A lgebra

For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given),the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box $\boxed{}$,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes $\frac{\boxed{}}{\boxed{}}$,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.

A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.If $3x + 2(x + 2) = 2x + 16$,then $x =$

(A) 3
(B) 4
(C) 20/3
(D) 10
(E) 12

2.If $x \neq 0$ and $\frac{3x+7}{x}=10$,w hat is the value of x?

$\boxed{}$

3.If $4(-3x - 8) = 8(-x + 9)$,w hat is x^2?

4.If $3x + 7 - 4x + 8 = 2(-2x - 6)$,w hat is the value of x?

5.If $2x(4 - 6) = -2x + 12$,w hat is the value of x?

6.If $x \neq 0$ and $\frac{3(6-x)}{2x} = -6$,w hat is the value of x?

7.If $x \neq -13$ and $\frac{13}{x+13} = 1$,w hat is the value of x?

8.If $x \neq 2$ and $\frac{10(-3x+4)}{10-5x} = 2$,w hat is the value of x?

9.If $x \neq 2$ and $\frac{8-2(-4+10x)}{2-x} = 17$,w hat is the value

of x? 10.

-5 is 7 m ore than $-z$.

Q uantity A	**Q uantity B**
z	-12

11.If $(x + 3)^2 = 225$,w hich of the follow ing could be the value of $x - 1$?

(A) 13
(B) 12
(C) -12
(D) -16
(E) -19

12.

$x = 2$

Quantity A	**Quantity B**
$x^2 - 4x + 3$	1

13.

$$p = 300c^2 - c$$
$$c = 100$$

Quantity A	**Quantity B**
p	$29{,}000c$

14. If $3(7 - x) = 4(1.5)$, then $x =$

15.

$$1{,}200x + 6{,}000 = 13{,}200$$
$$12y + 60 = 132$$

Quantity A	**Quantity B**
x	y

16.

$$-(x)^3 = 64$$

Quantity A	**Quantity B**
x^4	x^5

17. If $3t^3 - 7 = 74$, what is $t^2 - t$?

(A) -3
(B) 3
(C) 6
(D) 9
(E) 18

18. If $3x + 7 - 4x + 8 = 2(-2x - 6)$, what is the value of x?

19. If $y = 4x + 10$ and $y = 7x - 5$, what is the value of y?

20.If $2h - 4k = 0$ and $k = h - 3$,w hat is the value of $h + k$?

21.If $x - y = 4$ and $2x + y = 5$,w hat is the value of x?

22.If $x + 2y = 5$ and $x - 4y = -7$,w hat is the value of x?

23.$4x + y + 3z = 34$

$4x + 3z = 21$

W hat is the value of y?

24.

Q uantity A	**Q uantity B**
$(x + 2)(x - 3)$	$x^2 - x - 6$

25.

Q uantity A	**Q uantity B**
$(2s + 1)(s + 5)$	$2s^2 + 11s + 4$

26.

$xy > 0$

Q uantity A	**Q uantity B**
$(2x - y)(x + 4y)$	$2x^2 + 8xy - 4y^2$

27.

$$x^2 - 2x = 0$$

Quantity A	**Quantity B**
x	2

28.

Quantity A	**Quantity B**
$d(d^2 - 2d + 1)$	$d(d^2 - 2d) + 1$

29.

Quantity A	**Quantity B**
$xy^2z(x^2z + yz^2 - xy^2)$	$x^3y^2z^2 + xy^3z^3 - x^2y^4z$

30.

$a = 2b = 4c$ and a,b,and c are integers.

Quantity A	**Quantity B**
$a + b$	$a + c$

31.

$k = 2m = 4n$ and k,m ,and n are nonnegative integers.

Quantity A	**Quantity B**
km	kn

32.

For the positive integers a,b,c,and d,a is half of b,which is one-third of c.The value of d is triple that of c.

Quantity A	**Quantity B**
$\frac{a+b}{c}$	$\frac{a+b+c}{d}$

33.If $x^2 - y^2 = 0$ and $xy \neq 0$,which of the following MUST be true?

Indicate <u>all</u> such statements.

