P robability, C om binatorics, and O verlapping Sets

For questions in the Q uantitative C om parison form at (“Q uantity A ” and “Q uantity B ” given),the answ er choices are alw ays as follow s:

(A) Q uantity A is greater.
(B) Q uantity B is greater.
(C) The tw o quantities are equal.
(D) The relationship cannot be determ ined from the inform ation given.

For questions follow ed by a num eric entry box ☐,you are to enter your ow n answ er in the box.For questions follow ed by fraction-style num eric entry boxes ☐/☐,you are to enter your answ er in the form of a fraction.Y ou are not required to reduce fractions.For exam ple,if the answ er is 1/4,you m ay enter 25/100 or any equivalent fraction.

A ll num bers used are real num bers.A ll figures are assum ed to lie in a plane unless otherw ise indicated.G eom etric figures are not necessarily draw n to scale.Y ou should assum e,how ever,that lines that appear to be straight are actually straight,points on a line are in the order show n,and all geom etric objects are in the relative positions show n.C oordinate system s,such as *xy*-planes and num ber lines,as w ell as graphical data presentations such as bar charts,circle graphs,and line graphs, *are* draw n to scale.A sym bol that appears m ore than once in a question has the sam e m eaning throughout the question.

1.A num ber is random ly chosen from a list of 10 consecutive positive integers.W hat is the probability that the num ber is greater than the m ean?

(A) 3/10
(B) 2/5
(C) 1/2
(D) 7/10
(E) 4/5

2.A num ber is random ly chosen from the first 100 positive integers.W hat is the probability that it is a m ultiple of 3?

(A) 32/100
(B) 33/100
(C) 1/3
(D) 34/100
(E) 2/3

3.A restaurant m enu has several options for tacos.There are 3 types of shells,4 types of m eat,3 types of cheese,and 5 types of salsa.H ow m any distinct tacos can be ordered assum ing that any order contains exactly one of each of the above choices?

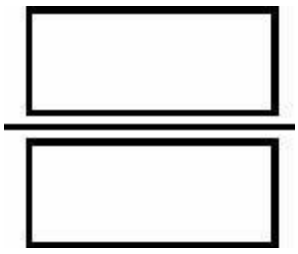
4.A history exam features 5 questions.3 of the questions are m ultiple-choice w ith four options each.The other tw o questions are true or false.If C aroline selects one answ er for every question,how m any different w ays can she answ er the exam ?

5.A certain com pany places a six-sym bol code on each of their products.The first tw o sym bols are one of the letters A –E and the last four sym bols are digits.If repeats are allow ed on both letters and num bers,how m any such codes are possible?

6.The probability is 1/2 that a coin w ill turn up heads on any given toss and the probability is 1/6 that a num ber cube w ith faces num bered 1 to 6 w ill turn up any particular num ber.W hat is the probability of turning up a heads and a 6?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/4
(E) 2/3

7.A n integer is random ly chosen from 2 to 20 inclusive.W hat is the probability that the num ber is prim e?



8.Five students in a classroom are lining up one behind the other for recess.H ow m any different lines are possible?

(A) 5
(B) 10
(C) 24
(D) 25
(E) 120

9.A n Italian restaurant boasts 320 distinct pasta dishes.Each dish contains exactly one pasta,one m eat,and one sauce. If there are 8 pastas and 4 m eats available,how m any sauces are there to choose from ?

10.A 10-student class is to choose a president,vice president,and secretary from the group.A ssum ing that no person can occupy m ore than one post,in how m any w ays can this be accom plished?

11.

Q uantity A	**Q uantity B**
The num ber of 4-digit positive integers w here all 4 digits are less than 5	625

12.B urgerTow n offers m any options for custom izing a burger.There are 3 types of m eats and 7 condim ents: lettuce, tom atoes,pickles,onions,ketchup,m ustard,and special sauce.A burger m ust include m eat,but m ay include as m any or as few condim ents as the custom er w ants.H ow m any different burgers are possible?

(A) 8!
(B) (3)(7!)
(C) (3)(8!)
(D) $(8)(2^7)$
(E) $(3)(2^7)$

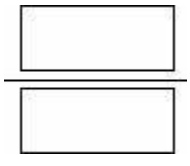
13.The probability of rain is 1/6 for any given day next w eek.W hat is the chance it rains on both M onday and Tuesday?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/3
(E) 2/3

14.H ow m any five-digit num bers can be form ed using the digits 5,6,7,8,9,0 if no digits can be repeated?

(A) 64
(B) 120
(C) 240
(D) 600
(E) 720

15.A bag contains 3 red,2 blue,and 7 w hite m arbles.If a m arble is random ly chosen from the bag,w hat is the probability that it is N O T blue?



16.A m an has 3 different suits,4 different shirts,2 different pairs of socks,and 5 different pairs of shoes.In how

m any w ays can the m an dress him self if he m ust w ear 1 suit,1 shirt,1 pair of socks,and 1 pair of shoes?

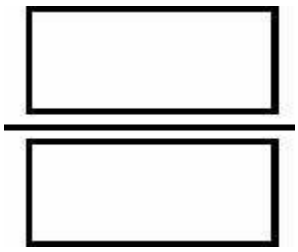
17.A state issues autom obile license plates using tw o letters selected from a 26-letter alphabet,as w ell as four num erals selected from the digits 0 through 9,inclusive.R epeats are perm itted.For exam ple,one license plate com bination could be G F3352.

Q uantity A	**Q uantity B**
The num ber of possible unique license plate com binations	6,000,000

18.A sm all nation issues license plates that consist of just one num ber (selected from the digits 0 through 9, inclusive) and four letters,selected from a 20-letter alphabet.R epeats are perm itted.H ow ever,there is one four-letter com bination that is not allow ed to appear on license plates.H ow m any allow able license plate com binations exist?

(A) 1,599,990
(B) 1,599,999
(C) 1,600,000
(D) 4,569,759
(E) 4,569,760

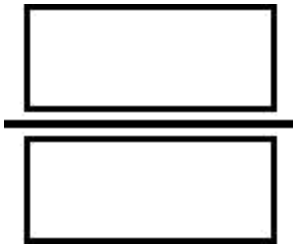
19.A bag contains 6 black chips num bered 1–6 respectively and 6 w hite chips num bered 1–6 respectively.If Pavel reaches into the bag of 12 chips and rem oves 2 chips,one after the other,w ithout replacing them ,w hat is the probability that he w ill pick black chip #3 and then w hite chip #3?



20.Tarik has a pile of 6 green chips num bered 1–6 respectively and another pile of 6 blue chips num bered 1–6 respectively.Tarik w ill random ly pick 1 chip from the green pile and 1 chip from the blue pile.

Q uantity A	**Q uantity B**
The probability that both chips selected by Tarik w ill display a num ber less than 4	1/2

21.A bag contains 6 red chips num bered 1–6 respectively and 6 blue chips num bered 1–6 respectively.If 2 chips are to be picked sequentially from the bag of 12 chips,w ithout replacem ent,w hat is the probability of picking a red chip and then a blue chip w ith the sam e num ber?



22.In a school of 150 students,75 take Latin,110 take Spanish,and 11 take neither.

Q uantity A	Q uantity B
The num ber of students w ho take only Latin	46

23.H ow m any 10-digit num bers can be form ed using only the digits 2 and 5?

(A) 2^{10}
(B) (22)(5!)
(C) (5!)(5!)
(D) 10!/2
(E) 10!

24.A 6-sided cube has sides num bered 1 through 6.If the cube is rolled tw ice,w hat is the probability that the sum of the tw o rolls is equal to 8?

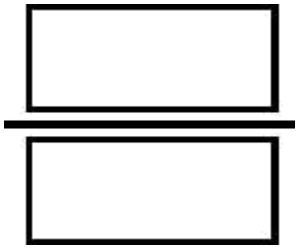
(A) 1/9
(B) 1/8
(C) 5/36
(D) 1/6
(E) 7/36

25.A coin w ith heads on one side and tails on the other has a 1/2 probability of landing on heads.If the coin is flipped 5 tim es,how m any distinct outcom es are possible if the last flip m ust be heads? O utcom es are distinct if they do not contain exactly the sam e results in exactly the sam e order.

26.In a class of 25 students,every student takes either Spanish,Latin,or French,or tw o of the three,but no students take all three languages.9 take Spanish,7 take Latin and 5 take exactly tw o languages.

Q uantity A	Q uantity B
The num ber of students w ho take French	14

27.B ob has a 24-sided die w ith an integer betw een 1 and 24 on each face.Every num ber is featured exactly once. W hen he rolls,w hat is the probability that the num ber show ing is a factor of 24?



