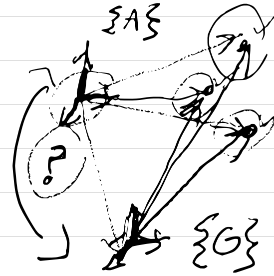
HW 5 Report (check part D)

Part A.)

$${}^{G}P_1 = {}^{G}_{A}R \cdot {}^{A}P_1 + {}^{G}P_{Aorg}$$



$$\Delta x_{12} = x_1 - x_2$$
$$\Delta y_{12} = y_1 - y_2$$
$$\Delta z_{12} = z_1 - z_2$$

• Finding each row

$$\begin{bmatrix} {}^{G}\Delta x_{12} \\ {}^{G}\Delta y_{12} \\ {}^{G}\Delta z_{12} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} {}^{A}\Delta x_{12} \\ {}^{A}\Delta y_{12} \\ {}^{A}\Delta z_{12} \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 5x + 3y \\ 2x + 2y \\ \cancel{4x + 5x} \end{bmatrix}$$

solving

Pt1 $0 = r_{11}\,{}^{A}\Delta x_{12} + r_{12}\,{}^{A}\Delta y_{12} + r_{13}\,{}^{A}\Delta z_{12} - {}^{G}\Delta x_{12}$

Pt2 $0 = r_{11}\,{}^{A}\Delta x_{23} + r_{12}\,{}^{A}\Delta y_{23} + r_{13}\,{}^{A}\Delta z_{23} - {}^{G}\Delta x_{23}$

Pt3 $0 = r_{11}\,{}^{A}\Delta x_{31} + r_{12}\,{}^{A}\Delta y_{31} + r_{13}\,{}^{A}\Delta z_{31} - {}^{G}\Delta x_{31}$

$$\underbrace{\begin{bmatrix} {}^{A}\Delta x_{12} & {}^{A}\Delta y_{12} & {}^{A}\Delta z_{12} \\ {}^{A}\Delta x_{23} & {}^{A}\Delta y_{23} & {}^{A}\Delta z_{23} \\ {}^{A}\Delta x_{31} & {}^{A}\Delta y_{31} & {}^{A}\Delta z_{31} \end{bmatrix}}_{\text{"}\mathbf{A}\text{"}} \underbrace{\begin{bmatrix} r_{11} \\ r_{12} \\ r_{23} \end{bmatrix}}_{\text{"}\underline{x}\text{"}} = \underbrace{\begin{bmatrix} {}^{G}\Delta x_{12} \\ {}^{G}\Delta x_{23} \\ {}^{G}\Delta x_{31} \end{bmatrix}}_{\text{"}\underline{b}\text{"}}$$

$$\underline{x} = A^{-1}\underline{b} \rightarrow \underline{x} = A^{T}\underline{b}$$

$$r_{11} = {}^{A}\Delta x_{12} \cdot {}^{G}\Delta x_{12} + {}^{A}\Delta x_{23}\,{}^{G}\Delta x_{23} + {}^{A}\Delta x_{31}\,{}^{G}\Delta x_{31}$$

$$r_{12} = {}^{A}\Delta y_{12} \cdot {}^{G}\Delta x_{12} + {}^{A}\Delta y_{23}\,{}^{G}\Delta x_{23} + {}^{A}\Delta z_{31}\,{}^{G}\Delta x_{31}$$

$$r_{13} = {}^{A}\Delta z_{12}\,{}^{G}\Delta x_{12} + {}^{A}\Delta z_{23}\,{}^{G}\Delta x_{23} + {}^{A}\Delta z_{31}\,{}^{G}\Delta x_{31}$$

- The next row (r_{21}, r_{22}, r_{23}) follows the same pattern:

$$r_{21} = {}^{A}\Delta x_{12}\,{}^{G}\Delta y_{12} + {}^{A}\Delta x_{23}\,{}^{G}\Delta y_{23} + {}^{A}\Delta x_{31}\,{}^{G}\Delta y_{31}$$

$$r_{22} = {}^{A}\Delta y_{12}\,{}^{G}\Delta y_{12} + {}^{A}\Delta y_{23}\,{}^{G}\Delta y_{23} + {}^{A}\Delta y_{31}\,{}^{G}\Delta y_{31}$$

$$r_{23} = {}^{A}\Delta z_{12}\,{}^{G}\Delta y_{12} + {}^{A}\Delta z_{23}\,{}^{G}\Delta y_{23} + {}^{A}\Delta z_{31}\,{}^{G}\Delta y_{31}$$

...