☐ $x = y$

☐ $|x| = |y|$

☐ $\frac{x^2}{y^2} = 1$

34.

$$3x + 6y = 27$$
$$x + 2y + z = 11$$

Q uantity A	**Q uantity B**
$z + 5$	$x + 2y - 2$

35.If $(x - y) = \sqrt{12}$ and $(x + y) = \sqrt{3}$,w hat is the value of $x^2 - y^2$?

(A) 3
(B) 6
(C) 9
(D) 36
(E) It cannot be determ ined from the inform ation given.

36.

$$a \neq b$$

Q uantity A	**Q uantity B**
$\frac{a-b}{b-a}$	1

37.

$$a = \frac{b}{2}$$

$$c = 3b$$

Q uantity A	**Q uantity B**
a	c

38.If $xy \neq 0$ and $x \neq -y$, $\frac{x^{36} - y^{36}}{\left(x^{18} + y^{18}\right)\left(x^{9} + y^{9}\right)} =$ =

(A) 1
(B) $x^2 - y^2$
(C) $x^9 - y^9$

(D) $x^{18} - y^{18}$

(E) $\dfrac{1}{x^9 - y^9}$

39.If $x \neq -y$, $\dfrac{x^2 + 2xy + y^2}{2(x+y)^2} =$

(A) 1
(B) 1/2
(C) $\dfrac{1}{x+y}$
(D) xy
(E) $2xy$

40.If $ab \neq 0$, $\dfrac{a^8 - b^8}{(a^4 + b^4)(a^2 + b^2)} =$

(A) 1
(B) $a - b$
(C) $(a + b)(a - b)$
(D) $(a^2 + b^2)(a^2 - b^2)$
(E) $\dfrac{a-b}{a+b}$

41.

$x > y$

$xy \neq 0$

Q uantity A	**Q uantity B**
$\dfrac{x^2}{y + \frac{1}{y}}$	$\dfrac{y^2}{x + \frac{1}{x}}$

42.If $x + y = -3$ and $x^2 + y^2 = 12$,w hat is the value of $2xy$?

43.If $x - y = 1/2$ and $x^2 - y^2 = 3$,w hat is the value of $x + y$?

44.If $x^2 - 2xy = 84$ and $x - y = -10$,w hat is the value of $|y|$?

45.$(x - 2)^2 + (x - 1)^2 + x^2 + (x + 1)^2 + (x + 2)^2 =$

(A) $5x^2$
(B) $5x^2 + 10$
(C) $x^2 + 10$
(D) $5x^2 + 6x + 10$
(E) $5x^2 - 6x + 10$

46.If $a = (x + y)^2$ and $b = x^2 + y^2$ and $xy > 0$,w hich of the follow ing m ust be true?

Indicate <u>all</u> such statem ents.

☐ $a = b$
☐ $a > b$
☐ a is positive

47.*a* is directly proportional to *b*.If $a = 8$ w hen $b = 2$,w hat is *a* w hen $b = 4$?

(A) 10
(B) 16
(C) 32
(D) 64
(E) 128

48.*a* is inversely proportional to *b*.If $a = 16$ w hen $b = 1$,w hat is *b* w hen $a = 8$?

(A) -2
(B) -1
(C) 2
(D) 4
(E) 8

49.The tim e it takes to erect a bonfire is inversely proportional to the num ber of students doing the w ork.If it takes 20 students 1.5 hours to do the job,how long w ill it take 35 students to do the job,to the nearest m inute?

(A) 51
(B) 52
(C) 53
(D) 54
(E) 55

50.

$$3a + 2b = 20 \text{ and } 2a + 3b = 5$$

Quantity A	**Quantity B**
$a + b$	a

51.

$$m + 2n = 10 \text{ and } m \text{ is } 50\% \text{ of } n$$

Quantity A	**Quantity B**
m^2	n

52.

For the integers a, b, and c, the sum of a and b is 75% of c.