28.A baby has *x* total toys.If 9 of the toys are stuffed anim als,7 of the toys w ere given to the baby by its grandm other,5 of the toys are stuffed anim als given to the baby by its grandm other,and 6 of the toys are neither stuffed anim als nor given to the baby by its grandm other,w hat is the value of *x*?

29.H ow m any integers betw een 2,000 and 3,999 have a ones digit that is a prim e num ber?

30.H ow m any integers betw een 2,000 and 6,999 are even and have a digit that is a prim e num ber in the tens place?

31.A group of 12 people w ho have never m et are in a classroom .H ow m any handshakes are exchanged if each pair shakes hands exactly once?

(A) 12
(B) 22
(C) 66
(D) 132
(E) 244

32.A classroom has 12 girls and 20 boys.O ne quarter of the girls in the class have blue eyes.If a child is selected at random from the class,w hat is the probability that he/she is a girl w ho does not have blue eyes?

(A) 3/32
(B) 9/32
(C) 3/8
(D) 23/32
(E) 29/32

33.A coin w ith heads on one side and tails on the other has a 1/2 probability of landing on heads.If the coin is flipped three tim es,w hat is the probability of flipping 2 tails and 1 head,in any order?

(A) 1/8
(B) 1/3
(C) 3/8
(D) 5/8

(E) 2/3

34.A 6-sided cube has sides num bered 1 through 6.If the cube is rolled tw ice,w hat is the probability that at least one of the rolls w ill result in a num ber higher than 4?

(A) 2/9
(B) 1/3
(C) 4/9
(D) 5/9
(E) 2/3

35.Tiles are labeled w ith the integers from 1 to 100 inclusive;no num bers are repeated.If A lm a chooses one tile at random ,replaces it in the group,and chooses another tile at random ,w hat is the probability that the product of the tw o integer values on the tiles is odd?

(A) 1/8
(B) 1/4
(C) 1/3
(D) 1/2
(E) 3/4

36.If the w ord “W O W ” can be rearranged in exactly 3 w ays (W O W ,O W W ,W W O),in how m any w ays can the w ord “M ISSISSIPPI” be rearranged?

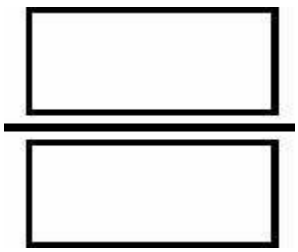
37.If *a*,*b*,and *c* are integers random ly chosen from the set of prim e num bers greater than 2 and less than 30,w hat is the probability that *ab* + *c* is equal to 23?

38.

The probability of rain is 1/2 on any given day next w eek.

Q uantity A	**Q uantity B**
The probability that it rains on A T LEA ST one out of the 7 days next w eek	127/128

39.Tw o fair dice w ith sides num bered 1 to 6 are tossed.W hat is the probability that the sum of the exposed faces on the dice is a prim e num ber?



40.Jack has a cube w ith 6 sides num bered 1 through 6.H e rolls the cube repeatedly until the first tim e that the sum of all of his rolls is even,at w hich tim e he stops.(N ote: it is possible to roll the cube just once.) W hat is the probability that Jack w ill need to roll the cube m ore than 2 tim es in order to get an even sum ?

(A) 1/8
(B) 1/4
(C) 3/8
(D) 1/2
(E) 3/4

41.Jan and 5 other children are in a classroom .The principal of the school w alks in and chooses tw o children at random .W hat is the probability that Jan is chosen?

(A) 4/5
(B) 1/3
(C) 2/5
(D) 7/15
(E) 1/2

42.The probability that G ary w ill eat eggs for breakfast on any given day is 3/7.The probability that G ary w ill eat cereal for breakfast on any given day is 4/7.G ary never has both eggs and cereal for breakfast on the sam e day.

Q uantity A	**Q uantity B**
Probability that G ary eats eggs or cereal for breakfast on a particular day	1

43.The probability that M aria w ill eat breakfast on any given day is 0.5.The probability that M aria w ill w ear a sw eater on any given day is 0.3.The tw o probabilities are independent of each other.

Q uantity A	**Q uantity B**
The probability that M aria eats breakfast or w ears a sw eater	0.8

44.The probability of rain in G reg's tow n on Tuesday is 0.3.The probability that G reg's teacher w ill give him a pop quiz on Tuesday is 0.2.The events occur independently of each other.

Q uantity A	**Q uantity B**
The probability that either or both events occur	The probability that neither event occurs

45.The probability of event *X* occurring is the sam e as the probability of event *Y* occurring.The events occur independently of each other.

Q uantity A	**Q uantity B**
The probability that both events occur	The probability that neither event occurs.

46.A certain city has a 1/3 chance of rain occurring on any given day.In any given 3-day period,w hat is the probability that the city experiences rain?

(A) 1/3
(B) 8/27
(C) 2/3
(D) 19/27

(E) 1

47.Five students,A dnan,B eth,C arol,D an,and Edm und are to be arranged in a line.H ow m any such arrangem ents are possible if B eth is not allow ed to stand next to D an?

(A) 24
(B) 48
(C) 72
(D) 96
(E) 120

48.A polygon has 12 edges.H ow m any different diagonals does it have? (A diagonal is a line draw n from one vertex to any other vertex inside the given shape.This line cannot touch or cross any of the edges of the shape.For exam ple,a triangle has zero diagonals and a rectangle has tw o.)

(A) 54
(B) 66
(C) 108
(D) 132
(E) 144

49.A student council is to be chosen from a class of 12 students consisting of a president,a vice president,and 3 com m ittee m em bers.H ow m any such councils are possible?

(A) $\frac{12!}{7!5!}$
(B) $\frac{12!}{7!3!}$
(C) $\frac{12!}{5!3!}$
(D) $\frac{12!}{7!}$
(E) 12!

50.

Q uantity A	**Q uantity B**
The num ber of possible pairings of 2 colors that can be selected from 5 possible options	The num ber of possible pairings of 8 colors that can be selected from 9 possible options

51.

Q uantity A	**Q uantity B**
The num ber of possible 4-person team s that can be selected from 6 people	The num ber of possible 2-person team s that can be selected from 6 people

52.

Q uantity A	**Q uantity B**

The num ber of w ays 1st,2nd,and 3rd place prizes could be aw arded to 3 out of 6 contestants	The num ber of w ays 1st,2nd,3rd,4th,and 5th place prizes could be aw arded to 5 contestants

53.A n inventory of coins contains 100 different coins.

Q uantity A	**Q uantity B**
The num ber of possible collections of 56 coins that can be selected w here the order of the coins does not m atter	The num ber of possible collections of 44 coins that can be selected w here the order of the coins does not m atter

54.A n office supply store carries an inventory of 1,345 different products,all of w hich it categorizes as “business use,” “personal use,” or both.740 products are categorized as “business use” O N LY and 520 products are categorized as both “business use” and “personal use.”

Q uantity A	**Q uantity B**
The num ber of products characterized as “personal use”	600

55.H ow m any distinct 4-letter “w ords” can be m ade from the nam e “C H R ISTY N A ”? (A “w ord” is any arrangem ent of 4 letters regardless of w hether it can be found in a dictionary.)

(A) 9 (B
) 24 (C)
36 (D)
504
(E) 3,024

56.Seiko has a 6-sided num ber cube w ith sides labeled 1 through 6.If she rolls the cube tw ice,w hat is the probability that the product of the tw o rolls is less than 36?

(A) 1/6
(B) 1/3
(C) 2/3
(D) 5/6
(E) 35/36

57.There is an 80% chance D avid w ill eat a healthy breakfast and a 25% chance that it w ill rain.If these events are independent,w hat is the probability that D avid w ill eat a healthy breakfast O R that it w ill rain?

(A) 20%
(B) 80%
(C) 85%
(D) 95%
(E) 105%

58.The probability of rain is 1/2 for every day next w eek.W hat is the chance that it rains on at least one day during the w orkw eek (M onday through Friday)?

(A) 1/2 (B
) 31/32 (C
) 63/64
(D) 127/128

(E) 5/2

59. Eight women and two men are available to serve on a committee. If three people are picked, what is the probability that the committee includes at least one man?

(A) 1/32
(B) 1/4
(C) 2/5
(D) 7/15
(E) 8/15

60. At Lexington High School, everyone takes at least one language — Spanish, French, or Latin — but no one takes all three languages. If 100 students take Spanish, 80 take French, 40 take Latin, and 22 take exactly two languages, how many students are there?

(A) 198
(B) 220
(C) 242
(D) 264
(E) 286

61.

