

# Design & Analysis of Algorithm

In case of any problem:

**Steps:**

- 1) Identify & write its algo
- 2) Translate it into Mathematical language (pseudocode)
- 3) Analysis phase

Running Time      Space

In running time, terms introduced are:

- Worst-case (Maximum running time of an algo on input size)

e.g. let code:

```
func(n)
{ for(i=n; i>1; i--)
  if (n==100)
    break; }
```

Let  $l$  be input size that varies so,

$$T_{\text{worst}}(l) = \max_{|l|=n} [T(l)]$$

- Average-case: Based on Probability

$$T_{\text{avg}}(I) = \sum_{i=1}^n P(l_i) \times T_i$$

## **Mathematical Tools:**

Summation:

Let there is sequence of  $a_1, a_2, \dots, a_n$

So,

$$a_1 + a_2 + \dots + a_n = \sum_{i=1}^n a_i$$

$$1 + 1 + \dots + 1 \quad (n \text{ times})$$

(pseudo code)

$$1) = \sum_{i=1}^n 1 = n(1) = n$$

$$2) \sum_{i=1}^n c a_i = c \sum_{i=1}^n a_i$$

$$3) \sum_{i=1}^n (a_i + b_i) = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i$$

### Arithmetic Series:

There should be a common difference in consecutive terms.

$$\cdot \sum_{i=1}^n i = 1 + 2 + 3 + \dots + n$$

$$= \frac{n(n+1)}{2} \quad (\text{when } d=1)$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

### Geometric Series:

There is common ratio ( $r$ )

$$\begin{array}{l} 1 + 2 + 4 + 8 + \dots + n \\ 2^0 + 2^1 + 2^2 + \dots + 2^n \end{array} = \frac{r^{n+1} - 1}{r - 1}$$

Let series is

$$a_1 + a_2 + a_3 + a^n = \frac{a(r^{n+1} - 1)}{r - 1} = \frac{a_1(1 - r^n)}{1 - r}$$

$(r > 1)$ 
 $(r < 1)$



He was an impostor  
rejected

$$\frac{a}{1-r} \quad (r < 1)$$

$$\frac{a_1(1-r^n)}{1-r} \quad (r > 1)$$

Harmonic Series:

$$\sum_{i=1}^n \frac{1}{i}$$

Quadratic Series:

$$\begin{aligned} \sum_{i=1}^n i^2 &= 1+4+9+\dots+n^2 \\ &= \frac{n(n+1)(2n+1)}{6} \end{aligned}$$

Selection Sort:

61	40	52	7	17
0	1	2	3	4

Pick minimum & swap i.e.

7	40	52	61	17
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7	17	52	61	40
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⋮  
similar

Code for this is:  
let arr be the array then  
{  
  for (int i=0; i<n; i++)  
  {  
    min=i;  
    for (int j=i; j<n; j++)  
  }}

```

    if (arr[j] < arr[min])
        min = j
    }
    if (min != i)
        swapping(arr[i], arr[min])
    }

```

$\text{for } (i=0; i < n; i+=5)$   $T(n) = 3n+2$   
 print { "SE" } — ②

Iteration#	1	2	3	4	k
i	0	5	10	15	5k
		↓ 5x1	↓ 5x2	↓ 5x3	↓ (k-1)(5)

Let print(body) for k time

Loop terminate when  $i = n$

$$5k = n$$

$$k = n/5$$