Quantity A	**Quantity B**
$(3/4)(a + b)$	$(4/3)(c)$

53. If $2a = 4b = 8c = 10$, then $64abc =$

(A) 64,000
(B) 16,000
(C) 8,000
(D) 4,000
(E) 1,000

54. If $4m^2 + 6n^3 - 9 = 16$, what is the value of $2m^2 + 3n^3$?

$\boxed{}$

55. If $a + b = 8$, $b + c = 11$, and $a + c = 5$, what is the value of $a + b + c$?

$\boxed{}$

A lgebra A nsw ers

1.**(B).**D istribute the 2 on the left side over the (x + 2),then com bine like term s and sim plify:

$3x + 2(x + 2) = 2x + 16$
$3x + 2x + 4 = 2x + 16$
$5x + 4 = 2x + 16$
$3x + 4 = 16$
$3x = 12$
$x = 4$

2.**1.**First,m ultiply both sides by x to get:

$3x + 7 = 10x$
$7 = 7x$
$1 = x$

The answ er is 1.B y the w ay,“$x \neq 0$” w as in the problem sim ply because the problem had x on the bottom of a fraction,and dividing by zero is illegal.This is just the problem w riter’s w ay of assuring you that the problem ,in fact, has an answ er.So,you generally don’t have to w orry about verbiage like “$x \neq 0$.”

3.**676.**D istribute,group like term s,and solve for x:

$4(-3x - 8) = 8(-x +$
$9)$ $-12x - 32 = -8x +$
72 $-32 = 4x + 72$
$-104 = 4x$
$-26 = x$

Then,m ultiply 26 by 26 in your calculator (or -26 by -26,although the negatives w ill cancel each other out anyw ay) to get x^2,w hich is 676.

4.-**9.** $3x + 7 - 4x + 8 = 2(-2x - 6)$
$-x + 15 = -4x - 12$
$3x + 15 = -12$
$3x = -27$
$x = -9$

5.-**6.** $2x(4 - 6) = -2x + 12$
$2x(-2) = -2x + 12$
$-4x = -2x +$
12 $-2x = 12$
$x = -6$

6.-**2.** $\frac{3(6-x)}{2x}=-6$

3(6 - x) = -6(2x)
18 - 3x = -12x
18 = -9x
-2 = x

7.**0.** $\frac{13}{x+13}=1$

13 = 1(x + 13)
13 = x + 13
0 = x

8.**1.** $\frac{10(-3x+4)}{10-5x}=2$

10(-3x + 4) = 2(10 - 5x)
-30x + 40 = 20 - 10x
40 = 20 + 20x
20 = 20x
1 = x

9.-**6.** $\frac{8-2(-4+10x)}{2-x}=17$

8 - 2(-4 + 10x) = 17(2 - x)
8 + 8 - 20x = 34 - 17x
16 - 20x = 34 - 17x
16 = 34 +
3x -18 = 3x
-6 = x

10.**(A).**Translate the question stem into an equation and solve for z:

- 5 = -z +
7 -12 = -z
12 = z

B ecause z = 12 > -12,Q uantity A is greater.

11.**(E).**B egin by square-rooting both sides of the equation,but rem em ber that square-rooting 225 w ill yield both 15 and -15 as results.(The calculator w ill not rem ind you of this! It's your job to keep this in m ind.) So:

x + 3 =
15 x = 12
so,x - 1 = 11

O R

$x + 3 = -15$
$x = -18$
so, $x - 1 = -19$

O nly -19 appears in the choices.

12. **(B).** To evaluate the expression in Q uantity A ,replace x w ith 2.

$x^2 - 4x + 3 =$
$(2)^2 - 4(2) + 3 =$
$4 - 8 + 3 = -1 < 1$

Therefore,Q uantity B is greater.

13. **(A).** To find the value of p,first replace c w ith 100 to find the value for Q uantity A :

$p = 300c^2 - c$
$p = 300(100)^2 - 100$
$p = 300(10,000) - 100$
$p = 3,000,000 - 100 = 2,999,900$

Since $c = 100$,the value for Q uantity B is 29,000(100) = 2,900,000.

Thus,Q uantity A is greater.

14. **5.** D istribute the 3 on the left side and m ultiply 4(1.5).Feel free to use the calculator:

$3(7 - x) = 4(1.5)$
$21 - 3x = 6$
$-3x = -$
$15\ x = 5$

15. **(C).** First,solve for x:

$1,200x + 6,000 = 13,200$
$1,200x =$
$7,200\ x = 6$

N ow ,solve for y:

$12y + 60 = 132$
$12y =$
$72\ y = 6$

The quantities are equal.A lternatively,you could have noticed that dividing both sides of the first equation by 100 w ould yield an equation identical to the second one,except w ith *x* in place of *y*.Thus,w ithout solving the equations, you could note that the tw o quantities m ust be the sam e.