Of 60 birds found in a certain location, 20 are songbirds and 23 are migratory. (It is possible for a songbird to be migratory, or not.)

Quantity A	**Quantity B**
The number of the 60 birds that are neither migratory nor songbirds	16

P robability, C om binatorics, and O verlapping Sets A nsw ers

1.**(C).**In a list of 10 consecutive integers,the m ean w ill be the average of the 5th and 6th num bers.Therefore,the 6th through 10th integers (five total integers) w ill be larger than the m ean.Since probability is determ ined by the num ber of desired item s divided by the total num ber of choices,the probability that the num ber chosen is higher than the m ean is 5/10 = 1/2.

A nother approach to this problem is to create a set of 10 consecutive integers;the easiest such list contains the num bers {1,2,3,4,5,6,7,8,9,10}.The m ean is one-half the sum of the first elem ent plus the last elem ent,or $\frac{1+10}{2}=5.5$.Therefore,there are 5 num bers higher than the m ean in the list: 6,7,8,9 and 10.A gain,the probability of choosing a num ber higher than the m ean is 5/10 = 1/2.

2.**(B).**The first 100 positive integers com prise the set of num bers containing the integers 1 to 100.O f these num bers,the only ones that are divisible by 3 are {3,6,9,… ,96,99},w hich adds up to exactly 33 num bers.This can be determ ined in several w ays.Y ou could sim ply count the m ultiples of 3,but that's a bit slow .A lternatively,you can com pute 99/3 = 33 and realize that there are 33 m ultiples of 3 up to and including 99.The num ber 100 is not divisible by 3,so the correct answ er is 33/100.

A lternatively,you can use the "add one and you're done trick," subtracting the first m ultiple of 3 from the last m ultiple of 3,dividing by 3 and then adding 1: $\frac{(99-3)}{3}+1=33$.Then,since probability is determ ined by the num ber of desired options divided by the total num ber of options,the probability that the num ber chosen is a m ultiple of 3 is 33/100.

3.**180.**This problem tests the fundam ental counting principle,w hich states that the total num ber of choices is equal to the product of the independent choices.Therefore,the total num ber of tacos is (3)(4)(3)(5) = 180 tacos.

4.**256.**This question tests the fundam ental counting principle,w hich states that the total num ber of choices is equal to the product of the independent choices.The five separate test questions give you five independent choices.For the three m ultiple choice questions there are four options each,w hereas for the tw o true/false questions there are tw o options each.M ultiplying the independent choices yields (4)(4)(4)(2)(2) = 256 different w ays to answ er the exam .

5.**250,000.**This question tests the fundam ental counting principle,w hich states that the total num ber of choices is equal to the product of the independent choices.There are 5 choices for each of the letters at the beginning of the code (A ,B ,C ,D ,or E) and 10 choices for each of the 4 digits at the end of the code (0,1,2,3,4,5,6,7,8,or 9). Therefore,there are (5)(5)(10)(10)(10)(10) = 250,000 possible codes.

6.**(B).**The probability of independent events A A N D B occurring is equal to the product of the probability of event A and the probability of event B .In this case,the probability of the coin turning up heads is 1/2 and the product of rolling a 6 is 1/6.Therefore,the probability of heads A N D a 6 is equal to (1/2)(1/6) = 1/12.A lternatively,you could list all the possible outcom es: H 1,H 2,H 3,H 4,H 5,H 6,T1,T2,T3,T4,T5,T6.There are 12 total outcom es and only 1 w ith heads and a 6.Therefore,the desired outcom e divided by the total num ber of outcom es is equal to 1/12.

7.**8/19.**A m ong the integers 2 through 20 inclusive there are 8 prim es: 2,3,5,7,11,13,17,and 19.From 2 to 20 inclusive there are exactly 20 - 2 + 1 = 19 integers;rem em ber to “add one before you’re done” to include both endpoints.A lternatively,there are 20 integers from 1 to 20 inclusive,so there m ust be 19 integers from 2 to 20 inclusive.Since probability is defined as the num ber of desired item s divided by the total num ber of choices,the probability that the num ber chosen is prim e is 8/19.

8.**(E).**This problem describes a specific arrangem ent of people,so this is an ordering problem .The total num ber of w ays to arrange *n* item s in order is *n*! (the exclam ation point m eans “factorial”).Therefore,since there are 5 students to be arranged in a line,the total num ber of possible orderings is 5! = (5)(4)(3)(2)(1) = 120.

A lternatively,ask,“H ow m any choices do I have for each place in the line?” C onsider the first place in line.Since no students have been chosen yet,there are 5 total options for the first place in line.Sim ilarly,for the second place there are 4 choices,because one student has already been chosen to occupy the first spot.A pplying the sam e logic,there are 3 choices for the third place,2 for the fourth,and 1 for the fifth.U sing the fundam ental counting principle,there are a total of (5)(4)(3)(2)(1) = 120 different lines.

9.**10.**This problem tests the fundam ental counting principle,w hich states that the total num ber of choices is equal to the product of the independent choices.Let the num ber of sauces be represented by the variable *S*.The total num ber of possible pasta dishes can be represented by each separate choice m ultiplied together: (8)(4)(*S*),or 32*S*.The problem also indicates that the total num ber of pasta dishes m ust be equal to 320.Therefore,32*S* = 320 and *S* = 10.

10.**720**.O ne possible approach is to ask,“H ow m any choices do I have for each of the class positions?” B egin by considering the president of the class.Since no one has been chosen yet,there are 10 students from w hom to choose. Then,for the vice president there are 9 options because now one student has already been chosen as president. Sim ilarly,there are 8 choices for the secretary.U sing the fundam ental counting principle,the total num ber of possible selections is (10)(9)(8) = 720.

A lternatively,you could use factorials.In this case order m atters because you are choosing people for specific positions.This problem is synonym ous to asking,“H ow m any different w ays can you line up 3 students as first, second,and third from a class of 10?” The num ber of w ays to arrange the entire class in line is 10!.H ow ever,the problem is only concerned w ith the first 3 students in line,so exclude rearrangem ents of the last 7.The w ay in w hich these “non-chosen” 7 students can be ordered is 7!.Thus the total num ber of arrangem ents for 3 students from a class of 10 is $\frac{10!}{7!} = (10)(9)(8) = 720$.

11.**(B).**This is a com binatorics problem — to calculate Q uantity A ,m ake four “slots” (since the num bers are all four-digit num bers),and determ ine how m any possibilities there are for each slot:

____ ____ ____ ____

Since all 4 digits m ust be less than 5,the possible options are lim ited to 0,1,2,3 and 4.H ow ever,a 4-digit num ber cannot begin w ith 0 (the sm allest four-digit num ber is 1,000).So,the first slot has only 4 possibilities,not 5:

4 ____ 5 ____ 5 ____ 5 ____

(4)(5)(5)(5) = 500,w hich is less than 625.The answ er is (B).

N ote that the value in Q uantity B com es from m ultiplying 5 × 5 × 5 × 5.N eglecting to note that 4-digit num bers cannot begin w ith 0 results in incorrect choice (C).

12.**(E).**This problem tests the fundam ental counting principle,w hich states that the total num ber of choices is equal to the product of the independent choices.The key to this problem is realizing how m any choices there are for each option.For the m eat,there are obviously 3 choices.For each of the condim ents there are exactly 2 choices: yes or no The only real choice regarding each condim ent is w hether to include it at all.A s there are 7 condim ents,the total num ber of choices is (3)(2)(2)(2)(2)(2)(2)(2) = (3)(2^7).

N ote: the condim ent options cannot be counted as 8! (0 through 7 = 8 options) because,in this case,the order in w hich the options are chosen does not m atter;a burger w ith lettuce and pickles is the sam e as a burger w ith pickles and lettuce.

13.**(A).**For probability questions,alw ays begin by separating out the probabilities of each individual event.Then,if you need all the events to happen (an “A N D question”),m ultiply the probabilities together.If you only need one of the m ultiple events to happen (an “O R question”),add the probabilities together.

In this case,there are tw o events: rain on M onday and rain on Tuesday.The question asks for the probability that it w ill rain on M onday A N D on Tuesday,so m ultiply the individual probabilities together:

$$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$$

14.**(D).**This problem relies on the fundam ental counting principle,w hich says that the total num ber of w ays for som ething to happen is the product of the num ber of options for each individual choice.The problem asks how m any five-digit num bers can be created from the digits 5,6,7,8,9,and 0.For the first digit,there are only five options (5, 6,7,8,and 9) because a five-digit num ber m ust start w ith a non-zero integer.For the second digit,there are 5 choices again,because now zero can be used but one of the other num bers has already been used,and num bers cannot be repeated.For the third num ber,there are 4 choices,for the fourth there are 3 choices,and for the fifth num ber there are 2 choices.Thus,the total num ber of choices is (5)(5)(4)(3)(2) = 600.

