi(1)^2(2) = 2\pi$.A gain,the volum e is cut in half,or reduced by 50% .

30.**(C).**N ote that a M IN O R arc is the “short w ay around” the circle from one point to another,and a M A JO R arc is the “long w ay around.” A rc *ABC* is thus the sam e as m ajor arc *AC* .

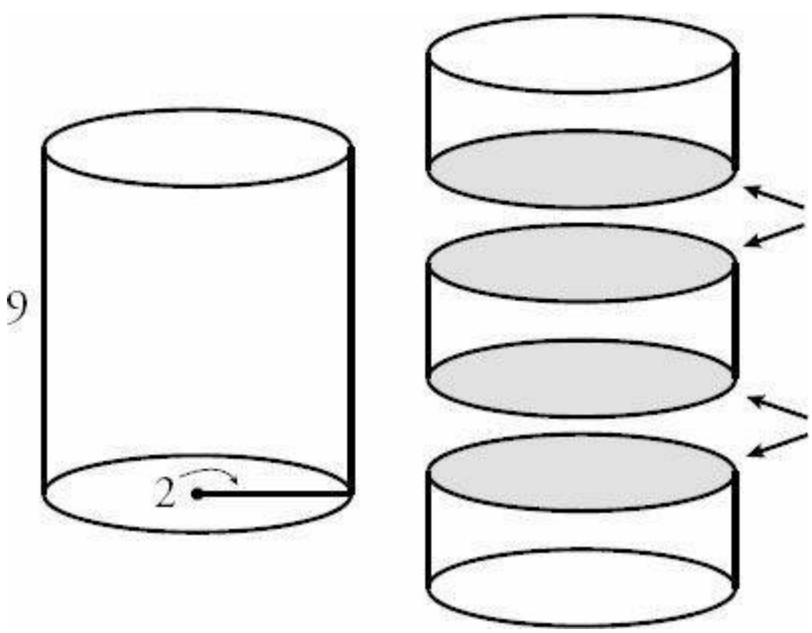
For a given arc,an inscribed angle is alw ays half the central angle,w hich w ould be 80° in this case.The m inor arc *AC* is thus $\frac{80}{360} = \frac{2}{9}$ of the circle.Since the circum ference is 36π:

$$\text{minor arc } AC = \frac{2}{9}(36\pi) = 8\pi$$

A rc *ABC* ,or m ajor arc *AC* ,is the entire circum ference m inus the m inor arc:

$$36\pi - 8\pi = 28\pi$$

31.**(C).**Y ou *could* find the surface area of the large cylinder,then the surface areas of the three new cylinders,then subtract the surface area of the large cylinder from the com bined surface areas of the three new cylinders.H ow ever, there is a m uch faster w ay.W hen the large cylinder is cut into three sm aller ones,only a few *new* surfaces are created — the bottom base of the top cylinder,the top and bottom bases of the m iddle cylinder,and the top surface of the bottom cylinder.



Thus,these four circular bases represent the new surface area created.Since the radius of each base is 2,use the area form ula for a circle,$A = \pi r^2$:

$$A = \pi(2)^2$$
$$A = 4\pi$$

Since there are 4 such bases,m ultiply by 4 to get 16π.

32.**(A).**If x w ere equal to 60°,arc *ABC* w ould have a central angle of 120°.(Inscribed angles,w ith the vertex at the far side of the circle,are alw ays half the central angle.) A 120° arc is 120/360 = 1/3 of the circum ference of the circle. Since x is actually greater than 60°,the arc is actually greater than 1/3 of the circum ference.Thus,the ratio of the arc length to the circum ference is greater than 1/3.

33.**(A).**Since the angle that determ ines the arc is equal to 60 and 60/360 = 1/6,m inor arc *AB* is 1/6 of the circum ference of the circle.(There are alw ays 360 degrees in a circle.M inor arc *AB* is the “short w ay around” from *A* to *B* ,w hile m ajor arc *AB* is the “long w ay around.”)

Since m inor arc *AB* is 1/6 of the circum ference,m ajor arc *AB* m ust be the other 5/6.Therefore,the ratio of the m inor arc to the m ajor arc is 1 to 5 (N O T 1 to 6!) Y ou could calculate this as $\frac{1/6}{5/6} = \frac{1}{6} \times \frac{6}{5} = \frac{1}{5}$,or you could just reason the ratio of 1 of *anything* (such as sixths) to 5 of the sam e thing (again,sixths) is a 1 to 5 ratio.

O f course,the trap answ er here is (C).This is a com m on m istake.1/6 of the total is not the sam e as a 1 to 6 ratio of tw o parts.

34.**(B).**The equation for the area of a circle is $A = \pi r^2$.N ote that the given area is just 25,*not* 25π! So:

$\pi r^2 = 25$

$r^2 = \frac{25}{\pi} \approx 8$

r = a bit less than 3.

So the diam eter of the circle is a bit less than 6.The diam eter is the chord w ith m axim um length,so w herever *AB* is on this circle,it’s significantly shorter than 10.