16.**(A).**First,solve for *x:*

$$-(x)^3 = 64$$
$$(x)^3 = -64$$

Y our calculator w ill not do a cube root for you.Fortunately,on the G R E,cube roots w ill tend to be quite sm all and easy to puzzle out.A sk yourself w hat num ber tim es itself three tim es equals -64? The answ er is $x = -4$.

Since *x* is negative,Q uantity A w ill be positive (a negative num ber tim es itself four tim es w ill be positive) and Q uantity B w ill be negative (a negative num ber tim es itself five tim es w ill be negative).N o further calculations are needed to see that Q uantity A is greater.

17.**(C).**First,solve for *t*:

$$3^3 - 7 = 74t$$
$$3^3 = 81t$$
$$^3 = 27t$$
$$t = 3$$

N ow ,plug = 3 into ^{2} - t: *t t*

$$(3)^2 - 3 = 9 - 3 = 6$$

18.-**9.**First,com bine like term s on each side:

$$3x + 7 - 4x + 8 = 2(-2x -$$
$$6)\ -x + 15 = -4x - 12$$
$$3x + 15 = -12$$
$$3x = -27$$
$$x = -9$$

19.**30.**Since each equation is already solved for *y*,set the right side of each equation equal to the other.

$$4x + 10 = 7x - 5$$
$$10 = 3x - 5$$
$$15 = 3x$$
$$5 = x$$

Substitute 5 for *x* in the first equation and solve for *y*.

$$y = 4(5) +$$
$$10\ y = 30$$

$x = 5$ and $y = 30$.B e sure to answ er for y,not x.

20.**9.**Since the second equation is already solved for k,plug (h - 3) in for k in the first equation:

$2h - 4k = 0$
$2h - 4(h - 3) = 0$
$2h - 4h + 12 =$
$0 -2h = -12$
$h = 6$

Substitute 6 for h in the second equation and solve for k.

$k = (6) -$
$3\ k = 3$

$h = 6$ and $k = 3$,so $h + k = 9$.

21.**3.**N otice that the first equation has the term $-y$ w hile the second equation has the term $+ y$.W hile you could use the substitution m ethod,adding the equations together w ill m ake $-y$ and y cancel,so this is the easiest w ay to solve for x.

$x - y = 4$
$2x + y =$
$5\ 3x = 9$
$x = 3$

22.**1.**B oth equations have the term $+ x$,so you can elim inate the variable x by subtracting the second equation from the first:

$x + 2y = 5\ -$
$(x - 4y = -7)$
$6y = 12$
$y = 2$

Plug this value for y into the first equation to get $x + 2(2) = 5$,or $x = 1$.

B e very careful to change the sign of each term in the second equation w hen subtracting.For exam ple,$-(-4y) = +4y$ and $-(-7) = +7$.

A lternatively,you could have m ultiplied the entire second equation by -1 to get $-x + 4y = 7$ and then added this equation to the first.Either w ay,$x = 1$.

23.**13.**This question contains only tw o equations,but three variables.To isolate y,both x and z m ust be elim inated. N otice that the coefficients of x and z are the sam e in both equations.Subtract the second equation from the first to elim inate x and z.

$4x + y + 3z = 34$
$\underline{-(4x + 3z = 21)}$

$y = 13$

24.**(C).**FO IL the term s in Q uantity A :

$$(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

The tw o quantities are equal.

25.**(A).**FO IL the term s in Q uantity A :

$$(2s + 1)(s + 5) = 2s^2 + 10s + s + 5 = 2s^2 + 11s + 5$$

Since $2s^2 + 11s$ appears in both quantities,elim inate it.B ecause 5 is greater than 4,Q uantity A is greater.