A lternatively,you can use the sam e logic and realize there are 5 choices for the first digit.(Separate out the first step because you have to rem ove the zero from consideration.) The rem aining five digits all have an equal chance of being chosen,so choose four out of the rem aining five digits to com plete the num ber.The num ber of w ays in w hich this second step can be accom plished is (5!)/(1!) = (5)(4)(3)(2).Thus,the total num ber of choices is again equal to (5)(5) (4)(3)(2) = 600.

15.**5/6.**In the bag of m arbles,there are 3 red m arbles and 7 w hite m arbles,for a total of 10 m arbles that are N O T blue.There are a total of 3 + 7 + 2 = 12 m arbles in the bag.Since probability is defined as the num ber of desired item s divided by the total num ber of choices,the probability that the m arble chosen is not blue is 10/12 = 5/6.

16.**120.**This problem utilizes the fundam ental counting principle,w hich states that the total num ber of choices is equal to the product of the independent choices.Since the m an m ust choose one suit,one shirt,one pair of socks,and one pair of shoes,the total num ber of outfits is the num ber of suits tim es the num ber of shirts tim es the num ber of socks tim es the num ber of shoes: (3)(4)(2)(5) = 120.

17.**(A).**This is a com binatorics problem .The license plates have 2 letters and 4 num bers,so m ake six “slots” and

determ ine how m any possibilities there are for each slot.There are 26 letters in the alphabet and 10 digits to pick from ,so:

$$\underline{26} \quad \underline{26} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10}$$

M ultiply 26 × 26 on your calculator to get 676.A dd four zeroes for the four 10's to get 6,760,000.Q uantity A is larger.

18.**(A).**First,calculate the total num ber of com binations,ignoring the one illegal "w ord." Since there are 10 possibilities for the first slot and 20 possibilities for the other 4 slots:

$$\underline{10} \quad \underline{20} \quad \underline{20} \quad \underline{20} \quad \underline{20}$$

M ultiply to get 1,600,000.

N ow ,consider all the license plates that contain the forbidden w ord.Say,for exam ple,that the forbidden w ord is G U R G .That m eans that the plates 0G U R G ,1G U R G ,2G U R G ,3G U R G ,etc.are forbidden,for a total of 10 forbidden plates.Y ou can also express this m athem atically as:

$$\underline{10} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1}$$

M ultiply to get 10 forbidden plates.Subtract: 1,600,000 - 10 = 1,599,990.

A lternatively,after finding the 1,600,000 figure,glance at the answ er choices: the correct answ er can't be (C),(D),or (E).A nsw er choice (B) subtracts just 1 plate,but m ore than 1 plate could have the forbidden "w ord." Therefore,(A) m ust be the correct answ er.

19.$\mathbf{\frac{1}{121}}$ The probability of picking black chip #3 is $\frac{1}{12}$.O nce Pavel has rem oved the first chip,only 11 chips rem ain,so the probability of picking w hite chip #3 is $\frac{1}{11}$.M ultiply $\frac{1}{12} \times \frac{1}{11} = \frac{1}{121}$.

20.**(B).**In this problem ,Tarik is N O T picking 1 chip out of all 12.R ather,he is picking 1 chip out of 6 green ones, and then picking another chip out of 6 blue ones.There are 3 green chips w ith num bers less than 4,so Tarik has a $\frac{3}{6}$ chance of selecting a green chip show ing a num ber less than 4.Likew ise,Tarik has a $\frac{3}{6}$ chance of selecting a blue chip show ing a num ber less than 4.Therefore,Q uantity A is equal to $\frac{3}{6} \times \frac{3}{6} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.Q uantity B is larger.

21.$\mathbf{\frac{1}{22}}$.The trap answ er in this problem is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.This is N O T the answ er to the question being asked—

rather,this is the answ er to the question,“W hat is the probability of picking a red chip and then a blue chip that both have #3?” (or any other specific num ber).This is a m ore specific question than the one actually asked.In the question, asked,there are six possible w ays to fulfill the requirem ents of the problem ,not one,because the problem does not specify w hether the num ber should be 1,2,3,4,5,or 6.

Thus,A N Y of the 6 red chips is acceptable for the first pick.H ow ever,on the second pick,only the blue chip w ith the sam e num ber as the red one that w as just picked is acceptable (you need w hatever chip is the “m atch” for the first one picked).Thus:

$$\frac{6}{12}\times\frac{1}{11}=\frac{1}{2}\times\frac{1}{11}=\frac{1}{22}$$

22.**(B).**U se the overlapping sets form ula for tw o groups: Total = G roup 1 + G roup 2 - B oth + N either.(A dding the tw o groups— in this case Latin and Spanish— double-counts the students w ho take both classes,so the form ula subtracts the “both” students.)

150 = 75 + 110 - *B* + 11
150 = 196 - *B*
46 = *B*

C areful! This is not the value of Q uantity A .Since 46 students take both Latin and Spanish,subtract 46 from the total w ho take Latin to find those w ho take only Latin:

75 - 46 = 29

Thus,Q uantity A is equal to 29 and Q uantity B is larger.

23.**(A).**This problem relies on the fundam ental counting principle,w hich says that the total num ber of w ays for som ething to happen is the product of the num ber of options for each individual choice.For any digit of the 10-digit num ber there are exactly tw o options,a 2 or a 5.Thus,since there are tw o choices for each digit and it is a 10-digit num ber,there are (2)(2)(2)(2)(2)(2)(2)(2)(2)(2) = 2^{10} total choices.

24.**(C).**The probability of any event equals the num ber of w ays to get the desired outcom e divided by the total num ber of w ays for the event to happen.Starting w ith the denom inator,use the fundam ental counting principle to com pute the total num ber of w ays to roll a cube tw ice.There are 6 possibilities (1,2,3,4,5,or 6) for the first roll and 6 for the second,giving a total of (6)(6) = 36 possibilities for the tw o rolls.For the num erator,determ ine the num ber of possible com binations that w ill add to 8.For exam ple,you m ight roll a 2 the first tim e and a 6 the second tim e.The full set of options is (2,6),(3,5),(4,4),(5,3),and (6,2).Thus there are 5 possible com binations that sum to 8,yielding a probability of 5/36.

25.**16.**This problem utilizes the fundam ental counting principle,w hich states that the total num ber of choices is equal to the product of the independent choices.For the first flip there are 2 options: heads or tails.Sim ilarly,for the second flip there 1 2 options,for the third there are 2 options,for the fourth there are 2 options,and for the fifth there is only one option because the problem restricts this final flip to heads.Therefore,the total num ber of outcom es is (2)(2)(2)(2)(1) = 16.A good rephrasing of this question is,“H ow m any different outcom es are there if the coin is flipped 4 tim es?” The fifth flip,having been restricted to heads,is irrelevant.Therefore,the total num ber of w ays to flip the coin five tim es w ith heads for the fifth flip is equal to the total num ber of w ays to flip the coin four tim es;either w ay,the answ er is 16.

26.**(C).**The problem specifies that no one takes all three languages.In addition,a total of 5 people take 2 languages. Thus,5 people have been double-counted.B ecause you know the total num ber of people w ho have been double-counted (5) and triple-counted (0),you can use the standard overlapping sets form ula:

Total = Spanish + French + Latin - (Tw o of the Three) - 2(A ll Three)

25 = 9 + French + 7 - 5 - 2(0)
25 = 11 + French
14 = French

The tw o quantities are equal.

$\frac{1}{3}$

27. Probability equals the num ber of desired outcom es divided by the total num ber of possible outcom es.A m ong the integers 1 through 24 there are 4 factor pairs of 24: (1,24),(2,12),(3,8),and (4,6),for a total of 8 factors.The total num ber of possible outcom es w hen rolling the die once is 24.The probability that the num ber chosen is a factor of 24 is 8/24 = 1/3.

28.**17.**U se the overlapping sets form ula for tw o groups: Total = G roup 1 + G roup 2 - B oth + N either.H ere,the groups are “stuffed anim al” and “given by the baby’s grandm other.” The problem indicates that the “both” category is equal to 5,and that the “neither” num ber is 6.The total is *x*.

