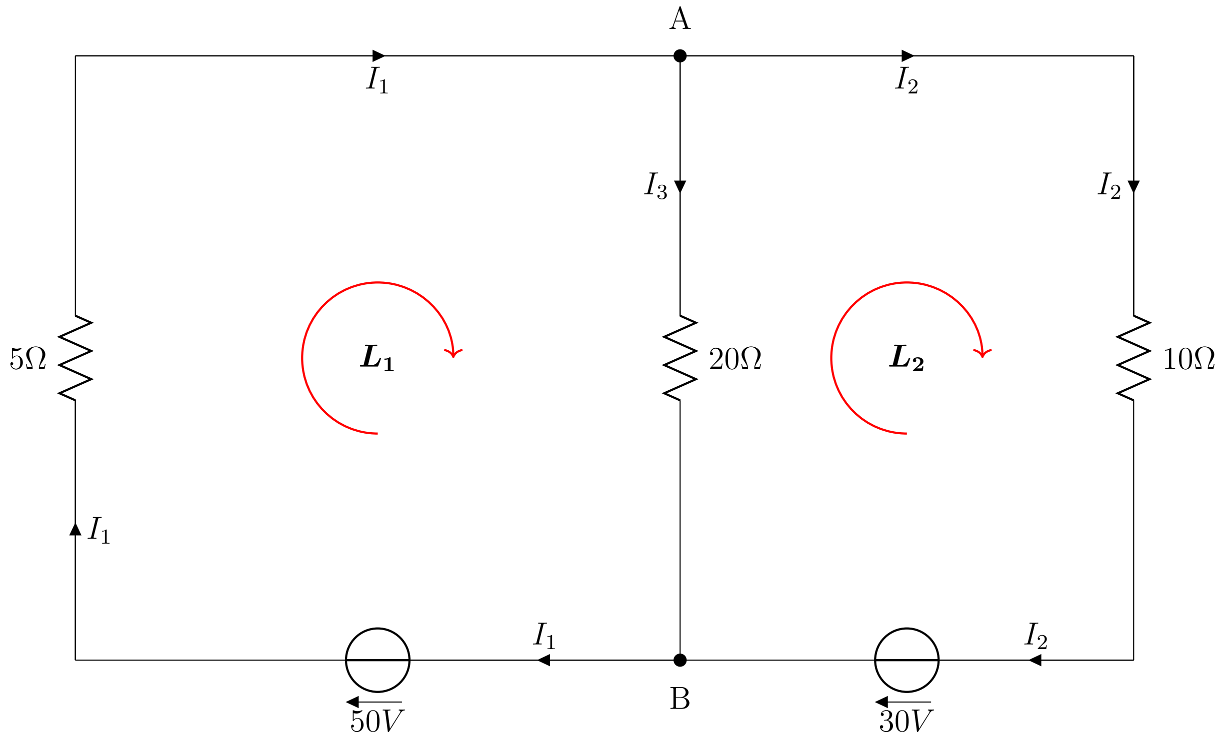
Lecture 2 (sections 1.9, 1.1)

$\Longrightarrow$ Find currents in EC:



- node A:

$$I_1 = I_2 + I_3$$

- node B:

$$I_2 + I_3 = I_1$$

- loop $\boldsymbol{L_1}$:

$$5 \cdot I_1 + 20 \cdot I_3 = 50$$

- loop $\boldsymbol{L_2}$:

$$10 \cdot I_2 + (-20 \cdot I_3) = 30$$

$$\begin{cases} I_1 = I_2 + I_3 & \Longrightarrow \quad I_1 - I_2 - I_3 = 0 \\ 5I_1 + 20I_3 = 50 & \Longrightarrow \quad 5I_1 = 50 - 20I_3 \quad \Longrightarrow \quad I_1 = 10 - 4I_3 \\ 10I_2 - 20I_3 = 30 & \Longrightarrow \quad 10I_2 = 30 + 20I_3 \quad \Longrightarrow \quad I_2 = 3 + 2I_3 \end{cases}$$

$\Longrightarrow$ From the first equation:

$$(10 - 4I_3) - (3 + 2I_3) - I_3 = 0$$

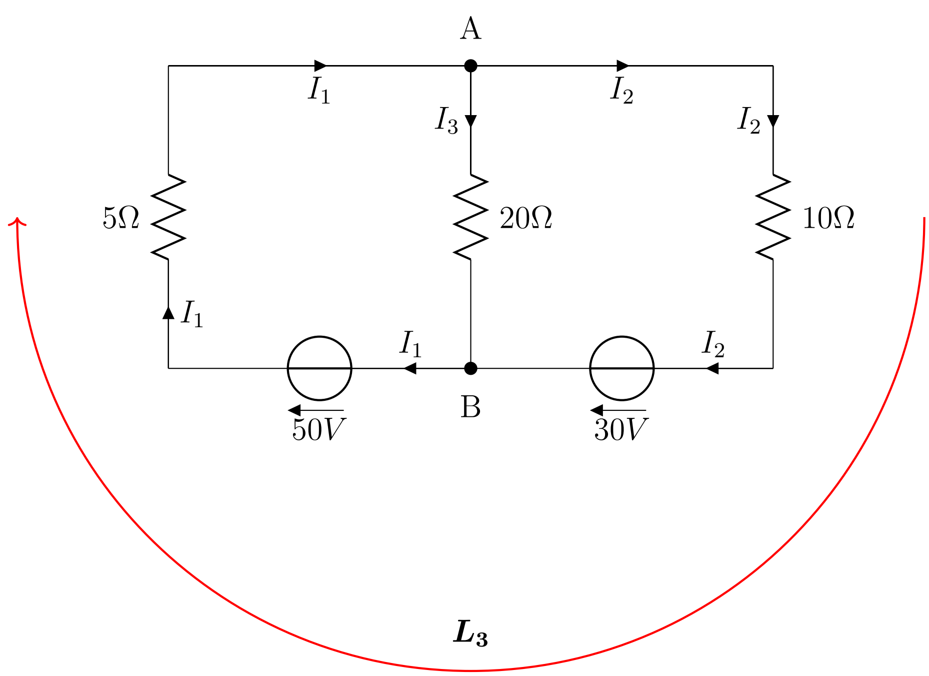
$$7 - 7I_3 = 0 \quad \Longrightarrow \quad I_3 = 1 \text{ amp}$$