26.**(B).**FO IL the term s in Q uantity A :

$$(2x - y)(x + 4y) = 2x^2 + 8xy - xy - 4y^2 = 2x^2 + 7xy - 4y^2$$

Since $2x^2$ and $-4y^2$ appear in both quantities,elim inate them .Q uantity A is now equal to $7xy$ and Q uantity B is now equal to $8xy$.B ecause $xy > 0$,Q uantity B is greater.(D on't assum e! If xy w ere 0,the tw o quantities w ould have been equal.If xy w ere negative,Q uantity A w ould have been greater.)

27.**(D).**Factor $x^2 - 2x = 0$:

$x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0$ O R $(x - 2) = 0$

$x = 0$ or 2.

Thus,Q uantity A could be less than or equal to Q uantity B .The answ er is (D).

(N ote that you C A N N O T sim ply divide both sides of the original equation by x.It is illegal to divide by a variable unless you have evidence that that variable does not equal zero.)

28.**(D).**In Q uantity A ,m ultiply d by every term in the parentheses:

$d(d^2 - 2d + 1) =$
$(d \times d^2) - (d \times 2d) + (d \times$
$1) = d^3 - 2d^2 + d$

In Q uantity B ,m ultiply d by the tw o term s in the parentheses:

$d(d^2 - 2d) + 1 =$
$(d \times d^2) - (d \times 2d) + 1 =$

$d^3 - 2d^2 + 1$

B ecause $d^3 - 2d^2$ is com m on to both quantities,it can be ignored.The com parison is really betw een *d* and 1.W ithout m ore inform ation about *d*,there is no w ay to know w hich quantity is greater.

29.**(C).**In Q uantity A ,the term xy^2z on the outside of the parentheses m ust be m ultiplied by each of the three term s inside the parentheses.Then sim plify the expression as m uch as possible.

Taking one term at a tim e,the first is $xy^2z \times x^2z = x^3y^2z^2$,because there are three factors of *x*,tw o factors of *y*,and tw o factors of *z*.Sim ilarly,the second term is $xy^2z \times yz^2 = xy^3z^3$ and the third is $xy^2z \times (-xy^2) = -x^2y^4z$.A dding these three term s together gives the distributed form of Q uantity A : $x^3y^2z^2 + xy^3z^3 - x^2y^4z$.

This is identical to Q uantity B ,so no m ore w ork is required.

30.**(D).**Since *a* is com m on to both quantities,it can be ignored.The com parison is really betw een *b* and *c*.B ecause $2b = 4c$,it is true that $b = 2c$,so the com parison is really betw een $2c$ and *c*.W atch out for negatives.If the variables are positive,Q uantity A is greater,but if the variables are negative,Q uantity B is greater.

31.**(D).**If the variables are positive,Q uantity A is greater.H ow ever,all three variables could equal zero,in w hich case the tw o quantities are equal.W atch out for the w ord “nonnegative,” w hich m eans “positive or zero.”

32.**(C).**The follow ing relationships are given: $a = \frac{b}{2}$, $b = \frac{c}{3}$, and $d = 3c$.Pick one variable and put everything in term s of that variable.For instance,variable *a*:

$b = 2a$
$c = 3b = 3(2a) = 6a$
$d = 3c = 3(6a) = 18a$

Substitute into the quantities and sim plify.

Quantity A : $\frac{a+b}{c} = \frac{a+2a}{6a} = \frac{3a}{6a} = \frac{1}{2}$

Quantity B : $\frac{a+b+c}{d} = \frac{a+2a+6a}{18a} = \frac{9a}{18a} = \frac{1}{2}$

The tw o quantities are equal.

33.**II and III only.**Since $x^2 - y^2 = 0$,add y^2 to both sides to get $x^2 = y^2$.It m ight look as though $x = y$,but this is not necessarily the case.For exam ple,*x* could be 2 and *y* could be -2.A lgebraically,w hen you square root both sides of $x^2 = y^2$,you do N O T get $x = y$,but rather $|x| = |y|$.Thus,statem ent I is not necessarily true and statem ent II is true. Statem ent III is also true and can be easily generated algebraically:

$x^2 - y^2 =$

$0\ x^2 = y^2$

$$\frac{x^2}{y^2} = 1$$

34.**(C).**This question m ay at first look difficult,as there are three variables and only tw o equations.H ow ever,notice that the top equation can be divided by 3,yielding $x + 2y = 9$.This can be plugged into the second equation:

$(x + 2y) + z = 11$

$(9) + z =$

$11\ z = 2$

Q uantity A is sim ply 2 + 5 = 7.