Total = G roup 1 + G roup 2 - B oth + N either

x = 9 + 7 - 5 +
6 *x* = 17

29.**800.**This is a com binatorics problem .M ake four “slots” (since the num bers are all four-digit num bers),and determ ine how m any possibilities there are for each slot:

____ ____ ____ ____

Since the num ber m ust begin w ith 2 or 3,there are 2 possibilities for the first slot.B ecause the ones digit m ust be prim e and there are only 4 prim e 1-digit num bers (2,3,5,and 7),there are 4 possibilities for the last slot.

2 ____ ____ 4

The other slots have no restrictions,so put 10,since there are 10 digits from 0–9:

2 10 10 4

M ultiply to get 800.

A lternatively,figure out the pattern and add up the num ber of qualifying 4-digit integers.In the first ten num bers, 2000–2009,there are exactly 4 num bers that have a prim e units digit: 2002,2003,2005,and 2007.The pattern then repeats in the next group of ten num bers,2010–2020,and so on.In any group of ten num bers,then,four qualify.In the

first one hundred num bers,2000–2099,there are ten groups of ten,or 10 × 4 = 40 num bers that have a prim e units digit.In the first one thousand num bers,2000–2999,there are ten groups of one hundred,or 100 × 4 = 400 num bers that have a prim e units digit.There are a total of tw o groups of one thousand num bers (2000–2999 and 3000–3999), so there are a total of 400 × 2 = 800 num bers that have a prim e units digit.

30.**1,000.**This is a com binatorics problem .M ake four “slots” (since the num bers are all four-digit num bers),and determ ine how m any possibilities there are for each slot:

____ ____ ____ ____

Since the num ber m ust begin w ith 2,3,4,5,or 6,there are 5 possibilities for the first slot.B ecause the digit in the tens place m ust be prim e and there are only 4 prim e 1-digit num bers (2,3,5,and 7),there are 4 options for the tens place.

5 ____ 4 ____

Since the num ber m ust be even,the final digit m ust be 0,2,4,6,or 8— thus,there are 5 possibilities for the units digit.(N ote that saying that a 4-digit num ber is even is sim ply saying that it ends in an even digit— for instance,3,792 is even.) N o restrictions w ere given for the hundreds slot,so there are 10 options,since there are 10 digits from 0–9.

5 10 4 5

M ultiply to get 1,000.

A lternatively,figure out the pattern and add up the num ber of qualifying 4-digit integers.In the first one hundred num bers,2,000–2,099,there are exactly four sets of ten num bers that have a prim e tens digit: 202-,203-,205-,and 207-.W ithin each of those four sets,there are five possibilities for num bers that also end in an even digit.For exam ple,the possibilities for the 202- set w ould be 2,020,2,022,2,024,2,026,and 2,028.O ut of the first one hundred num bers,then,there are 5 × 4 = 20 possibilities that have both a prim e tens digit and end in an even digit.

From 2,000 to 6,999,there are 50 groups of 100 num bers (6,999 - 2,000 + 1 = 5,000,and 5,000 ÷ 100 = 50). Therefore,there are 20 × 50 = 1,000 num bers.

31.**(C).**M ultiple approaches are possible here.O ne w ay is to im agine the scenario and count up the num ber of handshakes: you are the first person.H ow m any hands do you need to shake? There are 11 other people in the room , so you need to shake hands 11 tim es.N ow ,m ove to the second person: how m any hands m ust he shake? H e has already shaken your hand,leaving him 10 others w ith w hom to shake hands.The third person w ill need to shake hands w ith 9 others,and so on.Therefore,there are a total of 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 handshakes.The fastest w ay to find the sum of a group of consecutive num bers is to take the average of the first and last term s and m ultiply it by the num ber of term s.The average is (11 + 1)/2 = 6 and there are 11 - 1 + 1 = 11 term s (find the difference betw een the term s and “add one before you’re done”).The sum is 6 × 11 = 66.

A lternatively,rephrase the question as “H ow m any different w ays can any 2 people be chosen from a group of 12?” (This w orks because the problem ultim ately asks you to “choose” each distinct pair of 2 people one tim e.) The key here is to realize that handshakes are independent of order,i.e.,it doesn’t m atter if A shakes hands w ith B or if B shakes hands w ith A ;it’s the sam e outcom e.Thus,you only care about how m any pairs you can m ake.A ny tim e you recognize a group of order-independent item s being selected from a larger set,you can apply the form ula $\frac{\text{total!}}{\text{in!out!}}$

to arrive at the total num ber of com binations.Thus: $\frac{12!}{2!10!} = \frac{12 \cdot 11}{2} = 66$.

32.**(B).**The probability of any outcom e is equal to the num ber of desired outcom es divided by the total num ber of outcom es.There are 12 girls and 20 boys in the classroom .If one-quarter of the girls have blue eyes,then there are (12)(1/4) = 3 girls w ith blue eyes.Therefore,there are 12 - 3 = 9 girls w ho do N O T have blue eyes.The total num ber of w ays in w hich you could choose a child is sim ply the total num ber of children in the class,nam ely 12 + 20 = 32. Therefore,the probability of choosing a girl w ho does not have blue eyes equals the num ber of girls w ithout blue eyes divided by the total num ber of children,w hich is 9/32.

33.**(C).**There are only 2 possible outcom es for each flip and only 3 flips total.The m ost straightforw ard approach is to list all of the possible outcom es: {H H H ,H H T,H TH ,H TT,TTT,TTH TH H ,TH T}.O f these 8 possibilities,3 of the outcom es have one head and tw o tails,so the probability of this event is 3/8.

A lternatively,you can count the total num ber of w ays of getting 1 head w ithout listing all the possibilities.If the coin is flipped 3 tim es and you w ant only 1 head,then there are 3 possible positions for the single head: on the first flip alone,on the second flip alone,or on the third flip alone.Since there are 2 possible outcom es for each flip,heads or tails,there are (2)(2)(2) = 8 total outcom es.A gain,the probability is 3/8.

Finally,you can com pute the probability directly.The probability of flipping heads is 1/2 and the probability of flipping tails is also 1/2.The probability of getting heads in the first position alone,or H TT,is (1/2)(1/2)(1/2) = 1/8, w here you m ultiply because you have heads A N D tails A N D tails.This represents the probability of heads in position 1,but heads could also be in position 2 alone or in position 3 alone.Since there are 3 possible positions for the heads, m ultiply by 3 to get the total probability (3)(1/8) = 3/8.

34.**(D).**B ecause this problem is asking for an “at least” solution,you can use the 1 - *x* shortcut.The probability that at least one roll results in a num ber higher than 4 is equal to 1 m inus the probability that both of the rolls result in num bers 4 or low er.For one roll,there are 6 possible outcom es (1 through 6) and 4 w ays in w hich the outcom e can be 4 or low er,so the probability is 4/6 = 2/3.Thus,the probability that both rolls results in num bers 4 or low er is (2/3) (2/3) = 4/9.This is the result that you do N O T w ant;subtract this from 1 to get the probability that you do w ant. The probability that at least one of the rolls results in a num ber higher than 4 is 1 - (4/9) = 5/9.

A lternatively,w rite out the possibilities.The total num ber of possibilities for tw o rolls is (6)(6) = 36.H ere are the w ays in w hich at least one num ber higher than 4 can be rolled:

51,52,53,54,55,56
61,62,63,64,65,66
15,25,35,45 (note: 55 and 65 have already been counted above)
16,26,36,46 (note: 56 and 66 have already been counted above)

There are 20 elem ents (be careful not to double-count any options).The probability of at least one roll resulting in a num ber higher than 4 is 20/36 = 5/9.

35.**(B).**Y ou need to use both probability and num ber properties concepts in order to answ er this question.First,in order for tw o integers to produce an odd integer,the tw o starting integers m ust be odd.A n odd tim es an odd equals an odd.A n even tim es an odd,by contrast,produces an even,as does an even tim es an even.

W ithin the set of tiles,there are 50 even num bers (2,4,6,… ,100) and 50 odd num bers (1,3,5,… ,99).O ne random ly-chosen tile w ill have a 50/100 = 1/2 probability of being even,and a 1/2 probability of being odd.The probability of choosing an odd tile first is (1/2) and the probability of choosing an odd tile second is also (1/2),so the probability of "first odd A N D second odd" is (1/2)(1/2) = 1/4.

A lternatively,you can recognize that there are only four options for odd/even pairs if tw o tiles are chosen: O O ,O E, EO ,EE .The only one of these com binations that yields an odd product is O O .Since all of these com binations are equally likely,and since O O is exactly one out of the four possibilities,the probability of choosing O O is 1/4.

36.**34,650.**This is a com binatorics problem ,and the W O W exam ple is intended to m ake it clear to you that any W is considered identical to any other W — sw itching one W w ith another w ould N O T result in a different com bination, just as sw itching one S w ith another in M ISSISSIPPI w ould not result in a different com bination.