$$r_{31} = {}^{A}\Delta x_{12}\,{}^{G}\Delta z_{12} + {}^{A}\Delta x_{23}\,{}^{G}\Delta z_{23} + {}^{A}\Delta x_{31}\,{}^{G}\Delta z_{31}$$

$$r_{32} = {}^{A}\Delta y_{12}\,{}^{G}\Delta z_{12} + {}^{A}\Delta y_{23}\,{}^{G}\Delta z_{23} + {}^{A}\Delta y_{31}\,{}^{G}\Delta z_{31}$$

$$r_{33} = {}^{A}\Delta z_{12}\,{}^{G}\Delta z_{12} + {}^{A}\Delta z_{23}\,{}^{G}\Delta z_{23} + {}^{A}\Delta z_{31}\,{}^{G}\Delta z_{31}$$

with these, we have $\boxed{{}^{G}_{A}R}$ ✓

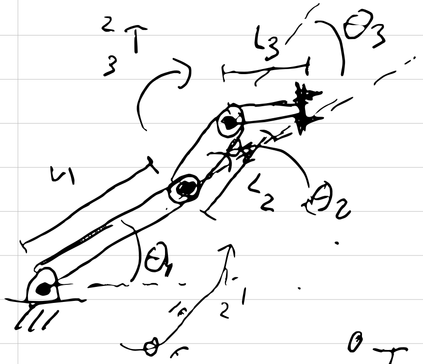
Finding ${}^{G}P_{A}$

$${}^{G}P_1 + {}^{G}P_2 = 2\,{}^{G}P_A + {}^{G}_{A}R\left({}^{A}P_1 + {}^{A}P_2\right)$$

$$\boxed{{}^{G}P_A} = \frac{1}{2}\left({}^{G}P_1 + {}^{G}P_2 - {}^{G}_{A}R\cdot\left({}^{A}P_1 + {}^{A}P_2\right)\right)$$

Part B) Craig 4.2 Derive the inverse Kinematics of a 3 link manipulator

RRR



DH Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

$${}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T$$

R_{13} R_{23}

r_{31} r_{32}

$$= \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & c_1(c_2L_2+L_1) \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & s_1(c_2L_2+L_1) \\ s_{23} & c_{23} & 0 & s_2L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_{13} = s_1$
$R_{23} = -c_1 \rightarrow \boxed{\theta_1} = \mathrm{atan2}(R_{13}, -R_{23})$

$P_x = c_1(c_2L_2 + L_1) \rightarrow c_2 = \dfrac{\left(\frac{P_x}{c_1} - L_1\right)}{L_2}$

∘ Now we need s_2:

$P_y = s_2L_2 \rightarrow s_2 = \dfrac{P_y}{L_2}$

$\rightarrow \boxed{\theta_2} = \mathrm{atan2}\left(\dfrac{P_y}{L_2}, \dfrac{\left(\frac{P_x}{c_1} - L_1\right)}{L_2}\right)$

$\boxed{\theta_3} = \mathrm{atan2}(r_{31}, r_{32}) - \theta_2$

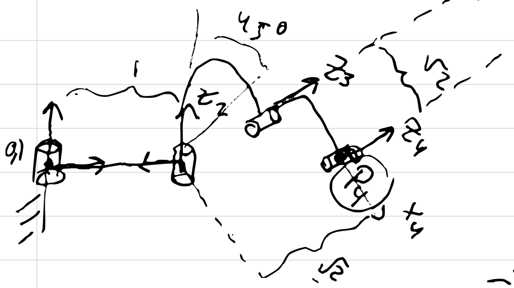
4.16 For the 4R manipulator, find all θ_3 options such that: ${}^0P_{4org} = \begin{bmatrix} 1.1 \\ 1.5 \\ 1.707 \end{bmatrix}$

$a_1 = 1 \quad \alpha_1 = 0 \quad d_1 = 0 \quad \theta_1 = 0$

$a_2 = 0 \quad \alpha_2 = 45° \quad d_2 = 0 \quad \theta_2 = 90$

$a_3 = \sqrt{2} \quad \alpha_3 = 0 \quad d_3 = \sqrt{2} \quad \theta_3 = -90$

$\theta_4 = 0$



$${}^0P_{4org} = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3P_{4org}$$

$${}^3P_{4org} = \begin{bmatrix} a_3 \\ -d_4 s\alpha_3 \\ d_4 c\alpha_3 \end{bmatrix} \rightarrow \begin{bmatrix} \sqrt{2} \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow {}^0P_{4z} = \cancel{c\alpha_1\, {}^1f_3} + \cancel{c\alpha_1\, d_2}$

$\rightarrow {}^0P_{4z} = f_3$

$\rightarrow = a_3 s_{\alpha_2} s_3 + d_3 c_{\alpha_2}$

$1.707 = \sqrt{2}\,\frac{\sqrt{2}}{2}\, s_3 + \sqrt{2}\,\frac{\sqrt{2}}{2}$

$1.707 - 1 = s_3$

$\underline{s_3 = 0.707}$

$$\boxed{\begin{array}{l}\text{Two sol:} \\ \theta_3 = \underline{135°, 45°} = \operatorname{asin}(0.707)\end{array}}$$