For Q uantity B ,rem em ber that $x + 2y = 9$.Thus,Q uantity B is 9 - 2 = 7.

The tw o quantities are equal.

35.**(B).**The factored form of the D ifference of Squares (one of the “special products” you need to m em orize for the exam) is com prised of the term s given in this problem .

$x^2 - y^2 = (x + y)(x - y)$

Substitute the values $\sqrt{12}$ and $\sqrt{3}$ in place of $(x - y)$ and $(x + y)$,respectively:

$x^2 - y^2 = \sqrt{12} \times \sqrt{3}$

C om bine 12 and 3 under the sam e root sign and solve:

$x^2 - y^2 = \sqrt{12 \times 3}$

$x^2 - y^2 = \sqrt{36}$

$x^2 - y^2 = 6$

36.**(B).**Plug in any tw o unequal values for a and b,and Q uantity A w ill alw ays be equal to -1.This is because you can factor a negative out of the top or bottom of the fraction to show that the top and bottom are the sam e,except for their signs:

$$\frac{a-b}{b-a} = \frac{a-b}{-(a-b)} = -1$$

37.**(D).**To com pare a and c,put c in term s of a.M ultiply the first equation by 2 to find that $b = 2a$.Substitute into the second equation: $c = 3b = 3(2a) = 6a$.If all three variables are positive,then $6a > a$.If all three variables are negative, then $a > 6a$.Finally,all three variables could equal 0,m aking the tw o quantities equal.

38.**(C).**The D ifference of Squares (one of the “special products” you need to m em orize for the exam) is $x^2 - y^2 = (x + y)(x - y)$.This pattern w orks for any perfect square m inus another perfect square.Thus,$x^{36} - y^{36}$ w ill factor according to this pattern.N ote that $\sqrt{x^{36}} = (x^{36})^{½} = x^{36/2} = x^{18}$,or $x^{36} = (x^{18})^2$.First,factor $x^{36} - y^{36}$ in the num erator,then cancel $x^{18} + y^{18}$ w ith the $x^{18} + y^{18}$ on the bottom :

$$\frac{x^{36}-y^{36}}{\left(x^{18}+y^{18}\right)\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{18}-y^{18}\right)}{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{9}+y^{9}\right)}=\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}$$

The $x^{18} - y^{18}$ in the num erator w ill also factor according to this pattern.Then cancel $x^9 + y^9$ w ith the $x^9 + y^9$ on the bottom :

$$\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{9}+y^{9}\right)}\left(x^{9}-y^{9}\right)}{\cancel{\left(x^{9}+y^{9}\right)}}=x^{9}-y^{9}$$

39.**(B)**.First,you need to know that $x^2 + 2xy + y^2 = (x + y)^2$.This is one of the “special products” you need to m em orize for the exam .Factor the top,then cancel:

$$\frac{x^{2}+2xy+y^{2}}{2(x+y)^{2}}=\frac{\cancel{(x+y)^{2}}}{2\cancel{(x+y)^{2}}}=\frac{1}{2}$$

40.**(C).**The D ifference of Squares (one of the “special products” you need to m em orize for the exam) tells you that $x^2 - y^2 = (x + y)(x - y)$.This pattern w orks for any perfect square m inus another perfect square.N ote that $\sqrt{a^8} = (a^8)^{½} = a^{8/2} = a^4$,or $a^8 = (a^4)^2$.Thus,$a^8 - b^8$ w ill factor according to this pattern:

$$\frac{a^{8}-b^{8}}{\left(a^{4}+b^{4}\right)\left(a^{2}+b^{2}\right)}=\frac{\cancel{\left(a^{4}+b^{4}\right)}\left(a^{4}-b^{4}\right)}{\cancel{\left(a^{4}+b^{4}\right)}\left(a^{2}+b^{2}\right)}=\frac{a^{4}-b^{4}}{a^{2}+b^{2}}$$

N ow ,factor $a^4 - b^4$ according to the sam e pattern:

$$\frac{a^{4}-b^{4}}{a^{2}+b^{2}}=\frac{\cancel{\left(a^{2}+b^{2}\right)}\left(a^{2}-b^{2}\right)}{\cancel{a^{2}+b^{2}}}=a^{2}-b^{2}$$

Since $a^2 - b^2$ does not appear in the choices,factor one m ore tim e to get $(a + b)(a - b)$,w hich is choice (C).