Therefore,solve this problem using the classic com binatorics form ula for accounting for subgroups am ong w hich order does not m atter:

Total N um ber of Item s!
First G roup! Second G roup! Etc…

B ecause M ISSISSIPPI has 11 letters,including one M ,four S's,four I's,and tw o P's:

$$\frac{11!}{1!4!4!2!}$$

N ow expand the factorials and cancel;use the calculator for the last step of the calculation:

$$\frac{11\times10\times9\times8\times7\times6\times5\times4!}{1!4!(4\times3\times2\times1)(2\times1)}=\frac{11\times10\times9\times8\times7\times6\times5}{(4\times3\times2\times1)(2\times1)} = 11 \times 10 \times 9 \times 7 \times 5 = 34{,}650$$

37.**0.**B ecause *a,b,*and *c* are prim e num bers greater than 2,they m ust all be odd.So,*ab* + *c* = (O D D)(O D D) + O D D . O D D tim es O D D yields another odd num ber,so the calculation sim plifies to O D D + O D D ,w hich w ill alw ays yield an even answ er.

Thus,it is im possible for *ab* + *c* to be odd.The probability is zero.(N ote: if you w ere ever asked to type a nonzero probability into a box,you w ould need to express it as a decim al betw een 0 and 1.)

38.**(C).**Since Q uantity A is an "at least" problem ,you can use the 1 - *x* shortcut.R ather than calculate the probability of rain on exactly 1 day next w eek,and then the probability of rain on exactly 2 days next w eek,and so on (after w hich you w ould still have to add all of the probabilities together!),instead calculate the probability of no rain at all on any day,and then subtract that num ber from 1.That w ill give the com bined probabilities for any scenarios that include rain on at least 1 day.

Probability of N O rain for any of the 7 days = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{128}$

Subtract this probability from 1:

$$1-\frac{1}{128}=\frac{128}{128}-\frac{1}{128}=\frac{127}{128}$$

Q uantities A and B are equal.

39.**5/12.**First think about the prim e num bers less than 12,the m axim um sum of the num bers on the dice.These prim es are 2,3,5,7,11.

The probability of rolling 2,3,5,7,or 11 = the num ber of w ays to roll any of these sum s,divided by the total num ber of possible rolls.The total num ber of possible dice rolls is 6 × 6 = 36.

Sum of 2 can happen 1 w ay: 1 + 1
Sum of 3 can happen 2 w ays: 1 + 2 or 2 + 1
Sum of 5 can happen 4 w ays: 1 + 4,2 + 3,3 + 2,4 + 1
Sum of 7 can happen 6 w ays: 1 + 6,2 + 5,3 + 4,4 + 3,5 + 2,6 + 1
Sum of 11 can happen 2 w ays: 5 + 6,6 + 5

That's a total of 1 + 2 + 4 + 6 + 2 = 15 w ays to roll a prim e sum .

Thus,the probability is 15/36 = 5/12.

40.**(B).**Jack w ill only continue to roll the cube if the sum of the individual rolls is odd.For the first roll,this w ill only occur if the num ber itself is odd;if Jack does not stop after the first roll,then,he m ust have rolled an odd num ber for the first roll.For the second roll,in order for the sum of the first and second to be odd,Jack m ust now roll an even (because odd + even = odd).Y ou can rephrase the question: "W hat is the probability that Jack w ill roll an odd first and an even second?" The probability of event A A N D event B equals the probability of A tim es the probability of B .Since the probability of odd = 1/2 and the probability of even = 1/2,the probability of the first num ber being odd A N D the second num ber being even is (1/2)(1/2) = 1/4.

41.**(B).**The probability of any event equals the num ber of w ays to get the desired outcom e divided by the total num ber of outcom es.

Start w ith the denom inator,w hich is the total num ber of w ays that the principal can choose tw o children from the classroom .U se the fundam ental counting principle.There are 6 possible options for the first choice and 5 for the second,giving (6)(5) = 30 possibilities.H ow ever,this double-counts som e cases;for exam ple,choosing Jan and then R obert is the sam e as choosing R obert and then Jan.D ivide the total num ber of pairs by $\frac{6\cdot 5}{2}=15$ .A lternatively,use the form ula for a set w here the order doesn't m atter: $\frac{\text{total!}}{\text{in! out!}}$.In this case: $\frac{6!}{2!4!}=\frac{6\times 5\times 4!}{(2)(4!)}=\frac{6\times 5}{2}=15$.

N ow com pute the num erator,w hich is the num ber of pairs that include Jan.Since the pair only includes tw o children and one is already decided (Jan),there are exactly 5 options for the other child.Thus,there are 5 total pairs that include Jan: Jan w ith each of the other students.

The probability of choosing a pair w ith Jan is 5/15 = 1/3.

A s a final alternative,you m ay sim ply list all the pairs of students and count how m any of them include Jan.Label the students in the class as J,1,2,3,4,and 5,w here J is Jan.Then all the pairs can be listed as (J1),(J2),(J3),(J4),(J5), (12),(13),(14),(15),(23),(24),(25),(34),(35),and (45).(B e careful not to include repeats.) There are 15 total elem ents in this list and 5 that include Jan,yielding a probability of 5/15 = 1/3.

42.**(C).**The probability that G ary w ill eat eggs is 3/7.The probability that he w ill eat cereal is 4/7.In order to calculate the probability of eating one O R the other,add: 3/7 + 4/7 = 7/7 = 1.Q uantities A and B are the sam e.

43.**(B).**The problem indicates that the events occur independently of each other.Therefore,in calculating Q uantity A , the first step is to calculate the “or” situation,but you cannot stop there.W hen you add 0.5 + 0.3 = 0.8,you double-count the occurrences w hen both events occur.N ext,you have to subtract out the probability of both events occurring in order to get rid of the “double counted” occurrences.

N otice that this is a Q uantitative C om parison.A t this point,you could conclude that,because the 0.8 figure includes at least one “both” occurrence,the real figure for Q uantity A m ust be sm aller than 0.8.Therefore,Q uantity B m ust be larger.

To do the actual m ath,find the probability of both events occurring (breakfast A N D sw eater): (0.5)(0.3) = 0.15. Subtract the “A N D ” occurrences from the total “or” probability: 0.8 - 0.15 = 0.65

Q uantity B is larger.

44.**(B).**The problem indicates that the events occur independently of each other.Therefore,in calculating Q uantity A , do not sim ply add both events,even though it is an “or” situation.A dding 0.3 + 0.2 = 0.5 is incorrect because the probability that both events occur is counted tw ice.(O nly add probabilities in an “or” situation w hen the probabilities are m utually exclusive.)

W hile Q uantity A ’s value should include the probability that both events occur,m ake sure to count this probability only once,not tw ice.Since the probability that both events occur is 0.3(0.2) = 0.06,you m ust subtract this value from the “or” probability.

Q uantity A : A dd the tw o probabilities (rain O R pop quiz) and subtract B O TH scenarios (rain A N D pop quiz):

0.3 + 0.2 - (0.3)(0.2) = 0.44

Q uantity B : M ultiply the probability that rain does N O T occur (0.7) and the probability that the pop quiz does N O T occur (0.8).

0.7(0.8) = 0.56

Y ou could also note that,collectively,Q uantities A and B include every possibility and are m utually exclusive of one another (Q uantity A includes “rain and no quiz,” “quiz and no rain,” and “both rain and quiz,” and Q uantity B includes “no rain and no quiz”).Therefore,the values of Q uantities A and B m ust add to 1.C alculating the value of either Q uantity A or Q uantity B w ould autom atically tell you the value for the other quantity.

If you do this,calculate Q uantity B first (because it’s the easier of the tw o quantities to calculate) and then subtract

Q uantity B from 1 in order to get Q uantity A 's value.That is,1 - 0.56 = 0.44.

45.**(D).**The probabilities of both events are the sam e *as each other,*but that doesn't indicate anything about the value of those probabilities.For instance,if both probabilities are equal to 0.99,then the probability of both events occurring (Q uantity A) is M U C H higher than the probability of neither occurring (Q uantity B).

B ut w hat if the probability of each event is m ore like 0.000001 ("1 in a m illion")? Then the chance of neither occurring (Q uantity B) w ould be m uch higher than the chance of both occurring (Q uantity A).

In the case that the probabilities are each equal to 0.5,then — and only then — w ould the tw o quantities be equal.