Then:

$$I_1 = 10 - 4 \cdot 1 = 6 \text{ amp}$$

$$I_2 = 3 + 2I_3 = 3 + 2 \cdot 1 = 5 \text{ amp}$$

$\Longrightarrow$ Electric circuits always lead to consistent equations:



$$\boldsymbol{L_3}: \quad \underbrace{5I_1 + 10I_2}_{\text{"drops"}} = \underbrace{30 + 50}_{\text{"rises"}}$$

$$\underbrace{5 \cdot 6 + 10 \cdot 5}_{\text{"drops"}} = \underbrace{80}_{\text{"rises"}}$$

$\Longrightarrow$ (last application): polynomial fitting

Problem: given

$$\{(0,0)\,,\,(-1,1)\,,\,(1,1)\}$$

find the quadratic polynomial that passes through the given points

$\Longrightarrow$ A general quadratic polynomial

$$P_2(x) = ax^2 + bx + c$$

- $_2$ — order of the polynomial
- (a, b, c) — constant coefficients of the polynomial
- $$\boxed{(x, y) \text{ is on the polynomial curve if } y = P_2(x)}$$

$$(0,0): \qquad P_2(0) = 0 \qquad \Longrightarrow \qquad a\ 0^2 + b\ 0 + c = 0$$

$$(-1,1): \qquad P_2(-1) = 1 \qquad \Longrightarrow \qquad a\ (-1)^2 + b\ (-1) + c = 1$$

$$(1,1): \qquad P_2(1) = 1 \qquad \Longrightarrow \qquad a\ 1^2 + b\ 1 + c = 1$$

$$\begin{cases} c = 0 \\ a - b + c = 1 \\ a + b + c = 1 \end{cases}$$

$\Longrightarrow$ corresponding linear system has 3 equations (3 points) on 3 unknowns (3 coefficients of the polynomial)

$$a = 1 + b - c = 1 + b \ \Longrightarrow (1 + b) + b + 0 = 1 \ \Longrightarrow b = 0$$

so that $a = 1 \Longrightarrow$ $\boxed{P_2(x) = 1\ x^2 + 0\ x + 0 = x^2}$

Section 1.1
Introduction to system of linear equations

$\Longrightarrow$ linear vs nonlinear equations:

- linear —

$$3\ x - 4\ y + \pi\ z = 1.2$$

unknowns (x, y, z) enter the equation *linearly* with constant coefficients

- nonlinear —

$$3\ x + \underbrace{\tan y}_{y \text{ enters nonlinearly}} = 7$$

- nonlinear —

$$x + \underbrace{y\ z}_{\text{nonlinear}} + 3 = 0$$

Definition: A system of linear equations in n-unknowns

$$x_1\ ,\ x_2\ ,\ \cdots x_n$$

is defined as

$$\begin{cases} a_{11}\ x_1 + a_{12}\ x_2 + \cdots + a_{1n}\ x_n = b_1 \\ a_{21}\ x_1 + a_{22}\ x_2 + \cdots + a_{2n}\ x_n = b_2 \\ \cdots \\ \boxed{a_{i1}\ x_1 + a_{i2}\ x_2 + \cdots + a_{ij}\ x_j + \cdots + a_{in}\ x_n = b_i} \\ \cdots \\ a_{m1}\ x_1 + a_{m2}\ x_2 + \cdots + a_{mn}\ x_n = b_m \end{cases}$$

- There are m equations in total
- the coefficients a_{ij} and b_i are constants

Note: for every equation i **not all** coefficients can be zero

$$\{a_{i1}\,,\, a_{i2}\,,\, \cdots\,,\, a_{in}\,,\, b_i\} \qquad \text{not all simultanuously zero}$$

$\Longrightarrow$ solution to the system of linear equations is an ordered set of numbers (ordered n-tuple):

$$(x_1\,,\, x_2\,,\, \cdots\,,\, x_n)$$

— If a system has a solution $\Longrightarrow$ *consistent*

— If a system has no solution $\Longrightarrow$ *inconsistent*

Examples:

- $$\begin{cases} x + y = 3 \\ x - y = 5 \end{cases}$$

  $(x, y) = (4, -1)$ — the unique solution $\Longrightarrow$ consistent

- $$\begin{cases} x + y = 3 \\ -2x - 2y = -6 \end{cases}$$

  $(x, y) = (a, 3-a)$, a is any number — infinitely many solutions $\Longrightarrow$ consistent

- $$\begin{cases} x + y = 3 \\ x + y = 2 \end{cases}$$

  no solution $\Longrightarrow$ system is inconsistent