41.**(D)**.Y ou could sim plify first and then plug in exam ples,or just plug in exam ples w ithout sim plifying.For instance if $x = 2$ and $y = 1$:

Quantity A : $$\frac{x^2}{y+\frac{1}{y}} = \frac{2^2}{1+\frac{1}{1}} = \frac{4}{2} = 2$$

Quantity B : $$\frac{y^2}{x+\frac{1}{x}} = \frac{1^2}{2+\frac{1}{2}} = \frac{1}{\frac{5}{2}} = \frac{2}{5}$$

In this case,Q uantity A is greater.Then,try negatives.If $x = -1$ and $y = -2$ (rem em ber,x m ust be greater than y):

Quantity A : $$\frac{x^2}{y+\frac{1}{y}} = \frac{(-1)^2}{-2+\frac{1}{-2}} = \frac{1}{\frac{5}{-2}} = \frac{-2}{5}$$

Quantity B : $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(-1)+\frac{1}{-1}} = \frac{4}{-2} = -2$$

Q uantity A is still greater.H ow ever,before assum ing that Q uantity A is *alw ays* greater,m ake sure you have tried every category of possibilities for x and y.W hat if x is positive and y is negative? For instance,$x = 2$ and $y = -2$:

Quantity A : $$\frac{x^2}{y+\frac{1}{y}} = \frac{2^2}{-2+\frac{1}{-2}} = \frac{4}{-\frac{5}{2}} = 4\times-\frac{2}{5} = -\frac{8}{5}$$

Quantity B : $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(2)+\frac{1}{2}} = \frac{4}{\frac{5}{2}} = 4\times\frac{2}{5} = \frac{8}{5}$$

42.-**3.**O ne of the “special products” you need to m em orize for the G R E is $x^2 + 2xy + y^2 = (x + y)^2$.W rite this pattern on your paper,plug in the given values,and sim plify:

$x^2 + 2xy + y^2 = (x + y)^2$
$(x^2 + y^2) + 2xy = (x + y)^2$
$(12) + 2xy = (-3)^2$
$12 + 2xy = 9$
$2xy = -3$

43.**6.** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. Write this pattern on your paper and plug in the given values:

$x^2 - y^2 = (x + y)(x - y)$
$3 = (x + y)(1/2)$
$6 = x + y$

44.**4.** One of the "special products" you need to memorize for the exam is $x^2 - 2xy + y^2 = (x - y)^2$. Write this pattern on your paper and plug in the given values:

$x^2 - 2xy + y^2 = (x - y)^2$
$84 + y^2 = (-10)^2$
$84 + y^2 = 100$
$y^2 = 16$
$y = 4$ or -4, so $|y| = 4$.

45.**(B)**. First, multiply out (remember FOIL = First, Outer, Inner, Last) each of the terms in parentheses:

$(x^2 - 2x - 2x + 4) + (x^2 - 1x - 1x + 1) + (x^2) + (x^2 + 1x + 1x + 1) + (x^2 + 2x + 2x + 4)$

Note that some of the terms will cancel each other out (e.g., $-x$ and x, $-2x$ and $2x$):

$(x^2 + 4) + (x^2 + 1) + (x^2) + (x^2 + 1) + (x^2 + 4)$

Finally, combine:

$5x^2 + 10$

46.**II and III only.** Distribute for a: $a = (x + y)^2 = x^2 + 2xy + y^2$. Since $b = x^2 + y^2$, a and b are the same except for the "extra" $2xy$ in a. Since xy is positive, a is greater than b. Statement I is false and statement II is true.

Each term in the sum for a is positive: xy is given as positive, and x^2 and y^2 are definitely positive, as they are squared and not equal to zero. Therefore, $a = x^2 + 2xy + y^2$ is positive. Statement III is true.