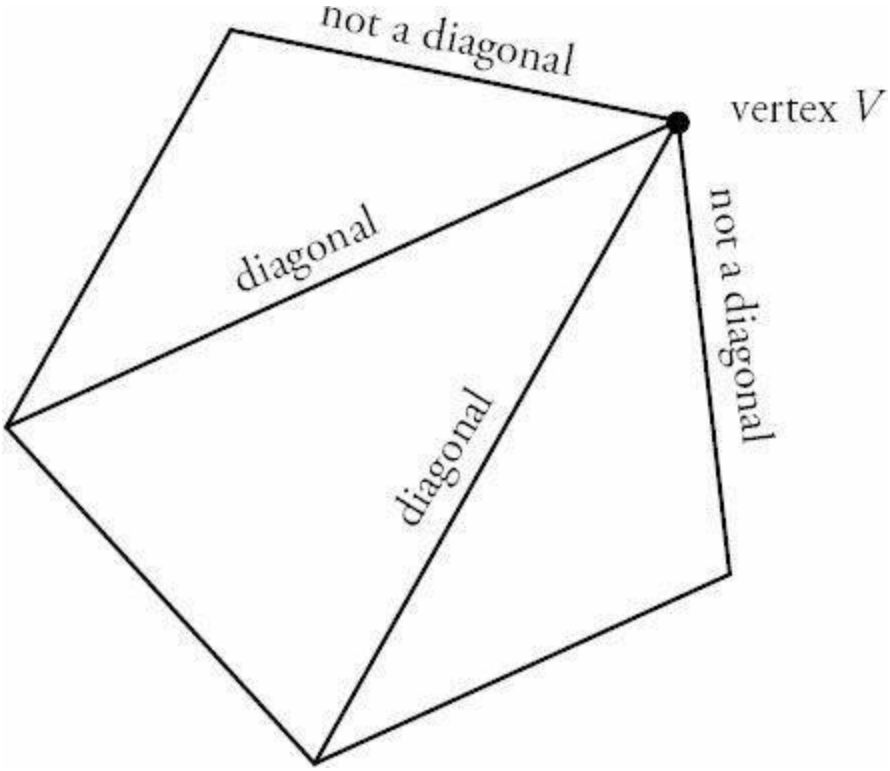
46.**(D).**In essence,the question is asking "W hat is the probability that one or m ore days are rainy days?" since any single rainy day w ould m ean the city experiences rain.In this case,em ploy the 1 - *x* shortcut,w here the probability of rain on one or m ore days is equal to 1 m inus the probability of no rain on any day.Since the probability of rain is 1/3 on any given day,the probability of no rain on any given day is 1 - 1/3 = 2/3.Therefore,the probability of no rain on three consecutive days is (2/3)(2/3)(2/3) = 8/27.Finally,subtract from 1 to find the probability that it rains on one or m ore days: P(1 or m ore days) = 1 - P(no rain) = 1 - 8/27 = 19/27.

47.**(C).**The num ber of w ays in w hich the students can be arranged w ith B eth and D an separated is equal to the total num ber of w ays in w hich the students can be arranged m inus the num ber of w ays they can be arranged w ith B eth and D an together.The total num ber of w ays to arrange 5 students in a line is 5! = 120.To com pute the num ber of w ays to arrange the 5 students such that B eth and D an are together,group B eth and D an as "one" person,since they m ust be lined up together.Then the problem becom es one of lining up 4 students,w hich gives 4! possibilities.H ow ever, rem em ber that there are actually tw o options for the B eth and D an arrangem ent: B eth first and then D an or D an first and then B eth.Therefore,there are (4!)(2) = (4)(3)(2)(1)(2) = 48 total w ays in w hich the students can be lined up w ith D an and B eth together.Finally,there are 120 - 48 = 72 arrangem ents w here B eth w ill be separated from D an.

A lternatively,com pute the num ber of w ays to arrange the students directly by considering individual cases.In this case,investigate how m any w ays there are to arrange the students if B eth occupies each spot in line and sum them to find the total.If B eth is standing in the first spot in line,then there are 3 options for the second spot (since D an cannot occupy this position),3 options for the next spot,2 options for the next spot,and finally 1 option for the last spot. This yields (3)(3)(2)(1) = 18 total possibilities if B eth is first.If B eth is second,then there are 3 options for the first person (D an cannot be this person),2 options for the third person (D an cannot be this person either),2 options for the fourth person,and 1 for the fifth.This yields (3)(2)(2)(1) = 12 possibilities.In fact,if B eth is third or fourth in line, you arrive at the sam e situation as w hen B eth is second.Thus there w ill be 12 possible arrangem ents w hether B eth is 2nd,3rd,or 4th in line,yielding 36 total arrangem ents for these 3 cases.U sing sim ilar logic,the situation in w hich B eth is last in line is exactly equal to the situation w here she is first in line.Thus,there are (18)(2) = 36 possibilities w here B eth is first or last.In total,this yields 36 + 36 = 72 possible outcom es w hen considering all of the possible placem ents for B eth.

48.**(A).**A diagonal of a polygon is an internal line segm ent connecting any tw o unique vertices;this line segm ent does not lie along an edge of the given shape.C onsider a polygon w ith 12 vertices.C onstruct a diagonal by choosing any tw o vertices and connecting them w ith a line.R em em ber that this is order independent;the line is the sam e regardless of w hich is the starting vertex.Therefore,this is analogous to choosing any 2 elem ents from a set of 12,and can be w ritten as $\frac{12!}{10!2!}=\frac{12\times11\times10!}{10!(2)(1)}=\frac{12\times11}{2}=6\times11=66$ .H ow ever,this m ethod includes the vertices connected to their adjacent vertices,w hich form edges instead of diagonals.In order to account for this,subtract the num ber of edges on the polygon from the above num ber: 66 - 12 = 54.

A lternatively,if you choose a random vertex of the 12-sided shape,then there are 12 - 1 = 11 lines that can be draw n to other vertices since no line can be draw n from the vertex to itself.H ow ever,the lines from this vertex to the tw o adjacent vertices w ill lie along the edges of the polygon and therefore cannot be included as diagonals (see the figure of a pentagon below for an exam ple).



Thus,there are 12 - 1 - 2 = 9 diagonals for any given vertex.Since there are 12 vertices,you m ight think that the total num ber of diagonals is equal to (12)(9) = 108.H ow ever,using this schem e you have counted each diagonal tw ice, using each side of the diagonal once as the starting point.Therefore,there are half this m any different diagonals: 108/2 = 54.

49.**(B).**C onsider each independent choice and then use the fundam ental counting principle to calculate the total num ber of possibilities.Start by choosing the president,for w hich there are 12 total options.N ext,for the vice president,there are 11 options.Finally,choose the com m ittee of 3 from the rem aining 10 students using the form ula $\frac{\text{total!}}{\text{in! out!}}$.The num ber of possible arrangem ents for the 3 com m ittee m em bers is $\frac{10!}{7!3!}$.Finally,using the fundam ental counting principle,the total num ber of w ays to choose a president, vice president,and com m ittee of 3 is $\frac{12\times11\times10!}{7!3!}$ w hich can be rew ritten as $\frac{12!}{7!3!}$.

50.**(A).**This is a classic com binatorics problem in w hich *order doesn't m atter*— that is,the pairing "blue,green" is the sam e as "green,blue." A color is either "in" or "out." U se the standard "order doesn't m atter" form ula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Q uantity A :

$$\frac{5!}{2!3!}=\frac{5\times4\times3!}{(2)(1)3!}=\frac{5\times4}{2}=10$$

For Q uantity B :

$$\frac{9!}{8!1!}=\frac{9\times8!}{8!(1)}=9$$

N ote that w hile the form ula is necessary for Q uantity A ,you could reason your w ay to the value for Q uantity B : every com bination that selects 8 out of 9 colors w ill leave out exactly 1 color.Since there are 9 colors,there are 9 com binations.

51.**(C).**This is a classic com binatorics problem in w hich *order doesn't m atter* — that is,picking Joe and Jane is the sam e as picking Jane and Joe.A person is either on the team or not.U se the standard "order doesn't m atter" form ula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Q uantity A :

$$\frac{6!}{2!4!}=\frac{6\times5\times4!}{(2)(1)4!}=\frac{6\times5}{2}=15$$

For Q uantity B :

$$\frac{6!}{4!2!}=\frac{6\times5\times4!}{4!(2)(1)}=\frac{6\times5}{2}=15$$

The quantities are equal.N ote the first line of each Q uantity: from that stage,you can already tell that the values w ill be the sam e.

This w ill alw ays w ork— w hen order doesn't m atter,the num ber of w ays to pick 4 and leave out 2 is the sam e as the num ber of w ays to pick 2 and leave out 4.Either w ay,it's one group of 4 and one group of 2.W hat actually happens to those groups (getting picked,not getting picked,getting a prize,losing a contest,etc.) is irrelevant to the ultim ate solution.

