Lecture 5 (sections 1.2, 1.3)

Section 1.2

— a homogeneous system has at least 1 solution (the zero solution)

Consider a homogeneous system with n-unknowns. If RREF of the corresponding augmented matrix has r non-zero rows, then the system has $(n - r)$ free variables.
▪ $n - r = 0 \Longrightarrow$ no free variables, only zero solution
▪ $n - r > 0 \Longrightarrow$ an infinite number of solutions

$\Longrightarrow$ Question: can $r > n$? (or $m > n$?)

No proof: If $m > n$, REF of the augmented matrix will have at least $(m - n)$ zero rows.

$\Longrightarrow$ Note:

$$m < n \quad \underset{\textbf{it must be}}{\Longrightarrow} \quad r < n \quad \Longrightarrow \quad \text{infinitely many solutions}$$

A matrix is a 2D array (table) of numbers:

$$
\begin{array}{c}
\begin{bmatrix} 2 & 1 & 3 & -7 \\ 4 & 0 & 2 & 13 \\ 0 & 1 & 0 & 99 \end{bmatrix} \\
 \uparrow \\
\text{column}
\end{array}
\qquad\qquad
\begin{array}{cc}
\begin{bmatrix} 2 & 1 & 3 & -7 \\ 4 & 0 & 2 & 13 \\ 0 & 1 & 0 & 99 \end{bmatrix} & \begin{array}{c} \\ \longleftarrow \quad \text{row} \\ \\ \end{array}
\end{array}
$$

$\#$ rows : 3 $\qquad$ $\#$ columns : 4

The dimension of this matrix is 3×4

$\Longrightarrow$ In general, $m \times n$ matrix has m rows and n columns:

$$
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
a_{m1} & m_{m2} & \cdots & a_{mn}
\end{bmatrix}
$$

- 1×1 matrix

$$[a_{11}]$$

is a single #

- 1×4 matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \end{bmatrix}$$

is a single row

- 4×1 matrix

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix}$$

is a single column

$\Longrightarrow$ Notation:

$$A = \Big[a_{ij}\Big] \qquad \begin{matrix} i = 1, \cdots m \\ j = 1, \cdots n \end{matrix}$$

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if they have the same dimension and

$$a_{ij} = b_{ij} \qquad \text{for all} \qquad \begin{matrix} i = 1, \cdots m \\ j = 1, \cdots n \end{matrix}$$

$\Longrightarrow$ Matrix operations:

- Multiplication by a scalar: if A is $m \times n$ matrix, $A = [a_{ij}]$, and $c \in \mathbb{R}$ (a real number), then

  $$c \cdot A = [c \cdot a_{ij}]$$

  Example:

  $$(-2) \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ -6 & -8 \end{bmatrix}$$

- Addition of matrices: if A and B are matrices of the same dimension,

  $$A + B = [a_{ij} + b_{ij}]$$

  Example:

  $$\begin{bmatrix} 1 & 2 & -2 \\ \sqrt{2} & \pi & e \end{bmatrix} + \begin{bmatrix} -1 & e & e \\ 0 & 0 & \pi \end{bmatrix} = \begin{bmatrix} 0 & 2+e & e-2 \\ \sqrt{2} & \pi & e+\pi \end{bmatrix}$$

A matrix $A = 0_{m \times n}$ means that all a_{ij}=0:

$$0_{3\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Longrightarrow$ Note: if A and B are $m \times n$ matrices,

- $$A + 0_{m\times n} = A$$

- $$A + (-1) \cdot A = 0_{m\times n}$$

- $$A - B = A + (-1) \cdot B$$