47.**(B)**. To answer this question, it is important to understand what is meant by the phrase "directly proportional." It means that $a = kb$, where k is a constant. In alternative form: $\frac{a}{b} = k$, where k is a constant.

So, because they both equal the constant, $\frac{a_{old}}{b_{old}} = \frac{a_{new}}{b_{new}}$. Plugging in values: $\frac{8}{2} = \frac{a_{new}}{4}$. Cross multiply and solve:

$32 = 2a_{new}$
$a_{new} = 16$

48.**(C)**. To answer this question, it is important to understand what is meant by the phrase "inversely proportional." It means that $a = \frac{k}{b}$,w here *k* is a constant.In alternative form ,*ab* = *k*,w here *k* is a constant.

So,because the product of *a* and *b* is alw ays constant: (16)(1) = (8)(*b*),or *b* = 2.

49.**(A).**To answ er this question,it is im portant to understand w hat is m eant by the phrase “inversely proportional.” It means that $\text{time} = \frac{k}{\#\text{ of students}}$,w here *k* is a constant.In alternative form ,(tim e)(# of students) = *k*,w here *k* is a constant.

So,because the product of (tim e) and (# of students) is alw ays constant:

(1.5 hours)(20 students) = (*t* hours)(35 students)

$$t = \frac{(1.5)(20)}{35} = \frac{30}{35} = \frac{6}{7}$$

R em em ber that *t* is in hours,so *t* is $\frac{6}{7} \times 60 = \frac{360}{7} \approx 51.43$ m inutes.To the nearest m inute,the tim e is 51 m inutes.

50.**(B).**B ecause variable *a* is com m on to both quantities,the real com parison is betw een + *b* and 0.Solve the system of equations for *b*.

Multiply the first equation by 2: 3*a* + 2*b* = 20 ⟶ 6*a* + 4*b* = 40
Multiply the second equation by 3: 2*a* + 3*b* = 5 ⟶ 6*a* + 9*b* = 15

Subtract the resulting second equation from the resulting first equation,canceling the *a* term s:

- 5*b* = 25
b = -5

B ecause *b* is negative,Q uantity B is greater.

51.**(C).**Since *m* + 2*n* = 10 and *m* = 0.5*n*,substitute 0.5*n* for *m* to get:

0.5*n* + 2*n* = 10
2.5*n* =
10 *n* = 4

Substitute *n* = 4 back into either equation to get *m* = 2.Since 2^2 = 4,the tw o quantities are equal.

52.**(D).**If *a,b,*and *c* are positive,Q uantity B is greater.If the variables are negative,Q uantity A is greater.For instance,*a* = 1,*b* = 2,and *c* = 4 are valid num bers to test (since 1 + 2 is 75% of 4).In such a case,Q uantity B is obviously greater. But $a = -1$, $b = -2$, and $c = -4$ are also valid numbers to test, in which case Quantity A is greater.

53. **(E).** First, divide the entire given equation by 2 to simplify: $a = 2b = 4c = 5$

Then, break up the equation into several smaller equations, setting each variable expression equal to 5:

$a = 5$
$2b = 5$ (so $b = 2.5$)
$4c = 5$ (so $c = 1.25$)

Thus, $64abc = 64(5)(2.5)(1.25) = 1{,}000$.

54. **12.5.** You do not need to solve for m and n to answer this question. (Nor is it possible to do so!) Simplify the equation:

$$4m^2 + 6n^3 - 9 = 16$$
$$4m^2 + 6n^3 = 25$$

Now divide both sides of the equation by 2:

$$2m^2 + 3n^3 = 12.5$$

This is exactly the quantity the problem is asking for. No further work is required.

55. **12.** While this problem can be solved by substitution, it is much easier and faster to simply stack and add all three equations. To keep them lined up properly, insert "placeholder terms"— for instance, instead of $a + b = 8$, write $a + b + 0c = 11$ or $1a + 1b + 0c = 11$ (since this equation does not use the variable c, it has "zero c"):

$1a + 1b + 0c = 8$
$0a + 1b + 1c = 11$
$1a + 0b + 1c = 5$
$2a + 2b + 2c = 24$

Divide both sides of the equation by 2 to get $a + b + c = 12$.