52.**(C).**In this problem ,order m atters;if Jane com es in 1st place and R ohit com es in 2nd,there is a different outcom e than w hen R ohit places 1st and Jane places 2nd.U se the fundam ental counting principle to solve.To determ ine Q uantity A m ake three slots (one for each prize).Six people are available to w in 1st,and then five people could w in 2nd,and four people could w in 3rd:

M ultiply: (6)(5)(4) = 120.

For Q uantity B ,m ake 5 slots,one for each prize.Five people can w in 1st prize,then 4 people for 2nd prize,and so on:

M ultiply (5)(4)(3)(2)(1) = 120.Q uantities A and B are equal.

53.**(C).**This is a classic com binatorics problem in w hich *order doesn't m atter* — in fact,the problem tells you that explicitly.U se the standard "order doesn't m atter" form ula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Q uantity A :

$$\frac{100!}{56!44!}$$

B ecause the num bers are so large,there m ust be a w ay to solve the problem w ithout actually sim plifying (even w ith a calculator,this is unreasonable under G R E tim e lim its).Try Q uantity B and com pare:

$$\frac{100!}{44!56!}$$

The quantities are equal.N ote that this w ill alw ays w ork — w hen order doesn't m atter,the num ber of w ays to pick 56 and leave out 44 is the sam e as the num ber of w ays to pick 44 and leave out 56.Either w ay,it's one group of 56 and one group of 44.W hat actually happens to those groups (being part of a collection,being left out of the collection, etc.) is irrelevant to the ultim ate solution.

54.**(A).**U se the overlapping sets form ula for tw o groups: Total = G roup 1 + G roup 2 - B oth + N either.B ut first,add 740 ("business use O N LY ") + 520 ("business use" and "personal use") to get 1260,the total num ber of products categorized as "business use."

A lso note that the problem indicates that *all* of the products fall into one or both of the tw o categories,so "neither" in this form ula is equal to zero.

Total = B usiness Personal - B oth + N
+ 1,345 = 1,260 + either - 520 + 0
P 1,345 = 740 +
P 605 = *P*

Q uantity A is larger.N ote that the question asked for the num ber of products characterized as “personal use” (w hich includes products in the “both” group).If the problem had asked for the num ber of products characterized as “personal use” O N LY ,you w ould have had to subtract the “both” group to get 605 - 520 = 85.In this problem ,how ever,Q uantity A equals 605.

55.**(E).**B ecause all of the letters of the nam e “C hristyna” are unique,there are 9 distinct choices of letters to form the 4-letter “w ord.” In addition,order does m atter: the “w ord” C H R I is different from the “w ord” C R H I.U se the fundam ental counting principle to solve.

B egin by considering the first choice,for w hich there are 9 total options.Sim ilarly,for the second choice there are 8 options,because one letter has already been chosen.Em ploying the sam e logic,there are 7 choices for the third letter and 6 choices for the fourth letter.U sing the fundam ental counting principle,(9)(8)(7)(6) = 3024 w ords.

A lternatively,try factorials.The total num ber of w ays to arrange all 9 letters is 9! H ow ever,the problem is only concerned w ith “w ords” using four of these letters,m eaning you m ust exclude rearrangem ents of the other 5.The num ber of w ays in w hich you can order the 5 “non-chosen” letters is 5!.Thus the total num ber of “w ords” w ith 4 letters that can be m ade from the nam e “C hristyna” is $\frac{9!}{5!}$ = (9)(8)(7)(6) = 3,024 “w ords.”

56.**(E).**U se the 1 - *x* shortcut for this problem .H ow do you know to use this shortcut? In this case,you’re being asked to solve for the probability that the product of the values from the tw o rolls w ill be less than 36.This w ill be tim e consum ing to calculate directly because there are m any different com binations that w ould produce a product less than 36 — (1)(6),(4)(5),(6)(3),and so on — the vast m ajority of com binations,actually! In fact,there’s only one case w hen the product of the tw o rolls w ill *not* be less than 36: (6)(6).It is m uch easier to solve for this one value and then apply the 1 - *x* shortcut.In other w ords:

(The probability of rolling less than 36) + (The probability of rolling 36) = 1
→ (The probability of rolling less than 36) = 1 - (The probability of rolling 36)

This is an “A N D question,” because you need to get a 6 on the first roll *and* on the second roll.M ultiply the tw o probabilities together:

$$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$$

Subtract from 1:

$$1-\frac{1}{36}=\frac{35}{36}$$

57.**(C).**For probability questions,begin by separating out the probabilities of each individual event.Then,if all of the events m ust happen (an “A N D question”),m ultiply the probabilities together.If only one of the m ultiple events needs to happen (an “O R question”),add.This question is an O R question,because it asks for the probability that D avid w ill eat a healthy breakfast *or* that it w ill rain.

A t first glance,this m ay seem strange,because if you add the tw o probabilities together,you’ll get som ething bigger

than 100% ,w hich is N EV ER possible: 0.8 + 0.25 = 1.05.This figure double-counts the cases w here D avid eats a healthy breakfast A N D it rains.Subtract out these cases in order to find the desired value.

In order to calculate the probability that D avid w ill eat a healthy breakfast A N D that it w ill rain,m ultiply the individual probabilities together:

0.8 × 0.25 = 0.2

Finally,subtract to find the probability that D avid w ill eat a healthy breakfast O R that it w ill rain:

1.05 - 0.2 = 0.85,or 85%

58.**(B).**B ecause this question uses “at least” language,use the 1 - *x* shortcut.In this case,the only outcom e you do *not* w ant is rain on zero days next w eek.It w ill be m uch faster to solve for that probability and subtract from 1 in order to find the probability of all of the other outcom es (that there w ill be rain on one or m ore days next w eek).

(The probability of rain on at least one day) + (The probability of no rain) = 1

H ow do you find the probability of no rain? Y ou w ant no rain on M onday A N D Tuesday A N D W ednesday A N D Thursday A N D Friday.M ultiply together the individual probabilities of no rain on each of the five days.

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{32}$$

Subtract this from 1 to get the desired answ er:

$$1-\frac{1}{32}=\frac{31}{32}$$

59.**(E).**B ecause this is an “at least” question,use the 1 - *x* shortcut.

(The probability of picking at least one m an) + (The probability of picking no m en) = 1

The probability of picking no m en is an A N D setup: w om an A N D w om an A N D w om an.

For the first choice,there are 8 w om en out of 10 people: 8/10 = 4/5
For the second choice,there are 7/9 (because one w om an has already been chosen)
For the third choice,there are 6/8 = 3/4

M ultiply the three probabilities together to find the probability that the com m ittee w ill be com prised of w om an A N D w om an A N D w om an:

$$\frac{4}{5}\times\frac{7}{9}\times\frac{3}{4}=\frac{1}{5}\times\frac{7}{3}\times\frac{1}{1}=\frac{7}{15}$$

To determ ine the probability of picking at least one m an,subtract this result from 1:

1 - 7/15 = 8/15

60.**(A).**This overlapping sets question can be solved w ith the follow ing equation:

Total # of People = G roup 1 + G roup 2 + G roup 3 - (# of people in 2 groups) - (2)(# of people in all 3 groups) + (# of people in no groups)

The problem indicates that everyone takes at least one language,so the num ber of people in no groups is zero.The problem also indicates that nobody takes all three languages,so that value is also zero.

Total # of Students = 100 + 80 + 40 - 22 - (2)(0) + 0 = 198.

61.**(A).**It is not possible to solve for a single value for Q uantity A ,but it is possible to tell that Q uantity A is greater than 17.Since 20 birds are songbirds and 23 are m igratory,the total of these groups is 43,w hich is less than 60.It is possible for the overlap (the num ber of m igratory songbirds) to be as little as 0,w hich w ould result in 20 songbirds, 23 non-songbird m igratory birds,and 60 - 20 - 23 = 17 birds that are neither songbirds nor m igratory.

It is also possible that there could be as m any as 20 birds that overlap the tw o categories.(Find this figure by taking the num ber of birds in the sm aller group;in this case,there are 20 songbirds).In the case that there are 20 m igratory songbirds,there w ould also be 3 m igratory birds that are not songbirds,in w hich case there w ould be 60 - 20 - 3 = 37 birds that are neither m igratory nor songbirds.

Thus,the num ber of birds that are neither m igratory nor songbirds is at least 17 and at m ost 37.N o m atter w here in the range that num ber m ay be,it is greater than Q uantity B ,w hich is only 16.